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VALIDATION OF AN INTERDISCIPLINARY MATHEMATICS-READING
CONCEPTUAL MODEL THROUGH AN ANALYSIS OF
INTERDISCIPLINARY RESEARCH IN
MATHEMATICS AND READING

by:

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Chapter I

Education, by design, exists to reduce inefficient trial and error information-gathering processes. Yet, ironically, within its own ranks, there is all too little significant exchange of information. Each area of specialization concentrates on its own expertise in a surprisingly isolated manner. Too little effort is made to communicate with other disciplines to minimize redundancy. It would seem particularly expedient to do more sharing of information. . . . Teachers and students are not isolated, and it is sheer folly to assume that subject matter is isolated. The dynamic learning process is the important factor that should guide the development of teaching theory. (Russell and Dunlap, 1977).

INTRODUCTION

For years, taxpayers have funded and teachers have instructed elementary students in a curriculum centered around "reading", "ritin", and "rithmetic." Each "R" has developed its own constituency of researchers who have sought to develop methods of teaching their own discipline in more effective ways. Their research has uncovered plentiful information about the initial stages of learning in each discipline. Concurrently, psychologists, philosophers and other educators have observed and recorded information about how cognitive development and growth affects the learning process. In the years following the second world war mathematicians and scientists established how information is communicated from one individual to another. In the spirit of seeking to establish whether there is any purpose in correlating research in the several fields of mathematics, reading, cognitive development and communication, this author has undertaken a review of literature which

discusses interdisciplinary approaches to the teaching of mathematics and reading. A brief examination of the theoretical work which explains human cognitive development and communication has also been made.

STATEMENT OF PROBLEM

This paper was undertaken to answer the general question: Does present research establish the authenticity of a conceptual model that justifies an interdisciplinary approach to the teaching of reading and mathematics in the primary grades? In order to answer fully this question, it is necessary to establish answers to several related but subordinate queries. The writer will seek answers to the questions below and then develop an answer to the primary questions from information gathered in that process.

1. Are children at some point in their development rather more than less capable of learning specific skills? Is it possible to relate the accomplishment of certain linguistic and quantitative tasks to the attainment of specific levels of cognitive growth?

An answer to this question will require a brief review of the ideas of educational theorists. The purpose of such a review will be to clarify the theoretical approach to learning that the writer will adopt.

2. How is it that information is communicated from one person to another? How does information theory contribute to our understanding of printed matter?

A cursory overview of the ideas contained in the general theory of information will be necessary to answer this question. It will also be pertinent to relate that theory to the developmental theories reviewed for

question one. These two broad theoretical strands may be interwoven to help form a framework for approaching the research which will be reviewed later.

3. What does recent research literature tell readers about an interdisciplinary mathematical/reading conceptual model? How does the research relate to the theoretical framework already developed?

Fifteen experimental studies or literary reviews will be cited as examples of recent research pertaining to an interdisciplinary approach to teaching reading and mathematics.

4. Should it prove possible to develop an interdisciplinary model for the teaching of mathematics and of reading what might the practical applications of such a model be?

Two school curriculae will be examined in an attempt to extract appropriate classroom approaches. These approaches will be based on the information provided by the research.

5. Does present research establish the authenticity of a conceptual model which will justify an interdisciplinary approach to the teaching of reading and mathematics in the primary grades?

In answer to this question, a general summary of information acquired while answering the previous questions will be presented.

RATIONALE

Why is a study of interdisciplinary approaches to teaching appropriate at this time? For several reasons: 1) compilations of research material relating to this subject exist dating back to 1931, but have not been updated within the last decade; 2) current research

indicates the possible existence of a conceptual basis for such interdisciplinary approaches; and 3) should such a basis be confirmed, its existence might be beneficial to educational systems.

Numerous studies have correlated the findings of developmental psychologists to separate areas of learning. In one year, 1975, twenty-four articles connecting the principles of Piaget to the teaching of mathematics appeared in the Journal for Research in Mathematics Education. For the period July, 1977, through July, 1978, The Cumulative Index of Journals in Education lists eleven articles whose titles suggest discussion of developmental level and reading success. A decade ago a review of research was published by Aiken (1971) covering publications that related verbal factors to mathematical learning. This is the most current compilation of research on this topic. Recent inquiries, published during the last ten years, indicate a continuing interest in examining the possible value of an interdisciplinary path to learning. Therefore, an appraisal of studies on the subject of interdisciplinary learning in the primary grades in mathematics and reading from 1971 to the present time, is appropriate. If that body of research can be attached to a theoretical foundation based upon information theory and cognitive/developmental theory it perhaps will become more useful.

SIGNIFICANCE

The significance of this study is two-fold, theoretical and practical.

The study will relate the cognitive developmental theories of Piaget, et.al., with the work on communication done by C.E. Shannon. Research

work which attempts to interrelate learning in the disciplines of reading and mathematics will be examined and discussed. In the course of that discussion, information synthesized from Piaget and Shannon will be applied to empirical conclusions. The combination of these three components may help shed new light on how mathematics and reading are related to one another.

The practical significance of this study will lie in its ability to suggest adequate approaches to classroom instruction which are compatible with the theories that are developed.

METHODOLOGY

In seeking to establish the existence of a mathematics/reading conceptual model which justifies interdisciplinary approaches to teaching primary students, the researcher has reviewed relevant materials with six objectives in mind. These objectives are:

1. to examine how cognitive developmental theories and the theory of information relate to one another and to interdisciplinary learning;
2. to determine what research exists in the area of interdisciplinary learning with special emphasis on research completed between 1967 and 1981;
3. to examine in what ways the results of existing research reinforce one another or how they may disagree;
4. to interpret in so far as is possible, the meaning of such agreements or disagreements;
5. to note briefly how the conclusions of cited works might be transposed into useful classroom techniques for teachers;

6. to assay the need for additional research on this topic and the form in which such research might be most productive.

An ERIC search was conducted utilizing key identifiers including: cognitive ability, cognitive development, intelligence, academic ability or aptitude, kindergarten and primary education, grades 1-3, mathematics and reading. The search was undertaken on March 24, 1982; it produced zero published articles or other materials that fit the parameters, but revealed 14 unpublished articles and studies.

In addition, the Educational Index, Psychological Abstracts, and the Journal for Research in Mathematical Education were searched for articles that bore seemingly relevant titles. Materials from 1972 through the most currently available issues were checked.

Of course, careful attention was paid to all bibliographies appended to materials reviewed. Any titles noted there or suggested in the body of work that seemed to be pertinent to this study were investigated.

In Chapter 3, the researcher will attempt to clarify the meaning of the body of research cited by noting where studies seem to be examining the same issues and factors. When such similarities of topic are noted, the similarity (or lack thereof) of results will be examined. The significance of such similarities will be discussed also.

