THEORY OF HEAVY NUCLEAR TRACKS IN ELECTRON SENSITIVE NUCLEAR EMULSIONS WITH AN APPLICATION TO THE TRACK OF A DIRAC MONOPOLE

by

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INTRODUCTION AND REVIEW OF LITERATURE

Tracks of Heavy Nuclei

In 1948, the Minnesota (5) and Rochester (11) groups discovered heavy nuclei in the cosmic radiation. These nuclei were detected by the heavy tracks they produced in electron sensitive emulsions which had been exposed at high altitudes. Two distinguishable features characterized these heavy tracks: the prolific number of secondary electrons (δ rays) projecting from the track; and the thickening and subsequent tapering near the end of the track.

The thickening of the track was explained by Frier, Lofgren, Ney, and Oppenheimer (11) to be the result of an increasing energy loss per unit of track length by the bare nucleus; the tapering of the track was the result of a decreasing energy loss per unit of track length. Increasing energy loss was predicted by the Bragg ionization curve. (At low velocities the energy loss per unit track length will increase.) The decrease in energy loss occurred because the effective charge of the nucleus was decreasing, due to the electron pickup by the nucleus. Electron pickup becomes very probable when the speed of the nucleus reaches the speed of an orbital electron. According to Bohr's theory, the speed of the K shell electron for a nucleus of charge Ze is $\frac{Zc}{137}$; consequently the track of a nucleus of charge Ze should begin tapering at a residual range corresponding to the velocity $\frac{Zc}{137}$. Observations proved that this taper length was too short.

The discrepancy between predicted taper length and actual taper length was first noted by Hoang (15). He postulated that the taper length was related to the δ ray density along the trajectory of the nucleus. In 1953, theory of track formation was given by Lonchamp (19). The width of the track was ascribed to δ rays that could not be individually resolved; the tapering of the track near the end was due to the decreased energy of the δ rays.

An extension to Lonchamp's theory was made by Skjeggestad (25). He made a confirmation of this extended theory for the last 150 microns of track in Ilford G-5 emulsions for nuclei lighter than oxygen. The track width predicted by Skjeggestad goes through a very gradual maximum before it tapers down. Observations made in the present work show this maximum to be more pronounced. Recently, a new theory has been proposed by Bizzeti and Della Corte (2). They were concerned with the explanation of the results of photodensitometer measurements of track width in the region of taper length. In the present work, visual measurements of track width were made; therefore the theory of Bizzeti and Della Corte was not used.

The shape of a Dirac monopole track in Ilford G-5 emulsions has been discussed by Katz and Parnell (16). They have pointed out that the track of a Dirac monopole might be mistaken for the track of a heavy nucleus, were it not for the difference in shape of the two tracks. The monopole would form a wedge shaped track; therefore, any track that does not go through a maximum in width is suspect as being a monopole track.

It must be noted that heavy nuclei form thick tracks in electron sensitive emulsions only. All reference to nuclear emulsions henceforth will refer to emulsions that will record the tracks of electrons with energies in the range of 10 kev to 200 kev. (The Ilford G-5 emulsion is such an emulsion.)

It will be useful to separate nuclear tracks into three regions: (A) the taper length region; (B) the maximum width region; (C) high energy region. Figure 1 shows these regions. (See plates V, VI, and VII for pictures of nuclear tracks.)

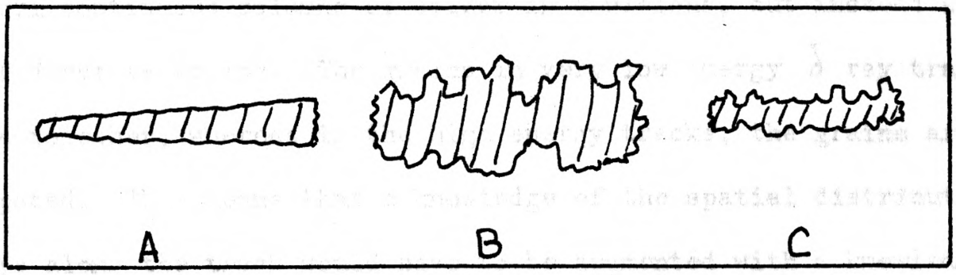


Figure 1. Regions of nuclear track.

Purpose

A Dirac monopole created by the cosmic radiation or existing in the primary cosmic radiation might be detected by means of nuclear emulsions. A simple and precise criterion is needed for distinguishing the tracks of Dirac monopoles from the tracks of heavy nuclei. Firstly, a simple definition of the track width that corresponds to observation must be formulated. Secondly, a theory of track formation must be obtained that is valid for region (C) of the nuclear track as shown in figure 1. The track width given by Katz and Parnell (16) is given as a function of βc, the velocity of the monopole; the range as a function of βc for Dirac monopoles of different mass was calculated for electron sensitive nuclear emulsions.

THEORY OF TRACK FORMATION

The key to understanding track formation lies in connecting the spatial distribution of δ rays along the trajectory of the nucleus to the energy distribution of δ rays ejected by the nucleus. Usually a very simple relationship is used for the connection; a δ ray with initial energy w will extend a perpendicular distance x from the trajectory of the nucleus. The validity of such a relationship will be discussed below. Delta rays do not form continuous columns of silver in emulsions, but instead are formed of discrete grains. The grains in very low energy δ ray tracks lie close together, whereas in the high energy tracks, the grains are widely spaced. This means that a knowledge of the spatial distribution of δ rays along the track would have to be augmented with a knowledge of how the rays become visible for a complete theory of track formation. A complete theory would also include a simple, precise definition of track width that corresponds to the observation process.

Plate II shows the validity of the premise that the δ ray distribution, and not the energy loss per unit of track length, is responsible for the character of the track. (The increased energy loss does not contribute to the formation of high energy δ rays, but contributes to the formation of large numbers of low energy δ rays.) The energy loss per unit of track length was obtained experimentally by Heckman, Perkins, Simon, Smith, and Barkas (12); whereas the track width versus range was determined in the present work.

Delta Ray Distribution Function

The dominant characteristic of the track formed by a nucleus in electron sensitive emulsions is a result of the spatial distribution of δ rays along the trajectory. This spatial distribution is related to the energy distribution which is derived below.

The interaction of a nucleus with matter can be treated by the semi-classical impact parameter method for interactions where Coulomb's law is valid, as shown in appendix II. (The following derivation follows that given by Rossi (23), page 17).

Consider a nucleus with charge Ze and velocity βc interacting electrically with a resting free electron. Since the mass of the nucleus is much greater than the mass of the electron, the nucleus will move by the electron and will be essentially undeflected. As the nucleus moves by the electron, it will give the electron an impulse. The impact parameter (b) is defined as the distance of closest approach between the nucleus and electron. (It is assumed that the electron does not move during the impact.) As shown in appendix I, the total impulse given the electron is

$$|\underset{\sim}{I}_{Ze}| = \frac{Ze^2}{b^2}\cdot\left(\frac{2b}{\beta c}\right) = \left(\begin{smallmatrix}\text{MAX.}\\\text{FORCE}\end{smallmatrix}\right)\cdot\left(\begin{smallmatrix}\text{IMPACT}\\\text{TIME}\end{smallmatrix}\right). \qquad (1)$$

The electron acquires a change in momentum equal to this impulse. (Note that the "impact time" introduces a $\frac{1}{\beta}$ term, and the force term introduces a Z.) The energy acquired by the electron is given by

$$W = \sqrt{m_e^2c^4 + c^2 I_{Ze}^2} - m_e c^2, \qquad (2)$$

where m_e is the mass of the electron. If W is small in comparison to $m_e c^2$, this expression reduces to

$$W = \frac{1}{2}\,\frac{I_{Ze}^2}{m_e}, \qquad (3)$$

Relativistic velocities of the nucleus introduce no correction to the impulse given the electron, because the relativistic contraction of the field is compensated by the impact time dilitation as shown in appendix I. The energy acquired by the electron is given by

$$W = \frac{2Z^2e^4}{m_e\beta^2c^2b^2}. \tag{4}$$

For the case of a magnetic pole of pole strength g_m and velocity βc, the impulse given to the electron is shown in appendix I to be

$$|\underset{\sim}{I}_{ze}| = \frac{e\,g_m\beta c}{c\,b^2}\cdot\left(\frac{2b}{\beta c}\right) = \left(\begin{matrix}\text{MAX.}\\ \text{FORCE}\end{matrix}\right)\left(\begin{matrix}\text{IMPACT}\\ \text{TIME}\end{matrix}\right). \tag{5}$$

(Note the absence of a $1/\beta$ term in this expression; physically, this is a result of the velocity dependence of the interaction force.) The energy acquired by the electron is

$$W = \frac{2g_m^2e^2}{m_ec^2b^2}. \tag{6}$$

There is no change in $|I_{g_m}|$ for the relativistic case as shown in appendix I. The probability that a particle has a collision with impact parameter between b and b + db while traveling a distance dx is given by

$$F(b)\,db\,dx = N(2\pi b)\,db\,dx = G(W)\,dW\,dx, \tag{7}$$

where N is the number of electrons per unit volume. This also gives the probability that the particle will have a collision in dx, for which an energy between W and $W+dW$ is given to a free electron. Solving equation (4) for b^2 and differentiating, one obtains, using absolute values,

$$2b\,db = \frac{2Z^2e^4}{m_ec^2\beta^2}\cdot\frac{dW}{W^2}. \tag{8}$$

Putting this result in equation (7), the result for the probability of a collision in dx for which an energy lying between W and $W+dW$ is transferred to an electron is

$$\frac{4\pi NZ^2e^4}{m_ec^2\beta^2}\cdot\frac{dW}{W^2}. \tag{9}$$

The number of collisions in a finite distance Δl, where an energy between W and W + dW is given to a free electron in each collision, is given by

$$G(W)\,dW\,\Delta l = \int_0^{\Delta l} G(W)\,dW\,dx = \frac{4\pi NZ^2e^4}{m_ec^2\beta^2}\cdot\frac{dW}{W^2}\,\Delta l. \tag{10}$$

The number of electrons given an energy lying between W and W + dW in a distance Δl of the trajectory of the nucleus is given by

$$dn = \frac{4\pi N z^2 e^4}{m_e c^2 \beta^2} \cdot \frac{dW}{W^2} \, . \tag{11}$$

For a monopole of pole strength gm, dn becomes

$$\frac{4\pi N g_m^2 e^2}{m_e c^2} \cdot \frac{dW}{W^2} \, . \tag{12}$$

(The important difference between the two distribution functions is the absence of a $1/\beta^2$ term in the monopole expression.) The $\frac{1}{W^2}$ dependence in both distribution functions means that collisions for which high energies are transferred are less common than low energy transfers.

Electrons encountered in materials are not free but are bound to atoms with a binding energy. If the energy given to the electron is large compared to its binding energy, a high energy secondary electron (δ ray) is created. Mechanical restrictions place an upper limit on the energy given to a secondary electron. Consider the collision from the rest frame of the nucleus; the greatest change in velocity the electron can have is given by $2\beta c$. This gives rise to an energy transfer of $2m_e\beta^2c^2$ to the secondary electron. (A more exact expression is given by Rossi (23), page 14, as $\frac{2m_e\beta^2c^2}{1-\beta^2}$.

Delta ray distribution functions for different values of β and for a charge $z=30$ are plotted in plate I. The energy cut off is taken as $2m_e\beta^2c^2$. (In plate I, note how the width of the energy spectrum of δ rays varies with β^2 and also how the number of δ rays per 100 microns of track varies with $1/\beta^2$.) The monopole distribution function has no $1/\beta^2$ dependence and, therefore, only the width of the energy spectrum is a function of β.

