DIRECT DISTRIBUTION WITH SIDE SWAY FOR MULTI-STORY ONE BAY FRAME

by

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INTRODUCTION

The moment-distribution method was developed by Professor Hardy Cross and is usually termed the "Cross Method". It is applicable to all types of rigid-joint structures and is probably the most popular and widely used method for the analysis of such structures. The general procedure of moment distribution is well known and quite familiar to us. But when the rigid frame is under the action of transverse loads or of unsymmetrical vertical loads, the translation of joints, or sidesway occurs.

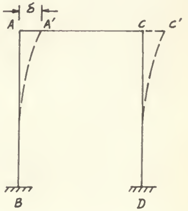
In the analysis of a structure where sidesway may occur we first put sufficient reactions to prevent the sidesway then distribute the moment by the general procedure. Next, after the reactions are determined, loads opposite in direction, but equal to the reactions, are put on the basic frame to replace the reactions. Then the moments are determined. The final moment is obtained by adding the two sets of moments together. Since the procedures are more complicated more labor is required.

A method of direct moment distribution with sidesway is introduced in the "Analysis of Statically Indeterminate Structures" written by Parcel and Moorman. By taking the translation and rotation into consideration at the same time, a modified form of moment distribution can distribute the moment directly where no temporary supports of reactions are necessary. In the Parcel and Moorman's book discussion and examples are given on single-story, one-bay symmetrical rigid frames. Here special advantages can be seen when compared with the moment distribution method. It is the purpose of this report to extend the same method into multi-story-one-bay symmetrical and unsymmetrical rigid frames and compare with moment distribution methods.

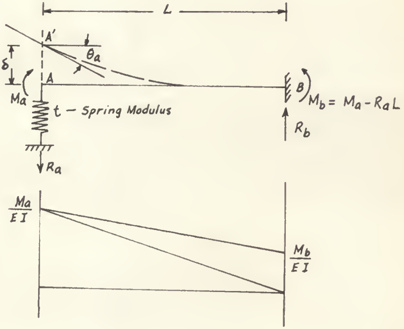
DIRECT DISTRIBUTION WITH SIDESWAY IN ONE-STORY-ONE-BAY FRAME

Modified Rotational Stiffness

Adopting the assumption that the girders are infinitely rigid in a rigid frame when it deflects sidewise, we consider the column AB in the frame shown at the right as a cantilever beam to be fixed at B and to be elastically restrained against translation at A.



The translational restraint at A, which actually is supported by the stiffness of the other column, has a spring modulus t. The stiffness S_{ab} of AB is $\frac{M_a}{\theta_a}$, and the carry-over



factor $r_{ab} = \frac{M_b}{M_a}$

From the moment diagram shown above, by use of the moment-area method, we find the rotation at A as

$$\theta_a = \frac{L}{2EI}(M_a + M_b) = \frac{L}{2EI}(2M_a - R_a L) \tag{1}$$

and deflection at A as

$$\delta_a = \frac{R_a}{t} = \frac{1}{EI}\left(\frac{M_a L^2}{6} + \frac{2(M_a - R_a L)L^2}{6}\right) \tag{2}$$

from (2)

$$R_a = \frac{t}{EI}\left(\frac{M_a L^2}{6} + \frac{2(M_a - R_a L)L^2}{6}\right)$$

which follows

$$R_a = \left(\frac{3EI}{3EI + L^3t}\right)\left(\frac{M_aL^2t}{2EI}\right) = \frac{3M_aL^2t}{6EI + 2L^3t} \qquad (2a)$$

substituting (2a) into (1)

$$\theta_a = \frac{M_aL}{EI}\left(1 - \frac{3L^3t}{12EI + 4L^3t}\right)$$

and the stiffness at A is

$$S_{ab} = \frac{M_a}{\theta_a} = \frac{EI}{L}\left(\frac{12EI + 4tL^3}{12EI + tL^3}\right) \qquad (3)$$

Now, the translational stiffness is $T = \frac{12EI}{L^3}$ (3a)

then $S_{ab} = \frac{EI}{L}\left(\frac{T + 4t}{T + t}\right)$ (3a)

if we neglect the effect of any axial loads. A tranlational distribution factor may be defined by the same expression in the Cross Method where the distribution factor is $\frac{K}{\Sigma K}$, that $\tau = \frac{t}{t + T}$. Substituting τ into (3a) we have the stiffness

$$S_{ab} = \frac{EI}{L}\left(1 + \frac{3t}{T + t}\right) = EK(3\tau + 1) \quad \text{where } K = \frac{I}{L}$$

The carry-over factor

$$r_{ab} = \frac{M_b}{M_a} = \frac{M_a - R_aL}{M_a} = 1 - \frac{3L^3t}{6EI + 2L^3t} = \frac{6EI - L^3t}{6EI + 2L^3t}$$

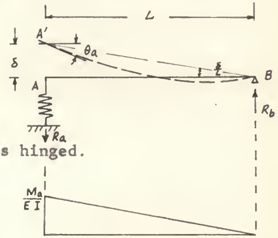
$$= \frac{12EI - 2tL^3}{12EI + 4tL^3} = \frac{T - 2t}{T + 4t} = 1 - \frac{6t}{T + 4t}$$

$$= 1 - \frac{6\left(\frac{t}{T + t}\right)}{\frac{T + 4t}{T + t}} = 1 - \frac{6}{1 + 3\tau} = \frac{1 - 3\tau}{1 + 3\tau} \qquad (4)$$

When statical moment sign convention is used

$$r_{ab} = \frac{3\tau - 1}{3\tau + 1} \qquad (4a)$$

Consider the column AB. When B end is not fixed but hinged, the carry-over factor is, by using area-moment theory again



$$\theta_a = \left(\frac{M_aL}{2EI}\right)\left(\frac{2L}{3L}\right) + \frac{\delta}{L}$$

$$= \frac{M_aL}{3EI} + \frac{\delta}{L}$$

But $\delta = \frac{R_a}{t} = \frac{M_a}{Lt}$ when B is hinged.

Hence $\theta_a = M_a \left(\frac{L}{3EI} + \frac{1}{L^2 t}\right)$

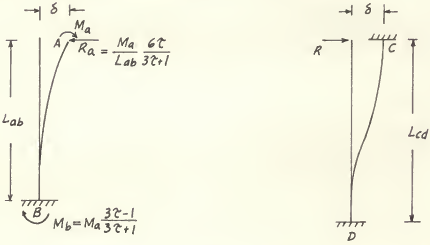
and the stiffness being

$$S_{ab} = \frac{M_a}{\theta_a} = \frac{3EIL^2 t}{L^3 t + 3EI} = \frac{12EIL^2 t}{4L^3 t + 12EI}$$

$$= \frac{EI}{L}\left(\frac{12t}{4t + T}\right) = EK\left(\frac{12\tau}{3\tau + 1}\right)$$

and it is clear that since B is hinged the moment is zero at B and the carryover on factor r_{ab} = o

Translational Carry-over Factor



Column CD of the above frame actually contributes to the translational stiffness of member AB. After the moments are balanced at A, moment will be induced at C and D. As the shear at A is transmitted to C through BC girder, the shear R_c, acting at C equal but opposite in direction to R_a, pushes the column CD through a same sidewise deflection δ. In column CD the induced moment $M_c = M_d$, taking moment about its point of inflection at the middle, $M_d = - R_a \frac{Lcd}{2}$ in which, from (2a)

$$R_a = \frac{3M_a L^2 t}{6EI + 2L^3 t} = \frac{M_a}{Lab} \left(\frac{6\tau}{3\tau + 1} \right)$$

Hence $M_d = M_a\left(- \frac{Lcd}{Lab} \frac{3\tau}{3\tau + 1}\right)$ (6)

Carry-over factor for the Girder

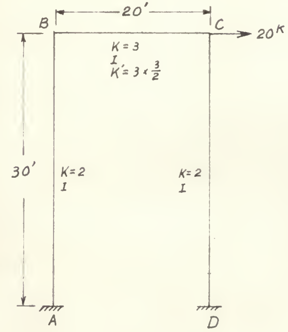
The carry-over factor for the girder will be the same as in the moment distribution method

$$r = \frac{M_{ba}}{M_{ab}} = \frac{1}{2}$$

Examples

Simple examples are worked out by applying this method on one-story one-bay frames.

