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EVALUATION OF PLASTIC SCINTILLATOR AND HIGH-PURITY
GERMANIUM DETECTORS FOR USE IN BETA SPECTROSCOPY

by

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I. Introduction

In recent years there has been an increased interest in the fields of beta dosimetry and beta spectroscopy. The reasons for this are principally twofold. First, it has become apparent that situations exist in which beta fields of appreciable energies are present. Reprocessing facilities have been cited as one example. Second, current dosimetry techniques have been shown to yield unsatisfactory results in the measurement of absorbed dose due to beta radiation. Consequently, studies are currently being performed whose purpose is the development and evaluation of new beta dosimeters. In conjunction, work is underway to develop a beta spectrometer applicable in field use to provide guidance in beta dosimeter design and implementation.

Work in the field of non-magnetic beta spectroscopy dates back into the mid 1950's. Numerous extensive studies were performed during the period 1955-1965. Following this period, interest began to decline, and few works have been published since.

There are two major differences between past and current work in the field of beta spectroscopy. One difference is the present emphasis on measurement of the total beta spectrum based upon the desire for accurate average energy determination. Past work was primarily concerned with measurement of the higher energy portions and endpoints of beta spectra. The second difference is that past work was solely of a laboratory orientation in contrast to the field orientation of present work.

The objective of this study was to provide a survey type evaluation of two detector types under consideration for use in beta spectroscopy,

plastic scintillator and high purity germanium (HPGe) detectors. Emphasis was placed on usability for both average energy and endpoint measurements.

Subjects covered include beta decay theory and detector responses to incident beta particle, gamma-ray and conversion electron radiation. Detector evaluation was based principally upon comparison of theoretical and measured beta particle energy distributions for selected beta-minus emitting isotopes. Gamma-ray discrimination and application data are presented for the plastic scintillator based spectrometer systems.

II. Beta decay and the beta spectra

Unlike alpha-particle or gamma-ray emission, beta particle decay exhibits a continuous energy spectrum rather than one composed of a series of monoenergetic particles. In the early 1930's,¹ the study of beta decay led to the discovery of the neutrino and the postulation of the negatron and positron decay reactions given by

$$n \rightarrow p + e^{-} + \bar{\nu} + Q, \text{ and} \quad (2.1)$$

$$p \rightarrow n + e^{+} + \nu + Q. \quad (2.2)$$

To fulfill the requirements of beta emission, the neutrino, ν , and anti-neutrino, $\bar{\nu}$, carry away energy, linear momentum, and angular momentum. Additionally, they must possess a spin of $1/2$.

Based upon reactions (2.1) and (2.2), Fermi developed the first satisfactory quantum mechanical theory of beta-decay, which has been in a state of constant development every since. To date, an "exact" theory does not exist. The theoretical models available consist of approximations to quantum mechanical solutions of the Dirac and Schrödinger equations for an electron in an electrostatic field.²

In general, the probability of beta emission with total relativistic energy W in units of electron rest mass energy, between W and $W + dW$ is given by³

$$N(W)dW = C|M|^2 F(\pm Z, W) S_{\pm}(W) W(W^2-1)^{1/2} (W_0-W)^2 dW, \text{ for } e^{\pm}, \quad (2.3)$$

in which

$F(\pm Z, W)$ = electron density ratio,

C = a constant,

$|M|^2$ = the square modulus of the transition matrix element,

$S_n(W)$ = unique shape factor, and

W_0 = maximum transmission energy.

Total relativistic energy W is related to kinetic energy E_k by

$$W = E_k/m_0c^2 + 1. \quad (2.4)$$

For allowed spectra, or non-unique forbidden spectra, $S_n \equiv S_0 = 1$.

For first, second and third unique forbidden spectra, however, the shape factors are, respectively,

$$S_1(W) = (W^2-1) + (W_0-W)^2, \quad (2.5)$$

$$S_2(W) = (W^2-1)^2 + (W_0-W)^4 + \frac{10}{3}(W^2-1)(W_0-W)^2, \text{ and} \quad (2.6)$$

$$S_3(W) = (W^2-1)^3 + (W_0-W)^6 + 7(W^2-1)(W_0-W)^2[(W^2-1) + (W_0-W)^2]. \quad (2.7)$$

A. The Electron Density Ratio

Specific evaluations of the electron density ratio $F(\pm Z, W)$, also known as the Fermi function, can be obtained from solutions of the non-relativistic Schrödinger or the relativistic Dirac equations for an electron in an electrostatic field.

Treating the nucleus as a point charge and using the non-relativistic Schrödinger equation for an electron in a point charge field, the electron density ratio becomes³

$$F(\pm Z, W) = \frac{2\pi\nu}{(1 - e^{2\pi\nu})}, \quad (2.8)$$

in which, for e^+ ,

$$\nu = \pm Ze^2/\hbar v, \quad (2.9)$$

$$= \pm Z\alpha(c/v), \quad (2.10)$$

where c is the speed of light, v is the electron's velocity, and α the fine structure constant approximately equal to $1/137$. The parameters e and $\hbar$ represent the electronic charge and Planck's constant, respectively.

Relativistic corrections may become important despite the fact that the high velocity particles ($v \rightarrow c$) are the ones that should be least affected by electrostatic forces. The relativistic effects may be taken into account by using a solution of the Dirac, rather than the Schrödinger, equation. The corresponding expression for $F(\pm Z, W)$, is

$$F(\pm Z, W) = \frac{4(1+\frac{S}{2})}{|\Gamma(3+2s)|^2} \left(\frac{2R}{\lambda_c}\right)^{2s} e^{\pi y} (W^2-1)^s |\Gamma(1+s+iy)|^2, \quad (2.11)$$

where

$$y = \pm Z\alpha W / (W^2-1)^{1/2}, \text{ for } e^{\mp}, \quad (2.12)$$

$$s = (1 - (\alpha Z)^2)^{1/2} - 1, \text{ and} \quad (2.13)$$

$$\lambda_c = \hbar/m_0 c \approx 386 \times 10^{-13} \text{ cm}, \quad (2.14)$$

= the rationalized Compton wavelength of an electron.

The nuclear radius, R , is given by

$$R = (1.123 A^{1/3} - 0.941 A^{-1/3}) \times 10^{-13} \text{ cm}. \quad (2.15)$$

To avoid complex gamma functions calculations, it is possible to utilize the Bethe-Bacher approximation to $|\Gamma(1+s+iy)|^2$:

$$|\Gamma(1+s+iy)|^2 \approx |\Gamma(1+iy)|^2 (y^2+1/4)^s, \quad (2.16)$$

$$= (\pi y / \sinh \pi y) [(\alpha^2 Z^2 + 1/4) W^2 - 1/4]^s. \quad (2.17)$$

When $(\alpha Z)^2 \ll 1$, as is the case for all but heavier nuclei the parameter s is approximately equal to zero and the complex gamma function is well represented by²

$$|\Gamma(1+iy)|^2 = (iy)!(-iy)!, \quad (2.18)$$

$$= \pi y / \sinh \pi y. \quad (2.19)$$

Under these conditions, Eq. (2.11), the Dirac solution, can be shown to coincide with the non-relativistic Schrödinger solution, Eq. (2.8).

B. Self-screening corrections

In dealing with the low energy portions of spectra from heavy elements, the assumption of a point charge coulomb field needs some correction for the self screening effect of the atomic electrons. To correct for the screening, $F(\pm Z, W)$ should be replaced by⁴

$$F(\pm Z, W \mp V_0) \left(\frac{(W \mp V_0)^2 - 1}{W^2 - 1} \right)^{1/2} \left(\frac{W \mp V_0}{W} \right), \text{ for } e^\mp, \quad (2.20)$$

where

$$V_0 = 1.13\alpha^2 Z^{4/3}. \quad (2.21)$$

C. Adequacy of the theoretical models

In general, two parameters of a beta spectrum are of interest, the average and spectrum endpoint energies. Although both relativistic and non-relativistic theoretical models exhibit identical endpoint energies, their average energies can be significantly different. Figure 2.1 is a comparison of the ^{36}Cl spectrum calculated using the non-relativistic model, with no self-screening correction, and the spectrum calculated using the relativistic model, incorporating both a self-screening correction and the Bethe-Bacher complex gamma function approximation. Table 2.1 contains a comparison of the calculated and published average energies for selected isotopes. The relativistic model demonstrates excellent results, except in the case of high Z nuclei, in which cases the nucleus may be over-idealized as a point charge.²

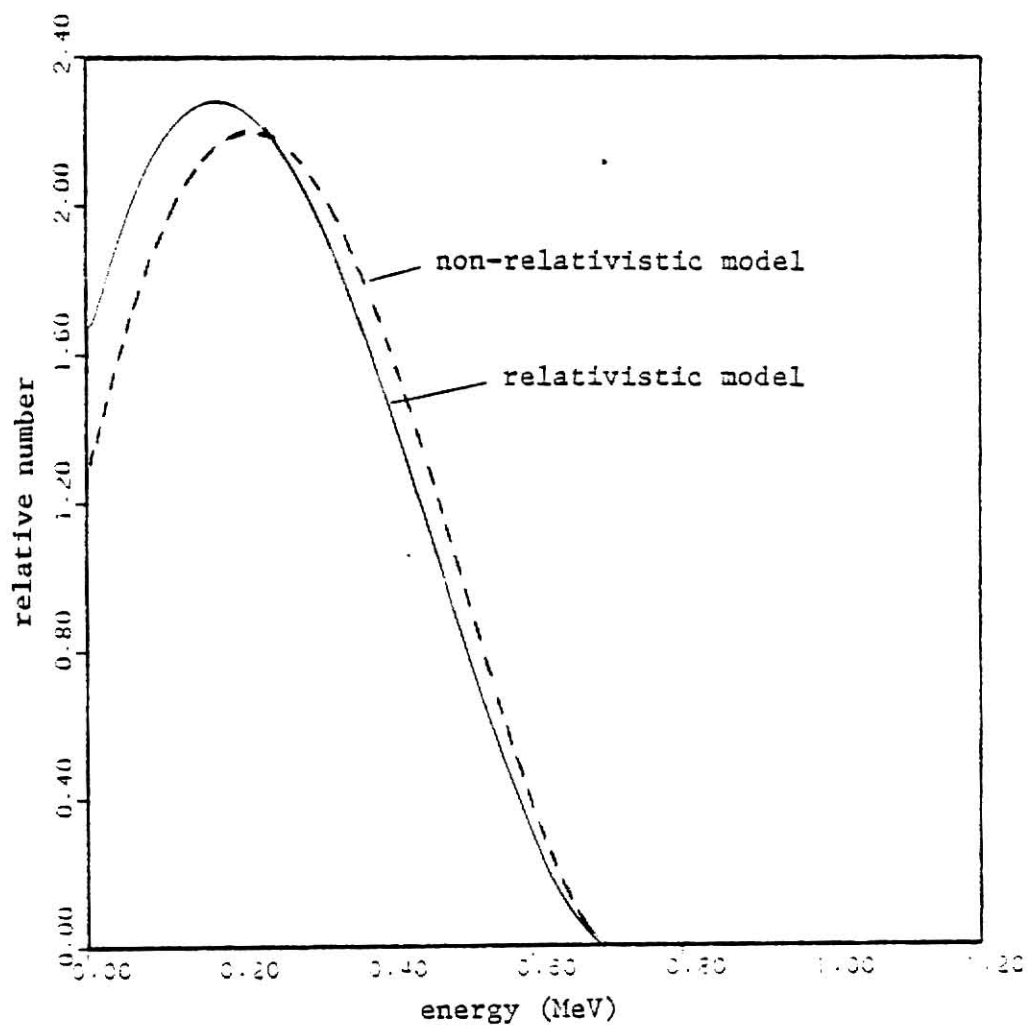


FIG. 2.1. Comparison of the ^{36}Cl beta spectra as calculated using the non-relativistic model, without self-screening correction, and the relativistic model, incorporating both a self-screening correction and the Bethe-Bacher complex gamma function approximation.

Table 2.1 Comparison of published with calculated average energies for the relativistic and non-relativistic theoretical models.

Isotope	A	Z	Published Average Energy ^a {MeV}	Relativistic Model Average Energy {MeV}	Non-relativistic Model Average Energy {MeV}
P	32	15	0.6950	0.6933	0.7630
S	35	16	0.0488	0.0485	0.0501
Cl	36	17	0.2525	0.2525	0.2695
Ca	45	20	0.0773	0.0775	0.0802
Y	90	39	0.9367	0.9270	1.1376
W	185	74	0.1440 ^b	0.1402	0.1706
Tl	204	81	0.2433	0.2352	0.3232

^aFrom M.J. Berger, J. Nucl. Med., 12, Supplement No. 5 (1970).

^bFrom Table of Isotopes, 7th ed., John Wiley and Sons (1978).

D. The Kurie Plot

It is often convenient to represent beta spectra using what is known as a Kurie plot. If Eq. (2.3) is rewritten in the form¹

$$\left(\frac{N(W)}{C|M|^2 F(\pm Z, W) S_n(W) W(W^2-1)^{1/2}} \right)^{1/2} = W_0 - W, \quad (2.22)$$

it is apparent that the value on the left-hand side of Eq. (2.22) is linear with respect to W , and possesses an intercept W_0 on the W axis.

A plot of the calculable quantity on the left-hand side of Eq. (2.22) versus relativistic total energy W for experimental data, is known as a Kurie plot. In such a form, experimental beta spectra are easily compared with their theoretical counterparts.

Some deviations from linearity in Kurie plots are seen in the low energy end.⁴ In nearly all cases, however, the high energy end is linear, allowing extrapolation of a fitted regression line for experimental endpoint determinations.^{4,5}

III. Beta spectrometer systems

A. Introduction

Two distinctly different beta particle spectrometer systems were chosen for evaluation, one based on the BC-400 plastic scintillator and the other on a HPGe detector.

The cylindrical BC-400 plastic scintillators utilized were of 50.8 mm diameter and ranged in thickness from nominally 5 to 20 mm. Each BC-400 scintillator was fitted with a 9 μm Al (2.43 mg/cm^2) entrance window.

The HPGe detector evaluated was 5 mm thick and had a 25 mm^2 active area. The detector assembly was factory equipped with a 2 mil thick Be (9.40 mg/cm^2) entrance window.

The specific detector types were selected for several reasons. First, both detectors can be designed and constructed in configurations possessing an ultra-thin entrance window. This is a critical requirement for beta particle studies which require minimal entrance losses. Second, both detectors are commonly available in thicknesses sufficient to stop even high energy (greater than 4 MeV) beta particles. This is not true of silicon detectors, normally available only in thicknesses sufficient to stop beta particles with kinetic energies less than 2.5 MeV. Additionally, the HPGe detector demonstrates a low ambient temperature sensitivity due to being liquid nitrogen cooled. The plastic scintillator has the advantage of being a low atomic number material, nearly tissue equivalent, and is readily available in nearly any geometry at a low cost.

Differences in the responses of the two detector materials, for incident electrons, can be predominantly attributed to their dissimilar atomic numbers. Germanium and plastic exhibit markedly different stopping powers, as shown in Fig. 3.1. This difference influences both electron ranges in the media, illustrated by Table 3.1, and the nature of the low-energy electron responses of the detectors.

Backscatter of incident electrons from the detector, a property strongly dependent upon atomic number, is also significantly different for the two materials. In the case of plastic scintillators, backscatter events are nearly negligible, whereas for germanium detectors, they can, unfortunately, represent a large fraction of all events.^{6,7} Backscatter coefficients p^- , defined as the fraction of normally incident electrons which are backscattered from the detector surface, are compiled in Table 3.2 for the two detector materials.

An important consideration for any beta spectrometer system is its sensitivity to X- and gamma-rays. The photon response of the two detectors evaluated is primarily determined by their individual mass absorption coefficients. Tables 3.3 and 3.4 and Figs. 3.2 and 3.3 illustrate that over the majority of the photon energy range of interest, i.e., 0.1 - 3.0 MeV, photon interactions in the plastic scintillator material will be almost entirely Compton events, whereas, in germanium, a mixture of Compton, photoelectric and pair production interactions will occur.

B. The BC-400 based beta particle spectrometer

1.0 System description

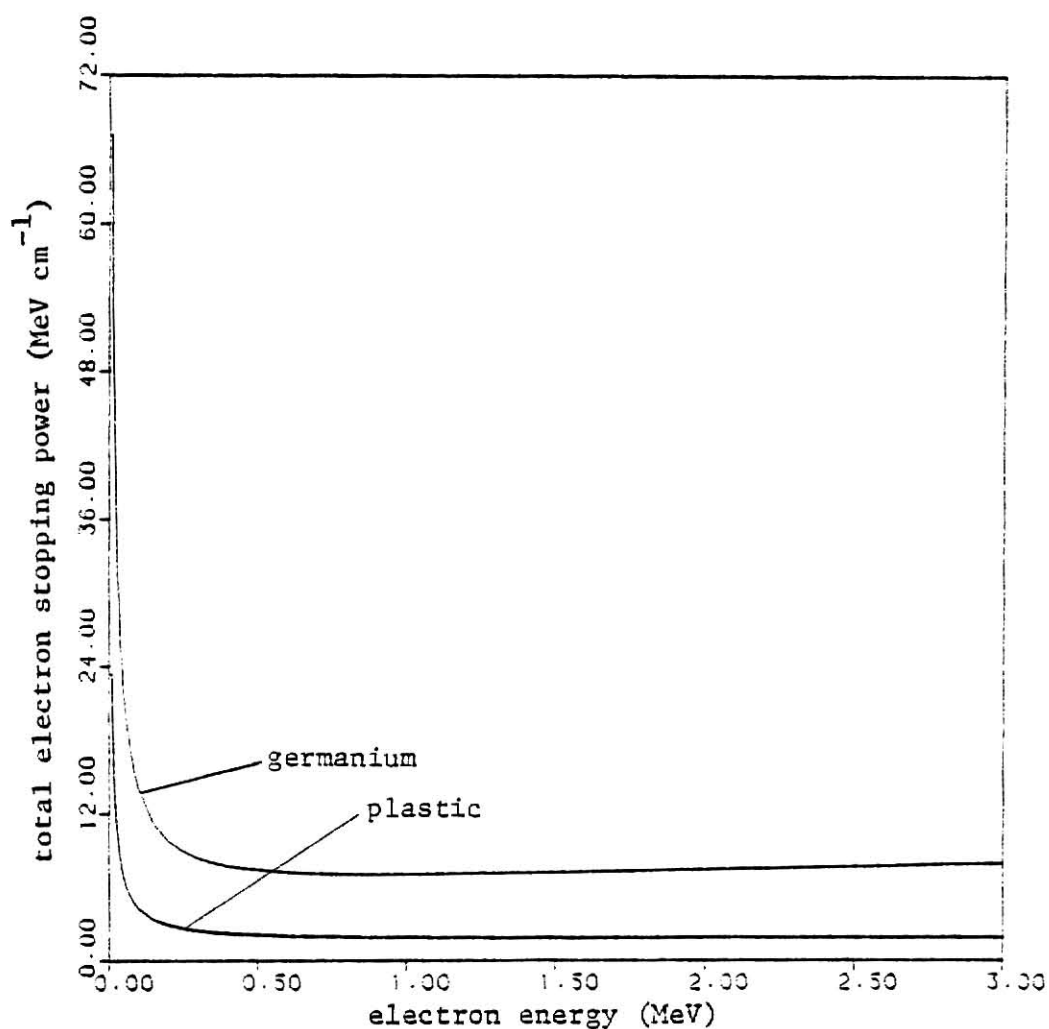


FIG. 3.1. Total electron stopping power for germanium and plastic materials. Plastic material stopping power is for polyethylene with a density equivalent to BC-400 plastic scintillators. (Berger and Seltzer, Tables of Energy Losses and Ranges of Electrons and Positrons, National Bureau of Standards, 1964)

TABLE 3.1. Electron energy versus zero transmission range.[†]

Electron Energy (meV)	Range in Germanium (mm)	Range in Plastic Scintillator (mm)
0.01	0.009	0.0024
0.05	0.0136	0.042
0.10	0.0434	0.140
0.50	0.501	1.73
0.80	0.919	3.24
1.00	1.203	4.29
1.20	1.486	5.36
1.50	1.907	6.96
1.70	2.183	8.02
2.00	2.591	9.61
2.20	2.853	10.7
2.60	3.384	12.77
2.80	3.642	13.81
3.00	3.896	14.85
3.50	4.519	17.42
4.00	5.123	19.97

[†] From M. J. Berger and S. M. Seltzer, Tables of Energy Losses and Ranges of Electrons and Electrons and Positrons, National Bureau of Standards, 1964.

Table 3.2. Tabulated backscatter coefficients for germanium and plastic scintillator material as a function of increasing electron energy.

Electron energy {MeV}	Backscatter coefficient p^-	
	Germanium ^a	Plastic scintillator ^b
0.10	0.296	-
0.25	-	0.080
0.50	0.276	0.053
0.75	-	0.040
0.80	0.258	-
1.00	0.245	0.032
1.25	-	0.030
2.00	0.186	-

^aReference 7.

^bReference 6.

TABLE 3.3. Compton, photoelectric and pair production mass absorption coefficients for Germanium

Photon Energy (MeV)	Compton Mass Absorption Coefficient (cm ² /g)	Photoelectric Mass Absorption Coefficient (cm ² /g)	Pair Production Mass Absorption Coefficient (cm ² /g)
0.001	0.00001211	1373.1	
0.001217	0.0000201	1178.2 (L3)	
		5841.1	
0.001248	0.0000213	5144.1 (L2)	
		7334.34	
0.001413	0.0000290	5323.3 (L2)	
		6272.5	
0.0015	0.0000334	5476.0	
0.002	0.0000647	2579.9	
0.003	0.0000151	943.86	
0.004	0.000264	443.53	
0.005	0.000401	243.10	
0.006	0.000538	143.19	
0.008	0.000929	54.88	
0.010	0.00134	34.68	
0.011104	0.00139	26.55 (K)	
		121.97	
0.015	0.00250	64.30	
0.020	0.00372	32.36	
0.030	0.00605	11.53	
0.040	0.00819	5.194	
0.050	0.0101	2.763	
0.060	0.0119	1.643	
0.080	0.0146	0.713	
0.100	0.0167	0.365	
0.150	0.0203	0.107	
0.200	0.0228	0.463	
0.300	0.0248	0.0141	
0.400	0.0256	0.00626	
0.500	0.0261	0.00343	
0.600	0.0258	0.00222	
0.800	0.0231	0.00112	
1.000	0.0241	0.000212	
1.500	0.0221	0.000326	0.0001361
2.000	0.0201	0.000204	0.000766
3.000	0.0168	0.000108	0.00281
4.000	0.0146	0.0000741	0.00503
5.000	0.0123	0.0000333	0.00707
6.000	0.0114	0.0000442	0.00895
8.000	0.00938	0.0000310	0.0119
10.00	0.00739	0.0000242	0.0144

* From Nuclear Data Tables, Section A, Vol. 7, Number 6, June, 1970.

TABLE 3.4. Compton, photoelectric and pair production mass absorption coefficients for a BC-400 type plastic scintillator.

Photon Energy (MeV)	Compton Mass Absorption Coefficient (cm ² /g)	Photoelectric Mass Absorption Coefficient (cm ² /g)	Pair Production Mass Absorption Coefficient (cm ² /g)
0.001	0.0000352	1966.7	
0.0015	0.0001033	633.4	
0.002	0.000209	274.5	
0.003	0.000491	81.70	
0.004	0.000823	33.96	
0.005	0.00117	17.03	
0.006	0.00152	9.913	
0.008	0.00222	3.667	
0.010	0.00292	1.304	
0.015	0.00470	0.486	
0.020	0.00639	0.190	
0.030	0.00945	0.0505	
0.040	0.0120	0.0194	
0.050	0.0143	0.0092	
0.060	0.0163	0.00510	
0.080	0.0194	0.00194	
0.100	0.0219	0.000913	
0.150	0.0261	0.000237	
0.200	0.0285	0.000095	
0.300	0.0311		
0.400	0.0320		
0.500	0.0322		
0.600	0.0319		
0.800	0.0312		
1.000	0.0302		
2.000	0.0252		0.000148
3.000	0.0214		0.000583
4.000	0.0188		0.00109
5.000	0.0168		0.00160
6.000	0.0152		0.00208
8.000	0.0123		0.00291
10.00	0.0111		0.00361

† From Nuclear Data Tables, Section A, Vol. 7, Number 6, June 1970.

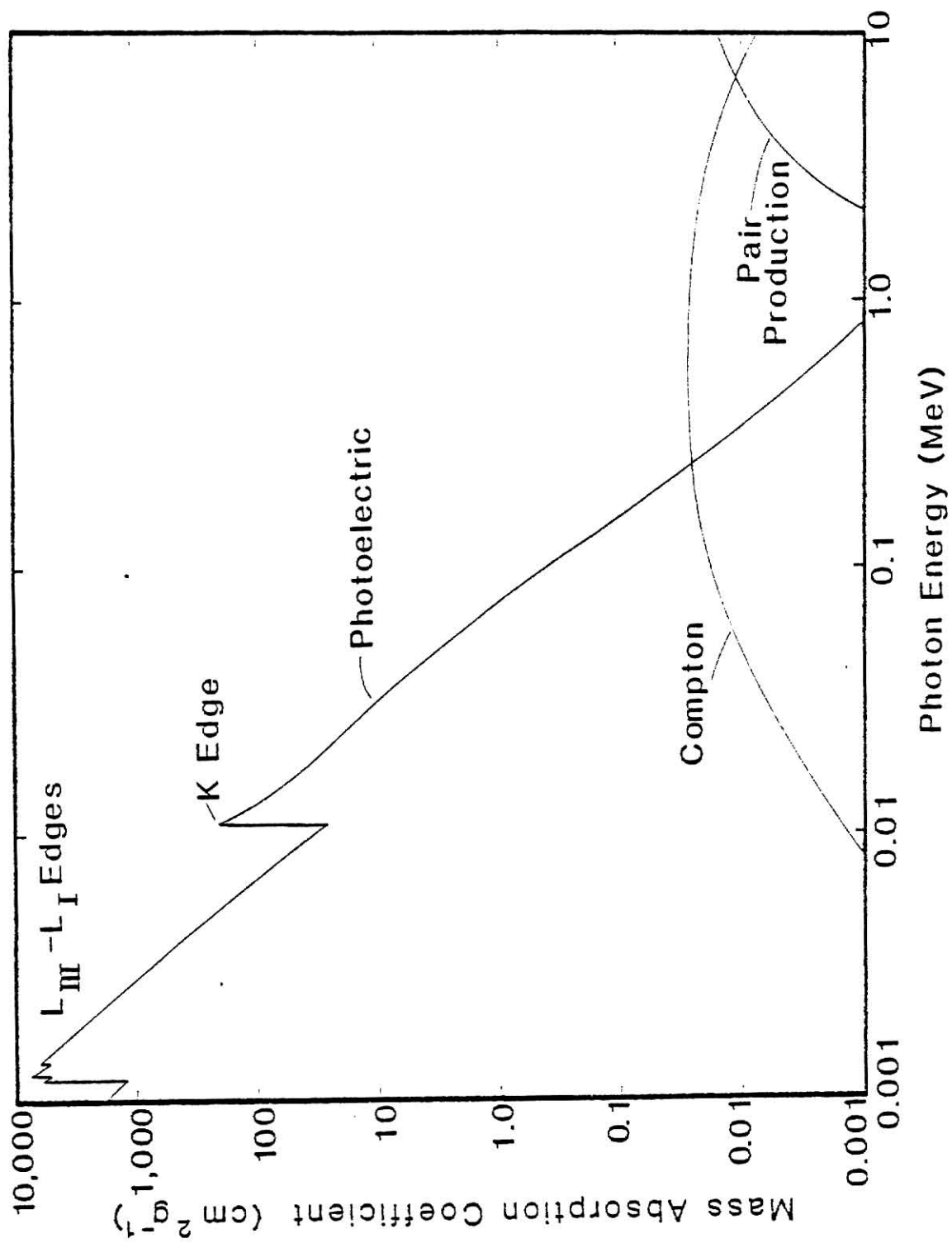


FIG. 3.2. Photoelectric, Compton and pair production absorption coefficients for germanium.

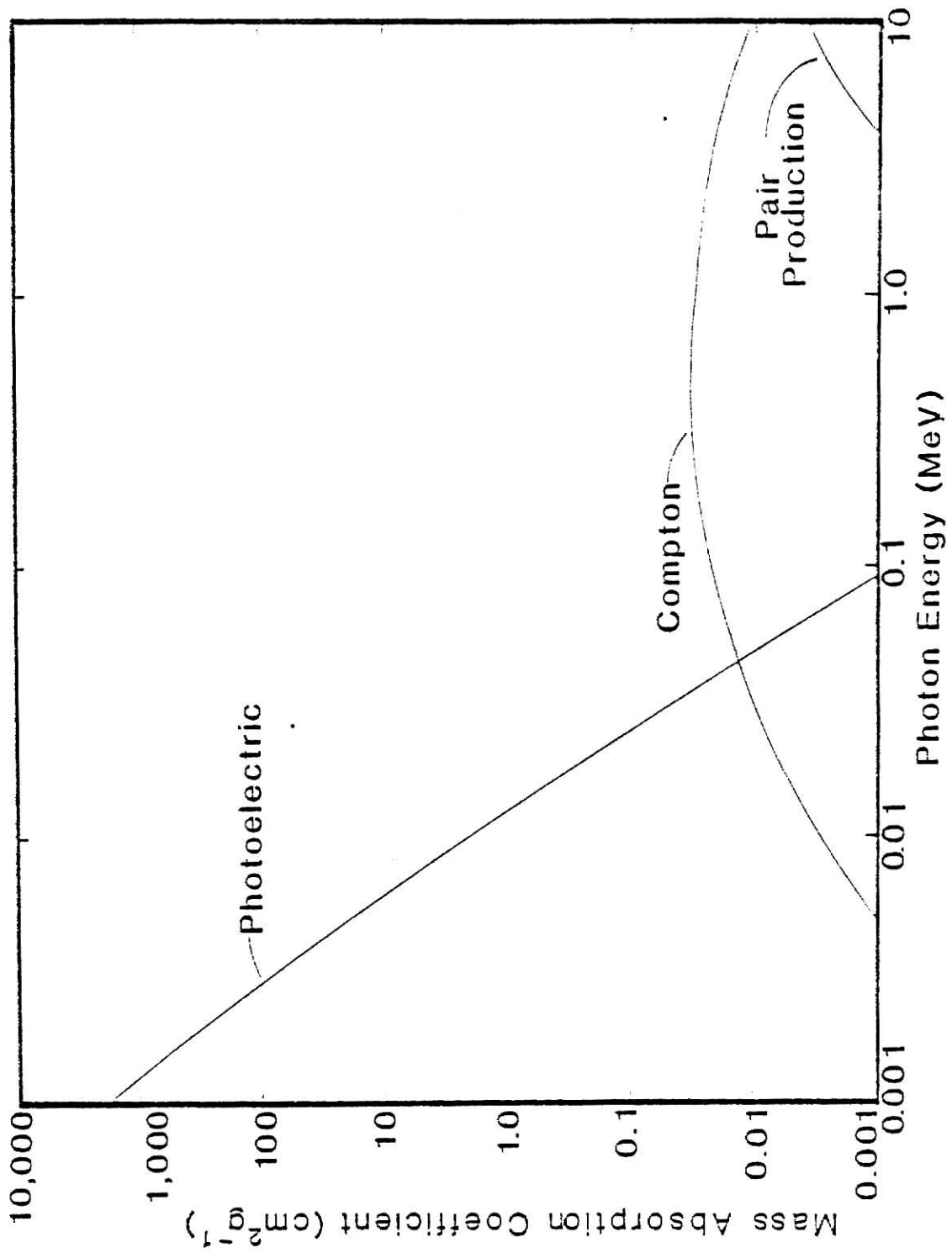


FIG. 3.3. Photoelectric, Compton and pair production absorption coefficients for a BC-400 type plastic scintillator.

Four factory machined and polished cylindrical BC-400 detectors were evaluated. The detectors had a diameter of 50.8 mm and thicknesses of 5.08, 10.2, 15.2 and 20.3 mm. Some characteristics of the polyvinyl-toluene based BC-400's are given in Table 3.5. During assembly, each detector was fitted with a light-tight 9 μ m Al foil window and coupled to a 50.8 mm diameter photomultiplier tube using optical coupling grease. The detector sides were covered with reflective 1 mil Al foil and then light sealed with 7 mil black vinyl plastic tape. The photomultiplier tube was coupled to a commercial photomultiplier tube base. The sides of the assembly were surrounded with a commercial magnetic shield. A schematic of the assembly is shown in Fig. 3.4.

The detector-phototube assembly was connected to the remaining components of the spectrometer system as shown in Fig. 3.5. Pulse height data were accumulated using a multichannel analyzer and then stored on magnetic tape for off-line data analysis.

