AN INVESTIGATION OF A STATISTICAL APPROACH FOR PROJECT SELECTION

by

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CHAPTER 1

INTRODUCTION

1.0 Purpose

Around 1970, it was discovered that the "capital asset pricing model" could be applied to capital budgeting problems faced by the firm. The capital asset pricing model is a mathematical model in which the random rate of return on an individual security in the market is related in a linear fashion to the "risk" of that security. The model was originally developed by Sharpe (17), Lintner (10) and Mossin (13), from a suggestion made in some earlier work by Markowitz (11) on the return from a portfolio of investments.

In analyzing projects, the rate of return for a project can be compared to the rate of return for the firm demanded at the risk level of the project. In 1976, Campbell (5) applied this methodology to the non-public firm; that is, those firms whose stock is not traded on a stock exchange. However, his project selection criterion used only point estimates because the methodology did not sample the future project returns. In order to make probability statements, it is necessary to sample the project so that at a specified level of confidence, the project can be compared to the firm or surrogate firm. It is the purpose of this thesis to develop a statistical approach to the capital budgeting problem that will provide probability statements concerning the "success" or "failure" of a project, in relation to the firm's goal of maximizing corporate profits.

1.1 Problem

The problem to be investigated by this thesis is how to make probability statements concerning the firm's decision to accept or reject a project. The capital asset pricing model can be used as a resource allocation method, and can be applied to privately-owned firms as well as publicly-owned companies. The capital asset pricing model is the basis for developing a selection criterion for comparing a prospective investment-type project to the market determined cost of raising the necessary capital. The method insures an increase in the expected net present value of the firm, if the selection of projects is made in accordance to the criterion. It is desirable to determine the probability distribution of the return criterion representing the project so that some measure of the attendant risk can be made. Only by determining this distribution for the project can statements be made concerning the possible outcomes of this future project with respect to the firm. Therefore, the examination of the concept of a statistical approach to the capital asset pricing model is the problem undertaken by this thesis.

1.2 Literature Survey

1.2.1 Development of the Capital Asset Pricing Model. In 1952 Markowitz (11) developed an analytical method that explicitly took into account the uncertainty that is associated with the future returns on a portfolio of investments. This "risk" was measured by the variance (or semivariance) of the return rate of a security, which also included a correlational term to express the covariance of the security's return rate with the return rates of all possible pairs of securities in a portfolio.

Sharpe (17), Lintner (10) and Mossin (13) each developed a mathematical model, based on Markowitz's theory, that related the risk premium required by a security to the risk premium required by the securities market itself. Fama (6), in 1968, showed that all of these models were essentially the same, and that a single capital asset model can be stated in the form:

$$E_i = R_f + \beta_i (E_M - R_f) \qquad (1.1)$$

where

E_i = expected return on the ith security

R_f = the "risk free" interest rate (assumed constant)

β_i = volatility of security i

E_M = expected return of the market portfolio.

The volatility β_i is the ratio of the covariance of the ith security and the market divided by the variance of the market portfolio. The market portfolio is the portfolio which contains all of the securities that are available for purchase. Thus, β_i relates each security to the variance of the market portfolio and thus establishes an appropriate measure of the risk encountered by each security.

R_f is the "risk-free" rate of interest assumed to be a constant. A more suitable term would be the "default free asset". An investor can invest capital at this rate of interest with virtually no risk of receiving a return on his investment any lower than the "risk-free" rate.

To date, there are two fundamental methodologies for evaluating investment opportunities. These are based on either deterministic or utilitarian assumptions. Both methods, being only mathematical models of a real situation, are approximations and have limitations. However, some of the more serious limitations can be overcome by capital asset pricing theory and some basic assumptions. Jensen (8) has reviewed the earlier works of Markowitz (11), Sharpe (17), Lintner (10) and Mossin (13), and states that the asset pricing model either explicitly or implicitly makes the following general assumptions:

1. All investors are single-period expected utility-of-terminal-wealth maximizers who choose among alternative portfolios on the basis of the mean and standard deviation of return. This means that there is no compounding of any factors involved.

2. All investors can borrow or lend an unlimited amount of money at a given risk-free rate of interest with no restrictions on the short sales of any assets.

3. All investors have identical subjective estimates of the means, variances and covariances of return among all assets.

4. All assets are perfectly divisible and perfectly liquid. That is, any amount of an asset can be held and it can be sold and converted into money at will.

5. There are no taxes.

6. All investors are price takers. That is, there is no one investor who can purchase enough of an asset or have enough influence in the market to control or significantly affect the price of an asset.

7. The quantities of all assets are assumed given.

The capital asset pricing theory is based on the idea that an investor holds or purchases a portfolio of investments. The portfolio is assumed to have a random return rate, with finite mean and finite variance of the random return rate. The return rate of the portfolio can be calculated from the following expression:

$$R_p = \sum_{i=1}^{n} X_i R_i \qquad (1.2)$$

where

R_p = return of the portfolio (dollars)

R_i = return of security i (dollars)

X_i = proportion of portfolio p held in security i in terms of dollar amount of total portfolio, $\sum X_i = 1$.

The variance of the portfolio return is found by assuming that each security is correlated with every other security in the portfolio. The expression is

$$\sigma_p{}^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j \rho_{ij} \sigma_i \sigma_j \tag{1.3}$$

where

$\sigma_p{}^2$ = variance of return rate of portfolio

n = number of securities in the portfolio

X_i = proportion held in security i

X_j = proportion held in security j

$\rho_{i,j}$ = correlation of return rates of security

$\sigma_{i,j}$ = standard deviation of return rate of i & j.

Using the preference ordering (or utilitarian) approach for evaluating investment decisions, Bussey (3) showed that an individual's expected utility function for money, based on the assumption that a person maximizes expected utility instead of expected monetary value, is a series of concave upward indifference curves, in which his utility increases as expected return rate increases and the standard deviation of the return rate decreases. This is illustrated in Figure 1.1.

Using this principle a person who maximizes his expected utility will choose among investment alternatives by always choosing the portfolio that lies on the indifference curve having the greatest possible expected utility.

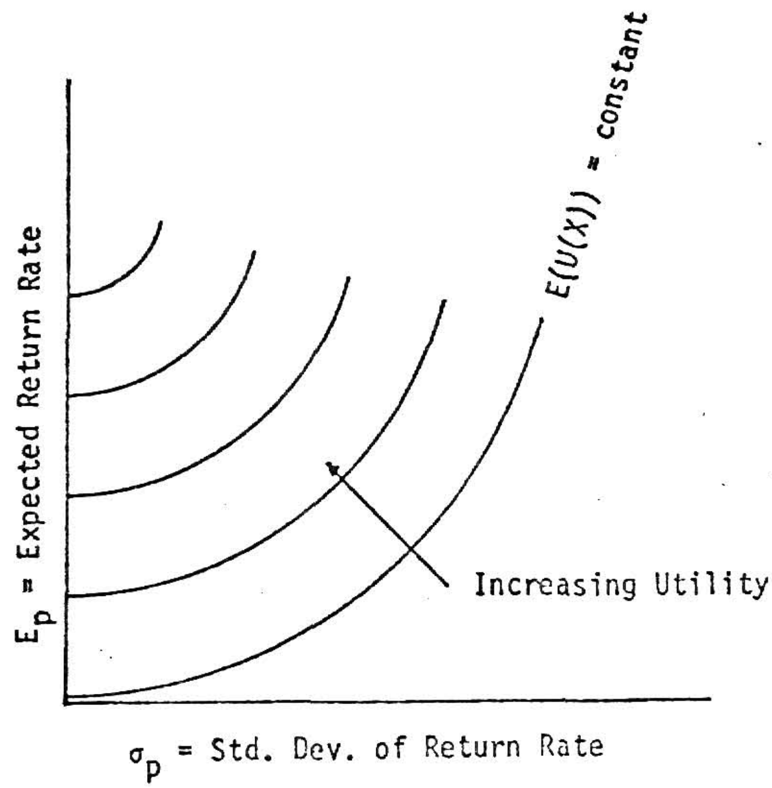

Figure 1.1. Typical decision maker's indifference curves

An "efficient" set of portfolios can be prescribed by recognizing that each security in the portfolio is related to every other security through the correlation coefficient. Since each security has its own expected return rate and standard deviation of return rate, a security can be represented on (E_p, σ_p) coordinates by a single point. Two securities can be combined to form a portfolio. As the portfolio "mix" of securities is altered, the correlation coefficient between each security in the E_p - σ_p coordinates will determine an infinite locus of points, representing all feasible portfolios.

The <u>efficient frontier</u> is a line that represents the dominant, or efficient, set of portfolios. Indifference curves, I_1, I_2, I_3, etc., can then be used to select the portfolio that will maximize an investors utility function. This set of indifference curves may vary from individual to individual due to different individual utility functions. The efficient frontier lies only between the horizontal and vertical tangents to the portfolio set boundary. (Figure (1.2)

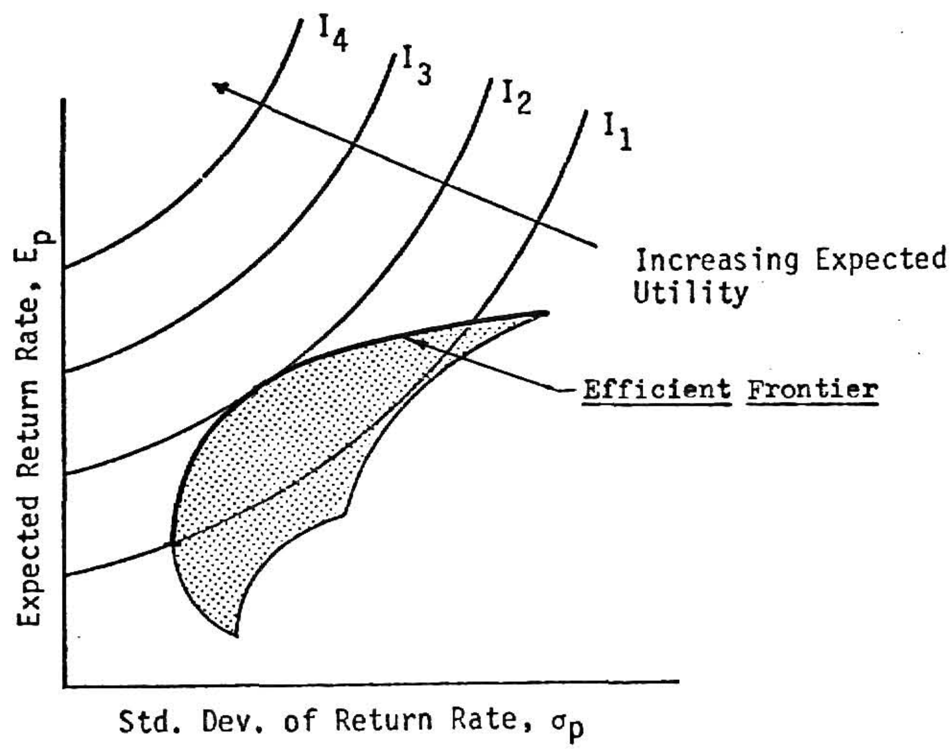

Figure 1.2. Maximization of Investor's Utility

The second major assumption underlying the capital asset pricing model states that all investors can borrow or lend unlimited amounts of money at a default free interest rate, denoted by R_f. Fama (6) points out that the combination of borrowing or lending money at R_f and on arbitrary security A must lie along a straight line through R_f and A, as shown in Figure 1.3. The accepted value of R_f lies between 5 & 6%. The interpretation of R_f is that at this rate, there is zero risk, or, the $\sigma_p = 0$. This default free rate of interest is approximated by short-term U. S. Treasury bills (91 days). In Figure 1.3, X is the proportion of available funds invested in the risk-free security at R_f, and (1-X) is the proportion invested in Security A.

