DETERMINATION OF THE APPROPRIATE CUTOFF FREQUENCY IN THE DIGITAL FILTER DATA SMOOTHING PROCEDURE

by

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DEDICATION

This thesis is dedicated to my parents, Dr. Gou-Rei Yu and Ming-Hua Lu, to my wife Wei Li, to her parents, Dr. Ping Li and Dr. Xiu-Zhang Yu, and to all of the other folks of my family and her family for their understanding of my absence when my son Charlse Alan Yu was born. Their constant support and encouragement are deeply appreciated.

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CHAPTER 1

INTRODUCTION

Cinematography is the most widely used technique in sport biomechanics research to provide a digital record and/or an image record of overt human body movements. These film records are analyzed quantitatively to obtain linear and angular coordinate-time data for total body or segmental movements. Typically, the basic coordinate-time function of a motion does not provide sufficient information to fully describe the activity. Therefore, these data have to be further treated mathematically to determine the corresponding velocity and acceleration functions. Knowledge of the velocity and acceleration patterns provides more descriptive information about the movement under investigation. It is therefore critical in most studies that precise estimates of velocity and acceleration be obtained.

For precise estimates of velocity and acceleration, various mathematical techniques have been employed to smooth raw data to eliminate or reduce the effect of errors which arise in filming and digitizing procedures. Winter (1974) advocated a second-order recursive digital filter for smoothing film analysis data. This digital filter has been suggested as the best among the smoothing techniques used in biomechanics studies (Li & Yu, 1983; Winter, 1979;

Wood, 1982), and is widely used in biomechanics studies at present.

In the digital filter smoothing procedure, there is a set of coefficients which control the "degree" of smoothing or the sharpness of cutoff. This set of coefficients is determined according to the cutoff frequency of the raw data. At the present time, the cutoff frequency is subjectively determined by the researcher. There is no method to objectively determine the cutoff frequency. The determination of cutoff frequency has a great influence on the smoothing quality of digital filter. Therefore, if a precise estimate of velocity and acceleration is expected from the data smoothed by the digital filter, it is essential that the problem of determination of cutoff frequency be solved.

Statement Of The Problem

The purpose of this study was to develop a method for objective determination of the proper cutoff frequency of the digital filter used in data smoothing procedures in sport biomechanics. A set of coordinate-time data of freely falling movement was used as the standard data in which no error was involved. Different sets of computer-generated random numbers were used as random errors, and mixed into the standard data. These sets of artificial raw data with random errors were smoothed by the digital filter procedure using varying cutoff frequencies. The filtered outputs were compared with the standard data to determine the best cutoff harmonic. Multiple regression procedures were used to analyze the relationship between the best cutoff frequency and the characteristic variables of raw data. It was expected that a multiple regression equation could be established to determine the best cutoff harmonic from the characteristic variables of raw data. The independent variables selected to reflect the characteristics of errors involved in raw data were sampling frequency, normalized harmonic amplitudes of the signal, and corresponding harmonics.

Definitions

Data smoothing

The mathematical procedure to eliminate or reduce the errors involved in the measured signal.

Data fitness

The mathematical procedure to seek mathematical expression of the measured signal.

Raw data

Also termed as unprocessed data or signal, they are coordinate-time data directly measured from film through the digitizing procedure.

Fourier series

The weighted sum of sine and cosine terms of increasing frequencies. Any continuous signal can be expressed as a Fourier series.

$$X(t) = A0 + \sum_{k=1}^{K} [Ak \sin(Fk\pi t) + Bk \cos(Fk\pi t)] \quad (1)$$

where A0 is the constant term; Ak and Bk are the amplitudes of sine and cosine functions at kth harmonic, respectively; Fk is the frequency at kth harmonic; and t is the independent variable, time; M is the highest harmonic of the signal.

Frequency

The number of cycles per second of a sine or cosine function. Frequency can be expressed in radian/s or Hz, and

$$\text{radian/s} = 2\pi \text{ Hz.}$$

Fundamental frequency

The frequency of the first sine and cosine terms in a Fourier series.

Harmonic

The frequencies of the sine and cosine functions in a Fourier series are expressed as the multiples of the fundamental frequency. These functional components are called harmonics.

Harmonic amplitude

The modulus of the amplitudes of the sine and cosine functions with the same harmonic. The kth harmonic amplitude is expressed as:

$$HAk = \sqrt{Ak^2 + Bk^2} \qquad (2)$$

Harmonic power

The ratio of a harmonic amplitude to the total harmonic amplitude of the signal. The kth harmonic power is expressed as:

$$HPk = \sqrt{Ak^2 + Bk^2} \Big/ \sum_{j=1}^{K} \sqrt{Aj^2 + Bj^2} \qquad (3)$$

Harmonic content

The sum of the harmonic powers below a specific harmonic. The kth harmonic content is expressed as:

$$HCk = \sum_{i=1}^{k} \sqrt{Ai^2 + Bi^2} \Big/ \sum_{j=1}^{K} \sqrt{Aj^2 + Bj^2} \qquad (4)$$

$(k < K)$

Cutoff frequency

The frequency at which the best compromise is obtained to retain most of the signal of interest and the least of the error signal.

Sampling frequency

The number of data points taken per second from the cine film recording.

Knot

When raw data are smoothed by spline functions, the raw data curve will be broken down into sections. The point which connects two adjacent sections is termed as knot.

Nyquit frequency

One-half of sampling frequency. Nyquit frequency can be expressed as:

$$Fn = Fs/2 \tag{5}$$

CHAPTER 2

REVIEW OF RELATED LITERATURE

The literature reviewed for this study is divided into four parts: (1) the nature of errors, (2) the sources of errors, (3) data smoothing techniques used in sport biomechanics, and (4) determination of cutoff frequency for the digital filter. A basic understanding of the nature of errors involved in the measured signal and how errors arise is very helpful for understanding of (a) the comparison of different smoothing techniques used in sport biomechanics, and (b) the principle for determining the appropriate cutoff frequency when using the digital filter. Thus, the first two sections of literature review concern the nature of errors and the sources of errors.

The Nature Of Errors

Human body movement is a process which is continuously changing with time. In sport biomechanics, cinematography is used to measure this continuous process at discrete points. These discrete points are considered as the measured signal of a human body movement. Any measured signal can be divided into two components---actual signal component and error, or noise component.

The error component can be broken down further into two parts---systematic error and random error. A measured signal is the sum of actual signal and error. It has been demonstrated that in human body movement the actual signal component of a cinematography measured signal is typified by large amplitude and low frequency. The total error, or noise on the other hand, is typified by small amplitude and high frequency even though systematic error has low frequency (Lees, 1980; Winter, Sidwall, & Hobson, 1974; Winter, 1979; Wood, 1982).

The effect of the errors in coordinate-time data or displacement data is not serious because of the small amplitude of these errors. But, when derivations are formed to calculate velocity and acceleration, these errors will be amplified because of their high frequencies. To illustrate this problem, consider a sinusoidal motion of frequency F and amplitude A, A sin(Ft), as shown in Figure 1, with some added measurement error in the form of another sinusoid of fequency 10F but amplitude A/10, (A/10)sin(10Ft). In communications-engineering terms, the ratio of signal to noise is 10:1. The measurement signal, f(t), can be written as:

$$f(t) = A \sin(Ft) + (A/10)\sin(10Ft)$$

Upon taking the first derivative,

$$df(t)/dt = AF \cos(Ft) + AF \cos(10Ft)$$

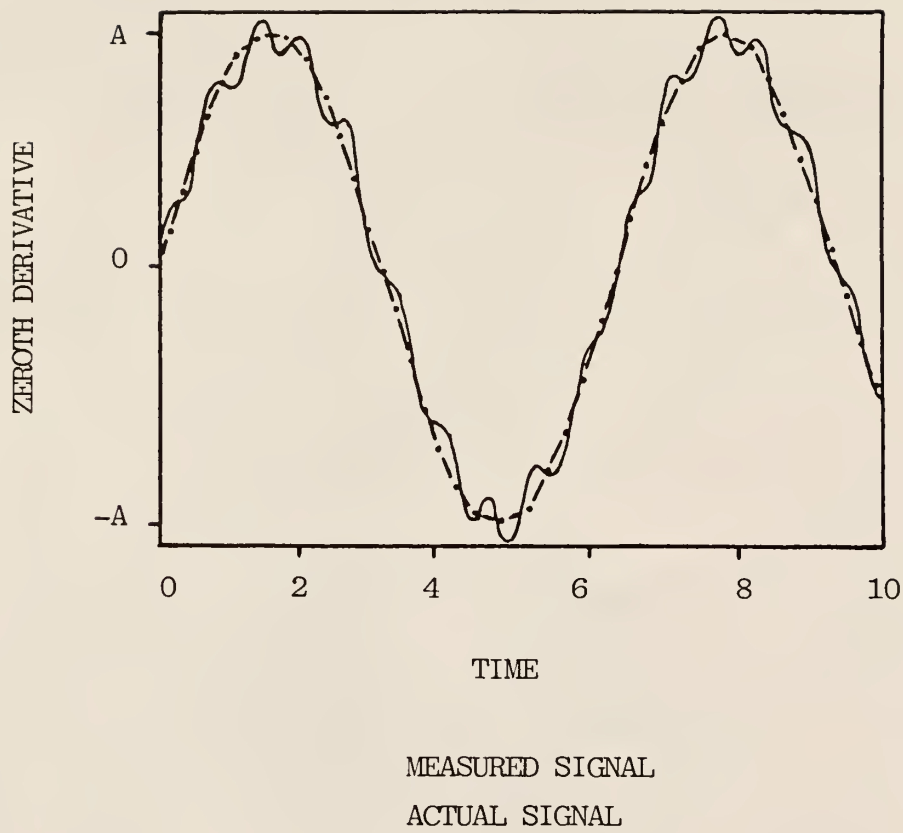

Figure 1. Measured and actual sinusoidal signals (Wood, 1982).

it can be found that the amplitude of noise is equal to that of the actual signal (Figure 2). When differentiated twice,

$$d^2f(t)/dt^2 = -AF^2 \sin(Ft) - 10AF^2 \sin(10Ft)$$

the noise is amplified $100F^2$ times, and the amplitude of noise became 10 times of that of actual signal (Figure 3).

From this example, it can be seen that the amplitude of each of the harmonics increases with its harmonic number in differentiations; for velocities the amplitudes increase linearly, and for accelerations the increase is proportional to the square of the harmonic number (Winter, 1979; Wood, 1982). With such a large amplitude, the noise has concealed the actual signal. In this case, it is very difficult to carry out any analysis of the actual signal, even a qualitative analysis.

Sources Of Errors

The sources of errors associated with the kinematic or kinetic data obtained by cinematography are numerous. Some investigators (McLaughlin, Dillman, & Lardner, 1978; Wood 1982) listed some potential sources of errors associated with kinematic and kinetic data obtained by cinematography as shown in table 1.

Errors arising from misalignment of the camera, perspective error due to the object being out side of the

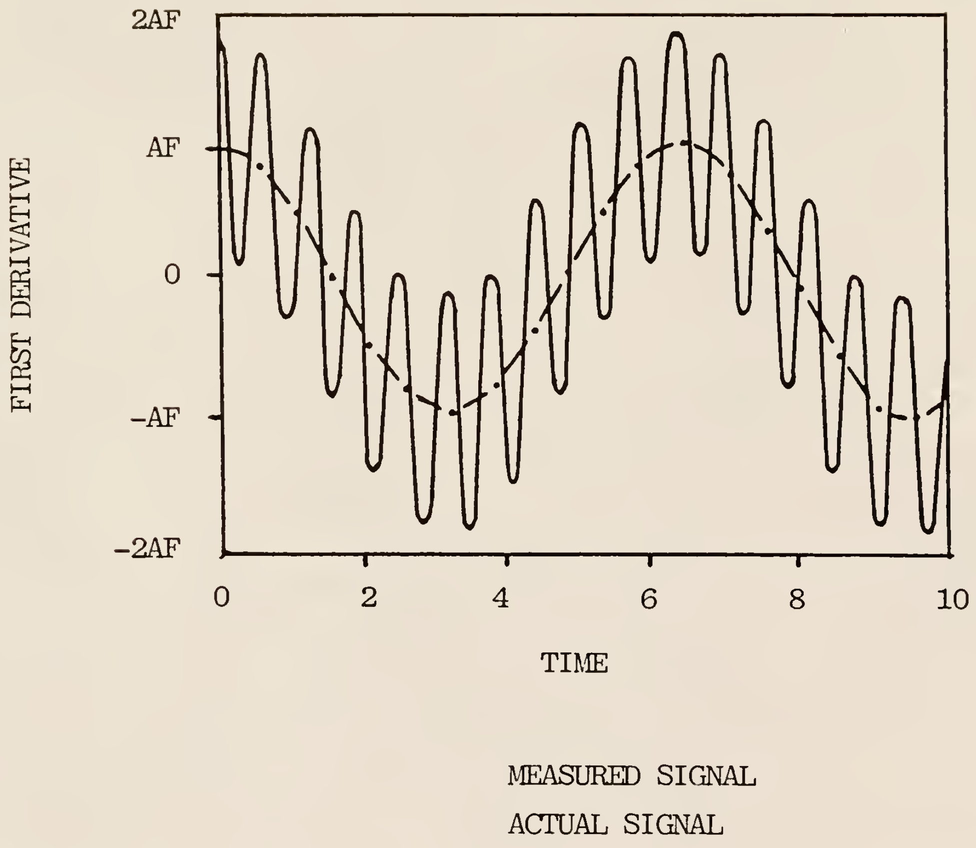

Figure 2. The first derivative of the measured and actual signals (Wood, 1982).

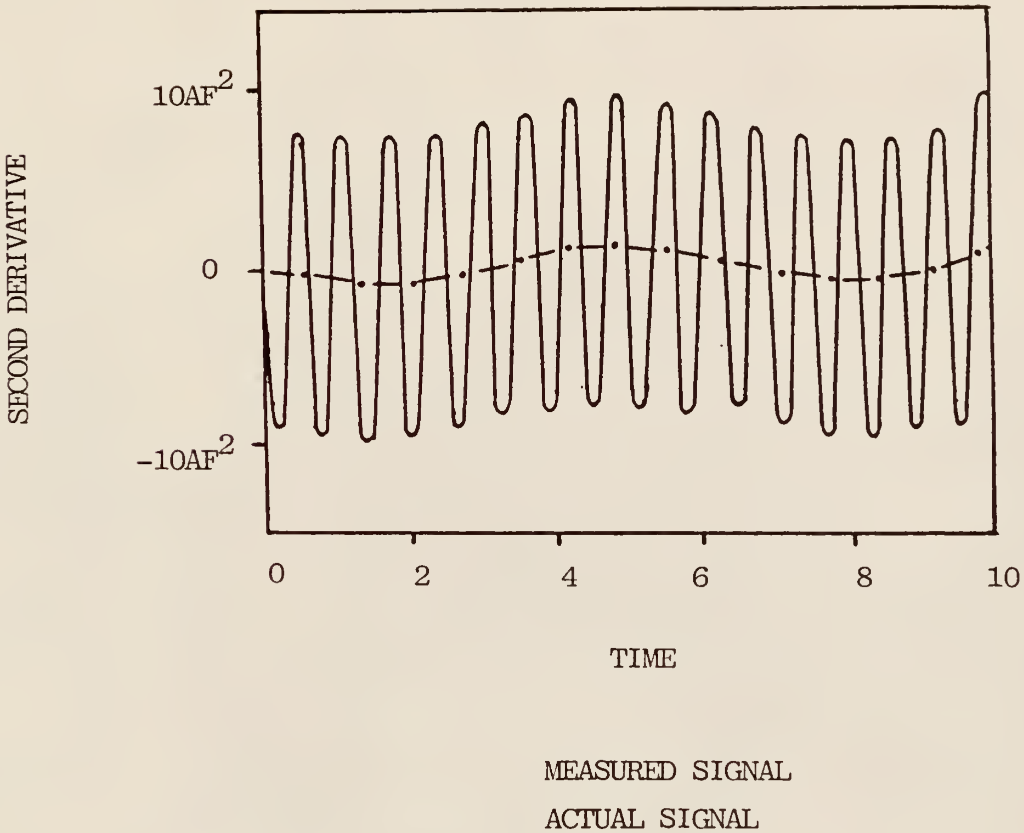

Figure 3. The second derivative of the measured and actual signals (Wood, 1982).

Table 1. Sources of errors in cinematographic analysis.

Filming Procedure	Camera	1. Misalignment of the camera 2. Movement of the camera 3. Imperfect registration of film in the camera
	Subject	Perspective error due to subject out of the photographic plane
Digitizing Procedure	Projector	1. The movement of projector 2. Imperfect registration of film in projector
	Film	1. Stretching of the film 2. Graininess of the film
	Digitizer	Precision limits of digitizer
	Operator	Errors of judgment and parallax in locating joint axes of rotation.

photographic plane, stretching of film or imperfect registration of film in camera, movement of camera, graininess of film are generally considered as systematic errors with low frequencies. These errors all arise in the photographic process and most of them can be mathematically corrected or avoided by selecting proper photographic equipment or methodology (Wood, 1982). For example, errors arising from the misalignment of the camera can be mathematically corrected by setting two spatial scales in the photographic plane with one parallel to the horizon and the other perpendicualr to the horizon. Perspective error due to the object being out side of the photographic plane can be avoided or neglected by using three dimensional cinematography or telescopic lens. Errors arising from stretching of the film or imperfect registratrion of the film and graininess of film can be avoided by using an infrared photography system or a video photography system.

Errors arising from operator errors of judgement and parallex in locating joint axes of rotation are random errors with high frequencies. As previously pointed out, the effect of such errors with small amplitudes but high frequencies on the differentiations of the measured signal is serious. To eliminate or reduce this kind of random error, body markers are used in cinematography. Unfortunately, they do not eliminate the error because: (a) the joint centers of rotation are not on the surface of the skin, (b) the positions of body markers keep changing with respect to the joint centers of rotation and anatomical landmarks in the movement, and (c) body markers can only be used in the laboratory environment. This means that the only "chance" to reduce the effect of these random errors is the mathematical reduction after the digital process, i. e., data smoothing process.

Data Smoothing Techniques Used In Sport Biomechanics

Random errors in measured signals can be smoothed in many ways. The smoothing techniques most often used in sport biomechanics include: (a) finite difference technique, (b) least square polynomial approximations, (c) spline functions, (d) Fourier series, and (e) digital filtering.

Finite difference technique

The derivative of the coordinate-time function y = f(t) is velocity

$$V = dy/dt = f'(t) \tag{6}$$

The derivative of the coordinate-time function at a given point t can be defined as

$$f'(t) = \lim_{\Delta t \to 0} [f(t + \Delta t) - f(t)]/\Delta t \tag{7}$$

This expression represents the slope of the chord of the curve represented by the function y = f(t) between points $[t + \Delta t,\ f(t + \Delta t)]$ and $[t,\ f(t)]$. As Δt approaches zero, the slope of the chord of the curve represented by the function $y = f(t)$ between points $[t + \Delta t, f(t + \Delta t)]$ and $[t, f(t)]$ will infinitely approximate the slope of the tangent at the point $[t, f(t)]$. For finite time intervals, however, a better approximation to the derviative of the function $y = f(t)$ at t is found in the slope of the chord of the curve represented by the function $y = f(t)$ between points $[t + \Delta t, f(t + \Delta t)]$ and $[t - \Delta t, f(t - \Delta t)]$, which can be given by the formula

$$V = f'(t) = [f(t + \Delta t) - f(t - \Delta t)]/(2\Delta t) \tag{8}$$

where $f(t + \Delta t) - f(t - \Delta t)$ is referred to as the first central difference.

Similarly, the acceleration at point t can be obtained by applying the central differences twice to the coordinate-time function

$$A = f''(t) = [f(t + 2\Delta t) - 2f(t) + f(t - 2\Delta t)]/4\Delta t^2 \tag{9}$$

Equations (8) and (9) are called the finite difference technique. These equations are used as a smoothing technique in sport biomechanics by some researchers. However, the poor results of this method have been shown by several studies (Li & Yu, 1983; Pezzack, Norman, & Winter, 1977; Winter, 1979; Wood, 1982). In fact, the finite difference technique is nothing more than an appropriate calculation method for velocity and acceleration in cinematography.

Least square polynomial approximations

In many scientific experiments, experimenters measure N + 1 data points of continuous processes and want to develop functions for these continuous processes through N + 1 data points obtained from the experiments. If these N + 1 data are accurate enough, a polynomial of degree n can be developed through these N + 1 data to mathematically describe the process from which these N + 1 data are obtained. This polynomial can be derived in such a way that it represents an analytical line of best fit. This procedure requires that the sum of the squares of the deviations be minimized, so it is called the least square polynomial approximation or fitness procedure. Wood (1982) described this procedure in detail.

This least square polynomial approximation procedure has been used by numerous researchers in biomechanics studies to develop some low order polynomials (n = 1-7) fitted to either N + 1 total data in the series, or a small number (N = 2-8) of data points at a time.

Some researchers (Li & Yu, 1983; Pezzack, et.al., 1977; Winter, 1979; Wood, 1982) evaluated the smoothing results of least square polynomial approximations. The results of their studies have shown that the smoothing results of least square polynomial approximations are not better than those of finite difference techniques. The poor smoothing results of least square polynomial are caused by two factors. First, in sport biomechanics, the movement of any human body point is an unknown function. It is very difficult to determine the degree n of the polynomial for a given body point in a given movement. The different degrees of the polynomials will cause great differences in smoothing results (Lees, 1980; Wood, 1982). Second, in least square polynomial approximations, the polynomials are developed to fit the experimental data. If the experimental data were accurate enough, the accurate function might be obtained through least square polynomial approximations. But, unfortunately, in sport biomechanics, the raw experimental data obtained through cinematography are not accurate enough and have serious errors. In fitting a polynomial of degree n to a set of N + 1 experimental data points which has serious errors, when the least square has been obtained, most errors may be retained in the signal.

Spline functions

The spline function method is a modification of the least square polynomial approximation technique. The essential principle of the spline function is to break the curve into sections, with special fitting being done between adjacent sections. Detailed descriptions of the mathematical formulae of spline functions can be found elsewhere (Dunfield & Read, 1972; Greville, 1969; Rice, 1969; Wold, 1974).

Some researchers (McLaughlin, Dillman, & Lardner, 1978; Soudan & Dierdkx, 1979) analyzed some human body movements with cubic spline functions. The results of their experiments suggested that the cubic spline can provide an accurate description of displacement-time data and the corresponding time derivatives. Wood (1982) stated that spline functions have extreme flexibility and pronounced local properties which make them well suited to biomechancial applications.

Despite the advantages of the spline function method mentioned above, there are still some problems in the procedure. The major problems with this technique are: (1) consideration must be given to the appropriate degree of the spline function, the number and positions of the knots or the junction of adjacent segments, required accuracy of least squares fit, and the management of end-conditions (Winter, 1979; Wood, 1982); (2) at least 20 extra points have to be taken at the endpoint regions for the confident interpretation of results when the acceleration at end point is not zero (McLaughlin, et.al., 1978); (3) for the best results, the number of data points should be at least 50 (McLaughlin, et.al., 1978; Wood, 1982); and (4) the complexity of the curve fitting requires much more computer time. These problems limit the accuracy and the application of spline function smoothing procedure.

Fourier series

It is known that a function, f(t), can be expressed as a weighted sum of polynomials in terms of t. This weighted sum of polynomials is called a series. The Fourier series is one such series. A Fourier series provides a means of expressing a periodic function as a weighted sum of sine and cosine terms of increasing frequency. Winter (1979) and Wood (1982) have described the Fourier series smoothing technique in detail.

Hatze (1981) outlined a Fourier sine series, a modification of the Fourier series, as a data smoothing technique. He introduced a set of transformation data in this approach. This transformation permits an odd extension of the measured signal in the form of a Fourier sine series.

It has been pointed out that Fourier series have a very good basis in periodic movement (Winter, 1979; Wood, 1982). Very satisfactory smoothing results of this technique have been shown in some studies (Hatze, 1981; Wood, 1982). However, this approach also can be generalized to nonperiodic data sequences by a suitable manipulation of the data when it is assumed that the data are repetitive, especially when transformation data are introduced in this smoothing procedure. Under these circumstances, the frequencies at which the data are analyzed have no meaningful relationship to physical phenomenon being studied and are only a mathematical convenience (Wood, 1982).

Fourier series can be used not only in data smoothing but also in harmonic analysis for human body movements. Winter et.al. (1974) applied this Fourier analysis approach in studying harmonic contents of human walking. Alexander (1980) did a Fourier analysis of forces exerted in walking and running.

Wood (1982) has pointed out that one of the major problems of the Fourier series smoothing technique is the high cost of computer time. The study done by Li and Yu (1983) identified another problem associated with this technique: the difficulty in determining the appropriate cutoff harmonic. Improperly determined cutoff harmonic will result in grossly over- or undersmoothed derivative estimates. Some poor smoothing results of the Fourier series smoothing procedure might be caused by a wrong selection of the cutoff harmonic.

Digital filter

A digital filter is a frequency-selective device that accepts as input a sequence of equispaced numbers and operates on them to produce as output another number sequence of limited frequency. The digital filter most often used in sport biomechanics is an alternative form called recursive or autoregressive filter. The output of this type of filter depends not only on present and past samples of unsmoothed original signal but also on past smoothed values of the original signal. This form of digital filter is said to have an infinite memory, or "feedback", in that some knowledge of all previous data is retained in the process (Winter, 1979; Wood, 1982). Winter et.al. (1974) advocated a second-order recursive filter.

The details of this procedure can be found in several studies (Li & Yu, 1983; Winter, et.al., 1974; Winter, 1979; Wood, 1982).