Lonchamp's Theory

The theory which Lonchamp (19) has developed makes use of the integral form of the δ ray distribution derived in a previous part. Lonchamp gives, for the number of δ rays with energy between W_0 and $2m_ec^2\beta^2$,

$$n = \frac{5.34 \times 10^{-3} z^2}{\beta^2}\left\{\frac{510}{W_0} - \frac{1}{2\beta^2}\right\} ,$$

where n is the number of δ rays along 100 microns of track. These rays were considered to be ejected in a direction perpendicular to the trajectory of the nucleus, and δ ray scattering was neglected. The author then assumed that 400 δ rays per 100 microns of track would cause the track to be opaque. The energy for which one gets 400 δ rays is given as

$$W_0 = \frac{1020\ (\text{kev})}{\left\{\frac{2(400)\beta^2}{5.34 \times 10^{-3} z^2} + \frac{1}{\beta^2}\right\}} .$$

From range-energy relations for low energy electrons, the range of δ rays with energy W_0 was found. Two times this range gave the track thickness of a nucleus with charge ze and velocity βc.

For the present work, a theory of track formation is needed that is valid for region C in figure 1 of the introduction; therefore, a critical examination of the limitations of Lonchamp's theory will be given.

The most significant limitation is that this theory neglects the energy spectrum of the δ rays; thus all δ rays with an energy greater than W_0 were given equal weight in the track formation process. It seems reasonable to assume that Lonchamp's theory will give poor results at positions on the track where the δ ray energy spectrum is wide (as in plate I for $\beta = .3$), but better results would be expected where the spectrum is narrow (as in plate I for $\beta = .2$).

EXPLANATION OF PLATE I

Delta ray distribution function

for

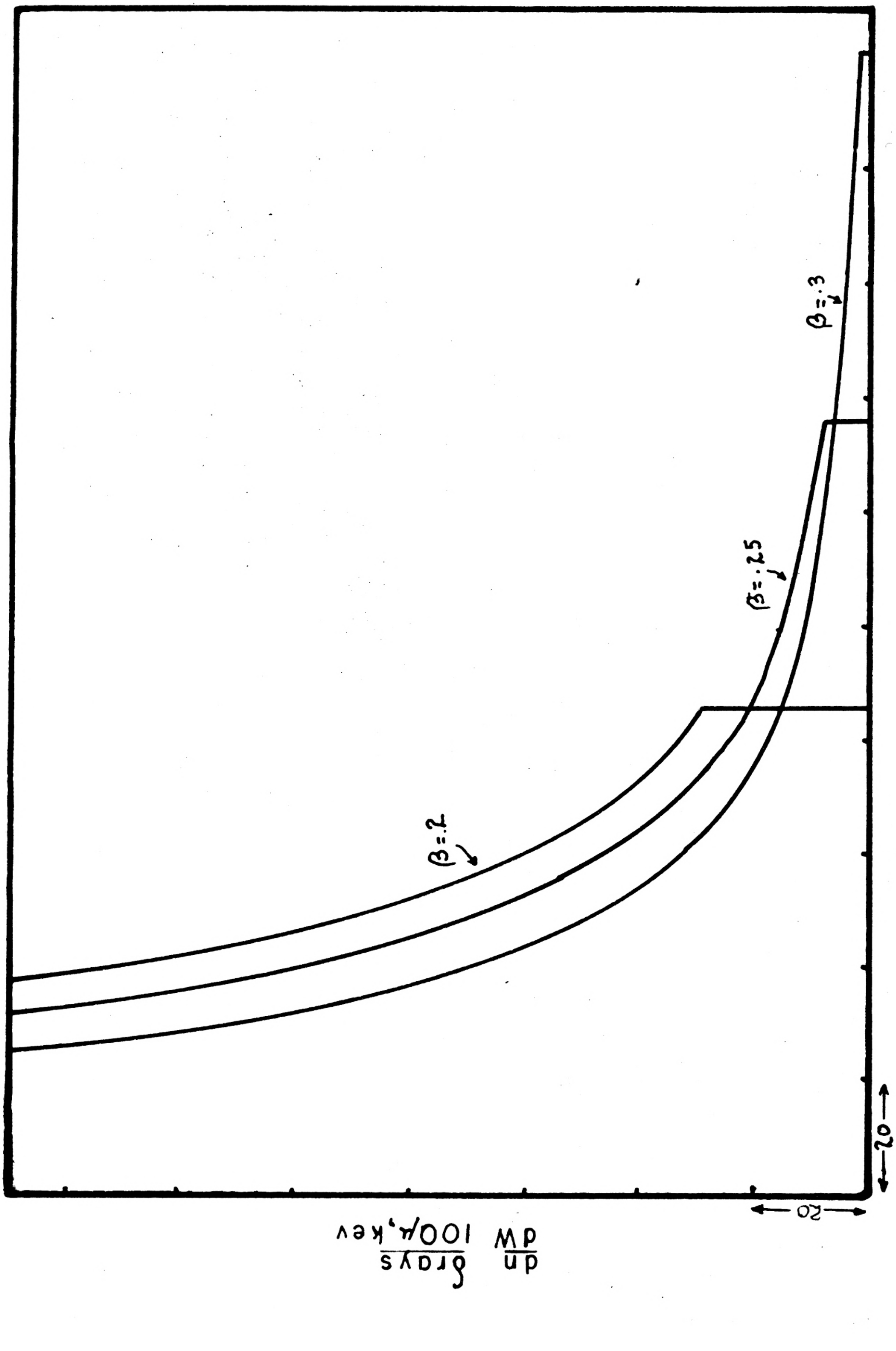
$Z = 30$, and several values of β

displaying the energy spectrum

of

delta rays.

PLATE I



EXPLANATION OF PLATE II

Energy loss per unit track length

as a function

of

residual range,

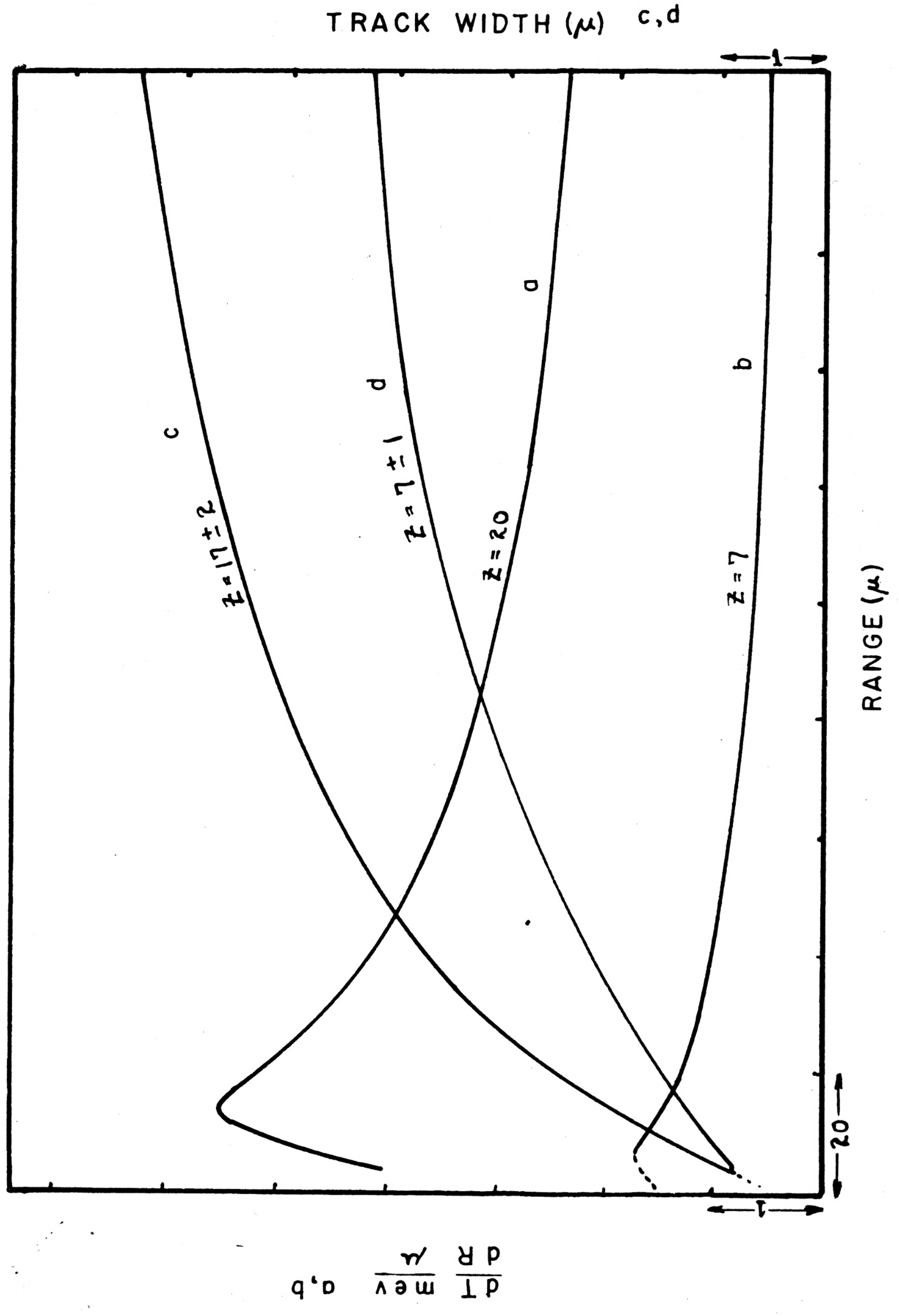
and

thickness of track as a function

of

residual range.

PLATE II



The assumption that δ rays are not scattered is a very weak assumption. A calculation was made of the root-mean-square multiple scattering angle of an electron with an initial energy W and a range R in an Ilford G-5 emulsion. R was taken as the end point range of the electron with energy W (kev). The multiple scattering angle for electrons with energies up to 200 kev, turns out to be approximately 200°. (The multiple scattering formula was taken from Fermi (10), and the range-energy data was taken from Demers (9).) The range of electrons with an energy less than 20 kev is less than 3 microns; therefore, the large multiple scattering of these low energy electrons will give δ rays that follow very random trajectories. Closely associated with the scattering problem is the uncertainty in range-energy relations for low energy electrons. Confusion in the literature arises from the different definitions of an electron's range (see L. Katz and Penfold (17)). At least three ranges are defined: average range; practical range; and end point range. Even if scattering were negligible, the question arises as to which range-energy relation should be used in Lonchamp's theory.

The width of a track where a prolific number of long δ rays is present is a difficult quantity to define. The width predicted by Lonchamp's theory probably has no meaning in this region of the track, since observations show that no well defined core is present.

A final problem inherent in the theory was pointed out by Dr. Katz. As the track gets wider, one would expect an increase in the number of δ rays needed to blacken the track. Consider the track to be a cylinder of a small radius (according to Lonchamp, 400 δ rays passing through a cylinder of 100 microns in length will cause the track to clog). This means that a certain area on the cylinder must have one δ ray passing through it; as the radius of the cylinder increases, the cylinder area will increase and, therefore, more δ rays will be needed to clog the track.