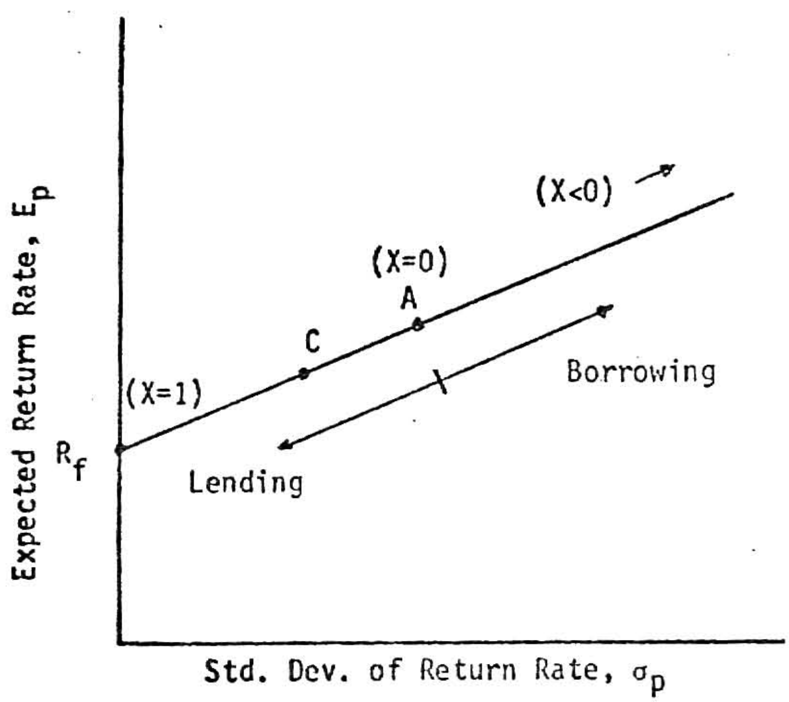

Figure 1.3. Combination of any portfolio C and the riskless asset

<u>1.2.2 The Capital Market Line.</u> Using the idea of the feasible set of portfolios and the concept of borrowing and lending at the risk-free rate of interest, the "capital market line" can be constructed by drawing a line from the point (R_f,0) (on E_p, σ_p coordinates) to a point that is tangent to the efficient frontier of portfolios. This point of tangency describes an "optimal" portfolio of risky securities. (See Figure 1.4)

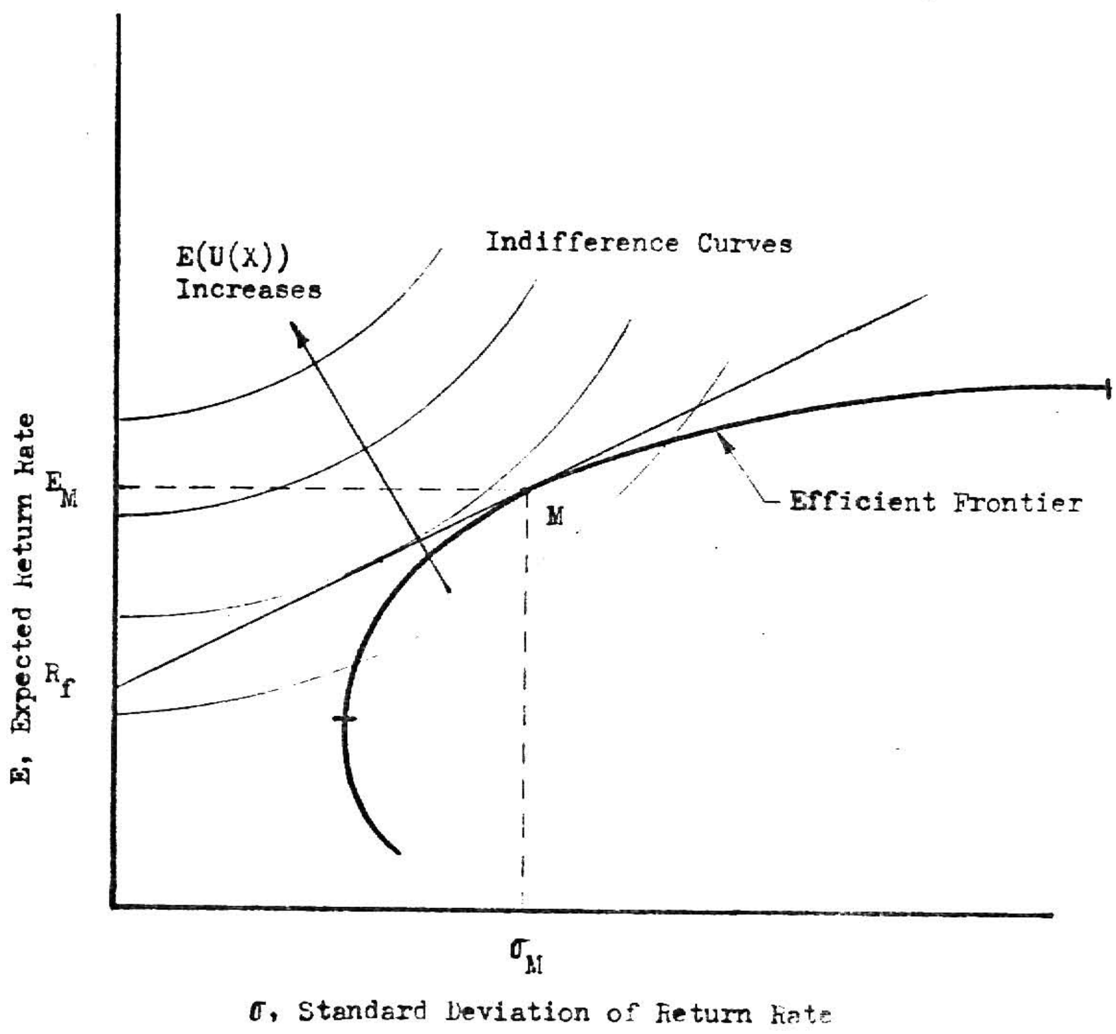

Figure 1.4. The Capital Market Line

This line, then, defines the optimal portfolio that all investors will want to hold. The indifference curves for a particular investor can now be placed on this graph to determine where his optimal investment occurs at along the market line, consisting of portfolio "M" and borrowing or lending at the risk free rate R_f. The main point here is that the proportion of funds invested in the portfolio M, (X_M), versus the proportion invested in the riskless asset R_f, ($1 - X_M$), is determined by each individual's indifference curve that is tangent to the capital market line.

The equation of this linear market line can be written as

$$E_M = R_f + \lambda \sigma_M \tag{1.4}$$

where

λ = a proportionality constant, which can be interpreted as the premium required for incurring additional risk for a finite variance portfolio; i.e., one having an assessable risk σ_p .

E_M = expected return of the market portfolio

σ_M = standard deviation or risk of the market portfolio.

This market line is an expression that simply states that one expects the return of the market portfolio to increase as the market uncertainty increases. The relationship was given by λ , the slope. This expression, however, fails to tell us anything about how the firm behaves with respect to its decisions concerning prospective investment projects. The firm does not invest in this "optimal" portfolio. On the contrary, the firm is interested in individual investments and/or individual portfolios.

Sharpe (18) has shown that if a security j is combined with the market portfolio M, the trade-off between expected return and risk for small changes in the amount of security j included in the market portfolio must equal the trade-off in the capital market as a whole.

This can be done by equating the slope of the capital market line and the curve joining the market portfolio M and the individual security j (the shape of the curve is determined by the correlation coefficient between security j and portfolio M).

The result is

$$E_j - R_f = (E_M - R_f) \cdot (C_{jM} / \sigma_M^2) \qquad (1.5)$$

where

E_j = Expected return of the jth project

C_{jM} = Covariance between the jth project and the market

σ_M^2 = Variance of the market portfolio.

The left hand side of the expression is the expected risk premium (above the risk-free rate) required for the jth security and the quantity $(E_M - R_f)$ is the expected risk premium of the market portfolio itself. The term (C_{jM} / σ_M^2) is denoted by the symbol β_j and is called the "volatility" of security j. β_j is taken as the measure of risk for an individual security as well as relating the individual security's expected risk premium to that of the market.

Substituting β_j into Equation (1.4), we have

$$(E_j - R_f) = \beta_j (E_M - R_f) \qquad (1.6)$$

Equation (1.6) is called the "Capital Asset Pricing Model".

The volatility of a security is an important concept in the CAPM. The market return, R_M, is taken to be a random variable with mean E_M and variance σ_M^2. Therefore, the expected market risk premium, $E_M - R_f$, is also a random variable. If a security has a β_j greater than 1, an increase in the random market risk premium will elicit an even greater increase in the expected risk premium of the security. Similarly, a larger decrease would occur in the expected return rate of the security in the event of a decrease in the market rate.

Since in the actual economy there are deviations away from this market line, one can treat the problem statistically and analyze it by the use of regression techniques. The market line can be estimated by a time series regression of the form:

$$(R_{jt} - R_f) = \alpha_j + \beta_j (R_{Mt} - R_f) + \epsilon_{jt} \tag{1.7}$$

where

R_{jt} = random return on sec. j at time t

R_{Mt} = return rate on market portfolio M at time t

R_f = the risk-free interest rate

α_j = an unknown intercept parameter

β_j = the volatility of security j with the market

ϵ_{jt} = normally distributed random error term representing the deviation away from the security line.

The parameter α_j & β_j must be estimated. The expression can be put in the form of a linear regression model and the parameters can be estimated by a linear regression function based on the least-squares method. This method gives point estimates of α & β for security j.

If the correct value of R_f is chosen, the estimate $\hat{\alpha}_j$ should be indistinguishable from zero. So, the capital asset pricing model is empirically testable, and the parameters R_f & β_j can be estimated by the above technique.

1.2.3 The Capital Market Line and Capital Budgeting. A proposed project selection criterion using the capital market line would compare the expected rate of return of the project to the expected rate of return of the firm. The firm tends to establish its own market "line" by the transactions occurring in the stock market. This market-established risk-return tradeoff line then becomes the selection criterion used to evaluate future projects. (Figure 1.5) The market line is established by passing a straight line through the points $(R_f,0)$ and $(\bar{R}_j, \sigma_j)$, on (E,σ) axis. The point $(\bar{R}_j, \sigma_j)$ is established by trading in the firm's own equity shares in the market; that is, the point is the mean return rate and standard deviation of the return rate of the firm itself, as established by the market. Proposed investment projects, to be examined for acceptance by the firm, have expected values $(\bar{R}_A, \sigma_A)$ lying above or below the market line. Those projects lying above the line would yield expected return rates greater than the rate required by the firm's shareholders in order to increase the value of the firm. Therefore the projects would be accepted. Projects having values $(\bar{R}_B, \sigma_B)$ lying below the line would yield expected return rates less than those required by the firm's shareholders, and hence would be rejected because they would fail to increase the value of the firm. The firm would be indifferent to those projects which lie on the firm's market line itself.

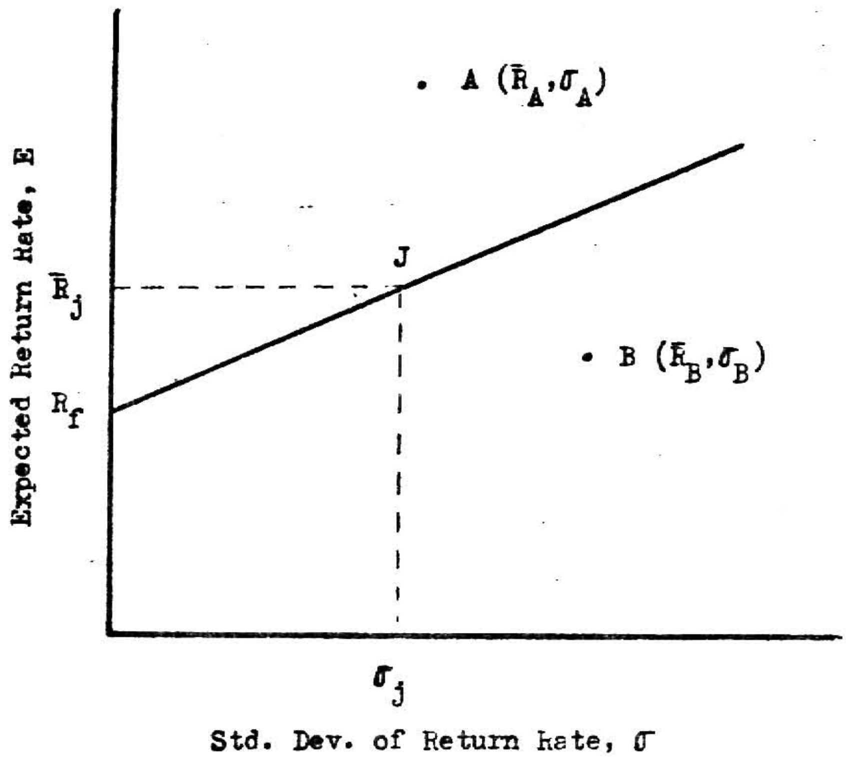

Figure 1.5. Combination of the firms own asset and the riskless asset.

Each project that the firm considers for acceptance has its own "riskiness". The market itself develops a current risk level also. The current risk level is representative of the current pricing of the firm's shares and its current and expected future dividend policies. As projects are accepted by the firm, the overall "riskiness" and equity price structure of the firm tends to be altered, thereby changing the risk level of the firm, which alters the firm's market trade-off point. Tuttle and Litzenberger (22) have proposed a method of risk adjustments to achieve risk-equivalency of the proposed project and the firm's residual return to equity. The purpose of this procedure is to leave the market price of the firm's shares unchanged. This approach utilizes the concept of borrowing or lending with equity capital to finance the candidate project.

As demonstrated in Figure 1.5, a project will be accepted if the slope of its line through the point (R_i , σ_i) and the riskless asset point (R_f,0) is greater than the slope of the firm's security line. The slopes of these lines can be calculated by the following equation

$$\frac{d\bar{R}_z}{d\,\sigma_z} = (\bar{R} - R_f)/\sigma_i \qquad (1.7)$$

Where

$\frac{d\,\bar{R}_z}{d\,\sigma_z}$ = rate of change of the return rate R_z with respect to the level of risk σ_z.

The slopes can now be compared and a decision rule for the firm can be established. The resulting criterion is:

Accept the project if $(R_i - R_f)/\sigma_i > (R_o - R_f)/\sigma_o$ (1.8)

Reject otherwise.

These relationships are known as "reward-to-variability" ratios and were originally developed by Sharpe (18). In order for a candidate project to be accepted by the firm, its reward-to-variability ratio must be equal to or greater than the ratio the firm currently exhibits from its existing projects.

