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A LINEAR PROGRAMMING REACTOR REFUELING MODEL

by 2214

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF FIGURES (continued)

LIST OF TABLES

NOMENCLATURE

a_{ij}	coefficient of variable j in equation i
b_i	right hand side coefficient
BFS	basic feasible solution
c_j	coefficient of variable j in the objective function
En_{max}	maximum enrichment
En_{min}	minimum enrichment
f	thermal utilization
I_m	amount of nuclide m
I_m^o	initial amount of nuclide m
I_{im}	amount of nuclide m in irradiation period i
k_{eff}	effective multiplication factor
k_∞	infinite multiplication factor
OF	objective function
p	resonance escape probability
P	non-leakage probability
t	time
U_{im}	amount of nuclide m added to or removed from the reactor after irradiation period i
V	volume of the reactor core
Z	objective function

Greek symbols

ε	fast fission factor
η_o	average number of neutrons liberated directly by fission for every thermal neutron absorbed in the fuel at the beginning of each irradiation period

η_f	average number of neutrons liberated by fission for every thermal neutron absorbed in the fuel at the end of each irradiation period
η_m	ratio of the neutrons produced by fission to the number of neutrons absorbed in fissile species m
ν_m	ratio of the number of neutrons produced by fission to the number of neutrons absorbed in fission by fissile species m
σ_{am}	microscopic absorption cross section for nuclide m
σ_{cm}	microscopic capture cross section for nuclide m
σ_{fm}	microscopic fission cross section for nuclide m
Σ_s	macroscopic cross section of reactor structural material
$\overline{\phi}(t)$	average neutron flux
θ	neutron fluence

Subcript (m) for symbols

0	Pu-240
1	Pu-241
5	U-235
6	U-236
7	fission product pairs
8	U-238
95	Pu-239 from thermal neutron absorption in U-235
98	Pu-239 from thermal neutron absorption in U-238
N7	Np-237
P8	Pu-238

1.0 INTRODUCTION

As the number of nuclear power plants increases, nuclear fuel management considerations take on greater urgency. The dollar magnitude of fuel in the nuclear picture was illustrated by Benedict (1): A 1000 Mwe pressurized water reactor (PWR) or boiling water reacter (BWR), over 30 years at 65 per cent capacity will spend 303 million dollars on fuel. This illustrates the importance of the optimization of fuel usage; an error of one per cent could lead to a loss of 3.03 million dollars for the electric utility.

Several techniques have been used for fuel optimization but most deal with the maximization of fuel burnup. Two such techniques appear in works by Fagan (2) and Tabak (3). These techniques are not best suited to a utility because the utility must schedule refueling shut down times according to their seasonal load curve and other system capacity. Hence, the utility must refuel during a proper refueling window. This is a predicted time of the year, usually in the spring or fall, when power demands are low enough that the utility can shut down a reactor to refuel and still maintain necessary power requirements with the operation of other generating units of the system. Thus, scheduling of refueling times is a very important consideration.

It is the purpose of this work to develop a reactor model to obtain the most economical use of nuclear fuel for the utility over a period of five refueling periods. The reactor was assumed to be a base load reactor, operating at a steady power, and refueling was assumed to take place once a year during the predicted refueling window. A linear programming model was developed to fulfill this objective.

2.0 CONCEPTS OF LINEAR PROGRAMMING

2.1 Definition of Linear Programming

Linear programming (LP) deals with those problems in which all relations among the variables are linear. This includes the objective function (OF), that function which is to be maximized or minimized, and all problem constraints. LP is a technique which can be used to optimize the OF, which must be a linear function of a parameter such as cost, profit, or products produced. The constraints developed for the model must be linear combinations of such things as the amount of time, money, man power, or raw materials available.

"The general linear programming problem can be described as follows: Given a set of m linear inequalities or equations in r variables, we wish to find non-negative values of these variables which will satisfy the constraints and maximize or minimize some linear function of the variables (4)."

Mathematically this can be written as, given

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{ir}x_r\{\leq = \geq\}b_i \;, \quad i = 1,\ldots,m, \tag{2.1.1}$$

where b_i and a_{ij}, $i=1,\ldots,m$ and $j=1,\ldots,r$, are known constants. The optimal values of x_j will satisfy Eq. (2.1.1) with

$$x_j \geq 0 \;, \quad j = 1,\ldots,r, \tag{2.1.2}$$

and maximize or minimize a linear objective function,

$$Z = c_1x_1 + \ldots c_rx_r \;, \tag{2.1.3}$$

where c_j, $j=1,\ldots,r$, are known constants.

2.2 Linear Programming Example

As an illustration of a simple LP problem consider a fuel fabricator who produces a type A and B fuel element. The company has three shops X, Y, Z in which the fuel elements are produced. Figure 2.2.1 shows: 1) the hours required in each shop per unit of each element; 2) the total available shop time per month; 3) the profit on the sale of one unit of any one of the elements. The company wishes to know the optimum production schedule for maximizing profit when the contribution to profit is 2000 dollars for a type A fuel element and 1000 dollars for a type B fuel element.

The OF selected in this problem is profit:

$$Z = 2000x_1 + 1000x_2 , \tag{2.2.1}$$

where x_1 is the number of type A elements produced and x_2 is the number of type B elements produced.

This OF is subject to the following constraints:

$$2x_1 + 3x_2 \leq 600 , \tag{2.2.2}$$

$$1x_1 + \frac{10}{3} x_2 \leq 500 , \tag{2.2.3}$$

$$3x_1 + x_2 \leq 600 , \tag{2.2.4}$$

Since we cannot produce negative quantities, i.e., to have a feasible solution to Eqs. (2.2.2 - 2.2.4), the following are additional implicit restrictions of the model:

$$x_1, x_2 \geq 0 . \tag{2.2.5}$$

In tableau form (4) the model is given as in Figure 2.2.2.

The constraints and OF (dotted line) are shown in Fig. 2.2.3. The shaded area is the feasibility polygon, i.e., convex space, which contains any point which will satisfy the constraints of Eqs. (2.2.2 - 2.2.5). Because the OF has a constant slope, it is maximized by passing through the extreme point D, $x_1 = 171.43$ and $x_2 = 85.71$. The optimal value of the OF at this point is Z = \$428,571, i.e.,

$$Z = (\$2000)\ (171.43) + (\$1000)\ (85.71) = \$428,571\ . \qquad (2.2.6)$$

This example shows the geometrical interpretation of a simple linear programming problem and that the set of feasible solutions described by Eqs. (2.2.2 - 2.2.5) formed a convex region. It was also shown that at least one extreme point of the convex region was optimal. When a problem involves more than three dimensions the solution becomes hard to visualize and nearly impossible to show graphically. The number of basic feasible solutions (BFS), extreme points, can soon become large (but finite) and the problem of examining these BFS's for the optimal solution may take an inordinate amount of time. Thus the simplex and revised simplex methods of finding the optimal solution were developed (2). The procedure used in each of these is a series of systematic steps from one initial BFS to other BFS's and finally, in a finite number of steps, to an optimal BFS, if one exists. The optimal BFS is found in such a way that the value of the OF at each step, for a maximization problem, is greater than the preceding step.

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Shop	Fuel Elements A	Fuel Elements B	Total hours available per month
X	3	3	600
Y	1	10/3	500
Z	2	1	600
unit profit	$2000	$1000	

Figure 2.2.1. The data for the example.

	C_j	2000	1000	0	0	0	
C_B	Vectors in basis	a_1	a_2	a_3	a_4	a_5	b
0	a_3	2	3	1	0	0	600
0	a_4	1	10/3	0	1	0	500
0	a_5	3	1	0	0	1	600
	$Z_j - C_j$	-2000	-1000	0	0	0	0

Figure 2.2.2. The initial tableau for the example.

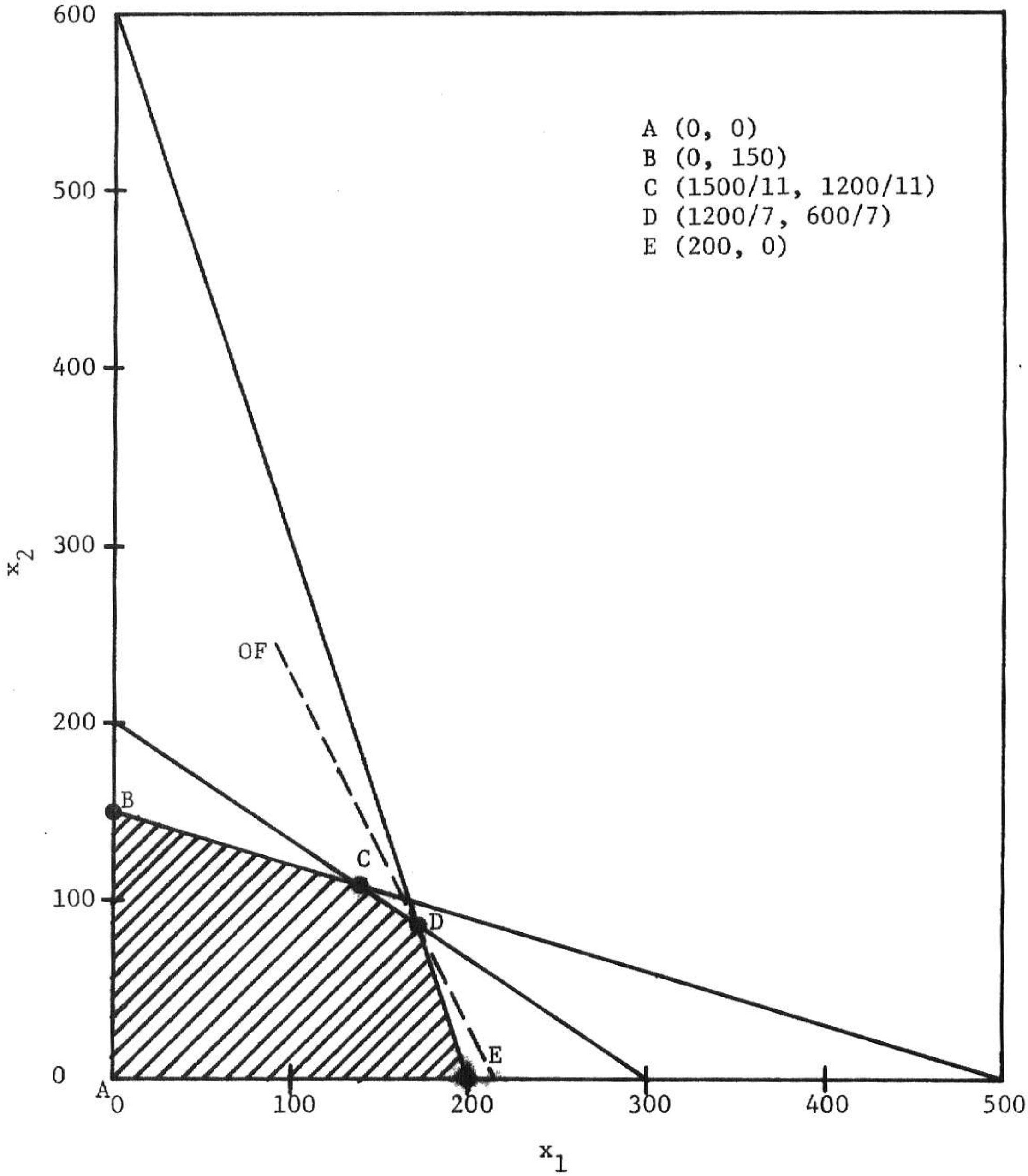

Figure 2.2.3 Graphical Solution to Example Problem.

3.0 MODEL DEVELOPMENT

3.1 The Reactor

Because of the limited supply of uranium much research is being done on fast breeder and plutonium recycle reactors. Due to difficult technical problems the fast breeder reactor does not seem to be an economically feasible reactor until at least 1985. But, the plutonium recycle reactor should be ready for the utility industry by 1975.

Because of this, a plutonium recycle reactor was chosen for this work. The effects of plutonium recycle were studied to determine its advantages or disadvantages to U-235 minimization during refueling.

3.2 Nuclide Production and/or Burnup

The reactor was assumed to be loaded initially with only U-235 and U-238. During operation of the reactor the production of the following nuclides was considered; U-236, Pu-238, Pu-239, Pu-240, Pu-241, Np-237, and fission products. The following assumptions were used in the development of a computer burnup code: 1) One homogeneous region, 2) a known neutron fluence for each operation period, 3) one neutron energy group (thermal), 4) no U-238 fission, 5) negligible decay of nuclides, and 6) batch irradiation.

With the above assumptions, nuclide concentrations during continuous irradiation can be expressed by a set of first order, linear, ordinary differential equations with constant coefficients. The independent variable in these equations is neutron fluence, Θ, i.e.,

$$\Theta = \int_0^t \bar{\phi}(t')dt' \quad . \tag{3.2.1}$$

$\bar{\phi}(t)$ is the average neutron flux at which the reactor operates over the irradiation period.

The concentration of U-235, I_5, during irradiation is

$$\frac{dI_5}{d\Theta} = -I_5\sigma_{a5}. \tag{3.2.2}$$

The solution to Eq. (3.2.2) is

$$I_5 = K_{11}I_5^o, \tag{3.2.3}$$

where

$$K_{11} = \exp(-\sigma_{a5}\Theta). \tag{3.2.4}$$

The concentration of U-238, I_8, is expressed by

$$\frac{dI_8}{d\Theta} = -I_8\sigma_{a8}, \tag{3.2.5}$$

which integrates to

$$I_8 = K_{33} I_8^o, \tag{3.2.6}$$

where

$$K_{33} = \exp(-\sigma_{a8}\Theta). \tag{3.2.7}$$

For build up of U-236, I_6, the differential equation is

$$\frac{dI_6}{d\Theta} = I_5\sigma_{c5} - I_6\sigma_{a6}. \tag{3.2.8}$$

The solution to Eq. (3.2.8) is

$$I_6 = K_{21} I_5^o + K_{22} I_6^o, \tag{3.2.9}$$

where

$$K_{21} = \frac{\sigma_{c5}}{A}(B - C), \tag{3.2.10}$$

$$K_{22} = C, \tag{3.2.11}$$

$$A = \sigma_{a6} - \sigma_{a5}, \tag{3.2.12}$$

$$B = \exp(-\sigma_{a5}\Theta), \tag{3.2.13}$$

$$C = \exp(-\sigma_{a6}\Theta). \tag{3.2.14}$$

The production of Pu-239 was assumed to come from two reactions, the thermal absorption of neutrons in U-238 and U-235. The differential equation for production of Pu-239 from thermal absorption in U-238 is

$$\frac{dI_{98}}{d\Theta} = I_8\sigma_{a8} + \varepsilon P_1(1 - p)(I_5\sigma_{a5}\eta_5 + I_{98}\sigma_{a9}\eta_9) - I_{98}\sigma_{a9}. \tag{3.2.15}$$

The first term of Eq. (3.2.15) is the thermal absorption of a neutron in U-238 to form U-239 which then decays by two betas to form Pu-239. The second term represents the resonance absorption of fast neutrons produced by U-235 and Pu-239 fission. The last term is the removal of Pu-239 due to thermal absorption.

The solution to Eq. (3.2.15) is

$$I_{98} = K_{41}I_5^o + K_{43}I_8^o + K_{44}I_{98}^o, \tag{3.2.16}$$

where

$$K_{41} = \frac{C5}{D}(E - B), \tag{3.2.17}$$

$$K_{43} = CB(1 - E), \tag{3.2.18}$$

$$K_{44} = E, \tag{3.2.19}$$

$$E = \exp(C9\Theta), \tag{3.2.20}$$

$$D = \sigma_{a5} + C9, \tag{3.2.21}$$

$$C5 = \varepsilon P_1(1 - p)\eta_5\sigma_{a5}, \tag{3.2.22}$$

$$C8 = -\sigma_{c8}/C9, \tag{3.2.23}$$

$$C9 = \varepsilon P_1(1 - p)\eta_9\sigma_{a9} - \sigma_{a9}. \tag{3.2.24}$$

Before considering the production of Pu-239 from thermal absorption in U-235, Np-237 and Pu-238 must be considered. The concentration of Np-237, I_{N7}, can be found from

$$\frac{dI_{N7}}{d\Theta} = \sigma_{c6}I_6 - \sigma_{cN7}I_{N7}. \tag{3.2.25}$$

The solution to this is

$$I_{N7} = K_{91}I_5^o + K_{92}I_6^o + K_{97}I_{N7}^o, \tag{3.2.26}$$

where

$$K_{91} = \frac{\sigma_{c6}\sigma_{c5}}{A}\left[\frac{B - F}{G} + \frac{F - C}{H}\right], \tag{3.2.27}$$

$$K_{92} = \frac{\sigma_{c6}}{G}(C - F), \tag{3.2.28}$$

$$K_{97} = F, \tag{3.2.29}$$

$$F = \exp(-\sigma_{cN7}), \tag{3.2.30}$$

$$G = \sigma_{cN7} - \sigma_{a5}, \tag{3.2.31}$$

$$H = \sigma_{cN7} - \sigma_{a6}, \tag{3.2.32}$$

Then for Pu-238, I_{P8}, the differential equation is

$$\frac{dI_{P8}}{d\Theta} = \sigma_{cN7}I_{N7} - \sigma_{cP8}I_{P8}, \tag{3.2.33}$$

which integrates to

$$I_{P8} = K_{81}I_5^o + K_{82}I_6^o + K_{87}I_{N7}^o + K_{88}I_{P8}^o, \tag{3.2.34}$$

where

$$K_{81} = \frac{\sigma_{cN7}\sigma_{c6}\sigma_{c5}}{A}\left[\frac{B - J}{LG} - \frac{C - J}{MH}\right] + \frac{\sigma_{cN7}\sigma_{c6}\sigma_{c5}}{NA}\left[\frac{1}{H} - \frac{1}{G}\right](F - J), \tag{3.2.35}$$

$$K_{82} = \frac{\sigma_{cN7}\sigma_{c6}}{H}\left[\frac{C - J}{M} - \frac{F - J}{N}\right], \tag{3.2.36}$$

$$K_{87} = \frac{\sigma_{cN7}}{N}(F - J), \tag{3.2.37}$$

$$K_{88} = J, \tag{3.2.38}$$

$$J = \exp(-\sigma_{cP8}\Theta), \tag{3.2.39}$$

$$L = \sigma_{cP8} - \sigma_{a5}, \tag{3.2.40}$$

$$M = \sigma_{cP8} - \sigma_{a6}, \tag{3.2.41}$$

$$N = \sigma_{cP8} - \sigma_{cN7}. \tag{3.2.42}$$

For Pu-239 produced from thermal fission in U-235, I_{95}, the differential equation is

$$\frac{dI_{95}}{d\Theta} = \sigma_{cP8}I_{P8} - \sigma_{a9}I_{95}, \tag{3.2.43}$$

which has the solution:

$$I_{95} = K_{101}I_5^o + K_{102}I_6^o + K_{107}I_{N7}^o + K_{108}I_{P8}^o + K_{109}I_{95}^o, \tag{3.2.44}$$

where

$$K_{101} = \frac{\sigma_{cP8}\sigma_{cN7}\sigma_{c6}\sigma_{c5}}{A}\left[\frac{B-0}{QLG} - \frac{C-0}{RMH} + \frac{F-0}{SNH} - \frac{F-0}{SNG} - \frac{J-0}{TLG} + \frac{J-0}{TMH} - \frac{J-0}{TNH} + \frac{J-0}{TNG}\right], \quad (3.2.45)$$

$$K_{102} = \sigma_{cP8}\sigma_{cN7}\sigma_{c6}\left[\frac{C-0}{RMH} - \frac{F-0}{SNH} + \frac{J-0}{THN} - \frac{J-0}{THM}\right], \quad (3.2.46)$$

$$K_{107} = \frac{\sigma_{cP8}\sigma_{cN7}}{N}\left[\frac{F-0}{S} - \frac{J-0}{T}\right], \quad (3.2.47)$$

$$K_{108} = \frac{\sigma_{cP8}}{T}(J - 0) \quad (3.2.48)$$

$$K_{109} = 0, \quad (3.2.49)$$

$$0 = \exp(-\sigma_{a9}\Theta), \quad (3.2.50)$$

$$Q = \sigma_{a9} - \sigma_{a5}, \quad (3.2.51)$$

$$R = \sigma_{a9} - \sigma_{a6}, \quad (3.2.52)$$

$$S = \sigma_{a9} - \sigma_{cN7}, \quad (3.2.53)$$

$$T = \sigma_{a9} - \sigma_{cP8}. \quad (3.2.54)$$

The remaining plutonium isotopes are Pu-240 and Pu-241. For the build up of Pu-240, I_0, the differential equation to solve is

$$\frac{dI_0}{d\Theta} = I_9\sigma_{c9} - I_0\sigma_{a0}. \quad (3.2.55)$$

The solution to Eq. (3.2.56) after substituting $I_{98} + I_{95}$ for I_9 is

$$I_0 = K_{51}I_5^o + K_{53}I_8^o + K_{54}(I_{98}^o + I_{95}^o) + K_{55}I_0^o, \quad (3.2.56)$$

where

$$K_{51} = \frac{C5\sigma_{c9}}{D}\left[\frac{E - V}{W} + \frac{B - V}{X}\right], \tag{3.2.57}$$

$$K_{53} = C8\sigma_{c9}\left[\frac{1 - V}{\sigma_{a0}} + \frac{V - E}{W}\right], \tag{3.2.58}$$

$$K_{54} = \frac{\sigma_{c9}}{W}(E - V), \tag{3.2.59}$$

$$K_{55} = V, \tag{3.2.60}$$

$$V = \exp(-\sigma_{a0}\Theta), \tag{3.2.61}$$

$$W = C9 + \sigma_{a0}, \tag{3.2.62}$$

$$X = \sigma_{a5} - \sigma_{a0}. \tag{3.2.63}$$

The concentration of Pu-241, I_1, is formed from

$$\frac{dI_1}{d\Theta} = I_0\sigma_{c0} - I_1\sigma_{a1}. \tag{3.2.64}$$

After substituting for I_0, the solution is given as

$$I_1 = K_{61}I_5^o + K_{63}I_8^o + K_{64}(I_{98}^o + I_{95}^o) + K_{65}I_0^o + K_{66}I_1^o, \tag{3.2.65}$$

where

$$K_{61} = \frac{\sigma_{c0}\sigma_{c9}\ C5}{WDZ}(E - Y) + \frac{B}{DX(AA)} - \frac{\sigma_{c0}\sigma_{c9}C5V}{(BB)Z}\left[\frac{1}{W} + \frac{1}{X}\right]$$
$$- \frac{\sigma_{c0}\sigma_{c9}C5Y}{D}\left[\frac{1}{X(AA)} - \frac{1}{W(BB)} - \frac{1}{X(BB)}\right], \tag{3.2.66}$$

$$K_{63} = \sigma_{c0}\sigma_{c9}C8\left[\frac{1 - Y}{\sigma_{a1}\sigma_{a0}} + \frac{V - Y}{W(BB)} - \frac{V - Y}{\sigma_{a0}(BB)} - \frac{E - Y}{WZ}\right], \tag{3.2.67}$$

$$K_{64} = \sigma_{c0}\sigma_{c9}\left[\frac{E - Y}{WZ} - \frac{V - Y}{W(BB)}\right], \tag{3.2.68}$$

$$K_{65} = \frac{\sigma_{c0}}{(BB)}(V - Y), \tag{3.2.69}$$

$$K_{66} = Y, \tag{3.2.70}$$

$$Y = \exp(-\sigma_{a1}\Theta), \tag{3.2.71}$$

$$Z = C9 + \sigma_{a1}, \tag{3.2.72}$$

$$AA = \sigma_{a1} - \sigma_{a5}, \tag{3.2.73}$$

$$BB = \sigma_{a1} - \sigma_{a0}. \tag{3.2.74}$$

Finally, the concentration for fission products, I_7, is found by solving

$$\frac{dI_7}{d\Theta} = 2(I_5\sigma_{f5} + I_9\sigma_{f9} + I_1\sigma_{f1}). \tag{3.2.75}$$

Upon substituting for I_9, the solution is

$$I_7 = K_{71}I_5^o + K_{73}I_8^o + K_{74}(I_{98}^o + I_{95}^o) + K_{75}I_0^o + K_{76}I_1^o, \tag{3.2.76}$$

where

$$K_{71} = \frac{2C5(E - 1)}{C9D}\left[\sigma_{f9} + \frac{\sigma_{f1}\sigma_{c0}\sigma_{c9}}{WZ}\right] + \frac{2(B - 1)}{\sigma_{a5}}\left[\frac{\sigma_{f9}C5}{D} - \sigma_{f5} - \frac{\sigma_{f1}\sigma_{c0}\sigma_{c9}C5}{DX(AA)}\right]$$
$$+ \frac{2\sigma_{f1}\sigma_{c0}\sigma_{c9}C5}{\sigma_{a0}D(BB)}\left[\frac{V - 1}{W} + \frac{V - 1}{X}\right] + \frac{2\sigma_{f1}\sigma_{c0}\sigma_{c9}C5(Y - 1)}{\sigma_{a1}D}\left[\frac{1}{WZ} + \frac{1}{X(AA)}\right.$$
$$\left. - \frac{1}{W(BB)} - \frac{1}{X(BB)}\right], \tag{3.2.77}$$

$$K_{73} = 2C8\left[\sigma_{f9} + \frac{\sigma_{f1}\sigma_{c0}\sigma_{c9}}{\sigma_{a0}\sigma_{a1}}\right]\Theta - 2C8(E - 1)\left[\frac{\sigma_{f9}}{C9} + \frac{\sigma_{f1}\sigma_{c0}\sigma_{c9}}{C9WZ}\right]$$

$$+ \frac{2\sigma_{f1}\sigma_{c0}\sigma_{c9}C8}{\sigma_{a1}}(Y - 1)\left[\frac{1}{\sigma_{a0}\sigma_{a1}} - \frac{1}{WZ} + \frac{1}{W(BB)} - \frac{1}{\sigma_{a0}(BB)}\right]$$

$$+ \frac{2\sigma_{f1}\sigma_{c0}\sigma_{c9}C8}{\sigma_{a0}(BB)}(V - 1)\left[\frac{1}{\sigma_{a0}} - \frac{1}{W}\right], \qquad (3.2.78)$$

$$K_{74} = \frac{2\sigma_{f9}(E - 1)}{C9}\left[1 + \frac{\sigma_{c0}\sigma_{c9}}{WZ}\right] + \frac{2\sigma_{f1}\sigma_{c0}\sigma_{c9}}{W}\left[\frac{V - 1}{\sigma_{a0}(BB)} + \frac{Y - 1}{\sigma_{a1}Z} - \frac{Y - 1}{\sigma_{a1}(BB)}\right], \qquad (3.2.79)$$

$$K_{75} = \frac{2\sigma_{f1}\sigma_{c0}}{(BB)}\left[\frac{Y - 1}{\sigma_{a1}} - \frac{V - 1}{\sigma_{a0}}\right], \qquad (3.2.80)$$

$$K_{76} = \frac{2\sigma_{f1}}{\sigma_{a1}}(1 - Y). \qquad (3.2.81)$$

The general form of the U-235 burnup for a fuel cycle of N irradiation periods is

$$I_{i5} = K_{11}I_{(i - 1)5} + U_{i5} \qquad i = 1, 2, \ldots N, \qquad (3.2.82)$$

where U_{i5} is the amount of U-235 added to the reactor at the end of irradiation period i, $I_{(i - 1)5}$ is the initial loading of irradiation period i, and I_{i5} is the initial amount of U-235 in the (i + 1) irradiation period. U_{N5} is zero because no U-235 will be added at the end of the last irradiation period. The concentration of U-235 as a function of fluence is shown in Fig. 3.2.1. U-235 and U-238 are the only nuclides considered which decrease during the irradiation period as shown in Fig. 3.2.1. The other nuclides build up during the irradiation period. The build up equation for U-236 is

$$I_{i6} = K_{21}I_{(i - 1)5} + K_{22}I_{(i - 1)6} - U_{i6}, \qquad i = 1, 2, \ldots N, \qquad (3.2.83)$$

where U_{i6} is the amount of U-236 removed at the end of fuel cycle i, and U_{N6} is zero. The concentration of U-236 as a function of fluence is shown in Fig. 3.2.2. For this model N was chosen as 5 years. Only the initial and final concentration of the nuclide during each irradiation period is of interest to this model.