Example 1. Horizontal force acting on a symmetrical frame with constant I



Sway moment $= \frac{20 \times 30}{4} = 150$

$\tau = \frac{1}{1+1} = 0.5$

$S_{ba} = S_{cD} = EK_{ba}(3\tau + 1) = 5E$

$S_{bc} = S_{cb} = 4EK' = 18E$

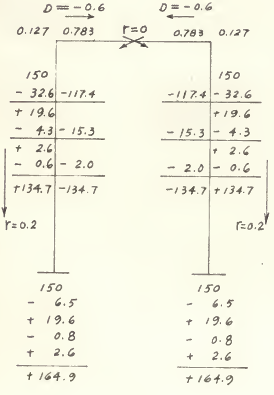
Distribution factor

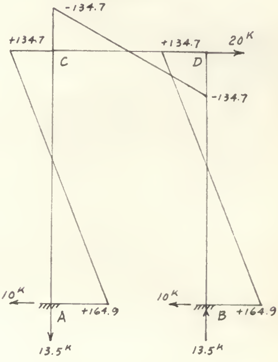
$\frac{5}{18+5} = 0.217 \qquad \frac{18}{18+5} = 0.783$

$C_{ba} = C_{cD} = \frac{3\tau - 1}{3\tau + 1} = 0.2$

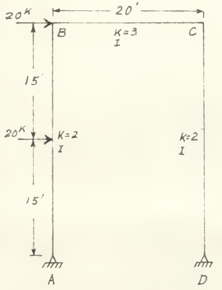
$C_{bc} = C_{cb} = 0$

$D = -\frac{L_{AB}}{L_{CD}} \frac{3\tau}{3\tau + 1} = -0.6$





Example 2. Horizontal force acting on a symmetrical frame with hinged end



$$\tau = \frac{\frac{1}{4}}{1+\frac{1}{4}} = 0.2$$

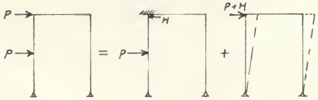
$$S_{ba} = S_{cd} = \frac{12\tau}{3\tau+1} E K_{ba} = 3E$$

$$S_{bc} = S_{cb} = 4 E K_{bc} = 12E$$

$$C_{ba} = C_{cd} = 0$$

$$D = -1$$

$$C_{bc} = C_{cb} = 0.5$$

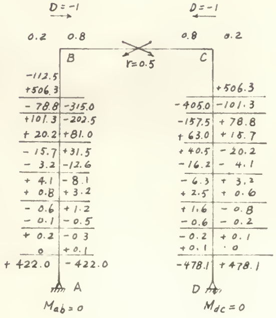


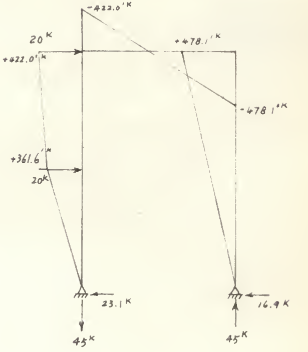
$$M_{bc\,(Fix)} = \frac{20 \times 15^2}{30} \times \frac{1}{2} = 75$$

$$M_{bc\,(hinged)} = 75 \times \frac{3}{2} = 112.5$$

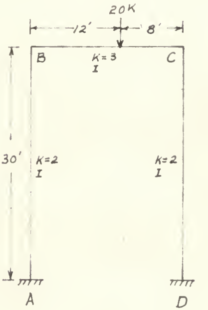
$$H = \frac{20}{2} + \frac{112.5}{30} = 13.75$$

$$M_{bc\,(sway)} = \frac{20 \times 30 + 13.75 \times 30}{2} = 506.3$$





Example 3. Vertical load acting on unsymmetrical frame



$$\tau = \frac{1}{1+1} = 0.5$$

$$S_{ba} = S_{cd} = EK(3\tau+1) = 5E$$

$$S_{bc} = S_{cb} = 4EK = 12E$$

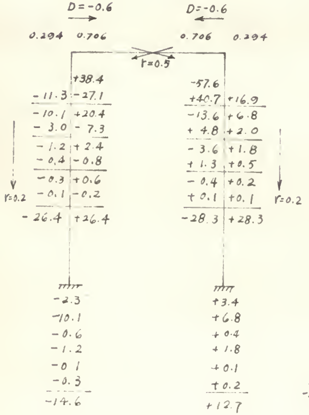
$$C_{ba} = C_{cd} = 0.2$$

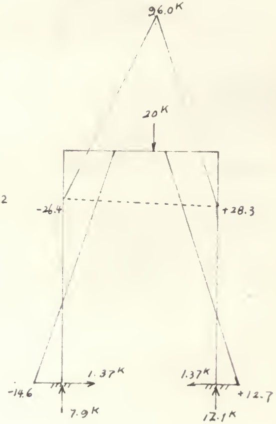
$$C_{bc} = C_{cb} = 0.5$$

$$D = -\frac{L_{cd}}{L_{ad}}\,\frac{3\tau}{3\tau+1} = -0.6$$

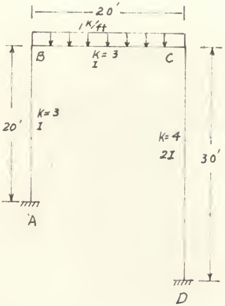
$$M_{bc(Fix)} = \frac{20\times12\times8}{20}\times\frac{8}{20} = 38.4$$

$$M_{cb(Fix)} = \frac{20\times12\times8}{20}\times\frac{12}{20} = 57.6$$





Example 4. Unsymmetrical frame with uniform load.



$$T_{ab} = \frac{12E \times 3}{20^2}$$

$$T_{cd} = \frac{12E \times 4}{30^2}$$

$$\frac{T_{ab}}{T_{cd}} = \frac{27}{16}$$

For AB.

$$\tau_{ab} = \frac{16}{27+16} = 0.372$$

$$S_{ab} = EK(3\tau+1) = 6.35E$$

$$C_{ab} = \frac{3\tau-1}{3\tau+1} = 0.055$$

$$D_{to\,CD} = -\frac{30}{20}\,\frac{3\times0.372}{3\times0.372+1} = -0.792$$

For CD.

$$\tau_{cd} = \frac{27}{27+16} = 0.628$$

$$S_{cd} = EK(3\tau+1) = 11.53E$$

$$C_{cd} = \frac{3\tau-1}{3\tau+1} = 0.306$$

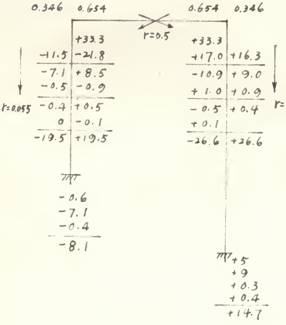
$$D_{to\,AB} = -\frac{20}{30} \times \frac{3\tau}{3\tau+1} = -0.436$$