Sharpe (18), however, questions the use of the reward-to-variability ratio as being the proper measure for evaluating a single security or a single project. He states:

> The reward-to-variability ratio is designed to measure the performance of a portfolio. The investor is presumed to have placed a substantial portion of his wealth in the portfolio in question. Variability is thus the relevant measure of the amount of risk actually borne. To evaluate the performance of a single security, or that of a portfolio constituting only part of an investors holdings, a different measure is needed. Variability will not adequately represent the risk actually borne. A more appropriate choice is volatility.

This, then, paved the way for the development of the reward-to-volatility criterion, which was originated by Treynor (21) and subsequently developed by Mossin (13). This criterion is similar to the reward-to-variability criterion, except that the volatility, β_i replaces the standard deviation, σ_i, in the denominator. Now, the reward-to-volatility criterion for project selection can be stated as follows:

$$\text{Accept the Project if } \frac{\bar{R}_i - R_f}{\beta_i} > \frac{\bar{R}_o - R_f}{\beta_o} \qquad (1.9)$$

otherwise, reject the project.

(The subscript "0" refers to the "present" known values for the firm and "i" represents the project under consideration by the firm).

This criterion is derived from the equation of the expected return rate for an individual security (or project), given by the following equation:

$$\bar{R}_i - R_f = \frac{\text{Cov}(R_i, R_M)}{\sigma_M^2} \cdot (\bar{R}_M - R_f) = \beta_i (\bar{R}_M - R_f) \qquad (1.10)$$

where

$$\beta_i = \frac{\text{Cov}(R_i, R_M)}{\sigma_M^2}$$

If we look at the expected market premium, $(\bar{R}_M - R_f)$, as the (assumed) known multiplying factor, then the risk premium for the ith project is a linear function of β_i, or

$$\bar{R}_i - R_f = (\bar{R}_M - R_f) \cdot \beta_i \tag{1.11}$$

Note that β_i is now the independent variable, and that since $(\bar{R}_M - R_f)$ is an expected value, it is viewed as a constant. If Equation (1.10) is plotted on rectangular coordinates of $((\bar{R} - R_f), \beta)$, as in Figure 1.6, the slope of the line is simply the constant $(\bar{R}_M - R_f)$, since the slope is $(\bar{R}_i - R_f) / \beta_i = (\bar{R}_M - R_f)$.

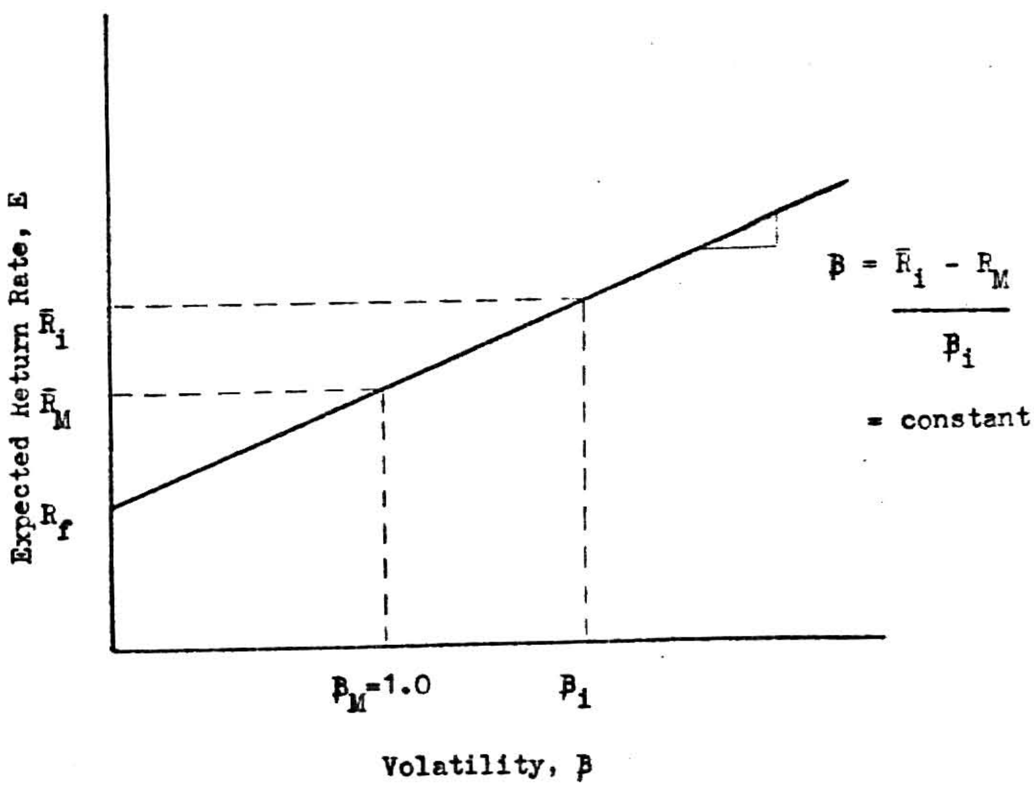

Figure 1.6. Reward - to - Volatility Plot.

The volatility of the market itself, β_M, is the "basing point" for comparisons involving ($\bar{R}_i$, β_i) points for investment projects. Noting that the covariance of the market rate with itself is equal to Cov (R_M, R_M) = σ_M^2, then β_M is

$$\beta_M = \frac{\text{Cov }(R_M, R_M)}{\sigma^2_M} = \frac{\sigma^2_M}{\sigma^2_M} = 1.0 \tag{1.12}$$

Therefore, when $\beta_M = 1$, the corresponding return rate is equal to the expected market return rate.

The use of the reward-to- volatility as a project selection criterion is supported by the fact that this criterion, when used to select project, will <u>result in an increase in the expected net present value of the firm</u> - this proof, originally given by Mossin (13), later outlined by Rubenstein (16), and finally shown in detail by Bussey (4), establishes the reward-to-volatility criterion as a bona-fide project selection criterion.

Without going into the mathematical details, the acceptance of a "favorable" project will temporarily throw the price of the firm's shares on the market into a disequilibrium condition, which causes the firm's reward-to-volatility ordered pair ((R_o - R_f), β_o) to plot above the market line indicated by Equation (1.9). This increase in E(R_o) causes individuals to demand more of the firm's shares, which causes the price of the share to go up. Since the market tends to move toward equilibrium after a disturbance, the E(R_o) is reduced due to the increased price paid for the firm's share. This, then, restores equilibrium. However, the whole cause is the <u>expected</u> increase in net worth of the firm caused by acceptance of the favorable project.

So, according to this criterion, projects A & B in Figure 1.7 should be accepted while project C should be rejected. However, this is not always the case, as Bussey (4) has shown. Here, then, is the situation.

The mathematical derivation that the reward-to-volatility selection criterion does increase the expected net present value of the firm, which can be shown by the following equation.

$$\frac{E(R_i) - R_f}{\beta_i} > E(R_M) - R_f \tag{1.13}$$

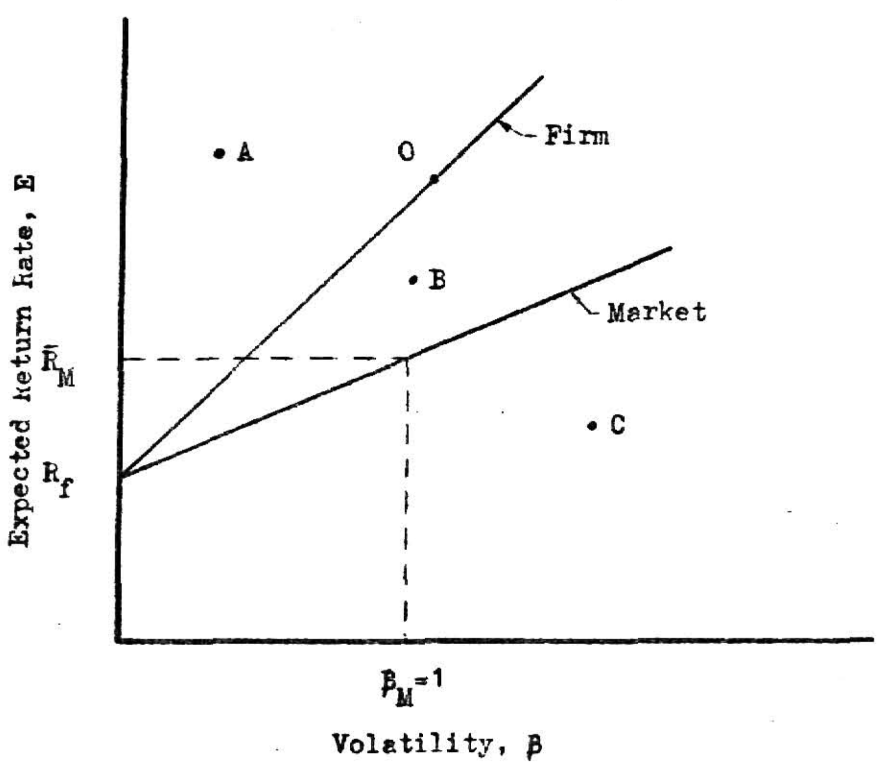

Figure 1.7. Reward - to - Volatility Plot for the Projects A, B, and C.

This criterion was based on the initial assumption that the price of the firm's equity shares was in equilibrium with the market. It has been demonstrated that generally the price of a single firm's shares will stabilize in equilibrium so that the expected risk premium for the firm $(E\ (R_o) - R_f)$, equalized for the firm's volatility, β_o, does not fall on the market line. One would expect, from the theory, that expected risk premium for the firm equalized for its volatility, should fall on the market line. This can be demonstrated as follows. From the equation developed earlier,

$$E(R_o) = R_f + \frac{Cov\ (R_o,\ R_M)}{\sigma_M^{\ 2}} \cdot \left[\ E(R_M) - R_f\ \right] \qquad (1.14)$$

and upon substituting

$$\beta_o = \frac{Cov\ (R_o,\ R_M),}{\sigma_M^{\ 2}}$$

and rearranging the terms,

we have:

$$\frac{E\ (R_o - R_f)}{\beta_o} = E\ (R_M) - R_f \qquad (1.15)$$

where

$\dfrac{E(R_o) - R_f}{\beta_o}$ = expected risk premium for the firm

$E(R_M) - R_f$ = expected return rate for the market above the risk-free rate.

An explanation of this phenomenon can be generated from the idea that the individuals who purchase a firm's equity stock think the firm will provide an expected return premium, $E(R_o) - R_f$, in relation to the firm's volatility, β_o, greater or smaller than the market itself, and will act according to their belief. So, it is the usual case that the firm itself will have an ordered pair $(E(R_o, \beta_o))$ that does not lie on the market line. This is shown in Figure 1.7 by the point "O", which represents the market's historical evaluation of the firm's ability to produce a higher expected rate of return than the market itself. Figure 1.7 shows that the firm "O" has consistently outperformed the market, since

$$\frac{E\ (R_o) - R_f}{\beta_o} > \frac{E\ (R_M) - R_f}{1} \tag{1.16}$$

This being the case, firm "O" would not increase its current net present value by the acceptance of project B, since it lies below the firm's historical market line $\overline{R_f O}$. Both projects B & C should be rejected on this basis, while project A should be accepted because

$$\frac{E\ (R_A) - R_f}{\beta_A} > \frac{E\ (R_o) - R_f}{\beta_o} \tag{1.17}$$

Therefore, the proper selection criterion should be one based on the idea of improving the current net present value of the firm, which can be done by accepting projects that will "out-perform" the firm's present performance line, $(E\ (R_o) - R_f) / \beta_o$. In conclusion, the proper selection criterion to be used to evaluate the ith project is

$$\frac{E\,(R_i) - R_f}{\beta_i} > \frac{E\,(R_o) - R_f}{\beta_o} \tag{1.18}$$

The application of this project selection will, in fact, increase the current expected worth of the firm.

1.2.4 Desirability of Interval Estimates - As was pointed out earlier in this text, the reward-to-volatility criterion for project selection uses only point estimates in the decision-making process. Bower and Lessard (1) state that management, in general, desire a simple, intuitive measure to be used as a capital budgeting criteria. The capital asset pricing theory satisfies this constraint, based on its simplistic assumption of a linear relationship between risk and expected return for individual securities or projects. However, by using this criterion, all that can be stated concerning the project is that on the average, it will perform as an investment in a manner superior to the level of return commanded by the firm, from both their past and present investments (an ex-post analysis). Since this method only deals with the expected value of the return, the expectancy is that 50% of the time the value will be higher than the point estimate, and 50% of the time the value will be lower. As Campbell (5) points out, this is only a survival criterion for the firm. This question, quite simply, is "how confident can the firm be that the project will, in fact, perform at a level superior to that of the firm?" This, then, demands the exploration of a statistical model to answer this question. If the firm wants to perform in a manner superior to the market, or in the case of the private firm, to its competitors, with some degree of confidence, a statistical model that generates interval estimates must be used.

