

Bin packing approach for identifying minimum cost coalition structure

by

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Abstract

Humanitarian logistics involves the crucial effort of delivering aid to affected populations during crises such as natural disasters or disease outbreaks. The success of these operations relies on managing goods, services, information, and resources across multiple planning stages. Timely response is essential for survival and recovery, and the required scale and complexity often exceed the capabilities of a single organization. When multiple organizations work together in a relief effort, they form what is known as a coalition, which can improve efficiency, effectiveness, and impact. The opportunity for cooperation raises an important question: how can these organizations best partner with one another to form coalitions? This challenge of determining the most effective arrangement of organizations into coalitions, called the optimal coalition structure, is critical for improving the effectiveness of humanitarian aid delivery.

This thesis focuses on solving the optimal coalition structure problem when the costs incurred by cooperating organizations have particular features that are common in humanitarian settings. Specifically, this work considers setting in which cost is a function of coalition size, and marginal costs decrease as size increases up to a point, after which marginal costs increase. Thus, the cost exhibits economies of scale in part of the function's range, and diseconomies at the upper end of the range. Such cost functions are non-subadditive. The goal of the optimal coalition structure problem is to find the set of coalitions that minimizes the total costs.

To address the challenge of efficiently organizing coalitions, this thesis introduces a heuristic solution approach inspired by the bin packing problem. Bin packing is a classical operations research problem that aims to fit objects of different sizes into the fewest number of containers, each with a specified capacity. Similarly, the bin packing approach to the optimal coalition structure problem views each organization as an item with a given size and attempts to

pack these organizations into as few coalitions (bins) as possible. Bin capacity is chosen to take advantage of the range in the cost function with economies of scale. The bin packing approach aims to determine an optimal, or near-optimal, coalition structure with less computation time than solving the optimal coalition structure problem directly.

The novel approach presented by this research contributes to the understanding of optimal coalition structure problems for scenarios with non-subadditive costs. This study provides a practical tool to enhance cooperation among relief organizations by reducing the time required to find a near-optimal solution to the coalition structure problem. Also, this study sets the stage for future research, particularly in developing strategies for equitable cost distribution among organizations within coalitions and further refining the bin packing approach to achieve results that are even closer to optimal solutions. Through its contributions, this thesis not only advances academic knowledge in the field of humanitarian logistics, but also presents actionable insights for improving disaster response operations.

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Dedication

To my parents, Dan and Sara, who were my first teachers in life. Your unconditional love and support have not only helped me navigate challenges in my academic journey but have also encouraged me to consistently apply myself. This thesis is not just a reflection of my studies, it is a result of your efforts in raising me as well as the values and lessons you have imparted upon me. I am extremely grateful for both of you, your belief in me, and for teaching me that with hard work and determination, anything is achievable. This accomplishment is as much yours as it is mine.

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Chapter 1 - Introduction

Humanitarian logistics is the non-profit practice of managing the delivery of aid in response to a crisis, such as a natural or human-made disaster or disease outbreak. The inherent challenges presented by humanitarian logistics require innovative solutions that can enhance the efficiency and effectiveness of relief operations. This thesis aims to contribute practical tools and valuable insights to humanitarian logistics by addressing the critical need for enhanced cooperation between actors overseeing these complex operations.

1.1 Research Motivation

In humanitarian settings, the success of a crisis-relief operation directly impacts the well-being of the affected populations. This thesis refers to conditions in humanitarian logistics as humanitarian settings. Efficiently orchestrating relief operations in humanitarian settings is crucial and has the potential to save and improve lives. While the principles of logistics and supply chain management are extensively researched, humanitarian settings introduce challenges that are uncommon in traditional commercial supply chains. The presence and timeliness of relief operations can mean the difference between life and death. However, efficient use of resources can help aid reach more people. Often, a single organization cannot handle a disaster alone without becoming overwhelmed. By delegating tasks and splitting financial burdens among multiple organizations involved in a relief operation, the chances of an operation being successful increase.

When multiple organizations work together, or cooperate, the resulting partnership is a coalition. In scenarios with many organizations participating in relief operations, multiple coalitions typically form, each consisting of a distinct set of members. A coalition structure details which organizations belong to each coalition when multiple coalitions are involved in

relief operations. In humanitarian settings, an optimal coalition structure incurs the least cost or achieves the greatest benefit to society, depending on the operation's objectives. The counterpart of the coalition structure problem is the issue of determining allocations, which concerns how to equitably distribute costs or benefits among coalition members. This study addresses the challenge of determining which organizations should cooperate. It specifically investigates what coalition structure should be utilized to minimize operational costs. Existing methods for determining optimal coalition structure are computationally demanding and time-consuming. This research aims to develop an approach for identifying optimal, or near-optimal, coalition structures more efficiently.

1.2 Proposed Methods and Research Contributions

Research in humanitarian logistics has extensively investigated the benefits of cooperation and the challenges faced by organizations in relief operations. However, a notable gap in research exists considering scenarios where forming the grand coalition is not the most effective strategy for minimizing operational costs. The grand coalition is formed when all organizations present in relief operations for a specific crisis cooperate and form a single coalition. Scenarios where the grand coalition is not desirable demand tailored approaches to determining a coalition structure that incurs minimal costs and equitably allocates costs among coalition members.

To bridge this gap, this thesis proposes a novel approach inspired by bin packing methodology. Bin packing is commonly used in logistics to optimize the arrangement of items in containers. In this thesis, bin packing serves as a heuristic for identifying optimal, or near-optimal, coalition structures by viewing organizations as items to pack into coalitions. This approach aims to develop a procedure that quickly suggests how organizations could best

cooperate in humanitarian settings that involve purchasing goods or services for relief operations. In the proposed method, each organization has an order quantity, and these quantities are used to strategically pack organizations in coalitions. To develop a robust approach, this research examines various scenarios characterized by differing numbers of organizations and total order quantities.

The primary contribution of this research lies in its development and validation of a novel approach that dramatically decreases the time required to identify near-optimal coalition structures. This research offers a practical tool that can be readily applied to real-world scenarios to facilitate the discovery of advantageous partnerships among organizations. The findings reveal the potential of the bin packing approach to expedite planning response strategies in relief operations, ultimately enabling organizations to serve affected populations more efficiently.

1.3 Thesis Outline

This thesis is structured to provide a comprehensive exploration of the proposed methodology and its implications in humanitarian settings. Chapter 2 reviews relevant literature in the context of humanitarian logistics and game theory applications. Chapter 3 describes the proposed bin packing approach and the computational study design for analyzing its performance. Chapter 4 discusses the computational results, including an evaluation of the bin packing approach's effectiveness. Finally, Chapter 5 concludes this thesis by summarizing key findings and suggesting areas for future research.

Chapter 2 - Literature Review

Chapter 2 presents a literature review of materials related to humanitarian logistics and game theory. The chapter begins with an introduction to humanitarian logistics, then transitions to an overview of cooperation in humanitarian logistics. The discussion then explores how game theory concepts intersect with relief operations, followed by an examination of bin packing and its applications in logistics. The chapter concludes with an identification of research gaps and how this thesis addresses them, laying the groundwork for the methodology and analyses that follow.

2.1 Introduction to Humanitarian Logistics

Humanitarian logistics is the practice of alleviating populations suffering from infectious disease outbreaks or disasters, both natural and human-made, as well as supporting long-term development operations such as routine immunization and health care provisions. This is achieved through strategic planning and control of the flow of goods, services, information, and resources. Supply chains in humanitarian logistics are often referred to as relief chains. Due to the nature of humanitarian logistics operations, there are many parallels between activities performed in commercial supply chains and relief chains. However, relief chains are shrouded in greater uncertainty compared to commercial supply chains, which makes planning operations immensely more difficult [1, 2]. In turn, adequate planning and proper resource management are critical components of successful humanitarian logistics operations [3].

2.1.1 Planning Stages

There are multiple ways to categorize supply chain decisions in humanitarian settings. One common approach is to divide humanitarian logistics operations into those activities that occur in two distinct stages: pre-disaster and post-disaster. Pre-disaster operations primarily

focus on procurement and resource pre-positioning, whereas focus shifts to distribution and additional procurement needs post-disaster [3]. These stages are often further broken down into phases such as “mitigation, preparedness, response, and recovery” [4]. In operations aimed at enhancing public health, long-term strategies within the mitigation and preparedness phases often include routine immunizations and the maintenance of sufficient healthcare supplies to manage and treat infectious disease outbreaks. Although the practices within these phases vary based on the nature of the disaster, these steps are present in nearly all humanitarian logistics operations.

2.1.2 Resource Management

Due to the uncertain nature of disasters, a common challenge faced by relief chains is matching supply to demand [3]. When disasters can occur anywhere at any time, globally, resource management is a top priority in humanitarian logistics operations. Proper resource management aids in minimizing harms and lasting effects wrought by disasters [5]. Through efficient use of supplies and strategic placement of resources, relief operations can maximize the effectiveness of their efforts and respond to disasters in a timelier manner [4, 5]. Clearly, time itself could be considered the most impactful resource on the success of a humanitarian logistics operation.

2.2 Cooperation in Humanitarian Logistics Operations

Without cooperation, organizations involved in humanitarian logistics would often find themselves spread too thin. The greater the disaster magnitude, the more effort required to meet the affected populations’ needs. Regardless of how many resources an individual organization may have at their disposal, it is nearly impossible for a single actor to respond effectively to a

widespread, catastrophic disaster [3]. Fortunately, many activities involved in facilitating a relief chain can be executed jointly.

2.2.1 Cooperative Activities

The most common cooperative activities in humanitarian logistics fall under the categories of procurement, transportation, and warehousing [3]. These three types of activities lend themselves to cooperation as their associated costs can be explicitly divided among organizations based on individual operational needs for goods or services. Splitting costs associated with these activities can result in cost savings that benefit all parties involved [1]. In particular, cost structures for procurement and transportation that exhibit economies of scale are especially advantageous for joint operations since greater collective needs translate to lower marginal costs [1, 2].

2.2.2 Advantages of Cooperation

While the main benefit from cooperating with other organizations in humanitarian logistics operations is realized cost savings, other advantages exist that make cooperation attractive. These benefits include improved operational efficiency and effectiveness, reduced uncertainty, risk sharing, data sharing, access to otherwise cost-prohibitive services, and quicker response time [2, 3, 6, 7, 8].

2.2.3 Drawbacks of Cooperation

Although there are advantages to cooperative relief operations, there are drawbacks as well. The main challenge presented by cooperating in humanitarian logistics is that the setting typically requires immediate response, but when there are more groups involved in decision making it may take longer to reach an agreement [1]. Additionally, lack of coordination can hinder operational effectiveness and result in wasted efforts [2, 6, 9]. Other inefficiencies

brought about by cooperation are the result of working with unfamiliar people from different backgrounds such as language barriers and cultural conflicts [6].

2.2.4 Barriers to Cooperation

Even though there are many possible advantages of cooperating in humanitarian logistics, there are barriers that reduce organizations' ability to cooperate. With most non-governmental organizations (NGOs) relying on donations to fund their efforts, NGOs occasionally compete against one another for funding [1]. Another barrier to cooperation is a lack of trust between organizations that are unfamiliar with each other [10]. Both reasons portray the difficulty of finding a suitable partner with whom to cooperate. The benefits of cooperation outlined in section 2.2.2 are not fully achievable in all cases due to limitations in existing approaches to cooperation [1].

2.2.5 Facilitating Cooperation

In commercial supply chains, the use of contracts to facilitate cooperation among organizations has been researched since the late 1980s [11]. Considering that commercial supply chains are concerned with reducing costs to generate profit, companies place great importance in optimizing their operations [12]. However, misaligned interests may emerge in cooperative partnerships as a result of differing risk tolerances or conflicting performance metrics [11]. When cooperating players have differing priorities, the performance of the partnership suffers [12]. To remedy this issue, contractual arrangements have been developed with the intent of aligning partnering firms' objectives using profit sharing as an incentive [12, 13]. Some common practices present in contractual agreements aimed at facilitating coordination between buyers and suppliers include quantity discounts, policies for processing returns, and backup agreements for addressing uncertainty [11]. Although these systems are not perfect, Cachon finds that

contractual agreements aid in mitigating the likelihood of selfish behavior in cooperative supply chain operations [13].

To facilitate cooperation among organizations in humanitarian logistics, a similar approach for cost sharing may be used in relief chains to make cooperation more attractive. Revenue sharing contracts also improved operational effectiveness in some instances, which would have desirable implications in humanitarian logistics [13].

2.3 Game Theory Applications in Humanitarian Logistics

Game theory is the mathematical study of scenarios in which two or more parties, or players, choose from among defined sets of strategic actions, and the outcomes for players depend on others' choices. Players are assumed to be rational decision makers who identify a favorable action from their strategy set to reach a desired outcome [14]. Payoffs are the result of the combined actions of all players in a game [14]. Each player maintains an objective, which can be expressed mathematically, that they wish to optimize using the payoffs associated with their actions [15]. Games are categorized by players' objectives, strategy sets, and payoffs associated with their actions.

2.3.1 Game Types Present in Humanitarian Logistics

Many scenarios in humanitarian logistics can be modeled using game theory. One way to classify the models employed in literature is as cooperative and non-cooperative games. Cooperative games are used to model situations in which players may enter binding agreements; doing so may enable them to achieve greater benefits or lower costs, and the focus of the models is on identifying which players cooperate and how they divide the benefits or costs among them [1]. Non-cooperative games are used to model scenarios where binding agreements between players are not possible, and players act independently to optimize their own payoffs [1].

2.3.2 Basics of Cooperative Games

This thesis uses cooperative game models, and the notation for such models is introduced here. Let N represent the set of players in a game [15]. Players may cooperate and form groups called coalitions. Coalitions are denoted as S , where S is a subset of N [15]. A coalition's worth is measured by its characteristic function value $v(S)$ [15]. This is interpreted as the total benefit or total cost generated by the coalition. If a coalition is formed, its worth $v(S)$ can be divided among its members in any way imaginable [15]. If a coalition is empty and contains no players, it is assigned a characteristic function value of zero [15]. In instances where all players cooperate, the group is known as the grand coalition. Using cooperative game theory models, coalitions in humanitarian logistics may be directly compared using their characteristic function values as a measure of their worth to society.

2.3.3 Characteristic Functions

A coalition's value is determined by its membership. The characteristic function maps players, or sets of players, to real numbers. A game is categorized as additive if all players' collective value is equal to the sum of the values of all players playing independently [16]. However, when synergies exist between players in a game, their cooperation yields a characteristic function value greater than the sum of players' individual values, resulting in a superadditive game [16]. Superadditive games are desirable in the context of benefits, monetary or otherwise, such that cooperation results in a group earning greater benefits than if each player had performed individually. On the other hand, a game is subadditive if a coalition's value is less than the sum of all member's individual values [16]. In games where the characteristic function represents costs, subadditivity is desirable since cooperation translates to lower total costs. In

games with superadditive benefits or subadditive costs, the grand coalition is optimal, meaning that the greatest total benefit (least total cost) is achieved when all players cooperate.

2.3.4 Coalition Structures

Superadditive and subadditive games, whose conditions are defined in section 2.3.3, promote formation of the grand coalition to optimize total costs or benefits [17]. However, in non-superadditive and non-subadditive games, the sum of all players' contributions does not always yield the least cost (greatest benefit) [15]. As a result, there is more importance placed on examining specific interactions among players to identify which cooperative partnerships among players yield the greatest benefit or the least cost. In cooperative game theory, such a cooperative arrangement is called a coalition structure, or mathematically, partition of the game's players [15]. The coalition structure problem consists of identifying the partition with least cost (greatest benefit). The value of a coalition structure is the sum of each coalition's corresponding characteristic function and is utilized to determine which coalition structure should be formed [18].

When examining coalition structures in the context of cost allocation games, an optimal coalition structure value is one that minimizes total costs. In this context, the optimal coalition structure yields a minimal value for the sum of coalitions' cost functions and is referred to as the least cost coalition structure [19]. Identifying an optimal coalition structure is computationally hard [20, 21]. This difficulty is attributed to the relationship between the number of players in a game and the number of possible coalitions [22]. Although for small problems, it is possible to enumerate all coalitions and compare coalition structure values, this quickly becomes infeasible for larger problems [22]. As the number of players in a game increases, the number of possible coalitions grows exponentially and results in a solution space that is too large for exhaustive

search [22, 23]. In essence, analyzing games with many players requires a substantial amount of time that may hinder the ability to make decisions in a timely manner. As a result, reducing computational time for identifying the least cost coalition structure enables humanitarian logistics entities to make quicker decisions.

Due to the computational time required for solving the coalition structure problem, many heuristic approaches and their performance have been analyzed in previous research [24]. To address the time intensive nature of this problem, Fatima and Woolridge presented polynomial-time solvable cases for determining coalition structure through use of player ordering and monotonicity measures that reduced the problem's complexity from exponential to polynomial for some instances [20]. Shen et al. proposed use of a particle swarm optimization (PSO) algorithm for efficient search of coalition structure values and found that PSO achieved best model fitness in far fewer iterations compared to traditional genetic algorithms for games with any number of players [25]. Prántare et al. explored Monte Carlo techniques and heuristic algorithms that found solutions that improved with each iteration within 100 seconds for games with up to 300 players [21]. Ghazanfar and Prugel-Benne developed novel heuristics, namely LocalSearch and GreedySearch, for determining optimal coalition structure and demonstrated their effectiveness at quickly producing “good enough” solutions for benefit-maximizing games with a large number of players [22]. For the LocalSearch heuristic, Ghazanfar and Prugel-Benne reported solutions with total benefits greater than 80% of those achieved by optimal coalition structure for eight to 15-player games and greater than 75% of optimal for 16 to 22-player games; these were found in under 400 milliseconds. Similarly, their GreedySearch heuristic returned solutions whose total benefits were 70-75% of those in the optimal coalition structure within 400 milliseconds for games with 15 to 27 players.