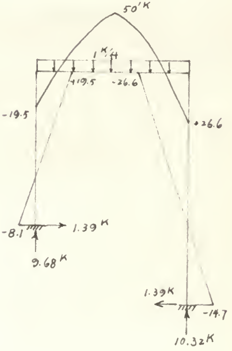
Fixend M. $= \frac{wl^2}{12} = 33.3'^{K}$

distribution factor are

$$\frac{6.35}{6.35+11.53} = 0.346,\quad \frac{11.53}{6.35+11.53} = 0.654$$

$D = -0.792 \rightarrow$ $\qquad D = -0.436 \leftarrow$

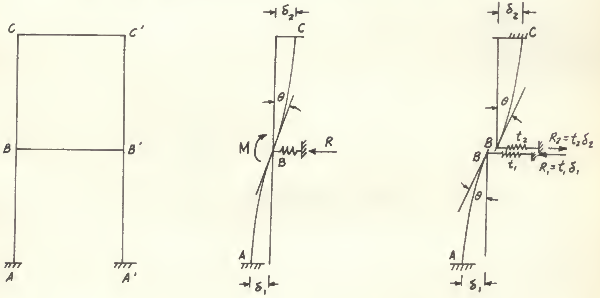




DIRECT MOMENT DISTRIBUTION OF MULTI-STORY-ONE-BAY-FRAME

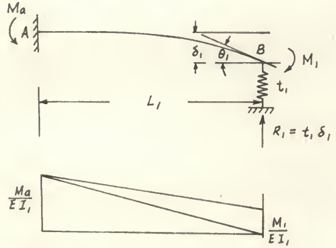
Rotational Stiffness and Carry-over Factor

Consider the two-story frame shown in Fig. below.



Point A is fixed against rotation and translation. Point B is supported by a spring where translation and rotation are both allowed. Point C is free to translate but not free to rotate. Now in column ABC shown above, bar AB and BC can be considered separately. It actually can be considered as two cantilever beams which we already discussed previously. If the cantilever beams are supported by a spring with spring modulus t_1 and t_2

and under the action of M_1 and M_2, respectively, we can obtain the stiffness for AB and BC. For AB,



$$M_a = M_1 - R_1L_1$$

$$\theta_1 = \frac{L_1}{2EI_1} (M_a + M_1) = \frac{L_1}{2EI_1} (2M_1 - R_1L_1) \qquad \text{(a)}$$

$$\delta_1 = \frac{R_1}{t_1} = \frac{1}{EI_1} \left(\frac{ML_1^{2}}{6} + \frac{2(M_1 - R_1L_1)L_1^{2}}{6} \right) \qquad \text{(b)}$$

Eliminating R_1 from (a) and (b).

$$S_1 = \frac{M_1}{\theta_1} = \frac{EI_1}{L_1} \left(\frac{12EI_1 + 4t_1L_1^{3}}{12EI_1 + t_1L_1^{3}} \right)$$

$$\text{Let } T_1 = \frac{12EK_1}{L_1^{2}}, \quad \tau_1 = \frac{t_1}{t_1 + T_1}$$

Then $S_1 = EK_1\ (3\tau_1 + 1)$

where $K_1 = \dfrac{I_1}{L_1}$

For BC,

$$M_c = M_2 - R_2L_2$$

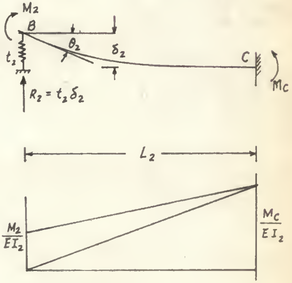
$$\theta_2 = \frac{L_2}{2EI_2}(M_2 + M_c) = \frac{L_2}{2EI_2}(2M_2 - R_2L_2)$$

$$\delta_2 = \frac{R_2}{t_2} = \frac{1}{EI_2} = \frac{M_2L^2{}_2}{6} + \frac{2(M_2 - R_2L_2)\ L^2 2}{6}$$

$$S_2 = \frac{M_2}{\theta_2} = \frac{EI_2}{L_2}\ \frac{12EI_2 + 4t_2L_2{}^3}{12EI_2 + t_2L_2{}^3}$$

Let $T_2 = \dfrac{12EK_2}{L_2{}^2}$, $\tau_2 = \dfrac{t_2}{t_2 + T_2}$

$S_2 = EK_2(3\tau_2 + 1)$ where $K_2 = \dfrac{I_2}{L_2}$



and the carry-over factor can be found as

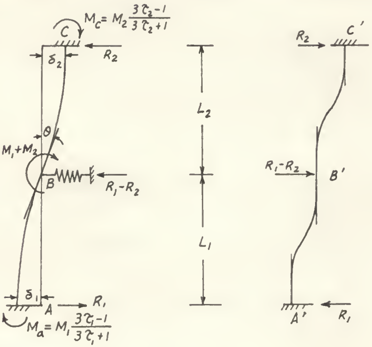
$$r_{ba} = \frac{M_a}{M_1} = \frac{M_1 - R_1L_1}{M_1} = \frac{12E_1 - 2t_1L_1{}^3}{12EI_1 + 4t_1I_1{}^3} = \frac{1 - 3\tau_1}{1 + 3\tau_1}$$

$$r_{bc} = \frac{M_b}{M_2} = \frac{M_2 - R_2L_2}{M_2} = \frac{1 - 3\tau_2}{1 + 3\tau_2}$$

By changing sign convention into static-moment expression

$$r_{ba} = \frac{3\tau_1 - 1}{3\tau_1 + 1} \quad \text{and} \quad r_{bc} = \frac{3\tau_2 - 1}{3\tau_2 + 1}$$

Superposing AB and BC we have the Fig. below



Translational Carry-over Factor

Since all the horizontal force or shear, transmit in to the other column through the rigid girder, the column A'B'C' deformed as shown above at right. The point of inflection is at the middle of AB and BC. Hence,

$$M_{A'} = M_1 \left(- \frac{L'_1}{L_1} \quad \frac{3\tau_1}{3\tau_1 + 1}\right)$$

$$M_{C'} = M_2 \left(- \frac{L'_2}{L_2} \quad \frac{3\tau_2}{3\tau_2 + 1}\right)$$

The carry-over factor D is

$$D_1 = \frac{-L'_1}{L_1} \left(\frac{3\tau_1}{3\tau_1 + 1}\right) \qquad D_2 = - \frac{L'_2}{L_2} \left(\frac{3\tau_2}{3\tau_2 + 1}\right)$$

Spring Modulus

Since the elastic restraint of a spring is actually supported by the other column

$$t_1 = T'_1 \left(\frac{12EI'_1}{L'^3_1}\right) = \frac{12E\,K'_1}{L'^2_1}$$

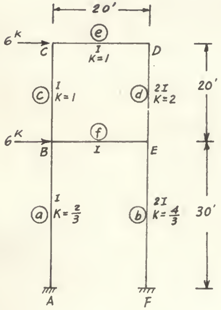
$$t_2 = T'_2 \left(\frac{12EI'_2}{L'^3_2}\right) = \frac{12E\,K'_2}{L'^2_2}$$

Examples

(See the following pages.)

Example 1. Horizontal force acting on two-story frame.

