

EFFECT OF THE SHAPE OF WAVE OPENINGS ON THE
FREQUENCY OF SPHERICAL RESONATORS

by

RALPH ABRAHAM SHENK

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INTRODUCTION

The smallness of the damping and sharpness of the tuning in spherical resonators renders them extremely sensitive detectors of sounds of a particular frequency. The quality of musical tones may be studied by means of such resonators to determine the presence, the number and character of upper partials in a complex musical sound.

As above mentioned, any particular resonator has a characteristic frequency. This frequency is dependent primarily upon two factors of the resonator as follows: (1) the volume of the resonator and (2) the conductivity of the wave opening. The conductivity of the wave opening in turn is

dependent upon the area and possibly the shape of the wave opening.

PURPOSE

The purpose of this problem was to determine the effect if any of the shape of the wave openings, of constant area upon the frequency of spherical resonators.

HISTORY

The frequency of a resonating spherical shell as given by Helmholtz is expressed by the following equation:⁽¹⁾

$$f = \frac{a \sqrt[4]{\sigma}}{4 \sqrt{\pi^5} \cdot \sqrt{2 S}}$$

where

a = velocity of sound in air at zero degrees centigrade

σ = area of circular wave opening

S = the volume of the sphere and

f is the frequency of the resonator.

(1) "Sensation of Tone", by Helmholtz.

The period of vibration of an elastic body is of the form⁽¹⁾

$$2 \pi \sqrt{\frac{M}{F}}$$

where

M = the inertia of the moving parts and

F = the elastic force

In a spherical resonator, resonance is caused by a periodical inward and outward rush of air through the wave opening, compressing and expanding the air inside of the resonator in accordance with the frequency.

In the resonator, "F" is the force of restitution for unit displacement or rise of pressure when unit volume of air is introduced. Now the rise of pressure resulting from the introduction of a given volume of air depends upon the volume of the resonator. That is, if the volume of gas introduced is small, the rise of pressure is inversely proportional to the volume "S" of the sphere. Then the period of vibration is directly proportional to the square root of "S".

We may call the property of the mouth in virtue of which it allows the air to pass more or less easily, the

(1) "Sound", by Kapstick.

"conductivity". The greater the conductivity, the greater the frequency. Then conductivity depends mainly upon the size and shape of the wave opening. So far a complete mathematical analysis to express conductivity is unknown except for the circular wave opening whose frequency is given by Rayleigh to be:

$$f = \frac{a}{2 \pi} \sqrt{\frac{C}{S}}$$

where

C = the conductivity of the circular wave opening and

S = the volume of the resonator.

METHOD OF ATTACKING THE PROBLEM

The general method of attack was that of making use of a variable speed siren as a sound source for determining the maximum resonance frequency of a spherical shell of known dimensions into which were fitted the wave openings to be studied.

The arrangement of the apparatus is shown in the accompanying photograph, plates 1 and 2. The details of the apparatus are shown on plate 3.