The statistical model can provide management with a tool with which they can assess the "chances" of a candidate project being successful and thereby increase the probability of an increased net present value of the firm. The main advantage of the statistical model, however, is that probability statements can be made concerning the probability of accepting not only good projects, but also the probabilities of making wrong decisions, such as accepting a bad project or rejecting a good one. If management can assess the relative costs of making these two types of errors as well as recognizing the possible gains from the firm's standpoint, of the acceptance of a project, they are then in a position to make a rational decision based on the facts. Of course, a certain degree of uncertainty will still be involved in this process since the outcome of future events cannot be predicted exactly. However, the statistical model allows the firm to recognize explicitly the probability that the project will or will not perform in a superior manner, as opposed to merely accepting or rejecting a project ignorant of the possible outcomes by using the previous reward-to-volatility project selection criterion.

1.3 Thesis Organization

This thesis is composed of four chapters. This, the first chapter, is the introduction and contains the purpose of the thesis, the problem to be studied, and a brief survey of the more important literature concerning the topic of capital asset pricing theory, its application to capital budgeting, and the concept of using a statistical approach in capital budgeting.

The second chapter applies the concept of statistical modeling to the capital budgeting problem. The reward-to-volatility ratio of the firm is discussed as being an ex-post performance measure from historical data that results in a single point estimate. The project's reward-to-volatility ratio can be described by the use of a probability distribution, by either deriving the theoretical distribution of the reward-to-volatility test statistic or by using the actual sample data and fitting the empirical distribution to a known distribution. Then, the mechanics of making probability statements concerning the chances of success or failure of the project are discussed as a managerial aid to the decision making process.

Chapter III applies this statistical methodology to the private firm. The idea of a "surrogate" is revealed, which is a combination of the private firm's publicly owned competitors. The brewing industry was chosen for this example. A simulation model of the project is designed and the results shown for a sample size of 20. A Kolmognov-Smirnov goodness of fit test is performed on the sample data and the distribution of the project's reward-to-volatility ratio is hypothesized. This distribution is then compared to the point estimate of the reward-to-volatility ratios for the surrogate firm and three of the public firms that make up a portion of the surrogate firm. The results of these comparisons, in the form of the probabilities of the project being a "successful" or an "unsuccessful" project, are discussed for each firm.

The final chapter forms the conclusion of the thesis. The results are examined and discussed and areas for possible further study are cited.

CHAPTER II

A STATISTICAL APPROACH TO PROJECT SELECTION

2.0 Introduction

As has been suggested earlier, the reward-to-volatility selection criterion proposed by Sharpe (18) can be used by the firm to select projects that will increase the net present value of the firm. This ratio, as given by Equation (1.17), now becomes a test statistic to be examined. The basic idea involved in making probability statements concerning two separate identities is that the probability distributions of both must be known in order to make interval estimates, or establish confidence intervals. The case of the project versus the firm, however, does not meet this constraint in its entirety. The project return rate can be sampled, via a simulation technique, and a resulting probability distribution of the reward-to-volatility test statistic can be calculated from these repeated samples. But, when the firm is considered, there is only one sample to analyze - that being the historical performance of the firm. This ex-post examination of the firm results in only a single point estimate of the reward-to-volatility statistic used in the comparison between the firm and the project. Obviously, since only a single point is known, no probability distribution can be established. However, some statements of probability can be made from the comparison of a single point to the probability distribution of the project, as we shall see later.

2.1 Assumptions of The Statistical Approach

There are several points concerning the statistical approach of the capital asset pricing model to capital budgeting that need to be emphasized and highlighted. As mentioned in the introduction to this chapter, the performance level of the firm is an ex-post performance measure; that is, it is a measure based on the historical performance of the firm with respect to the market. Since we cannot take more than one sample of the historical performance of the firm for the same period of time, we are left with only one estimate of the firm's performance level to work with.

The project's reward-to-volatility ratio, on the other hand, is an ex-ante estimate since we are sampling a future event. Typically, a mathematical model would be built to simulate the likely occurance of a project, with respect to the market, and then the model would be repeatedly sampled to obtain estimates of what the project's performance level might be. A probability distribution of that expected performance level is one of the desirable products that results from the sampling procedure.

An important consideration here is the fact that although we looked at the firm's performance level from a historical standpoint, we must assume that the firm continues on into the future, or else there would not be a necessity to evaluate candidate projects. How the firm continues on in the future, however, is a matter of great importance. For lack of better evidence, one must assume that the firm will perform at least at its present level or better if it follows the selection criterion detailed in section 1.2.3. The reasoning is as follows.

If the firm's historical performance level is used as a base for the acceptance of candidate projects (in accord with the reward-to-volatility project selection criterion), the firm's expected net present value will at least remain stable. If this procedure is adhered to, the firm's performance level will likewise remain at least stable. At some time in the future, the firm's performance level can be re-evaluated and up-dated, so to speak, in order to take into account the firm's then ex-post performance level, due to its successful application of the reward-to-volatility selection criterion. Generally speaking, therefore, the firm's ex-post evaluation of its performance level can be used to judge the acceptance of candidate projects for the coming year. Then, the next year's performance criterion used by the firm will take into account the previous year's performance based on the acceptance of successful projects (i.e., projects where expected rate of return is higher than that previously demanded by the firm at that level of risk).

2.2 Statistical Decision Making

Very often in practice one is called upon to make decisions about certain statistical populations on the basis of sample information. Such decisions are called "statistical decisions". In attempting to make such decisions, certain assumptions can be made about the population distributions which may or may not be true. Such assumptions are called "statistical hypotheses" and these assumptions are the basis for making statistical tests of significance. Procedures which enable us to decide whether to accept or reject hypotheses or to determine whether or not one sample distribution is statistically different from another are called "tests of hypotheses" or "tests of significance". It is therefore the goal of a statistical model to be able to make such tests and to supply a decision rule to the decision maker.

The statistical approach to project selection developed in this thesis is one designed to test the hypothesis that the <u>project's</u> reward-to-volatility level (a random variable) is statistically different from the reward-to-volatility level of the firm, already established from the historical performance of the firm with respect to the market. This comparison of the project with the firm, however, cannot be made with complete certainty. Any decision made as to the acceptance or rejection of a project is subject to error. Herein lies the benefit of using a statistical approach to decision making. What are the inherent errors possible in any decision making process and how can they be estimated?

As has been suggested earlier, the reward-to-volatility selection criterion can be used by the firm to select projects. The firm's reward-to-volatility ratio was shown to be an <u>ex-post</u> performance measure. Therefore, only a <u>single</u> reward-to-volatility ratio representing the firm can be used in the selection criterion. This ratio is an <u>expectation</u> obtained from a regression analysis, which is discussed in detail in section 2.3. This single, point estimate needs to be considered as a constant. There is no probability distribution associated with the firm's reward-to-volatility because of the single sample.

In the case of the project, however, repeated samples of the project's return rate can be taken, and a probability distribution of the reward-to-volatility test statistic can be arrived at.

The project can now be compared to the firm, in accordance to the reward-to-volatility selection criterion proposed by Sharpe (18). The statistical testing method developed in this thesis uses the single point estimate of the firm's reward-to-volatility ratio as a constant criterion against which the project's reward-to-volatility probability distribution is compared to. It should be noted that the project's reward-to-volatility ratio is a random variable and has a definite probability distribution with a mean and variance, while the firm's reward-to-volatility ratio is not a random variable.

The results of this type of comparison (a point estimate to a probability distribution) enables certain probability statements to be made. Figure 2.1 shows a typical example of this type of comparison. The probability distribution, representing the projects reward-to-volatility ratio, is distributed with a mean of $\bar{A}$ and a standard deviation of $\sigma_{\bar{A}}$. The single point, representing the firm's expectation of its reward-to-volatility ratio, is denoted as $\bar{B}$.

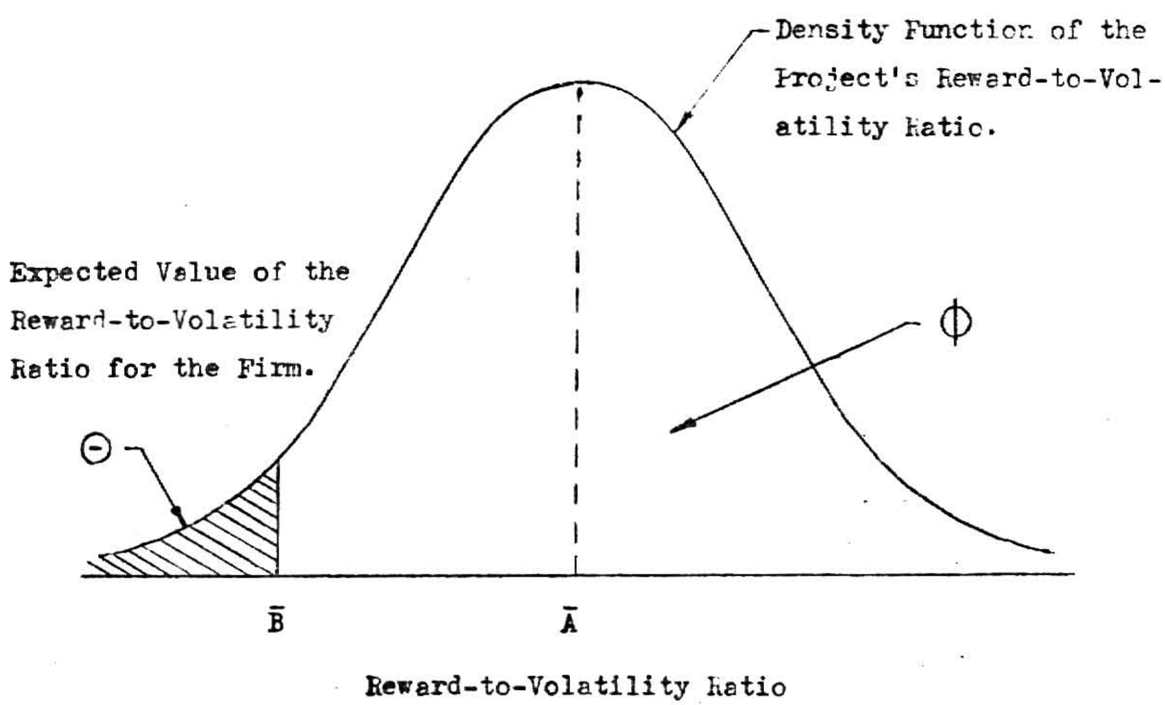

Figure 2.1. Comparison of the Probability Distribution A and the Point $\bar{B}$.

It can now be easily seen that the probability of the reward-to-volatility ratio of the project exceeding the constant value of the reward-to-volatility ratio of the firm is given by the area under the density funtion curve to the right of $\overline{B}$. This area is denoted by the symbol Φ. Similarly, the probability that the project's reward-to-volatility ratio does not exceed the established value of the firms reward-to-volatility ratio is the area under the curve to the left of the point $\overline{B}$, which is denoted by the symbol $\ominus$.

The ability to make these probability statements concerning the chances that the project's return rate may or may not exceed the level demanded by the firm is an important tool in the project selection process. A decision maker is now in a better position to choose the projects that are the most likely to improve the financial position of the firm.

2.3 The firm's Reward-To-Volatility Function

The basis of the firm's project selection criterion is its historically established reward-to-volatility ratio. The reward-to-volatility criterion stems from a single variable model, called the Sharpe-Lintner model. In the case of the firm, the firm's expected return rate is compared to the return rate displayed by the market for any given period, as shown in Equation (1.5). This relationship is

$$(E_0 - R_f) = \beta_0 \, (E_M - R_f) \tag{2.1}$$

where

$(E_0 - R_f)$ = expected risk premium (above the risk-free rate for the firm

$(E_M - R_f)$ = expected risk premium of the market itself

β_0 = volatility of the $_0$th project, security.

The market return rate, R_M is taken to be a random variable with mean E_M and standard deviation σ_M. Hence, $E_M - R_f$, the market risk premium, is also a random variable.

Equation (2.1), the equation for the firm's market line, is unfortunately never observable because of the deviations away from the market line that occur in actuality. Using regression techniques, however, relationship can still be inferred. This procedure was mentioned briefly in Section 1.2.2, but will be presented in detail here.

The volatility, β_0, can be estimated from the relationship of the return of the firm, R_0, and the market return R_M. β_0 can be estimated if

1. both R_0 and R_M are assumed to be bivariate random variables with finite means and variances and are correlated with a covariance C_{0M}. The returns occur in a time series t = 1,2....n with observable values R_{0t} and R_{Mt}

2. the nature of the correlative relationship can be examined using regression techniques and taking R_{0t} as being conditionally distributed upon fixed values of R_{Mt}.

If this is done, Equation (2.1) can be estimated by a time-series regression.

$$(R_{0t} - R_f) = \alpha_0 + \beta_0 (R_{Mt} - R_f) + \epsilon_{0t} \tag{2.2}$$

where

R_{0t} = random return on the firm at time t

R_{Mt} = return rate of the market portfolio M at time t

R_f = the risk-free rate of interest

α_0 = an unknown intercept parameter

β_0 = an unknown proportionality factor (volatility of the firm with respect to the market)

ϵ_{0t} = normally distributed random error term representing the deviation away from the firm's market line.

By assumption, ϵ_{0t}, the random error term, has the following three properties:

1. $E(\epsilon_{0t}) = 0$
2. $C(\epsilon_{0t}, \epsilon_{0,t-1}) = 0$; i.e., there is no timewise correlation between error terms
3. for two securites, i and j, $C(\epsilon_{it}, \epsilon_{jt}) = 0$.

The third property is based on the assumption by Markowitz (11) that the covariance between two securities is zero. The covariance is assumed to exist only between each security (or firm) and the market.