In the digital filter smoothing procedure, there is a set of coefficients which need to be determined by the user according to the cutoff and the sampling frequencies. Winter (1979) has calculated and published some sets of these coefficients for different sampling and cutoff frequencies. Those coefficients published by Winter (1979) can be directly used in the digital filter smoothing procedure by approximating the ratio of cutoff frequency to sampling frequency to one of those published by Winter (1979). The best smoothed output, however, results when the coefficients are calculated for the given measured signal.

Pezzack et.al. (1977) and Li et.al. (1983) compared several smoothing techniques. Their results have suggested that the digital filter is the best of all the smoothing techniques used in sport biomechnics studies. Wood (1982) has pointed out that the computer time for this smoothing procedure is the shortest of that of all the smoothing techniques used in sport biomechanics. For these reasons, such a device has attracted the interest of biomechanists in spite of the problem associated with selecting the cutoff frequency.

Determination Of Cutoff Frequency For The Digital Filter

The quality of the smoothed output of the digital filter depends on the proper determination of cutoff frequency. Figure 4a (Winter, 1979) showns a schematic plot of a signal and the noise spectrum in the measured signal. From this plot, it can be seen that, as previously pointed out, the signal occupies the lower end of the frequency spectrum and the error, or noise, occupies the higher end of the frequency spectrum. There is an overlap of actual signal and noise. When the measured signal is smoothed by the digital filter, as can be seen in Figure 4b, the response at lower frequencies is 1.0. This means that the input signal passes through the filter unattenuated. However, there is a sharp transition around the cutoff frequency, Fc, and the signals above Fc are severely attenuated. The result of the filtering process is shown in Figure 4c. Winter (1979) pointed out that two things should be noted: (1) the higher frequency noise has been severely reduced but not completely eliminated; and (2) the signal, especially in the region where the signal and noise overlap (around Fc), also is slightly attenuated. This attenuation results in a slight distortion of the signal.

Thus, a compromise has to be made in the selection of the cutoff frequency. If the cutoff frequency is set too high, less signal distortion occurs but too much noise will be allowed to pass. Conversely, if the cutoff frequency is too low, the noise may be completely eliminated, but too much signal will be lost (Lees, 1980; Winter, 1979; Wood, 1982).

Pezzack, et.al. (1977) compared the peaks of the

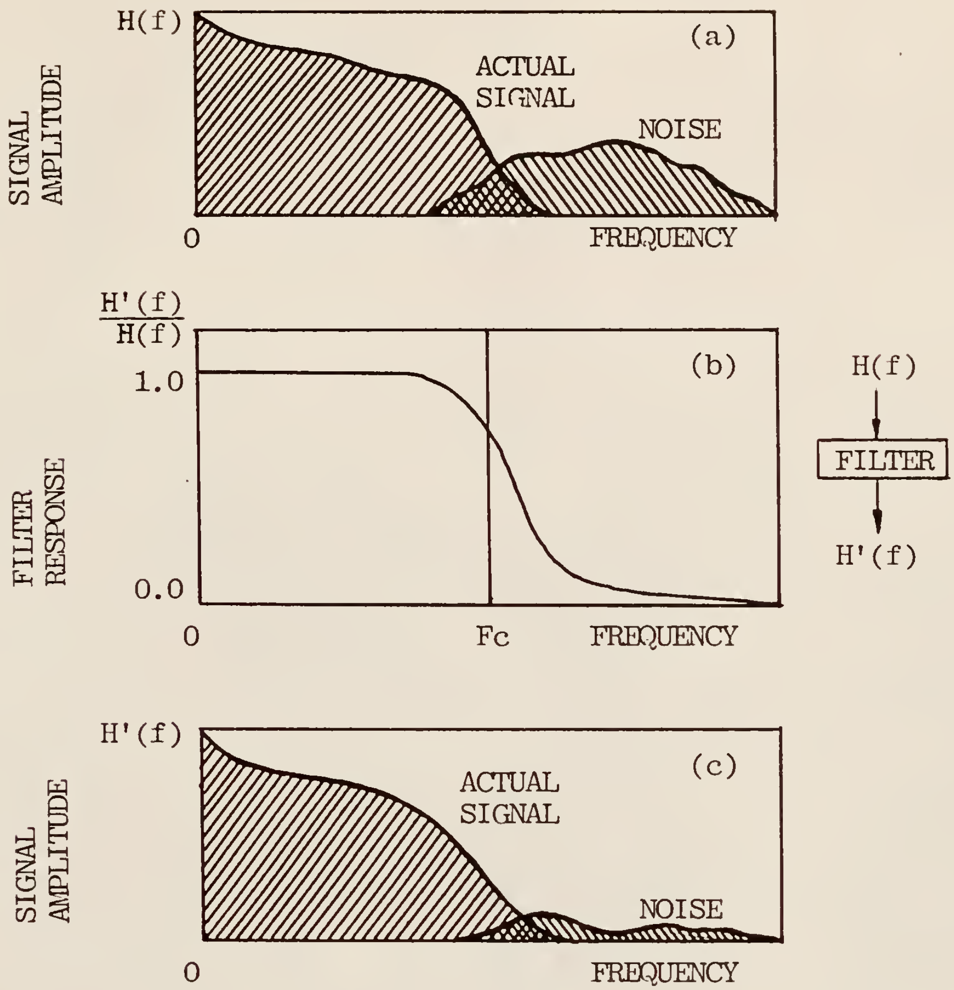

Figure 4. Frequency spectrum of measured signal before and after filtered and filter response (Winter, 1979).

filtered curves to the raw curves in his digital filter procedure. If the peaks of the filtered curves were too low, the cutoff frequency of the filter would be raised; if the oscillations of the filtered curves were too close to those seen in the unfiltered curves, the cutoff frequency would be reduced. Here the question remains: what are "too low" and "too close"? On the other hand, in the case where error characteristics are not known, it is very difficult to give out a proper cutoff frequency through the comparision between the filtered and unfiltered signals.

Harmonic analysis (Cook & Rabinowicz, 1963) is one of the best ways to gain insight into the frequency spectrum of the measured signal as a forerunner to the design of digital filters to separate signal from noise (Wood, 1982). Some researchers (Li, et.al., 1983; Winter, 1974; Winter, 1979) have applied this method in determination of the cutoff frequency for the digital filter.

In the harmonic analysis, the harmonic amplitudes can be normalized as the percentage of fundamental harmonic amplitude or as the harmonic power. When a harmonic amplitude is normalized as the percentage of the fundamental harmonic amplitude, it can be expressed mathematically as:

$$NHAk = \sqrt{Ak^2 + Bk^2} \Big/ \sqrt{A1^2 + B1^2} \qquad (10)$$

When NHAk is less than a specific value AD, k will be selected as the cutoff fequency.

When a harmonic amplitude is normalized as the harmonic power, it can be mathematically expressed as:

$$HPk = \sqrt{Ak^2 + Bk^2} \Big/ \sum_{j=1}^{K} \sqrt{Aj^2 + Bj^2} \qquad (11)$$

when HPk is less then a specific value PD, k will be selected as the cutoff frequency.

Two questions now must be answered to complete these selection processes: (1) how to determine the highest harmonic M of the measured signal, and (2) how to determine the proper value of AD and PD.

Summary

In cinematography, there are some random errors which can not be avoided and must be eliminated or reduced in the data smoothing process. The digital filter is one of the best smoothing techniques used in sports biomechanics. Research suggests that very satisfactory smoothing results can be obtained through the use of this smoothing technique if appropriate cutoff frequency is selected. Further research designed to develop a procedure to determine the appropriate cutoff frequency is needed.

CHAPTER 3

METHOD

The procedures for this study are presented in this chapter. Data collection, harmonic analysis, Fourier series fitness, digital filter procedure, resemblance evaluation, statistical analysis, and all pertinent mathematical formulas are included. All of the procedures of this study are shown in Figure 5.

Standard Data

The only way to evaluate the smoothed results is to compare the smoothed data to the actual data (here called standard data). Pezzack *et.al.* (1977) designed an aluminium "arm" to simulate the arm motion of the human body, and positioned an accelerometer on the free end of this "arm" to record the actual acceleration data which were used as the standard data. McLaughlin *et.al.* (1978) used force platform data as the standard data. In this study, the vertical coordinate-time data of free fall body movement, and corresponding velocity and acceleration data was employed as the standard data. These standard data were obtained from the following equations:

$$Sa(t) = G\,t^2/2 \quad (m) \qquad (12)$$

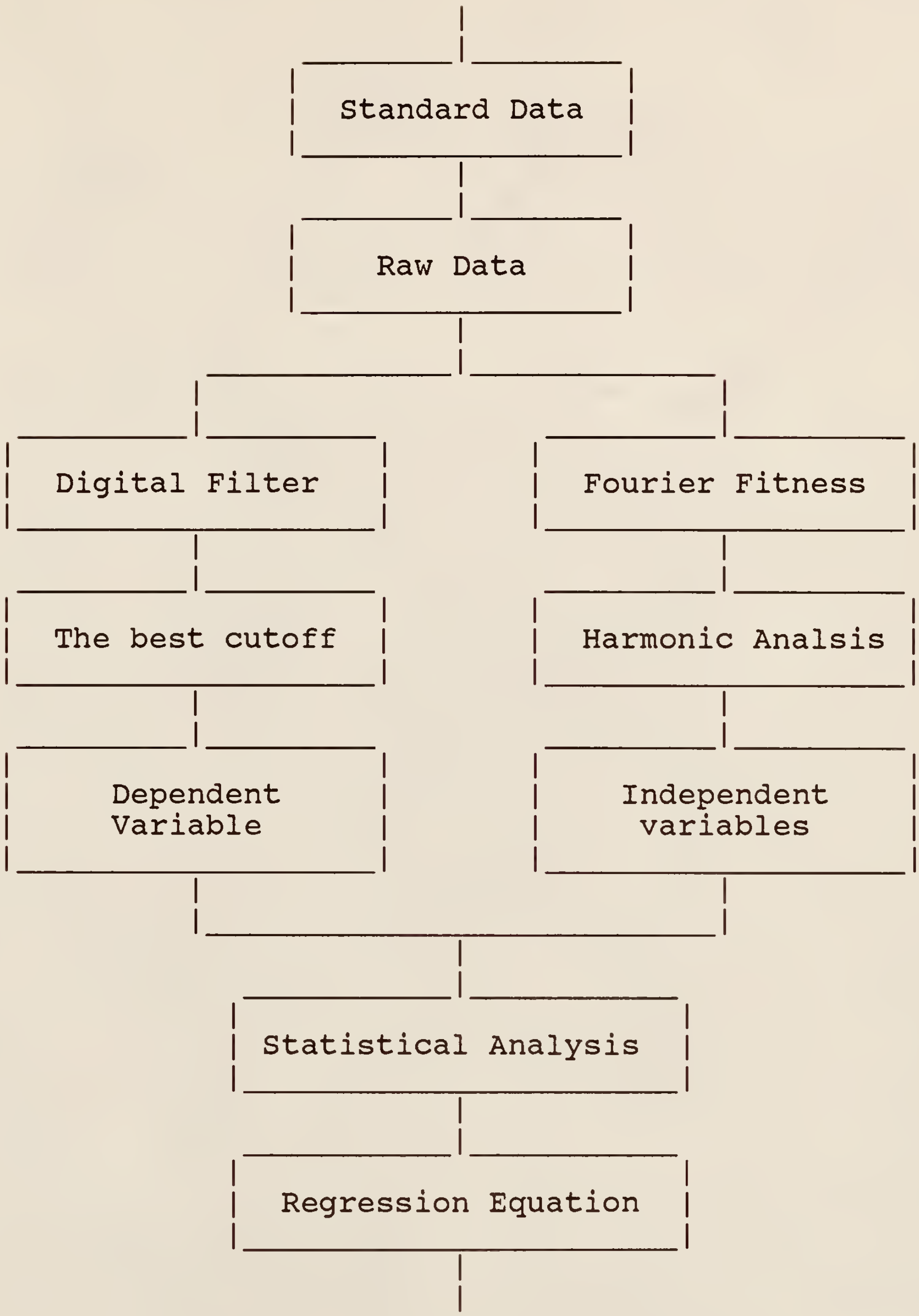

Figure 5. Flow chart of the procedure.

$$V(t) = G\ t \quad (m/s) \tag{13}$$

$$G = 9.8 \quad (m/s^2)$$

Raw Data

As previously pointed out, any measured signal from film represents the actual signal plus or minus the error or noise inherent in the procedure used to record and measure the sample data. This is represented by the following expression:

$$Sm(t) = Sa(t) + e(t) \tag{14}$$

where Sm, Sa, and e are measured signal, actual signal, and error, respectively. Systematic errors can be minimized by conducting well-controlled experimental and measurement procedures. If controls are appropriate, the total error associated with obtaining measures from film can be reduced to random errors of digitizing. In this study, different sets of random real numbers were generated by the computer from the following equation:

$$e(t) = [R(t) - 0.5]/C \tag{15}$$

where R(t) was a random real number which was less than 1.0 and greater than 0; C was a constant. This constant, C, was given different values for each sampling frequency. The values of C controlled the mean absolute values of different sets of random real numbers at each sampling frequency. The different values of the constant, C, and the corresponding mean absolute values are shown in Table 2.

Table 2. C values and corresponding mean absolute values.

C	5.0	10.0	20.0	40.0	80.0
MAV	0.05000	0.02500	0.01250	0.00625	0.00312

MAV --- the mean absolute values, the unit of MAV is m.

These different sets of random real numbers with different mean absolute values were mixed with the standard data as the random errors to comprise different sets of raw data with different mean random errors at each sampling frequency.

Table 3 contains the sampling frequencies used in this study.

Table 3. Sampling frequencies (Hz).

Fs	30	60	100	150	200	250	300	350	400	450	500

These sampling frequencies included all of those widely used in cinematography in sport biomechanics.

Digital Filter Process

To determine the most appropriate cutoff frequency for a given measured signal, the raw data were smoothed by a digital filter with different cutoff harmonics, and the filtered outputs were compared to the standard data.

The raw data were filtered both forward and backward to overcome the phase characteristic of the digital filter. When this was done, the digital filter was written as

$$Y1_0 = y_0$$

$$Y1_1 = y_1$$

$$Y1_i = A0\ y_i + A1\ y_{i-1} + A2\ y_{i-2} + B1\ Y1_{i-1} + B2\ Y1_{i-2} \quad (16)$$

$(i = 2, 3, \ldots, N-2, N-1, N)$

$$Y2_N = y_N$$

$$Y2_{N-1} = y_{N-1}$$

$$Y2_i = A0\ Y1_i + A1\ Y1_{i+1} + A2\ Y1_{i+2} + B1\ Y2_{i+1} + B2\ Y2_{i+2} \quad (17)$$

$(i = N-2, N-3, \ldots, 2, 1, 0)$

where Y1 and Y2 were forward and backward filtered outputs, respectively. A0, A1, A2, B1, and B2 were filter coefficients defined as:

$$A0 = 1/(C^{2} + \sqrt{2}\ C + 1) \qquad (18)$$

$$A1 = 2\ A0 \qquad (19)$$

$$A2 = A0 \qquad (20)$$

$$B1 = (2\ C^{2} - 2)/(C^{2} + \sqrt{2}\ C + 1) \qquad (21)$$

$$B2 = (\sqrt{2}\ C - C^{2} - 1)/(C^{2} + \sqrt{2}\ C + 1) \qquad (22)$$

where C was defined as:

$$C = 1/\tan(\pi\ Fc/Fs) \qquad (23)$$

where Fc and Fs were, respectively, cutoff frequency and sampling frequency.

The cutoff frequency at which a set of raw data was best smoothed was considered as the best cutoff frequency for this set of raw data.

Evaluation Of Filtered Results

To determine the best cutoff frequency of the measured signal for the digital filter, the filtered results with different cutoff frequencies were evaluated in order to determine the best filtered result. In this study, error energy analysis was employed to evaluate the filtered results with different cutoff frequencies.

Error energy of the two sets of data is defined as the mean square of deviations of two sets of data. The error energy Q of two sets of data can be written as

$$Q = \sum_{i=0}^{N} (X1i - X2i)^2 / (N + 1) \tag{24}$$

where X1 and X2 represent two curves which have been digitized. Generally, when Q reaches its minimum, there is the strongest resemblance between the two compared curves. In the evaluation of filtered results, when error energy between filtered data and the standard data reached its minimum, the filtered result was considered as the best.

Fourier Fitness

Before harmonic analysis of the measured signal, Fourier series must be used to fit the measured signal. When a measured continuous signal f(t) is fitted with a Fourier series, it can be expressed as

$$f(t) = A0 + \sum_{k=1}^{K} [Ak \cos(2\pi kt/T) + Bk \sin(2\pi kt/T)] \tag{25}$$

where t is the time point at which the signal f(t) is measured; T is the total time interval during which the signal occurs. A0, Ak, and Bk are Fourier coefficients and can be expressed as

$$A0 = 1/T \int_{-T/2}^{T/2} f(t)dt \tag{26}$$

$$Ak = 1/T \int_{-T/2}^{T/2} f(t) \cos(2\pi kt/T)dt \tag{27}$$

$$Bk = 1/T \int_{-T/2}^{T/2} f(t) \sin(2\pi kt/T)dt \tag{28}$$

(k = 1, 2, ... , K)

In cinematography, the measured signal is a set of discrete data, y0, y1, y2, ... , yN. In this case, the measured signal can be fitted by a Fourier series as

$$fi = A0 + \sum_{k=1}^{K} [Ak \cos(2\pi ki/N) + Bk \sin(2\pi ki/N)] \tag{29}$$

$(i = 0, 1, 2, \ldots, N)$

$(k = 1, 2, \ldots, K; \quad M < K/2)$

where N + 1 is the total number of frames digitized.

Considering that Fourier series have a very good basis in periodic movements (Winter, 1979; Wood, 1982), the data transformation outlined by Hatze (1981) was used to make the data have the characteristics of periodic data. Hatze defined this data transformation as the following:

$$g_i = y_i - y_0 - [(y_N - y_0)/N]i \tag{30}$$

$(i = 0, 1, 2, \ldots, N)$

Because of this data transformation, the measured signal fitted by Fourier series was expressed as

$$f_i = y_0 + [(y_N - y_0)/N]i + A_0 + \sum_{k=1}^{K} [A_k \cos(2\pi ki/N) + B_k \sin(2\pi ki/N)] \tag{31}$$

where

$$A_0 = 1/T \int_{-T/2}^{T/2} g(t)dt \tag{32}$$

$$Ak = 1/T \int_{-T/2}^{T/2} g(t)\cos(2\pi kt/T)\,dt \tag{33}$$

$$Bk = 1/T \int_{-T/2}^{T/2} g(t)\sin(2\pi kt/T)\,dt \tag{34}$$

$(k = 1, 2, \ldots, K)$

where g(t) denoted an unknown continuous function of t expressed by the discrete values of g0, g1, g2, ... , gN.

The integrations in equations 32, 33, and 34 were accomplished by different numerical integral analysis methods. In this study, the Simpson integration rule (Davis, 1975) was employed to calculate A0, Ak, and Bk.

When the measured signal was best fitted, K was be considered as the highest harmonic of the measured signal.

Evaluation Of Fourier Fitness Results

To determine the highest harmonic of the measured signal, Fourier fitnesses of the measured signal with different values of K have to be evaluated in order to determine the best Fourier fit of the measured signal. The error energy analysis used in evaluation of filtered results was used to evaluate the Fourier fitness results. When error energy between fitted data and the raw data reached its minimum, the fitness result was considered as the best.

Harmonic Analysis

As previously pointed out, harmonic analysis is one of the best ways to reveal the characteristics of random errors in a measured signal. There are two ways to normalize harmonic amplitudes in harmonic analysis. In this study, considering that Fourier fitness and the evaluation of fourier fitness results was very time consuming and would reduce calculation speed in future application, harmonic amplitudes of raw data were normalized as the percentages of fundamental harmonic amplitude by equation 10.

Statistical Analysis

From the above procedures, the best cutoff frequency and the normalized harmonic contents for each set of raw data were obtained. A multiple regression analysis was carried out in which the best cutoff frequency were treated as the dependent variable. The following regression equation is expected

$$Fc = F(X1, X2, ..., Xp) \tag{36}$$

where Fc is the best cutoff frequency; X1, X2, ..., Xp are some independent variables selected based on the results of analysis of the raw data.

Application Of Regression Equation

The regression equations obtained by the procedure described above were used to determine the proper cutoff frequency for the film angular displacement data collected by Pezzack, *et*.*al*. (1977). The filtered angular displacement data and the corresponding angular acceleration data were compared with the analog data (Pezzack, *et*.*al*., 1977) to examine the filtered results at the cutoff frequencies determined by the regression equations.

CHAPTER 4

RESULTS AND DISCUSSION

110 sets of raw data were generated by the computer. There were 2 sets of raw data for each mean error controllor C at each sampling frequency. Harmonic spectrums were analyzed for all of these raw data. Then, based on the harmonic analysis of measured signals, independent variables were selected to establish regression equations to estimate the proper cutoff frequency. Different models for estimating the proper cutoff frequency were tested. Two regression equations were established based on the results of statistical analysis. Because the harmonic analysis was the key in selection of possible independent variables, the result of harmonic analysis is shown in the first section of this chapter.

Harmonic Analysis Of Measured Signal

A typical harmonic spectrum is shown in Figure 6. In this study, it was found that the normalized harmonic amplitudes of a measured signal kept decreasing before a special harmonic but began to oscillate after this special harmonic. The existence of this special harmonic had not been reported before this study. Considering the overlap of actual signal and error pointed out by Winter (1979) and comparing Figure 6 to frequency spectrum in Figure 4, it is

Figure 6. Harmonic spectrum of measured signal.

very reasonable to consider this special harmonic as the indicator of the beginning of the overlap of actual signal and error. Therefore, in this study, this special harmonic was named as overlap harmonic which was denoted by Ho and the normalized harmonic amplitude of this special harmonic was named as normalized overlap harmonic amplitude which was denoted by NHAo.

Selection Of Variables

Dependent variable

Figure 7 is the symmetry plot for the best cutoff frequencies. It can be found that the best cutoff frequencies were skewed to the region of the distribution above the median, and a power transformation was needed to make the distribution of the best cutoff frequencies nearly symmetric. When The 0.5th power power transformation was given to Fc, the data were nearly symmetrically distributed as shown in Figure 8. Therefore the square root of the best cutoff frequency was determined as the dependent variable in regression analysis procedures of this study. The square root of Fc was denoted by PFc and expressed as:

$$PFc = Fc^{1/2}$$

Independent variables

Figure 9 shows the relationship between PFc and sampling frequency Fs. It was found that PFc is highly

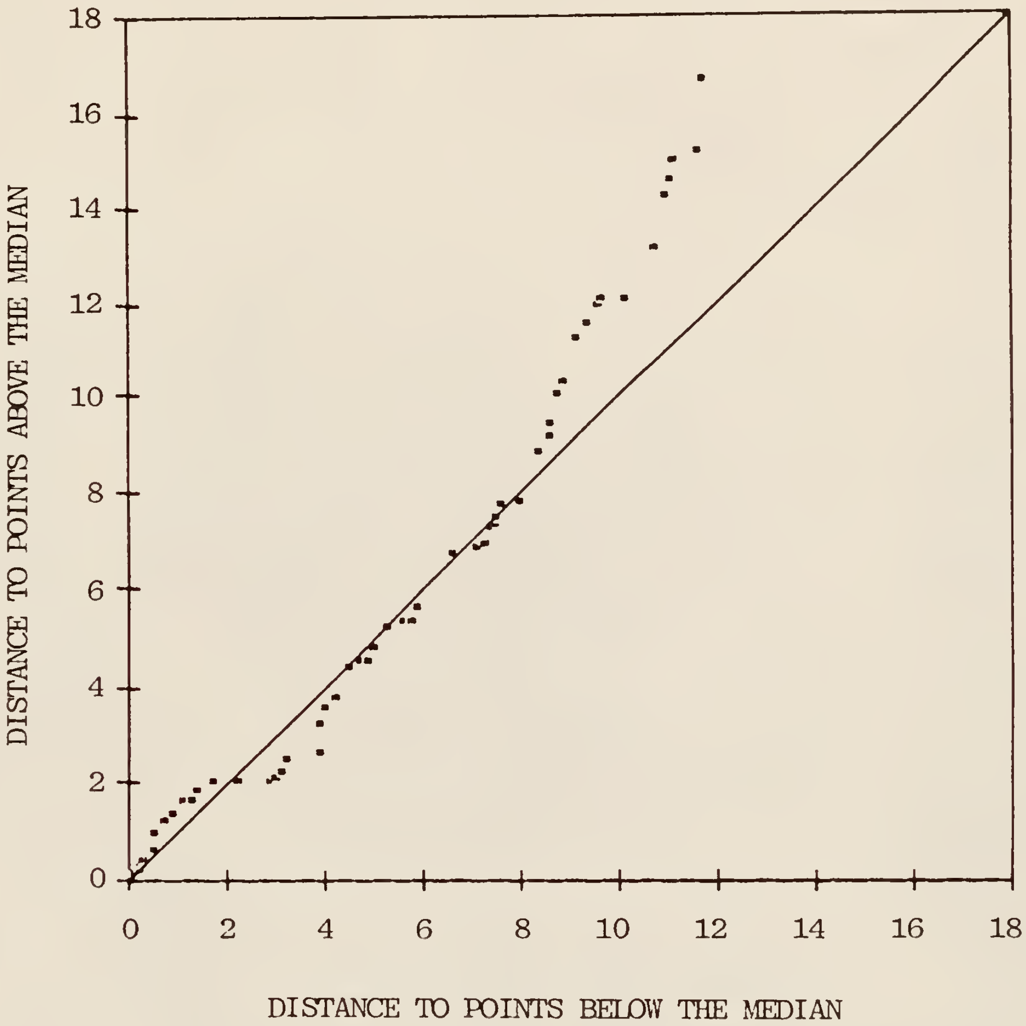

Figure 7. A symmetry plot for the best cutoff frequency (Fc).