The photograph shows the relative position of the ac-

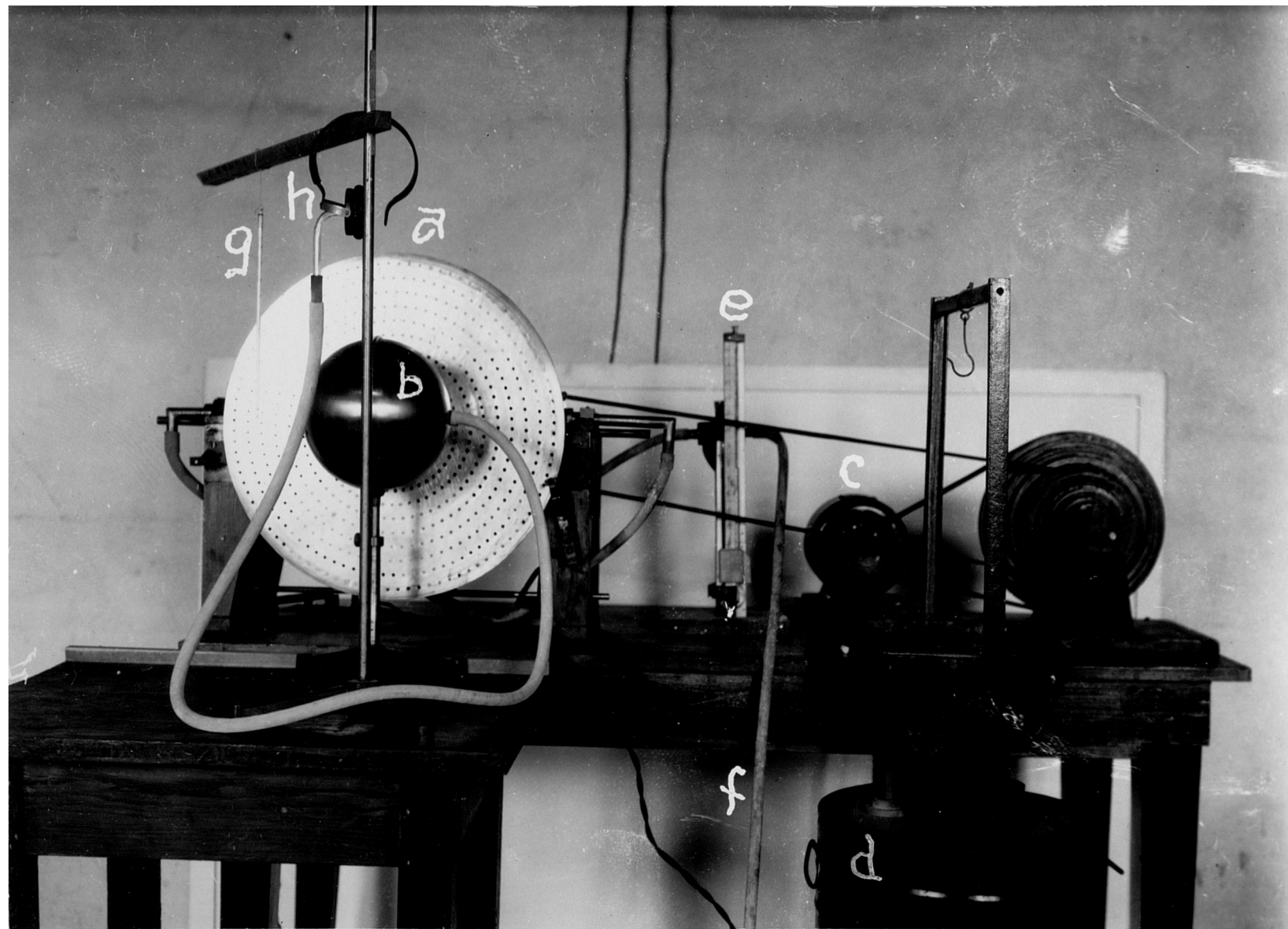


PLATE II.