An Extension of Lonchamp's Theory

For a theory to be valid in region B or C (see fig. 1 in the introduction), it must have the following conditions imposed upon it: the energy spectrum of the δ rays must be taken into account; the problem of scattering and range-energy relations must be overcome; and an observational track width must be obtained. In short, this amounts to finding a more realistic connection between the energy and spatial distributions of δ rays than used by Lonchamp. The following theory is proposed with the above conditions in mind.

The track is divided into two parts: the core and extra-core. The core part will be dealt with first. Consider a series of concentric cylinders with increasing radii as being centered about the trajectory of the nucleus. One wants to calculate the number of δ rays that pass out of a given cylinder that is 100 microns long. Both scattering and absorption will restrict the number of δ rays passing through a cylinder of t/2 radius. The relative importance of scattering and absorption is summarized by the following quotation from L. Katz and Penfold (17):

"There is little experimental evidence to decide whether absorption or scattering is the major cause of electron removal in the absorber."

There is a strong analogy between a δ ray passing through a cylinder of radius t/2 and an electron passing through a thin foil of thickness t/2; therefore, transmission curves for thin foils might be used to take into account both the scattering and absorption of the δ rays. Aluminum foils bear a close resemblance in their scattering and absorption properties to nuclear emulsions. (A similar conclusion was given by Tidman, George, and Herz (26).) Transmission curves for electrons through thin aluminum foils were taken from Schonland (24) and Carlvik (6). The thickness of the foils were transformed to the corresponding emulsion thickness by calculating the mass per unit area of the foils and finding the thickness of emulsions with the same mass per unit area. (Schönland has pointed out that the thickness of metal needed to stop electrons of different velocities is dependent only on the mass per unit area. R. H. Herz (14) has found agreement with this assumption.)

Plate III shows the transmission curves for flat sheets of emulsions of different thickness. These curves were then used without alteration as the best available approximation for transmission through emulsion cylinders of different radii. The energy distribution function for δ rays is given by

$$\rho(W, \beta, z) = \frac{dn}{dW} = 2.735\,(z/\beta)^2 \frac{1}{W^2},$$

where $\rho(W,\beta,z)$ is the number of δ rays with an energy between W and W + dW ejected by a nucleus of charge ze and velocity βc along 100 microns of track. The number of δ rays that pass through a cylinder of radius t/2 is given by

$$f(z,\beta,t/2) = \int_0^{2m_e\beta^2c^2} \varphi(W,t/2)\,\rho(W,\beta,z)\,dW,$$

where the $\varphi(W,t/2)$ is given in plate III. If one defines

$$\rho(W,\beta,z) = z^2 \rho_0(W,\beta),$$

one obtains

$$f(z,\beta,t/2) = z^2 \int_0^{2m_e\beta^2c^2} \varphi(W,t/2)\,\rho_0(W,\beta)\,dW = z^2 F(\beta,t/2).$$

This integral was evaluated graphically for 8 values of β and 6 values of t/2. Continuous curves were drawn through these points for interpolative purposes; these are given in appendix three.

A δ ray passing outside the core will form a certain number of grains. Some of these grains will make up the extra-core part of the track. At this point in the theory, a connection between the observed track width and theoretically derived width must be made. The track width will be defined as consisting of all connected portions of developed silver. This width will be made up of the core and certain of the grains lying outside the core. There is no way of theoretically predicting which grains will lie close enough to be considered rigidly attached; however, visual observations of the track give the approximate maximum number of grains in a δ ray that appears to be connected to the rest of the track as 15. Information as to the number of grains in the track of an electron with initial energy W was taken from Skjeggestad (25), Tidman, George, and Herz (26). If we denote the number of grains in a track of energy W as $\Gamma(W)$, then the number of grains that will be formed outside a cylinder of thickness t/2 will be given by

$$\Theta(W, t/2) = \left(1 - \frac{t/2}{R(W)}\right) \Gamma(W),$$

when R(W) is the range of the electrons with energy W as given by Demers (9). Although R(W) is the curved path range, it seems reasonable that the above will have some validity. $\Theta(W, t/2)$ is cut off at 15 grains in accordance with what is stated above. The number of grains lying outside a cylinder of radius t/2 and length 100 microns is given by

$$g(z, \beta, t/2) = z^2 \int_0^{2 m_e \beta^2 c^2} \Theta(W, t/2)\, \varphi(W, t/2)\, \rho_0(W, \beta)\, dW.$$

Since the integral will be a function of the total number of δ rays passing through a thickness t/2, one can write

$$g(z, \beta, t/2) = z^2 G\left\{F(\beta, t/2)\right\}.$$

Graphical integration was performed for 8 values of β and several values of t/2. Continuous curves were drawn through these points for interpolation purposes.

It remains to formulate some criterion for the number of δ rays needed to form the core. Recent pictures (12) of the end views of tracks indicate that a solid pencil of silver forms the core. If one takes into account primary ionization and the δ rays with an energy below 10 kev, it seems reasonable to ascribe a minimum thickness to the core as 1.4 microns. (This will be a better approximation for nuclear tracks of large z.) The number of δ rays needed to form the core as a function of core thickness was arrived at by geometrical considerations of the area of a grain and the area of annular regions between cylinders. (An implicit assumption in this statement is that δ rays have continuous tracks; clearly, this is a weak assumption.) In calculating the mean thickness of a track of nucleus ze at the residual range where the velocity is βc, one first found the value of $z^2 F(\beta, t/2)$ that corresponded to the number of δ rays needed to clog a core of radius t/2. The corresponding value of $G\left\{F(\beta, t/2)\right\}$ was found and the mean track width was given by

$$\bar{t} = t + z^2 \frac{G\left\{F(t/2, \beta)\right\} \cdot A}{100},$$

where A is the area of the grain. (The 1/100 comes in because $\rho(W, \beta, z)$ was defined as the number of δ rays with energy between W and W + dW/100 .) It should be mentioned that the calculation of the mean thickness assumes that all the grains counted outside the core will form part of the observed area. There should be a cylindrical symmetry of grains about the core in the undeveloped emulsion; however, the development of nuclear emulsions causes a shrinkage of about 2.5 times and, hence one would expect the cylindrical symmetry to be destroyed, and for most of the grains to lie in a flattened cylinder. This effect would bring most of the grains into the field of view of the microscope where they would form part of the track width. The theories developed above will only be good for tracks with a dip angle of less than 20°; for greater dip angles, the track may buckle in the developing process.

Plate IV shows the theoretical calculations of track width versus range for several nuclei; the first several hundred microns of track width for $Z = 17$ are calculated from Lonchamp's theory using Demers (9) range-energy relation for slow electrons. The grain diameter used in calculating the grain area was taken as 0.72 microns. Range-energy relations were taken from Demers (9). The energy-velocity relation was determined from

$$E = \frac{1}{2} A (931) \beta^2 \text{ (Mev)},$$

where A was the atomic mass number of the nucleus.

It is doubtful that the theory cited here will give any meaningful results in the taper length region. The large number of δ rays in this region would cause grain saturation which cannot be accounted for in this theory.

EXPERIMENTAL PROCEDURES AND RESULTS

All measurements on the tracks were done with a Leitz Ortholux microscope under bright field illumination. Photographs of the tracks were taken with a Leitz Aristophot equipped with a bellows camera; a Polaroid Land camera was attached to the bellows camera. Five Ilford G-5 nuclear track pellicles which had been exposed to the cosmic radiation at 100,000 feet were used in this experiment. These plates had been previously

100,000 feet were used in this experiment. These plates had been previously scanned for heavy tracks by Parnell (21). Three tracks were picked to be measured in detail. Track one was followed through two pellicles; track two and three were each followed through three pellicles.

Residual range measurements were made using Leitz precision stage micrometers. Residual range is defined as

$$R = \left\{ \Delta x^2 + \Delta y^2 + S^2 n^2 \Delta z^2 \right\}^{1/2}$$

where Δx and Δy were the horizontal distances from the end of the track, Δz the vertical distance from the end of the track, S the shrinkage factor, and n the index of refraction of the emulsion. (For Ilford G-5 emulsions, S is reported as 2.7 and n as 1.50 (20).)

Charge determination was made by counting delta rays with 4 or more grains in their track. Delta ray density measurements were made at two points on the track in order to get an upper and lower limit on the charge at a residual of 2000 microns and of 5000 microns. The charge was then determined by using delta ray density versus residual range curves taken from Dainton, Fowler, and Kent (8). The charge of track one and two was determined as 17 ± 1; the charge of track three was 7 ± 1. Parnell (21) made similar measurements but arrived at a charge of between 29 to 32 for track one, 23 to 31 for track two, and 18 to 21 for track three. A comparison of photographs of tracks one, two, and three with published photographs shows agreement with the lower charge estimates.

The track width was defined as the projected area of a unit length of track comprising all rigidly attached portions of developed silver. Two methods were used to measure this width. The first method made use of a Leitz eyepiece micrometer; the observer estimated the width of the track visually. Two observers made such measurements on the tracks. The second

EXPLANATION OF PLATE III

Probability of transmission of electrons

through several thicknesses

of

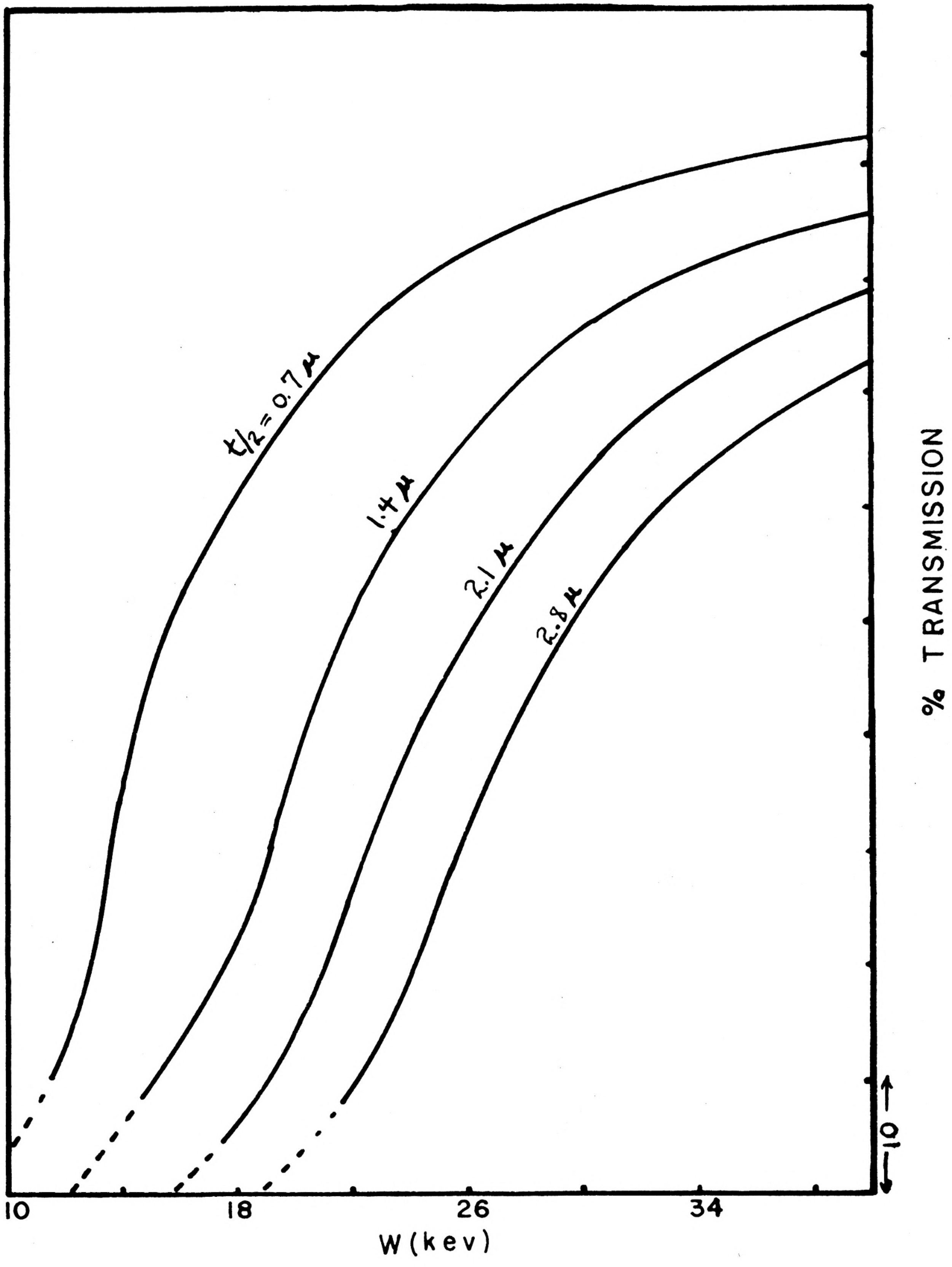
Ilford G-5 emulsions

as a function

of

electron energy.