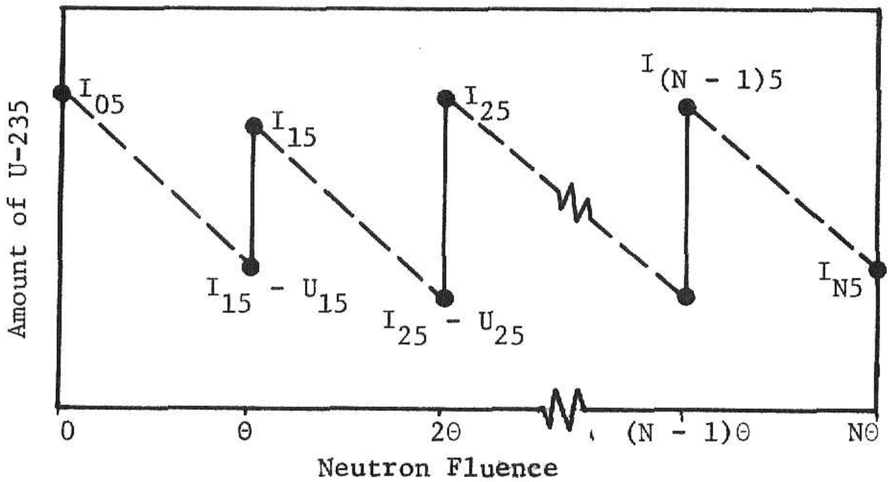

Figure 3.2.1 The burnup of U-235 during N irradiation periods.

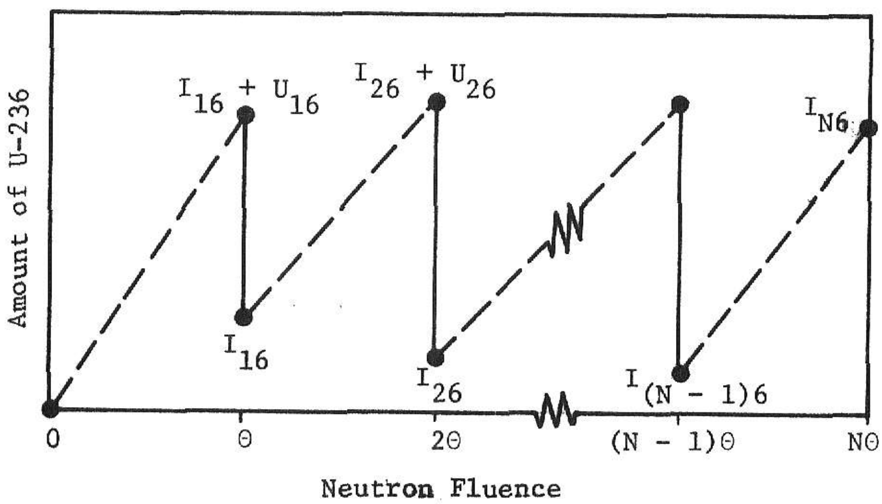

Figure 3.2.2 The buildup of U-236 during N irradiation periods.

3.3 Model Constraints

The nuclides are not entirely free to increase or decrease during the irradiation periods. Certain constraints must be satisfied and in order to satisfy them some of the nuclides must be added or removed at the end of each irradiation period. One such constraint, the multiplication factor constraint, must be equal to or greater than some minimum value at the beginning and end of each irradiation period. The multiplication factor, k_{eff}, is

$$k_{eff} = Pk_{\infty} = P(\eta\varepsilon pf) \ , \tag{3.3.1}$$

$$k_{eff} = PK_1\eta \ , \tag{3.3.2}$$

where

$$K_1 = \varepsilon pf \ , \tag{3.3.3}$$

P = non-leakage probability.

Equation (3.3.2) can be written as

$$\eta = \frac{k_{eff}}{PK_1} = \frac{\sum_{i=1}^{10} \nu_i \sigma_{fi} I_i}{\sum_{i=1}^{10} \sigma_{ai} I_i + \Sigma_s} \ , \tag{3.3.4}$$

where Σ_s is the macroscopic cross section representing all the materials present in the reactor except those mentioned in Section 3.2.

Equation (3.3.4) can be rearranged to the form

$$Z_{11}I_{i5} + Z_{12}I_{i6} + Z_{13}I_{i8} + Z_{14}(I_{i98} + I_{i95}) + Z_{15}I_{i0} + Z_{16}I_{i1} + Z_{19}I_{iN7} + Z_{18}I_{iP8} + Z_{17}I_{i7} \geq Z_{110} \ , \tag{3.3.5}$$

where

$$Z_{11} = \nu_5\sigma_{f5} - \eta_o\sigma_{a5} \ ,$$

$$Z_{12} = \qquad - \eta_o\sigma_{a6} \ ,$$

$$Z_{13} = \qquad \eta_o\sigma_{a8} \ ,$$

$$Z_{14} = \nu_9\sigma_{f9} - \eta_o\sigma_{a9} ,$$

$$Z_{15} = - \eta_o\sigma_{a0} ,$$

$$Z_{16} = \nu_1\sigma_{f1} - \eta_o\sigma_{a1} , \qquad (3.3.6)$$

$$Z_{19} = - \eta_o\sigma_{aN7} ,$$

$$Z_{18} = - \eta_o\sigma_{aP8} ,$$

$$Z_{17} = - \eta_o\sigma_{a7} ,$$

$$Z_{110} = \eta_o\Sigma_s ,$$

η_o = minimum value of η at the beginning of each irradiation period.

The corresponding equation for the end of each irradiation period is

$$Z_{21}(I_{i5} - U_{i5}) + Z_{22}(I_{i6} + U_{i6}) + Z_{23}(I_{i8} - U_{i8}) + Z_{24}(I_{i98} + U_{i98}) + Z_{24}(I_{i95} + U_{i95}) + Z_{25}(I_{i0} + U_{i0}) + Z_{26}(I_{i1} + U_{i1}) + Z_{29}(I_{iN7} + U_{iN7}) + Z_{28}(I_{iP8} + U_{iP8}) + Z_{27}(I_{i7} + U_{i7}) \geq Z_{210} , \qquad (3.3.7)$$

where

$$Z_{21} = \nu_5\sigma_{f5} - \eta_f\sigma_{a5} ,$$

$$Z_{22} = - \eta_f\sigma_{a6} ,$$

$$Z_{23} = - \eta_f\sigma_{a8} ,$$

$$Z_{24} = \nu_9\sigma_{f9} - \eta_f\sigma_{a9} ,$$

$$Z_{25} = - \eta_f\sigma_{a0} , \qquad (3.3.8)$$

$$Z_{26} = - \eta_f\sigma_{a1} ,$$

$$Z_{29} = - \eta_f\sigma_{aN7} ,$$

$$Z_{28} = - \eta_f\sigma_{aP8} ,$$

$$Z_{27} = - \eta_f\sigma_{a7} ,$$

$$Z_{210} = \eta_f \Sigma_s ,$$

η_f = minimum value of η at the end of each irradiation period.

The next constraint which must be satisfied is the fuel enrichment constraint. In most cases considered there will be a maximum enrichment constraint and in one specific case there will be a minimum enrichment constraint added. For the maximum enrichment constraint,

$$\frac{I_{i5}}{I_{i5} + I_{i8}} \leq En_{max} , \tag{3.3.9}$$

which reduces to the form

$$Z_{41} I_{i5} + Z_{42} I_{i8} \leq 0 , \tag{3.3.10}$$

where

$$Z_{41} = 1 - En_{max} , \tag{3.3.11}$$

$$Z_{42} = - En_{max} . \tag{3.3.12}$$

For the minimum enrichment constraint the equation is

$$Z_{81} I_{i5} + Z_{82} I_{i8} \geq 0 , \tag{3.3.13}$$

where

$$Z_{81} = 1 - En_{min} , \tag{3.3.14}$$

$$Z_{82} = - En_{min} . \tag{3.3.15}$$

Another constraint to the model is the volume constraint. The volume of the nuclides in the core cannot exceed the volume of the core, which was assumed to be approximately the volume of the fuel at the initial loading. The volume constraint is then written as

$$Z_{61} I_{i5} + Z_{62} I_{i6} + Z_{63} I_{i8} + Z_{64} (I_{i98} + I_{i95}) + Z_{65} I_{i0} + Z_{66} I_{i1} + Z_{69} I_{iN7} +$$
$$Z_{68} I_{iP8} + Z_{67} I_{i7} \leq V , \tag{3.3.16}$$

where Z_{6j} is the volume coefficient, in units of liter/kgm-atom, of the corresponding nuclides, and V is the assumed volume of the core in liters.

The final restrictions on the model will be those which control the amount of certain nuclides which will be added to or removed from the reactor during refueling.

Since it is desired to minimize the amount of U-235 added to the reactor the objective function will be

$$\min Z = I_{55} + U_{15} + U_{25} + U_{35} + U_{45} . \tag{3.3.17}$$

This has a dual purpose: Not only does it minimize the amount of U-235 added during the refueling periods but also it obtains the best burnup of the fuel in the reactor by minimizing the amount of U-235 left after all the irradiation periods.

3.4 Model Formulation

The burnup equations and constraint equations must be rewritten for each irradiation period. A typical set of equations for any irradiation period will be similar to those for the first irradiation period:

$$K_{11}I_{05} - I_{15} + U_{15} = 0 \ ,$$

$$K_{21}I_{05} + K_{22}I_{06} - I_{16} - U_{16} = 0 \ ,$$

$$K_{33}I_{08} - I_{18} + U_{18} = 0 \ ,$$

$$K_{41}I_{05} + K_{43}I_{08} + K_{44}I_{098} - I_{198} - U_{198} = 0 \ ,$$

$$K_{101}I_{05} + K_{102}I_{06} + K_{109}I_{095} + K_{107}I_{0N7} + K_{108}I_{0P8} - I_{195} - U_{195} = 0 \ ,$$

$$K_{51}I_{05} + K_{53}I_{08} + K_{54}I_{098} + K_{54}I_{095} + K_{55}I_{00} - I_{10} - U_{10} = 0 \ ,$$

$$K_{61}I_{05} + K_{63}I_{08} = K_{64}I_{098} + K_{64}I_{095} + K_{65}I_{00} + K_{66}I_{01} - I_{11} - U_{11} = 0 \ ,$$

$$K_{91}I_{05} + K_{92}I_{06} + K_{97}I_{0N7} - I_{1N7} - U_{1N7} = 0 \ , \qquad (3.4.1)$$

$$K_{81}I_{05} + K_{82}I_{06} + K_{87}I_{0N7} + K_{88}I_{0P8} - I_{1P8} - U_{1P8} = 0 \ ,$$

$$K_{71}I_{05} + K_{73}I_{08} + K_{74}I_{098} + K_{74}I_{095} + K_{75}I_{00} + K_{76}I_{01} - I_{17} - U_{17} = 0 \ ,$$

$$Z_{11}L_{15} + Z_{12}I_{16} + Z_{13}I_{18} + Z_{14}I_{198} + Z_{14}I_{195} + Z_{15}I_{10} + Z_{16}I_{11} + Z_{19}I_{1N7} + Z_{18}I_{1P8} + Z_{17}I_{17} \geq Z_{110} \ ,$$

$$Z_{21}(I_{15} - U_{15}) + Z_{22}(I_{16} + U_{16}) + Z_{23}(I_{18} - U_{18}) + Z_{24}(I_{198} + U_{198}) + Z_{24}(I_{195} + U_{195}) + Z_{25}(I_{10} + U_{10}) + Z_{26}(I_{11} + U_{11}) + Z_{29}(I_{1N7} + U_{1N7}) + Z_{28}(I_{1P8} + U_{1P8}) + Z_{27}(I_{17} + U_{17}) \geq Z_{210} \ ,$$

$$Z_{41}I_{15} + Z_{42}I_{18} \leq 0 \ ,$$

$$Z_{61}I_{15} + Z_{62}I_{16} + Z_{63}I_{18} + Z_{64}I_{198} + Z_{64}I_{195} + Z_{65}I_{10} + Z_{66}I_{11} + Z_{69}I_{1N7} + Z_{68}I_{1P8} + Z_{67}I_{17} \leq V \ ,$$

Other equations can be added to constrain the model further. For example, to make sure that all the U-236 is removed after the first irradiation period the equation $I_{16} = 0$ is added. It can be observed then from Eq. (3.4.1) that

$$U_{16} = K_{21}I_{05} + K_{22}I_{06}, \tag{3.4.2}$$

where U_{16}, the amount of U-236 removed at the end of the first irradiation period, is equal to the amount of U-236 produced during the first irradiation period. To make sure that no U-236 is removed at the end of the irradiation period add the equation

$$U_{16} = 0. \tag{3.4.3}$$

Similar equations can be added to restrict the addition or removal of other nuclides. All of the equations can be put into the mini-matrix form of Fig. 3.4.1. The complete matrix is shown on page 13 of the MPS Computer Program listing in Appendix B.

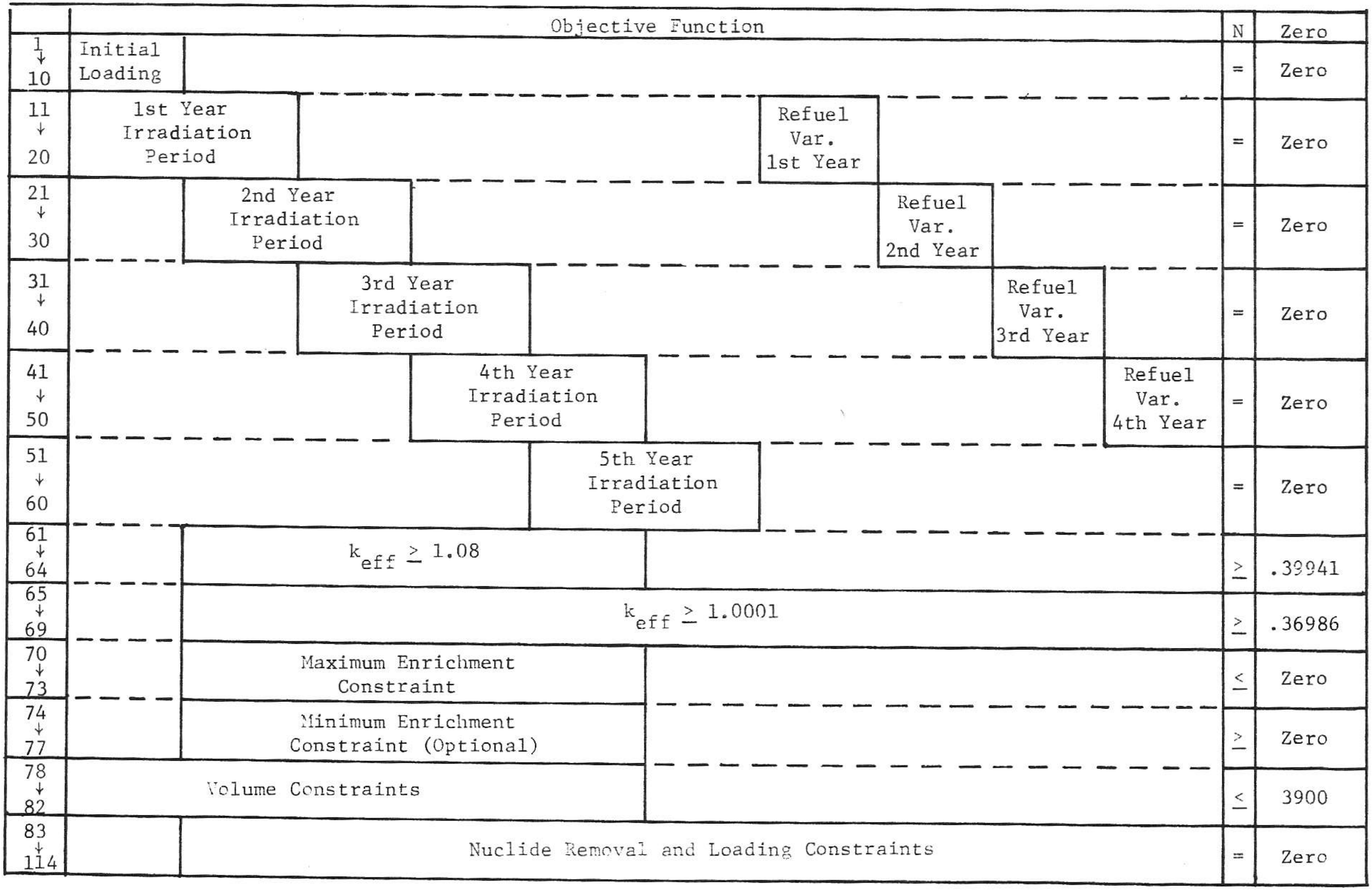

Figure 3.4.1 The mini-matrix of the linear programming nuclear reactor refueling model.

4.0 RESULTS OF MODEL ANALYSIS

4.1 Evaluation of Constants

The reactor was assumed to operate at 1000 Mwe with an average flux of $5(10^{13})$ neutrons/cm^{2}sec. The irradiation period was chosen as 360 days, therefore, from Eq. (3.2.1), θ is 1.5552 neutrons/kb. With that value of θ and the data given in Table 4.1.1 the constants in the burnup equations were easily calculated and are given in Table 4.1.2.

The constants represented in Eqs. (3.3.5 - 3.3.8) were calculated after η_o, η_f, Σ_s were defined. The minimum values of k_{eff} at the beginning and end of each irradiation period were chosen arbitrarily, but in agreement with Tabak (3), as 1.08 and 1.0001, respectively. P was assumed to be 0.80 and K_1 was assumed to be 0.85 (5). Therefore, from Eq. (3.3.4) η_o and η_f were calculated to be 1.58824 and 1.4707, respectively. Σ_s was calculated from data given for a similar reactor (6) and was found to be .25148 cm^{-1}. The calculated constants are listed in Table 4.1.3.

The reactor was initially loaded with 10.6383 kg -atoms of U-235 and 297.89916 kg -atoms of U-238, which corresponds to an initial enrichment of 3.448%. For the enrichment constraints in Section 3.3, En_{max} was chosen as 3.5% and En_{min} was chosen as 3.0%. Therefore from Eqs. (3.3.9 - 3.3.15), Z_{41} = .965, Z_{42} = - .035, Z_{81} = .97, and Z_{82} = - .03.

The constants for the volume constraints were calculated and are listed in Table 4.1.4. For the initial loading the volume of the fuel was calculated to be 3873.35 liters. Hence, the maximum volume of the fuel was fixed at V = 3900 liters.

Table 4.1.1 Cross Section Data

Nuclide	σ_a(kb)†	σ_c(kb)*	σ_f(kb)†	ν*	η*
U-235	.6780	.0980	.580	2.47	1.97
U-236	.0055	.0055			
U-238	.00273	.00273			
Pu-238	.5400	.5400			
Pu-239	1.0130	.2710	.742	2.905	1.81
Pu-240	.2800	.2800			
Pu-241	1.3640	.3640	1.000	3.06	
Np-237	.1690	.1690			
fission product	.0319				

† Data taken from the Chart of the Nuclides, USAEC, May 1970.

* Data taken from revised Chapter 3 of reference 6.

Table 4.1.2 Burnup Equation Constants

Constant	Value	Constant	Value	Constant	Value
K_{11}	.34839	K_{108}	.25673	K_{92}	.00749
K_{21}	.09371	K_{51}	.05026	K_{97}	.76887
K_{22}	.99148	K_{53}	.00059	K_{81}	.00003
K_{33}	.99576	K_{54}	.22178	K_{82}	.00079
K_{41}	.19893	K_{55}	.64697	K_{87}	.15355
K_{43}	.00284	K_{61}	.00537	K_{88}	.43179
K_{44}	.42152	K_{63}	.00006	K_{71}	1.44436
K_{101}	.00001	K_{64}	.03117	K_{73}	.00380
K_{102}	.00017	K_{65}	.13615	K_{74}	1.77141
K_{104}	.20692	K_{66}	.11988	K_{75}	.31801
K_{107}	.04684	K_{91}	.00043	K_{76}	1.29050

Table 4.1.3 Multiplication Factor Constraint Constants

Constant	Value	Constant	Value
Z_{11}	.35578	Z_{21}	.43544
Z_{12}	- .00874	Z_{22}	- .00809
Z_{13}	- .00434	Z_{23}	- .00402
Z_{14}	.54663	Z_{24}	.66566
Z_{15}	- .44471	Z_{25}	- .41181
Z_{16}	.89365	Z_{26}	1.05392
Z_{19}	- .26841	Z_{29}	- .24855
Z_{18}	- .85765	Z_{28}	- .79420
Z_{17}	- .05066	Z_{27}	- .04692
Z_{110}	.39941	Z_{210}	.36986

Table 4.1.4 Volume Constraint Constants

Constant	Value 1/kgm-atom	Constant	Value 1/kgm-atom
Z_{61}	12.401	Z_{66}	12.147
Z_{62}	12.454	Z_{69}	12.474
Z_{63}	12.559	Z_{68}	11.996
Z_{64}	12.046	Z_{67}	1.7187
Z_{65}	12.097	V	3900.00

4.2 Refueling Schemes

Twelve refueling schemes were considered. Two such schemes included no plutonium recycle, one with no minimum enrichment constraint and the other with a minimum enrichment of 3%. The other ten schemes considered different degrees of plutonium recycle varying from 10% to 100% by increments of 10% with no minimum enrichment constraint. The nuclide flow diagram of Fig. 4.2.1 shows the general flow of the nuclides from one irradiation period to the next for a fuel cycle of five irradiation periods. The amount of plutonium removed from the system depends on the percent of plutonium recycle. When plutonium recycle is considered, each isotope of plutonium is recycled at the same percent. This is due to their identical chemical and similar physical properties which constrains their separation. All of the U-236 is recycled because it cannot be easily separated from U-235 due to their identical chemical properties.

As shown in Fig. 4.2.1, all of the Np-237 and fission products are removed after each irradiation period, which emphasizes the assumption of complete reprocessing of the reactor core after each irradiation period. This assumption also considers that the reprocessing and refueling is completed in an amount of time which is insignificant when compared to the year operating time. Another assumption made is that U-235 can be added to the reactor in a pure form, i.e., 100% enriched uranium.

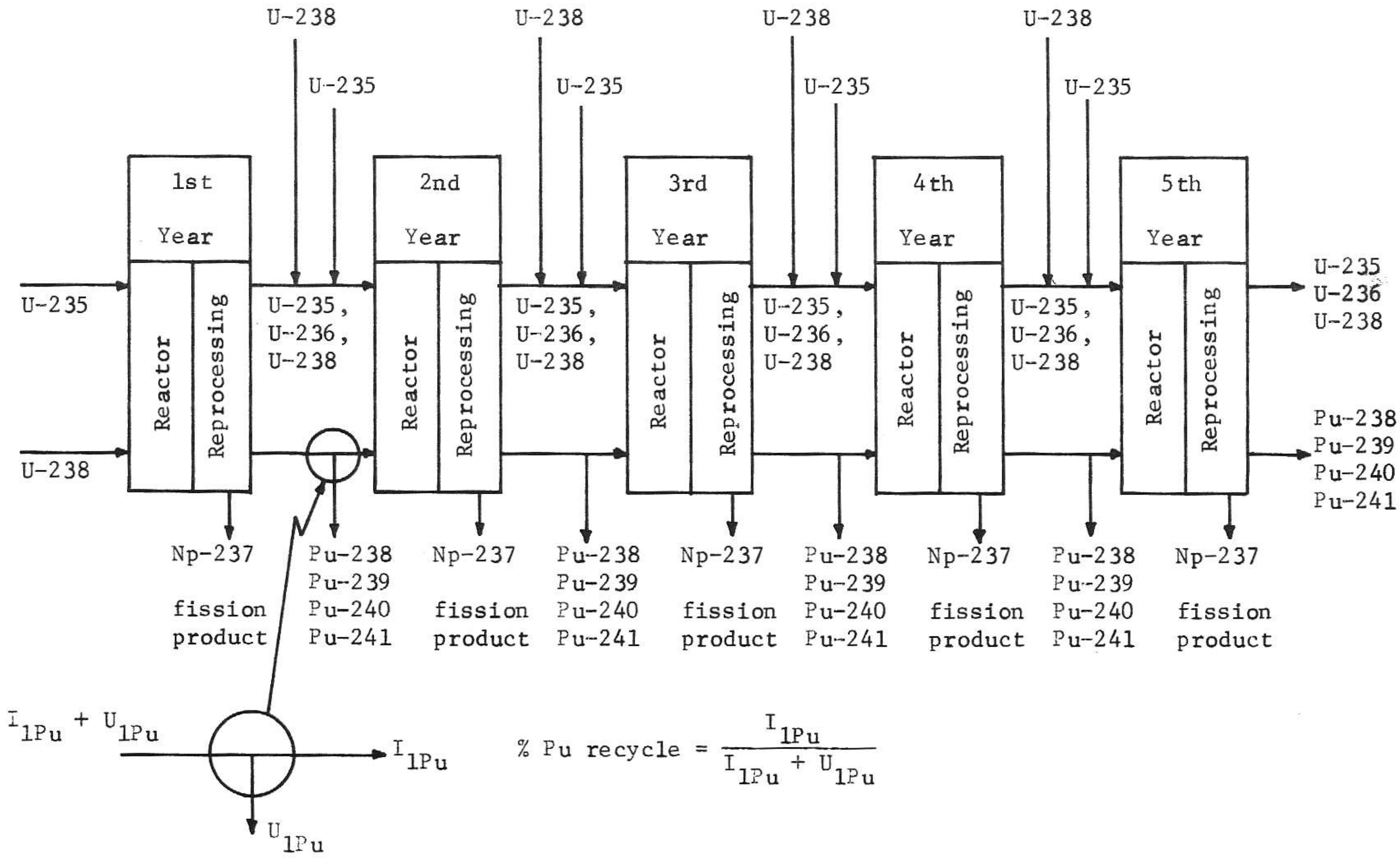

Figure 4.2.1 Nuclide Flow Diagram for a Five Irradiation Period Reactor Fuel Cycle

4.3 Graphical Results

The IBM-MPS/360 which uses the revised simplex method of solving LP problems was used to obtain the solutions for the cases considered. Figures 4.3.1 through 4.3.24 are plots of the results obtained for the 12 refueling cases considered.

Figures 4.3.1, 4.3.2, and 4.3.3 are plots of the kgm-atoms of U-235 versus the time of operation of the reactor for the three different cases of 0%, 50%, and 100% Pu recycle. Figures 4.3.4 through 4.3.11 are plots of the amount of the eight other nuclides versus the years of reactor operation for the case of 50% Pu recycle. Plots of the total amount of plutonium produced in and removed from the reactor versus the time of reactor operation for the cases of 0%, 50%, and 100% Pu recycle are shown in Figs. 4.3.12 through 4.3.14. These values were found by summing the contributions from each plutonium isotope at each refueling time. Figures 4.3.15, 4.3.16 and 4.3.17 are plots of the amounts of fissionable plutonium versus the time of reactor operation for the case of 0%, 50%, and 100% Pu recycle.

Figure 4.3.18 is a plot of the objective function, i.e., the total U-235 requirements, versus the percent of Pu recycled. The amount of U-235 added to the reactor during the fuel cycle, which is found by subtracting the amount of U-235 left at the end of the fuel cycle from the value of the objective function, versus the percent of Pu recycle is shown in Fig. 4.3.19. Figure 4.3.20 shows the total amount of plutonium recycled and the amount of fissionable plutonium recycled versus the percent of Pu recycle. Figure 4.3.21 shows only the fissionable plutonium recycled and the individual contributions of Pu-239 and Pu-241 to this curve. Figures 4.3.22 and 4.3.23 each show the ratio of the change in the U-235 inventory to the change in the total and fissionable plutonium inventory. The change in inventories plotted in

Fig. 4.3.22 was found by using the 0% Pu recycle case as the reference. In Fig. 4.3.23 the change was found by using the previous percent Pu recycle case as the reference to the case being considered. The final figure, Fig. 4.3.24, shows the amount of fission product pairs removed during the fuel cycle for the different Pu recycle cases considered.

