DESIGN OF A THICK FILM LOWPASS ACTIVE FILTER

by

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CHAPTER I

INTRODUCTION

The current interest in the area of active filters is due primarily to the advent of integrated circuits where many network elements can be placed on a single chip or substrate. Since attempts to build miniature inductors for lower frequency applications have not been successful, passive LC filters cannot be constructed using the present integrated circuit techniques (1). Active filters, on the other hand, do not require the use of inductors. They consist only of active devices in combination with RC networks. Since RC networks are easily made in integrated form, active filters provide a method for the construction of integrated filters.

At the present time hybrid thick film technology furnishes the most economical method for making integrated circuits. The smaller initial capital investment, the higher yields in production, and the simplicity of the process are the main reasons for its economic superiority compared to the thin film process (2). If active filters are to be economically feasible and produced on a large scale, then it is clear that the thick film process should be used to construct them.

Naturally, if thick film circuits are to be used, their properties must be recognized and incorporated into the design process. The advantages and disadvantages in circuit design resulting from the use of thick film hybrid circuits are:

1. The size of the circuit is reduced and the temperature can be considered approximately uniform over the smaller surface area.

2. If precision resistors are required, trimming is necessary. Tolerances before trimming are generally 20% to 30% but can be improved with better process control.
3. Resistors designed in the top hat configuration can be trimmed to values greater than twice their initial values.
4. If the thick film resistors are made at the same time and with the same ink, they tend to vary in the same manner with changes in the circuit's operating environment.
5. If different inks and multiple printing are to be avoided, the range of resistor values on one substrate should be limited to about 10 to one.
6. Ceramic chip capacitors, which can be mounted directly on the substrates, are available in the range of 1.2 pF to 2.5 Mfd.
7. Variable ceramic capacitors are available with tuning ranges up to 1-50 pF.

The purpose of this thesis is to show: (a) the complete design of an active filter, (b) the compatibility of this design and the above characteristics of thick film circuits, and (c) the advantages resulting from the use of the thick film circuits.

Definitions and concepts found in the current literature on active filters are presented in Chapter II. Three classes of active realizations and their applications are also given. The active networks are limited to the type containing RC networks and operational amplifiers as the active devices.

Chapter III contains the design of a 5th order Chebyshev active filter which can be constructed using hybrid thick film circuits. The characteristics of thick film circuits are considered throughout the design and the final thick film layouts are shown. Other topics such as the temperature stability of the filter's response and the tuning procedure are also discussed.

Chapter IV contains the measurements taken from the active filter and a discussion of the results.

CHAPTER II

LITERATURE SURVEY

Several methods of active filter synthesis using operational amplifiers and RC networks are presented in this chapter. Each of these methods has a useful but limited area of application. However, before examining each synthesis approach, a discussion of definitions and other background material is necessary.

The Sensitivity Function

The parameters of active and passive components change from their nominal values due to aging and variations in the external environment. Since an active filter is constructed using the above components, its performance will also vary. The sensitivity function provides a means to determine the change in performance of an active filter when a capacitor, resistor, or active element varies.

The sensitivity function S of a network variable T due to the variation of a parameter K is

$$S_K^T = \frac{(\partial T/T)}{(\partial K/K)} = \frac{(\partial T/\partial K)}{(T/K)} \tag{2.1}$$

which is the percentage change in T, due to a change in K, divided by the percentage change in K (3). The function T can be a voltage transfer function, a pole or zero, or any function which contains the variable K. K is generally a resistor, capacitor, or gain of an active device.

The pole-position sensitivity of a network function is defined as

$$S_K^p = \frac{\partial a/a}{\partial K/K} + j\,\frac{\partial b/b}{\partial K/K}\ ;\ p = a + jb \tag{2.2}$$

where p is a particular pole of the network function (4). Given a one percent change in the parameter K, the real part of S_K^p gives the percentage change in the pole position parallel to the real axis, and the imaginary part of S_K^p gives the percentage change in the pole position parallel to the imaginary axis.

The sensitivity function has two important practical applications. It can be used as a sorting tool to eliminate active filter configurations whose performance changes drastically with changes in the active and passive elements. It can also be used to determine the changes in the performance of a filter with changes in temperature, provided the temperature coefficients of the resistors and capacitors are known.

The Cascade Approach

A filter synthesis problem begins with the determination of a voltage transfer function, a set of poles and zeros, which will meet the filter's specifications. Passive filter design handbooks contain the information needed to transform a set of specifications into a voltage transfer function (5,6). Several lowpass transfer functions, their poles and zeros, and their attenuation characteristics are tabulated. Highpass and bandpass transfer functions can be obtained from the lowpass forms with the correct frequency transformations. A general transfer function is given by

$$T(S) = \frac{b_m S^m + b_{m-1} S^{m-1} + \ldots.. b_1 S + b_0}{a_n S^n + a_{n-1} S^{a-1} + \ldots.. a_1 S + a_0} \tag{2.3}$$

where $S = \sigma + j\omega$ and $n \geq m$, or in factored form

$$T(S) = K \frac{(S + z_m)(S + z_{m-1}) \ldots..(S + z_0)}{(S + p_n)(S + p_{n-1}) \ldots..(S + p_0)} \tag{2.4}$$

Once the transfer function T(S) is determined, it can be factored into the product of second order functions.

$$T(S) = \prod_{j=1}^{n/2} T_j(S) = \prod_{j=1}^{n/2} K_j \frac{S^2 + (w_{zj}/q_{zj})S + w_{zj}^2}{S^2 + (w_{pj}/q_{pj})S + w_{pj}^2} \tag{2.5}$$

Note that if n is an odd number then (2.5) will also contain a first order term. It has been shown that the best way to realize an nth order transfer function T(S), is to factor it into the second order terms $T_j(S)$, realize each second order term separately, and then cascade all of the second order sections to obtain the product T(S) (3).

The cascade approach requires each second order section to have a very low output impedance and a very large input impedance if additional buffering stages are to be avoided. These conditions are easily met if the output of each stage is taken from the output terminal of an operational amplifier (op-amps in a feedback configuration have a very small output impedance) and the input impedance is increased by scaling all impedances in the filter section (the voltage transfer function is not affected by scaling of the impedance levels).

In decomposing T(S) in (2.4) to obtain second order functions like $T_j(S)$ in (2.5), the problem arises as to which poles should be grouped with which zeros in forming each $T_j(S)$. Moschytz has discussed this problem and has shown that networks can be divided into two categories with respect to their transmission sensitivities $S_x^{T(S)}$ (7).

The class 1 type networks have sensitivities independent of the particular zeros, thus the pole-zero grouping is arbitrary. The class 2 type networks have sensitivities which are dependent upon the zeros, but the choice of pole-zero grouping is dependent upon the particular application and reference (7) can be consulted for the correct criterion for the pole-zero grouping. The types of active networks, to be discussed later, which are in class 1 are the Sallen and Key networks and the frequency emphasizing networks or FEN. All of the state variable or analog type active filters are in the class 2 category.

The Q Factor

The Q factor is associated with a set of complex poles, and is frequently used in the literature on active filters. It may be defined by observing the use of Q in the following second order transfer function

$$T_j(S) = \frac{AS^2 + BS + C}{(S + p)(S + p^*)} = \frac{AS^2 + BS + C}{S^2 + (w_n/Q)S + w_n^2} \tag{2.6}$$

where p is the complex pole $p = a + jb$, and p^* is the complex conjugate of p. Figure 1 shows that the pole position is completely defined by the Q factor and w_n, and that the Q factor is a measure of how close a pole is to the $j\omega$ axis.

Geffe has shown the importance of the Q factor by stating that an active filter design has to have low sensitivities of Q with respect to the passive elements of the circuit if the design is to be practical (8). This is a necessary condition but not completely sufficient because other practical features must also be achieved. Later it will be shown that the sensitivity of Q with respect to the active element's gain will be a factor in deciding which type of active realization should be used to construct the filter.

Operational Amplifiers

Active networks encountered in the current literature contain some type of ideal amplifier as represented in Fig. 2. These ideal amplifiers are assumed to have infinite input impedance and zero output impedance. One problem concerned with the construction of active circuits is the replacement of the ideal amplifier with a nonideal or practical amplifier which has finite input impedance and nonzero output impedance. An operational amplifier along with some type of feedback network can be used to give a good approximation of an ideal amplifier.

The symbol and simple circuit model for a differential input operational amplifier are shown in Fig. 3. The parameters A_o, Z_i, and Z_o are respectively the differential open loop gain, the input impedance, and the output impedance of the particular operational amplifier. A set of typical values

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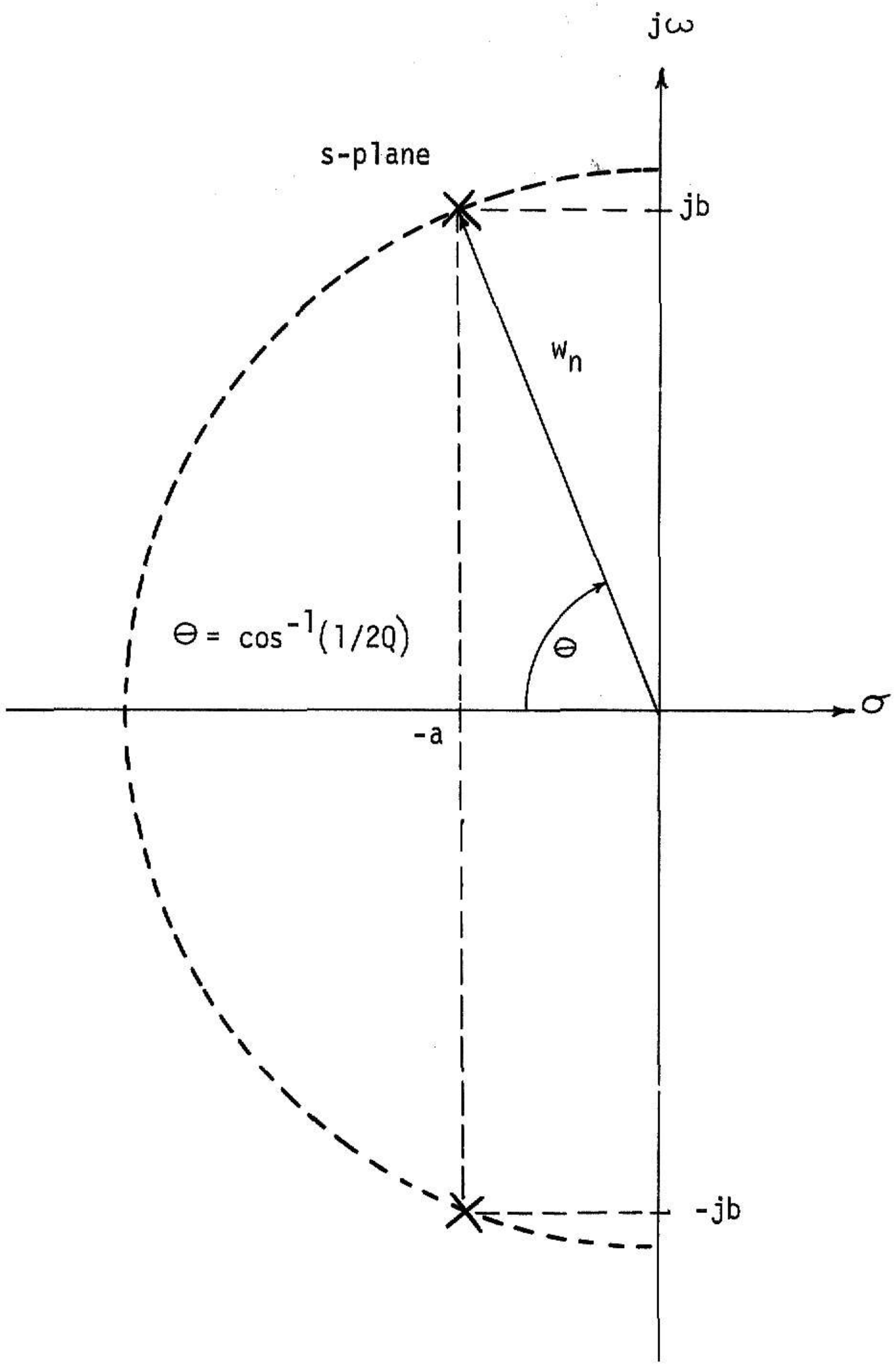


Figure 1. Relationship of the Q factor and the pole position.

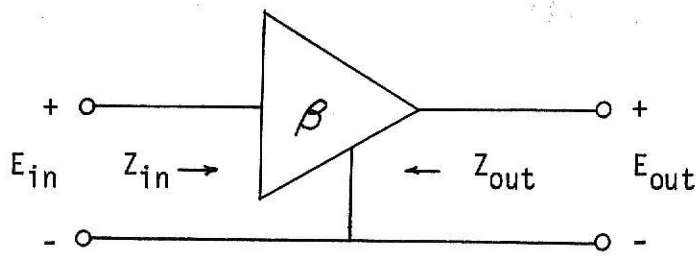
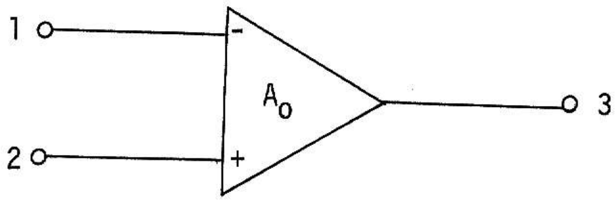
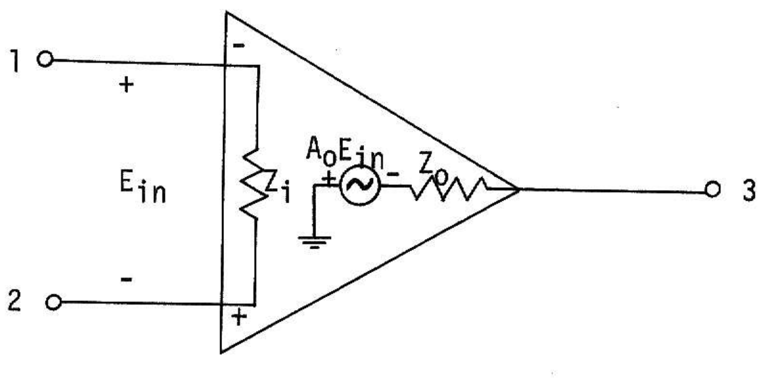


Figure 2. An ideal amplifier.



(a)



(b)

Figure 3. (a) Circuit symbol of a differential input operational amplifier. (b) Simple model of an operational amplifier.

might be $A_o = 10^5$, $Z_i = 10^5\Omega$, and $Z_o = 10^3\Omega$. The terminal marked (+) is the noninverting input terminal and the terminal marked (-) is the inverting input.

Operational amplifiers with the correct type of feedback can be used as voltage amplifiers, voltage followers, integrators, and summing devices, all of which will be needed when the different types of active filter realizations are discussed.

Operational amplifiers operating in the inverting and noninverting modes are shown in Fig. 4. V_{out}/V_{in}, Z_{in}, and Z_{out} are the parameters of the feedback amplifier and not the parameters of the operational amplifier (A_o, Z_i, and Z_o). If A_o and Z_i are very large compared to the other circuit elements, the expressions for V_{out}/V_{in}, Z_{in}, and Z_{out} reduce to simple forms (9).

For the inverting mode circuit, the expressions are

$$\frac{V_{out}}{V_{in}} = -\frac{Z_B}{Z_A} , \tag{2.7a}$$

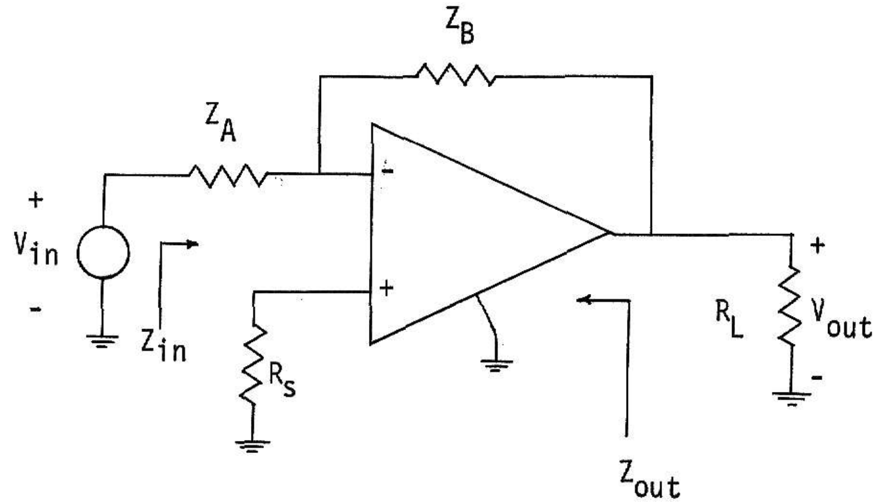
$$Z_{in} = Z_A , \tag{2.7b}$$

and

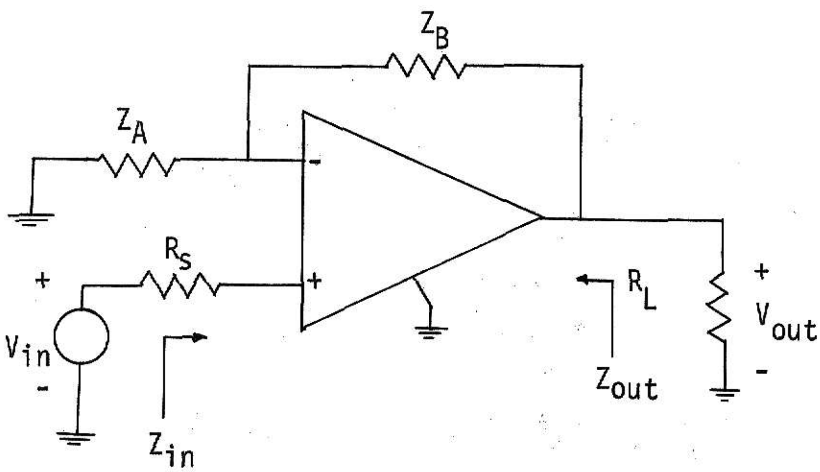
$$Z_{out} = Z_o \frac{1 + (Z_B/Z_A)}{A_o} . \tag{2.7c}$$

The expressions for the noninverting mode are

$$\frac{V_{out}}{V_{in}} = 1 + \frac{Z_B}{Z_A} , \tag{2.8a}$$



(a)



(b)

Figure 4. Operational amplifier in (a) the inverting mode (b) the noninverting mode.

$$Z_{in} = \frac{A_o Z_i}{(1 + Z_B/Z_A)} \quad , \tag{2.8b}$$

and

$$Z_{out} = Z_o \frac{1 + Z_B/Z_A}{A_o} \quad . \tag{2.8c}$$

The resistor R_s, present in both circuits of Fig. 4., is chosen so that the dc paths to ground from each input terminal of the operational amplifier are equal. This is necessary to minimize the offset voltage at the output caused by biasing currents at each of the input terminals. In Fig. 4 the resistor R_s is equal to the parallel combination of Z_A with the series combination of Z_B and Z_o.

As an example, take the case where $Z_A = 10^3\Omega$, $Z_B = 10^4\Omega$, and the operational amplifier parameters are the typical ones given before; $A_o = 10^5$, $Z_i = 10^5\Omega$, and $Z_o = 10^3\Omega$. Then for the operational amplifier operating in the inverting mode $V_{out}/V_{in} = -10$, $Z_{in} = 10^3\Omega$, and $Z_{out} = .10\Omega$. These same expressions for the noninverting mode amplifier are $V_{out}/V_{in} = +11$, $Z_{in} = 9.1 \times 10^8\Omega$, $Z_{out} = .10\Omega$.

The example above shows first that the inverting mode gives a negative voltage gain whereas the noninverting mode gives a positive gain. Secondly, for the same approximate gains, the noninverting mode configuration has a much larger input impedance Z_{in}. The input impedance Z_{in} of the inverting mode amplifier is equal to Z_A, which must be kept less than Z_i and A_o if the approximations in (2.7a-2.7c) are to hold true; thus when using the inverting mode configuration, care must be taken to insure that the impedance Z_{in} does not load down the preceding circuit. The third conclusion is that the output impedances of both the inverting and noninverting mode amplifiers are equal.

Figure 5a shows an operational amplifier used as a voltage follower. The expressions for Z_{in}, Z_{out}, and V_{out}/V_{in} are similar to the ones given for the noninverting mode amplifier, and are given as

$$\frac{V_{out}}{V_{in}} = 1 \quad , \tag{2.9a}$$

$$Z_{in} = Z_i A_o \quad , \tag{2.9b}$$

and

$$Z_{out} = \frac{Z_o}{A_o} \quad . \tag{2.9c}$$

An integrator is shown in Fig. 5b, where the output voltage is given by

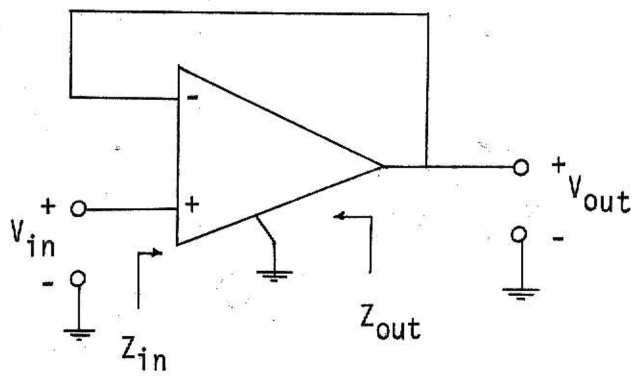
$$V_{out} = - \frac{V_1}{SR_1C} - \frac{V_2}{SR_2C} \quad , \; S = \sigma + j\omega \; . \tag{2.10}$$

The output impedance of the circuit is the same as in (2.7c), but the input impedances are R_1 for the terminal corresponding to V_1 and R_2 for the terminal corresponding to V_2.

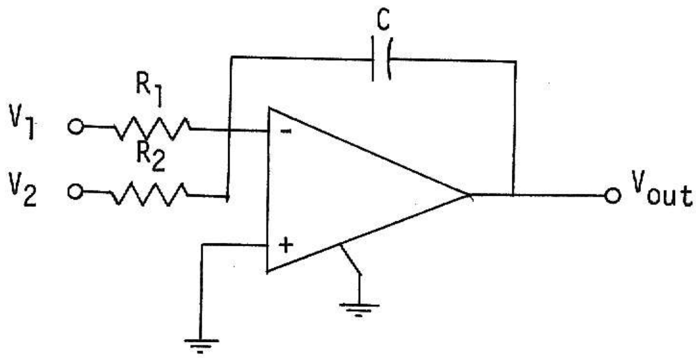
Figure 5c shows a summing amplifier where the output voltage is given by the expression

$$V_{out} = - \frac{R_B V_1}{R_1} - \frac{R_B V_2}{R_2} - \frac{R_B V_3}{R_3} \quad . \tag{2.11}$$

Again as for the integrator, the input impedances are R_1, R_2, and R_3 corresponding respectively to the voltages V_1, V_2, and V_3. The output impedance of the summer is proportional to Z_o/A_o.



(a)



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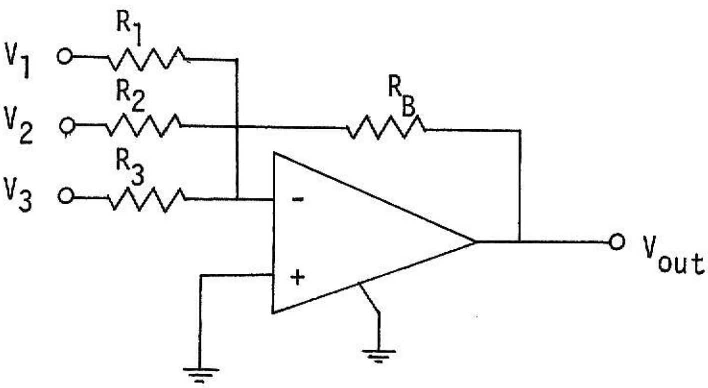


(c)

Figure 5. (a) Voltage follower (b) integrator (c) summing amplifier.