PLATE III



EXPLANATION OF PLATE IV

Track width as a function

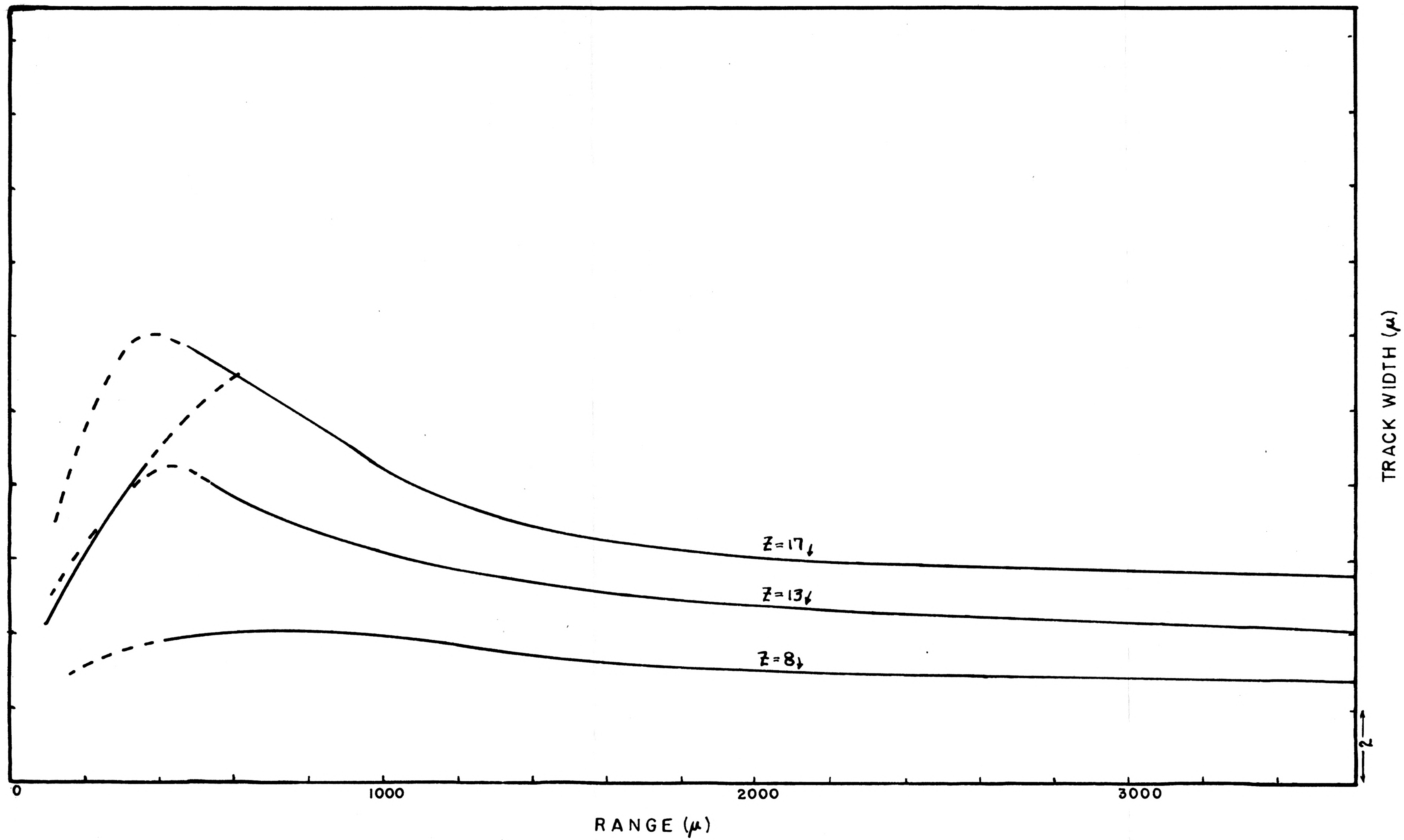
of

residual range

for

several values of Z.

PLATE IV



method consisted of photographing parts of the track, projecting the photographs on a screen using an opaque projector, and drawing outlines of all parts of the track rigidly attached. Because of the resolution of the system, it is estimated that portions of developed silver with a separation of less than .7 micron were considered as rigidly attached. Each outline was broken up into cells; the area of eanh cell was measured using an Ott planimeter. The width of the track was equal to the area of a cell divided by the length of this cell. Cells of 10, 20, and 30 microns were used. The larger cells were used where the track contour was very irregular. The mean grain diameter was measured and found to be 0.72 micron. Table 1 indicates the optical system used for the various measurements. Photographs of tracks two and three are shown in plates V, VI, and VII.

Table 1. The optical system used for various measurements.

Measurement	Objective Immersion	Eyepiece	Mag.
ray count	40X air	Leitz eyepiece 25X	1250
Visual width	40X air	Leitz micrometer eyepiece 12.5X	625
Photographic width	40 air	Leitz eyepiece 10X	950
Grain diameter	100X oil	Leitz micrometer eyepiece 12.5X	1560

A COMPARISON OF THEORY AND EXPERIMENT

Plate VIII shows the results of the experimental observations performed on track one and two. Also plotted on plate VI are the results of the

EXPLANATION OF PLATE V

Photographs of the tracks of heavy nuclei
in an Ilford G-5 emulsion.

Fig. 1. The end of a $Z = 17 \pm 2$ track.

Fig. 2. Continuation of the $Z = 17 \pm 2$ track.

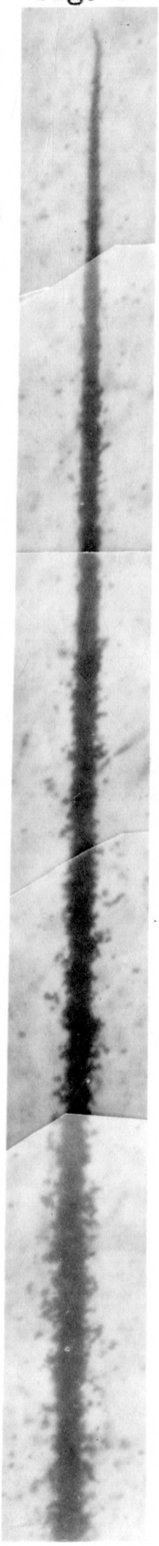
Fig. 3. The end of a $Z = 7 \pm 1$ track.

Fig. 4. Continuation of the $Z = 7 \pm 1$ track.

(Residual range is given in microns and is placed beside each portion of the track.)