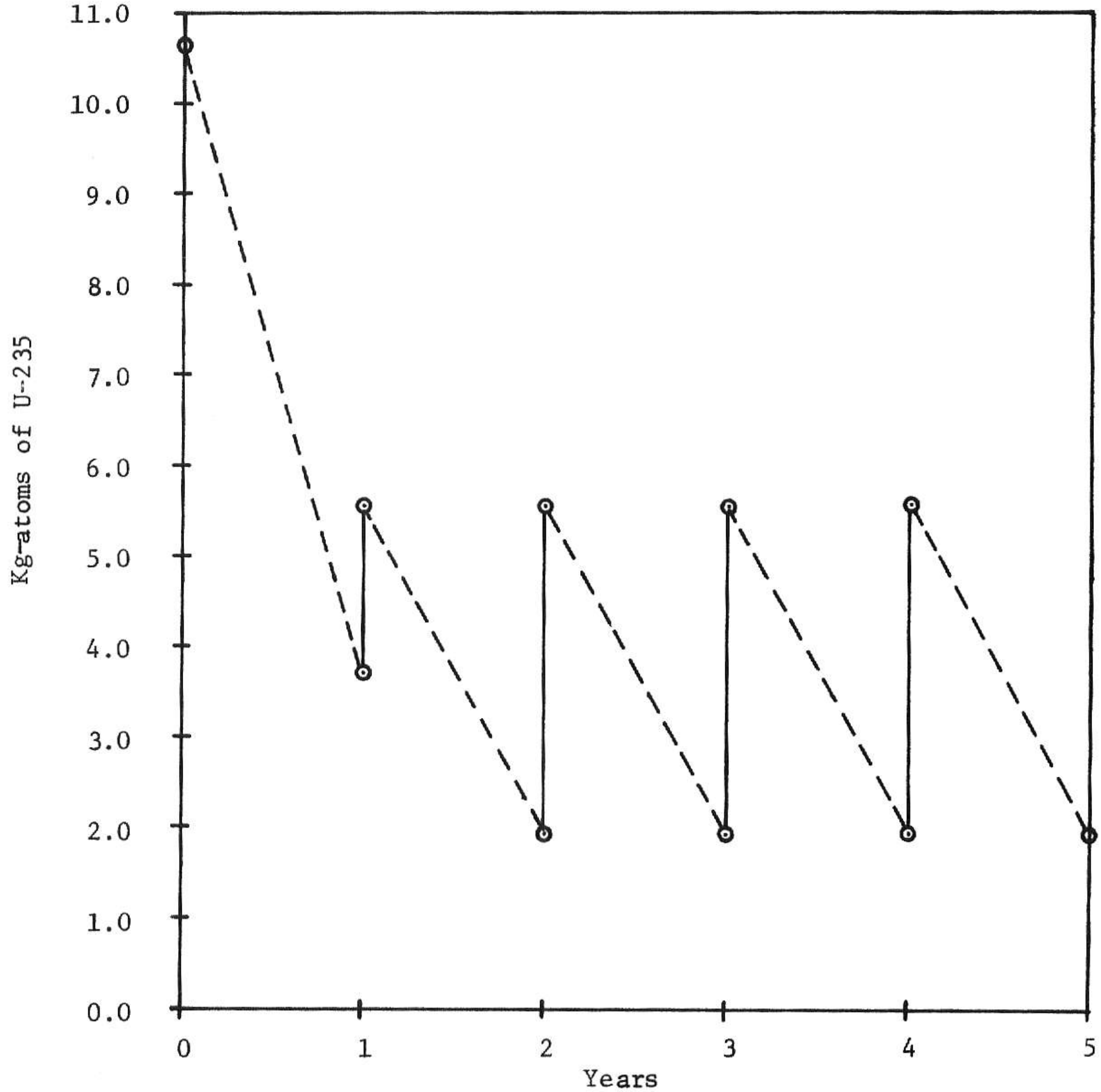

Figure 4.3.1 The amount of U-235 in the reactor during the fuel cycle vs the number of irradiation periods for the 0% Pu recycle case.

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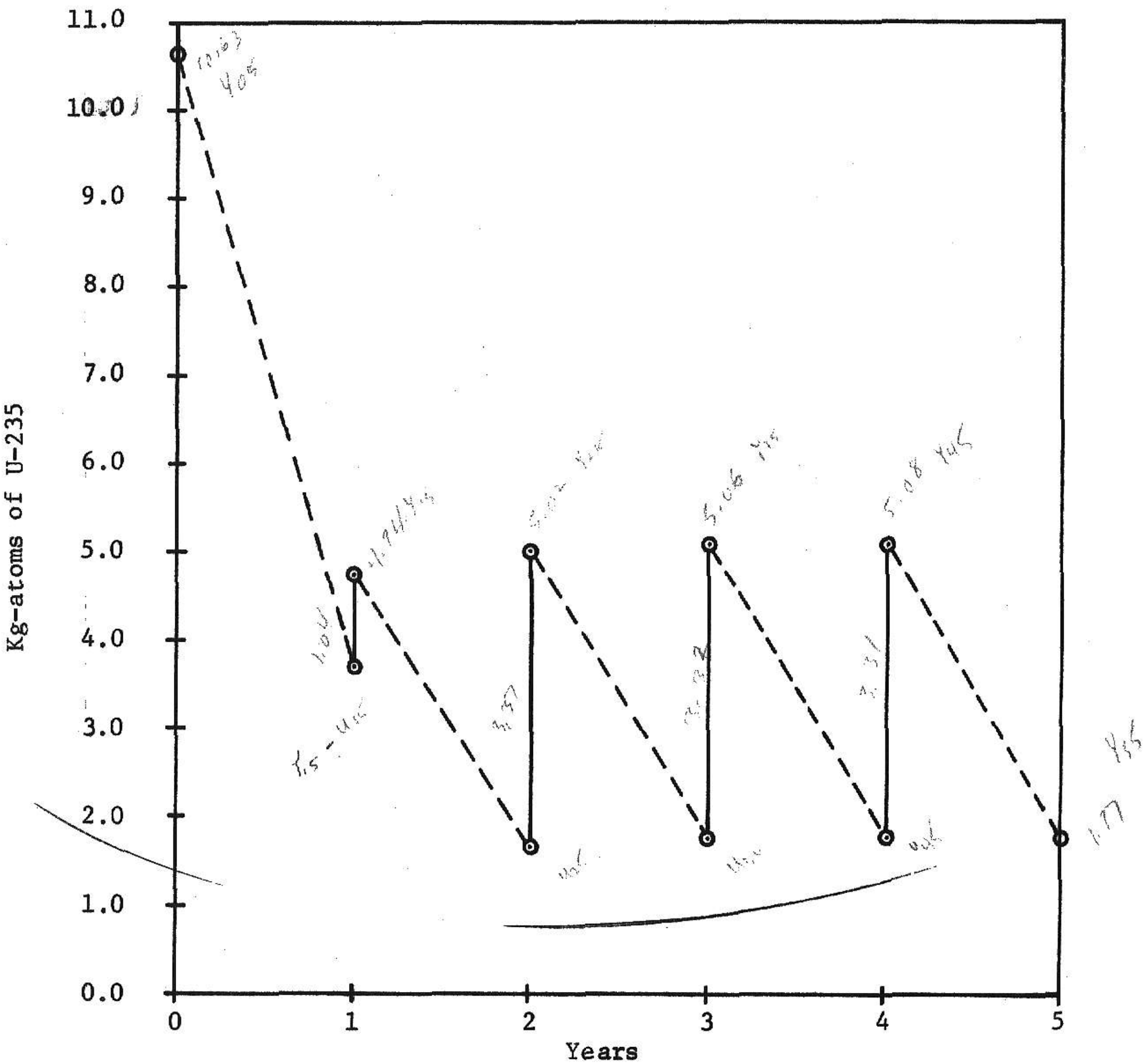

Figure 4.3.2 The amount of U-235 in the reactor during the fuel cycle vs the number of irradiation periods for the 50% Pu recycle case.

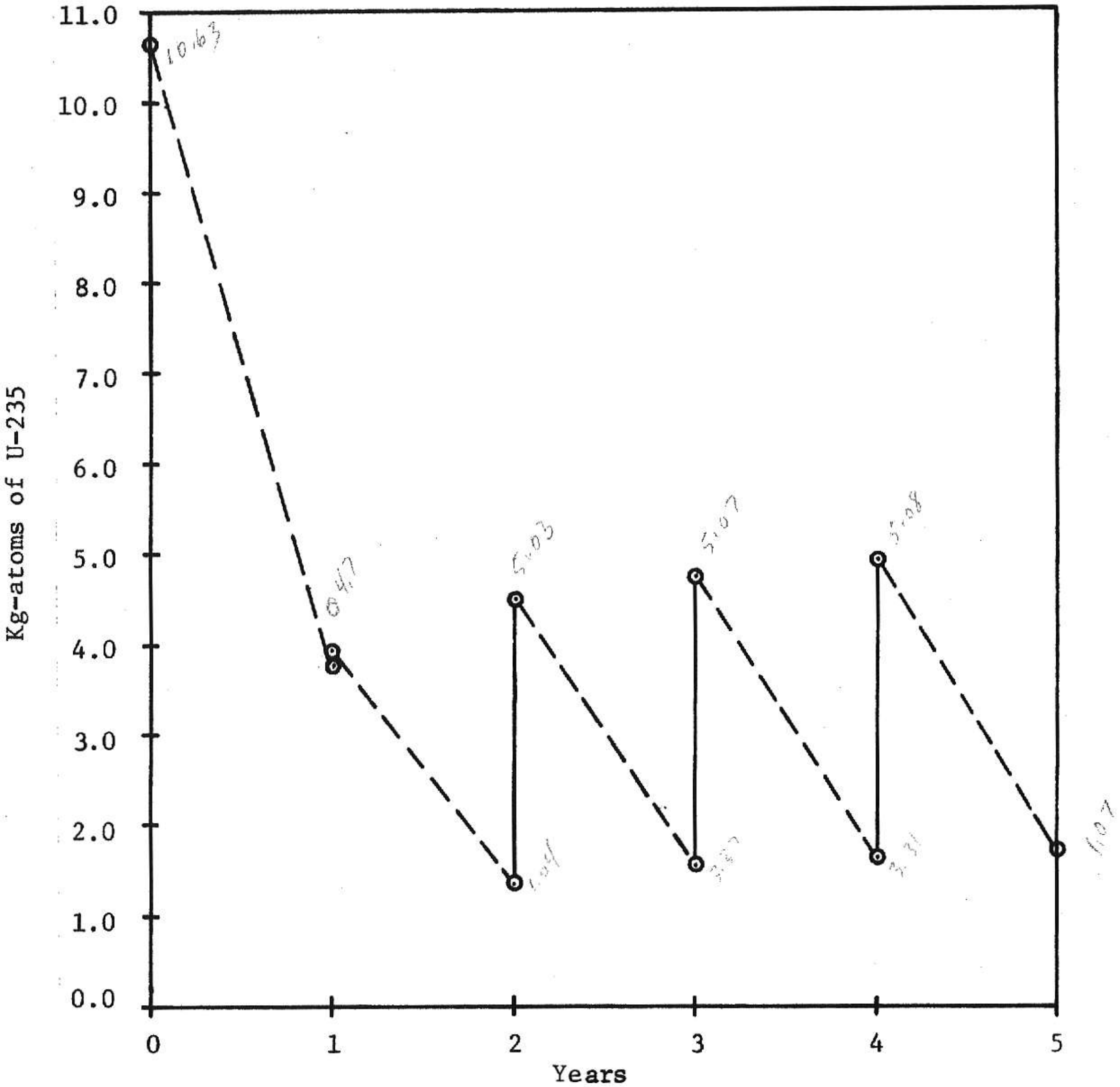

Figure 4.3.3 The amount of U-235 in the reactor during the fuel cycle vs the number of irradiation periods for the 100% Pu recycle case.

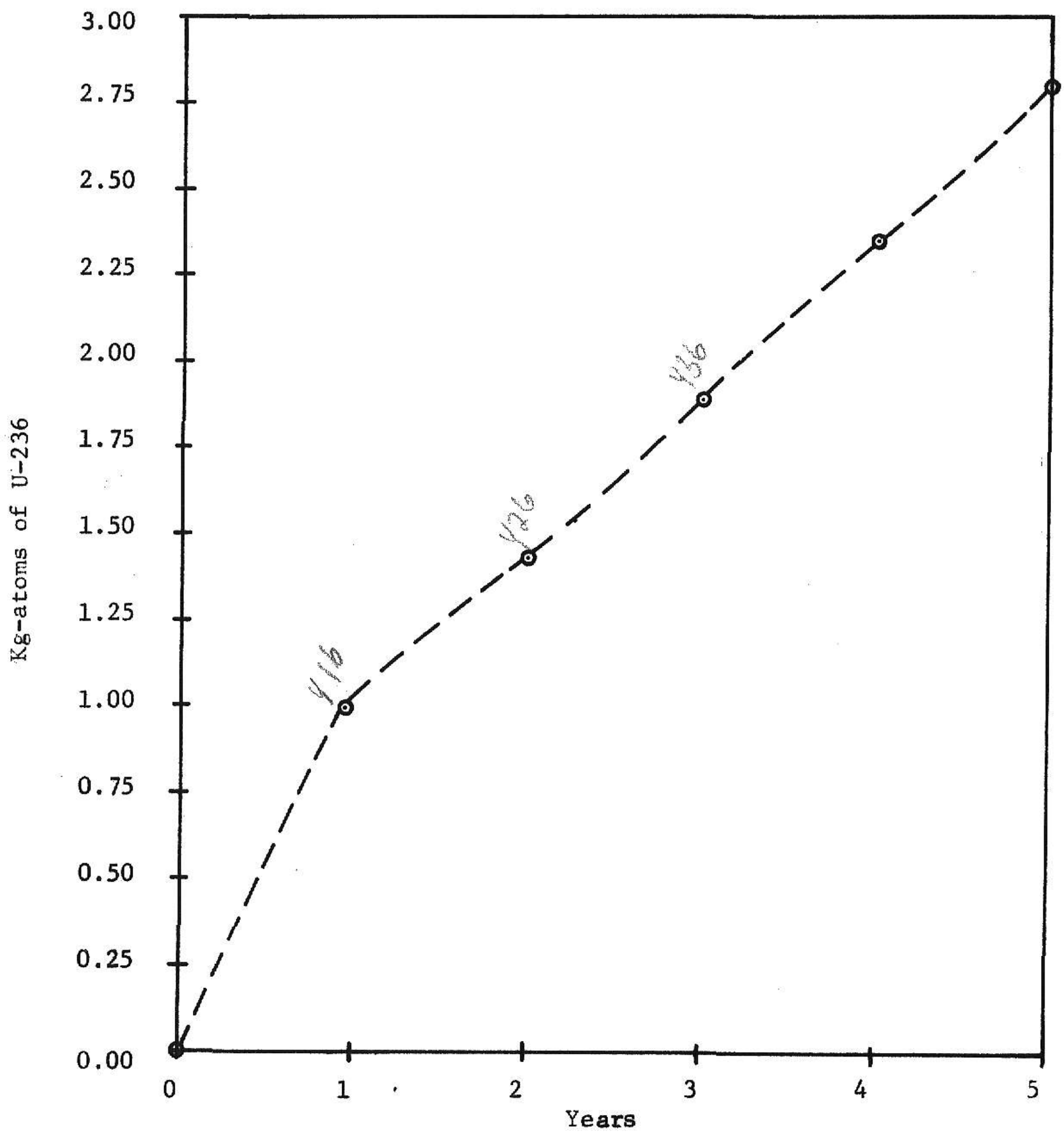

Figure 4.3.4 The amount of U-236 in the reactor during the fuel cycle vs the number of irradiation periods for the 50% Pu recycle case.

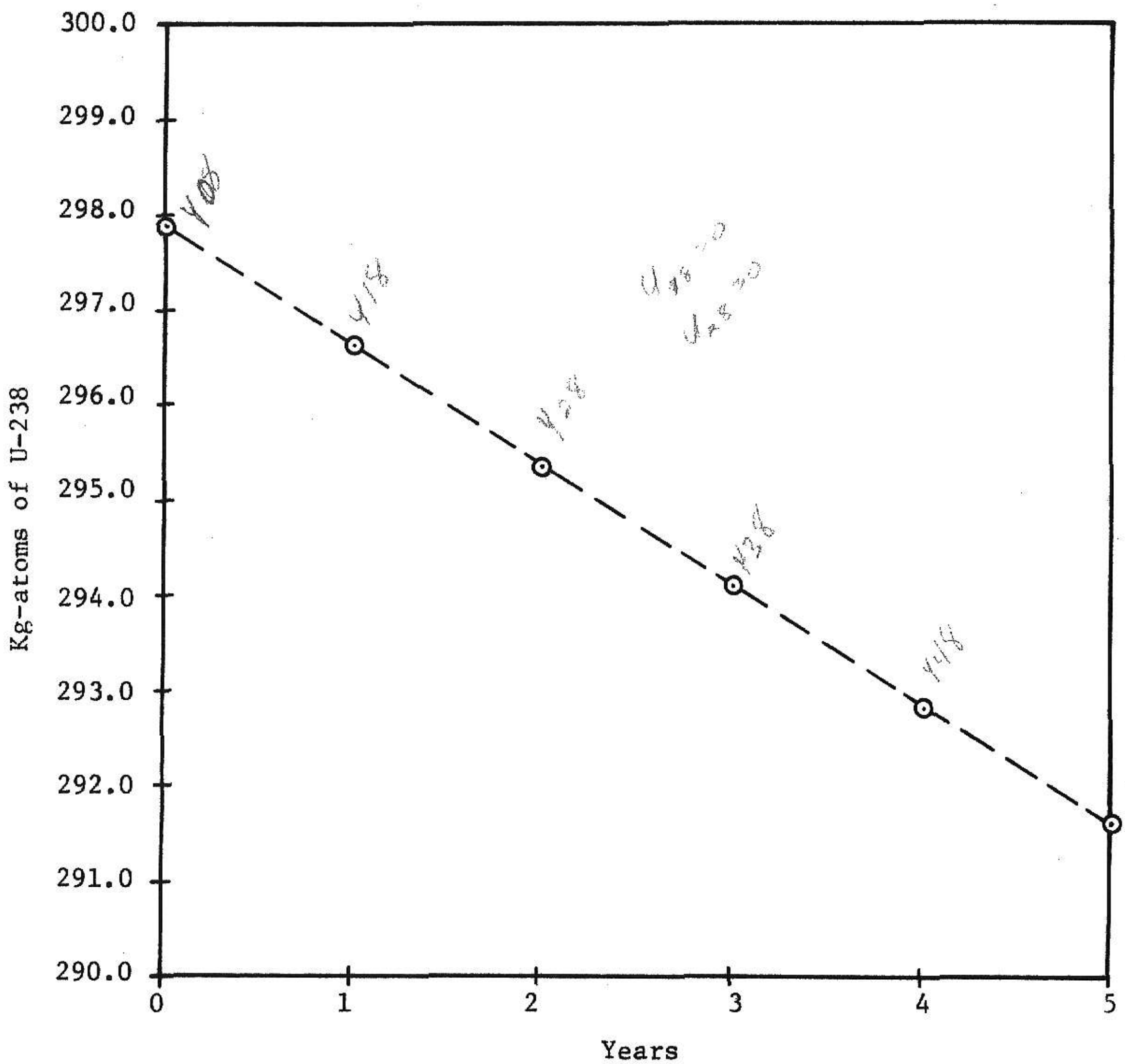

Figure 4.3.5 The amount of U-238 in the reactor during the fuel cycle vs the number of irradiation periods for the 50% Pu recycle case.

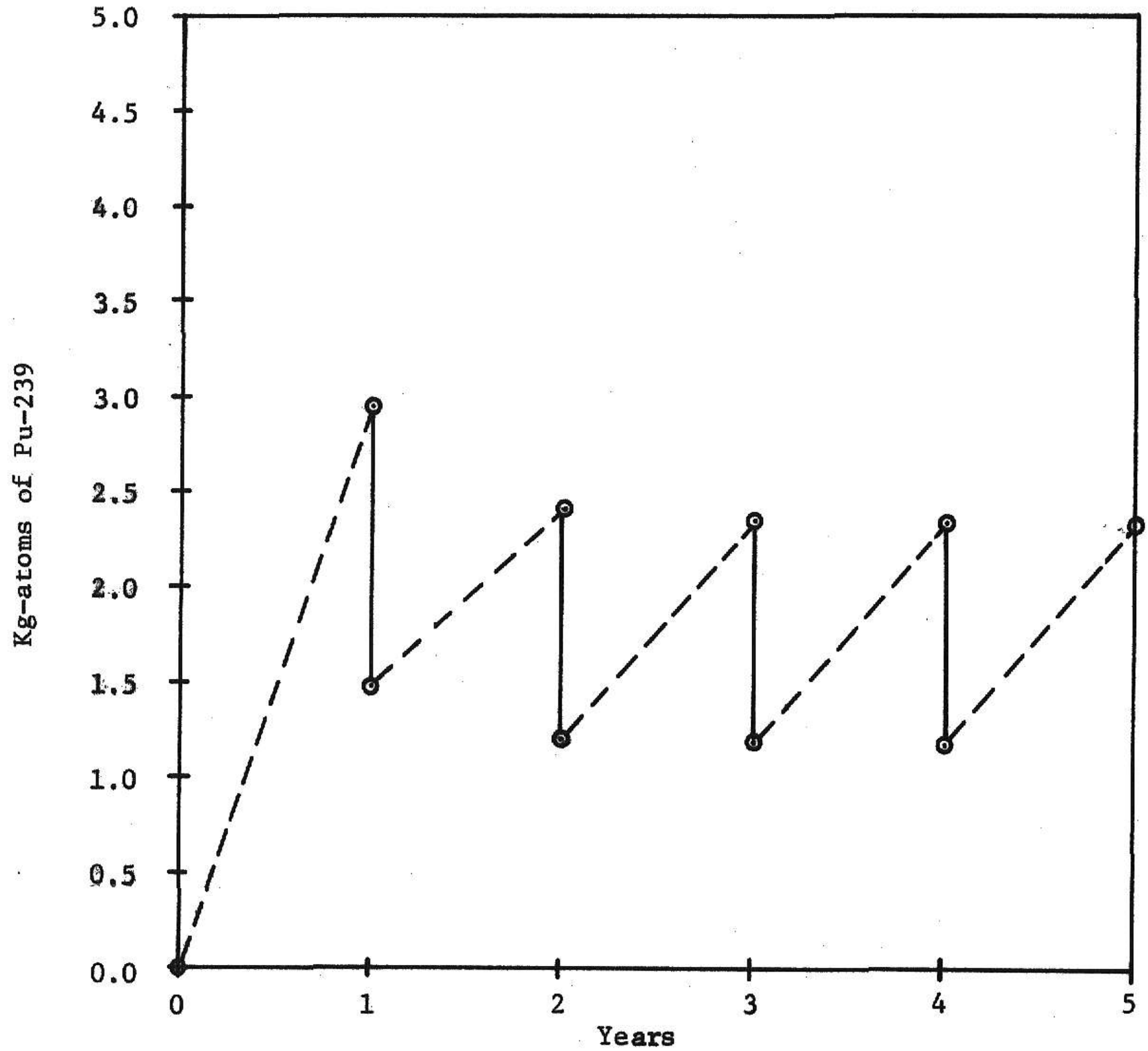

Figure 4.3.6 The amount of Pu-239 in the reactor during the fuel cycle vs the number of irradiation periods for the 50% Pu recycle case.

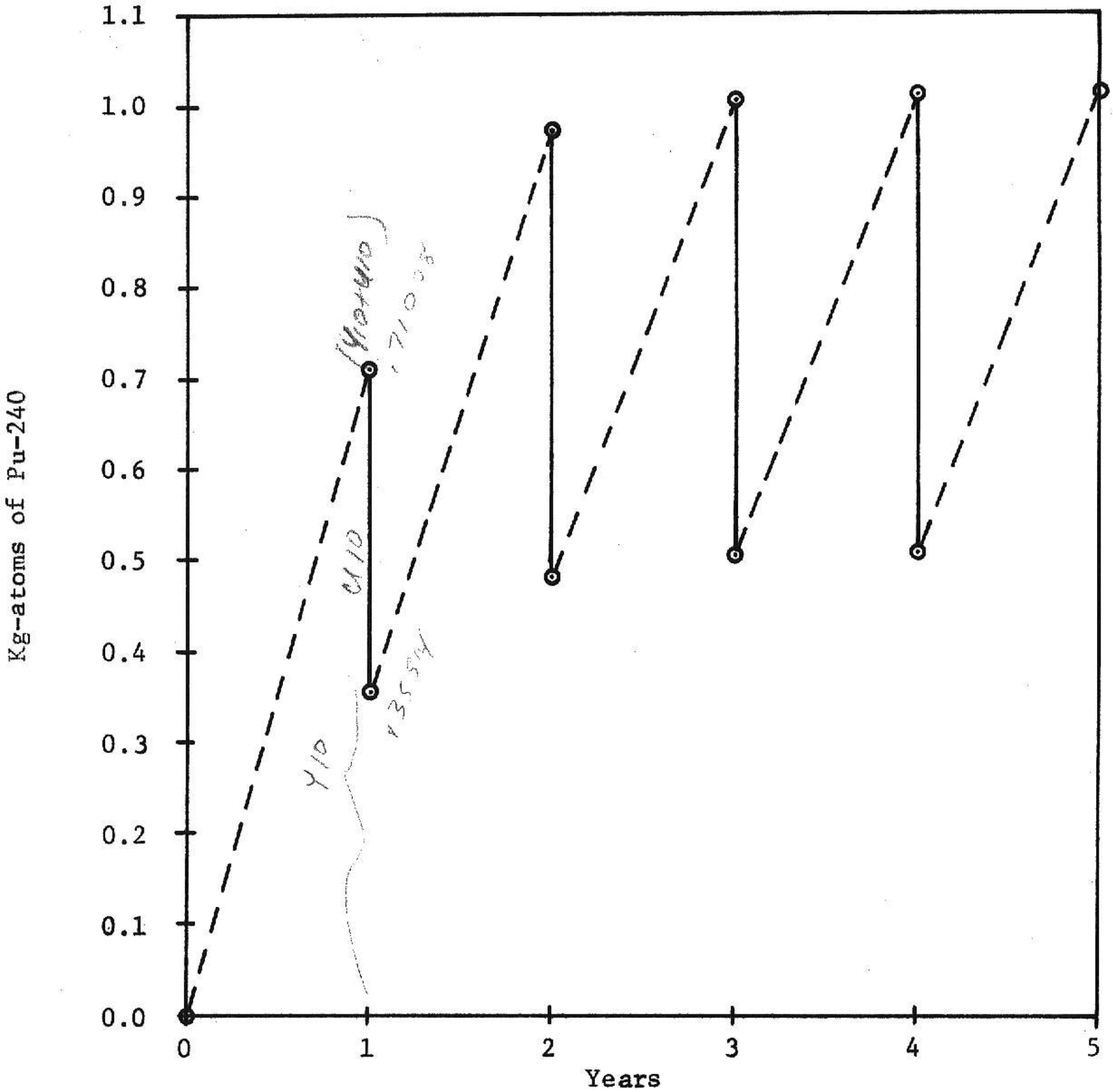

Figure 4.3.7 The amount of Pu-240 in the reactor during the fuel cycle vs the number of irradiation periods for the 50% Pu recycle case.