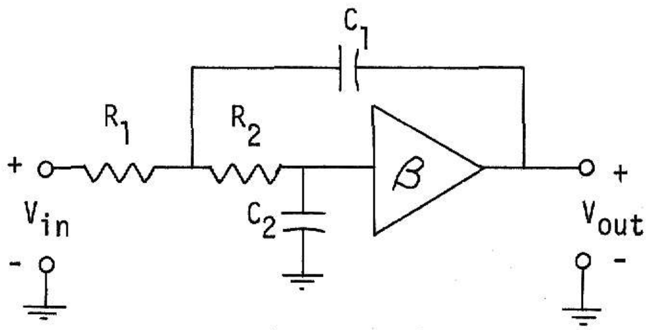
All of the circuits presented in Fig. 5 will be used in the different types of active filter building blocks presented in the next section. Since the objective of this section was to present just the types of operational amplifier circuits which will be needed in the development which follows, no further information will be presented. If additional information is desired, the references (9,10,11) can be consulted. It should also be noted that all the operational amplifier circuits generally need some form of compensation network and this information is usually given in the specification data for the particular operational amplifier.

Active RC Networks

This section presents three classes of active RC networks which realize second order transfer functions such as $T_j(S)$ in (2.5). These second order networks can be cascaded to give an nth order transfer function. It will be shown that the Q-factor of a second order function $T_j(S)$ will determine which of the three classes of active networks to use for the realization.

The first class of active networks is the Sallen and Key type (4). The various forms of this class are shown in Fig. 6 along with their voltage transfer functions and Q sensitivities with respect to the amplifier gain β. A modified Sallen and Key circuit given by Moschytz and Thelen (12) produces zeros on the $j\omega$-axis and is shown in Fig. 7. The forms of the Sallen and Key circuits in Figs. 6 and 7 provide realizations for most of the factored second order transfer functions encountered in filter design.

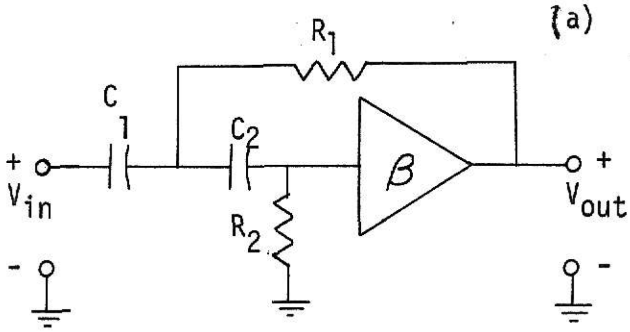
In all of the Sallen and Key networks, the sensitivity of Q with respect to amplifier gain is directly proportional to the Q itself. If a transfer function $T_j(S)$ has poles with a large Q-factor, then the Sallen and



$$S^Q_\beta = \sqrt{\frac{C_1R_1}{C_2R_2}}\;\beta Q$$

$$T_j(S) = \frac{\beta/C_1C_2R_1R_2}{S^2 + (1/C_1R_1 + 1/C_1R_2 + (1-\beta)/C_2R_2)\;S + 1/C_1C_2R_1R_2}$$

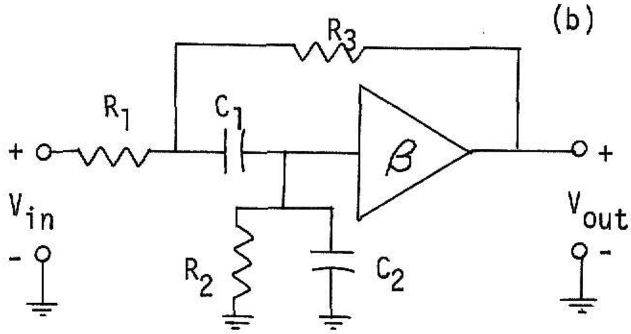
(a)



$$S^Q_\beta = \sqrt{\frac{C_2R_2}{C_1R_1}}\;\beta Q$$

$$T_j(S) = \frac{\beta S^2}{S^2 + (1/C_1R_2 + 1/C_2R_2 + (1-\beta)/C_1R_1)\;S + 1/C_1C_2R_1R_2}$$

(b)

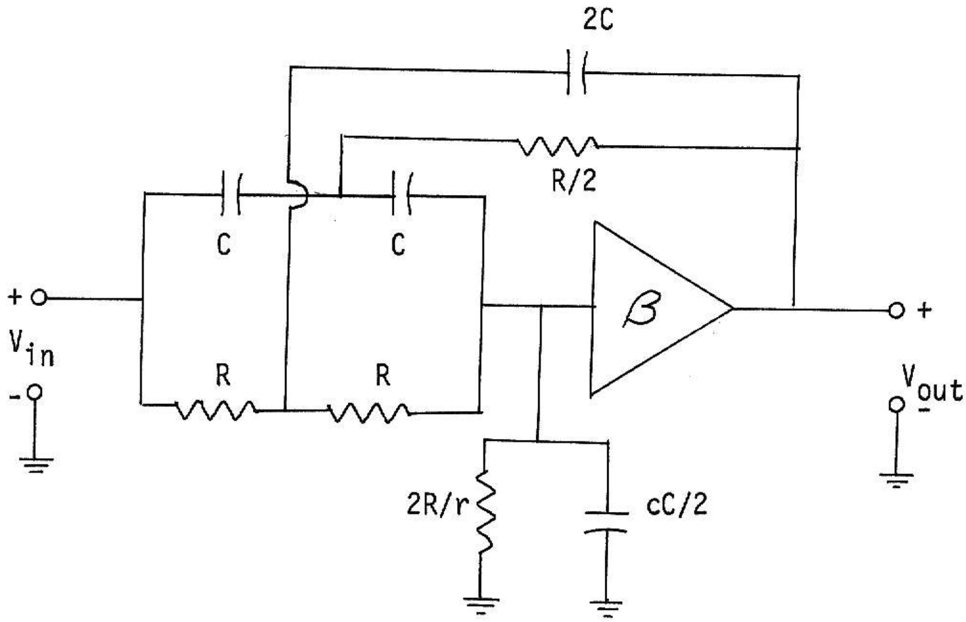


$$S^Q_\beta = \sqrt{\frac{C_1R_1R_2}{(R_1+R_3)C_2R_3}}\;\beta Q$$

$$T_j(S) = \frac{(\beta/R_1C_2)S}{S^2 + \left[\frac{1}{C_2R_2} + \frac{1}{C_1R_3} + \frac{1}{C_1R_1} + \frac{1}{C_2R_1} + \frac{(1-\beta)}{C_2R_3}\right] S + \frac{R_1+R_3}{C_1C_2R_1R_2R_3}}$$

(c)

Figure 6. Sallen and Key type circuits (a) lowpass form (b) highpass form (c) bandpass form.



$$T_j(S) = \frac{\beta}{1 + c} \; \frac{S^2 + \left(\frac{1}{RC}\right)^2}{S^2 + S\left(\frac{4 - 4\beta + c + r}{1 + c}\right) + \left(\frac{1}{RC}\right)^2\left(\frac{1 + r}{1 + c}\right)}$$

$$S_{\beta}^{Q} = \frac{4\beta}{\sqrt{(1 + r)(1 + c)}} \quad Q$$

Figure 7. Modified Sallen and Key network for realizing $j\omega$ axis zeros.

Key type circuits are not desirable because a very small change in amplifier gain causes a large change in the Q of the pole pair. Referring back to Fig. 1, the effect of a change in the Q of a pair of complex poles is to move the poles on a circle of constant radius w_n, and since most filter specifications limit the movement of the poles, it is desirable to keep the change in the Q-factor as small as possible. Moschytz has suggested that the Sallen and Key circuits be used only to realize transfer functions which have a Q-factor smaller than 10.

Frequency emphasizing networks or FEN's form the second class of active networks. They are used to realize second order transfer functions which have a Q in the range of 10 to 50. The FEN's can be used to obtain higher Q's than the Sallen and Key type networks without reducing the Q stability (13,14).

The FEN realizations are based on pole-zero cancellations. A desired second order transfer function

$$T_j(S) = K \frac{S^2 + (w_z/q_z)S + w_z^2}{S^2 + (w_p/q_p)S + w_p^2} \tag{2.12}$$

is decomposed into the product of two functions. Thus,

$$T_j(S) = (Y_{21})\ (Z_{21}) \tag{2.13}$$

where

$$Y_{21} = K_r \frac{S^2 + (w_z/q_z)S + w_z^2}{S^2 + (w_p/q_r)S + w_p^2} \tag{2.14}$$

and $$Z_{21} = K_a \frac{S^2 + (w_p/q_r)S + w_p^2}{S^2 + (w_p/q_p)S + w_p^2} \quad . \tag{2.15}$$

The zeros of Z_{21} cancel with the poles of Y_{21} to give the desired transfer function $T_j(S)$.

Y_{21} is the transfer admittance of a passive RC network and is required to have negative real poles. Z_{21} is the transfer impedance of an active correcting network which can have complex poles.

Figure 8 shows a FEN which will realize a general second order transfer function. The type of response (lowpass, highpass, etc.) is completely determined by the transfer admittance of the passive RC network. Y_{21} is left in block form so that the FEN can be as general as possible. In reference (13) Moschytz has a table of RC networks used to realize the different forms of Y_{21}.

Referring to Fig. 8, Z_{21} can be expressed as

$$Z_{21} = \frac{1}{1 + \beta(R_F/R_Q)} \; \frac{S^2 + (4/RC)S + (1/RC)^2}{S^2 + S\left[(4/RC)/(1 + \beta(R_F/R_Q)\right] + (1/RC)^2} \; . \tag{2.16}$$

The ratio R_F/R_Q can be interpreted as the magnitude of the gain of the operational amplifier operating in the inverting mode; thus, let $\mu = R_F/R_Q$.

The Q-factor of the second order transfer function realized by the FEN is determined by the poles of Z_{21} and is given by

$$Q = (1/4)(1 + \mu\beta) \quad . \tag{2.17}$$

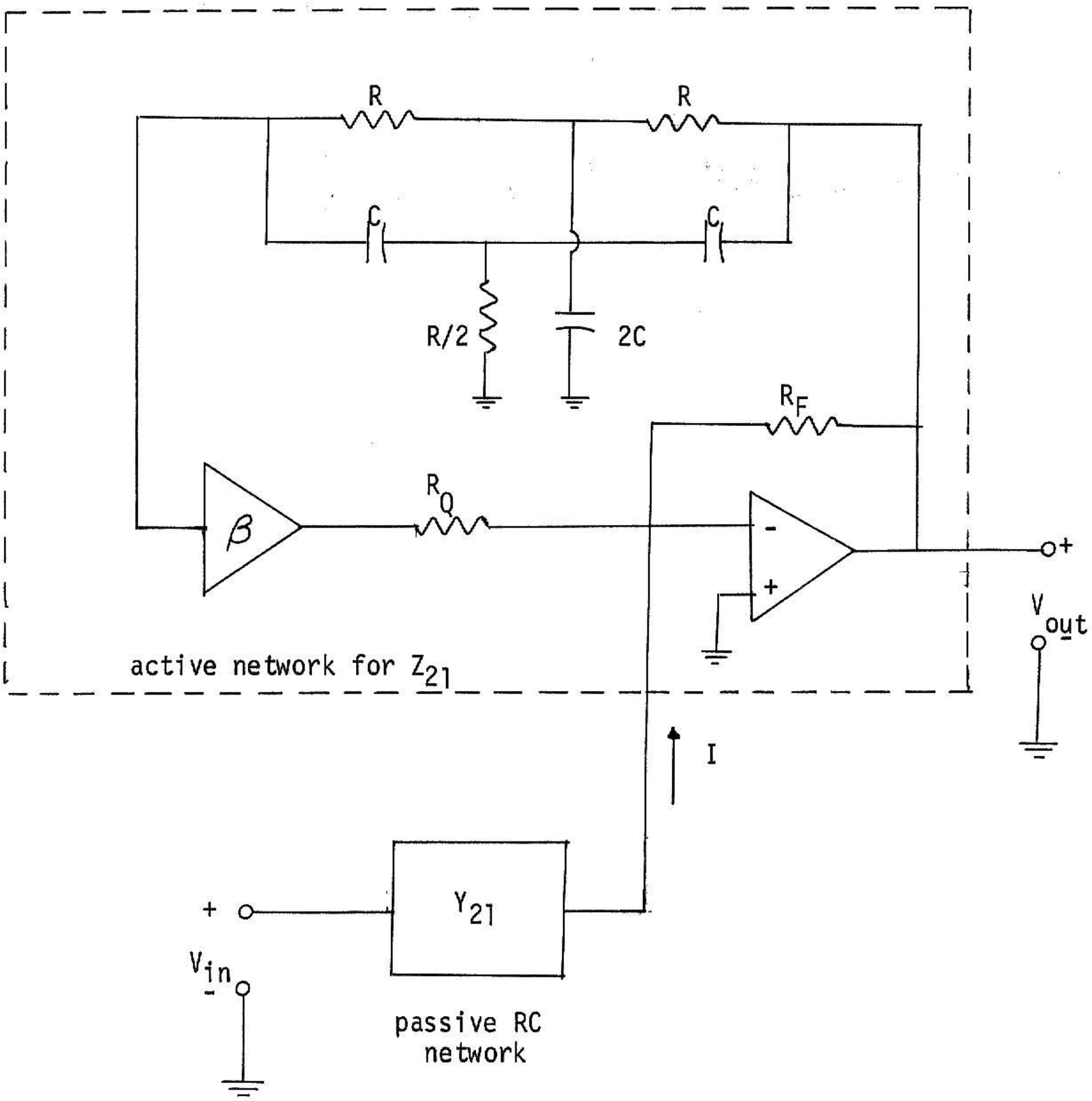


Figure 8. FEN realization for second order transfer functions.

The sensitivities of Q with respect to the amplifier gains μ and β are

$$S_{\mu}^{Q} = S_{\beta}^{Q} = \frac{\mu\beta}{(1+\mu\beta)} \tag{2.18}$$

and these sensitivity functions are always less than one, regardless of the value of Q. The FEN can then be used to realize higher Q transfer functions without loss of Q stability. Q's larger than 50 cannot be achieved due to practical limits on the amplifier gains (13,14).

The analog or state variable realizations are useful for the synthesis of second order transfer functions with Q's greater than 50. With this method, the Q-factor is not a function of the open loop gains of the op-amps, as long as the gains are considered to be very large. The integrators and summers will be considered to be ideal in what follows, but Kerwin, Huelsman, and Newcomb have modified the approach to include finite gain operational amplifiers (15).

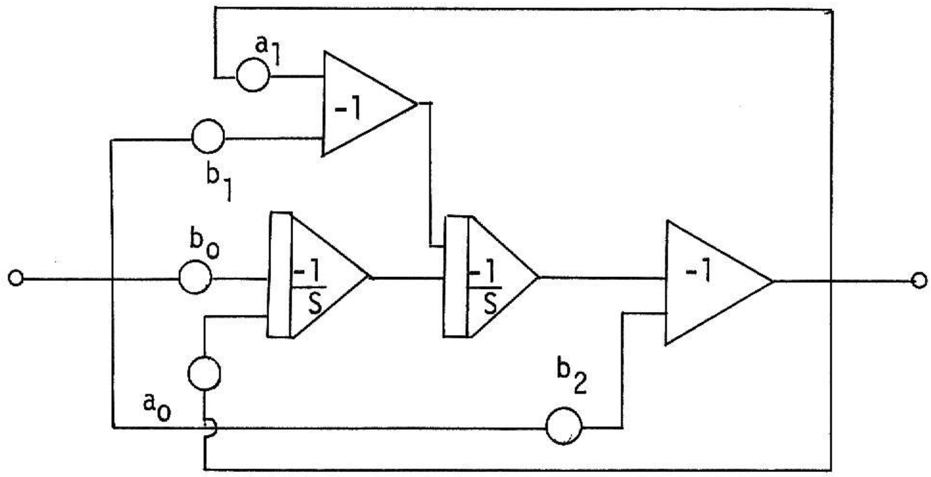
Consider the following second order voltage transfer function

$$T_j(S) = \frac{V_{out}}{V_{in}} = -\frac{b_2S^2 + b_1S + b_0}{S^2 + a_1S + a_0} \quad , \tag{2.19}$$

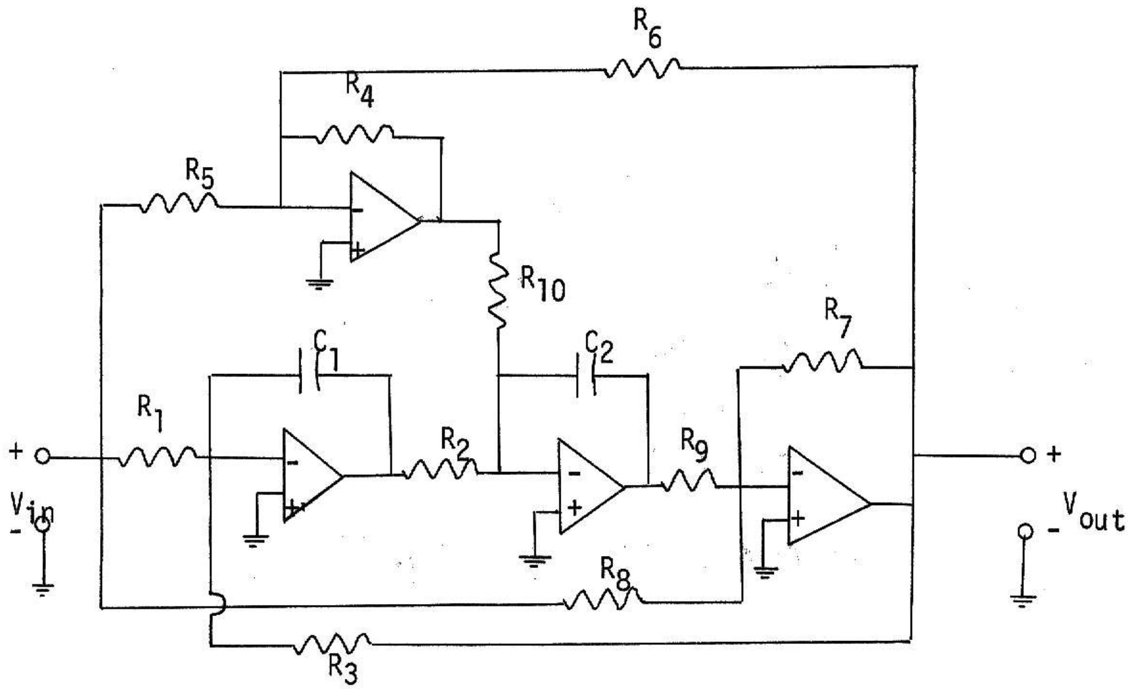
which can be rewritten as

$$V_{out} = -V_{in}(b_2 + b_1/S + b_0/S^2) - V_{out}(a_1/S + a_0/S^2) \quad . \tag{2.20}$$

Figure 9 (a) shows the block diagram which satisfies (2.20). The circuit realization of (2.20) is shown in Fig. 9 (b) where op-amps are used for the integrators and summers. With regard to the coefficients in (2.19), the



(a)



(b)

Figure 9. State variable realization (a) block diagram form
(b) circuit using operational amplifiers.

resistors and capacitors in Fig. 9 (b) have the following values (16):

$$R_1 = \frac{1}{b_0 C_1}, \quad R_2 = \frac{1}{C_2},$$

$$R_3 = \frac{1}{a_0 C_1}, \quad R_5 = \frac{R_4}{b_1},$$

$$R_6 = \frac{R_4}{a_1}, \quad R_8 = \frac{R_7}{b_2}, \tag{2.21}$$

$$R_9 = R_7, \text{ and } R_{10} = \frac{1}{C_2}.$$

The various cases such as lowpass, highpass, and bandpass are derived as follows: for the lowpass case, $b_2=b_1=0$ which requires $R_8=R_5=\infty$ (open circuits); for the highpass case, $b_1=b_0=0$ which requires $R_5=R_1=\infty$; for the bandpass case, $b_2=b_0=0$ which requires $R_8=R_5=\infty$.

As stated before, the Q of the state variable realization is independent of the open loop gains if the operational amplifiers are assumed to have large gains. From (2.19) and (2.21), the Q-factor of the state variable realization is given by

$$Q = \frac{\sqrt{a_0}}{a_1} = \frac{R_6}{R_4 \sqrt{R_3 C_1}}. \tag{2.22}$$

Thus, the sensitivity of Q with respect to the gains of the op-amps is zero. The ability of the state variable synthesis to realize high Q transfer functions is based on the fact that the Q depends only on passive components.

Summary

This chapter can be summarized by stating the approach taken to realize a general nth order transfer function. Given an nth order transfer function, the method of design is as follows:

1. The nth order transfer function is factored into the product of second order functions as in (2.5).
2. The Q-factor for each second order transfer function is calculated. Sallen and Key circuits are used to realize low Q transfer functions ($Q \leq 10$). FEN circuits are used to realize medium Q functions ($10 \leq Q \leq 50$). State variable circuits are used to realize high Q functions ($Q \geq 50$).
3. Referring back to the first step, the pole-zero grouping is important only when the state variable networks are used. Thus, high Q poles deserve special attention in that it is best to place zeros with them according to the criterion given by Moschytz in (7).
4. Values of the capacitors, resistors, and amplifier gains are chosen to give the active networks the correct poles and in addition to minimize the movement of these poles when component values vary (the design example in the next chapter will show how the pole sensitivities can be used to minimize the pole movement).

The next chapter will be concerned with the practical aspects of a particular active filter design. The thick film process will be incorporated into the design so that the filter can be constructed using hybrid thick film circuits.

CHAPTER III

AN ACTIVE FILTER DESIGN

The design procedure given in the summary of chapter two was, of necessity, very general since many of the practical and detailed aspects of a filter design can only be discussed when considering a particular design problem.

The objective of this chapter is to present the complete design of a lowpass active filter from the initial filter specifications to the final thick film layouts.

Filter Specifications

The following specifications are placed on the performance of the lowpass active filter which is to be designed.

1. The filter is required to have .5 db (or less) ripple in its voltage transfer function over the passband frequencies of 0-8 KHz.
2. The filter is required to attenuate all frequencies greater than 12 KHz by not less than 20 db. These requirements on the magnitude of the voltage transfer function are shown in Fig. 10 along with the normalized response (i.e. the cutoff frequency is normalized to 1 rad per sec).
3. The source and load impedances terminating the filter are set at 5 KΩ.
4. The filter's response should remain reasonably stable over a small temperature range centered at room temperature (25°C).