All literature reviewed was concerned with optimal coalition structure in the context of maximizing total benefits. The approaches used in the literature mentioned could potentially be modified to aid in identifying the optimal coalition structure for non-subadditive games concerned with the least cost coalition structure. However, these contributions collectively emphasize the need for simple, innovative heuristic approaches to solving the coalition structure problem. Of the heuristic approaches to the optimal coalition structure problem reviewed, only the work of Ghazanfar and Prugel-Benne offers performance statistics suitable for direct comparison. This presents an opportunity for this thesis to contribute to the literature by directly comparing heuristic and optimal approaches.

2.3.5 Allocations

In the context of cooperative game theory, an allocation refers to an approach to dividing a coalition's costs (benefits) among the players in the coalition [26]. Since players are autonomous, their decisions about whether to participate in a coalition are likely to depend on whether the benefit or cost allocated to them is attractive compared to alternative allocations they might achieve outside of the coalition. The core is a widely used concept for determining whether an allocation will be acceptable to players in superadditive and subadditive games. Core allocations must satisfy two conditions: stability and budget balance [27]. An allocation is considered stable if there does not exist a better option for an individual, such as operating alone or forming a different coalition, where they would receive a more desirable allocation of greater benefits or less costs [16]. The budget balance condition means that the total cost (benefit) incurred by the grand coalition must equal total allocations to players [16]. It is important to note that the core of a game may be empty, suggesting that there are no budget-balanced, stable allocations for the scenario [16].

In games with non-superadditive benefits or non-subadditive costs, the grand coalition may not be the optimal coalition structure. The coalition structure core generalizes the definition of core allocations [28]. The coalition structure core is defined as the set of allocations in which the budget balance condition is met for each coalition, and the stability condition ensures that no players would be better off participating in a coalition other than the one they are assigned [29].

Allocations may be generated using many methods. One practical approach for determining an individual's allocation is proportionality, where an individual's share of cost or benefits is proportional to their contribution to the coalition's value [26]. However, proportionality does not guarantee stable allocations and as a result, such allocations may not belong to the core [26]. A widely studied allocation mechanism is Shapley value, which prioritizes fairness by allocating payoffs to players based on their marginal contributions to a coalition [15, 26]. In games with a non-empty core, Shapley value-based allocations are not guaranteed to belong to the core and become increasingly computationally intense to find as the number of players in a game grows [27]. Another common allocation mechanism discussed in literature is the nucleolus, which has the objective of minimizing total dissatisfaction among players [15]. In games where the core is non-empty the nucleolus is guaranteed to be in the core, but nucleolus-based allocations are also computationally intense and are impractical to calculate for large games [27]. In 2008, Özener and Ergun attempted to develop a simple algorithmic approach that consistently produced stable, budget-balanced allocations, but found that their models had limitations in satisfying the conditions required of cost allocations in the core [27].

Cost allocation games in humanitarian logistics are designed to distribute costs among coalition members equitably and stably, ensuring that no player has an incentive to operate outside of their assigned coalition [15]. These games aim to efficiently deliver aid to affected

populations at the least cost. The most well-studied cost allocation games are subadditive with characteristic functions referred to as cost functions [15].

2.3.6 Cooperative Game Applications in Logistics

Some examples of cooperative activities in humanitarian logistics are listed in section 2.2.1. However, an example more relevant to this research is joint procurement that leverages economies of scale to reduce overall procurement costs. Economies of scale is a term that refers to cost structures that exhibit smaller unit costs as the quantity of goods purchased or produced increases [30]. In one such cost structure, a given unit cost holds for procurement or production quantities in a range of values, and different quantity ranges have different unit costs. For example, quantities between one to 25 units would have a greater unit cost compared to quantities between 26 to 50 units. Economies of scale are present in many industries such as manufacturing, finance, and utilities [30]. In supplier-buyer relationships, specifically in manufacturing and procurement, economies of scale and associated savings are the result of reductions in operating costs achieved by larger production lot and order sizes [30]. Cooperation between buyers in procurement for humanitarian logistics operations enable organizations to leverage economies of scale and reduce unit cost through purchasing goods in bulk and splitting transportation costs [1].

To reap the benefits of cooperation, it is necessary for players to determine what coalitions to form and how to allocate costs or benefits within coalitions. In their 2021 study, Ben Jouida et al. discussed the formation of profit-maximizing coalitions with shared capacities in distribution networks by utilizing cooperative game theory [31]. They used a Collaborative Warehousing Algorithm to determine optimal coalition structures, from which they made allocations using three approaches: Shapley value, egalitarian, and proportional allocations [31].

Their analysis revealed the potential for substantial distribution cost savings through strategic cooperation and coalition formation in logistics operations [31]. In a 2010 study, Frisk et al. applied game theory and operations research principles to improve logistics efficiencies among Swedish forestry companies [32]. They made use of linear programming models to optimize operations for cooperation among eight players and computed allocations using Shapley value, the nucleolus, and the equal profit method [32]. Their analysis found that there were potential cost savings of 5-15% for each of the eight companies through strategic cooperation [32]. A 2015 study by Guajardo and Rönnqvist identified coalition structure using service level requirements and the concept of stability, and allocated costs in a fair manner that encouraged participation in a resource pool [33]. Although this study took Shapley value, the nucleolus, and equal profit method allocations into consideration, they proposed a novel approach they called the Minimum Deviation from Service Level Referential Cost Method that achieved the most desirable results. The proposed method led to roughly 20% in savings in the form of reduced inventory costs. Another practical application of cooperative game theory was presented by a 2022 study of Chilean wine grape harvesting which details use of game theory concepts to foster horizontal collaboration in supply chain operations. Varas et al. performed their analysis by modeling characteristic functions in the game using mixed integer programming and utilized three approaches for determining appropriate coalition structures and cost allocations: entropy, equal profit method, and Gini index method [34]. They found that the entropy method produced comparatively more stable and fair cost allocations among coalitions, and that the Gini index method favored use of smaller coalitions. A particular model configuration of their experiment, involving four permutations of blocks owned by different wineries that represent various cooperation scenarios, revealed that strategic coalition formation achieves an average cost

savings of 13.68%, demonstrating the potential of cooperative strategies in optimizing operational efficiencies. The literature highlights the application of cooperative game theory in forming strategic alliances that reduce costs by streamlining logistics operations and leveraging existing synergies between operations.

2.4 Bin Packing

This thesis makes use of parallels between the bin packing problem and the coalition structure problem to propose a heuristic approach for solving the latter. Bin packing is a classic operations research problem that is concerned with determining the minimum number of capacitated bins required for packing a set of items, each with a given size [35]. The bin packing problem shares characteristics with the coalition structure problem in the sense that both problems are concerned with finding the best way to partition a set with respect to an objective.

2.4.1 Bin Packing Applications in Logistics

In supply chain logistics, bin packing is a critical tool for minimizing shipping costs while maximizing space utilization. A common application of bin packing in logistics views vehicle trailers as bins and aims to pack them such that the number of trailers required for transporting goods is minimized [36]. By minimizing the number of trailers used to transport goods, companies are able to minimize resource usage and improve operational efficiency [36]. Bin packing is also utilized in humanitarian logistics scenarios, such as identifying minimum fleet size in vehicle routing problems [37]. For both commercial and humanitarian supply chains, minimizing resource use is desirable and aids in improving operational effectiveness.

2.5 Contributions in Context

The literature review identified two main gaps in the field of humanitarian logistics: the optimization of coalition structures beyond the grand coalition to minimize operational costs and

the challenge of equitable cost allocations among coalition members. While the primary focus of this thesis is to introduce a novel bin packing-inspired methodology for identifying optimal or near-optimal coalition structures, it acknowledges but does not directly tackle the allocation gap. By significantly reducing the time and computational resources required for identifying effective coalitions, this research contributes a practical tool to improve strategic planning and operational efficiency in the delivery of humanitarian aid. Though the issue of fair and stable allocations remains for future exploration, this research sets a foundation for enhancing the effectiveness of relief operations and opens avenues for future work to build upon, including the development of strategies for equitable cost distribution within coalitions.

Chapter 3 - Methods

This chapter introduces the proposed bin packing approach for efficiently generating near-optimal solutions to the coalition structure problem. The focus is on procurement settings where costs exhibit both economies and diseconomies of scale. This problem is critical in humanitarian logistics, where efficient use of time and resources may significantly impact the success of a relief operation. Economies of scale in cost occur when the unit cost of goods or services decreases as order quantity increases, which presents a unique opportunity for pooling resources through cooperation. Identifying the most cost-efficient way to procure goods and services is critical in humanitarian logistics, where operations seek to minimize their crisis response time while maximizing the impact of their financial resources. The ability to leverage economies of scale through coalition formation has great potential for helping relief operations stretch limited resources while fostering cooperation across multiple organizations. The outlined approach identifies procurement strategies that leverage volume discounts while minimizing fixed ordering costs.

The coalition structure problem is computationally hard. This thesis introduces a bin packing solution approach for coalition formation and demonstrates its usefulness as a faster, less computationally intense avenue for finding optimal, or near-optimal, coalition structures. Computational time and proximity to objective value are the key performance metrics used to assess the bin packing methodology. This thesis compares factors present in instances where the bin packing approach generates high quality solutions with those whose solutions have less-desirable attributes. The resulting insights have implications for current practice in humanitarian and commercial supply chains.

Chapter 3 begins by summarizing research questions. Next, in section 3.2, an optimal coalition structure model is defined and formulated. Section 3.3 provides details on cubic characteristic functions and explains their use in this research. Then in Section 3.4, the bin packing model definition and formulation as well as the methodology for setting bin capacity are presented. Lastly, Section 3.5 details the computational study design.

3.1 Research Questions

Given the motivation for this research and the problem context, this research seeks to answer the following questions:

- How does the total cost of the coalition structure generated by the bin packing approach compare to that of the optimal coalition structure?
- How does the computational time required to generate an optimal or near-optimal coalition structure using the bin packing approach compare to that required to solve the optimal coalition structure problem?
- How do factors including the cost function, number of players, and player order quantity impact the effectiveness of a bin packing approach in forming optimal coalition structures?

These questions guide this research towards meaningful insights that can enhance relief operations in humanitarian logistics.

3.2 Optimal Coalition Structure Model

The problem of finding the least cost coalition structure is formulated as a binary integer programming problem. Table 3.1 details the necessary sets, parameters, and binary decision variables for the formulation of the optimal coalition structure model [38].

Table 3.1 - Optimal coalition structure model

Sets		
N	The set players	
S	The set of coalitions	
Parameters		
c_s	Cost of coalition s	$s \in S$
$x_{n,s}$	$\begin{cases} 1 & \text{if player } n \text{ is in coalition } s \\ 0 & \text{otherwise} \end{cases}$	$n \in N, s \in S$
Binary decision variables		
y_s	$\begin{cases} 1 & \text{if coalition } s \text{ is formed} \\ 0 & \text{otherwise} \end{cases}$	$\forall s \in S$

In terms of the notation above in Table 3.1, formulation of an optimal coalition structure model proceeds as follows:

$$\text{Minimize } \sum_{s \in S} c_s y_s \quad (3.1)$$

Subject to:

$$\sum_{s \in S} x_{n,s} y_s = 1 \quad \forall n \in N \quad (3.2)$$

$$y_s \in \{0,1\} \quad \forall s \in S \quad (3.3)$$

Equation (3.1) represents the objective function of minimizing total coalition structure cost. Constraint (3.2) requires that each player belongs in exactly one coalition within the coalition structure. Lastly, constraint (3.3) states that y_s is an element of a set containing zero and one, thus defining it as a binary decision variable.

Although the optimal coalition structure model may be solved exactly for small instances, it is an *NP*-hard problem [38]. The number of possible coalitions in a game can be found by

solving the combination expression $2^n - 1$ where n is the total number of players [18]. Two is used as the base of the expression since each player is either in a particular coalition, or not. One is subtracted from the number of combinations since the empty set is not considered a valid coalition. The number of players in games analyzed in this thesis ranges from two to 16. As a result, the number of coalitions ranges from three and 65,535. This thesis, therefore, proposes a heuristic to find optimal or near-optimal coalition structures. The heuristic solution objective function value is compared to that of the optimization model as a measure of the heuristic's effectiveness.

3.3 Cubic Characteristic Functions

For scenarios assessing the cost associated with procuring goods or services, cubic characteristic functions, or cubic cost functions, use a third-order equation to express total cost as a function of order quantity in units. Cubic cost functions are of the form: $C(x) = ax^3 + bx^2 + cx + d$, where a , b , c , and d are constants that determine the shape and position of the cubic function [39]. Games with a cubic cost function are non-subadditive, meaning that the grand coalition may not be the least cost coalition structure.

3.3.1 Relation to Economies of Scale

A procurement cost function takes the quantity to be purchased as input and outputs the total procurement cost. The cost-quantity relationship is visualized by graphing the cost function. In most cases, the y-axis corresponds to cost in monetary units such as dollars, while the x-axis denotes quantity of goods in units. Price ranges under economies of scale are often specified as piece-wise linear cost functions [40, 41]. Cost structures that utilize economies of scale may also be generalized using a cubic cost function [42]. Unlike piece-wise linear functions, which introduce discontinuities in price at the boundaries of each segment, cubic cost functions provide

smooth transitions between price ranges throughout the cost curve. For continuous non-linear cost functions, inflection points aid in identifying where unit cost transitions from decreasing to increasing [42]. Cubic functions have only one inflection point, which can be identified by setting the second derivative of the cost function equal to zero ($C''(x) = 0$) [39]. In this experiment, multiples of the cubic cost function's inflection point are used to determine where one price range ends and another begins.

3.3.2 Conditions of Cubic Cost Functions Characterizing Economies of Scale

Some additional conditions are necessary to ensure that a cubic cost function is relevant to the procurement context with economies and diseconomies of scale. The function must be increasing to ensure that total costs do not decrease as additional units are purchased [30]. For a cubic function to be monotonically increasing, the leading coefficient and the first derivative of the cost function must both be greater than zero ($a > 0, C'(x) > 0$) [39]. Additionally, to account for fixed order costs, the cost function must have a positive, non-zero y-intercept ($d > 0$). It is important to note that coefficients b and c shift the function along the x-axis. Shifting the cost function too far to the right may lead to a positive x-intercept and a negative y-intercept, which are undesirable for this study. From the customer's standpoint, procuring goods deals in positive cost and units, translating to a cost function that resides within the first quadrant of a graph. Through experiment, it was determined that monotonically increasing cost functions residing in the first quadrant are produced when $c \gg b$ where $b < 0$ and $c > 0$.

3.4 Bin Packing Approach to Least Cost Coalition Structure

This thesis proposes solving a related bin packing problem to identify optimal or near-optimal coalition structures. Applying the bin packing framework to the concept of optimal coalition structure requires that coalitions be viewed as bins that contain players. Each player has

an associated order quantity. All players must be assigned to bins such that each player is housed in only one bin, and the sum of order quantities associated with all players in a bin must not exceed the bin's capacity [35]. Capacities for the bin packing problem are determined by utilizing information from the characteristic function of the coalition structure problem. This section details the formulation of a bin packing model and the methodology used for determining bin capacity.

3.4.1 Bin Packing Model

To approach the optimal coalition structure problem using bin packing, a bin packing model in terms of player order quantity must first be defined. Table 3.2 contains all relevant information for constructing a bin packing model [35].

Table 3.2 - Sets, parameters, and decision variables for bin packing model

Sets		
I	The set of players to be packed, where n is the total number of players	
J	The set of bins, where k is the total number of bins	
Parameters		
a_i	Order quantity of player i	$a_i \in \mathbb{Z}^+, \forall i \in I$
Ω	Capacity of each bin	$\Omega \in \mathbb{Z}^+$
Binary decision variables		
x_{ij}	$\begin{cases} 1 & \text{if player } i \text{ is placed in bin } j \\ 0 & \text{otherwise} \end{cases}$	$\forall i \in I, \forall j \in J$
y_j	$\begin{cases} 1 & \text{if bin } j \text{ is used} \\ 0 & \text{otherwise} \end{cases}$	$\forall j \in J$

Using the notation specified in Table 3.2, formulation of the bin packing model is as follows:

$$\text{Minimize } \sum_{j \in J} y_j \quad (3.4)$$

Subject to:

$$\sum_{i \in I} a_i x_{ij} \leq \Omega \quad \forall j \in J \quad (3.5)$$

$$\sum_{j \in J} x_{ij} = 1 \quad \forall i \in I \quad (3.6)$$

$$x_{ij} \leq y_j \quad \forall i \in I, \forall j \in J \quad (3.7)$$

$$x_{ij}, y_j \in \{0,1\} \quad \forall i \in I, \forall j \in J \quad (3.8)$$

Equation (3.4) is the objective function which aims to minimize the total number of bins used to pack all items. By minimizing the number of bins (coalitions) used, the approach minimizes the number of times fixed order costs are incurred. Constraint (3.5) dictates that the total number in units of order quantity associated with players placed in bin j must not exceed the bin's capacity. Constraint (3.6) enforces that all players must be placed in exactly one bin. Constraint (3.7) states that players may only be assigned to bins in use. Finally, constraint (3.8) defines x_{ij} and y_j as binary decision variables.