A. Solve By Direct Distribution Method



$$T_a = \frac{12E \times \frac{2}{3}}{30^2} \qquad t_a = \frac{12E \times \frac{4}{3}}{30^2}$$

$$\tau_a = \frac{2}{1+2} = \frac{2}{3}$$

$$S_a = EK_a(3\tau_a + 1) = E \times \frac{2}{3} \times 3 = 2E$$

$$r_a = \frac{3\tau_a - 1}{3\tau_a + 1} = \frac{1}{3}$$

$$D_a = -\frac{3 \times \frac{2}{3}}{3 \times \frac{2}{3} + 1} = -\frac{2}{3}$$

$$T_b = \frac{12E \times \frac{4}{3}}{30^2} \qquad t_b = \frac{12E \times \frac{2}{3}}{30^2}$$

$$\tau_b = \frac{t_b}{T_b + t_b} = \frac{1}{1+2} = \frac{1}{3}$$

$$S_b = EK_b\,(3\tau_b + 1) = \frac{8}{3}E$$

$$r_b = \frac{3\tau_b - 1}{3\tau_b + 1} = 0$$

$$D_b = -\frac{30}{30} \quad \frac{3 \times \frac{1}{3}}{3 \times \frac{1}{3} + 1} = \frac{-1}{2}$$

Fixend Moment - By Mooris Assumption

$M_{CB} = M_{BC} = \frac{6 \times 20}{2} \times \frac{1}{3} = +20$ ft.-K

$M_{DE} = M_{ED} = \frac{6 \times 20}{2} \times \frac{2}{3} = 40$ ft.-K

$M_{EF} = M_{FE} = \frac{12 \times 30}{2} \times \frac{2}{3} = 120$ ft.-K

$M_{BA} = M_{AB} = \frac{12 \times 30}{2} \times \frac{1}{3} = 60$ ft. -K

Total unbalanced moment at B and C

$M_{unb_B} = 20 + 60 = 80$

$M_{unb_E} = 40 + 120 = 160$

$\tau_c = \frac{2}{3}$

$S_c = E \times 1 \times (3 \times \frac{2}{3} + 1) = 3E$

$r_c = \frac{1}{3}$

$D_c = -\frac{2}{3}$

$\tau_d = \frac{1}{3}$

$S_d = 4E$

$r_d = 0$

$D_d = -\frac{1}{2}$

$S_e = 4EK = 4E$

$r_e = \frac{1}{2}$

$S_f = 4EK = 4E$

$r_f = \frac{1}{2}$

At Point A

A_1 Fixend moment $M_{AB} = +\ 60$

A_2 Carry-over From M_{BA} $-\ 17.8 \times \frac{1}{3} = -\ 5.9$

A_3 C.O. From M_{EF} $-\ 40 \times -\ \frac{1}{2} = +\ 20$

A_4 C.O. From M_{BA} $-\ 6.0 \times \frac{1}{3} = -\ 2.0$

At Point B

B_1 Distribution of unbalanced moment $+\ 20 + 60 = 80$

$$M_{BC} = -\ 80 \times \frac{3}{3 + 4 + 2} = -\ 26.7$$

$$M_{BA} = -\ 80 \times \frac{2}{9} = -\ 17.8$$

$$M_{BE} = -\ 80 \times \frac{4}{9} = -\ 35.5$$

B_2 Translational carry-over from M_{EF}, $-\ 40 \times -\ \frac{1}{2} = +\ 20$

Carry-over from M_{EB} $\frac{1}{2} \times -\ 60 = -\ 30$

B_3 Carry-over from M_{CB} $-\ 8.6 \times \frac{1}{3} = -\ 2.9$

B_4 Translational C.O. from M_{ED} and M_{DE}

$$(-\ 60 \times -\ \tfrac{1}{2}) + (-\ 20 \times -\ \tfrac{1}{2}) = +\ 40$$

B_5 Redistribution of unbalanced moment

(Unbalanced moment =-2.9 + 40 + 20 - 30 = + 27.1)

$$M_{BC} = - 27.1 \times \frac{3}{9} = - 9$$

$$M_{BD} = - 27.1 \times \frac{2}{9} = - 6$$

$$M_{BE} = - 27.1 \times \frac{4}{9} = - 12.1$$

At Point C

Fix-end moment = + 20

C_1 Distribution of unbalanced moment

$$M_{CB} = - 20 \times \frac{3}{7} = - 8.6$$

$$M_{CD} = - 20 \times \frac{4}{7} = - 11.4$$

C_2 Carry-over from M_{BC} $- 26.7 \times \frac{1}{3} = - 8.9$

Carry-over from M_{DC} $- 20 \times \frac{1}{2} = - 10.0$

C_3 T.C.O. from M_{DE} and M_{ED} $(- 20 - 60) \times - \frac{1}{2} = +40$

C_4 Redistribution of unbalanced moment

(Unbalanced moment = -8.9 - 10 + 40 = + 21.1)

$-21.1 \times \frac{3}{7} = 9.0 \quad - 21.1 \times \frac{4}{7} = - 12.1$

At Point D

D_1 Fix-end moment = + 40

Distribution of unbalanced moment $- \frac{1}{2} \times 40 = -20$

D_2 C.O. Factor = 0 in Member DE. $M_{DE} = 0$ from M_{ED}

C.O. Factor = $\frac{1}{2}$= in Member CD. $M_{DC} = \frac{1}{2} \times - 11.4 = -5.7$

D_3 T.C.O. from M_{CB} and M_{BC} $(-8.6 - 26.7) \times - \frac{2}{3} = + 23.5$

D_4 Re-distribution of unbalanced moment

(unbalanced moment)$- \frac{-5.7 + 23.5}{2} = - 8.9$

At Point E

E_1 Unbalanced moment = 40 + 120 = 160

$M_{EF} = - 160 \times \frac{8}{32} = - 40$

$M_{ED} = - 160 \times \frac{12}{32} = - 60$

$M_{EB} = - 160 \times \frac{12}{32} = - 60$

E_2 Carry-over from $M_{DE} = 0$

Carry-over from $M_{BE} = - 35.5 \times \frac{1}{2} = - 17.8$

T.C.O. from $M_{BA} = - 17.8 \times - \frac{2}{3} = + 11.9$

T.C.O from M_{CB} and $M_{BC} = -(8.6 + 26.7) \times -\frac{2}{3} - 23.5$

E_3 Re-distribution

Unbalanced Moment = $-17.8 + 11.9 + 23.5 = 17.6$

$$M_{ED} = M_{EB} = -17.6 \times \frac{12}{32} = -6.6$$

$$M_{EF} = -17.6 \times \frac{8}{32} = -4.4$$

At Point F

Fix-end moment = + 120

F_1 C.O from $M_{EF} = 0$

T.C.O. from $M_{BA} = -17.8 \times -\frac{2}{3} = +11.9$

$r = 0.5$

	M_{CB}	M_{CD}	M_{DC}	M_{DE}	
	+20.0			+40.0	
C_1	− 8.6	−11.4	−20.0	− 20.0	D_1
C_2	− 8.9	−10.0	−5.7	0	D_2
C_3	+40.0			+23.5	D_3
C_4	− 9.0	−12.1	− 8.9	− 8.9	D_4
	− 3.0	− 4.5	− 6.1	0	
	+ 7.8			+12.0	
	− 0.1	− 0.2	− 2.9	− 3.0	
	− 0.4	− 1.5	− 0.1	0	
$r = \frac{1}{3}$	+ 3.3			+ 0.9	$r = 0$
	− 0.6	− 0.8	− 0.4	− 0.4	
	+ 0.3	− 0.2	− 0.4	0	
	+ 0.3			+ 1.0	
	+ 0.1	− 0.1	− 0.3	− 0.3	
	0	− 0.1	0	0	
	+ 0.3			− 0.1	
	− 0.1	− 0.1	0	+ 0.1	
	+40.8	−40.8	−44.8	+44.8	

