

GRAPHING CALCULATOR USE BY HIGH SCHOOL MATHEMATICS TEACHERS OF
WESTERN KANSAS

by

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B.S., Fort Hays State University, 1983
M.A., Fort Hays State University, 1990

AN ABSTRACT OF A DISSERTATION

submitted in partial fulfillment of the requirements for the degree

DOCTOR OF PHILOSOPHY

Department of Curriculum and Instruction
College of Education

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Manhattan, Kansas

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Abstract

Graphing calculators have been used in education since 1986, but there is no consensus as to how, or if, they should be used. The National Council of Teachers of Mathematics and the National Research Council promote their use, and ample research supports the positive benefits of their use, but not all teachers share this view. Also, rural schools face obstacles that may hinder them from implementing technology. The purpose of this study is to determine how graphing calculators are used in mathematics instruction of high schools in western Kansas, a rural region of the state. In addition to exploring the introduction level of graphing calculators, the frequency of their use, and classes in which they are used, this study also investigated the beliefs of high school mathematics teachers as related to teaching mathematics and the use of graphing calculators. Data were collected through surveys, interviews, and observations of classroom teaching. Results indicate that graphing calculators are allowed or required in almost all of the high schools of this region, and almost all teachers have had some experience using them in their classrooms. Student access to graphing calculators depends more on the level of mathematics taken in high school than on the high school attended; graphing calculator calculators are allowed or required more often in higher-level classes than in lower-level classes. Teachers believe that graphing calculators enhance student learning because of the visual representation that the calculators provide, but their teaching styles have not changed much because of graphing calculators. Teachers use graphing calculators as an extension of their existing teaching style. In addition, nearly all of the teachers who were observed and classified as non-rule-based based on their survey utilized primarily rule-based teaching methods.

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Major Professor
Dr. Jennifer Bay-Williams

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Dedication

I would like to dedicate this dissertation to my wife, Pam, and children Jenna, Levi, and Darci for their patience and understanding throughout this process. I also dedicate this paper to my parents, Marvin and Kathleen, for the work ethic and determination that they instilled in me.

CHAPTER 1 - Introduction

Graphing calculators have been used in mathematics education since 1986, but researchers and teachers have not reached a consensus on how to best implement them in instruction (Milou, 1999). A range of opinions exist about when students should first be introduced to graphing calculators, to what extent students should use them in their classes, how students should use them in lessons and homework, and if graphing calculators should be used on tests. These issues parallel those involving calculator use in general, and the discussion of these issues revolves mainly around whether students learn better with calculators or fail to learn necessary skills because they rely on calculators too much.

Ample researched-based support for students' use of graphing calculators in learning mathematics exists, but there are certain factors that limit calculator use. One factor involves teachers' beliefs in how mathematics should be taught and how graphing calculators should be used in learning mathematics. A teacher's beliefs guide his or her instructional decisions and the incorporation of graphing calculators in teaching mathematics (Arvanis, 2003). One can identify teacher beliefs about teaching mathematics by determining the level at which the teacher focuses on procedures, facts, and rules compared to the level of focus on exploring problems, discovering patterns, and making generalizations (Merriweather & Tharp, 1999). In this way a teacher's beliefs can be identified along a continuum from rule-based to non-rule-based instruction. A teacher's use of graphing calculators can also be described according to the amount, level, and type of use. To determine how teacher beliefs and graphing calculator use are related, this researcher will investigate the relationship between the beliefs of teachers in western Kansas in teaching and learning mathematics and the way in which they use graphing calculators in their classes.

In this chapter the researcher will provide an overview of the issues that influence graphing calculator use. Included are support for graphing calculator use, obstacles that limit the use of graphing calculators, prior research on graphing calculators, the problem statement and research questions, delimitations, limitations, and definitions.

Support for the Use of Graphing Calculators

National Council of Teachers of Mathematics' Position on Technology Use

The National Council of Teachers of Mathematics (NCTM) strongly supports the use of technology in education. NCTM's position statement on this issue reads, "Technology is an essential tool for teaching and learning mathematics effectively; it extends the mathematics that can be taught and enhances students' learning" (NCTM, 2003). NCTM recommends that all students have access to technology, that professional development on the use of technology be provided for teachers and pre-service teachers, and that teachers make informed curricular decisions that incorporate technology (NCTM, 2003).

NCTM's Equity Principle emphasizes "high expectations and strong support for all students" (NCTM, 2000, p. 11). Differences in student abilities must be accommodated so that every student can learn mathematics. Technology can help to address the equity issue by providing students who have special needs with support to practice skills and the tools needed to explore complex problems.

NCTM realizes the impact that technology has on education. Not only does technology change the way the mathematics curriculum is taught, technology also changes the content of the mathematics curriculum. "Because of technology, some mathematics becomes less important (like paper-and-pencil arithmetic and symbol-manipulation techniques); some mathematics becomes more important (like discrete mathematics, data analysis, parametric representations, and nonlinear mathematics); some new mathematics becomes possible (like fractal geometry)" (Waits & Demana, 2000, pp. 55-56). The graphing calculator is one type of technology used in education, and its use can impact the mathematics curriculum in several ways.

Impact on Curriculum

Graphing calculators allow students to represent, analyze, and explore functions. Equations that cannot be solved using algebraic methods can be solved with graphing calculators for approximate solutions and sometimes exact solutions. Graphing calculators allow students to graph functions quickly, manipulate the graphs, and develop generalizations about the functions. More time can be spent on analyzing the graphs and less time on the actual development of the graphs. Students build deeper understanding of functions and the graphs of the functions since less time is spent performing calculations. According to Pomerantz (1997), "By reducing the

time that, in the past, was spent on learning and performing tedious paper-and-pencil arithmetic and algebraic algorithms, calculator use today allows students and teachers to spend more time developing mathematical understanding, reasoning, number sense, and applications” (p. 3). In this way, students are forming understanding instead of memorizing processes and algorithms.

Graphing calculators allow students to form linked multiple-representations of mathematical concepts (Waits & Demana, 2000) and to “explore, estimate, and discover graphically and to approach problems from a multirepresentational perspective” (Hollar & Norwood, 1999, p. 222). “As an instructional resource, graphing calculators provide opportunities for students to learn about the connections between algebraic and graphing representations, an important skill in the visualization process” (Smith & Shotsberger, 1997, p. 368). These representations relate tables of values, graphs, and paper-and-pencil algebra so that several methods of solution are possible for any given problem. The student can choose the best method to solve the problem.

Supporting Research

An ample body of research supports the use of graphing calculators in mathematics education. One area of research focuses on student understanding and achievement when using graphing calculators, and another area focuses on the effect of graphing calculators on student attitudes.

Several researchers have identified positive effects of graphing calculators on student understanding and achievement. The Third International Mathematics and Science Study (TIMSS) provided evidence that students who were allowed daily use of calculators performed considerably better on the TIMSS tests than those students who rarely used calculators (The International Study Center, 1998). Hollar and Norwood (1999) found that students in graphing-approach classes demonstrated better understanding of functions than students in traditionally taught classes. They have higher graphical understanding of functions and a better understanding among graphical, numerical, and algebraic representations (Dunham & Dick, 1994). Students are able to visualize concepts more easily when using a graphing calculator (Smith & Shotsberger, 1997).

In addition to their positive effect on student understanding, graphing calculators have no negative effect on the learning of basic skills. Students who learn basic skills with technology do

no worse than students who learn basic skills without technology (Dunham, 1999; Hembree & Dessart, 1992; Hollar & Norwood, 1999; Stick, 1997).

Graphing calculator use has a positive effect on students' problem-solving abilities. Students work longer on solving problems when they use calculators (Dunham & Dick, 1994; Wheatley & Shumway, 1992), they improve their problem solving abilities (Dick, 1992; Hembree & Dessart, 1992), and they are able to solve problems that they were not able to solve before (Merriweather & Tharp, 1999).

Another area of research reports on the positive effects of graphing calculators on student attitudes. Student confidence improves (Smith & Shotsberger, 1997), student interest in mathematics increases (Waits & Demana, 1994), and student attitude improves when using graphing calculators (Hembree & Dessart, 1992; Hollar & Norwood, 1999).