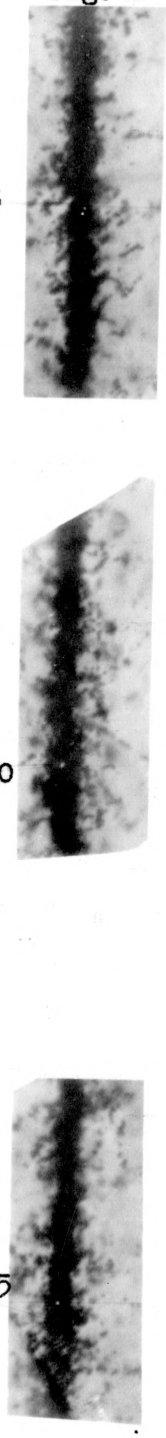
PLATE V

Fig. 1



295

Fig. 2

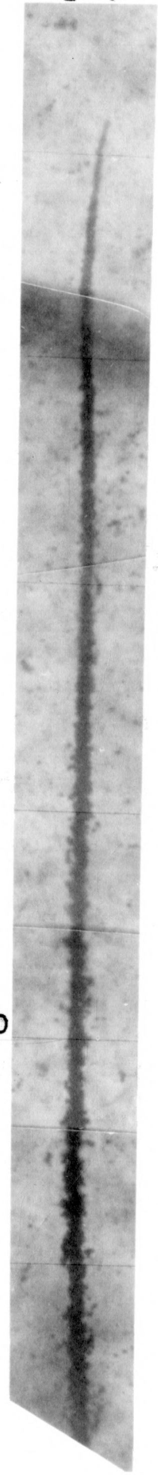


445

540

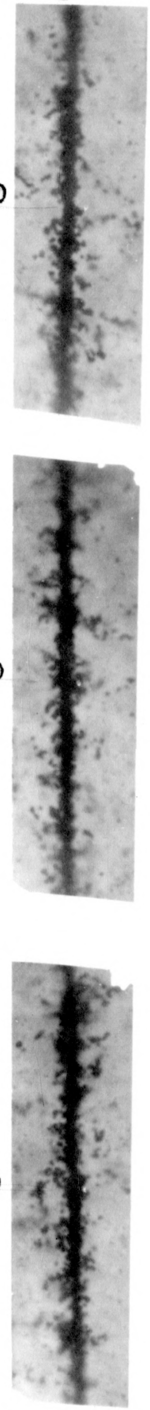
645

Fig. 3



200

Fig. 4



360

440

620

EXPLANATION OF PLATE VI

Continuation of $Z = 17 \pm 2$ track

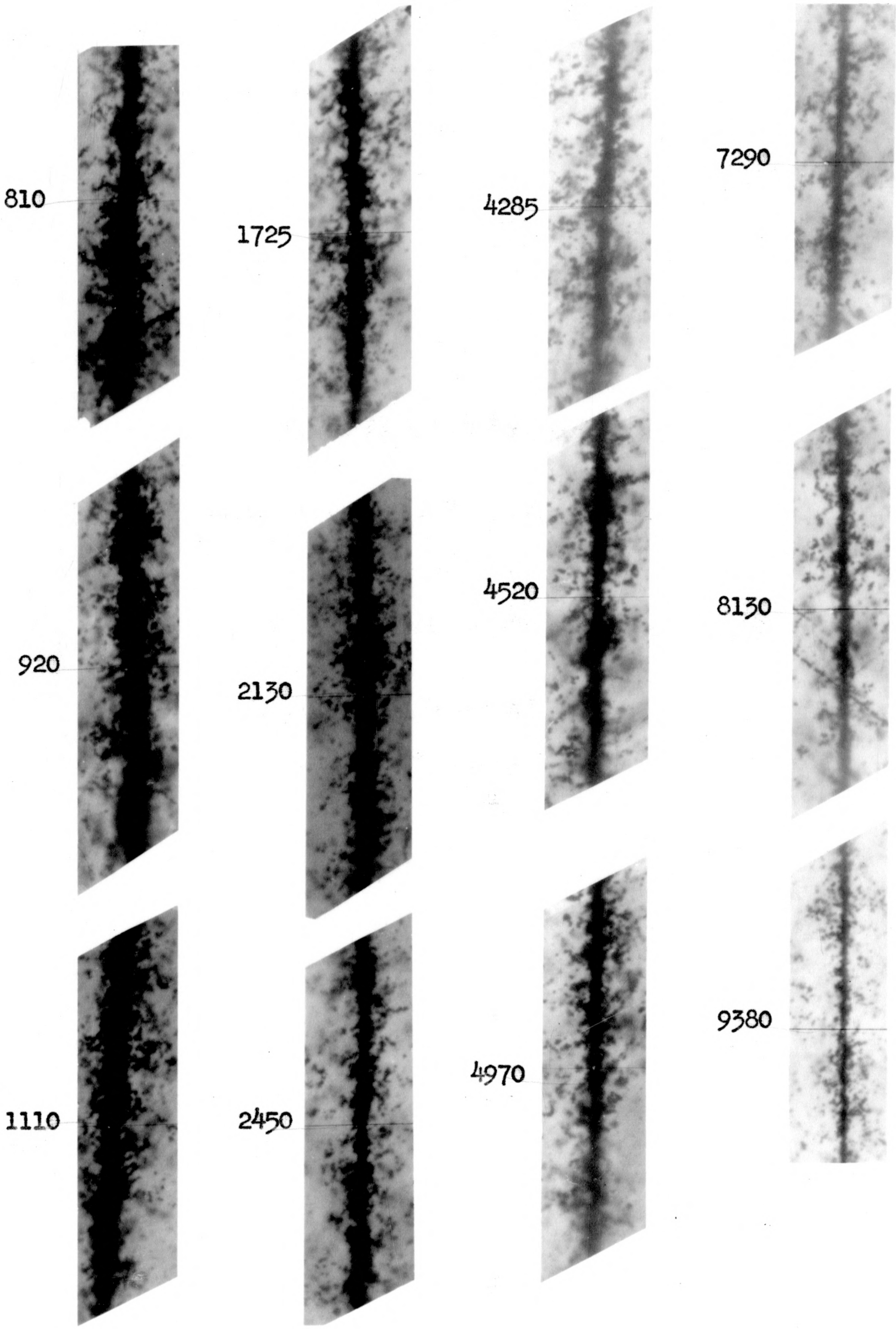
to a

residual range of 9380 microns.

(Residual range is

given in microns.)

PLATE VI



EXPLANATION OF PLATE VII

Continuation of $Z = 7 \pm 1$ track

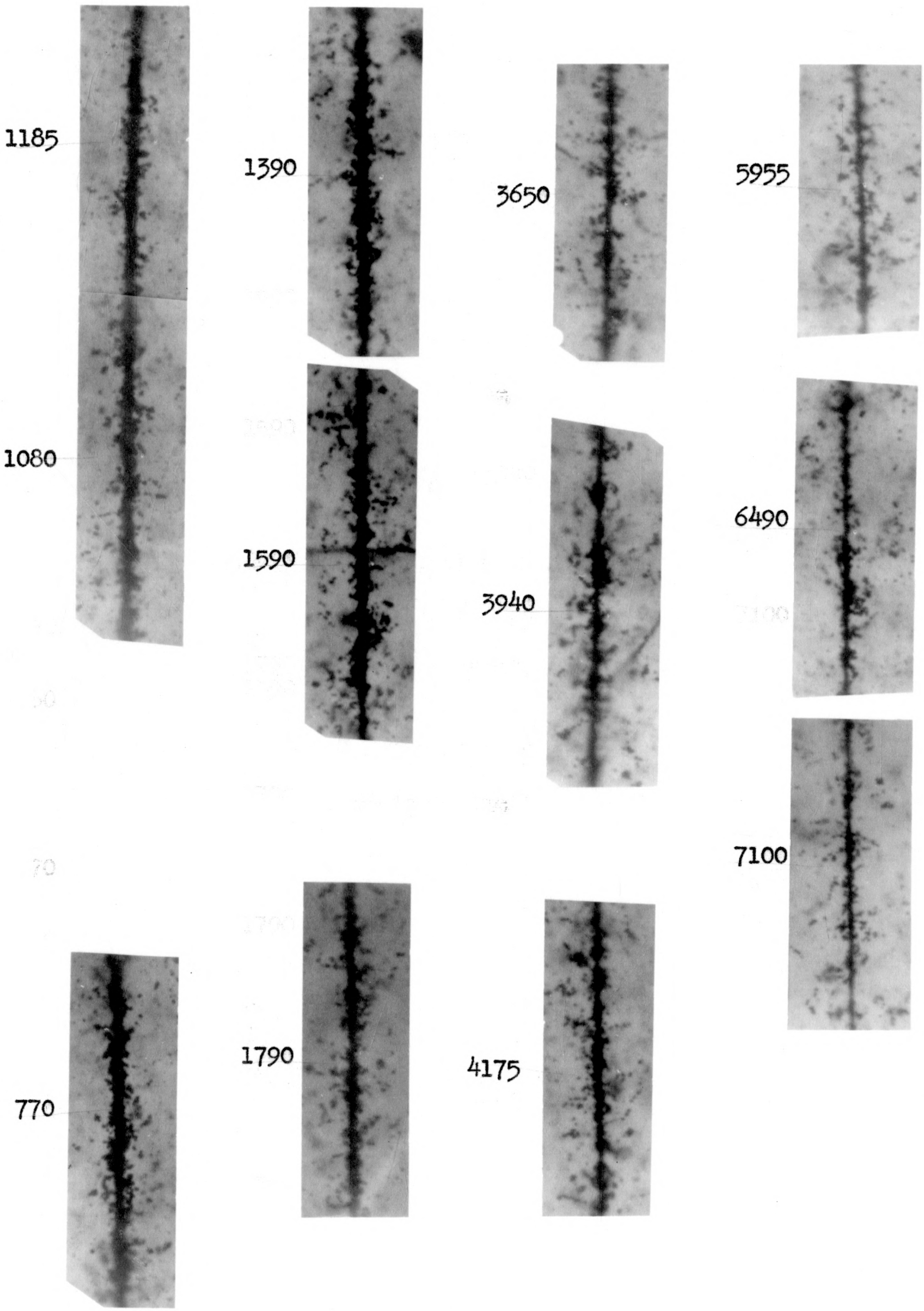
to a

residual range of 7100 microns.

(Residual range is

given in microns.)

PLATE VII



theories presented in section two. The photographic method gives a larger thickness than the visual. Poor resolution in the photographs could be one reason; a lack of correlation in what is being measured by each method could be another. The uncertainty of z makes a comparison of theory and experiment difficult, as do the wide fluctuations of width. It is evident that a large subjective factor is involved in the visual width observations made by the two observers. This subjective factor is less important at the end of the track where the contour of the track is more regular. Near the end of the track where the contour of the track is regular, it is likely that one is measuring the width of Lonchamp's theory. When the contour gets very irregular, it is difficult to know what width corresponds to his theory. The second theory is obviously not valid at the end of the track. There is an implicit assumption in the second theory that grains individually make up the extra-core track width. If a large number of δ rays with low energy are ejected, it is conceivable that several δ rays may pass through the same grain. This would make the actual track width smaller than the predicted one. An adequate theory of track formation would be valid for all of the track and would correspond to a simple observational method. Neither of the previously mentioned theories fulfill these criteria. The second theory fits the observation beyond the maximum and will be of some use, as shown in the next section.

Plate IX gives the experimental points and theoretical curves for track three. Lonchamp's curve was not plotted, because W_0 was below 10 kev for several points, and Demers does not give electron ranges for below this energy.

THE TRACK OF A DIRAC MONOPOLE

The track that a Dirac monopole would make in Ilford G-5 emulsions has recently been investigated by Katz and Parnell (16). They took the distribution function for the monopole derived in the introduction, and applied Lonchamp's theory of track formation to this distribution function. In accordance with Dirac's original work, g_m was taken as

$$g_m = \frac{1}{2}\frac{\hbar c}{e} = \frac{1}{2}(137)e = 3.3 \times 10^{-8} \text{ gauss cm}^2 \text{ (esu)},$$

where e is the charge of the electron. The lowest energy of the δ ray spectrum containing the four hundred δ rays needed to clog the track became

$$W_0 = \frac{1020 \text{ (kev)}}{32 + 1/\beta^2}.$$

It was pointed out that no maximum for W_0 exists and, therefore, the track of a Dirac monopole should not go through a maximum in width.

The second theory has also been applied to the track of a Dirac monopole. It is doubtful that the results give a meaningful width prediction because the large number of δ rays ejected by a monopole would cause saturation of grains along the track. The second theory is not valid when there is grain saturation, as was pointed out in the last part of section three. The track widths predicted by the two theories given in this paper will be upper and lower bounds on the possible track width of a Dirac monopole. The lower bound is given by the Lonchamp theory. The lower bound track of the Dirac monopole would be the least distinguishable from the track of a heavy nucleus; therefore, it will be sufficient to discuss the track predicted by Lonchamp's theory.

Range-energy curves for the monopoles in air of different masses are given by Cole (7). It was decided to calculate range-velocity relations for monopoles of different masses in Ilford G-5 emulsions directly from the theory. The distribution function derived in the previous part can be used to compute the average energy loss per unit track length as follows:

$$\left(-\frac{dE}{dR}\right)_{g_m} = \int_{E_{min}}^{E_{max}} E\, d\eta = \frac{4\pi N g_m^2 e^2}{m_e c^2} \int_{E_{min}}^{E_{max}} \frac{dE}{E} = \frac{4\pi N g_m^2 e^2}{m_e c^2} \ln \frac{E_{max}}{E_{min}} .$$

The maximum energy transferred from the monopole in one collision is given by $\frac{2 m_e \beta^2 c^2}{1-\beta^2}$, whereas the minimum energy given up in one collision is taken as $\bar{I}$, the mean ionization potential of the medium through which the monopole is traveling. The range-velocity relation is obtained from

$$R(E_i) = \int_0^{R(E_i)} dR = \int_0^{E_i} \frac{dE}{\left(-\frac{\partial E}{\partial R}\right)_{g_m}} = \frac{m_e c^2 M}{4\pi N e^2 g_m^2} J(\beta_i) ,$$

where

$$J(\beta_i) = c^2 \int_0^{\beta_i} \frac{\beta\, d\beta}{\ln\left(\frac{2 m_e c^2}{\bar{I}\,(1/\beta^2 - 1)}\right)} ,$$

and M is the mass of the monopole in units of the proton mass. The value of $J(\beta)$ was calculated graphically for values of β up to .95. The mean ionization potential of Ilford G-5 emulsions was calculated from knowledge of the composition of Ilford emulsions and the mean ionization potential of each element in the emulsion. (A more complete treatment would involve finding the energy loss per unit length of track for each element in the absorber, and by taking the composition of the material into account, find the mean energy loss per unit length of track.) $\bar{I}$ was taken as 24.77 electron volts. For a monopole of mass M and velocity βc, the range in Ilford G-5 emulsions is given as

$$R(\beta, M) = 3.72 \times 10^{-1} M J(\beta) \text{ cm} .$$

Values of R for several values of M and β are given in table two.