Letting

$R_{0't} = R_{0t} - R_f$ in (2.2), then

and $R_{M't} = R_{Mt} - R_f$

$$R_{0't} = \alpha_0 + \beta_0 R_{M't} + \epsilon_{0t} \tag{2.3}$$

where

$R_{0't}$ = random return rate of the firm above the risk free rate for period t

$R_{M't}$ = return rate on the market protfolio M above the risk free rate for period t.

Equation (2.3) is a linear regression model with parameters α_0 & β_0. Since these parameters are not directly observable, they must be estimated. This regression model can be estimated by a linear regression function, using the least squares method. The estimate of (2.3) is

$$\hat{R}_{0'} = \hat{\alpha}_0 + \hat{\beta}_0 R_{M'} \tag{2.4}$$

where $\hat{R}_0$, $\hat{\alpha}_0$ and $\hat{\beta}_0$ are estimates of the corresponding parameters. In (2.3) this method gives point estimates of α & β for the firm "0", using the normal equations. These estimates are

$$\hat{\beta}_0 = (\sum (R_{M't} - E_{M'})(R_{0't} - E_{0'}))/\sum (R_{M't} - E_{M'})^2 \tag{2.5}$$

and

$$\hat{\alpha}_0 = E_{0'} - \beta_0 E_{M'} \tag{2.6}$$

Therefore, the point estimate of the firm's reward-to-volatility ratio can now be calculated using the expected value of the firm's return rate above the risk-free rate and the estimate of the volatility of the firm supplied from the regression analysis.

2.4 The Project Reward-To-Volatility Distribution

As discussed in previous sections, the estimation of the project reward-to-volatility distribution is conceived of as an ex-ante estimation. Therefore, we can "sample" future projects and estimate the probability distribution for the project selection criterion by two separate approaches, both of which will be discussed here. One method deals with the theoretical derivation of the probability distribution of the reward-to-volatility ratio while the other method uses an empirical approach based on the data accumulated by the sampling procedure.

The method which utilizes the empirical approach to discover the project's reward-to-volatility distribution seems to be the easier of the two methods. Basically, there are two problems that should be considered when a sampling procedure is used. The sampling procedure must be designed and conducted so that the extracted samples are representative of the population being studied, and then, having studied the samples, the investigation must be able to make correct inferences about the population.

Using the data gathered from the samples, Hogg (7) states that an empirical distribution can be fitted to these data points by a variety of statistical tests, such as the Chi-square test, the Kolmogrov-Smirnov test and David's Empty cell test. All of these tests are goodness-of-fit tests and require that the hypothesized distributions of the sampled population be completely specified prior to the testing. Therefore, after the samples are taken, a distribution can often be fitted to the data. An important point should be made here concerning the fitting of these sample points to a probability distribution. If a goodness of fit test is performed, all that can be stated is that, at a given level of significance, the distribution of the sample points can (or cannot) be distinguished from the hypothesized distribution. It doesn't mean that if the test is successful that the sample is distributed according to the hypothesized probability distribution. This point should always be kept in mind when dealing with empirical distributions.

Of the goodness of fit tests mentioned, the K-S test is superior to the Chi-square test in the following ways. The K-S tests uses ungrouped data and every observation represents a point at which "goodness of fit" is examined. On the other hand, the Chi-square test loses this information (if the hypothesized distribution is continuous) because of a requirement that data be grouped into cells. The K-S test requires only relatively modest assumptions such as that sampling be random and that the sampled population be continuous, whereas the Chi-square test assumes, among other things, conditions that can be completely fulfilled only when the sample size is infinite. So, the exact distribution of K (test statistic) for the K-S test is known and tabled for small sample sizes, whereas the Chi-square test statistic is known and tabled only for large sized samples.

Therefore, the Chi-square test is only an approximate test while the K-S test is an exact test. The David's Empty Cell Test can be considered as an alternative to the K-S test. The K-S test is used more often because of its simplicity and its characteristics of producing an exact and powerful test.

In this manner, the empirical approach can be used to give an estimate of useable probability distribution. From the distribution, then, the point estimate of the reward-to-volatility for the firm can be compared to the projects reward-to-volatility probability distribution.

The other approach that can be taken to derive information about the project's reward-to-volatility probability distribution is the theoretical development of a test statistic from the capital asset pricing model. Recalling from Equation (1.17) that the project will be accepted if

$$\frac{E\ (R_i) - R_f}{\beta_i} > \frac{E\ (R_0) - R_f}{\beta_0} \tag{2.7}$$

where

R_i = random return on project i,

R_0 = random return on the firm "0",

R_f = a constant risk free rate of interest,

β_i = estimated slope of the regression of $\tilde{R}_i$ on R_M ,

β_0 = estimated slope of the regression of $\tilde{R}_0$ on R_M ,

We can make certain statements about this criterion. We know, for example, that the variables $E\ (R_i)$ and $E\ (R_0)$ are independent and normally distributed variables because of the central limit theorem concerning expected values. The central limit theorem states that the sampling distribution of means is approximately a normal distribution, irrespective of the population sampled, provided that the population mean and variance are finite and the population size is "large" (e.g., at least twice the sample size). In fact, the general form of the central limit theorem shows that the accuracy of the approximation improves as the sample size becomes larger. (approaches infinitely large size). This is sometimes indicated by saying that the sampling distribution is asymptotically normal.

The two estimated parameters $\hat{\beta}_i$ and $\hat{B}_o$ are results of the regression analysis of the return rates of the project and the firm with respect to the return on the market. The statistic $\hat{\beta}$, in general is a linear function of the independent random variable R_{M1}, $R_{M2}, \cdots R_{Mn}$, and has a normal distribution with mean derived as follows:

$$E\,(\hat{\beta}) = \frac{\sum_{i=1}^{n} (R_i - \bar{R}) \quad E\,(R_{Mi})}{\sum_{i=1}^{n} (R_i - \bar{R})^2} \tag{2.8}$$

substituting into Equation (2.8) an expression for $E\,(R_{Mi})$ we have

$$E\,(\hat{\beta}) = \frac{\sum_{i=1}^{n} (R_i - \bar{R}) \quad (\alpha + \beta\,(R_i - \bar{R})\,)}{\sum_{i=1}^{n} (R_i - \bar{R})^2} \tag{2.9}$$

where

α, β = constants from the regression analysis.

simplifying the expression we have

$$E\,(\hat{\beta}) = \frac{\alpha \sum_{i=1}^{n} (R_i - \bar{R}) + \beta \sum_{i=1}^{n} (R_i - \bar{R})^2}{\sum_{i=1}^{n} (R_i - \bar{R})^2} \tag{2.10}$$

the variance is

$$\sigma_p^2 = \sum_{i=1}^{n} \left[\frac{R_i - \bar{R}}{\sum_{i=1}^{n} (R_i - \bar{R})^2} \right] \left(\sigma_{RM}^{\ 2} \right) \tag{2.11}$$

$$= \frac{\sigma^2}{\sum_{i=1}^{n} (R_i - \bar{R})^2}$$

Now, the random reward-to-volatility ratio can be expressed as the ratio of two random-normal variates. The ratio of two such variates is distributed Cauchy (7) with a probability density function

$$g_1(c_1) = \frac{1}{\pi \left(1 + c_1^{\ 2} \right)} \quad ; \quad (-\infty < c_1 < \infty) \tag{2.12}$$

The Cauchy distribution is actually the ratio of two standardized random normal variates, i.e., the two normals are both distributed with a mean equal to zero and a variance equal to one.

Therefore, to determine the Cauchy form of the exact distribution of the reward-to-volatility ratio for the project, a transformation of variables must be made in order to standardize the ratio. The transformation can be made as follows:

Let

$$Z_i = R_i - R_f$$

$$\text{Then} \quad E(Z_i) = E\,(R_i) - R_f, \tag{2.13}$$

and the variance of the variable Z_i is:

$$V\,(Z_i) = V\,(R_i - R_f)$$

But, since R_i and R_f are independent and $V(R_f) = 0$ (R_f = constant), we have

$$V(Z_i) = V(R_i) \tag{2.14}$$

or, the corresponding standard deviation

$$\sigma_{Zi} = (V(R_i))^{\frac{1}{2}} = \sigma_{Ri}$$

However, what is needed is the standard deviation of the distribution of sample means and this is related to the population standard deviation by

$$\sigma_{\bar{Z}i} = \sigma_{\bar{R}i} = \frac{\sigma_{Ri}}{\sqrt{n}} \tag{2.15}$$

where $\sigma_{\bar{R}i}$ is the variance of a sample distribution of means.

At this point, a peculiar property of the Cauchy distribution forces a slight change in the test statistic. The Cauchy distribution does not have a mean or a variance (or any other higher moments). The reason for this is the fact that the Cauchy distribution has characteristic function, and therefore no moment generating function. (The moment generating function for a distribution is unique and completely determines the distribution of the random variable). For any moment, $M(i)$, $i=o,1,\ldots.n$, the moment generating function exist; i.e., as an expanded series, it must converge to a finite value in order for it to exist. In the case of the Cauchy distribution, the moment generating function does not converge for any value of $M(i)$, $i=0,1,2,\ldots n$, (i.e., it is an indeterminent series.)

So, in order to use the Cauchy distribution, the "average" of the data points examined is used in order to standardize the variable.

Therefore, the test statistic becomes

$$E\,(Z_i) - \bar{Z}_i \,/\, \sigma_{\bar{Z}i} \tag{2.16}$$

where

$$E(Z_i) = E\,(R_i) - R_f$$

$\bar{Z}_i$ = "average" of the variable $R_i - R_f$

$\sigma_{\bar{Z}i}$ = variance of the variable $R_i - R_f$.

This statistic (Equation (2.16)) is distributed normally with a mean of zero and a variance of 1.

Substituting the actual variables into (2.16) we have

$$\frac{E\,(R_i) - R_f - (\bar{R}_i - R_f)}{\dfrac{\sigma_{Ri}}{\sqrt{n}}} = \frac{E\,(R_i) - \bar{R}_i}{\dfrac{\sigma_{Ri}}{\sqrt{n}}} \tag{2.17}$$

This ratio is also distributed normally with a mean of zero and a variance of one.

This takes care of the standardization of the numerator of the reward-to-volatility ratio expressed in Equation (2.7)

A similar procedure can be used to show that

$$\hat{\beta}_i - \hat{\beta}_i \,/\, \sigma_{\hat{\beta}i} \tag{2.18}$$

is also distributed normally with a mean of zero and a variance of one. Now, let Q_i equal the right hand side of Equation (2.17) and B_i equal the right hand side of Equation (2.18). We now have the ratio of two standardized normal deviates.

$$C_i = \frac{Q_i}{B_i} = \frac{(E(R_i) - \bar{R}_i) \,/\, \sigma_{Ri} \,/\sqrt{n}}{(\hat{\beta}_i - \bar{\hat{\beta}}_i) \,/\, \sigma_{\hat{\beta}i}} \tag{2.19}$$

The variable C_i is now distributed Cauchy and can be described by the probability density function given in Equation (2.12).

CHAPTER III

CAPITAL BUDGETING FOR THE PRIVATE FIRM

AN APPLICATION

3.0 Introduction

A statistical model using the reward-to-volatility ratio can be used by publicly held firms for use in capital budgeting decisions. However, since this ratio and the resulting probability distribution requires a relationship between the return of the firm and the market, this selection criterion cannot be used directly for many privately owned and financed firms that operate in the industrial sector. The concept of volatility with the market for a private firm is meaningless, since there is no market for the firm's stock. Campbell (5) developed a method by which the private firm could indirectly apply this project selection criterion to its candidate projects. Campbell (5) originated the concept of the "surrogate" firm in order to use such a selection procedure. However, his methodology for selecting projects in the reward-to-volatility criterion used only point estimates of both the firm and the project's reward-to-volatility ratio. This, then, was a point value model of the capital asset pricing model. It was therefore incapable of making probability statements concerning the likely outcome of a project's return rate as compared to the firm's return rate. This thesis is an expansion of this previous work in an attempt to develop a statistical model that will generate probability statements concerning the project's return rate. This chapter presents an application of a statistical model and project selection methodology presented in Chapter II. The example will be carried along as each step of the methodology is illustrated.

3.1 The Surrogate Firm

There are many privately owned firms in competition with publicly owned firms in today's economy. No matter what type of industry is involved, one measure of the success of a firm would be the comparison of that firm with its competitors. If the firm has a higher return rate on its investments than its competitors, it is reasonable to assume that the firm is performing well in its investment decision making. Thus, an "average" of the competing, publicly owned firm could be calculated and used as a comparative standard. This average, then, could serve as a "surrogate" or pseudo firm, against which the private firm could be compared. The private firm can now evaluate all the candidate investment alternatives against a security line constructed from the surrogate firm. The project selection criterion (reward-to-volatility) developed earlier could then be used to select the candidate projects that will improve the private firm's position with respect to its competitors.

The weighted average of the returns of selected individual public firms can be calculated as follows:

$$R_0 = \sum X_j R_j \tag{3.1}$$

where X_j is the proportion investment in firm j using the net worth of the firm as the weighting factor, X_j. Thus, the larger firms be weighted greater than the smaller firms, acknowledging their importance in that industry. Since the return of the surrogate is given by Equation (3.1), these values could be used in a time series regression model, such as the one developed in Chapter II. The estimates of R_0 and β_0 could be then used to find the point estimate of the historical performance level of the surrogate.