Knowing whether or not a specific conceptual model exists is of interest to educational theorists. Knowing whether or not such a model has practical application in a primary classroom is of interest to educational practitioners (i.e., teachers.) In Chapter 4, it would seem appropriate to examine approaches which conform with the theoretical

research. If the research should suggest methods of improving classroom teaching, that will be noted.

In Chapter 5, the author will offer suggestions for research which might further elucidate questions related to interdisciplinary teaching approaches and summarize the findings of the paper.

LIMITATIONS

The scope of this study is to review literature pertaining to the conceptual skills a child will develop at the time he or she learns to read and begins to work simple mathematical problems. Its purpose is to identify any areas common to the teaching of mathematics and reading which a teacher might address, thereby strengthening a student's skills in both disciplines. In consequence, only those reports, articles, dissertations, etc., that addressed both areas have been included. Moreover, only those materials exploring interdisciplinary approaches to teaching in the primary grades (kindergarden through third grade) have been examined.

A broad body of observation and experimentation has been accumulated by philosophers, psychologists and educators which seeks to explain the developmental process in children's conceptual growth. Selected material from that literature is included to provide a conceptual basis for the study of interdisciplinary approaches to learning. Only authors whose work has been accepted by educators (over the relatively long time frame of approximately a quarter century) as setting a standard in the field of cognitive development are included.

This paper does not define "readiness" for reading or math. The researcher acts on the supposition that each child, at some point in his

or her development, is prepared intellectually to succeed at certain tasks. The developmental theories of Piaget are accepted in this review as a satisfactory definition of the developmental level a student may have attained.

The material included in this review is derived exclusively from educational and psychological publications printed in English. Materials from countries other than the United States were not methodically examined.

The author is aware of recent medical findings concerning the possibility that each hemisphere of the brain addresses discrete cognitive tasks. While such inquiries may yet prove to be related to the topic of this thesis, no attempt is made to review medical literature on the subject of hemispheric specificity.

Research materials which examine the effects of teacher behaviors, classroom social climates, teacher preparation, etc., were deemed outside the scope of this review. Any of these factors may, of course, affect a child's achievement and are, therefore, perhaps, proper topics for another study.

Chapter II

AN OVERVIEW OF THE THEORIES OF COGNITIVE DEVELOPMENT AND COMMUNICATION OF INFORMATION

INTRODUCTION

The purpose of this chapter is to establish a theoretical foundation for the proposed development of an interdisciplinary model of teaching reading and mathematics. In order to build such a foundation portions of the writings of Jean Piaget, C.E. Shannon, and Warren Weaver will be briefly examined. That part of each author's work which pertains to signs and symbols and the use of the semiotic function to communicate information will be emphasized.

A DEVELOPMENTAL THEORY OF COGNITION

Cognitive development results essentially from an interaction between the subject and his environment. (Inhelder, et.al. p. 25).

Jean Piaget and his associates, Inhelder, Bovet, and Sinclair, have outlined the pattern of cognitive growth through which normal humans pass, from the age of infancy to adolescence. Their conclusions are based on a series of observations of young children (basically Piaget's own) and on tests performed with children from schools in Geneva.

Piaget argues that children approach cognitive tasks with different tools at different ages. These tools differ in quality as well as quantity. When compared to an older child, a younger child not only has fewer tools with which to attack problems, those tools are less appropriate to solving problems. As the child matures, each month brings

greater sophistication in his or her approach to learning and problem solving. Specific characteristics of thought may be ascribed to specific ages or stages of growth. There is some disagreement by other theoreticians as how best to describe various traits and as to how the individual progresses through such mental development. Broad accord exists, however, concerning the presence of a discernable pattern of cognitive growth and the impact of this growth on a child's learning techniques. Piaget observed that many children pass a watershed in their cognitive development between the ages of 6 and 7 years. He noted qualitative differences occurring at this age in the way children solved problems and explained phenomena. Specifically, before this change in a child from the stage of pre-operativity to concrete operativity, the child is more reliant on physical cues and less able to utilize logical constructs. Prior to the change, a child is less able to mentally remove himself from a situation (decenter) and less able to imagine that a substance could revert back to another shape (reverse actions). Once past this developmental benchmark a child can reason in a more logical way about ideas, feelings and his/her physical surroundings. It is important to note at this point, that Piaget is not saying that a child will develop cognitive maturity unaffected by the information and experiences he/she assimilates. Inhelder (1974) tells us,

Experience, particularly experience of discrepancies between one's predictions and ideas and the actual outcome of their realization, is an important factor in the acquisition of knowledge. . . (p. 267)

Piaget describes a system of cognitive growth which is developmental and interactionist.

What specifically has Piaget to say about language and the development of the use of signs and symbols by children? (Piaget differentiates between symbols and signs. For him a sign is a numeral or word or letter: a representative form which is developed and utilized by members of an entire society. For Piaget a symbol is developed by an individual to represent an idea: for example, a painting. This author utilizes the word symbol for both cases.)

Children use many symbolic forms to express their ideas. These range from dramatic play, to drawing, speaking, and writing. In each of these cases a child is attributing meaning to a "signifier" which is differentiated from that which is signified. In the first two cases, dramatic play and drawing, the forms of the "symbols" evolve out of the child's own inner figurative set. In the latter two cases, speaking and writing, the "signs" are fashioned by the conventions of society and are therefore more arbitrary in their form.

Piaget (1969) observed three major differences that acquiring the ability to use language may cause in a child's cognitive structure:

- 1) a long chain of actions may be represented verbally very quickly;
- 2) language liberates the user from his immediate surroundings and time;
- 3) language is capable of representing many elements of an event or structure simultaneously rather than serially.

What is the connection between thought and language according to Piaget and his associates? On the basis of experiments done by Hermina Sinclair, Hans Furth and others, Piaget (1969) concluded that there is a close connection between the level of development of cognition of a child

and the type and complexity of language used by the child. He states that the sophistication of the child's language is controlled by the level of cognitive growth rather than the other way around.

In 1969 Hermina Sinclair de Zwart reported the results of experiments performed to research the parallelism of the development of operativity and a certain linguistic sophistication in children. She found that among other things, pre-operative children could not create a passive affirmative sentence; "Peter is washed by Mary." Neither could they make sense out of passive sentences uttered to them. "The blue marble is pushed by the red marble," was misinterpreted to mean the red and blue marble pushed one another.

In another experiment Sinclair de Zwart found that children who had developed into the stage of concrete operativity were able to use comparative words to describe a situation: John is taller than Fred. Pre-operative children used only descriptive words: John is tall. Fred is short. In addition, concrete operative children could compare 2 aspects of an object (this pencil is longer and sharper) while pre-operative children could not.

According to Piaget a symbol is differentiated by its user from that which it represents. It allows its user to represent actions or objects not physically present. If it is a conventional symbol, (i.e., language related) it will allow its user to represent social or informational material to any other person who understands that set of symbols.