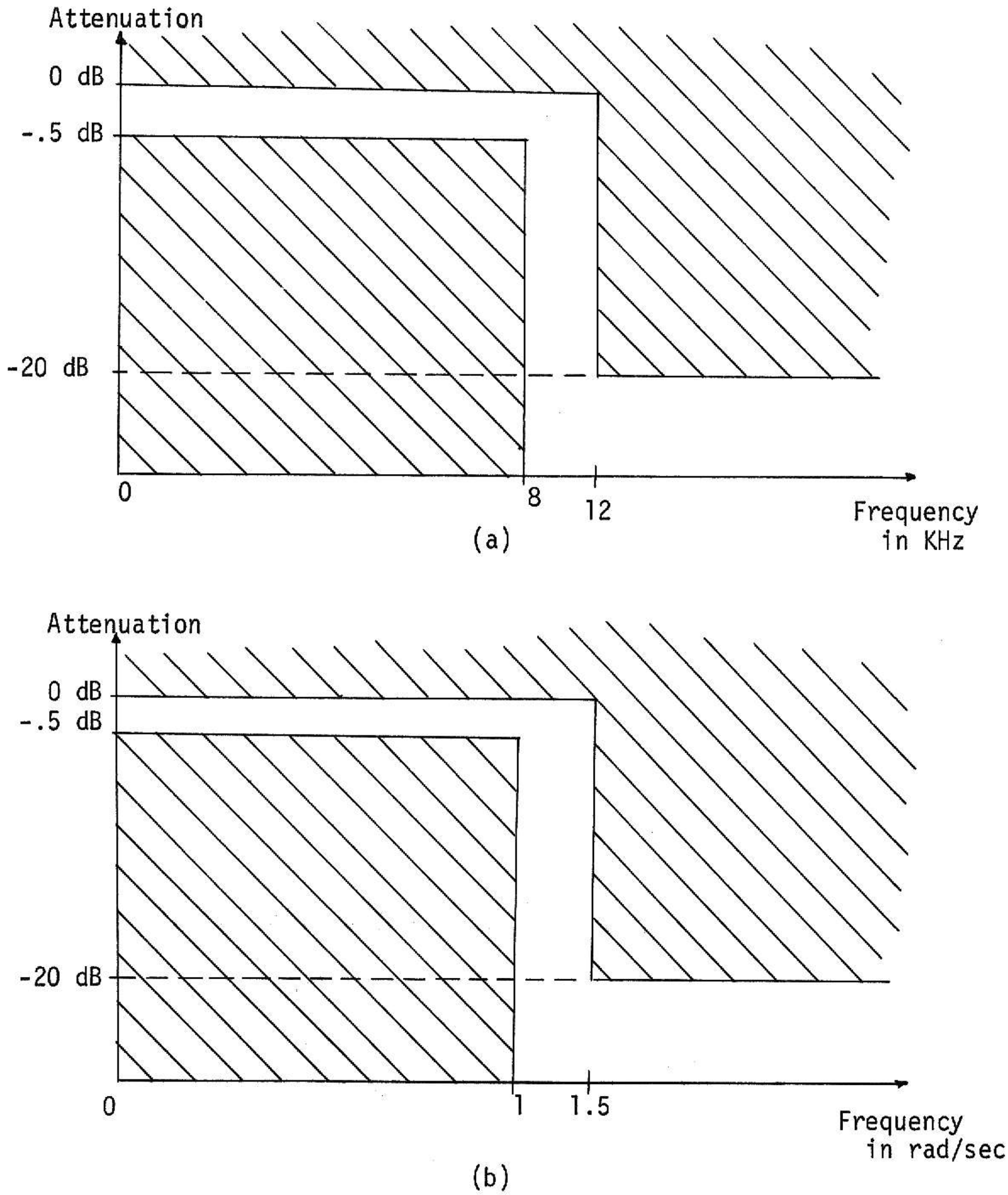


Figure 10. (a) Attenuation characteristics required by the filter specifications (b) the same characteristics in normalized form.

Voltage Transfer Function

The first step in the design of any active filter is to obtain a transfer function which will satisfy the specifications. For economic reasons it is desirable to keep the order of the transfer function as small as possible (higher order transfer functions require more stages and hence more resistors, capacitors, and op-amps).

Since a ripple is allowed in the passband (.5 db or less), it is advantageous to use a Chebyshev transfer function rather than a Butterworth type. A Chebyshev transfer function can satisfy a set of attenuation characteristics (with a ripple in the passband) with a lower order transfer function than can a Butterworth type (17).

It can be shown that a 4th order Chebyshev transfer function will not satisfy the attenuation characteristics stated in the specifications (17). For a .5 db ripple in the passband, the 4th order response is only attenuated 18.5 db at the normalized frequency $\omega = 1.5$ rps.

A 5th order Chebyshev transfer function with .5 db ripple will attenuate the response by 26.6 db at $\omega = 1.5$ rps and thus the normalized response specifications are satisfied. To allow for element tolerances, temperature changes, and other variations which will change the 5th order response, the ripple should be reduced to some value less than .5 db.

One of the possible choices, the one which will be used to design the filter, is a 5th order Chebyshev transfer function with a .141 db ripple in the passband and an attenuation of 21.0 db at $\omega = 1.5$ rps. The transfer function is

$$T(S) = \frac{.3431}{(S + p_1)(S + p_2)(S + p_3)(S + p_4)(S + p_5)} \tag{3.1}$$

where $p_{1,2} = -.1546 \pm j\ 1.0634$,

$p_{3,4} = -.4046 \pm j\ .6572$,

and $p_5 = -.5001$.

Factoring T(S) into the product of second order functions and one first order function gives $T(S) = T_1(S)\ T_2(S)\ T_3(S)$, where

$$T_1(S) = \frac{1.1546}{S^2 + .3091\ S + 1.1546} \ , \tag{3.2}$$

$$T_2(S) = \frac{.5956}{S^2 + .8092\ S + .5956} \ , \tag{3.3}$$

and

$$T_3(S) = \frac{.5001}{S + .5001} \ . \tag{3.4}$$

Figure 11 shows the position of the poles of the transfer function T(S) along with their respective permitted movements. The shaded areas indicate the possible positions that the poles may take and still have T(S) satisfy the specifications. The shaded areas of Fig. 11 will be useful later when tuning the filter.

Circuit Topology

The choice of which type of active network to use in synthesizing the second order functions $T_1(S)$ and $T_2(S)$ is determined by their respective

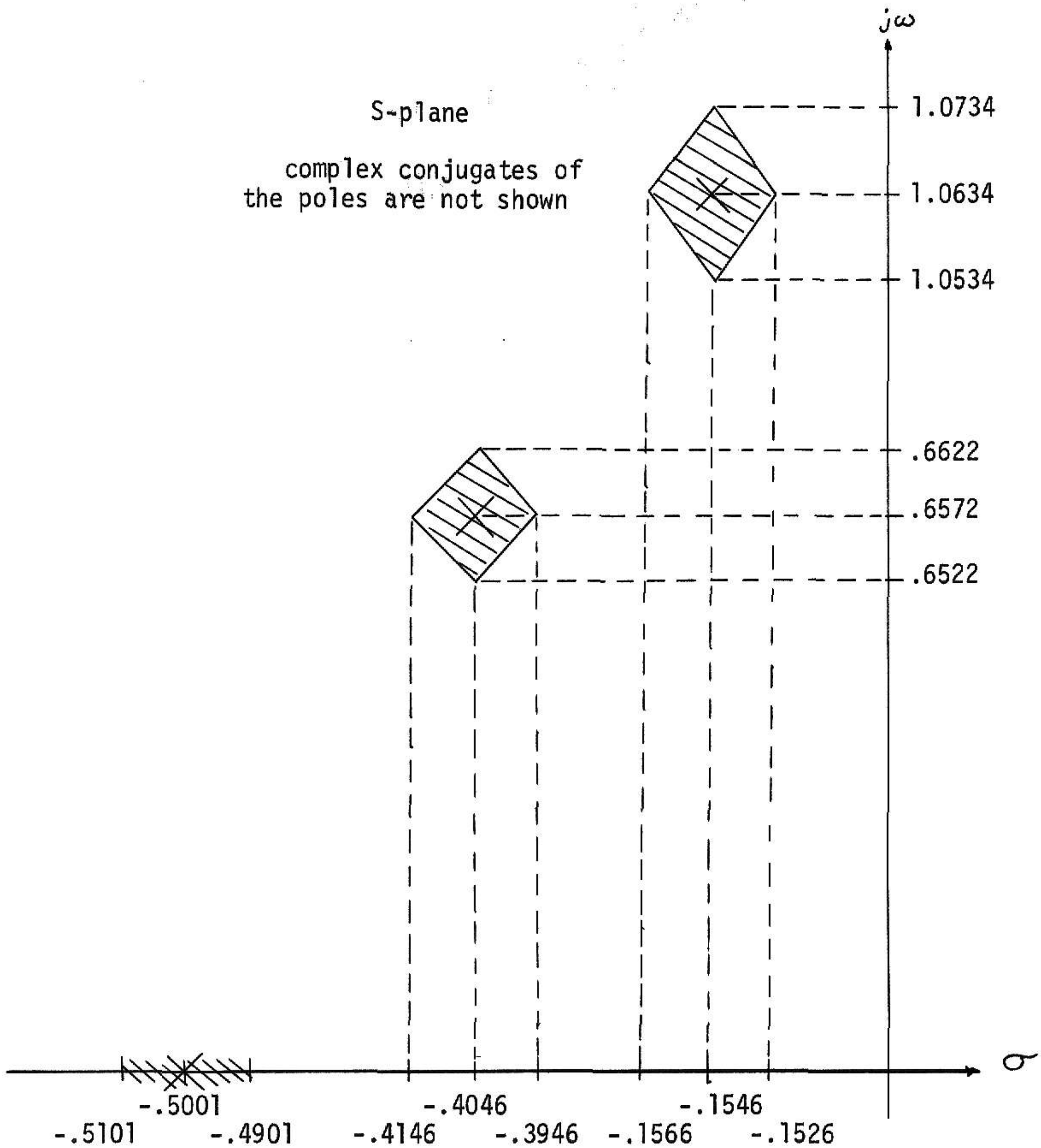


Figure 11. Poles of the transfer function of (3.1) with shaded areas showing permitted pole positions.

Q's, Q_1 and Q_2. By using (2.6), (3.2), and (3.3) the values for the Q's are

$$Q_1 = \frac{Q_1}{w_{n1}} \cdot \sqrt{w_{n1}^2} = \frac{1}{.3091} \cdot \sqrt{1.1546} = 3.48 \tag{3.5}$$

$$Q_2 = \frac{Q_2}{w_{n2}} \cdot \sqrt{w_{n2}^2} = \frac{1}{.8092} \cdot \sqrt{.5956} = .955 \quad . \tag{3.6}$$

Both $T_1(S)$ and $T_2(S)$ have Q's in the low Q range ($Q \leq 10$) and can be synthesized by the Sallen and Key lowpass network shown in Fig. 12, repeated for convenience from Fig. 6.

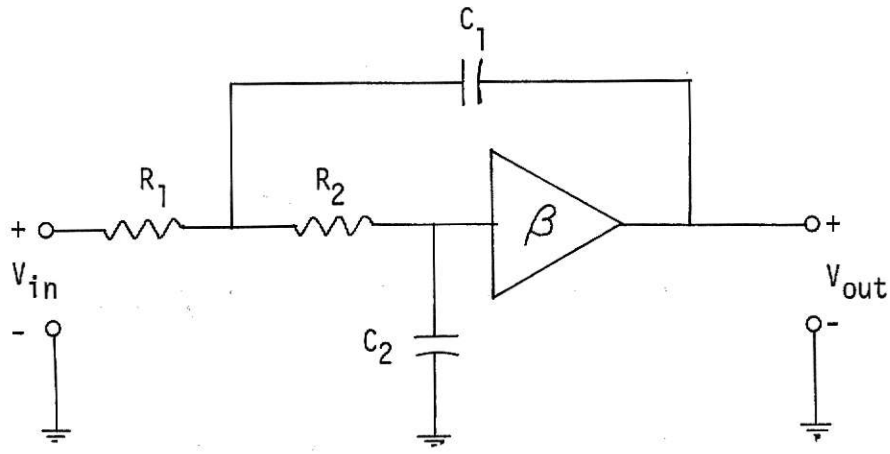
$T_3(S)$ contains only a real pole and Fig. 13 shows the network which is commonly used to synthesize this type of transfer function. The amplifier is used to isolate this stage from the next cascaded stage.

Element Values

The previous section showed that the Sallen and Key lowpass network should be used to realize $T_1(S)$ and $T_2(S)$. The circuit which will realize the real pole of $T_3(S)$ was also presented.

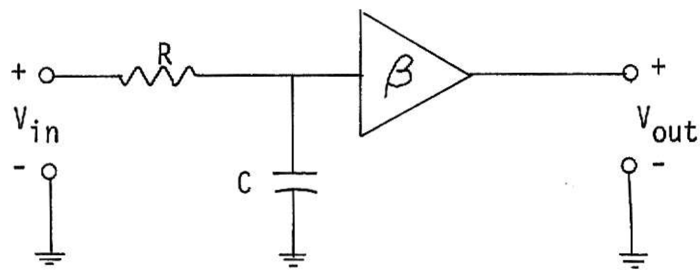
The next step in the design is the calculation of the values for capacitors, resistors, and amplifier gains which will give the active networks the desired transfer functions. Only the pole positions will be important due to the fact that the constant multiplying each transfer function can be obtained by additional amplification or resistive attenuation of the filter's response.

In addition to requiring the correct pole positions for each transfer function, it is also desirable to make each pole as stable as possible with



$$T(S) = \frac{V_{out}}{V_{in}} \qquad \frac{\beta / C_1 C_2 R_1 R_2}{S^2 + \left(\frac{1}{C_1 R_1} + \frac{1}{C_1 R_2} + \frac{(1-\beta)}{C_2 R_2}\right) S + \frac{1}{C_1 C_2 R_1 R_2}}$$

Figure 12. Lowpass Sallen and Key network.



$$T(S) = \frac{\beta / RC}{S + 1/RC}$$

Figure 13. Network used to realize a real pole.

respect to changes in component values. This is where the sensitivity aspect of the design enters. Define the function $M(C_1, C_2, R_1, R_2, \beta)$, for the Sallen and Key circuit, as

$$M = \left|S^a_{R_1}\right| + \left|S^b_{R_1}\right| + \left|S^a_{R_2}\right| + \left|S^b_{R_2}\right| + \left|S^a_{C_1}\right| + \left|S^b_{C_1}\right| + \left|S^a_{C_2}\right| + \left|S^b_{C_2}\right| + \left|S^a_{\beta}\right| + \left|S^b_{\beta}\right| \qquad (3.7)$$

$S^a_{R_1}$ is the sensitivity of the real part of a particular pole with respect to the component R_1, $S^b_{R_1}$ is the sensitivity of the imaginary part of the same pole with respect to the component R_1, and the other terms are defined likewise. The sensitivity functions give the percentage change in the real or imaginary part of a pole when a component changes by one percent.

Now if the element values are chosen to give the correct pole positions and in addition to minimize the function $M(R_1, R_2, C_1, C_2, \beta)$, then the average of the absolute values of the pole sensitivities will be minimized. The absolute values of the sensitivity functions are used because the positive terms could cancel with the negative terms giving a small value for M even though the individual terms were very large. For example, if $S^a_{R_1} = +100$, $S^b_{R_1} = +100$, $S^a_{R_2} = -100$, and $S^b_{R_2} = -100$, these terms would cancel each other even though their magnitudes were large. The use of the absolute values of these terms eliminates this difficulty. Since the sensitivity functions are a measure of the movement of a pole when a component changes in value, the minimization of M tends to stabilize a pole with respect to component variations.

The desired transfer function for $T_1(S)$ from (3.2) is

$$T_1(S) = \frac{1.1546}{S^2 + .3091\ S + 1.1546} \tag{3.8}$$

and the transfer function for the Sallen and Key lowpass active filter is

$$T(S) = \frac{\beta/C_1C_2R_1R_2}{S^2 + [1/C_1R_1 + 1/C_1R_2 + (1-\beta)/C_2R_2]\ S + 1/C_1C_2R_1R_2} \tag{3.9}$$

If the transfer functions in (3.8) and (3.9) are to have the same poles, then the following two equations must be satisfied.

$$1/C_1R_1 + 1/C_1R_2 + (1-\beta)/C_2R_2 = .3091 \tag{3.10}$$

and

$$1/C_1C_2R_1R_2 = 1.1546 \tag{3.11}$$

The multiplying term, $\beta/C_1C_2R_1R_2$, does not have to be equal to 1.1546 because this can be corrected, if necessary, by additional amplification or resistive attenuation once the filter has the correct poles.

Table 1 gives values for R_1, R_2, C_1, C_2, and β which satisfy equations (3.10) and (3.11). The value of the function M is also given for each set of element values. The table gives the minimum value for M (M = 25.21) when R_1, R_2, and β are restricted to the following limits: $1 \leq R_1 \leq 10$, $1 \leq R_2 \leq 10$, and $1 \leq \beta \leq 10$. To conserve space, only a few of the possible sets of elements near the minimum value of M are given (values were calculated using a simple computer program).

The reasons for selecting the specific limits on R_1, R_2, and β are given below.

1. Once the values for R_1, R_2, and β are given, C_1 and C_2 can be determined from (3.10) and (3.11).
2. The resistors are to be constructed using the thick film process and if multiple masks and different resistor inks are to be avoided then the largest resistor range should be about 10 to 1. This 10 to 1 ratio will be satisfied if $1 \leq R_1 \leq 10$ and $1 \leq R_2 \leq 10$.
3. The smallest positive gain which can be achieved using an operational amplifier is $\beta = 1$ corresponding to the voltage follower configuration. When the operational amplifier is used in the noninverting mode, the gain is given by $\beta = 1 + Z_B/Z_A$. If Z_A and Z_B are allowed to have values such that $.1Z_B \quad Z_A \quad 10Z_B$, then multiple masks and inks can be avoided if the limits on β are $1.1 \leq \beta \leq 11$. By using the limits $1.0 \leq \beta \leq 10$, both the voltage follower and noninverting operational amplifier can be considered for the design.

TABLE 1

VALUES OF R_1, R_2, C_1, C_2, AND β WHICH SATISFY (3.10) AND (3.11) ALONG WITH THEIR ASSOCIATED VALUE OF THE FUNCTION M. ALL ELEMENT SETS ARE CENTERED ABOUT THE MINIMUM VALUE FOR M.

R_1	R_2	C_1	C_2	β	M
1.000	5.500	3.823	.04118	1.000	25.24
1.100	5.500	3.529	.04056	1.000	25.22
1.200	5.500	3.284	.03996	1.000	25.26
1.000	5.900	3.784	.03880	1.000	25.26
1.100	5.900	3.489	.03824	1.000	25.21
1.200	5.900	3.244	.03770	1.000	25.25
1.000	6.300	3.749	.03667	1.000	25.29
1.100	6.300	3.455	.03618	1.000	25.24
1.200	6.300	3.219	.03569	1.000	25.27

The element values which minimize M and give the active network in Fig. 12 the correct poles (the poles of the transfer function $T_1(S)$) are $R_1 = 1.100$, $R_2 = 5.900$, $C_1 = 3.489$, $C_2 = .03824$, and $\beta = 1.000$.

The desired transfer function for $T_2(S)$ is

$$T_2(S) = \frac{.5956}{S^2 + .8092\ S + .5956} \quad . \tag{3.12}$$

If the active network in Fig. 12 is to have the same poles as $T_2(S)$, then the following two equations must be satisfied.

$$1/C_1R_1 + 1/C_1R_2 + (1-\beta)/C_2R_2 = .8092 \tag{3.13}$$

and

$$1/C_1C_2R_1R_2 = .5956 \quad . \tag{3.14}$$

Table 2 gives values for R_1, R_2, C_1, C_2, and β which satisfy equations (3.13) and (3.14). Again the sets of values are centered around the set which minimizes the function M. From Table 2 the minimum value for M is M = 7.51. Thus, the best element values for the active network in Fig. 12 are $R_1 = .9000$, $R_2 = 2.000$, $C_1 = 1.991$, $C_2 = .4685$, and $\beta = 1.000$.

The desired transfer function for $T_3(S)$ is

$$T_3(S) = \frac{.5001}{S + .5001} \quad . \tag{3.15}$$

The transfer function for the network which realizes the real pole is

$$T(S) = \frac{\beta/RC}{S + 1/RC} \quad . \tag{3.16}$$

TABLE 2

VALUES OF R_1, R_2, C_1, C_2, AND β WHICH SATISFY (3.13) AND (3.14) AND THEIR ASSOCIATED VALUE OF THE FUNCTION M. ALL ELEMENT SETS ARE CENTERED ABOUT THE MINIMUM VALUE FOR M.

R_1	R_2	C_1	C_2	β	M
.6000	1.600	2.832	.6176	1.000	7.91
.9000	1.600	2.145	.5435	1.000	7.58
1.2000	1.600	1.802	.4852	1.000	7.66
.6000	2.000	2.667	.5226	1.000	7.86
.9000	2.000	1.991	.4685	1.000	7.51
1.2000	2.000	1.648	.4246	1.000	7.58
.6000	2.400	2.574	.4529	1.000	7.91
.9000	2.400	1.888	.4117	1.000	7.56
1.2000	2.400	1.545	.3774	1.000	7.63

If the transfer functions of equations (3.15) and (3.16) are to have the same pole, then the following equation must be satisfied.

$$1/RC = .5001 \tag{3.17}$$

Since $T_3(S)$ has only a real pole, the M function must be redefined as

$$M(R,C) = \left|S_R^a\right| + \left|S_C^a\right| \tag{3.18}$$

where $a = -1/RC$, the real pole. The sensitivity functions S_R^a and S_C^a are independent of the values for R and C as shown below.

$$S_R^a = \frac{\partial a}{\partial R} \cdot \frac{R}{a} = \frac{1}{R^2C}\left[\frac{R}{-1/RC}\right] = -1 \tag{3.19}$$

and

$$S_C^a = \frac{\partial a}{\partial C} \cdot \frac{C}{a} = \frac{1}{RC^2}\left[\frac{C}{-1/RC}\right] = -1 \tag{3.20}$$

Therefore, $M(R,C) = 2$ and cannot be minimized by choosing values for R and C.

The only equation that R and C must satisfy is (3.17). Let $R = 1.000$ which makes $C = 2.000$, and $\beta = 1.000$.

The complete active filter is shown in Fig. 14, with each section containing the appropriate element values decided on above. Note that since $\beta = 1.000$ for each section, the transfer function is exactly as desired. If β was not equal to 1.000 then it would have been necessary to modify the design to achieve the constant multiplier for the transfer function. This could have been achieved by resistive attenuation at the output of the filter, if the filter's transfer function had a constant multiplier which was larger than desired. If the filter's constant multiplier was smaller than desired, the β of the stage which realized the real pole could have been increased

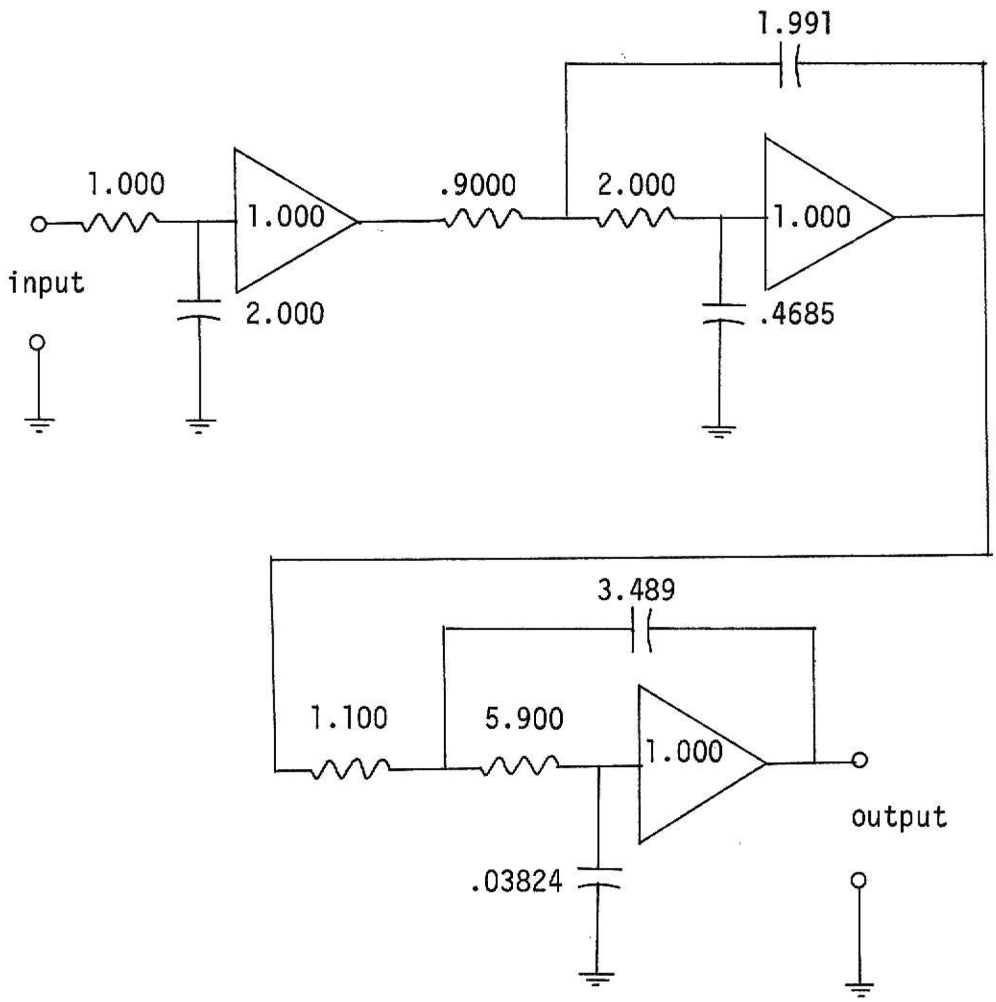


Figure 14. The active filter, with normalized elements, which satisfies the normalized attenuation characteristics of Fig. 10.

to correct for this.