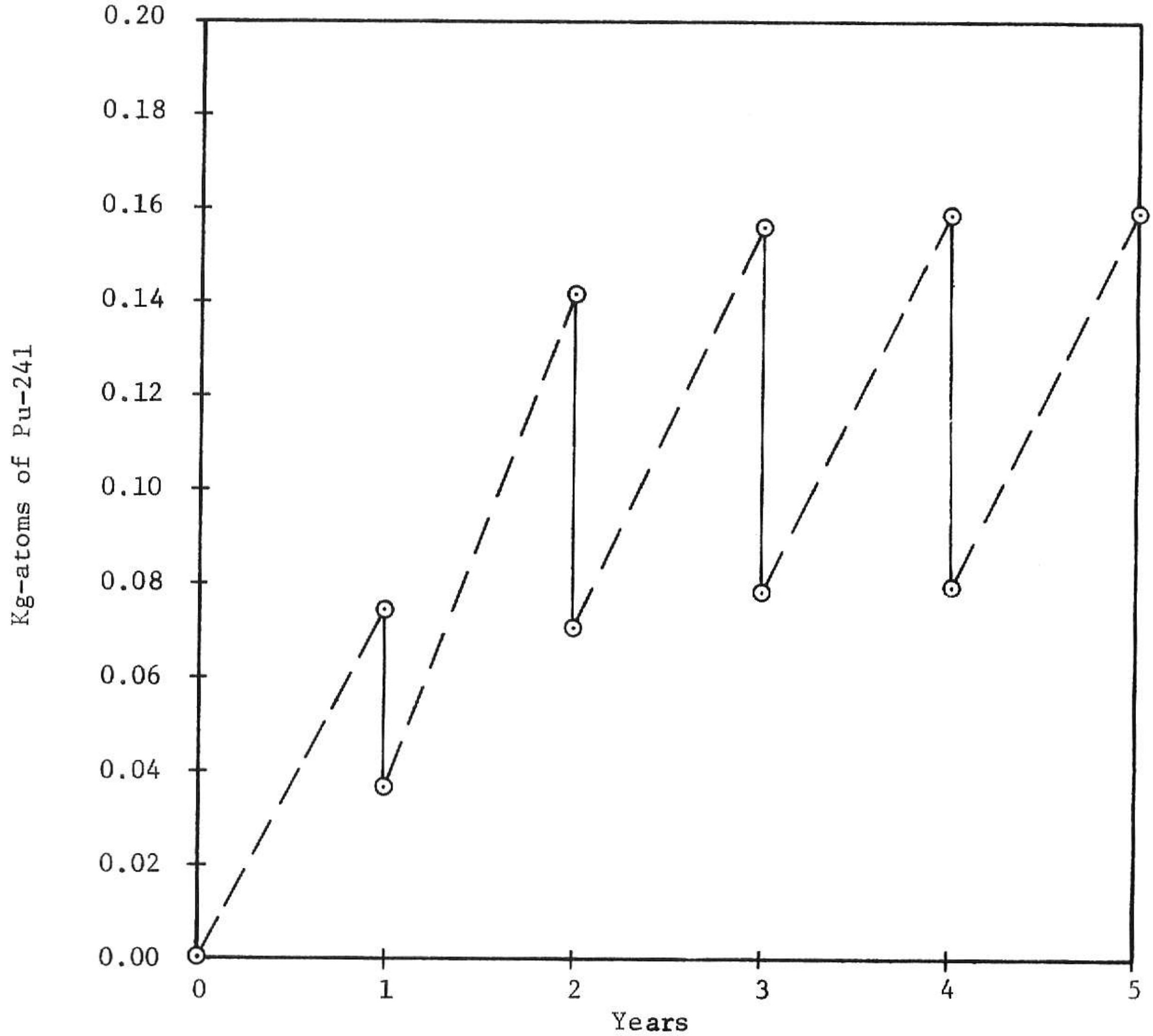

Figure 4.3.8 The amount of Pu-241 in the reactor during the fuel cycle vs the number of irradiation periods for the 50% Pu recycle case.

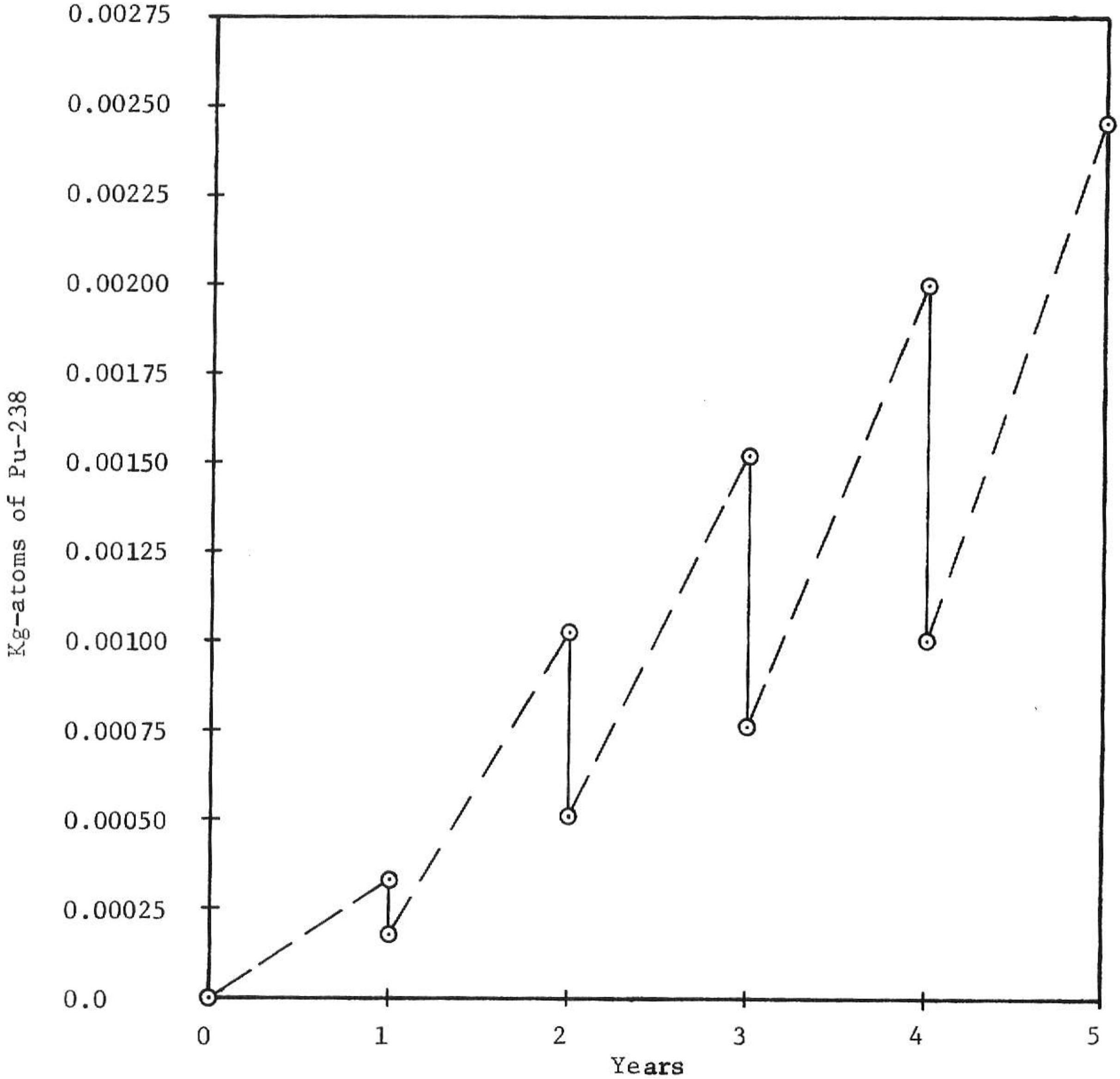

Figure 4.3.9 The amount of Pu-238 in the reactor during the fuel cycle vs the number of irradiation periods for the 50% Pu recycle case.

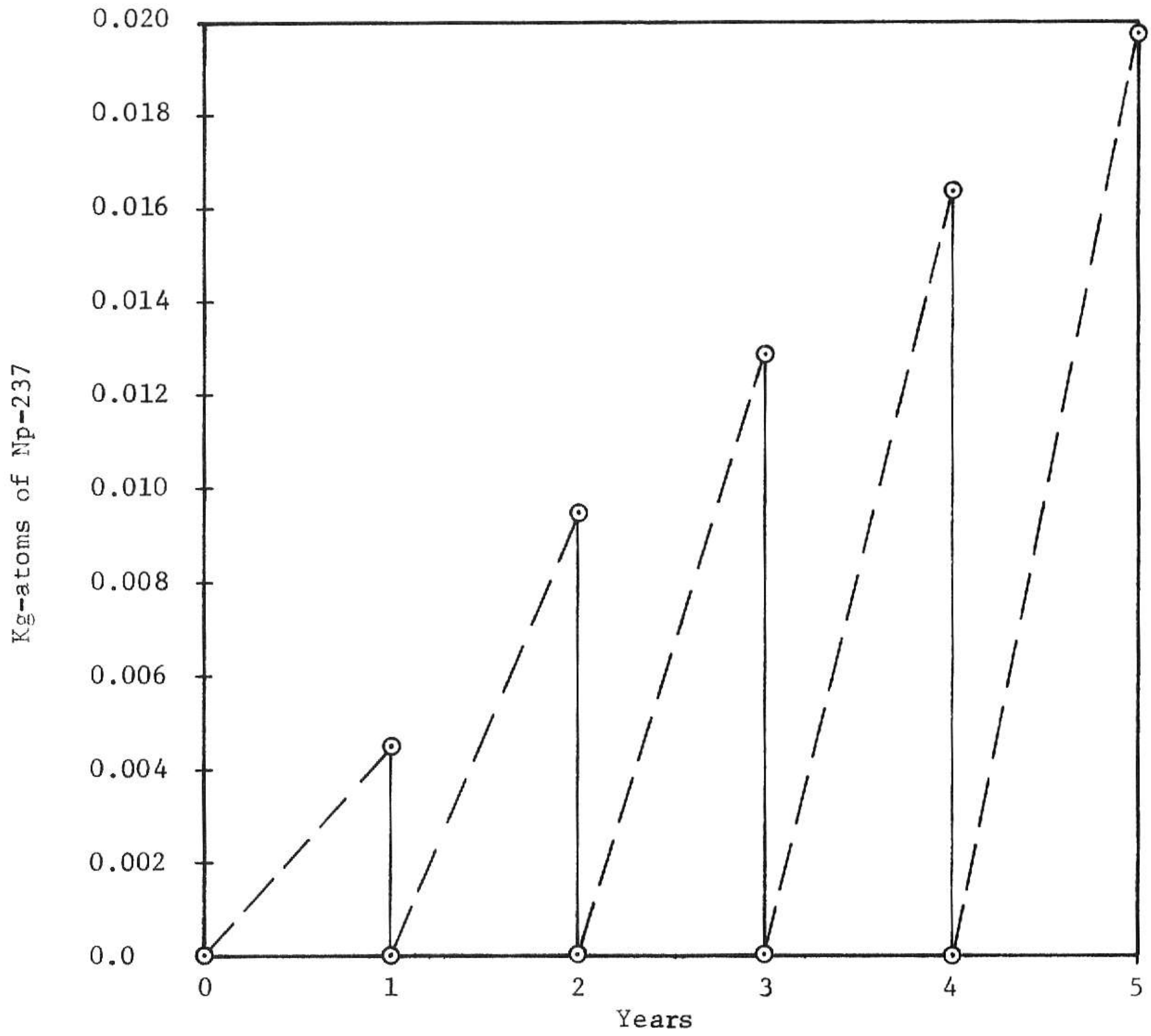

Figure 4.3.10 The amount of Np-237 in the reactor during the fuel cycle vs the number of irradiation periods for the 50% Pu recycle case.

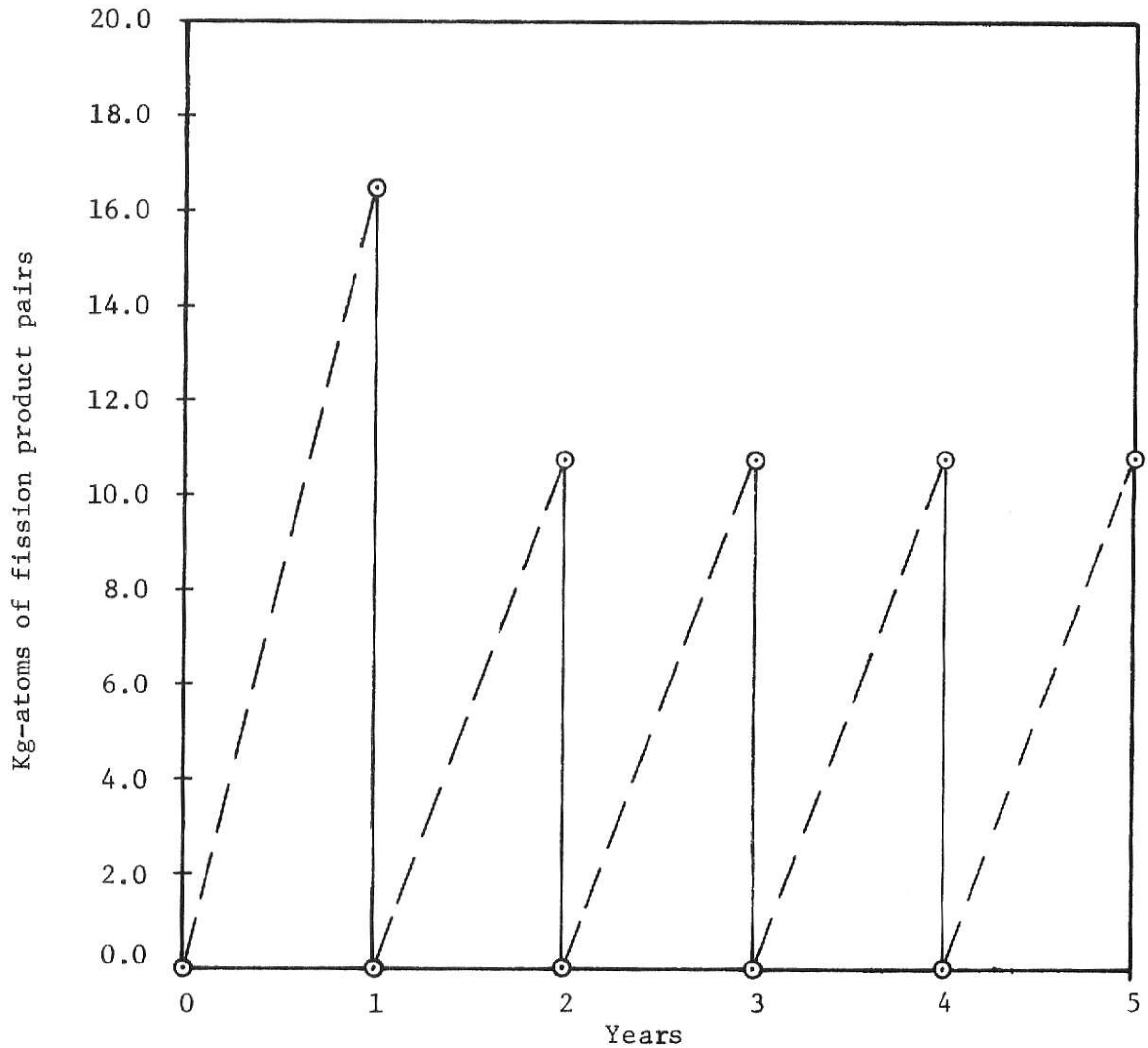

Figure 4.3.11 The amount of fission product pairs in the reactor during the fuel cycle vs the number of irradiation periods for the 50% Pu recycle case.

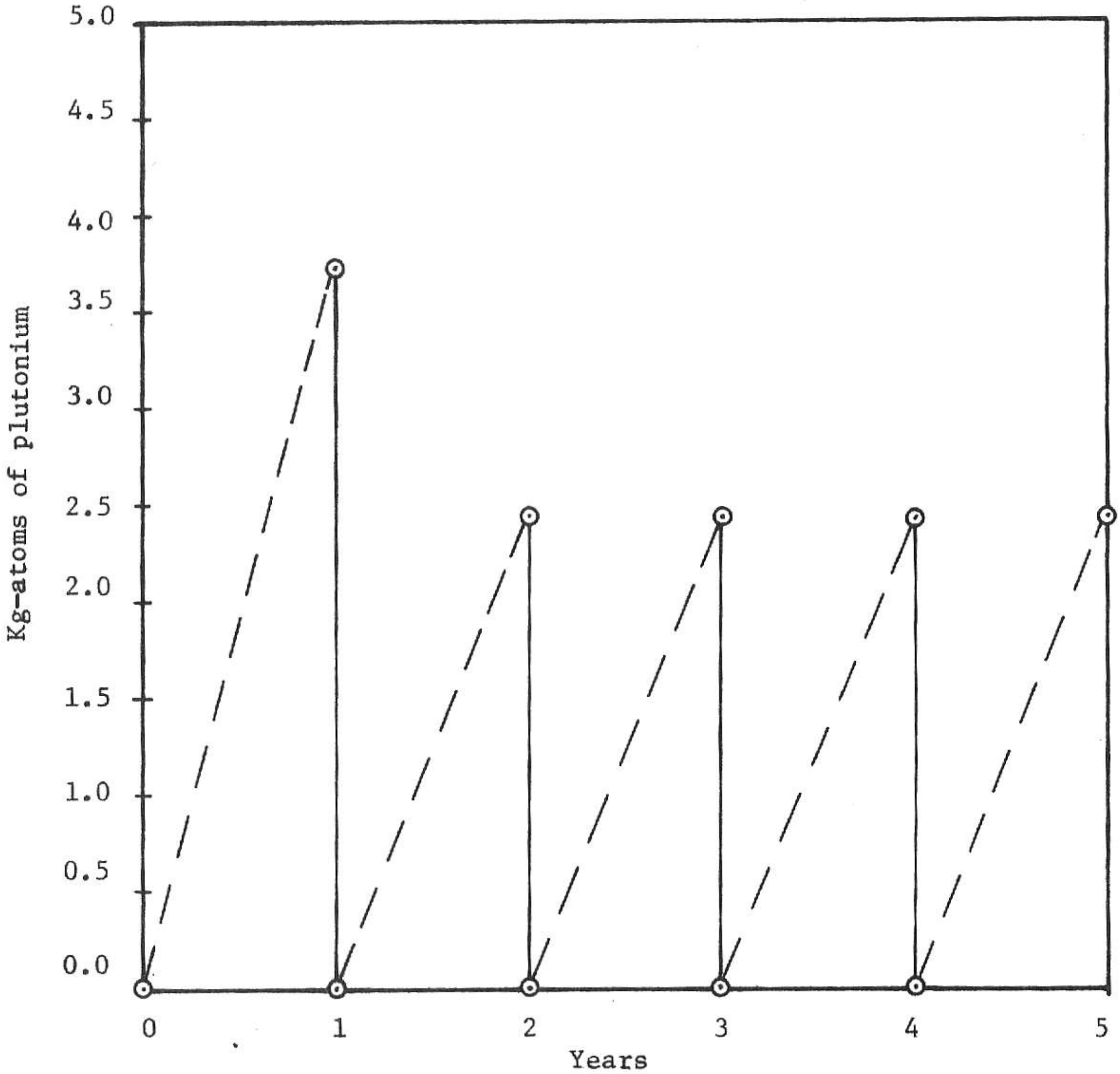

Figure 4.3.12 The total amount of plutonium in the reactor during the fuel cycle vs the number of irradiation periods for the 0% Pu recycle case.

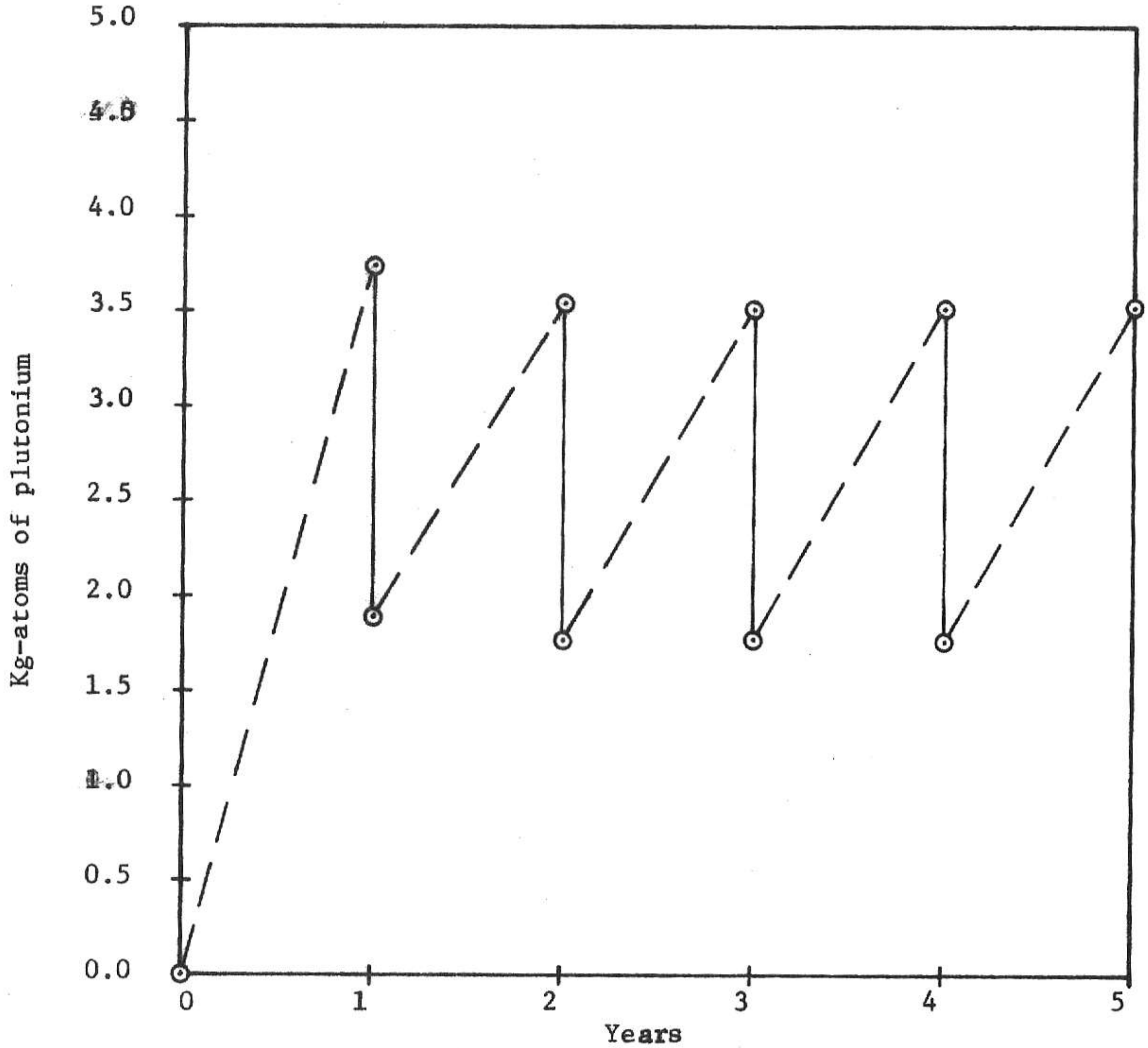

Figure 4.3.13 The total amount of plutonium in the reactor during the fuel cycle vs the number of irradiation periods for the 50% Pu recycle case.

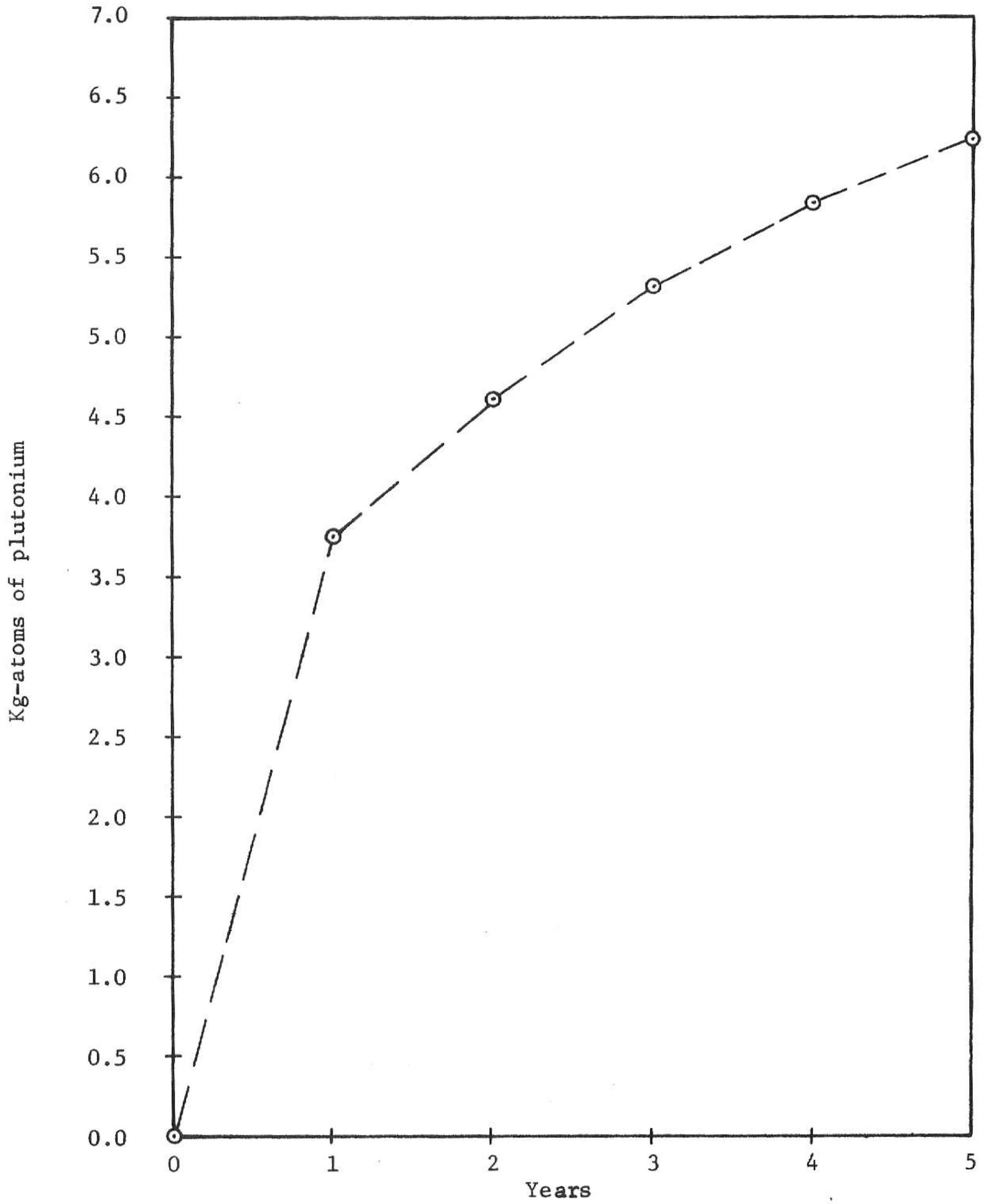

Figure 4.3.14 The total amount of plutonium in the reactor during the fuel cycle vs the number of irradiation periods for the 100% Pu recycle case.