The use of the "surrogate" concept cannot be considered as a generally applicable concept. For example, in industries which are dominated by conglomerate firms, which are diversified in several or many different industries, the concept of a surrogate firm is unrealistic. The return of the surrogate would be distorted by the return of divisions of those firms whose product or service is unrelated to the industry for which the surrogate is being developed. Other requirements necessary to keep the example simple for illustrative purposes while still exhibiting characteristics of the actual industrial economy were requiring that the selected industry have between five and twenty firms and that these firms be easily identifiable and familiar to the general public. Campbell (5) selected the brewing industry and this thesis will use this example to develop the statistical model, which is an expansion from Campbell's .

The Standard Industrial Classification (S.I.C.) Manual (23) categorizes types of industry by function, giving each particular category a unique S.I.C. number. Thus "2082" is the S.I.C. number for Malt Beverages. Standard & Poor's publishes a list of corporations by S.I.C. number. From this list, the publicly held firms with listing on the NewYork and American stock exchanges or "over-the-counter" price information were selected. These six firms are Anheuser-Busch, Carling O'Keefe Ltd., Falstaff Brewing, Pabst Brewing, F & M Schaefer, and Joseph Schlitz Brewing. Closing prices, dividends paid, and net worth for the years 1961-1975 were obtained using issues of the Wall Street Journal and Value Line (24). For the market, Standard & Poor's 500 Composite Index was selected as the indicator of market return. This index is a composite of 500 common stocks traded on the New York Exchange. The dividend yield of the market was obtained from various issues of the Federal Reserve Bulletin.

The return rate for each of the breweries was then calculated according to the following relationship:

$$R_i = (P_t + D_t - P_{t-1}) / P_{t-1} \tag{3.2}$$

where

R_i = return rate for security i for year t

P_t = closing price for security i at year t (dollars)

P_{t-1} = closing price for security i at year t-1 (dollars)

D_t = dollar amounts of dividends paid by firm during year t.

These returns were weighted by net worth and summed using Equation (3.1) to obtain a surrogate return of the brewing industry.

The return of the market for each year was calculated in a similar manner:

$$R_M = (P_t - P_{t-1}) / P \quad + \quad (D_t/P_t) \cdot (P_t/P_{t-1}) \tag{3.3}$$

where

R_M = return rate for the market for year t

P_t = closing market price (relative) of Standard & Poor's 500 Composite Index at year t

P_{t-1} = closing market price (relative) of Standard & Poor's 500 Composite Index at year t-1

D_t/P_t = current dividend-to-price ratio for the market at close of year t.

The results of this calculation are shown in Table 3.1. The raw data used in obtaining these results can be found in the Appendix.

Using the information from Table 3.1, the linear regression analysis developed in Chapter II was performed. The market premium ($R_M - R_f$) is the independent variable, with the surrogate security premium ($R_O - R_f$) the dependent variable of the analysis. The analysis was performed using the computer system, AARDVARK, developed by the Statistics Department at K.S.U. (9). The results of the regression analysis are found in Table 3.2. The risk-free rate of return was assumed to be 5% in this analysis.

Therefore, from Table 3.2, the point estimate of the reward-to-volatility for the surrogate is:

$$\frac{E\ (R_O) - R_f}{\beta_O} = \frac{.08122}{1.425} = .057 \qquad (3.4)$$

3.2 Modeling The Project

When evaluating candidate projects for acceptance by the firm, cash flow streams are one of the main factors to be considered. It is generally recognized that probabilistic cash flow formulations give more insight into the problems of project evaluation than does the deterministic approach. Bussey (4) examines some of the additional problems that may be encountered in probabilistic formations of cash flow streams. These problems are mainly concerned with independence, cross-correlation, and auto-correlation of the individual cash flow elements between projects. The capital asset pricing model, however, does not attempt to compare one project to another project. The comparison is made between the firm or surrogate and the project, with the firm always having the option not to invest in any project if such a project does not provide a return

TABLE 3.1

RETURN RATES FOR INDIVIDUAL SECURITIES, BREWING SURROGATE SECURITY AND THE MARKET

	1961	1962	1963	1964	1965	1966	1967	1968
Anheuser-Busch	44.27	-13.64	1.79	49.37	47.06	26.30	43.93	56.00
Carling O'Keefe	31.49	- 8.90	-5.25	11.23	-15.13	-12.38	13.85	68.29
Falstaff	13.91	-16.97	12.48	34.68	- 5.84	-30.47	4.98	32.24
Pabst	91.43	-20.15	64.08	60.00	28.08	- 9.40	78.08	51.56
F & M Schaefer								
Schlitz			70.08	11.49	4.70	-58.51	90.95	42.98
Surrogate	44.39	-13.33	22.47	27.00	12.66	-12.44	48.85	53.24
Market (S&P 500)	26.66	- 8.82	22.64	16.42	12.40	- 9.93	23.79	12.10

	1969	1970	1971	1972	1973	1974	1975
Anheuser-Busch	15.31	5.81	48.02	-0.30	-39.40	-25.18	43.29
Carling O'Keefe	-27.24	- 4.44	2.10	-15.99	-33.09	-46.29	32.98
Falstaff	-37.82	-41.54	52.75	-39.65	-56.14	-44.44	- -
Pabst	- 4.00	10.51	54.00	0.26	-64.85	-31.72	35.70
F & M Schaefer		-47.57	-38.89	-46.18	-57.77	-36.53	21.01
Schlitz	47.54	- 6.12	60.22	62.21	- 3.03	-72.02	32.07
Surrogate	6.55	- 0.47	40.62	7.43	-35.81	-41.36	37.03
Market (S&P 500)	-10.35	4.82	14.10	17.80	-13.83	-25.79	36.96

TABLE 3.2

RESULTS OF REGRESSION ANALYSIS USING AARDVARK SUBROUTINE

Variable	Mean	Std. Dev.	Parameter	Estimate	Std. Dev.
$R_0 - R_f$	8.122	29.818	Alpha	3.900	4.252
$R_M - R_f$	2.965	17.814	Beta	1.425	.243

ANALYSIS OF VARIANCE

	DF	Sum of Squares	Mean Squares	F
Total (Uncorrected)	15	13,437.143		
Corrections: Mean	1	989.503	989.503	
Total (Corrected)	14	12,447.640	889.117	
Due to Regression	1	9,023.363	9,023.363	34.256
Deviations from Regression	13	3,424.277	263.406	

$r^2 = .7249$

$r = .8514$

rate on investment high enough to satisfy the firm. Therefore, these additional problems will not alter the approach to project selection that has been developed earlier, and the model of the project can use cash flow streams as an indicator of the project's return rate for any particular period.

A convenient form of the cash flow element for any period t is given by the following:

$$Y_t = \left[G.I._t - O.E_t - D_t\right]\left[1 - t\right] + D_t \tag{3.5}$$

where

Y_t = Net cash flow increment for period t (dollars)

$G.I._t$ = Gross Income from sales, for period t (dollars)

$O.E._t$ = Operating expenses for the period t (dollars)

D_t = Depreciation expense for period t (dollars)

t = Effective tax rate.

There are certain assumptions that will be made with respect to this model. The initial investment, the life of the project and the salvage value of the assets are all assumed to be known with certainty. In reality, of course, these variables would not be known with absolute certainty. However, for demonstrative purposes, these assumptions will be made in order to simplify the building of the model. These assumptions should be recognized, but are not, however, severe limitations to the model. These three variables can usually be estimated quite accurately, relative to the estimation of the sales or the operating expenses, for example. In order for the sampling of the future project to be completely a random sample, ex ante distributions for each of these variables would need to be established and then sampled randomly. This could be done via a simulation procedure. In effect, the assigning of values to these three variables, coupled with an assumption of a straight-line depreciation method (for example), establishes the value of the depreciation expense per period. Therefore, the depreciation expense per period was the same for all the samples of the project. Also, the tax rate was assumed to be 50%, which is a simplifying but reasonable assumption.

One of the major problems facing the firm when an attempt is made to analyze a future project is the accurate prediction of the future income generated by the sales of the project. The prediction of a future event is often based on the occurrences of the past. The key question here is how accurately can past data be used to predict the future.

One method of predicting, or forecasting the value of a future occurrence based on its past performance, is through time series analysis. The class of forecasting techniques that depend only on past history are classified as ad hoc forecasting formulas. These types of formulas look at a series of observations of a variable at $t= 1,2,3....$ and attempt to develop a mathematical model that will satisfactorily predict what the value of the variable will be at some time t. This kind of model is referred to as a stochastic process model, since it depends upon the process becoming "stationary" in time so that satisfactory predictions can be made.

To use time series analysis for forecasting purposes, it is necessary for the firm to be able to establish a mathematical expression that will accurately predict the sales of the project. This task might be the most difficult problem faced by the firm. Often sales models predict the sales of a firm based on certain predicted levels of several commonly known economic indicators. It is, however, not the purpose of this thesis to explain the methodology of arriving at such an expression. For our purpose, it is sufficient to assume that the firm can establish its own predictive sales (or income) equation for a project under consideration.

For demonstration purposes, a sales equation was formulated using three exdogenous variables that have already been studied and an appropriate mathematical time-series model established (See Nelson (14), pp 202). The model is an "Auto Regressive Integrated Moving-Average" process (ARIMA) for a time series. Nelson states that many output measures, such as sales of a firm or gross national product, tend to display random behavior, such as is found in an ARIMA model. These types of series can be analyzed by looking at the differences in successive values of the series; that is, the successive incremental changes in the series sometimes become stationary. Thus a model of a "non-stationary" time series can be formulated by observing that the time series of differences is stationary time-wise.

Three exdogenous variables were chosen to be used in the equation to predict the sales for the project. The variables were the expenditures on producer's structures,[1] non-farm inventory investment and housing expenditures. These three models were chosen from a list of fourteen ARIMA models listed in Nelson (14), because they are nationally recognized economic indicators and their historical data appear in several of the leading statistical reports published on the economy. The ARIMA models for these three variables are Nelsons Equations 3.6, 3.7, and 3.8 and are:

Expenditures on Producer's Structure:

$$Z_t = Z_{t-1} + .303\,(Z_{t-1} - Z_{t-2}) + .216\,(Z_{t-2} - Z_{t-3}) + .297\,(Z_{t-3} - Z_{t-4}) - .442\,(Z_{t-4} - Z_{t-5}) + .159 + \mu_t \qquad (3.6)$$

Non-farm Inventory Investment:

$$Z_t = .531\ Z_{t-1} + \mu_t + .0013\mu_{t-1} + .742\mu_{t-2} + 1.69 \qquad (3.7)$$

Housing Expenditures:

$$Z_t = Z_{t-1} + .639\ (Z_{t-1} - Z_{t-2}) + .076\ (Z_{t-2} - Z_{t-3}) - .286\ (Z_{t-3} - Z_{t-4}) + \mu_t \qquad (3.8)$$

where

Z_t = observed value of the time series at time t

μ_t = "disturbance", or error term, associated with each respective variable.

Using these three models, the sales for the project were calculated by the following equation, using arbitrary fractional weights of 0.2, 0.4, and 0.4:

$$S_t = .2X_{1t} + .4X_{2t} + .4X_{3t} \qquad (3.9)$$

where

S_t = sales for period t (dollars)

X_{1t} = observed value of the time series at time t for the variable "non-farm inventory investment"

X_{2t} = observed value of the time series at time t for the variable expenditures on producer's structures"

X_{3t} = observed value of the three series at time t for the variable "Housing Expenditures".

Separate computer routines were written to sample Equation (3.9) for 100 periods. The sales equation was then used to calculate the sales for the project. Since each of the models had an associated random error term, the <u>mean</u> sales value, $E(S_t) = \mu_t$, for the project was also a random variable. Sales were generated twenty different times, which resulted in a random sample of sales figures for the project. A linear regression analysis was formed to determine if these sets of sales figures were correlated with the return on the market. The tests indicated that the sales were independent of the return on the market.

As was noted in Chapter I, in order to use the capital asset pricing model, the expected return on a security or for a firm must have some relationship to the return on the market. This is what "ties" the firm and the project to the overall activity of the economy.

It was decided that the operating expenses for the project were the factors that would be the most likely ones to exhibit a relationship with the market. It was assumed that as the random market rate of return increased, the expenses of the firm would decrease, due to the favorable general economic condition signified by the market. Conversely, if the market return rate fell during a given period, then it was assumed that the change in the economic condition would force the expenses of the project up. (It should be emphasized that this is merely an assumption made by this model for demonstration purposes only. There are many factors that influence expenses and sales, for that matter, to the project. The purpose of this thesis is to demonstrate a methodology; it is not intended to explore the interrelations of economic factors that affect the project).

The operating expenses for the project were assumed to be given by the following autoregressive expressions

$$O.E._t = O.E._{t-1} - .25\,(R_{Mt} - R_{Mt-1}) + 1.6 \tag{3.10}$$

where

$O.E_t$ = operating expenses at the end of period t (\$ x 1000)

$O.E_{t-1}$ = operating expenses at the end of period t-1 (\$ x 1000)

R_{Mt} = return on the market at end of period t

R_{Mt-1} = return on the market at end of period t-1.