The specification of a 5 KΩ source and load impedance will not cause any difficulty. Note that the output of the filter is taken from the output terminal of an operational amplifier and hence the 5 KΩ load impedance will not load down the circuit. The input of the filter contains a resistor which will be in series with the source and by reducing this resistor (after scaling the impedances) the 5 KΩ source impedance can be accounted for.

The Unity Gain Amplifier

Each of the active networks used to realize the above filter requires a unity gain amplifier (β = 1.000). A Motorola MC 1539G operational amplifier can be used to closely approximate an ideal unity gain amplifier.

The model which approximates the MC 1539G operational amplifier is shown in Fig. 15(a). The parameters for the MC 1539G, as given by its specifications, are $Z_i = 3\text{x}10^5\Omega$, $Z_0 = 4\text{x}10^3\Omega$, and $A_0 = 1\text{x}10^5$.

Figure 15(b) shows the operational amplifier in the voltage follower configuration. Z_{in}, Z_{out}, and β represent the parameters of the voltage follower and not the parameters of the MC 1539G. Figure 15(c) gives the circuit model for the voltage follower and includes the parameters of the operational amplifier.

Using (2.9b) and (2.9c), the approximate values for the input and output impedances of the voltage follower are

$$Z_{in} = Z_i A_0 = (3\text{x}10^5\Omega)(1\text{x}10^5) = 3\text{x}10^{10}\Omega \qquad (3.21)$$

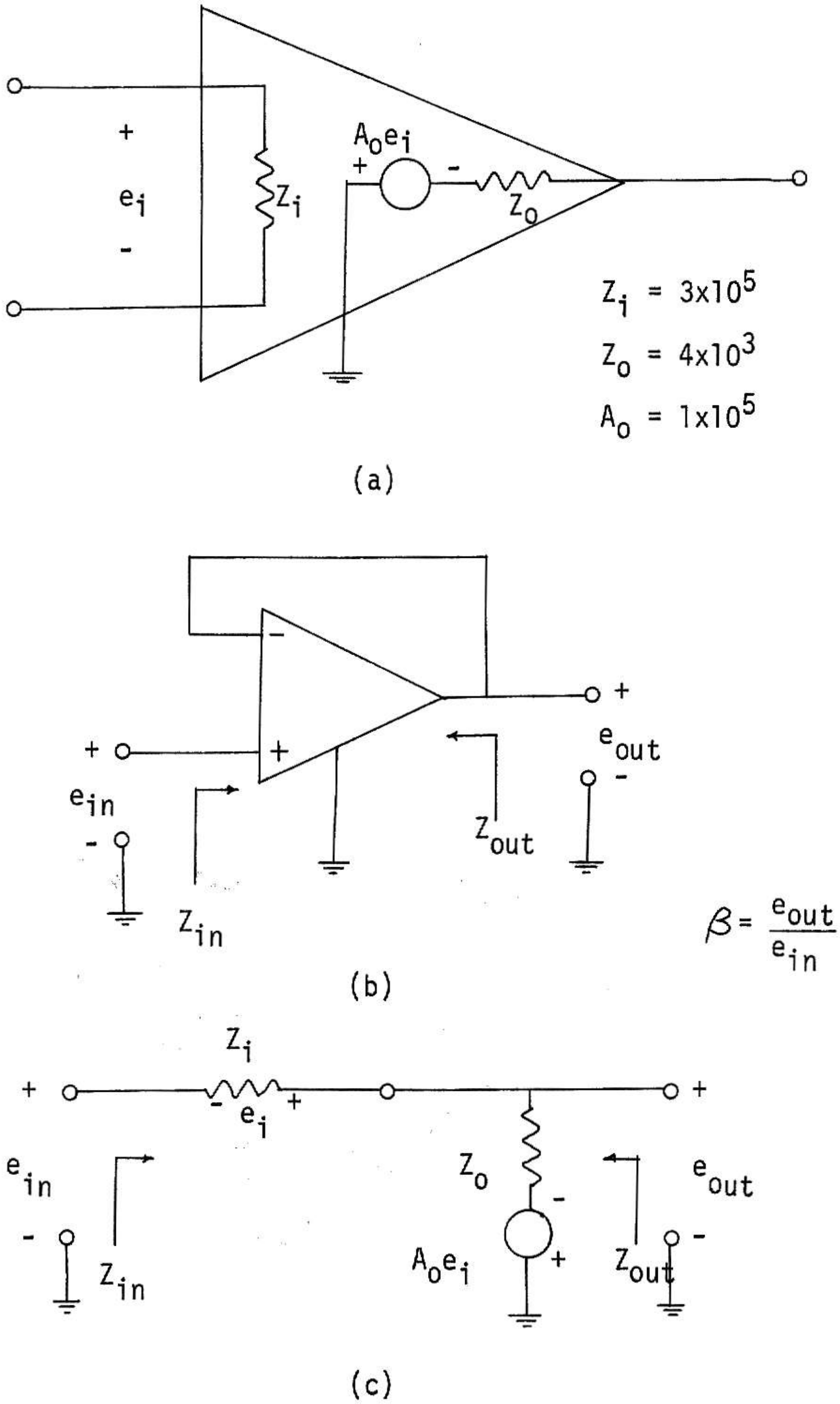


Figure 15. (a) Circuit model for the MC 1539G operational amplifier (b) voltage follower configuration using the MC 1539G (c) circuit model for the voltage follower.

and
$$Z_{out} = \frac{Z_o}{A_o} = \frac{(4\times10^3\Omega)}{(1\times10^5)} = .04\,\Omega\ . \tag{3.22}$$

The exact expression for the gain, β, of the voltage follower (using Fig. 15(c) is

$$\beta = \frac{e_{out}}{e_{in}} = \frac{Z_o + Z_iA_o}{Z_o + Z_i(1 + A_o)} = \frac{4\times10^3 + (3\times10^5)(1\times10^5)}{4\times10^3 + (3\times10^5)(1 + 1\times10^5)} =$$

$$.999990\ . \tag{3.23}$$

An ideal unity gain amplifier would have $Z_{in} = \infty\,\Omega$, $Z_{out} = 0\,\Omega$, and $\beta = 1.000$. Equations (3.21), (3.22), and (3.23) show that the voltage follower using the MC 1539G closely approximates the ideal amplifier and can be used as such.

The gain of the voltage follower must be stable if it is to be used in the active filter networks. The parameters of the operational amplifier (Z_i, Z_o, and A_o) will vary with changing temperature, tolerances, etc., and since β is a function of these parameters, the sensitivity of β with respect to Z_i, Z_o, and A_o must be small if β is to be stable. These sensitivity functions are given below (calculations are simplified if $1+A_o$ is set equal to A_o).

$$S^{\beta}_{Z_i} = \frac{\partial\beta}{\partial Z_i}\cdot\frac{Z_i}{\beta} = \frac{Z_i^2A_o}{[Z_o + Z_iA_o]^2} =$$

$$\frac{(9\times10^{10})(10^5)}{[3\times10^4 + (3\times10^5)(10^5)]^2} = 1.00\times10^{-5} \tag{3.24}$$

$$S^{\beta}_{Z_o} = \frac{\partial \beta}{\partial Z_o} \cdot \frac{Z_o}{\beta} = \frac{Z_i^2\, Z_o}{[Z_o + Z_i A_o]^2} =$$

$$\frac{(4\text{x}10^3)(3\text{x}10^5)}{[4\text{x}10^3 + (3\text{x}10^5)(10^5)]^2} = 1.33\text{x}10^{-12} \tag{3.25}$$

$$S_{A_o} = \frac{\partial \beta}{\partial A_o} \cdot \frac{A_o}{\beta} = \frac{Z_o^2\, A_o}{[Z_o + Z_i A_o]^2} =$$

$$\frac{(16\text{x}10^6)(10^5)}{[4\text{x}10^3 + (3\text{x}10^5)(10^5)]^2} = 1.78\text{x}10^{-9} \tag{3.26}$$

The above equations indicate good stability of β with respect to changes in Z_i, Z_o, and A_o. Since S^{β}_{k} can be interpreted as the percentage change in β divided by the percentage change in k, a one percent change in Z_i would only change β by $1.00\text{x}10^{-5}$ percent. Likewise, a one percent change in Z_o and A_o would only change β by $1.33\text{x}10^{-12}$ percent and $1.78\text{x}10^{-9}$ percent respectively.

Figure 16 shows the MC 1539G operational amplifier in the voltage follower configuration. The compensation network connected to pins 1 and 8 is given in the specifications of the MC 1539G and must be used when the device is used as a voltage follower.

Procedure for Tuning the Filter

Tolerances on the values of the capacitors and resistors result in

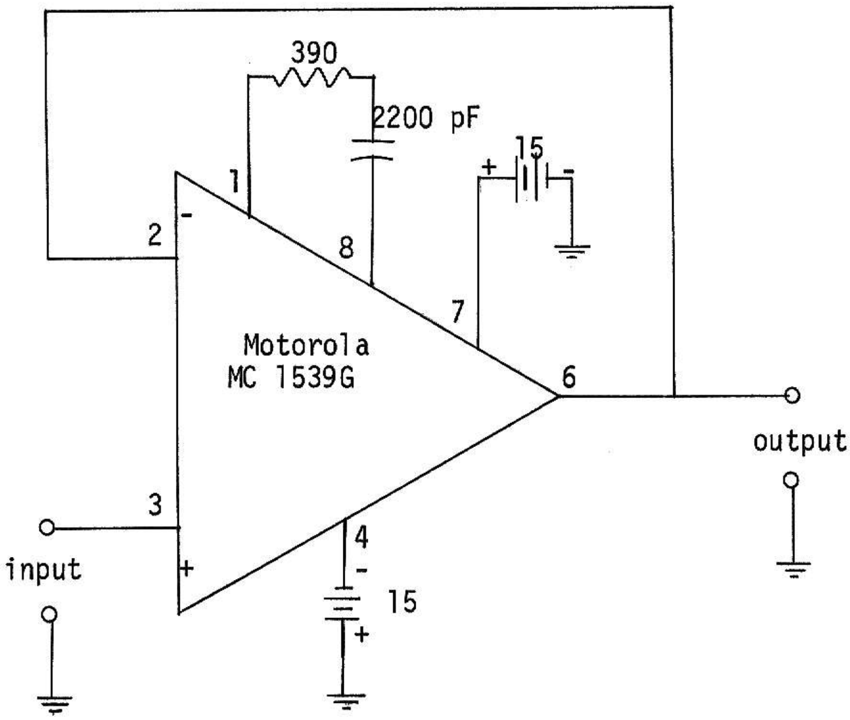


Figure 16. Connections necessary to use the MC 1539G operational amplifier as a unity gain amplifier.

differences between the actual elements and the elements required by the design. Thus, each of the cascaded filter sections must be tuned for the correct response.

The resistors will eventually be constructed using the thick film process and trimming them to tolerances of 1.0 percent to .1 percent is possible. Variable ceramic capacitors with a maximum tuning range of 1-50 pF are also available for changing the values of the fixed capacitors.

The transfer function of the Sallen and Key active lowpass network is repeated below for convenience.

$$T(S) = \frac{\beta/R_1R_2C_1C_2}{S^2 + [1/C_1R_1 + 1/C_1R_2 + (1-\beta)/C_2R_2]\,S + 1/R_1R_2C_1C_2} \tag{3.27}$$

For both $T_1(S)$ and $T_2(S)$, $\beta = 1.000$ and it has been shown that β is very stable. Equation (3.27) can then be reduced to

$$T(S) = \frac{1/R_1R_2C_1C_2}{S^2 + [1/C_1R_1 + 1/C_1R_2]\,S + 1/R_1R_2C_1C_2} \tag{3.28}$$

The poles of the transfer function in (3.28) are

$$\begin{aligned} p_{1,2} &= -.5(1/C_1R_1 + 1/C_1R_2) \\ &\quad \pm j\sqrt{1/R_1R_2C_1C_2 - .25(1/C_1R_1 + 1/C_1R_2)^2}\,. \end{aligned} \tag{3.29}$$

The real parts of the complex conjugate poles p_1 and p_2 are independent of the value of C_2. This suggests the following *tuning procedure*. Component values from Fig. 12 are slightly changed (to allow for tolerances) so that one of the resistors R_1 or R_2 can be trimmed to give the real parts of the poles the correct value. A variable capacitor can then be used to change the value of C_2 until the imaginary parts of the poles achieve the correct values.

To realize the transfer function $T_1(S)$, it was shown that the best component values for the Sallen and Key lowpass circuit were $R_1 = 1.100$, $R_2 = 5.900$, $C_1 = 3.489$, $C_2 = .03824$, and $\beta = 1.000$. The two equations which these elements must satisfy are

$$1/C_1R_1 + 1/C_1R_2 = .3091 \tag{3.30}$$

and

$$1/R_1R_2C_1C_2 = 1.1546 \quad . \tag{3.31}$$

In equation (3.30), β has been eliminated by setting it equal to unity as in the reduced transfer function of equation (3.28).

Ceramic chip capacitors with tolerances of $\pm$ 10 percent and thick film resistors (5 $K\Omega/\Box$ ink) with tolerances controlled to $\pm$ 20 percent will be used to construct the active networks. Substituting the worst case values for R_2 and C_1 into equations (3.30) and (3.31), the ranges over which R_1 and C_2 must be tunable can be determined. Equations (3.30) and (3.31) can be satisfied for $R_2 = 5.900 \pm 20$ percent and $C_1 = 3.489 \pm 10$ percent only if it is possible to tune R_1 and C_2 over the ranges

$$.955 \leq R_1 \leq 1.320 \tag{3.32}$$

and

$$.0321 \leq C_2 \leq .0468 \quad . \tag{3.33}$$

If R_1 is designed to have the value of .7640 $\pm$ 20 percent, then it can be trimmed up in value over the range $.955 \leq R_1 \leq 1.320$. The trimming process only increases the value of the thick film resistors and thus it is necessary to set R_1 = .7640 to allow for a possible 20 percent increase in its value due to tolerance limits.

The normalized component values are R_1 = .7640 $\pm$ 20 percent (R_1 must also have the ability to be trimmed to 1.320), R_2 = 5.900 $\pm$ 20 percent, C_1 = 3.489 $\pm$ 10 percent, and the range for C_2 is $.0321 \leq C_2 \leq .0468$. With these normalized components, the filter's cutoff frequency is ω = 1 rad/sec. Scaling of the components is necessary if the specified cutoff frequency of 8000 Hz is to be obtained.

If all of the resistors and capacitors are replaced by scaled values using

$$R^* = AR \tag{3.34}$$

and

$$C^* = C/AB, \tag{3.35}$$

(R and C are the components to be scaled, R* and C* are the scaled components, A is the impedance scaling constant, and B is the frequency scaling constant) then the scaled transfer function becomes

$$T^*(S) = T(S/B) \quad . \tag{3.36}$$

If $B = 5.027\text{x}10^4$ then T*(S) will have the specified cutoff frequency f_c = 8000 Hz or $\omega_c = 2\pi f_c = 5.027\text{x}10^4$ rad/sec. This value for B will be used to scale each of the networks realizing $T_1(S)$, $T_2(S)$, and $T_3(S)$. The value of A can change for each network since it has no effect on the transfer function.

With reference to the network realizing $T_1(S)$, let $C_1{}^*$ be equal to a standard capacitor value, then A can be solved for using equation (3.35). Let $C_1^* = 3900$ pF, then

$$A = C_1/C_1^*B = (3.489)/(3900\text{x}10^{-12})(5.027\text{x}10^4)$$

$$= 1.780\text{x}10^4 \ .$$

$$R_1^* = AR_1 = (1.780\text{x}10^4)(.7640) = 13.53 \text{ K}\Omega \pm 20\%,$$

$$R_2^* = AR_2 = (1.780\text{x}10^4)(5.900) = 105.0 \text{ K}\Omega \pm 20\%,$$

$$\frac{C_{2min}}{AB} \leq C_2^* \leq \frac{C_{2max}}{AB} \ ,$$

$$\frac{.0321}{(1.780\text{x}10^4)(5.027\text{x}10^4)} \leq C_2^* \leq \frac{.0468}{(1.780\text{x}10^4)(5.027\text{x}10^4)} \ ,$$

$$35.8 \text{ pF} \leq C_2^* \leq 52.5 \text{ pF}$$

The capacitor C_2^* can be tuned over the above range if it is composed of a fixed 22pF capacitor in parallel with a variable 9-35 pF capacitor. The scaled resistor R_1^* must also be capable of being trimmed to a maximum value of $AR_{1max} = (1.780\text{x}10^4)(1.320) = 23.5$ KΩ. Figure 17 shows the final circuit for realizing the transfer function $T_1^*(S)$. This is the circuit which will be constructed by the thick film process.

The network which realizes $T_2(S)$ is also a Sallen and Key lowpass network and the design for the scaled transfer function $T_2^*(S)$ follows that

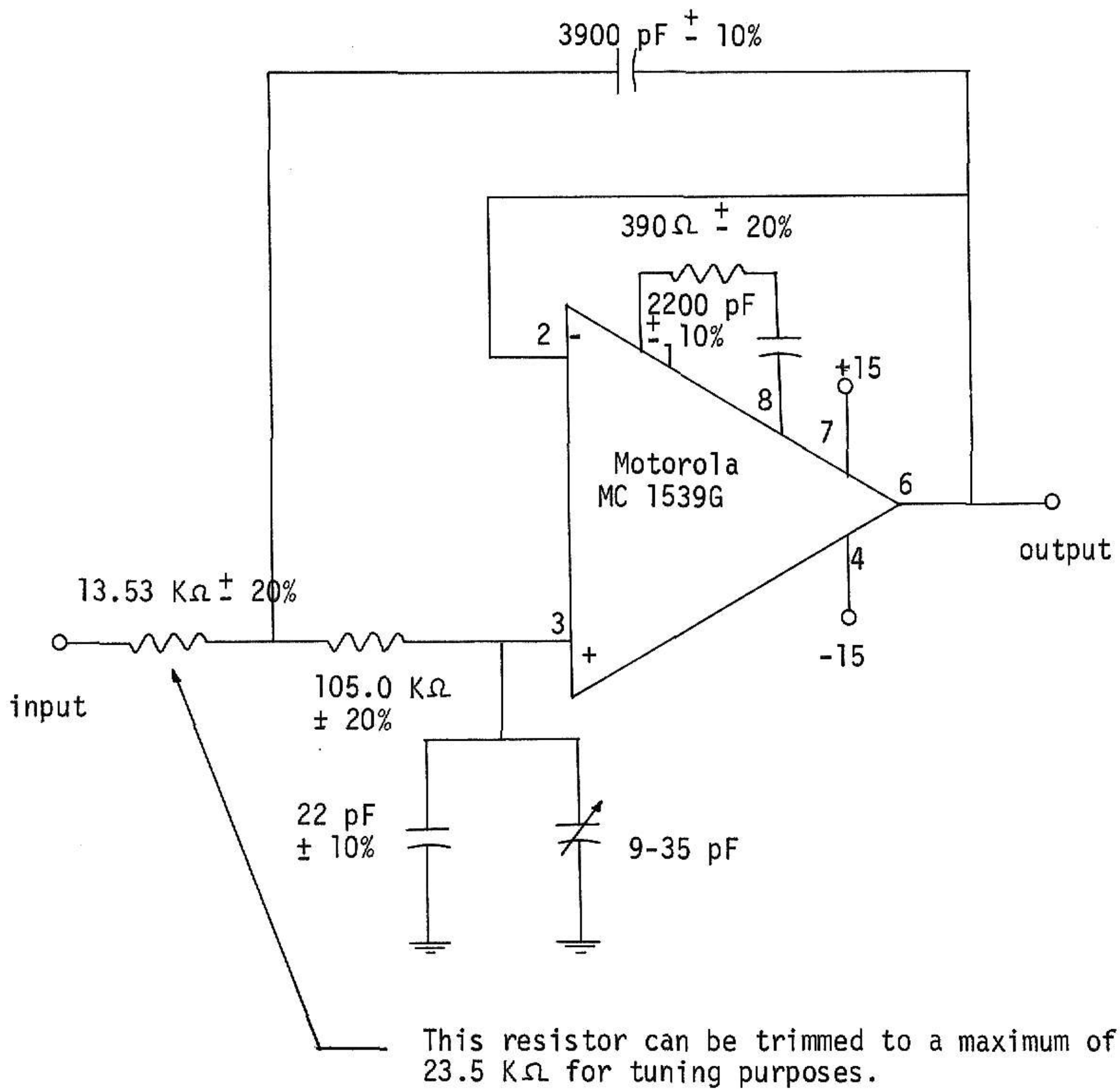


Figure 17. Active network for $T_1^*(S)$ which can be tuned for the correct response.

of $T_1^*(S)$.

From the sensitivity calculations, the best normalized component values for the Sallen and Key circuit (Fig. 12) when realizing T_2 were found to be $R_1 = .9000$, $R_2 = 2.000$, $C_1 = .4685$, and $\beta = 1.000$.

Because of its good stability, β can be set equal to 1.000 in equation (3.13) and the resulting equations that the components must satisfy are

$$1/C_1R_1 + 1/C_1R_2 = .8092 \tag{3.37}$$

$$1/R_1R_2C_1C_2 = .5956 \quad . \tag{3.38}$$

Equations (3.37) and (3.38) can be satisfied for $R_2 = 2.000 \pm 20$ percent and $C_1 = .4685 \pm 10$ percent only if it is possible to tune R_1 and C_2 over the ranges

$$.7400 \leq R_1 \leq 1.210 \tag{3.39}$$

and

$$.4030 \leq C_2 \leq .5470 \quad . \tag{3.40}$$

Because R_1 will have a tolerance of $\pm$ 20 percent, it is designed to equal .5920 so that it can be trimmed to values over the range of equation (3.39).

The normalized component values are $R_1 = .5920 \pm 20$ percent (also must be designed so that it can be trimmed to $R_{1max} = 1.210$), $R_2 = 2.000 \pm 20$ percent, $C_1 = .4685 \pm 20$ percent, and the range of C_2 is $.4030 \leq C_2 \leq .5470$.

The components are scaled according to equations (3.34) and (3.35). The frequency scaling constant B is again equal to 5.027×10^4 because it was determined to scale the entire cascaded filter's response from 1 rad/sec to 5.027×10^4 rad/sec = 8000 Hz.