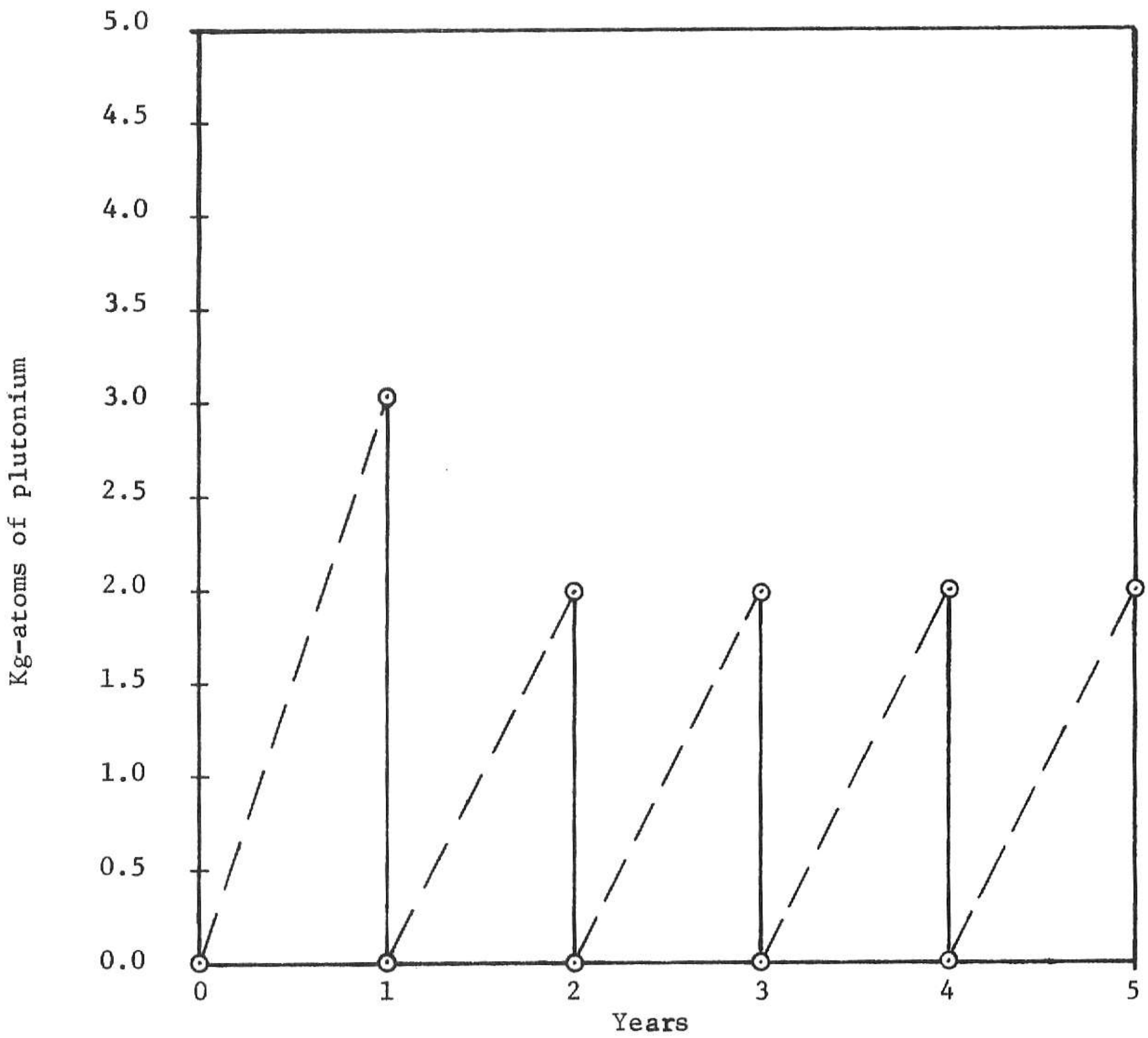

Figure 4.3.15 The amount of fissionable plutonium in the reactor during the fuel cycle vs the number of irradiation periods for the 0% Pu recycle case.

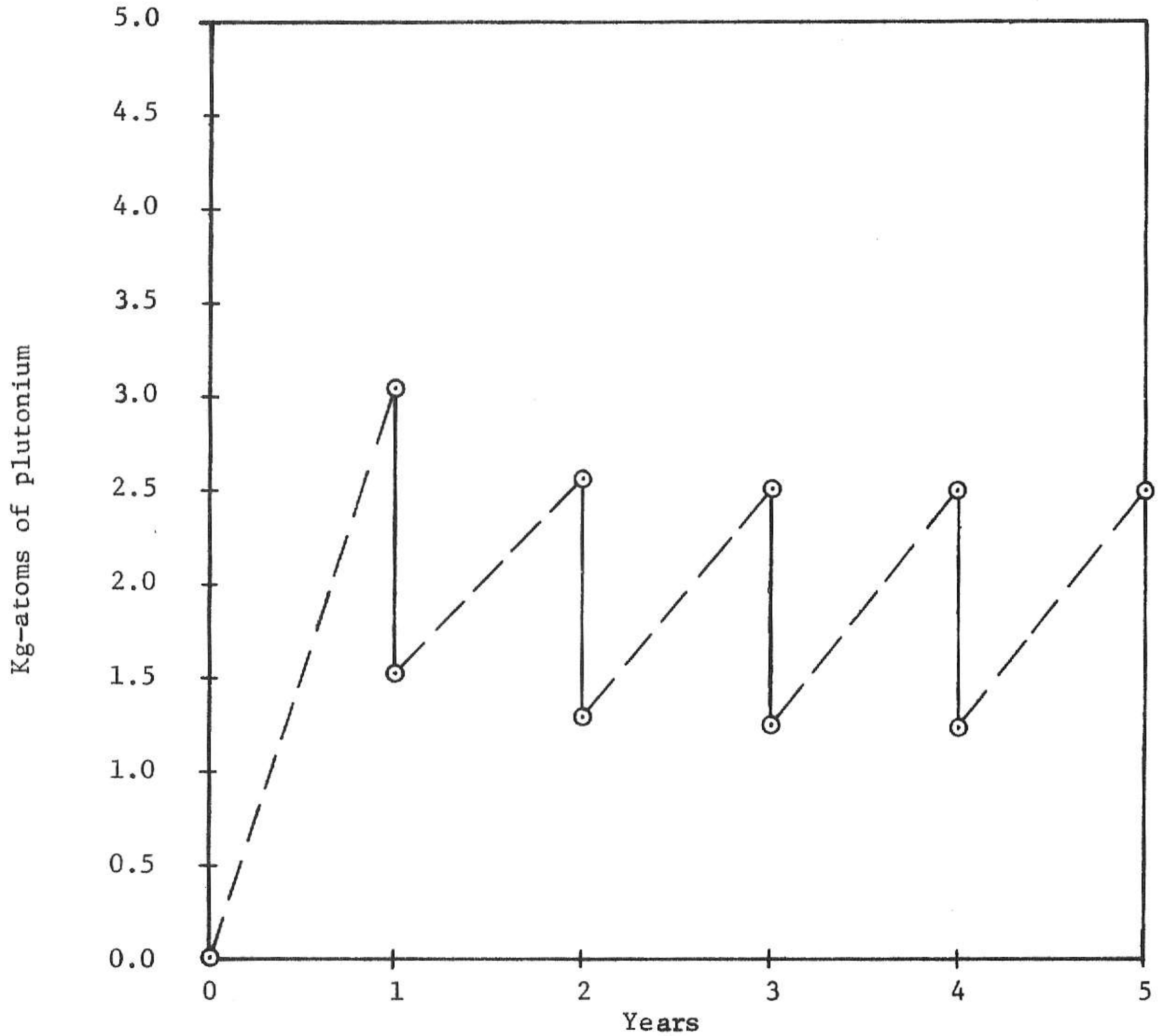

Figure 4.3.16 The amount of fissionable plutonium in the reactor during fuel cycle vs the number of irradiation periods for the 50% Pu recycle case.

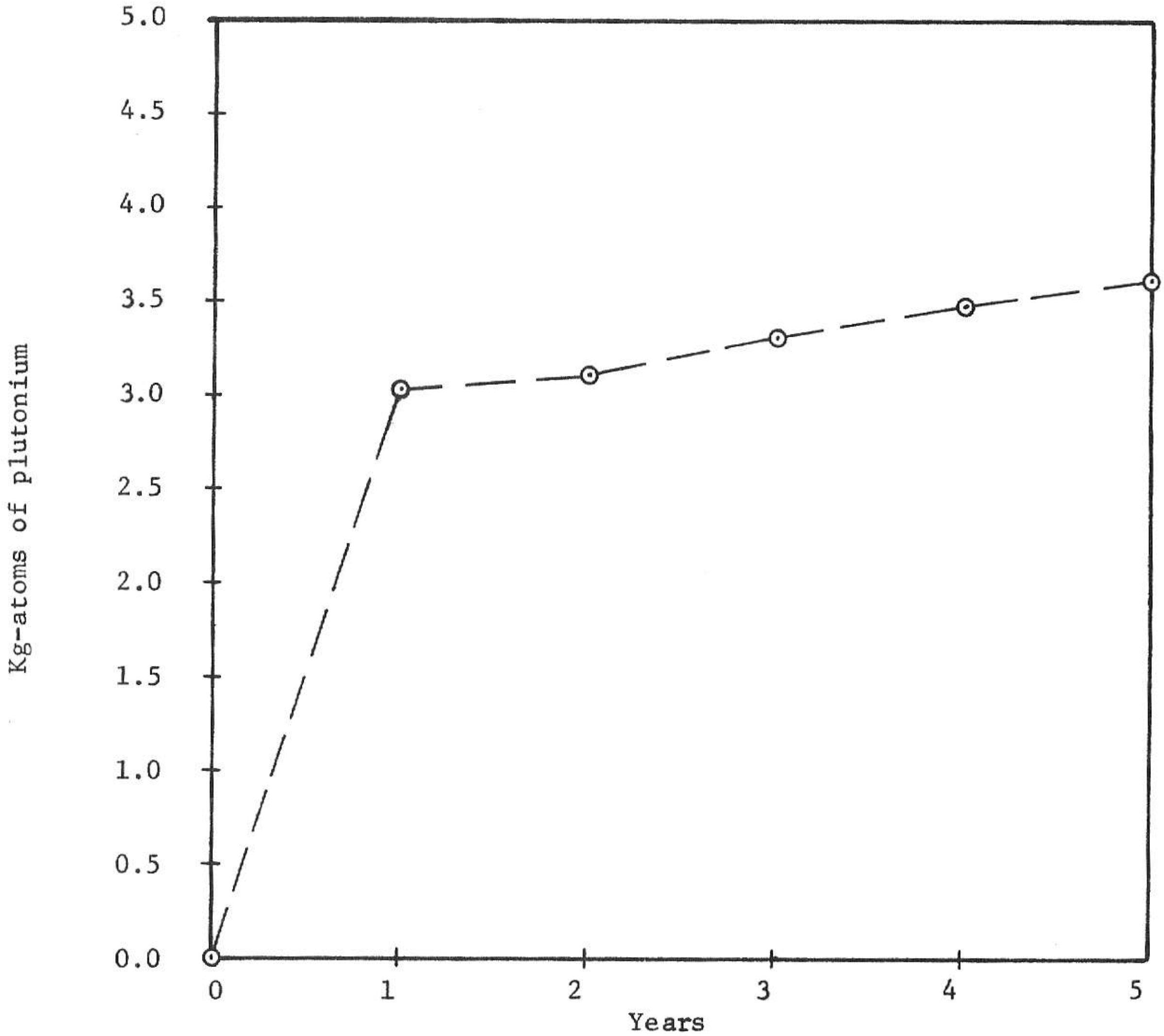

Figure 4.3.17 The amount of fissionable plutonium in the reactor during the fuel cycle vs the number of irradiation periods for the 100% Pu recycle case.

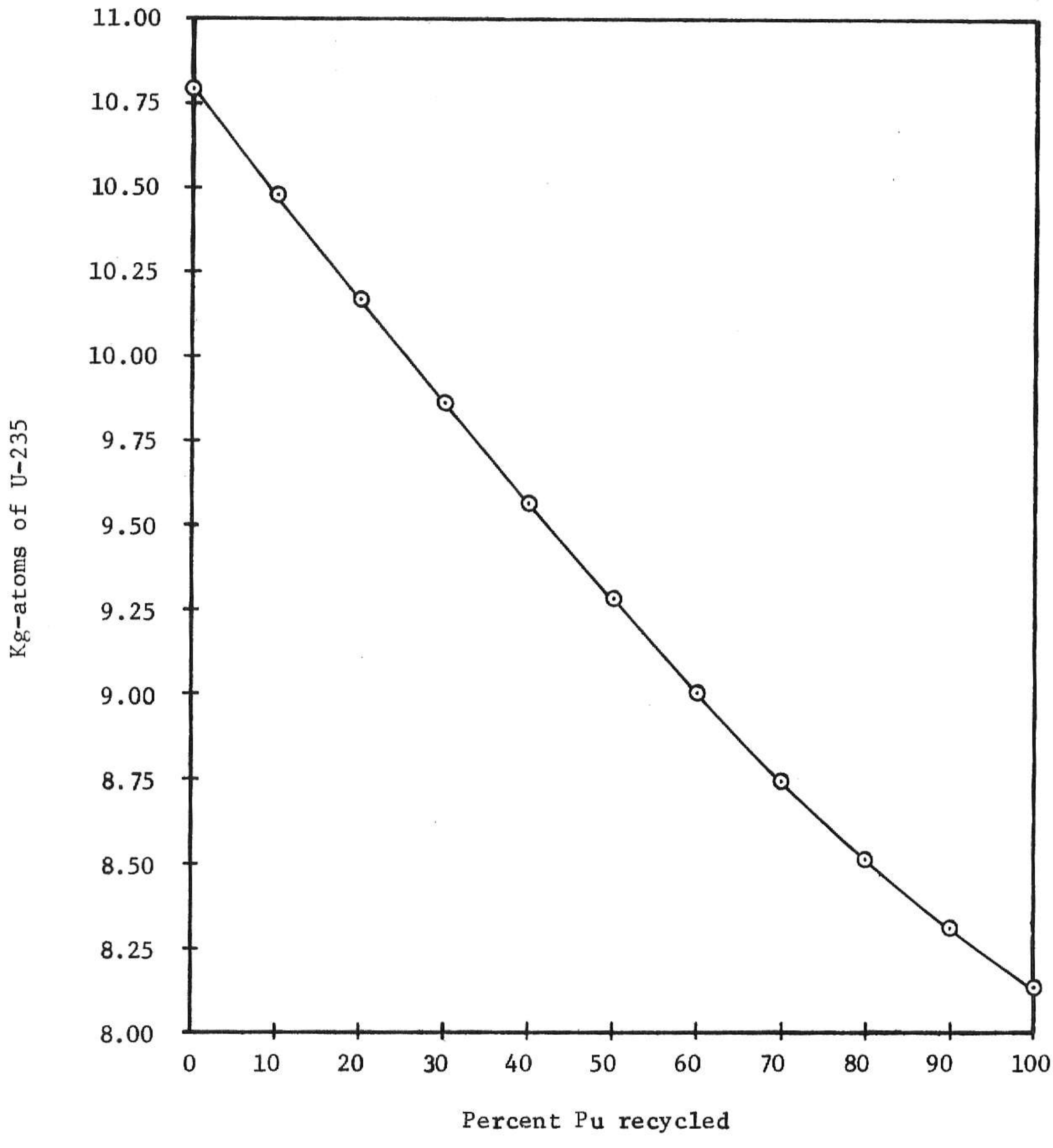

Figure 4.3.18 The value of the objective function vs the percent of Pu recycled.

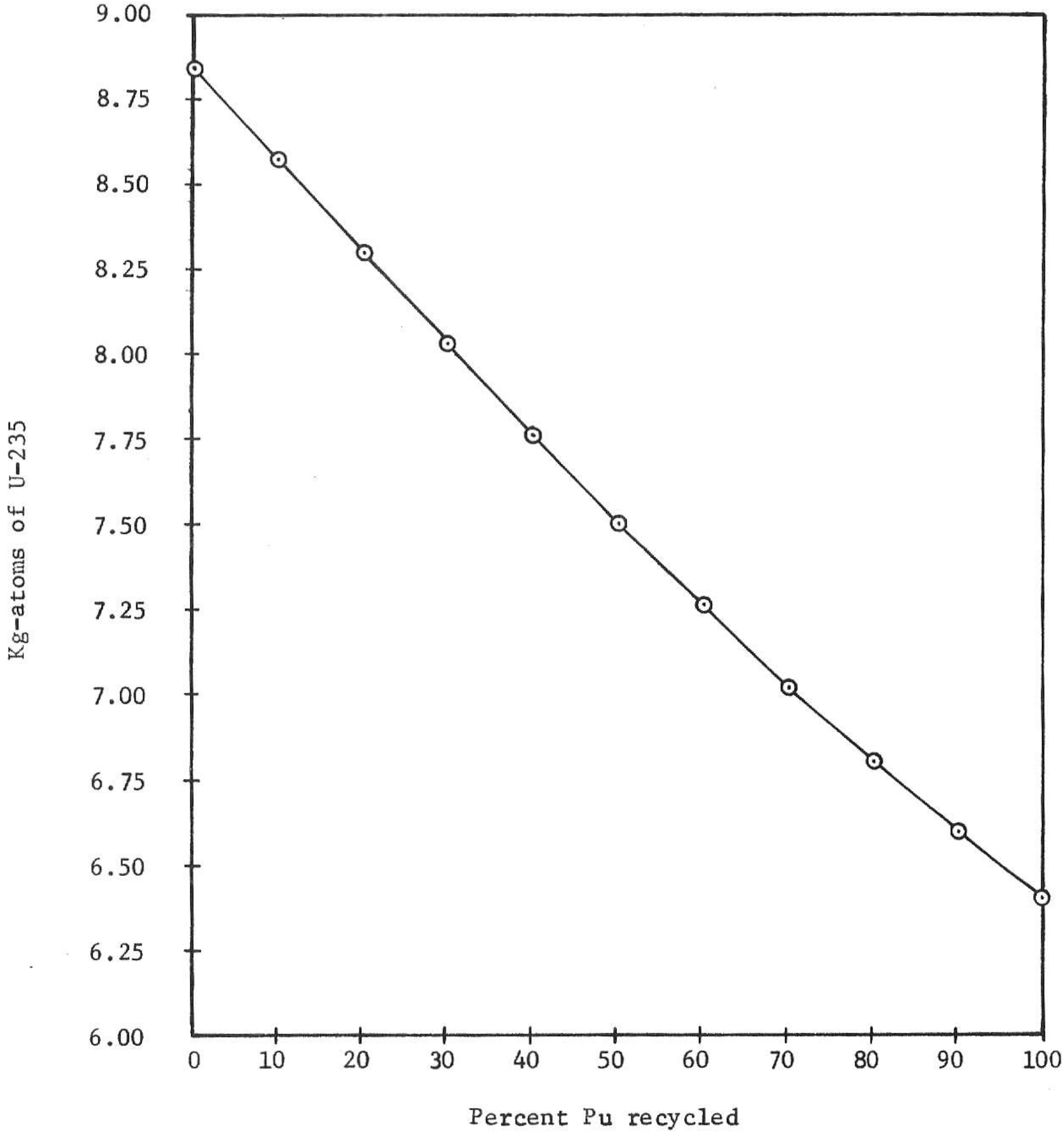

Figure 4.3.19 The amount of U-235 added to the reactor during the fuel cycle vs the percent of Pu recycled.

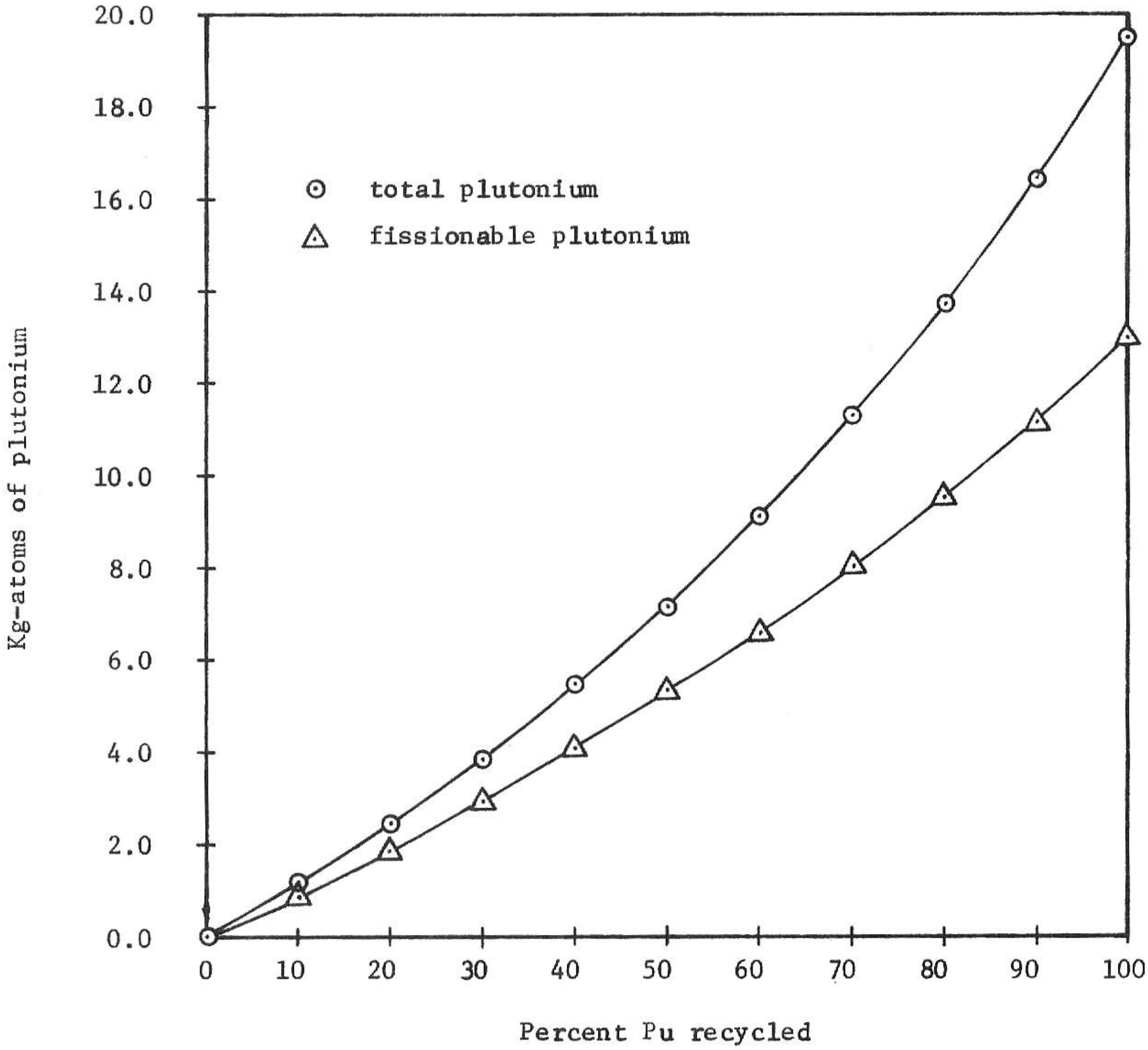

Figure 4.3.20 The amount of plutonium added to the reactor during the fuel cycle vs the percent fo Pu recycled.

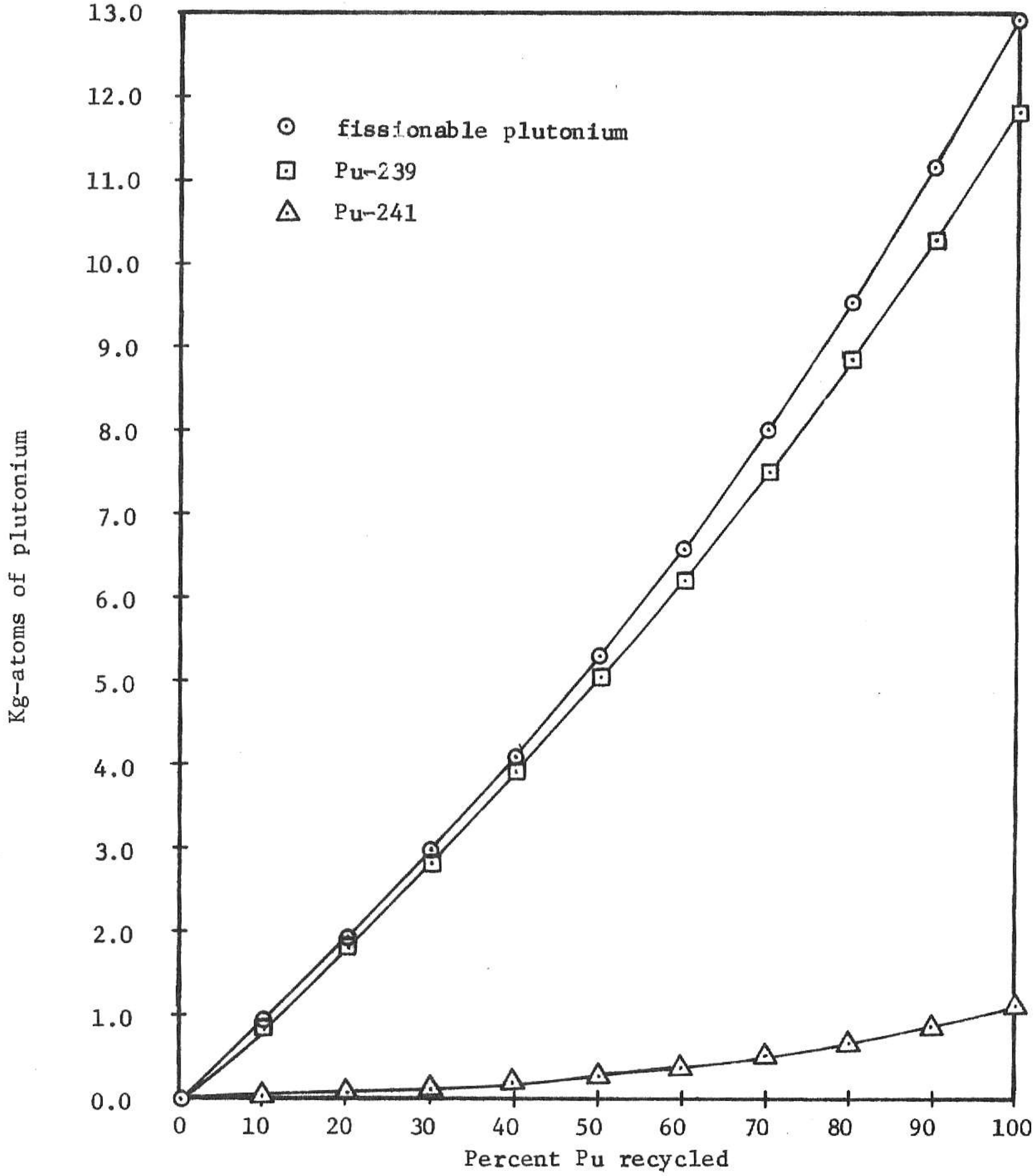

Figure 4.3.21 The amount of fissionable plutonium added to the reactor during the fuel cycle vs the percent of Pu recycled.

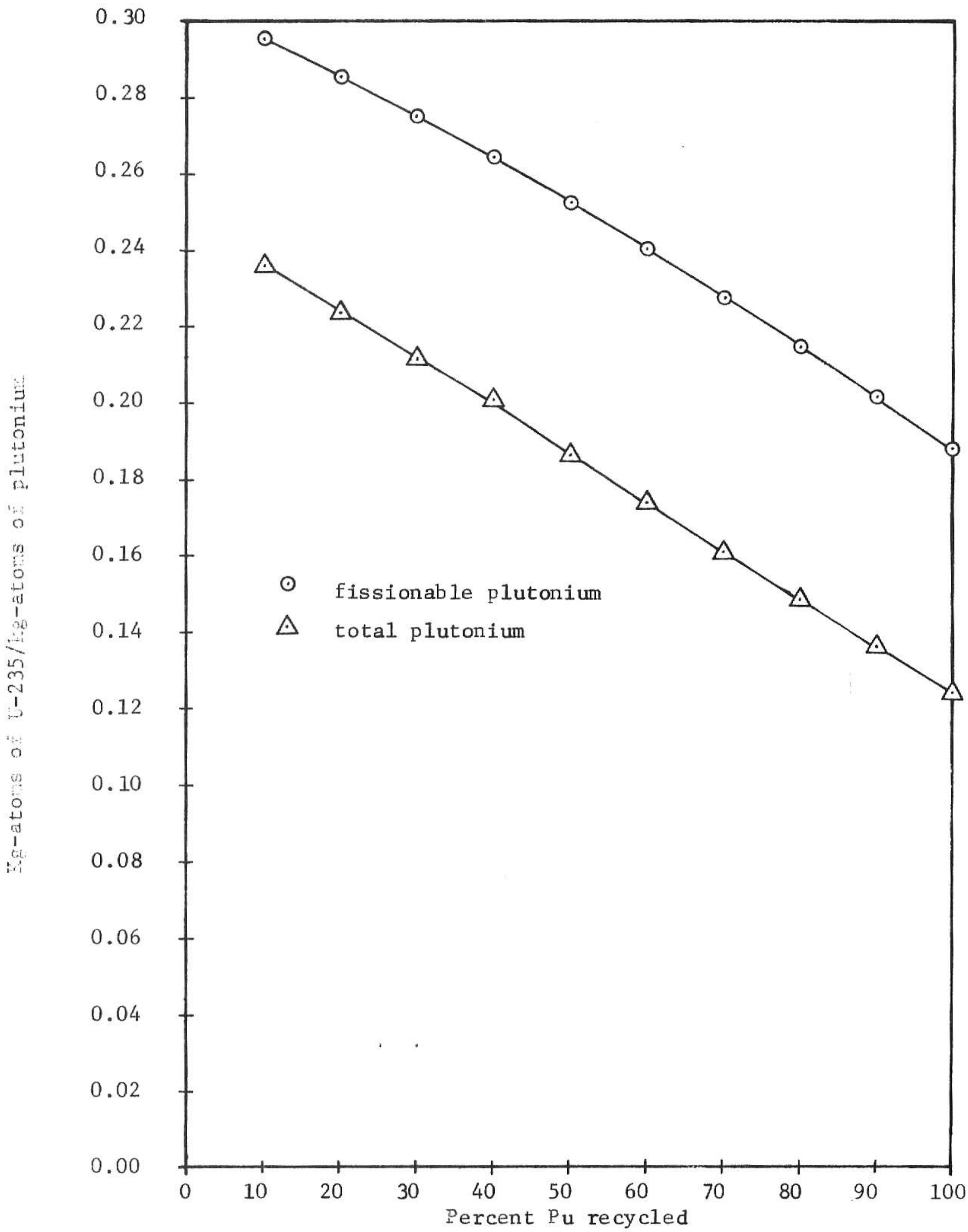

Figure 4.3.22 The ratio of change in the U-235 inventory to change in the Pu inventory vs the percent of Pu recycled.