Additional Issues That Affect Graphing Calculator Use

While NCTM strongly supports the use of technology, and solid research evidence exists that graphing calculators have positive effects on student understanding, achievement, and attitudes, graphing calculators are not available to all high school mathematics students because competing issues, beliefs, and arguments influence the use of graphing calculators in mathematics instruction. These issues involve teachers' beliefs that graphing calculator use will prevent the students' development of basic skills, problems with the implementation of graphing calculator use, and equity issues that arise when some students have access to graphing calculators and others do not.

Basic Skills

Opponents of NCTM's stance on calculator use argue that students will not learn basic skills or understand what they are doing unless they work problems with paper and pencil (Smith & Shotsberger, 1997; Pomerantz, 1997). Relating this to the graphing calculator, they claim that students will not learn to graph a function properly because they will depend on the calculator to do it for them. Others believe that the only proper use of calculators is to check answers once problems have been worked on paper, and students must master the paper-and-pencil procedures before being allowed to use a calculator (Yoder, 2000; Milou, 1999).

In these arguments the calculator is described as a tool for graphing and computation rather than an instrument for conceptual development. These arguments are supported by Smith

(1996) who found no significant difference in achievement in graphing functions and a negative effect on achievement in graphing functions at the twelfth grade level when using graphing calculators to learn to graph functions. Results from a study by Ritz (1999) reveal that the control group outperformed the experimental group that used graphing calculators to learn algebra and geometry concepts. A possible explanation was that the students in the experimental group had to learn to use the graphing calculator in addition to learning the content.

Implementation

Deciding how to incorporate graphing calculators in the classroom is another issue affecting graphing calculator use. A teacher may believe that not enough time exists to teach the use of technology in addition to mathematical concepts. In the current era of No Child Left Behind with its increased emphasis on test scores, teachers may be less willing to teach concepts and processes that are not directly assessed in standardized tests. Moreover, teachers may not feel knowledgeable about the graphing calculator and may not be able to justify the time needed to learn how to use it.

Teachers may not want to incorporate teaching methods that work well with the graphing calculator but are unfamiliar to them. A teaching method that works well with the graphing calculator is the discovery approach. Using the graphing calculator in a discovery approach shifts the role of the teacher from lecturer to facilitator. This role may be uncomfortable for teachers who have a teacher-centered approach to teaching (Simonsen & Dick, 1997). The teacher retains control of the direction of the lesson when using a teacher-centered approach, and the teacher can be prepared for many of the questions and issues that arise during the class. A discovery approach, with its many possible directions of thought by the students, increases the chance that the teacher will be unprepared for questions asked. Even though valuable learning experiences can arise out of situations that have not been planned by teachers, they may not be comfortable with the unexpected turns that a problem may take. The discovery approach removes a teacher from the comfort of teaching one discrete topic at a time, and it requires that the teacher develop a greater depth of mathematical understanding, flexibility in thinking, and the ability to address unexpected questions. The teacher must be able to diagnose technical and mathematical problems, facilitate group work, and adapt to the different paths taken by students as they solve problems (Heid, 1997).

Equity Issues

Another issue that affects graphing calculator use is equity. Although calculators may improve equity among students by enabling students who are weak in computation to solve difficult problems, calculators may also contribute to inequity. Not every student can afford to buy a graphing calculator, and not all schools can afford to supply an adequate number of calculators for their students (Simonsen & Dick, 1997). This can lead to inequity in which some students have access to technology while others do not. It can also lead to inequity across schools since some schools are more technologically advanced than other schools. Students from schools that lack technology will not be as well prepared for higher education and the workforce as their counterparts from technologically oriented schools. The isolation of rural schools may contribute to the inequity between schools since rural teachers may not be able to attend as many calculator workshops as urban and suburban teachers. Also, rural teachers may not have other mathematics teachers with whom they can share calculator knowledge and ideas (Arvanis, 2003).

Issues that affect graphing calculator use include teachers' belief that students' basic skills will suffer, teachers' unwillingness or inability to incorporate technology in teaching, and inequity caused by requiring technology that may not be affordable by every student. Not everyone is convinced by research supporting graphing calculator use that the benefits are worth the cost in time, money, and energy. As a result, a range of levels of graphing calculator use exist across and within high schools.

Statement of Problem

Despite support by NCTM for calculator use in mathematics education, and despite evidence supporting the use of graphing calculators, parents, teachers, and administrators do not yet agree on the role of graphing calculators in mathematics education (Dunham, 1999). As a result, students enter college with a wide range of prior knowledge and experience with graphing calculators. Some students have a solid background while others have very little or no prior experience. According to the National Center for Education Statistics (NCES), 68% of twelfth grade students reported that they use calculators almost every day on class work, 14% use calculators a few times a week, 3% use calculators a few times a month, and 14% never or hardly ever use calculators (NCES, 2005). These data do not specifically target how students use graphing calculators, but one can infer from the data that graphing calculators are used daily by

no more than 68% of the twelfth grade students and never or hardly ever used by at least 14% of the twelfth grade students. These data reveal the potential roots of the wide range of differences in college bound students' knowledge of and experience with graphing calculators.

Many factors contribute to the use, or the lack of use, of graphing calculators at the high school level. In this study, the investigator will identify the factors affecting graphing calculator use in high schools of western Kansas. The results of this study will inform the professional development needs of teachers, which can in turn lead to increased achievement by their students.

Goal of Study

The goal of this study is to describe how graphing calculators are used in high schools of western Kansas and to determine the relationship between teachers' beliefs in teaching and learning mathematics and the way in which graphing calculators are used in their classes. The researcher investigated the frequency of graphing calculator use by the students, the ways in which graphing calculators are used to learn mathematics, and the grade and mathematical areas in which they are used. The researcher also investigated teachers' beliefs and how they are related to graphing calculator use.

Research Questions

This investigator gathered information to answer the following question: What is the relationship between teachers' backgrounds and beliefs and the way in which they implement graphing calculators in mathematics instruction in high schools in western Kansas? To answer this overall question, the following specific research questions were put forth:

1. What are characteristics of mathematics teachers in western Kansas, including their level of education, teaching experience, preparation in the use of graphing calculators, and experience with graphing calculators?
2. In what mathematics courses do students use graphing calculators, how often do they use them, what calculator functions do they use, and to what extent are graphing calculators being used to learn mathematical concepts?
3. What beliefs do teachers have regarding teaching and learning mathematics?
4. What beliefs do teachers have regarding the use of graphing calculators?

5. What is the relationship between teachers' beliefs about the teaching and learning of mathematics and how graphing calculators are used?

To answer the first question, the researcher gathered information about mathematics teachers of western Kansas. Teacher responses to a survey revealed their backgrounds and experiences as mathematics teachers in this geographical area.

Answering the second question, the researcher gained understanding about the extent to which graphing calculators are used at the high school level. In addition to identifying the level at which students are first introduced to graphing calculators and the types of classes in which students are allowed to use graphing calculators, a major focus of this question was on the specific ways in which graphing calculators are used. Information was collected on whether the students use the graphing calculator only for calculations and graphing or if they use graphing calculators to explore concepts. Information was also collected as to whether the students are only learning to use the calculator to perform mathematical tasks or if they are using the calculator to learn mathematical concepts.

Answers to the third question provided insights about teachers' beliefs as to how mathematics should be taught. Information was collected on whether a teacher believes that mathematics is best learned through the memorization of rules, facts, and procedures or whether a teacher believes that mathematics should be learned through exploring problems, discovering patterns, and making generalizations.

Answers to the fourth question provided insights about teachers' beliefs about how graphing calculators should be used in the learning of mathematics. Information was collected on whether a teacher believes that a graphing calculator should be used as a tool to learn more mathematics or if it should be used only as a way to make mathematics processes easier.

Answers to the fifth question combined findings of the third and fourth questions when comparing individual teacher responses to those questions. The relationship between a teacher's belief in the way in which mathematics is best learned and the way in which graphing calculators are used in the classroom were investigated. The goal of this comparison was to determine if a relationship exists between the level of rule-based beliefs by teachers and the level and type of graphing calculator use in their teaching of mathematics.