3.4.2 Determining Bin Capacity

To perform bin packing using players' order quantities, bin capacity Ω is set equal to one-and-a-half times the inflection point of the characteristic function. If this calculation yields a decimal, bin capacity is rounded down to the nearest integer. The proposed bin packing approach aims to minimize the number of coalitions by utilizing as much of each bin's capacity as possible, such that each player's entire order quantity fits within a single bin. All cubic cost

functions analyzed in this experiment exhibit economies of scale up to a certain point, beyond which diseconomies of scale set in and marginal costs begin increasing as the number of units increases. This approach places emphasis on procuring goods or services within the portion of the cost function that exhibits economies of scale. Defining bin capacity as a scaled value of the inflection point aims to set bin capacity at the point where costs shift from economies to diseconomies of scale. This choice, motivated by the lack of marginal benefit beyond this threshold, enables the bin packing approach to minimize the number of coalitions by methodically grouping players into coalitions with total order quantities not exceeding this critical point. Additionally, scaling bin capacity in relation to the inflection point ensures that bin capacity is specifically adapted to each cubic cost function under analysis.

3.5 Computational Study Design

This section describes the instance characteristics and computational environment used to compare the proposed bin packing approach to the optimal coalition structure model. Randomly generated instances are solved using both models, and the objective function values and computation times are analyzed. An instance is specified by the characteristic function for the optimal coalition structure model, the total number of players, and the numbers of those players whose order quantities are small and medium (as defined in this section). This computational experiment was executed in Python using Gurobi version 11.0.0 on a machine powered by an Intel Xeon processor with a clock rate of 3.5 Gigahertz, equipped with 16 gigabytes of RAM.

3.5.1 Cubic Cost Functions Analyzed

Table 3.3 lists the five cost functions analyzed in this experiment. Each equation is assigned a letter, A through E, used to reference each cost function. In this experiment, cost

functions are denoted using $C(x)$ to display costs with purchasing as a function of order quantities in units (x). A visual representation of each cost function can be found in this section.

Table 3.3 - Cubic cost functions analyzed

ID	Cost Function
A	$C(x) = 0.0025x^3 - 0.245x^2 + 3x + 50$
B	$C(x) = 0.0015x^3 - 0.109x^2 + 3.818x + 4.388$
C	$C(x) = 0.25x^3 - 3.85x^2 + 20.75x + 9.25$
D	$C(x) = 0.0085x^3 - 0.202x^2 + 1.608x + 0.356$
E	$C(x) = 0.01x^3 - 0.4x^2 + 6.05x + 7.25$

Figure 3.1 depicts the graphical visualization of cost function A. Refer to Table 3.3 for the equation of cost function A.



Figure 3.1 - Cost function A visualized with price ranges

As shown in Figure 3.1, price range one for cost function A spans from zero units to 16 units, which is half of the inflection point's x -value. This first range uses a pricing strategy similar to minimum base charges found in less-than-truckload (LTL) shipping cost functions [43]. Price range two spans from 16 units to 49 units, with the upper range being equal to one-and-a-half times the inflection point's x -value. In the bin packing approach, bin capacity is set equal to the upper limit of unit quantity for price range two. This price range is the most desirable as it has the lower marginal increase in cost when adding additional units compared to price ranges one and three. The second price range has similarities to breakpoint discounts in LTL shipping, where costs decrease beyond specific volume thresholds [43]. Lastly, price range

three spans from 49 units and extends upwards. In this range, unit costs rise with each additional purchase, making coalition formation to procure goods or services in this range undesirable.

In all cost functions analyzed in this thesis, prices ranges one and two exhibit economies of scale, whereas price range three exhibits diseconomies of scale. A similar phenomenon is observed in LTL shipping when shipping capacity is exceeded, which leads to significantly higher shipping rates due to the premium associated with additional space beyond what is readily available. In cases where the inflection point's corresponding x-value is non-integer, all values for determining price ranges are rounded down to the nearest integer due to fractional units of goods being an undesirable measure in procurement settings. By utilizing the inflection point as a measure of marginal cost for price ranges, unit values associated with price ranges are uniquely tailored to each cost function. In turn, this approach for identifying price ranges enables detailed analysis of any increasing cubic characteristic function with a single inflection point.

Visualized in Figure 3.2, with its equation provided in Table 3.3, cost function B's first price range has an upper limit of 12 units. From there, it transitions into a second price range which spans up to 36 units. The third and final price range begins at 36 units.



Figure 3.2 - Cost function B visualized with price ranges

Displayed in Figure 3.3, with its equation in Table 3.3, the pricing structure of cost function C starts with a range up to three units. The second price range extends from three to seven units. Price range three's domain for cost function C begins at seven units.



Figure 3.3 - Cost function C visualized with price ranges

Cost function D's pricing intervals are graphically represented in Figure 3.4, with its equation shown in Table 3.3. Pricing within the first range stops at four units, followed by a second interval that stretches to 12 units. For cost function D, the third and final price range commences at 12 units and continues onward.



Figure 3.4 - Cost function D visualized with price ranges

The graph in Figure 3.5, alongside the equation in Table 3.3, maps out cost function E's structure. The initial price range ends at seven units, with a second price range extending to 20 units. Following 20 units, price range three onsets and continues across the spectrum of increasing unit purchases for cost function E. A graph depicting all five cost functions analyzed is located on page 64 in Figure A.1.



Figure 3.5 - Cost function E visualized with price ranges

3.5.2 Player Order Quantities

The characteristic function specifies the cost for each individual player and for each coalition in an instance. This cost depends on the order quantity. The computational analyses examine instances in which players' order quantities are randomly generated and belong to

ranges that depend on the shape of the characteristic function. In this experiment, two classifications of player order quantity were utilized. Small players are defined as those whose order quantity is between one unit and one-quarter of the unit upper limit for price range two. Medium players have order quantities between one-quarter and one-half of the unit upper limit for price range two. By inspecting games with small and medium players, the experiment neglects large players since they already have an order quantity that is advantageous for procuring goods within price range two. Table 3.4 presents the order quantity ranges used to classify small and medium players. Bin capacity, defined as Ω , is equal to the upper limit for purchasing units in price range two.

Table 3.4 - Order quantity ranges by player size

Player size	Order Quantity Lower Limit	Order Quantity Upper Limit
Small	1	$\frac{\Omega}{4}$
Medium	$\frac{\Omega}{4}$	$\frac{\Omega}{2}$

3.5.3 Player Size Dynamics

Player size dynamic refers to the number of small and medium players in an instance of the optimal coalition structure problem. While holding the number of total players in an instance constant, the number of small and medium players present in each game was varied. For example, Table 3.5 summarizes all six-player game configurations utilized in this experiment. The scope of this research is limited to instances where players of both sizes are present. Games consisting of purely small or medium players, where only one class of player type is present, were not of interest in this experiment.

Table 3.5 - Six-player game dynamics

Small Players Present	Medium Players Present	Total Players
1	5	6
2	4	6
3	3	6
4	2	6
5	1	6

The approach outlined above for varying the number of small and medium players present in a game is used for each n -player game inspected ranging from two to 16 total players. In total, 120 player size dynamics were utilized in analysis.

3.5.4 Comparing Computational Runtime

For each replicated instance of player size dynamic, the runtime required to return a solution using the bin packing approach and the optimal coalition structure model is stored. To facilitate a direct comparison, computational runtime is summed and divided by the number of replicated trials to compute an average runtime for both models using each player size dynamic. Furthermore, average runtimes for both models are aggregated by total players in a game to allow comparison of model runtimes by number of players.

3.5.5 Instances Solved

It is important to note that the optimal coalition structure problem is considered computationally intense, meaning that runtime associated with finding a solution increases rapidly as the total number of players in a game grows [20]. As a result, many instances with a small number of total players may be solved in the time required to return one solution for a game with a relatively large number of players. To facilitate timely generation of results for the five cost functions analyzed, the number of instances solved decreases as the total number of

players grows. Table 3.6 holds the number of instances solved for each player size dynamic based on the total number of players. In this computational experiment, 6,465 instances were solved across 120 distinct player size dynamics, for all five cost functions analyzed, for a total of 32,325 solved instances.

Table 3.6 - Number of instances solved by total number of players

Total Players (n)	Number of Player Size Dynamics	Number of Instances Solved for Each Dynamic	Total Number of Instances Solved
2	1	100	100
3	2	100	200
4	3	100	300
5	4	100	400
6	5	100	500
7	6	100	600
8	7	100	700
9	8	100	800
10	9	100	900
11	10	50	500
12	11	50	550
13	12	25	300
14	13	25	325
15	14	10	140
16	15	10	150
	120		6,465

Chapter 4 - Results

Chapter 4 delves into the findings of the computational experiment outlined in Chapter 3. The chapter begins with an examination of bin packing approach's performance and ends with a discussion of results.

4.1 Bin Packing Performance

This section evaluates performance with respect to two key metrics: proximity to minimum cost and computational runtime, by comparing the solutions identified by the bin packing approach and the optimal coalition structure model. These metrics are used to analyze the performance of the bin packing approach from two perspectives: 1) breakdown by player size dynamic and 2) an aggregate view by total number of players in a game.

First, the overall performance with respect to cost is summarized. For each cost function analyzed, the average cost ratio is determined by dividing coalition structure cost found through the bin packing approach by the optimal coalition structure cost across all solved instances for a specific player size dynamic. Table 4.1 displays the overall average and maximum of the average cost ratio, computed across all solved instances of each player size dynamic for the respective cost functions analyzed.

Table 4.1 - Average and maximum of average cost ratios relative to the bin packing approach's solutions for each cost function

Cost Function	Average of All Solved Instances'	Maximum of All Solved Instances'
	Average Cost Ratio	Average Cost Ratio
A	1.087	1.300
B	1.035	1.118
C	1.115	1.467
D	1.056	1.234
E	1.060	1.233

As shown in Table 4.1, cost function B has the lowest mean of average cost ratio and cost function C has the highest mean of average cost ratio. Using the maximum values for average cost ratio, cost functions B and C returned solutions that were at most 11.8% and 46.7% above optimal, respectively, representing the best and worst performances observed by cost function using the bin packing approach. Cost functions B and C are fully explored in this chapter to further analyze the bin packing approach from the perspective of best and worst cases of performance observed.

4.1.1 Proximity to Minimum Cost by Player Size Dynamic

Shown in Figure 4.1 are the average cost ratios associated with the number of small and medium players present in a game. The cost ratio for each instance is computed by dividing the coalition structure cost identified by the bin packing approach by the optimal coalition structure cost. The average cost ratio is then computed for each player size dynamic as the average cost ratio across all instances with that dynamic. The horizontal and vertical axes represent the number of small and medium players, respectively, while the cells within the graph denote the average cost ratio for each player size dynamic. The average cost ratio is color-coded by relative

magnitude, with lighter colors indicating lower values (bin packing solution is closer to optimal) and darker colors signifying higher values (bin packing solution is farther from optimal).

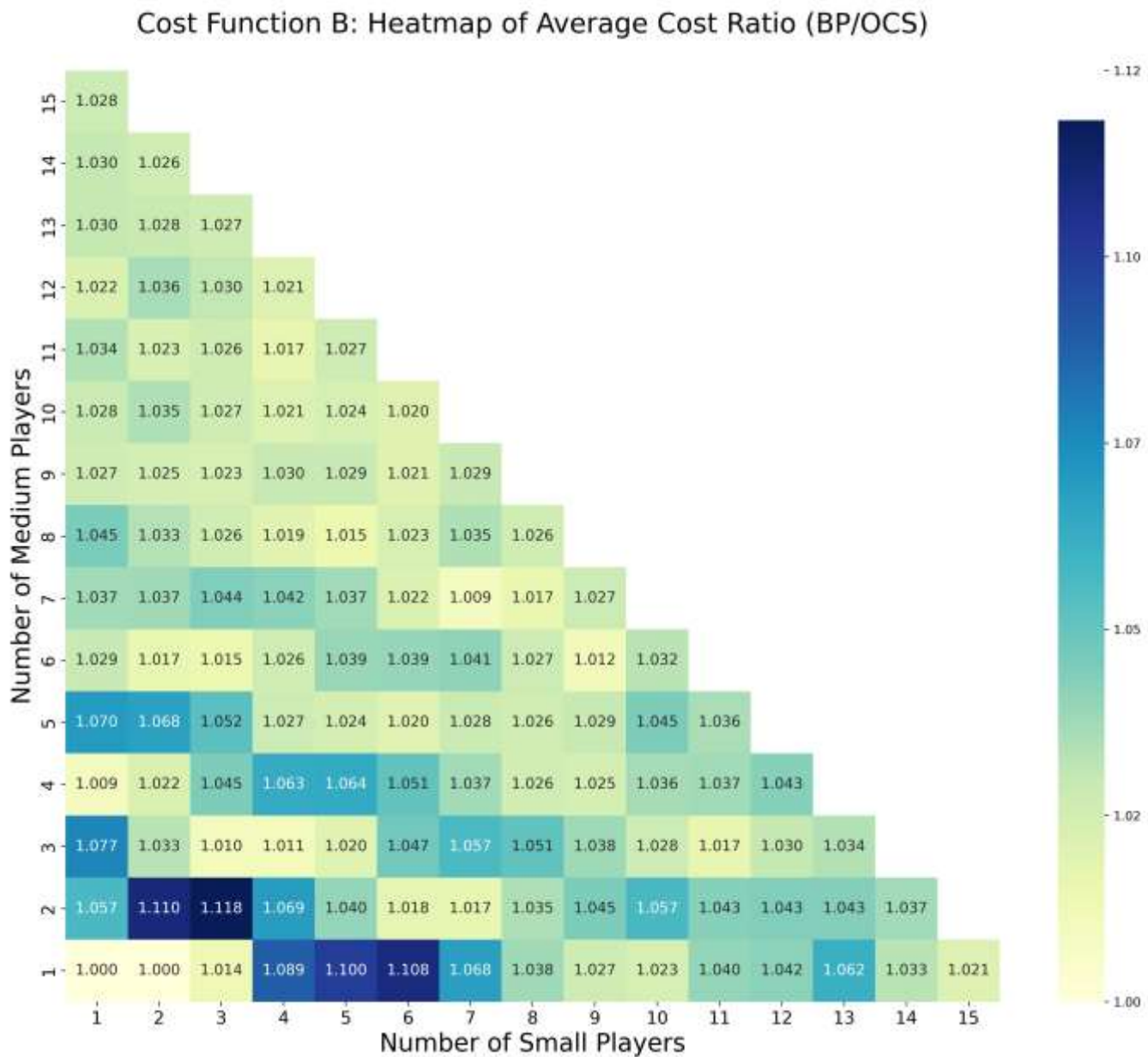


Figure 4.1 - Cost function B average cost ratio by player size dynamic

Of the five cost functions analyzed, replications completed for cost function B yielded the most impressive results. For cost function B, the greatest average cost ratio is equal to 1.118 and occurs in five-player game configurations consisting of two medium players and three small players. In this setting, the bin packing solution total cost is, on average, 11.8% greater than the optimal coalition structure cost. More notably, the average cost ratio for 16-player games is between 1.020 and 1.043 for all 16-player game configurations. The magnitude of the range in average cost ratio for 16-player games under cost function B is equal to 0.023. For 5-player games under cost function B, the difference between the maximum and minimum average cost ratios observed is equal to 0.109. Under cost function B, there is generally less variation in proximity to the optimal coalition structure cost when using the bin packing approach for games with a larger number of players.