THE PLACE OF SYMBOLS IN THE TRANSMISSION OF INFORMATION

Piaget has shown how the use of conventional sets of languages may

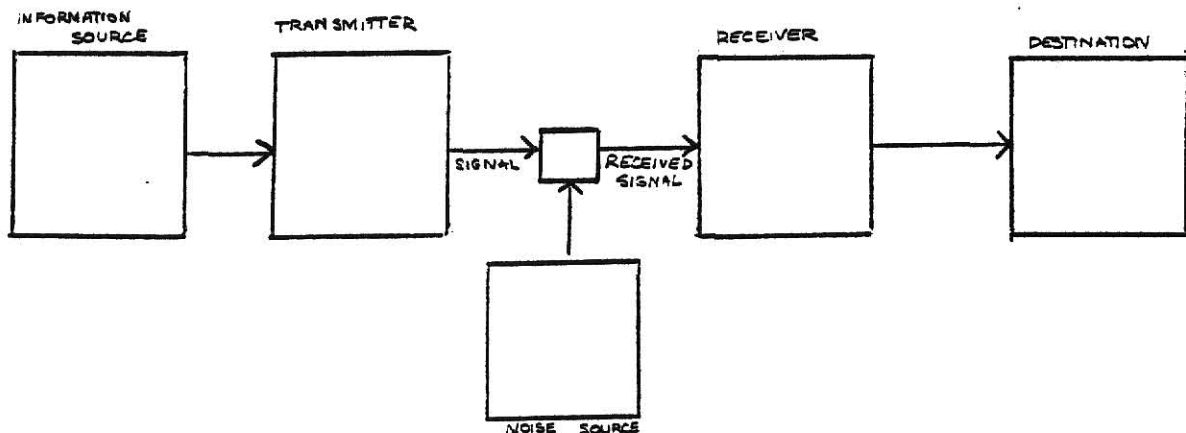
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develop. It will be used now to look at how such symbolic languages may be used to transmit information.

During the Second World War, scientists and engineers were involved with developing ways to transmit information that were as efficient as possible but at the same time capable of accurately transmitting information. Out of this work for the Allied defense effort, came a theoretical work by C.E. Shannon in 1949 which mathematically validated just how an information transmission system had to be organized in order to function most efficiently

Shannon (1949, p. 5) diagrams a communication event in the following way:



A person, (the information source) generates and transmits a message. The message may be transmitted by book, T.V., computer, or some other mode, (signal). The signal is seen/heard (received) by a second person, (the receiver). Understanding on the part of the receiver may or may not occur.

The information source may choose to transmit any of the number of

messages. (There is some argument as to whether in the English language the choice is from an infinite number or just such a large number that it is unknowable by any one person at any given time.) The choice of message is not, however, random. Neither is its form. Once chosen and transmitted, the message may be affected by outside interference (noise) in such a way that it is received in a form other than that intended by the sender. An example may serve to clarify the concept of noise. When text of a book is photocopied, the letters toward the center (or gutter) are sometimes disfigured or deleted due to the shape of the page. This disfigurement is noise. If the message has been transmitted in the English language it may not be unintelligible even though disfigured. Any message transmitted in English contains more information than is needed for it to be understood by the receiver. Therefore, there is a high probability that messages sent in English will contain enough clues as to their original state, that they may be accurately repaired by their recipients. Returning to the example of disfigured text, let us examine the following lines:

he is so informed, and proceeds to guess the second letter. If not, he is the correct first letter and proceeds to his next guess. This is contin through the text. As the experiment progresses, the subject writes down

In the case of the second line, "contin" can be the beginning of only a limited number of English words and their derivatives: continence, contingent, continent, and continue. Thus spelling limits the choices to one of these words. Knowledge of preceeding and succeeding words will cause the word "continue" to be chosen. Knowledge of syntax will cause the reader to predict with much certainty that "continued" is the word

intended by the writer. The constraints of spelling, context, and syntax make the English language predictable. This predictability makes it possible to reclaim the original message.

Shannon (1951) also produced a mathematical method which quantified how predictable the appearance of a given letter is once the preceeding letter or letters are known. He verified that as one or more letters are known to the receiver, uncertainty as to the following letter or letters is reduced. One method he used to verify this was to play guessing games with people. Shannon asked them to supply one-by-one the letters of words in a previously unheard sentence. When the listener missed a letter it was supplied and the guessing continued. Shannon found that people's guesses were accurate roughly 69% of the time.

What all this predictability in the language means to the recipient of a message is that either not every part of the message must be attended to carefully, or that if parts of the message are deleted, sense may still be reclaimed from the message. Shannon termed this characteristic of a symbolic code, *redundancy*. In 1949, he calculated a mathematical basis for the idea of redundancy in languages. He stated that in any symbolic code, that part of the message which was uncertain should be termed its *entropy*. That which reduced entropy in a message was already known and therefore was termed *redundant*. Everything which was received including noise was *information*. (Information could be useful, then, or confusing.) The length of text, nature of the subject matter being transmitted, and the familiarity of the recipient with the subject matter would all affect the percentage of entropy of any given passage and therefore, its

redundancy. Shannon defined mathematically the redundancy of any message as: $1 - \text{entropy}$.

Entropy in a message may be increased through unfamiliarity on the part of the recipient with either the code or the subject matter. For example, a daily newspaper printed in Japanese would be highly entropic to an Englishman or American because of the code (language) used to transmit the message. Conversely, the code may be familiar, but the subject matter not and the message will still remain full of entropy to its recipient. An example of this case would be a person trying to read the technical manual for the development of some component of the space shuttle without the proper technical training.

Redundancy has two important functions. It makes the transmission of an accurate message more likely under adverse conditions, and it increases the probability that the recipient will understand the message.

Many messages which are sent are bombarded with noise, interference from influences outside of the message. The interferences may occur at any point in the transmission. The sender may purposely or accidentally convey a wrong or incomplete message. The signal may be interfered with during transmission by outside sources or technical problems. The receiver may cause noise by misinterpreting the message. A certain amount of redundancy of the part of the code and subject matter will aid the recipient to repair more accurately the message that is received.

Why is redundancy able to simplify the cognitive act of understanding a message? Perhaps a quote from George Miller (1951) will help answer:

Redundancy has its advantages, and a large degree of interdependence among the successive units of a language means that parts of the

message can be lost or distorted without causing a disruption of communication. Any missing portions can be supplied by the receiver on the basis of the surrounding portions, on the basis of contextual clues. Although a redundant language is necessarily more verbose than a language of independent units, it is more dependable under adverse circumstances. (p. 103).

A message which becomes more redundant becomes at the same time more verbose and more cumbersome. The art in transmitting an accurate message lies in developing ways of communicating which are redundant enough to be understood but not so redundant as to be cumbersome and inefficient.

A THEORETICAL FRAMEWORK

In many ways the work of Piaget and Shannon are complementary. Piaget provides the reader with a clear picture of how the use of symbols and signs develops in children. Shannon clarifies how symbols are used to transmit messages. What is the relationship of symbols and symbolic reasoning to mathematical and linguistic learning?