Let $C_1^* = 150$ pF, then A is given by

$$A = C_1/C_1^*B = (.4685)/(150 \times 10^{-12})(5.027 \times 10^4) = 2.640 \times 10^5 \quad .$$

$$R_1^* = AR_1 = (2.640 \times 10^5)(.5920) = 156.0 \text{ K} \pm 20\% \; ,$$

$$R_2^* = AR_2 = (2.640 \times 10^5)(2.000) = 528.0 \text{ K} \pm 20\% \; ,$$

$$\frac{C_{2min}}{AB} \leq C_2^* \leq \frac{C_{2max}}{AB} \quad ,$$

$$\frac{.4030}{(2.640 \times 10^5)(5.027 \times 10^4)} \leq C_2^* \leq \frac{.5470}{(2.640 \times 10^5)(5.027 \times 10^4)} \quad ,$$

$$30.2 \text{ pF} \leq C_2^* \leq 41.2 \text{ pF} \; .$$

The tuning range for C_2^* (above) can be satisfied if a fixed 22 pF capacitor in parallel with a 7-25 pF variable capacitor is used. The resistor R_1^* must also be designed so that it can be trimmed to a maximum value of $AR_{1max} = (2.640 \times 10^5)(1.210) = 319.4$ KΩ. Figure 18 shows the final circuit for $T_2^*(S)$.

The active network which realizes the transfer function $T_3(S)$ (Fig. 13) can be tuned by trimming the resistor R. The equation which the resistor and capacitor must satisfy is

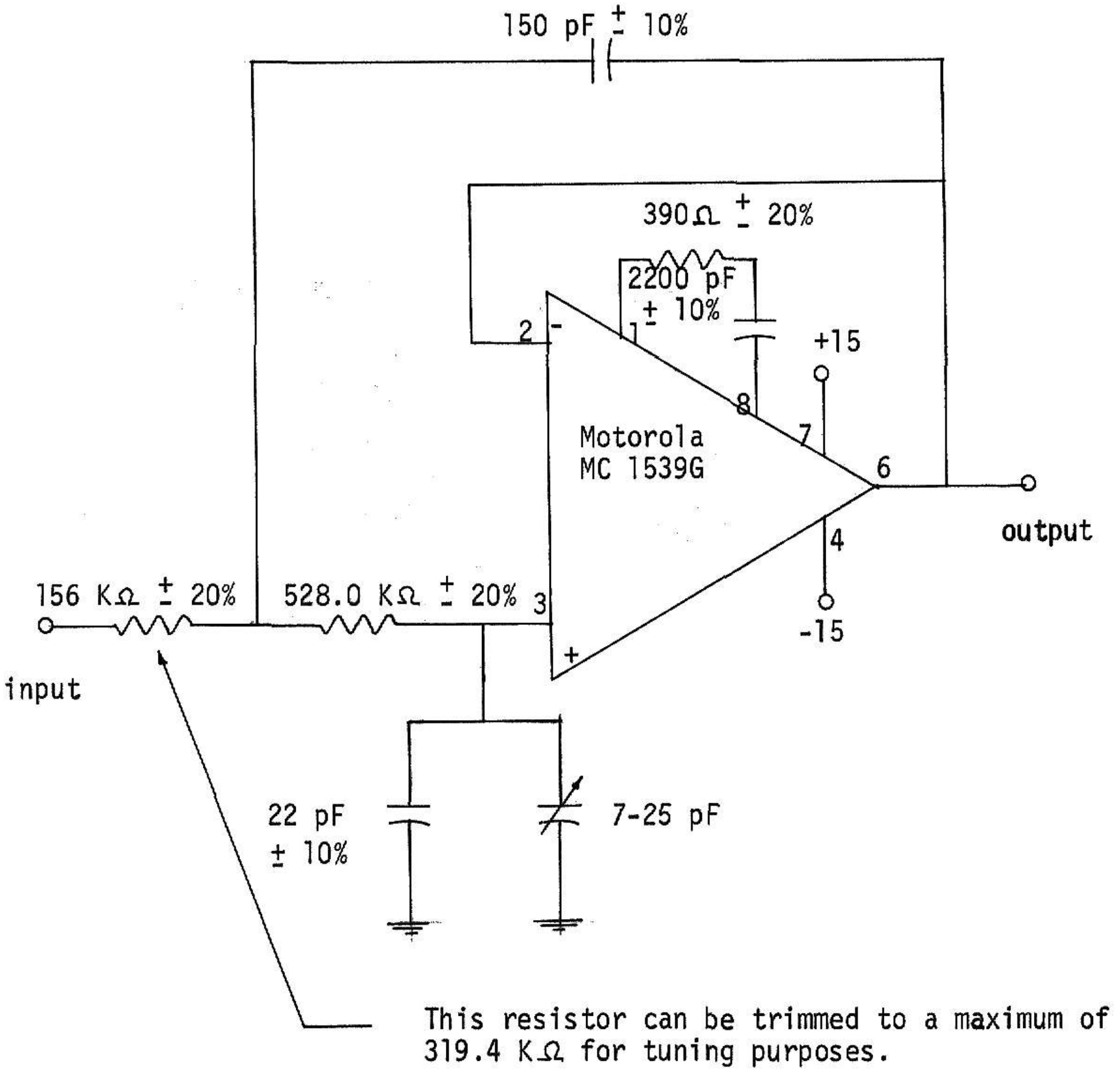


Figure 18. Active network for $T_2^*(S)$ which can be tuned for the correct response.

$$1/RC = .5001 \quad . \tag{3.41}$$

The values for R and C, determined previously, are R = 1.000 and C = 2.000. Again the capacitor C has $\pm$ 10 percent tolerance. The limits on R, such that it can be trimmed for the correct response, are

$$.9091 \leq R \leq 1.111 \quad . \tag{3.42}$$

To allow for the $\pm$ 20 percent tolerance on R, it is given the value R = .7273 $\pm$ 20 percent.

Let C^* = 100 pF $\pm$ 10 percent, then

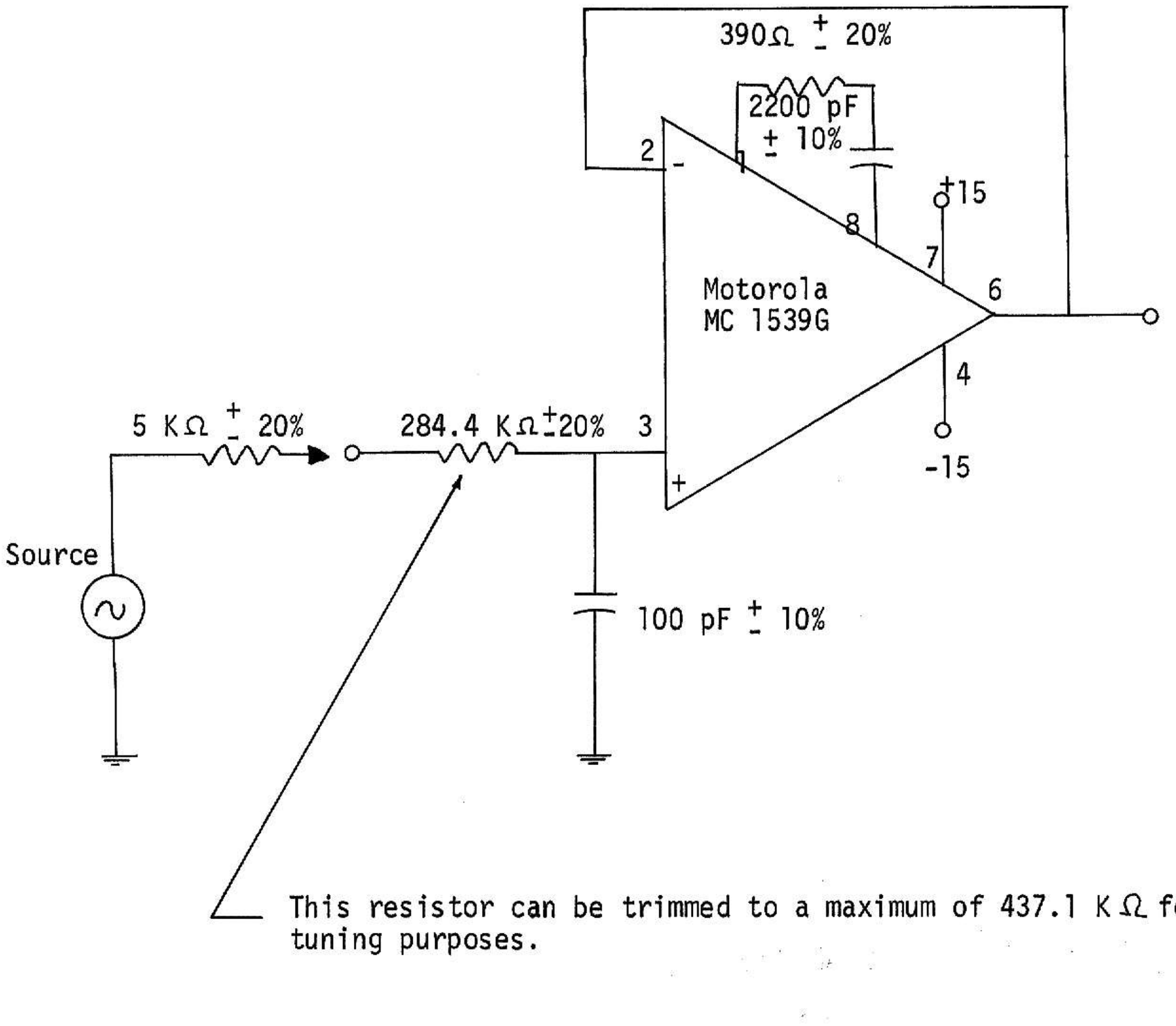
$$A = C/C^*B = (2.000)/(100 \times 10^{-12})(5.027 \times 10^4) = 3.979 \times 10^5 \quad .$$

$$R^* = AR = (3.979 \times 10^5)(.7273) = 289.4 \text{ K}\Omega \quad .$$

$$R^*_{max} = AR_{max} = (3.979 \times 10^5)(1.111) = 442.1 \text{ K}\Omega \quad .$$

Figure 19 shows the active network which realizes $T_3^*(S)$. The resistor R^* has been decreased by 5 KΩ to allow for the specified source impedance. Note that the source impedance can change by 20 percent or 1000 Ω without much change in the designed resistance R^* (approximately .3 percent).

When the components were scaled the transfer functions changed according to the equation $T^*(S) = T(S/B)$. The scaled transfer functions corresponding to $T_1(S)$, $T_2(S)$, and $T_3(S)$ are (see equations 3.2, 3.3, and 3.4)



This resistor can be trimmed to a maximum of 437.1 KΩ for tuning purposes.

Figure 19. Active network for $T_3^*(S)$ which can be tuned for the correct response.

$$T_1^*(S) = T_1(S/B) = \frac{2.919\times10^9}{S^2 + 1.554\times10^4\, S + 2.919\times10^9}, \tag{3.43}$$

$$T_2^*(S) = T_2(S/B) = \frac{1.505\times10^9}{S^2 + 4.068\times10^4\, S + 1.505\times10^9}, \tag{3.44}$$

and

$$T_3^*(S) = T_3(S/B) = \frac{2.514\times10^4}{S + 2.514\times10^4}. \tag{3.45}$$

Figure 20 shows the poles of the above transfer functions and the areas in which they can move and still satisfy the specifications. Figure 20 gives the same information for the scaled transfer functions as Fig. 11 gives for the normalized transfer functions.

As stated previously, the tuning procedure for the Sallen and Key type circuits is to trim the resistor R_1^* until the real part of each pole has the correct value then tune the capacitor C_2^* to obtain the correct imaginary part of each pole. It is necessary to trim R_1^* before tuning C_2^* since the imaginary parts of the complex poles depend upon the value of R_1^*. The real parts of the poles are independent of the value for C_2^* and once the correct values for the real parts of the poles are fixed by R_1^*, the imaginary parts can be tuned to the correct values using C_2^* without affecting the real parts of the poles.

Figure 21 shows a method for measuring the real part of the complex poles of the Sallen and Key circuit which realizes $T_1^*(S)$. The exact pole positions are at $-7772 \pm j\, 53440$ on the complex plane. If the real part of the complex poles can be placed between the lines AA' and BB', then the capacitor C_2^* can be tuned to place the poles in the shaded areas. The real

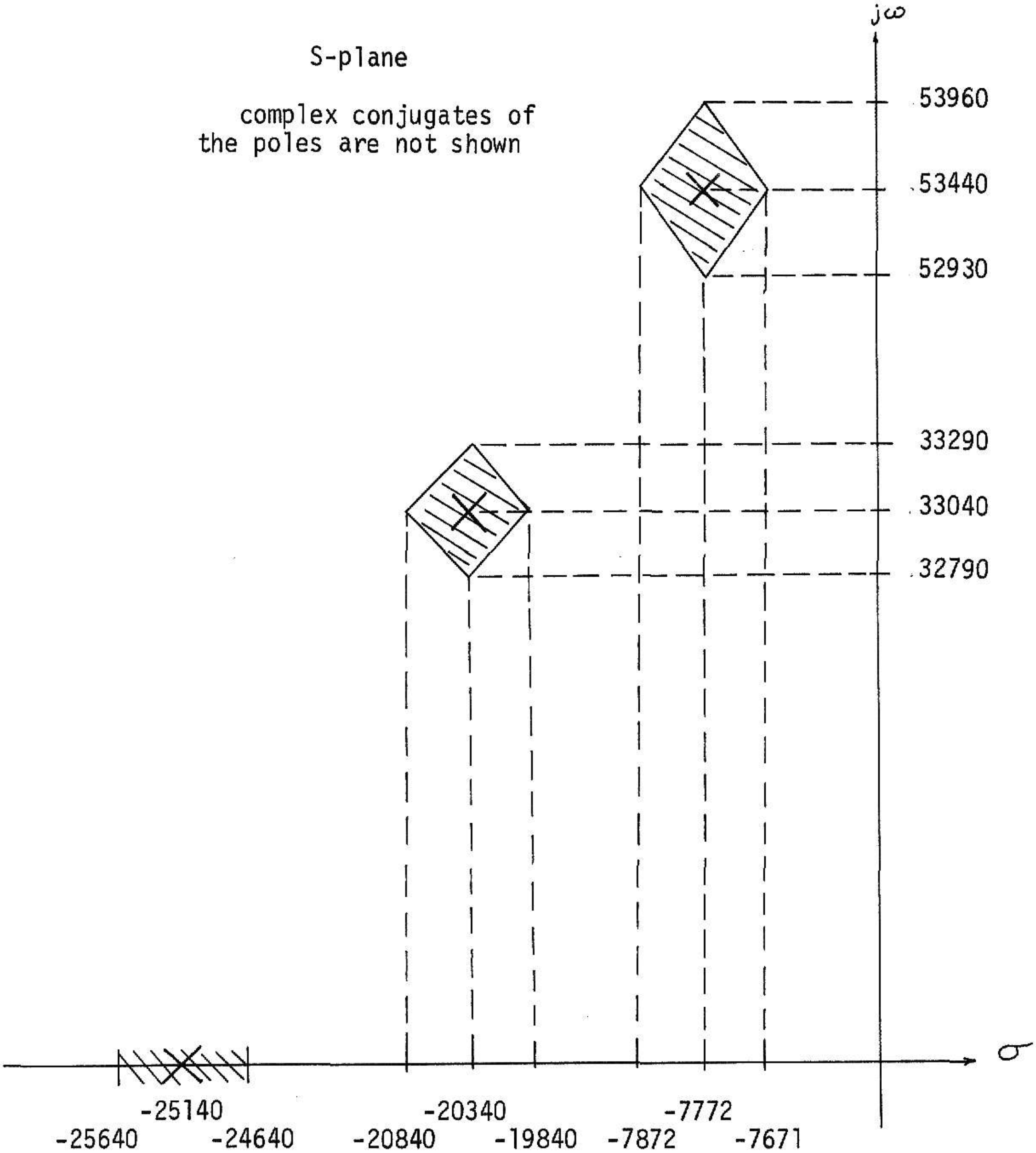


Figure 20. Poles of the scaled transfer function which satisfies the design specifications. Shaded areas show the permitted pole movements which will still meet the specifications.

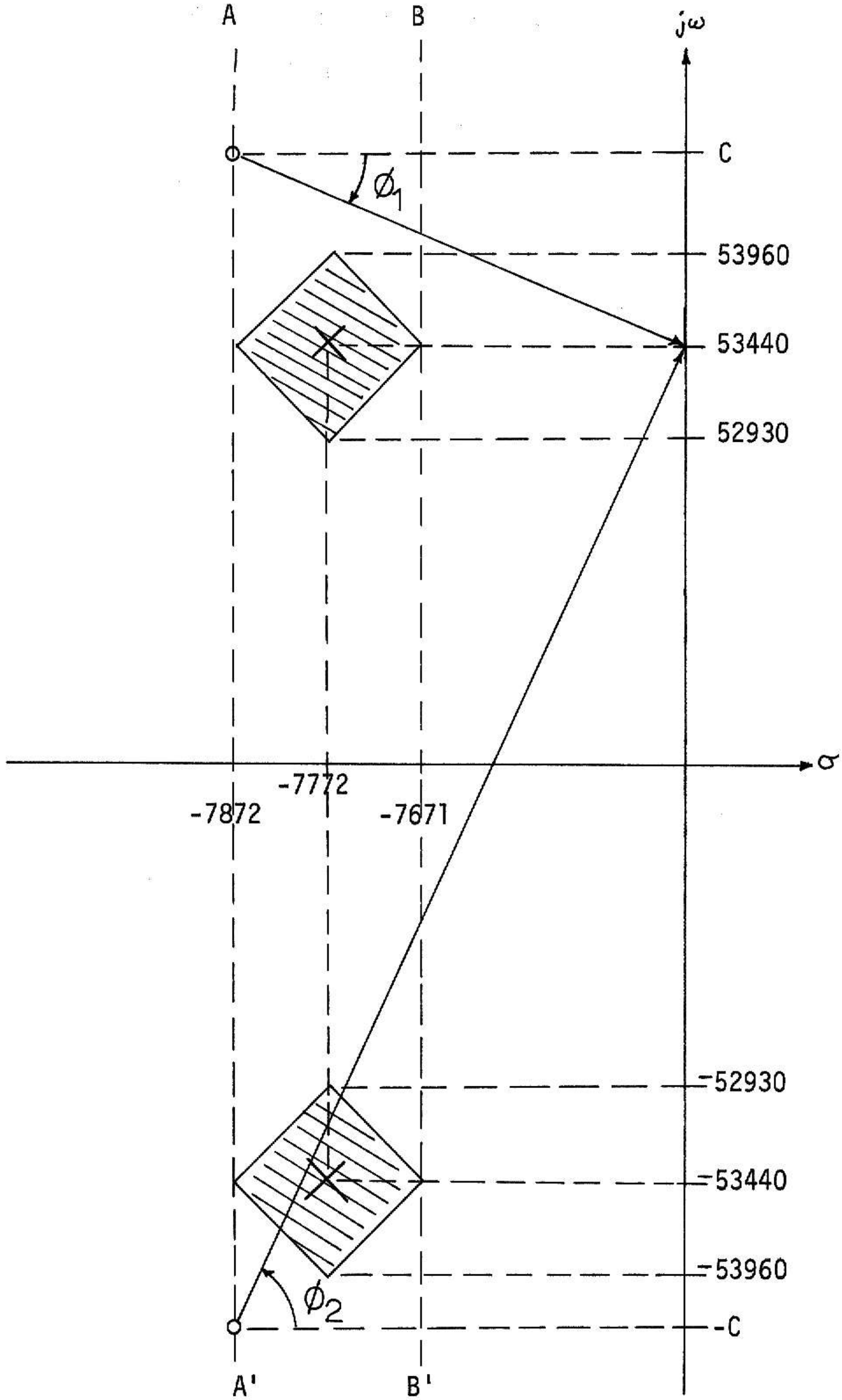


Figure 21. Method for obtaining a tuning chart for the scaled poles S = -7772 ± j53440.

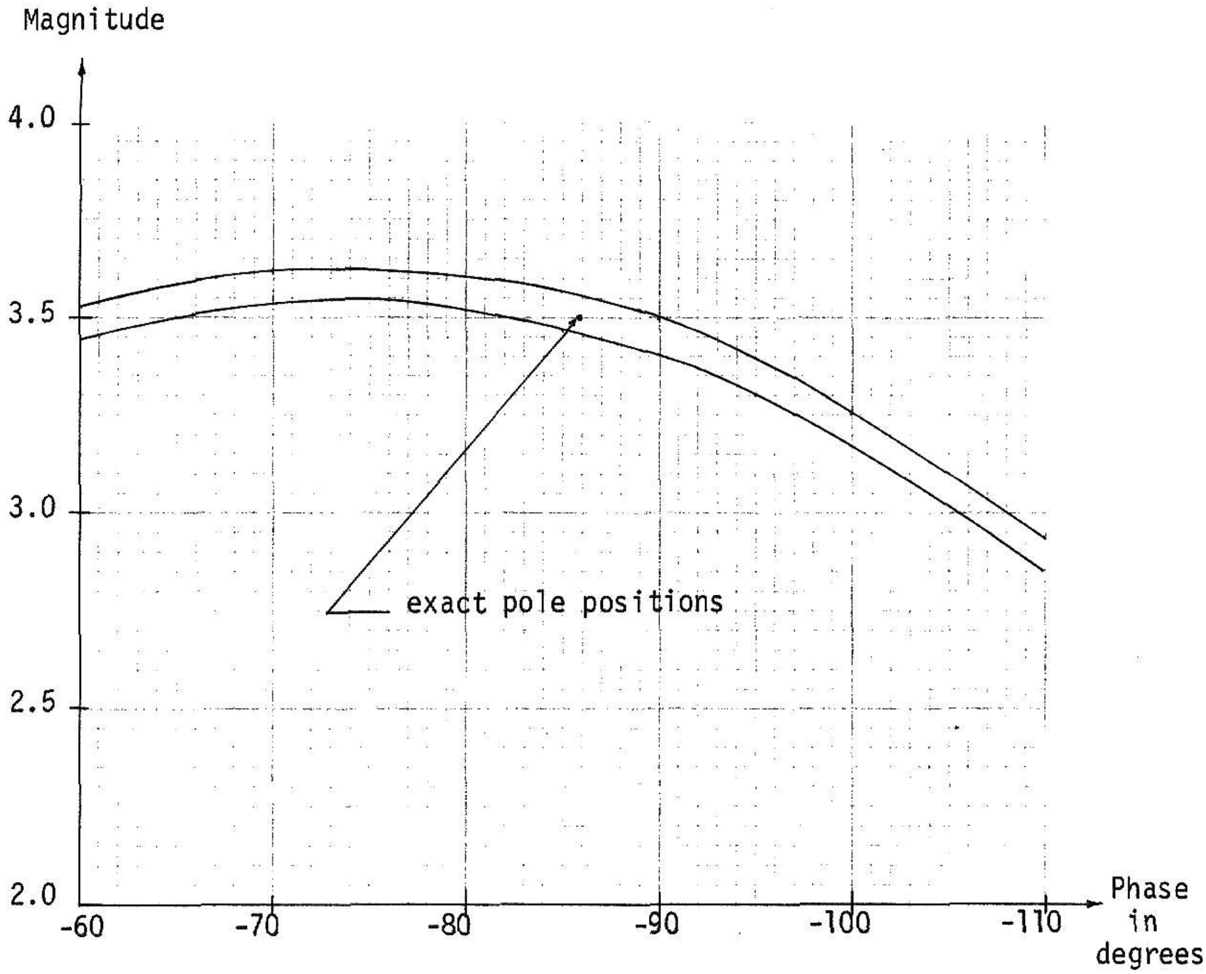


Figure 22. Tuning plot for the poles of $T_1^*(S)$.

part of the poles is not dependent upon the value for C_2^* and hence, changing its value only moves the poles in a direction parallel to the jw axis.