2.0 Gamma-ray response characteristics

As discussed previously, gamma-ray interactions within the BC-400 detectors are predominantly Compton events. Consequently, a source of monoenergetic gamma rays incident upon an organic scintillator will produce recoil electrons with a continuous energy distribution described by the Compton scattering cross section. This continuous distribution is then subject to the broadening effect of the energy and dimensionally dependent resolution inherent in the detection system.⁸ An example pulse-height distribution for a 10.2 mm thick BC-400 detector exposed to 0.511 and 1.2746 MeV gamma rays is shown in Fig. 3.6.

Obviously, gamma-ray response is a function of both gamma-ray energy and detector thickness. For a given detector thickness,

Table 3.5. Manufacturer quoted characteristics
of BC-400 plastic scintillators.

Description	Value
Density	1.032 g/cm ³
Refractive index	1.581
Melting point	75°C
Light output	65% Anthracene
Decay constant (main component)	2.4 ns
Wavelength of maximum emission	423 nm
H/C atomic ratio	1.104
Coefficient of linear expansion	$\sim 7.8 \times 10^{-5}$ below 67°C

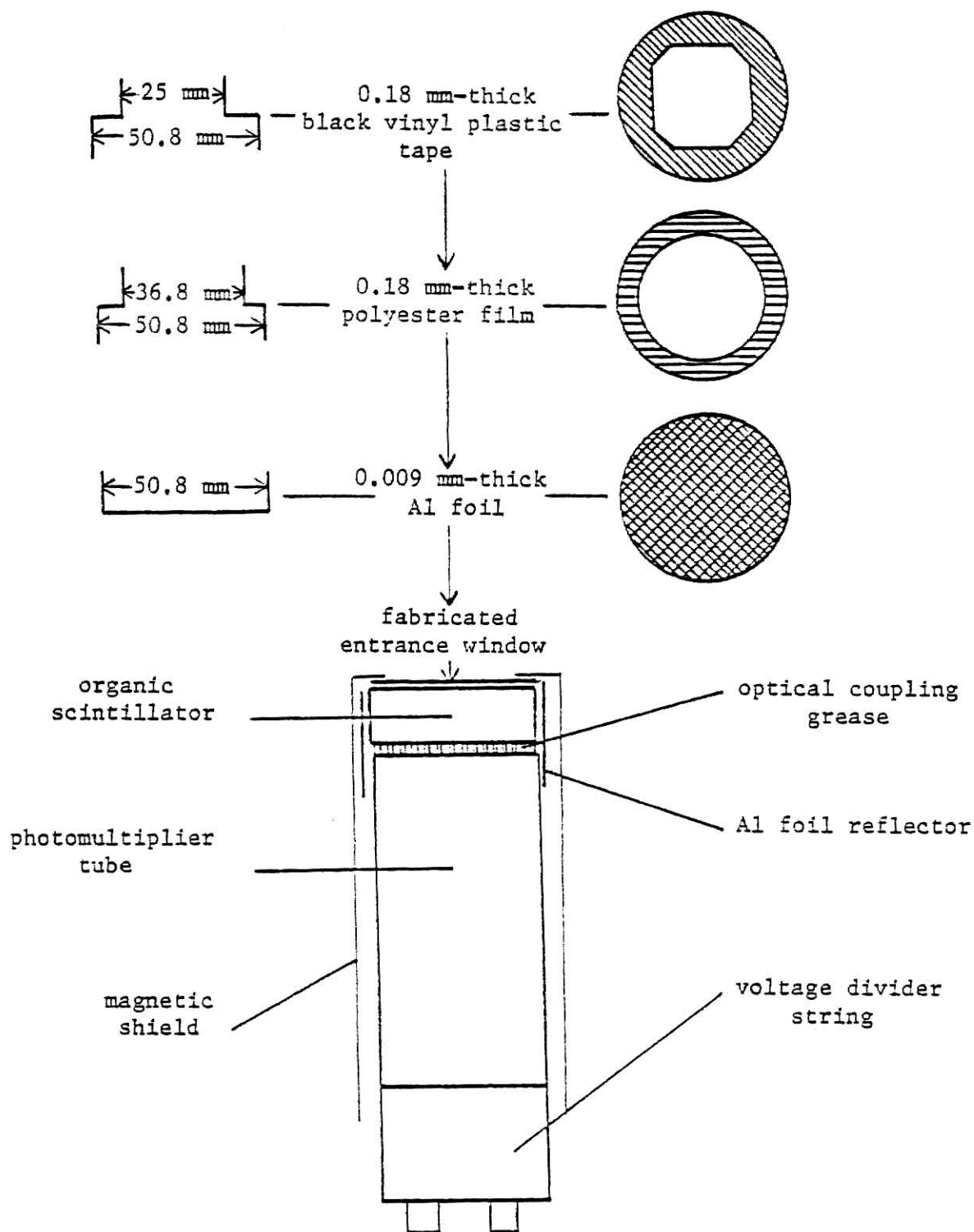


FIG. 3.4. Schematic of BC-400 detector assembly.

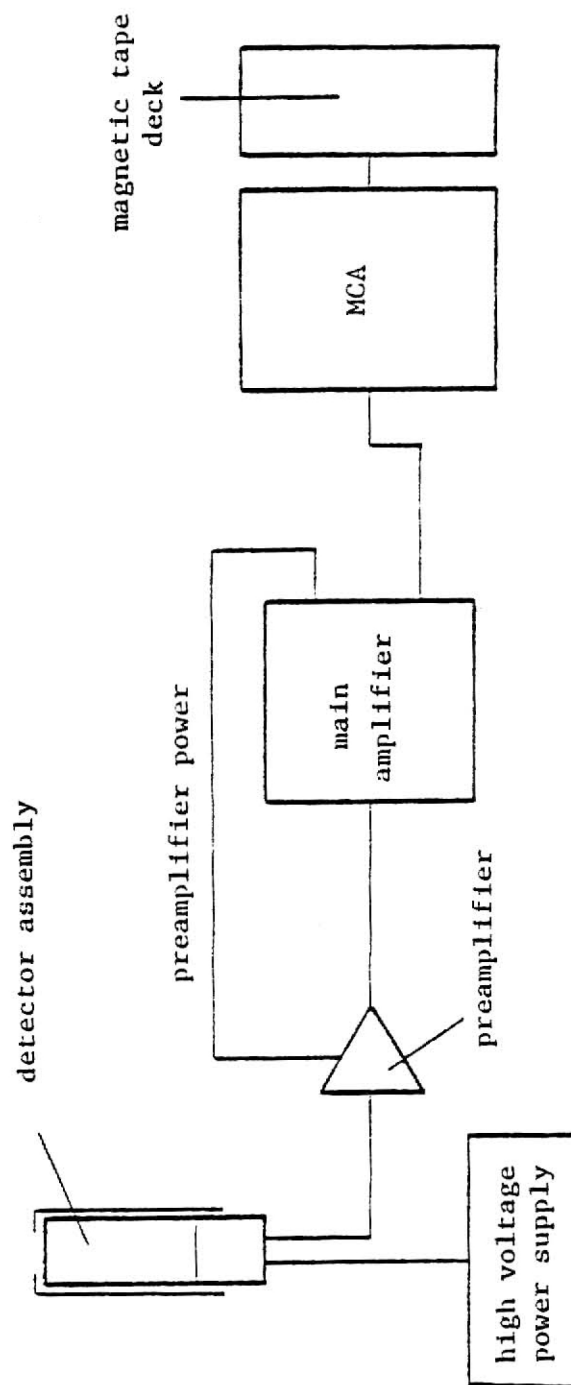


FIG. 3.5. BC-400-based spectrometer system component diagram.

gamma-rays above a certain energy will produce electrons with kinetic energies large enough to allow them to escape the scintillator with only partial energy deposition. When a sufficient energy loss occurs in this manner, the effect upon the measured distribution is so momentous that the result is nearly unrecognizable as a Compton distribution. Consequently, there is a practical "upper-limit" in terms of measurable gamma-ray energies for a detector of a given thickness. This fact is apparent when dealing with detectors which have thicknesses less than 50 mm.

The gamma-ray resolution of organic-scintillators can be estimated using the high energy portion of the Compton edge. In organic scintillators, resolution is defined as twice the half-width at half the peak value ($2 \times \text{HWHM}$). Values of this parameter measured for the 20.3 mm thick BC-400 detector assembly are tabulated in Table 3.6. Typically, the relationship between the half-width at half the peak height and photon energy is given by⁹

$$\left(\frac{\text{HW}}{E}\right)^2 = a + b/E \quad (3.1)$$

in which HW is the half-width, E is energy and a and b are constants with $a \ll b$.

In a similar manner, the actual Compton-edge energy is related to the fraction of the broadened Compton peak height. This fractional height has been shown to demonstrate a slight energy dependence. It is also dependent upon the thickness of the detector. For this work, a value of 0.7 was chosen as corresponding to the Compton-edge energy for the 10 mm thick BC-400 scintillator. The selection draws support from results reported by Cramer,¹⁰ and Nardi⁸ as well as from experimental

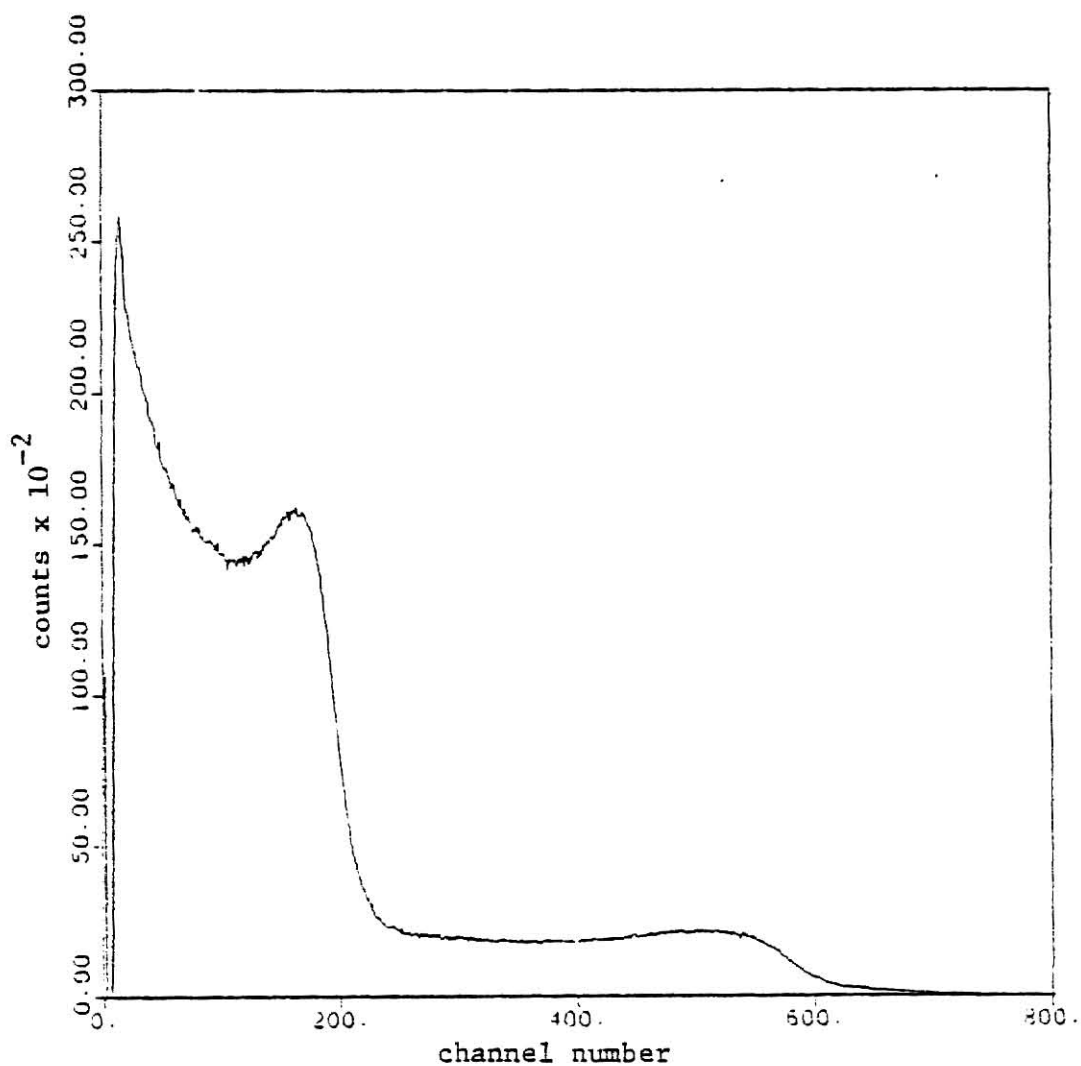


FIG. 3.6. Pulse height distribution measured with a 10.2 mm thick BC-400 plastic scintillator for the 0.511 and 1.275 MeV Na-22 gamma rays.

Table 3.6. Resolution of measured Compton distributions for a 20.3 mm-thick BC-400 scintillator.

Source	Gamma-ray energy {MeV}	Compton edge energy {MeV}	Resolution ^a {%}
¹¹³ Sn	0.393	0.238	41.2
¹³⁷ Cs	0.662	0.478	25.7
⁵⁴ Mn	0.835	0.640	23.5
²² Na	0.511	0.341	36.8
	1.275	1.062	18.7

^aResolution defined in terms of percentage by
 $2 \times \text{HWHM \{channels\}} / \text{Compton peak channel number}.$

results and Gaussian-broadening calculations. To demonstrate the utility of this procedure, theoretical Compton-edge energies versus the corresponding measured channel numbers are shown in Fig. 3.7. This figure illustrates the existence of a linear relationship over the energy range examined. Previous investigators have reported similar results for gamma ray energies above 125 keV.^{11,12}

3.0 Beta-particle response characteristics

In contrast to its gamma-ray response, a reasonably thick BC-400 detector is nearly 100% efficient for beta particles. Exceptions to this general case include very low energy beta particles -- the electrons lose a non-negligible portion of their initial energy upon passing through the entrance window, or very high energy beta particles--their kinetic energy is sufficient to allow only a fraction of the total electron energy to be deposited within the detector. For the detection assemblies investigated, the low energy cutoff, created by the 9 μ m Al entrance window, was nominally 30 keV. The upper limit varied with detector thickness, being nominally 4.2 and 2.3 MeV for the 20 and 10 mm thick BC-400 detectors, respectively.

Resolution broadening of beta spectra is not as significant a problem as it is with gamma-ray Compton distributions. Tests using modified Gaussian broadening of selected theoretical beta spectra indicate only slight spectral distortions near the spectrum endpoints. Additionally, the linearity of Kurie plots, based upon experimental data, indicates an exaggeration of the spectral distortion near the endpoint by the simple modified Gaussian technique.

As a measure of the detectors energy response for beta particles, a plot of theoretical endpoint energies versus measured pulse height

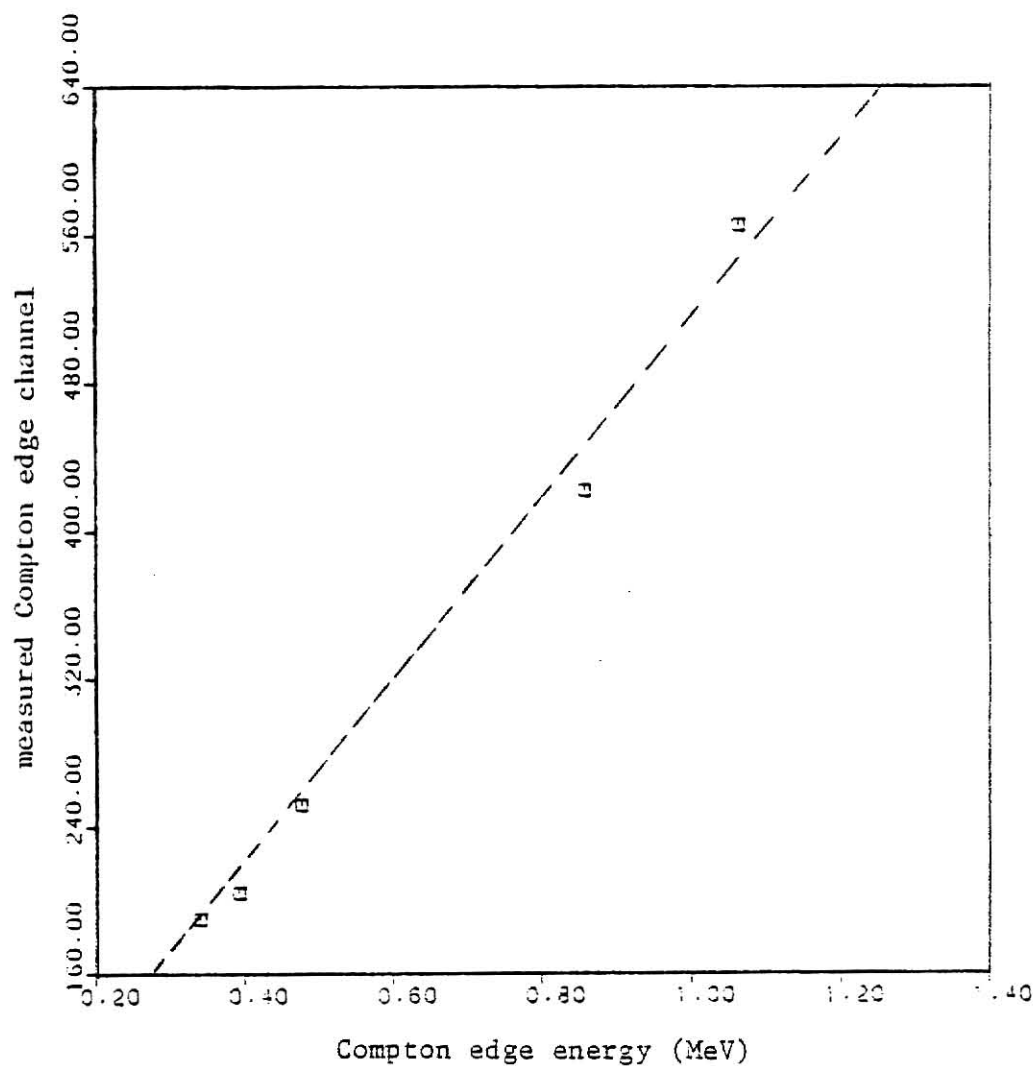


FIG. 3.7. Compton edge energy versus measured Compton edge channel for a 20.3 mm thick BC-400 organic scintillator based spectrometer system. Dashed line is based on a least squares fit to the data.

distribution endpoint channels, calculated from Kurie plots, can be used. Figure 3.8 is a plot of theoretical endpoint energy versus measured endpoint channel for a 10 mm thick BC-400 detector. A least squares linear regression fit to the data yields a fitted line with mean square error 0.0013 and multiple correlation coefficient 0.992, demonstrating a definitely linear relationship over the energy range examined, 0.167-2.28 MeV.

4.0 Nonlinearities in the BC-400 electron response

Nonlinearities in the response of organic scintillators to electrons with energies below about 125 keV have been reported.¹³ These studies were performed using both externally incident electrons and Compton recoil electrons produced by gamma rays. This low energy non-linearity has been directly attributed to the dependence of the organic scintillators fluorescent light yield upon dE/dx , its stopping power. This relationship is described by the semiempirical relation¹¹

$$dL/dE = S[1 + kB(dE/dx) + C(dE/dx)^2]^{-1}, \quad (3.2)$$

in which dL/dE is the quantity of fluorescent light emitted when an electron of energy E loses a quantity of energy dE along a path increment dx within the scintillator. The parameter S is defined as the absolute scintillator efficiency, and B is a constant such that the product $B(dE/dx)$ represents the specific density of ionized and excited particles along the particle track. The parameter k is denoted as the relative quenching probability, generally considered a constant, and C is an empirically determined constant.

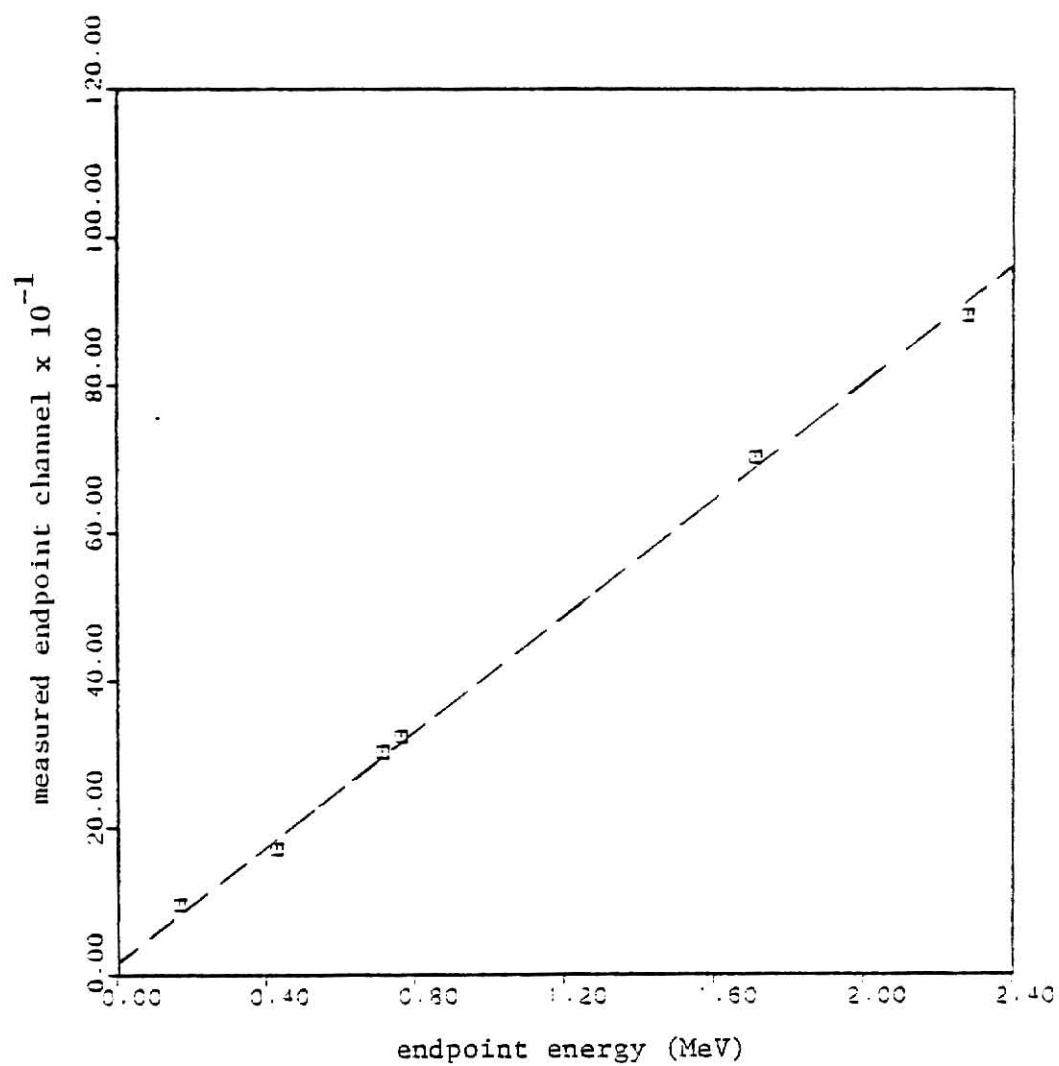


FIG. 3.8. Measured endpoint channel as a function of increasing beta spectrum endpoint energy. Measurements were performed using a 10.2 mm thick BC-400 organic scintillator.

The integral of Eq. (3.2), with respect to particle energy, is

$$L(E) = S \int_0^E dE [1 + kB (dE/dx) + C(dE/dx)^2]^{-1} \quad (3.3)$$

where $L(E)$ is the total light emitted by an electron expending all of its energy within the scintillator. For electrons with kinetic energies >125 keV, dE/dx is approximately equal to zero, and Eq. (3.3) reduces to the linear expression

$$L(E) \approx SE + L_0. \quad (3.4)$$

From Eqs. (3.2) and (3.3), it can be seen that, for large values of dE/dx , dL/dE and therefore $L(E)$ are reduced in magnitude, accounting for the nonlinearities exhibited by organic scintillators for low energy electrons.

5.0 Energy calibration procedures for BC-400 spectrometers.

Several differing energy calibration procedures were explored for the BC-400 based spectrometers. Initially, gamma-ray pulse height distributions were examined for use in energy calibration. Due to the uncertainty in accurately determining the channel number corresponding to the maximum kinetic energy of the Compton recoil electrons, and the poor definition of the Compton edge for thin scintillators, gamma rays were not used to perform energy calibrations. Moreover, data measured using a ^{207}Bi conversion electron source were rejected because the pulse height distributions measured with the organic scintillators showed that it was impossible to separately resolve the K- and L-shell conversion electron peaks. Thus, the observed single peaks were located in channels higher than should correspond to the K-shell electron and lower than that corresponding to L-shell conversion electrons.

Ultimately, a procedure utilizing beta spectra endpoint energies was adopted. The procedure involved two steps. First, a preliminary calibration was made, based upon either previous measurements or alternate techniques deemed accurate within about 20%. Second, a selected series of known beta spectra were measured, and their endpoint energies calculated through the use of Kurie plots. It was then possible to obtain the multichannel analyzer endpoint channel numbers through backcalculation. Accurate calibrations were obtained from the known endpoint energies and the measured endpoint channels of the selected beta spectra.

To account for the nonlinearities present for low energy electrons, an additional correction to the fitted calibration line was required. Ideally, spectra with endpoints <125 keV should be represented with a polynomial fitted to the data to describe the scintillator energy response in the low energy end. Practical considerations, however, such as source fabrication, air scattering and increasing backscatter for low energy electrons, make measurement of low energy spectra difficult. Additionally, fitting a polynomial to experimental data and matching it to a straight line fit at an unknown transition energy is typically a computer problem requiring an iterative solution.¹⁴ To avoid these difficulties, a straight line was fit between the origin and the channel corresponding to 100 keV, as calculated using the previously determined calibration equation. Tests indicated that, for all but the lowest energies, calculated beta-spectra average and endpoint energies were relatively insensitive to the type of fit and transition energy selected for this region, provided that some fit, other than a mere extrapolation of the calculated calibration line, was used.

6.0 Measurement of beta spectra

Beta spectra measurements were made for eight negatron emitting isotopes, ^{35}S , ^{32}P , ^{36}Cl , ^{45}Ca , $^{90}\text{Sr}/^{90}\text{Y}$, ^{185}W , ^{198}Au and ^{204}Tl , with endpoint energies 0.167-2.28 MeV (see Appendix A). Measurements were performed with the sources less than 3 mm from the detector face along the detector centerline. All four detector thicknesses were utilized although most measurements were made using the 10.2 mm-thick detector, as its thickness was sufficient to stop the highest energy beta particle measured.

7.0 Gamma-ray discrimination

A simple yet effective means of gamma-ray discrimination was utilized. The procedure involved two steps. First, the combined beta-gamma pulse height distribution was measured. Once this was completed, a thickness of high density polyethylene (1.4 g/cm^3) sufficient to stop the measured field's highest energy beta particle was placed in front of the detector. The multichannel analyzer was then operated in the subtract mode for a time equivalent to that of the initial accumulation. The resulting distribution was attributed to the beta component of the measured field.

C. HPGe detector based beta spectrometer

1.0 System description

The detector utilized was a planar intrinsic germanium detector. Manufacturer specifications are given in Table 3.7. The detector assembly was liquid nitrogen cooled with a 31 liter liquid nitrogen dewar. The detector assembly included an RC feedback preamplifier. Detector assembly signals were processed and analyzed using the system

Table 3.7. HPGe detector specifications.

Density	5.36 g/cm ³
Atomic number	32
Detector type	planar
Active area	25 mm ²
Thickness	5 mm
Detector to cryostat front face distance	~3 mm
Recommended bias	-1500V
Leakage current at bias	<10pA
Photon full energy resolution	170 eV at 5.9 keV peak 50 eV at 122 keV

illustrated in Fig. 3.9.

illustrated in Fig. 3.9. As with the BC-400 systems, pulse-height data were accumulated using a multichannel analyzer and recorded on magnetic tape.

2.0 Gamma-ray response of the HPGe beta spectrometer

The 5 mm-thick HPGe detector exhibited the photon detection efficiency shown in Fig. 3.10. Being nearly 100% efficient for photons up to 100 keV, the detector is an ideal X-ray and low energy gamma-ray detector.

The HPGe detector system exhibited excellent linearity of response for photons over the energy range 30-1770 keV (see Fig. (3.11)). Previous literature has reported a linear response extending down into the eV range for similar systems.¹⁵

The energy resolution of full energy peaks is typically a few keV. Figure 3.12 is a plot of full energy peak resolution versus photon energy measured for the HPGe detector purchased for this research.

3.0 Conversion electron response.

The thin Be window of the HPGe detector, in combination with its higher atomic number and good resolution, allowed measurement of conversion electrons despite their usually low frequency of emission. As shown in Fig. 3.13, even with entrance losses, the system demonstrated a linear response to conversion electrons over the energy range of 480 - 1770 keV. In a manner similar to that experienced by organic scintillators, nonlinearities can be expected to exist for low energy electrons incident upon the detector. This occurs as the thickness of the entrance window becomes sufficient, relative to the incident electrons kinetic energies, to prevent the electrons from

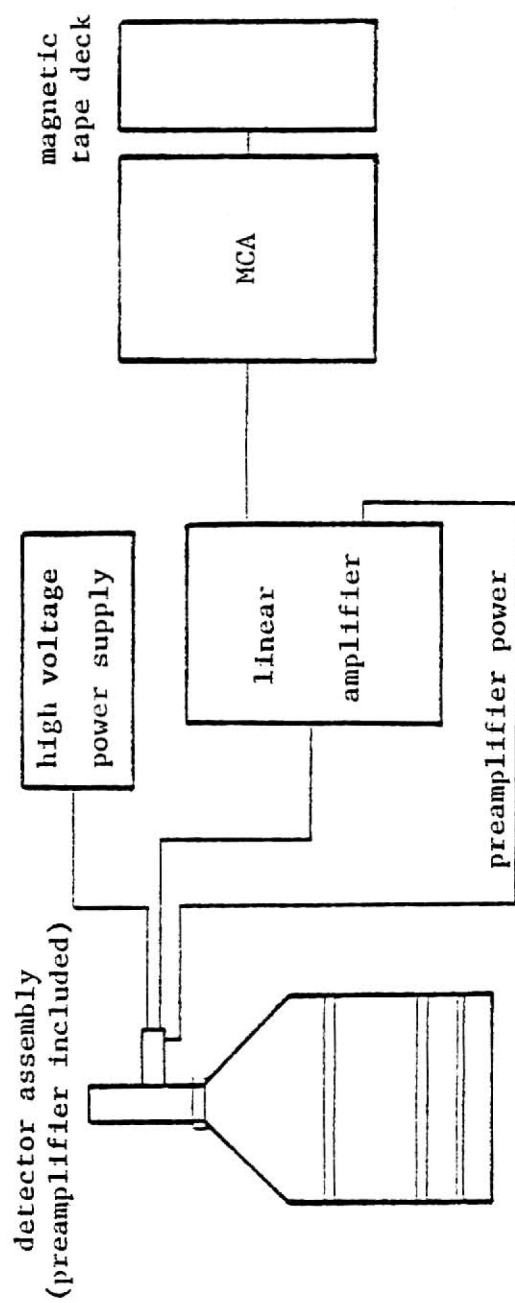


FIG. 3,9, HPGe detector based beta spectrometer system component diagram.