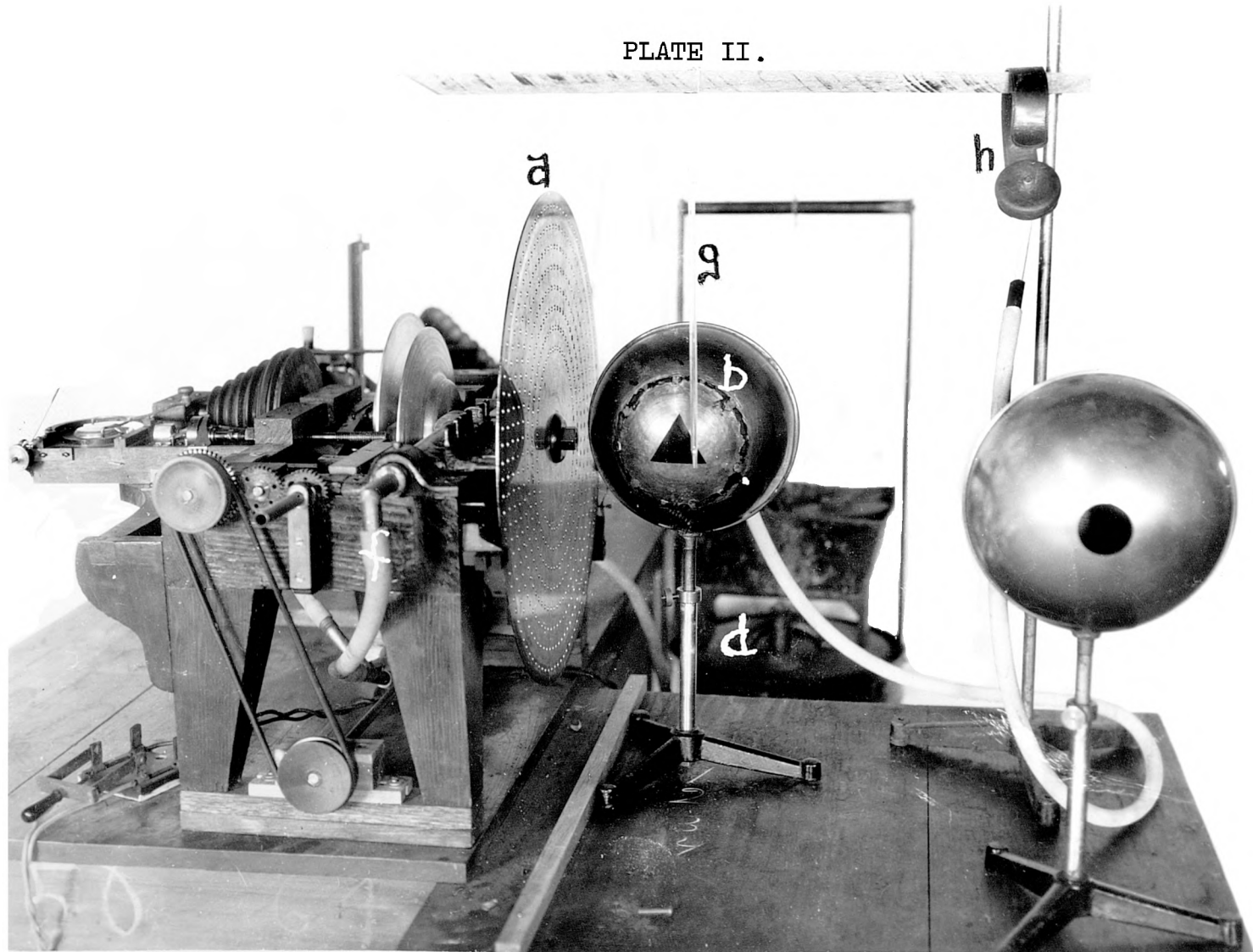
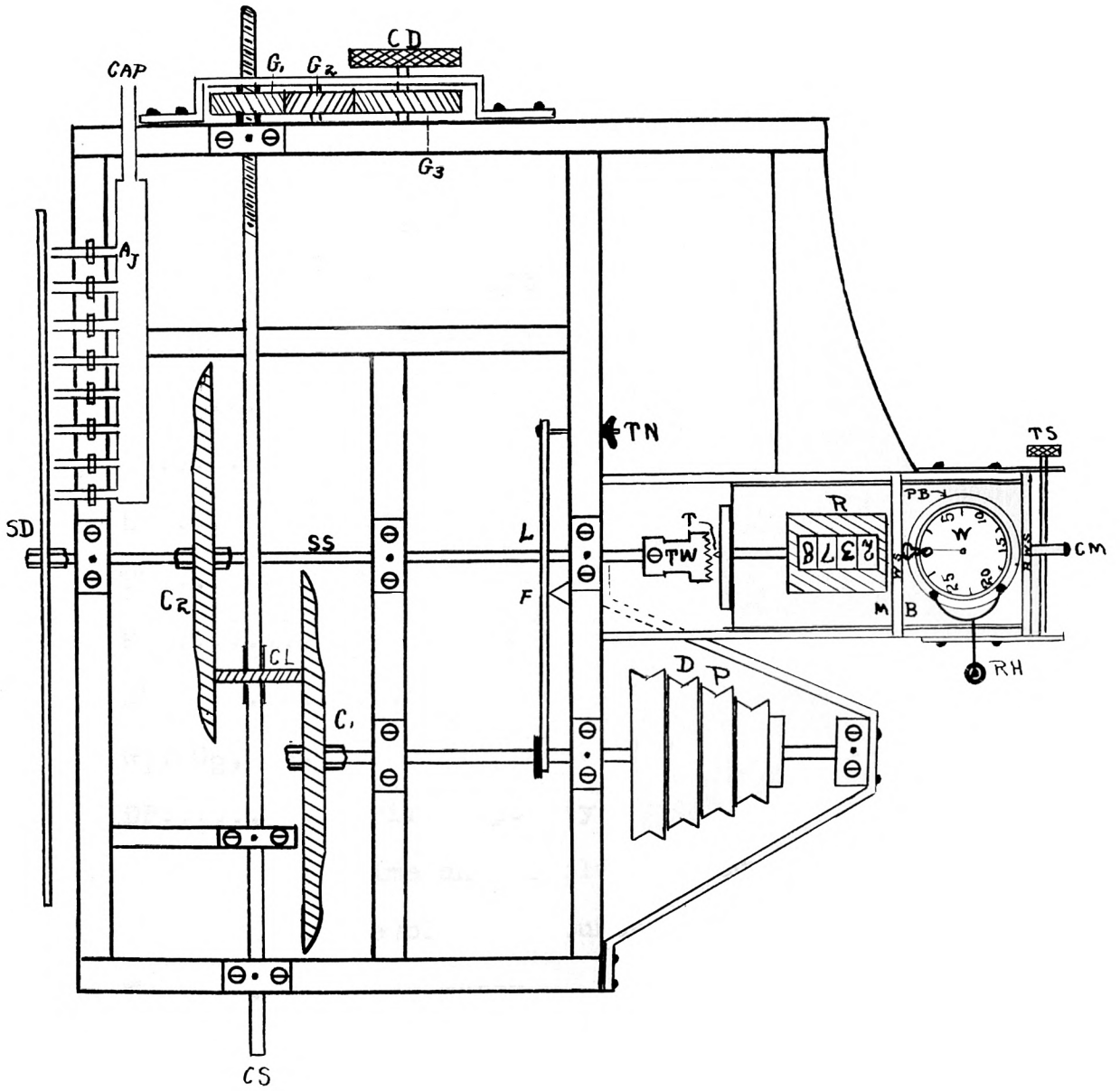


PLATE III.



cesory equipment with that of the siren machine. The position of the resonator with respect to the siren is especially of importance, as the wave opening of the resonator must not be obstructed.

Key to Plate 3

SD.....	Siren disc
SS.....	Siren shaft
C ₁ , C ₂	Aluminum clutch plates
CL.....	Rubber clutch roller
CS.....	Clutch roller shaft
L.....	Clutch pressure lever
TN.....	Clutch tightening nut
F.....	Pressure lever fulcrum
CD.....	Speed control dial
G ₁ , G ₂ , G ₃ ..	Gears for controlling position of clutch roller
DP.....	Driving pulley
TRC.....	Time and revolution counter
R.....	Revolution counter
W.....	Stop watch
T.....	Tooth on revolution counter
TW.....	Toothed wheel
MB.....	Movable block
PB.....	Pivoted block

CM..... Cam for operating time and revolution
counter

TS..... Thumb screw for operating cam

WS..... Watch stop

RWS..... Rear watch stop

RH..... Rotating handle

AJ..... Air jets

CAP..... Constant air pressure

Description of Essential Features

An aluminum siren disc (SD) is provided with eight rings containing holes of the same size and of such number per ring that the tones of a true diatonic scale may be produced. This siren is mounted on the siren shaft, (SS) by means of a nut and washer, and in turn is mounted on the sturdy wood frame as shown in plates 1-3.