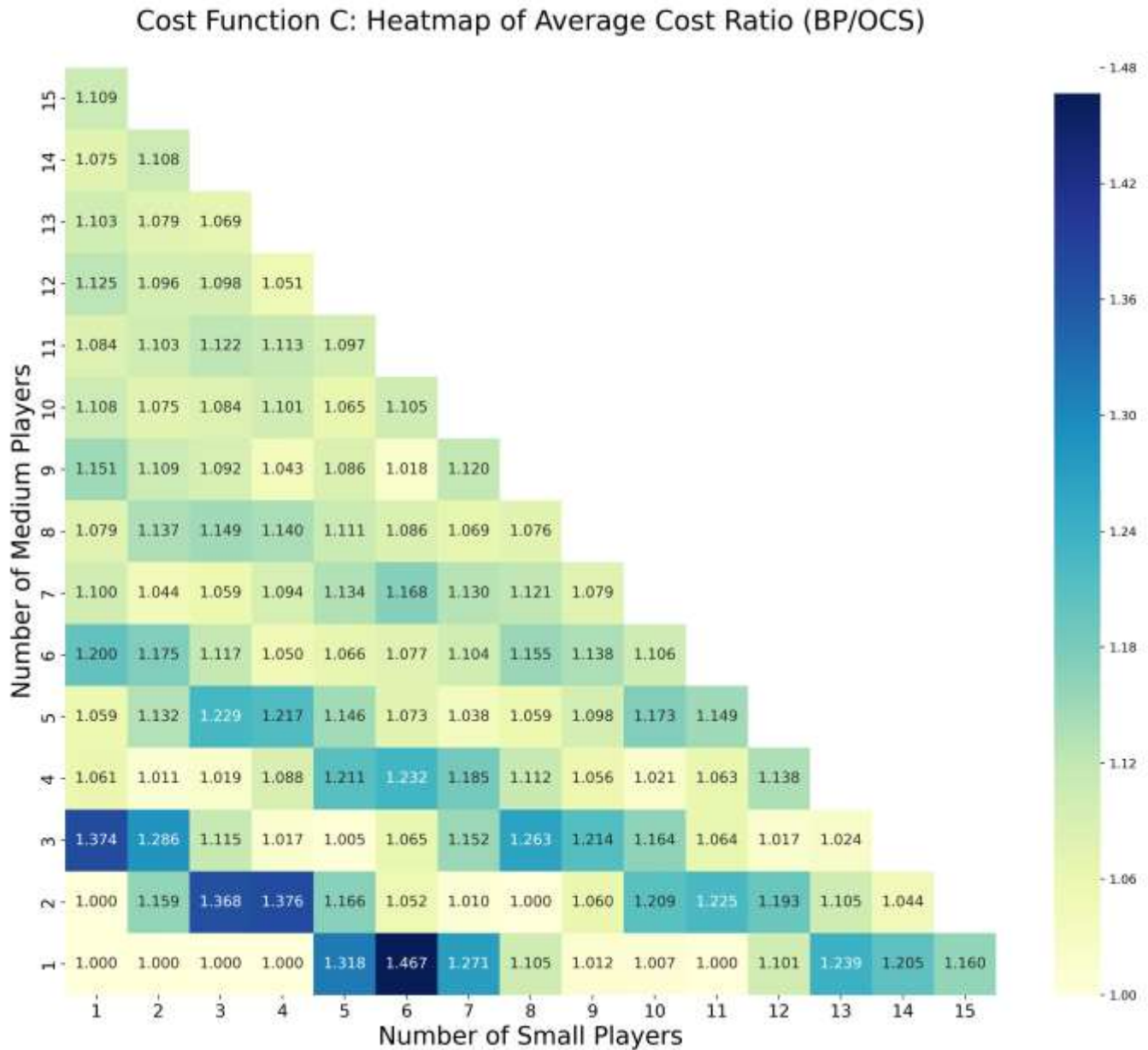


Figure 4.2 - Cost function C average cost ratio by player size dynamic

Figure 4.2 summarizes the average cost ratio by number of small and medium players for cost function C, which generated the least impressive results of the five cost functions analyzed. As shown in Figure 4.2, the bin packing approach performed the worst for seven-player games involving six small players and one medium player, returning solutions that were 46.7% above

optimal on average. However, the bin packing approach identified an optimal coalition structure in all 50 instances solved for 12-player games involving 11 small players and one medium player. The average cost ratio for 16-player games using cost function C is between 1.024 and 1.160, spanning a difference equal to 0.136. For seven-player games, the gap between the maximum and minimum average cost ratios observed is equal to 0.450. Although cost function C yielded the highest average cost ratio average of all cost functions analyzed, its performance in generating quality solutions for larger games remains impressive.

For both cost functions B and C, the difference between the maximum and minimum average cost ratios observed is significantly smaller in games with a relatively large number of players compared to games with a small number of players. The minimum average cost ratio for 16-player games using cost function C is close to that of cost function B. However, the maximum average cost ratio for 16-player games under cost function C is significantly higher than that of cost function B. This performance gap is due to the difference in the steepness of the cost functions. As shown in Figure A.1 on page 64, cost function C has the sharpest increase in cost per unit of the five cost functions analyzed. In contrast, cost function B is the flattest and features the most gradual increase in cost per unit. Figures B.1-3 on pages 65-67 provide average cost ratio heatmaps for cost functions A, D, and E, respectively.

4.1.2 Proximity to Minimum Cost by Total Number of Players

This subsection considers the average cost ratio in relation to the total number of players. Although variability in player size dynamic impacts solution quality for games with a small number of players, there is observably less variation in solution quality for larger games.

Figure 4.3 depicts the average cost ratio averaged across all solved instances for player size dynamics with the same number of total players using cost function B. The average cost

ratio under cost function B, averaged across solved instances with the same number of total players, peaks in four-player games, where the bin packing approach yields solutions with objective function values 6.69% greater than optimal on average. After this point, the aggregate average cost ratio abruptly decreases until it reaches 9-player games and then is stable between 9 and 16-player games.

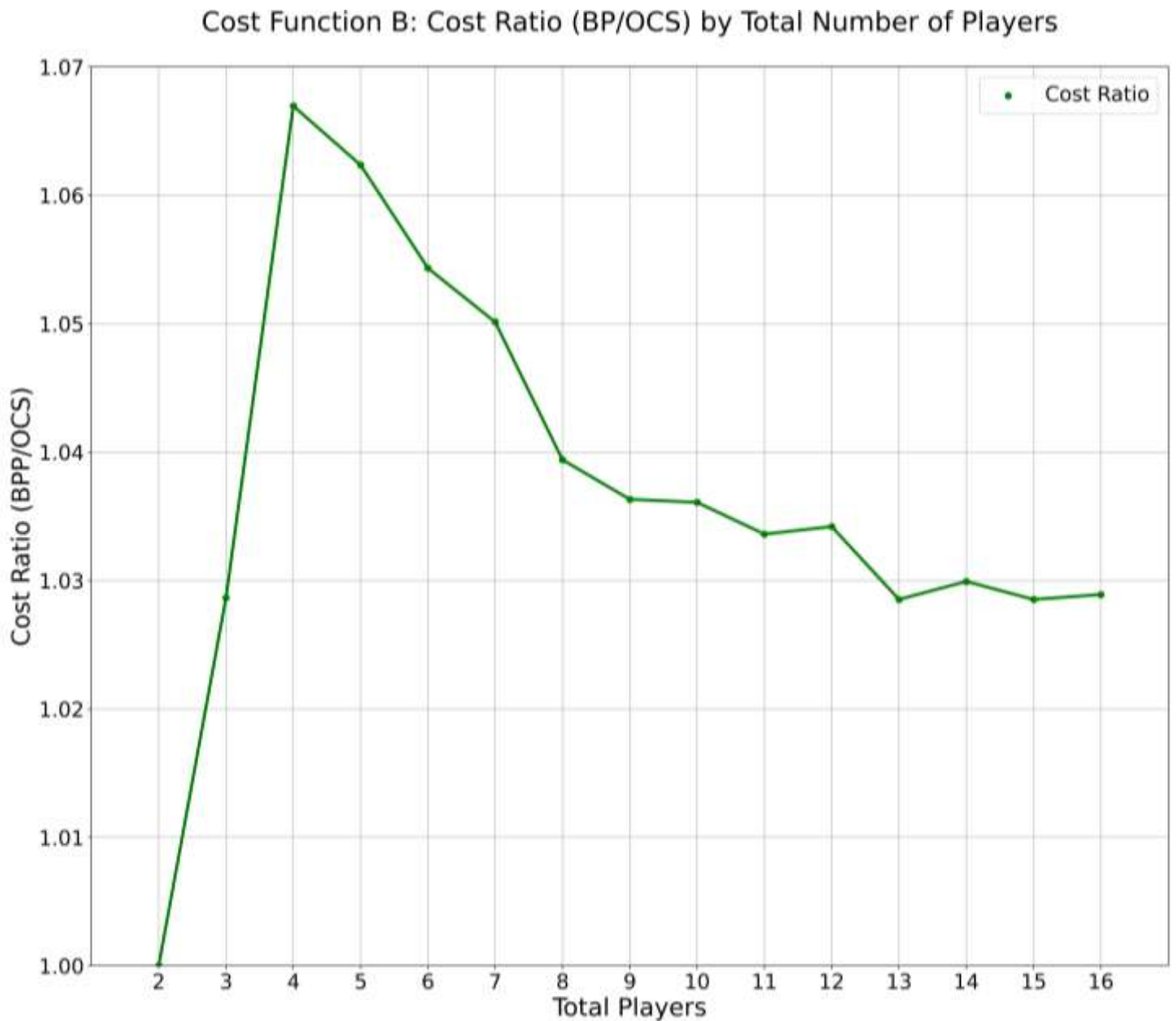


Figure 4.3 - Cost function B average cost ratio aggregated by total number of players

Similarly, Figure 4.4 charts average cost ratio, aggregated by total number of players, for cost function C. While the aggregate average cost ratio plots for cost functions B and C are not identical, they display similar patterns. The aggregate average cost ratio for cost function C is at its maximum in five-player games before it starts to decline. However, there is a slight increase in the cost ratio between 10 and 14-player games before decreasing once again.

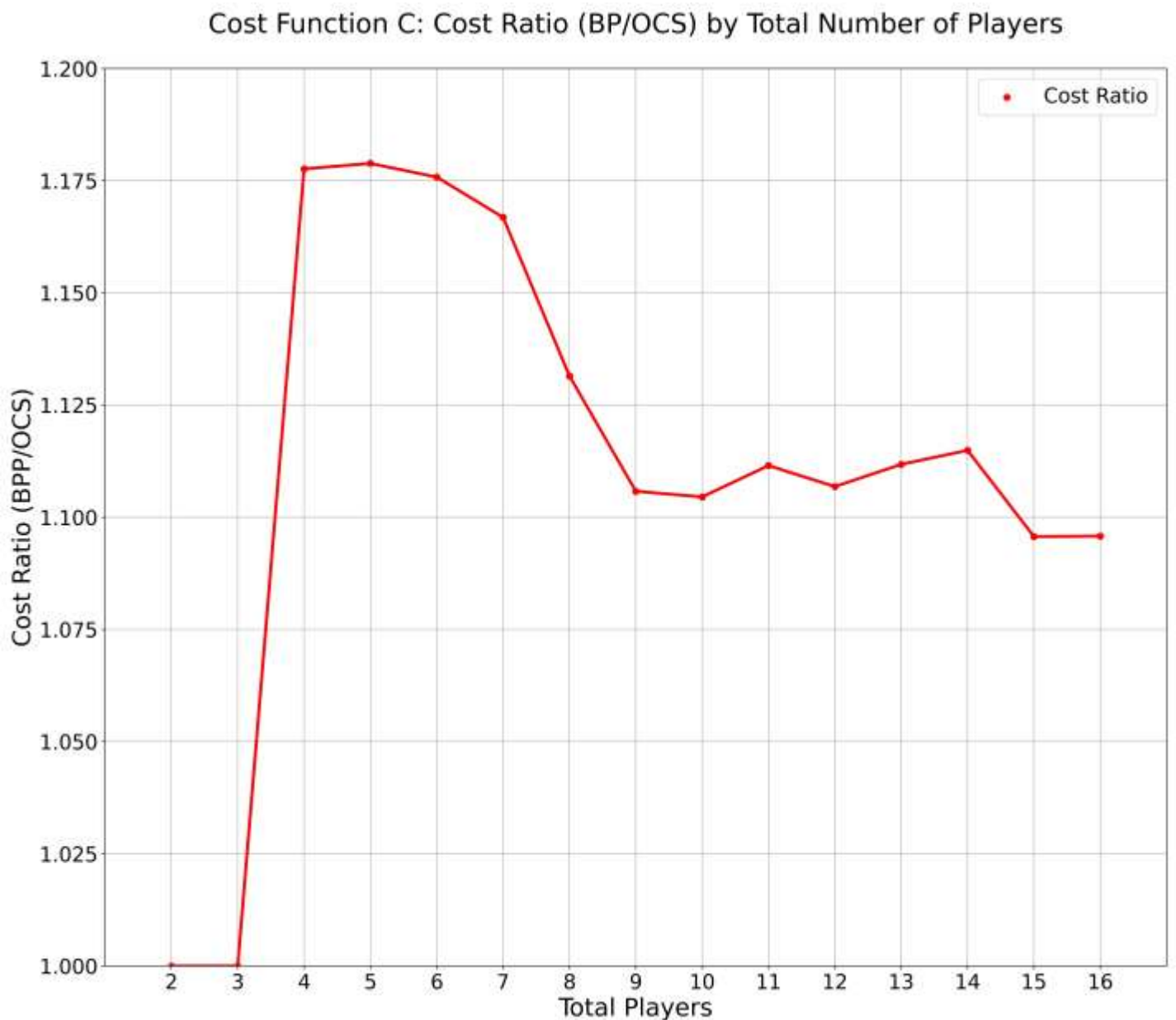


Figure 4.4 - Cost function C average cost ratio aggregated by total number of players

Additional line charts visualizing average cost ratio aggregated by total number of players using cost functions A, D, and E are located on pages 68-70 in Figures B.4-6, respectively.

4.1.3 Comparison of Computation Times by Player Size Dynamic

Analyzing the average computation time for the bin packing approach versus the optimal coalition structure model by player size dynamics reveals a significant disparity. For the optimal coalition structure model, games with a small number of players are generally less computationally intense. However, as the number of players increases, the time required to identify the least cost coalition structure drastically increases. The runtime ratio for each instance is computed by dividing the computation time for the optimal coalition structure model by the computation time for the bin packing approach. For each player size dynamic, the average runtime ratio is computed for all instances of that dynamic.

Figure 4.5 depicts a heatmap of the average runtime ratio for cost function B by number of small and medium players. Figures 4.5 and 4.6 utilize a base 10 logarithmic scale to visualize average runtime ratio values on a heatmap. This logarithmic scaling enhances the figure's ability to visualize the dramatic differences in values. The scale achieves this by compressing the values into a narrower range, which allows for greater contrast and more distinct visualization when representing the values using a color scale. The values obtained from base 10 logarithms represent the powers to which 10 must be raised to retrieve the original values before the application of logarithmic scaling.

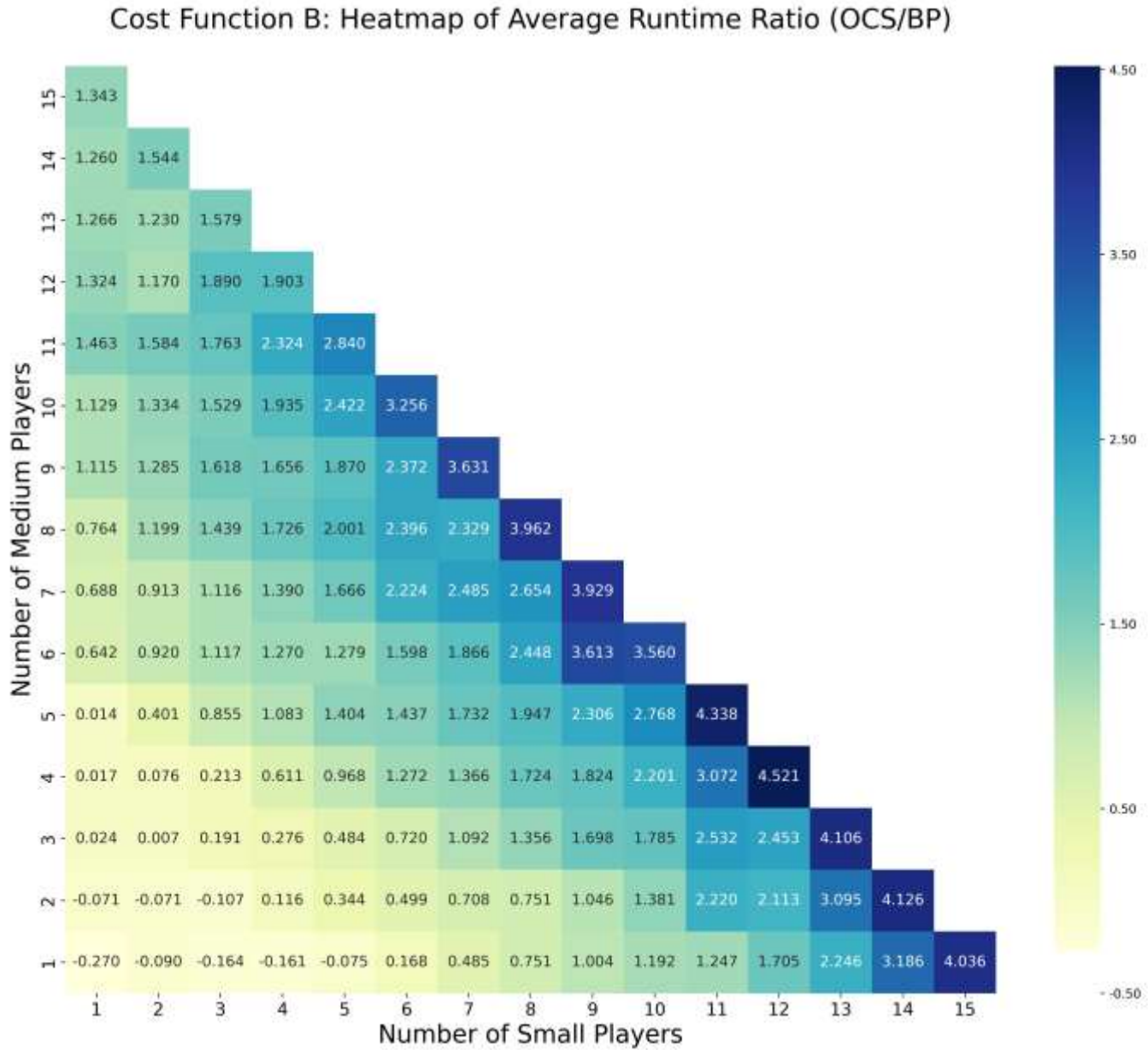


Figure 4.5 - Cost function B average runtime ratio by player size dynamic using a base 10 logarithmic scale

In games with 15 small players and one medium player using cost function B, dividing the average computation time for the optimal coalition structure model by the bin packing model's average computation time returns a number that is equal to 10 to the power of 4.036,

which results in a runtime ratio of 10,864.256. This value can be interpreted as the optimal coalition structure model requiring roughly 10,864 times the computation time required by the bin packing approach to return a solution. Looking at the color scales in Figure 4.5, one can observe that as the total number of players in a game grows, the average runtime ratio increases. Nearly all average runtime ratios are greater than zero, except for a few games with a relatively small number of total players, since the bin packing approach is less computationally intense than the optimal coalition structure model. An average runtime ratio less than one is attributable to a quick optimal coalition structure model runtime since the occurrence of ratios less than one is only observed in games with a small number of players. Figure 4.5 shows that for cost function B, 16-player games have a computational runtime ratio between 22.029 and 33,189.446. These ratios translate to the bin packing approach returning a solution roughly 22 to 33,189 times as fast as the optimal coalition structure model.