$D = -\frac{2}{3}$ $D = -\frac{1}{2}$

	M_{BC}	M_{BA}	M_{BE}	M_{EB}	M_{EF}	M_{ED}	
	+20.0	+60.0			+120.0	+40.0	
B_1	−26.7	−17.8	−35.5	−60.0	− 40.0	+60.0	E_1
B_2		+20.0	−30.0	−17.8	+ 11.9	0	E_2
B_3	− 2.9					+23.5	E_2
B_4	+ 40.0			− 6.6	− 4.4	− 6.6	E_3
B_5	− 9.0	− 6.0	−12.1				
	− 3.0	+ 2.2	− 3.3	− 6.1	+ 4.0	0	
	+ 7.8					+12.0	
	− 1.2	− 0.8	− 1.7	− 3.7	− 2.5	− 3.7	
	0	+ 1.2	− 1.9	− 0.9	+ 0.5	0	
	+ 3.3					+ 0.9	
	− 0.9	− 0.6	− 1.1	− 0.2	− 0.1	− 0.2	
$r = \frac{1}{3}$	− 0.2	+ 0.1	− 0.1	− 0.5	+ 0.4	0	$r = 0$
	+ 0.3					+ 1.0	
	0.0	0.0	− 0.1	− 0.4	− 0.2	− 0.4	
	0.0	+ 0.1	− 0.2	0.0	0.0	0	
	+ 0.3					− 0.1	
	− 0.1	− 0.1	0.0	+ 0.1	0.0	0.0	
	+ 27.7	+ 58.3	− 86.0	− 96.1	+89.6	+ 6.5	

$D = -\frac{2}{3}$ $D = -\frac{1}{2}$

	M_{AB}	M_{FE}	
A_1	+60.0	+120.0	F_1
A_2	− 5.9	0	F_1
A_3	+20.0	+ 11.9	F_2
A_4	− 2.0	0	
	+ 2.2	+ 4.0	
	− 0.3	0	
	+ 1.2	+ 0.5	
	− 0.2	0	
	+ 0.1	+ 0.4	
	0	0	
	+ 0.1	0	
	+75.2	+136.2	

Example 1. Solved By Morris Method

	M_{CB}	M_{CD}		M_{DC}	M_{DE}	
D.F. →	$\frac{1}{2}$	$\frac{1}{2}$		$\frac{1}{3}$	$\frac{2}{3}$	
	+20.0				+40.0	
C_0	−10.0	−10.0		−13.3	−26.7	D_0
C_1	−15.0	−6.7		−5.0	−36.9	D_1
C_2	+35.1				+70.1	D_2
C_3	−6.7	−6.7		−9.4	−18.9	D_3
	−5.5	−4.7		−3.4	−17.7	
	+18.0				+36.0	
	−3.9	−3.9		−5.0	−9.9	
	−2.5	−2.5		−2.0	−8.4	
	+8.9				+17.8	
	−1.9	−2.0		−2.4	−5.0	
	−1.2	−1.2		−1.0	−4.1	
	+4.4				+8.7	
	−1.0	−1.0		−1.2	−2.4	
	−0.6	−0.6		−0.5	−2.0	
	+2.1				+4.3	
	−0.5	−0.4		−0.6	−1.2	
	−0.3	−0.3		−0.2	−1.0	
	+1.1				+2.3	
	−0.2	−0.3		−0.4	−0.7	
	−0.2	−0.2		−0.2	−0.5	
	+0.6				+1.1	
	−0.1	−0.1		−0.1	−0.3	
	+40.8	−40.8		−45.0	+45.0	

	M_{BC} 3/8	M_{BA} 2/8	M_{BE} 3/8		M_{EB} 3/13	M_{ED} 6/13	M_{EF} 4/13	
B_0	+20.0	+60.0				+40.0	+120.0	E_0
B_1	−30.0	−20.0	−30.0		−36.9	−73.6	−49.6	E_1
B_2	−5.0		−18.4		−15.0	−13.3		E_2
B_3	+35.1	+17.4				+70.1	+34.8	E_3
B_4	−10.9	−7.3	−10.9		−17.7	−35.4	−23.5	E_5
	−3.4		−8.9		−5.5	−9.5		
	+18.0	+7.7				+36.0	+15.4	
	−5.0	−3.4	−5.0		−8.4	−16.8	−11.2	
	−2.0		−4.2		−2.5	−5.0		
	+9.0	+3.6				+17.8	+7.3	
	−2.4	−1.6	−2.4		−4.1	−8.1	−5.4	
	−1.0		−2.0		−1.2	−2.5		
	+4.4	+1.7				+8.7	+3.5	
	−1.2	−0.7	−1.2		−2.0	−3.9	−2.6	
	−0.5		−1.0		−0.6	−1.2		
	+2.1	+0.9				+4.3	+1.6	
	−0.6	−0.3	−0.6		−0.9	−1.9	−1.3	
	−0.2		−0.5		−0.3	−0.8		
	+1.1	+0.4				+2.2	+0.8	
	−0.3	−0.2	−0.3		−0.4	−0.9	−0.6	

− 0.1		− 0.2
+ 0.5	+ 0.2	
− 0.1	− 0.1	− 0.2
+ 27.5	+ 58.3	− 85.8

− 0.2	− 0.4	
	+ 1.1	+ 0.4
− 0.2	− 0.4	− 0.3
− 95.8	+ 6.2	+ 89.3

	M_{AB}	M_{FE}	
A_0	+ 60.0	+ 120.0	F_0
A_1	− 10.0	− 24.8	F_1
A_2	+ 17.4	+ 34.8	F_2
A_3	− 3.7	− 11.8	F_3
A_4	+ 7.8	+ 15.4	F_4
	− 1.7	− 5.6	
	+ 3.7	+ 7.3	
	− 0.8	− 2.7	
	+ 1.8	+ 3.5	
	− 0.4	− 1.3	
	+ 0.9	+ 1.6	
	+ 75.0	+ 136.4	

REMARKS:

F_0, A_0 Fix-end Moment

F_1, A_1 Carry-over from $B_1 - M_{BA}$ $E_1 - M_{EF}$

F_2, A_2 Correctional Moment (See B_3)

F_3, A_3 C.O. from $B_4 - M_{BA}$, $E_4 - M_{EF}$

F_4, A_4 Correctional moment (Second correction)

B_0, E_0 Fix-end Moment $M_B = +80$. $M_E = +160$

B_1, E_1 distribution

B_2, E_2. Carry-over from $C_0 - M_{CB}$ $B_1 - M_{EB}$. $B_1 - M_{BE}$, $D_0 - M_{DE}$

B_3, E_3. correctional moment

For uper frame BCDE

External force $= +6^K$

Column shear $= \dfrac{-5-15}{20} + \dfrac{-23.6-46.9}{20} = -4.525^K$

Total correctional sway-moment $= 20 \times (6+4.525) = 210.5$

$M_{BC} = M_{CB} = \dfrac{210.5}{6} = 35.1$ $M_{DE} = M_{ED} = \dfrac{210.5}{3} = 70.1$

For lower frame ABEF

External force $= 12^K$

Column Shear $= \dfrac{40+50}{30} + \dfrac{70.4+95.2}{30} = 8.52^K$

correctional sway moment $= 30\,(12 - 8.52) = 104.4$

$M_{BA} = M_{AB} = \dfrac{104.4}{6} = 17.4$

$M_{EF} = M_{FE} = \dfrac{104.4}{3} = 34.8$

B_4, E_4 Distribution of the correctional moment.