Two aluminum clutch plates (C_1 , C_2), one mounted on the siren shaft and the other on the driving shaft, are provided with a rubber clutch roller (CL) for rolling on the plates in order to vary the speed of the siren disc. Also the driving shaft is provided with driving pulleys of various diameters for still greater speed variations.

The machine is provided with a simultaneous time and revolution counter (TRC). This part of the apparatus is mounted at the end of the siren shaft opposite the siren

disc. On this end of the siren shaft is attached a toothed wheel (TW) which engages a tooth (T) of the revolution counter (R), when the latter, with the stop watch (W), is thrust forward by means of a cam (CM) mounted on a knurled thumb screw shaft (TS).

The time and revolution recording apparatus consists of a stop watch (W) and revolution counter (R) mounted on a to and fro sliding block which is operated by the thumb cam. The stop watch is mounted on a pivoted block (PB) which is provided with a handle (RH) for operating so that the watch may be rotated through 180° . The operating position of the watch is stem toward the cam preparatory to starting and away from the cam preparatory to stopping the watch and the revolution counter.

The position of the stop watch and toothed wheel is adjustable so that they start and stop simultaneously. When the stem of the watch is pointing away from the cam, the adjustment is made such that the watch starts at the same instant that the tooth on the revolution counter engages a tooth of the wheel. This is accomplished by the stem of the watch being pressed against a "stop" (WS) during the forward movement of the sliding block which is operated by the cam.

After simultaneously starting the watch and the revolution counter the watch is rotated counterclockwise through

180° (stem toward the cam) by means of the handle provided. When the watch hand reaches the point which ends the approximate desired time interval, the cam is quickly turned backward through 180° thrusting the watch stem against a rear "stop" (RS) which has previously been adjusted so that the watch is stopped at the same instant that the tooth on the revolution counter disengages the toothed wheel on the siren shaft.

The speed of the siren is regulated by adjusting the position of the clutch roller (CL) on the surfaces of the clutch plates (C_1 and C_2). The position of the roller, whose position on its shaft is constant but rotates, is varied by drawing its shaft (CS) to and fro through the threaded gear (G_1) which in turn is operated by the milled head (CD). The pressure of the clutch plate surfaces on the clutch roller is adjusted by means of the adjustable clutch pressure lever (L). The fulcrum of the lever being at (F) and the thumb nut to tighten the clutch at (TN).

Wave Openings

The photograph and line drawings of the wave openings used are shown on plates 4-6. These wave opening dimensions were computed from the circular wave opening. The drawings

are actual size to a rather high degree of approximation, except the one shown mounted. A form or pattern was not made for the circular opening as it was directly cut with a washer cutting tool.

The radius of the copper sphere was computed by determining the mass of water required to fill it. From this computed value the radius of the circular wave opening was found by use of the five to one ratio used by Helmholtz. The area of the circular wave opening was found to be 12.5664 square centimeters. The dimensions of the other geometric wave openings were determined from this value since they all have constant areas.

The circular orifice was cut directly in the sphere used as a standard (Plate 7-B) by the use of a cutting tool whose cutting edge was a distance from its center equal to that of the radius of the circular opening.

Patterns for the other wave openings were computed and plotted on plate brass of a uniform thickness. The forms were then cut out in the rough and filed to calipered dimensions, the final measurements being checked with accuracy to the second decimal place. (Plates 5-6).

These forms were then centered and mounted on mild steel rods which were put in the lathe to turn out a concave spherical face on the forms of an arc equal to that of the

PLATE IV.