Figure 4.6 contains the average runtime ratios for cost function C by number of small and medium players. The ratio values for cost function C are slightly different than that of cost function B, but the findings remain the same. As observed in Figure 4.6, cost function C replications for 16-player games yielded an average runtime ratio between 50.350 and 27,227.013. Using cost function C, the bin packing approach identified a solution 50 to 27,227 times as fast as the optimal coalition structure model.

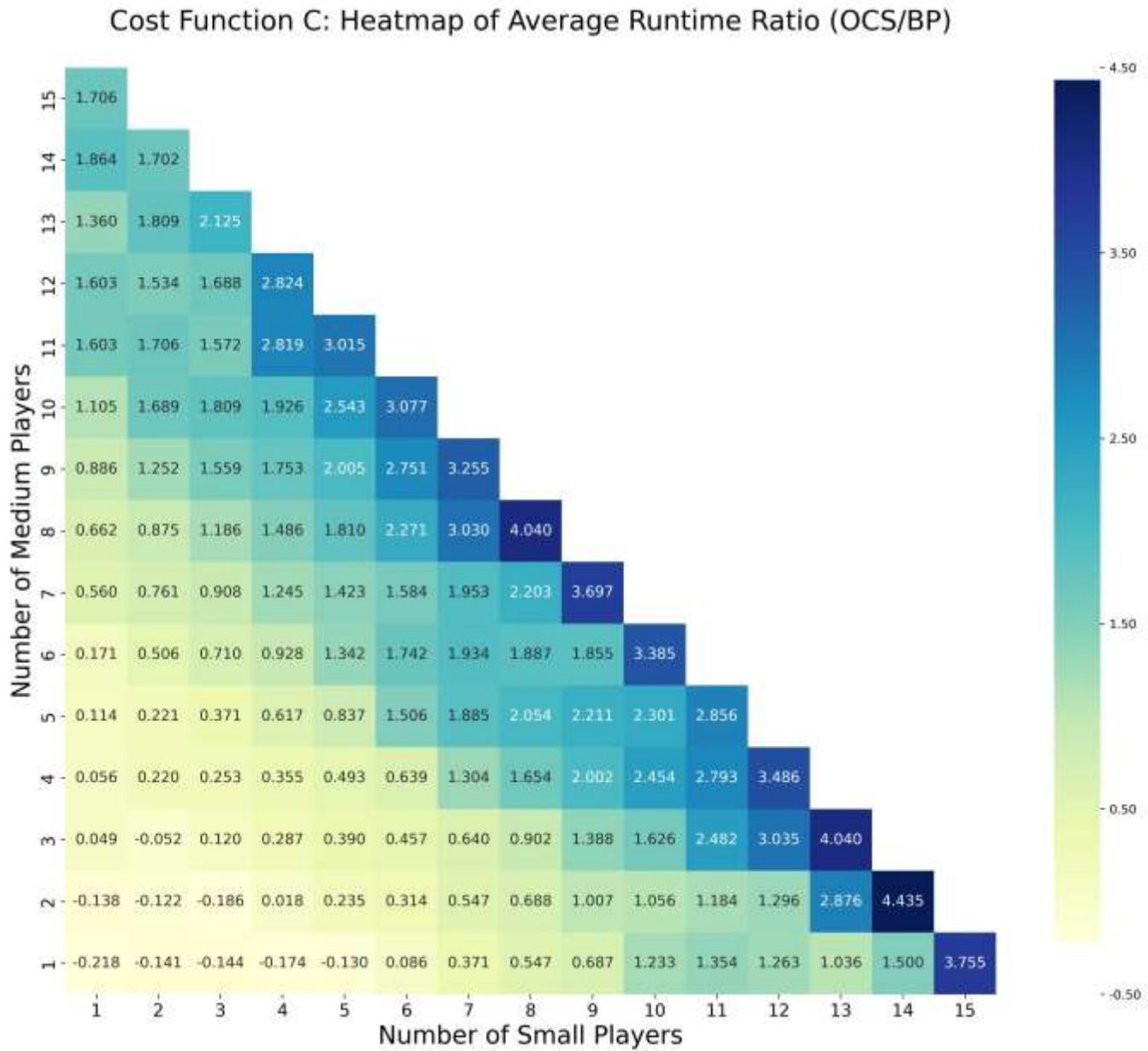


Figure 4.6 - Cost function C average runtime ratio by player size dynamic using a base 10 logarithmic scale

Figures 4.5 and 4.6 show that large games with a greater number of small players than large players typically achieve a greater average runtime ratio. By inspecting results for singular solved instances for cost functions B and C regarding computation time, it was determined that

lower runtime ratios for games with more medium players than small players are caused by longer runtimes for the bin packing approach. This suggests that the bin packing approach requires greater computational time to identify a near-optimal coalition structure for games with more medium players than small players. Heatmaps of cost functions A, D, and E for runtime ratio by player size dynamic are shown in Figures B.7-9 on pages 71-73, respectively.

4.1.4 Comparison of Computation Times by Total Number of Players

This subsection provides a view of the computation time required for generating solutions using the bin packing and the optimal coalition structure models, averaged across all solved instances of player size dynamics under cost function B with the same number of total players, with runtime in terms of seconds.

Table 4.2 - Cost function B bin packing (BP) and optimal coalition structure (OCS) runtime in seconds averaged across all solved instances by total number of players

Total Players (<i>n</i>)	Average BP Runtime (seconds)	Average OCS Runtime (seconds)	Runtime Ratio Averaged by Total Number of Players (OCS/BP)
2	0.001007	0.000541	0.536875
3	0.003169	0.002634	0.830950
4	0.003780	0.003277	0.853592
5	0.004907	0.004327	0.881647
6	0.006468	0.007785	1.203498
7	0.009027	0.021052	2.332187
8	0.012494	0.058604	4.690561
9	0.018502	0.149570	8.083929
10	0.025415	0.358904	14.121456
11	0.032002	0.628882	19.651277
12	0.053408	1.967145	36.832396
13	0.117945	8.436440	71.528425
14	0.230382	37.718293	163.720474
15	0.357167	196.085665	549.002822
16	0.356299	1,038.287491	2,914.090415

Table 4.2 details average runtime for simulating the bin packing and optimal coalition structure models under cost function B, aggregated by number of total players. Table 4.2 reveals that runtime for the bin packing approach and optimal coalition structure are similar for games with up to six total players. Up to this point, optimal coalition structure occasionally achieves a quicker runtime than the bin packing approach. However, for seven-player games under cost function B, the bin packing approach achieves a notably quicker runtime with an average runtime ratio between 1.471 and 4.389. Beyond six-player games, the aggregate average runtime

ratio using cost function B is consistently greater than one. As the number of total players exceeds 10, the runtime required to find a solution using the optimal coalition structure model begins to rapidly grow. For 16-player games, the average time to solve a single instance for the optimal coalition structure model using cost function B exceeds 1,000 seconds or about 16-and-a-half minutes. The bin packing approach returned solutions for cost function B in 356.3 milliseconds on average. This translates to the bin packing approach being 2,914.090 times as fast as the optimal coalition structure model, on average, for 16-player games.

Table 4.3 presents the average computational runtime for the bin packing and optimal coalition structure models using cost function C. Runtime for the bin packing and optimal coalition structure models are relatively close for games with up to only six total players, with the optimal coalition structure model sometimes outpacing the bin packing approach. Under cost function C, seven-player games maintain an average runtime ratio between 1.219 and 1.936. Like cost function B, all games with seven or more players using cost function C yield an average runtime ratio greater than one. Cost function C optimal coalition structure model replications for 16-player games achieved an average runtime slightly over 800 seconds whereas cost function B 16-player replications averaged a runtime greater than 1,000 seconds. The bin packing approach returned solutions for 16-player games under cost function C in an average of 227.3 milliseconds. For cost function C, the bin packing approach returned solutions 3,618.394 times as fast as the optimal coalition structure model for 16-player games, on average.

Table 4.3 - Cost function C bin packing (BP) and optimal coalition structure (OCS) runtime in seconds averaged across all solved instances by total number of players

Total Players (<i>n</i>)	Average BP Runtime (seconds)	Average OCS Runtime (seconds)	Runtime Ratio Averaged by Total Number of Players (OCS/BP)
2	0.001384	0.000837	0.604855
3	0.004787	0.003473	0.725474
4	0.004785	0.003963	0.828029
5	0.008439	0.006912	0.819015
6	0.007958	0.009687	1.217152
7	0.011425	0.018833	1.648395
8	0.016692	0.043462	2.603720
9	0.020249	0.082938	4.095897
10	0.023183	0.147632	6.368027
11	0.030304	0.526298	17.367258
12	0.038326	1.468766	38.322731
13	0.079277	4.783281	60.336573
14	0.193740	23.411420	120.839278
15	0.264575	116.371388	439.842877
16	0.227345	822.622072	3,618.393641

Instances solved for cost functions B and C were identical apart from the cost function used for procurement pricing. These results suggest that the cost function shape influences computation time for both the bin packing and optimal coalition structure models. Although the bin packing approach generated solutions quicker for cost function C, the solutions generated were the furthest from optimal on average. Average runtime, aggregated by total number of players, for cost functions A, D, and E, for the bin packing and optimal coalition structure models, is presented in Tables B.1-3 on pages 74-76, respectively.

4.2 Discussion of Bin Packing Performance

This subsection further discusses performance trends and observations regarding proximity to objective value and computation time by player size dynamic and total number of players.

4.2.1 Performance Related to Player Size Dynamic

Referring to Figures 4.1 and 4.2, there is no discernable trend between player size configuration and the bin packing approach's proximity to minimum cost. Although there are a few outliers where the bin packing approach yielded comparably high or low average cost ratios, they are present in player size configurations that are nearly all small or medium players as well as player size dynamics where the number of small and medium players are nearly equal.

However, as seen in Figures 4.5 and 4.6, there is a clear correlation between player size dynamic and computation time. When comparing average runtime ratios for 16-player size dynamics, it is evident that the ratio is dramatically smaller for games with more medium players than small players. For 16-player games under cost function C, with 15 medium players and one small player, the average runtime ratio is 50.816. But for 16 player games with one medium player and 15 small players, the average runtime ratio is equal to 5,688.529. Average runtime ratio, which is expressed as the average computation time for the optimal coalition structure model divided by that of the bin packing approach. The average runtime ratio values for 16-player games suggests that there is a dramatic difference in runtime for either the optimal coalition structure model or the bin packing model. Upon close examination of singular solved instances for 16-player games, it was determined that the difference observed in runtime ratio for 16-player games is due to the bin packing approach requiring greater computational time to generate solutions for player size dynamics with many medium players and few small players.

The computational time required by the optimal coalition structure model is fairly consistent across player size dynamics with the same number of total players.

4.2.2 Performance Related to Game Size

In procurement scenarios, adding more players to a game typically results in a higher total order quantity for the grand coalition. Given the cubic nature of the analyzed cost functions, there is a specific range where the marginal increase in cost per unit is relatively low. This phenomenon is characterized by viewing the cost function as having three price ranges. The first two price ranges demonstrate economies of scale, where marginal costs decrease as the quantity increases, while the third price range shows diseconomies of scale, where marginal costs increase with quantity. In games that deal with procurement under a cubic cost function, the formation of coalitions becomes progressively more beneficial as more players are involved.

When players in a game unite to form the grand coalition, games with a small number of players yield lower total order quantities, while games with a larger number of players see higher totals. In scenarios where there is a fixed cost associated with placing an order, each existing coalition incurs the fixed cost, resulting in the formation of fewer coalitions to balance fixed costs and unit costs. Increasing fixed costs further incentivizes players to form fewer coalitions.

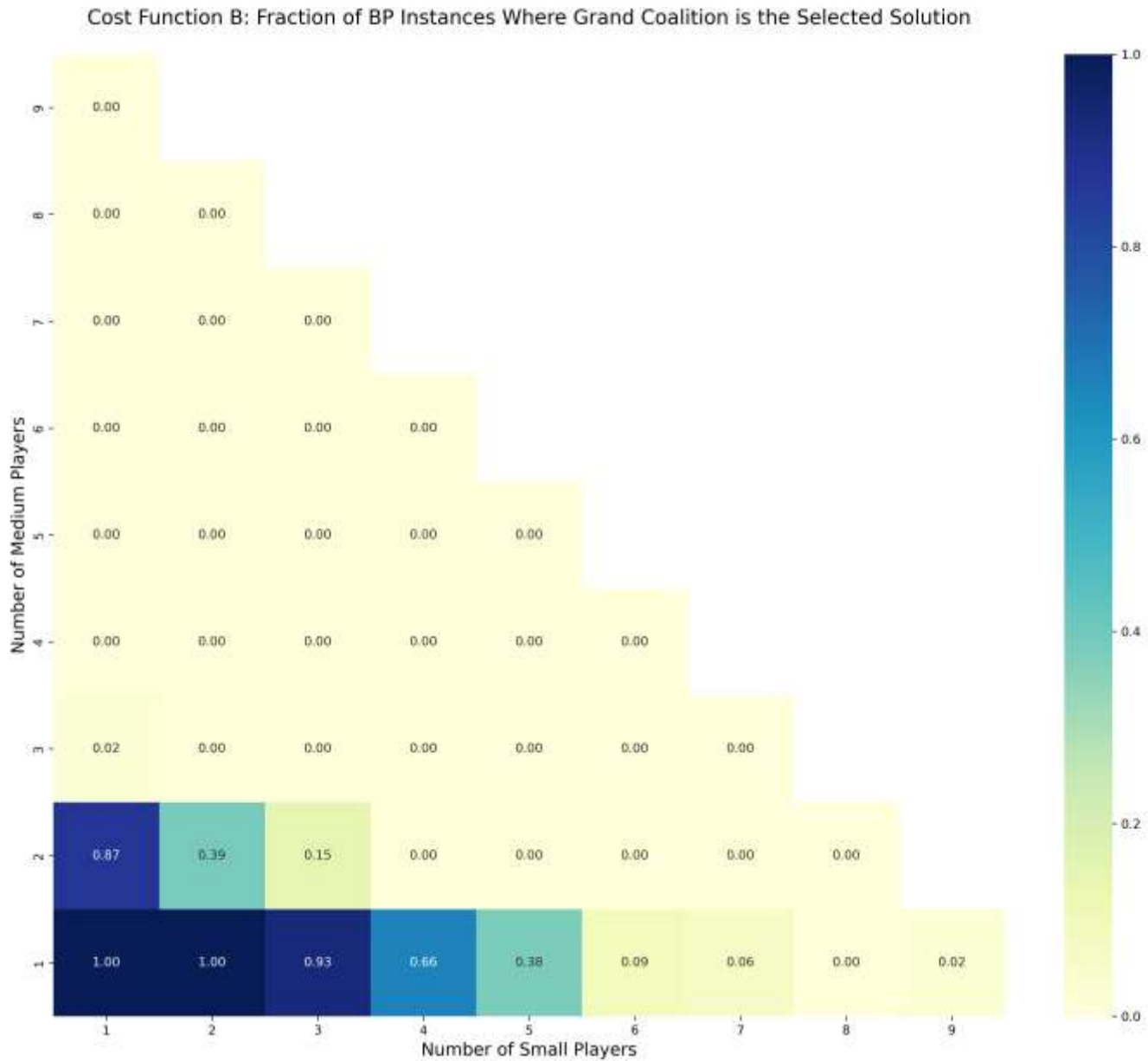


Figure 4.7 - Occurrence of grand coalition formation in bin packing (BP) solutions by player size dynamic under cost function B as a fraction of solved instances

Figure 4.7 illustrates the fraction of instances solved by player size dynamic for cost function B using the bin packing approach, where the solution recommends forming the grand coalition. This fraction is calculated by dividing the number of solved instances where the

solution suggests forming the grand coalition by the number of solved instances for each player dynamic. The number of instances solved for each player size dynamic, by total number of players, can be found in Table 3.6 on page 35. These values serve as the denominator in calculating the fraction. As shown in Figure 4.7, the bin packing approach predominantly advises the formation of the grand coalition in smaller games, with up to three players, due to the total order quantity not exceeding bin capacity. For games involving more than three players, the bin packing approach increasingly suggests forming multiple coalitions, indicating a shift in strategy as the number of players in a game grows. This is especially true for games with more than one medium player due to total order quantity exceeding the favorable ordering capacity of a single coalition. Although the bin packing approach commonly utilizes the grand coalition for games with up to six players featuring a single medium player, the grand coalition is rarely selected by the heuristic beyond this point. Beyond 10-player games, the bin packing approach does not suggest forming the grand coalition.