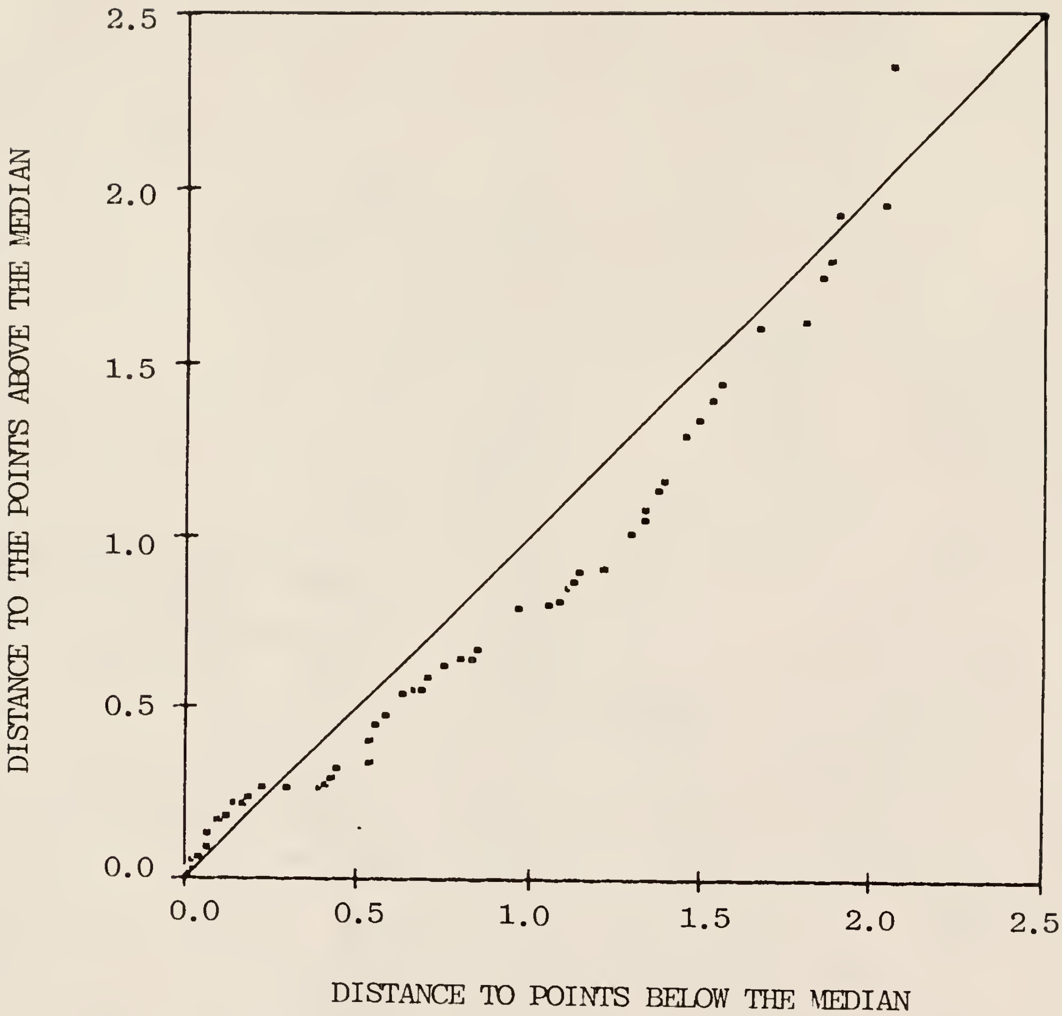

Figure 8. A symmetry plot for the square root of the best cutoff frequency (PFc).

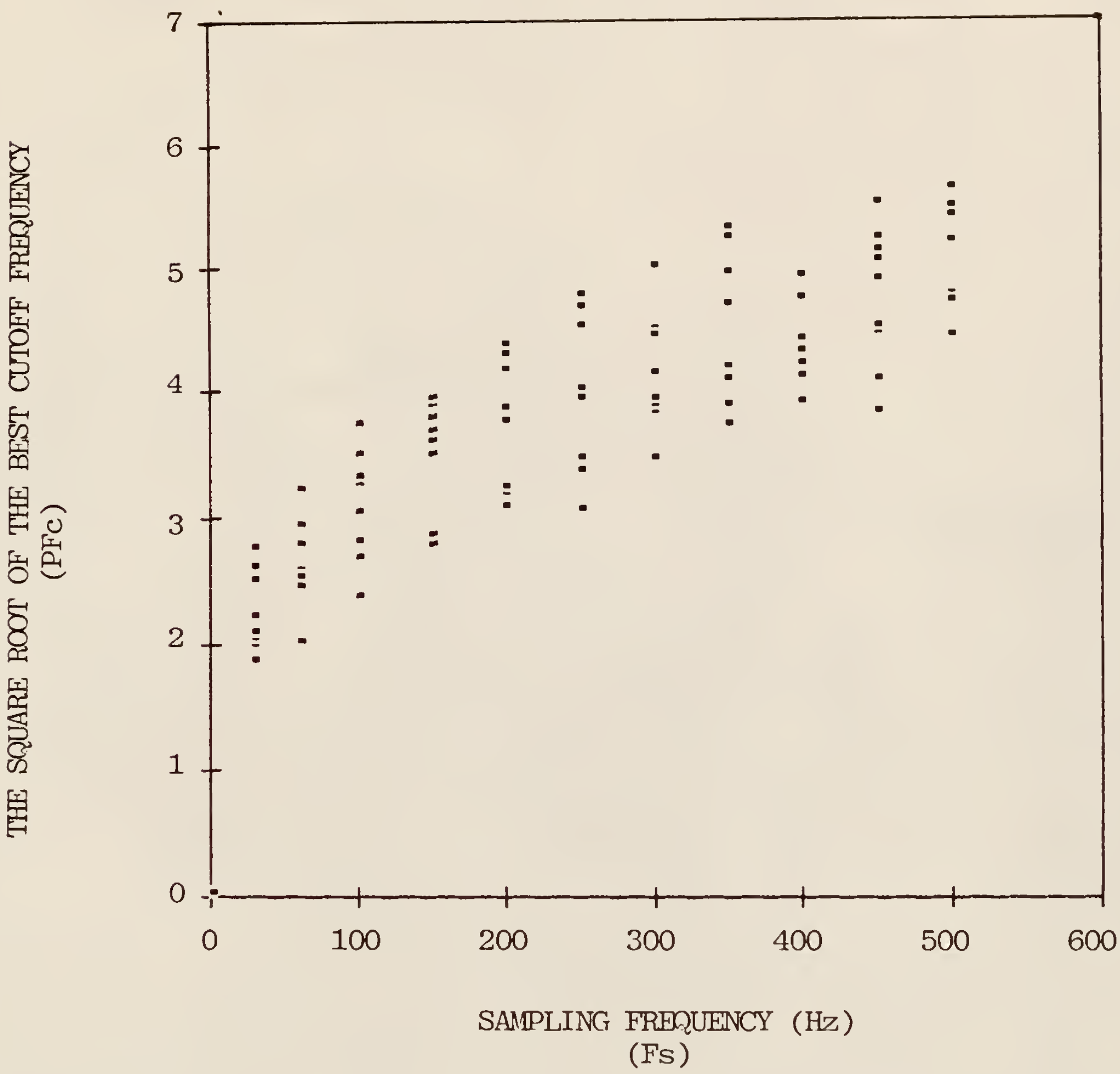

Figure 9. The relaitonship between the square root of the best cutoff frequency (PFc) and sampling frequency (Fs).

correlated to Fs. However, the relationship between PFc and Fs was not linear. According to the relationship between PHc and Fs shown in Figure 9, a 0.5th power transformation was given to Fs

$$PFs = Fs^{1/2}$$

Figure 10 shows the relationship between PFc and PFs. It can be seen that the relationship between PFc and PFs was quite linear.

Previous studies didn't report the influence of sampling frequency on cutoff frequency. However, Winter (1974) analyzed the harmonic spectrum of joint angles in walking with a sampling frequency of 60 Hz. He demonstrated that the proper cutoff frequencies of the measured signals he analyzed were 5 to 7 Hz. This is just in the range of the variation of the best cutoff frequencies of the measured signals at the sampling frequency of 60 Hz found in this study.

The finding of the influence of sampling frequency on cutoff frequency in this study indicates that if the sampling frequencies and other factors are the same, the cutoff frequencies should be the same no matter what the highest frequency present in the signal is. This is in agreement with the results of Winter's study (1974). The results of that study indicate that there is no significant influence of movement speed on cutoff frequency. However,

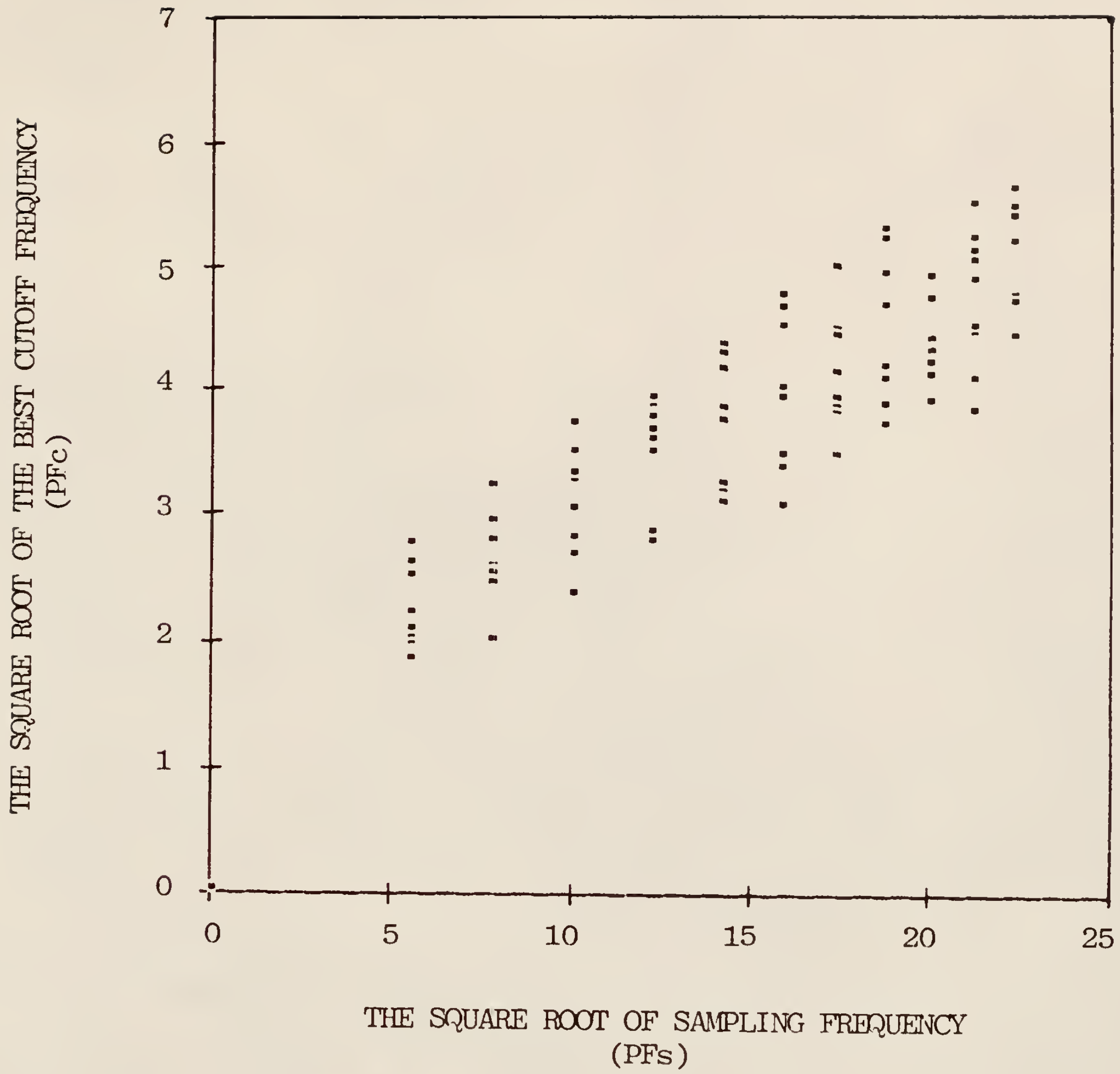

Figure 10. The relationship between the square root of the best cutoff frequency (PFc) and the square root of sampling frequency (PFs).

the following facts should be noticed: (1) the higher the speed of movement is, the higher the highest frequency present in the actual signal itself is, (2) the higher the sampling frequency is, the higher the highest frequency present in the measured signal is and the more details of the actual signal could be picked up, (3) according to the sampling theorem (Winter, 1979), the higher the highest frequency present in the actual signal itself is, the higher the sampling frequency should be in order to pick up the details of the signal, and (4) the positive correlation between sampling frequency and the best cutoff frequency found in this study indicates that the higher the sampling frequency is, the higher the cutoff frequency should be. When these facts are considered, it can be said that the speed of movement has an indirect influence on the cutoff frequency. The non-significant influence of speed of movement on cutoff frequency found by Winter (1974) is due to the same sampling frequency used for all of the signals analyzed in his study.

Considering the relationship between PFs and PFc, PFs was selected as the first independent variable for the regression analysis procedures in this study.

Examining Figure 10, it also can be seen that PFc at the same sampling frequency varied in a relatively wide range. This means that the best cutoff frequency is also influenced by some other factors besides sampling frequency, and some other variables besides PFs are needed to estimate the best cutoff frequency.

As previously pointed out, if the best smoothing result of the digital filter is expected, a compromise has to be made in determination of cutoff frequency to remain as more actual signal and less error as possible. To make this compromise in determination of cutoff frequency the following factors should be considered: (1) the relative amplitude of actual signal before the overlap of actual signal and error, (2) the position in the harmonic spectrum of the actual signal the overlap of the actual signal and error begins, and (3) the relative amplitude of error. So, after sampling frequency has been considered, the other variables selected to estimate best cutoff harmonic should be able to predict these three factors. The finding of overlap harmonic provides a basis for selection of possible variables to predict these factors.

To detect the effects of the other possible independent variables on cutoff frequency, the difference between each PFc and the mean PFc at each sampling frequency were calculated as the following:

$$DPFcij = PFcij - MPFcj$$

$$(i = 1, 2, \ldots , 10; j = 30, 50, 100, \ldots , 450, 500)$$

where DPFcij was the difference between ith PFc at j sampling frequency and the mean PFc at j sampling frequency; MPFcj was the mean PFc at j sampling frequency.

It has been pointed out in harmonic analysis of measured signals that overlap harmonic is a predictor of the beginning of the overlap of actual signal and error. However, the overlap harmonic was not considered as a possible independent variable in the expected regression equation. What the overlap harmonic indicates is the absolute position in the harmonic spectrum of the measured signal the error begins to mix with the actual signal. This absolute beginning position of overlap in the harmonic spectrum of the measured signal may be different for different kinds of data. If the overlap harmonic were employed as an independent variable in regression analysis procedures of this study, the use of the regression equations to the data of other kinds of movements would be limited. The possible independent variables considered for the regression analysis procedures in this study should be able to indicate the common charicteristics of all kinds of data.

The normalized harmonic amplitude before the overlap harmonic was the first one considered as a predictor of the above mentioned factors. This normalized harmonic amplitude was denoted by NHA1. Here, the hypothesis was that NHA1 could predict not only the relative amplitude of the signal before the overlap of the signal and error but also the relative position of the overlap in harmonic spectrum of the actual signal. The higher NHA1 was, the lower the overlap harmonic would be and the more the errors there would be in the overlap, therefore, the cutoff frequency would be lower. Contrarily, the lower NHA1 was, the higher the overlap harmonic would be and the less the errors there would be in the overlap, therefore, the cutoff frequency would be higher. The functions of NHA1 was partially demonstrated by the relationship between NHA1 and Ho shown in Figure 11. Figure 12 is the symmetry plot for NHA1. This symmetry plot shows that the distribution of NHA1 was skewed to the left. A -0.125th power transformation was given to NHA1, and PNHA1 was used to denote this transformation

$$PNHA1 = NHA1^{-1/8}$$

As shown in Figure 13, the distribution of PHNA1 was nearly symmetric.

The relationship between PNHA1 and DPFc was shown in Figure 14. Some correlation between PNHA1 and PFc can be detected from this figure. Therefore, PNHA1 was considered as the second possible independent variable.

The third possible independent variable considered was the -0.125th power of the normalized overlap harmonic amplitude. The -0.125th power of NHAo was denoted by PNHAo. The reason to consider PNHAo as one of the possible independent variables in expected regression equation was the same as that for PNHA1. Figure 15 illustrates the relationship between NHAo and Ho. The relationship between these two variables was quite similar to that between NHA1

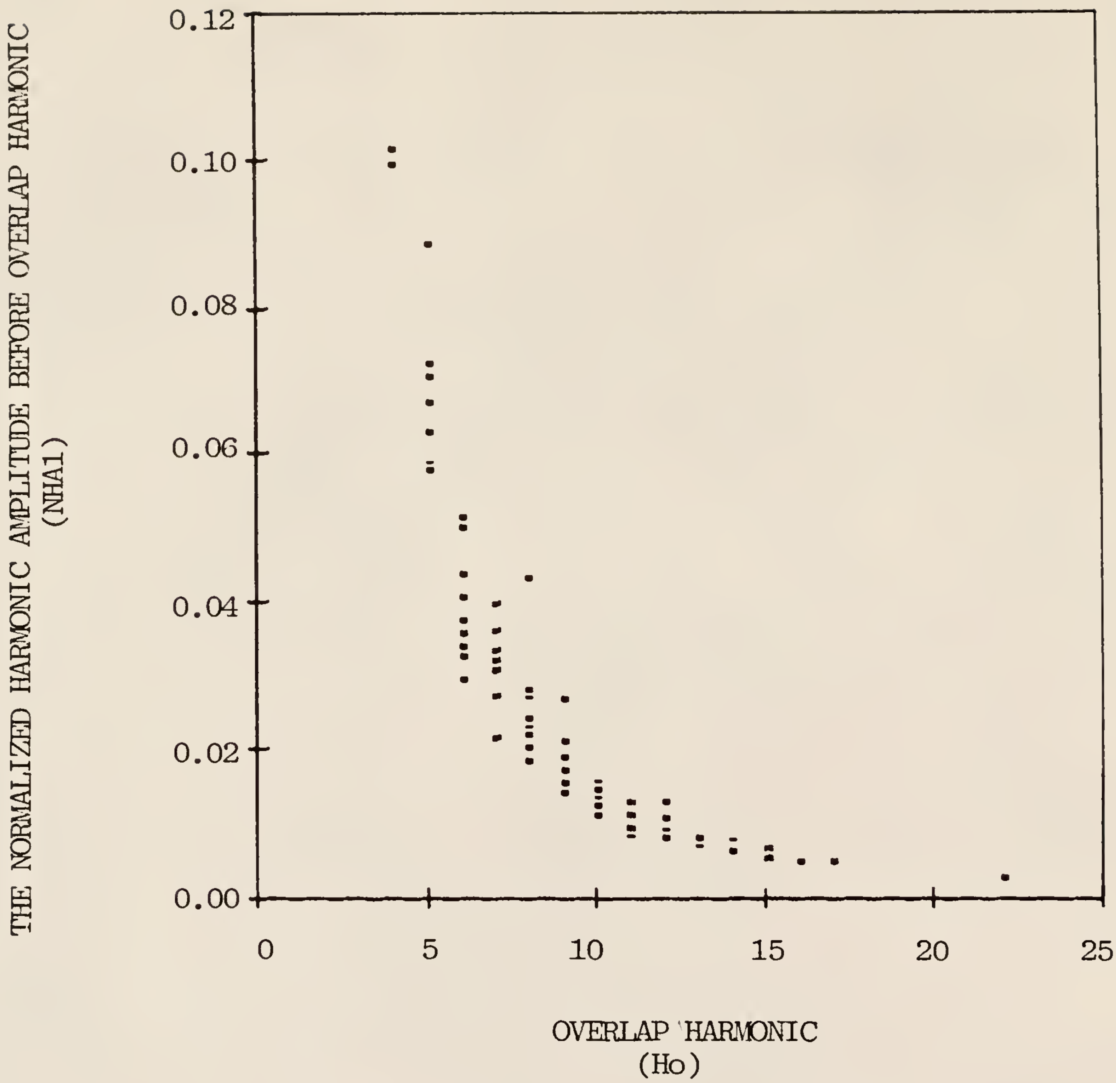

Figure 11. The relationship between the normalized harmonic amplitude before the overlap harmonic (NHA1) and the overlap harmonic (Ho).

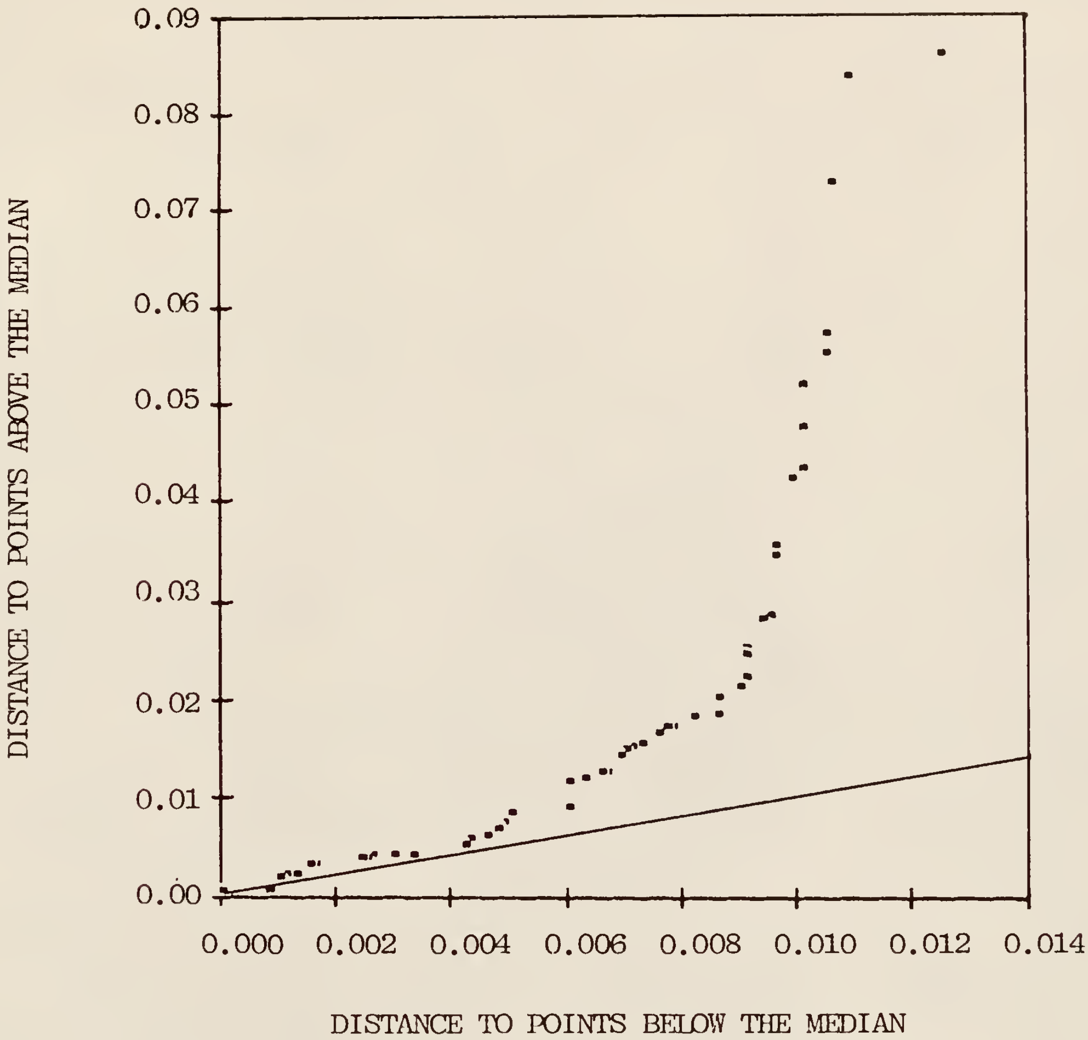

Figure 12. A symmetry plot for the normalized harmonic amplitude before the overlap harmonic (NHA1).