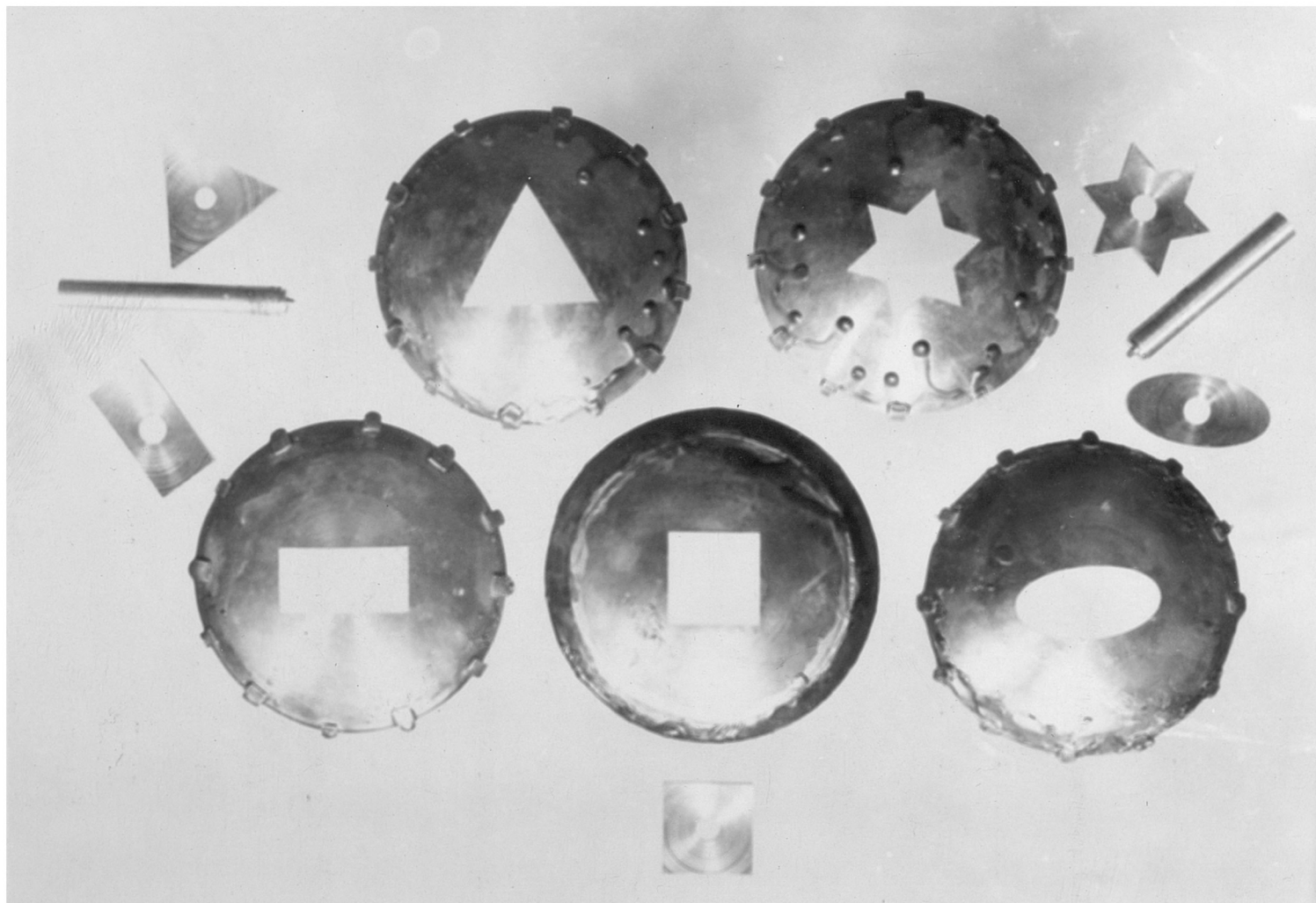


PLATE V.

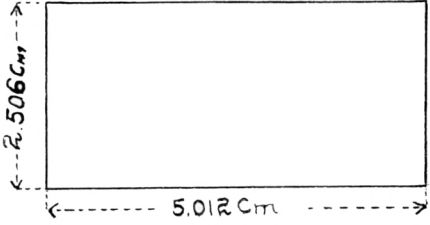
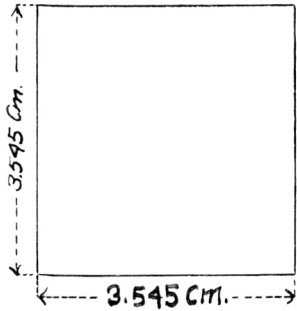
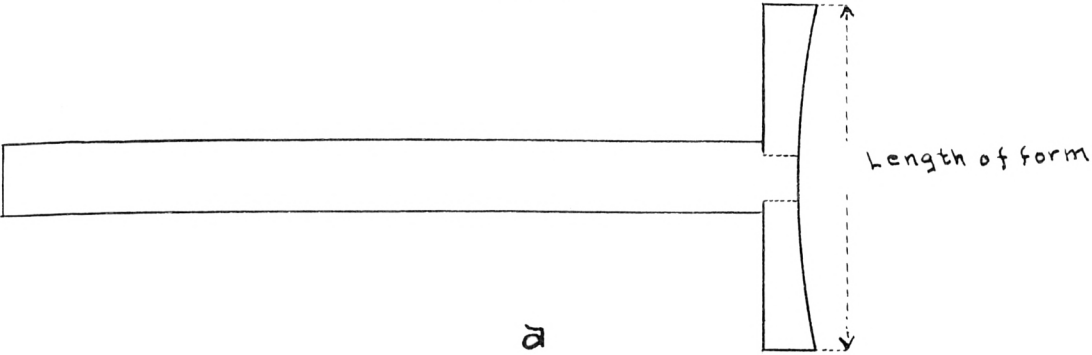
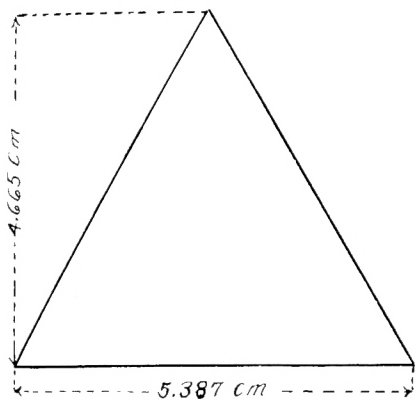
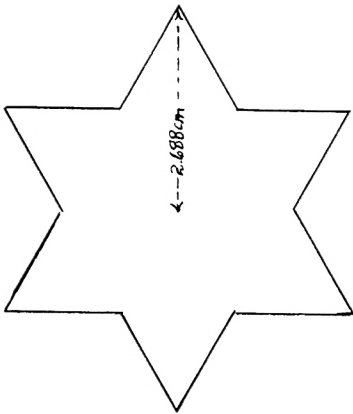


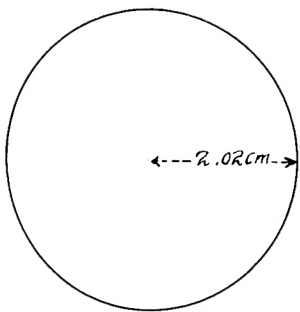
PLATE VI.