C_0, D_0. Fix-end moment. and distribution

C_1, D_1 Carry-over value

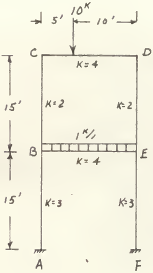
C_2, D_2 Correctional moment

C_3, D_3 Distribution of the correctional moment.

Example 2. Vetical force acting on two story frame

Frame Symmetry , load unsymmetry

A. Solve by Direct Distribution method.



Since frame symmetry , for columes

$\tau = \frac{1}{2}$

$S = 2.5\,EK$

$r = \frac{1}{5}$

$D = -0.6$

for beams

$S = 4EK$

$r = \frac{1}{2}$

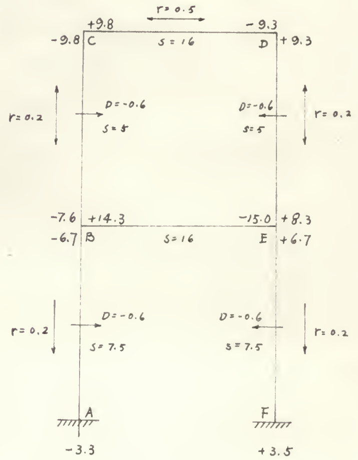
M_{CB}	M_{CD}	M_{DC}	M_{DE}
	+22.2	-11.1	
-5.3	-16.9	+8.4	+2.7
-0.7	+4.2	-8.5	+0.7
-3.6			+5.2
0	+0.1	+2.0	+0.6
+0.1	+1.0	+0.1	-0.1
0			-0.2
-0.3	-0.8	+0.1	+0.1
0	0	-0.4	0
0			+0.1
0	0	+0.1	+0.2
-9.8	+9.8	-9.3	+9.3

M_{BC}	M_{BA}	M_{BE}	M_{EB}	M_{EF}	M_{ED}
		+18.8	-18.8		
-3.3	-5.0	-10.5	+10.5	+5.0	+3.3
-1.1	-3.0	+5.3	-5.3	+3.0	+0.5
-3.6					+5.2
+0.4	+0.6	+1.4	-1.9	-0.9	-0.6
+0	+0.5	-0.9	+0.7	-0.2	+0.1
0					-0.2
+0.1	+0.1	+0.2	-0.2	-0.1	-0.1

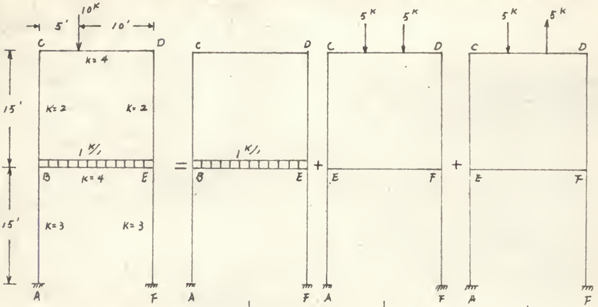
-0.1	+0.1	-0.1		+0.1	-0.1	+0.1
0						0
0	0	+0.1		-0.1	0	0
-7.6	-6.7	+14.3		-15.0	+6.7	+8.3

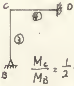
M_{AB}	M_{FE}
-1.0	+1.0
-3.0	+3.0
+0.1	-0.2
+0.5	-0.2
0	0
+0.1	-0.1
-3.3	+3.5

Final Result:



B: Salve By Modified Stiffness Factor, and cantilever Momement Distribution method.





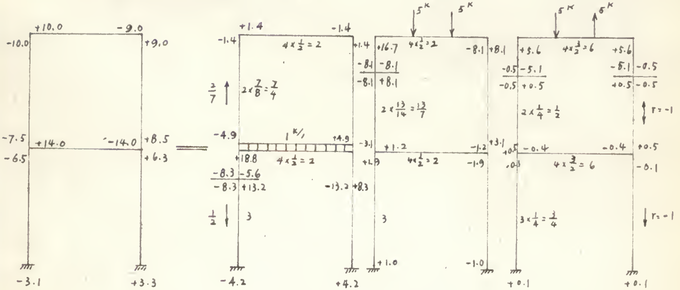
$$\frac{M_c}{M_B} = \frac{1}{2}\cdot\frac{4}{7} = \frac{2}{7}$$

$$j = \frac{3/4}{1-\frac{1}{2}\cdot\frac{2}{7}} = \frac{7}{8}$$

$$\frac{M_b}{M_c} = \frac{1}{2}\cdot\frac{10}{13} = \frac{5}{13}$$

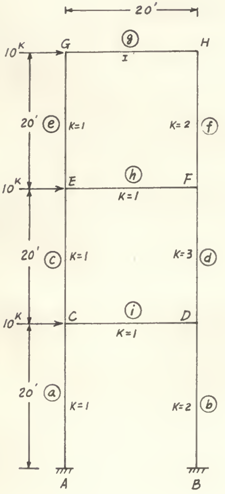
$$j = \frac{3/4}{1-\frac{1}{2}\cdot\frac{5}{13}} = \frac{13}{14}$$

final results



Example 3, Horizontal force acting on three story frame.

A. Solve by Direct Distribution Method



$$T_a = \frac{12EK_a}{20^2} \qquad t_a = \frac{12EK_b}{20^2}$$

$$\tau_a = \frac{2}{1+2} = \frac{2}{3}$$

$$S_a = EK_a(3\tau_a + 1) = E \times 1 \times 3 = 3E$$

$$r_a = \frac{3\tau_a - 1}{3\tau_a + 1} = \frac{1}{3}$$

$$D_a = -\frac{3 \quad a}{(3 \quad_a + 1)} = -\frac{2}{3}$$

$$T_b = \frac{12EK_b}{L_b^{\ 2}} \qquad t_b = \frac{12EK_a}{L_a^{\ 2}}$$

$$\tau_b = \frac{1}{1+2} = \frac{1}{3}$$

$$S_b = 4E$$

$$r_b = 0$$

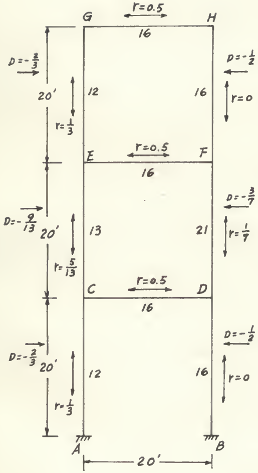
$$D_b = -\frac{1}{2}$$

$$\tau_c = \frac{3}{1+3} = \frac{3}{4}$$

$$S_c = \frac{13}{4} E$$

$$r_c = \frac{5}{13}$$

$$D_c{}^2 = -\frac{9}{13}$$



$$\tau_d = \frac{1}{4}$$

$$S_d = \frac{21}{4} E$$

$$r_d = -\frac{1}{7}$$

$$D_d = -\frac{3}{7}$$

$$\tau_e = \frac{2}{3}$$

$$S_e = 3E$$

$$r_e = \frac{1}{3}$$

$$D_e = -\frac{2}{3}$$

$\tau_f = \frac{1}{3}$

$S_f = 4E$

$r_f = 0$

$D_f = -\frac{1}{2}$

$S_g,\ S_h,\ S_i = 4EK = 4E$

$r_g,\ r_h,\ r_i = \frac{1}{2}$

Fix end moment - By Moaris Assumption

$$M_{GE} = M_{EG} = \frac{1}{3}\ \frac{10x20}{2} = 33.3 \text{ ft-K}$$

$$M_{HF} = M_{FH} = \frac{2}{3}\ \frac{10x20}{2} = 66.7 \text{ ft-K}$$

$$M_{EC} = M_{CE} = \frac{1}{4}\ \frac{20x20}{2} = 50.0 \text{ ft-K}$$

$$M_{FD} = M_{DF} = \frac{3}{4}\ \frac{20x20}{2} = 150.0 \text{ ft-K}$$

$$M_{CA} = M_{AC} = \frac{1}{3}\ \frac{30x20}{2} = 100 \text{ ft-K}$$

$$M_{DB} = M_{BD} = \frac{2}{3}\ \frac{30x20}{2} = 200 \text{ ft-K}$$

Total Unbalanced Moment at E,F,C,D.