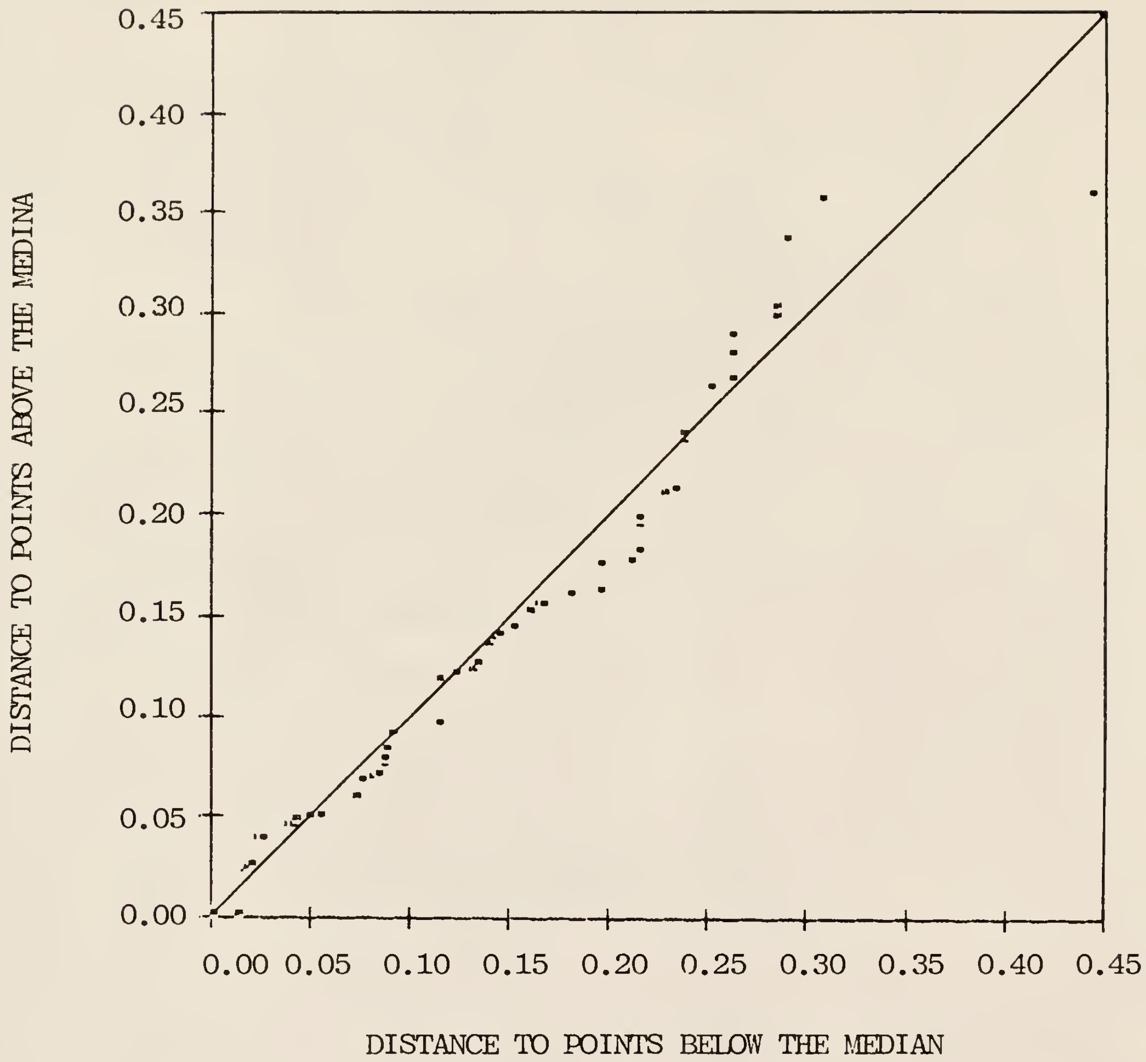

Figure 13. A symmetry plot for the -0.125th power transformation of the normalized harmonic amplitude before the overlap harmonic (PNHA1).

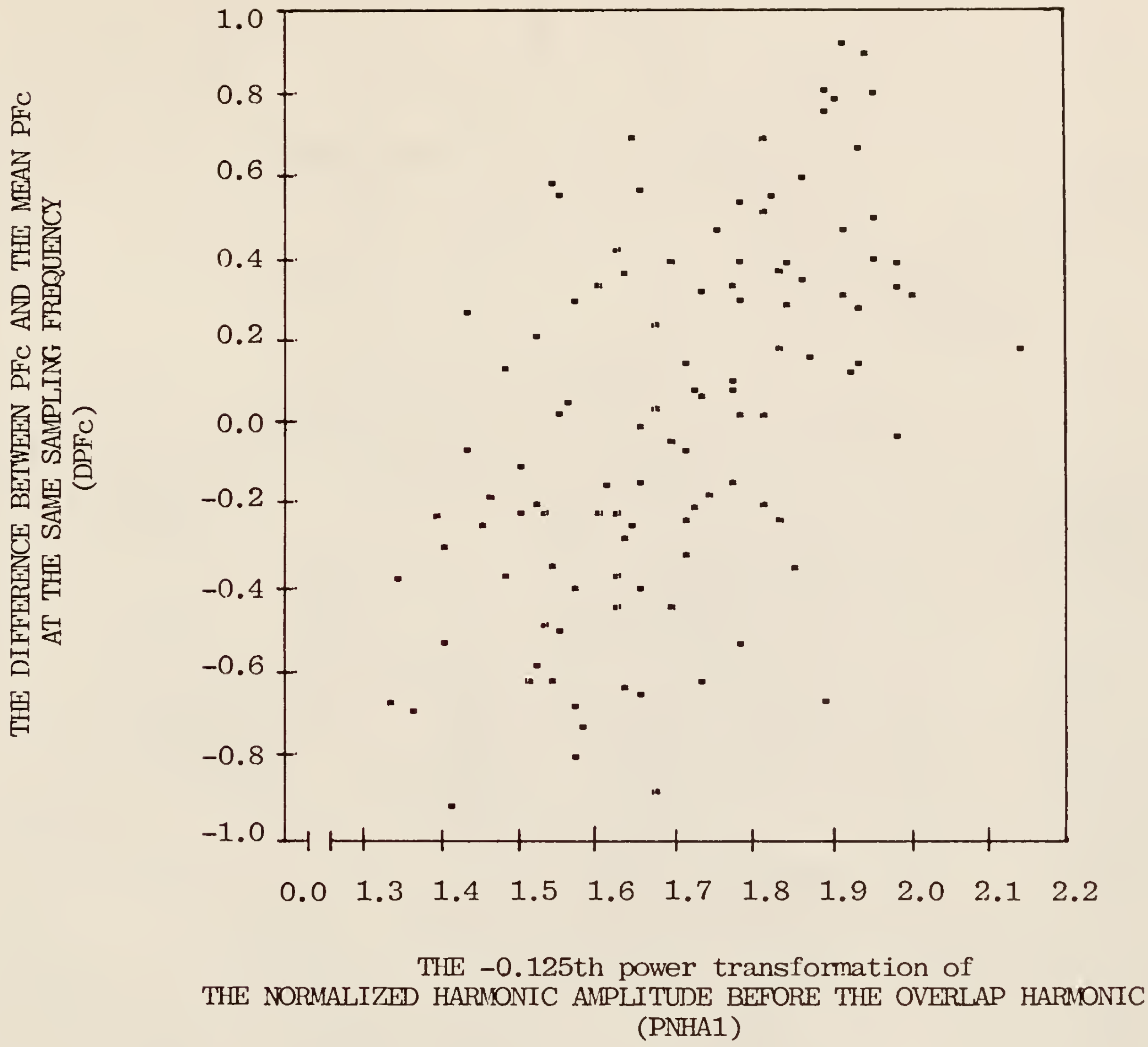

Figure 14. The relationship between the -0.125th power transformation of the normalized harmonic amplitude before the overlap harmonic (PNHA1) and the difference between the square root of the best cutoff frequency and the mean of the square root of the best cutoff frequency at the same sampling frequency (DPFc).

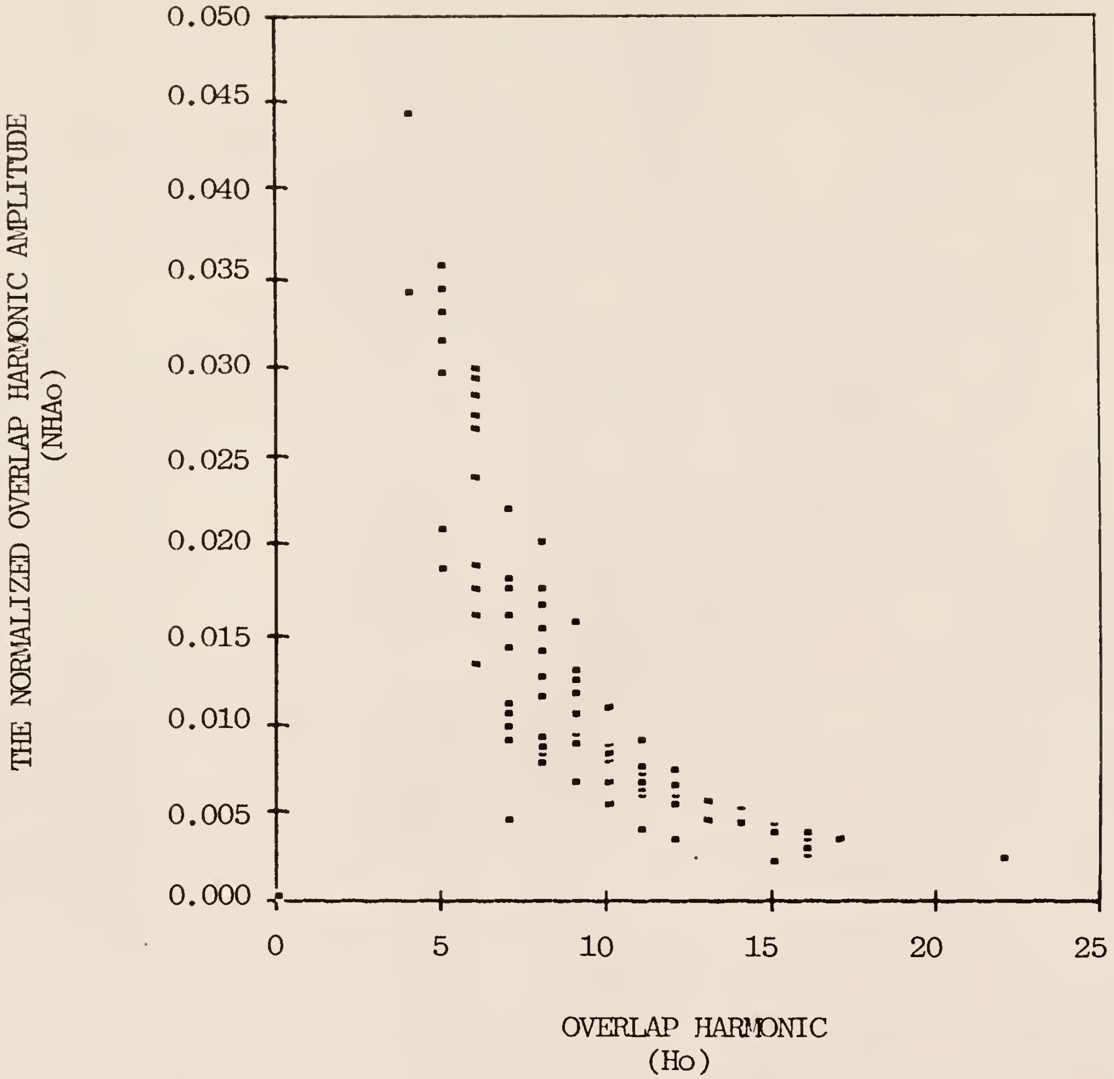

Figure 15. The relationship between the normalized overlap harmonic amplitude (NHAo) and the overlap harmonic (Ho).

and Ho. Figures 16 and 17 are symmetry plots for NHAo and PNHAo respectively. Figure 17 shows that PNHAo was nearly symmetrically distributed. Figure 18 shows the relationship between PNHAo and DPFc. Some correlation between these two variables can be seen in this figure.

Here it should be noticed that error begins to mix into actual signal at Ho. NHAo might have included some error. Therefore, the function of PNHAo as a predictor of the relative harmonic amplitude of the signal before the overlap of the signal and error and the relative position of the beginning of the overlap in the spectrum of actual signal might not be as effective as that of PNHA1.

To predict the relative amplitude of error, the standard deviation of the five normalized harmonic amplitudes after Ho was calculated as the following

$$ME = \sqrt{\sum_{i=Ho+1}^{Ho+n} NHAi^{2} - \left(\sum_{i=Ho+1}^{Ho+n} NHAi \right)^{2} /n \,]/(n-1)}$$

(If Hm - Ho >= 5, n = 5;
 if Hm - Ho < 5, n = Hm - Ho)

where ME is the standard deviation of the five normalized harmonic amplitudes after Ho. The correlation between ME and the mean error can be detected in Figure 19. Therefore, it was expected that ME could act as a predictor of the relative amplitude of error in the overlap of tha actual

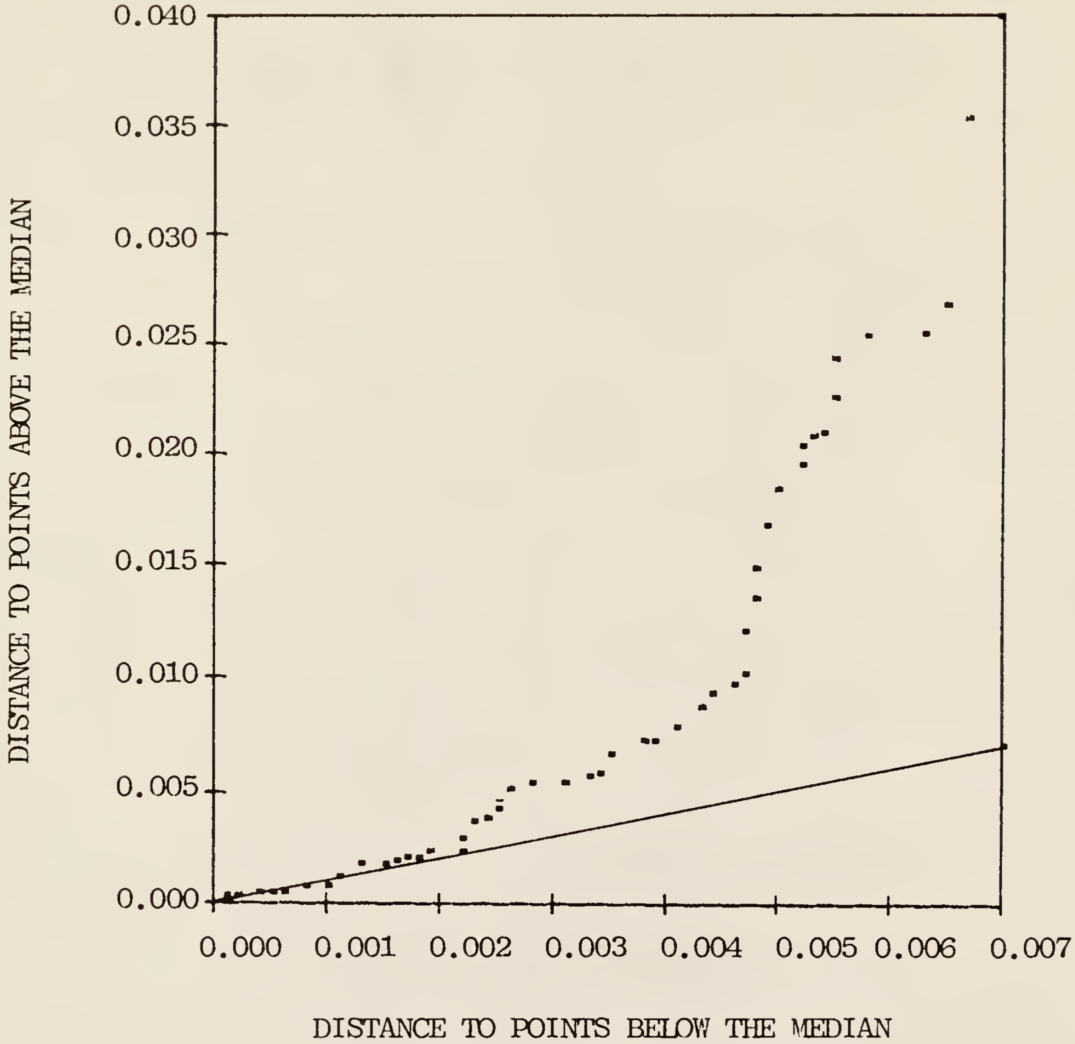

Figure 16. A symmetry plot for normalized overlap harmonic amplitude (NHAo).

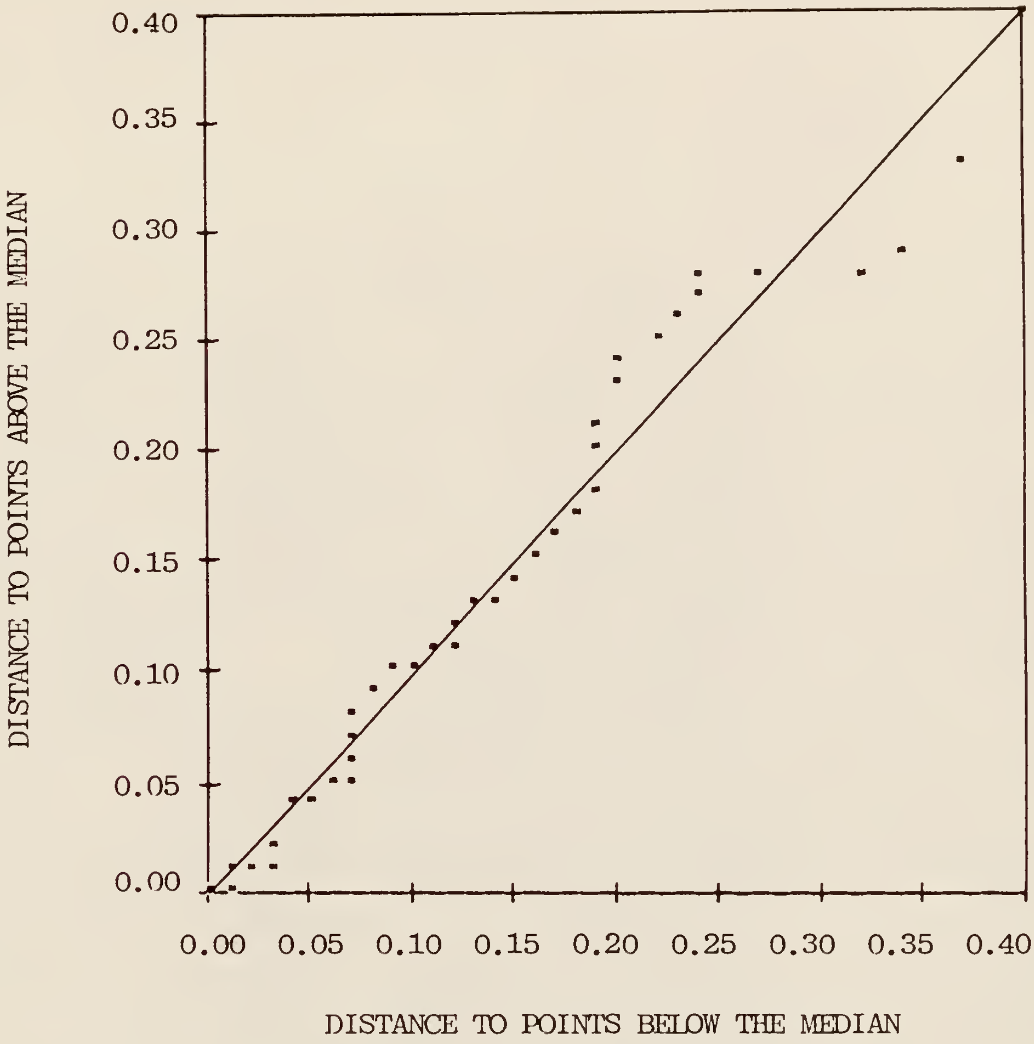

Figure 17. A symmetry plot for the -0.125th power transformation of the normalized overlap harmonic amplitude (NHAo).

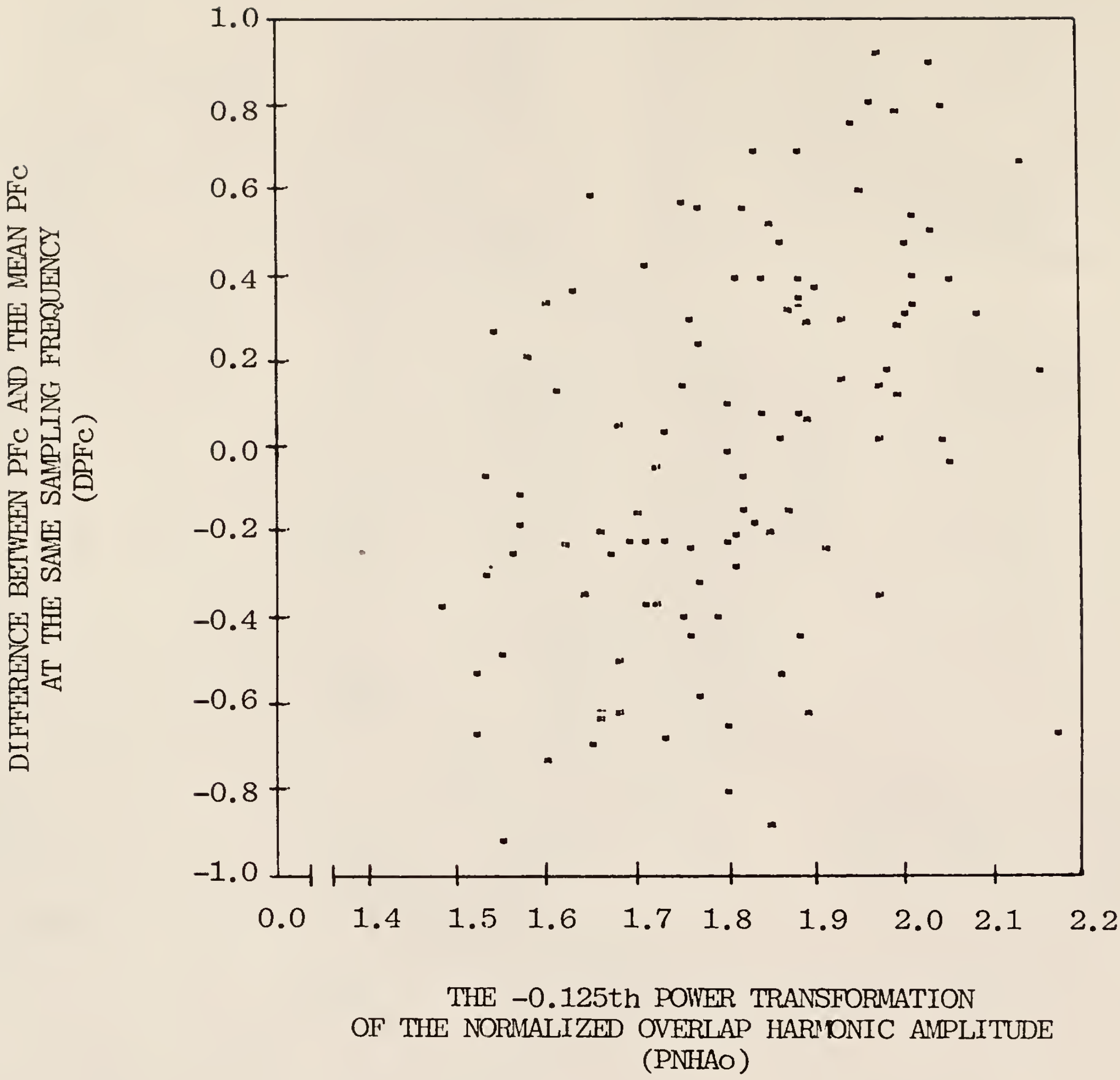

Figure 18. The relationship between the -0.125th power transformation of the normalized overlap harmonic amplitude (PNHAo) and the difference between the square root of the best cutoff frequency and the mean of the square root of the best cutoff frequency at the same sampling frequency (DPFc).

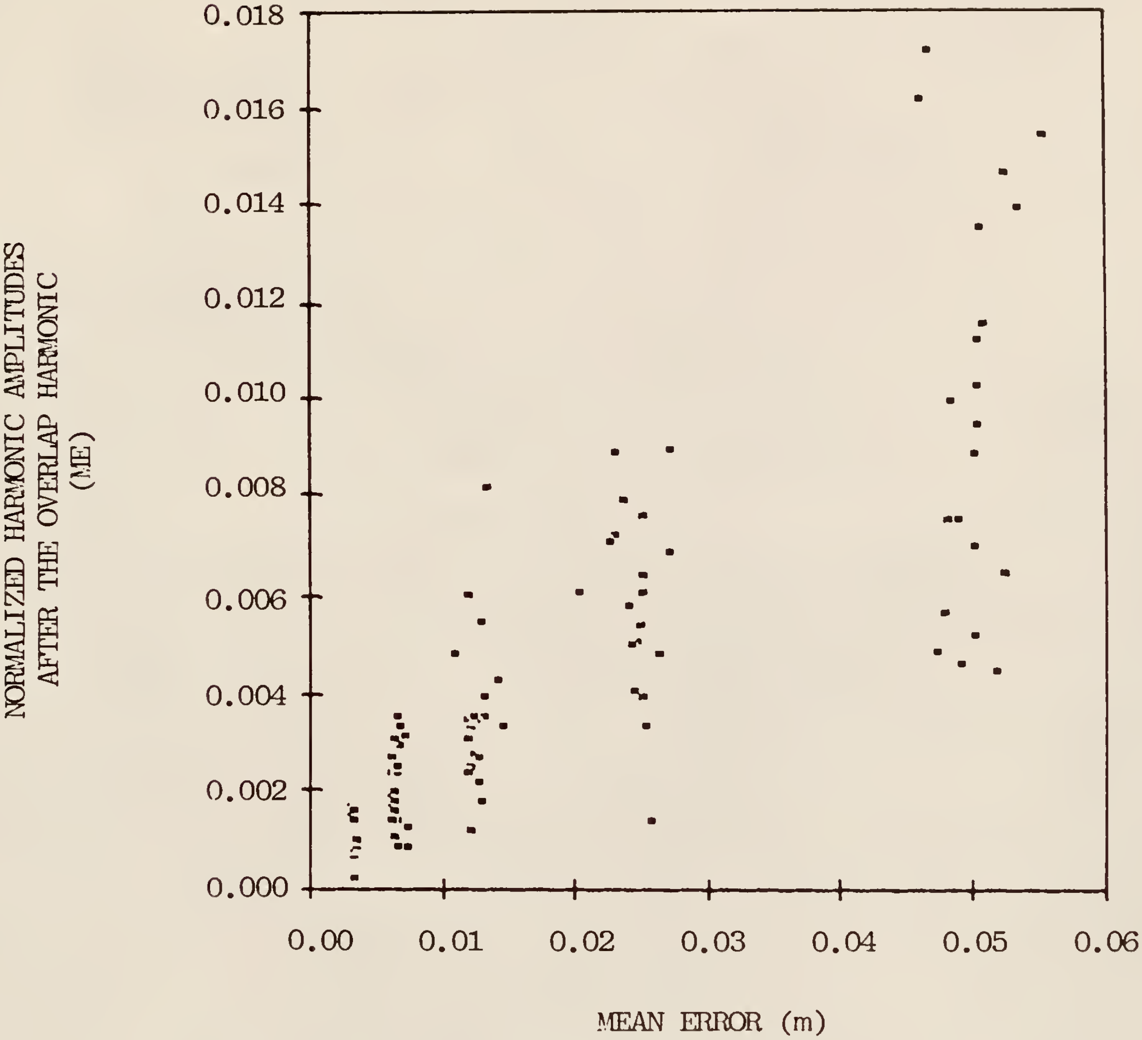

Figure 19. The relationship between mean error and the standard deviation of the five normalized harmonic amplitude after the overlap harmonic (ME).

signal and error, and that the higher ME was, the lower the cutoff frequency would be, and the lower the ME was, the higher the cutoff frequency would be.