Having developed a definition of a symbol as a sign which is differentiated from but representative of an action or object or thought, it is possible to state that both numerical notational systems and linguistic systems (specifically English) are symbolic in nature. That is, they are both sets of signifiers which are separate from but representative of objects or actions. The word cat signifies the four-legged, fuzzy, animal that meows and purrs. At the same time it is easy to differentiate the word cat from the animal. In the same way, the symbol 5 stands for five objects, but is easily discernable from the objects which it represents.

Do these two symbolic sets share the characteristic of being redundant? Redundancy may be related to the structure of the code or to

the subject matter of the message. Structural redundancy will be examined first. Shannon has proved that the English language is redundant. Miller (1951) and Taylor (1954) applied the mathematical findings of Shannon more specifically to the English language. Miller buttressed Shannon's finding about the redundancy of letters by expanding his analysis to spoken language and to written words. Taylor used Shannon's findings to develop the readability measure known as the cloze procedure. Why is English redundant? It is redundant because it is based upon syntactic rules, spelling rules, and it is verbose enough to provide the reader or hearer abundant context clues.

Are mathematical statements also redundant? They are much less so than messages conveyed in English. Of the three bases of redundancy in English languages, syntax, spelling and context, mathematical statements have only a syntactic structure in common with English. There are certain accepted algorithms for stating mathematical facts; i.e., $4 + 3 = 7$. One's awareness of that structure will allow one to fill in any missing piece from an arithmetic sentence. Mathematical statements do not share any set of rules analogous to linguistic spelling rules. In addition, mathematical statements are very precise, that is, not verbose at all. This low level of verbosity provides very few context clues.

Redundancy may be based not only on the code but on the information that a recipient brings to the interpretation of the message. It is this kind of redundancy that Smith (1978) discussed in this quote:

Put another way, there is no utility in redundancy in the text if it does not reflect something the reader already knows, whether it involves the visual, orthographic, syntactic, or semantic structure of written language. Redundancy, in other words, can be equated with

prior knowledge. In making use of redundancy, the reader makes use of nonvisual information, using something that is already known to eliminate some alternatives and thus reduce the amount of visual information that is required. Redundancy represents information you do not need because you have it already. (p. 19)

This prior knowledge is a type of redundancy that is available to readers of mathematical notation or English language statements, provided the reader has proper preparation for the message. A reader of either type of message may bring previously formed concepts to the cognitive foreground in order to extract meaning from the symbolic text. What is comprehended from the text is affected by the reader's previous knowledge or experiences.

Both English and mathematical notation are what Piaget terms conventional codes. That is, their form is dictated by the society. The child is not free to invent their form from his/her individual imagination. Their use may be dictated by the individual. But their form must be learned.

We may conclude that theoretically the English language and mathematical notation are similar in these ways:

- 1) they are both symbolic in nature;
- 2) they both are conventional in their form (as opposed in idiosyncratic);
- 3) they both contain syntactic structures;
- 4) they are both used to convey messages to people;
- 5) messages sent in both codes may be made more easily accessible by prior knowledge on the part of the recipient of the subject matter.

They are dissimilar because mathematical notation is less redundant

by reason of containing fewer contextual clues than English. In addition, mathematical notation has no set of clues analogous to the spelling clues in English.

In addition to the many similarities between English and mathematics, there is some proof in the experiments of Hermina Sinclair de Zwart (1969) that use of both codes appears to develop congruently. It seems safe to assume that these codes are comparable enough in their structure and use that children may utilize similar cognitive techniques while becoming facile manipulators of both codes.

In summary, the theories of Piaget and Shannon have been combined to form a theoretical base for the interdisciplinary model of teaching mathematics and reading. The theories work well in tandem to provide information about children's cognitive growth. Piaget explained how it is that children become users of symbols, either linguistic or quantitative. Shannon explained why the ability to use one set of symbols reinforces the ability to use the other. Shannon also delineated the use and limits of use of symbolic communication in a technical society.

Now it is appropriate to examine research pertinent to multidisciplinary teaching with the objective of substantiating the theoretical findings.

CHAPTER III
STUDIES IN WHICH THE LEARNING OF READING AND MATHEMATICAL
SKILLS ARE RELATED TO ONE ANOTHER

INTRODUCTION

Many studies are done each year on how children learn to read and how they learn to solve mathematical problems. Each of these studies must address (to a greater or lesser degree) how it is that children derive meaning from symbolic text--be it mathematical or linguistic in form. In this chapter studies were reviewed which were completed predominantly during the last 15 years, 1967-1982, to note how the development of the ability to utilize one code (i.e., linguistic) seems to be connected to the use of the other. Some experimenters examined how verbal facility affects mathematical ability. Others approached the question from the opposite direction and inquired how the acquisition of certain logico-mathematical skills may affect linguistic learning.

A decade ago a review of research was published in the Journal for Research in Mathematics Education, (Aiken, 1971) covering publications that correlated verbal factors and mathematical learning. He cites experiments done by Gilmary (1967) with elementary children during a summer school remedial arithmetic program. The children were divided into two groups, one of which received instruction in both reading and arithmetic and the other receiving instruction only in arithmetic. On the basis of scores on the Metropolitan Achievement Test, children in the first group gained 1/3 of a grade more than children in the second group. He also cites conflicting results from an experiment undertaken by Henney

(1969). Here children given special help on verbal problems did no better on test scores than children who were not given such help. (It is not clear whether the types of instruction given in the verbal field in each of these experiments were comparable). Still Aiken (1971) concluded,

It seems reasonable to suppose that attempting to cultivate the skill of reading carefully and analytically in order to understand meanings, thinking about what one is reading, and translating what is read into other symbols would improve performance on many types of mathematics problems. (p. 309-310).

A similar review of literature undertaken by Aiken the following year, 1972, covers a broader range of factors relating mathematics to linguistic factors. He examined intellectual abilities, readability of texts, specific teaching procedures, teacher-student verbal interactions, language influences on mathematical development, and stages in learning mathematics. He suggested that further studies would be fruitful in the following areas: the relationship between mathematical development and language development, the degree of emphasis in the home on syntactic thinking and linguistic encoding, and controlled experiments concerned with the effects of instruction in vocabulary and reading on mathematics learning. (p. 379).

Aiken's reviews (1971, 1972) outlined a need for seeking a clearer understanding of the connection between linguistic and mathematical development and learning. The research cited below will examine that relationship.

RESEARCH

Application of verbal ability in mathematical tasks.