$$M_{unb_E} = 50 + 33.3 = 83.3 \text{ ft-K}$$

$$M_{unb_F} = 150 + 66.7 = 216.7 \text{ ft-K}$$

$$M_{unb_C} = 100 + 50 = 150.0 \text{ ft-K}$$

$$M_{unb_D} = 200 + 150 = 350.0 \text{ ft-K}$$

M_{GE}	M_{GH}	M_{HG}	M_{HF}
+33.3			+66.7
-14.3	-19.0	-33.4	-33.3
- 8.1	-16.7	- 9.5	0
+49.4			+25.8
- 10.5	- 14.1	- 8.1	- 8.2
- 8.8	- 4.1	- 7.0	0
+ 16.3			+24.5
- 1.5	- 1.9	- 8.8	- 8.7
- 1.4	- 4.4	- 1.0	0
+ 11.8			+ 3.9
- 2.6	- 3.4	- 1.5	- 1.4
- 1.7	- 0.8	- 1.7	0
+ 2.2			+ 5.2
+ 0.1	+ 0.2	- 1.7	- 1.8
- 0.1	- 0.9	+ 0.1	0
+ 2.3			+ 0.2
- 0.5	- 0.8	- 0.2	- 0.1
- 0.4	- 0.1	- 0.4	0
+ 0.2			+ 1.1
+ 0.1	+ 0.2	- 0.4	- 0.3
0	- 0.2	+ 0.1	0
+ 0.4			+ 0.1
- 0.1	- 0.1	- 0.1	- 0.1
+66.1	-66.1	-73.6	+73.6

M_{EG}	M_{EC}	M_{EF}	M_{FE}	M_{FD}	M_{FH}
+33.3	+50.0			+150.0	+66.7
-24.4	-26.4	-32.5	-65.4	-85.9	-65.4
- 4.8	-18.3	-32.7	-16.3	+19.8	0
+49.4	+96.2			+51.2	+25.8
-26.3	-28.5	-35.0	-24.3	-31.9	-24.3
- 3.5	-10.5	-12.2	-17.5	+ 3.6	0
+16.3	+24.5			+38.6	+24.5
- 4.3	- 4.6	- 5.7	-14.9	-19.5	-14.8
- 0.5	- 1.6	- 7.5	- 2.9	+ 2.5	0
+11.8	+15.7			+ 6.2	+ 3.9
- 5.2	- 5.7	- 7.0	- 2.9	- 3.9	- 2.9
- 0.9	- 1.7	- 1.5	- 3.5	+ 0.5	0
+ 2.1	+ 3.2			+ 7.0	+ 5.2
- 0.4	- 0.4	- 0.4	- 2.8	- 3.6	- 2.8
0	- 0.1	- 1.4	- 0.2	+ 0.4	0
+ 2.3	+ 2.8			+ 0.5	+ 0.2
- 1.1	- 1.1	- 1.4	- 0.3	- 0.3	- 0.3
- 0.2	- 0.4	- 0.2	- 0.7	+ 0.1	0
+ 0.2	+ 0.3			+ 1.4	+ 1.1
+ 0.1	+ 0.1	+ 0.1	- 0.6	- 0.7	- 0.6
0	0	- 0.3	0	+ 0.1	0
+ 0.4	+ 0.5			0	+ 0.1
- 0.2	- 0.2	- 0.2	- 0.1	- 0.1	0
+44.1	+93.8	-137.9	-152.4	+136.0	+16.4

M_{CE}	M_{CA}	M_{CD}	M_{DC}	M_{DB}	M_{DF}
+50.0	+100.0			+200.0	+150.0
-47.6	-43.9	-58.5	-105.7	-105.7	-138.6
-10.1		-52.9	-29.3		+12.3
+96.2	+52.9			+25.3	+51.2
-27.3	-25.2	-33.6	-19.1	-19.2	-25.2
-11.0		-9.6	-16.8		+4.5
+24.5	+9.6			+16.8	+38.6
-4.3	-4.0	-5.2	-13.0	-13.0	-17.1
-1.8		-6.5	-2.6		+2.8
+15.7	+6.5			+2.7	+6.2
-4.4	-4.1	-5.4	-2.8	-2.7	-3.6
-2.2		-1.4	-2.7		+0.6
+3.2	+1.4			+2.7	+7.0
-0.3	-0.3	-0.4	-2.3	-2.3	-3.0
-0.1		-1.2	-0.2		+0.5
+2.8	+1.2			+0.2	+0.5
-0.9	-0.8	-1.0	-0.3	-0.3	-0.4
-0.4		-0.2	-0.5		0
+0.3	+0.2			+0.5	+1.4
0	0	+0.1	-0.4	-0.4	-0.6
0		-0.2	0	0	+0.1
+0.5	+0.2			0	0
-0.2	-0.1	-0.2	0	0	-0.1
+82.6	+93.8	-137.9	-152.4	+136.0	+87.1

M_{AC}

+100.0
- 14.6
+ 52.8
- 8.4
+ 9.6
- 1.3
+ 6.5
- 1.4
+ 1.4
- 0.1
+ 1.2
- 0.3
+ 0.2
0
+ 0.2

+145.8

M_{BD}

+200.0
+ 29.3
+ 16.8
+ 2.7
+ 2.7
+ 0.2
+ 0.5
0

+252.2

B. *Solve By Morris Method*

M_{GE}	M_{GH}	M_{HG}	M_{HF}
+33.3			+66.7
-16.6	-16.7	-22.3	-44.4
-13.9	-11.1	-8.4	-36.1
+40.2			+80.5
-7.6	-7.6	-12.0	-24.0
-9.4	-6.0	-3.8	-26.7
+26.0			+52.0
-5.3	-5.3	-7.2	-14.3
-6.7	-3.6	-2.7	-17.5
+17.0			+34.0
-3.3	-3.4	-4.6	-9.2
-4.2	-2.3	-1.7	-11.4
+10.9			+21.9
-2.2	-2.2	-2.9	-5.9
-2.6	-1.5	-1.1	-7.2
+7.0			+14.0
-1.4	-1.5	-1.9	-3.8
-1.7	-1.0	-0.8	-4.6
+4.4			+8.9
-0.8	-0.9	-1.2	-2.3
-1.0	-0.6	-0.5	-2.9
+2.7			+5.5
-0.5	-0.6	-0.7	-1.4
-0.6	-0.4	-0.3	-1.8
+1.7			+3.4
-0.3	-0.4	-0.5	-0.8
-0.4	-0.2	-0.2	-1.1
+1.0			+2.0
-0.2	-0.2	-0.3	-0.4
-0.2	-0.1	-0.1	-0.7
+0.6			+1.2
-0.1	-0.2	-0.1	-0.3
-0.1	0	-0.1	-0.4
+0.4			+0.8
-0.2	-0.1	-0.1	-0.2
+65.9	-65.9	-73.5	+73.5