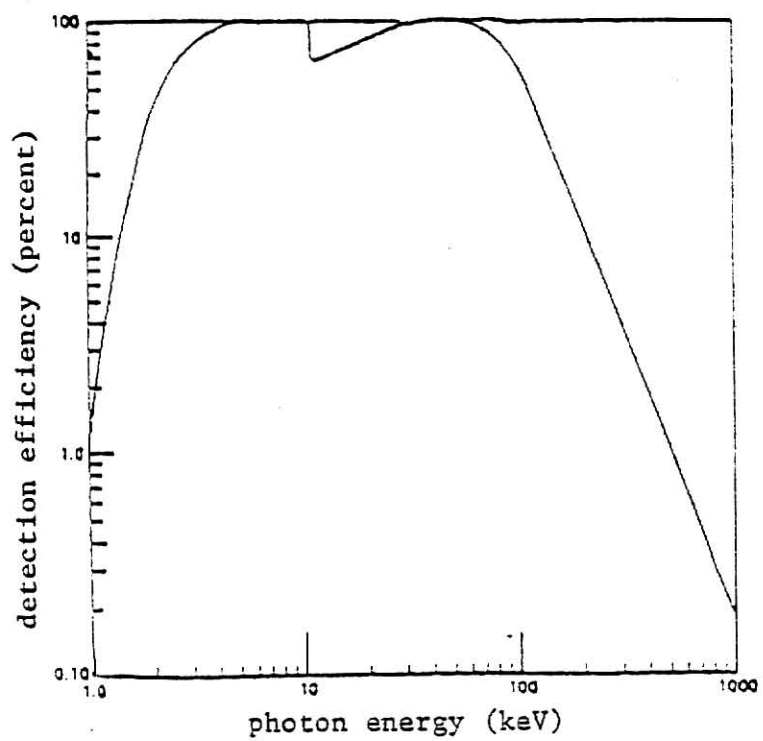


FIG. 3.10. Photon efficiency curve for a 5 mm thick planar intrinsic germanium detector with a 2 mil thick Be entrance window. (Princeton Gamma-Tech Corporation)

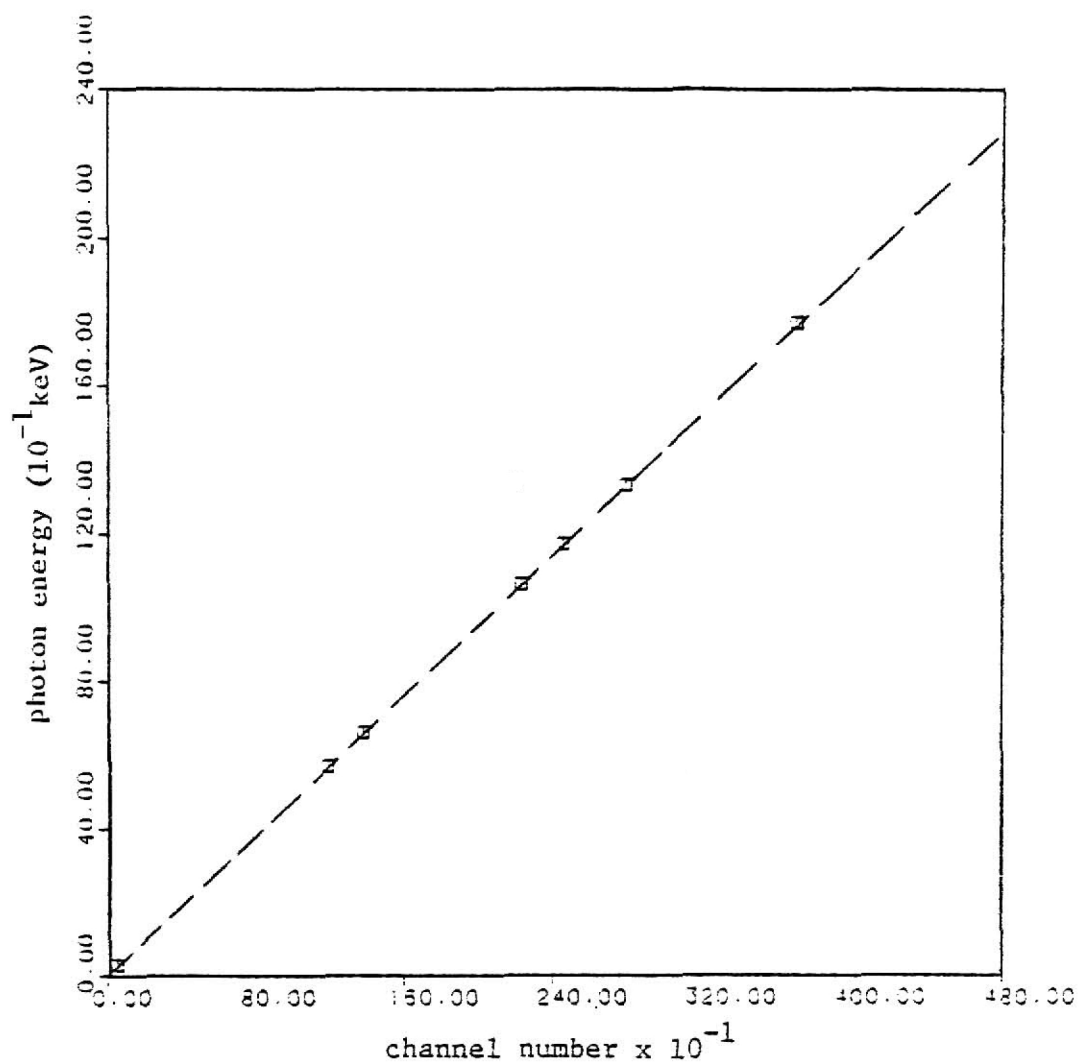


FIG. 3.11. Measured channel number of the photon full-energy peak versus photon peak energy for the HPGe detector spectrometer system.

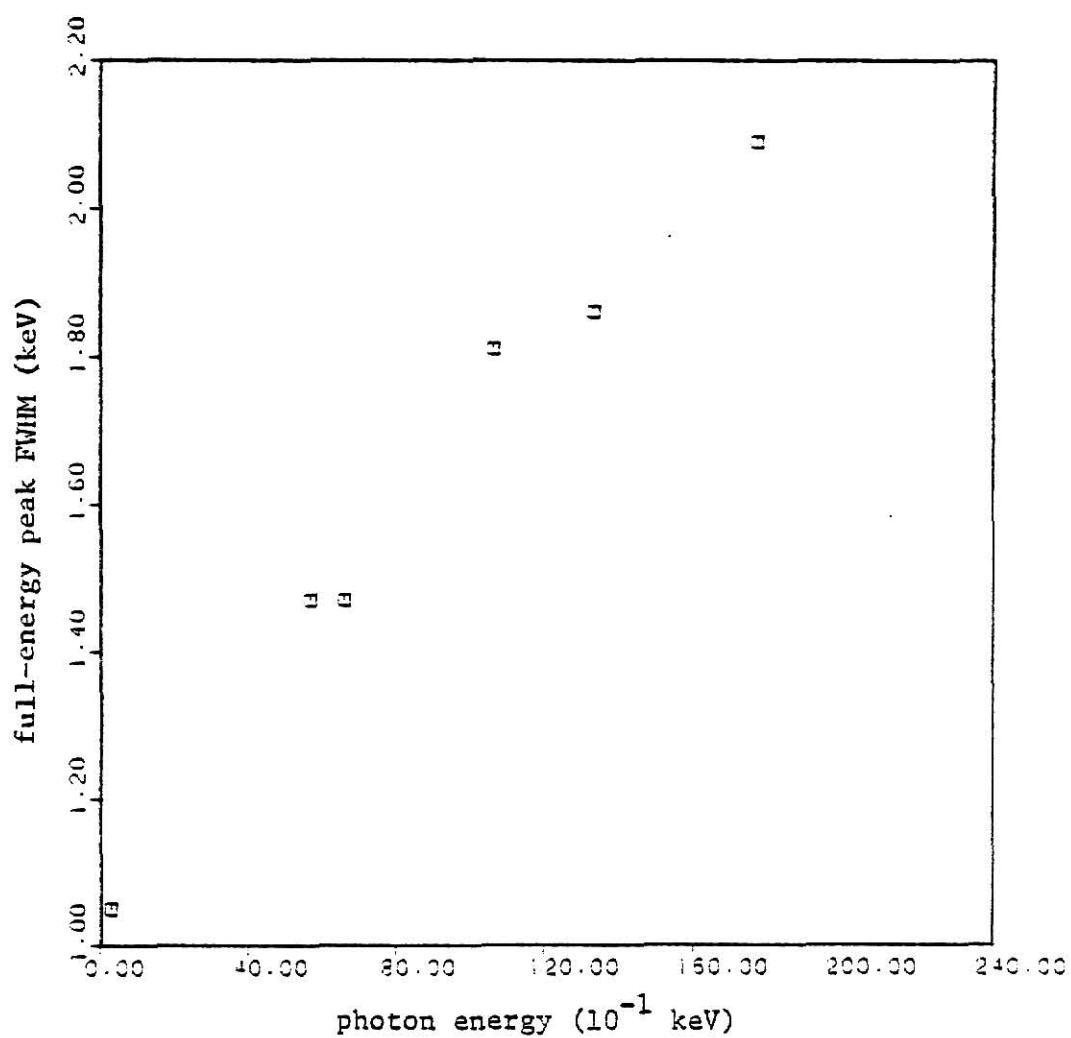


FIG. 3.12. Photon full-energy peak resolution as a function of photon energy for the HPGe spectrometer system.

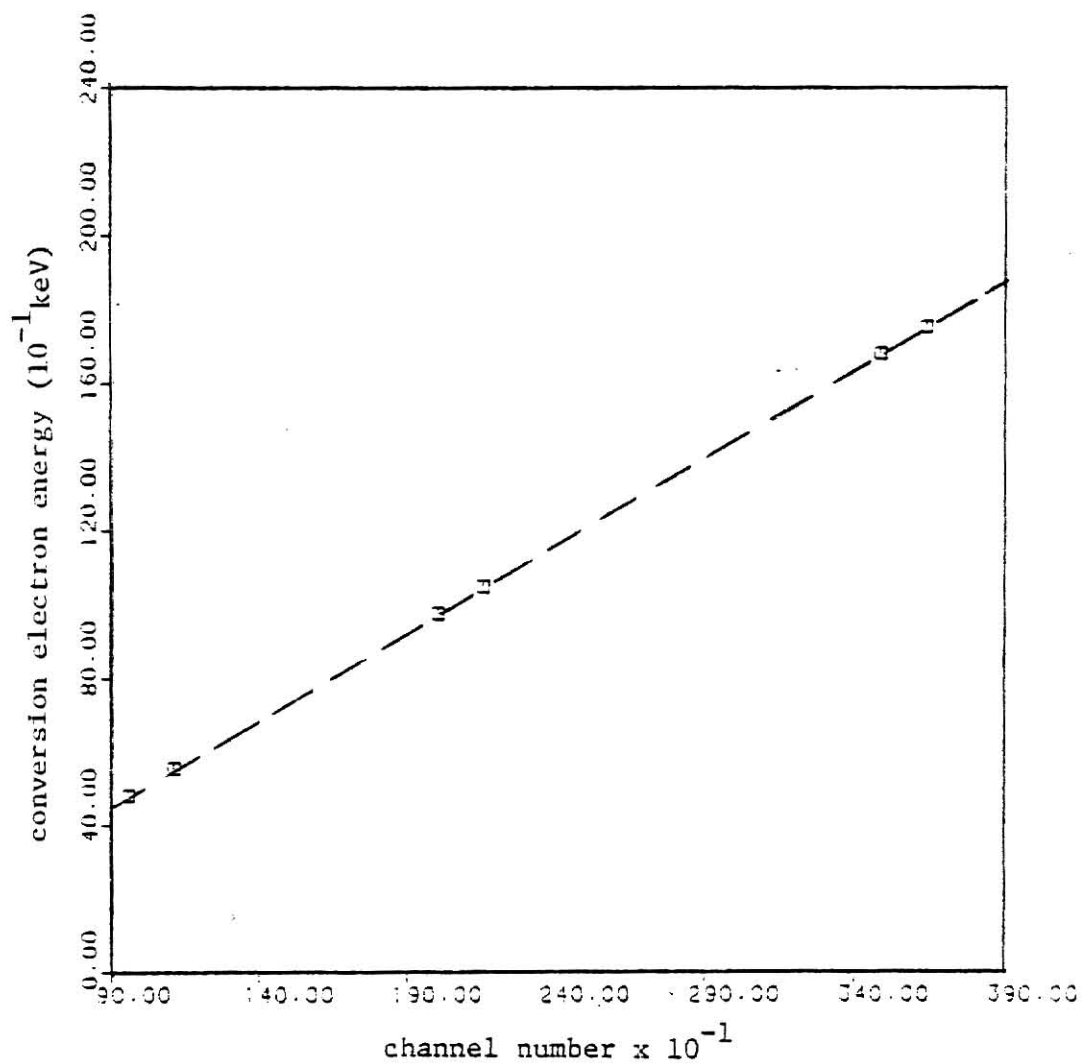


FIG. 3.13. Measured conversion electron peak channel number as a function of increasing conversion electron kinetic energy for the HPGe detector spectrometer system. Line drawn for aid in visualization.

reaching the detector. Over the energy range that was examined (480 - 1770 keV), conversion electrons exhibited average entrance window losses of approximately 18.5 keV. Typical resolutions were on the order of 10 keV.

4.0 Beta particle response

The response of the HPGe detector to beta particles can be expected to be identical to that for conversion electrons. However, the continuous beta particle distribution more clearly demonstrated the distortive effects of backscatter and bremsstrahlung production. As expected, and shown in Fig. 3.14, measured endpoint channels of known beta spectra exhibit a linear relationship to spectrum endpoint energy.

Radiation yields are relatively high for germanium, as compared to organic scintillators (see Table 3.8). However, the predominantly forward peaked nature of bremsstrahlung given by⁷

$$\langle \theta \rangle = \frac{mc^2}{T_0 + mc^2}, \quad (3.5)$$

in which

$\langle \theta \rangle$ = angle of emitted photon relative to the direction of the incident electron,

mc^2 = rest mass energy of an electron, and

T_0 = initial electron kinetic energy,

coupled with the high detector efficiency for low energy photons significantly reduces the probability of bremsstrahlung escape. From the work of Rehfield and Moore,⁷ it is possible to deduce that for 1 MeV electrons the ratio of bremsstrahlung escape, to the total of backscatter and radiation escape events, is approximately 0.04. Considering that backscatter events account for nearly 24% of all events

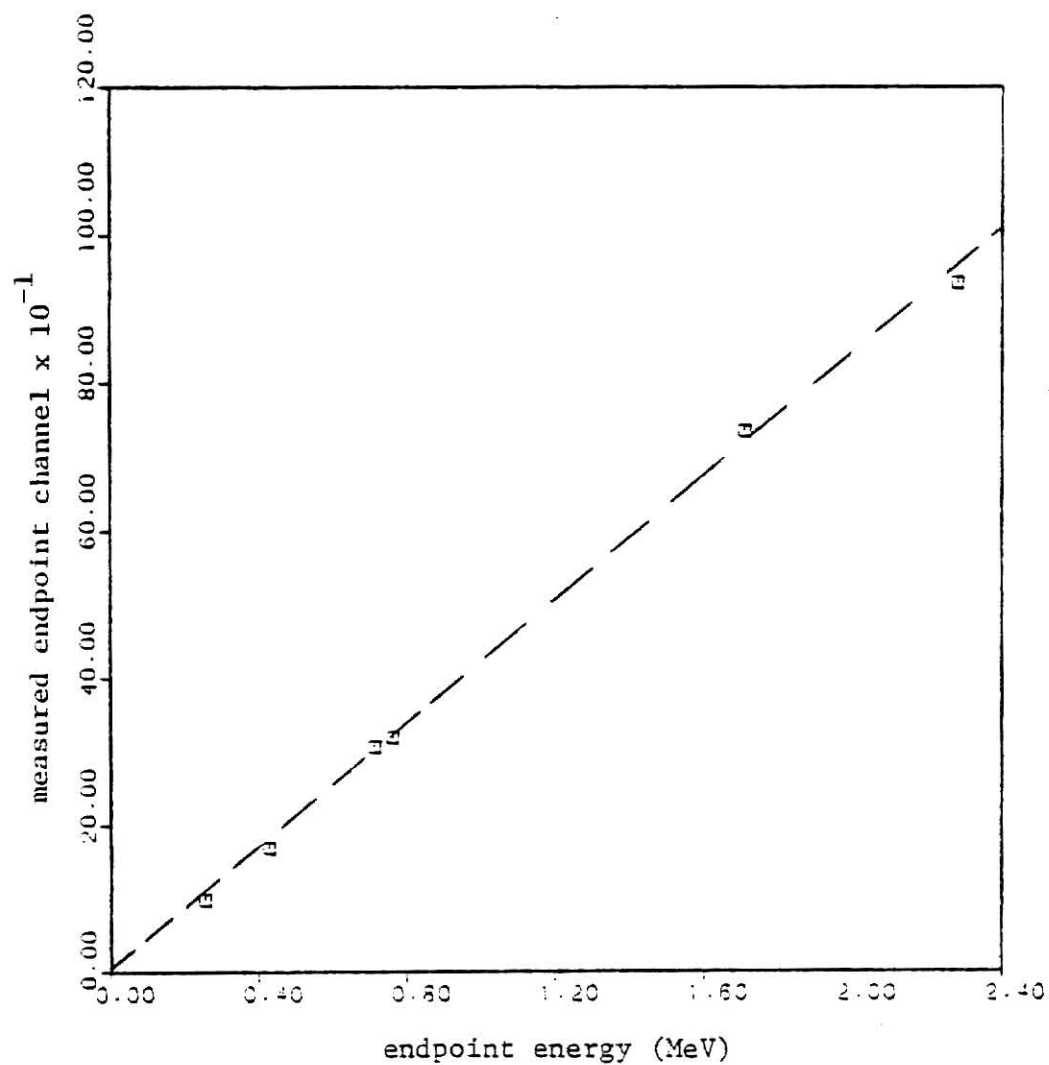


FIG. 3.14. Measured beta endpoint channel as a function of increasing spectrum endpoint energy for the HPGe detector based spectrometer system. Line is based on a least squares fit to the data.

TABLE 3.8: Radiation yields versus electron energy for plastic scintillator and germanium materials.[†]

Electron Energy (MeV)	Radiation Yield (MeV)	
	Plastic Scintillator	Germanium
0.01	1.04(-4)	9.04(-4)
0.10	5.73(-4)	4.96(-3)
0.50	2.18(-3)	1.75(-2)
1.00	4.05(-3)	2.96(-2)
2.00	7.55(-3)	4.93(-2)
5.00	1.79(-2)	9.917(-2)

[†] From M. J. Berger and S. M. Seltzer, Tables of Energy Losses and Ranges of Electrons and Electrons and Positrons, National Bureau of Standards, 1964

for incident 1 MeV electrons, as shown previously in Table 3.1, bremsstrahlung losses should result in much less distortion than backscatter events in the measured beta particle energy distribution.

5.0 Calibration procedures

Typically, calibration of the HPGe detector system was performed using gamma-ray sources. Some experiments were completed utilizing a conversion electron calibration. Lack of an accurate response function for low energy electrons, however, incorrectly skewed the energies derived from these calibrations toward higher than actual values in the lower channels of the analyzer.

6.0 Measurement of beta pulse height distributions

Using the HPGe detector system, measurements of beta pulse-height distributions were obtained. Nominal detector to source distance was approximately 4 mm.

IV. Data analysis

Analysis of recorded data was performed off-line using the KSU-developed BETAPLUS-II computer code. The code was designed to facilitate calculation of theoretical beta spectra and to provide a means for theoretical to experimental intercomparisons.

BETAPLUS-II performs a series of functions. Principally, for a measured pulse height distribution of known isotopic origin, BETAPLUS-II calculates the theoretical spectrum for the isotope, area normalizes it to the measured distribution, and then provides comparative output in both linear and Kurie plot formats. Output data are both plotted and tabularized. Theoretical calculations are based on the relativistic model, incorporating a self-screening correction and the Bethe-Bacher complex gamma function approximation (see Section II).

BETAPLUS-II is designed to provide analysis and output for any negatron emitter with an allowed, first forbidden or second forbidden transition. A special routine has been incorporated to allow analysis of the combined $^{90}\text{Sr}/^{90}\text{Y}$ beta spectra. The code is designed to accept either linear or bilinear energy calibration format.

In addition to theoretical to experimental comparisons, BETAPLUS-II provides unity normalized relativistic and non-relativistic model based beta spectrum calculations for the isotope of interest. Relativistic model calculations include a self-screening correction and the Bethe-Bacher complex gamma function approximation. Non-relativistic calculations are based on the simple non-relativistic model with no self-screening correction. Both plotted and tabular output are used.

Tabular data also includes kinetic energy, total relativistic energy, Fermi function values and calculated Kurie plot data points. From this output, theoretical and experimental Kurie plots are both easily obtained. A listing of the BETAPLUS computer code is given in Appendix B.

V. Results of the beta spectrometer studies

Results presented are divided into three sections. The first section involves general beta spectra measurements, conversion electron responses and a gamma-ray discrimination technique for the 10.2 and 5.08 mm thick BC-400 detector systems. The second section is concerned with beta spectra measurements using the HPGe detector system. Additionally, during the course of the spectrometer evaluations, selected application type experiments pertinent to the general fields of beta spectroscopy and beta dosimetry were performed. Results of these studies are presented in the third section.

A. Organic scintillator based spectrometer

1.0 Measured beta particle distributions

As stated previously, measured beta particle energy distributions were obtained using the 10.2 mm thick BC-400 detector for ^{35}S , ^{32}P , ^{36}Cl , $^{90}\text{Sr}/^{90}\text{Y}$, ^{185}W and ^{204}Tl . A comparison of theoretical and measured distributions for the ^{36}Cl and $^{90}\text{Sr}/^{90}\text{Y}$ sources are presented in Figs. 5.1 and 5.2. Selected Kurie plots are available in Appendix C. A tabulation of the theoretical to experimental comparisons of average and endpoint energies for the isotopes measured is given in Table 5.1. Standard deviation estimates for the measured endpoint energies ranged from 2-10 keV, or, in percent, ranged from 0.2 - 3.0% of endpoint energy values.

In all measured distributions, the higher energy portion closely matched the theoretical spectra. A small degree of distortion was present, however, in the lower one-half to one-third of the measured distributions. This was principally due to the combination of detector

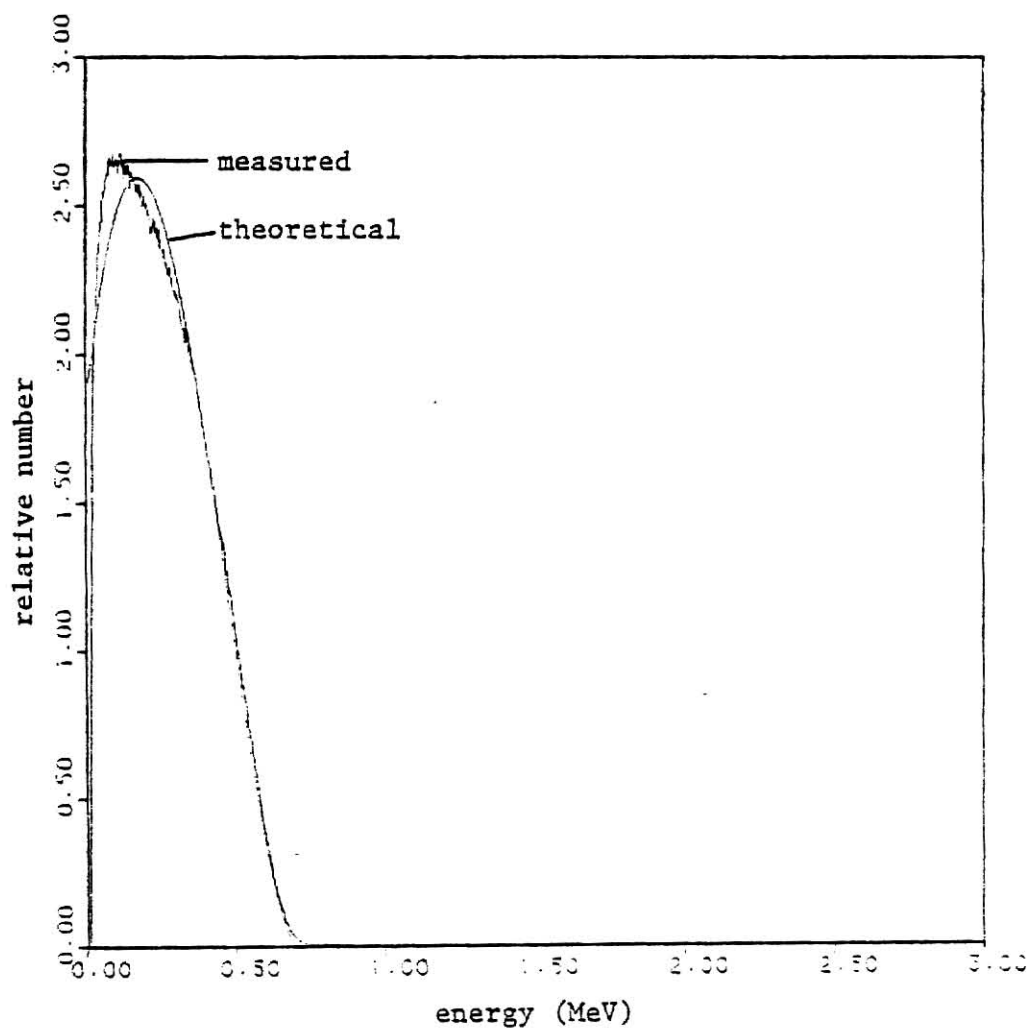


FIG. 5.1. Theoretical and measured beta particle energy distributions for Cl-36, as measured with a 10.2 mm-thick BC-400 based spectrometer system.

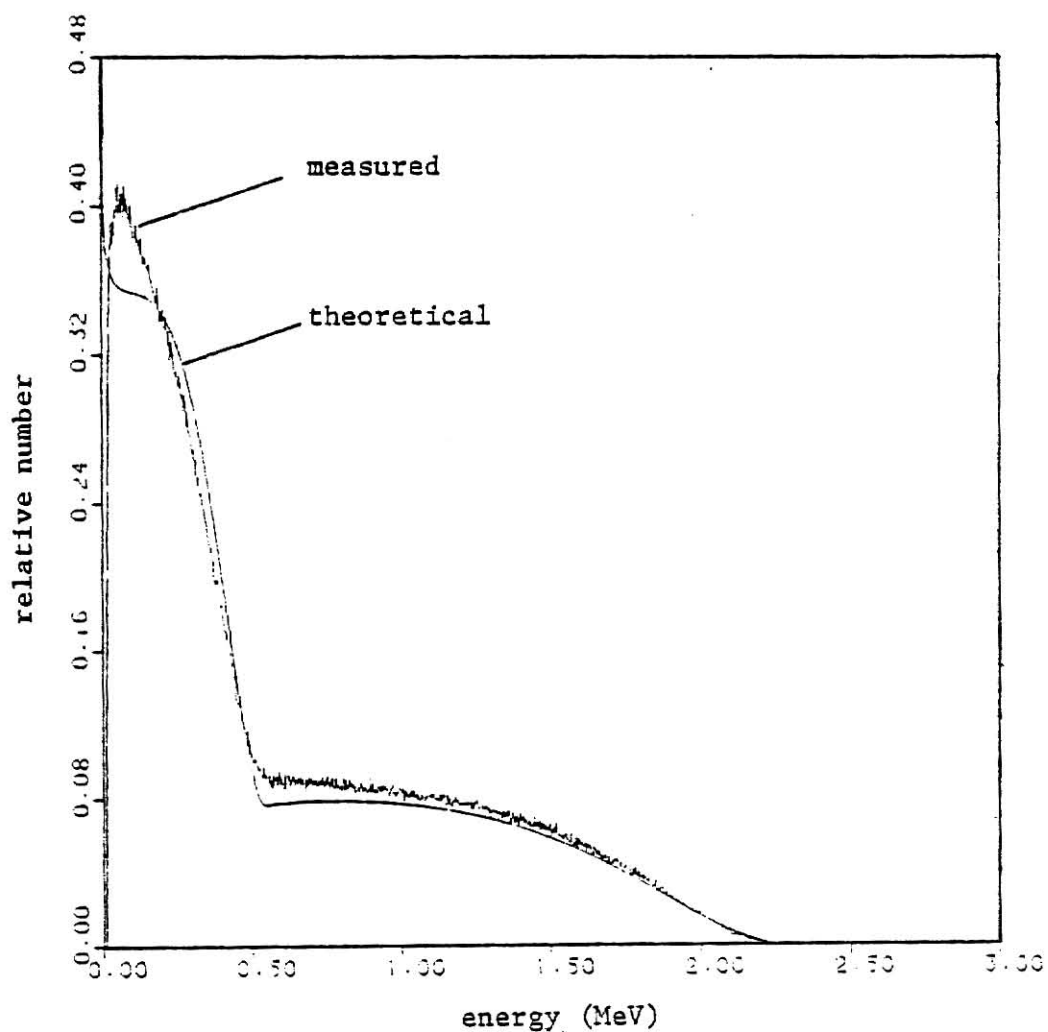


FIG. 5.2. Theoretical and measured beta particle energy distributions for the combined Sr-90 and Y-90 spectra. Measurements were made with a 10.2 mm-thick BC-400 based spectrometer system.

Table 5.1. Comparison of theoretical and measured average and endpoint energies for the 10.2 mm thick BC-400 based spectrometer

Isotope	Theoretical endpoint energy (MeV)	Measured endpoint energy (MeV)	Theoretical average energy (MeV)	Measured average energy (MeV)	Δ (keV)	Δ (%)
^{35}S	0.1674	0.1869	0.0488	0.0656	+16.8	+34.8
^{185}W	0.4326	0.4018	0.1440	0.1039	-40.1	-27.8
^{36}Cl	0.7087	0.7215	0.2525	0.2373	-15.2	-6.02
^{204}Tl	0.7635	0.7756	0.2433	0.2411	- 2.20	- 0.90
^{32}P	1.711	1.7504	0.6950	NA	NA	NA
$^{90}\text{Sr}/^{90}\text{Y}$	2.288	2.249	0.5665	0.5635	- 3.00	- 0.13

NA - not available

resolution and electron backscattering. Extremely low energy distributions, such as ^{35}S , were somewhat unduly influenced by the presence of low level discriminators in the associated electronics. As a consequence, the ^{35}S measured average energy is uncharacteristically higher than the expected value. The ^{185}W source also presented a somewhat unique situation. Although of low frequency, the 125 keV gamma ray was of significant influence in the measured distribution due to the detector's increased efficiency for the lower energy photons. The net effect of the gamma ray was a reduction in the value of the measured average energy.

In addition to data accumulated with the 10.2 mm thick detector, similar results are presented for the 5.08 mm thick BC-400 detector. Figures 5.3-5.5 demonstrate the theoretical and measured distributions for ^{36}Cl , ^{32}P and $^{90}\text{Sr}/^{90}\text{Y}$. Selected Kurie plots are presented in Appendix C. Theoretical and measured average and endpoint energies for several isotopes are given in Table 5.2. Standard deviation estimates for the endpoint energy measurements valued from 2.5-11.0 keV, or 0.2-2% of endpoint energy.

Since the 5.08 mm thickness of the detector is sufficient to stop totally only those electrons below approximately 1.2 MeV, its response is not linear for those particles with kinetic energies above this threshold. Consequently, beta spectra with higher endpoint energies are distorted, as is illustrated by Figs. 5.4 and 5.5. An additional representation is provided in Figure 5.6 through a plot of theoretical versus measured endpoint energy over the energy range 0.1674-2.288 MeV. This phenomena illustrates the importance of utilizing a detector

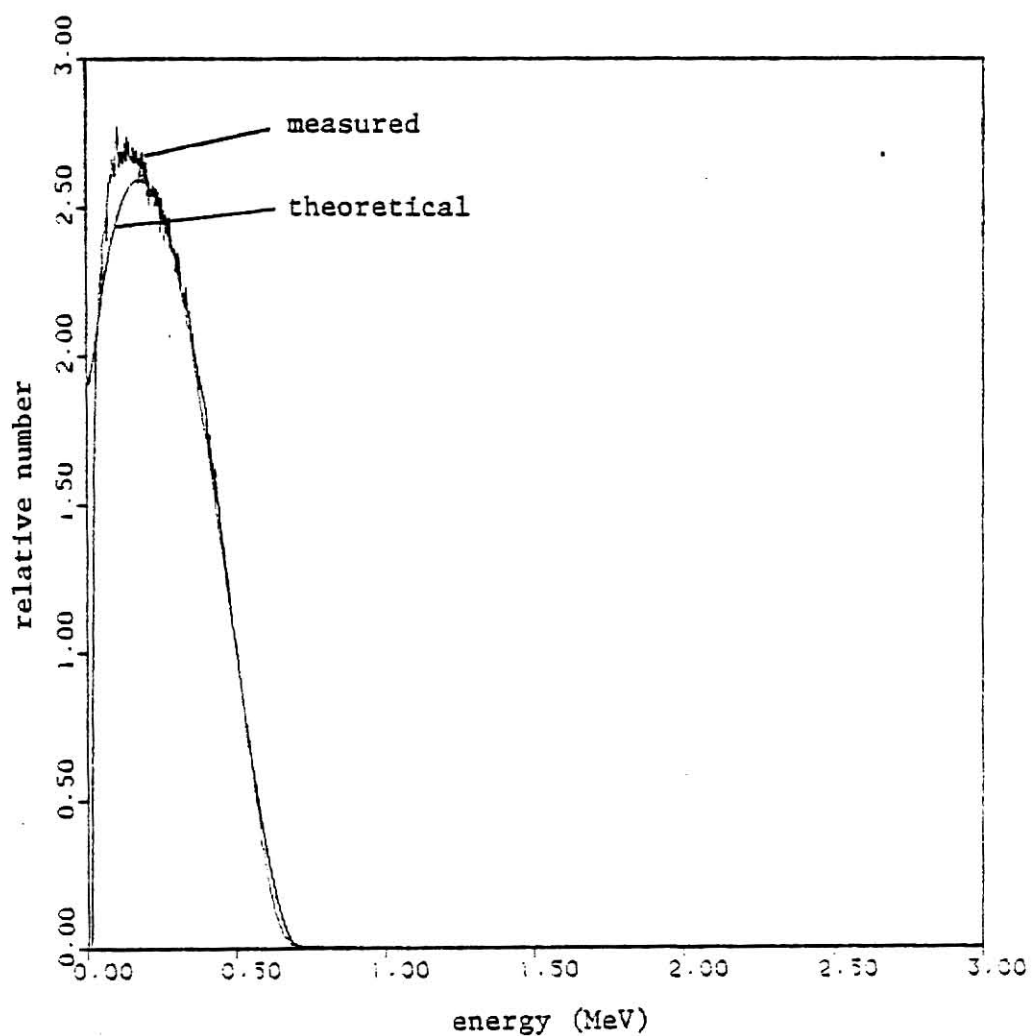


FIG. 5.3. Theoretical and measured beta particle energy distributions for Cl-36, as measured with a 5.08 mm-thick BC-400 based spectrometer system.