The constants were added to the expression in order for the resulting project rate of return to be approximately those that would be found in the industrial sector today. The initial value for the operating expenses (at time t=0) was randomly chosen around a mean value of $30,000 in order to simulate the varying expense patterns that the project might face. The model for the project now has a random set of sales and a correlated expense stream that ties the projects rate of return to that of the market.

As mentioned earlier, the initial investment, the life of the project, the salvage value of the assets and the tax rate were assumed to be constant for each sample of the project. The initial investment for the project was set at $20,000, the life of the project was set at 15 years, the salvage value of the project was set at $5,000 and the tax rate faced by the project was set at 50%. The depreciation method chosen for use in the project was the straight line depreciation method, which is given by:

$$D_t = \frac{I_o - S}{N} \tag{3.11}$$

where

D_t = depreciation at the end of period t (dollars/year)

I_o = initial investment in the project (dollars)

S = salvage value at the end of the life of the project (dollars)

N = life of the project (years).

Using Equation (3.11), the depreciation per year for the project was equal to $1,000. Since all the factors were the same for each sample, the same depreciation expense was used for the calculation of all the sampled projects cash flow streams.

The return rate for the project for each period is a function of the cash flow Y_t and the current investment level I_t that the firm has in the project. The cash in-flow for each period can be considered as a "dividend" to the firm from the project. The investment level of the firm changes each period as the amount of depreciation is accounted for. The depreciation expense reduces the taxable income of the firm, which reduces the taxes payable by the firm. This, then, acts as a benefit to the firm from the project. Therefore, the amount of capital invested in the project is reduced each period by the amount of the depreciation that the firm is legally able to "write off". So, in order to be consistent, the return rate for the project should be the ratio of the dividends received in period t to the amount invested in the project at the <u>beginning</u> of the previous period. The return rate for the project is thus given by the following expression:

$$R_{it} = Y_{it}/I_{t-1} \tag{3.12}$$

where

R_{it} = return on the ith project at the end of period t

Y_{it} = cash flow for the ith project at the end of period t

I_{t-1} = investment level for the ith project at the end of period t-1 (which is equal to the investment level at the beginning of period t).

Recalling that the cash flow can be calculated by Equation (3.4), the return on the project for each year was then calculated for the life of the project. Table 3.3 is an example of how the return rates were calculated for one of the samples of the project.

The sampling procedure produced twenty sets of possible return rates for the project. Using these data, the linear regression analysis developed in Chapter II was performed, similar to the one performed on the surrogate firm in Section (3.1) and described in detail in Chapter II. The market premium $(R_M - R_f)$ is the independent variable and the project's premium $(R_i - R_f)$ is the dependent variable of the analysis. Table 3.4 presents the results of these regression analyses of the twenty samples. The reward-to-volatility ratio is also calculated for each sample shown in the table.

3.3 Statistical Analysis of The Project Reward-To-Volatility Distribution

From the results in Table 3.4, an assumed distribution can be fitted to the twenty sample points of the reward-to-volatility ratio. The fit of the assumed distribution can be tested by the use of the Kolmogrov-Smirnov goodness of fit test. In order to use this test, a completely specified theoretical distribution must be chosen to test the sample points against. It was shown earlier that the theoretical distribution of the reward-to-volatility test statistic is a Cauchy distribution. The shape of the density function of the Cauchy distribution is very similar to the shape of the density function of the Normal distribution.

TABLE 3.3

RETURN RATE OF THE PROJECT FOR SAMPLE NO. 6

TIME	SALES	OPERATING EXPENSES	DEPRECIATION D_t	PROFIT	PROFIT AFTER TAXES	CASH FLOW Y_t	ITERATION I_{t-1}	R_i	$R_i - R_f$
1	40.43	30.50	1.0	8.93	4.47	5.47	20	.27	.22
2	46.18	40.98	1.0	4.20	2.10	3.10	19	.16	.11
3	45.23	34.83	1.0	9.40	4.70	5.70	18	.32	.27
4	45.71	37.98	1.0	6.73	3.37	4.37	17	.26	.21
5	47.03	40.58	1.0	5.45	2.73	3.73	16	.23	.18
6	48.62	47.78	1.0	-.16	-.08	.92	15	.06	.01
7	54.29	40.93	1.0	12.36	6.18	7.18	14	.51	.46
8	52.16	45.43	1.0	5.73	2.87	3.87	13	.30	.25
9	54.93	52.66	1.0	1.27	.64	1.64	12	.14	.09
10	57.61	50.57	1.0	6.04	3.02	4.02	11	.37	.32
11	55.08	49.61	1.0	4.47	2.24	3.24	10	.32	.27
12	56.25	50.29	1.0	4.96	2.48	3.48	9	.39	.34
13	54.98	59.39	1.0	-5.41	-2.71	-1.71	8	-.21	-.26
14	55.73	63.99	1.0	-9.26	-4.63	-3.63	7	-.52	-.57
15	54.88	50.24	1.0	3.64	1.82	2.82	6	-.47	.42

Both curves are unimodal and symmetric about the mean. Because of this similarity, the Normal distribution was chosen as the hypothetical distribution to test the sample points against.

The following test of hypotheses was made using the Kolmogrov-Smirnov goodness-of-fit test.

H_0: Sample distribution $\sim$ Normal, with a mean = 0.207, standatd deviation = 0.036

H_1: Sample distribution is not $\sim$ Normal, with mean = 0.207, standard deviation 0.036.

The level of significance: = 0.05

Critical values for the K-S test statistic are tabled for various levels of significance. (See, for example, Hogg (7)). At the 5% level of significance, the critical value for the K-S test statistic is 0.29408. The test statistic for the K-S test is:

$$\text{MAX } D = \left| F(X) - S_N(X) \right|; \quad S_N(X) = \frac{K}{N} \qquad (3.13)$$

where

MAX D = Maximum absolute difference between $F(X) - S_N(X)$

F(X) = Completely specified theoretical cumulative distribution function under the null hypothesis

K = Cumulative observed frequency

N = Sample size.

The maximum absolute deviation found in the test was 0.1821. Since $0.1821 < 0.29408$, the null hypothesis fails to be rejected and the sample distribution of points cannot be distinguished from the Normal distribution, at a 5% level of significance. So, for our purposes we can accept the sample distribution of the reward-to-volatility ratio as being distributed

TABLE 3.4

SUMMARY OF LINEAR REGRESSION ANALYSIS

PERFORMED ON THE SAMPLES OF THE PROJECT

SAMPLE #	β	σ_B	REWARD-TO-VOLATILITY	$\overline{R}_i$	r^2
1	0.818	0.232	0.243	0.25	0.489
2	0.897	0.241	0.181	0.21	0.516
3	0.680	0.196	0.201	0.19	0.482
4	0.852	0.223	0.195	0.20	0.529
5	0.838	0.218	0.285	0.29	0.532
6	0.847	0.242	0.183	0.20	0.480
7	0.803	0.243	0.211	0.22	0.460
8	0.786	0.250	0.170	0.17	0.430
9	0.813	0.220	0.295	0.29	0.505
10	0.874	0.234	0.178	0.21	0.517
11	0.881	0.235	0.146	0.18	0.518
12	0.817	0.222	0.165	0.18	0.511
13	0.714	0.193	0.203	0.19	0.513
14	0.806	0.217	0.196	0.21	0.515
15	0.823	0.236	0.223	0.23	0.483
16	0.811	0.237	0.216	0.23	0.475
17	0.725	0.240	0.210	0.20	0.412
18	0.797	0.224	0.223	0.23	0.498
19	0.811	0.226	0.233	0.24	0.497
20	0.803	0.225	0.188	0.20	0.494

E (Reward-To-Volatility) = 0.207

V (Reward-To-Volatility) = 0.036

3.4 Comparing The Project To The Surrogate

Having arrived at an acceptable estimate of the probability distribution of the reward-to-volatility ratio for the project, a comparison can now be made with the point estimate of the reward-to-volatility ratio of the surrogate. Figure 3.1 shows the point estimate of the reward-to-volatility ratio for the surrogate in relation to the probability distribution of the reward-to-volatility ratio of the sample project. The point estimate of this ratio for the surrogate is equal to 0.057. For the project, this ratio is accepted as being distributed Normally, with a mean of 0.207 and a standard deviation of 0.036.

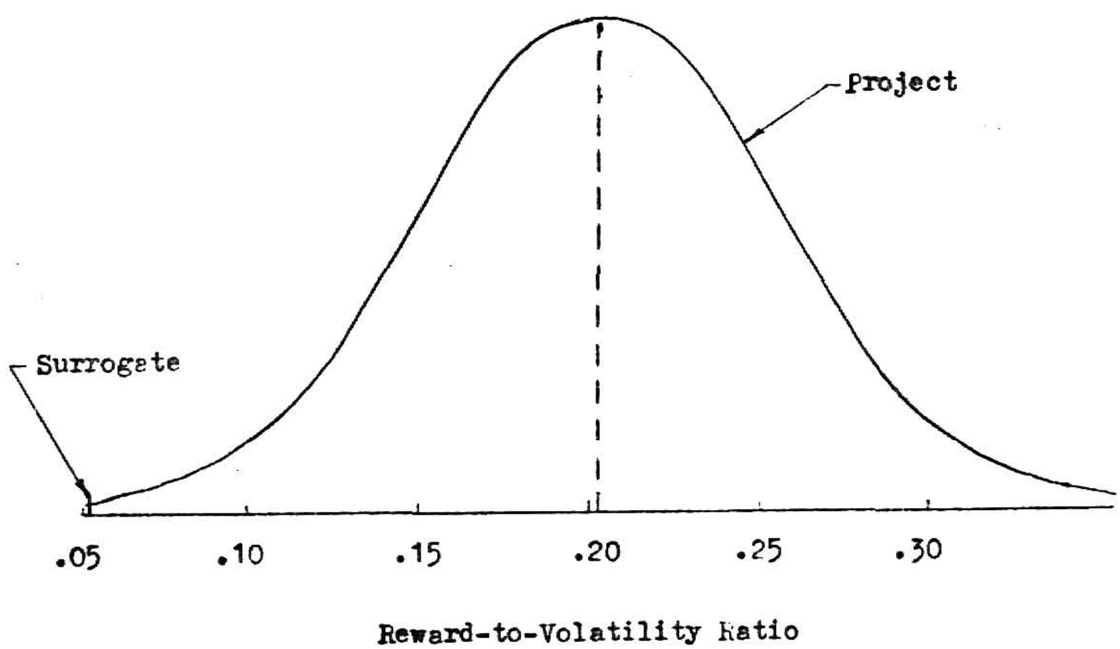

Figure 3.1. Comparison of the sampled project's Reward-to-Volatility Distribution to the point estimate of the Surrogate's Reward-to-Volatility Ratio.

The probability of the "success" or "failure" of the project can now be determined. The area under the curve to the left of 0.057 (the point estimate of the reward-to-volatility ratio of the surrogate) represents the probability that the projects reward-to-volatility ratio is less than the expectation of the surrogate's reward-to-volatility ratio. In this case, this probability is approximately 0.0. This probability represents the likelihood of an "unsuccessful" project.

The area under the curve to the right of 0.057 represents the probability that the project's reward-to-volatility ratio exceeds that of the firm's. In this case, this probability is approximately 1.0. This probability represents the likelihood of a "successful" project (i.e., one that satisfies the reward-to-volatility criterion stated in Equation (1.17)).

This example is not a realistic situation usually encounted in the economic sector. The analysis shows that the project's return rate will be higher than the return rate demanded by the firm because the probability of the project's reward-to-volatility ratio being greater than the expectation of the firm's reward-to-volatility ratio is equal to 1.0. This is a case of a "no-risk" investment opportunity. This is probably a very rare occurence in industry. It was pointed out earlier that the surrogate firm is composed of publicly owned firms. The private firm uses this "surrogate", or combination of its competitors, to select it's projects. In this example, the private firm would have accepted the project. But, how would this project be evaluated by some of the publicly-owned firms comprising the surrogate firm, assuming that this project were available to them for consideration? The next section evaluates the project from the standpoint of some of the publicly-owned firms economic position.

3.5 Comparing The Project To Some Publicly-Owned Firms

As was mentioned in Section 3.1, six firms were chosen to represent a surrogate, or pseudo firm, for the brewing industry. The candidate project was then compared to the surrogate in order to determine whether or not a private firm should accept or reject the project. The private firm would have accepted this project with 100% certainty that it would return a rate higher than that demanded by the private firm's competitors. However, how would some of the private firms competitors react to this project? Since the surrogate is constructed by weighing the relative strength of the public firm's financial position, some of the firms are better off than the others and thus would demand a higher expected rate of return from the project than might be demanded by the other firms. Three of the publicly owned firms that made up part of the surrogate were selected to compare the project's reward-to-volatility distribution to. The three firms were Anheuser-Busch, Pabst Brewing, and Schlitz Brewing. Using the data given in the appendix for the return rates for each of these securities, a time series regression analysis was performed similar to the one performed on the brewing surrogate. Table 3.5 shows the results of these regression analyses and the resulting point estimates of each of the firms reward-to-volatility ratios.