Figure 4.8 depicts the fraction of solved instances in which the grand coalition is selected using the optimal coalition structure model by player size dynamic for cost function B. For games with up to four players, this model recommends forming the grand coalition nearly every time. However, for games with more than three medium players, the optimal coalition structure model forms the grand coalition only four times out of the 2,100 solved instances. In games featuring one medium player, the optimal coalition structure model selects the grand coalition 686 times out of 900 solved instances, or 76.2% of the time. Clearly, the optimal coalition structure model favors the grand coalition for games with few medium players. But for player size dynamics with three or more medium players, the optimal coalition structure model is more likely to suggest forming multiple coalitions.

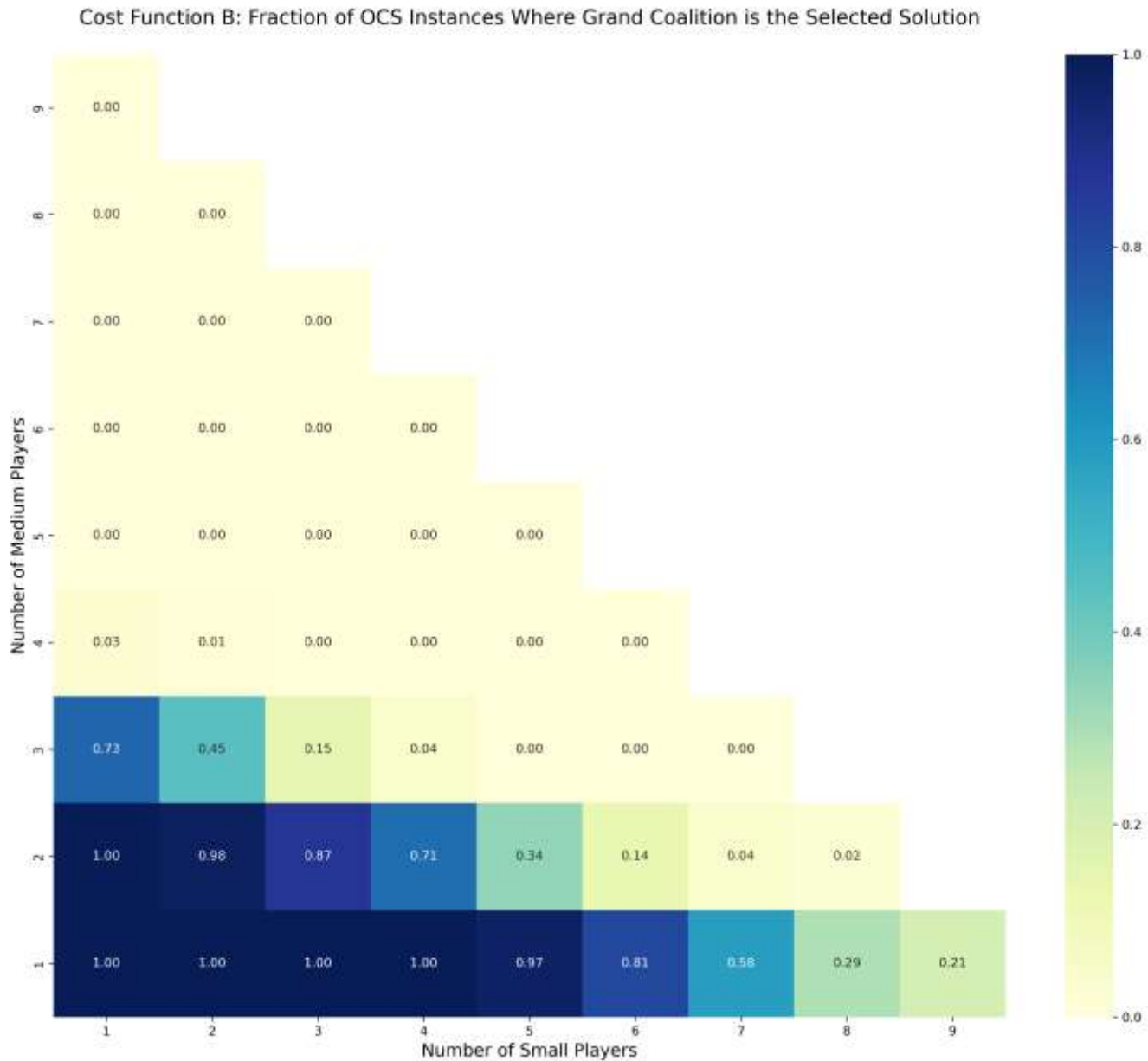


Figure 4.8 - Occurrence of grand coalition formation in optimal coalition structure (OCS) solutions by player size dynamic under cost function B as a fraction of solved instances

By analyzing the fraction of solved instances where both models recommend forming the grand coalition, the inability of the bin packing approach to identify optimal coalition structures is explained. Although the bin packing approach is capable of generating near-optimal solutions,

it is limited by bin capacity in its ability to form the grand coalition for games with greater than four players with more than one medium player. However, for games with greater than four medium players, both models agree that the grand coalition is not the most effective strategy for minimizing coalition structure costs. Additionally, both models' results agree that the grand coalition should not be used for 16-player games, except for those with one medium player.

As for computation time, Tables 4.2 and 4.3 showcase the vast difference in the time required to generate solutions between the bin packing and optimal coalition structure models, by the number of players in a game. Using the bin packing approach, the computation time required to return solutions for 16-player games was 356.299 and 227.345 milliseconds on average for cost function B and C, respectively. These computation times are comparable to that of LocalSearch and GreedySearch, which both returned solutions for games with up to 27 players in under 400 milliseconds. On average, the bin packing approach generated solutions for 16-player games at a rate of 2,914.090 and 3,618.394 times as fast as the optimal coalition structure model for cost function B and C, respectively.

Chapter 5 - Conclusions and Future Work

Chapter 5 concludes this thesis by summarizing results and recommendations for future work. Hopefully, this study encourages researchers to consider an “outside of the box” approach for furthering research in the field of humanitarian logistics.

5.1 Conclusions

This thesis contributes to the literature by directly comparing heuristic and optimal approaches and providing performance statistics for multiple metrics. By presenting this research, future works aimed at developing heuristic approaches to the coalition structure problem will have benchmarks that enable direct comparison. The research undertaken in this thesis presents the potential of utilizing a bin packing approach to the least cost coalition structure problem to enhance cooperative operations in humanitarian logistics. The proposed bin packing approach is shown to identify near-optimal coalition structures in a fraction of the time needed to solve the problem optimally. Instances analyzed for cost function B involving 12 small players and four medium players found a solution that was, on average, only 4.03% above the optimal coalition structure cost at a rate 33,189.446 times as fast as solving an optimization model to find the least cost coalition structure. The average runtimes for 16-player games under cost functions B and C were 356.3 and 227.3 milliseconds, respectively. These times are comparable to previously published data on LocalSearch and GreedySearch, which returned solutions in under 400 milliseconds for games with as many as 27 players. The bin packing approach outperformed these heuristics in solution quality regarding proximity to objective value for cost function B, but underperformed them when applied to cost function C. Although this report focuses on the best (cost function B) and worst (cost function C) performances observed

using the bin packing approach, comprehensive results for cost functions A, D, and E can be found in Appendix B.

This research found that the bin packing approach to the coalition structure problem required more computation time to generate solutions for larger games with many medium players and few small players. Additionally, the performance gap observed between cost functions B and C using the bin packing approach is due to their respective rates of increase in cost per unit; cost function B is the flattest of the five cost functions analyzed and features the most gradual increase in cost per unit whereas cost function C has the sharpest increase in cost per unit. The noticeable difference in how close the solutions generated by the bin packing approach are to the optimal coalition structure cost using cost functions B and C highlights the need to refine the methodology, tailoring it more closely to the characteristic function used in defining the optimal coalition structure problem.

This significant reduction in computational time, without substantial compromise in coalition structure cost, highlights the potential impact of this research. These findings and experiment details provided in Chapter 3 offer actionable insights that can be directly applied to both humanitarian logistics and commercial supply chain practices. Additionally, this research enhances understanding of non-subadditive games, which are important in modeling cooperative logistics operations.

5.2 Future Work

There are several promising directions for further advancements in the research domain of humanitarian logistics. Developing a quick procedure for determining fair and stable allocations among coalitions would address a significant challenge in humanitarian logistics. Although this topic has been widely researched for games where the grand coalition is the

optimal coalition structure, much less work has been done on non-subadditive and non-superadditive games. Formulation of such a mechanism would have significant implications in both humanitarian logistics and commercial supply chain. Such advancements would not only further the methodology presented in this thesis but would also extend its applicability and potential for use in practice.

To further develop the bin packing approach presented in this study, the approach would significantly benefit from a formal mathematical proof. Such a proof would affirm the validity of the bin packing approach by offering a generalized framework that may reveal further applications and use cases. Evaluating the efficacy of various bin packing heuristics in solving the coalition structure problem presents another avenue for further research. Such an analysis could potentially yield even further reductions in computational effort. Additionally, the bin packing approach could be further tailored to the coalition structure problem's characteristic function by formulating the approach as a two-stage optimization model that minimizes coalition structure cost, where the multiplier for the inflection point to determine bin capacity is defined as a decision variable. While such an adaptation is expected to increase the computational time required, it could yield solutions that are closer to the optimal coalition structure cost for a wider variety of cost function shapes. This trade-off is worth investigating and suggests a promising path for further research on the approach presented by this research.

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Appendix A - Graphical Comparison of Cost Functions

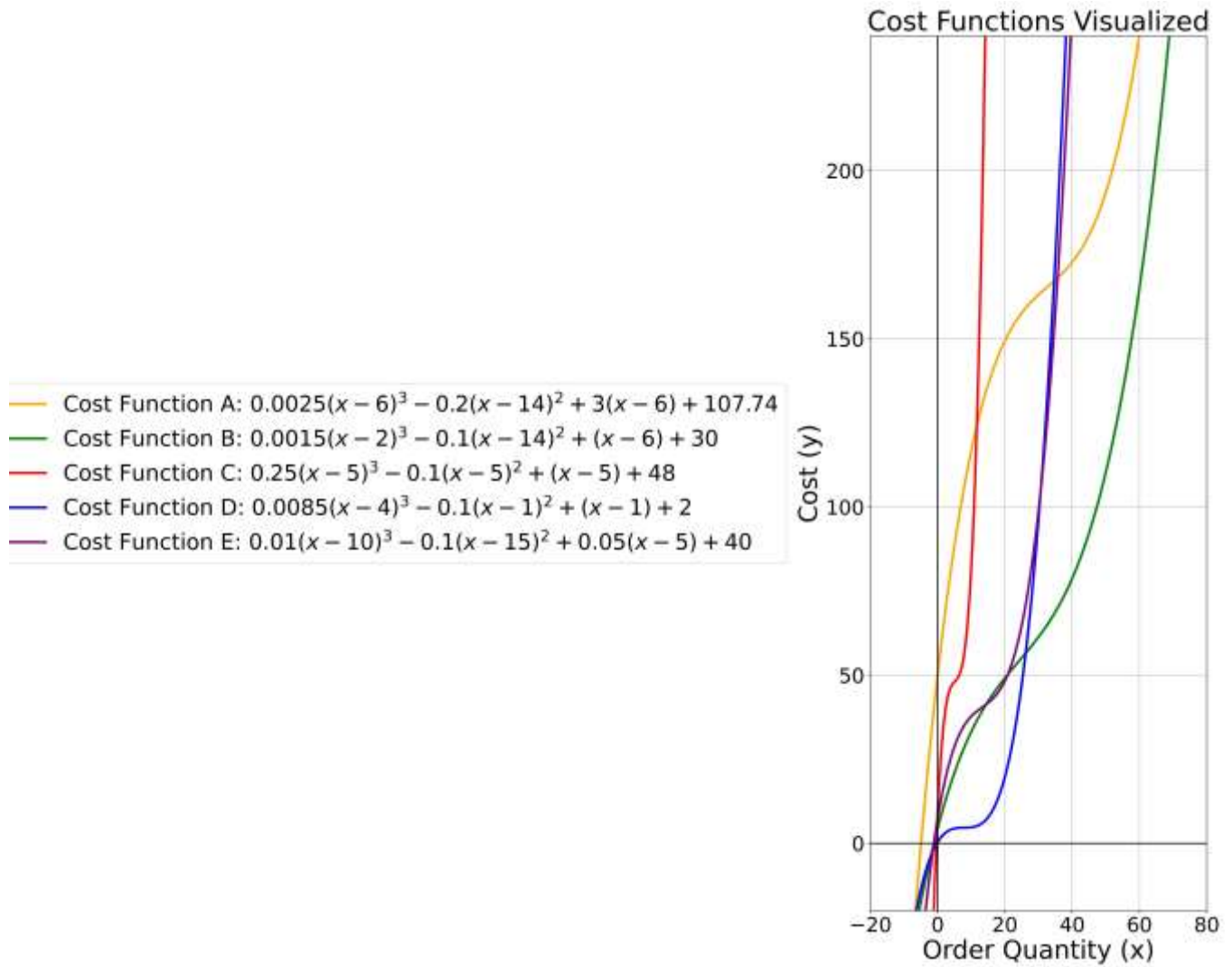


Figure A.1 - All cost functions visualized

Appendix B - Supplementary Results Tables and Figures

Cost Function A: Heatmap of Average Cost Ratio (BP/OCS)