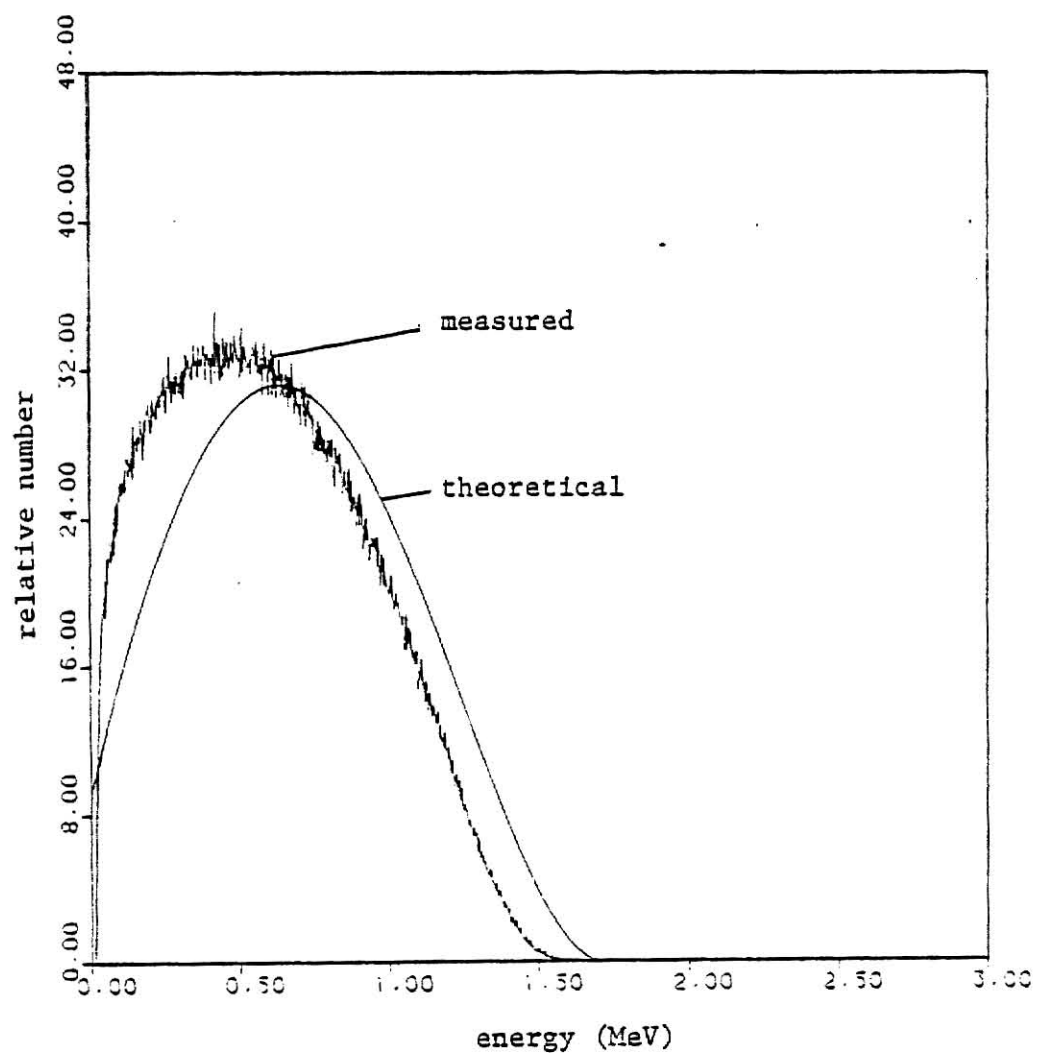


FIG. 5.4. Theoretical and measured beta particle energy distributions for P-32, as measured with a 5.08 mm-thick BC-400 based spectrometer system.

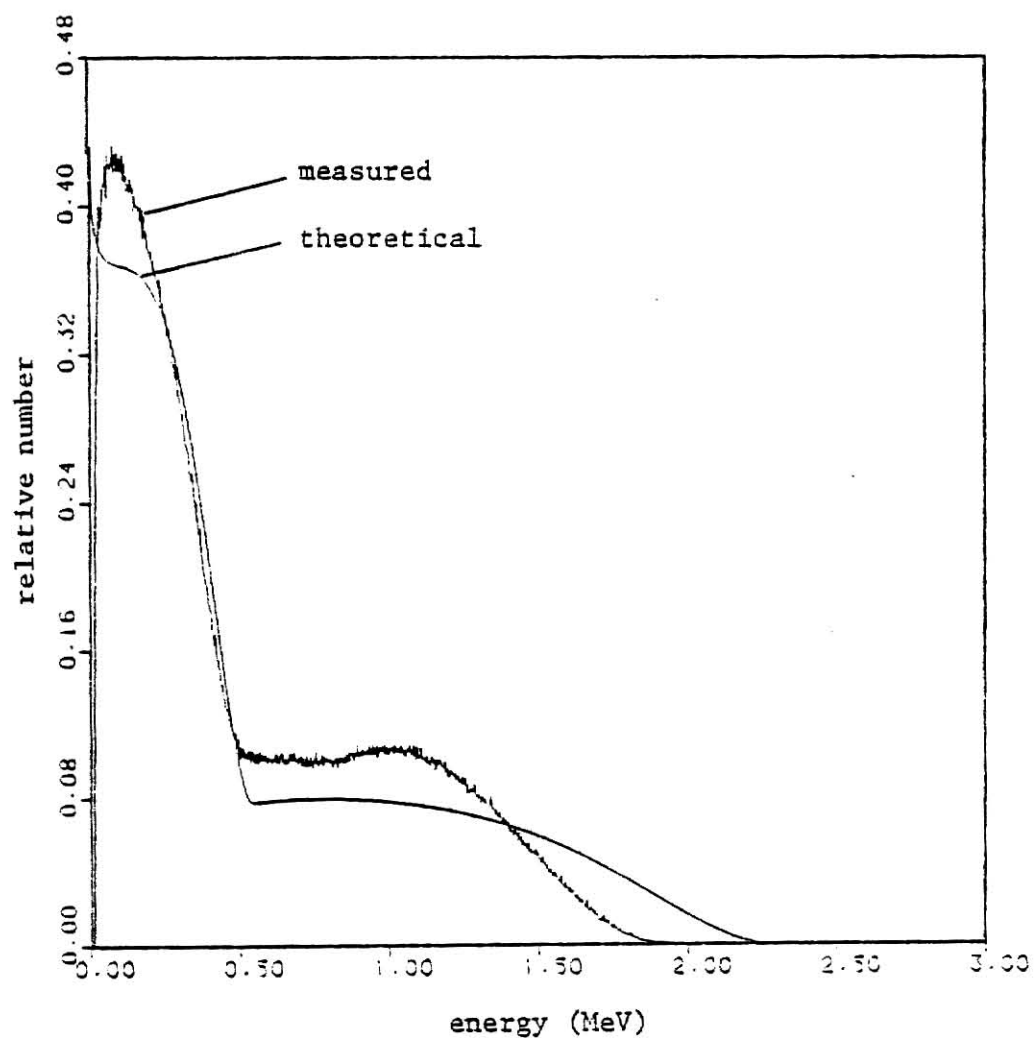


FIG. 5.5. Theoretical and measured beta particle energy distributions for the combined Sr-90 and Y-90 spectra. Measurements were made with a 5.08 mm-thick BC-400 based spectrometer system.

Table 5.2. Comparison of theoretical and measured average and endpoint energies for the 5.08 mm thick BC-400 based spectrometer

Isotope	Theoretical endpoint energy (MeV)	Measured		Theoretical		Measured	
		endpoint energy (MeV)	Δ (%)	average energy (MeV)	Δ (keV)	average energy (MeV)	Δ (%)
^{35}S	0.1674	0.1934	+15.5	0.0488	+26.0	0.0696	+42.7
^{185}W	0.4326	0.4036	- 6.70	0.1440	-29.0	0.1096	-23.9
^{36}Cl	0.7087	0.6921	+ 2.34	0.2525	-16.6	0.2414	- 4.40
^{204}Tl	0.7635	0.8020	+ 5.04	0.2433	+38.5	0.2587	+ 6.33
^{32}P	1.711	1.5870	- 7.25	0.6950	-124.0	NA	NA
$^{90}\text{Sr}/^{90}\text{Y}$	2.288	1.9258	-15.8	0.5665	-162.2	0.5108	- 2.43

NA - not available

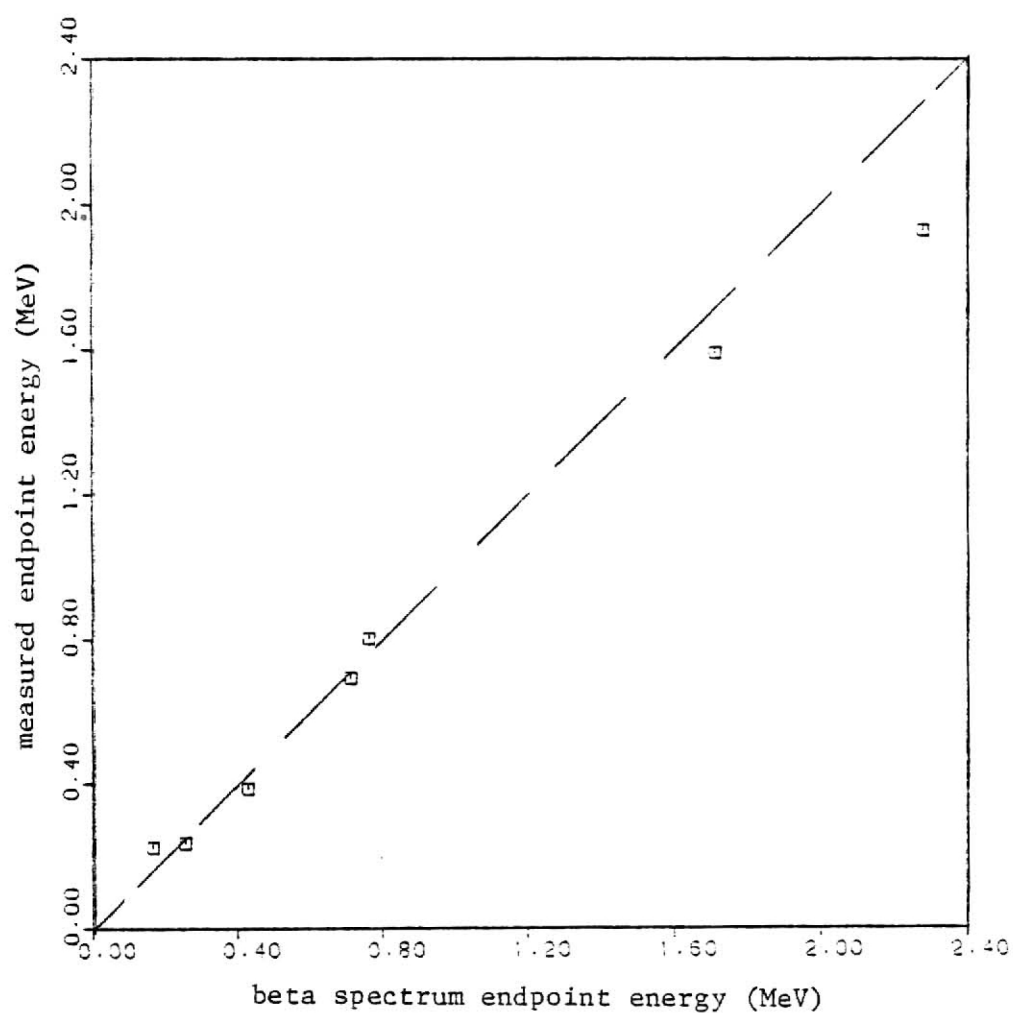


FIG. 5.6. Measured beta spectrum endpoint energy as a function of beta spectrum endpoint energy for the 5.08 mm thick BC-400 organic scintillator. The dashed line represents an ideal one-to-one correspondence.

sufficiently thick to stop the highest energy beta particle in the field of interest. Failure in this could obviously lead to erroneous results.

2.0 Conversion electron response

Figures 5.7 and 5.8 show the measured ^{207}Bi emission distribution for the 10.2 mm and 5.08 mm thick BC-400 detectors, respectively. Ideally conversion electron peaks should be visible at 0.4817, 0.5549, 0.9756 and 1.0481 MeV. These peaks should appear on top of the Compton distributions of the 0.5697 and 1.063 MeV gamma-rays, having Compton edge energies of 0.3933 and 0.8576 MeV, respectively. In neither measured distribution, however, can the 0.9756 and 1.0481 MeV conversion electrons be resolved separately, and only hints of the 0.4459 MeV peaks are visible in the two measured distributions. The figures do demonstrate the higher β/γ detection efficiency of the 5.08 mm detector over the 10.2 mm detector through intercomparison of the conversion electron peak areas to the total area of the measured distributions.

3.0 Gamma-ray discrimination

Figures 5.9 - 5.11 demonstrate the effectiveness of the gamma-ray discrimination technique described in Section III.7. Figure 5.9(a) is the ^{36}Cl beta particle energy distribution measured in the presence of a ^{60}Co , 1.17 and 1.33 MeV gamma-ray field for the 10.2 mm thick BC-400 detector. Figure 5.9(b) is the same distribution after a 427 mg/cm^2 high density polyethylene filter was placed over the detector face, and the system operated in subtract mode to remove the penetrating gamma-rays from the combined beta-gamma distribution. Figure 5.9(c) is a ^{36}Cl beta particle energy distribution measured without interfering

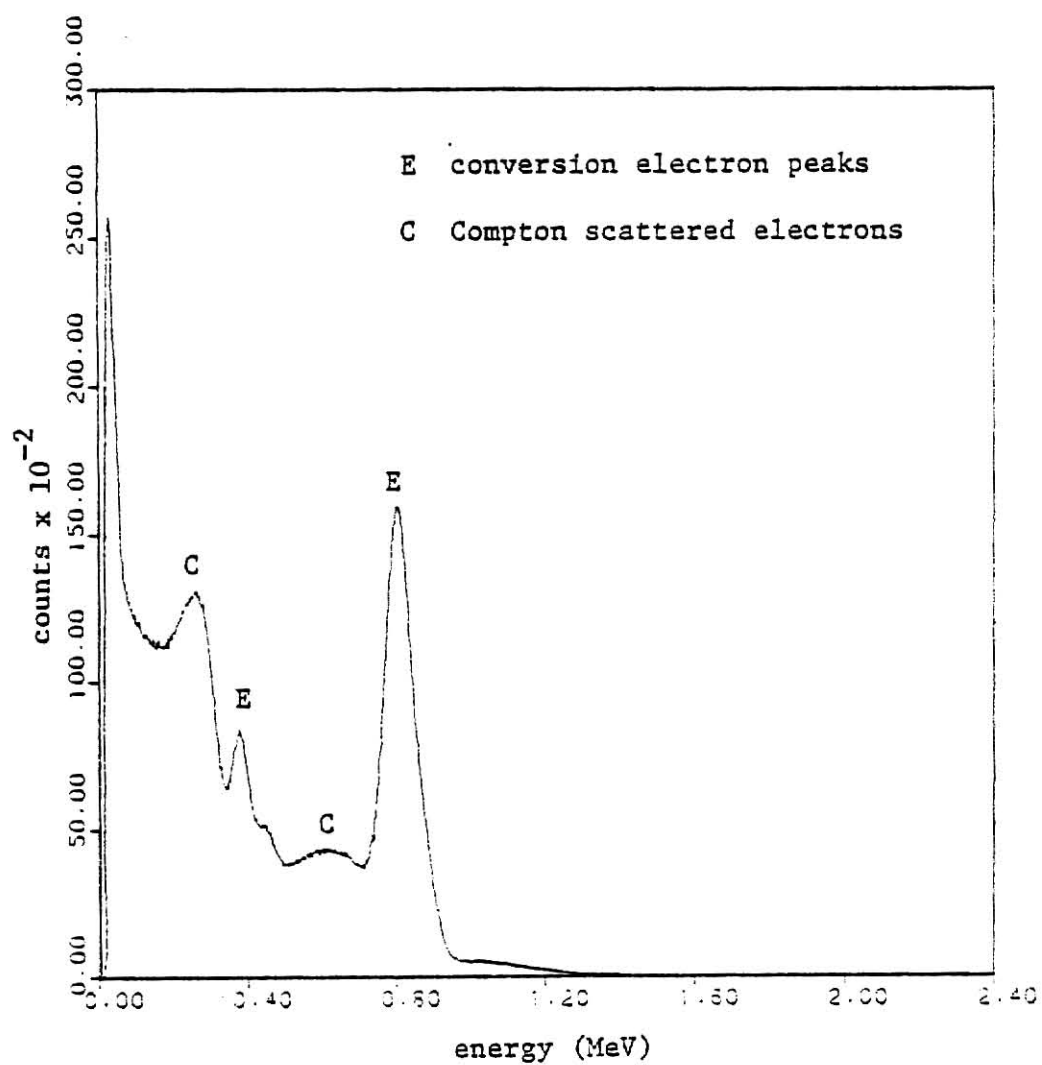


FIG. 5. 7. Measured ^{207}Bi emission distribution for the 10.2 mm-thick BC-400 detector.

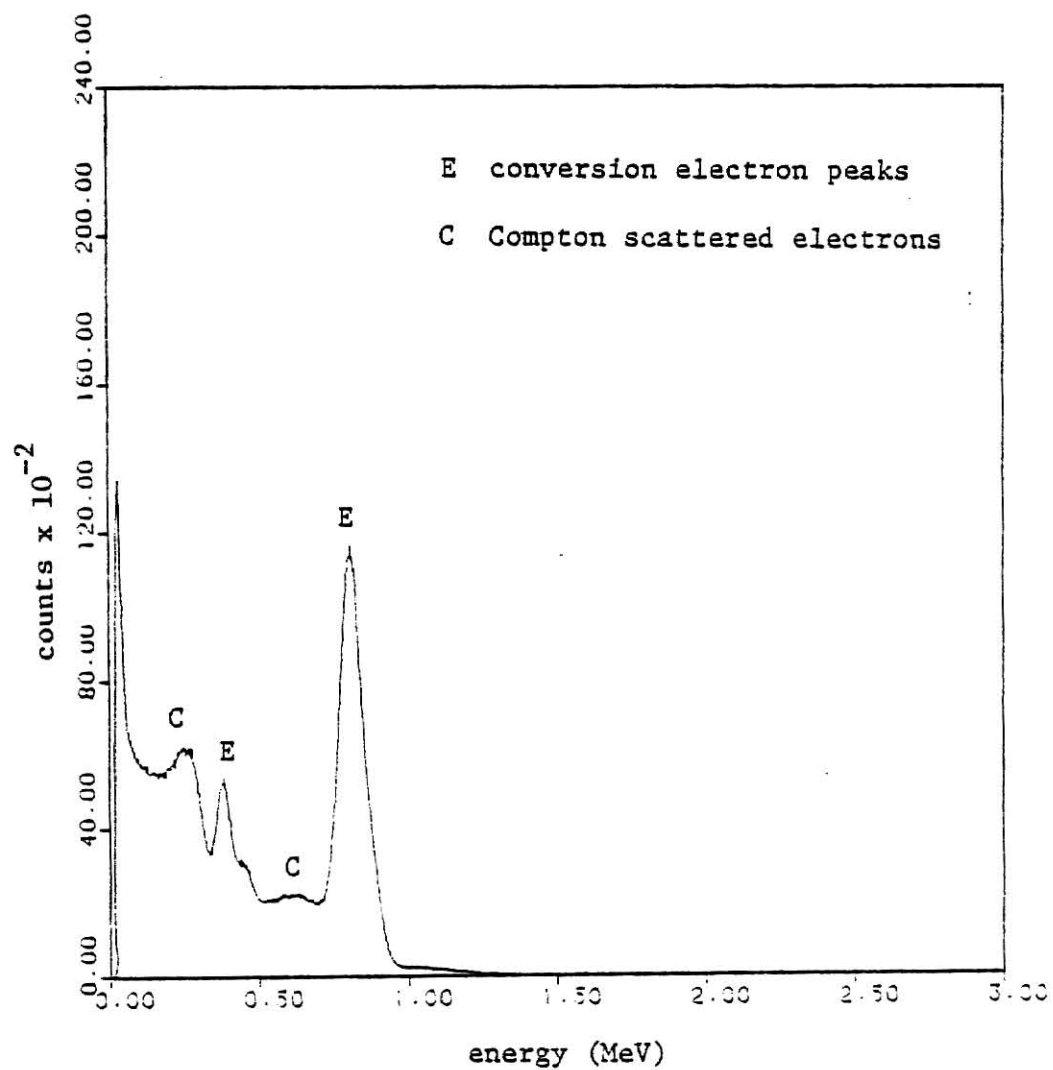


FIG. 5.8. Measured ^{207}Bi emission distribution for the 5.08 mm-thick BC-400 detector.

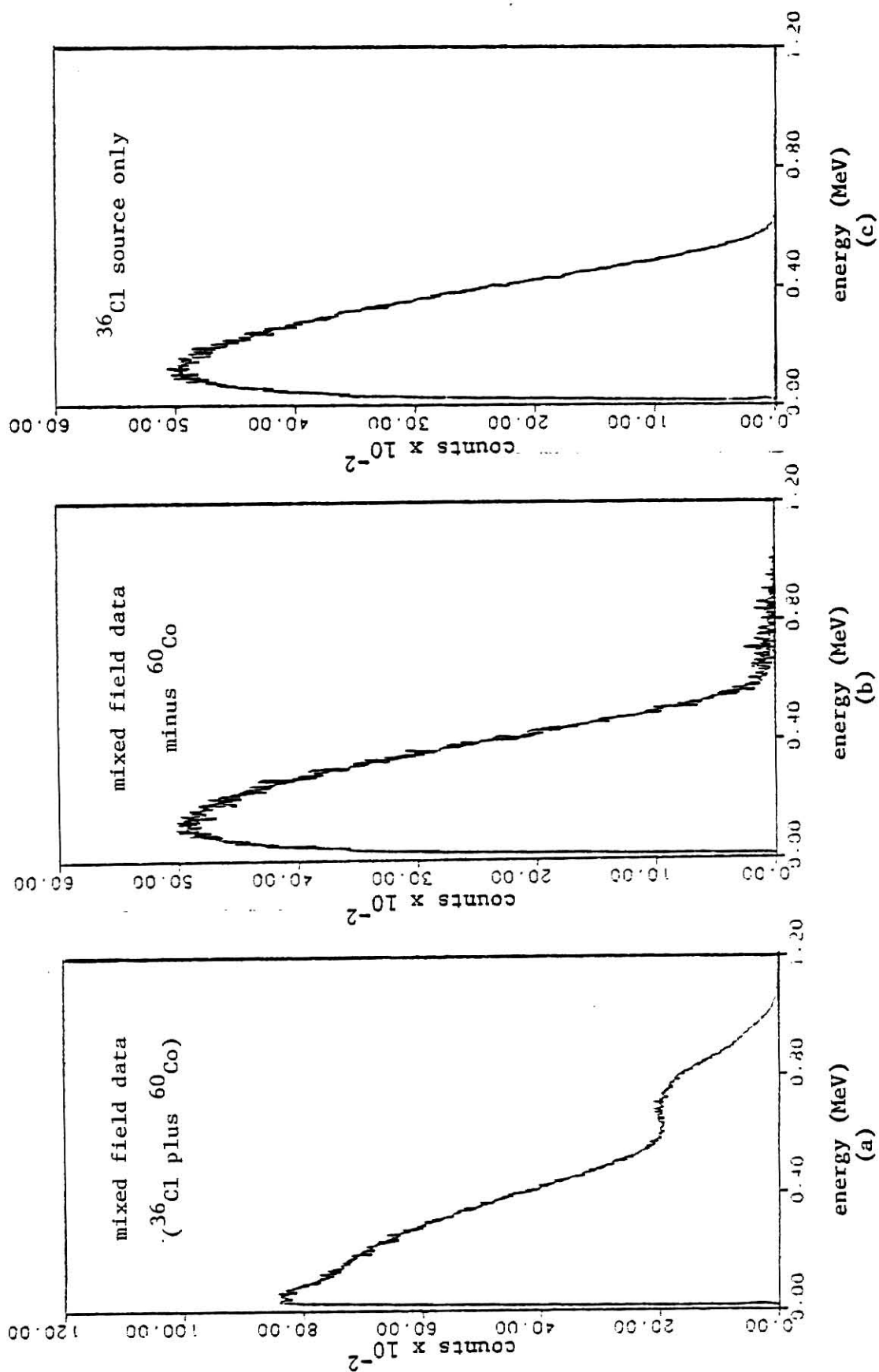


FIG. 5.9. Measured distributions for (a) combined ^{36}Cl beta particles and ^{60}Co 1.17 and 1.33 MeV gamma rays, (b) beta particles measured in the beta plus gamma mixed field after gamma-ray subtraction and (c) ^{36}Cl measured without interfering gamma rays. Measurements were performed with a 10.2 mm-thick BC-400 detector.

gamma-rays. Figures 5.9(b) and (c) have measured average energies of 0.2508 and 0.2397 MeV, respectively, showing a difference of only 11.1 keV. The endpoint energies were not significantly different.

The identical technique, applied to a small sample of ^{198}Au , yielded the results shown in Figs. 5.10(a) - (c). ^{198}Au decays through emission of two beta particles with endpoints of 0.286 and 0.962 MeV and frequencies of 0.011 and 0.99, respectively. A small percentage of internal conversion electrons are also emitted. The principal gamma ray has an energy of 0.412 MeV and a corresponding Compton edge energy of 0.254 MeV. Figure 5.10(a) shows the measured distribution of the combined beta, electron and gamma fields. Figure 5.10(b) is the measured distribution after the gamma rays were subtracted. The measured average energy of Fig. 5.10(b) was 0.2873 MeV as compared to the published average beta energy of 0.3143 MeV, showing a difference of 26.9 keV. In this case, a slightly lower than theoretical average would be predicted due to the backscatter caused by the finite thickness of the gold foil. Moreover, the sharp peak in Fig. 5.10(b) was caused by the presence of the internal conversion electrons.

Figure 5.10(c) shows the measured Compton distribution of the 0.4118 MeV gamma-ray measured when the 427 mg/cm^2 filter was in place. The large number of low energy counts in the distribution is an indication of the occurrence of forward scattering Compton events within the polyethylene filter.

The $^{36}\text{Cl} - ^{60}\text{Co}$ mixed field experiment was repeated for the 5.08 mm thick BC-400 detector assembly [see Fig. 5.11(a) - (c)]. The average energy of the beta particle energy distribution measured from the mixed field was 0.2417 MeV as compared to the value of 0.2422 MeV measured

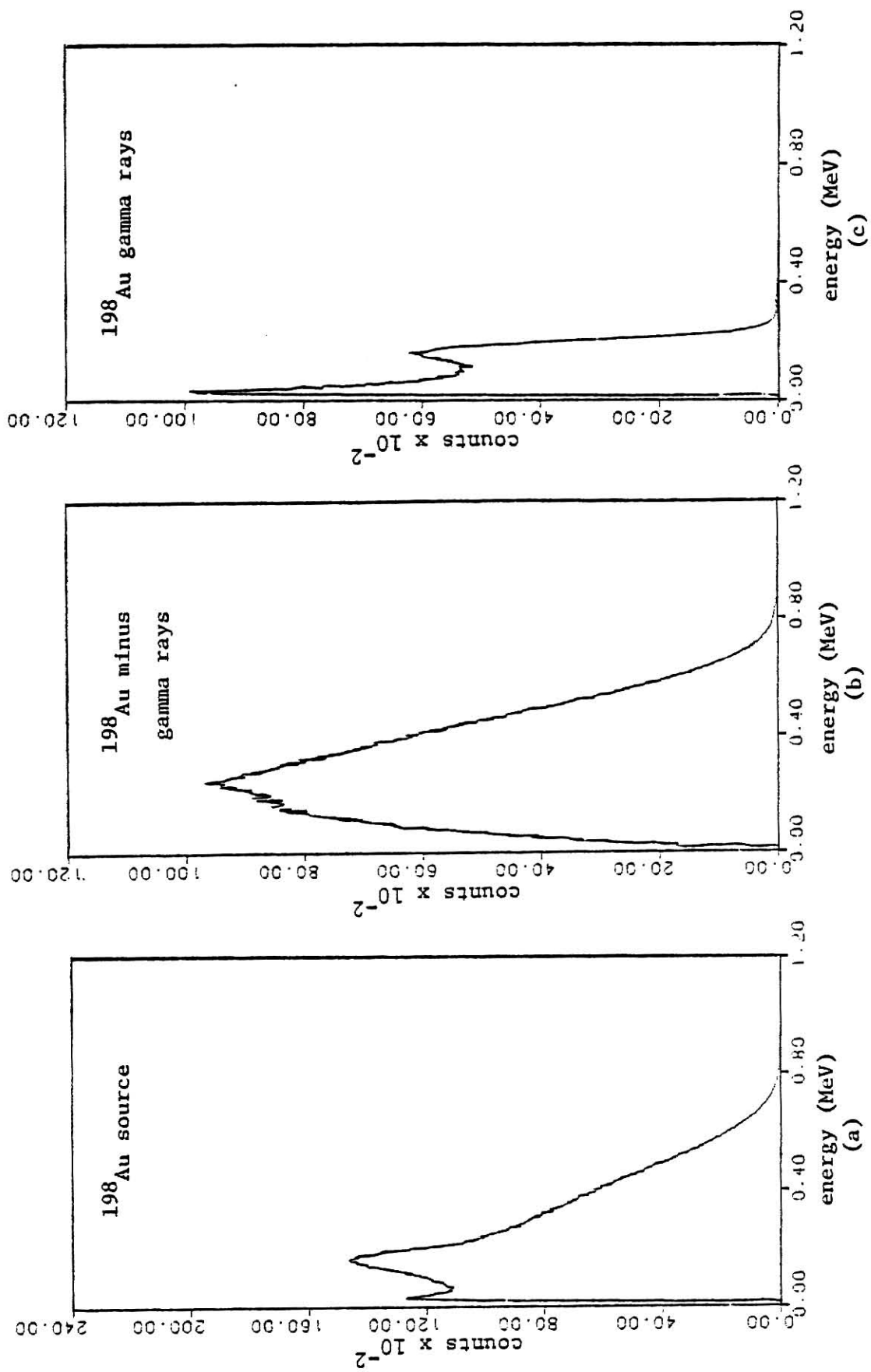


FIG. 5.10. Measured distributions for ^{198}Au (a) mixed beta and gamma-ray field, (b) beta particle and electron distribution following gamma ray subtraction, and (c) 0.412 MeV gamma-ray Compton distribution. Measurements were performed with a 10.2 mm-thick BC-400 detector.