A paper by K. Fell and B. Newnham (1978) looked for evidence in

research work which suggested that language abilities and mathematical reasoning abilities are linked in important ways. They cited research data by Harasym (1971), Donaldson and Balfour (1968) and Palermo (1973) which concluded that children need a basic vocabulary facility (specifically in these experiments, with the words "more" and "less") before they can understand elementary mathematical concepts. Holland and Palermo (1975) indicated that merely teaching the words "more" and "less" to children would not improve their mathematical conceptual thinking. (This result implies to this writer that further developmental growth was necessary before the child could incorporate his/her knowledge of "more" and "less" into a useful mathematical model. Fell and Newnham specifically reject that conclusion for reasons which are unclear.) Their review of literature led Fell and Newnham to conclude that linguistic and mathematical skills do develop congruently, based on the findings of the research they cite. They admit in their conclusion that the reasons why such development is congruent are vague but pervasive.

As a part of Project for Mathematical Development in Children undertaken by a consortium of southern universities, Anna Hamrick (1977) studied 38 first graders to test the assumption that written mathematical symbols are similar to written language. To do so she tested all 38 students with mathematical and language readiness tests. On the basis of the student's scores, she identified 22 children who were not ready, and 16 others who were ready to work with symbols. She then divided the not-ready students into two groups. One group was given regular work with arithmetic concepts and symbols introduced together, the other "not-ready"

children were delayed in their use of symbols until each student had mastered the concepts of the mathematical topic verbally. Those students who were found ready to proceed with symbolic work were also divided into two groups. One group was given instruction with mathematical concepts and symbols being introduced simultaneously (much as one group of the not-ready students). The other group had symbolic work delayed for an arbitrary five week period. Hamrick found that those students who were ready to work with symbols and were given symbolic work were academically successful. Those students who were initially ready to work with symbols were bored when such work was delayed. Students who were not ready to work with symbols but were allowed to develop verbal facility with the concepts before continuing with symbolic processes were also successful in their learning. Hamrick, therefore, concluded that high verbal language facility was a significant predictor of success in comprehending written mathematical symbols.

A study by Kamhi (1981) examined the claim that language impaired children with normal nonverbal intelligence may suffer from symbolization deficits. He compared one group of language impaired children with two groups of normally developing children. Children in all groups were tested with six Piagetian tasks designed to assess nonlinguistic symbolic abilities. One of the conclusions that Kamhi drew from this experiment was the conceptual delays experienced by language impaired children seemed to be related to the involvement of the symbol in developing a particular concept. This finding is related to Hamrick's finding that conceptual and verbal facility was predictive of success in comprehending symbols.

E.H. Irish (1964) and R.J. Call and N.A. Wiggin (1966) undertook, independently, experiments similar to one another. Irish worked with students of elementary age while Call and Wiggin carried out their work with secondary algebra students. Both experiments were structured to test whether instruction in vocabulary and other reading skills would improve mathematical scores. In both cases the students who received the reading-based instruction scored higher on mathematical post-tests than those students receiving standard mathematical instruction.

Rose and Rose (1961) studied 456 third grade children, 100 of whom were suburban children from the upper socio-economic class. The other 356 were inner-city children from Detroit. The major concern of these two researchers was the connection of family position, I.Q., and socio-economic status of a child to his/her academic achievement in mathematics. In the course of their study they found an interesting link between language and arithmetic ability.

Rose and Rose found that younger children and over-protected children tended to do significantly less well on mathematical tests than older children or less "smothered" children, even when matched for socio-economic status and I.Q. They attributed this difficulty with arithmetic to the quality of language that this class of children tend to use in their pre-school years. Rose and Rose hypothesized that children who are over-protected or babies of a family would tend to receive more help finishing sentences, hear more baby-talk, be provided more of their wants without verbal requests. Therefore the researchers suggested that the language that such children develop will be less precise, less

elaborated, less developed than other children of comparable I.Q. or socio-economic class.

This study by Rose and Rose (1961) provokes the thought that the quality of language a child is surrounded with in early years may affect his/her ability in mathematics during the school years.

Application of logico-mathematical skills to reading success.

Seeking to gauge whether success on nine Piagetian tasks would predict academic success in first grade, Arlin (1981) worked with 192 kindergarteners. She tested them twice, once in kindergarten, and once in first grade. Children were interviewed on the following Piagetian tasks: simple seriation, double seriation, simple classification, two-way classification, class inclusion, three-way classification, conservation of number, conservation of continuous quantities, and conservation of discontinuous quantities. Arlin attempted to correlate success in any one Piagetian task with success in mathematics and reading in first grade. Her theory was that the development of logical thinking skills is requisite to success in beginning reading and mathematics. Arlin concluded that general concrete operativity, expressed by success with a majority of the tasks, was an important component of a child's readiness for achievement in both fields. The twenty kindergarteners scoring the lowest in success in the Piagetian tasks remained below grade level in their work in first grade in both mathematics and reading. These same kindergarteners were not identified by any other method as being unlikely to succeed academically. Arlin concluded that while success or failure with one or two tasks was not predictive of academic success, general

logical operativity on the part of a child was predictive of academic achievement in first grade.

Utilizing methods similar to Arlin's, Brekke (Brekke, Williams and Harlow, 1973) found that a child's score on Piaget's conservation tasks was only a little less valuable than the I.Q. scores as a predictor of success in reading in first grade. In a later study, Brekke and Williams (1975) verified these results with another report that showed that children who succeed in Piagetian conservation tasks tend to score higher on reading readiness tests. To reach this conclusion the researchers tested 81 first graders from North Dakota. The subjects were pre-tested on a Gates-MacGinitie Reading Test and a SRA Primary Mental Abilities Test. They were then divided into three groups on the basis of their responses to five Piagetian tasks, conservation of inequality of number, conservation of equality of number, conservation of equality of substance, (administered in two different ways) conservation of inequality of substance. They were designated as conservers, transitional conservers, and nonconservers. The conservers were found to have scored significantly higher on the reading readiness scores than either the transitional group or non-conservers group. Dimitrovsky and Almy (1975) studied 121 kindergarteners over a period of two and a half years. They were tested on Piagetian conservation tasks and given Metropolitan Achievement Tests to assess their developmental level and their academic achievement. The authors found that 90% of the children who succeeded with the Piagetian conservation tasks in kindergarten were at or above grade level by the time they reached second grade. Conversely, only 56% of the children who

were designated as non-conservers in kindergarten were achieving at grade level in the second grade.

In an earlier book Young Children's Thinking, Almy, Chittenden, and Miller (1966) report the findings of another longitudinal study. This was performed with 655 students from metropolitan New York. These subjects were also observed from their kindergarten year until the end of their second grade year. In this study children were also asked to perform Piagetian conservation tasks. The results of this study also indicated that the ability to conserve, in a Piagetian sense, was a useful cognitive skill for children and generally led to greater success in reading in grades one and two than if the child had not reached that level of development by kindergarten.

Another longitudinal study reported in 1979 by Tomlinson-Keasey et.al., followed 38 students from kindergarten through the third grade. Its purpose was to obtain as complete a picture as possible of the development of cognitive growth. Each child was interviewed each of the four years. Students were tested on the Piagetian tasks of seriation, numeration, class inclusion, hierarchical classification and conservation of mass, weight and volume. The authors concluded that while each child will develop cognitive skills according to his/her own time-table, all children tend to develop such skills in similar order.