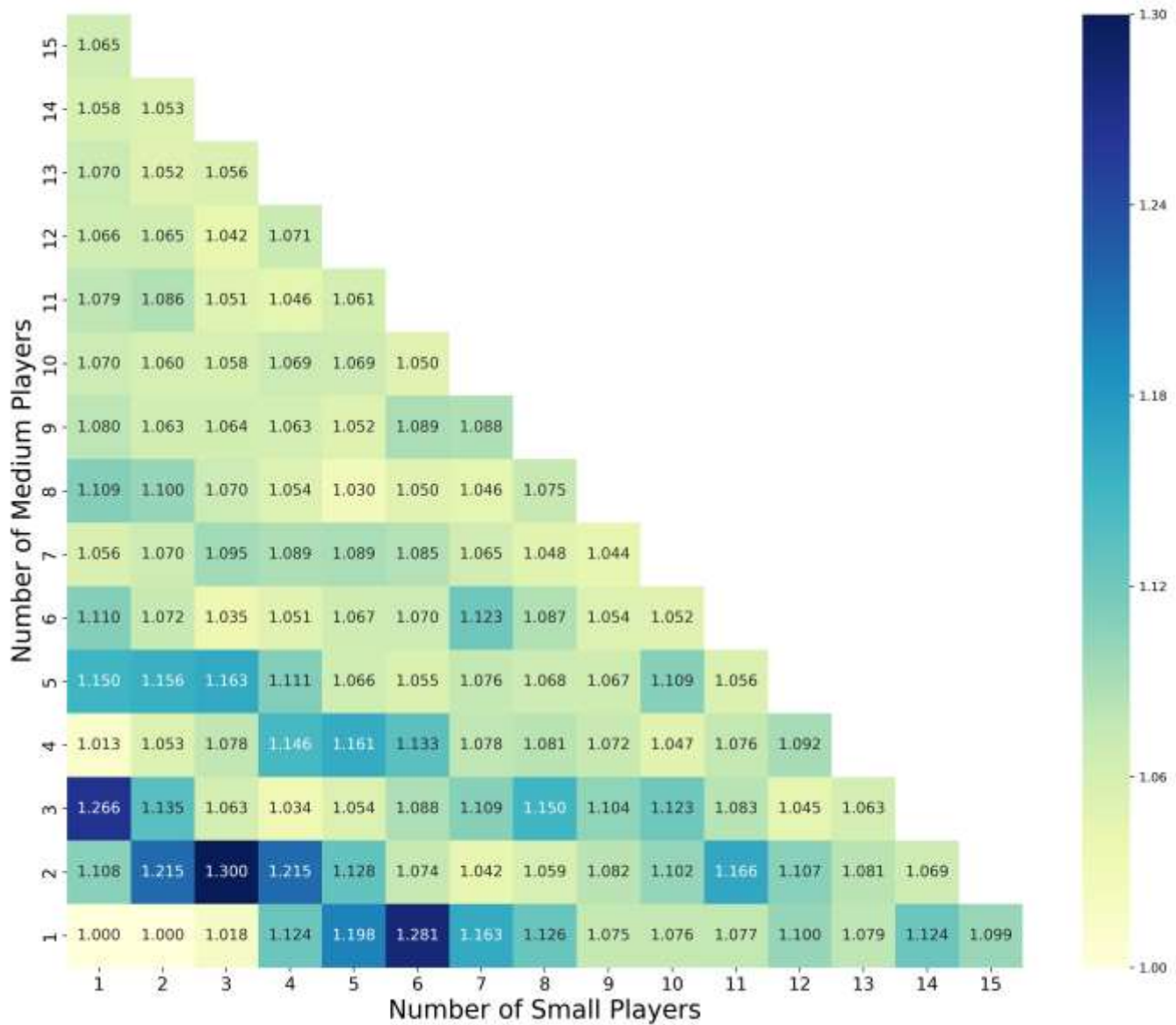


Figure B.1 - Cost function A average cost ratio by player size dynamic

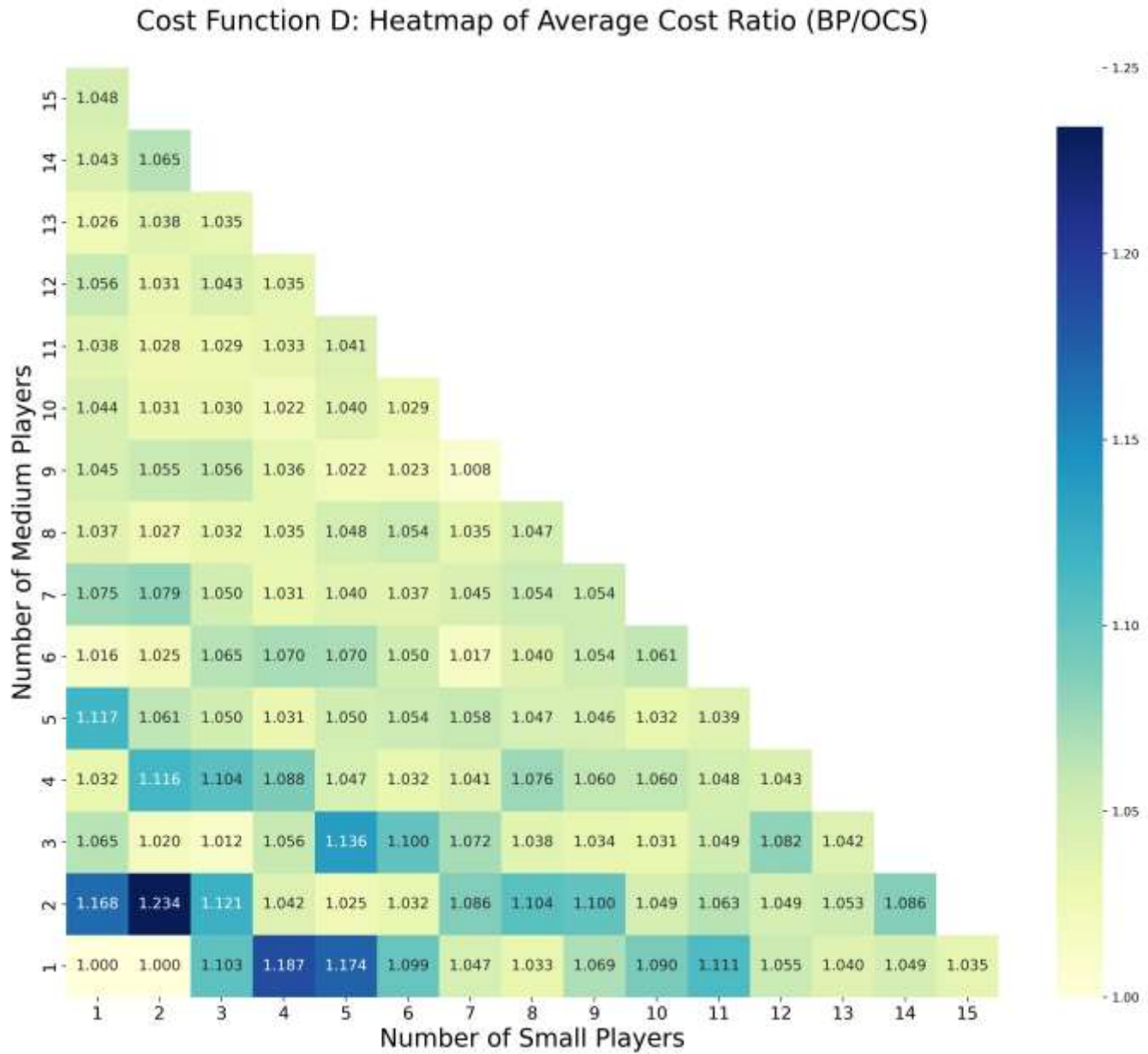


Figure B.2 - Cost function D average cost ratio by player size dynamic

Cost Function E: Heatmap of Average Cost Ratio (BP/OCS)

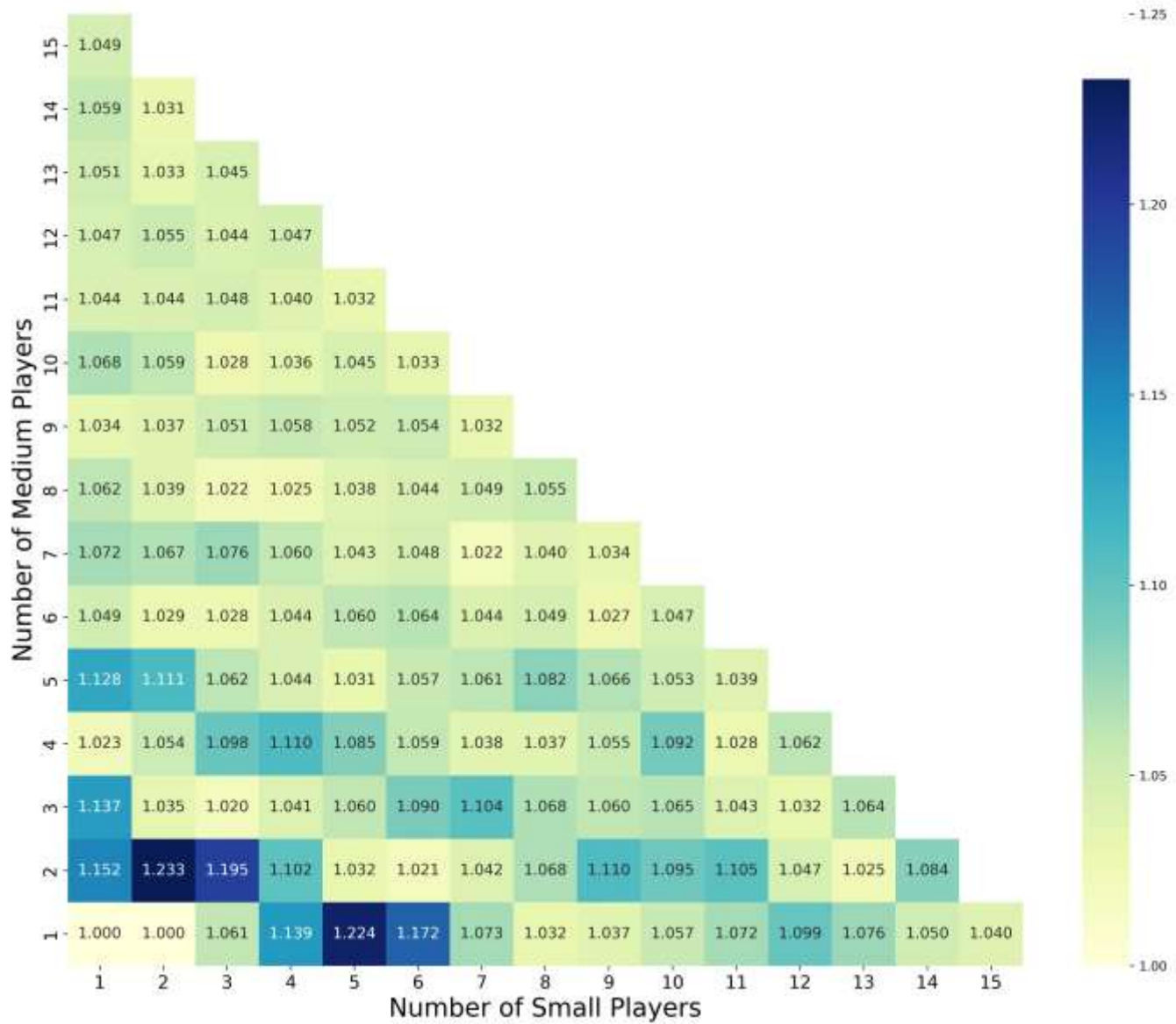


Figure B.3 - Cost function E average cost ratio by player size dynamic

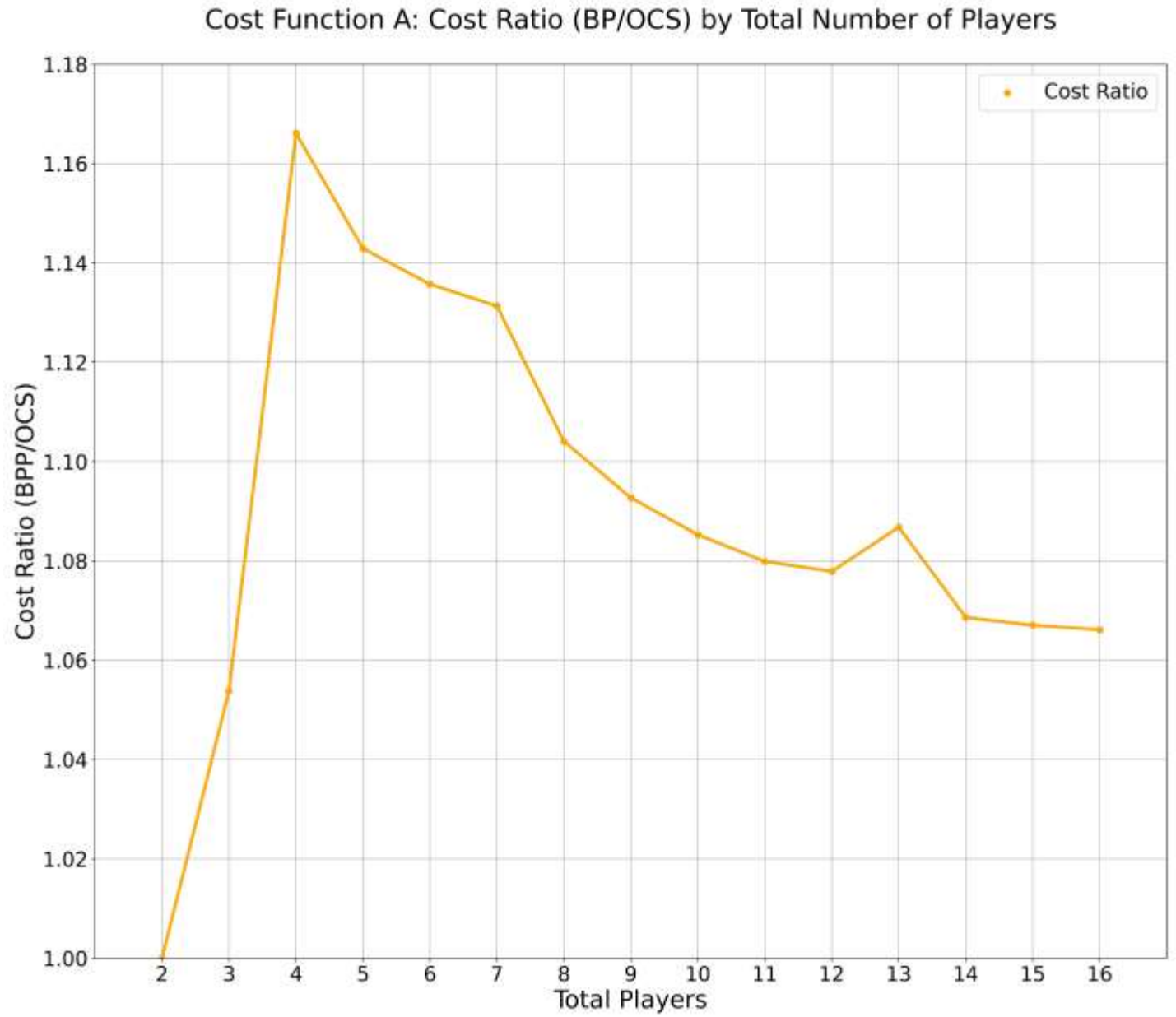


Figure B.4 - Cost function A average cost ratio aggregated by total number of players

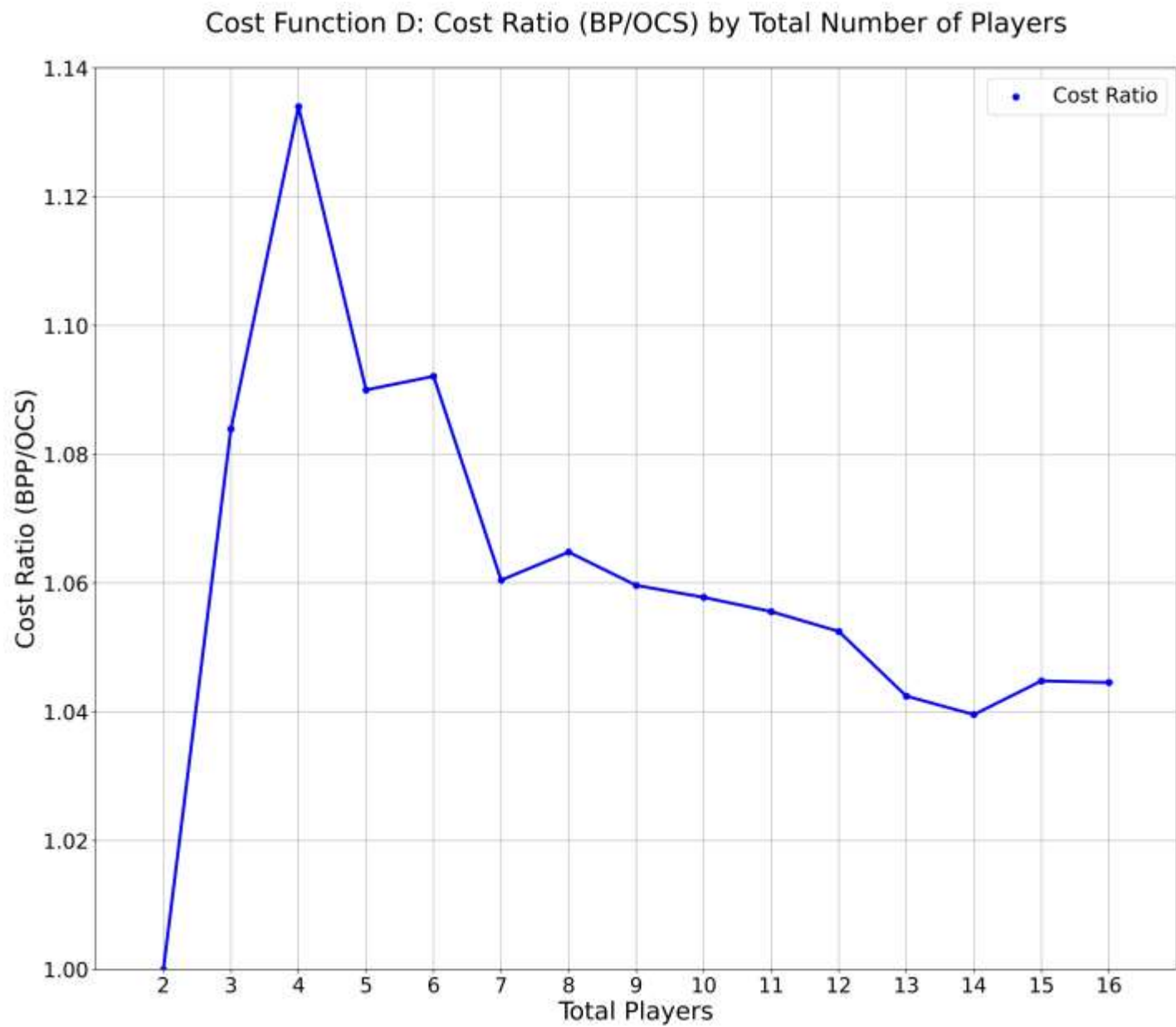


Figure B.5 - Cost function D average cost ratio aggregated by total number of players