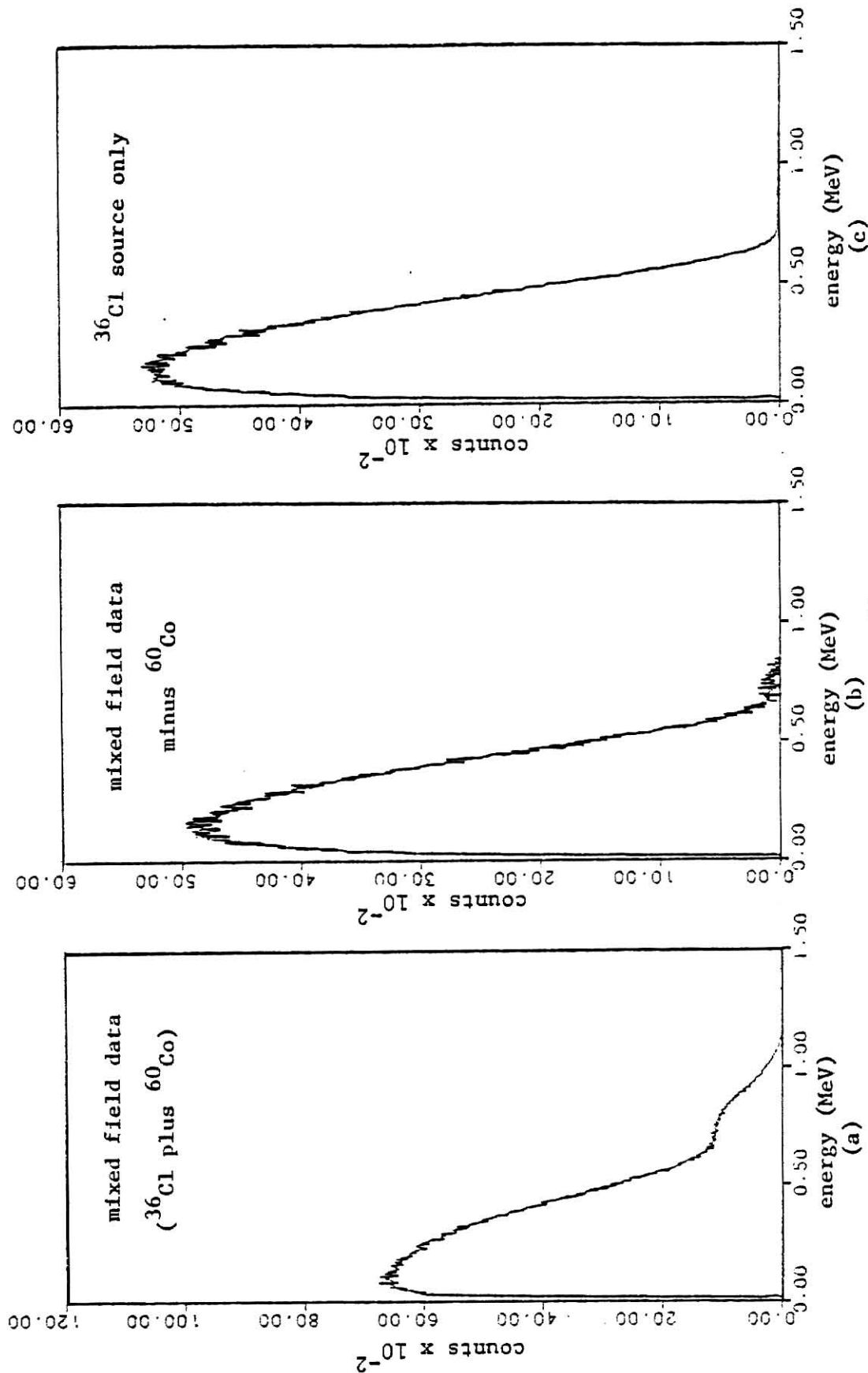


FIG. 5.11. Measured distributions for (a) combined ^{36}Cl beta particles and ^{60}Co 1.17 and 1.33 MeV gamma rays, (b) beta particles measured in the beta plus gamma mixed field after gamma-ray subtraction and (c) ^{36}Cl measured without interfering gamma rays. Measurements were performed with a 5.08 mm-thick BC-400 detector.

from a "beta-only" ^{36}Cl field. A comparison of the two is shown in Figs. 5.11(b) and 5.11(c). The difference of 5 keV was slightly lower than that obtained using the 10.2 mm BC-400 detector. This result is consistent with that expected due to the thinner BC-400's lower gamma-ray efficiency.

B. Beta particle energy distributions measured with the HPGe detector based spectrometer

Beta particle energy distributions were obtained for ^{45}Ca , ^{36}Cl , ^{185}W , ^{204}Tl and $^{90}\text{Sr}/^{90}\text{Y}$ using the HPGe detector based spectrometer. Measured and theoretical distributions for ^{36}Cl and $^{90}\text{Sr}/^{90}\text{Y}$ are shown in Figs. 5.12 and 5.13. Selected Kurie plots are available in Appendix C. A tabulation of theoretical and experimental average and endpoint energies is provided in Table 5.3.

An examination of the deviation between measured and theoretical endpoint values as a function of energy reveals a trend somewhat contrary to that which would be predicted. Since backscatter and window losses decrease as electron energy increases, the deviations in endpoint energies would be expected to decrease accordingly. This, however, is not the effect experienced. Additionally, it was noted that the increase in deviation was paralleled by an increase in the estimated standard deviation of the measured endpoint energies. Normally, values for the standard deviation estimates ranged from 2-64 keV or, in percent terms, from 0.5-6.0% of endpoint energy. Results seem to indicate that the induced distortion in the measured distributions is not linear in nature, thus decreasing the accuracy of the endpoint measurements.

In addition to the decrease experienced in the measured endpoint energies, the measured average energies were significantly lower than

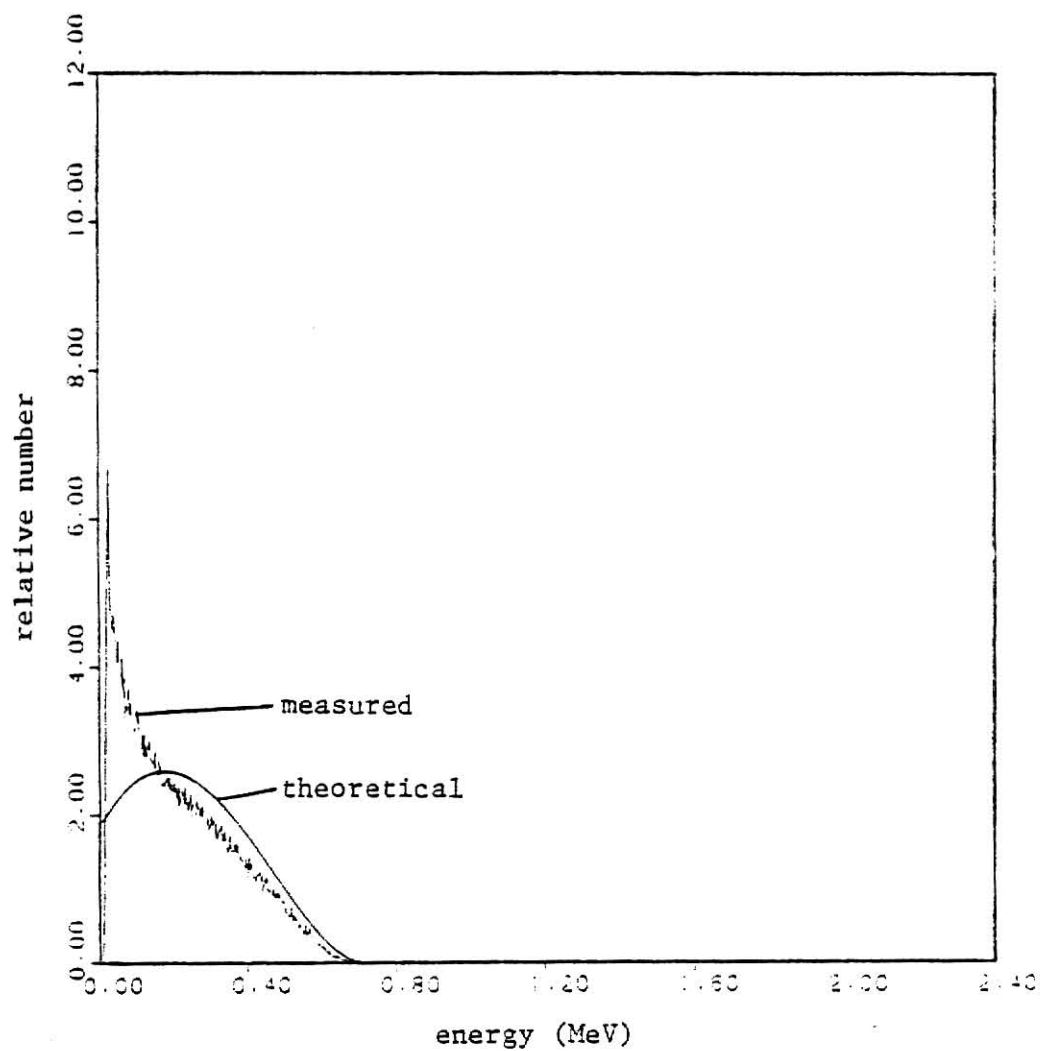


FIG. 5.12 Theoretical and measured beta particle energy distributions for Cl-36, as measured with a 5 mm-thick planar HPGe detector.

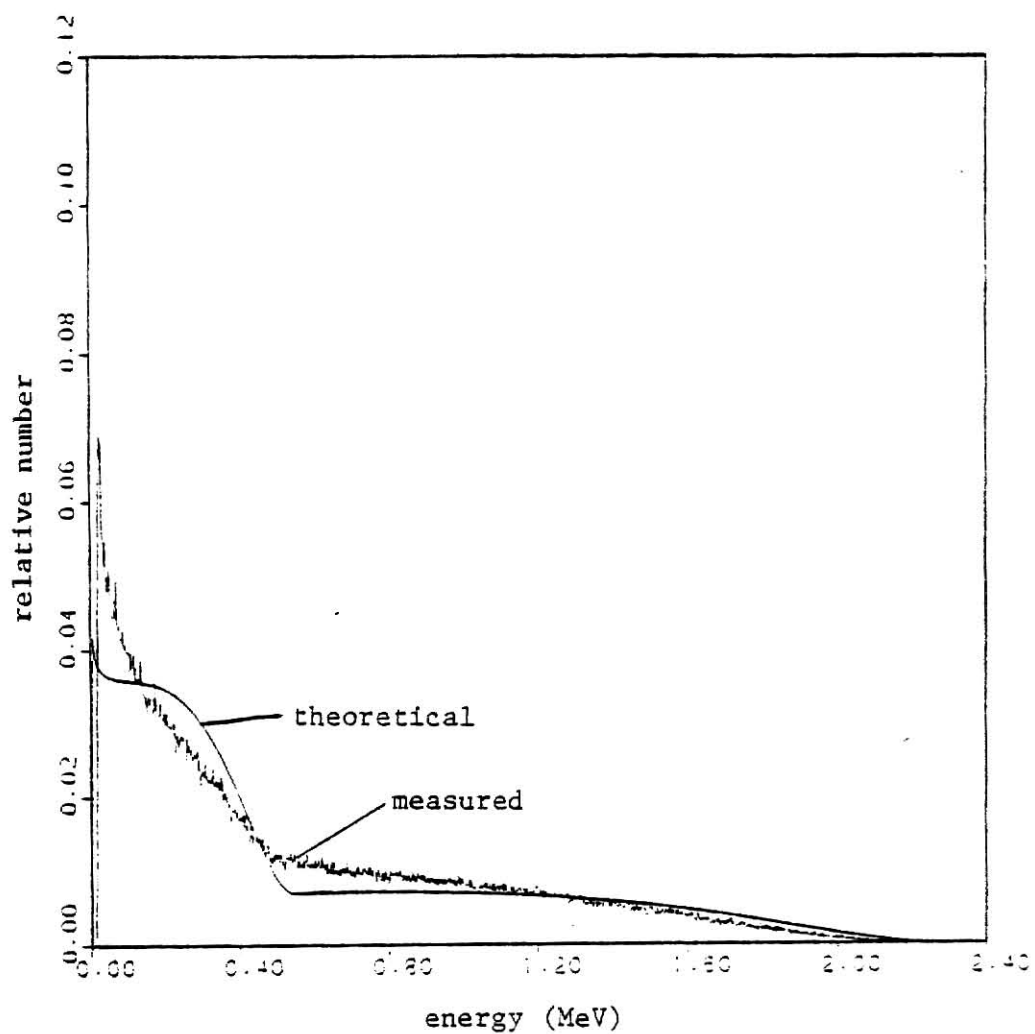


FIG. 5.13. Theoretical and measured beta particle energy distributions for Sr-90/Y-90, as measured with a 5 mm thick planar HPGe detector.

Table 5.3. Comparison of theoretical and measured endpoint and average energies. Measurements were made with a 5 mm planar HPGe detector.

Isotope	Theoretical endpoint energy (MeV)	Measured endpoint energy (MeV)	Δ (keV)	Δ (%)	Theoretical average energy (MeV)	Measured average energy (MeV)	Δ (keV)	Δ (%)
^{45}Ca	0.2560	0.2301	-25.9	-10.1	0.0773	0.0807	+ 3.40	+ 4.2
^{185}W	0.4326	0.3870	-45.6	-10.5	0.1440	0.1255	-18.5	-12.8
^{36}Cl	0.7087	0.6593	-49.4	- 6.97	0.2525	0.2149	-37.6	-14.9
^{204}Tl	0.7635	0.7290	-34.5	- 4.52	0.2433	0.2151	-27.4	-11.3
$^{90}\text{Sr}/^{90}\text{Y}$	2.288	2.165	-123.0	- 5.38	0.5665	0.5426	-23.9	- 4.22

the theoretical values. This was principally attributed to the high frequency of backscatter for electrons incident upon the detector surface. As would be predicted, results improve with increasing endpoint energy.

C. Application studies with a beta spectrometer

1.0 Effect of increasing absorber thickness upon transmitted beta spectra

Absorber experiments were performed utilizing three beta sources, ^{185}W , ^{36}Cl and $^{90}\text{Sr}/^{90}\text{Y}$. This study was designed to demonstrate the effect on beta spectra due to increasing the absorber thickness. The sources and the 10.2 mm-thick detector were maintained at a fixed distance from each other. Figure 5.14 demonstrates the effect of increasing absorber thickness, over the range 6.0 - 66.4 mg/cm^2 , upon the general shape of the measured ^{185}W beta particle energy distribution. Similar results are shown for ^{36}Cl in Fig. 5.15 for the absorber range 2.9 - 88.3 mg/cm^2 . For ease of visualization, the results for $^{90}\text{Sr}/^{90}\text{Y}$ are broken into three absorber ranges, 2.4 - 35.0, 20.2 - 144.7 and 144.7 - 254.4 mg/cm^2 , as shown in Figs. 5.16 - 5.18. To better illustrate the effect of increasing absorber upon the measured distributions, plots of integral counts versus absorber thickness, for the three beta sources, are shown in Figs. 5.19 - 5.21. Each isotope demonstrated an apparently exponential decrease in integral counts as is predicted by beta particle attenuation theory. Additionally, the effect upon measured average energy for the three sources is shown in Figs. 5.22 - 5.24. All three demonstrate an initial increase followed by a decrease in average energy as absorber thickness increases. For ^{36}Cl ,

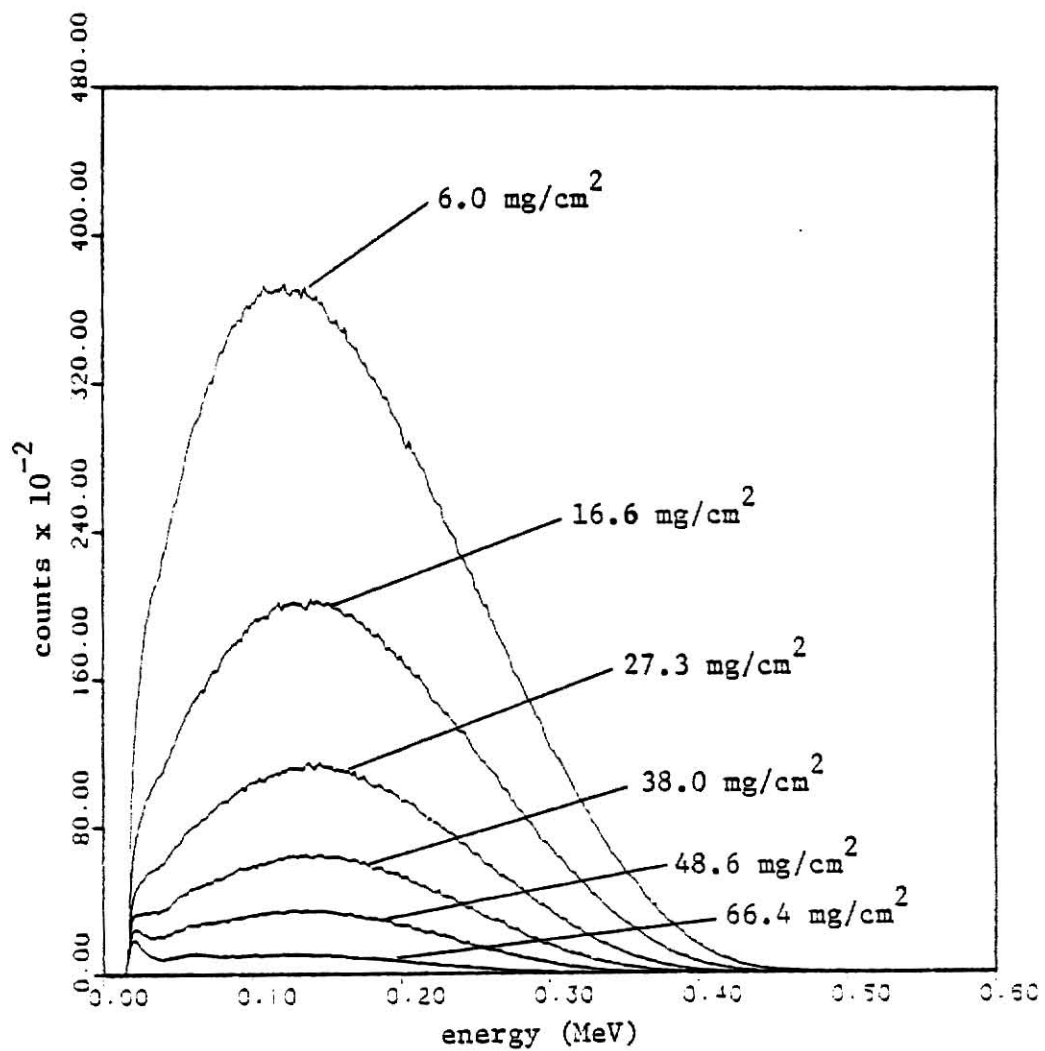


FIG. 5.14. Measured ^{185}W beta particle energy distribution as a function of increasing absorber thickness. Measurements were performed with a 10.2 mm-thick BC-400 detector.

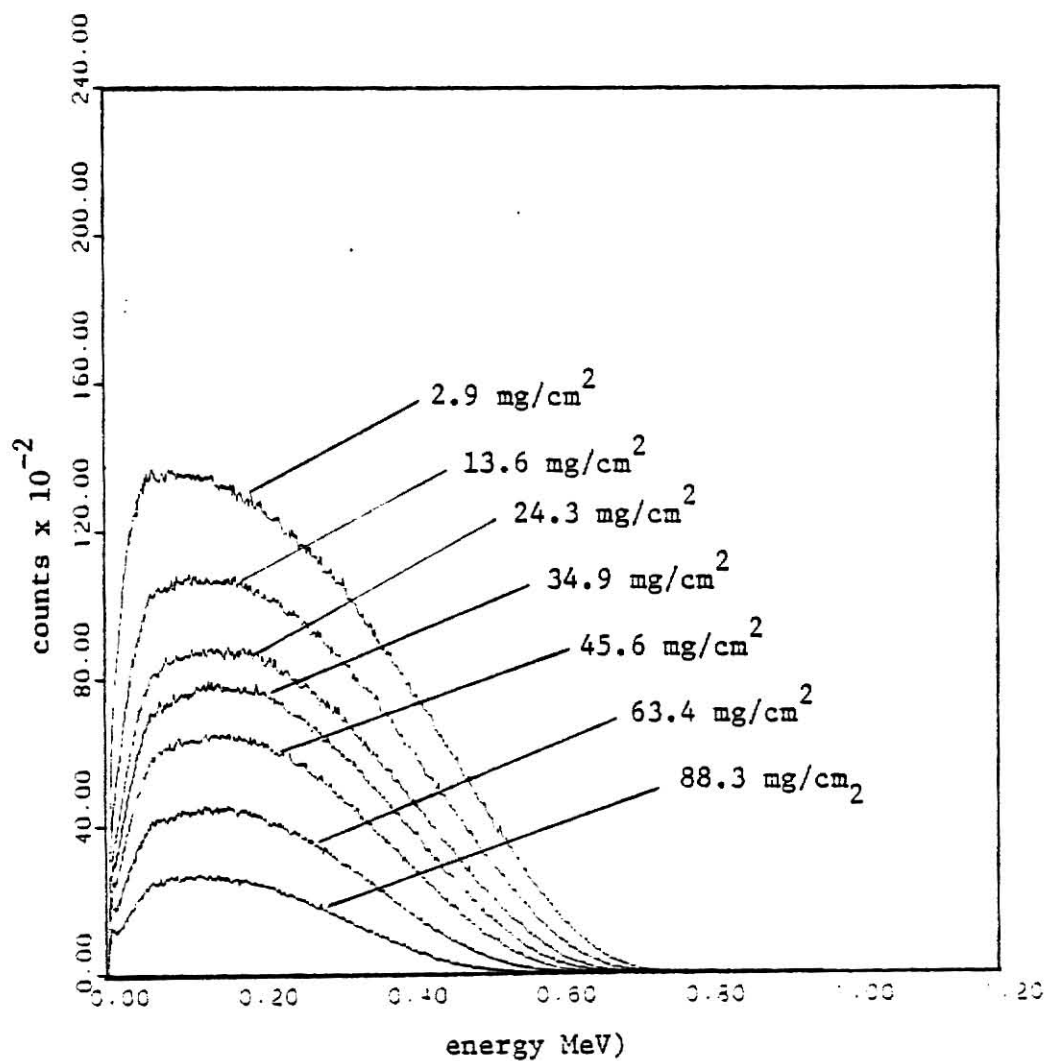


FIG. 5.15. Measured ^{36}Cl beta particle energy distribution as a function of increasing absorber thickness. Measurements were performed with a 10.2 mm-thick BC-400 detector.

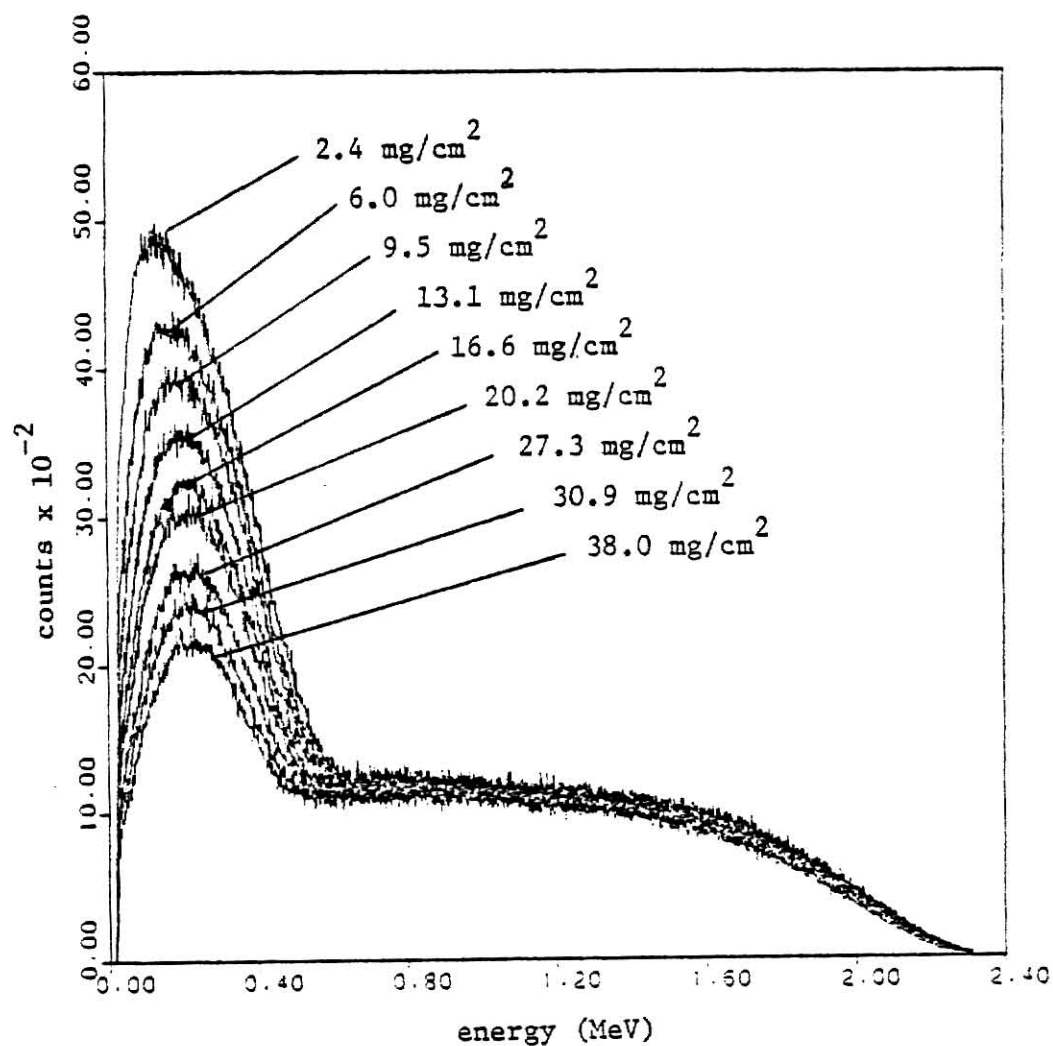


FIG. 5.16. Measured $^{90}\text{Sr}/^{90}\text{Y}$ beta particle energy distributions as a function of increasing absorber thickness. The measurements were made with a 10.2 mm-thick BC-400 detector.

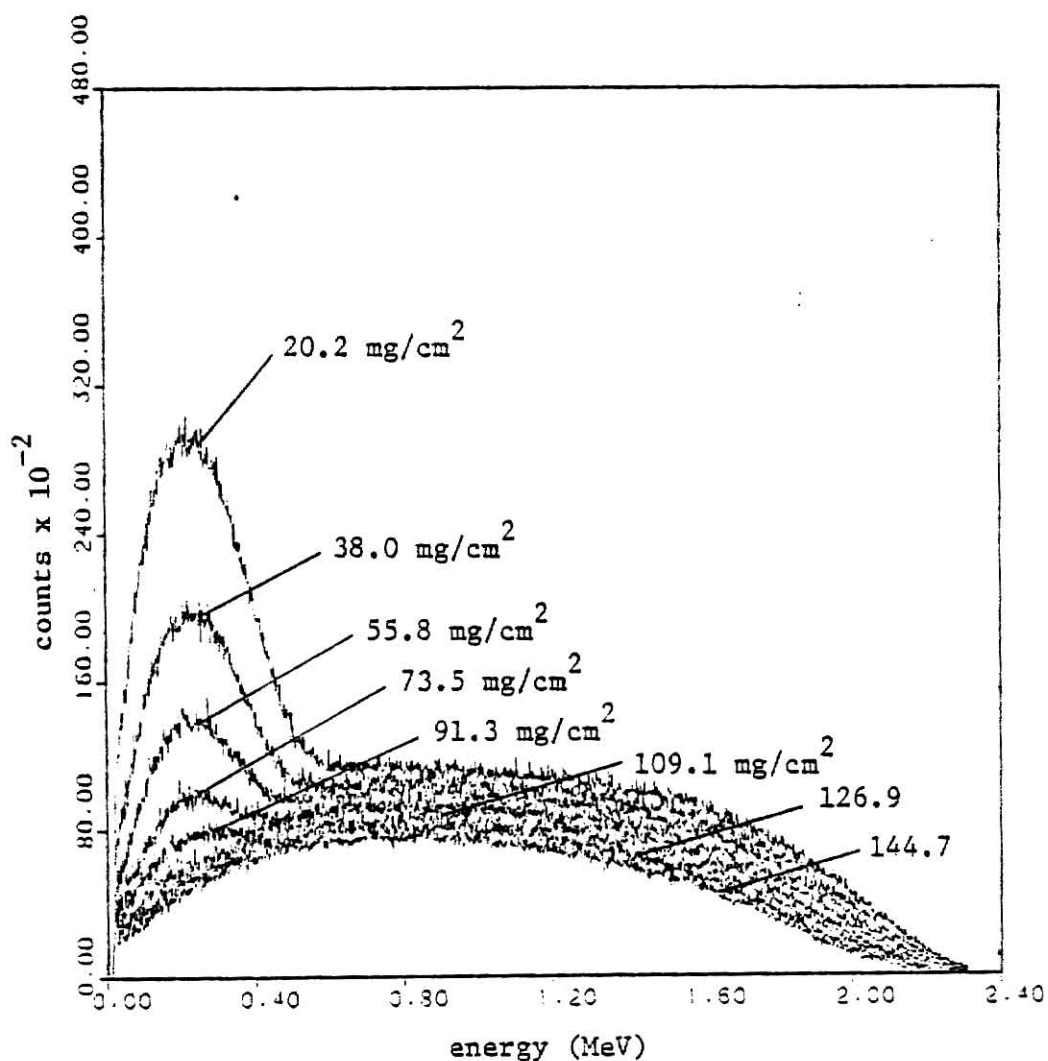


FIG. 5.17. Measured $^{90}\text{Sr}/^{90}\text{Y}$ beta particle energy distributions as a function of increasing absorber thickness. The absorber range shown is 20.2-144.7 mg/cm^2 . Measurements were made with a 10.2 mm-thick BC-400 detector.

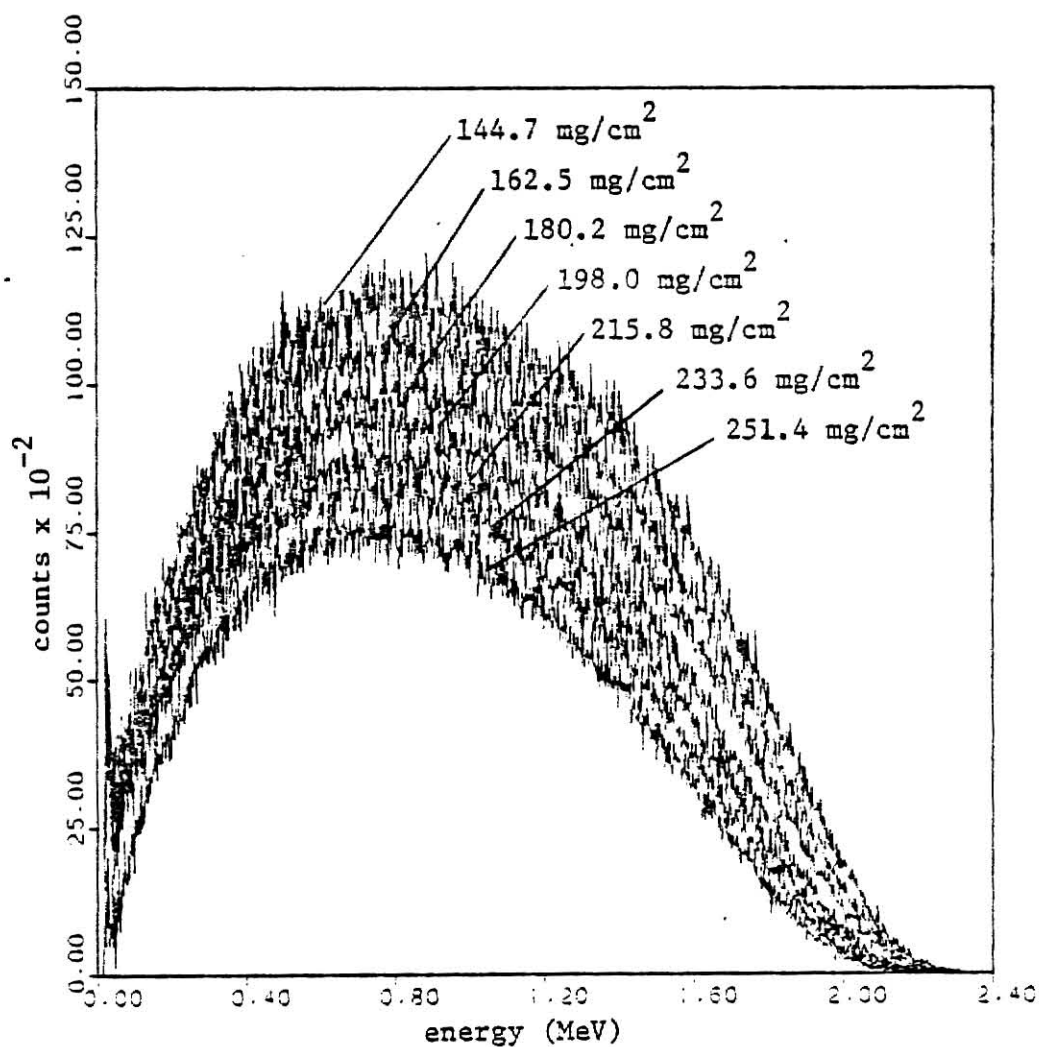


FIG. 5.18. Measured $^{90}\text{Sr}/^{90}\text{Y}$ beta particle energy distributions as a function of increasing absorber thickness. The absorber range shown is 144.7-251.4 mg/cm^2 . Measurements were performed with a 10.2 mm-thick BC-400 detector.