Not all researchers have found that Piagetian conservation tasks are predictive of academic achievement however. As an example of one team who did not, note the work of Orpet, Meyers and Grein (1976). They administered a conservation of liquids test to 52 middle-class children in

California. They then divided the children into two groups, conservers and nonconservers, on the basis of these tests. Next they administered the Stanford Achievement, Reading, to both groups. They found little significant difference between the reading scores of the conservers and nonconservers. It is interesting to note here the findings of Arlin that success on any one task was not sufficient to predict academic success.

CONCLUSIONS

The several researchers cited here are all struggling with ways of proving the cognitive connection between reading and mathematics. What were they able to prove with certainty?

1) That verbal facility affects a student's ability to perform well on mathematical tasks (Hamrick, Kamhi);

2) That instruction in verbal skills seems to improve mathematical scores (Irish, Call and Wiggin);

3) That success in performing Piagetian conservation tasks in kindergarten may be predictive of success academically in reading in the early primary years (Arlin, Brekke, Brekke, Williams and Harlow; Dimitrovsky, Almy and Tomlinson-Keasey, Eisart, Kahle, Hardy-Brown, and Keasey;

4) That success on any one Piagetian task was not necessarily predictive of academic success (Arlin, Orpet, et.al.).

On the basis of the theoretical works cited previously, it is interesting to examine these findings.

1. Verbal facility affects a student's ability to perform well on mathematical tasks.

Piaget posited the need for a child to develop a certain cognitive sophistication before he/she will be expected to succeed in certain mathematical tasks. Piaget (1969) then stated that a child's language is tied to this developmental level and will often reflect his/her developmental level. (Sinclair de Zwart's (1969) experiments buttressed this latter argument.) Specifically, Piaget (1969) stated that "the semiotic function (language) makes thought possible...Reciprocally, it evolves under the guidance of thought..."(p.91). Theoretically then, one would be able to predict that verbal facility would indicate an advanced enough developmental state that a child would succeed in mathematical tasks. The child's use of language indicates the ability to utilize symbols successfully.

There is a tie here also to the work of Shannon and others on redundancy. A child who has verbal facility with a mathematical concept can carry that prior knowledge to the mathematical task. Redundancy of subject matter will simplify the cognitive act. The more knowledge a student has about a problem - whether it is in linguistic or quantitative symbolic form - the lower the level of uncertainty in the problem.

2. Instruction in verbal skills seems to improve mathematical scores.

Redundancy as a concept seems to be relevant here, also. Giving a child information about the mathematical task in verbal form will increase the student's store of information which he/she may apply to the problem.

There has been considerable conflict over whether instruction in specific vocabulary may hasten a child's movement toward Piagetian "operativity." Perhaps, it will when the child receiving the instruction

has the developmental sophistication and sufficient information to perceive how such vocabulary is related to a general concept.

3. Success in performing Piagetian tasks in kindergarten may be predictive of success academically in reading in the early primary years.

A part of the test of whether a child can conserve or not is being able to explain why a certain answer is correct. This implies a certain level of verbal facility on the part of the respondent. Therefore, the theoretical applications stated under number one seem to apply here also.

4. No one Piagetian task passed individually will be consistently predictive of academic success.

Operativity in the Piagetian sense is a more general concept than the ability to perform one specific task. One might theoretically predict this outcome of research.

Is there an area of cognitive interface for young children between reading and mathematics? Based on the research a tentative yes seems in order. Yes, because enough commonality in learning function between reading and mathematics exists that researchers Almy, Dimitrovsky, Brekke, Arlin, and Tomlinson-Keasey can verify that those young children who perform best on reading tests are the same children who indicate attainment of Piaget's concrete operational stage of development. Conversely, it appears that students who are instructed in reading will have improved scores on mathematical tests (Hamrick, Irish, and Call and Wiggin).

This finding is tentative because no one paper cited has approached the question head-on. Comparison of research cited here is difficult

because of the diversity of methods utilized to test the relationship between mathematics and reading. The relationship between the two disciplines is stated as a justification for undertaking some of the research cited, but no one researcher encountered by the author addressed directly what that relationship might be. Because the methods were tangential to the problem, the answers were also.

The writer has been seeking a conceptual model embracing the cognitive skills children utilize while reading and solving mathematical problems. In the process of that search the reader's attention has been directed to the developmental theories of Jean Piaget and the theory of information developed by C.E. Shannon. Research done recently which is applicable to such a conceptual model has been examined. Thus far the writer has been able to conclude that on a theoretical basis, mathematical and linguistic codes have much in common. Also the ability to elaborate the use of such codes seems to develop congruently in children. On the basis of studies done thus far it appears that children do in fact utilize cognitive skills learned in one discipline while working in the other. Moreover, certain abilities developed in one discipline seem to be predictive of similar development in the other.

Chapter IV

CLASSROOM CURRICULA THAT ARE COMPATIBLE WITH MULTIDISCIPLINARY APPROACHES TO TEACHING

THEORETICAL CONSIDERATIONS

Before examining a total curriculum that is implicitly multidisciplinary in its approach, this writer will review the characteristics of a lesson which is compatible with developmental learning theory.

R. Case (1974, 1975, 1977) has established the characteristics of a lesson which would be developmentally appropriate to young children. His characteristics are supported by the developmental theories of Jean Piaget. Based on Piaget's thinking, Case contends that young children:

- 1) cannot keep many aspects of a problem in their short-term memory, and
- 2) use simplified and therefore inadequate strategies to solve problems.

To help students overcome these cognitive shortcomings, Case concluded that lessons which are appropriate for early primary students must meet the following criteria:

- 1) They must provide the student with some meaningful procedure for determining whether or not his or her strategy has been successful.

- 2) They must reduce to a bare minimum the number of items of information that require the student's attention.

- 3) They must insure that all cues to which the student must attend and all responses which he or she must exhibit are familiar ones.

- 4) They must ensure that all stimuli to which the student must attend are conspicuous, either because their physical characteristics make

them stand out from their context, or because they are verbally noted by the instructor.

These characteristics have been noted to help distinguish the outlines of a lesson which would conform with the theories of Piaget. Would a lesson which contains these characteristics also agree with the guidelines for a clear message as they were stated by Shannon? It appears that each of Case's characteristics are designed to reduce entropy or increase redundancy. Case does not utilize Shannon as a source for his ideas, but the methods he advocated seem compatible with the theories of Shannon.

TWO CURRICULA

Programs that approach the learning of mathematics and reading through interdisciplinary means exist even though they are not labeled in that way. In the descriptions of their goals, both of the programs outlined below stress the need for children to become independent thinkers. The activities outlined in both curricula are explicitly directed toward strengthening symbolic reasoning. The use of such thinking appears to be one interface between the learning of mathematics and reading. Research, theory, and observations of children reinforce the idea that symbolic thinking is important to both disciplines. Because both of the following curricula emphasize the need for representational thinking, this writer has chosen them to exemplify the characteristics one might expect of interdisciplinary learning approaches.