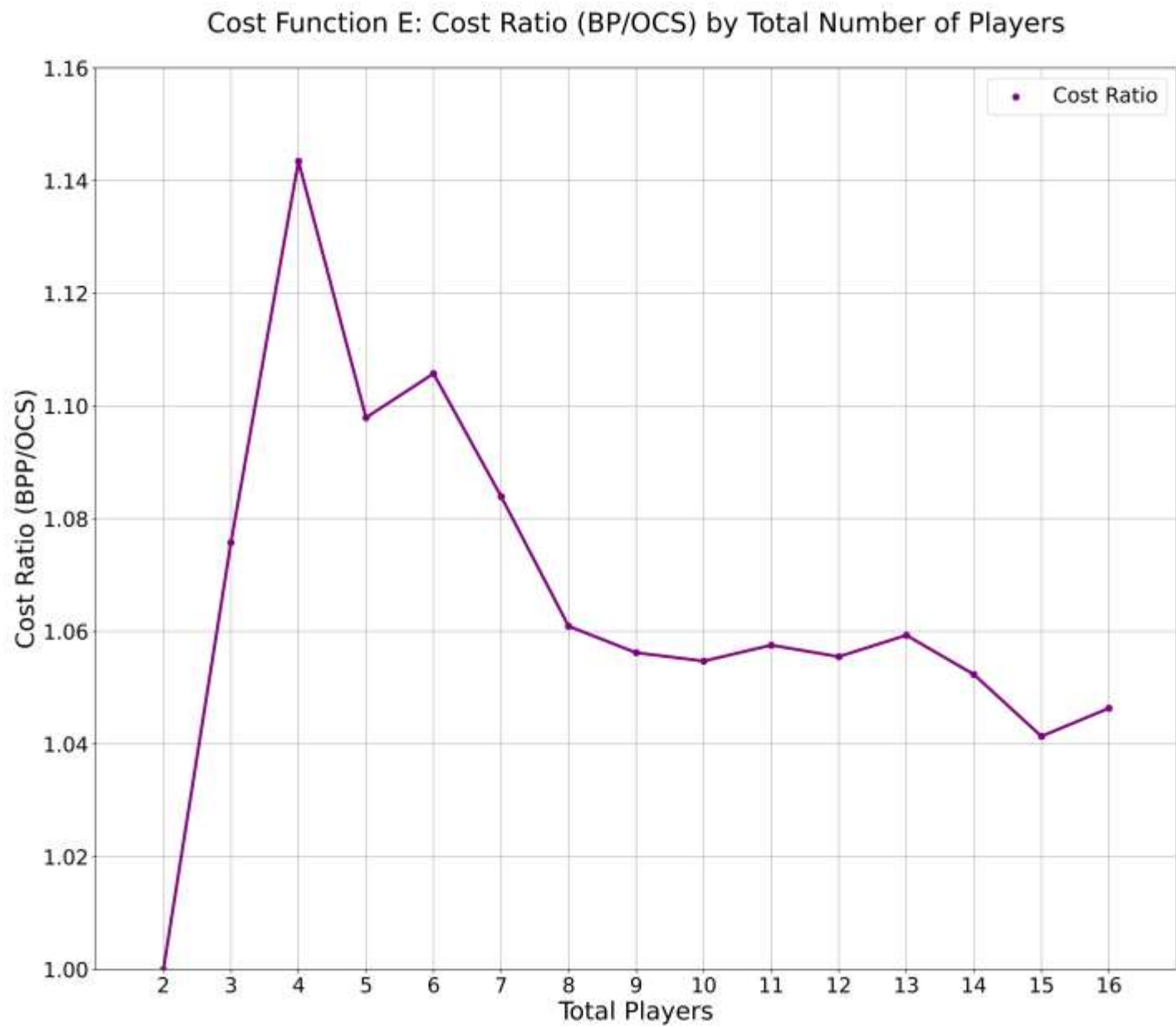


Figure B.6 - Cost function E average cost ratio aggregated by total number of players

Cost Function A: Heatmap of Average Runtime Ratio (OCS/BP)

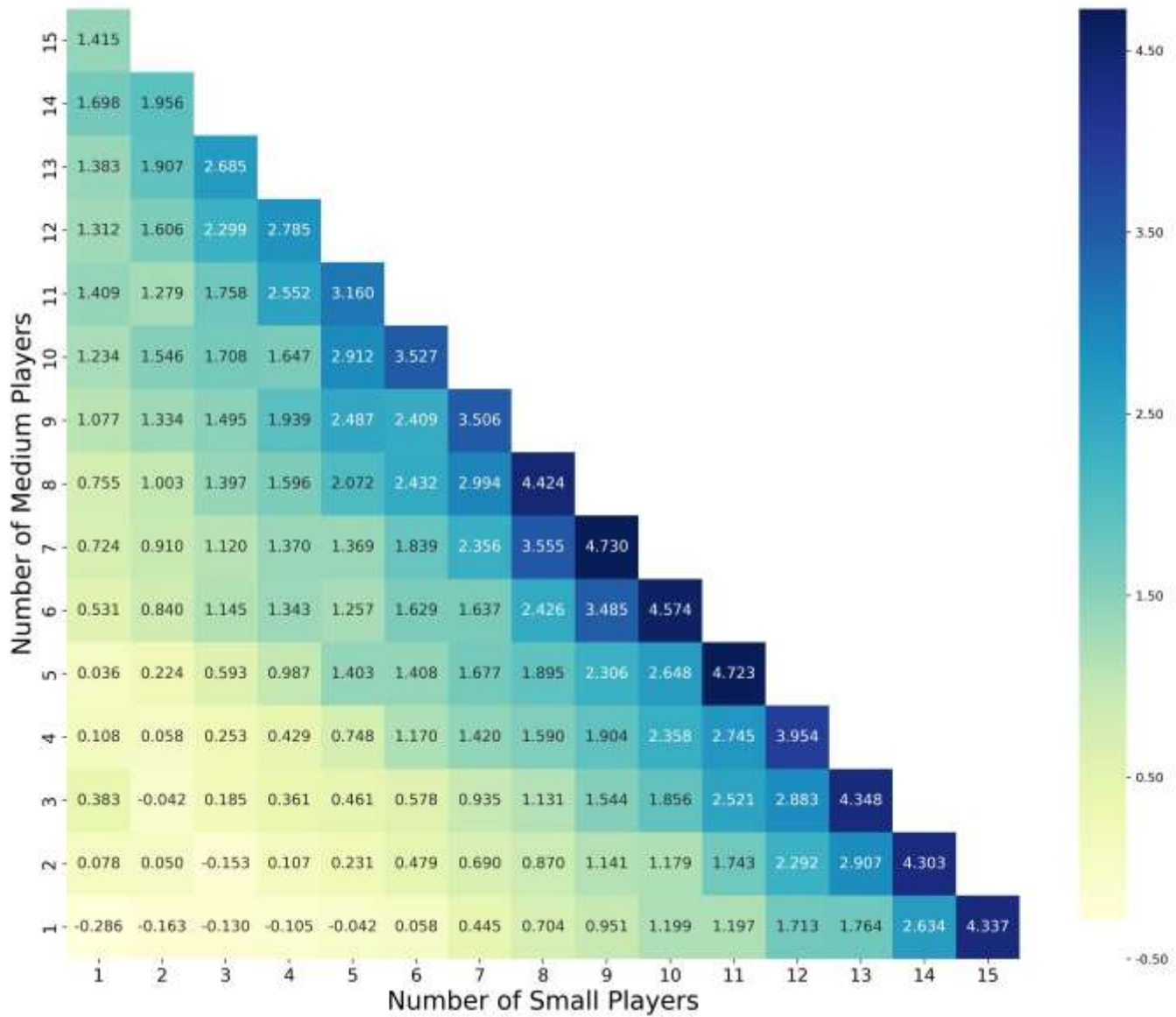


Figure B.7 - Cost function A average runtime ratio by player size dynamic using a base 10 logarithmic scale

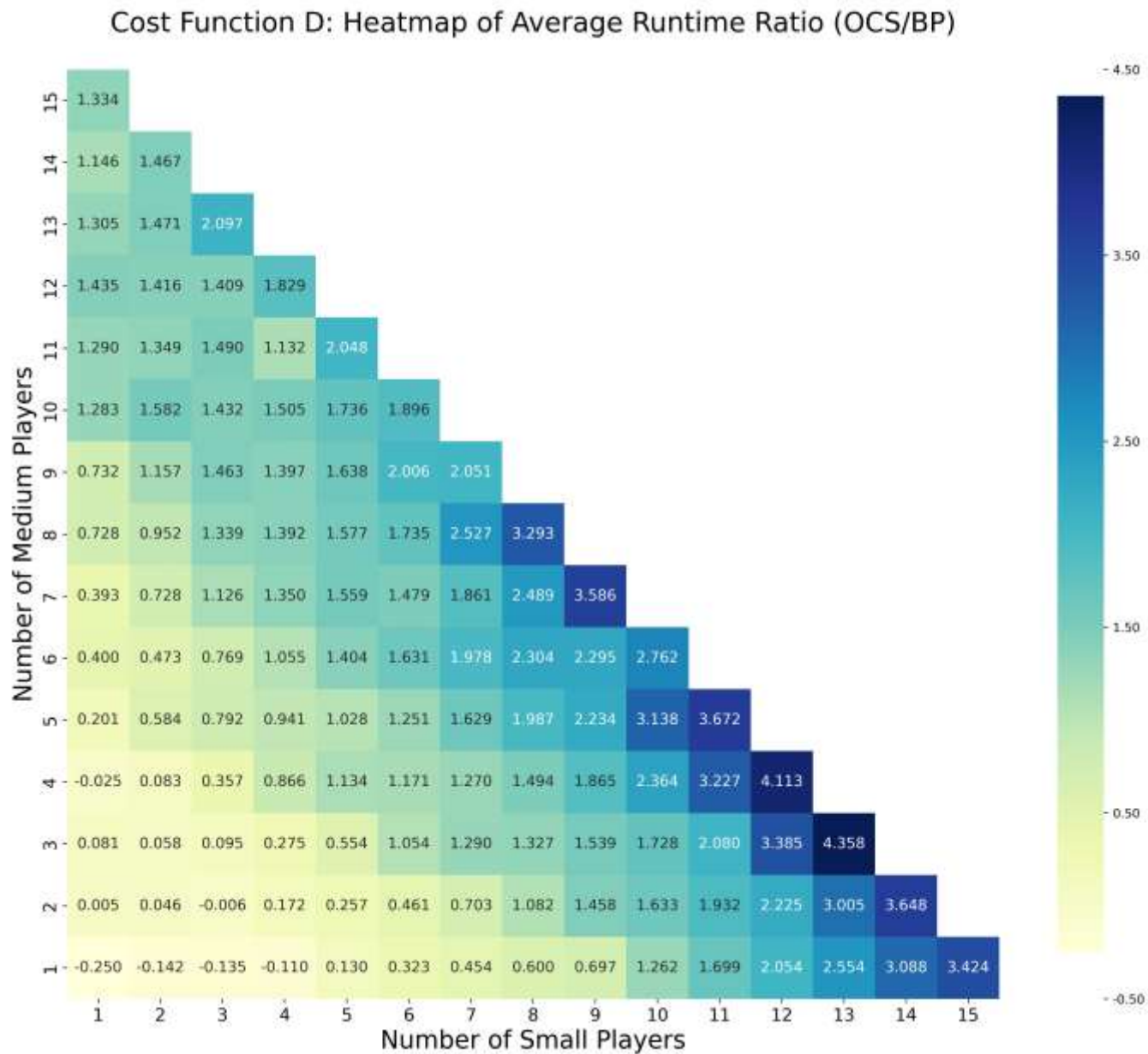


Figure B.8 - Cost function D average runtime ratio by player size dynamic using a base 10 logarithmic scale

Cost Function E: Heatmap of Average Runtime Ratio (OCS/BP)

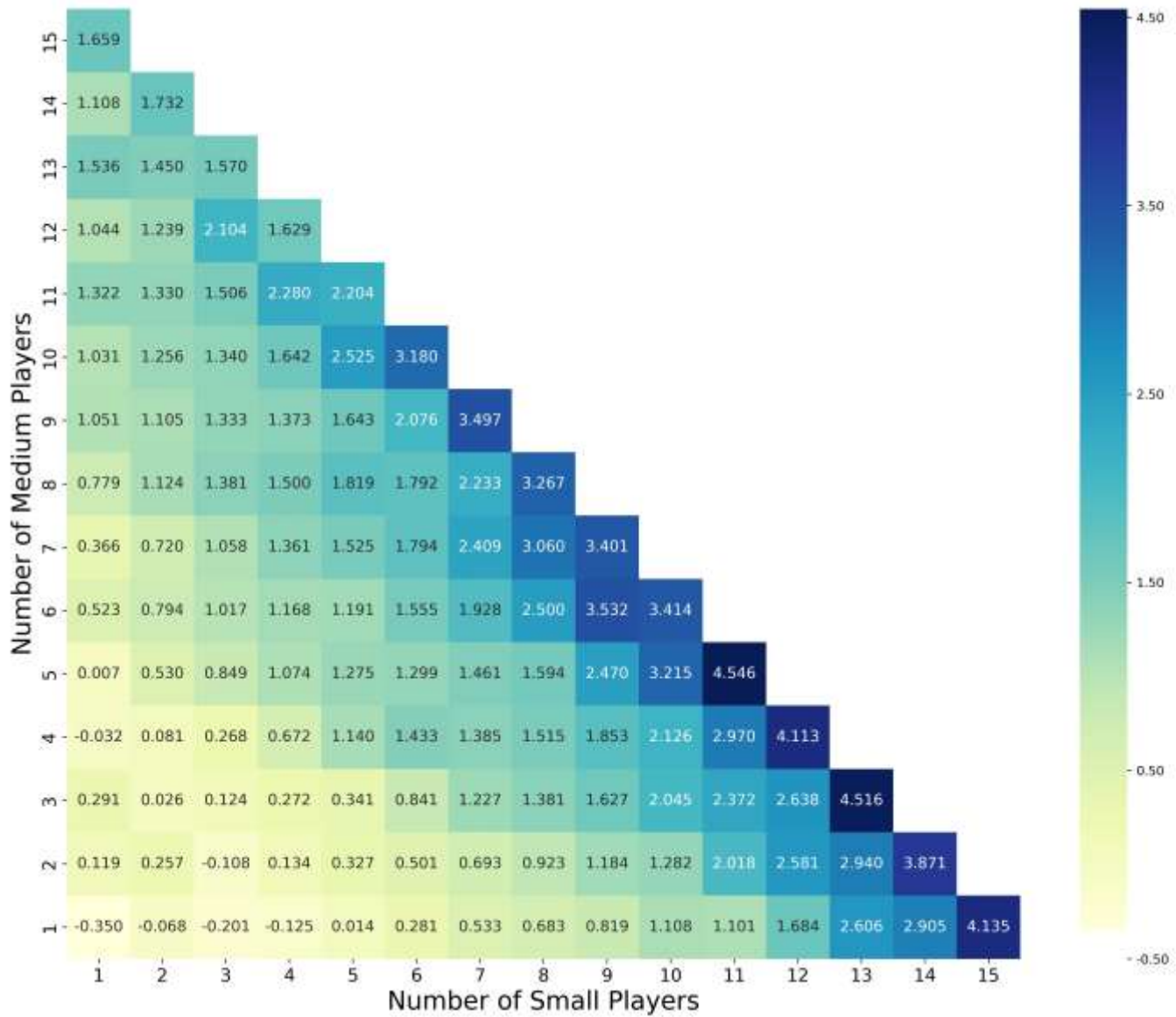


Figure B.9 - Cost function E average runtime ratio by player size dynamic using a base 10 logarithmic scale

Table B.1 - Cost function A bin packing (BP) and optimal coalition structure (OCS) runtime in seconds averaged across all solved instances by total number of players

Total Players (n)	Average BP Runtime (seconds)	Average OCS Runtime (seconds)	Runtime Ratio Averaged by Total Number of Players (OCS/BP)
2	0.001500	0.000776	0.517335
3	0.010870	0.009525	0.876208
4	0.006278	0.007968	1.269107
5	0.011485	0.010090	0.878530
6	0.011660	0.013715	1.176240
7	0.015283	0.030808	2.015785
8	0.019275	0.075199	3.901422
9	0.024644	0.181944	7.382927
10	0.026401	0.352430	13.349197
11	0.038876	0.764942	19.676639
12	0.086198	2.479694	28.767379
13	0.134387	8.140919	60.578213
14	0.263552	42.247247	160.299618
15	0.323784	253.438697	782.739112
16	0.260406	1,304.862353	5,010.879347

Table B.2 - Cost function D bin packing (BP) and optimal coalition structure (OCS) runtime in seconds averaged across all solved instances by total number of players

Total Players (<i>n</i>)	Average BP Runtime (seconds)	Average OCS Runtime (seconds)	Runtime Ratio Averaged by Total Number of Players (OCS/BP)
2	0.001277	0.000718	0.561858
3	0.004047	0.003459	0.854803
4	0.004375	0.004318	0.986960
5	0.009163	0.008767	0.956808
6	0.011758	0.016161	1.374475
7	0.014269	0.033880	2.374377
8	0.017658	0.071168	4.030473
9	0.017559	0.128663	7.327610
10	0.027410	0.307274	11.210380
11	0.031591	0.646837	20.475636
12	0.054713	1.921158	35.113369
13	0.114968	6.899977	60.016612
14	0.191248	25.480306	133.232006
15	0.321669	152.140307	472.971479
16	0.275291	730.302018	2,652.837139

Table B.3 - Cost function E bin packing (BP) and optimal coalition structure (OCS) runtime in seconds averaged across all solved instances by total number of players

Total Players (<i>n</i>)	Average BP Runtime (seconds)	Average OCS Runtime (seconds)	Runtime Ratio Averaged by Total Number of Players (OCS/BP)
2	0.001688	0.000754	0.446716
3	0.008235	0.009273	1.125940
4	0.007219	0.008877	1.229712
5	0.014895	0.013331	0.894988
6	0.015378	0.018666	1.213812
7	0.017105	0.041134	2.404783
8	0.026709	0.105188	3.938252
9	0.019313	0.154421	7.995660
10	0.026588	0.372556	14.012184
11	0.042593	0.757493	17.784402
12	0.092322	2.379437	25.773294
13	0.152996	7.297759	47.699089
14	0.222312	39.110170	175.924387
15	0.309900	200.565275	647.193418
16	0.349775	987.757665	2,823.983578