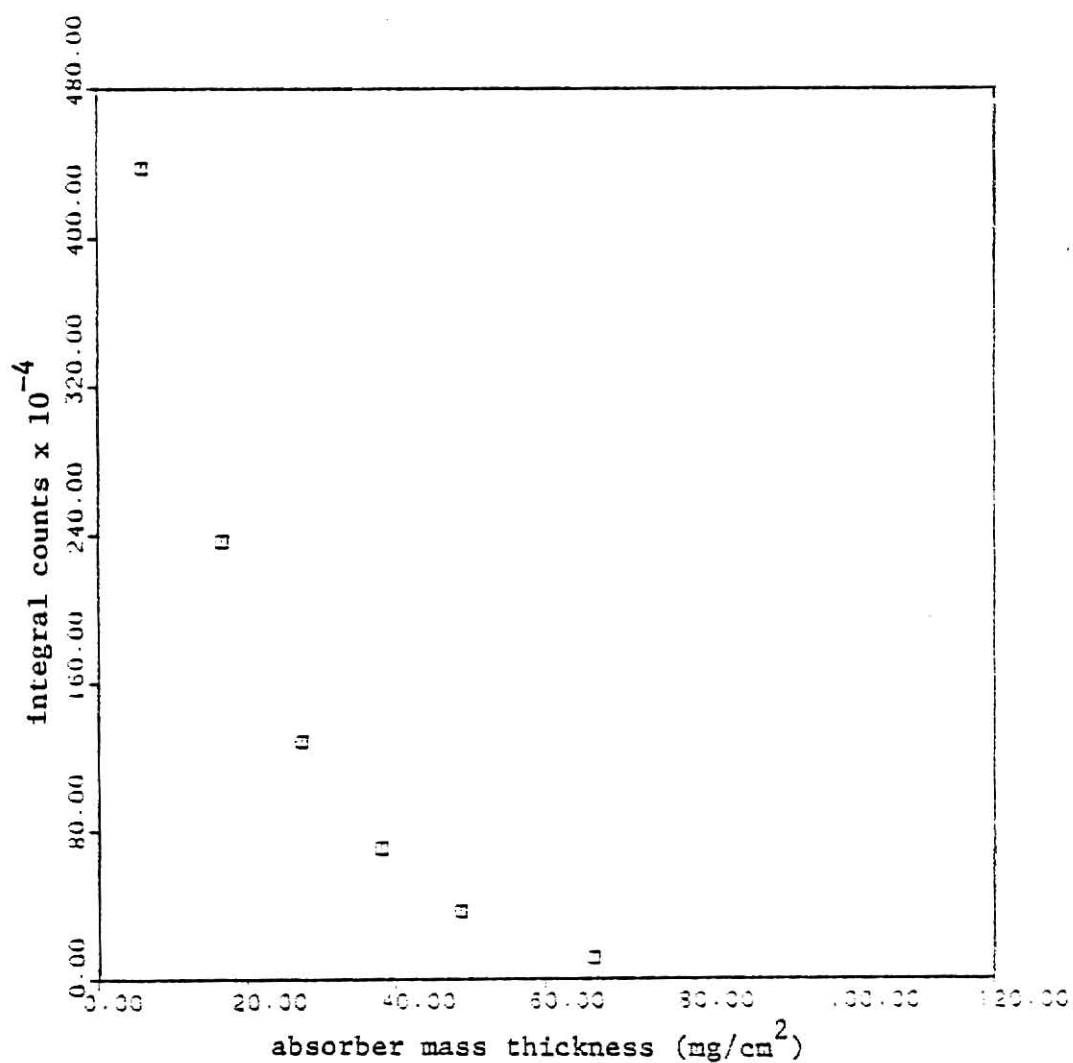


FIG. 5.19. Integral number of counts in the measured W-185 beta particle energy distribution as a function of increasing absorber thickness.

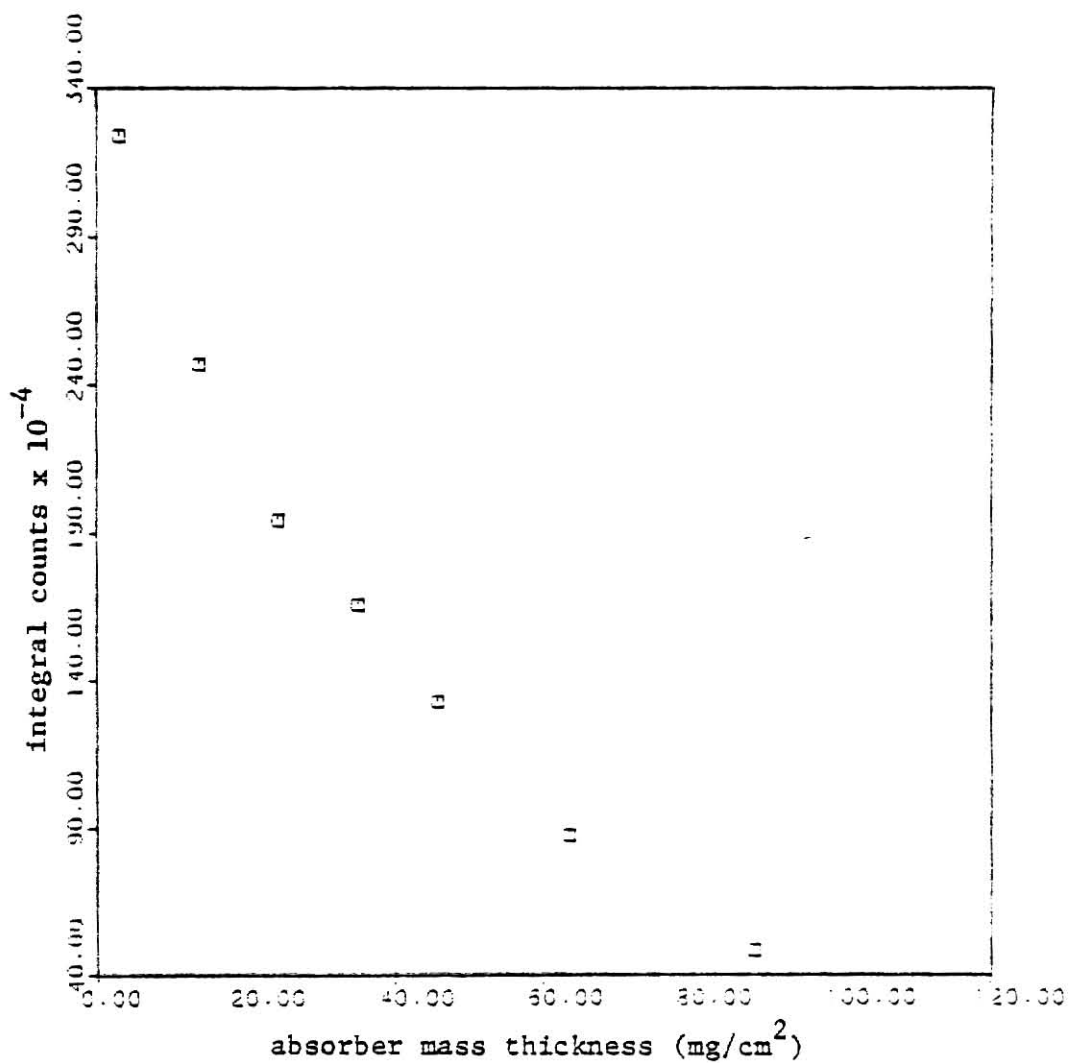


FIG. 5.20. Integral number of counts in the measured Cl-36 beta particle energy distribution as a function of increasing absorber thickness.

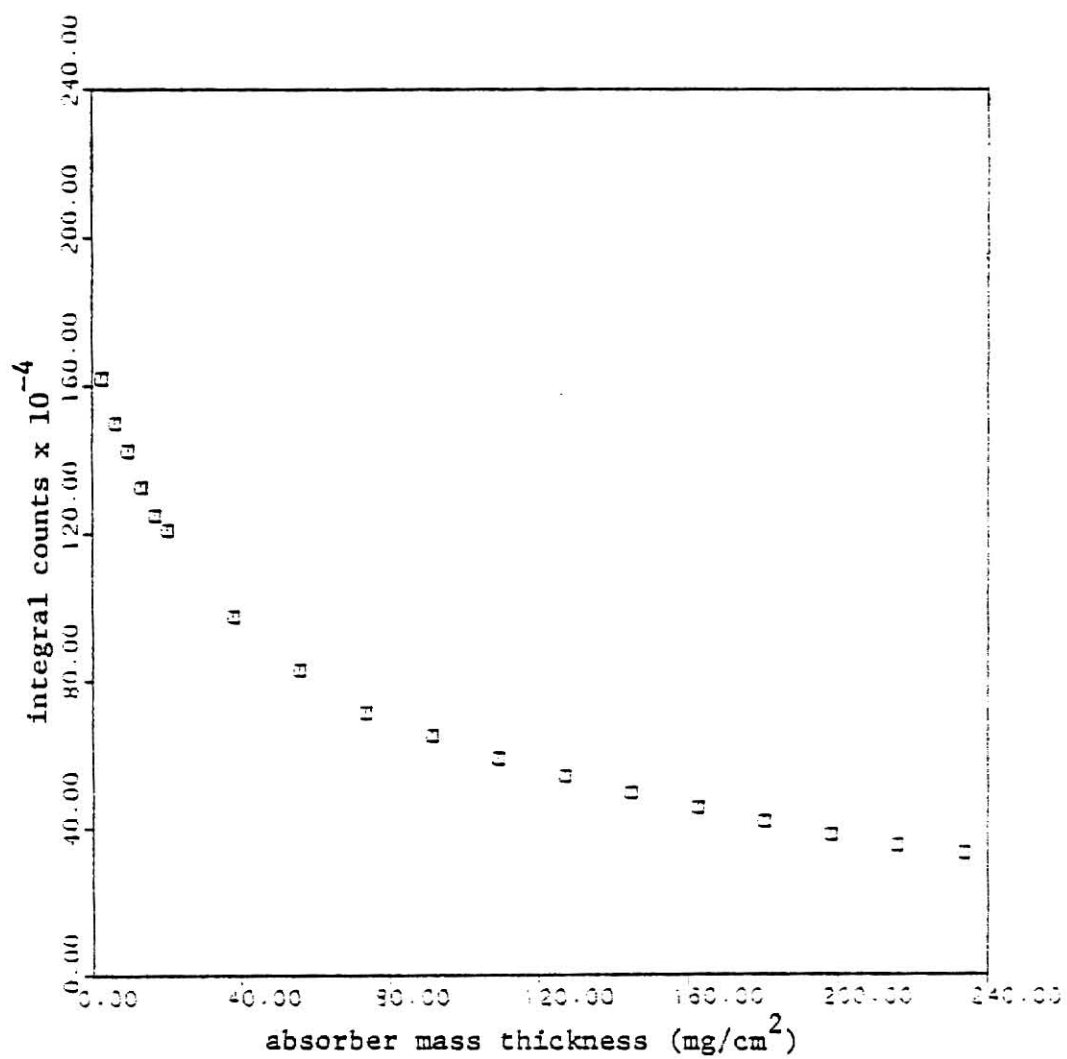


FIG. 5. 21. Integral number of counts in the measured $^{90}\text{Sr}/^{90}\text{Y}$ beta particle energy distribution as a function of increasing absorber thickness.

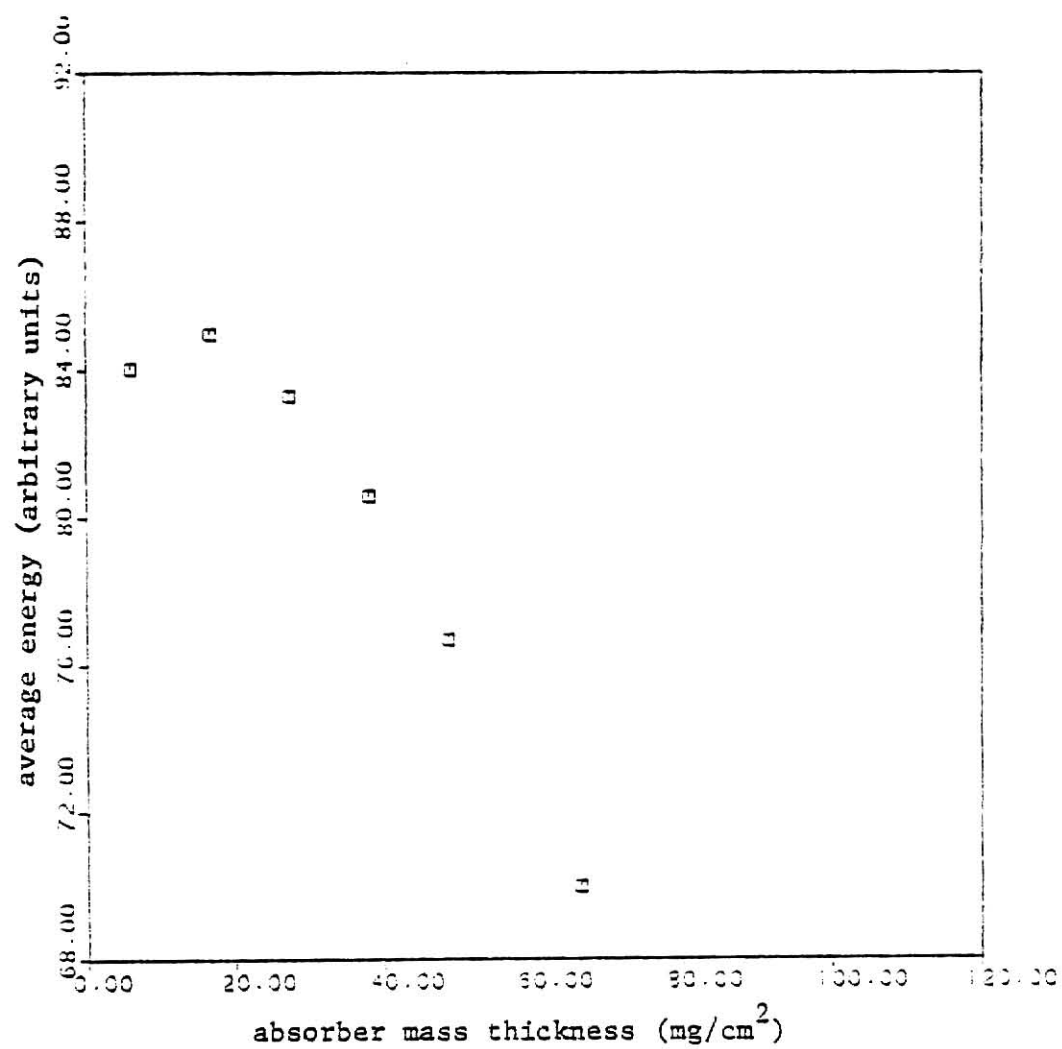


FIG. 5.22. Average energy in the measured ^{185}W beta particle energy distribution as a function of increasing absorber thickness.

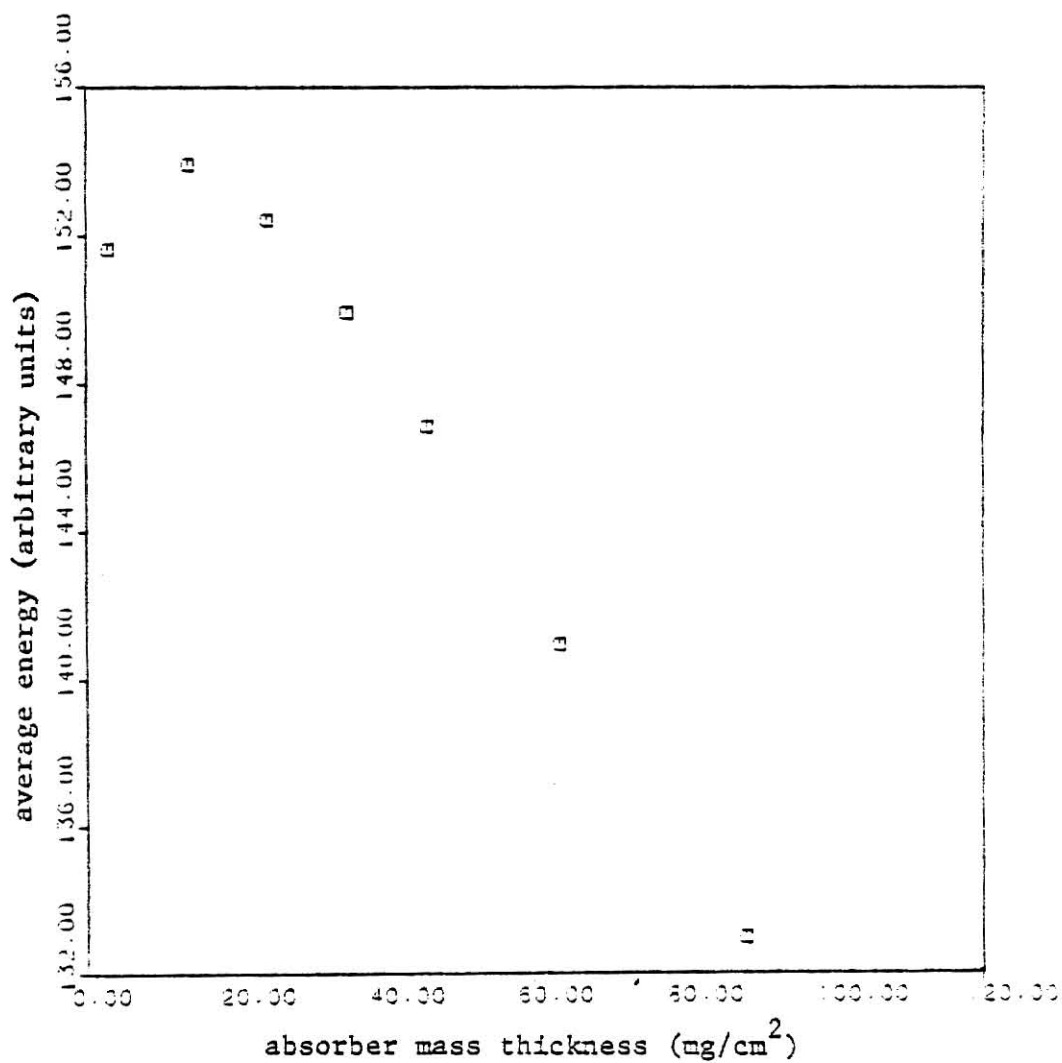


FIG. 5.23. Average energy in the measured ^{36}Cl beta particle energy distribution as a function of increasing absorber thickness.

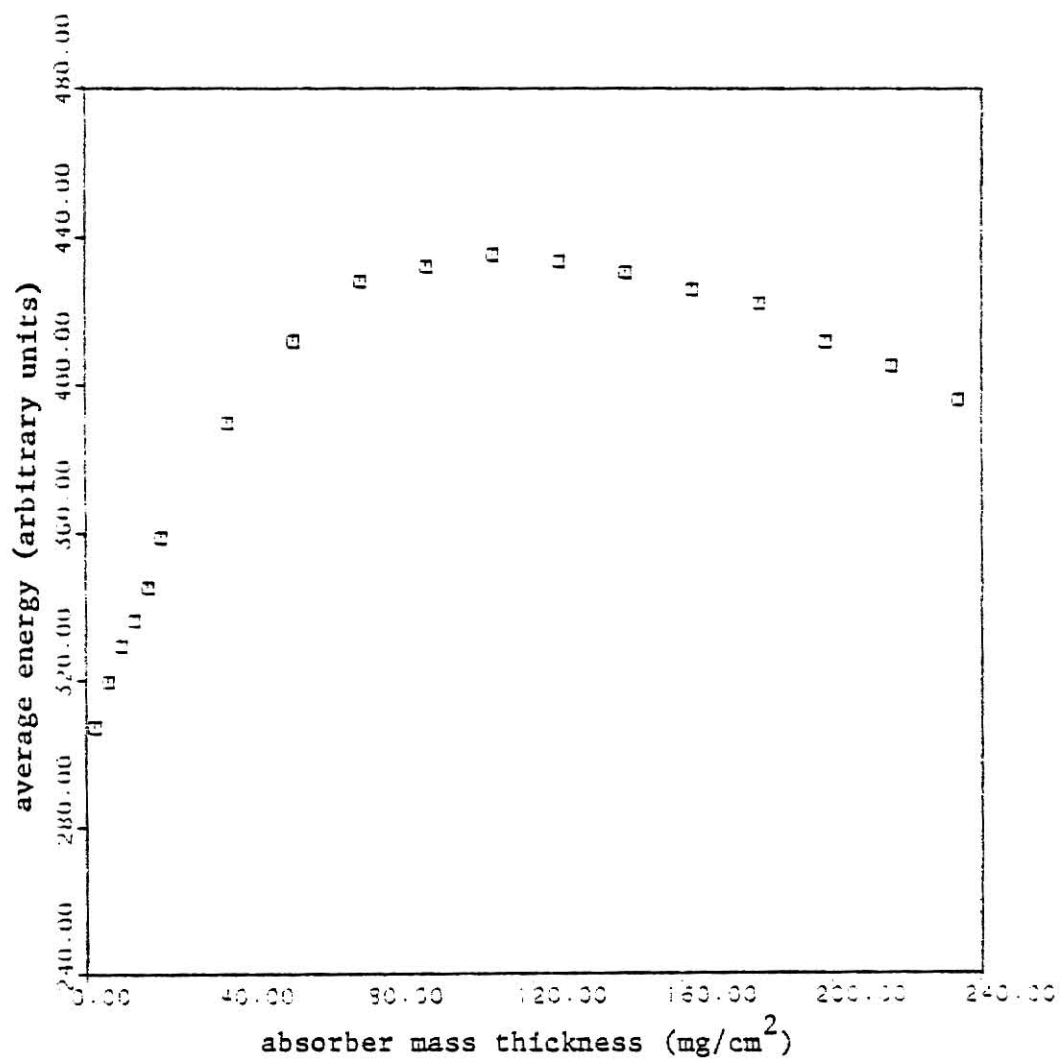


FIG. 5.24. Average energy in the measured $^{90}\text{Sr}/^{90}\text{Y}$ beta particle energy distribution as a function of increasing absorber thickness.

the effect of increasing absorber thickness upon the measured endpoint energy was determined and is shown in Fig. 5.25. As is expected, endpoint energy decreases as absorber thickness increases.

Beta dosimetry studies often rely upon $^{90}\text{Sr}/^{90}\text{Y}$ beta particle sources with cover thicknesses of 100 mg/cm^2 . Therefore, the effect of a 102.5 mg/cm^2 low Z cover was investigated. Figure 5.26 is a comparison of the measured distribution from the $^{90}\text{Sr}/^{90}\text{Y}$ source with a 102.5 mg/cm^2 cover and the theoretical ^{90}Y beta spectra. The measured distribution possesses an average energy of 0.8221 MeV as compared to 0.9637 MeV for the theoretical distribution, showing a difference of 141.6 keV in average energy. The measured decrease in endpoint energy was approximately 180 keV.

2.0 Natural uranium beta particle energy distribution

Measurements were performed upon a natural uranium source in the form of a paint matrix deposited upon a ceramic backing. The detector utilized was the 10.2 mm thick BC-400 organic scintillator. The resulting distribution, shown in Fig. 5.27, possessed an endpoint energy of $2.209 \pm 0.055 \text{ MeV}$ (95% confidence interval) and an average energy of 0.6814 MeV. Some distortion is present in the low energy end due to the presence of several low energy photons ($< 110 \text{ keV}$). HPGe detector measurements confirmed the photons to be those expected to originate from the decay of natural uranium.

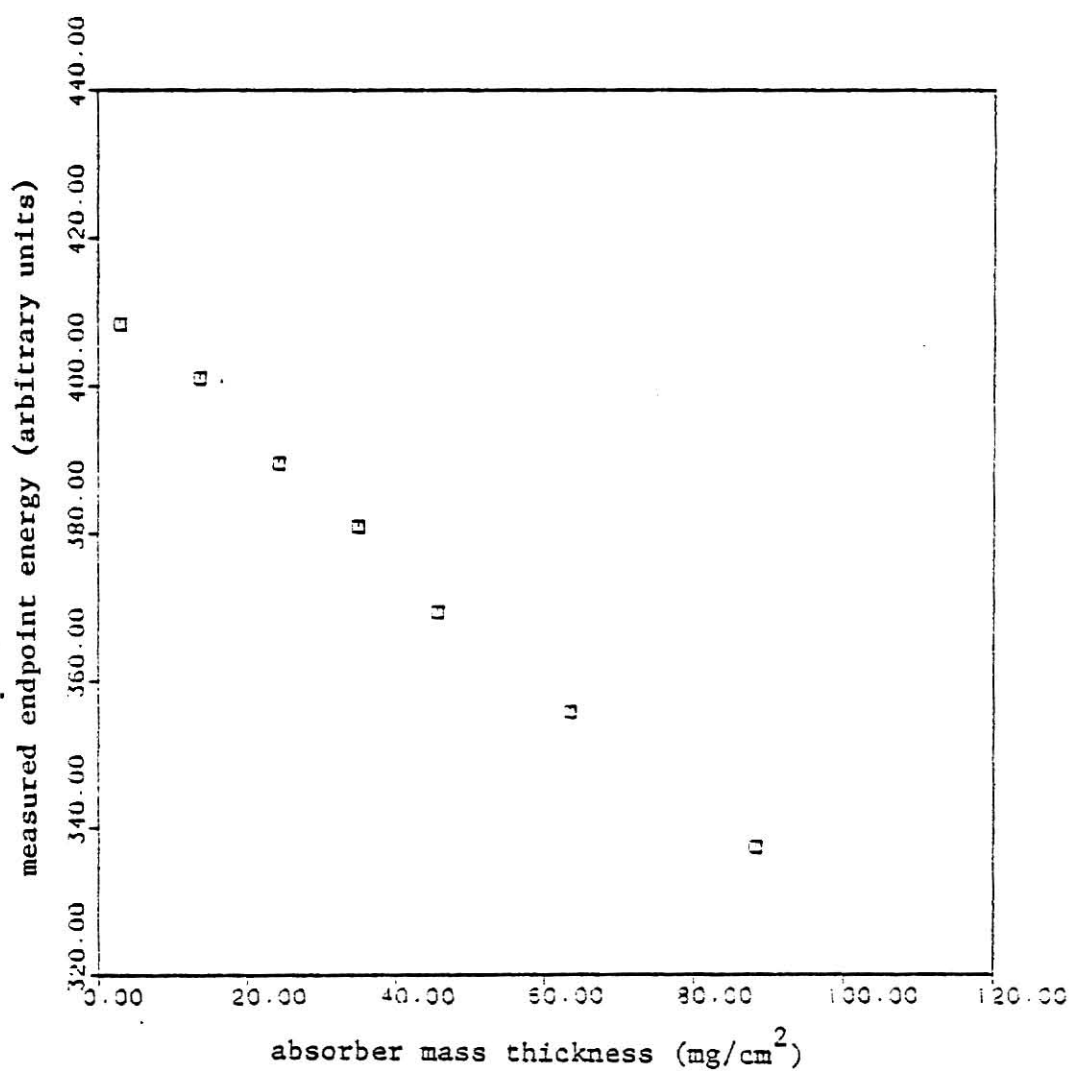


FIG. 5.25. Endpoint energy of the measured ^{36}Cl beta particle energy distribution as a function of increasing absorber thickness.

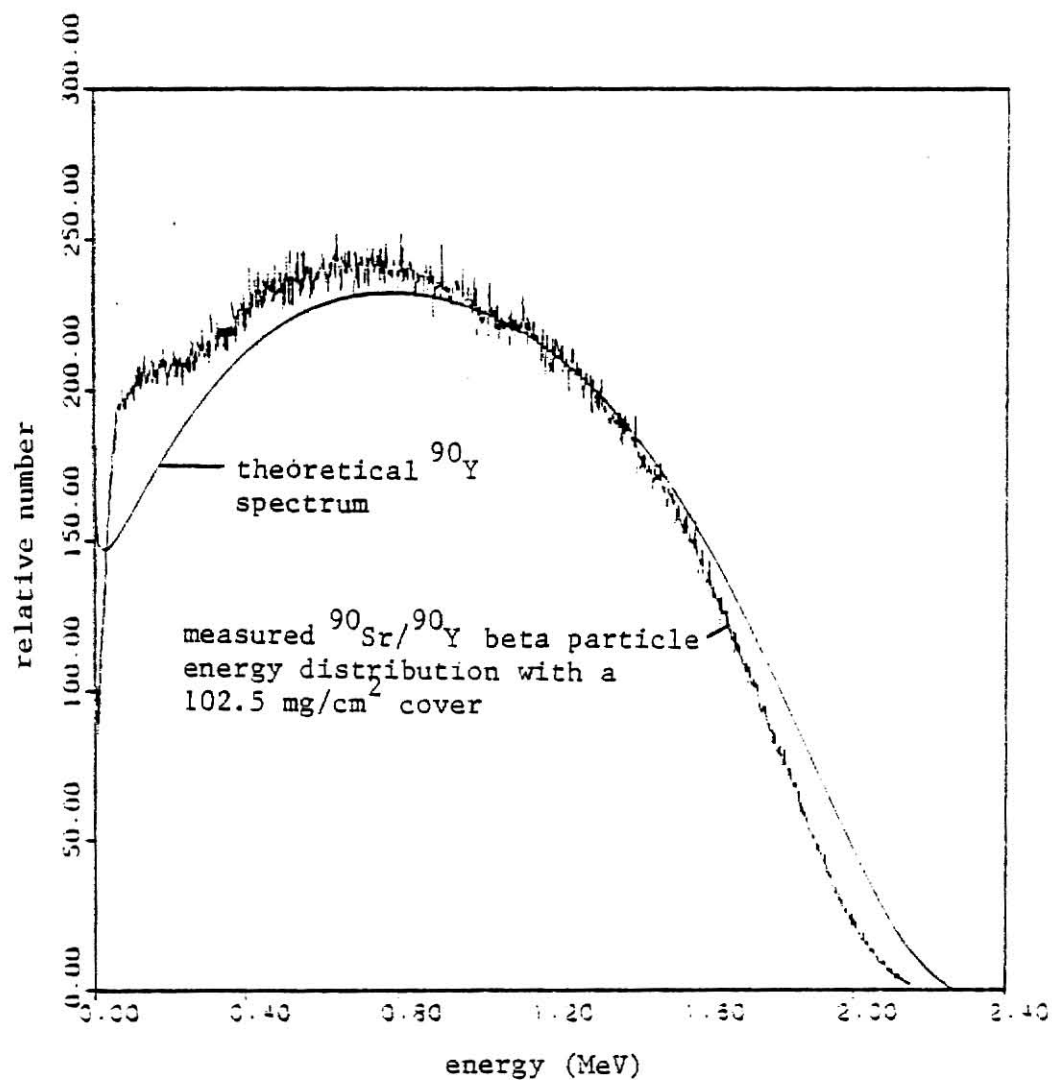


FIG. 5.26. A comparison of the measured $^{90}\text{Sr}/^{90}\text{Y}$ beta particle energy distribution when the source is covered with a 102.5 mg/cm^2 cover and the theoretical Y-90 beta-ray spectrum.