In plate X, the track width versus range for Dirac monopoles of different masses is plotted. The width of the track for different values of β is taken from Katz and Parnell (16). The range for different values of β is

Table 2. Range-velocity values for five values of M.

β	$R_{M\pi}^{*}$ (μ)	R_{1Mp}(μ)	R_{4Mp}(μ)	R_{25Mp}(μ)	R_{50Mp}(μ)
.1	.537	3.63	14.55	91.0	181.5
.15	1.06	7.2	28.8	180	360
.2	1.72	11.6	46.4	290	580
.25	2.50	16.9	67.7	423	847
.3	3.43	23.2	93.0	582	1160
.4	5.62	38.0	149.0	930	1860
.5	8.22	55.6	222.0	1390	2770
.6	11.15	75.5	302.0	1888	3780
.7	14.60	98.7	394.9	2463	4930
.8	18.35	124.1	497.0	3108	6210
.9	22.3	150.8	603.0	3770	7535
.95	24.23	164.1	657.0	4110	8208

*$M\pi$ refers to the mass of a π meson which is .1478 times the mass of the proton (Mp).

taken from Table 2. Also plotted in Plate X is the width versus range for nuclei of charges 17 and 26 from the second theory given in section three. It will be recalled that this theory gave a fair fit to experimental track width beyond the maximum in track width.

Examination of Plate X will reveal that the track for a Dirac monopole with the mass of Mp or 4 Mp would be easily distinguishable from the track of a heavy nucleus. The track of a monopole with mass 50 Mp might be confused with an iron nucleus track until the residual range of 2000 microns was reached. This range for a monopole of mass 50 Mp corresponds to a kinetic energy of about 3.5 Bev. (If one assumes poles to be created in pairs, the energy needed to create a pole of mass 50 Mp would be about 100 Bev.) A lower limit on the mass of the monopole can be set at approximately the π meson mass (4). (The existence of virtual Dirac monopoles with mass smaller than the π meson mass would result in a sizable increase in the Lamb shift.) The track of a monopole with mass .1478 Mp (π meson mass) would be very short in comparison to most tracks seen in cosmic ray exposed emulsions. For β's greater than .99 "Bremsstrahlung" results in appreciable energy loss. The expression for monopole "Bremsstrahlung" has been worked out by Bauer (1). The energy loss due to "Bremsstrahlung" is inversely proportional to the mass and is, therefore, much greater for poles of small mass. ("Bremsstrahlung" will become important at energies between $10 Mc^2$ and $1000 Mc^2$.) One would expect the track of any pole with a mass between .1478 Mp and 4 Mp to be easily distinguishable from a heavy nuclear track.

CONCLUSION

Two possibilities arise in connection with the detection of a Dirac monopole with electron sensitive nuclear emulsions. Firstly, the Dirac monopole may have a mass between the mass of a π meson and 4 Mp. A monopole with a mass in this range would form an odd shaped, arrowhead track. (The track of a monopole with a mass of the π meson would be shaped more like a blunt arrowhead.) The track would be easily distinguished by a trained scanner, but, because of its small range in the emulsion, the track would lie near the edge of the pellicle and might, therefore, be overlooked. The second possibility is that a massive Dirac monopole might be mistaken for a heavy nucleus track. If the mass of the Dirac monopole is 50 Mp or less, the criterion for distinguishing the monopole track from the heaviest track normally seen in the cosmic ray primaries ($z = 26$) is that one investigate the trackat a residual range of 2000 microns. If the track does not show a decrease in width at this range, and if it is a very wide track, it is suspect as being a Dirac monopole track.

EXPLANATION OF PLATE VIII

Experimental and theoretical determination

of the track width

versus range for two nuclei with $Z = 17 \pm 2$.

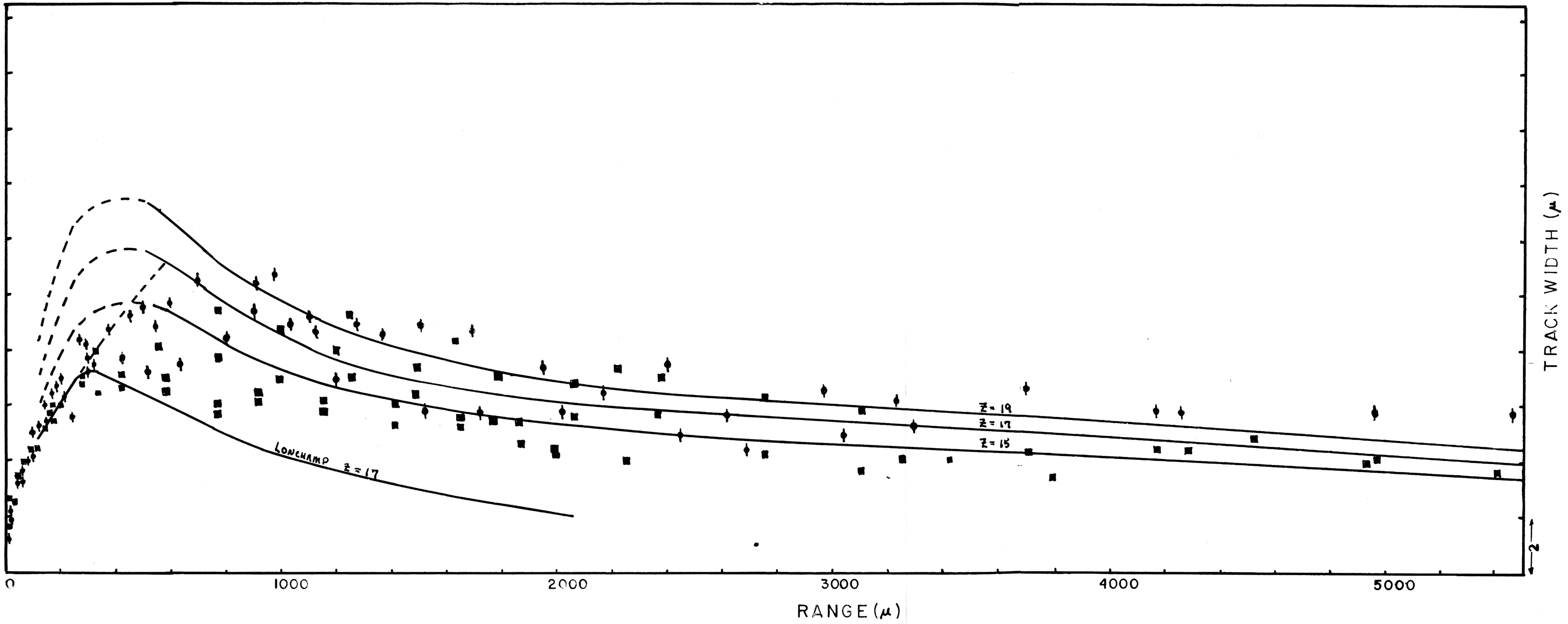
Visual measurements of track width

are denoted by symbol ■.

Photomicrographic measurements of track width

are denoted by symbol ⧫.

PLATE VIII


TRACK WIDTH (μ)
RANGE (μ)
0
1000
2000
3000
4000
5000
Z = 19
Z = 17
Z = 15
LONCHAMP Z = 17
2

EXPLANATION OF PLATE IX

Experimental and theoretical determination

of the track width

versus range for two nuclei with $Z = 7 \pm 1$.

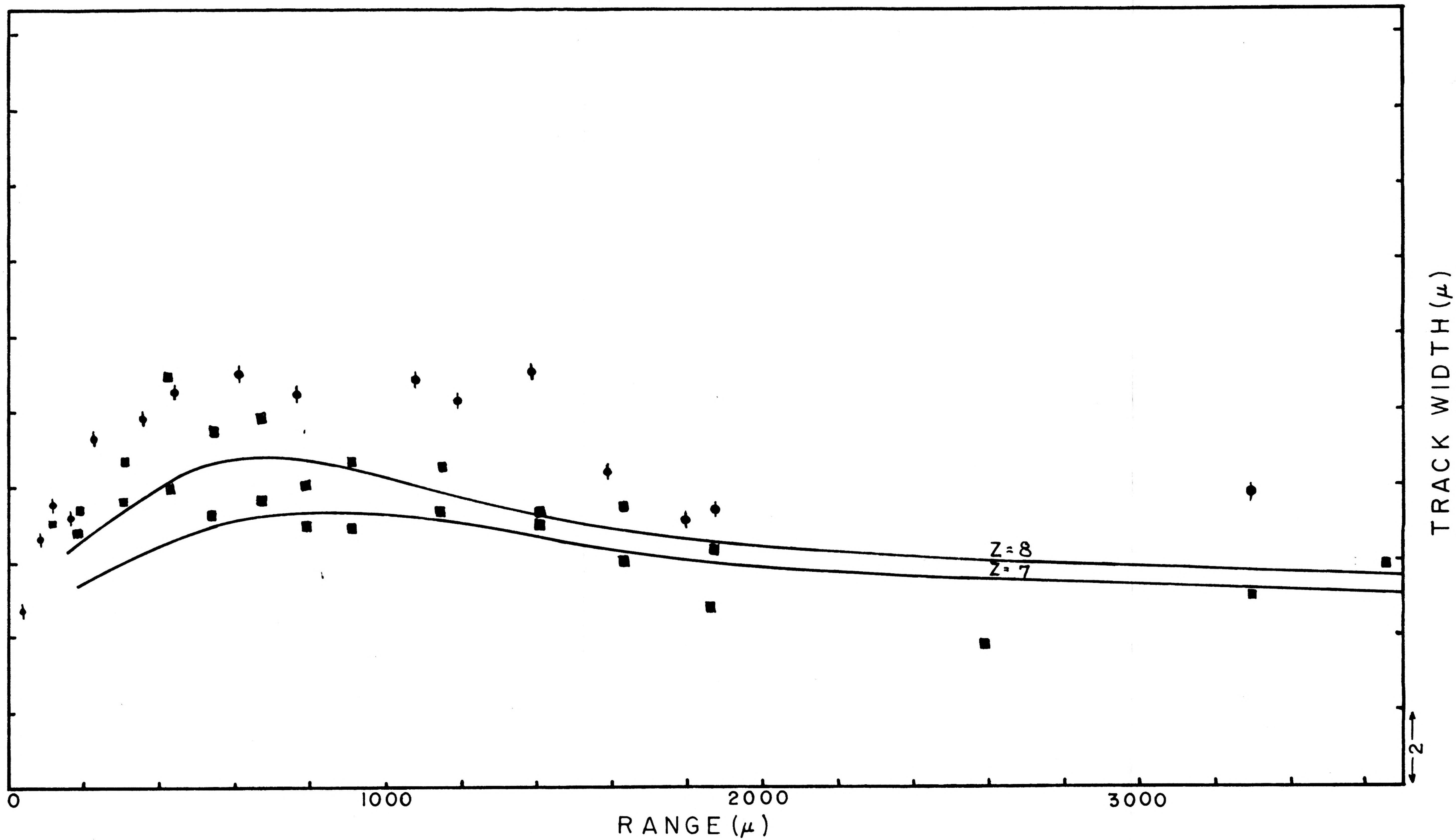
Visual measurements of track width

are denoted by symbol ■.

Photomicrographic measurements of track width

are denoted by symbol ●.

PLATE IX



EXPLANATION OF PLATE X

Track width versus residual range

for Dirac monopoles

with pole strength one

and various masses.