Figure 20 is the symmetry plot for ME. This symmetry plot shows that the distribution of ME was skewed to the region of the distribution below the median. Figure 21 is the symmetry plot for the 0.25th power transformation of ME. This transformation was expressed as

$$PME = ME^{1/4}$$

The distribution of PME was nearly symmetric as seen in Figure 21. The relationship between PME and DPFc is shown in Figure 22. There seems to be some correlation between these two variables. Therefore, PME was considered as the fourth possible independent variable in regression analysis procedures.

Regression Analysis

The Backward elimination method was used to build the regression equation. The first model to be tested was

$$PFc = L0 + L1\ PFs + L2\ PNHAl + L3\ PNHAo + L4\ PME + e \tag{35}$$

The statistics for this model are shown in Table 4. The analysis of variance shows that overall regression was

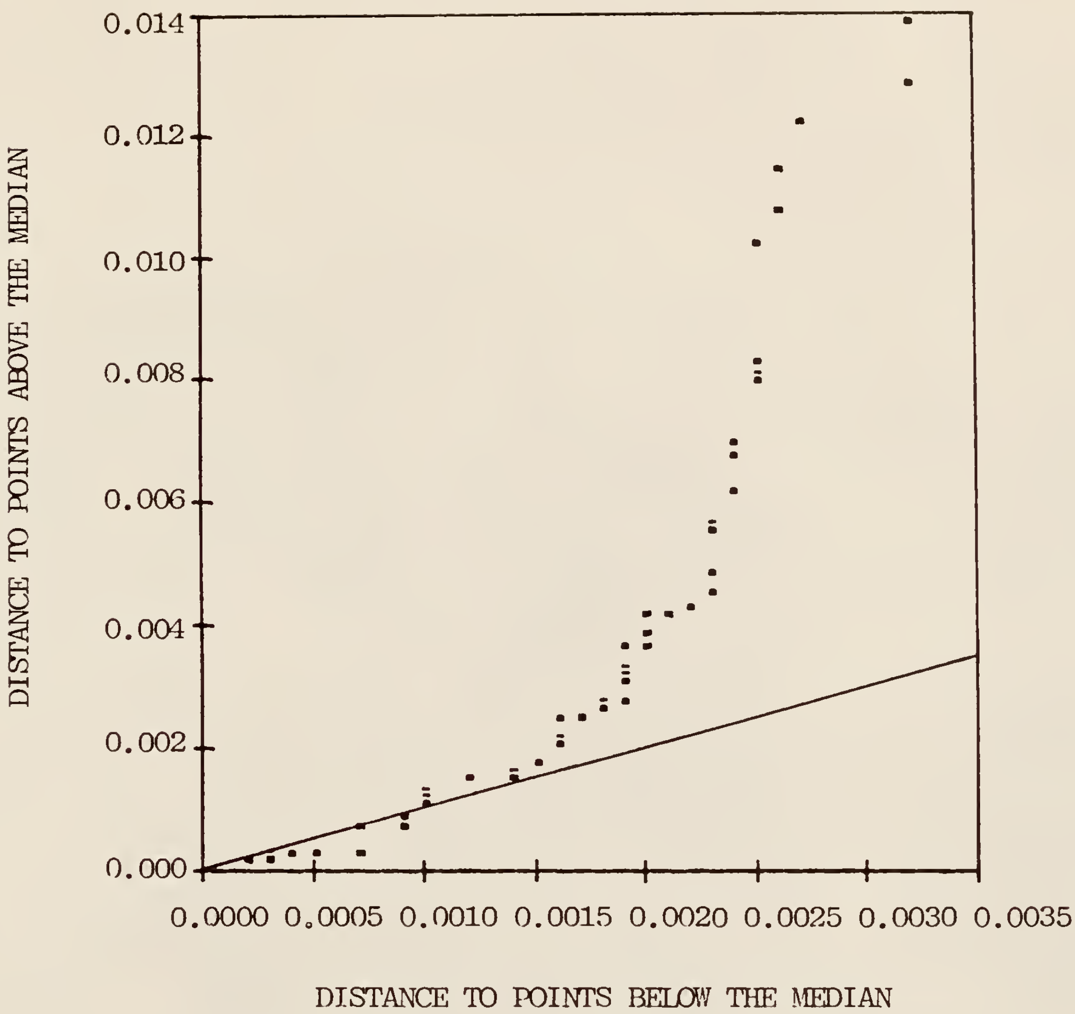

Figure 20. A symmetry plot for the standard deviation of the five normalized harmonic amplitudes after the overlap harmonic (ME).

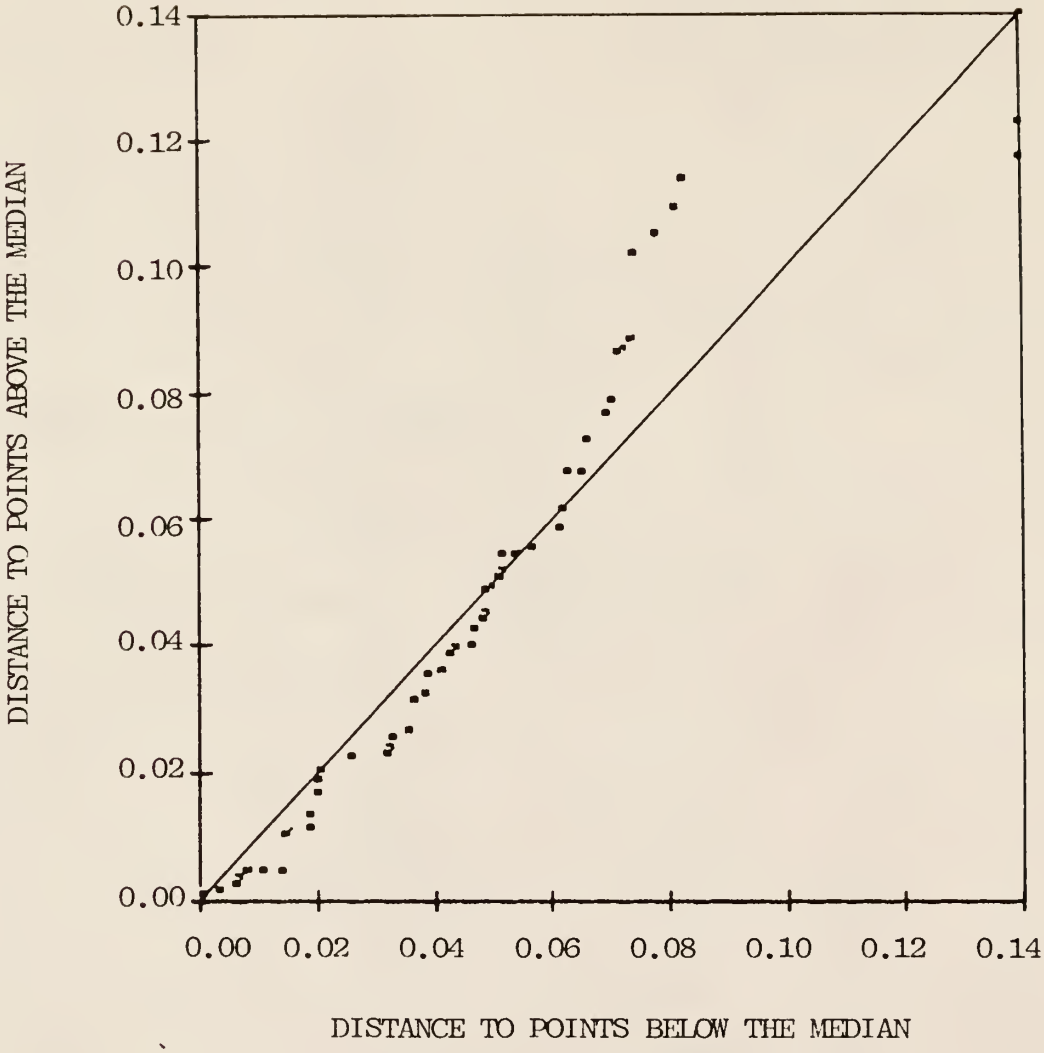

Figure 21. A symmetry plot for the 0.25th power transformation of the standard deviation of the five normalized harmonic amplitude after the overlap harmonic (PME).

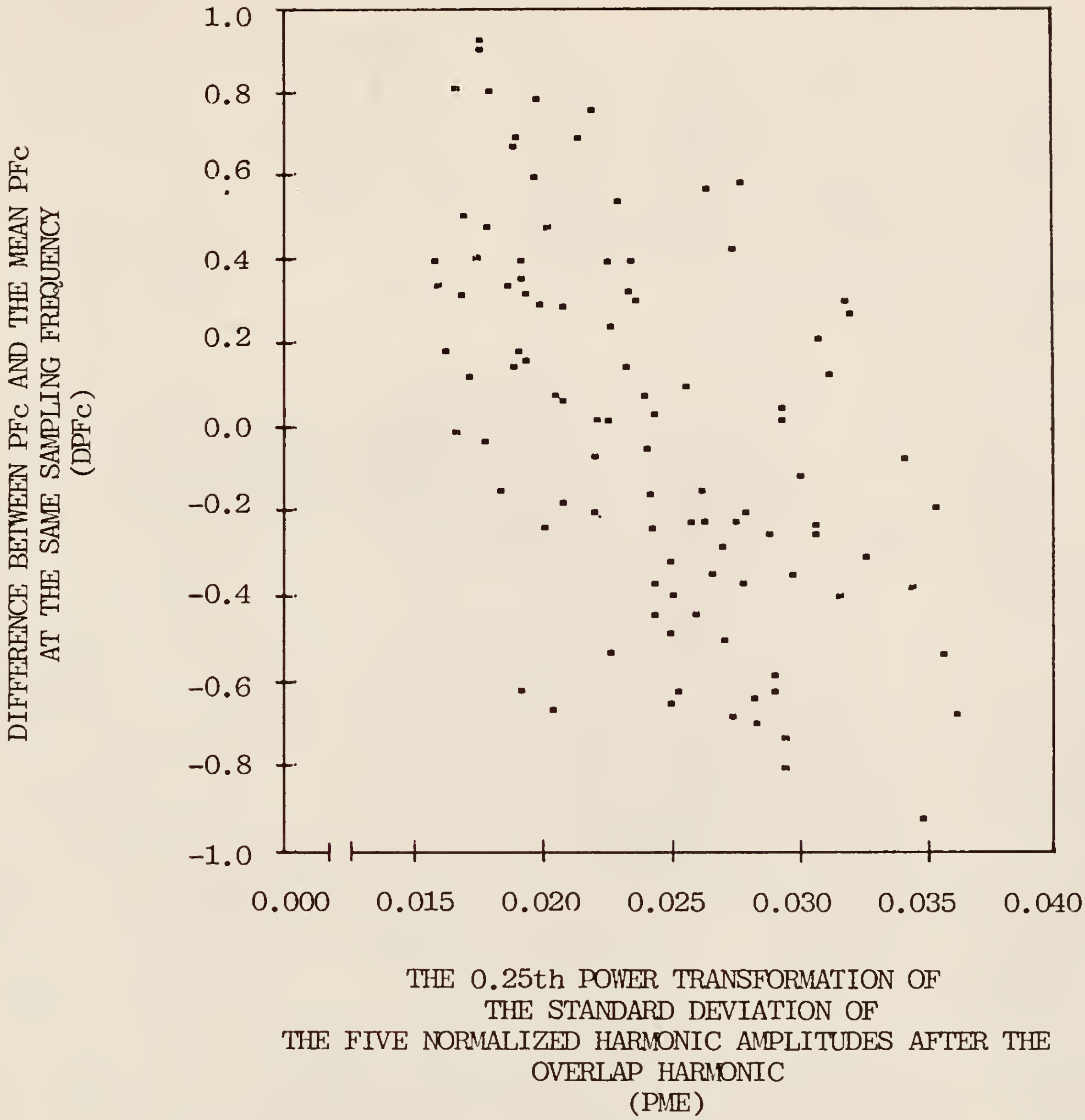

Figure 22. The relationship between the 0.25th power transformation of the standard deviation of the five normalized harmonic amplitudes after the overlap harmonic and the difference between the square root of the best cutoff frequency and the mean of the square root of the best cutoff frequency at the same cutoff frequency (DPFc).

Table 4. Statistical results for Equation 35.

DEP VARIABLE: PFc

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	P VALUE
MODEL	4	76.4299	19.1074	158.831	0.0001
ERROR	105	12.6345	0.1203		
C TOTAL	109	89.0645			

ROOT MSE	0.3468	R-SQUARE	0.8581
DEP MEAN	3.7372	ADJ R-SQ	0.8525
C.V.	9.5111		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T VALUE	P VALUE
INTERCEP	1	-0.7563	0.8167	-0.9261	0.3566
PFs	1	0.1361	0.0071	19.1690	0.0001
PNHAl	1	1.8562	0.5808	3.1959	0.0019
PNHAo	1	-0.3356	0.5396	-0.6219	0.5353
PME	1	-0.2098	0.9985	-0.2101	0.8340

significant but PME and PNHAo had no significant contribution to the model and the P value for PME was the highest. That means that the contibution of PME to the model was the smallest among the independent variables. Therefore, PME was deleted from the first model.

The second model to be tested was

$$PFc = L0 + L1\ PFs + L2\ PNHA1 + L3\ PNHAo + e \qquad (36)$$

The statistics for this model are shown in Table 5. The analysis of variance shows that the overall model was significant but PNHAo still had no significant contribution to the model. Therefore, PNHAo was deleted from the second model.

The third model to be tested was

$$PFc = L0 + L1\ PFs + L2\ PNHA1 + e \qquad (37)$$

The statistics for this model are shown in Table 6. The analysis of variance shows that the overall regression was significant and all of the independent variables in the model had significant contribution to the model. The R square and adjusted R square for overall regression were 0.8547 and 0.8575 respectively. These results indicate that over 85% of the variation of the square root of cutoff frequency can be explained by Equation 37 and the correlation between the actual best cutoff frequency and the estimated best cutoff frequency from the corresponding

Table 5. Statistical results for Equation 36.

DEP VARIABLE: PFc

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	P VALUE
MODEL	3	76.4299	25.4747	213.554	0.0001
ERROR	106	12.6401	0.1192		
C TOTAL	109	89.0645			

ROOT MSE	0.3452	R-SQUARE	0.8581
DEP MEAN	3.7372	ADJ R-SQ	0.8539
C.V.	9.4659		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T VALUE	P VALUE
INTERCEP	1	-0.9057	0.4001	-2.2636	0.0257
PFs	1	0.1357	0.0069	19.6667	0.0001
PNHAl	1	1.9205	0.4913	3.9090	0.0002
PNHAo	1	-0.3380	0.5369	-0.6295	0.5304

Table 6. Statistical results for Equation 37.

DEP VARIABLE: PFc

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	P VALUE
MODEL	2	76.3747	38.1873	322.255	0.0001
ERROR	107	12.6897	0.1185		
C TOTAL	109	89.0645			

ROOT MSE	0.3442	R-SQUARE	0.8575
DEP MEAN	3.7372	ADJ R-SQ	0.8547
C.V.	9.4378		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T VALUE	P VALUE
INTERCEP	1	-1.0432	0.3343	-3.1205	0.0024
PFs	1	0.1358	0.0068	19.9705	0.0001
PNHA1	1	1.6401	0.2066	7.9385	0.0001

regression equation of Equaiton 37 was 0.9245. Figures 23, 24, 25 and 26 are residual plots of the equation 37. No pattern in the distribution of residuals can be detected from these residual plots. Therefore, Equation 37 was considered as the best model for estimating the proper cutoff frequency and the resulting regression equation can be expressed as

$$Fc = (0.1358\ Fs^{1/2} + 1.6401\ NHA1^{-1/8} - 1.0432)^2 \tag{38}$$

This regression equation shows that Fs has a positive effect on Fc but NHA1 has a negative effect on Fc. The higher Fs is, the higher Fc will be. The higher NHA1 is, the lower Fc will be.

In the above procedure of model building, the non-significant contribution of PNHAo to the second model in which both of PNHAo and PNHA1 were included, the higher significance of the contribution of PNHA1 to the third model after PNHAo was deleted, and the effect of PNHA1 on Fc in the final regression equation further demonstrated the hypotheses about the functions of NHA1 and NHAo.

The rejection of PME in the above procedure of model building was unexpected. Examining the relationships between NHA1 and mean error and between NHA1 and ME shown in Figures 27 and 28, it can be found that NHA1 is correlated with mean error and ME. This means that NHA1 is

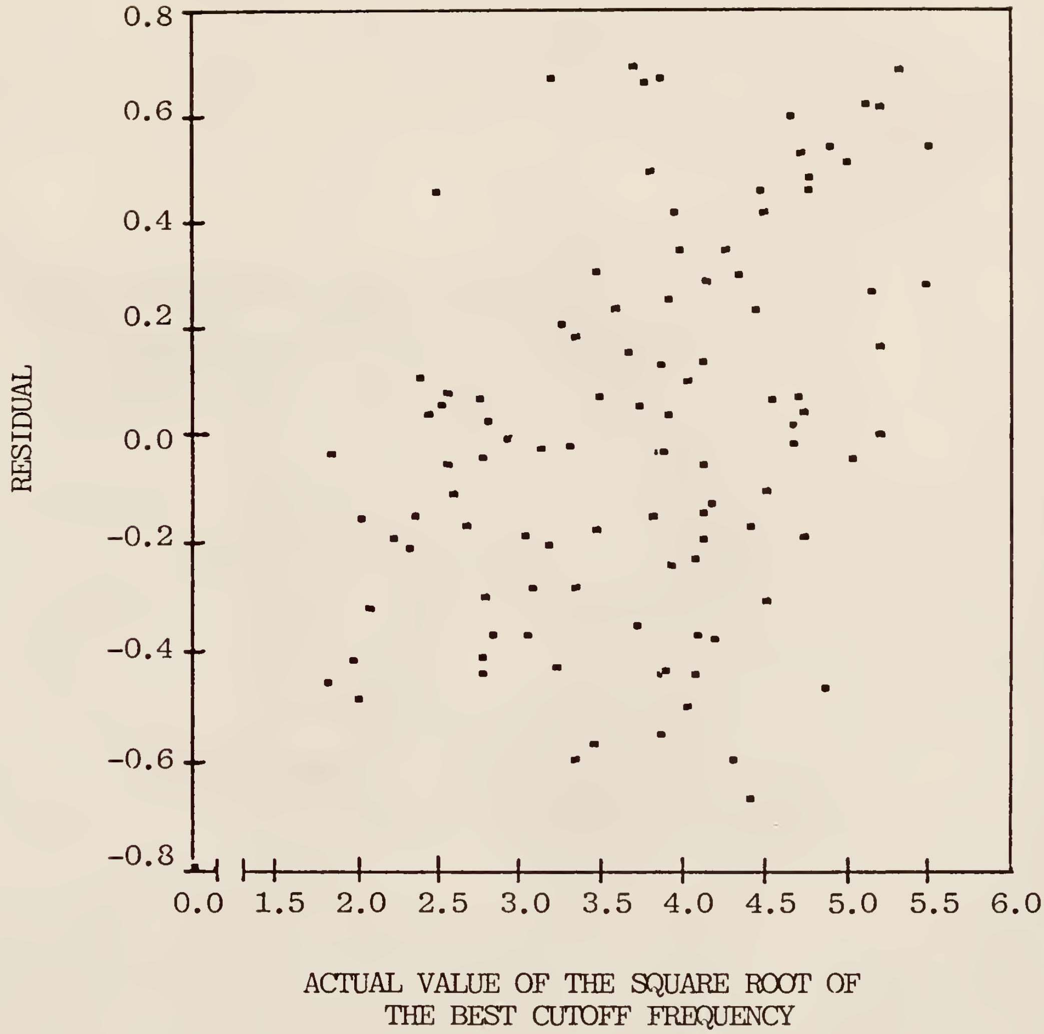

Figure 23. Residuals vs the actual value of the square root of the best cutoff frequency for the first best model.

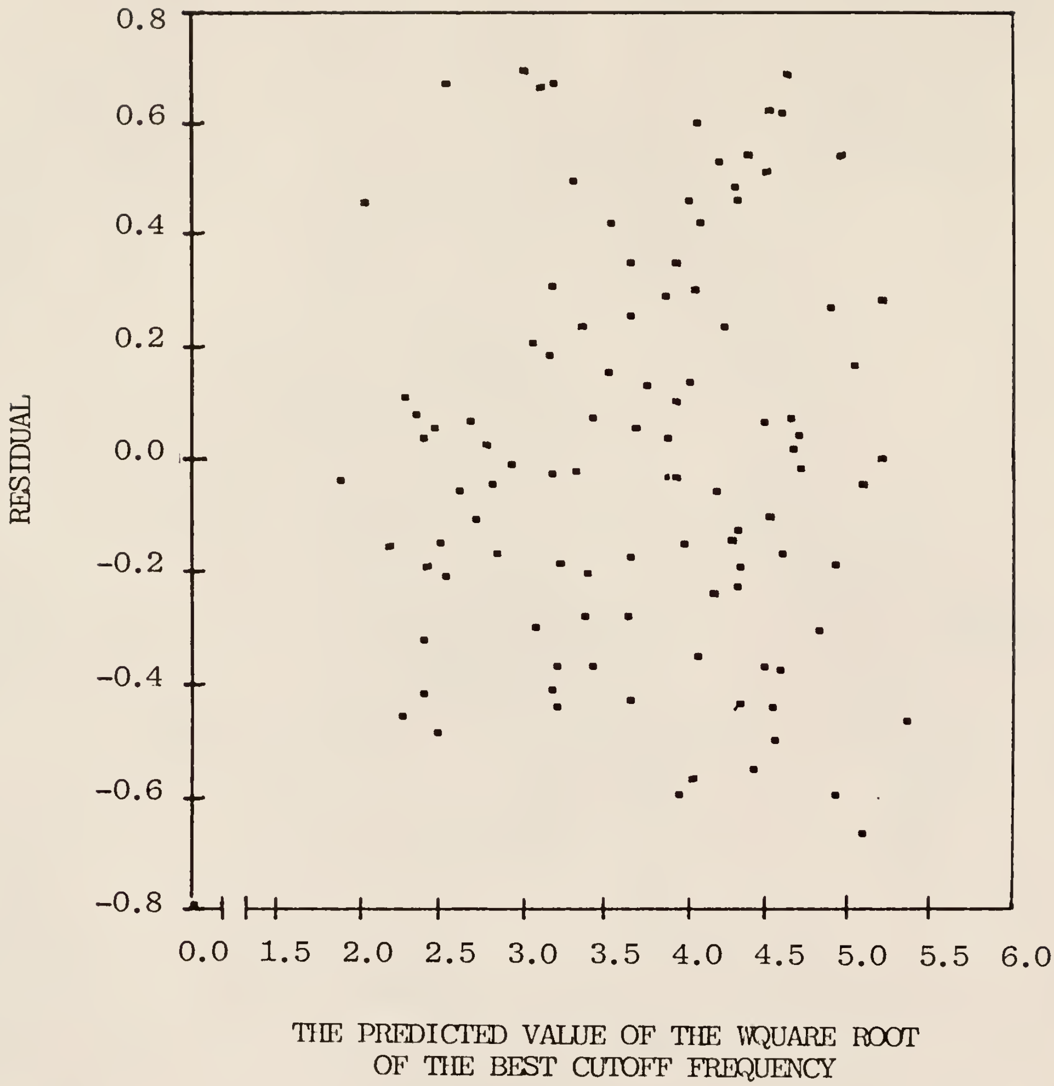

Figure 24. Residuals vs the predicted value of the square root of the best cutoff frequency for the first best model.