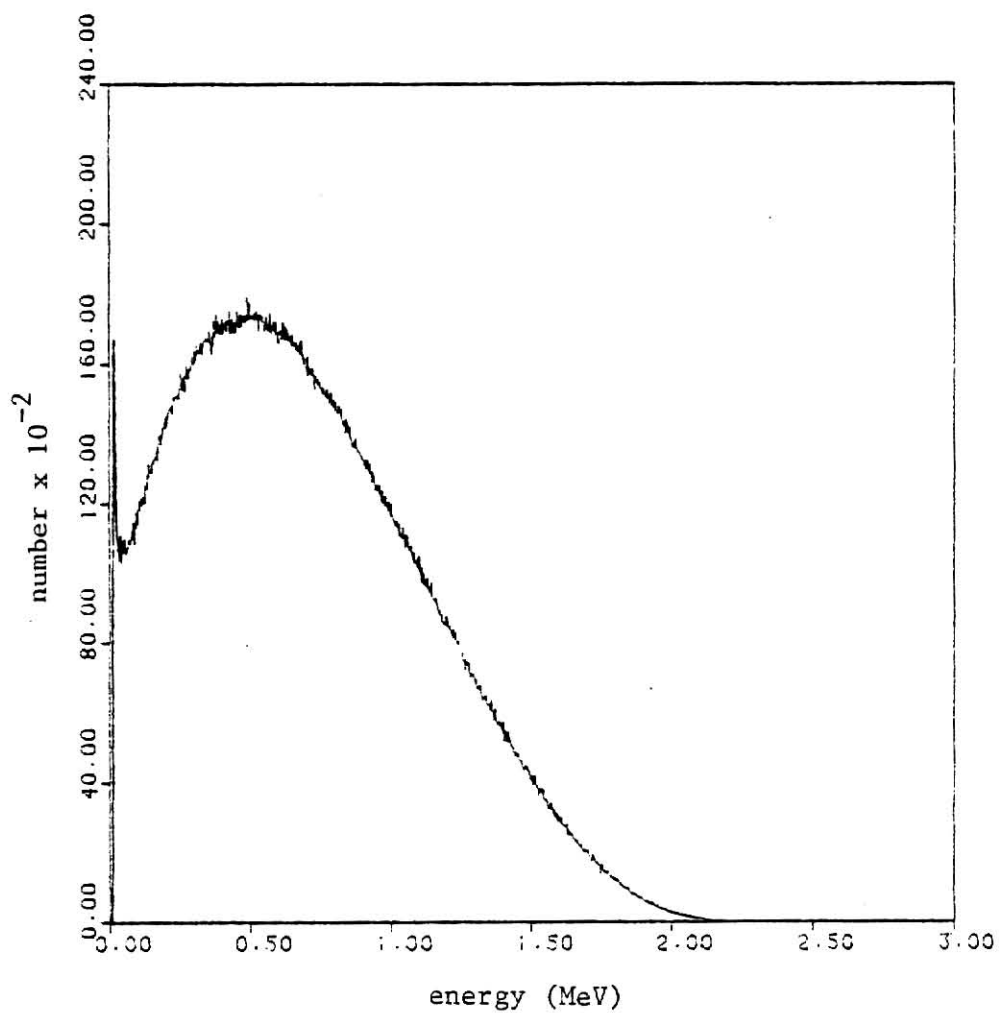


FIG. 5.27. Measured beta particle energy distribution for natural uranium. Measurement made with 10.2 mm thick BC-400 detector.

VI. Conclusions concerning the beta spectrometers

In general, the BC-400 organic scintillator based spectrometer systems provided endpoint measurements within 3% of the actual values, and average energy measurements within 6%, and often within 1%, of theoretical, for beta spectra with endpoint energies greater than 500 keV. These limits were obtainable provided that the system was calibrated using known beta spectra in a manner which accounted for the nonlinear response of the scintillators for electrons with energies below 125 keV. An additional restraint imposed was that the detector utilized must be of sufficient thickness to stop all incident beta particles of interest. For beta spectra with endpoints below 500 keV, endpoint measurements were obtained within 7-12% of actual values. Average energies, however, were subject to an increased incidence of backscatter events and, additionally, were more significantly influenced by the low level discriminators present in the associated electronics. Consequently, average energies were measurable only within 35% (~25 keV) for beta spectra with endpoints below 500 keV.

For the organic scintillators, gamma-ray discrimination was possible through the use of beta-particle filters, provided that a reasonable estimate of the mixed field's maximum beta particle energy was available. X-ray discrimination was more difficult, however, due to the easily attenuable nature of the photons involved.

One drawback of the BC-400 based systems is their inability to indicate the presence of gamma-rays in a measured field without the use of filters. Due to the smooth nature of recorded Compton distributions, they blend quite readily with measured beta-particle energy

distributions. This seems to indicate that the scintillators could be utilized most efficiently in conjunction with a high resolution gamma-ray detector.

In contrast to the plastic scintillators, the HPGe detector appeared usable only for endpoint measurements. The endpoint energy measurements were nominally within 4-11% of actual values over the endpoint energy range 0.256-2.288 MeV. Results could be improved if a thinner entrance window could be utilized. The average energy measurements made were considerably in error due to the high incidence of backscatter for electrons incident upon the detector. Nominally, measured average energies can only be assumed to be within 15% of the theoretical values for beta spectra with endpoint energies from 0.256 - 2.288 MeV. Obviously, average energy results improve as the endpoint energy increases and the incidence of backscatter thus lessens.

VII. Suggestions for further study

In the course of this investigation, several ideas have presented themselves as candidates for future research.

An examination of the effect of detector geometry upon low energy electron response should be performed. More specifically, a well detector with a collimated entrance port should be evaluated as a possible method of reducing low energy electron backscatter from the detector.

The possibility of utilizing two phosphors of differing decay times to allow active gamma-ray discrimination seems probable. Two techniques appear feasible. In one, a thin scintillator is mounted upon the face of a thicker scintillator possessing a longer decay constant. In the other, the thicker detector should take the form of a well detector with the thin scintillator recessed into the well. In both cases, the goal is for the thicker scintillator to provide a type of Compton shield, allowing gamma-ray discrimination through pulse shape analysis.

Additionally, work appears to be needed in the minaturization of the support equipment involved, and in the adaptation of phototube based detector assemblies for higher temperature reactor containment environments.

VIII. Acknowledgements

I would like to express my sincere appreciation to Dr. G. G. Simons for his advise and support during the course of this research. Additionally, I would like to thank Tim DeBey for his cooperation and assistance, and Kevin Stansbury for sharing his knowledge gained from related work.

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APPENDIX A: BETA SOURCES

Eight beta-minus emitting isotopes were used in the course of this study, with endpoint energies ranging from 0.1674-2.288 MeV. Table A.1 characterizes each of the sources utilized. Two of the eight sources were purchased with the remainder being produced in the Kansas State University TRIGA Mk II nuclear reactor.

The commercial sources were encapsulated in differing manners. The $^{90}\text{Sr}/^{90}\text{Y}$ source, produced by the Nuclear Enterprises Corporation, was encapsulated in a thin aluminum ring with a 0.9 mg/cm^2 aluminized polyester film window. The American Nuclear Products ^{36}Cl source was deposited on an aluminum backing with a thin aluminized polyester film window.

In general, the KSU produced beta sources were encapsulated by placing the active material between two pieces of 1 mil thick transparent tape. An exception was the ^{198}Au source which was unencapsulated due to the metal being in a foil configuration.

TABLE A.1. Characterization of the beta-minus emitting isotopes
used in the beta spectroscopy studies

Isotope	Half-life	Activity	Endpoint Energy (MeV)	Average Energy (MeV)	% Yield	Other Emissions
^{32}P	14d	<1.0 μCi	1.711	0.6950	100	None
^{35}S	88d	<0.5 μCi	0.1674	0.0488	100	None
^{36}Cl	$3 \times 10^5 \text{ y}$	0.07 μCi	0.7087	0.2525	98.1	None
^{45}Ca	165d	<0.5 μCi	0.2560	0.0773	100	γ :0.0125 MeV, $3 \times 10^{-6}\%$
$^{90}\text{Sr}/$ ^{90}Y	28.5y	0.02 μCi	0.5461 2.288	0.1963 0.9367	100 100	γ :1.761 MeV, $3 \times 10^{-3}\%$ γ :2.186 MeV, $1.4 \times 10^{-6}\%$
^{185}W	75d	<1.0 μCi	0.4326	0.144	100	γ :0.125 MeV, 0.021%
^{198}Au	2.7d	<1.5 μCi	0.286 0.962	0.0800 0.3166	1.1 99	γ :0.4118 MeV, 100% γ :0.6759 MeV, 1.0% γ :1.0877 MeV, 0.2%

APPENDIX B: THE BETAPLUS COMPUTER CODE

In general, the code listing is self-explanatory. There are two points, however, that require additional comment. First, the required JCL is identical to that of the modified ASPPP code available in the KSU Neutron Activation Analysis Laboratory. Second, operation is terminated upon execution of a read statement which acquired a value of zero for the number of channels in the measured distribution. This is most easily accomplished through insertion of a blank card at the rear of the data set.

To provide greater clarity, a sample data set is included below. The set shown is for analyses of two measured distributions ^{36}Cl and ^{32}P .

12000

1024bbb.000000Eb00bbb.154456E-02b-0.565753E-01bbb.241495E-02665

CHLORINE-36

10.2mm thick BC-400

18.0bbbbbb2.397315bb36.0bbbbbb00

1024bbb.000000Eb00bbb.154456E-026-0.565753E-01bbb.241495E-02b65

PHOSPHORUS-32

10.2 mm thick BC-400

16.0bbbbbb4.34847bbb.32.0bbbbbb00

(blank card)

```

***** BETAPLUS *****
**
**
**
**
**      WRITTEN BY:          *
**              BOB STUEWE
**              AUGUST 1982
**
*****
**
**
**      BETAPLUS IS DESIGNED TO PERFORM A
**      SERIES OF CALCULATIONS AND THEORETICAL
**      INTERCOMPARISONS UPON MEASURED BETA
**      SPECTRA. FOR BETAPLUS TO PERFORM PROPERLY,
**      A SERIES OF CONDITIONS MUST BE MET.
**
**
**      1) PULSE HEIGHT DISTRIBUTION DATA MUST BE
**         RECORDED ON MAGNETIC TAPE USING 256, 512 OR 1024
**         CHANNELS. 1024 IS OPTIMUM.
**
**      2) THE SPECTRA MUST BE OF SINGLE NEUTRON EMITTERS,
**         WHOSE TRANSITIONS ARE EITHER ALLOWED, FIRST OR SECOND
**         FORBIDDEN.
**
**      3) THE MEASURED DISTRIBUTION CONTAINS NO CHANNELS WITH COUNTS
**         GREATER THAN 45000. THE COUNTS IN ANY SUCH CHANNELS WILL BE CON-
**         sidered as negative and thus will be set to zero.
**
**      4) THE ANALYZER IS CALIBRATED IN A LINEAR OR BI-LINEAR FORM COM-
**         PATIBLE WITH THE CALIBRATION PROCEDURE TO BE DESCRIBED.
**
**      IN ADDITION TO THE TAPE/RECORDED DISTRIBUTION, THE FOLLOWING INFORMATION
**      IS REQUIRED IN THE FOLLOWING FORM:
**
**      CARD ONE: INPUTS THE FOLLOWING PARAMETERS WITH THE FOLLOWING FIELDS-
**
**      ND(14) = NUMBER OF DATA POINTS IN THE NUMERICAL INTEGRATION, MUST BE AN EVEN NUMBER.
**      NCR(11)= 0, IF NO FERMI PLOT DATA CARDS ARE TO BE PUNCHED. ANY OTHER ENTRY WILL RESULT IN CARDS BEING PUNCHED.

```


[illegible]

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C      **      ISTIC AND NONRELATIVISTIC MODELS IS      **
C      **      ALSO PRODUCED. TABULAR DATA INCLUDES    **
C      **      MEASURED AND THEORETICAL SPECTRA, FER-    **
C      **      MI PLOT VALUES AND FERMI FUNCTION        **
C      **      VALUES. THEORETICAL CALCULATIONS ARE     **
C      **      PROVIDED BASED UPON BOTH RELATIVISTIC     **
C      **      AND NONRELATIVISTIC MODELS, DENCTED      **
C      **      BY POSTSCRIPTS R AND N RESPECTIVELY.      **
C      **      PUNCHED CARDS CONTAINING THE ENERGY,    **
C      **      MEASURED FERMI PLOT VALUES AND CALC-    **
C      **      ULATED FERMI PLOT VALUES ARE ALSO       **
C      **      AVAILABLE. THE RESPECTIVE OUTPUT FIE-    **
C      **      LDS ARE (F10.7,1X,F10.7,1X,F10.7).      **
C      **      **
C      *****
C      **
C      **      SPECIAL NOTE: A SPECIAL ROUTINE HAS      **
C      **      BEEN INCORPORATED TO ALLOW SOME          **
C      **      ANALYSIS ON COMBINED SR-90/Y-90          **
C      **      SPECTRA. ROUTINE IS INITIATED           **
C      **      THROUGH INPUT OF THE Z AND A VAL-       **
C      **      UES FOR THE SR-90 TRANSITION.           **
C      **      OUTPUT IS LIMITED.                      **
C      **      **
C      *****
C
C      **
C      **      DIMENSION AND INPUT PARAMETERS          **
C      **
C      REAL*8 XARRAY(1500),YARRAY(1500),ZARRAY(1500),UARRAY(1500),VARRAY
1500),WARRAY(1500),XEN(1500),XSPECT(1500),DIST(1500)
C      REAL*8 W,E,RAV,FAV,RSINT,SINT,F
C      REAL*8 DIV
C      DIMENSION FR(1500),FN(1500),Y(1500),X(1500),R(1500)
C      DIMENSION FC(1500),XKM(1500),XKC(1500)
C      DIMENSION WM(1500)
C      DIMENSION DIST1(1500),DIST2(1500)
C      DIMENSION YKC(130),YKM(130),YWM(130)
C      DIMENSION XA(500),ZA(500),YA(500)
C      DIMENSION BUF(1000)
C      DIMENSION TITLE(9)
C      DIMENSION TITLE2(17)
C      INTEGER SPECT(4098),ID,NCHNLS
C      REAL*8 CAV,CAVG
C      REAL*8 TCT
C      REAL*8 SUM
C      INTEGER U
C      IPLCT=0
C      REAC(5,300)NO,NCP
300  FORMAT(I4,I1)
11  READ(5,99)NCHNLS,ZB,ZM,Z3P,ZMP,NPR
99  FORMAT(I4,4E14.6,I3)
      IF(NCHNLS.EQ.0)GO TO 999

```

```

      READ(5,101)(TITLE(I),I=1,8)
101 FCRMAT(8A4)
      READ(5,91)(TITLE2(I),I=1,16)
      91 FORMAT(16A4)
      READ(5,100)Z,W0,A,U,NP
100 FCRMAT(3F10.5,2I1)
C      **
C      **
C      **          INPUT SPECTRA
C      **
      READ(8,93)ID,(SPECT(I),I=1,NCHNLS)
      98 FCRMAT(I6,(I7,16(16I6)))
      IF(Z.EQ.0.0)GO TO 11
C      **
C      **          CALCULATE AVERAGE CHANNEL AND ENERGY
C      **
      SPECT(1)=0
      SPECT(2)=0
      DO 12 I=1,NCHNLS
      IF(SPECT(I).GT.45000)GO TO 181
      GO TO 12
181 SPECT(I)=0
      12 CCNTINUE
      SUM=0.000
      DO 251 I=8,NCHNLS
251 SUM=SUM+SPECT(I)
      CAV=0.0
      DO 121 I=1,NCHNLS
      XSPECT(I)=FLOAT(SPECT(I))/SUM
      CAV=CAV+FLOAT(SPECT(I))*FLOAT(I-1)
121 CCNTINUE
      CAV=CAV/SUM
      IF(CAV.GE.NPR)GO TO 120
      CAVG=ZB+ZM*CAV
      GO TO 119
120 CAVG=ZBP+ZMP*CAV
119 DCW=0.0
      DO 124 I=1,NCHNLS
      IF(I.GE.NPR)GO TO 122
      XEN(I)=ZB+ZM*FLCAT(I-1)
      GO TO 123
122 XEN(I)=ZBP+ZMP*FLOAT(I-1)
123 IF(XEN(I).GT.0.0) GO TO 124
      XEN(I)=0.0
124 CCNTINUE
C      **
C      **          CALCULATE THEORETICAL SPECTRA AND
C      **          FERMI PLOT VALUES
C      **
      IF(Z.NE.39.C.CR.A.NE.90.0)GO TO 301
      CALL SRY(Z,A,W0,XEN,1500,NCHNLS,DIST,XSPECT,DIST1,DIST2,U,FD,WM)
      EAVG=0.00
      FAVG=0.00
      SINT=0.00
      RSINT=0.00
      N=NCHNLS+2
      WC=5.46397
      GO TO 302
301 N=NCHNLS+2

```

```

      NCHN=NCHNLS+1
      NN=N-2
      DIV=0.005
      ND=((WC-1.0)*0.511)/DIV)+1
      NDD=ND-1
      IF(ND.LE.1500)GO TO 18
      WRITE(6,41)
41  FORMAT('1',//////////,35X,'ENERGY DIVISION TOO SMALL FOR ARRAY DIM
      ENSIONS---',/,35X,'---INCREASE VARIABLE CIV',///)
      GO TO 11
18  E=0.0
      F=1.0
      DO 20 J=1,ND
      W=(E/0.51098)+1.0
      XRR=XR(Z,WC,A,E,F,U)
      FR(J)=F
17  XNN=XN(Z,WC,E,F,U)
      FN(J)=F
19  WARRAY(J)=W
      XARRAY(J)=E
      YARRAY(J)=XNN
      ZARRAY(J)=XRR
      IF(J.LT.NDD)GO TO 16
      E=(WC-1.0)*0.51098
      GO TO 20
16  E=E+DIV
20  CONTINUE
      DO 21 J=1,ND
      IF(WARRAY(J).GT.1.0)GO TO 22
      VARRAY(J)=1.0
      UARRAY(J)=1.0
      GO TO 21
22  VARRAY(J)=(ZARRAY(J)/((WARRAY(J)*FR(J))*(((WARRAY(J)**2.0)-1.0)**0
      .5))))**0.5
      UARRAY(J)=(YARRAY(J)/((WARRAY(J)*FN(J))*(((WARRAY(J)**2.0)-1.0)**0
      .5))))**0.5
21  CONTINUE
C      **
C      **      CALCULATE THEORETICAL AVERAGE ENERGIES      **
C      **
C      **
C      **      NORMALIZE THEORETICAL SPECTRA TO UNITY      **
C      **
      EMAX=(WC-1.0)*0.51098
      ANO=ND
      H=EMAX/ANO
      SINT=0.0
      PSINT=0.0
      RAV=0.0
      FAV=0.0
      E=0
      F1=XN(Z,WC,E,F,U)
      G1=XR(Z,WC,A,E,F,U)
      GS1=E*G1
      FS1=E*F1
      DO 2 I=2,ND,2
      E=E+H
      F2=XN(Z,WC,E,F,U)

```

```

G2=XR(Z,WC,A,E,F,U)
GS2=E*G2
FS2=E*F2
E=E+H
F3=XN(Z,WC,E,F,U)
G3=XR(Z,WC,A,E,F,U)
GS3=E*G3
FS3=E*F3
SINT=SINT+F1+4.0*F2+F3
RSINT=RSINT+G1+4.0*G2+G3
RAV=RAV+GS1+4.0*GS2+GS3
FAV=FAV+FS1+4.0*FS2+FS3
F1=F3
G1=G3
GS1=GS3
FS1=FS3
2 CONTINUE
SINT=SINT*H/3.0
RSINT=RSINT*H/3.0
RAV=RAV*H/3.0
FAV=FAV*H/3.0
ZC=1.0/RSINT
ZW=1.0/SINT
EAVG=RAV/RSINT
FAVG=FAV/SINT
DO 30 JJ=1,ND
ZARRAY(JJ)=ZARRAY(JJ)*ZC
YARRAY(JJ)=YARRAY(JJ)*ZW
30 CONTINUE
302 DZR=0.0
C      **
C      **      OUTPUT TITLES AND CALCULATED AVERAGES      **
C      **
WRITE(6,102)ID,(TITLE(I),I=1,8),(TITLE2(J),J=1,16)
102 FORMAT('1',//,50X,'TAG NUMBER',16,/,40X,'ISOTOPE : ',8A4,/,40X,'DE
TECTOR : ',16A4,///)
WRITE(6,103)Z,A,EMAX
103 FORMAT(21X,'Z = ',F5.2,11X,'A = ',F5.1,11X,'EMAX = ',F7.4,/)
WRITE(6,104)ZB,ZM,ZBP,ZMP,NPR
104 FORMAT(21X,'ENERGY CALIBRATION:',/,42X,'E = ',E14.7,' + ',E14.7,'*
N FOR N<NPR',/,42X,'E = ',E14.7,' + ',E14.7,'*N FOR N>NPR',/,42X,'
WHERE N IS THE CHANNEL NUMBER AND NPR = ',I3,/)
WRITE(6,108)CAV,CAVG
108 FORMAT(21X,'AVERAGE CHANNEL NUMBER IN MEASURED DISTRIBUTION = ',F1
0.4,/,21X,'MEASURED AVERAGE ENERGY = ',F6.4,/)
WRITE(6,105)EAVG,SUM
105 FORMAT(21X,'CALCULATED AVERAGE ENERGY = ',F6.4,/,21X,'TOTAL NUMBER
OF COUNTS IN PULSE HEIGHT DISTRIBUTION = ',D17.10,/)
WRITE(6,106)FAVG
106 FORMAT(/,21X,'SIMPLE NONRELATIVISTIC MODEL AVERAGE ENERGY = ',F6.4,
/,/)
WRITE(6,107)SINT,RSINT
107 FORMAT(/,21X,'NONRELATIVISTIC INTEGRAL = ',E14.7,/,21X,'RELATIVIST
IC INTEGRAL = ',E14.7,/)
WRITE(6,50)
50 FORMAT('1',//,17X,'ENERGY',29X,'THEORETICAL',29X,'MEASURED',///)
IF(Z.EQ.35.C.AND.A.EQ.90.0)GO TO 303
C      **
C      **

```

```

C      **      NORMALIZE THEORETICAL SPECTRA TO THE      **
C      **      MEASURED PULSE HEIGHT DISTRIBUTION      **
C      **
      DO 191 I=1,NCHNLS
      E=XEN(I)
      WM(I)=(E/C.51098)+1.0
      IF(E.LT.EMAX)GO TO 189
      DIST(I)=0.0
      GO TO 191
189 IF(E.GE.0.0)GO TO 190
      E=0.0
190 DIST(I)=XR(Z,W0,A,E,F,U)
191 CCNTINUE
      CALL FFD(Z,W0,A,XEN,FD,1500,NCHNLS)
      TOT=0.0
      DO 192 J=8,NCHNLS
192 TCT=TCT+DIST(J)
      DO 193 K=1,NCHNLS
193 XSPECT(K)=XSPECT(K)*TOT
303 CZR=0.0
C      **
C      **      CALCULATE THE FERMI PLOT VALUES      **
C      **
      DO 195 I=1,NCHNLS
198 IF(WM(I).GT.1.0)GO TO 187
      XKM(I)=1.0
      XKC(I)=1.0
      GO TO 195
187 IF(WM(I).LE.WC)GO TO 194
      XKC(I)=0.0
      GO TO 188
194 XKC(I)=(DIST(I)/((WM(I)*FD(I))*(((WM(I)**2.0)-1.0)**0.5)))**0.5
188 XKM(I)=(XSPECT(I)/((WM(I)*FD(I))*(((WM(I)**2.0)-1.0)**0.5)))**0.5
195 CCNTINUE
      I=1.0
      DO 196 J=8,NCHNLS,8
      YKC(I)=XKC(J)
      YKM(I)=XKM(J)
      YWM(I)=WM(J)
196 I=I+1
      NF=(NCHNLS/8)+2
C      **
C      **      OUTPUT TABULAR INFORMATION      **
C      **
      DO 80 K=9,NCHNLS,8
      80 WRITE(6,200)XEN(K),DIST(K),XSPECT(K)
200 FORMAT(15X,F10.8,25X,E14.7,25X,E14.7,/)
      IF(Z.EQ.39.C.AND.A.EQ.90.0)GO TO 304
      JC=1
      NPAGE=(EMAX/DIV)/23+1
      DO 82 I=1,NPAGE
      WRITE(6,52)
252 FORMAT('1',/)
      WRITE(6,51)
251 FORMAT(8X,'E',12X,'W',10X,'FR(Z,W)',12X,'AR(E)',12X,'FERMI R.',10X,'
      FN(Z,W)',12X,'NN(E)',11X,'FERMI N',/)
      J=JC
      DO 81 JZ=1,23

```

```

      WRITE(6,201)XARRAY(J),WARRAY(J),FR(J),ZARRAY(J),VARRAY(J),FN(J),YA
      RRAY(J),UARRAY(J)
201  FORMAT(5X,F6.4,5X,F10.8,6(3X,E14.7),/)
      J=J+1
      IF(J.GT.ND)GO TO 83
81  CCNTINUE
      JC=J
82  CCNTINUE
83  DDR=0.0
304  DZR=0.0
      WRITE(6,53)
53  FORMAT('1',/,18X,'W',17X,'CALCULATED FERMI PLCT VALUE',10X,'MEASUR
      ED FERMI PLCT VALUE',///)
      DO 85 J=8,NCHNLS,8
85  WRITE(6,203)WM(J),XKC(J),XKM(J)
203  FORMAT(14X,F10.8,21X,E14.7,23X,E14.7,/)
      IF(Z.EQ.39.C.AND.A.EQ.90.0)GO TO 305
      I=1
      DO 86 J=8,NCHNLS,8
      YKC(I)=XKC(J)
      YKM(I)=XKM(J)
      YWM(I)=WM(J)
      I=I+1
86  CCNTINUE
      DO 87 J=1,ND
      XA(J)=XARRAY(J)
      YA(J)=YARRAY(J)
87  ZA(J)=ZARRAY(J)
      IF(NCP.EQ.0)GO TO 901
      CALL CARDS(YKM,YKC,YWM,130,IC,TITLE)
901  DRT=0.0
305  DZR=0.0
C      **
C      **          PLOT TABULAR INFORMATION          **
C      **
      IF(NP.NE.0)GO TO 404
      IF(IPLCT.GT.0)GO TO 401
      CALL PLCTS(8UF,1000)
      CALL PLCT(2.0,2.0,-3)
      CALL PLCT(2.0,2.0,23)
401  IPLCT=1
      DO 403 I=1,NCHNLS
      X(I)=XEN(I)
      R(I)=XSPECT(I)
403  Y(I)=DIST(I)
      CALL SCALE(Y,6.0,NCHNLS,1)
      CALL SCALE(X,6.0,NCHNLS,1)
      CALL SCALE(R,6.0,NCHNLS,1)
      IF(R(N).GT.Y(N))GO TO 995
      R(N-1)=Y(N-1)
      R(N)=Y(N)
      CALL AXIS(0.0,0.0,6HENERGY,-6,6.0,0.0,X(N-1),X(N))
      CALL AXIS(0.0,0.0,6HNUMBER,6,6.0,90.0,Y(N-1),Y(N))
      GO TO 996
995  Y(N-1)=R(N-1)
      Y(N)=R(N)
      CALL AXIS(0.0,0.0,6HENERGY,-6,6.0,0.0,X(N-1),X(N))
      CALL AXIS(0.0,0.0,6HNUMBER,6,6.0,90.0,R(N-1),R(N))

```

```
996 CALL PLCT(0.0,6.0,3)
      CALL PLCT(6.0,6.0,2)
      CALL PLCT(6.0,0.0,2)
      CALL LINE(X,Y,NCHNLS,1,0,0)
      CALL LINE(X,R,NCHNLS,1,0,0)
      CALL PLCT(12.0,0.0,-3)
      CALL KURIE(YKC,YKM,130,YWM)
      IF(Z.EQ.39.C.AND.A.EQ.90.C)GO TO 11
      NCN=NC+2
      CALL MCDELS(XA,ZA,YA,NCN,500)
404 DDQ=0.0
      GC TO 11
999 DCZ=0.0
      IF(NP.NE.0)GO TO 997
      CALL PLCT(0.0,0.0,999)
997 DCY=0.0
      STOP
      END
```



```

C      *****
C      **                                     **
C      **               SUBROUTINE MODELS               **
C      **                                     **
C      **               PLOTS THEORETICAL DISTRIBUTIONS **
C      **                                     **
C      *****
C
SUBROUTINE MODELS(X,Z,Y,N,M)
  DIMENSION X(M),Z(M),Y(M)
  NC=N-2
  CALL SCALE(X,6.0,NC,1)
  CALL SCALE(Y,6.0,NC,1)
  CALL SCALE(Z,6.0,NC,1)
  IF(Z(N).GT.Y(N))GO TO 20
  Z(N-1)=Y(N-1)
  Z(N)=Y(N)
  CALL AXIS(0.0,0.0,6HENERGY,-6,6.0,0.0,X(N-1),X(N))
  CALL AXIS(0.0,0.0,15HRELATIVE NUMBER,15,6.0,90.0,Y(N-1),Y(N))
  GO TO 21
20 Y(N-1)=Z(N-1)
  Y(N)=Z(N)
  CALL AXIS(0.0,0.0,6HENERGY,-6,6.0,0.0,X(N-1),X(N))
  CALL AXIS(0.0,0.0,15HRELATIVE NUMBER,15,6.0,90.0,Z(N-1),Z(N))
21 CALL PLCT(0.0,6.0,3)
  CALL PLCT(6.0,6.0,2)
  CALL PLCT(6.0,0.0,2)
  CALL NEWPEN(2)
  CALL LINE(X,Y,NC,1,0,0)
  CALL SYMBCL(3.1,5.5,0.14,'BLUE-NONRELATIVISTIC',0.0,20)
  CALL NEWPEN(1)
  CALL LINE(X,Z,NC,1,0,0)
  CALL SYMBCL(3.1,5.2,0.14,'BLACK-RELATIVISTIC',0.0,18)
  CALL PLCT(12.0,0.0,-3)
  RETURN
END

```

```

C      *****
C      **                                     **
C      **               SUBROUTINE KURIE               **
C      **                                     **
C      **               CONSTRUCTS FERMI PLOTS               **
C      **                                     **
C      *****
C
SUBROUTINE KURIE(A,B,N,C)
DIMENSION A(N),B(N),C(N)
NC=N-2
C(N-1)=1.0
C(N)=C.9
CALL SCALE(A,6.0,NC,1)
CALL SCALE(B,6.0,NC,1)
IF(A(N).GT.B(N))GO TO 20
A(N-1)=B(N-1)
A(N)=B(N)
CALL AXIS(0.0,0.0,1HW,-1,6.C,0.0,C(N-1),C(N))
CALL AXIS(0.0,0.0,17HFERMI PLCT VALUES,17,6.0,90.C,B(N-1),B(N))
GO TO 21
20 B(N-1)=A(N-1)
   B(N)=A(N)
   CALL AXIS(0.0,0.0,1HW,-1,6.0,0.0,C(N-1),C(N))
   CALL AXIS(0.0,0.0,17HFERMI PLCT VALUES,17,6.0,90.0,A(N-1),A(N))
21 CALL PLCT(0.0,6.0,3)
   CALL PLCT(6.0,6.0,2)
   CALL PLCT(6.0,0.0,2)
   CALL LINE(C,A,NC,1,0,0)
   CALL LINE(C,B,NC,1,-1,2)
   CALL PLCT(12.0,0.0,-3)
RETURN
END

```

```

C      *****
C      **                                     **
C      **               SUBROUTINE FFD               **
C      **                                     **
C      **          CALCULATES FERMI FUNCTION          **
C      **                                     **
C      *****
C
SUBROUTINE FFD(Z,WC,A,XEN,FD,M,NCHNLS)
DIMENSION XEN(M),FD(M)
ALPHA=0.0072972
PI=3.141592654
LAMBDA=386.0
VC=1.13*(ALPHA**2)*(Z**(4.0/3.0))
S=(1.0-(ALPHA*Z)**2)**0.5-1.0
R=((1.123*A**(1.0/3.0))-(0.941*A**(-1.0/3.0)))
DO 52 I=1,NCHNLS
E=XEN(I)
W=(E/C.51098)+1.0
W=W-VC
IF(W.GT.1.0)GO TO 51
FD(I)=1.0
W=1.0
GO TO 52
51 V=(ALPHA*Z*W)/((W**2-1.0)**0.5)
P=4.0*(1.0+S/2.0)/([ABS(GAMMA(3.0+2.0*S))]**2)
B=(2.0*R/LAMBDA)**(2.0*S)
C=EXP(PI*V)*((W**2-1.0)**S)
D=((PI*V)/SINH(PI*V))*(((ALPHA*Z)**2+0.25)*(W**2)-0.25)**S)
FD(I)=P*B*C*D
52 CONTINUE
RETURN
END

```

0000000000

```

*****
**
**
**          FUNCTION XR
**
**
**  CALCULATES BETA SPECTRA BASED ON THE
**  ASSUMPTION THAT THE PARTICLES ARE REL-
**  ATIVISTIC
**
**
*****

```

```

FUNCTION XR(Z,W0,A,E,F,U)
INTEGER U
C=2.9579EC8
XE=1.6021E-19
ALPHA=0.CC72972
PI=3.141592654
W=(E/0.51098)+1.0
LAMBDA=386.C
VC=1.13*(ALPHA**2)*(Z**(4.0/3.0))
S=(1.0-(ALPHA*Z)**2)**0.5-1.C
R=((1.123*A**((1.0/3.0)))-(0.941*A**(-1.C/3.0)))
W=W-VC
IF(W.GT.1.0)GO TO 51
F=1.0
W=1.0
GC TO 52
51 V=(ALPHA*Z*W)/((W**2-1.0)**0.5)
P=4.0*(1.C+S/2.0)/((ABS(GAMMA(3.0+2.0*S)))**2)
B=(2.0*R/LAMBDA)**(2.0*S)
C=EXP(PI*V)*((W**2-1.0)**S)
D=((PI*V)/SINH(PI*V))*(((ALPHA*Z)**2+0.25)*(W**2)-0.25)**S)
F=P*B*C*D
W=W+V0
XR=F*((W-VC)**2-1.0)**0.5*(W-VC)*((WC-W)**2)
GC TO 53
52 XR=0.0
53 DDQ=0.C