Method

Two hundred fifty-three mathematics teachers from 90 high schools in western Kansas were asked to participate in the study, and 157 participated. The high schools were chosen because the majority of students who attend the researcher's university are graduates of these high schools, and this area of Kansas is regarded as the university's service area for teacher preparation and professional development.

The schools are located in a rural setting; school sizes range from 10 students to nearly 2,000 students. The median school size is 123 students, and half of the schools have student populations between 75 and 183 students. Only four of the cities involved have populations over 20,000, and only four high schools have over 1,000 students. Seventy-two percent of the students are Caucasian; 23% are Hispanic; 2% are African American; and 2% are Asian. Thirty-eight percent of the students are from low socio-economic status families (KSDE, 2006).

A survey instrument was used to gather information about the teachers, their schools, the use of graphing calculators at the schools, obstacles to graphing calculator use, teachers' beliefs of how students should use graphing calculators, and teachers' beliefs on the effect that graphing calculators have on student learning. In addition, stratified purposeful sampling was used to choose nine teachers as subjects of case studies. Responses to certain questions on the survey were used to determine the level of rule-based teaching beliefs held by the respondents. Three teachers were chosen whose beliefs indicated a rule-based approach to teaching, and six teachers were chosen whose beliefs indicated a non-rule-based approach to teaching. The teachers were interviewed by the researcher, and their responses were recorded on cassette tape. Six teachers were observed teaching three lessons so that additional information could be gathered about the ways in which graphing calculators are used in their teaching. Three teachers were chosen whose beliefs indicated a rule-based approach to teaching, and three teachers were chosen whose beliefs indicated a non-rule-based approach to teaching.

Qualitative survey data were analyzed descriptively to determine characteristics of mathematics teachers in western Kansas, the classes that they teach, the classes in which graphing calculators are used, and the frequency and ways in which graphing calculators are used. Quantitative survey data were analyzed descriptively to determine how graphing calculators are used in mathematics classrooms of western Kansas. Also, comparisons were

made between graphing calculator use and teaching experience, educational training, graphing calculator training, and teachers' beliefs in teaching mathematics.

Qualitative interview and observational data were collected, analyzed, and reported following Creswell's (1998) recommendations for qualitative research. The qualitative approach provided insight as to how teacher beliefs influence decisions regarding graphing calculator use.

Significance of Study

Many studies have been reported on a variety of issues regarding graphing calculator use, but additional research-based information about graphing calculator use is needed. Dunham and Dick (1994) claim that prior studies have investigated teaching practices in high school mathematics classrooms, and they suggest that future studies should investigate why graphing calculators are used. Dunham (1999) recommends that researchers investigate how, how often, and when calculators are used by students. Simmt (1997) suggests that researchers investigate teachers' views of technology and mathematics and how those views influence mathematics instruction. Tharp, Fitzsimmons, and Ayers (1997) ask for more research regarding teacher beliefs toward implementing graphing calculators in instruction and teacher change in instruction toward a discovery approach. This researcher intended to respond to the needs identified above. In addition to collecting information about practices in high school mathematics classes, the researcher gathered information via a survey and interviews to understand why mathematics teachers are or are not using graphing calculators.

This study is significant in identifying professional development needs of pre-service and in-service teachers. Simonsen and Dick (1997) identify the need to design curriculum materials, in-service programs, and support systems for teachers based on feedback gathered from the teachers. Dildine (1997) suggests more research that explores pre-service and continued professional development of teachers' use of technology in teaching mathematics. Because this investigator has examined the extent of graphing calculator use as it relates to teachers' backgrounds and beliefs, the results provided insights into the professional development needs of high school mathematics teachers. Information on how teacher beliefs relate to graphing calculator use should help to determine the specific focus of professional development. In particular, this study provides significant guidance for the researcher and the researcher's university. As an institution that prepares pre-service teachers and provides professional

development to current teachers, the results of this study help to determine the type, amount, and level of graphing calculator training needed in future workshops. These results also inform about better preparation of pre-service teachers in using graphing calculators.

Delimitations of the Study

This study was bounded by the following delimitations:

1. The study involved schools in rural communities of western Kansas where the student population is predominantly White, middle class.
2. The participants are high school mathematics teachers.
3. The only technology considered was the graphing calculator.
4. Interviews and observations were used to document the link between teachers' beliefs and their practices.

Limitations of the Study

As a consequence of the delimitations of the study, there were corresponding limitations. One limitation involved the number of teachers who actually completed and returned the survey, thereby precluding accurate data as to how most teachers and schools are using graphing calculators. It is possible that teachers who do not use graphing calculators in their classes do not hold opinions as strong as those held by frequent graphing calculator users and subsequently did not feel as obligated to complete and return the survey.

The case study design limited the quantity of teachers that were used in interviews and observations, so the information gathered from the interviews and observations was only used to generate themes from each case and across cases. The number of teachers interviewed may not have been sufficient to adequately represent the range of opinions that may have been evident from the surveys. Since interviewees were chosen from survey respondents, care was taken to include a combination of beliefs and experiences, but variety in beliefs and experiences may have been influenced by teachers declining to be interviewed. Interpretation of the interviews could have been influenced by the number of teachers declining to be interviewed and the characteristics of the teachers declining to be interviewed. Teachers who agreed to be interviewed may hold stronger beliefs than those who did not agree to an interview giving the impression that teachers in general hold stronger beliefs than they actually hold.

Care was taken to prevent the teachers interviewed from knowing researcher biases prior to the interviews, and teachers who were initially invited to participate in the interviews had no prior classes or workshops that would have revealed the researcher's biases; however, after initial invitations yielded too few interview participants, two teachers who had participated in classes that discussed graphing calculator use and were taught by the researcher prior to the study were invited to participate. It is uncertain how this affected their interview responses.

Definition of Terms

The term *graphing calculator* is defined to be any hand-held calculator that has the ability to graph a function on its screen.

The term *low socio-economic status* is defined as students who receive free or reduced lunches. This is consistent with the state of Kansas identification category of low socio-economic status in school reports.

The term *western Kansas* as used in this study is defined to be the area contained in the 46 Kansas counties to the west of the eastern boundaries of the counties stretching from Smith County to Barber County.

The term *high school* is defined as a school consisting of grades 9 through 12. Many of the smaller schools in western Kansas combine grades 7 and 8 with the high school, and the high school mathematics teacher teaches these students. Some of these schools include grades 7 and 8 when reporting data to the Kansas State Department of Education, and others include data from grades 7 and 8 with the elementary school data. The researcher included mathematics teachers who teach students in grades 9 through 12 even if they also teach students in grades 7 and 8. The survey questions were to be answered in reference to classes taught only to students in grades 9 through 12.

The term *rule-based* is defined to be the belief and/or practice of teaching mathematics as a set of rules, facts, and procedures (Merriweather & Tharp, 1999).

The term *non-rule-based* is defined to be the belief and/or practice of teaching mathematics through exploring problems, discovering patterns, and making generalizations (Merriweather & Tharp, 1999).

CHAPTER 2 - Literature Review

The Role of Graphing Calculators in Mathematics Education

The purpose of this study is to describe how graphing calculators are used in high schools of western Kansas and to determine the relationship between teachers' beliefs in teaching and learning mathematics and the way in which graphing calculators are used in their classes. Several factors affect the way in which graphing calculators are used. These factors include access to technology, issues with implementing technology in teaching, teachers' beliefs of teaching mathematics, and teachers' beliefs of the use of graphing calculators. Reformers of mathematics education encourage the use of technology in teaching and learning of mathematics, but not all teachers agree with the reform movement. There is ample research that shows that student achievement improves and attitudes improve with the use of graphing calculators, but some teachers fear that student computational skills will diminish. Because of factors that affect graphing calculator use, beliefs in how mathematics should be taught and how graphing calculators should be used, and issues with implementing technology in the classroom, all students do not have the same access to graphing calculators.