At least two programs have already been developed whose implicit intent was to teach children in a multidisciplinary way. The programs

were put into practice in the United States during the 1960's and 1970's. A description of how each program operated and the philosophical basis for its development will be examined.

The Tyler Thinking School

The Tyler Thinking School was established in Charleston, West Virginia in 1970 by Hans G. Furth and Harry Wachs. It was established to aid the normally developing processes of thinking in early primary age children. One basic goal of the program was the prevention of academic failures. A second goal was to make the students more receptive to educational opportunities in many different learning situations. (This goal was based on the belief that the child who was most actively involved in thinking would be the most flexible in his learning experiences and techniques). The program's minimum goal was to have all children at "grade level" in reading, mathematics, writing, and other subjects by the fourth grade.

The goals set for the school were an outgrowth of the founders' work with Piaget and Piagetian concepts. Furth and Wachs were attempting to develop a practical curriculum based on the theories of Jean Piaget. Their program was planned for children of kindergarten and first grade ages and was adapted to the developmental needs of children of those ages. Activities were selected that encouraged children to utilize "high-level" thinking. High-level thinking was defined as the most sophisticated thinking that a child was capable of at a particular developmental stage. Activities utilized in the school were specifically directed toward physical awareness and agility; ability to physically manipulate and

utilize objects; ability to utilize and process information visually, tactilely, and auditorily; and the ability to comprehend and solve problems logically and symbolically. Examples of some of the activities used include: parquet block designs (including attempting reversals and mirror images of designs), making up group stories, and various movement activities such as swimming on the floor, skipping, etc.

The school was in operation for two years. It did not last longer, according to the founders, because there was not enough support in the community to continue a school which emphasized thinking rather than learning specific academic skills. Nonetheless, the founders felt they were successful in putting the educational theories of Jean Piaget into practical operation. They also thought that the program contributed to the academic growth of the children who were associated with it. It is not known whether it helped prevent academic failures because no follow-up studies were reported.

The Cognitively Oriented Program

The second program to be examined is called the Cognitively Oriented Curriculum. This program is an outgrowth of work done by Weikart (1979) on the need for early intervention with children who are developmentally delayed. Its original purpose was to help such children attain success in school.

This program is still in use by several public school systems in numerous states. It is used as an alternative to current kindergarten programs (prior to enrollment in regular kindergarten) or as a basis for pre-school programs administered by public school systems.

The goal of the program is to "help children consolidate their abilities, through direct and representational experience, without trying to accelerate their development. Each child is recognized as an individual who builds his or her own knowledge." (Hohman, et.al., p. xvi). The program encourages students to utilize their skills in logico-mathematical and linguistic areas in ways appropriate to their developmental level. The program is based upon fifty key concepts through which the child is led. The basis of all these activities is that through active participation the student will arrive at a point where he/she is capable of making, using and interpreting representational statements. The structure of the program is based on the Piagetian principle of the necessity for the student to be actively involved with the learning process in order that effective learning may take place.

Children who attend classrooms utilizing the Cognitively Oriented Curriculum begin their day by planning with a teacher what they are going to do for the day. Then they are encouraged to record that intention in some way that is appropriate to their developmental level. For some children the recording will be a picture of the intended activity, for others an unintelligible scribble, and for some a word or sentence. The children then become involved with their chosen activity. At a circle or class meeting time children discuss what they did, how they did it, and display any finished projects they want to share. The projects from which children may choose on any given day or week are designed by the teacher to lead the children through the prescribed key experiences of the program. (These experiences include verbal, mathematical and social

concepts necessary for success in standard first-grade reading and arithmetic programs.)

The organizers of this program have kept careful statistics on the success level of students whom they have instructed. Eighty percent of their previous students achieve at or above grade level in elementary school. Of children followed into secondary school fifty percent still achieve at or above grade level. One-hundred percent of their students had been identified as being academic risks for developmental or socio-cultural reasons before entering the Cognitively Oriented Curriculum.

DISCUSSION

Both of these programs share the basic premise that children who think better will be better students in mathematics and reading. They are also dedicated to the type of teaching which involves the child's active participation. Other similarities in the programs are also notable.

They are for children who are in the age-group from roughly four years to seven and one-half years. These limits are based on Piaget's findings which indicate that children of these ages think differently (and therefore learn differently) than younger or older children.

They are both activity oriented, rather than pencil-to-paper oriented.

Both programs are planned to help children develop cognitive structures related to mathematics and reading. In fact, both specifically target academic success in these disciplines as a major goal. However, neither program taught directly the skills assumed necessary by

traditional programs in either discipline. The approach to teaching was implicitly interdisciplinary in its methods.

The curricula discussed here are different in content and in teaching method than those used for a subject-by-subject approach. Such differences are not trivial. The form and content of these curricula are based on a strong philosophical foundation. Both programs take advantage of what is known about how children learn at ages 4-8. They also conform to guidelines for efficient transmission of knowledge as they are outlined by Shannon.

A child's natural tendency at the pre-operative stage is to learn about the world in an integrated manner. A child does not separate information into various categories--mathematical, musical, scientific, etc.--until taught to do so by someone else. These two programs seem to take advantage of that cognitive inclination.

An activity approach to learning seems to have intrinsic interest for a child. Children become more involved in their learning when they help to structure the learning experiences. Insofar as learning is a process, and not the sum of a certain number of activities, the more the child is involved in the process, the more effective the learning will be.

These curricula have common goals: 1) academic achievement, 2) successful symbolic reasoning; common content: 1) activity-based experiences, 2) developmentally appropriate activities; and a common teaching method: involve the child.

What might the characteristics of a multidisciplinary curriculum be? From examining these two curricula it is probable that they would include

the following:

- 1) an emphasis on symbolic reasoning;
- 2) a bias toward activity-oriented learning;
- 3) abundant opportunities for children to test the appropriateness of learning strategies;
- 4) activities which are at once challenging to a student's reasoning strategies, and developmentally appropriate.

This kind of a teaching approach would differ from traditional programs because children taught in an interdisciplinary way would be learning techniques to utilize symbols as a means of communicating. Students would practice those techniques with the English language and mathematical notation. Strategies perfected by the children could be applied to linguistic and quantitative symbol sets, or to computer languages, musical notation, etc. Information that the child learned in one field could be used to reduce the entropy of material in another discipline.

Children have been instructed in ways which meet philosophical guidelines set up in this paper for interdisciplinary programs. Still, there is no scientific way to compare what results have been accomplished by teaching young students in this fashion with results obtained from teaching in more traditional ways because inadequate research has been attempted. It would be predicted that multidisciplinary curricula would produce children who did not know as many facts as soon as children who go through more customary programs. It is probably true that the kind of learning that these programs develop may not be easily quantifiable.