CHECK FOR DEGREE OF FORBIDDENNESS

IF(U.EQ.0)GC TO 55
IF(U.EQ.1)GC TO 54
SN=(W**2-1.0)**2+(WC-W)**4+(10.0/3.0)*(W**2-1.0)*((WC-W)**2)
GC TO 56
54 SN=(W**2-1.0)+(WC-W)**2
GC TO 56
55 SN=1.0
56 XR=XR*SN
RETURN
END

```

```
C
C
C *****
C **                                     **
C **                                FUNCTION XN                               **
C **                                **                                       **
C **    CALCULATES BETA SPECTRA BASED ON THE                             **
C **    ASSUMPTION THAT THE PARTICLES ARE NOT                            **
C **    RELATIVISTIC                                                       **
C **                                **                                       **
C *****
C
C FUNCTION XN(Z,W0,E,F,U)
C INTEGER U
C ALPHA=0.0072972
C C=2.9979EC8
C PI=3.1416
C W=(E/0.51C98)+1.0
C IF(W.GT.1.0) GO TO 51
C F=0.0
C W=1.0
C GO TO 52
51 ETA=(W**2.0-1.0)**0.5
C V=(C*ETA)/(1.0+ETA**2.0)
C XNU=(Z*ALPHA*C)/V
C F=(Z*PI*XNU)/(1.0-EXP(-2*PI*XNU))
52 XN=F*W*((W**2.0-1.0)**0.5)*((WC-W)**2.0)
C
C CHECK FOR DEGREE OF FORBIDDENNESS
C
C IF(U.EQ.0)GC TO 55
C IF(U.EQ.1)GC TO 54
C SN=(W**2-1.C)**2+(WC-W)**4+(10.0/3.0)*(W**2-1.C)*((WC-W)**2)
C GC TO 56
54 SN=(W**2-1.C)+(WC-W)**2
C GC TO 56
55 SN=1.0
56 XN=XN*SN
C RETURN
C END
```

C
C
C
C
C
C
C
C
C
C
C

```
*****
**                                     **
**                               SUBROUTINE CARDS                               **
**                                     **
**       PUNCHES CARDS USING FERMI PLOT VALUES                             **
**                                     **
*****
```

```
      SUBROUTINE CARDS(YKM,YKC,YWM,M,ID,TITLE)
      DIMENSION YWM(M),YKM(M),YKC(M),TITLE(9)
      N=M-2
      WRITE(7,100)ID,(TITLE(J),J=1,8)
100  FORMAT(16,5X,8A4)
      DO 20 J=1,N
      20  WRITE(7,450)YWM(J),YKM(J),YKC(J)
450  FORMAT(F10.7,1X,F10.7,1X,F10.7)
      RETURN
      END
```

```
C
C
C *****
C **                                     **
C **                               SUBROUTINE SRY                                **
C **                                     **
C **      CALCULATES THEORETICAL SPECTRUM FOR                                **
C **      SR-90 AND Y-90 COMBINED                                           **
C **                                     **
C *****
C
SUBROUTINE SRY(Z,A,WQ,XEN,M,NCHNLS,DIST,XSPECT,DIST1,DIST2,U,FD,WM)
REAL*8 XEN(M),XSPECT(M),DIST(M)
DIMENSION DIST1(M),DIST2(M)
DIMENSION FD(M),WM(M)
REAL*8 TCT
REAL*8 TOD1,TOD2
INTEGER U
EMAX=(WC-1.0)*0.51098
DO 10 I=1,NCHNLS
E=XEN(I)
IF(E.LT.EMAX)GO TO 9
DIST2(I)=0.0
GO TO 10
9 DIST2(I)=XR(Z,WQ,A,E,F,U)
10 CONTINUE
TOD2=0.0
DO 60 I=1,NCHNLS
60 TOD2=TOD2+DIST2(I)
DO 70 I=1,NCHNLS
70 DIST2(I)=DIST2(I)/TOD2
A1=90.0
Z1=40.0
WQ1=5.46357
EMAX1=(WQ1-1.0)*0.51098
DO 20 I=1,NCHNLS
E=XEN(I)
IF(E.LT.EMAX1)GO TO 19
DIST1(I)=0.0
GO TO 20
19 DIST1(I)=XR(Z1,WQ1,A1,E,F,U)
20 CONTINUE
TOD1=0.0
DO 80 I=1,NCHNLS
80 TOD1=TOD1+DIST1(I)
DO 90 I=1,NCHNLS
90 DIST1(I)=DIST1(I)/TOD1
DO 40 I=1,NCHNLS
40 DIST(I)=(0.9998*DIST2(I))+DIST1(I))/1.9998
CALL FFD(Z1,WQ1,A1,XEN,FD,1500,NCHNLS)
DO 50 I=1,NCHNLS
50 WM(I)=(XEN(I)/0.51098)+1.0
RETURN
END
```

APPENDIX C: KURIE PLOTS

To provide illustration, seven Kurie plots are presented, the plots shown are for the measured and theoretical beta particle energy distributions provided in Section V.

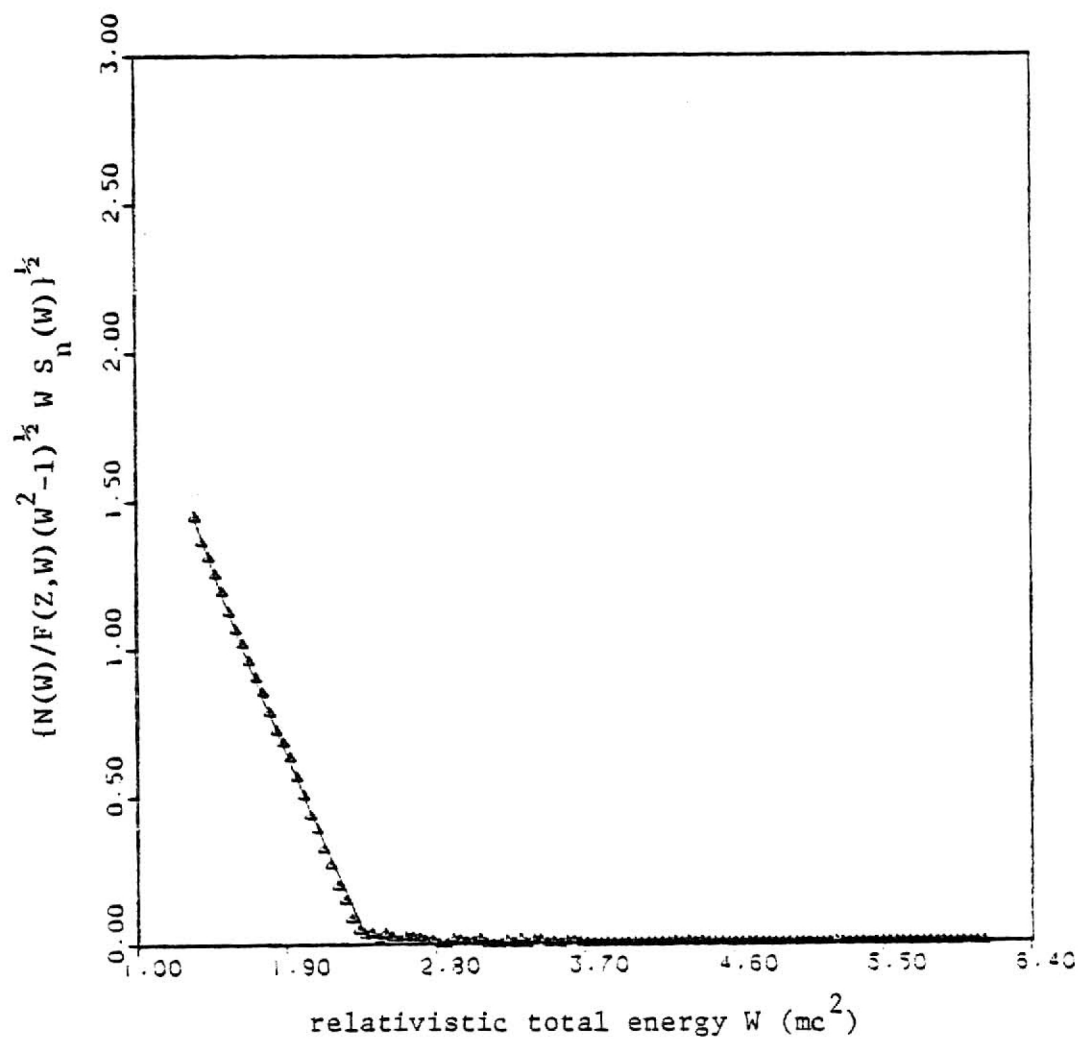


FIG. C.1. Theoretical and measured Kurie plots of ^{36}Cl . Measurements were performed using a 10.2 mm thick BC-400 organic scintillator. Solid line is based on theoretical calculations and discrete points are drawn from the measured distribution.

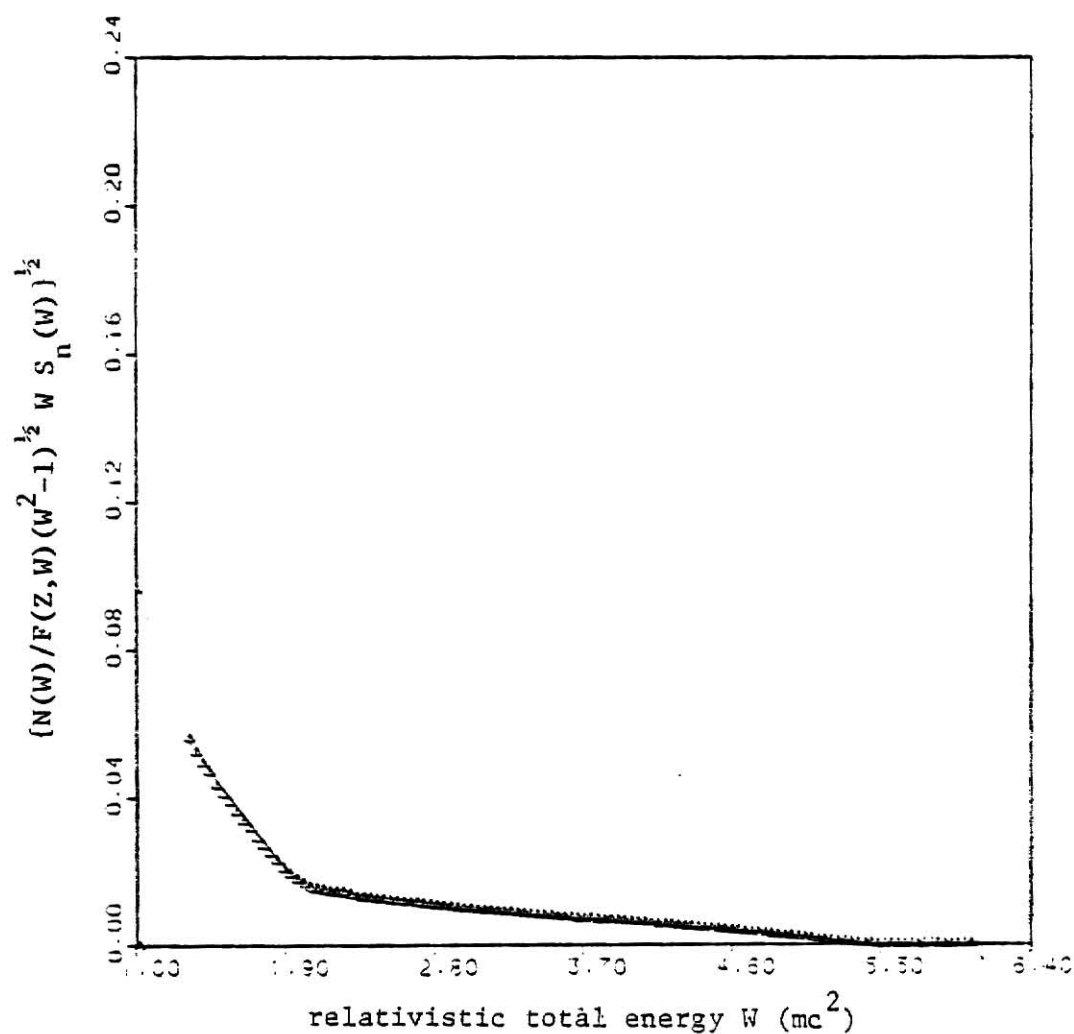


FIG. C.2. Theoretical and measured Kurie plots of the combined Sr-90/Y-90 beta particle energy distributions. Measurements were performed using a 10.2 mm thick BC-400 organic scintillator. Solid line is based on theoretical calculations and discrete points are drawn from the measured distribution.

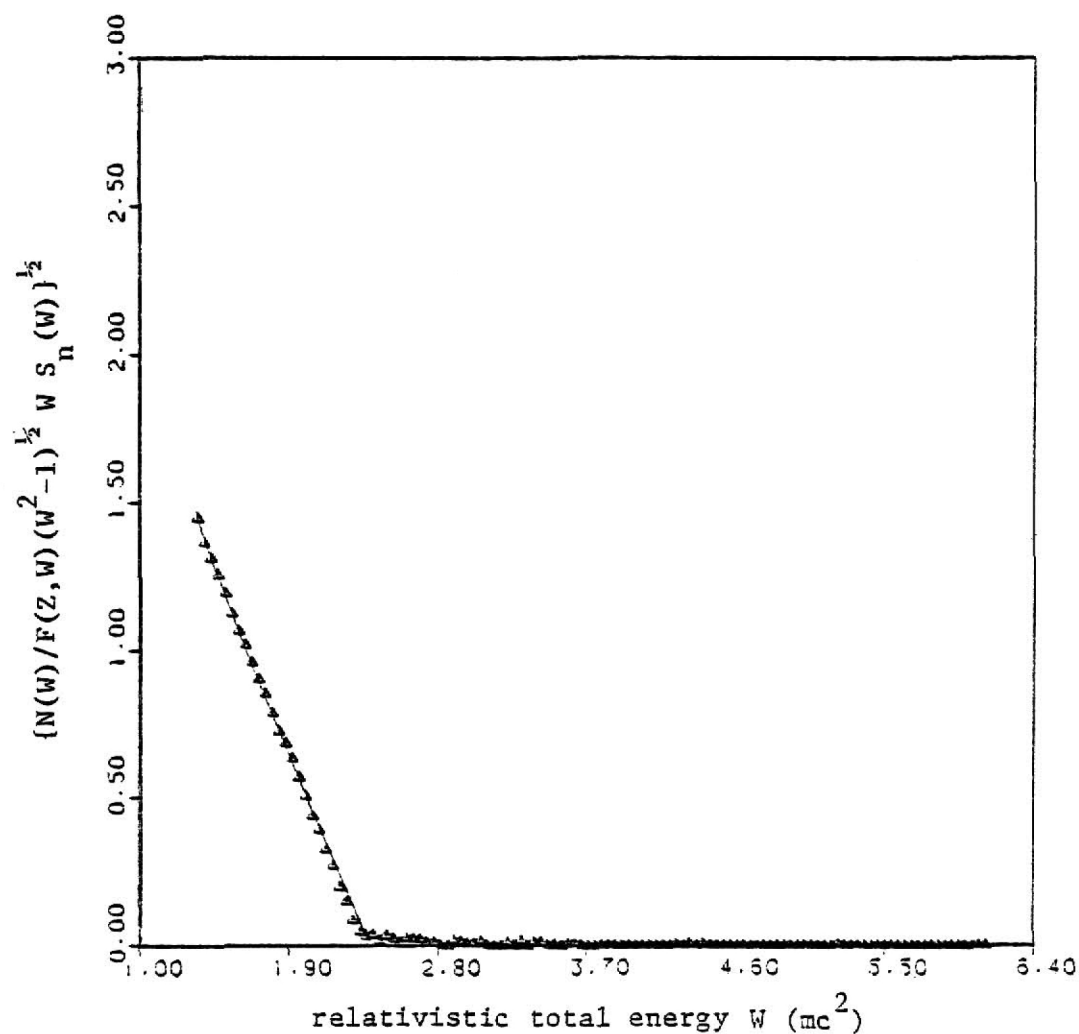


FIG. C.3. Theoretical and measured Kurie plots of ^{36}Cl . Measurements were performed using a 5.08 mm thick BC-400 organic scintillator. Solid line is based on theoretical calculations and discrete points are drawn from the measured distribution.

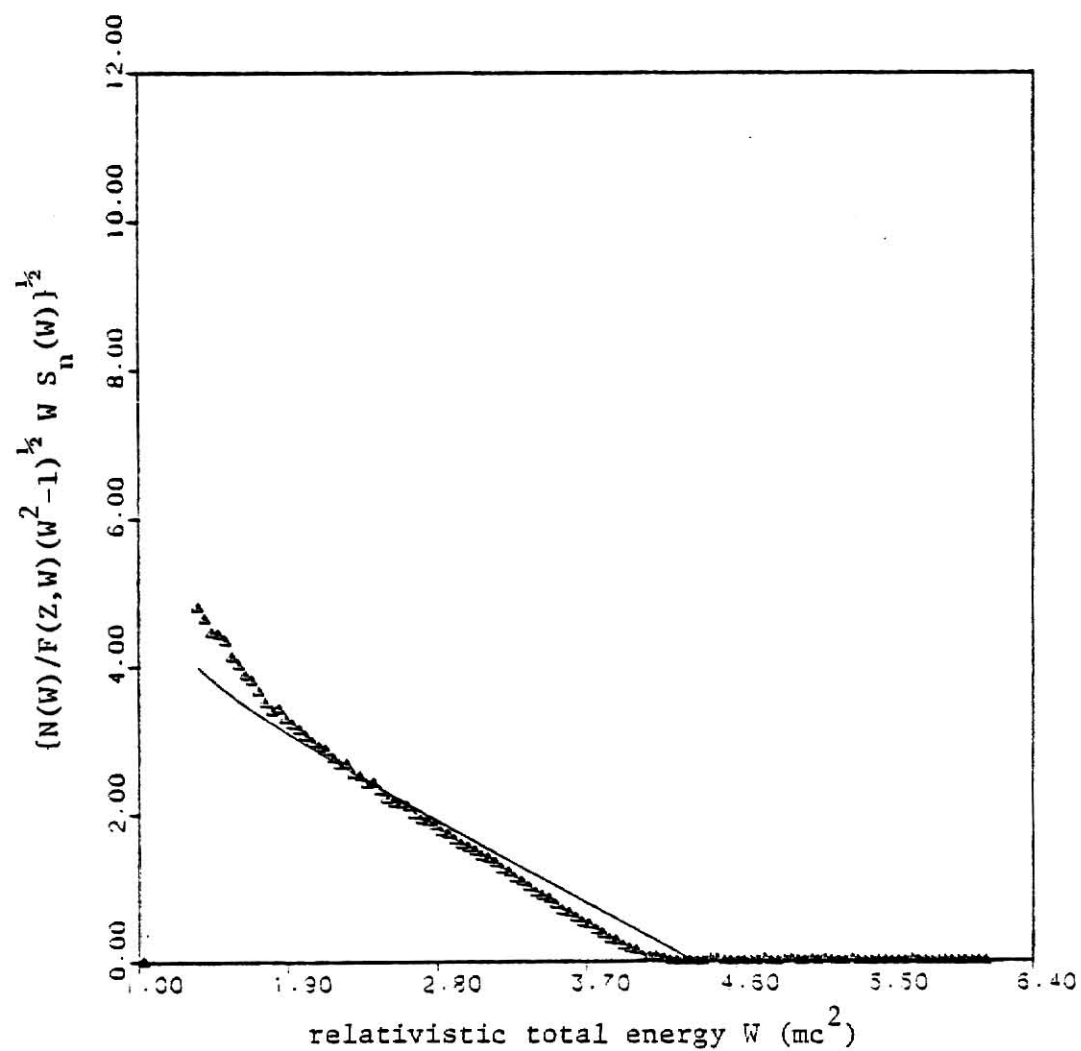


FIG. C.4. Theoretical and measured Kurie plots of ^{32}P . Measurements were performed using a 5.08 mm thick BC-400 organic scintillator. Solid line is based on theoretical calculations and discrete points are drawn from the measured distribution.

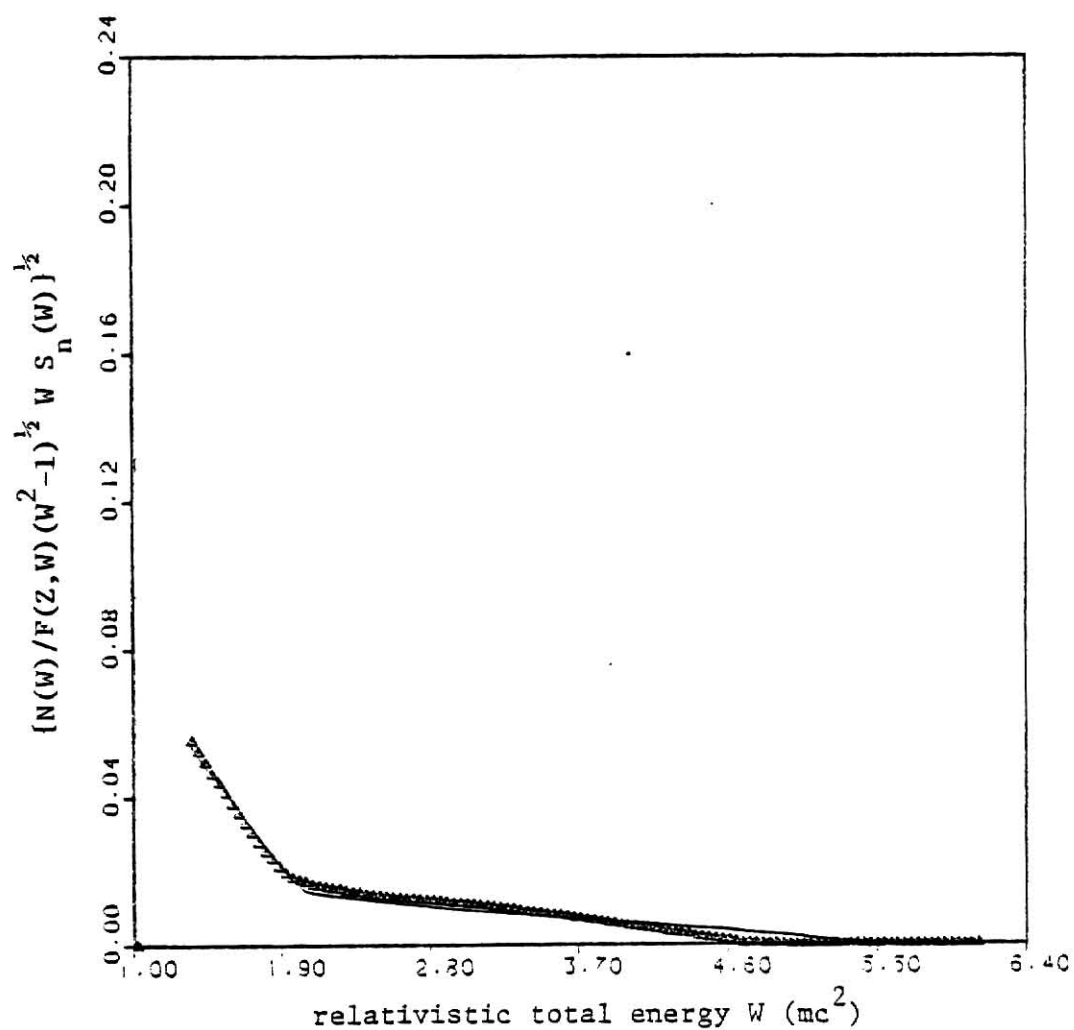


FIG. C.5. Theoretical and measured Kurie plots of the combined Sr-90/Y-90 beta particle energy distributions. Measurements were performed using a 5.08 mm thick BC-400 organic scintillator. Solid line is based on theoretical calculations and discrete points are drawn from the measured distribution.

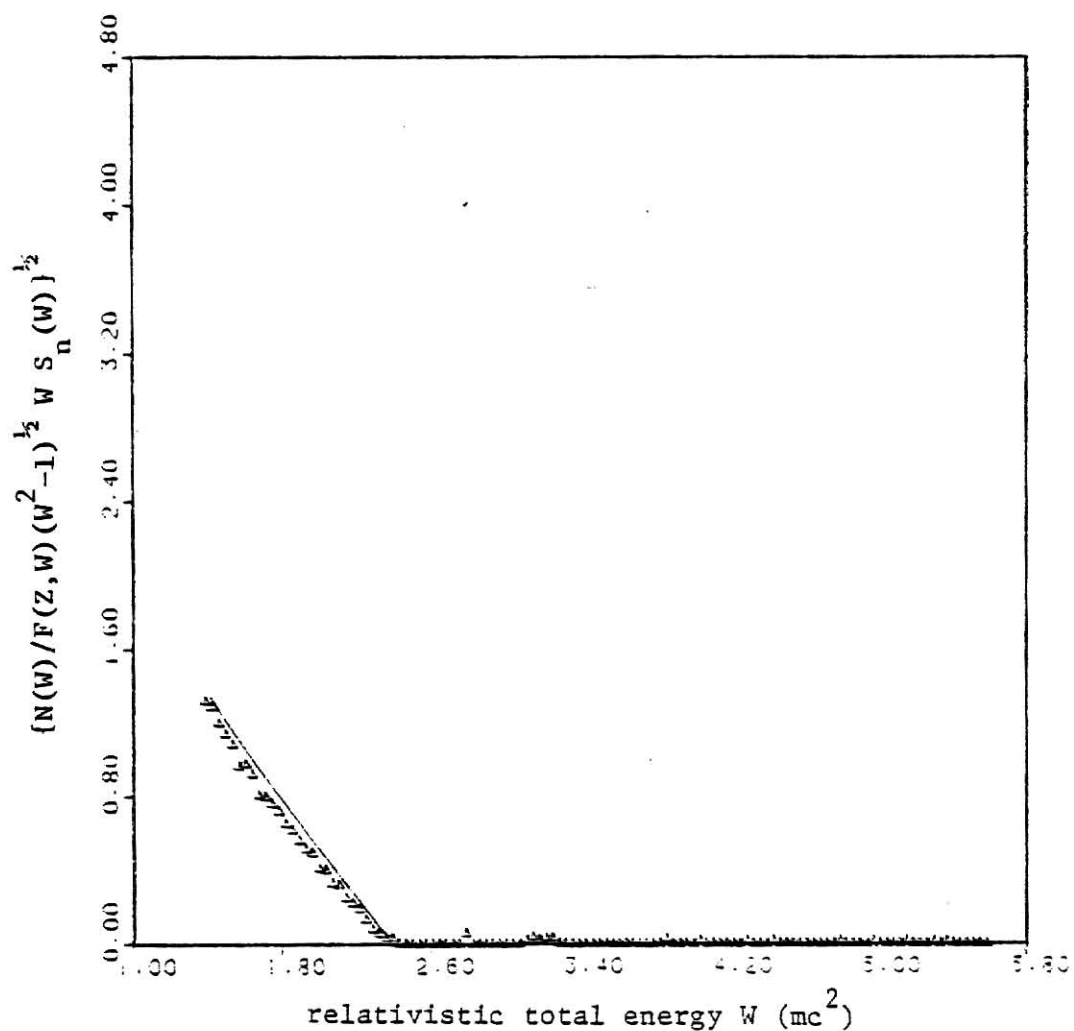


FIG. C.6. Theoretical and measured Kurie plots of ^{36}Cl . Measurements were performed using a 5 mm thick planar HPGe detector. Solid line is based on theoretical calculations and discrete points are drawn from the measured distribution.

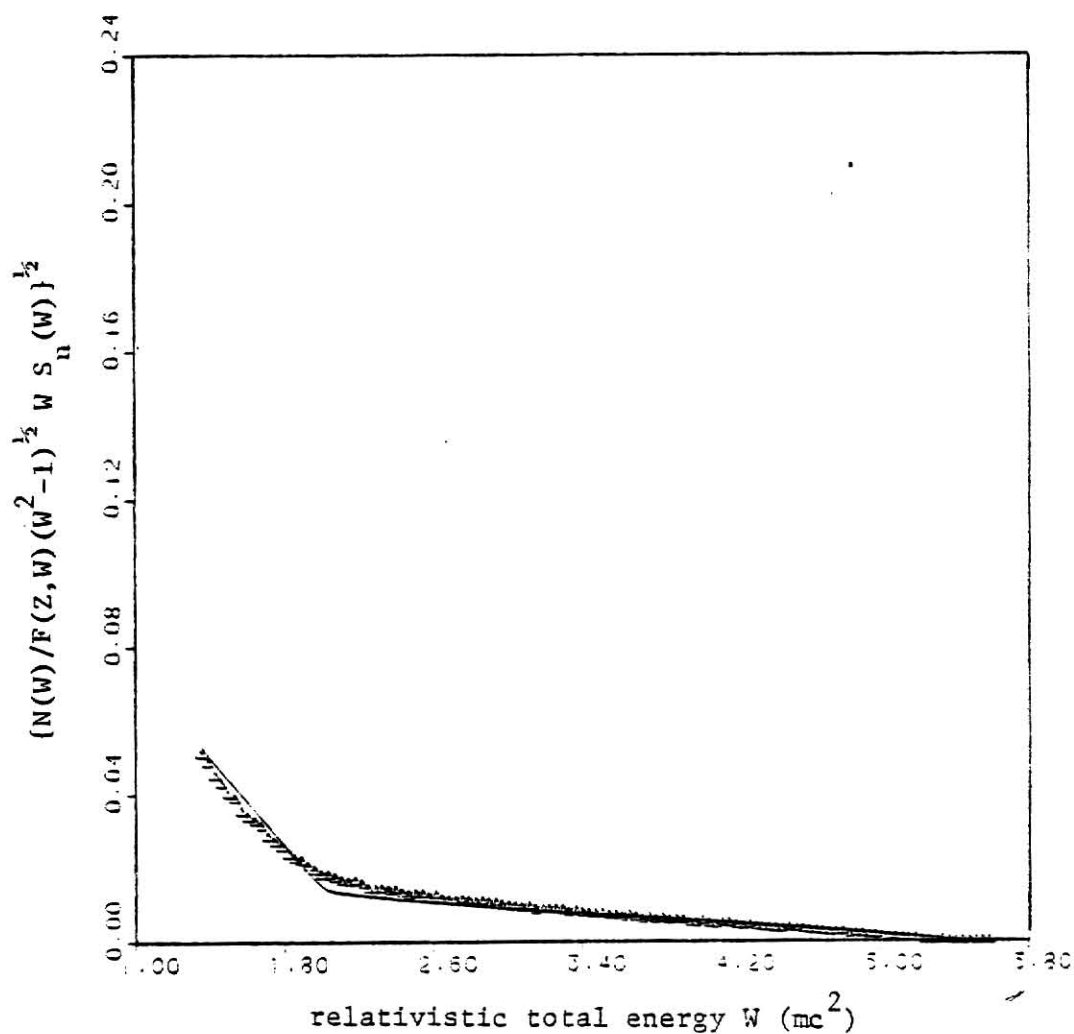


FIG. C.7. Theoretical and measured Kurie plots of the combined Sr-90/Y-90 beta particle energy distributions. Measurements were performed using a 5 mm thick planar HPGe detector. Solid line is based on theoretical calculations and discrete points are drawn from the measured distribution.

EVALUATION OF PLASTIC SCINTILLATOR AND HIGH-PURITY
GERMANIUM DETECTORS FOR USE IN BETA SPECTROSCOPY

by

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B.S., Kansas State University, 1981

AN ABSTRACT OF A MASTER'S THESIS

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ABSTRACT

Two beta spectrometer systems utilizing different detector types were constructed and evaluated. One system utilized BC-400 plastic scintillators with thicknesses ranging from 5.08-20.32 mm, and a diameter of 50.8 mm. The other system incorporated a planar high-purity germanium detector of 5 mm diameter and 25 mm² active area. The detectors possessed entrance windows of 9µm Al and 2 mils Be, respectively. Detector responses to incident beta, gamma and conversion electron radiations were obtained. Detector evaluation was based principally upon comparison of theoretical and measured beta particle energy distributions for selected beta minus emitting isotopes with endpoint energies of 0.1674-2.288 MeV. Results indicate that plastic scintillator based systems can yield accurate average and endpoint energy measurements when calibrated using the endpoints of known beta spectra. HPGe detector data were most useful for endpoint energy determination, since backscatter induced distortion limited the direct measurement of accurate average energies. Examinations of the effect of increasing absorber thickness upon transmitted beta spectra, and of the natural uranium beta particle energy distribution, were performed.