The complex poles are first considered to be at the positions -7872 $\pm$ j C. Now consider the magnitude and phase of the response when the frequency is $f = 53440/2\pi = 8505$ Hz. The magnitude of the response is related to the product of the lengths of the solid lines shown in Fig. 21. The phase angle of the response is given by the sum of the angles $+ \phi_1$ and $- \phi_2$ also shown in the figure. If C is allowed to take on different values along the line AA', then a plot of magnitude vs. phase angle for poles on the line AA' can be obtained. If the same procedure is repeated for the poles moving along the line BB', the plot shown in Fig. 22 results. The frequency at which these measurements are taken remains constant at the value f = 8505 Hz.

If the resistor R_1^* is trimmed until the measured phase angle and magnitude of the response falls between the two lines in Fig. 22, then the real part of the poles will have a value between the lines AA' and BB' in Fig. 21. Once the real part of the poles falls in this region, C_2^* can then be tuned to place the poles in the shaded areas of Fig. 21. The phase and magnitude for the exact pole positions are shown by the point in Fig. 22, and tuning of C_2^* will place the phase angle and magnitude of the response on or near this point.

The magnitude vs. phase angle plot for tuning the second set of complex poles at - 20340 $\pm$ j 33040 (see Fig. 20) is obtained by the same procedure as above. The only change is in the frequency at which the response is measured. For this set of poles the measuring frequency is given by

$f = 33040/2\pi = 5254$ Hz. The plot for the second set of poles (the poles of $T_2^*(S)$) is given by Fig. 23.

The tuning setup for the Sallen and Key type circuits is given in Fig. 24(a). The procedure for tuning the poles of the active network which realizes $T_1^*(S)$ is described below.

1. The audio oscillator is set at the frequency $f = 8505$ Hz.
2. The resistor R_1^* is trimmed until the measured magnitude and phase angle of the response fall inside the two curves shown in Fig. 22.
3. The capacitor C_2^* is then tuned to move the magnitude and phase angle as close as possible to those of the exact pole positions shown in Fig. 22.

The tuning setup of Fig. 24(a) is also used to tune the network realizing $T_2^*(S)$. The audio oscillator is set at $f = 5258$ Hz, and the magnitude vs. phase angle plot of Fig. 23 is used when trimming the resistor R_1^* and the capacitor C_2^*.

The unity gain amplifier used in the tuning setup of Fig. 24(a) contains an operational amplifier so that the source impedance is approximately zero. The unity gain amplifier of the preceding cascaded stage can be used in the tuning setup. This will be described later.

The tuning procedure for $T_3^*(S)$ is simplier than the one used for $T_1^*(S)$ and $T_2^*(S)$ because only a single pole is involved. The exact pole position for $T_3^*(S)$ is $p = -25140$ but the filter specifications are satisfied if the pole falls in the range $-25640 \leq p \leq -24640$ (see Fig. 20).

The transfer function for $T_3^*(S)$ is

$$T_3^*(S) = \frac{2.514\text{x}10^4}{S + 2.514\text{x}10^4} ,$$

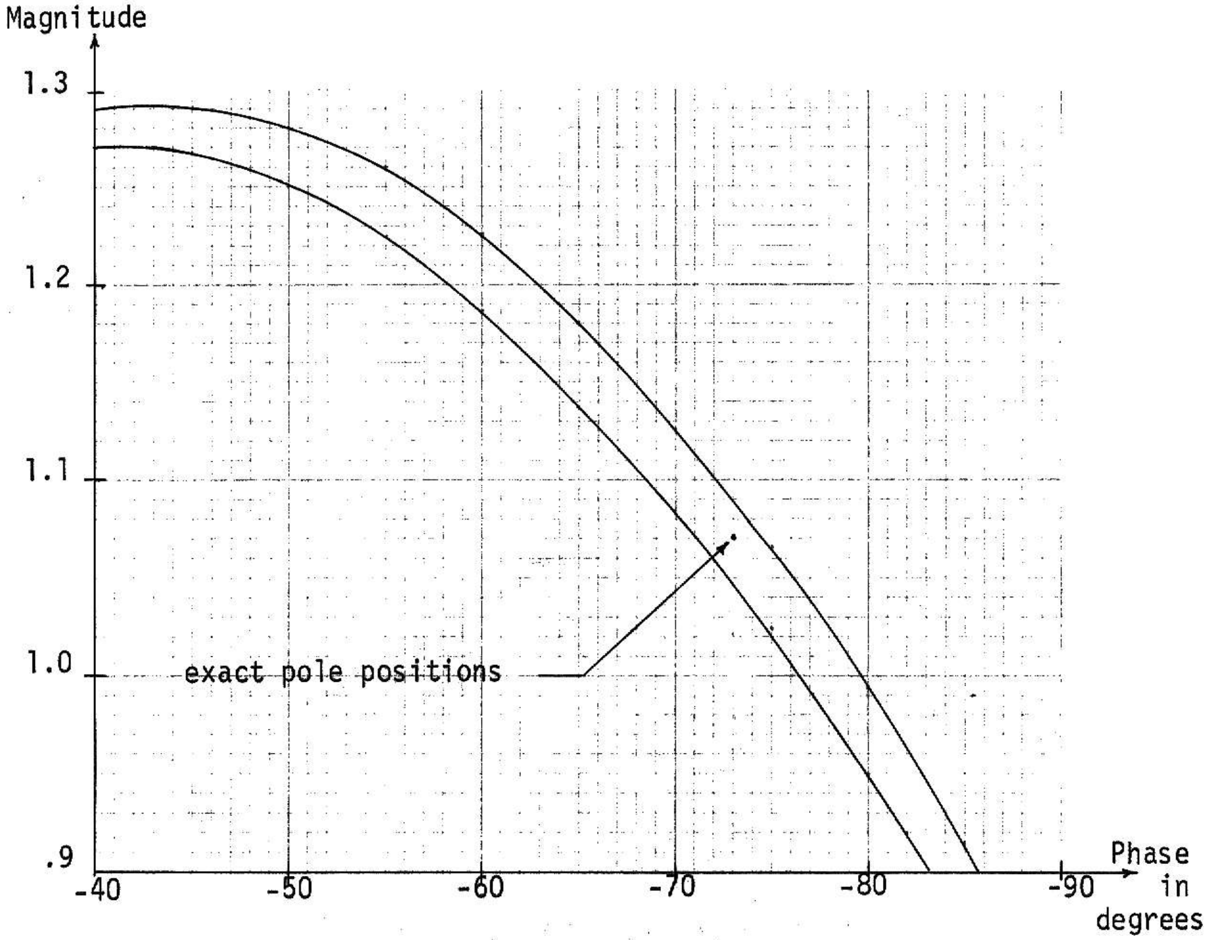


Figure 23. Tuning plot for the poles of $T_2^*(S)$.

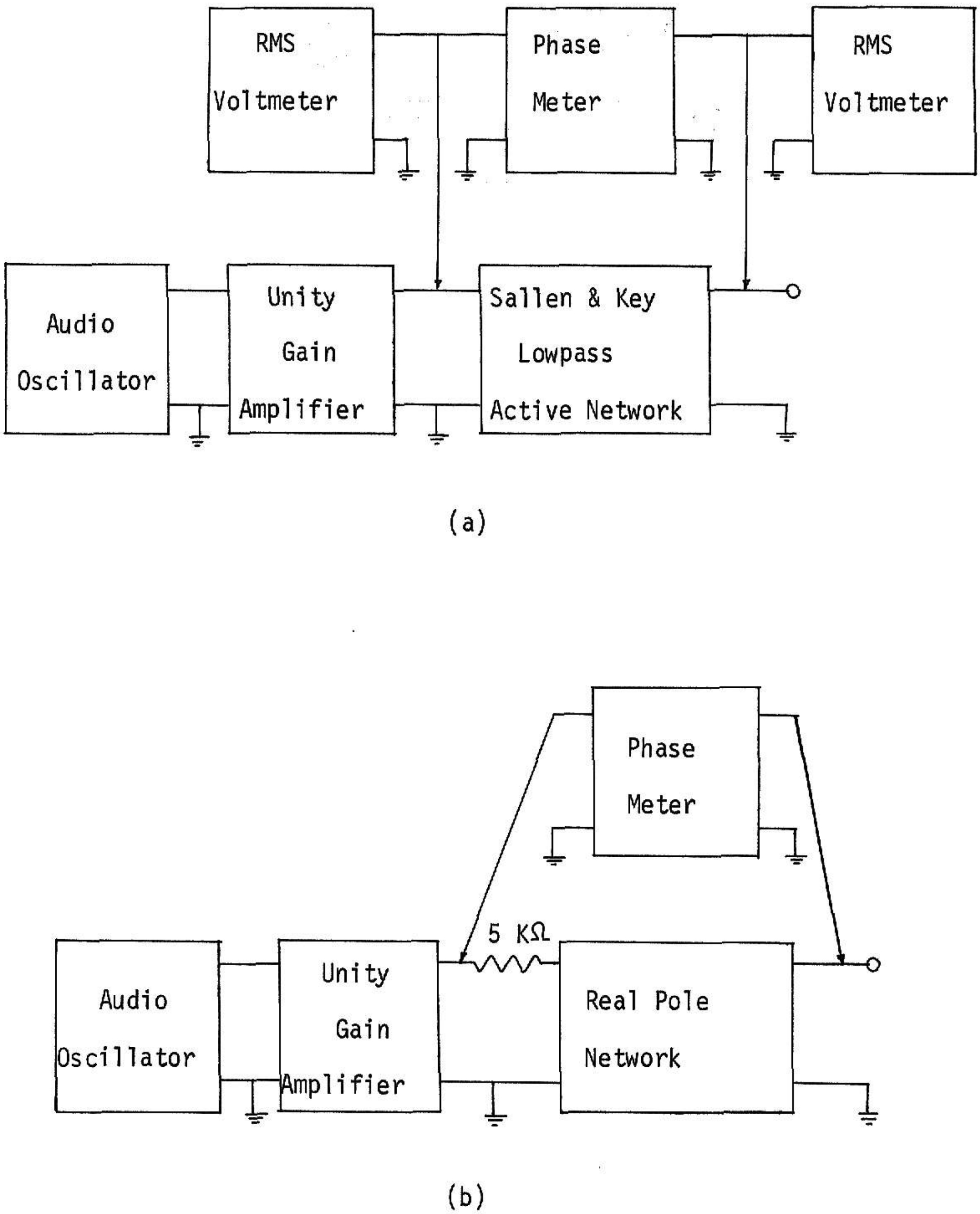


Figure 24. Tuning setups (a) for the Sallen and Key circuits (b) for the real pole network.

and if the phase angle of the response of this transfer function is measured at the frequency $f = 2.514\times10^4/2\pi = 4001$ Hz, it is found to be -45°.

The transfer functions corresponding to the limiting pole positions given in Fig. 20 (i.e. $-25640 \leq p \leq -24640$) are

$$T_3^*(S)' = \frac{2.564\times10^4}{S + 2.564\times10^4} \tag{3.46}$$

and

$$T_3^*(S)'' = \frac{2.464\times10^4}{S + 2.464\times10^4} \quad . \tag{3.47}$$

Now if the transfer functions in (3.46) and (3.47) are measured at the same frequency as $T_3^*(S)$, $f = 4001$ Hz, then the resulting phase angles are -44.4° and -45.6°.

The tuning setup of Fig. 24(b) is used to tune the network which realizes $T_3^*(S)$. The resistor R^* is trimmed until the measured phase angle falls in the range -44.4° to -45.6°. The closer the phase angle is to -45° the closer the pole is to the exact position on the real axis, -2.514×10^4.

The Effect of Changing Temperature

The ceramic capacitors and thick film resistors used to construct the active filter will change in value when the temperature changes from 25° C (room temperature). If the active filter is tuned for the correct response at room temperature, then the response will change when the temperature deviates from room temperature.

This section of the paper shows how the response of the active filter is changed when the temperature varies. The next chapter uses the results of this section to calculate the temperature range over which the filter's response satisfies the specifications.

The results of this section are based on two assumptions. First, it is assumed that all the resistors and capacitors of the filter are at the same temperature. The temperature is approximately uniform over the reduced surface area obtained by the use of thick film circuits. Since the thick film resistors are all made from the same ink and the capacitors are all of the ceramic type, the second assumption is that all the thick film resistors have the same percentage change and all the capacitors have the same percentage change when the temperature deviates from room temperature.

A general lowpass second order transfer function and the reduced Sallen and Key lowpass transfer function (β = 1.000) are given below.

$$T(S) = \frac{w_n^2}{S^2 + (w_n/Q)\,S + w_n^2} \tag{3.48}$$

$$T(S) = \frac{1/C_1C_2R_1R_2}{S^2 + [1/C_1R_1 + 1/C_1R_2]\,S + 1/C_1C_2R_1R_2} \tag{3.49}$$

Using equations (3.48) and (3.49) the Q factor and w_n for the Sallen and Key lowpass network are

$$Q = \frac{\sqrt{C_1C_2R_1R_2}}{C_2R_2 + C_2R_1} \tag{3.50}$$

and

$$w_n = \frac{1}{\sqrt{C_1 C_2 R_1 R_2}} \tag{3.51}$$

The following sensitivity functions can be obtained from equations (3.50) and (3.51):

$$S^Q_{C_1} = - S^Q_{C_2} = 1/2 \quad , \tag{3.52}$$

$$S^Q_{R_1} = - S^Q_{R_2} = - 1/2\left[\frac{R_1 - R_2}{R_1 + R_2}\right] \quad , \tag{3.53}$$

and

$$S^{w_n}_{C_1} = S^{w_n}_{C_2} = S^{w_n}_{R_1} = S^{w_n}_{R_2} = - 1/2 \tag{3.54}$$

If the temperature changes from room temperature by the amount ΔT, denote the changes in the components R_1, R_2, C_1, and C_2 by ΔR_1, ΔR_2, ΔC_1, and ΔC_2. The term ΔQ is defined as the change in Q when the temperature changes by the amount ΔT or since Q is a function of R_1, R_2, C_1, and C_2,

$$Q = Q(R_1 + \Delta R_1, R_2 + \Delta R_2, C_1 + \Delta C_1, C_2 + \Delta C_2) - Q(R_1, R_2, C_1, C_2) \quad .$$

Using the first order terms of a Taylor series expansion, ΔQ can be approximated by

$$\Delta Q = \frac{\partial Q}{\partial R_1}\Delta R_1 + \frac{\partial Q}{\partial R_2}\Delta R_2 + \frac{\partial Q}{\partial C_1}\Delta C_1 + \frac{\partial Q}{\partial C_2}\Delta C_2 \quad . \tag{3.55}$$

Using the definition of the sensitivity function, equation (3.55) can be expressed as

$$\frac{\Delta Q}{Q} = S_{R_1}^{Q}\frac{\Delta R_1}{R_1} + S_{R_2}^{Q}\frac{\Delta R_2}{R_2} + S_{C_1}^{Q}\frac{\Delta C_1}{C_1} + S_{C_2}^{Q}\frac{\Delta C_2}{C_2} \quad . \tag{3.56}$$

Likewise $\Delta w_n/w_n$ can be expressed as

$$\frac{\Delta w_n}{w_n} = S_{R_1}^{w_n}\frac{\Delta R_1}{R_1} + S_{R_2}^{w_n}\frac{\Delta R_2}{R_2} + S_{C_1}^{w_n}\frac{\Delta C_1}{C_1} + S_{C_2}^{w_n}\frac{\Delta C_2}{C_2} \tag{3.57}$$

If the sensitivity functions of equations (3.52) through (3.54) are substituted into equations (3.56) and (3.57), the results are

$$\frac{\Delta Q}{Q} = 1/2\ \frac{R_1 - R_2}{R_1 + R_2}\left[\frac{\Delta R_2}{R_2} - \frac{\Delta R_1}{R_1} + 1/2\ \frac{\Delta C_1}{C_1} \quad \frac{\Delta C_2}{C_2}\right] \tag{3.58}$$

and

$$\frac{\Delta w_n}{w_n} = -1/2\left[\frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta C_1}{C_1} + \frac{\Delta C_2}{C_2}\right] \quad . \tag{3.59}$$

R_1, R_2, C_1, and C_2 are all assumed to be at the same temperature ($25^{o}C$ + ΔT) and since the percentage change in R_1 is equal to the percentage change in R_2 for changes in temperature,

$$\frac{\Delta R_1}{R_1} = \frac{\Delta R_2}{R_2} \quad . \tag{3.60}$$

Also C_1 and C_2 have the same percentage changes for changes in temperature thus

$$\frac{\Delta C_1}{C_1} = \frac{\Delta C_2}{C_2} \quad . \tag{3.61}$$

Using equations (3.60) and (3.61), the equations (3.58) and (3.59) reduce to the simple expressions

$$\frac{\Delta Q}{Q} = 0 \tag{3.62}$$

and

$$\frac{\Delta w_n}{w_n} = - \left[\frac{\Delta R}{R} + \frac{\Delta C}{C} \right] \quad . \tag{3.63}$$

The above equations are approximations since the second order terms were neglected in (3.56) and (3.57). Note that in equation (3.63) the subscripts are dropped on R and C since $\Delta R/R = \Delta R_1/R_1 = \Delta R_2/R_2$ and $\Delta C/C = \Delta C_1/C_1 = \Delta C_2/C_2$.

Equations (3.62) and (3.63) are important because they show that for a change in temperature ΔT (from room temperature), the change in the Q of a Sallen and Key lowpass network is approximately zero. Only the w_n term is affected and the change Δw_n is proportional to the sum of the changes $\Delta R/R$ and $\Delta C/C$ of the resistors and capacitors.

$T'(S)$ is defined as the transfer function of a Sallen and Key lowpass network which is altered by a change in temperature ΔT from room temperature. Using equations (3.48), (3.62) and (3.63), $T'(S)$ can be expressed as

$$T'(S) = \frac{(w_n + \Delta w_n)^2}{S^2 + (w_n + \Delta w_n)/Q \;\; S + (w_n + \Delta w_n)^2} \tag{3.64}$$

If $w' = 1 + \Delta w_n/w_n$, then the expression for $T'(S)$ above can be written as

$$T'(S) = \frac{w_n^2}{(S/w')^2 + [w_n/Q]\,(S/w') + w_n^2} \quad . \tag{3.65}$$

Equation (3.65) indicates that the transfer function $T'(S)$, which is the transfer function of a Sallen and Key lowpass network at the temperature $25^oC + \Delta T$, is equal to $T(S/w')$ where $T(S)$ is the transfer function of the same Sallen and Key network at 25^oC.

The term $w' = 1 + \Delta w_n/w_n = 1 - [\,\Delta R/R + \Delta C/C]$ acts like a frequency scaling term and shifts the response of $T(S)$ in frequency but the shape of the response remains the same.

It can be shown that the real pole network acts in the same manner as the Sallen and Key network with respect to changes in temperature. If $T_p(S)$ represents the transfer function of a real pole network like

$$T_p(S) = \frac{w_p}{S + w_p} \quad ,$$

then using the same type of analysis that was used for the Sallen and Key network, it can be shown that $T_p'(S) = T_p(S/w')$. $T_p'(S)$ is the transfer function of the real pole network at the temperature $25^oC + \Delta T$, $T_p(S)$ is the transfer function of the same network at 25^oC, and w' is the same term used for the Sallen and Key network i.e. $w' = 1 -[\Delta R/R + \Delta C/C]$.

Now the transfer function of the designed active filter is $T^*(S) = T_1^*(S)\, T_2^*(S)\, T_3^*(S)$, where the terms $T_1^*(S)$, $T_2^*(S)$ and $T_3^*(S)$ are given by equations (3.43), (3.44) and (3.45). The transfer function for the active filter at the temperature $25^oC + \Delta T$ can then be represented by $T^{*\prime}(S) = T_1^{*\prime}(S)\, T_2^{*\prime}(S)\, T_3^{*\prime}(S)$, where $T_1^{*\prime}(S)$, $T_2^{*\prime}(S)$ and $T_3^{*\prime}(S)$ are the transfer functions of the networks which realize $T_1^*(S)$, $T_2^*(S)$ and $T_3^*(S)$ when the temperature is $25^oC + \Delta T$. The effect of a change in temperature of ΔT is only to scale the transfer functions $T_1^*(S)$, $T_2^*(S)$ and $T_3^*(S)$ by the term w', thus

$$T^{*\prime}(S) = T_1^*(S/w')\, T_2^*(S/w')\, T_3^*(S/w')$$

or

$$T^{*\prime}(S) = T^*(S/w') \quad . \qquad (3.66)$$

Equation (3.66) has the following interpretation. If $T^*(S)$ is the transfer function of the active filter at 25^oC and $T^{*\prime}(S)$ is the transfer function of the same filter at $25^oC + \Delta T$, then $T^{*\prime}(S) = T^*(S/w')$ where $w' = 1 - [\Delta R/R + \Delta C/C]$.

Equation (3.66) provides a practical way to determine the response of the active filter at different temperatures. The results of this section are used in the next chapter to determine the temperature range over which the active filter will satisfy the specifications.

Final Thick Film Circuits

Because the thick film process for making circuits depends on materials and equipment available, individual choice of layout, etc., the process will be summarized instead of giving the details of the procedure for the particular active filter circuits. A summary of the thick film process for making circuits is given below.

1. Design a suitable layout consistent with available substrate sizes, thick film resistor and conductor inks, and ceramic chip capacitors.
2. Prepare Rubylith master plates of the resistor and conductor patterns.
3. Photoreduce the Rubylith master plates to the size of the substrates on which the thick film circuits are to be constructed.
4. Prepare stencils and transfer them to the silk screens.
5. Print the resistor and conductor inks onto the substrates using the prepared silk screen and a silk screen printer.
6. Allow the substrates to air dry, then fire them in a furnace.
7. Attach the leads, operational amplifiers, and ceramic chip capacitors to the substrates.
8. Trim the thick film resistors.

There are three topics involved with the above process which deserve special attention with regard to the active filter design. These topics are: the use of top hat thick film resistors to allow for large trimming ranges; the thick film circuits used to realize each of the transfer functions $T_1^*(S)$, $T_2^*(S)$, and $T_3^*(S)$; and the final connection of these circuits to complete the active filter.