There is a wide range of perspectives among educators regarding calculator use in the mathematics classroom. For example, there are teachers who believe that calculators are a crutch for students and prevent students from learning basic mathematics. They discourage the use of calculators until basic skills and routines are mastered. There are also teachers who believe that calculators should be available at all times for all students. These teachers believe that calculators enable students to learn mathematics more deeply. It is unknown, however, how many high school mathematics teachers fit into either of these belief types or where they fit along a continuum between the two types. In addition to investigating the relationship between teacher beliefs and graphing calculator use, this study investigated factors influencing graphing calculator use that may be particularly challenging to teachers in rural communities.

This chapter discusses the literature pertaining to graphing calculator use. The chapter begins with mathematics reform as it relates to technology use, providing a rationale for incorporating graphing calculators in the classroom. Appropriate calculator use and specific classroom uses are discussed as well as research on the impact of graphing calculators on student

understanding and attitudes. The chapter ends with a discussion of the factors that influence calculator use such as access to technology, learning issues, and teachers' beliefs of mathematics and graphing calculator use.

Mathematics Reform and Technology Use

The current mathematics reform movement began in the 1980s when the mathematics community recognized the need for students to develop problem solving skills in addition to strengthening basic skills. Publications such as *An Agenda for Action* (NCTM, 1980) and *A Nation at Risk* (National Commission on Excellence in Education, 1983) called for mathematics teachers to place an increased emphasis on the teaching of problem solving. With the input from mathematics educators, mathematicians, and mathematics teachers from all levels of education, NCTM published *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) that outlined the goals of mathematics reform. According to these goals, students in a standards-based classroom are to learn to value mathematics, gain confidence in their mathematical ability, become problem solvers, and learn to communicate and reason mathematically.

NCTM reiterated and clarified these goals 11 years later in *Principles and Standards for School Mathematics* by identifying six principles (equity, curriculum, teaching, learning, assessment, and technology), five content standards (number and operations, algebra, geometry, measurement, and data analysis and probability), and five process standards (problem solving, reasoning and proof, communication, connections, and representation) to help students realize these goals (NCTM, 2000). The principles reflect ideas that promote a high-quality mathematics education, the content standards identify the content that should be present throughout the curriculum, and the process standards provide ways to learn and use the content knowledge.

Classrooms that incorporate the curriculum and teaching methods outlined first in the *Curriculum and Evaluation Standards* and later in the *Principles and Standards* are referred to as standards-based classrooms. These classrooms are structured to be student-centered with students engaged in real mathematical activity. The focus of the classroom is on the students' actions and not on the teacher's actions. A teacher must be able to create a setting conducive to learning by selecting appropriate instructional materials, tools, and techniques that help the students to actively participate in the learning process. Students are encouraged to make and

verify conjectures and to communicate mathematical ideas to others. Students are responsible for their learning and are given ample opportunities for reflection (Heid, 1997).

Changes in the mathematics curriculum and the pedagogy needed to teach the curriculum are necessary for mathematics classrooms to become student-centered. To guide these changes the National Research Council (National Research Council, 1990) proposes a shift from a dualistic mission of tracking students to different levels of mathematics to a singular mission of mathematics for all students, from a teacher-centered model to a student-centered model of teaching, from learning routines to gaining mathematical power, from emphasis of mathematics for future courses to emphasis of mathematical topics relevant to students' current and future needs, and from emphasis on paper-and-pencil manipulations to full implementation of calculators and computers.

A major part of the mathematics reform movement is the increased use of technology in the mathematics classroom. "The single most important catalyst for today's mathematics education reform movement is the continuing exponential growth in personal access to powerful computing technology" (Heid, 1997, p. 5). NCTM addresses the use of technology in mathematics education in the Technology Principle. The Technology Principle provides information on how to best implement technology in mathematics education. The Technology Principle states, "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances student learning" (NCTM, 2000, p. 24). The use of technology should not replace basic understanding of mathematical concepts, but technology should be used so that students learn more mathematics at a deeper level than can be taught without technology. Technology provides visual images of concepts, enables students to generate and explore more examples than can be generated by hand, allows for the solution of more complicated problems, and provides options for teachers to adapt instruction to students with special needs. NCTM warns, however, that the use of technology does not necessarily improve instruction. Teachers must develop lessons that help students to attain conceptual understanding. Mathematical content must change to take advantage of technology, and less emphasis should be placed on certain procedures and skills that were once necessary but now obsolete because of technology.

Some types of technology have been widely accepted in education. Computers, for example, have been regarded as valuable educational tools ever since they were first used in

education. Word processing, spreadsheet and data base applications, tutorials, drill-and-practice programs, applets, graphing software, the Internet, and presentation software lend themselves well to educational uses. However, because of the high cost of computers and computer software, the time needed for teachers to learn to use software, and the unavailability of computer labs to students, teachers and students do not use computers as fully as they should. In order to increase the availability of technology in the classroom, graphing calculators can be used as a relatively inexpensive alternative to computers (Demana & Waits, 1992).

Graphing calculators possess features similar to those in computers, such as computer processors, display screens, built-in software, and programming capabilities. Because they are portable, easy to use, and relatively inexpensive, graphing calculators can provide students with better access to technology at all times (Demana & Waits, 1992). The large screen display allows a student to view the keystrokes made in a calculation, and the student can view multiple calculations at the same time (Embse, 1992). The student is provided with immediate feedback of an answer, can recognize that the answer is not correct, can see the keystrokes on the large screen, and can make the corrections needed (Merriweather & Tharp, 1999). Basically, the graphing calculator is a computer that has specific mathematical functions.

For significant changes in mathematics to occur, calculators must be used on a regular basis for in-class work and for homework outside of class (Demana & Waits, 1992), they must be used in testing as well as instruction (Hembree & Dessart, 1992), and they must be integrated into the curriculum so that they play a central role in the learning process (Hembree & Dessart, 1992). Jones (1995) advises that we must distinguish between the two ways in which graphing calculators have an effect on student achievement. One way is a change in performance *with* technology in which the student shows improvement only when using the technology, and the other way is a change in performance *of* technology in which the student continues to show improvement without the calculator. Students must build understanding so that they can show improvement *of* technology and not just *with* technology.

In addition to the recommendations made by NCTM, the Panel on Educational Technology has recommendations for the use of technology in education. The Panel on Educational Technology was organized in 1995 as part of the President's Committee of Advisors on Science and Technology (PCAST) to provide independent advice to the President on technology use in K-12 education (President's Committee of Advisors on Science and

Technology, 1997). The panel's recommendations are based on a review of literature from education, industry, and professional organizations involved with technology in education. The panel's recommendations include that schools should focus on learning *with* technology, not *about* technology; emphasize content and pedagogy, not just hardware; provide professional development and adequate financial resources; ensure equitable, universal access; and initiate experimental research (PCAST, 1997).

Supporters of mathematics reform stress that technology should be incorporated into teaching mathematics at all grade levels, but technology will not necessarily improve education unless it is used to learn mathematical concepts and not be used to replace mathematical understanding. Students must learn how to use technology, but more importantly, they must learn mathematics more deeply through the use of technology. The graphing calculator is one type of technology that is supported by the mathematics reform movement, and one area of discussion involves the appropriate use of graphing calculators in mathematics instruction.

Appropriate Use

Much of the disagreement concerning the use of calculators in education centers on how the calculators are to be used. At the heart of the disagreement is the topic of appropriate use of calculators and the importance of paper-and-pencil skills and mental arithmetic. Since graphing calculators are used mainly in algebra classes, opponents of graphing calculators fear that students will lose the ability to perform algebraic processes, but proponents of graphing calculators stress that algebraic thinking is more important than algebraic processes. Regardless of one's stance on the use of graphing calculators, it is important to consider the potential that graphing calculators have in learning mathematics and ways in which graphing calculators can be used appropriately.

There are varying opinions on what constitutes appropriate use of graphing calculators, and these opinions range from complete use of graphing calculators to never using them. Waits and Demana (1998) define appropriate use of graphing calculators in teaching and learning of mathematics to mean that the student:

1. Solves analytically using traditional paper and pencil algebraic methods, and supports the results using a graphing calculator.

2. Solves using a graphing calculator, and then confirms analytically the result using traditional paper and pencil algebraic methods.
3. Solves using graphing calculator when appropriate (because traditional analytic paper and pencil methods are too tedious and/or time consuming or there is simply no other way!) (p. 73).