M_{EG}	M_{EC}	M_{EF}	M_{FE}	M_{FD}	M_{FH}
+33.3	+50.0			+150.0	+66.6
-27.8	-27.7	-27.8	-36.1	-108.4	-72.2
-8.3	-25.0	-18.1	-13.9	-87.5	-22.2
+40.2	+67.7			+203.2	+80.5
-18.8	-18.8	-18.9	-26.7	-80.0	-53.4

-3.8	-11.0	-13.4	-9.5	-51.8	-12.0
+26.0	+42.1			+126.3	+52.0
-13.3	-13.3	-13.3	-17.5	-52.5	-35.0
-2.6	-6.4	-8.8	-6.7	-30.2	-7.1
+17.0	+26.1			+78.3	+34.0
-8.4	-8.4	-8.5	-11.4	-34.1	-22.8
-1.6	-3.7	-5.7	-4.3	+8.0	-4.6
+10.9	+16.1			+48.4	+21.9
-5.3	-5.3	-5.4	-7.2	-21.7	-14.5
-1.1	-2.3	-3.6	-2.7	-10.8	-3.0
+7.0	+10.0			+30.0	+14.0
-3.3	-3.3	-3.4	-4.6	-13.7	-9.2
-0.7	-1.4	-2.3	-1.7	-6.6	-1.9
+4.4	+6.2			+18.6	+8.9
-2.1	-2.0	-2.1	-2.9	-8.6	-5.8
-0.4	-0.8	-1.5	-1.1	-4.0	-1.1
+2.7	+3.8			+11.4	+5.5
-1.2	-1.3	-1.3	-1.8	-5.3	-3.6
-0.3	-0.5	-0.9	-0.7	-2.5	-0.7
+1.7	+2.4			+7.1	+3.4
-0.8	-0.8	-0.8	-1.1	-3.3	-2.2
-0.1	-0.3	-0.6	-0.4	-1.5	-0.4
+1.0	+1.4			+4.4	+2.0
-0.5	-0.4	-0.5	-0.7	-2.0	-1.4
-0.1	-0.2	-0.4	-0.3	-0.9	-0.2
+0.6	+0.9			+2.6	+1.2
-0.2	-0.3	-0.3	-0.4	-1.2	-0.8
-0.1	-0.1	-0.2	-0.2	-0.6	-0.2
+0.4	+0.5			+1.6	+0.8
-0.2	-0.1	-0.2	-0.2	-0.7	-0.5
+44.2	+93.8	-138.0	-152.1	+136.0	+16.1
M_{CE}	M_{CA}	M_{CD}	M_{DC}	M_{DB}	M_{DF}
+50.0	+100.0			+200.0	+150.0
-50.0	-50.0	-50.0	-58.3	-116.7	-175.0
-13.9		-29.2	-25.0	+	-54.2
+67.7	+41.7			+83.4	+203.2
-22.1	-22.1	-22.1	-34.6	-69.1	-103.7
-9.4		-17.3	-11.1		-40.0
+42.1	+22.8			+45.6	+126.3
-12.7	-12.7	-12.8	-20.1	-40.3	-60.4
-6.7		-10.1	-6.4		-26.2
+26.1	+13.2			+26.5	+78.2
-7.5	-7.5	-7.5	-12.1	-24.0	-36.0
-4.2		-6.1	-3.8		-17.0
+16.1	+7.9			+15.7	+48.4
-4.6	-4.5	-4.6	-7.3	-14.4	-21.6

-2.6		-3.7
+10.0	+4.7	
-2.8	-2.8	-2.8
-1.7		-2.2
+6.2	+2.9	
-1.7	-1.7	-1.8
-1.0		-1.4
+3.8	+1.8	
-1.0	-1.1	-1.1
-0.7		-0.8
+2.7	+1.1	
-0.6	-0.7	-0.7
-0.4		-0.5
+1.4	+0.7	
-0.4	-0.4	-0.4
-0.2		-0.3
+0.9	+0.4	
-0.2	-0.3	-0.3
-0.2		-0.2
+0.5	+0.3	
-0.1	-0.1	-0.2
+82.5	+93.6	-176.1

-2.3		-10.9
	+9.5	+30.0
-4.4	-8.8	-13.1
-1.4		-6.9
	+5.8	+18.6
-2.7	-5.4	-8.0
-0.9		-4.3
	+3.6	+11.4
-1.6	-3.3	-4.9
-0.6		-2.7
	+2.2	+7.1
-1.0	-2.0	-3.0
-0.4		-1.6
	+1.4	+4.4
-0.6	-1.3	-1.9
-0.2		-1.0
	+0.8	+2.6
-0.4	-0.7	-1.1
-0.2		-0.6
	+0.5	+1.6
-0.3	-0.4	-0.6
-195.7	+108.6	+87.1

M_{AC}	M_{BD}
+100.0	+200.0
-25.0	-58.4
+41.7	+83.4
-11.0	-34.5
+22.8	+45.6
-6.3	-20.1
+13.2	+26.5
-3.7	-12.0
+7.9	+15.7
-2.3	-7.2
+4.7	+9.5
-1.4	-4.4
+2.9	+5.8
-0.8	-2.7
+1.8	+3.6
-0.5	-1.6
+1.1	+2.2
-0.3	-1.0
+0.7	+1.4
-0.2	-0.6
+0.4	+0.8
-0.2	-0.4
+0.3	+0.5
+145.8	+252.1

CONCLUSION

Each of the four examples was solved by the Direct-moment distribution method and the general moment distribution method. The following conclusions are based on the solutions of the four problems.

1. Results obtained by using the direct moment distribution method are very close to the results obtained by the moment distribution method. Since the answers obtained by both methods are approximations, the difference in the answers is insignificant.

2. For symmetrical frames, some other modified method (such as "Modify Stiffness Method," or "Cantilever Moment Distribution Method") would give a simpler solution than the Direct-moment distribution method or the general moment distribution method due to the symmetry of the frame and the loading. The choice of the method to use would be a matter of individual preference.

3. For unsymmetrical multi-story-one-bay rigid frames, the total work required for the solution obtained by Direct-moment distribution method is less than that in the moment distribution method. However, the operational procedures of the moment distribution method are simpler and can be performed by a person with less technical training.

DIRECT DISTRIBUTION WITH SIDESWAY
FOR MULTI-STORY ONE BAY FRAME

by

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For introducing the Direct Moment Distribution method with sidesway for multi-story-one-bay frame, the introduction gives the history of this method and its general breakdown. Also, in this introduction, a discussion of the sidesway of the rigid frame, and its well known Morris moment distribution method, are given.

In the first chapter, the general assumption of the direct moment distribution method is given and the simple one-story-one-bay frame is analyzed. The analysis includes the definition and determination of the Modified Rotational Stiffness, the Translational carry-over factor, and the carry-over factor for the girder. This chapter also gives the derivation of formulas. Four problems are given as examples with their complete solutions solved by direct moment distribution method.

In the second chapter, the same method is extended to solve the multi-story-one-bay frame and a few necessary assumptions are made. Also, the rotational stiffness and carry-over factor, the translational carry-over factor, the carry-over factor for the girder, and the spring modulus for the column are determined. Examples of multi-story-one-bay frame deflect with sidesway are solved by the direct moment distribution method as well as Morris moment distribution method to give a numerical proof for the extended use of the direct moment distribution method. From the examples, through comparison and discussion, a general conclusion is made.