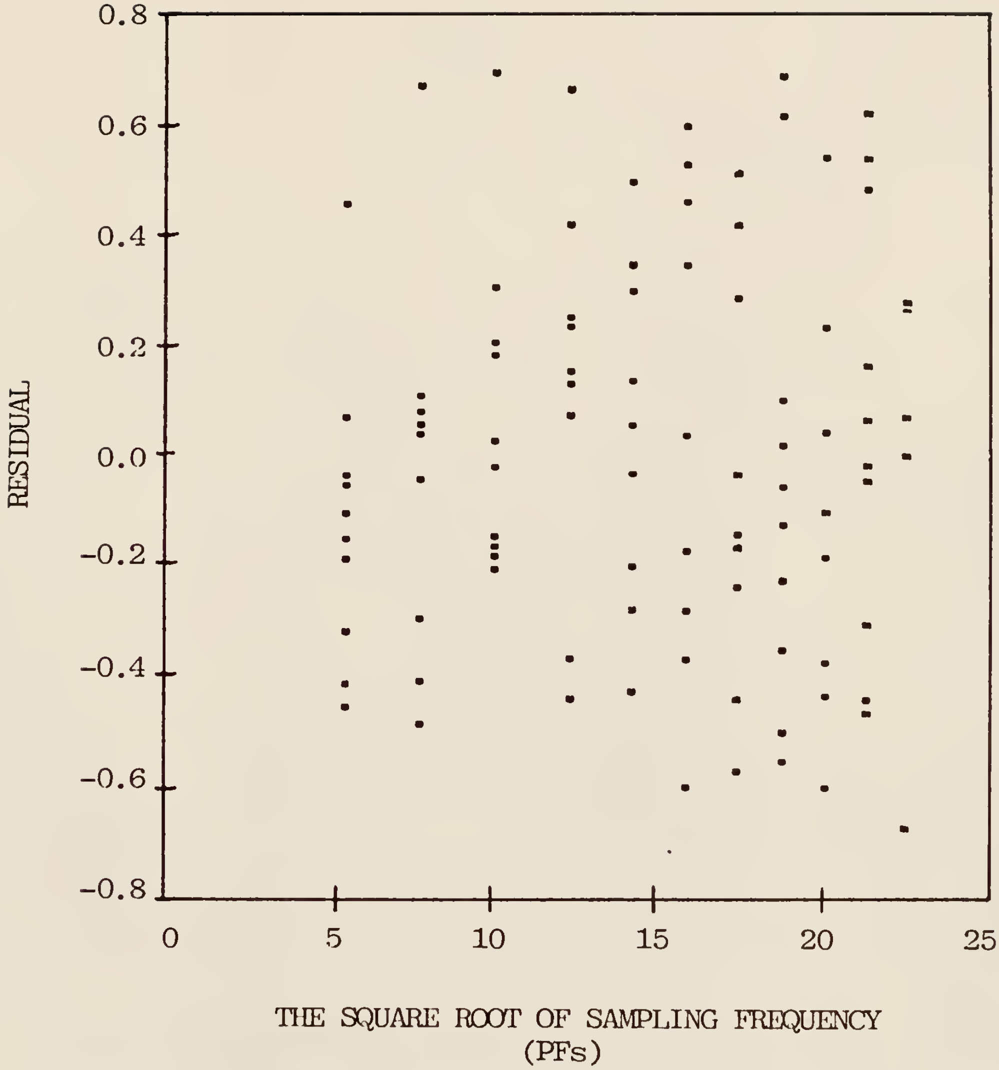

Figure 25. Residuals vs the square root of sampling frequency for the first best model.

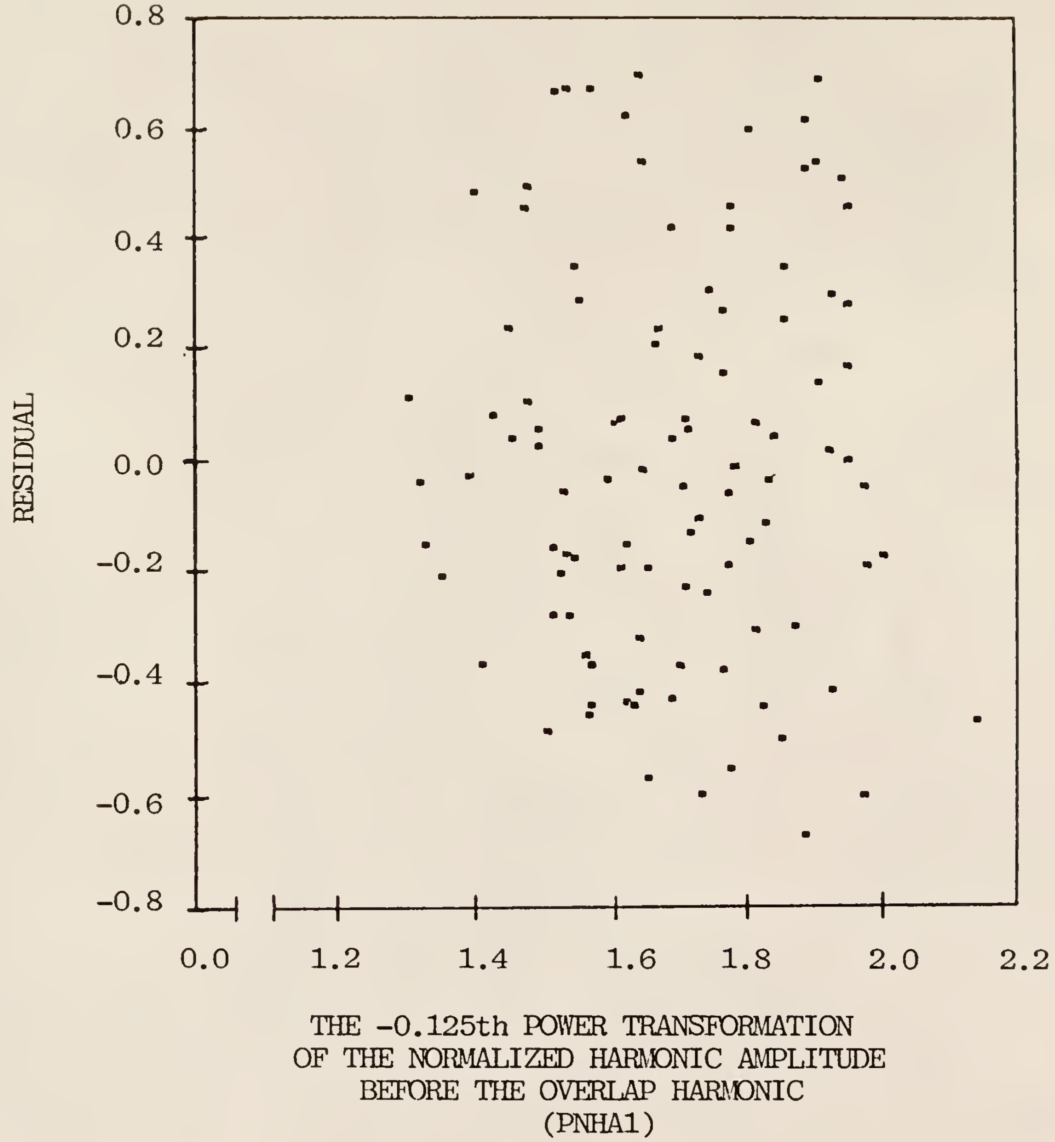

Figure 26. Residuals vs the -0.125th power transformation of normalized harmonic amplitude before the overlap harmonic for the first best model.

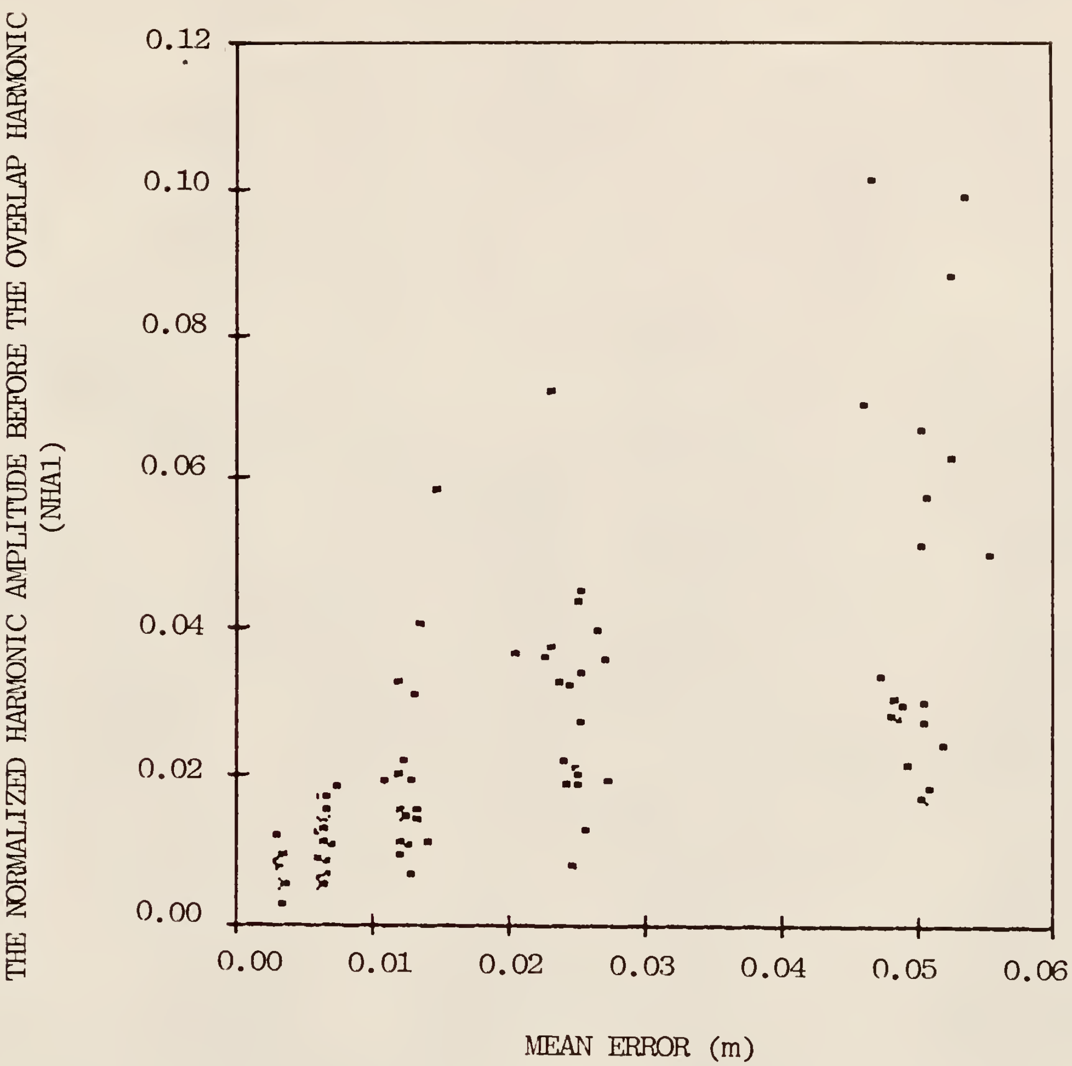

Figure 27. The relationship between the normalized harmonic amplitude before the overlap harmonic (PNHA1) and mean error.

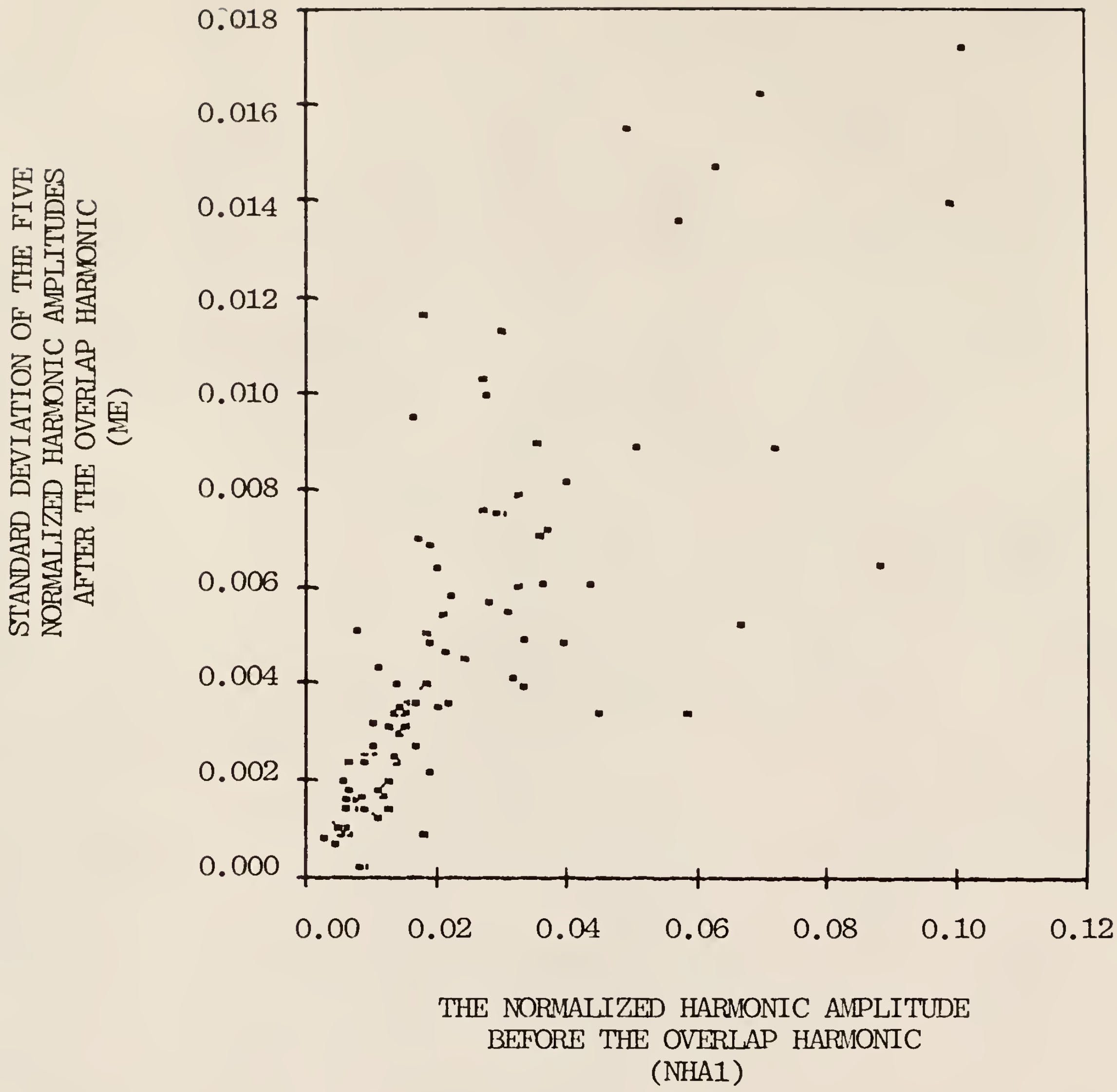

Figure 28. The relationship between the normalized harmonic amplitude before the overlap harmonic (NHA1) and the standard deviation of the five normalized harmonic amplitudes after the overlap harmonic (ME).

also a predictor of the relative amplitude of error and functions as well as ME. Thus, the reason why ME was rejected in regression analysis procedure can be explained as that although ME is a predictor of the relative amplitude of error in the overlap, NHA1 can function not only as a predictor of relative amplitude of error in the overlap as well as ME but also as a predictor of the relative amplitude of the signal before the overlap and the relative position of the beginning of the overlap on harmonic spectrum of the signal, so the function of NHA1 in estimating proper cutoff frequency is much stronger than that of ME.

In applicaton of Equation 38, Fourier analysis has to be carried out to calculate that NHA1. Considering that Fourier analysis procedure was very time consuming and would increase computer time spent in digital filter procedure, the fourth model was tested. In this model, only sampling frequency was remained as independent variable. This model was expressed as the following

$$PFc = L0 + L1\ PFs + e \tag{39}$$

The statistics for this model are shown in Table 8. The analysis of variance shows that the overall regression was significant and PFs had significant contribution to the model. The R square and adjusted R square of overall regression were 0.7695 and 0.7673 respectively. These results indicate that about 77% of the variation of cutoff

Table 7. Statistical results for Equation 39.

DEP VARIABLE: PFc

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	P VALUE
MODEL	1	68.5367	68.5367	360.582	0.0001
ERROR	108	20.5278	0.1900		
C TOTAL	109	89.0645			

ROOT MSE	0.4358	R-SQUARE	0.7695
DEP MEAN	3.7372	ADJ R-SQ	0.7673
C.V.	11.9454		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T VALUE	P VALUE
INTERCEP	1	1.4845	0.1290	11.5077	0.0001
PFs	1	0.1532	0.0082	18.6829	0.0001

frequencies can be explained by Equation 39 and the correlation between the actual best cutoff frequencies and the estimated best cutoff frequencies from the corresponding regression equation of equation 39 was 0.8772. Figures 29, 30, and 31 are residual plots for the corresponding regression equation of Equation 39. Some linear pattern of distribution of residual can be detected when the residual was plotted as the function of actual best cutoff frequency as shown in Figure 28. This linear pattern in the distribution of residual is due to the absence of PNHA1 from the model. No pattern can be detected from the other two residual plots. Therefore, equation 39 was selected as the second best model for estimating the best cutoff frequency, and the resulting regression equation was

$$Fc = (1.4845 + 0.1532\ Fs^{1/2})^2 \tag{40}$$

This equation avoids using Fourier analysis, therefore, it has no influence on computer time spent in digital filter procedure and is suited for fast feedback. However, The variation of the square root of the best cutoff frequency explained by Equation 40 is about 9% less than that explained by Equation 38, which will cause some errors in determination of the proper cutoff frequency, and these errors may cause some under- or over-filtering effect in filtered outputs.

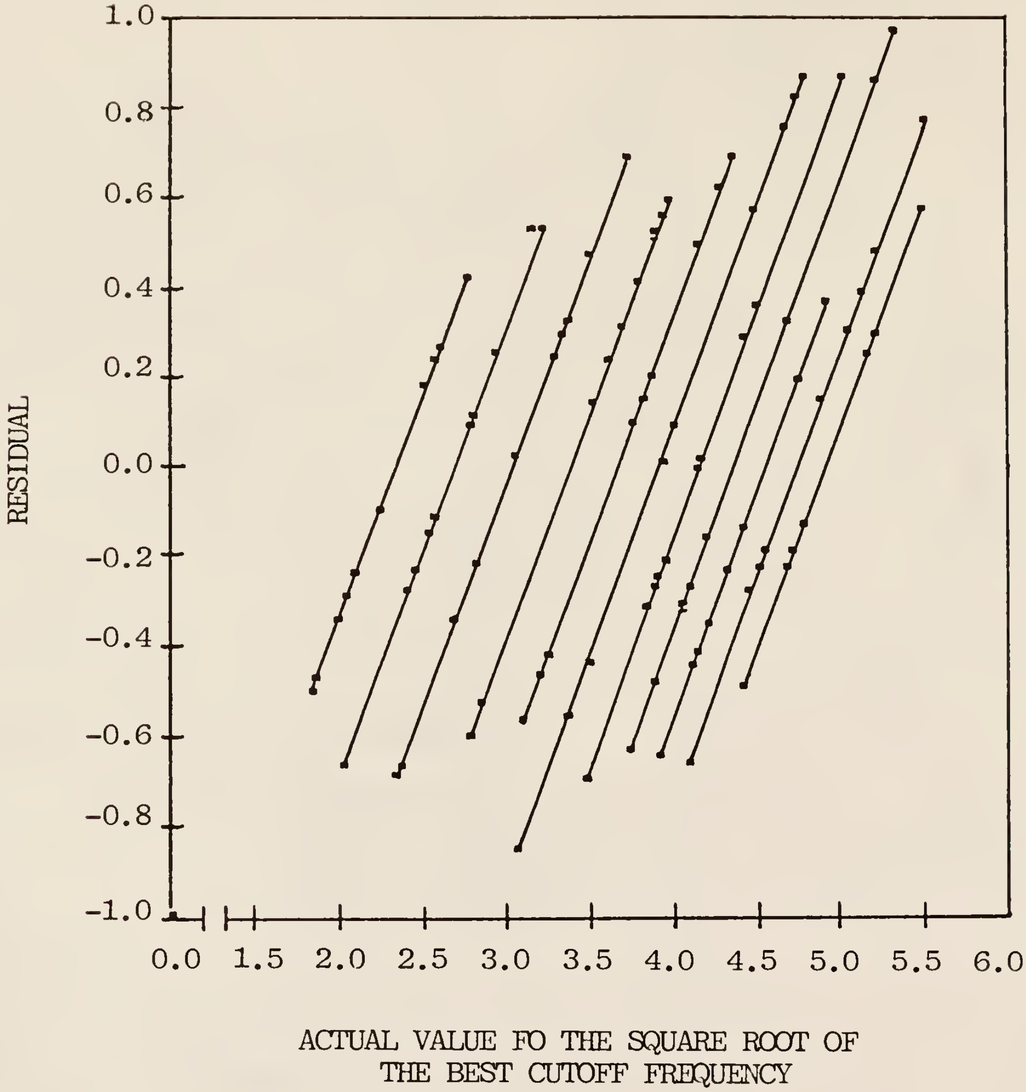

Figure 29. Residuals vs the actual value of the square root of the best cutoff frequency for the second best model.

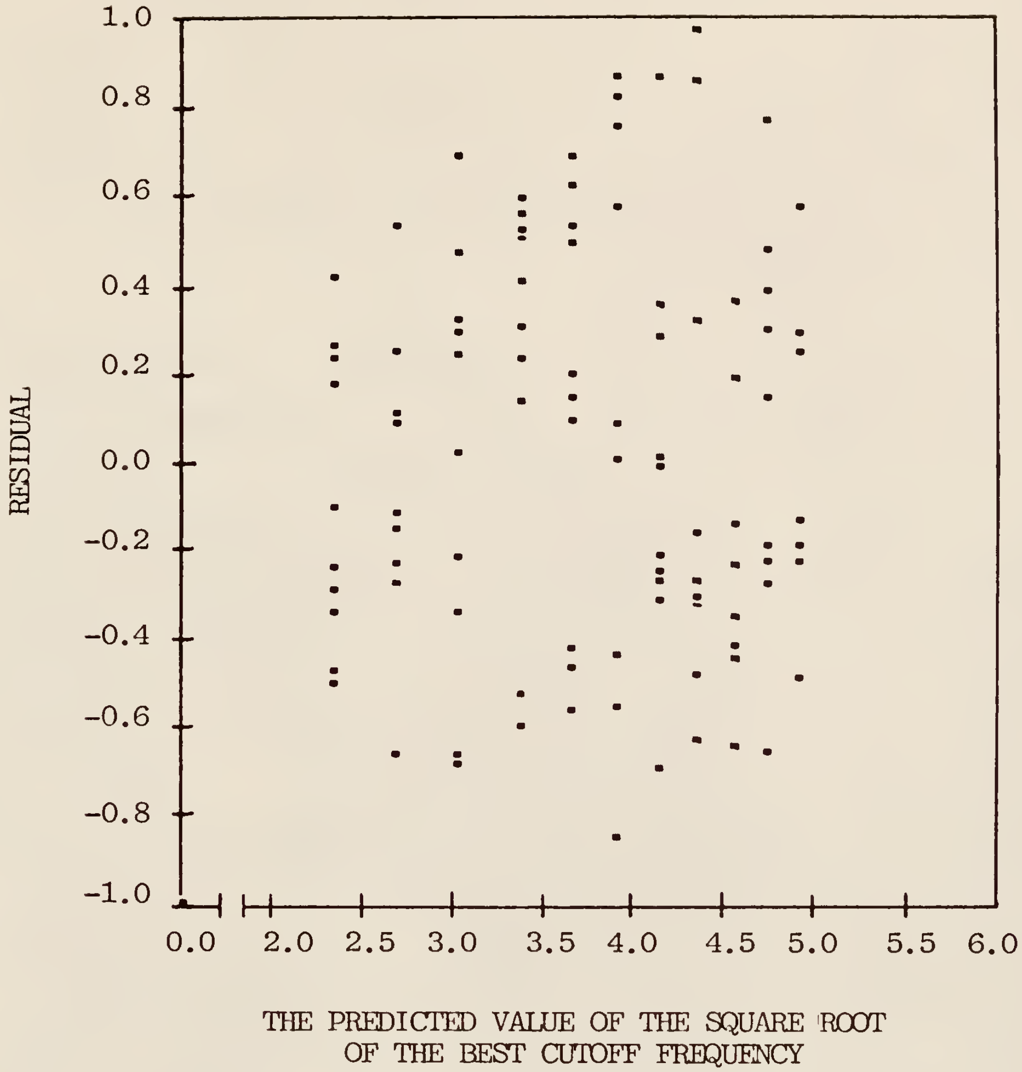

Figure 30. Residuals vs the predicted value of the sqaure root of the cutoff frequency for the second best model.

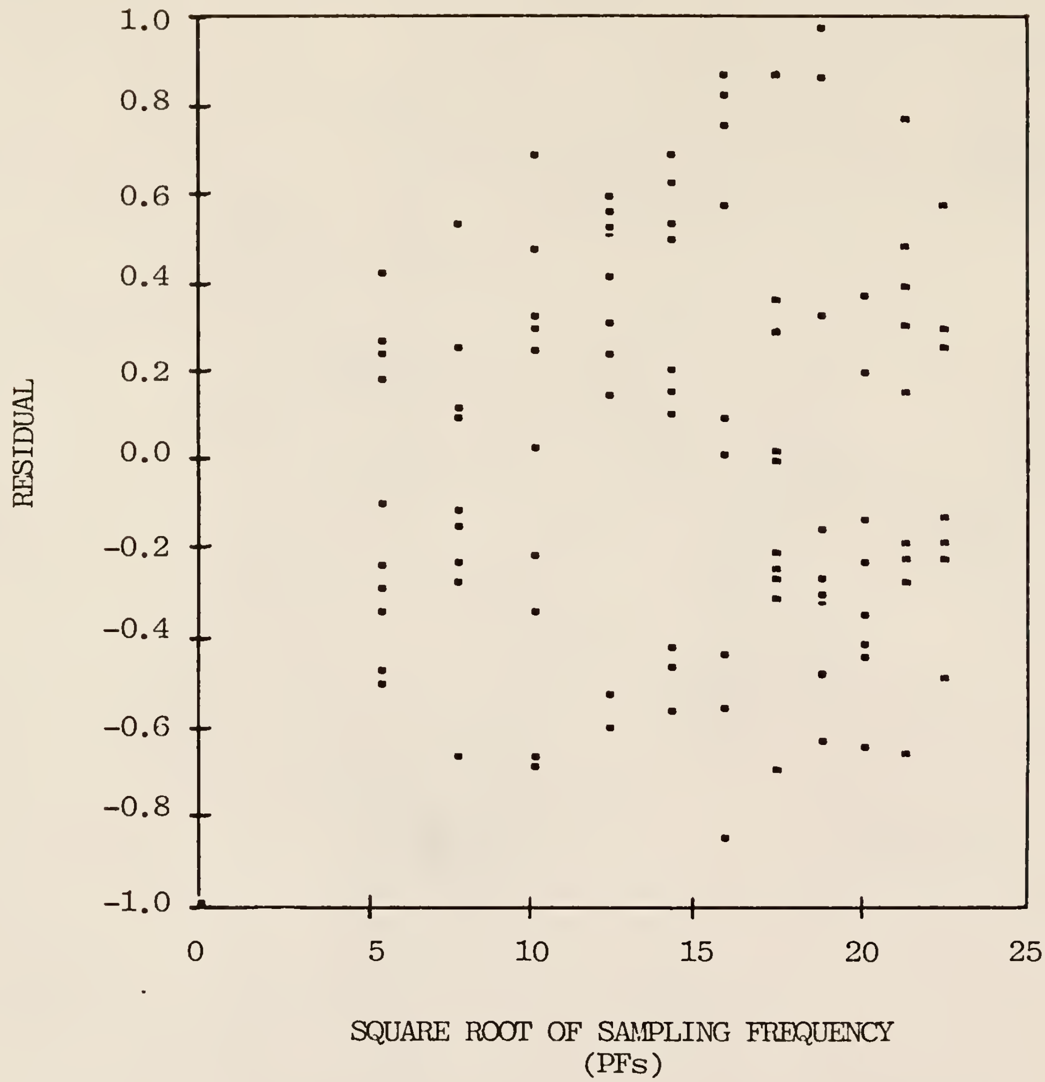

Figure 31. Residuals vs the square root of sampling frequency for the second best model.