Dessart, DeRidder, and Ellington (1999) recommend that calculators be used at all levels of mathematics education including computational skill development. They stress that teachers should help students to develop the ability to correctly decide when to use mental arithmetic, paper and pencil arithmetic, or a calculator. “We would chastise any student who reaches for the calculator to find 3×4 ; we would suggest pencil and paper for calculating 27×340 ; and we would insist on using the calculator for 2.7568×345.8972 after the student estimates mentally an answer of 900 (3×300)” (Dessart et al., 1999, p. 6). Through proper training and adequate experience using calculators, students should be able to decide what the best method of solution is before automatically using a calculator (Lott, 1999). To gain that experience, Montoya and Graber (1999) believe that the key to teaching students appropriate calculator use is to allow students access to calculators at all times. “Our position is: Let the teacher explain. Let the student think. Let the computer do mindless work” (Baggett & Ehrenfeucht, 1992, p. 61). Dick (1992) insists that people do mathematics; calculators do not. “Understanding comes from asking the right questions, making precise definitions, setting up problems, and interpreting the results. One needs to know what is involved in applying mathematics, and how to interpret results” (Baggett & Ehrenfeucht, 1992, p. 71).

Kutzler (2000) discusses appropriate use of calculators by comparing the use of mental mathematics, paper-and-pencil techniques, and calculator use to walking, riding a bicycle, and driving a car, respectively. Just as a person should choose which mode of transportation is the most sensible for a given situation, a student must choose the most sensible method of calculation in mathematics. Even though some people misuse their car by driving extremely short distances, cars should not be abolished. In the same way, misuse of calculators should not be reason for the elimination of calculators from education. Eliminating cars would limit the distance that a person would be able to travel, and eliminating calculators would limit the amount and level of mathematics that a student would be able to do. Also, just as a wheelchair can help a person with a disability to move from one place to another, a calculator can help a person with a

mathematical weakness to do higher-level thinking despite the weakness. “If technology is used properly, it leads to more efficient teaching and learning, more independent productive student activity, more student creativity, and an increased importance of the teacher” (Kutzler, 2000, p. 11).

Increased calculator use does not decrease the importance of mental mathematics. Graphing calculators produce answers and graphs very quickly, and without adequate number sense, mental mathematics skills, and estimation skills a student may not realize that the calculator is displaying an incorrect result when key stroking errors occur. Strong ability to mentally check calculator results is important to the proper use of graphing calculators.

Just as mental mathematics is important when using calculators, paper-and-pencil skills continue to be important since they promote student confidence and allow for appropriate calculator use (Harvey, Waits, & Demana, 1995). However, the role of paper-and-pencil mathematics will diminish as technology causes some paper-and-pencil algorithms to become obsolete. Algorithms that continue to be taught are those that are useful to mathematics (Usiskin, 1998). Regardless of technology use, the learning of an algorithm does not necessarily equate to the learning of the concept, and learning may actually be hindered by practicing an algorithm that is not understood (Burrill, 1992). Algebraic thinking must be taught so that “the students gain a deeper understanding of the role of algebra as a language that links technology to problems” (Brunner, Coskey, & Sheehan, 1998, p. 238). The student will have a better understanding of why a procedure is used, why it works, and when to use it. In this way, a student is forming conceptual understanding instead of merely memorizing processes and algorithms.

These descriptions of appropriate use include a combination of mental, paper-and-pencil, and calculator methods, and they are consistent with NCTM’s Position Statement on Computation, Calculators, and Common Sense:

School mathematics programs should provide students with a range of knowledge, skills, and tools. Students need an understanding of number and operations, including the use of computational procedures, estimation, mental mathematics, and the appropriate use of the calculator. A balanced mathematics program develops students’ confidence and understanding of when and how to use these skills and tools. Students need to develop

their basic mathematical understandings to solve problems both in and out of school.
(NCTM, 2005)

Even though graphing calculators can replace procedures that have traditionally been done mentally or on paper, a balance of the three methods is needed so that mathematical thinking is not replaced by mere button-pushing. The method used depends upon the situation, and the decision of which method to use is made by the student.

Uses of Graphing Calculators

Graphing calculators can be used to enhance conceptual understanding and algebraic thinking. Students can analyze the graphs of functions quickly, represent functions in different ways, and solve problems through alternative approaches when using graphing calculators. Students can explore the graphs of functions and “discover” properties of functions. Incorporating graphing calculators in mathematics instruction necessitates a change in the way in which students are assessed.

Analyzing Graphs in a Dynamic Environment

A graphing calculator allows a student to analyze the graph of a function by enabling the student to generate the graphs of several similar functions quickly. This allows the student to spend more time analyzing the graph and less time performing calculations and developing the graph. This is similar to the way in which mathematicians from earlier times used groups of humans to do their calculations for them. With the help of these “human calculators” enough examples were generated so that the mathematicians could recognize patterns and make conjectures without doing all of the calculations themselves (Kutzler, 2000). Electronic calculators can be used in the same way for students to explore, conjecture, and verify mathematical concepts without having to spend an excessive amount of time performing repeated calculations.

The graphs of functions can be analyzed in a dynamic environment. Paper-and-pencil techniques are static, and multiple instances of the function must be presented spatially to show variation. Static techniques may not provide enough examples to explore, or because of calculation errors, they may provide incorrect examples that prevent the student from recognizing patterns (Kutzler, 2000). In comparison, the student may more easily notice properties that remain the same and properties that change in a function when working with a

dynamic medium than when working with a static medium (Kaput, 1992). “The graphics calculator is ideal for visually illustrating the changing behavior of magnitudes in their mutual relation and, therefore, encourages a dynamic manner of observing analytical models” (Drijvers & Doorman, 1996, p. 430).

Multiple Representations of Functions

Another benefit to using graphing calculators is to help the student make connections between algebraic and graphical representations (Smith & Shotsberger, 1997). These representations form a linked multiple-representation of tables of values, graphs, and paper-and-pencil procedures so that several methods of solution are possible for any given problem (Waits & Demana, 2000) and connections can be made between areas of mathematics such as algebra and geometry (Embse, 1992). A student who makes connections between mathematical ideas creates a deeper understanding of those ideas, and different representations of a problem allow a student to represent the problem in a way that best makes sense to the student (NCTM, 2000).

Graphing calculators provide students with a graph that can be used to make sense of the function’s characteristics before calculations are performed. The graph can aid in the understanding of properties and characteristics of the function such as the roots, extrema, and asymptotes. Paper-and-pencil methods and graphing calculator methods should support each other, providing the student with instant feedback. Graphing calculators can also be used to solve for approximate solutions, and possibly exact solutions, to equations that cannot be solved using algebraic methods.

Alternative Approaches to Problem Solving

Graphing calculators allow a variety of approaches to solving problems, and they can be used to show realistic applications to students before the students learn the computational techniques for solving equations of that type (Barrett & Goebel, 1990). Realistic applications include problems that are likely to be encountered by students and problems in which a variety of solutions are possible. Problems such as this place the concept in context and provide rationale for the student to learn the concept. Graphing calculators allow other methods of solution such as solving by graphing or analyzing tables. Students are active in the solution process and use informal strategies, reflect on what they have learned, and generalize and formalize their findings (Drijvers & Doorman, 1996).

There are several ways in which graphing calculators can be used to enhance student understanding. The graphing calculator can aid in analyzing the graphs of functions in a dynamic environment, it can be used to represent a function in different ways, and it can allow for multiple solutions to realistic problems. These graphing calculator uses work well within a discovery learning environment.

Discovery Learning

Graphing calculators fit well with the idea of discovery learning in which a student is allowed to explore relationships, estimate results, and discover mathematical concepts graphically (Hollar & Norwood, 1999). Graphing calculators help students to become actively involved in problem solving, to talk and read about mathematics, and to make generalizations (Waits & Demana, 1998). Instead of learning through direct instruction, a student makes generalizations and constructs knowledge in a way that makes sense to the student. “In the constructivist perspective, one agrees that mathematical knowledge is constructed, at least in part, through a process of reflective abstraction, and that cognitive structures are under continual development” (Lauten, Graham, & Ferrini-Mundy, 1994, p. 227). A graphing calculator provides the student with a tool to explore, generalize, and verify mathematical concepts.