Also, the

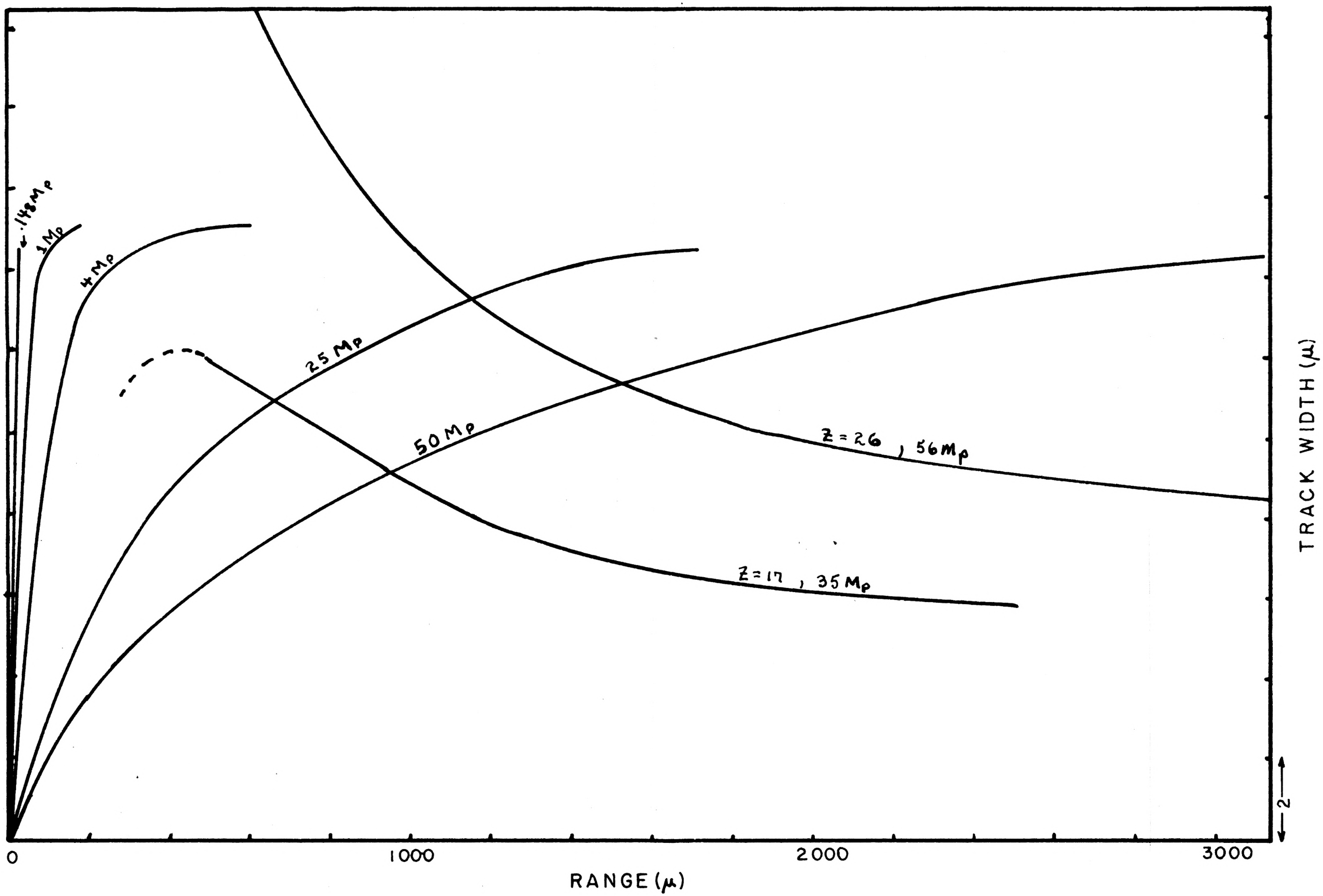
predicted track width

versus range for nuclei

of Z = 17 and Z = 26.

(Mp refers to proton mass.)

PLATE X



.148 Mp
1 Mp
4 Mp
25 Mp
50 Mp
Z = 26 , 56 Mp
Z = 17 , 35 Mp
TRACK WIDTH (μ)
2
0
1000
2000
3000
RANGE (μ)

It is interesting to speculate on the possible production of Dirac monopoles in the upper atmosphere. If we assume pair production of monopoles, the maximum energy needed to create and detect any of the monopoles discussed in this work would be about 10^{11}ev. There are about 7×10^{-3} primary cosmic rays with energy 10^{11}ev passing through a square cm of the upper atmosphere every second, according to Rossi (22). (Almost no photons are found in the primary cosmic radiation, but the pair production could occur by a high energy proton colliding with a nucleus. An analogous process for electron-positron pair production is discussed by Heitler (13).) Assuming the cosmic rays to be directed along the vertical direction and that the plates are exposed with the long sides in the vertical direction, about 75 cosmic ray primaries, each with an energy of 10^{11}ev, will enter the plates every hour. (Calculated for 5 pellicles, each with a thickness of 600 microns and a length of 10 cm.)

If free magnetic poles exist somewhere in the universe, one would expect them to show up in the primary cosmic radiation. A cosmic Dirac monopole with a mass of Mp that enters the atmosphere with a kinetic energy of 10^{12} ev will lose this energy and come to rest at a distance of 70 km above sea level. This estimate was made using Cole's (7) range-energy curve, and Newell's (20) air density versus height data. (Bremsstrahlung was neglected in this estimate; however, it will account for a significant energy loss and, therefore, the above estimate is conservative.) The average balloon only goes up to about 36 km; therefore, one would not expect to see cosmic Dirac monopoles with a mass Mp in balloon-carried nuclear emulsions.

In order to detect a cosmic monopole with a mass as small as the meson mass, nuclear emulsions would have to be sent aloft in rockets. Preferably no material would separate the stack of emulsions from the cosmic radiation. (One centimeter of steel would probably be sufficient to stop any monopole with a π meson mass.) The feasibility of such an experiment is unknown to the author.

In conclusion, the search for a Dirac monopole should be carried out with electron sensitive emulsions carried in rockets to distances above the atmosphere. Trained scanners should investigate every aggregation of developed silver, keeping in mind that the shape of a Dirac monopole track may range from a short arrowhead shaped thing to something resembling a heavy nuclear track.

ACKNOWLEDGMENTS

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APPENDICES

APPENDIX I

Impulse Given to a Free Electron by a Moving Nucleus or Monopole

The most significant force felt by an electron at rest in the laboratory frame when a nucleus or monopole with velocity $\underset{\sim}{v} = \beta c \underset{\sim}{i}$ (with respect to the laboratory frame) moves by is given by the Lorentz force,

$$\underset{\sim}{f} = (-e)\left(\mathcal{E}_x \underset{\sim}{i} + \frac{\mathcal{E}_y}{\sqrt{1-\beta^2}}\underset{\sim}{j} + \frac{\mathcal{E}_z}{\sqrt{1-\beta^2}}\underset{\sim}{k} + \frac{\underset{\sim}{v} \times \underset{\sim}{B}}{\sqrt{1-\beta^2}}\right), \tag{1}$$

where $\mathcal{E}_x$, $\mathcal{E}_y$, $\mathcal{E}_z$, and $\underset{\sim}{B}$ are measured in the rest frame of the moving nucleus or monopole. (See Landau and Lifshitz (18).) It should be noted that $\underset{\sim}{B}$ is tied to real magnetic poles or dipoles that exist in the rest frame of the nucleus or monopole; and that $\underset{\sim}{B}$ is related to the force on the charge carried by the electron. Other interaction forces exist between the electron and nucleus or monopole that are small and can be neglected. (For instance, in the case of a moving nucleus, the intrinsic magnetic moment of the electron (Bohr magneton) experiences a torque due to the existence of an $\underset{\sim}{H}$ vector created by the electric current, $Ze\beta c$. In a classical sense, this torque causes the polarization of the spin axis predicted by the Mott scattering formula.)

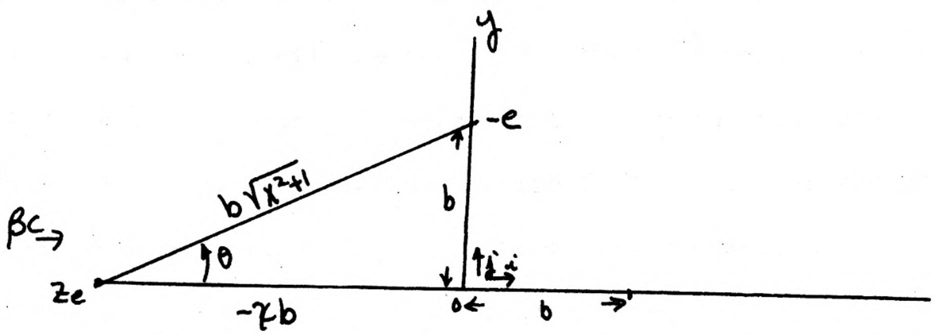


Fig. 1. Nucleus moving by an electron. (The unit of length along the X axis is the impact parameter b.)

Consider first the case of a nucleus with velocity βc and charge Ze moving past an electron as shown in Fig. 1. The following assumptions are made: 1) $\underset{\sim}{B}=0$ (the intrinsic magnetic dipole of the nucleus is neglected); 2) $\beta c \ll c$ and $\sqrt{1-\beta^2}\sim 1$. The Lorentz force reduces to Coulomb's force and one obtains from equation (1) and Fig. 1 (symmetry considerations give $\mathcal{E}_z=0$)

$$\underset{\sim}{f}_{Ze} = -\frac{Ze^2 X}{b^2(X^2+1)^{3/2}}\,\underset{\sim}{i} - \frac{Ze^2}{b^2(X^2+1)^{3/2}}\,\underset{\sim}{j}\,. \tag{2}$$

An increment of the impulse time is given by

$$d\tau = \frac{b\,dx}{\beta c}$$

therefore, the total impulse given to the electron is

$$\underset{\sim}{I}_{Ze} = \left\{\frac{-e^2 Z(1-\beta)}{b\beta c}\int_{-\infty}^{\infty}\frac{X\,dx}{(X^2+1)^{3/2}}\right\}\underset{\sim}{i} + \left\{\frac{-e^2 Z}{b\beta c}\int_{-\infty}^{\infty}\frac{(1-\beta)\,dx}{(X^2+1)^{3/2}}\right\}\underset{\sim}{j}\,. \tag{3}$$

The absolute value of the impulse becomes

$$|\underset{\sim}{I}_{Ze}| = \frac{2Ze^2}{b\beta c}\,,$$

which may be written

$$|\underset{\sim}{I}_{Ze}| = \left(\begin{matrix}\text{MAX.}\\ \text{FORCE}\end{matrix}\right)\cdot\left(\begin{matrix}\text{IMPACT}\\ \text{TIME}\end{matrix}\right) = \left(\frac{Ze^2}{b^2}\right)\left(\frac{2b}{\beta c}\right). \tag{4}$$

The "impact time" is defined as the time during which the force on the electron is greater than $\frac{1}{2}$ the maximum value. For most calculations, one may consider the force to be a significant value only when the nucleus is within a distance b on either side of $X=0$.