Application Of The Regression Equations

Figures 32 and 33 show the digitized analog angular displacement data (Pezzack, _et_ _al_., 1977) and the digitized film angular displacement data filtered at the cutoff frequencies determined by Equations 38 and 40. The cutoff frequencies determined by Equations 38 and 40 for this set of data were 8.61 Hz and 6.60 Hz respectively. The cutoff frequency determined by Equation 38 for this set of data is very close to 9 Hz used by Pezzack in his study (1977). Figure 34 and 35 depict the analog angular acceleration data measured by the accelerometer (Pezzack _et_ _al_., 1977) and the angular accelerations calculated from the filtered film angular displacement data. These figures demonstrate that, although the over-filtering effect appeared at the peaks of the angular acceleration curve calculated from the angular displacement data fitlered at the cutoff frequency determined by equation 40, the angular displacement data filtered at the cutoff frequencies determined by both equations fitted the true angular displacement function very well, and provided enough details of the true acceleration function. These results indicate that both Equations 38 and 40 were reasonably successful in estimating the proper cutoff frequency for the second-order low pass recursive digital filter to estimate velocity and acceleration from the film analysis data which are severely contaminated by noise.

A computer program has been written in PASCAL for the

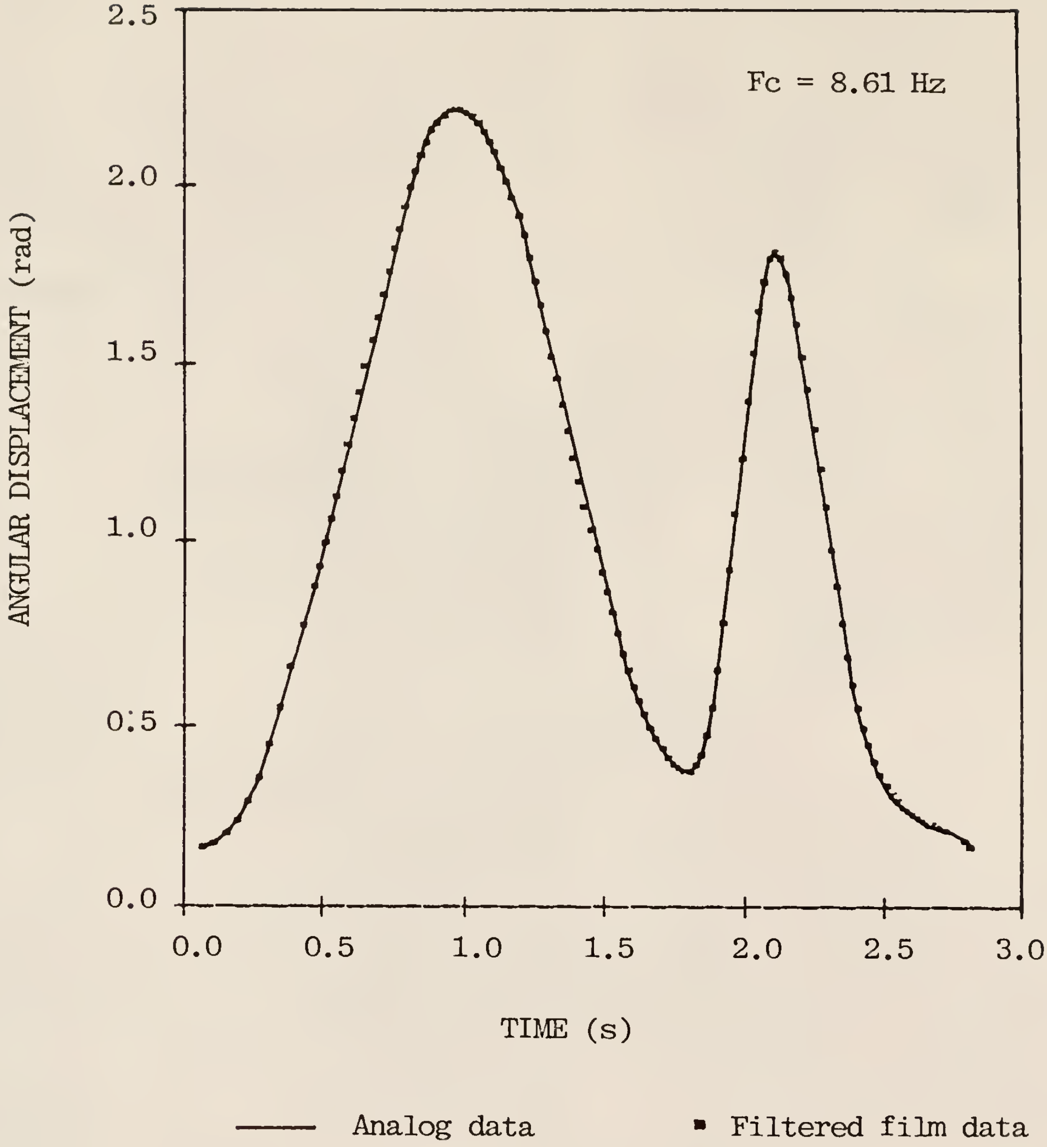

Figure 32. A comparison of analog and filtered film data of angular displacement for the first best model.

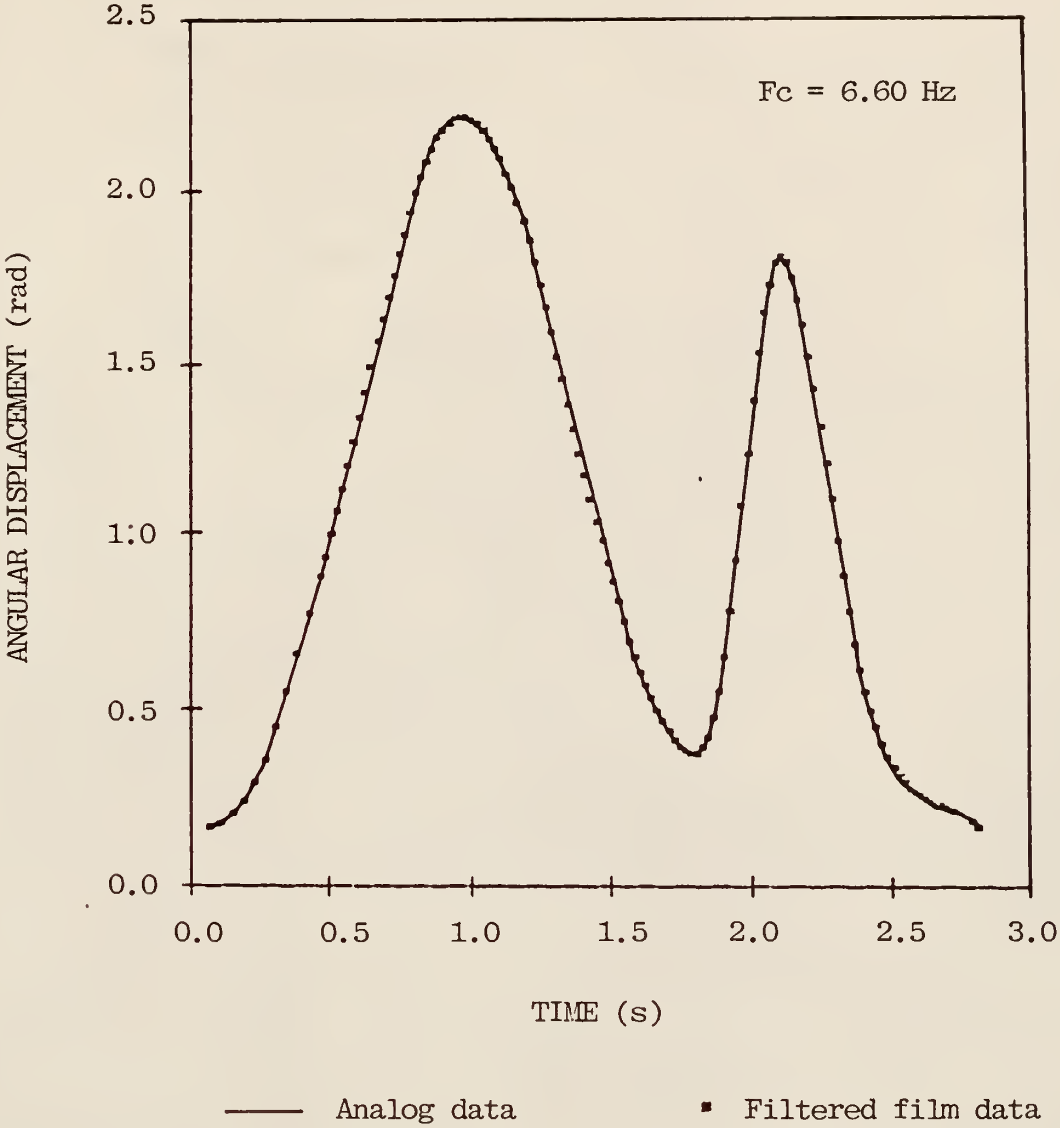

Figure 33. A comparison of analog and filtered film data of angular displacement for the second best model.

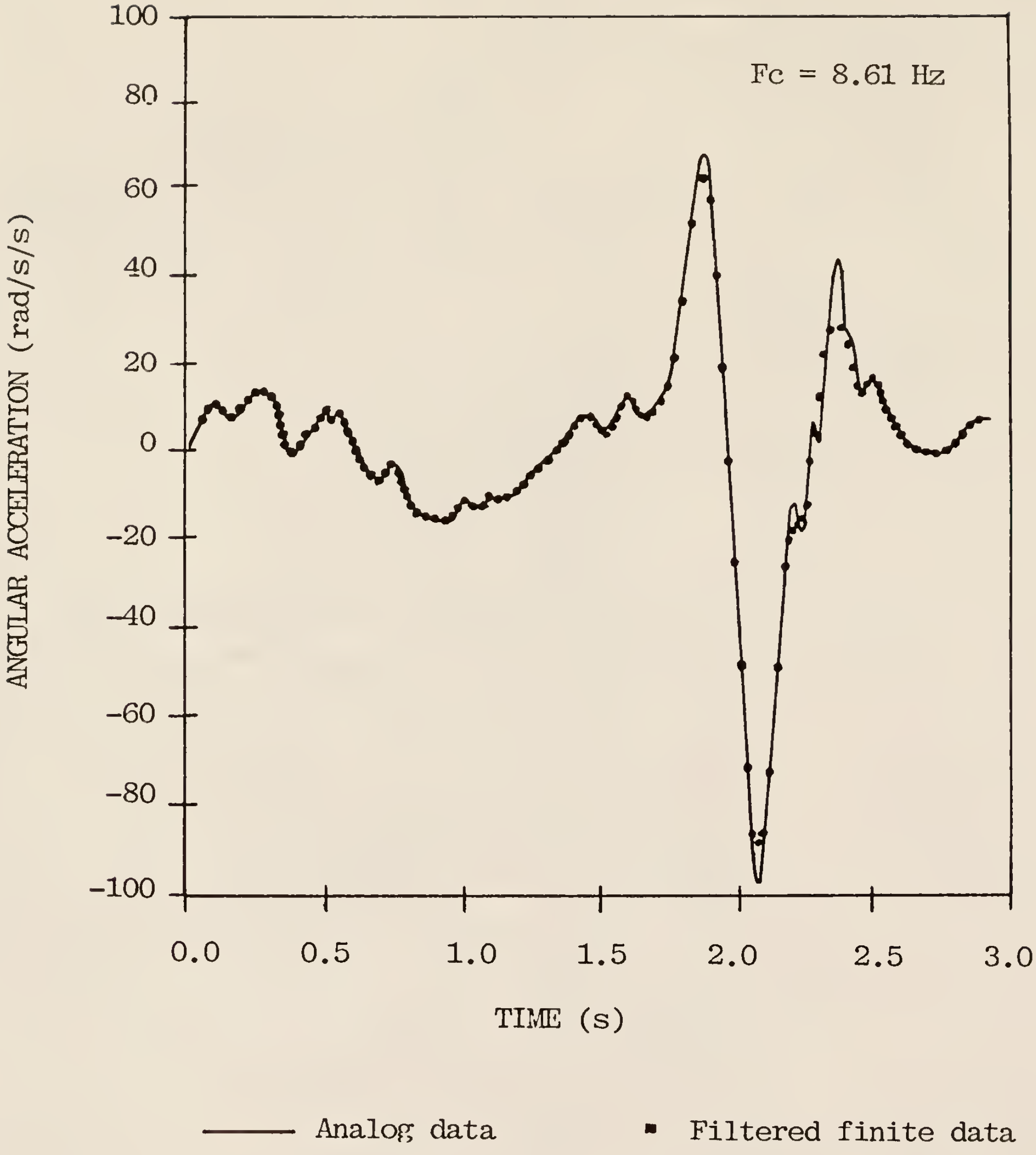

Figure 34. A comparison of analog and filtered finite angular acceleration data for the first best model.

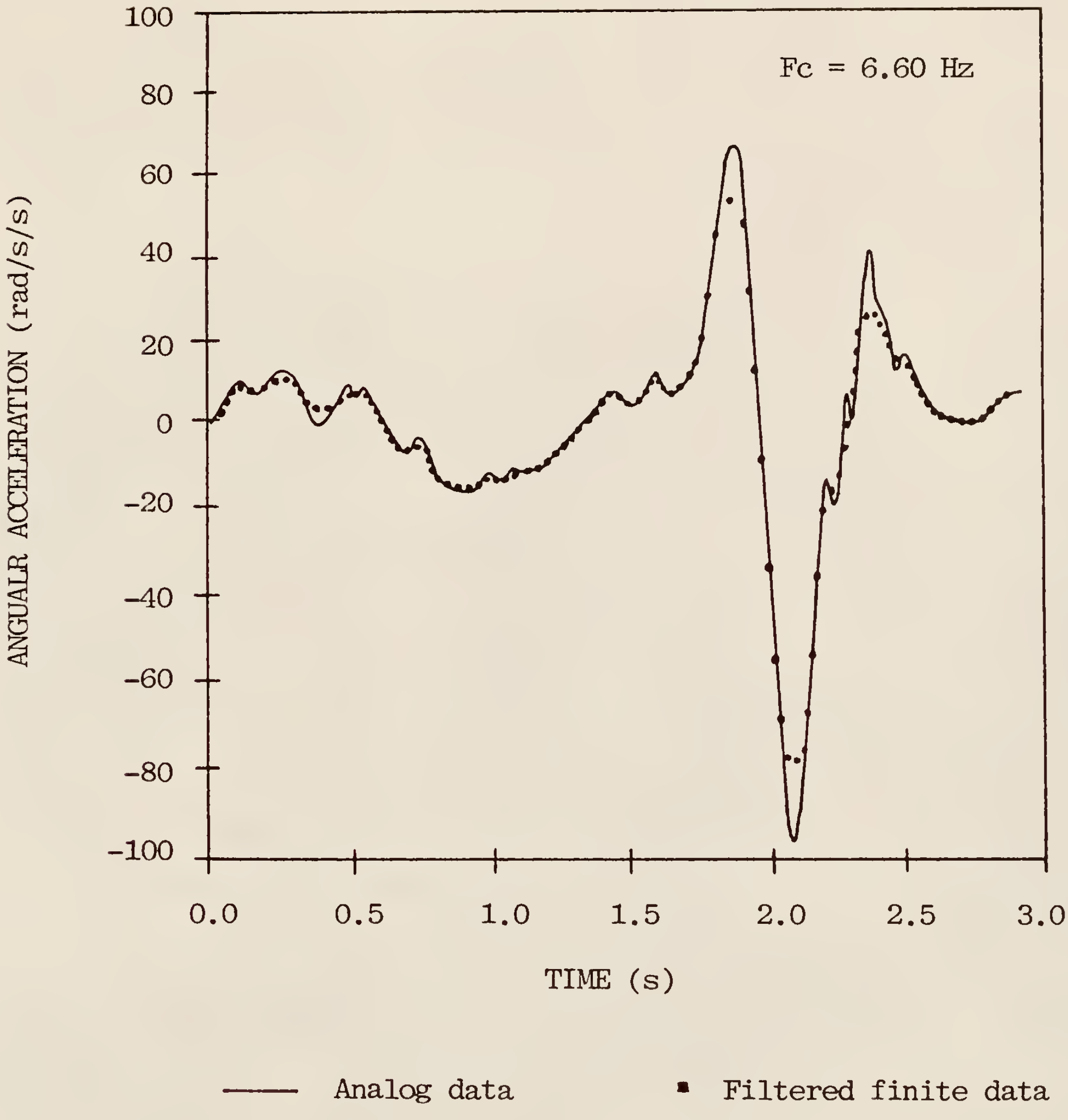

Figure 35. A comparison of analog and filtered finite angular acceleration data for the second best model.

new digital filter procedure in sport biomechanics analysis of human body movement. In this new digital filter procedure, the regression equations established in this study are applied to objectively determined the proper cutoff frequency either from the sampling frequency or from the sampling frequency and the normalized harmonic amplitude before the overlap harmonic.

CHAPTER 5

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary

Although the second-order recursive digital filter has been widely used in cinematography in sport biomechanics, the problem of determination of proper cutoff frequency has not been solved. The purpose of this study was to develop a method for accurate determination of the cutoff frequency in the digital filter.

A set of vertical coordinate-time data of freely falling movement was used as the standard data. Different sets of computer-generated random numbers were used as random errors and mixed into the standard data to generate different sets of artificial raw data. The mean absolute value of each set of random errors was controlled.

The artificial raw data were filtered by the digital filter at different cutoff frequencies. The filtered outputs were evaluated by the error energy of the filtered data and standard data to determine the best cutoff frequency for each set of raw data.

Harmonic analysis was then carried out to analyze the characteristics of the signal and error of each set of raw data. In the harmonic analysis, the highest harmonic of the raw data was defined as the harmonic at which the raw data were best fitted by Fourier series. Harmonic amplitudes were normalized as the percentages of fundamental harmonic amplitude of each set of raw data. Based on the results of harmonic analysis, independent variables were selected and added to sampling frequency develop regression models.

Two regression equations were then developed from these models. The proper cutoff frequency for a set of raw data can be estimated either from the sampling frequency and the normalized harmonic amplitude before the overlap harmonic using the relationship:

$$Fc = (0.1358\ Fs^{1/2} + 1.6401\ NHA1^{-1/8} - 1.0432)^{2}$$

and from only the sampling frequency using the following formula:

$$Fc = (1.4845 + 0.1532\ Fs^{1/2})^{2}$$

These two regression equations were used to determine the cutoff frequency for the film angular displacement data collected by Pezzack, et al. (1977). The cutoff frequency of this set of data estimated from the sampling frequency and the harmonic amplitude before the overlap was very similar to that used by Pezzack, et al. (1977). The filtered outputs with the cutoff frequencies determined by either of these two equations provided enough details of the true angular acceleration function although there were some over-filtering effects on the peaks of the estimated acceleration curve calculated from the displacement data filtered at the cutoff frequency estimated from only the sampling frequency.

The following are the findings of this study:

1. The cutoff frequency of digital filter is influenced by the sampling frequency. Over 77% of the variation of the square root of the best cutoff frequency is due to the sampling frequency.

2. The harmonic amplitude of raw data keeps decreasing below a particular harmonic. After this particular harmonic, the harmonic amplitude of raw data begins to oscillate. This particular harmonic can be considered as the indicator of the beginning of the overlap of signal and error on the frequency spectrum. In this study, this particular harmonic was named as overlap harmonic.

3. The normalized harmonic amplitude before the overlap harmonic can be used as an indicator of the relative amplitude of signal at the beginning of the overlap, the relative beginning position of the overlap of signal and error on the frequency spectrum of the signal, and the relative amplitude of error in the overlap. About 9% of the variation of the square root of the best cutoff frequency can be explained by the normalized harmonic amplitude before the overlap harmonic.

A Computer program for a new digital filter procedure has been written in PASCAL. In this program, the regression equations established in this study have been applied to objectively determine the proper cutoff frequency for the digital filter procedure.

Conclusions

The results of statistical analysis and application of the regression equations have demonstrated that the proper cutoff frequency of the digital filter can be estimated from the sampling frequency and the normalized harmonic amplitude of the signal before the overlap harmonic, and that either of the two regresion equations established in this study can be successfully applied to objectively determine the proper cutoff frequency of the second-order low pass recursive digital filter. It is suggested that the second equation be used if the speed of the feedback is more important than the accuracy, and the first equation be used if the accuracy is more important than the speed of the feedback.

Recommendations

The present study provides the basis for further studies on the determination of the cutoff frequency of the digital filter. Although two regression equations for estimating the proper cutoff frequency of the digital filter have been established in this study, there is still about 14% of the variation of cutoff frequency unexplained. This unexplained part of variation of cutoff frequency has some negative effect on the estimate of cutoff frequency, and may have caused some over- or under-fitlering effect in estimated velocity and acceleration curves. Further studies are needed to provide a more complete explanation of the variation of the cutoff frequency when using the digital filter.

The findings from this study should be applicable to human movement data with different frequency contents, because the harmonic analysis variables were normalized relative to the fundamental harmonic amplitude. However, further research is recommended to verify if the results of this study are fully applicable to human movement data with widely varying frequency content.