The use of graphing calculators is consistent with teaching practices grounded in constructivism, the belief that a student constructs knowledge on previous knowledge and experiences. A student constructs knowledge in a way unique to that student (von Glasersfeld, 1995). When used in a cooperative learning setting, students construct knowledge socially through interactions with other students (Gergen, 1995). Through the use of a graphing calculator, a student is allowed to “discover” relationships and make generalizations instead of learning about relationships through direct instruction.

It is worth noting that technology in its best pedagogical use encourages discovery learning.... It can help to guide each individual student to the “aha!” of discovery, guided by the inductive knowledge of the student’s pattern of thought and individual strengths and instincts (Pollak, 1986, pp. 350-351).

Problems can be represented in ways that are meaningful to each student and solved in ways understood by each student, and knowledge is constructed in a way that makes sense to each student. Constructivist learning theory underlies various instructional practices associated with graphing calculator use: teaching for understanding, teaching through multiple-representations,

discovery learning, and cooperative learning. These instructional practices are described as effective in the teaching and learning of mathematics in the NCTM process standards of communication, connections, representation, and problem solving (NCTM, 2000).

Discovery learning and cooperative learning can aid in the social construction of meaning in a classroom. Students are encouraged to discuss concepts, make conjectures, and verify their findings using the graphing calculator. Because of the multiple-representations provided by the graphing calculator, students working together may more fully discuss the relationships within a problem (Farrell, 1996).

Graphing Calculators in Assessment

One aspect of implementing graphing calculators into the classroom is adapting assessment to align with changes in instruction. "Not only has hand-held graphing technology changed what we teach and how we teach, it has also changed how we test our students" (Laughbaum, 1998, 184). By using calculators in assessment, teachers can more easily measure student growth in conceptual understanding and problem solving ability through the use of problems that are more open-ended and non-routine (Branca, Breedlove, & King, 1992).

Allowing graphing calculators on tests requires teachers to think carefully about the types of questions that can be asked. Beckman, Thompson, and Senk (1999) identify three levels of calculator problems in assessment. A problem may be calculator inactive in which there is no advantage and possibly a disadvantage in using a calculator. A problem may be calculator neutral in which it can be solved without the calculator, but the calculator may be helpful in solving the problem. A problem may be calculator active and require the use of a calculator. In their study, nine algebra II or post-algebra II courses that used technology in instruction were surveyed. It was found that up to 16% of the test items were graphing calculator active, and 67 to 82% were calculator inactive. This showed that technology use in instruction does not mean that technology is necessarily incorporated in testing. Beckman, et al. recommend a balance of graphing calculator active, graphing calculator inactive, and graphing calculator neutral questions on tests.

As another way of identifying test objectives as they relate to calculator use, Hopkins (1992) divided test objectives into two categories, calculator-specific objectives and mathematics objectives. Calculator-specific problems test the student's ability to manipulate the calculator. Mathematics objectives encourage students to explore number patterns, solve problems through

the use of guess-and-check strategies, formulate and verify hypotheses, and solve problems that involve realistic data. To clarify their position of testing students with the use of calculators, the College Board and the Mathematical Association of America recommend that mathematics achievement tests contain no questions that only test calculator skills. The Connecticut State Board of Education, the first state board to sanction the use of calculators on a standardized test, recommends a balance of calculator items and traditional computational skill items on the state-mandated mathematics mastery test (Leinwand, 1992).

The ability for students to test with graphing calculators has become increasingly important. There has been a trend in the last thirteen years to allow calculators on standardized tests. The Preliminary Scholastic Aptitude Test/National Merit Scholarship Qualifying Test (PSAT/NMSQT) has allowed a scientific or graphing calculator to be used since 1993. The Scholastic Aptitude Test (SAT) has allowed the use of calculators in testing since 1994 and has required the use of a scientific or graphing calculator since 2000 (Dion et al., 2001). Students who do not know how to properly use graphing calculators during assessment may be at a disadvantage when taking these types of tests.

Graphing calculators can be used to analyze functions dynamically. Properties and characteristics can be observed before calculations are performed, and immediate feedback for paper-and-pencil mathematics can be provided. Functions can be represented algebraically and graphically, and tables of values for the function can be generated quickly allowing for the “discovery” of properties and relationships. To assess student understanding, teachers must develop ways in which they can assess the students’ understanding of the concepts and not just their ability to use the calculator.

Impact of Graphing Calculator Use on Students

The uses discussed above provide ideas on how graphing calculators can be implemented in the classroom, but not everyone agrees that students benefit from using graphing calculators in these ways. One viewpoint is that graphing calculators help students to learn mathematical concepts more deeply. Another viewpoint is that students do not learn as much mathematics because the calculator does too much for the students, and the students become dependent on them. This section will discuss research on the impact that graphing calculators have had on students’ understanding, basic skills, problem solving ability, and attitudes.

Student Understanding

Despite beliefs that graphing calculators will hinder student learning, there is much evidence that graphing calculators positively impact student understanding even when students are tested without graphing calculators. The effects of calculator use on student understanding will be discussed along three main themes. The first is the effect that calculators, scientific and graphing, have on student understanding. The second theme is the effect that graphing calculators have on student understanding of mathematics in general. The third theme is the effect that graphing calculators have on students' understanding of functions and their graphs.

Two meta-analyses summarized findings of previous studies on student understanding when instruction included calculators. The first meta-analysis was conducted by Hembree and Dessart (1986) in 1984. They focused on 79 studies involving precollege mathematics students that compared treatment groups that used calculators in instruction to control groups that did not use calculators. Their research indicated that calculator use enhances learning and increases understanding of mathematical concepts. The additional nine studies that Hembree and Dessart (1992) included in 1992 supported their previous conclusions. The other meta-analysis was conducted by Smith (1996) on 24 studies, and his research indicated that students who used graphing calculators showed higher achievement than students who did not use graphing calculators on problem solving, computation, and conceptual understanding.

National and international tests have provided results showing that students who use calculators on a daily basis scored higher than students who rarely or never use calculators. Third International Mathematics and Science Study (TIMSS) tested half a million students from 41 countries at five grade levels in 1994-1995. The study reported that students who were allowed daily use of calculators performed considerably better on the TIMSS tests than those students who rarely used calculators (The International Study Center, 1998). Results from the 1999 TIMSS (now known as Trends in International Mathematics and Science Study) show that there is a positive correlation for most of the countries that report high calculator use between calculator use and achievement (NCES, 2005). Test data from the National Assessment of Educational Progress (NAEP) data collected in 2003 also showed that students who used calculators daily scored higher on the NAEP exam than students who used calculators less frequently (NCES, 2005). Tarr, Mittag, Uekawa, and Lennex (2000) studied data from TIMSS for Japan, Portugal, and the United States and found that the use of graphing calculators

produced a positive effect when combined with higher order instructional activities such as solving complex problems and exploring number concepts. They also found that the use of graphing calculators to merely check answers and perform routine computations does not contribute to mathematics achievement. They concluded that “the greatest attribute of handheld calculators lies in their ability to foster students’ learning of mathematics concepts” (Tarr et al., 2000, p. 149).

Research specific to graphing calculators shows positive effects of graphing calculators on student understanding. Smith and Shotsberger (1997) divided 114 college algebra students into two experimental classes and two control classes. Each of two instructors taught one experimental class and one control class. As part of the final exam the students solved six problems and explained if their approach was algebraic, graphical, or both. The two experimental classes scored higher than the control classes suggesting that the use of graphing calculators in learning the concepts did not hinder student understanding. Interview responses conducted after the testing revealed that students were able to visualize concepts more easily with the use of the graphing calculator. Waits and Demana (1994) also studied the effects of graphing calculators on student understanding. They developed the Computer and Calculator in PreCalculus Project (C²PC) in 1984 in which students in remedial college mathematics classes learned elementary and intermediate algebra through the use of graphing calculators. The project eventually included high school pre-calculus courses that taught content through the use of technology. Teachers for C²PC were trained in the use of graphing calculators during a 1-week summer institute. Results from C²PC showed that on Advanced Placement Calculus exams where calculators were not allowed, C²PC students did as well as students who took the AP exam before the C²PC project and did not learn with calculators (Dunham & Dick, 1994).