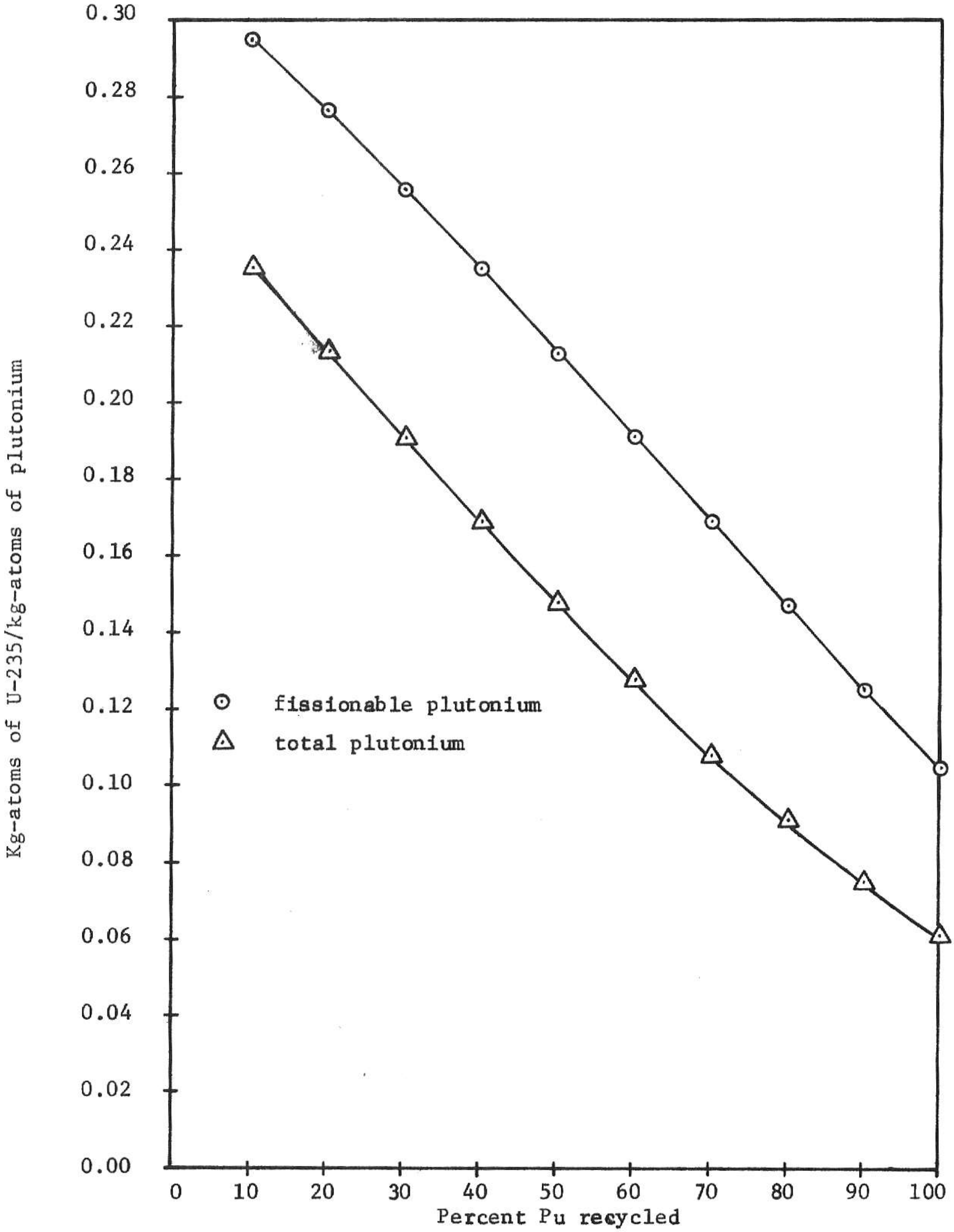

Figure 4.3.23 The ratio of change in the U-235 inventory to the change in the Pu inventory vs the percent of Pu recycled.

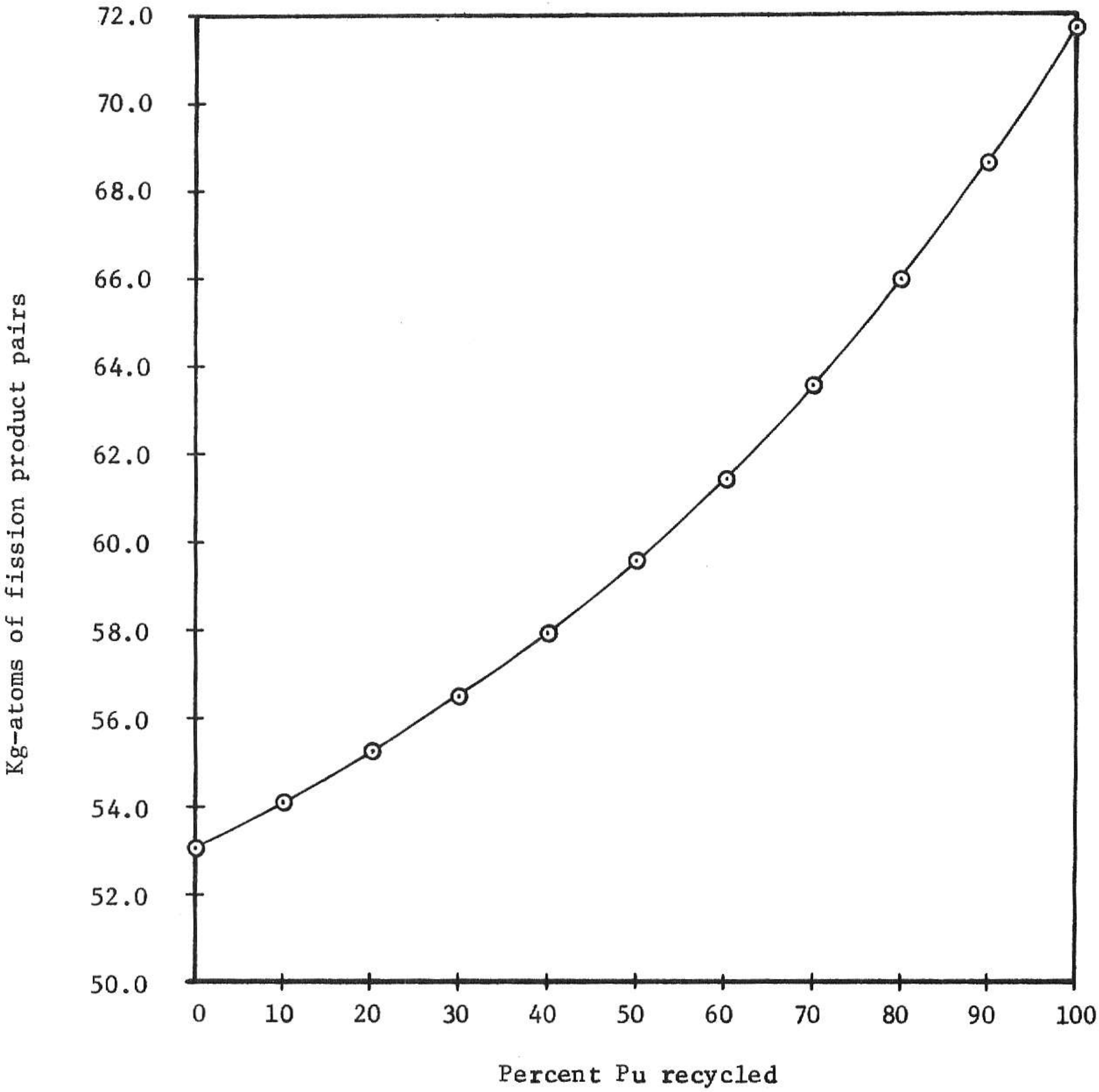

Figure 4.3.24 The amount of fission product pairs removed from the reactor during the fuel cycle vs the percent of Pu recycled.

4.4 Conclusions

As shown in Figs. 4.3.1, 4.3.2, and 4.3.3 the amount of U-235 in the reactor at the start of the second, third, fourth, and fifth irradiation periods is approximately half of the initial loading of U-235. An excess of reactivity is mandatory in the initial loading of a reactor to compensate for build up of fission products and temperature increases, which accounts for the large initial loading of U-235. Also, a result of the assumption of the complete reprocessing of the fuel after each irradiation period is that all fission product pairs are removed. Thus, the amount of U-235 added to the reactor during refueling will be less than the amount of U-235 required when only a fraction of the core is processed. This is particularly evident for refueling at the end of irradiation period one.

The nuclides U-236, Pu-239, Pu-240, Pu-241, and the fission product pairs are dependent on U-235, as shown in Section 3.2, and show a marked increase in production during the first irradiation period. This increase is attributed to the large value of U-235 during that irradiation period. U-238 which is independent of U-235 is used at a constant rate as shown in Fig. 4.3.5. It is also noted that no U-238 is added to the reactor, which is allowed because there is no constraint of minimum enrichment for uranium added.

Figures 4.3.12 through 4.3.17 show the build up of plutonium in the reactor for different percent Pu recycle cases. These figures illustrate the decrease in the removal of plutonium and the increase in the build up of plutonium as the percent Pu recycle increases.

From Fig. 4.3.18 it can be observed that the value of the objective function, i.e., the U-235 requirements, decreases from 10.78938 kg-atoms for 0% Pu recycle to 8.12989 kg-atoms for 100% Pu recycle. The amount of U-235 being added to the reactor during the fuel cycle also decreases from

8.84609 kg-atoms for 0% Pu recycle to 6.40865 kg-atoms for 100% Pu recycle. A 3% minimum enrichment contraint for 0% Pu recycle yields a value for the objective function of 20.05444 kg-atoms and a value of 16.89862 kg-atoms for the amount of U-235 added; almost double the values given for no minimum enrichment constraint. The minimum enrichment of 3% is evidently much larger than is needed to maintain the reactor operation under the assumptions used for this reactor model.

As illustrated in Fig. 4.3.20 the amount of plutonium added to the reactor during the fuel cycle increases from 0 kg-atoms for 0% Pu recycle to 19.51697 kg-atoms for 100% Pu recycle, while the amount of fissionable plutonium added to the reactor increases from 0 kg-atoms for 0% Pu recyle to 12.94503 kg-atoms for 100% Pu recycle. (All of this plutonium is produced by the reactor.) Figure 4.3.21 shows the individual contributions of Pu-239 and Pu-241 to the fissionable plutonium curve. For 100% Pu recycle there are 1.12469 kg-atoms of Pu-241 and 11.82034 kg-atoms of Pu-239 of which 11.81882 kg-atoms are produced from thermal absorption in U-238 and .00152 kg-atoms are produced from thermal absorption in U-235. In every case considered, the majority of fissionable plutonium produced was Pu-239. More than 99.9% of this Pu-239 was produced from thermal absorption in U-238.

Pu-239 increases in a non linear fashion because of the I^0_{98}, i.e., the amount of Pu-239 resulting from thermal absorption in U-238 in the previous irradiation period, term in Eq. 3.2.16. I^0_{98} will vary depending on the percent of Pu recycle. The other plutonium isotopes considered have a similar dependency on the value of the isotope in the previous period. The constraints of the model allow for plutonium to be used as a fissile material, thus reducing the demand for U-235. Therefore, the amount of plutonium increases non-linearly. Hence, the amount of U-235 decreases non-linearly. A plot of the ratio of change in the U-235 inventory to the change in the plutonium inventory versus the percent of Pu recycled amplifies these non-linearities. Figures 4.3.22 and 4.3.23 illustrate these results. In Fig. 4.3.22 the ratio decreases from .29536 kg-atom U-235/ kg-atom fissionable plutonium for 10% Pu recycle to .18829 kg-atom U-235/ kg-atom fissionable plutonium for 100% Pu recycle. Also in Fig. 4.3.22, when the total amount of plutonium is considered, the ratio decreases from .23615 kg-atom U-235/ kg-atom plutonium for 10% Pu recycle to .12489 kg-atom U-235/ kg-atom plutonium for 100% Pu recycle. In Fig. 4.3.23 the curves have a greater rate of decrease: from .29536 kg-atom U-235/ kg-atom fissionable plutonium to .10408 kg-atom U-235/ kg-atom fissionable plutonium and from .23615 kg-atom U-235/ kg-atom plutonium to .06057 kg-atom U-235/ kg-atom plutonium for 10% and 100% Pu recycle, respectively. The negative slope of these curves is attributed to the fact that the U-235 inventory decreases while the plutonium inventory increases, as shown in Figs. 4.3.19 and 4.3.20. The increasing negative slope of the change in the required U-235 ratioed to the change in plutonium used shown in Fig. 4.3.23 and is explained as follows. The rate of decrease in the U-235 inventory is greater as the percent of Pu recycle increases. Also, the rate of increase in the plutonium inventory is greater as the percent of Pu recycle increases.

This ratio is important in relating the value of plutonium to the cost of U-235. The value of U-235 is a fixed and known quantity, while the value of plutonium has not been firmly established. Figures 4.3.22 and 4.3.23 illustrate the result that the value of plutonium is a function of the percent of Pu recycle considered. Voila: Its value decreases as the percent of Pu recycle increases.

In summary, the results presented above indicate that this model is a fair representation of a power reactor refueling scheme. It can be made more complex and accurate by reevaluating the assumptions and adding more constraints. This model is quite versatile in the manner in which many different studies can be made of the same reactor. For example, by changing the objective function, it is possible to maximize the amount of plutonium removed, or to maximize the plutonium removed while minimizing the U-235 requirements. Thus, the model's versatility and availability make it an attractive addition to the already existing refueling codes.

5.0 SUGGESTIONS FOR FURTHER STUDY

Further study could be completed on this model by comparing different refueling schemes, such as comparing a four thirds or five fourths core refueling scheme with an unspecified fractional core refueling philosophy (7). Another study could be to divide the core into three or more zones, thus a flux profile could be simulated in the reactor. This model could be combined with a fuel element shuffling technique in such a manner as to resemble the actual refueling process (8). Another study may be to compare the results of the burnup code used in this work to more complex codes such as FEVER (2).

Other studies may be completed by including a fuel enrichment constraint for the addition of uranium to the reactor, or by considering the reactivity holddown requirements for a plutonium recycle reactor (1).

6.0 ACKNOWLEDGEMENTS

I wish to express my gratitude to Dr. N. Dean Eckhoff whose encouragement and direction made this work possible. Also, the financial support of the Department of Nuclear Engineering, and the National Science Foundation Traineeship program is greatly appreciated. A note of thanks is extended to Dr. R. E. Faw, Professor, Department of Nuclear Engineering, and Dr. L. E. Grosh, Professor, Department of Industrial Engineering for their suggested improvements.

A special thank you is extended to Maureen Beaudet for her assistance in typing this manuscript, and Donald Prigel for his assistance in drawing the figures. Finally, I wish to express my love and appreciation to my wife, Linda, for the long hours spent typing this thesis and her continued support and encouragement.

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8.0 APPENDICES

APPENDIX A

A.1 Nuclide Burnup Code

This code was written so that the constants for the burnup and production equations listed in Section 3.2 could be calculated using the assumptions stated in Section 3.2. The code is similar to one written by Hawk (7), with the exception that this code includes the nuclides Np-237, Pu-238, and Pu-239 produced from thermal absorption in U-235. The input data are included in the code and comprize the initial amounts of each nuclide, the absorption, fission, and capture cross sections for each nuclide, and ε, P_1, p, η_5, η_9, θ.

The code is written for the IBM 360 Model 50 computer and requires approximately .23 seconds execution time.

A.2 Nuclide Burnup Code Listing

THIS BOOK WAS BOUND WITH MULTIPLE PAGES THAT HAVE PRINTING EXTENDING INTO THE BINDING, CAUSING THE PRINTING TO BE CUT OFF.

THIS IS AS RECEIVED FROM THE CUSTOMER.

```
$JOB
      Y05= 10.6383
      Y06=  .00
      Y08= 297.5375
      Y09=  .00
      Y095= 0.0
      Y00=  .00
      Y01=  .00
      Y07=  .00
      Y0N7= 0.0
      Y0P8= 0.0
      ANU5=2.47
      ANU8=2.55
      ANU9=2.905
      ANU1=3.060
      SIGS= .25148
      SA5= .678
      SA6= .0055
      SA8= .00273
      SA9= 1.013
      SA0= .280
      SA1= 1.364
      SAF= .0319
      SAP8= .540
      SAN7= .169
      SF5= .580
      SF8= .000
      SF9= .742
      SF1= 1.0
      SC5= .098
      SC8= .00273
      SC9= .271
      SC0= .280
      SCN7= .169
      SC6= .0055
      SCP8= .540
      EPS = 1.0584
      P1 = .9742
      P = .758
      ETA5 = 1.97
      ETA9 = 1.81
      T= 1.5552
      R11=EXP(-SA5*T)
      R21=SC5/(SA6-SA5)*(EXP(-SA5*T)-EXP(-SA6*T))
      R22=EXP(-SA6*T)
      R33=1.0
      C9 = EPS*P1*(1.0-P)*SA9*ETA9
      C5 = EPS*P1*(1.0-P)*SA5*ETA5
      C91 = C9 - SA9
      R41=(C5/(SA5+C91))*(EXP(C91*T)-EXP(-SA5*T))
      R44=EXP(C91*T)
      C8=-SC8/C91
      R40=C8*(1.0-EXP(C91*T))
      A=(C91+SA0)
      B=(SA5+C91)
      C=(SA5-SA0)
      R51=(C5*SC9/(A*B))*(EXP(C91*T)-EXP(-SA0*T))+(C5*SC9/(B*C))
     1*(EXP(-SA5*T)-EXP(-SA0*T))
      R54=SC9*(EXP(C91*T)-EXP(-SA0*T))/A
      R55=EXP(-SA0*T)
```

```
      R50=C8*SC9*(1.0-EXP(-SA0*T))/SA0+C8*SC9*(EXP(-SA0*T)-EXP(C91*T))/A
      D=(SA1-SA0)
      E=(C91+SA1)
      F=(SA1-SA5)
      R61=SC0*SC9*C5*(EXP(C91*T)/(E*A*B)+EXP(-SA5*T)/(F*C*B))
     1-(SC0*SC9*C5*EXP(-SA0*T)/(D*B))*(1.0/A+1.0/C)-(SC0*SC9*C5
     2*EXP(-SA1*T)/B)*(1.0/(E*A)+1.0/(F*C)-1.0/(D*A)-1.0/(D*C))
      R64=SC0*SC9*EXP(C91*T)/(E*A)-SC0*SC9*EXP(-SA0*T)/(D*A)
     1-(SC0*SC9*EXP(-SA1*T)/A)*(1.0/E-1.0/D)
      R65=(SC0/D)*(EXP(-SA0*T)-EXP(-SA1*T))
      R66=EXP(-SA1*T)
      R60=SC0*C8*SC9*(1.0-EXP(-SA1*T))/(SA1*SA0)-SC0*C8*SC9*(EXP(C91*T)
     1-EXP(-SA1*T))/(A*E)+SC0*C8*SC9*(EXP(-SA0*T)-EXP(-SA1*T))*(1.0/(A*
     2D)-1.0/(SA0*D))
      R71=2.0*C5*(EXP(C91*T)-1.0)/(C91*B)*(SF9+SF1*SC0*SC9/(E*A))
     1+(2.0*(EXP(-SA5*T)-1.0)/SA5)*(SF9*C5/B-SF5-SF1*SC0*SC9*C5
     2/(F*C*B))+(2.0*SF1*SC0*SC9*C5*(EXP(-SA0*T)-1.0)/(SA0*D*B))*(1.0
     3/C+1.0/A)+(2.0*SF1*C5*SC0*SC9*(EXP(-SA1*T)-1.0)/(SA1*B))
     4*(1.0/(A*E)+1.0/(F*C))-(2.0*SF1*SC0*SC9*C5*(EXP(-SA1*T)-1.0)/
     5(SA1*B*D))*(1.0/A+1.0/C)
      R74=(2.0*SF9*(EXP(C91*T)-1.0)/C91)*(1.0+SC0*SC9/(A*E))
     1+2.0*SF1*SC0*SC9*(EXP(-SA0*T)-1.0)/(SA0*D*A)+(2.0*SF1*SC0*SC9*
     2(EXP(-SA1*T)-1.0)/(SA1*A))*(1.0/E-1.0/D)
      R75=2.0*SF1*SC0*(EXP(-SA1*T)-1.0)/(SA1*D)-2.0*SF1*SC0*
     1(EXP(-SA0*T)-1.0)/(SA0*D)
      R76=-2.0*SF1*(EXP(-SA1*T)-1.0)/SA1
      BB=(EXP(-SA0*T)-1.0)
      CC=(EXP(-SA1*T)-1.0)
      R70=2.0*C8*(SF9+SF1*SC0*SC9/(SA0*SA1))*T-2.0*C8*(EXP(C91*T)-1.0)
     1*(SF9/C91+SF1*SC0*SC9/(C91*A*E))+(2.0*SF1*SC0*SC9*C8/(SA0*D))
     2*(1.0/SA0-1.0/A)*BB+(2.0*SF1*C8*SC0*SC9/SA1)*(1.0/(SA1*SA0)
     3-1.0/(A*E))*CC+(2.0*SF1*SC0*SC9*C8/(SA1*D))*(1.0/A-1.0/SA0)*CC
      DD=EXP(-SA5*T)-EXP(-SCN7*T)
      EE=EXP(-SCN7*T)-EXP(-SA6*T)
      G=SCN7-SA5
      Q=SCN7-SA6
      V=SA6-SA5
      R91=SC6*SC5*(DD/G+EE/Q)/(SA6-SA5)
      R92=SC6*(-EE)/Q
      R93=EXP(-SCN7*T)
      FF=EXP(-SA5*T)-EXP(-SCP8*T)
      GG=EXP(-SA6*T)-EXP(-SCP8*T)
      QQ=EXP(-SCN7*T)-EXP(-SCP8*T)
      R=SCP8-SA5
      S=SCP8-SA6
      U=SCP8-SCN7
      R81=SCN7*SC6*SC5*FF/(R*G*V)-SCN7*SC6*SC5*GG/(S*Q*V)
     1+SCN7*SC6*SC5*QQ*(1.0/Q-1.0/G)/(U*V)
      R82=SCN7*SC6*(GG/S-QQ/U)/Q
      R83=SCN7*QQ/U
      R84=EXP(-SCP8*T)
      RR=EXP(-SA5*T)-EXP(-SA9*T)
      SS=EXP(-SA6*T)-EXP(-SA9*T)
      UU=EXP(-SCN7*T)-EXP(-SA9*T)
      VV=EXP(-SCP8*T)-EXP(-SA9*T)
      W=SA5-SA9
      Z=SA6-SA9
      WW=SCN7-SA9
      ZZ=SCP8-SA9
      R101=SCP8*SCN7*SC6*SC5*(SS/(Z*S*Q*V)-RR/(W*R*G*V))
```

```
     1+SCP8*SCN7*SC6*SC5*UU*(1.0/G-1.0/Q)/(WW*U*V)+SCN7*SCP8*SC6*SC5*
     2VV*(1.0/(R*G)-1.0/(S*Q)+1.0/(U*Q)-1.0/(U*G))/(ZZ*V)
      R102=SCP8*SCN7*SC6*(UU/(WW*U)-SS/(Z*S))/Q+SCP8*SCN7*SC6*(1.0/S-
     11.0/U)*VV/(ZZ*Q)
      R103=SCP8*SCN7*(VV/ZZ-UU/WW)/U
      R104=-SCP8*VV/ZZ
      R105=EXP(-SA9*T)
      Y5=R11*Y05
      Y6=R21*Y05+R22*Y06
      Y8=R33*Y08
      Y9=R41*Y05+R44*Y09+R40*Y08
      Y95=R101*Y05 +R102*Y06+R103*Y0N7+R104*Y0P8+R105*Y095
      Y0=R51*Y05+R54*Y09+R55*Y00+R50*Y08
      Y1=R61*Y05+R64*Y09+R65*Y00+R66*Y01+R60*Y08
      YN7=R91*Y05+R92*Y06+R93*Y0N7
      YP8=R81*Y05+R82*Y06+R83*Y0N7+R84*Y0P8
      Y7=R71*Y05+R74*Y09+R75*Y00+R76*Y01+R70*Y08
      S1=ANU5*SF5*Y5+ANU8*SF8*Y8+ANU9*SF9*(Y9+Y95)+ANU1*SF1*Y1
      S2=SA5*Y5+SA6*Y6+SA8*Y8+SA9*(Y9+Y95)+SA0*Y0+SA1*Y1+SAF*Y7
     1+SAN7*YN7+SAP8*YP8
      FK=S1/(S2+SIGS)
      WRITE(6,20) R11,R21,R22,R33,R41,R44,R40,R51,R54,R55,R50,R61,R64,
     1R65,R66,R60,R71,R74,R75,R76,R70,T,FK
   20 FORMAT(1X,4E15.8,/1X,4E15.8,/1X,4E15.8,/1X,4E15.8,/1X,4E15.8,
     1/1X,3E15.8)
      WRITE(6,21)  R91,R92,R93,R81,R82,R83,R84,R101,R102,R103,R104,R105
   21 FORMAT(1X,4E15.8,/1X,4E15.8,/1X,4E15.8)
      STOP
      END

              $ENTRY
       0.34839260E 00 0.93714230E-01 0.99148280E 00 0.10000000E 01
       0.19892650E 00 0.42151620E 00 0.28430070E-02 0.50264770E-01
       0.22177940E 00 0.64696960E 00 0.58927640E-03 0.53725830E-02
       0.31170630E-01 0.13614940E 00 0.11987660E 00 0.58577620E-04
       0.14443610E 01 0.17714070E 01 0.31800720E 00 0.12905020E 01
       0.38007550E-02 0.15551990E 01 0.16089070E 01
       0.42914150E-03 0.74883840E-02 0.76887350E 00 0.33830750E-04
       0.78768790E-03 0.15354850E 00 0.43179340E 00 0.58242580E-05
       0.16650740E-03 0.46836150E-01 0.25672500E 00 0.20692150E 00
```

APPENDIX B

B.1 MPS Computer Program

This program uses the revised simplex method to solve LP problems. The following two pages (9,10), obtained from Dr. Said Ashour (11) in his Linear Programming course during the spring semester 1971, describe the control cards on page 1 of the printout. The input data include a list of the rows, columns, and RHS (right hand side) of the matrix in Fig. 3.4.1, and are printed in the format shown on pages 5-12 of the printout. This printout is for the 50% Pu recycle case.

Short Notes On MPS Computer Program

PROGRAM. PROGRAM is a mandatory statement at the beginning of each program.