Figure 25(a) shows a typical top hat thick film resistor. If the resistor is made from 5 $K\Omega/\square$ ink, then the approximate resistance before trimming is

$$R_{BT} = (5\ K\Omega/\square)(\,2\square + 1\square + 2\square\,) = 25\ K\Omega\ . \qquad (3.67)$$

Most of the current passing through the resistor passes through only the 3x3 square of the top hat portion of the resistor thus giving only one

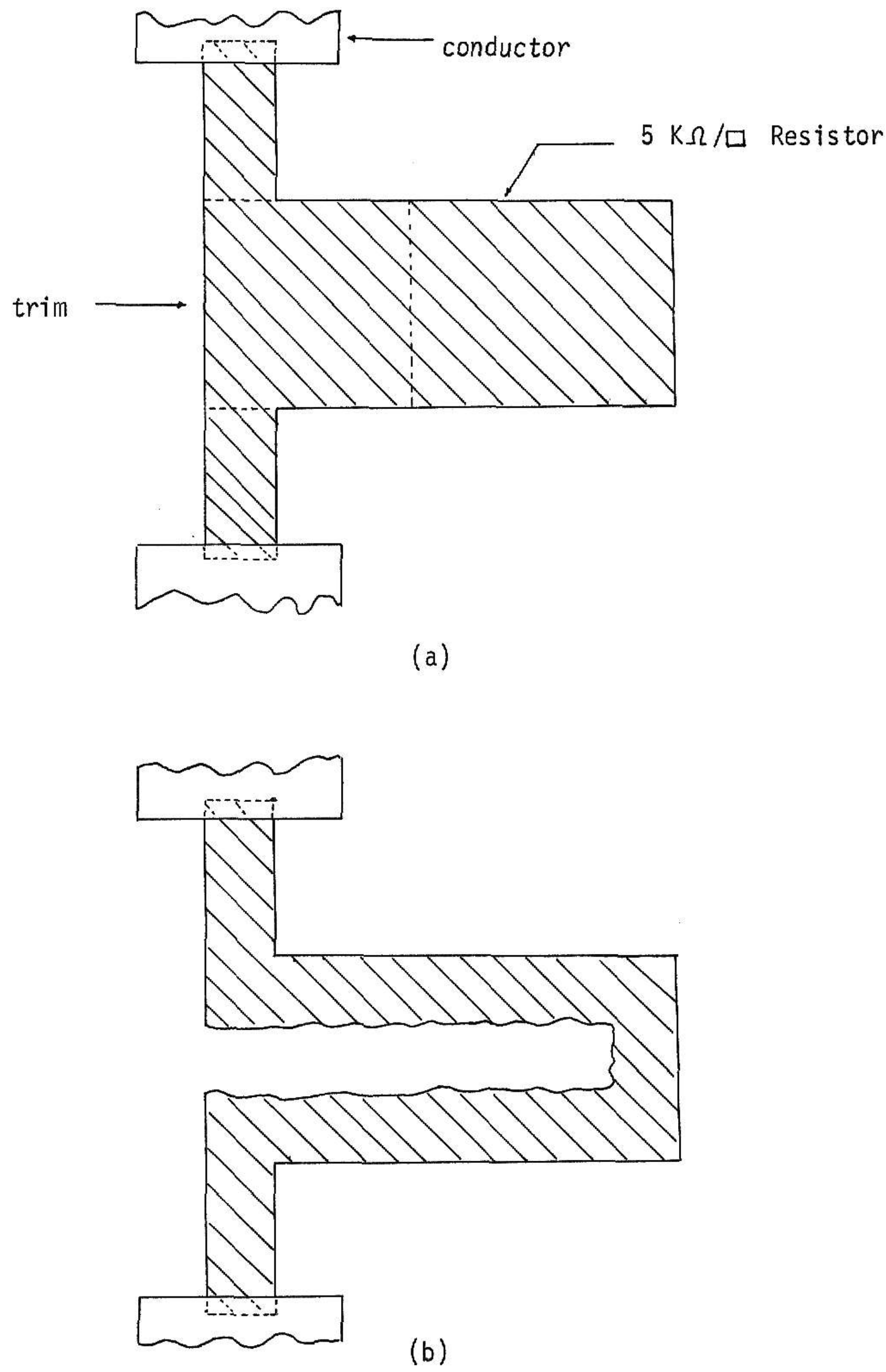


Figure 25. (a) Tophat resistor before trimming (b) after trimming.

square in equation (3.67).

Figure 25(b) shows the top hat resistor after trimming and its approximate resistance is

$$R_{AT} = (5\ K\Omega/\square)\ [15\,\square + 4(1/2)\,\square] = 85\ K\,\Omega\ . \tag{3.68}$$

The resistors at the corners are counted only as one half of a square.

The difference in the values of R_{BT} and R_{AT} indicates the large range of trimming available when top hat resistors are used. Top hat resistors are used to make the resistors R_1^* of the Sallen and Key type circuits and R^* of the real pole circuit so that the designed range of resistance needed for tuning can be achieved.

Figure 26(a) shows the unity gain amplifier used in each of the active networks for $T_1^*(S)$, $T_2^*(S)$, and $T_3^*(S)$. The substrate is 1" x 1". The operational amplifier is not shown connected to the substrate but the circled numbers on the conductors indicate the proper connections.

Figures 26(b), 27(a), and 27(b) show the thick film circuits which contain the passive components for $T_1^*(S)$, $T_2^*(S)$, and $T_3^*(S)$ respectively. The ceramic chip capacitors are shown and the positions of the variable ceramic capacitors are indicated by the dashed lines. The resistors which are to be trimmed, for tuning the network, are also shown. Each of the substrates containing the passive components are 1" x 1".

The resistor ink used to make the resistors for the passive networks was measured to be 15.5 $K\Omega/\square$ instead of the 5 $K\Omega/\square$ as labeled. The resistors in Figs. 26(b), 27(a), and 27(b) are designed using the measured value rather than the labeled value of the resistor ink.

Figure 28 shows the final active filter. Each of the networks of

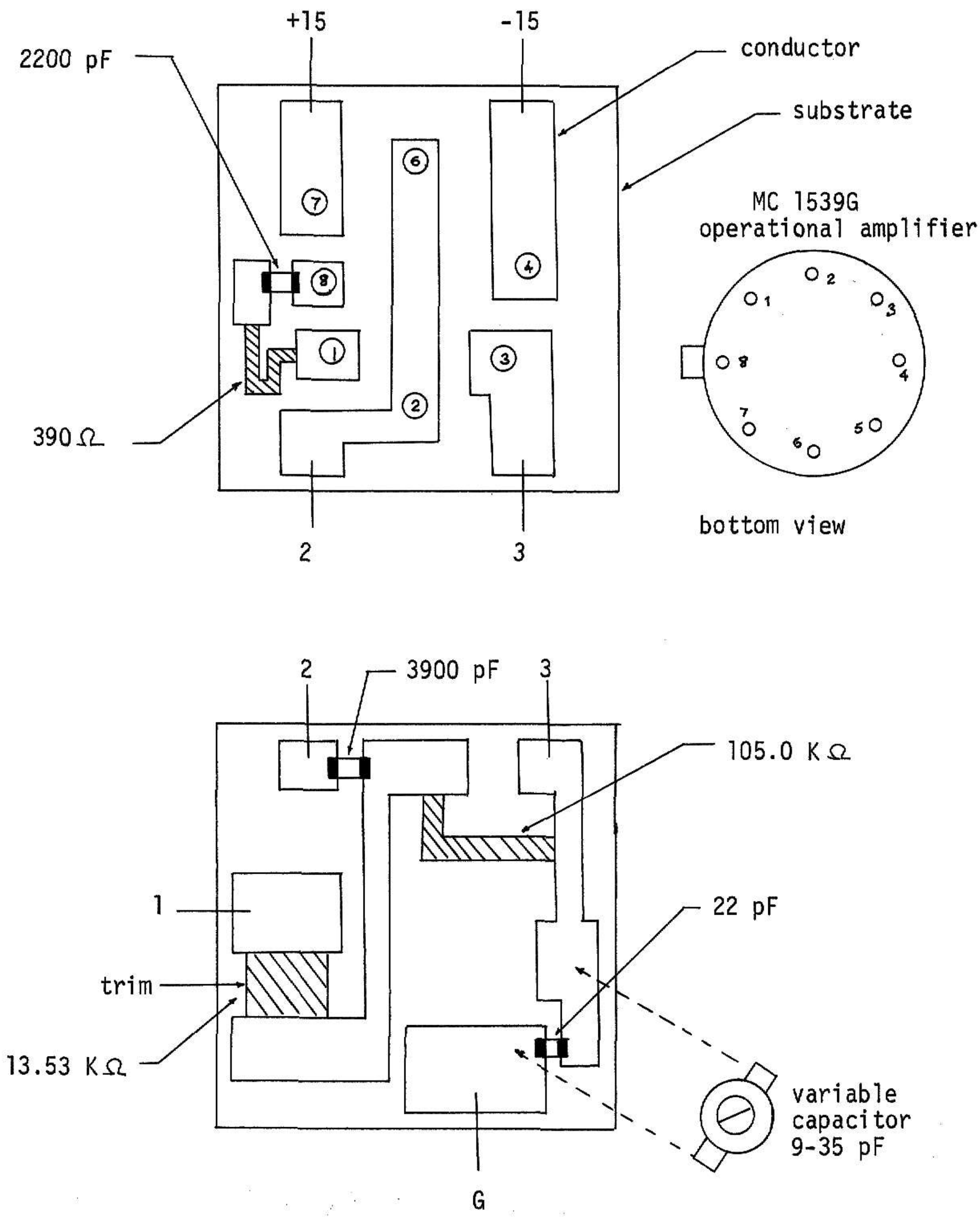


Figure 26. (a) Unity gain amplifier (b) passive network for $T_1^*(S)$.

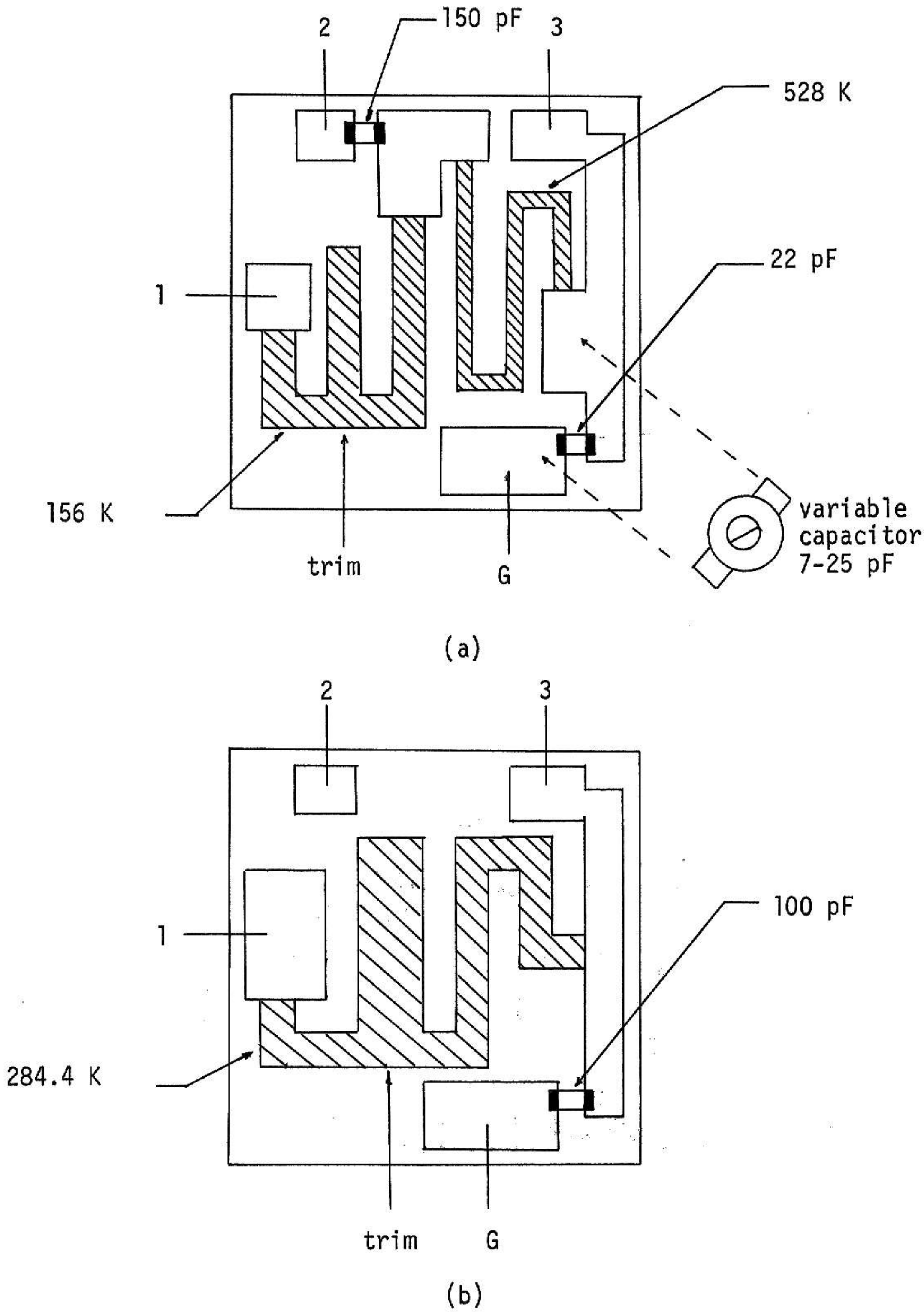


Figure 27. (a) Passive network for $T_2^*(S)$ (b) passive network for $T_3^*(S)$.

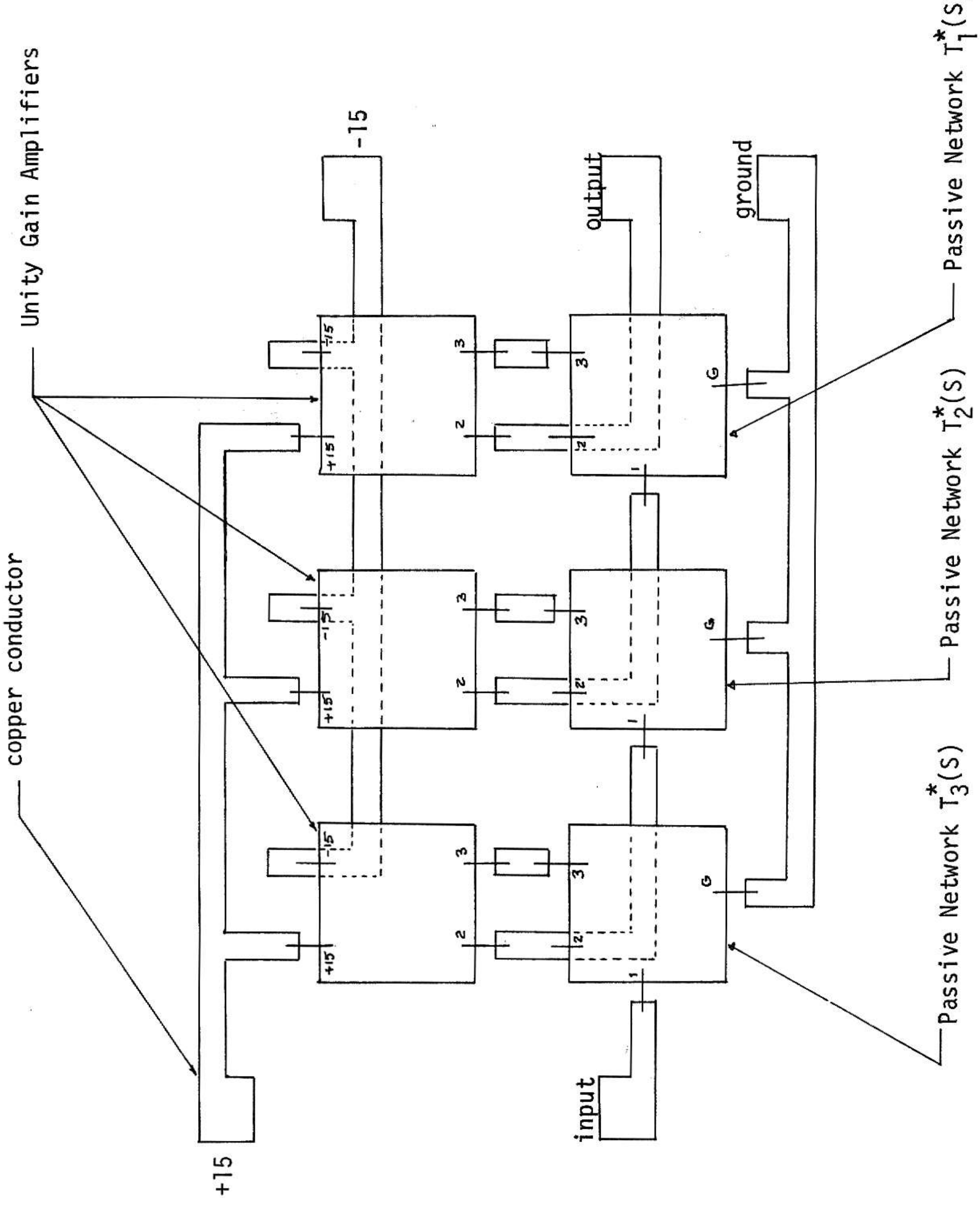


Figure 28. Printed circuit board with substrates connected.

Figs. 26 and 27 are mounted on a printed circuit board as indicated by the numbers on the external leads for each circuit. The substrates lie on top of the copper conductors and the leads are soldered directly to the copper. The input, output, ground and power supply terminals of the filter are indicated on the copper conductors of the printed circuit board.

It was mentioned in the section on tuning the active filter that the unity gain amplifier of each preceding stage could be used as a low impedance source when tuning the filter. Figure 29 indicated how this can be done. The unity gain amplifier for both $T_1^*(S)$ and $T_2^*(S)$ are soldered to the printed circuit board. Then the passive network for $T_1^*(S)$ is soldered into place. If the oscillator of the tuning setup, as shown in Fig. 24(a), is connected to the unity gain amplifier of $T_2^*(S)$ at pin 3, as indicated in Fig. 29(a), then the circuit of Fig. 29(b) is the result. Figure 29(b) shows the unity gain amplifier of $T_2^*(S)$ being used as a low impedance source for the active network which realizes $T_1^*(S)$. If this procedure is used, then there is no need to use the amplifier shown in the tuning setup of Fig. 24(a). After the network for $T_1^*(S)$ is tuned, the unity gain amplifier of $T_3^*(S)$ can be used to tune the network for $T_2^*(S)$. $T_3^*(S)$ can also be tuned without the use of an additional amplifier by placing the appropriate impedance in parallel or series with the oscillator so that the total source impedance for the oscillator is 5 KΩ as specified.

This completes the design of the active filter which satisfies the specifications on pages 25 and 26. It is self evident that the use of thick film circuits to build the filter presents no problem in the design. In fact, the top hat resistors provide the large trimming range necessary to tune the filter.

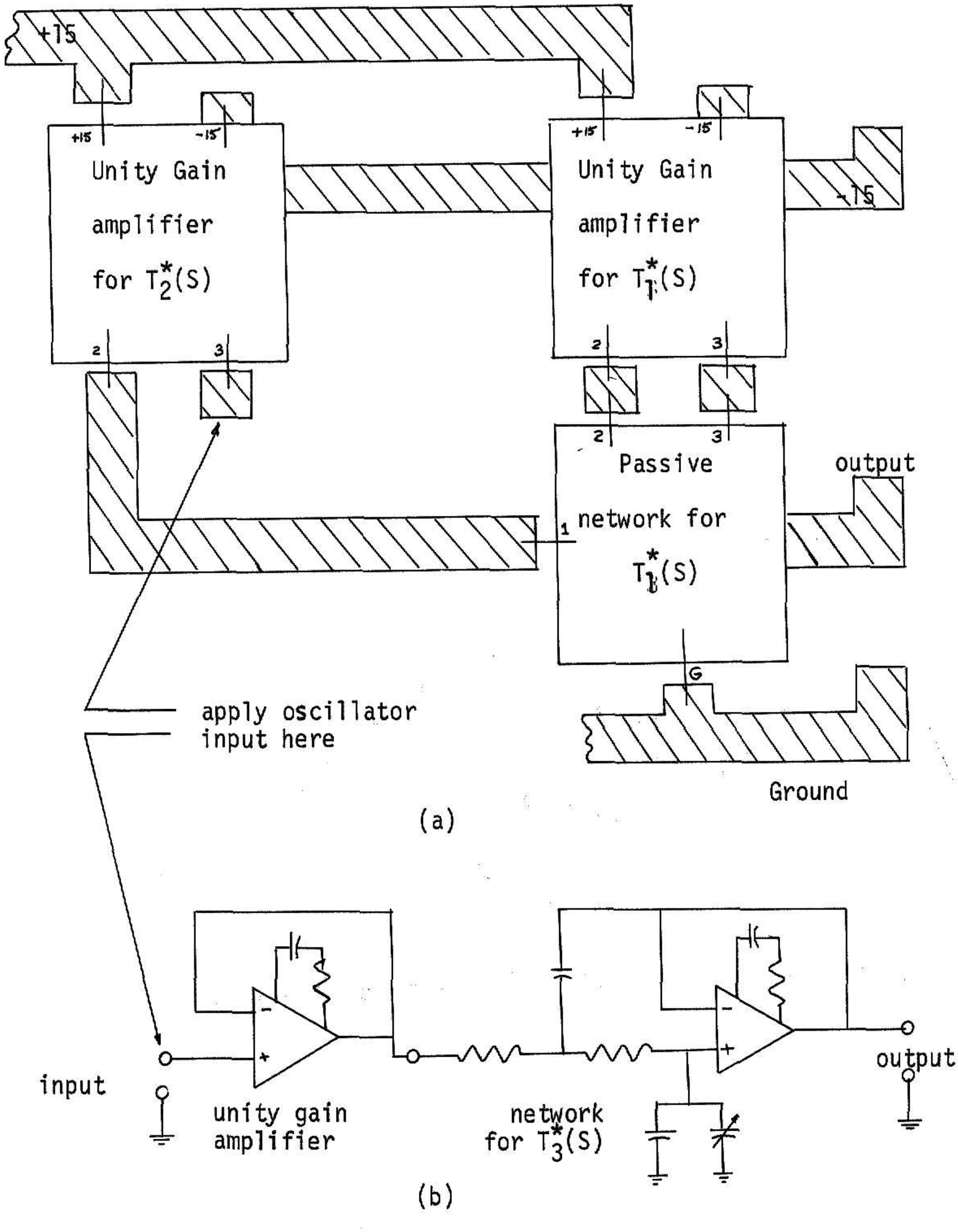


Figure 29. (a) Method of using the unity gain amplifier of $T_2^*(S)$ to tune $T_1^*(S)$ (b) circuit diagram for part (a).

CHAPTER IV

MEASUREMENTS AND RESULTS

This chapter presents the measurements made on the performance of the fabricated thick film filter which verify that the specifications on pages 25 and 26 are satisfied. A discussion of the advantages and design considerations resulting from the use of thick film circuits concludes the paper.

Equipment

Below is a list of the equipment used in tuning the filter and in making the final measurements.

General Radio oscillator model 1309A

AD-YU phase meter model 3800

Eldorado Electronics frequency meter model 225

Dana digital multimeter model 3800

Dremel hand grinder model 270

Tuning

The tuning procedure described in the last chapter was used to tune the filter. The equipment listed above was used as shown in the tuning setup of Fig. 24. The thick film resistors were trimmed using the Dremel hand grinder. Frequency measurements were made using the Eldorado

Electronics frequency meter which has a four place digital readout with accuracy $\pm$ one digit.

No trouble was encountered in the tuning process. Using the Dremel grinder, it was possible to remove small enough portions of the thick film resistors so that the poles could be placed within the limiting curves of the magnitude phase plots of Figs. 22 and 23. In fact, the poles were easily placed much closer to the desired positions than required.

Measured Performance of the Filter

Figure 30 shows the measured frequency response of the active filter. The input voltage V_{in} was kept constant at 1 volt rms and the output voltage V_{out} was measured across a 5 KΩ load impedance as required by the specifications.

Figure 30(a) shows the response of the filter over the passband frequencies 0-8 KHz. The maximum ripple in the passband is shown as .26 dB. Figure 30(b) shows that the response is down 21.8 dB at the frequency 12 KHz. Thus the response is well within the limits given by the specification (i.e. less than .5 dB ripple in the passband and down more than 20 dB at the frequency 12 KHz).

Two other measured quantities are important to the user of the filter. These are the power drain on the power supplies used to bias the operational amplifiers and the maximum input voltage which can be applied to the filter before clipping occurs at the output.

The power drain on the power supplies under operating conditions was measured to be .32 watts. The maximum input voltage which can be applied to the filter was measured to be 8.2 volts rms.