Another study that showed positive gains in achievement due to graphing calculator use was conducted by Shore (1999) with 180 elementary and intermediate algebra students. The experimental group, using graphing calculators in the classroom and on all tests, scored significantly higher on conceptual understanding items than the control group whose members used scientific calculators, and they scored higher on procedural skills tests than students who used scientific calculators. Ottinger (1993) divided first-year high school algebra students into a control group of 54 students and an experimental group of 39 students. The experimental group used graphing calculators and computer software to develop concepts for 18 weeks, and then

they learned procedural algebraic skills for six weeks. The control group was allowed the use of a scientific calculator and spent the entire 24 weeks learning procedural skills. The experimental group scored significantly higher than the control group on the algebra concepts posttest.

Several studies have shown positive effects of graphing calculator use on the understanding of functions. Chandler (1992) studied 173 high school pre-calculus students who were divided into four control group classes and five experimental classes. The students studied transformations of trigonometric functions for a two-week period. The posttest indicated that the students who used graphing calculators had a better understanding of the relationship between functions and their graphs. Hollar and Norwood (1999) separated 90 intermediate algebra students into a graphing calculator group and a control group. The experimental group was allowed to use graphing calculators for homework and tests but not on the final exam or the testing instrument for the study. The control group was not allowed to use graphing calculators. Results show that the experimental group had a significantly better understanding of functions than the control group.

Research has shown that the level of use and the experience that students have in using graphing calculators affects student understanding. The College Board conducted a study in 1990 in which 7800 Calculus AB students and 2900 Calculus BC students were tested. Twenty of the 30 test items did not require the use of a calculator, and the remaining 10 items required the use of a calculator. It was found that students who used calculators regularly and became familiar with a particular model of calculator performed significantly better on the test than those students who did not regularly use calculators or used a different calculator on the test than they were familiar. Students who used more sophisticated calculators such as graphing calculators outperformed students who used scientific calculators (Greenes & Rigol, 1992). Devantier (1992) obtained similar findings in her study of 151 high school pre-calculus students. Students were divided into an experimental group that used graphing calculators and a control group that did not use graphing calculators. Students were identified as experienced users, new users, and non-users of graphing calculators. The students were given the same 25-question test as a pretest and a posttest. It was found that experienced users outperformed new users who outperformed non-users on their understanding of functions, but the only significant difference was between the experienced users and the non-users.

Several studies have shown no significant difference in conceptual understanding for students who use graphing calculators, and some studies have shown negative effects of graphing calculator use. Hunter (2005) taught one section of college algebra with graphing calculators and one section without graphing calculators. Both sections used the same textbook, received the same homework, and took the same tests. Results from the researcher-designed posttest showed no significant difference in the students' overall achievement or understanding of functions or graphing. Autin (2001) separated 58 trigonometry students of an all male, private high school into an experimental group that used graphing calculators and a control group that did not use them. A posttest showed no significant difference in understanding of inverse trigonometric functions. Lesmeister (1996) found no significant difference in posttest scores between the control group and experimental group of 139 algebra II students at a large, public, urban high school. The experimental group was provided with graphing calculators for classroom use, and they were encourage to buy their own for use outside of class. Ritz (1999) conducted a two-year study of high school students in algebra and geometry. Twenty-eight students were randomly selected to be in the control group and not allowed graphing calculators. Thirty-six students were randomly selected to be in the experimental group and were allowed to use graphing calculators. The control group outperformed the experimental group on the Individual Test of Academic Skills (ITAS), but the difference was not significant.

As demonstrated by the studies reviewed here, there is sufficient evidence that graphing calculators can increase students' conceptual understanding of mathematics, specifically the understanding of functions in algebra. There is also significant evidence that graphing calculator use is not detrimental to conceptual understanding, although some research exists to the contrary. One possible explanation for the conflicting results is that the way in which calculators were used in these classrooms varied in significant ways.

Basic Skills

Probably the greatest concern with the use of graphing calculators is that they will weaken students' abilities to perform basic calculations and graphs. It is feared that students will use the graphing calculator to perform calculations that should be done mentally, and they will graph functions on the calculator without knowing how to draw the graph on paper or without the number sense to know if the calculator's graph is reasonable. In response to these fears,

research has shown that the use of calculators does not necessarily cause students' basic skills to diminish.

The discussion of the effects of calculator use initially focuses on elementary students. A common fear is that students who use calculators before learning basic mathematics facts will not learn their facts. To investigate the effect of calculator use on elementary students, Shuard (1992) studied the effects of the calculator-aware number (CAN) curriculum in England and Wales. Originally, 20 classes of 6-year-old students and teachers were included in the study. They were to continue in the study until they were 12 years old so that the long-term effects of calculator use could be investigated. The teachers did not teach the students paper-and-pencil arithmetic, but focused on the understanding of number and mental arithmetic. The students were to decide how and when to use the calculators. Since teachers did not have to spend great amounts of time teaching pencil-and-paper algorithms, they were able to spend more time developing their own teaching styles, engage in problem solving, and allow time for the students to conduct investigations. Shuard found that these changes in teaching practices increased the teachers' confidence in their own mathematical thinking. The students were able to work more independently, they understood topics previously thought to be too difficult for their age level, and they developed strong mental mathematics skills. These results are similar to those found by Dunham (1999) in her review of 96 studies of calculator use. She found that students who learn paper-and-pencil skills in conjunction with technology-based instruction and tested without calculators perform as well or better than students who did not learn with technology. The meta-analysis by Hembree and Dessart (1992) resulted in similar findings. Students of average ability scored slightly higher on computation assessments when calculators were used in instruction but not on tests. They did, however, find a negative effect on basic skills at the fourth grade level.

Basic skills and mental ability retain their importance even when calculators are used. Students who have a weak foundation of basic skills and mental mathematics may not realize that calculator results are incorrect, or they may lack the confidence to challenge calculator results that appear incorrect. To study student confidence in estimation skills as compared to student trust in calculator-generated answers, Glasgow and Reys (1998) provided twenty-five college students with calculators that were programmed to give incorrect answers that ranged from 10% overestimates on the first few problems to 50% overestimates for the last few problems. As a student estimated answers to seven problems and calculated the answers to the

problems on a calculator, the interviewer noted when the student verbally challenged the calculator result. They found that students doubted their estimated answers when in conflict with calculator results. They concluded that students must develop strong number sense and estimation skills so that they can question their own mathematical thinking as well as technology-produced results.

Studies have also been conducted to study the effects of graphing calculators on the acquisition of basic skills. Hollar and Norwood (1999) found that students who learned concepts with graphing calculators performed better, but not significantly better, on algebra skills tests than students who learned the concepts without graphing calculators. Their study involved 90 intermediate algebra college students with about half of the students placed in a treatment group that used graphing calculators and a textbook based on a graphing approach for class exercises, homework, and tests. The rest of the students formed the control group that did not use graphing calculators but memorized facts and performed paper-and-pencil procedures to simplify expressions and solve equations. Neither group used graphing calculators on the final exam that tested algebra skills. Hollar and Norwood determined that traditional skills are not diminished through the use of calculators. Another study was conducted by Stick (1997) who randomly assigned students to one section of calculus I and calculus II and taught concepts through the use of graphing calculators. The other sections were taught using traditional methods. He observed the behavior of students in class, administered a questionnaire, and used class grades to measure performance. Students were introduced to concepts graphically before analytic methods were shown. Students in the experimental class had higher concept retention and performed better on skill tests. Ottinger (1993) also found no significant difference in traditional algebraic skills between the experimental group and the control group. Hembree and Dessart (1992) found that using a calculator during instruction may improve paper and pencil skills on tests that do not allow calculators. In his meta-analysis, Smith (1996) found a significant positive difference in computation by students who used calculators and those who did not.

Research supports the idea that basic skills are not hindered by the use of calculators, and in some cases, calculators can aid in the learning of basic skills. This holds true for elementary students as well as older students. It is important for students to develop number sense and conceptual understanding so that calculator results are not blindly trusted. Students must use

calculators appropriately so that they develop a balance of paper-and-pencil skills, mental mathematics, and calculator literacy.