INITIAL Z. INITIAL Z is used to establish initial settings for tolerance, frequencies and demands. This is a system macro.

TITLE ('EXAMPLE'). This statement is for giving a suitable name to the problem.

MOVE (XDATA, 'EXAMPLE') and MOVE (XPBNAME, 'PBFILE'). The first statement moves input data set name EXAMPLE into the cell XDATA, and the second moves the problem file name, PBFILE, into the cell XPBNAME.

CONVERT. This routine is the master of the convert procedure, which is used to convert external input data into a binary model and transfers this to PBFILE.

SETUP ('MAX'). This routine is the master of the setup procedure, which set up the problem name in XPBNAME by

1. Searching for the problem
2. Opening the matrix, eta, and scratch files
3. Making the storage allocation
4. Building the work matrix
5. Setting the logical basis and structural bounded variables at upper bound
6. Building the inverse of the logical basis

For minimization 'MAX' should be changed to 'MIN'.

BCDOUT. This converts the specified binary problem into the external input data format. The output may be listed and/or punched out and is in the order of the input data sections, ROWS, COLUMNS, RHSS, RANGES, AND BOUNDS. The NAME and ENDATA cards may also be produced.

MOVE (XOBJ, 'PROFIT') and MOVE (XRHS, 'LIMITS'). These statements are for identifying the objective function and the right hand side limits. For minimizing 'PROFIT' becomes 'COST'. Objective function and right hand side have to be named because a program can work with many objective functions and limits.

PICTURE. PICTURE procedure produces a "picture" of the current work matrix in condensed format. 45 rows and up to 55 columns for an output page are given. The pages are numbered using matrix notation for ease of identification. The output of PICTURE gives a quick visual check to see whether the structure of the matrix is correct and whether any coefficients are missing.

The range on the problem right hand side, and bounds on variables are indicated if they exist.

PRIMAL. PRIMAL optimizes the current problem using a composite primal algorithm. It can work with a composite objective function and/or a composite right hand side. It can work with a composite restraint row if neither the objective function nor the right hand side is composite.

SOLUTION. The output of a summary of the solution corresponding to the current basis. This output may be either printed or filed depending on a parameter in the procedure card.

EXIT. EXIT is a procedure linked to by the executor before return to the operating system. It is mainly used to close data sets and print the time of executor job step.

PEND. This is equivalent to END statement in FORTRAN PROGRAM.

B.2 MPS Computer Program Listing

```
C001              PROGRAM
C002              INITIALZ
C064              MOVE(XDATA,'CHART')
C065              MOVE(XPBNAME,'PBFILE')
C066              CONVERT('SUMMARY')
C067              SETUP
C068              BCDOUT
C069              MOVE(XOBJ,'OBJ')
C070              MOVE(XRHS,'RHS')
C071              PICTURE
C072              PRIMAL
C073              SOLUTION
C074              EXIT
C075      SV1      DC(0.0)
C076      SV2      DC(0.0)
C077              PEND
```

```
          EXECUTOR.    MPS/360 V2-M8                                    PAGE    1 - 71/207
CONVERT CHART    TO PBFILE
TIME =    0.06
    SUMMARY

1- ROWS SECTION.

   0 MINOR ERROR(S) -    0 MAJOR ERROR(S).
2- COLUMNS SECTION.

   0 MINOR ERROR(S) -    0 MAJOR ERROR(S).
3- RHS'S SECTION.
  RHS
   0 MINOR ERROR(S) -    0 MAJOR ERROR(S).
```

NUMBER OF ELEMENTS BY COLUMN ORDER

112	Y05	11	Y06	6	Y08	7	Y098	6	Y095	6	Y00	5	Y01	.....
119	Y0N7	5	Y0P8	4	Y07	2	Y15	14	Y16	8	Y18	10	Y198	.....
126	Y195	9	Y10	8	Y11	7	Y1N7	8	Y1P8	7	Y17	5	Y25	.....
133	Y26	8	Y28	10	Y298	9	Y295	9	Y20	8	Y21	7	Y2N7	.....
140	Y2P8	7	Y27	5	Y35	14	Y36	8	Y38	10	Y398	9	Y395	.....
147	Y30	8	Y31	7	Y3N7	8	Y3P8	7	Y37	5	Y45	14	Y46	.....
154	Y48	10	Y498	9	Y495	9	Y40	8	Y41	7	Y4N7	8	Y4P8	.....
161	Y47	5	Y55	3	Y56	2	Y58	2	Y598	2	Y595	2	Y50	.....
168	Y51	2	Y5N7	2	Y5P8	2	Y57	2	U15	3	U16	3	U18	.....
175	U198	3	U195	3	U10	3	U11	3	U1N7	2	U1P8	3	U17	.....
182	U25	3	U26	3	U28	2	U298	3	U295	3	U20	3	U21	.....
189	U2N7	2	U2P8	3	U27	2	U35	3	U36	3	U38	2	U398	.....
196	U395	3	U30	3	U31	3	U3N7	2	U3P8	3	U37	2	U45	.....
203	U46	3	U48	2	U498	3	U495	3	U40	3	U41	3	U4N7	.....
210	U4P8	3	U47	2										

NUMBER OF ELEMENTS BY ROW ORDER, EXCLUDING RHS'S, INCLUDING SLACK ELEMENT

1	N	OBJ	6	E	E05	2	E	E06	2	E	E08	2	E	E098	2	E	E099	2	E	E00	.....
8	E	E01	2	E	E0N7	2	E	E0P8	2	E	E07	2	E	E15	4	E	E16	5	E	E18	.....
15	E	E1S8	6	E	E195	8	E	E10	8	E	E11	9	E	E1N7	6	E	E1P8	7	E	E17	.....
22	E	E25	4	E	E26	5	E	E28	4	E	E298	6	E	E295	8	E	E20	8	E	E21	.....
29	E	E2N7	6	E	E2P8	7	E	E27	9	E	E35	4	E	E36	5	E	E38	4	E	E398	.....
36	E	E395	8	E	E30	8	E	E31	9	E	E3N7	6	E	E3P8	7	E	E37	9	E	E45	.....
43	E	E46	5	E	E48	4	E	E498	7	E	E495	7	E	E40	8	E	E41	9	E	E4N7	.....
50	E	E4P8	7	E	E47	9	E	E55	3	E	E56	4	E	E58	3	E	E596	5	E	E595	.....
57	E	E50	7	E	E51	8	E	E5N7	5	E	E5P8	6	E	E57	8	G	C12	11	G	C13	1
64	G	C14	11	G	C15	11	G	C21	21	G	C22	21	G	C23	21	G	C24	11	G	C25	1
71	L	C42	3	L	C43	3	L	C44	3	L	C45	3	L	V11	11	L	V12	11	L	V13	1
78	L	V14	11	L	V15	11	E	C01	3	E	C02	3	E	C03	3	E	C04	3	E	C31	.....
85	E	C32	3	E	C33	3	E	C34	3	E	C51	3	E	C52	3	E	C53	3	E	C54	.....
92	E	C61	2	E	C62	2	E	C63	2	E	C64	2	E	C71	3	E	C72	3	E	C73	.....
99	E	C74	3	E	C91	2	E	C92	2	E	C93	2	E	C94	2	E	C101	2	E	C102	.....
106	E	C103	2	E	C104	2	E	C111	3	E	C112	3	E	C113	3	E	C114	3			

PROBLEM STATISTICS - 111 ROWS, 211 VARIABLES, 636 ELEMENTS, DENSITY = 2.71

THESE STATISTICS INCLUDE ONE SLACK VARIABLE FOR EACH ROW.

0 MINOR ERRORS, 0 MAJOR ERRORS.

```
SETUP  PBFILE

TIME =    0.51

 MATRIX1  ASSIGNED TC MATRIX1

 ETA1     ASSIGNED TC ETA1

 SCRATCH1 ASSIGNED TC SCRATCH1
 SCRATCH2 ASSIGNED TO SCRATCH2

 MAXIMUM PRICING NCT REQUIRED - MAXIMUM POSSIBLE   7

 NO CYCLING

     POOLS          NUMBER    SIZE     CORE
 M.REG-BITS MAP                         456
 WORK   REGIONS       9        912     8208
 MATRIX BUFFERS       4       3504    14016
 ETA    BUFFERS       4      14304    57216

                          TOTAL  NORMAL  .FREE.   FIXED BOUNDED
 ROWS     (LOG.VAR.)       111     18       1       92       0
 COLUMNS  (STR.VAR.)       100    100       C        0       0

      636 ELEMENTS - DENSITY =   2.71 -   3 MATRIX RECORDS (WITHOUT RHS'S)


BCDCUT - USING PBFILE

TIME =    0.57
```

```
NAME          CHART
ROWS
 N  OBJ
 E  E05
 E  E06
 E  E08
 E  E098
 E  E095
 E  E00
 E  E01
 E  E0N7
 E  E0P8
 E  E07
 E  E15
 E  E16
 E  E18
 E  E198
 E  E195
 E  E10
 E  E11
 E  E1N7
 E  E1P8
 E  E17
 E  E25
 E  E26
 E  E28
 E  E298
 E  E295
 E  E20
 E  E21
 E  E2N7
 E  E2P8
 E  E27
 E  E35
 E  E36
 E  E38
 E  E398
 E  E395
 E  E30
 E  E31
 E  E3N7
 E  E3P8
 E  E37
 E  E45
 E  E46
 E  E48
 E  E498
 E  E495
 E  E40
 E  E41
 E  E4N7
 E  E4P8
 E  E47
 E  E55
```

```
E  E56
E  E58
E  E598
E  E595
E  E50
E  E51
E  E5N7
E  E5P8
E  E57
G  C12
G  C13
G  C14
G  C15
G  C21
G  C22
G  C23
G  C24
G  C25
L  C42
L  C43
L  C44
L  C45
L  V11
L  V12
L  V13
L  V14
L  V15
E  C01
E  C02
E  C03
E  C04
E  C31
E  C32
E  C33
E  C34
E  C51
E  C52
E  C53
E  C54
E  C61
E  C62
E  C63
E  C64
E  C71
E  C72
E  C73
E  C74
E  C91
E  C92
E  C93
E  C94
E  C101
E  C102
E  C1C3
```

```
 E  C104
 E  C111
 E  C112
 E  C113
 E  C114
COLUMNS
    Y05       E05          1.00000   E15          .34839
    Y05       E16           .09371   E198         .19893
    Y05       E195          .00001   E10          .05026
    Y05       E11           .00537   E1N7         .00043
    Y05       E1P8          .00003   E17         1.44436
    Y05       V11         12.40100
    Y06       E06          1.00000   E16          .99148
    Y06       E195          .00017   E1N7         .00749
    Y06       E1P8          .00079   V11        12.45400
    Y08       E08          1.00000   E18          .99576
    Y08       E198          .00284   E10          .00059
    Y08       E11           .00006   E17          .00380
    Y08       V11         12.55900
    Y098      E098         1.00000   E198         .42152
    Y098      E10           .22178   E11          .03117
    Y098      E17          1.77141   V11        12.04600
    Y095      E095         1.00000   E195         .20692
    Y095      E10           .22178   E11          .03117
    Y095      E17          1.77141   V11        12.04600
    Y00       E00          1.00000   E10          .64697
    Y00       E11           .13615   E17          .31801
    Y00       V11         12.09700
    Y01       E01          1.00000   E11          .11988
    Y01       E17          1.29050   V11        12.14700
    Y0N7      E0N7         1.00000   E195         .04684
    Y0N7      E1N7          .76887   E1P8         .15355
    Y0N7      V11         12.47400
    Y0P8      E0P8         1.00000   E195         .25673
    Y0P8      E1P8          .43179   V11        11.99600
    Y07       E07          1.00000   V11         1.71870
    Y15       E15       -  1.00000   E25          .34839
    Y15       E26           .09371   E298         .19893
    Y15       E295          .00001   E20          .05026
    Y15       E21           .00537   E2N7         .00043
    Y15       E2P8          .00003   E27         1.44436
    Y15       C12           .35578   C21          .43544
    Y15       C42           .96500   V12        12.40100
    Y16       E16       -  1.00000   E26          .99148
    Y16       E295          .00017   E2N7         .00749
    Y16       E2P8          .00079   C12      -   .00874
    Y16       C21       -   .00809   V12        12.45400
    Y18       E18       -  1.00000   E28          .99576
    Y18       E298          .00284   E20          .00059
    Y18       E21           .00006   E27          .00380
    Y18       C12       -   .00434   C21      -   .00402
    Y18       C42       -   .03500   V12        12.55900
    Y198      E198      -  1.00000   E298         .42152
    Y198      E20           .22178   E21          .03117
```

Y198	E27	1.77141	C12	.54663
Y198	C21	.66566	V12	12.04600
Y198	C01	5.00000		
Y195	E195	-1.00000	E295	.20692
Y195	E20	.22178	E21	.03117
Y195	E27	1.77141	C12	.54663
Y195	C21	.66566	V12	12.04600
Y195	C31	5.00000		
Y10	E10	-1.00000	E20	.64697
Y10	E21	.13615	E27	.31801
Y10	C12	-.44471	C21	-.41181
Y10	V12	12.09700	C51	5.00000
Y11	E11	-1.00000	E21	.11988
Y11	E27	1.29050	C12	.89365
Y11	C21	1.05392	V12	12.14700
Y11	C71	5.00000		
Y1N7	E1N7	-1.00000	E295	.04684
Y1N7	E2N7	.76887	E2P8	.15355
Y1N7	C12	-.26841	C21	-.24855
Y1N7	V12	12.47400	C101	1.00000
Y1P8	E1P8	-1.00000	E295	.25673
Y1P8	E2P8	.43179	C12	-.85765
Y1P8	C21	-.79420	V12	11.99600
Y1P8	C111	5.00000		
Y17	E17	-1.00000	C12	-.05066
Y17	C21	-.04692	V12	1.71870
Y17	C61	1.00000		
Y25	E25	-1.00000	E35	.34839
Y25	E36	.09371	E398	.19893
Y25	E395	.00001	E30	.05026
Y25	E31	.00537	E3N7	.00043
Y25	E3P8	.00003	E37	1.44436
Y25	C13	.35578	C22	.43544
Y25	C43	.96500	V13	12.40100
Y26	E26	-1.00000	E36	.94148
Y26	E395	.00017	E3N7	.00749
Y26	E3P8	.00079	C13	-.00874
Y26	C22	-.00809	V13	12.45400
Y28	E28	-1.00000	E38	.99576
Y28	E398	.00284	E30	.00059
Y28	E31	.00006	E37	.00380
Y28	C13	-.00434	C22	-.00402
Y28	C43	-.03500	V13	12.55900
Y298	E298	-1.00000	E398	.42152
Y298	E30	.22178	E31	.03117
Y298	E37	1.77141	C13	.54663
Y298	C22	.66566	V13	12.04600
Y298	C02	5.00000		
Y295	E295	-1.00000	E395	.20692
Y295	E30	.22178	E31	.03117
Y295	E37	1.77141	C13	.54663
Y295	C22	.66566	V13	12.04600
Y295	C32	5.00000		
Y20	E20	-1.00000	E30	.64697

Y20	E31	.13615	E37	.31801
Y20	C13	-.44471	C22	-.41181
Y20	V13	12.09700	C52	5.00000
Y21	E21	-1.00000	E31	.11988
Y21	E37	1.29050	C13	.89365
Y21	C22	1.05392	V13	12.14700
Y21	C72	5.00000		
Y2N7	E2N7	-1.00000	E395	.04684
Y2N7	E3N7	.76887	E3P8	.15355
Y2N7	C13	-.26841	C22	-.24855
Y2N7	V13	12.47400	C102	1.00000
Y2P8	E2P8	-1.00000	E395	.25673
Y2P8	E3P8	.43179	C13	-.85765
Y2P8	C22	-.79420	V13	11.99600
Y2P8	C112	5.00000		
Y27	E27	-1.00000	C13	-.05066
Y27	C22	-.04692	V13	1.71870
Y27	C62	1.00000		
Y35	E35	-1.00000	E45	.34839
Y35	E46	.09371	E498	.19893
Y35	E495	.00001	E40	.05026
Y35	E41	.00537	E4N7	.00043
Y35	E4P8	.00003	E47	1.44436
Y35	C14	.35578	C23	.43544
Y35	C44	.96500	V14	12.40100
Y36	E36	-1.00000	E46	.99148
Y36	E498	.00017	E4N7	.00749
Y36	E4P8	.00079	C14	-.00874
Y36	C23	-.00609	V14	12.45400
Y38	E38	-1.00000	E48	.99576
Y38	E498	.00284	E40	.00059
Y38	E41	.00006	E47	.00380
Y38	C14	-.00434	C23	-.00402
Y38	C44	-.03500	V14	12.55900
Y398	E398	-1.00000	E498	.42152
Y398	E40	.22178	E41	.03117
Y398	E47	1.77141	C14	.54663
Y398	C23	.66566	V14	12.04600
Y398	C03	5.00000		
Y395	E395	-1.00000	E495	.20692
Y395	E40	.22178	E41	.03117
Y395	E47	1.77141	C14	.54663
Y395	C23	.66566	V14	12.04600
Y395	C33	5.00000		
Y30	E30	-1.00000	E40	.64697
Y30	E41	.13615	E47	.31801
Y30	C14	-.44471	C23	-.41181
Y30	V14	12.09700	C53	5.00000
Y31	E31	-1.00000	E41	.11988
Y31	E47	1.29050	C14	.89365
Y31	C23	1.05392	V14	12.14700
Y31	C73	5.00000		
Y3N7	E3N7	-1.00000	E495	.04684
Y3N7	E4N7	.76887	E4P8	.15355

Y3N7	C14	-	.26841	C23	-	.24855
Y3N7	V14		12.47400	C103		1.00000
Y3P8	E3P8	-	1.00000	E495		.25673
Y3P8	E4P8		.43179	C14	-	.85765
Y3P8	C23	-	.79420	V14		11.99600
Y3P8	C113		5.00000			
Y37	E37	-	1.00000	C14	-	.05066
Y37	C23	-	.04692	V14		1.71870
Y37	C63		1.00000			
Y45	E45	-	1.00000	E55		.34839
Y45	E56		.09371	E598		.19893
Y45	E595		.00001	E50		.05026
Y45	E51		.00537	E5N7		.00043
Y45	E5P8		.00003	E57		1.44436
Y45	C15		.35578	C24		.43544
Y45	C45		.96500	V15		12.40100
Y46	E46	-	1.00000	E56		.99148
Y46	E595		.00017	E5N7		.00749
Y46	E5P8		.00079	C15	-	.00874
Y46	C24	-	.00809	V15		12.45400
Y48	E48	-	1.00000	E58		.99576
Y48	E598		.00284	E50		.00059
Y48	E51		.00006	E57		.00380
Y48	C15	-	.00434	C24	-	.00402
Y48	C45	-	.03500	V15		12.55900
Y498	E498	-	1.00000	E598		.42152
Y498	E50		.22178	E51		.03117
Y498	E57		1.77141	C15		.54663
Y498	C24		.66566	V15		12.04600
Y498	C04		5.00000			
Y495	E495	-	1.00000	E595		.20692
Y495	E50		.22178	E51		.03117
Y495	E57		1.77141	C15		.54663
Y495	C24		.66566	V15		12.04600
Y495	C34		5.00000			
Y40	E40	-	1.00000	E50		.64697
Y40	E51		.13615	E57		.31801
Y40	C15	-	.44471	C24	-	.41181
Y40	V15		12.09700	C54		5.00000
Y41	E41	-	1.00000	E51		.11986
Y41	E57		1.29050	C15		.89365
Y41	C24		1.05392	V15		12.14700
Y41	C74		5.00000			
Y4N7	E4N7	-	1.00000	E595		.04684
Y4N7	E5N7		.76887	E5P8		.15355
Y4N7	C15		.26841	C24	-	.24855
Y4N7	V15		12.47400	C104		1.00000
Y4P8	E4P8	-	1.00000	E595		.25673
Y4P8	E5P8		.43179	C15	-	.85765
Y4P8	C24	-	.79420	V15		11.99600
Y4P8	C114		5.00000			
Y47	E47	-	1.00000	C15	-	.05066
Y47	C24	-	.04692	V15		1.71870
Y47	C64		1.00000			

Y55	OBJ	-	1.00000	E55	-	1.00000
Y55	C25		.43544			
Y56	E56	-	1.00000	C25	-	.00809
Y58	E58	-	1.00000	C25	-	.00402
Y598	E598	-	1.00000	C25		.66566
Y595	E595	-	1.00000	C25		.66566
Y50	E50	-	1.00000	C25	-	.41181
Y51	E51	-	1.00000	C25		1.05392
Y5N7	E5N7	-	1.00000	C25	-	.24855
Y5P8	E5P8	-	1.00000	C25	-	.79420
Y57	E57	-	1.00000	C25	-	.04692
U15	OBJ		1.00000	E15		1.00000
U15	C21	-	.43544			
U16	E16	-	1.00000	C21	-	.00809
U16	C91		1.00000			
U18	E18		1.00000	C21		.00402
U198	E198	-	1.00000	C21		.66566
U198	C01	-	5.00000			
U195	E195	-	1.00000	C21		.66566
U195	C31	-	5.00000			
U10	E10	-	1.00000	C21	-	.41181
U10	C51	-	5.00000			
U11	E11	-	1.00000	C21		1.05392
U11	C71	-	5.00000			
U1N7	E1N7	-	1.00000	C21	-	.24855
U1P8	E1P8	-	1.00000	C21	-	.79420
U1P8	C111	-	5.00000			
U17	E17	-	1.00000	C21	-	.04692
U25	OBJ		1.00000	E25		1.00000
U25	C22	-	.43544			
U26	E26	-	1.00000	C22	-	.00809
U26	C92		1.00000			
U28	E28		1.00000	C22		.00402
U298	E298	-	1.00000	C22		.66566
U298	C02	-	5.00000			
U295	E295	-	1.00000	C22		.66566
U295	C32	-	5.00000			
U20	E20	-	1.00000	C22	-	.41181
U20	C52	-	5.00000			
U21	E21	-	1.00000	C22		1.05392
U21	C72	-	5.00000			
U2N7	E2N7	-	1.00000	C22	-	.24855
U2P8	E2P8	-	1.00000	C22	-	.79420
U2P8	C112	-	5.00000			
U27	E27	-	1.00000	C22	-	.04692
U35	OBJ		1.00000	E35		1.00000
U35	C23	-	.43544			
U36	E36	-	1.00000	C23	-	.00809
U36	C93		1.00000			
U38	E38		1.00000	C23		.00402
U398	E398	-	1.00000	C23		.66566
U398	C03	-	5.00000			
U395	E395	-	1.00000	C23		.66566
U395	C33	-	5.00000			

```
    U30       E30       -   1.00000   C23      -      .41181
    U30       C53       -   5.00000
    U31       E31       -   1.00000   C23            1.05392
    U31       C73       -   5.00000
    U3N7      E3N7      -   1.00000   C23      -      .24855
    U3P8      E3P8      -   1.00000   C23      -      .79420
    U3P8      C113      -   5.00000
    U37       E37       -   1.00000   C23      -      .04692
    U45       OBJ           1.00000   E45            1.00000
    U45       C24       -    .43544
    U46       E46       -   1.00000   C24      -      .00809
    U46       C94           1.00000
    U48       E48           1.00000   C24             .00402
    U498      E498      -   1.00000   C24             .66566
    U498      C04       -   5.00000
    U495      E495      -   1.00000   C24             .66566
    U495      C34       -   5.00000
    U40       E40       -   1.00000   C24      -      .41181
    U40       C54       -   5.00000
    U41       E41       -   1.00000   C24            1.05392
    U41       C74       -   5.00000
    U4N7      E4N7      -   1.00000   C24      -      .24855
    U4P8      E4P8      -   1.00000   C24      -      .79420
    U4P8      C114      -   5.00000
    U47       E47       -   1.00000   C24      -      .04692
RHS
    RHS       E05          10.63830   E08          297.89916
    RHS       C12            .39941   C13             .39941
    RHS       C14            .39941   C15             .39941
    RHS       C21            .36986   C22             .36986
    RHS       C23            .36986   C24             .36986
    RHS       C25            .36986   V11         3900.00000
    RHS       V12         3900.00000  V13         3900.00000
    RHS       V14         3900.00000  V15         3900.00000
ENDATA
```

```
PICTURE - USING PBFILE
TIME =   0.71
```

		Y05	Y06	Y08	Y098	Y095	Y00	Y01	Y0N7	Y0P8	Y07	Y15	Y16	Y18	Y198	Y195	Y10	Y11	Y1N7	Y1P8	Y17	Y25	Y26	Y28	Y298	Y295	Y20	Y21	Y2N7	Y2P8	Y27	Y35	Y36	Y38	Y398	Y395	Y30	Y31	Y3N7	Y3P8	Y37	Y45	Y46	Y48	Y498	Y495	Y40	Y41	Y4N7	Y4P8	Y47	Y55	Y56	Y58	Y598	Y595
CBJ	N																																																			-1				
E05	E	1																																																						
E06	E		1																																																					
E08	E			1																																																				
E098	E				1																																																			
E095	E					1																																																		
E00	E						1																																																	
E01	E							1																																																
E0N7	E								1																																															
E0P8	E									1																																													
E07	E										1																																												
E15	E	T										-1																																												
E16	E	U	T										-1																																											
E18	E			T										-1																																										
E198	E	T		V	T										-1																																									
E195	E	Y	W			T			U	T						-1																																								
E10	E	U		W	T	T	T										-1																																							
E11	E	V		X	L	U	T	T										-1																																						
E1N7	E	W	V						T										-1																																					
E1P8	E	X	W						T	T										-1																																				
E17	E	A		V	A	A	T	A													-1																																			
E25	E											T										-1																																		
E26	E											U	T										-1																																	
E28	E													T										-1																																
E298	E											T		V	T										-1																															
E295	E											Y	W			T			U	T						-1																														
E20	E											U		W	T	T	T										-1																													
E21	E											V		X	U	U	T	T										-1																												
E2N7	E											W	V						T										-1																											
E2P8	E											X	W						T	T										-1																										
E27	E											A		V	A	A	T	A													-1																									
E35	E																					T										-1																								
E36	E																					U	T										-1																							
E38	E																							T										-1																						
E398	E																					T		V	T										-1																					
E395	E																					Y	W			T			U	T						-1																				
E30	E																					U		W	T	T	T										-1																			
E31	E																					V		X	U	U	T	T										-1																		
E3N7	E																					W	V						T										-1																	
E3P8	E																					X	W						T	T										-1																
E37	E																					A		V	A	A	T	A													-1															
E45	E																															T										-1														
E46	E																															U	T										-1													
E48	E																																	T										-1												
E498	E																															T	W	V	T										-1											
E495	E																															Y				T			U	T						-1										
E40	E																															U		W	T	T	T										-1									

2,

		Y05	Y06	Y08	Y0598	Y0595	Y00	Y01	Y0N7	Y0P8	Y07	Y15	Y16	Y18	Y198	Y195	Y10	Y11	Y1N7	Y1P8	Y17	Y25	Y26	Y28	Y298	Y295	Y20	Y21	Y2N7	Y2P8	Y27	Y35	Y36	Y38	Y398	Y395	Y30	Y31	Y3N7	Y3P8	Y37	Y45	Y46	Y48	Y498	Y495	Y40	Y41	Y4N7	Y4P8	Y47	Y55	Y56	Y58	Y598	Y595
E41	E																															V		X	U	U	T	T										-1								
E4N7	E																															W	V						T										-1							
E4P8	E																															X	W						T	T										-1						
E47	E																															A		V	A	A	T	A													-1					
E55	E																																									T										-1				
E56	E																																									U	T										-1			
E58	E																																											T										-1		
E598	E																																									T		V	T										-1	
E595	E																																									Y	W			T			U	T						-1
E5C	E																																									U		W	T	T	T									
E51	E																																									V		X	U	U	T	T								
E5N7	E																																									W	V						T							
E5P8	E																																									X	W						T	T						
E57	E																																									A		V	A	A	T	A								
C12	G											T-	V-	V	T	T-	T	T-	T-	T-	U																																			
C13	G																					T-	V-	V	T	T-	T	T-	T-	T-	U																									
C14	G																															T-	V-	V	T	T-	T	T-	T-	T-	U															
C15	G																																									T-	V-	V	T	T-	T	T-	T-	T-	U					
C21	G											T-	V-	V	T	T-	T	A-	T-	T-	U																																			
C22	G																					T-	V-	V	T	T-	T	A-	T-	T-	U																									
C23	G																															T-	V-	V	T	T-	T	A-	T-	T-	U															
C24	G																																									T-	V-	V	T	T-	T	A-	T-	T-	U					
C25	G																																																			T-	V-	V	T	T
C42	L											T		-U																																										
C43	L																					T		-U																																
C44	L																															T		-U																						
C45	L																																									T		-U												
V11	L	B	B	B	B	B	B	B	B	B	A																																													
V12	L											B	B	B	B	B	B	B	B	B	A																																			
V13	L																					B	B	B	B	B	B	B	B	B	A																									
V14	L																															B	B	B	B	B	B	B	B	B	A															
V15	L																																									B	B	B	B	B	B	B	B	B	A					
C01	E														A																																									
C02	E																								A																															
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C54	E																																														A									
C61	E																				1																																			
C62	E																														1																									
C63	E																																								1															