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APPENDIX A

A Computer Program For Digital Filter Procedure
With Different Methods
Of Determination Of Cutoff Frequency

```
(************************************************************)
(*                                                          *)
(* Author          :   Bing Yu                              *)
(*                     research assistant                   *)
(*                     Department of Physical Education,    *)
(*                       and Leisure Studies                *)
(*                     Kansas State University              *)
(*                                                          *)
(* Major Professor :   Dr. Larry Noble                      *)
(*                                                          *)
(************************************************************)

Program DigitalFilter(input, output, lst);

(************************************************************)
(*                                                          *)
(* This program is one the programs in the KSU film         *)
(* analysis system. This program is designed to smooth      *)
(* the raw data of joint center coordinates generated by    *)
(* the DATAIN program, generate a data file which           *)
(* contains the smoothed data of joint center               *)
(* coordinates, and print out the raw data and smoothed     *)
(* data if the user requires. The cutoff frequency can      *)
(* be automatically determined from sampling frequency      *)
(* and results of harmonic analysis, from only sampling     *)
(* frequency, or determined by user.                        *)
(*                                                          *)
(************************************************************)

const
   maxframes = 120;

(************************************************************)
(*                                                          *)
(* Limit of number of frames analyzed for each joint.       *)
(*                                                          *)
(************************************************************)

   possible  = 19;

(************************************************************)
(*                                                          *)
(* Limit of number of joints.                               *)
(*                                                          *)
```

```
(**********************************************************)

   pi        = 3.14159265;

type
   dataarray = array[1..possible,1..maxframes] of real;
   cogarray  = array[1..10,1..maxframes] of real;
   temparray = array[1..maxframes] of real;
   coearray  = array[1..5] of real;
   cutarray  = array[1..2,1..19] of real;
   realfile  = file of real;
   namelab   = string[20];

var
   data1    ,
   data2    ,
   data3    ,
   data4    : dataarray;

(**********************************************************)
(*                                                        *)
(* data1, data2, data3, and data4 will contain raw data  *)
(* of x and y coordinates of joint centers, and filtered *)
(* data of x and y coordinates of joint centers,         *)
(* respectively.                                          *)
(*                                                        *)
(**********************************************************)

   cutoff   : cutarray;

(**********************************************************)
(*                                                        *)
(* cutoff will contain cutoff frequencies for x and y    *)
(* coordinates of each joint center.                      *)
(*                                                        *)
(**********************************************************)

   deltatime,
   camspeed ,
   frameint ,
   impreal  : real;
   segnum   ,
   frame    : integer;
   answer   ,
   filename : namelab;
   ch       : char;

(**********************************************************)
(*                                                        *)
(* deltatime : the time interval between digitizing.     *)
(*  camspeed : filming rate.                              *)
(*  frameint : the number of frames between digiting.    *)
(*   impreal : flag for if user digitized an implement   *)
(*             or not, and the number of points on the   *)
(*             implement.                                 *)
```

```
(*    segnum : the number of points digitized.           *)
(*     frame : the number of frames digitized.           *)
(*                                                       *)
(*********************************************************)

{$I graph.p}
{$I util.pas}
```

```
Procedure introduce;

(*********************************************************)
(*                                                       *)
(* This procedure describes to the user what is going on.*)
(*                                                       *)
(*********************************************************)

var
   ch   : char;

begin
   clrscr;
   graphcolormode;
   graphbackground(red);
   textcolor(white);
   gotoxy(3,6);
   writeln('***********************************');
   gotoxy(3,7);
   writeln('___________________________________');
   gotoxy(3,12);
   writeln('WELCOME TO KSU FILM ANALYSIS SYSTEM');
   gotoxy(3,16);
   writeln('___________________________________');
   gotoxy(3,18);
   writeln('***********************************');
   gotoxy(7,22);
   writeln('(PRESS ANY KEY TO CONTINUE)');
   read(kbd, ch);
   clrscr;
   graphcolormode;
   graphbackground(blue);
   textcolor(white);
   gotoxy(4,7);
   writeln('_________________________________');
   gotoxy(4,12);
   writeln('NOW YOU ARE IN THE FILTER PROGRAM');
   gotoxy(4,17);
   writeln('^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^');
   gotoxy(4,18);
   writeln('_________________________________');
   gotoxy(7,22);
   writeln('(PRESS ANY KEY TO CONTINUE)');
   read(kbd, ch);
   clrscr;
   textmode(c80);
   graphbackground(blue);
   textcolor(yellow);
   gotoxy(1,5);
   writeln('      This program will smooth the joint center
coordinate data by using ');
   writeln('  digital filter. The joint center coordinates
data             were generated by DATAIN ');
   writeln('  program which should have been run previously
to             this program.');
```

```
   writeln;
   writeln('        The filtered data will be printed out
upon              request. The filtered data');
   writeln('  will be the following:');
   writeln('              --- right and left foottoes;');
   writeln('              --- right and left ankles;');
   writeln('              --- right and left knees;');
   writeln('              --- right and left hips;');
   writeln('              --- right and left shoulders;');
   writeln('              --- right and left elbows;');
   writeln('              --- right and left wrists;');
   writeln('              --- right and left hands;');
   writeln('              --- head;');
   writeln('              --- implement (if available).');
   writeln;
   writeln('  (Press any other key to continue.)');
   read(kbd,ch);
end;
```

```
Procedure readin(var data1,data2:dataarray;
                 var segnum,frame:integer;
                 var filename:namelab;
                 var camspeed,frameint,impreal:real);

(***********************************************************)
(*                                                         *)
(* This procedure reads raw data from data file            *)
(* established by the DATAIN program.                      *)
(*                                                         *)
(***********************************************************)

var
   framenum,
   tempio  ,
   index   : integer;
   actfile : realfile;

begin
   repeat

(***********************************************************)
(*                                                         *)
(* Enter file name.                                        *)
(*                                                         *)
(***********************************************************)

      clrscr;
      textmode(c80);
      graphbackground(blue);
      textcolor(yellow);
      gotoxy(1,5);
      writeln('  Enter the name of your files. DO NOT inclu
de the file number and file type.');
      gotoxy(20,12);
      readln(filename);

(***********************************************************)
(*                                                         *)
(* Check if the file exists or not.                        *)
(*                                                         *)
(***********************************************************)

      assign(actfile,'A:'+filename+'2.RAW');
      (*$I-*)
      reset(actfile);
      (*$I+*)
      tempio:=ioresult;
      if tempio <> 0 then
      begin
         clrscr;
         textmode(c80);
         graphbackground(red);
         textcolor(white);
         gotoxy(1,5);
```

```
         writeln('      The file named ',filename,' is not
found. Make sure the following things:');
         writeln;
         writeln('      1) you have run DATAIN program befo
re running this program;');
         writeln('      2) the disk with the file on it is
in drive A;');
         writeln('      3) file name is correctly entered.
');
         gotoxy(1,15);
         writeln('      If you have NOT run DATAIN program,
press ESC key to quit this program ');
         writeln('  and run DATAIN program. If you have alr
eady run DATAIN program, press any ');
         writeln('  other key to re-enter the file name.');
         read(kbd,ch);
         if ch = chr(27) then
         begin
            clrscr;
            graphcolormode;
            clrscr;
            textmode(c80);
            halt;
         end;
      end;
   until tempio = 0;

(**********************************************************)
(*                                                        *)
(* Read raw data.                                         *)
(*                                                        *)
(**********************************************************)

   clrscr;
   textmode(c40);
   graphbackground(magenta);
   textcolor(yellow+blink);
   gotoxy(15,12);
   writeln('READING DATA');
   read(actfile,camspeed, frameint, impreal);
   segnum:=17+trunc(impreal);
   deltatime:=frameint/camspeed;
   frame:=0;
   while not eof(actfile) do
   begin
      frame := frame+1;
      for index:=1 to segnum do
         read(actfile, data1[index,frame],
                       data2[index,frame]);
   end;
   close(actfile);
end;
```

```
Procedure filterdata(var data1,data2,data3,data4:dataarray;
                     var cutoff:cutarray;
                     var segnum,frame:integer;
                     var deltatime:real);

(**********************************************************)
(*                                                        *)
(* This procedure will filter rawdata of coordinates of  *)
(* jointer centers contained in DATA1 and DATA2. In this *)
(* procedure, DATA3 and DATA4 will contain filtered x    *)
(* and y coordinates of joint centers.                   *)
(*                                                        *)
(**********************************************************)

var
   index    ,
   framenum ,
   tempio   : integer;
   ch       : char;
   option   : namelab;
   tempcut  : real;

(**********************************************************)
(*                                                        *)
(*  tempcut : the cutoff frequency for a set of data.    *)
(*                                                        *)
(**********************************************************)

   trandata : temparray;

(**********************************************************)
(*                                                        *)
(* trandata contains the data of a joint center and      *)
(* transfer the data to and out of the FILTER procedure. *)
(*                                                        *)
(**********************************************************)

Procedure filter(var trandata:temparray; var frame:integer;
                 var option:namelab;
                 var deltatime,tempcut:real);

var
   count    ,
   subframe : integer;
   temp     : temparray;

(**********************************************************)
(*                                                        *)
(* temp contains raw data.                                *)
(*                                                        *)
(**********************************************************)

   coef     : coearray;
```

```
(***********************************************************)
(*                                                         *)
(* coef contains filter coefficients.                      *)
(*                                                         *)
(***********************************************************)

   wa        ,
   wb        ,
   wc        ,
   x1        ,
   x2        : real;
```

```
Procedure cutoff1(var trandata:temparray;
                  var frame:integer;
                  var tempcut,deltatime:real);

(***********************************************************)
(*                                                         *)
(* This procedure do harmonic analysis for each set of     *)
(* data, then calculate the cutoff frequency from the      *)
(* sampling frequency and the result of harmonic           *)
(* analysis.                                               *)
(*                                                         *)
(***********************************************************)

var
   subframe,
   harmonum,
   stop    : integer;
   temp    ,
   A, B, C : temparray;
   d1      ,
   d2      ,
   d3      ,
   d4      ,
   d5      ,
   d6      ,
   s1      ,
   s2      ,
   s3      ,
   x1      ,
   x2      ,
   NHA1    : real;

begin

(***********************************************************)
(*                                                         *)
(* Eliminate linear component of the raw data.             *)
(*                                                         *)
(***********************************************************)

   x1:=trandata[1];
   x2:=(trandata[frame]-trandata[1])/(frame-1);
   for subframe:=1 to frame do
      trandata[subframe]:=trandata[subframe]-x1-x2*(subfram
e-1);

(***********************************************************)
(*                                                         *)
(* Harmonic analysis.                                      *)
(*                                                         *)
(***********************************************************)

   d1:=2.0*pi/frame;
   harmonum:=0;
   stop:=0;
```

```
   repeat
      harmonum:=harmonum+1;
      d2:=d1*harmonum;
      d3:=0.0;
      d4:=0.0;
      d5:=0.0;
      d6:=0.0;
      s1:=trandata[1];
      s2:=trandata[frame]*cos((frame-1)*d2);
      s3:=trandata[frame]*sin((frame-1)*d2);
      for subframe:=2 to frame-1 do
      begin
         if (subframe-1)/2 = trunc((subframe-1)/2) then
         begin
            d4:=d4+trandata[subframe]*cos((subframe-1)*d2);
            d6:=d6+trandata[subframe]*sin((subframe-1)*d2);
         end
         else
         begin
            d3:=d3+trandata[subframe]*cos((subframe-1)*d2);
            d5:=d5+trandata[subframe]*sin((subframe-1)*d2);
         end;
      end;
      A[harmonum]:=((d1/3.0)*(s1+s2+4.0*d3+2.0*d4))/
                   (2.0*pi);
      B[harmonum]:=((d1/3.0)*(s3+4.0*d5+2.0*d6))/(2.0*pi);
      C[harmonum]:=sqrt(A[harmonum]*A[harmonum]
                  +B[harmonum]*B[harmonum]);

(***********************************************************)
(*                                                         *)
(* Search overlap harmonic.                                *)
(*                                                         *)
(***********************************************************)

      if harmonum > 2 then
         if (C[harmonum] > C[harmonum-1]) or
            (harmonum > 1/(2*deltatime)) then
         begin
            NHA1:=C[harmonum-2]/C[1];
            stop:=1;
         end;
   until stop = 1;

(***********************************************************)
(*                                                         *)
(* Remove linear component back to raw data.               *)
(*                                                         *)
(***********************************************************)

   for subframe:=1 to frame do
      trandata[subframe]:=trandata[subframe]
                          +x1+x2*(subframe-1);
```

```
(*****************************************************)
(*                                                   *)
(* Calculate cutoff frequency.                       *)
(*                                                   *)
(*****************************************************)

   if NHA1 < 0.001 then NHA1:=0.001;
   tempcut:=sqr(sqrt(1/deltatime)*0.1358+
            sqrt(sqrt(sqrt(1/NHA1)))*1.6401-1.0432);
end;
```

```
(**********************************************************)
(*                                                        *)
(* Main procedure of filter                               *)
(*                                                        *)
(**********************************************************)

begin
   if option[1] = '1' then cutoff1(trandata,frame,tempcut,
                               deltatime);

(**********************************************************)
(*                                                        *)
(* Calculate filter coefficients.                         *)
(*                                                        *)
(**********************************************************)

   wc:=sin(pi*tempcut*deltatime)/cos(pi*tempcut*deltatime);
   wa:=wc*2*sqrt(0.5);
   wb:=sqr(wc*sqrt(0.5))*2;
   coef[1]:=wb/(1+wa+wb);
   coef[2]:=2*coef[1];
   coef[3]:=coef[1];
   coef[4]:=-(2*wb-2)/(1+wa+wb);
   coef[5]:=-(wb-wa+1)/(1+wa+wb);
   for subframe:=1 to frame do
      temp[subframe]:=trandata[subframe];
   x1:=trandata[frame];
   x2:=trandata[frame-1];

(**********************************************************)
(*                                                        *)
(* Filter forward and backward.                           *)
(*                                                        *)
(**********************************************************)

  for count:=1 to 2 do
  begin
     if count = 2 then

(**********************************************************)
(*                                                        *)
(* Keep the last two points of a set of data unfiltered  *)
(* when filtering backward.                               *)
(*                                                        *)
(**********************************************************)

      begin
         trandata[1]:=x1;
         trandata[2]:=x2;
      end;
      for subframe:=3 to frame do
         trandata[subframe]:=coef[1]*temp[subframe]+
                             coef[2]*temp[subframe-1]+
                             coef[3]*temp[subframe-2]+
                             coef[4]*trandata[subframe-1]+
```

```
                                    coef[5]*trandata[subframe-2];

(*********************************************************)
(*                                                       *)
(* Reverse the order of the data.                        *)
(*                                                       *)
(*********************************************************)

      for subframe:=1 to frame do
         temp[subframe]:=trandata[frame+1-subframe];
   end;
   for subframe:=1 to frame do
      trandata[subframe]:=temp[subframe];
end;
```

```
(**********************************************************)
(*                                                        *)
(* Main procedure of filterdata                           *)
(*                                                        *)
(**********************************************************)

begin

(**********************************************************)
(*                                                        *)
(* Select the method of determination of cutoff           *)
(* frequency.                                             *)
(*                                                        *)
(**********************************************************)

   repeat
      clrscr;
      textmode(c80);
      textcolor(yellow);
      graphbackground(blue);
      gotoxy(3,4);
      writeln('OPTION FOR DETERMINATION OF CUTOFF FREQUENCY
:');
      gotoxy(3,8);
      writeln('     1) CUTOFF FREQUENCIES ARE AUTOMATICALLY
DETERMINED FROM SAMPLING');
      gotoxy(3,9);
      writeln('        FREQUENCY AND THE RESULT OF HARMONIC
ANALYSIS');
      gotoxy(3,11);
      writeln('     2) CUTOFF FREQUENCY IS AUTOMATICALLY DE
TERMINED FROM SAMPLING');
      gotoxy(3,12);
      writeln('        FREQUENCY ');
      gotoxy(3,14);
      writeln('     3) CUTOFF FREQUENCY IS DETERMINED BY US
ER');
      gotoxy(3,16);
      writeln('  Enter the number of you choice.');
      gotoxy(3,18);
      readln(option);

(**********************************************************)
(*                                                        *)
(* Check if the option is valid or not.                   *)
(*                                                        *)
(**********************************************************)

      if (option[1] <> '1') and (option[1] <> '2') and
         (option[1] <> '3') then
      begin
         clrscr;
         textcolor(white);
         graphbackground(red);
         gotoxy(3,10);
```

```
         writeln('There are only three choices as listed.
');
         writeln;
         writeln('  Only 1, 2 and 3 are valid numbers of ch
oices.');
         writeln;
         writeln('  Press any key to re-enter a valid numbe
r of choice.');
         read(kbd, ch);
      end;
   until option[1] in ['1','2','3'];

(*****************************************************)
(*                                                   *)
(* Calculate cutoff frequency from sampling frequency.   *)
(*                                                   *)
(*****************************************************)

   if option[1]='2' then tempcut:=sqr(1.4845+
                              0.1532*sqrt(1/deltatime));

(*****************************************************)
(*                                                   *)
(* Read cutoff frequency from screen.                *)
(*                                                   *)
(*****************************************************)

   if option[1] = '3' then
   begin
      repeat
         clrscr;
         gotoxy(3,10);
         writeln('Enter cutoff frequency.');
         gotoxy(3,12);
         (*$I-*)
         readln(tempcut);
         (*$I+*)
         tempio:=ioresult;
      until tempio = 0;
   end;
   clrscr;
   textmode(c40);
   graphbackground(magenta);
   textcolor(yellow+blink);
   gotoxy(15,12);
   writeln('FILTERING');
   for index:=1 to segnum do
   begin

(*****************************************************)
(*                                                   *)
(* Filter x coordinates.                             *)
(*                                                   *)
(*****************************************************)
```

```
      for framenum:=1 to frame do
         trandata[framenum]:=data1[index,framenum];
      filter(trandata,frame,option,deltatime,tempcut);
      cutoff[1,index]:=tempcut;
      for framenum:=1 to frame do
         data3[index,framenum]:=trandata[framenum];

(***********************************************************)
(*                                                         *)
(* Filter y coordinates.                                   *)
(*                                                         *)
(***********************************************************)

      for framenum:=1 to frame do
         trandata[framenum]:=data2[index,framenum];
      filter(trandata,frame,option,deltatime,tempcut);
      cutoff[2,index]:=tempcut;
      for framenum:=1 to frame do
         data4[index,framenum]:=trandata[framenum];
   end;
end;
```

```
Procedure printer(var data1,data2,data3,data4:dataarray;
                  var frame,segnum:integer;
                  var camspeed,deltatime:real;
                  var cutoff:cutarray;
                  var filename:namelab);

(**********************************************************)
(*                                                        *)
(* This procedure print out the raw data and filtered     *)
(* data on digital printer.                               *)
(*                                                        *)
(**********************************************************)

var
   answer      : namelab;
   segmentnum  ,
   option      : integer;


Procedure printout(var option, frame : integer;
                   var deltatime : real;
                   var data1,data2,data3,data4 : dataarray;
                   var cutoff : cutarray);

(**********************************************************)
(*                                                        *)
(* This procedure print out the data according to user's *)
(* requirement.                                           *)
(*                                                        *)
(**********************************************************)

var
   framenum    : integer;
   whichseg    : namelab;

begin
   whichseg := '';
   case option of
      1  : whichseg := 'RIGHT FOOTTIP';
      2  : whichseg := 'RIGHT ANKLE';
      3  : whichseg := 'RIGHT KNEE';
      4  : whichseg := 'RIGHT HIP';
      5  : whichseg := 'RIGHT SHOULDER';
      6  : whichseg := 'RIGHT ELBOW';
      7  : whichseg := 'RIGHT WRIST';
      8  : whichseg := 'RIGHT HAND';
      9  : whichseg := 'LEFT FOOTTIP';
     10  : whichseg := 'LEFT ANKLE';
     11  : whichseg := 'LEFT KNEE';
     12  : whichseg := 'LEFT HIP';
     13  : whichseg := 'LEFT SHOULDER';
     14  : whichseg := 'LEFT ELBOW';
     15  : whichseg := 'LEFT WRIST';
     16  : whichseg := 'LEFT HAND';
     17  : whichseg := 'HEAD';
```

```
      18  : whichseg := 'IMPLEMENT-1';
      19  : whichseg := 'IMPLEMENT-2';
   end;
   begin
      writeln(lst,'Data for ',whichseg);
      writeln(lst);
      writeln(lst);
      writeln(lst,'                       HORIZONTAL COORDI
NATE   VERTICAL COORDINATE');
      writeln(lst,'                     RAW DATA    FILTERE
D DATA     RAW DATA    FILTERED DATA ');
      writeln(lst);
      writeln(lst,'No.     TIME         CUTOFF FREQUENCY =
',cutoff[1,option]:5:2,'         CUTOFF FREQUENCY = ',cutof
f[2,option]:5:2);
      writeln(lst);
      writeln(lst);
      for framenum:=1 to frame do
      begin
         writeln(lst,framenum:3,' ':4,
                     deltatime*framenum:5:2,
                     data1[option,framenum]:17:4,
                     data3[option,framenum]:14:4,
                     data2[option,framenum]:19:4,
                     data4[option,framenum]:14:4);
         writeln(lst);
      end;

(***********************************************************)
(*                                                         *)
(* Go to the beginning of the next piece of paper.         *)
(*                                                         *)
(***********************************************************)

      writeln(lst,chr(12));
   end;
end;
```

```
(***********************************************************)
(*                                                         *)
(* Main procedure of PRINT.                                *)
(*                                                         *)
(***********************************************************)

begin

(***********************************************************)
(*                                                         *)
(* Set printing.                                           *)
(*                                                         *)
(***********************************************************)

 repeat
      clrscr;
      textmode(c80);
      textcolor(yellow);
      graphbackground(blue);
      gotoxy(3,8);
      write('OUTPUT OPTION-1:');
      gotoxy(8,10);
      write('1) PRINT OUT THE DATA FOR ALL OF THE SEGMENTS'
);
      gotoxy(8,11);
      write('2) PRINT OUT THE DATA FOR SELECTED SEGMENTS');
      gotoxy(8,12);
      write('3) QUIT');
      gotoxy(3,15);
      readln(answer);
   until answer[1] in ['1','2','3'];
   if answer[1] <> '3' then
   begin
      writeln(lst);
      writeln(lst);
      writeln(lst);
      writeln(lst);
      writeln(lst,'                    ********************
**********************');
      writeln(lst,'                    ********************
**********************');
      writeln(lst,'                    ***
                  ***');
      writeln(lst,'                    ***
                  ***');
      writeln(lst,'                    ***         FILTERED
DATA OF JOINT     ***');
      writeln(lst,'                    ***
                  ***');
      writeln(lst,'                    ***           CENTER C
OORDINATES        ***');
      writeln(lst,'                    ***
                  ***');
      writeln(lst,'                    ***
                 ***');
```

```
      writeln(lst,'                    ********************
*********************');
      writeln(lst,'                    ********************
*********************');
      writeln(lst);
      writeln(lst);
      writeln(lst);
      writeln(lst);
      writeln(lst);
      writeln(lst,'                 Event _________________
       Result _______________');
      writeln(lst);
      writeln(lst);
      writeln(lst);
      writeln(lst,'               Subject _________________ ');
      writeln(lst);
      writeln(lst);
      writeln(lst);
      writeln(lst);
      writeln(lst,'        Filming rate:  ',camspeed:6:2,
' (fra/sec)');
      writeln(lst);
      writeln(lst);
      writeln(lst);
      writeln(lst,'        Sampling frequency:   ',1/deltatim
e:6:2,' (fra/sec)    ');
      writeln(lst);
      writeln(lst);
      writeln(lst);
      writeln(lst);
      writeln(lst,'        Data file name:  ',filename);
      writeln(lst);
      writeln(lst);
      writeln(lst);
      writeln(lst);
      writeln(lst);
      writeln(lst);
      writeln(lst);
      writeln(lst);
      writeln(lst);
      writeln(lst,'
Researcher _____________________');
      writeln(lst,chr(12));
      if answer[1] = '1' then
         for option:=1 to segnum do
            printout(option,frame,deltatime,data1,data2,
                     data3,data4,cutoff);
      if answer[1] = '2' then
      repeat
         repeat
            clrscr;
            textmode(c80);
            textcolor(yellow);
            graphbackground(blue);
            gotoxy(3,3);
```

```
            writeln('OUTPUT OPTION-2:');
            writeln;
            writeln('      1) RIGHT FOOTTIP    9) LEFT FOOT
TIP');
            writeln('      2) RIGHT ANKLE     10) LEFT ANKL
E');
            writeln('      3) RIGHT KNEE      11) LEFT KNEE
');
            writeln('      4) RIGHT HIP       12) LEFT HIP'
);
            writeln('      5) RIGHT SHOULD    13) LEFT SHOU
LD');
            writeln('      6) RIGHT ELBOW     14) LEFT ELBO
W');
            writeln('      7) RIGHT WRIST     15) LEFT WRIS
T');
            writeln('      8) RIGHT HAND      16) LEFT HAND
');
            writeln('                         17) HEAD');
            if segnum > 17 then
               writeln('                      18) IMPLEM
ENT-1');
            if segnum > 18 then
               writeln('                      19) IMPLEM
ENT-2');
            writeln('                          20) PAGE ADV
ANCE AND QUIT');
            repeat
               gotoxy(3,18);
               writeln('  Enter the option number in which
you wish to see.');
               gotoxy(3,20);
               (*$I-*)
               read(option);
               (*$I+*)
            until ioresult = 0;
            if (option > segnum) and (option <> 20) then
            begin
               clrscr;
               textcolor(white);
               graphbackground(red);
               gotoxy(3,12);
               writeln(option,' is not an invalid option. P
ress any key to re-enter the option.');
               gotoxy(3,14);
               read(kbd,ch);
               option := 0;
            end;
         until (option <= 20) and (option >= 1);
         if option <= 19 then
            printout(option,frame,deltatime,data1,data2,
                     data3,data4,cutoff);
      until option = 20;
   end;
end;
```

```
Procedure generatefile(var data3,data4:dataarray;
                       var camspeed,frameint,impreal:real;
                       var frame,segnum:integer;
                       var filename:namelab);

(**********************************************************)
(*                                                        *)
(* This procedure save the filtered data into a data      *)
(* file.                                                  *)
(*                                                        *)
(**********************************************************)

var
   actfile       : realfile;
   framenum      ,
   segment       : integer;
   answer        ,
   sfilename     : namelab;

begin
   assign(actfile,'A:'+filename+'2.ACT');
   rewrite(actfile);
   write(actfile,camspeed,frameint,impreal);
   for framenum:=1 to frame do
      for segment:=1 to segnum do
         write(actfile,data3[segment,framenum],
                       data4[segment,framenum]);
   close(actfile);
end;
```

```
(*********************************************************)
(*                                                       *)
(* Main program.                                         *)
(*                                                       *)
(*********************************************************)

begin
   introduce;
   readin(data1, data2, segnum, frame, filename,camspeed,
          frameint,impreal);
   filterdata(data1,data2,data3,data4,cutoff,segnum,frame,
              deltatime);
   generatefile(data3,data4,camspeed,frameint,impreal,
                frame,segnum,filename);
   printer(data1,data2,data3,data4,frame,segnum,camspeed,
           deltatime,cutoff,filename);
   clrscr;
   graphcolormode;
   clrscr;
   textmode(c80);
   gotoxy(10,12);
   writeln('     ******     END OF DIGITAL FILTER PROCESS
   ******');
end.
```

DETERMINATION OF THE APPROPRIATE CUTOFF FREQUENCY IN THE DIGITAL FILTER DATA SMOOTHING PROCEDURE

by

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AN ABSTRACT OF A MASTER'S THESIS

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The purpose of this study was to develop a method for objective determination of cutoff frequency in the Butterworth low pass digital filter widely used in sport biomechanics. Computer generated vertical coordinate data of free fall body movement was used as the standard data. Different sets of computer generated random real numbers were mixed in to the standard data to comprise different sets of artificial raw data. The best cutoff frequency for each set of raw data was determined. Harmonic analysis was carried out for each set of raw data. Two regression equations were developed for estimating appropriate cutoff frequency in the digital filter. About 87% of the variation of the square root of the best cutoff frequency has been explained. The following were the findings of this study: (1) cutoff frequency was influenced by the sampling frequency; (2) an overlap harmonic existed in the harmonic spectrum of measured signal which can be used as an indicator of the beginning of the overlap of actual signal and error; and (3) the relative harmonic amplitude before the overlap harmonic can be used as an indicator of the relative amplitude of the actual signal, the relative position of the beginning of the overlap on the harmonic spectrum, and the relative amplitude of error. The regression equations have been successfully applied to determine the appropriate cutoff frequency for the film data collected by Pezzack (1977). Further studies are recommended to provide more complete explanation of the variation of cutoff freqeuency.