However, the proponents of interdisciplinary curricula would contend that the eventual academic success of students instructed in this fashion would exceed what might have been expected had the students not had the opportunity to learn in this manner.

Chapter V

SUMMARY AND CONCLUSIONS

SUMMARY

This study sought to establish whether or not enough information exists to develop an interdisciplinary conceptual model for teaching reading and mathematics. In order to do that the researcher:

- 1) reviewed theoretical works which explain the development of cognitive structures in young children.
- 2) reviewed theoretical works which explain how communication takes place.
- 3) reviewed research relevant to how the learning of mathematical and linguistic skills may be compatible.
- 4) examined two curricula.

DISCUSSION

The results this researcher developed in this study are ambiguous. On the basis of developmental theory it appears that such a model would help address the need to teach children in ways consistent with their development. If linguistic and quantitative abilities develop in an integrated fashion, why not teach concepts from those two fields in an integrated way?

Children appear to develop linguistic and logico-mathematical skills concurrently. The experiments done by Hermina Sinclair de Zwart (1969) on children's language abilities suggest the validation of that statement. Her conclusion that linguistic abilities change in consistent and predictable ways that mirrored the changes in the ability to perform

conservation tasks suggests the validation of that statement.

Children appear to develop the use of symbols in an integrated way. They seem, according to Piaget, to develop an ability to use the semiotic function in order to "allow the representative evocation of objects and events not perceived at that particular moment." (Piaget, 1969, p. 91). Instructors ought to be able to take advantage of the highly symbolic nature of both mathematics and reading and the use children want to make of symbols to communicate.

On the basis of information theory, it appears that communication of knowledge in both fields would occur in a similar manner. One could conclude that redundancy (not repetition) might be an important teaching concept in both fields. Redundancy is a reciprocal of uncertainty in a message. Any factor which reduces uncertainty will increase redundancy. Increased familiarity with either the code or the structure or the subject of a message will reduce the entropy in the message. Specifically, children ought to be able to use information gained in one symbolic mode when they are using another mode.

The theoretical concept of noise suggests how comprehension might be thwarted in either subject. That construct ought to suggest the need for messages that are redundant enough, and predictable enough in form to reduce the potential damage from the noise. Any message must also contain enough entropy that there is something to be learned by the recipient.

Research studies cited in this paper present a tantalizing fact. Instruction in one discipline seemed to improve scores in the other. Hamrick, Call and Wiggin, and Irish all were able to improve children's

mathematical test scores by increasing linguistic instruction. While that is a potent indicator of a cognitive tie, no one study was able to establish the exact nature of the connection.

The curricula presented evidence that children whose thinking and symbolic skills are strengthened by developmentally appropriate and multidisciplinary experiences do succeed in school. The records kept by the Cognitively Oriented Curriculum group suggest strongly that the graduates of their programs do much better than one might expect them to do without special help. Nevertheless, the lack of an experimental control group, statistically matched to their students, makes it difficult to make definitive judgments.

Aiken, Fell and Newnham, and Hamrick all made reference in the introductions to their studies to the basic interrelatedness of mathematics and reading. Still, there was no one study that addressed directly the idea of an interdisciplinary teaching model for mathematics and reading. This lack of a straightforward examination of the question has resulted in an abundance of information which is suggestive that a multidisciplinary approach would be useful, but no empirical data are included.

CONCLUSIONS

Studies of cognitive developmental theory, communication theory, and instructional theory would support an interdisciplinary model of instruction.

In fact, curricula based implicitly on interdisciplinary approaches have been developed and remain in use.

At this time inadequate research exists which addresses this instructional approach directly. Therefore, regardless of the theoretical arguments which might be made for such a model, it is not possible at this time to validate an interdisciplinary reading/mathematics conceptual model.

FUTURE RESEARCH

The theories of cognitive development and information, and some classroom programs, suggest strongly enough the possible use of the development of an interdisciplinary model that this reviewer would recommend further research into this field is justified.

What form might this research take? It ought to be experimental in nature so that it can produce pragmatic information. Theoretical foundations already exist. It ought to directly address interdisciplinary approaches to teaching mathematics and reading.

Experiments might be fruitful which explore theoretical issues such as:

- 1) is it possible to teach children mathematics as though it were a second language, albeit a specialized one?

- 2) is there a human developmental learning pattern for mathematical language and concepts that is roughly analagous to the linguistic trait that Chomsky describes? (Chomsky outlines a theory of linguistic develoment which suggests that human children have an innate ability to generate language. He insists language is not imitative. If it were imitatative how would children utter sentences they had never heard? Why would all human languages be so similar in structure?)

3) is training in some kinds of linguistic styles more useful than other modes? As Rose and Rose suggest, does precise and elaborate language really affect quality of mathematical thought?

Curriculum studies which might be pursued include:

1) Is it possible to develop a curriculum for young children that is multidisciplinary in nature? What would be contained in such a curriculum?

2) What would one expect a child to know after a certain number of years in such a program?

3) At what ages might a multidisciplinary teaching approach be appropriate? At what ages might it be inappropriate? For instance, would children from ages 8-12 benefit from this approach as much as children aged 4-8?

4) What would be the nature of appropriate classroom materials?

The educational establishment owes the public the most efficient and efficacious system of instruction that it is humanly possible to produce. In order to provide that system every reasonable teaching method should be examined to ascertain its usefulness and adequacy.

In 1972 Aiken concluded that there had been "a mass of unreplicated studies. . . but no serious attempt to determine either approach." (p.380) Our understanding of the connection between mathematical and linguistic languages has improved in the last decade. What seems not to have advanced much is our understanding of how best to instructionally address that connection.

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VALIDATION OF AN INTERDISCIPLINARY MATHEMATICS
READING CONCEPTUAL MODEL THROUGH AN ANALYSIS OF
INTERDISCIPLINARY RESEARCH IN
MATHEMATICS AND READING

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ABSTRACT

This paper reviewed theoretical and research studies seeking to establish the existence of a conceptual model which justifies interdisciplinary approaches to teaching mathematics and reading to primary students. In the process of that search the developmental theories of Jean Piaget and the theory of information validated by C.E. Shannon were briefly outlined. Research studies published during the last 15 years that were applicable to such a conceptual model were examined. Two primary curriculae were reviewed to ascertain what aspects of both programs supported the idea of a multidisciplinary approach to teaching.

The works of Piaget and Shannon were found to be supportive of an interdisciplinary approach to teaching mathematics and reading. Aggregate results from research experiments performed with elementary school children were ambiguous. Research indicated that the information children gain in one discipline may be utilized when seeking to solve problems in the other discipline. The existing multidisciplinary curriculae were found not to have conclusive results concerning the efficacy of their programs.

Therefore, this author concluded that more research was necessary before a conceptual model which justifies the interdisciplinary approach to teaching reading and mathematics to primary students could be validated.