Figure 3.2 shows the reward-to-volatility distribution of the project and its relationship to the three publicly-owned firms. By using the standard normal tables, the probabilities of the project's reward-to-volatility ratio being greater, or smaller, than the expectation of the firm's reward-to-volatility ratio, can be calculated. These calculations are shown in Table 3.6. The probability associated with the occurance of the project's reward-to-volatility ratio exceeding the firm's reward-to-volatility ratio is the probability of a "successful project. The probability of an "unsuccessful" project will always be equal to 1.0 minus the probability of a "successful" project. This situation occurs when the project's reward-to-volatility ratio is less than that of the firm.

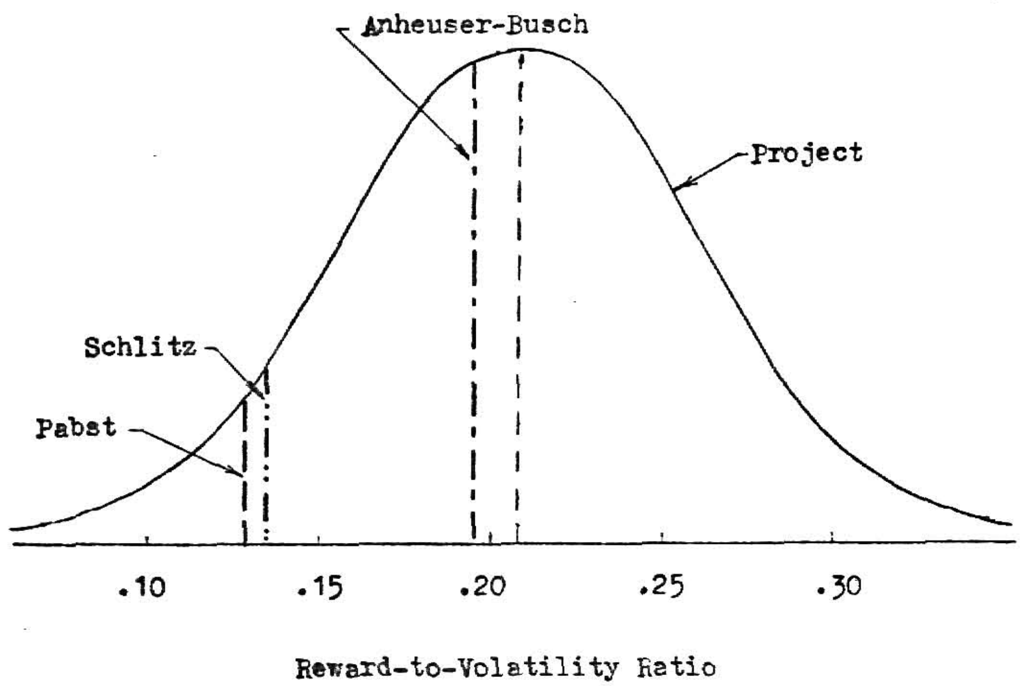

Figure 3.2. Relationship of the Project to the Publicly - Owned Companies.

The results of the comparisons between the project and these firms are a bit more realistic. As Table 3.6 shows, both the Pabst Brewing Company and the Schlitz Brewing Company would almost certainly accept this project because the probability of success is very high. Anheuser-Busch, on the other hand, might not accept this project. The probability that the project will demonstrate a return rate higher than the return rate demanded by the firm is 0.60. So, the project has a 60% probability of being a "successful" project.

The decision as to the acceptance or rejection of this project would probably be made in light of some of the characteristics of the project, (such as the initial investment, length of project, etc.). The statistical model can supply the decision maker with the probabilities concerning the likely outcome of his decisions. But here is where capital budgeting no longer is a science; it now becomes an art. The actual decision to accept or reject a project is a human decision, subject to human errors. This model provides no automatic decisions. The experience and judgement of the decision maker is now the crucial factor in whether the firm operates as a successful and profitable venture.

TABLE 3.5

RESULTS OF THE REGRESSION ANALYSIS ON THE THREE PUBLIC FIRMS

FIRM	β	σ_B	$\bar{R}_{Firm}$	REWARD-TO-VOLATILITY	r^2
Anheuser-Busch	0.763	0.322	0.201	0.198	0.301
Pabst	1.369	0.417	0.229	0.131	0.454
Schlitz	1.207	0.535	0.217	0.138	0.317

TABLE 3.6

PROBABILITIES OF THE SUCCESS OR FAILURE OF THE PROJECT WITH RESPECT TO THE PUBLICLY OWNED FIRM

FIRM	PROBABILITY OF SUCCESS	PROBABILITY OF FAILURE
Anheuser-Busch	0.5987	0.4013
Schlitz	0.9726	0.0274
Pabst	0.9826	0.0174

CHAPTER IV

CONCLUSION

Perhaps the most significant result of this thesis was the use of a statistical approach to make capital budgeting decisions. The use of the statistical approach to project selection developed in this thesis enabled the decision maker to assess the probabilities of making a wrong decision, concerning the acceptance or rejection of the candidate project. The ability to make statements of probabilities such as these comes from the idea that a simulation model of the project can be constructed and this model can then be used to sample a future event - the project. The use of these interval estimates allows the firm or surrogate firm to perform in a manner superior to its competitors, with a certain degree of confidence. From the capital asset pricing theory, the reward-to-volatility ratio was developed as the criterion upon which the projects would be evaluated, based on the fact that project selection in accordance to this criterion would increase the expected net present value of the firm. This test criterion ratio, then, became the statistic of interest and was used in making the statements of probability concerning the comparison of the firm's reward-to-volatility ratio versus the project's reward-to-volatility distribution.

An example was developed to demonstrate how a statistical approach to capital budgeting might be used by either the private firm or the publicly-owned firm. The statistical model of the capital asset pricing model is applicable as far as the private firm is concerned, to homogeneous industries in which there are publicly owned firms competing with a private firm. The only restriction occurs in industries which are dominated by conglomerate firms, because these firms would distort the return rate of the surrogate firm if they were included. All publicly owned firms can use such a model without restrictions.

The approach taken by this thesis emphasized the use of the empirical distribution of the sample data rather than the theoretical distribution of the reward-to-volatility ratio, which was shown to be distributed Cauchy. The Cauchy distribution is very interesting, due to the fact that it does not possess a mean, a variance, or any other higher moments. Therefore, it becomes more difficult to work with. Since the Cauchy and the normal distributions are quite similar, it is questionable whether the use of the Cauchy distribution would produce significantly different results from those obtained by assuming normality of the reward-to-volatility distribution.

The statistical approach developed in this thesis uses the point estimate of the firm's reward-to-volatility ratio from the historical data of the firm. Intuitively, a probability distribution of this ratio for the firm would present a better view of the firm's position in the economy. If this were possible, the two probability distributions (one from the firm and one from the project) could statistically be compared and the result would be the development of an operating characteristic curve for the project in relation to the firm. This would result in a far superior decision making tool than is presently provided by the model that only uses a point estimate of the reward-to-volatility ratio for the firm. The problem arises when trying to determine the distribution of the test ratio for the firm. The firm has only one opportunity to perform in a time period, and this is given by the historical data for the firm. In order to predict the future performance level of the firm, it too must be modeled like that of the project. In order to use the capital asset pricing theory, both the firm and the project must have a relationship with the market. Therefore, if a time series analysis could be done on the market, and a suitable predictive model constructed, the market too could be sampled, as well as the firm and the project in relation to that sample of the market. Through the sampling procedures the distributions could be discovered and compared.

The model of the return on the stock market might possibly be very difficult to construct. As part of the investigation for this thesis, a time series analysis was performed on the return rates for the market over the past fifteen years and the results showed that the return on the market was a random variable. Many leading economists support this finding. This problem however, is the next logical step in developing a superior statistical model of the capital asset pricing model and study in this area is recommended and encouraged.

FOOTNOTES

1. Expenditures on Producers Structures are the amounts of capital invested in buildings, manufacturing plants, offices, etc., by the various firms and companies.

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APPENDIX

DATA OF PUBLICLY OWNED FIRMS IN BREWING INDUSTRY

		1961			1962			1963			1964			1965		
FIRM	PRICE	P	D	NW	P	D	NW	P	D	NW	P	D	NW	P	D	NW
Anheuser-Busch	4.97	7.00	.17	153.4	5.88	.17	163.1	5.59	.18	170.9	8.16	.19	182.6	11.78	.22	197.6
Carling-O'Keefe	8.88	11.35	.32	164.4	10.00	.34	190.9	9.13	.35	216.1	9.75	.40	221.6	7.83	.40	221.8
Falstaff	17.25	19.00	.65	43.9	15.13	.65	47.2	16.31	.70	49.7	21.25	.72	53.7	19.25	.76	57.9
Pabst	4.38	8.38	--	75.5	6.44	.25	79.9	10.31	.25	83.9	16.25	.25	92.1	20.56	.25	100.4
F&M Schaefer	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
Schlitz	--	--	--	--	8.79	.20	--	14.71	.24	144.9	16.17	.23	147.2	16.67	.26	163.6

	1966			1967			1968			1969			1970		
Anheuser-Busch	14.63	.25	220.5	20.75	.30	255.4	32.00	.37	285.3	36.50	.40	314.1	38.19	.43	358.5
Carling O'Keefe	6.50	.40	221.8	7.00	.40	222.0	11.38	.40	191.9	7.88	.40	195.2	7.13	.40	172.6
[illegible]	12.63	.76	58.6	12.50	.76	54.5	16.13	.40	51.8	9.63	.40	51.6	5.63	-	52.8
Pabst	18.25	.38	112.2	32.00	.50	125.7	48.00	.50	141.3	45.40	.58	158.3	49.63	.65	175.4
F&M Schaefer	--	--	--	--	--	--	--	--	--	51.50	--	43.9	27.00	--	50.7
Schlitz	6.63	.29	172.7	12.33	.33	186.9	17.25	.38	186.7	25.00	.45	194.6	23.00	.47	208.1

P = price in dollars

D = dividend per share in dollars

NW = net work (dollars X 10^6)

FIRM	1971 P	1971 D	1971 NW	1972 P	1972 D	1972 NW	1973 P	1973 D	1973 NW	1974 P	1974 D	1974 NW	1975 P	1975 D	1975 NW
Anheuser-Busch	56.00	.53	414.0	55.25	.58	462.0	32.88	.60	50[illegible].8	24.00	.60	537.8	33.75	.64	590.0
Carling O'Keefe	6.88	.40	167.6	5.38	.40	163.2	3.50	.10	166.8	1.88	--	162.3	2.50	--	158.4
Falstaff	8.50	.10	53.7	5.13	--	47.7	2.25	--	41.8	1.25	--	37.9	--	--	--
Pabst	75.63	.80	193.1	75.00	.83	213.1	25.50	.86	226.1	16.50	.91	223.9	21.38	1.01	236.0
F&MSchaefer	16.50	--	55.0	8.88	--	48.6	3.75	--	49.6	2.38	--	50.3	2.88	--	49.0
Schlitz	36.33	.52	230.3	58.38	.55	252.5	56.00	.61	285.5	15.00	.67	315.3	19.13	.68	330.0

P = price in dollars

D = dividend per share in dollars

NW = net work (dollars X 10^6)

INDEX AND DIVIDEND-PRICE RATIO OF STANDARD & POOR'S 500 COMPOSITE INDEX

I	D/P	I	D/P	I	D/P	I	D/P	I	D/P	I	D/P	I	D/P	I	D/P
1960		1961		1962		1963		1964		1965		1966		1967	
58.11	3.37	71.55	2.86	63.10	3.39	75.02	3.15	84.75	3.05	92.43	3.06	80.33	3.64	96.47	3.08
1968		1969		1970		1971		1972		1973		1974		1975	
105.04	2.96	90.97	3.51	92.15	3.47	102.09	2.99	117.09	2.71	97.35	3.64	68.56	5.37	90.19	4.11

I = index

D/P = dividend-price

AN INVESTIGATION OF A STATISTICAL APPROACH FOR PROJECT SELECTION

by

ROGER DEAN BAKER

Roger Baker

AN ABSTRACT OF A MASTER'S THESIS

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ABSTRACT

This thesis examines the concept of using a statistical approach to the capital asset pricing model for use in capital budgeting. The theory supporting the capital asset pricing model is introduced and a project selection criterion developed from the model. This criterion is known as the "reward-to-volatility" criterion. This criterion is shown to be a valid selection criterion that, if adhered to by the firm, will increase the net present value of the firm. Previous work in this area has been limited to point-value models using this criterion. In this case, the project is either accepted or rejected by the selection criterion. This thesis expands the use of this selection criterion to include the probabilities relating to the project's chances of "success" or "failure", if accepted by the firm. The advantages to the decision-maker of this type of statistical approach are discussed.

In order to use this approach, the concept of "sampling" a future project is introduced. The sampling procedure leads to a probability distribution of the reward-to-volatility ratio used in the selection criterion. This distribution is arrived at from either the theoretical derivation of the reward-to-volatility ratio or from an empirical approximation of the data sampled. This distribution can now be compared to the firm's reward-to volatility ratio arrived at through the examination of the firm's historic economic performance. The probability statements concerning the success or failure of the project are arrived at through this comparison. This methodology is demonstrated by an example using

a private firm and several public firms that are participating in the brewing industry. Problems associated with a statistical model of the capital asset pricing model are discussed and areas of further investigation are suggested.