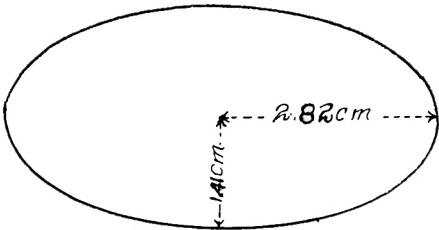
a



b



c



d

sphere. (Plate 5-A) This made it possible to place the forms on the spherical segments and thus get a very accurate pattern of the projection of the plane, constant area upon the spherical segments. (Plate 4).

These wave openings were cut on spherical segments which were spun from sheet copper of the same thickness of that of the copper shell. The spinning block form (Plate 7-C) was constructed so that its radius of curvature was equal exactly to that of the copper shell. Each segment with its wave opening cut in it was made to accurately fit the hole in the sphere which was made of the same dimensions as the segments to be inserted. Each segment to be used was waxed in position in the spherical wall. (Plate 7-A).

PROCEDURE

Source of Sound Energy

The constant speed power source for driving the siren disc was that of an alternating current motor which was practically noiseless. (Plate 1-2-C).

The air pressure used was also kept at an optimum constant by the use of a pressure tank (Plate 1-2-d) in the compressed air line (Plate 1-f). The constant air pressure could be varied only by the addition or removal of weights

from the pressure tank. The constant air pressure was read from a manometer (Plate 1-2-e) and was found to be that of 9.42 inches of water for optimum operation.

The temperature of the air near the wave opening was kept constant within $.5^{\circ}$ C. and recorded by a thermometer located near the wave opening. Plate 1-2-g).

All the bearings on the apparatus were oiled well and regularly. Special attention was also paid to electrical connections and to the dressing of belts. Any alteration of the speed of the siren disc could be detected by a change of pitch of the siren.

Resonators

Two copper shells of the same volume were used for resonators and mounted on tripods capable of vertical adjustment. (Plate 7). The location of the resonators on the table opposite the siren disc was such that a clear response could be heard relatively free from extraneous noises. The response of the resonator was heard through tubing which extended from the ear piece of the resonator to the ear of the observer. (Plate 1-2-h). The position of the resonator with respect to the siren disc was quite important as the conductivity of the orifice cannot be altered without also alternating its frequency. For this reason the orifice was

kept completely uncovered and the energy source unobstructed. To insure these conditions the resonator was placed to the right or left of the air jet and the wave opening faced at right angles to the longitudinal direction of the wave energy line of projection as shown on plate 1-2.

Method for Detecting the Maximum Resonance

The maximum resonance was detected by the response through the ear piece while varying the speed of the siren disc. When the maximum resonance point was found the adjustment was then left undisturbed while the speed of the siren disc was taken by the revolution counter and stop watch. The number of holes per revolution and the number of revolutions being known, the frequency was calculated by substituting values in the following equation:

$$F = \frac{H N}{T}$$

where

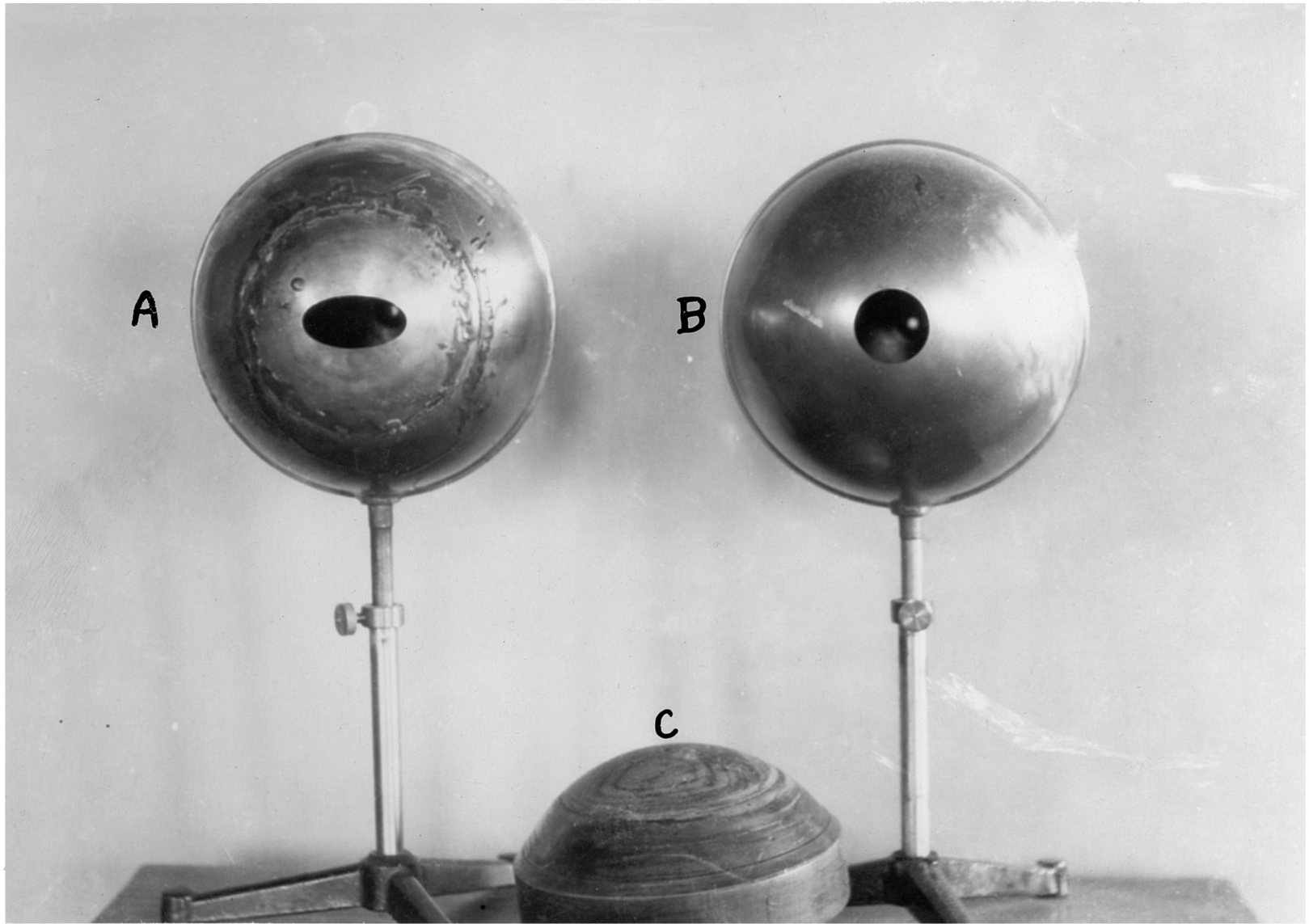
F = the frequency

H = the number of holes per revolution

N = the number of revolutions counted and

T = the time interval in seconds

PLATE VII



The apparatus was then moved out of adjustment, readjusted and the frequency computed again. Ten such values were taken for each wave opening. For each wave opening the mean value and probable error were computed.

THE SPHERICAL OPENING

The theoretical and experimental values for the fundamental frequency of the standard resonator, with a circular wave opening, are given in the two following paragraphs for comparison. These values check and calibrate the apparatus and method of procedure.

Theoretical Value for "f"

The theoretical frequency of the spherical shell used with circular wave opening was computed from the Helmholtz-Rayleigh equation and used as a means of calibration for the apparatus and method.

From the Helmholtz-Rayleigh equation:

$$f = \frac{a \sqrt[4]{\sigma}}{\sqrt[4]{\pi^5} \cdot \sqrt{2S}}$$

where

a = the velocity of sound in air

σ = the area of the circular wave opening

S = the volume of the sphere and

f = the frequency of the resonator

$$f = \frac{a \sigma^{1/4}}{\pi^{5/4} (2S)^{1/2}} = \frac{a (\pi r^2)^{1/4}}{\pi^{5/4} (2 \frac{4}{3} \pi R^3)^{1/2}} =$$

$$\frac{a \pi^{1/4} r^{1/2}}{8^{1/2} \pi^{7/4} R^{3/2}} = \frac{(3)^{1/2} a \pi^{1/4} r^{1/2}}{8^{1/2} \pi^{7/4} R^{3/2}} =$$

$$\frac{a (3r)^{1/2}}{(8\pi R)^{3/2}} = a \left\{ \frac{3r}{8\pi R} \right\}^{1/2}$$

We derive

$$f = a \frac{3r}{8\pi R}$$

Since the speed of sound "a" is a function of the temperature it is necessary to find an expression for the speed at any temperature.

Let $A_{t^{\circ}C}$ = speed of sound at any temperature and

$A_{0^{\circ}C}$ = speed of sound at 0° centigrade

Then

$$A_{t^{\circ}\text{C}} = A_{0^{\circ}\text{C}} \sqrt{1 + .00367t^{\circ}\text{C}}$$

Substituting

$$A_t = 33226 \sqrt{1 + .00367t^{\circ}\text{C}}$$

Therefore

$$f_{t^{\circ}\text{C}} = 33226 \sqrt{1 + .00367t^{\circ}\text{C}} \sqrt{\frac{3r}{8\pi r^3}}$$

By using this equation and substituting the actual values for the resonator dimensions and surrounding conditions as given below:

$r = 2.02$ centimeters (radius of circular opening)

$R = 10.09$ centimeters (radius of the sphere)

$t = 25^{\circ}$ centigrade (working temperature)

we get

$$f = 33226 \sqrt{1 + .00367 \times 25} \sqrt{\frac{3 \times 2.02}{8\pi \times (10.09)^3}}$$

$$f = 33226 \sqrt{1.09175} \sqrt{\frac{6.06}{8 \times 30.98 \times 1027.16}}$$

$$f = 33226 \times .1045 \sqrt{.0000238047}$$

Finally

$$f(25^{\circ}\text{C}) = 34721.17 \times .004879 \text{ or } 169.40 \text{ vibrations/sec.}$$

Experimental Value for "f"

The spherical resonator with the circular wave opening was placed on the table opposite the siren disc so that the orifice faced at right angles to the air jet. The siren disc was then regulated with the regulating dial until the speed of the disc was such that the resonator spoke with maximum loudness. This meant that the frequency which small air puffs were being emitted from the siren was equal to the natural frequency of the spherical resonator.