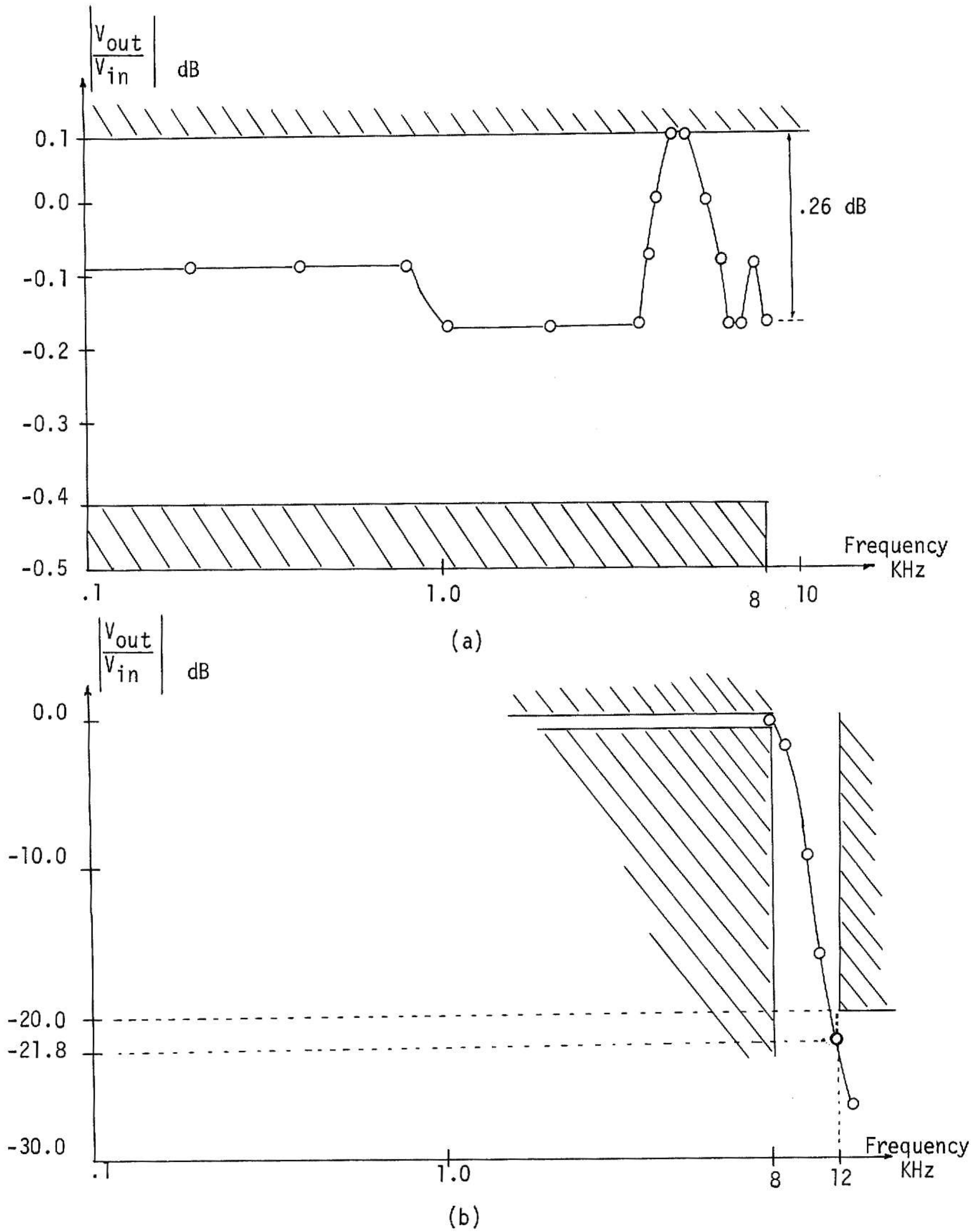


Figure 30. (a) Measured response of the filter in the passband 0-8 KHz (b) measured response near 12 KHz.

A photograph of the thick film active filter, which was tuned and tested, is shown in Fig. 31.

Temperature Stability

Using the results of the analysis on the effect of temperature changes on the filter's transfer function presented in Chapter III, it is possible to determine the temperature range over which the filter can operate and still satisfy the specifications.

It was shown in Chapter III that if the temperature changes by an amount ΔT, the transfer function of the filter at $25^{\circ}C$ changes from $T^*(S)$ to $T^*(S/w')$. $T^*(S)$ is the transfer function which gives the measured response shown in Fig. 30 and the term w' act like a frequency scaling constant and shifts the response in frequency.

Figure 32 shows expanded portions of the measured response near the frequencies of 8 KHz and 12 KHz. Figure 32(a) indicates that the response can be shifted to the left by scaling the frequency 8.25 KHz to the frequency 8 KHz by means of the term w'. The value for w' which will accomplish this shift is determined below.

$$8.25/w' = 8.00$$

or

$$w' = 1.03 \tag{4.1}$$

Figure 32(b) indicates that the response can be shifted to the right by scaling the frequency 11.65 KHz to 12.00 KHz. The value of w' which accomplishes this is

$$11.65/w' = 12.00$$

or

$$w' = .97 \quad . \tag{4.2}$$

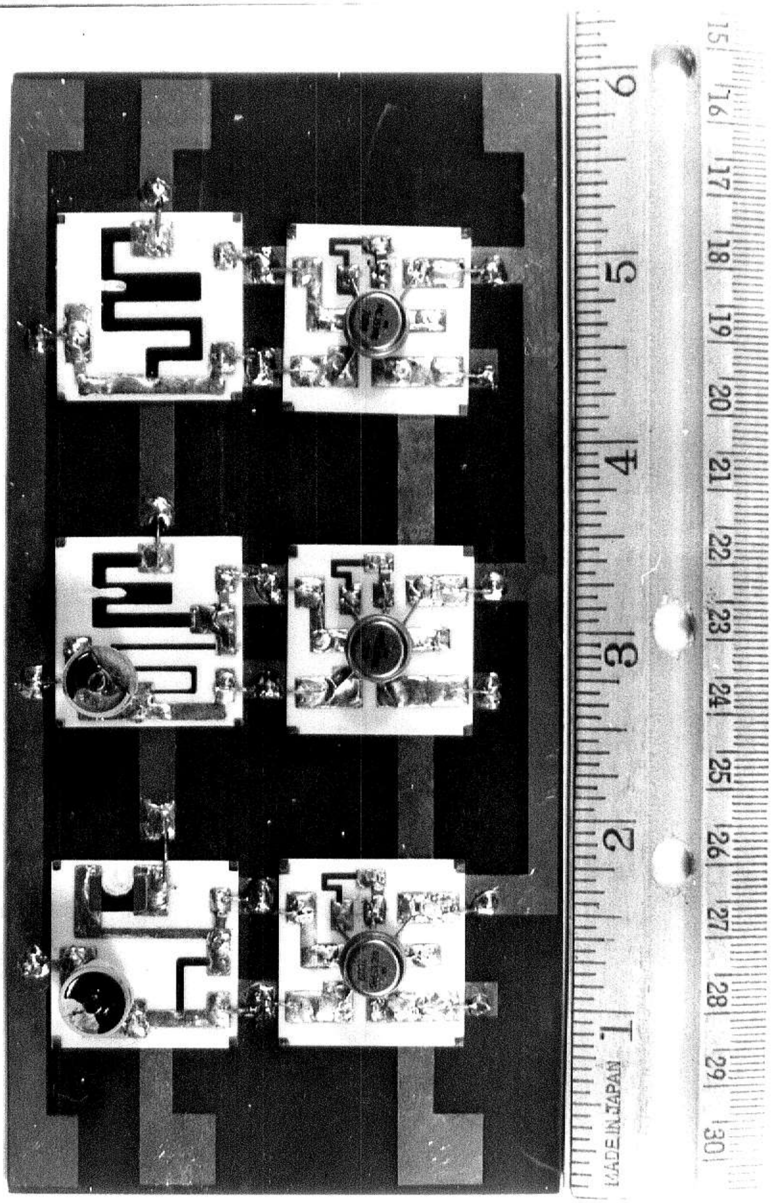


Figure 31. Photograph of the tuned thick film filter.

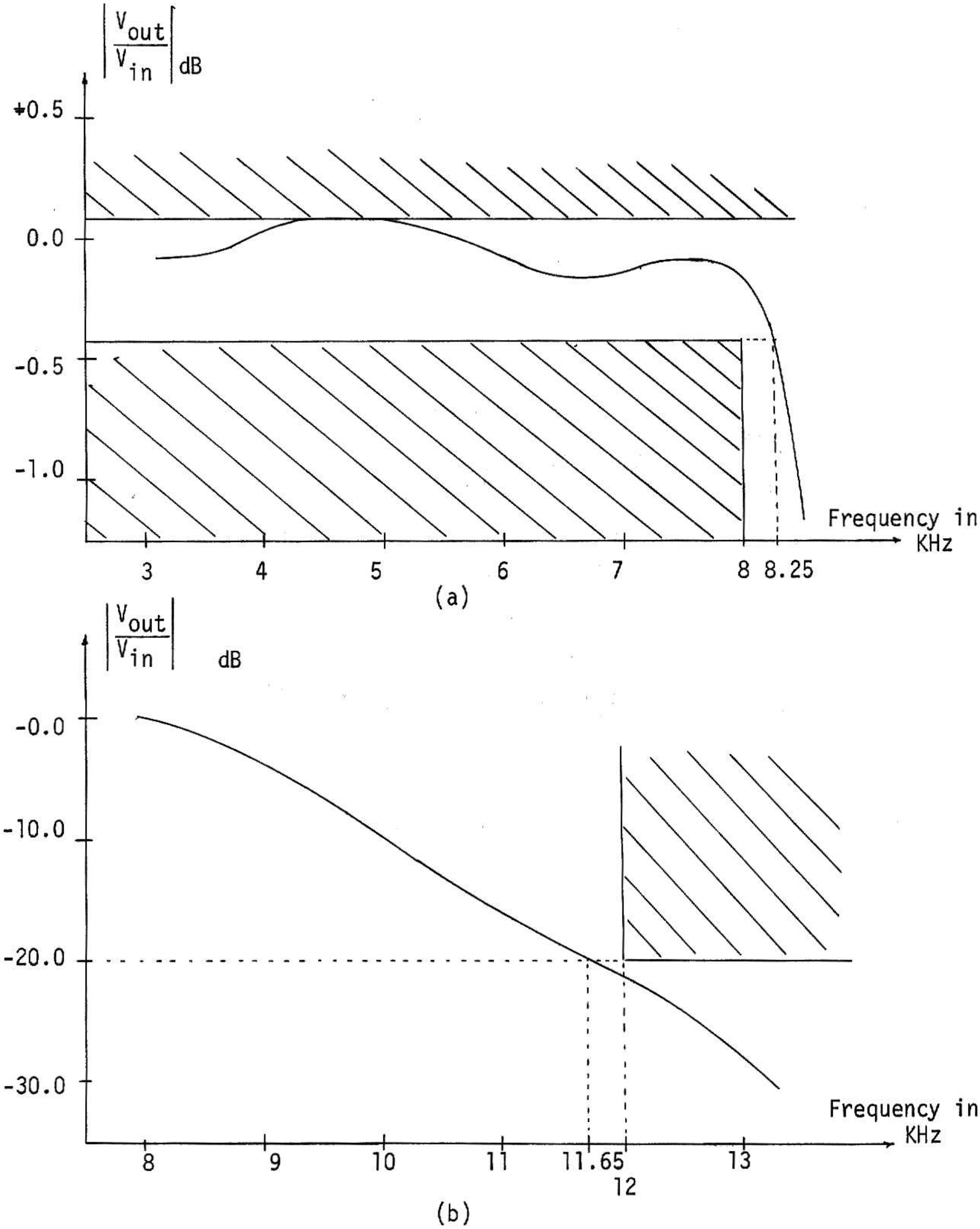


Figure 32. (a) Measured response near the frequency 8 KHz.
(b) measured response near the frequency 12 KHz.

In other words, if w' is in the range $.97 \leq w' \leq 1.03$, the measured response will be shifted in frequency but it will still satisfy the required attenuation characteristics.

The term w' is equal to $1 - [\Delta R/R + \Delta C/C]$. The limits placed on w' also limit the values that the quantity $[\Delta R/R + \Delta C/C]$ can have.

$$.97 \leq 1 - [\Delta R/R + \Delta C/C] \leq 1.03$$

or

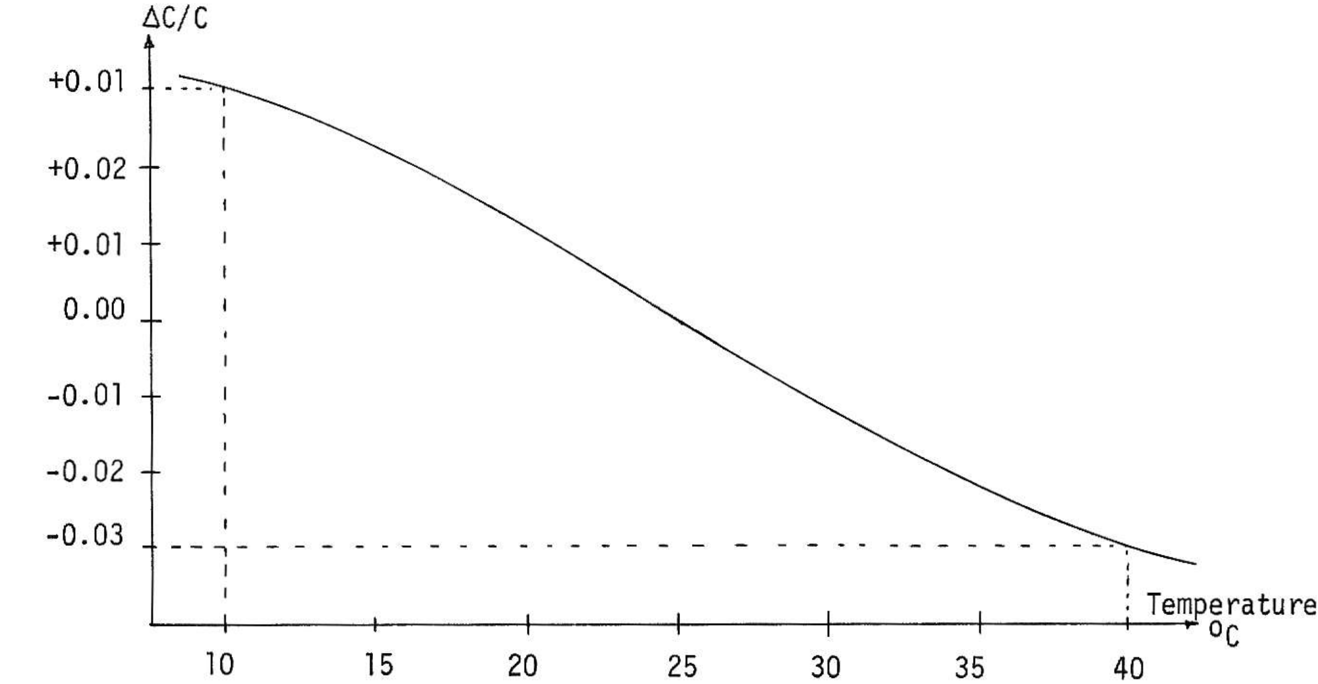
$$-.03 \leq [\Delta R/R + \Delta C/C] \leq +.03 \tag{4.3}$$

The quantity $\Delta C/C$ for the ceramic capacitors as a function of temperature is shown in Fig. 33(a). This information was given in the capacitors' specification. The quantity $\Delta R/R$ for the thick film resistors (labeled as $5\ K\Omega/\square$) as a function of temperature is shown in Fig. 33(b). The temperature characteristics of the thick film resistors were obtained from the measurement of eight resistors at three different temperatures $0^{\circ}C$, $25^{\circ}C$ and $100^{\circ}C$.

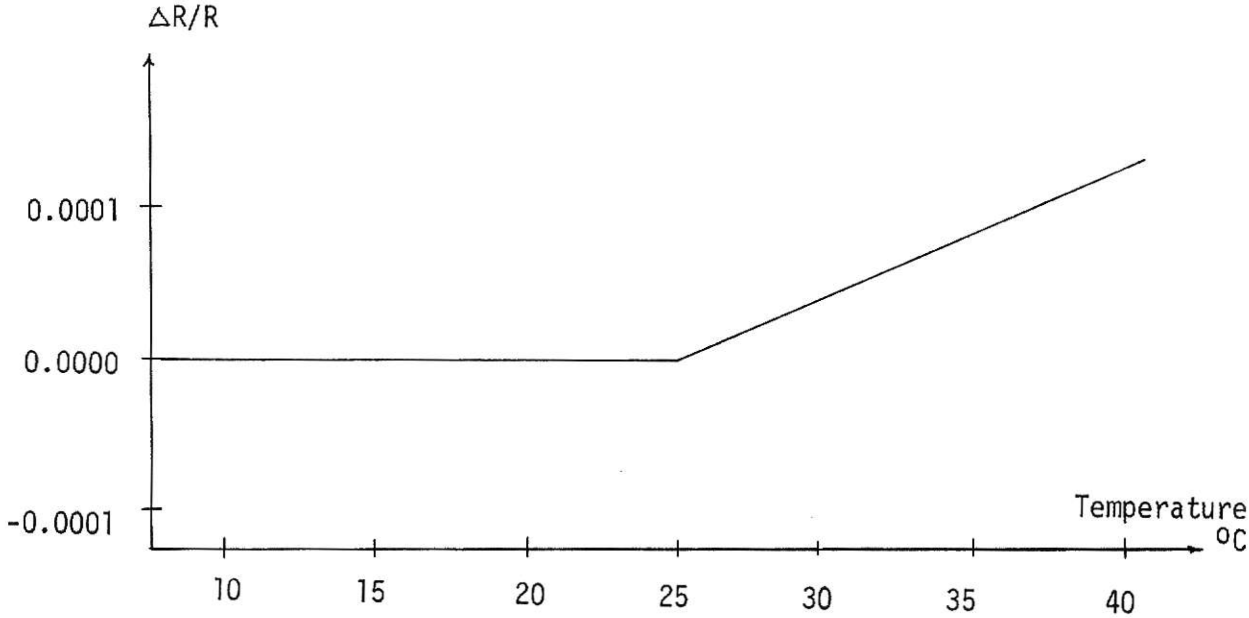
The rough approximation of the resistor temperature characteristics is sufficient because the magnitude of $\Delta R/R$ is all that is required. Figure 33 shows that $\Delta C/C$ is much greater than $\Delta R/R$ over the temperature range of interest. Thus $[\Delta R/R + \Delta C/C] \approx \Delta C/C$. Using this approximation in equation (4.3) gives the following limits on $\Delta C/C$:

$$-.03 \leq \Delta C/C \leq +.03 \quad . \tag{4.4}$$

Now referring to Fig. 33(a), the temperature range over which the quantity $\Delta C/C$ satisfies the limits in equation (4.4) is found to be $10^{\circ}C$ to $40^{\circ}C$. Thus the active filter is stable over the temperature range from $10^{\circ}C$ to



(a)



(b)

Figure 33. (a) ΔC/C for ceramic capacitors (b) ΔR/R for 5 KΩ/□ thick film resistors.

40^oC (i.e., it satisfies the specified attenuation characteristics over this temperature range).

It has been shown that the thick film active filter designed in Chapter III satisfies all of the original specifications stated on page 26. The temperature stability of the filter could not be checked because a temperature controlled chamber was not available but from the above analysis, it is reasonable to assume that the filter has good temperature stability over the range 10^oC to 40^oC.

Conclusions

A thick film active filter which satisfies a set of initial specifications has been designed and constructed. Once the filter realization and normalized components were found, the following considerations were necessary in order to build the filter using thick film techniques.

1. The components were scaled so that their final values were compatible with the range of ceramic chip capacitors and thick film inks available.
2. A tuning procedure, which allowed for the tolerances on the capacitors and thick film resistors, was determined. This procedure was based on the large trimming range of thick film resistors designed in the top-hat configuration.

Obviously the use of thick film circuits in the design of the filter presented no problems. In fact, definite advantages resulted from the use of thick film circuits. These are:

1. The filter was reduced in size and made more compact.
2. The large trimming range of the tophat resistors simplified the tuning procedure.

3. The high temperature stability of the thick film resistors made the filter's temperature stability dependent only upon the capacitors. If a more stable response is desired, it is only necessary to use better capacitors.

The simplicity of the temperature stability analysis was based on the fact that $\Delta Q/Q$ was approximately zero for changes in temperature. If different types of thick film resistor inks are used to fabricate the active filter, $\Delta R/R$ is not necessarily the same for each thick film resistor and the simplified expression for $\Delta Q/Q$ and $\Delta w_n/w_n$ given by equations (3.62) and (3.63) can no longer be used. Instead, equations (3.58) and (3.59) can be used to calculate the changes in Q and w_n and hence the change in the filter's response with changing temperature.

The design procedure followed in Chapter III can be applied to any filter problem which uses Sallen and Key type circuits for the realization. In applying the procedure to other types of circuits (frequency emphasizing networks and state variable realizations), the same steps of the design procedure can be followed but modifications will be necessary.

BIBLIOGRAPHY

1. Newell, W. E. "The Frustrating Problem of Inductors in Integrated Circuits." Electronics, 37 (March 13, 1964), 50-52.

2. Thun, R. E. "Thick Films or Thin?" IEEE Spectrum, 6 (October 1969), 73-79.

3. Mitra, S. K. Analysis and Synthesis of Linear Active Networks. New York: Wiley and Sons, 1969.

4. Huelsman, L. P. Active Filters: Lumped, Integrated, Digital, and Parametric. New York: McGraw-Hill, 1970.

5. Geffe, P. R. Simplified Modern Filter Design. New York: J. F. Rider, 1963.

6. Zverev, A. I. Handbook of Filter Design. New York: Wiley, 1967.

7. Moschytz, G. S. "Second-Order Pole-Zero Pair Selection for nth-Order Minimum Sensitivity Networks." IEEE Trans. Circuit Theory, CT-17 (November 1970), 527-534.

8. Geffe, P. R. "Toward High Stability in Active Filters." IEEE Spectrum, 7 (May 1970), 63-66.

9. RCA Linear Integrated Circuits. Technical Series IC-41. New Jersey: RCA, 1967.

10. Moschytz, G. S. "The Operational Amplifier in Linear Active Networks." IEEE Spectrum, 7 (January 1970), 42-50.

11. Moschytz, and Wyndrum. "Active Filters: Part 5, Applying the Operational Amplifier." Electronics, 41 (January 1970), 42-50.

12. Moschytz, and Thelen. "Design of Hybrid Integrated-Filter Building Blocks." IEEE J. Solid-State Circuits, SC-5 (June 1970), 99-107.

13. Moschytz, G. S. "FEN-Filter Design Using Tantalum and Silicon Integrated Circuits." Proc. IEEE, 58 (April 1970), 550-566.

14. Moschytz, G. S. "Active RC Filter Building Blocks Using Frequency Emphasizing Networks," IEEE J. Solid-State Circuits, SC-2 (June 1967), 59-62.

15. Kerwin, Huelsman, and Newcomb. "State Variable Synthesis for Insensitive Integrated Circuit Transfer Functions." IEEE J. Solid-State Circuits, SC-2 (September 1967), 87-92.

16. Salerno, J. "Active Filters: Part 7, Analog Blocks Ensure Stable Design." Electronics, 42 (February 17, 1969), 100-105.

17. Karni, S. Network Theory: Analysis and Synthesis. Boston, Mass.: Allyn and Bacon, 1966.

18. Moschytz, G. S. "Gain-Sensitivity Product-A Figure of Merit for Hybrid-Integrated Filters Using Single Operational Amplifiers" IEEE J. Solid-State Circuits, SC-6 (June 1971), 103-110.

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DESIGN OF A THICK FILM LOWPASS ACTIVE FILTER

By

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ABSTRACT

Three types of second order active RC realizations, which are present in the current literature, are discussed and it is shown that the Sallen and Key type, the frequency emphasizing type, and the state variable type, should be used to realize low, medium, and high Q poles respectively.

A set of lowpass filter specifications is approximated by a 5th order Chebyshev transfer function and a filter realization is achieved using a real pole network and two Sallen and Key lowpass networks. Component values are chosen to give the correct pole positions and in addition to stabilize pole movements. A tuning procedure which allows the active filter to be constructed using ceramic chip capacitors and thick film resistors is described. Thermal effects on the filter's response are discussed and the final thick film circuits are shown.

The fabricated thick film active filter was easily tuned to meet the required specifications and the response was shown to be stable over the range of temperatures from $10^{o}C$ to $40^{o}C$.