		Y05	Y06	Y08	YC98	Y095	Y00	Y01	Y0N7	Y0P8	Y07	Y15	Y16	Y18	Y198	Y195	Y10	Y11	Y1N7	Y1P8	Y17	Y25	Y26	Y28	Y298	Y295	Y20	Y21	Y2N7	Y2P8	Y27	Y35	Y36	Y38	Y398	Y395	Y30	Y31	Y3N7	Y3P8	Y37	Y45	Y46	Y48	Y498	Y495	Y40	Y41	Y4N7	Y4P8	Y47	Y55	Y56	Y58	Y598	Y595
C64	E																																																		1					
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C111	E																			A																																				
C112	E																													A																										
C113	E																																							A																
C114	E																																																	A						

		Y50	Y51	Y5N7	Y5P8	Y57	U15	U16	U18	U198	U195	U10	U11	U1N7	U1P8	U17	U25	U26	U28	U298	U295	U20	U21	U2N7	U2P8	U27	U35	U36	U38	U398	U395	U30	U31	U3N7	U3P8	U37	U45	U46	U48	U498	U495	U40	U41	U4N7	U4P8	U47	RHS
OBJ	N						1										1										1										1										
E05	E																																														B
E06	E																																														
E08	E																																														C
E098	E																																														
E095	E																																														
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E0N7	E																																														
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E18	E								1																																						
E198	E									-1																																					
E195	E										-1																																				
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E11	E												-1																																		
E1N7	E													-1																																	
E1P8	E														-1																																
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E298	E																			-1																											
E295	E																				-1																										
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E2N7	E																							-1																							
F2P8	E																								-1																						
E27	E																									-1																					
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E36	E																											-1																			
E38	E																												1																		
E398	E																													-1																	
E395	E																														-1																
E3C	E																															-1															
E31	E																																-1														
E3N7	E																																	-1													
E3P8	E																																		-1												
E37	E																																			-1											
F45	E																																				1										
E46	E																																					-1									
E48	E																																						1								
E498	E																																							-1							
E495	E																																								-1						
E40	E																																									-1					

		Y50	Y51	Y5N7	Y5P8	Y57	U15	U16	U18	UU198	UU195	U10	U11	UU1N7	UU1P8	U17	U25	U26	U28	UU298	UU295	U20	U21	UU2N7	UU2P8	U27	U35	U36	U38	UU398	UU395	U30	U31	UU3N7	UU3P8	U37	U45	U46	U48	UU498	UU495	U40	U41	UU4N7	UU4P8	U47	RHS
E41	E																																										-1				
E4N7	E																																											-1			
E4P8	E																																												-1		
E47	E																																													-1	
E55	E																																														
E56	E																																														
E58	E																																														
E598	E																																														
E595	E																																														
E5C	E	-1																																													
E51	E		-1																																												
E5N7	E			-1																																											
E5P8	E				-1																																										
E57	E					-1																																									
C12	G																																														T
C13	G																																														T
C14	G																																														T
C15	G																																														T
C21	G						-T	-V	V	T	T	-T	A	-T	-T	-U																															T
C22	G																-T	-V	V	T	T	-T	A	-T	-T	-U																					T
C23	G																										-T	-V	V	T	T	-T	A	-T	-T	-U											T
C24	G																																				-T	-V	V	T	T	-T	A	-T	-T	-U	T
C25	G	-T	A	-T	-T	-U																																									T
C42	L																																														
C43	L																																														
C44	L																																														
C45	L																																														
V11	L																																														D
V12	L																																														D
V13	L																																														D
V14	L																																														D
V15	L																																														D
C01	E									-A																																					
C02	E																			-A																											
C03	E																													-A																	
C04	E																																							-A							
C31	E										-A																																				
C32	E																				-A																										
C33	E																														-A																
C34	E																																								-A						
C51	E											-A																																			
C52	E																					-A																									
C53	E																															-A															
C54	E																																									-A					
C61	E																																														
C62	E																																														
C63	E																																														

		Y50	Y51	Y5N7	Y5P8	Y57	U15	U16	U18	U198	U195	U10	U11	U1N7	U1P8	U17	U25	U26	U28	U298	U295	U20	U21	U2N7	U2P8	U27	U35	U36	U38	U398	U395	U30	U31	U3N7	U3P8	U37	U45	U46	U48	U498	U495	U40	U41	U4N7	U4P8	U47	RHS
C64	E																																														
C71	E												-A																																		
C72	E																						-A																								
C73	E																																-A														
C74	E																																										-A				
C91	E							1																																							
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C101	E																																														
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C103	E																																														
C104	E																																														
C111	E														-A																																
C112	E																								-A																						
C113	E																																		-A												
C114	E																																												-A		

SUMMARY OF MATRIX

SYMBOL	RANGE			COUNT (INCL.RHS)
Z	LESS THAN		.000001	
Y	.000001	THRU	.000009	5
X	.000010		.000099	10
W	.000100		.000999	20
V	.001000		.009999	46
U	.010000		.099999	42
T	.100000		.999999	175
1	1.000000		1.000000	117
A	1.000001		10.000000	74
B	10.000001		100.000000	46
C	100.000001		1,000.000000	1
D	1,000.000001		10,000.000000	5
E	10,000.000001		100,000.000000	
F	100,000.000001		1,000,000.000000	
G	GREATER THAN		1,000,000.000000	

PRIMAL OBJ = CBJ RHS = RHS

TIME = 0.82 MINS. PRICING 7
SCALE = .

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
M	1	11	80	125	1.21228-	311.984
M	2	11	15	175	1.87794-	311.984
M	3	11	82	145	1.21228-	311.984
M	4	11	35	195	1.87794-	311.984
M	5	11	84	126	1.21228-	311.984
M	6	11	16	176	1.87794-	311.984
M	7	11	83	155	1.21228-	311.984
M	8	11	45	205	1.87794-	311.984
M	9	11	86	146	1.21228-	311.984
M	10	11	36	196	1.87794-	311.984
M	11	11	87	156	1.21228-	311.984
M	12	11	46	206	1.87794-	311.984
M	13	11	59	208	1.05392-	311.984
	14		48	158	3.00148-	311.984
M	15	11	96	178	1.05392-	311.984
	16		18	128	3.00148-	311.984
M	17	11	58	168	1.05392-	311.984
M	18	11	22	132	1.02291-	311.984
M	19	11	12	122	1.14759-	311.984
M	20	11	32	142	.98968-	311.984
M	21	11	33	182	.93227-	311.984
M	22	11	37	193	9.93991-	311.984
M	23	11	38	197	18.1203-	311.984
	24		39	148	171.665-	311.984
	25		42	152	.79638-	311.984
M	26	11	40	199	2374.27-	311.984
	27		43	192	.83187-	311.984

INVERT DEMANDED AFTER 22 MAJOR/ 27 MINOR ITERATIONS - CLOCK CONTROL

INVERT CALLED TIME 1.19 CURRENT INVERSE ---- ETA-VECTORS ....36 ELEMENTS ...515 RECORDS .....1 ITERATION 27
BASIS ---- NO.OF ROWS ...111 LOGICALS ....84 STRUCTURALS ....27 ELEMENTS ...255
INVERSE -- NUCLEUS ....16 TRANSFORMED ......6 ETA-VECTORS ....52 ELEMENTS ...202 RECORDS .....1 TIME TAKEN 0.15

PRIMAL OBJ = CBJ RHS = RHS

TIME = 1.37 MINS. PRICING 7
SCALE = .

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
M	28	11	41	200	21550.6-	311.984
	29		47	203	8.86852-	311.984
	30		28	138	1.87800-	311.984
M	31	11	49	207	16.1228-	311.984
	32		56	166	.66566-	311.984
	33		55	165	.66566-	311.984
M	34	11	50	209	2060.73-	311.984

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
	35		13	112	1.53159-	311.984
M	36	11	51	210	26140.0-	311.984
	37		17	173	16.3354-	311.984
M	38	11	19	177	30.0442-	311.984
	39		52	211	.56534-	311.984
M	40	11	20	179	3518.73-	311.984
	41		53	162	7.16282-	311.984
M	42	11	21	180	44635.1-	311.984
	43		57	163	26.6205-	311.984
M	44	11	59	167	21.5520-	311.984
	45		23	172	.51585-	311.984
	46		192	201	.44835-	311.984
M	47	11	60	169	18136.5-	311.984
	48		25	183	5.52955-	311.984
	49		26	135	4.17006-	311.984
	50		27	136	142460.-	311.984
M	51	11	61	170	230065.-	311.984
	52		29	187	14.9384-	311.984
	53		172	181	.90611-	311.984
	54		8	118	.13448-	311.964
	55		44	204	.00402-	311.984
	56		24	184	.00402-	311.984
M	57	11	30	189	10503.1-	311.984
	58		14	114	1.00307-	311.984
	59		5	115	.37986-	311.984
	60		6	116	.09412-	311.984
	61		34	194	.00402-	311.984
M	62	11	31	190	133234.-	311.984
M	63	11	81	141	2.20200-	311.984
M	64	11	182	185	26.4284-	311.984
M	65	11	211	198	2.19795-	311.984
	66		148	171	47.2554-	311.984
M	67	11	85	161	15.4096-	311.984

INVERT DEMANDED AFTER 16 MAJOR/ 40 MINOR ITERATIONS - CLOCK CONTROL

INVERT CALLED TIME 1.96 CURRENT INVERSE ---- ETA-VECTORS ....92 ELEMENTS ..2350 RECORDS .....2 ITERATION 57
BASIS ---- NO.OF ROWS ...111 LOGICALS ....49 STRUCTURALS ....62 ELEMENTS ...345
INVERSE -- NUCLEUS ....22 TRANSFORMED .....4 ETA-VECTORS ....93 ELEMENTS ...344 RECORDS .....1 TIME TAKEN 0.04

PRIMAL OBJ = OBJ RHS = RHS

TIME = 2.02 MINS. PRICING 7
SCALE = .

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
M	68	11	93	191	.05066-	311.984
M	69	11	95	211	.05066-	311.984
M	70	11	91	192	.82076-	311.984
	71		142	182	.28006-	311.984

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   ITER   NUMBER  VECTOR  VECTOR  REDUCED      SUM
  NUMBER  INFEAS   OUT      IN     COST      INFEAS

M    72     11      74     202    .55621-   311.984
M    73     11      54     154    .01737-   311.984
M    74      9      79     164    .01343-   311.215
M    75      9      98     148    .95944-   311.215
M    76      9      90     157   56.8445-   311.215
     77            182     186    .10E+07-  311.215
M    78      9     103     142    .55621-   311.215
M    79      9     111     153   5.93516-   311.215
M    80      9      88     147  172.287-    311.215
M    81      9     102     182    .61234-   311.215
M    82      9      89     172    .63930-   311.215
     83            122     127   31.5335-   311.215
 INVERT DEMANDED AFTER  13 MAJOR/  16 MINOR ITERATIONS - CLOCK CONTROL

 INVERT CALLED      TIME    2.34   CURRENT INVERSE ---- ETA-VECTORS ...109   ELEMENTS ...988   RECORDS .....1   ITERATION .....83
  BASIS ---- NO.OF ROWS ...111     LOGICALS ........36  STRUCTURALS ....75   ELEMENTS ...388
  INVERSE -- NUCLEUS .......32     TRANSFORMED .....9   ETA-VECTORS ...116   ELEMENTS ...474   RECORDS .....1   TIME TAKEN   0.04

PRIMAL       OBJ = OBJ         RHS = RHS

TIME =     2.40  MINS.        PRICING    7
 SCALE =     .

   ITER   NUMBER  VECTOR  VECTOR  REDUCED      SUM
  NUMBER  INFEAS   OUT      IN     COST      INFEAS
M    84      9     172     137   66.0952-   311.215
     85            182     122   4.25644-   311.215
M    86      9     211     143  123.933-    311.215
     87            192     160  397744.-    311.215
M    88      9      97     211   13.9014-   311.215
M    89      9     110     182    .80433-   311.215
M    90      9      73     192    .55621-   311.215
M    91      7     204     144    .01337-   310.445
M    92      7      71     188  338.779-    310.445
M    93      7     184     124   1.15044-   310.445
M    94      7     182     134   1.14854-   310.445
M    95      7     100     150    .54E+07-  310.445
M    96      7      72     182    .55621-   310.445
 INVERT DEMANDED AFTER  11 MAJOR/  13 MINOR ITERATIONS - CLOCK CONTROL

 INVERT CALLED      TIME    2.69   CURRENT INVERSE ---- ETA-VECTORS ...129   ELEMENTS ..1422   RECORDS .....2   ITERATION .....96
  BASIS ---- NO.OF ROWS ...111     LOGICALS ........30  STRUCTURALS ....81   ELEMENTS ...452
  INVERSE -- NUCLEUS .......41     TRANSFORMED ....11   ETA-VECTORS ...131   ELEMENTS ...600   RECORDS .....1   TIME TAKEN   0.21

PRIMAL       OBJ = OBJ         RHS = RHS

TIME =     2.91  MINS.        PRICING    7
 SCALE =     .
```

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
M	97	4	4	71	28.8589-	14.2815
M	98	4	101	172	.55621-	14.2815
	99		122	123	7.50473-	14.2815
M	100	4	108	133	13.5044-	14.2815
M	101	4	109	122	.55621-	14.2815
M	102	2	78	140	32882.6-	11.7424
M	103	2	67	130	70968.4-	10.0580
M	104	0	2	72	10.9007-	.

FEASIBLE SOLUTION

PRIMAL OBJ = OBJ RHS = RHS

TIME = 3.15 MINS. PRICING 7
SCALE = .
SCALE RESET TO 1.00000

	ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
M	105	11	68	74	8.66896-	16.3913
M	106	4	69	73	1.16007-	9.91488
	107		70	79	.05254-	9.39484
M	108	2	194	78	.00119-	9.27799

OPTIMAL SOLUTION

SOLUTION (OPTIMAL)

TIME = 3.31 MINS. ITERATION NUMBER = 108

...NAME...	...ACTIVITY...	DEFINED AS
FUNCTIONAL	9.27799	OBJ
RESTRAINTS		RHS

SECTION 1 - ROWS

	NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
	1	OBJ	BS	9.27799	9.27799-	NONE	NONE	1.00000
	2	E05	EC	10.63830	.	10.63830	10.63830	.36853
	3	E06	BS	.	.	.	.	.
	4	E08	EC	297.89916	.	297.89916	297.89916	.02887-
	5	EC98	EC	.	.	.	.	.03594
	6	EC95	EC	.	.	.	.	.07269-
	7	E0C	BS	.	.	.	.	.
	8	E01	EC	.	.	.	.	.01314
	9	E0N7	BS	.	.	.	.	.
	10	E0P8	BS	.	.	.	.	.
	11	E07	BS	.	.	.	.	.
	12	E15	EC	.	.	.	.	1.00000-
	13	E16	EC	.	.	.	.	.12993
	14	E18	EC	.	.	.	.	.02930
	15	E198	EC	.	.	.	.	.23652-
	16	E195	EC	.	.	.	.	.04125
	17	E10	EC	.	.	.	.	.30466
	18	E11	EC	.	.	.	.	.10958-
A	19	E1N7	EC	.	.	.	.	.
	20	E1P8	EC	.	.	.	.	.34826
A	21	E17	EC	.	.	.	.	.
	22	E25	EC	.	.	.	.	2.36413-
	23	E26	EC	.	.	.	.	.11326
	24	E28	EC	.	.	.	.	.03475
	25	E298	EC	.	.	.	.	2.32284-
	26	E295	EC	.	.	.	.	2.04706-
	27	E2C	EC	.	.	.	.	1.58723
	28	E21	EC	.	.	.	.	3.41049-
	29	E2N7	EC	.	.	.	.	.77850
	30	E2P8	EC	.	.	.	.	2.53015
	31	E27	EC	.	.	.	.	.14698
	32	E35	EC	.	.	.	.	2.35468-
	33	E36	EC	.	.	.	.	.09104
	34	E38	EC	.	.	.	.	.02759
	35	E398	EC	.	.	.	.	2.31048-
	36	E395	EC	.	.	.	.	2.04109-
	37	E3C	EC	.	.	.	.	1.55975
	38	E31	EC	.	.	.	.	3.36704-
	39	E3N7	EC	.	.	.	.	.77325
	40	E3P8	EC	.	.	.	.	2.79317
	41	E37	EC	.	.	.	.	.14576
	42	E45	EC	.	.	.	.	2.34811-
	43	E46	EC	.	.	.	.	.05564
	44	E48	EC	.	.	.	.	.03644
	45	E498	EC	.	.	.	.	2.26675-
	46	E495	EC	.	.	.	.	2.05496-
	47	E4C	EC	.	.	.	.	1.49362
	48	E41	EC	.	.	.	.	3.36982-
	49	E4N7	EC	.	.	.	.	.76951

	NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
	50	E4P8	EC	.	.	.	.	2.73645
	51	E47	EC	.	.	.	.	.14525
	52	E55	EC	.	.	.	.	2.41563-
	53	E56	EC	.	.	.	.	.02650
	54	F58	EC	.	.	.	.	.01305
	55	E598	EC	.	.	.	.	2.16406-
	56	E595	EC	.	.	.	.	2.16405-
	57	E50	EC	.	.	.	.	1.33879
	58	E51	EC	.	.	.	.	3.42630-
	59	E5N7	EC	.	.	.	.	.80803
	60	E5P8	EC	.	.	.	.	2.58195
	61	E57	EC	.	.	.	.	.15253
	62	C12	BS	1.08010	.68069-	.39941	NONE	.
	63	C13	BS	1.00159	.60218-	.39941	NONE	.
	64	C14	BS	.99628	.59887-	.39941	NONE	.
	65	C15	BS	1.00159	.60218-	.39941	NONE	.
	66	C21	BS	1.39797	1.02811-	.36986	NONE	.
	67	C22	LL	.36986	.	.36986	NONE	3.13275-
	68	C23	LL	.36986	.	.36986	NONE	3.11104-
	69	C24	LL	.36986	.	.36986	NONE	3.09595-
	70	C25	LL	.36986	.	.36986	NONE	3.25102-
	71	C42	BS	5.79880-	5.79880	NONE	.	.
	72	C43	BS	5.48724-	5.48724	NONE	.	.
	73	C44	BS	5.40367-	5.40367	NONE	.	.
	74	C45	BS	5.34717-	5.34717	NONE	.	.
	75	V11	BS	3873.24111	26.75889	NONE	3900.00000	.
	76	V12	BS	3819.38132	80.61868	NONE	3900.00000	.
	77	V13	BS	3811.15472	88.84528	NONE	3900.00000	.
	78	V14	BS	3801.59192	98.40808	NONE	3900.00000	.
	79	V15	BS	3791.82412	108.17588	NONE	3900.00000	.
	80	C01	EC	.	.	.	.	.04730
	81	C02	EC	.	.	.	.	.04750
	82	C03	EC	.	.	.	.	.04792
	83	C04	EC	.	.	.	.	.04519
	84	C31	EC	.	.	.	.	.00925-
	85	C32	EC	.	.	.	.	.00763-
	86	C33	EC	.	.	.	.	.00575-
	87	C34	EC	.	.	.	.	.00125-
	88	C51	EC	.	.	.	.	.06073-
	89	C52	EC	.	.	.	.	.05744-
	90	C53	EC	.	.	.	.	.05572-
	91	C54	EC	.	.	.	.	.04432-
	92	C61	BS	.	.	.	.	.
A	93	C62	EC	.	.	.	.	.
	94	C63	BS	.	.	.	.	.
A	95	C64	EC	.	.	.	.	.
	96	C71	EC	.	.	.	.	.02172
	97	C72	EC	.	.	.	.	.02177
	98	C73	EC	.	.	.	.	.02165
	99	C74	EC	.	.	.	.	.02179
	100	C91	EC	.	.	.	.	.12995

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
101	C92	EC	.	.	.	.	.09752
102	C93	EC	.	.	.	.	.06588
103	C94	EC	.	.	.	.	.03360
104	C101	BS	.	.	.	.	.
105	C102	BS	.	.	.	.	.
106	C103	BS	.	.	.	.	.
107	C104	BS	.	.	.	.	.
108	C111	EC	.	.	.	.	.06965-
109	C112	EC	.	.	.	.	.06943-
110	C113	EC	.	.	.	.	.06550-
111	C114	EC	.	.	.	.	.05593-

SECTION 2 - COLUMNS

	NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
	112	Y05	BS	10.63830	.	.	NONE	.
	113	Y06	LL	.	.	.	NONE	.12911
	114	Y08	BS	297.89916	.	.	NONE	.
	115	Y098	BS	.	.	.	NONE	.
	116	Y095	BS	.	.	.	NONE	.
	117	Y00	LL	.	.	.	NONE	.10219
	118	Y01	BS	.	.	.	NONE	.
	119	Y0N7	LL	.	.	.	NONE	.05541
	120	Y0P8	LL	.	.	.	NONE	.16096
A	121	Y07	LL	.	.	.	NONE	.
	122	Y15	BS	4.74974	.	.	NONE	.
	123	Y16	BS	.99696	.	.	NONE	.
	124	Y18	BS	296.63705	.	.	NONE	.
	125	Y198	BS	1.48159	.	.	NONE	.
	126	Y195	BS	.00003	.	.	NONE	.
	127	Y10	BS	.35514	.	.	NONE	.
	128	Y11	BS	.03730	.	.	NONE	.
	129	Y1N7	LL	.	.	.	NONE	.93738
	130	Y1P8	BS	.00018	.	.	NONE	.
A	131	Y17	LL	.	.	.	NONE	.
	132	Y25	BS	5.02701	.	.	NONE	.
	133	Y26	BS	1.43359	.	.	NONE	.
	134	Y28	BS	295.38029	.	.	NONE	.
	135	Y298	BS	1.20635	.	.	NONE	.
	136	Y295	BS	.00012	.	.	NONE	.
	137	Y20	BS	.48595	.	.	NONE	.
	138	Y21	BS	.07095	.	.	NONE	.
	139	Y2N7	LL	.	.	.	NONE	.92561
	140	Y2P8	BS	.00051	.	.	NONE	.
	141	Y27	BS	.	.	.	NONE	.
	142	Y35	BS	5.06823	.	.	NONE	.
	143	Y36	BS	1.89248	.	.	NONE	.
	144	Y38	BS	294.12686	.	.	NONE	.
	145	Y398	BS	1.17414	.	.	NONE	.
	146	Y395	BS	.00021	.	.	NONE	.
	147	Y30	BS	.50436	.	.	NONE	.
	148	Y31	BS	.07829	.	.	NONE	.
	149	Y3N7	LL	.	.	.	NONE	.91591
	150	Y3P8	BS	.00076	.	.	NONE	.
A	151	Y37	LL	.	.	.	NONE	.
	152	Y45	BS	5.08158	.	.	NONE	.
	153	Y46	BS	2.35133	.	.	NONE	.
	154	Y48	BS	292.88273	.	.	NONE	.
	155	Y498	BS	1.16962	.	.	NONE	.
	156	Y495	BS	.00013	.	.	NONE	.
	157	Y40	BS	.50741	.	.	NONE	.
	158	Y41	BS	.07956	.	.	NONE	.
	159	Y4N7	LL	.	.	.	NONE	.91[illegible]
	160	Y4P8	BS	.00100	.	.	NONE	.

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
161	Y47	BS	.	.	.	NONE	.
162	Y55	BS	1.77039	1.00000-	.	NONE	.
163	Y56	BS	2.80752	.	.	NONE	.
164	Y58	BS	291.64187	.	.	NONE	.
165	Y598	BS	2.33663	.	.	NONE	.
166	Y595	BS	.00070	.	.	NONE	.
167	Y50	BS	1.01577	.	.	NONE	.
168	Y51	BS	.15955	.	.	NONE	.
169	Y5N7	BS	.01979	.	.	NONE	.
170	Y5P8	BS	.00245	.	.	NONE	.
171	Y57	BS	10.78932	.	.	NONE	.
172	U15	BS	1.04343	1.00000	.	NONE	.
173	U16	BS	.	.	.	NONE	.
174	U18	LL	.	.	.	NONE	.02930
175	U198	BS	1.48159	.	.	NONE	.
176	U195	BS	.00003	.	.	NONE	.
177	U10	BS	.35514	.	.	NONE	.
178	U11	BS	.03730	.	.	NONE	.
179	U1N7	BS	.00457	.	.	NONE	.
180	U1P8	BS	.00018	.	.	NONE	.
181	U17	BS	16.49779	.	.	NONE	.
182	U25	BS	3.37224	1.00000	.	NONE	.
183	U26	BS	.	.	.	NONE	.
184	U28	LL	.	.	.	NONE	.02213
185	U298	BS	1.20635	.	.	NONE	.
186	U295	BS	.00012	.	.	NONE	.
187	U20	BS	.48595	.	.	NONE	.
188	U21	BS	.07095	.	.	NONE	.
189	U2N7	BS	.00950	.	.	NONE	.
190	U2P8	BS	.00051	.	.	NONE	.
191	U27	BS	10.77340	.	.	NONE	.
192	U35	BS	3.31686	1.00000	.	NONE	.
193	U36	BS	.	.	.	NONE	.
194	U38	LL	.	.	.	NONE	.01509
195	U398	BS	1.17414	.	.	NONE	.
196	U395	BS	.00021	.	.	NONE	.
197	U30	BS	.50436	.	.	NONE	.
198	U31	BS	.07829	.	.	NONE	.
199	U3N7	BS	.01289	.	.	NONE	.
200	U3P8	BS	.00076	.	.	NONE	.
201	U37	BS	10.76675	.	.	NONE	.
202	U45	BS	3.31585	1.00000	.	NONE	.
203	U46	BS	.	.	.	NONE	.
204	U48	LL	.	.	.	NONE	.00801
205	U498	BS	1.16982	.	.	NONE	.
206	U495	BS	.00013	.	.	NONE	.
207	U40	BS	.50741	.	.	NONE	.
208	U41	BS	.07956	.	.	NONE	.
209	U4N7	BS	.01635	.	.	NONE	.
210	U4P8	BS	.00100	.	.	NONE	.
211	U47	BS	10.77994	.	.	NONE	.

A LINEAR PROGRAMMING REACTOR

REFUELING MODEL

by

STEVEN KAY CLARK

B.S., Kansas State University, 1970

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Nuclear Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1971

ABSTRACT

A linear programming reactor refueling model was developed to minimize the U-235 requirements of a power reactor. The model consisted of a fuel cycle of five, one year, irradiation periods.

A fuel burnup code was used which made the following assumptions: 1) one homogeneous region, 2) known neutron fluence for each period, 3) one neutron energy group (thermal), 4) no U-238 fission, and 5) batch irradiation. The burnup equations were linearized for the model by using only nuclide concentrations at the end points of each irradiation period.

The constraints of the model include a maximum and minimum enrichment constraint, a multiplication factor constraint, a volume constraint, and nuclide refueling and removal constraints.

Several different refueling cases with varying degrees of Pu recycle were studied. The effect of the percent of Pu recycle on the burnup and buildup of the nuclides in the reactor was also studied. This study revealed that the value of plutonium decreases as the percent of Pu recycled increases.