As βc approaches the speed of light, relativistic effects must be considered; the impact will be considered from the rest frame of the electron (primed frame). Equation (1) gives the force on the electron in the electron's rest frame, however, we want the electric fields $\mathcal{E}_X, \mathcal{E}_Y$ expressed in terms of primed coordinates. The following Lorentz transformation gives the relation between the coordinates of both systems:

$$b = b'\,,\quad X\,b = \frac{X'b'}{\sqrt{1-\beta^2}}\,,\quad \tau = \sqrt{1-\beta^2}\,\tau'\,,$$

and

$$d\tau = \sqrt{1-\beta^2}\left(\frac{X'b'}{\sqrt{1-\beta^2}}\,\frac{1}{\beta c}\right) = \frac{X'b'}{\beta c}\,.$$

The electric fields expressed in the coordinates of the primed frame are

$$\mathcal{E}_x' = \frac{1}{\sqrt{1-\beta^2}} \cdot \frac{Zex'}{b'\left(\frac{x'^2}{\sqrt{1-\beta^2}}+1\right)^{3/2}}, \quad \mathcal{E}_y' = \frac{1}{\sqrt{1-\beta^2}} \cdot \frac{Ze}{b'\left(\frac{x'^2}{\sqrt{1-\beta^2}}+1\right)^{3/2}}.$$

The impulse seen by the electron is

$$\underset{\sim}{I}_{Ze}' = \left\{ -\frac{Ze^2(1-\beta^2)}{b\beta c}\int_{-\infty}^{\infty} \frac{x'\,dx'}{(x'^2+1-\beta^2)^{3/2}} \right\} \underset{\approx}{i}' + \left\{ -\frac{Ze^2}{b\beta c}(1-\beta^2)\int_{-\infty}^{\infty} \frac{dx'}{(x'^2+1-\beta^2)^{3/2}} \right\} \underset{\sim}{j}'.$$

The first integral goes to zero; the second one becomes

$$\lim_{A\to\infty} 2\int_0^A \frac{dx'}{(x'^2+1-\beta^2)^{3/2}} = \lim_{A\to\infty} \frac{2A}{(1-\beta^2)\sqrt{A^2+1-\beta^2}} = \frac{2}{1-\beta^2},$$

therefore $|I_{Ze}'|$ is given as $(b'=b)$

$$\frac{2be^2}{b\beta c}$$

which agrees with equation (4).

Consider a free magnetic pole with pole strength g_m moving with velocity $\underset{\sim}{v}=\beta c \underset{\sim}{i}$ (assumed small incomparison to the velocity of light) by an electron at rest. Neglecting any intrinsic electric moment of the pole, the interaction force becomes

$$\underset{\sim}{f}_{g_m} = -\frac{e\underset{\sim}{v} \times \underset{\sim}{B}}{c} = \frac{-e\underset{\sim}{v}}{c} \times \frac{g_m \mathbf{1}_r}{r^2}, \tag{5}$$

which is a velocity dependent force. Figure 1 will describe the interaction if Ze is replaced by g_m. From Fig. 1 and equation (5), one obtains

$$|\underset{\sim}{f}_{g_m}| = \frac{e g_m \beta c \sin\theta}{cb^2(x^2+1)} = \frac{e g_m \beta}{b(x^2+1)^{3/2}}$$

As before, the total impulse given to the electron is given by

$$|\underset{\sim}{I}_{g_m}| = \frac{e g_m}{cb}\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^{3/2}} = \frac{2e g_m}{cb},$$

which can be written as

$$(\text{Max. force}) \times (\text{Impact time}) = \left(\frac{e g_m \beta c}{cb^2}\right)\cdot\left(\frac{2b}{\beta c}\right)$$

Note that as the velocity βc increases, the maximum force increases; but the "impact time " decreases. The relativistic considerations follow the same treatment as given for the moving nucleus.

No reference has been made to the masses of any of the particles discussed on the preceding page(s). The considerations given are valid as long as the mass of the nucleus or monopole is much larger than the electron mass.

APPENDIX II

Validity of the Semi-Classical Impact Parameter Method

Consider the impact between an electron and a heavy nucleus from the rest frame of the heavy nucleus. The uncertainty in momentum of the electron when it is localized within the distance of significant force ($2b$) mentioned in Appendix I is given by

$$\Delta p \geq \hbar/2b. \tag{1}$$

If one is to describe the impact classically, the uncertainty in momentum of the electron must be much less than the momentum given to the electron by the nucleus. In Appendix I, it was shown that the momentum given the electron is

$$|I_{ze}| = \frac{2Ze^2}{\beta c b}. \tag{2}$$

For a classically defined impact $|I_{ze}| \gg \Delta p$, which results in

$$\frac{4e^2}{\hbar c}(Z/\beta) \gg 1. \tag{3}$$

This inequality is satisfied for values of (Z/β) on the order of 10^4; values of (Z/β) of this order can be obtained for $Z \approx 50$ and $\beta \approx .001$. For the values of (Z/β) used in this work, the inequality is actually reversed, and one obtains

$$\frac{4e^2}{\hbar c}(Z/\beta) \ll 1. \tag{4}$$

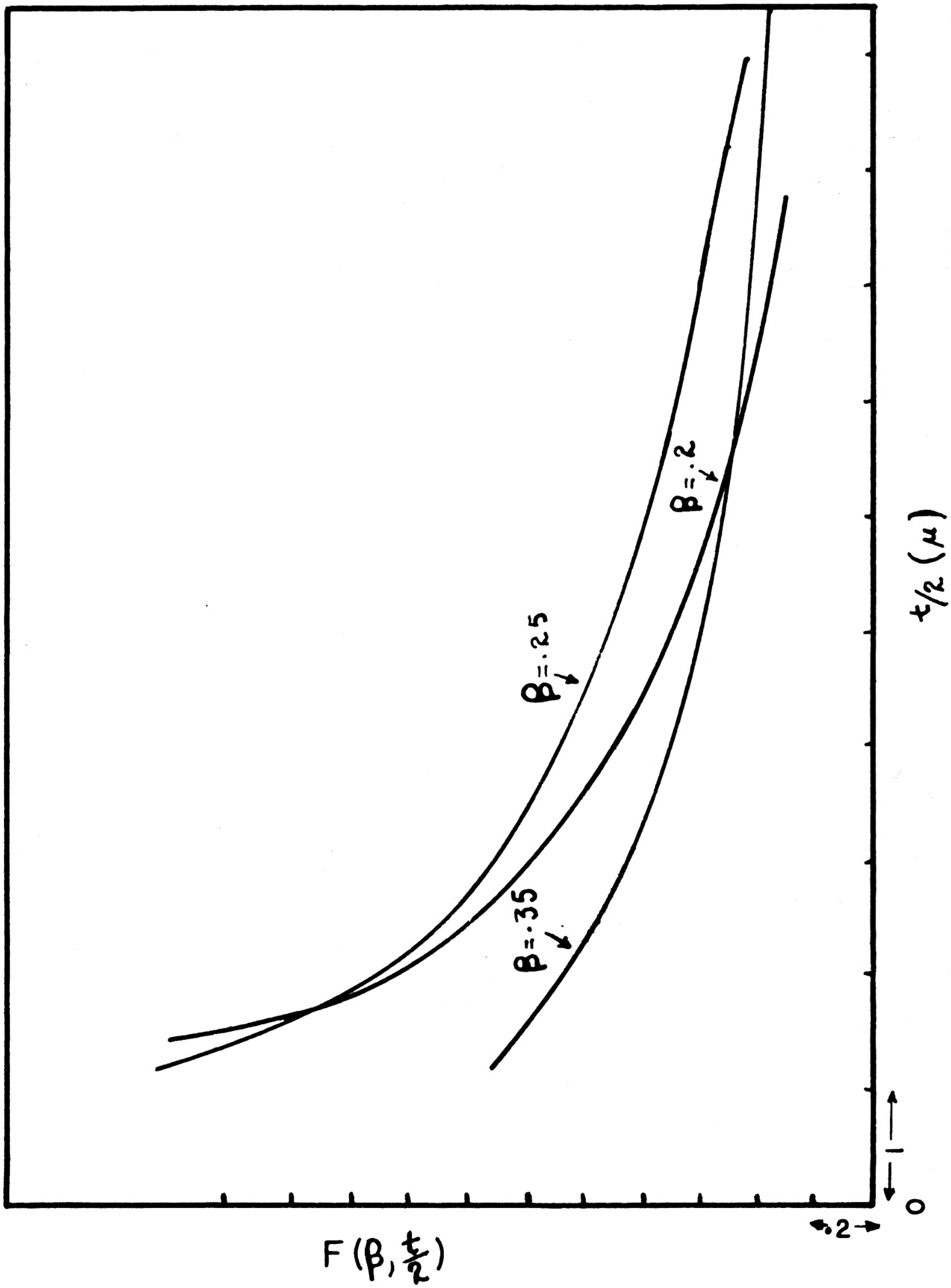
A quantum mechanical treatment will be needed to describe the impact. However, when a quantum mechanical treatment is used, the inequality (4) insures that the Born approximation will be valid. Fortunately, the Born approximation method gives the same result as the semi-classical impact parameter method when the interaction force is Coulombic (see Bohm (3)). The derivation of the δ ray distribution function given in the introduction was not correct because the semi-classical impact parameter method was valid, but because the interaction force was an inverse square force.

It is interesting to note that the Rutherford scattering formula as originally used by Rutherford in his α scattering experiments was valid classically, since

$$\frac{4Zze^2}{\hbar\beta c} \sim 38$$

(Values of Z=50, z=2, and β=.05 were used.) However, when the δ ray distribution function is used as it is in this work, it is valid fortuitously. Williams (27) has discussed the validity of the classical approach in detail.

APPENDIX III



THEORY OF HEAVY NUCLEAR TRACKS IN ELECTRON SENSITIVE NUCLEAR EMULSIONS WITH AN APPLICATION TO THE TRACK OF A DIRAC MONOPOLE

by

DAVID BRUCE CLINE

B. S., Kansas State University, 1959

AN ABSTRACT OF A THESIS

submitted in partial fulfillment of the

requirements for the degree

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AN ABSTRACT OF A THESIS

A Dirac monopole created by the cosmic radiation or existing in the cosmic ray primaries might be detected by electron sensitive nuclear emulsions carried into the upper atmosphere by rockets. A simple and precise criterion is needed to distinguish the track of a Dirac monopole from the tracks of nuclei commonly seen in the primary cosmic radiation. It has been pointed out by Katz and Parnell that the track of a Dirac monopole would not have a maximum in width; whereas the track of a heavy nucleus would have a maximum in width. To distinguish between a Dirac monopole track and a heavy nuclear track, it is important to know at what residual range the two tracks would be noticably different. A simple theory of track formation has been given by Lonchamp; this theory was extended in order to be valid in the region where the monopole track would look different from the nuclear track.

The character of a nuclear track is the result of the spacial distribution of secondary electron (δ rays) tracks along the trajectory of the nucleus. A simple method was used to relate the energy distribution of δ rays to the spacial distribution of δ rays. Observational procedures were taken into account in this theory of track formation.

The track width versus residual range of three heavy nuclei was measured. Two methods were used: visual observation through the microscope using an eyepiece micrometer, and photomicrographic measurements. For the photomicrographic measurements, the tracks were photographed, and the track width was measured using an Ott planimeter. The results of these measurements agreed with the predicted track width in the region of interest.

The tracks of Dirac monopoles of different mass were investigated using track width versus velocity data taken from Katz and Parnell and range velocity data calculated in this work. The masses investigated ranged from the mass of a π meson to fifty proton masses. A Dirac monopole with a mass less than four proton masses would create an arrowhead shaped track that would be easily distinguishable. A Dirac monopole with a mass between 25 and 50 proton masses would create a track that might be mistaken for a heavy nuclear track.

It was found that at a residual range of 2000 microns, the track of a Dirac monopole with a mass of fifty proton masses or less would be distinguishable from the track of the heaviest nucleus (iron) commonly found in the primary cosmic radiation.