This resonating frequency was then calculated by substituting in the formula for experimental frequency:

$$f(25^{\circ}\text{C}) = \frac{N \quad R}{T}$$

as previously explained.

The values for the circular orifice were found to be as follows:

"f" (25°C)

169.37
 169.90
 169.49
 169.97
 169.70
 169.33
 169.41
 169.62
 169.83
169.83

1697.45

Pressure used 9.42 inches of water

Temperature 25°C

Holes /revolution 60

The Mean Value of "f" = 169.745 vib./sec.

Maximum value of "f" = 170.33 vib./sec.

Minimum value of "f" = 169.37 vib./sec.

Maximum variation of "f" = .96 vib./sec.

Maximum deviation from mean "f" = .59 vib./sec.

From the above it can be seen that the experimental value for the frequency of the spherical resonator, with a circular wave opening, checks very closely with the theoretical value as calculated by substituting values in the Helmholtz-Rayleigh equation (for spherical resonators with circular wave openings having a five to one ratio of radii of sphere and circular wave opening).

Comparing the calculated value of the frequency of the resonator, with circular wave opening, and the experimental value we have:

Experimental frequency 169.745 vib./sec.

Theoretical frequency 169.405 vib./sec.

A difference of .340 vib./sec.

giving an error of $\frac{.34}{169.405} \times 100$ or .2%.

TABULATED DATA FOR ALL WAVE OPENINGS

Data taken for the frequency for all the other wave openings used were taken under exactly the same conditions as those of the circular opening and are tabulated, with the circular wave opening value, on the following page for comparison.

<u>Shapes of Wave Open- ings</u>	<u>Circle</u>	<u>Square</u>	<u>Elipse</u>	<u>Rec- tangle</u>	<u>Star</u>	<u>Equi- lateral Triangle</u>
Individual trials for resonance peaks *	169.37	173.16	174.56	175.61	176.21	177.16
	169.90	173.76	173.91	175.22	176.51	177.09
	169.49	173.61	174.36	175.07	176.06	177.45
	169.97	173.59	173.90	175.51	176.51	177.01
	169.70	173.65	174.42	175.58	176.55	177.53
	170.33	173.04	174.31	175.03	176.80	177.33
	169.41	173.12	174.26	175.16	176.08	177.54
	169.62	173.41	174.54	175.01	176.47	177.41
	169.83	173.71	174.59	175.64	176.79	177.20
	169.83	173.18	174.68	175.72	176.00	177.05
Mean frequency	169.745	173.423	174.353	175.355	176.398	177.277
Maximum frequency	170.33	173.76	174.68	175.72	176.80	177.54
Minimum frequency	169.37	173.04	173.90	175.01	176.00	177.01
Maximum variation	.96	.72	.78	.71	.80	.53
Maximum deviation from mean	.59	.36	.45	.355	.42	.27
Probable error	.176	.187	.181	.189	.196	.134
Relative conductivi- ty "K"		1.02167	1.02714	1.03305	1.03919	1.04437

*Second decimal place is approximated.

The probable error for the mean values for the resonating frequency for each of the wave openings was calculated by means of the probable error formula:

$$P.E. = .67 \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2}{n - 1}}$$

where $r_1, r_2, r_3, \dots, r_n$ are the residuals obtained by taking the difference of each of the individual trials and mean frequency values for each of the wave openings. "n" stands for the number of determinations for each of the wave openings.

CONCLUSION

It can be seen from the experimental tabulated data that as the shape of the wave opening (circle 1/5 ratio) of constant area is varied from circular to the various geometric forms given, the fundamental frequency of the resonator is slightly increased.

The greatest variation of frequency from that of the circular opening is found in the equilateral triangle. This means according to the Kapstick equation, that the conductivity of the triangular wave opening is greater than that for the other wave openings used.

A "conductivity constant" may be computed from the data for each of the wave openings. Let the conductivity constant of the circular opening be 1.000. The use of these conductivity constants allows the Helmholtz-Rayleigh equation to be used for other wave openings whose area is equal to that of a circular opening having a radius of one-fifth that of the resonator, by using the constant as a factor in the right hand member of the equation.

ACKNOWLEDGMENTS

This research was suggested, directed and approved by Professor E. V. Floyd, of the Kansas State College. To him the writer is very much indebted for his valuable suggestions, sincere interest, and faithful guidance. The writer is also indebted and grateful to Professor Floyd for having been able to use his siren machine and supplementary apparatus.

Acknowledgments are also made to Professor J. O. Hamilton for valuable suggestions and the making of the photographs in this thesis, and to others of the department for suggestions and comments.