Problem Solving

Supporters of graphing calculators suggest that student ability and motivation to solve problems increases when using graphing calculators. Students are exposed to problems that model real world situations, and the graphing calculator allows the student several methods for solving problems. Research supports the claim that graphing calculators improve students' ability to solve real world problems.

To study the benefits of students using graphing calculators to solve realistic problems, Drijvers and Doorman (1996) conducted a study of 18-year-old students in their sixth class of their college preparatory mathematics sequence. The students had extensive previous experience using graphing calculators, and they were asked to solve problems by several methods. Drijvers and Doorman found that this approach was confusing to the students at first. The students chose one method of solution and did not recognize the connection to the other methods, but as they gained experience, they were able to make connections between the methods. Alexander (1993) also found graphing calculators to be beneficial to students' understanding of algebraic modeling through real-world applications. She divided 68 college algebra students at a large urban university into a control group and an experimental group. The experimental group used a computer-assisted instructional module and the graphing calculator to investigate functions through concrete visualization. The control group was taught without the use of the module and graphing calculator. She found a significant difference between the groups in their ability to visualize functions and model real-world situations. The meta-analysis by Hembree and Dessart (1992), the meta-analysis by Smith (1996), and the study by Autin (2001) also found that calculator use improves student ability in problem solving. Students who use graphing calculators had more flexible approaches to problem solving, were more willing to engage in problem solving, worked longer on any single problem, concentrated on the mathematics of the problem and not on the algebraic manipulation, solved nonroutine problems inaccessible by algebraic techniques, and believed calculators improved their ability to solve problems (Dunham & Dick, 1994). Wheatley and Shumway (1992) attribute some of the success that students have in problem solving to their persistence in looking for a solution when allowed to use a calculator.

Research has shown that graphing calculators aid in problem solving. Students are motivated to try different methods, and they are provided with alternative ways to solve problems through the multiple representations offered by graphing calculators. Students are aided in their ability to visualize functions and model real-world situations.

Student Attitudes

In addition to impacting students' achievement, graphing calculators also impact students' attitudes toward mathematics and the way in which students approach mathematics. Research shows that students' attitudes about mathematics improve and confidence increases, but the evidence is inconclusive. Some research shows no difference in attitude when students are allowed to use graphing calculators and that some students prefer not to use them.

Several researchers have found positive effects of graphing calculator use on student attitudes. Dildine (1997) examined the attitudes of middle school students regarding the use of graphing calculators. Twenty-three basic mathematics students and 19 algebra students took pre- and post-tests, completed an attitudinal survey, and participated in interviews. He found that students who used graphing calculators had more positive views of mathematics and transferred concepts from the classroom to problems encountered in life outside of the classroom. The students were more involved in the lessons by communicating ideas, reasoning, making connections, and exploring. Stick (1997) found that his calculus I and II students who used graphing calculators had better attitudes, interacted better in class, and attended class more regularly than students that he previously taught without graphing calculators. Calculator use was found to increase student confidence (Smith & Shotsberger, 1997), help students to understand concepts more fully (Smith & Shotsberger, 1997), increase student interest in mathematics (Waits & Demana, 1994), and improve student attitudes (Hembree & Dessart, 1992; Hollar & Norwood, 1999; Merriweather & Tharp, 1999).

Most students are willing to learn to use the graphing calculator, but they must be allowed to use calculators consistently for a prolonged period of time for them to be comfortable with using them. If students are not allowed enough time to become comfortable with using graphing calculators, then they become confused and focus too much on the calculator procedures and not enough on the mathematics to be learned (Hunter, 2005; Merriweather & Tharp, 1999; Ritz, 1999).

Not all studies show that student attitudes improve when students are allowed to use graphing calculators. Some studies found that there is no significant difference in attitude between students who use graphing calculators and students who do not use them (Alexander, 1993; Smith, 1996). Hembree and Dessart (1992) found that even though most students favor the use of calculators, some students feel that using a calculator is "cheating." Hunter (2005) found that 78% of the control group and 72% of the experimental group claim that they learn best through lecture and not through discovery activities using the graphing calculator.

Although some research has found no significant differences, much research supports the claim that the use of graphing calculators improves students' attitudes, increases students' confidence, and increases classroom interaction. This may not happen if students are not allowed enough time to become familiar with calculators.

Factors that Affect Graphing Calculator Use

There are many factors that affect graphing calculator use. One factor is access to technology such as community and administrative support, financial resources, and time for training and learning how to use technology. Other factors include issues of student learning such as the acquisition of basic skills and time needed to teach content as well as technology use. Another factor is that of teacher beliefs. This can be divided into two areas, teachers' beliefs in the use of graphing calculators and teachers' beliefs in teaching mathematics.

Access

Not all schools or students have equal access to graphing calculators. There may be a lack of resources, a lack of administrative and community support, a lack of access to technology, problems in establishing and maintaining security of technology, and perceptions of student dependency on calculators (Simonsen & Dick, 1997). There may not be enough money and time to provide adequate training for teachers to properly implement technology in their teaching (Heid, 1997). There may be a lack of curricular materials, a lack of training and in-service opportunities, a lack of planning time, and few incentives for teachers to use calculators (Dunham, 1999).

Access to graphing calculators can provide equity between high and low achieving students (Dunham, 1999). Students who have difficulty in performing calculations can use calculator-produced results to solve problems that they otherwise would not be able to solve.

Students who have trouble visualizing the graph of a function are aided by the use of a graphing calculator to actually see the graph of the function. Even though graphing calculators can improve equity, they can also contribute to inequity if all students do not have access to the same technology. Not all students can afford to buy a calculator, and the gap widens between the students who can afford calculators and those who cannot. Calculator use can also widen the gap between schools that can afford technology and those that cannot (Heid, 1997). Not all schools can afford the same amount of technological tools, and students in those schools that cannot afford technology are at a disadvantage later when knowledge of how to use technology is needed in higher education or the workplace.

Another type of access that affects calculator use is availability of training and networking with other teachers. The isolation of rural schools may limit the training sessions that teachers are able to attend, and there may be fewer teachers with whom to network. Also, rural teachers are less likely to join state and national mathematics organizations (Arvanis, 2003).

Learning Issues

Issues related to student learning can also be barriers to calculator use. Technology may not always improve the way in which a concept is taught. Computer simulations may deprive students of real experiences that could be used to teach a concept better than the simulation. New routines used with technology may simply replace current routines and not provide any learning advantage. Students may play technology games that have no educational value. Teachers may overestimate students' abilities to make generalizations about the examples generated with technology and plan activities that do not enhance student learning. Students may also focus too much on keystrokes and not enough on the mathematics to be learned (Dion et al., 2001). Students may make certain errors because of using the calculator that they would not make otherwise (Simonsen & Dick, 1997).

A common fear among teachers and parents is that students will become dependent on calculators. They feel that students should be taught in the same way in which they were taught and that the use of calculators "dumbs down" the curriculum. They believe that students will not be challenged if they are allowed to use calculators (Pomerantz, 1997), and they will lose their computational skills (Dunham, 1999). These arguments are similar to arguments between the acceptance of paper-and-pencil algorithms developed hundreds of years ago and the acceptance

of technology used today (Usiskin, 1998). People rejected these algorithms, the advanced technology of that time, for the same reason that people reject technology use today. They feared then, and fear today, that students will lose mental power when new techniques and technology are used.

Also, many parents incorrectly believe that paper-and-pencil arithmetic is the only important aspect of mathematics and that to be allowed calculators too early will result in students being taught to push buttons and not learn math facts. They fear that the calculator will do the work for the student, and the student will lose mathematical ability (Dick, 1992). Another fear is that students will misuse technology, and student learning will decrease as the newness of technology wears off (Heid, 1997).

Ralston (1999) provides arguments against perceptions that graphing calculators hinder students' basic skills. Parents and teachers believe that students have poor paper-and-pencil skills, but Ralston argues that paper-and-pencil skills have not been that good in the past. Secondary and university teachers recognize that students lack mathematical technique, but the lack of technique is caused by deficiencies in a student's number sense and symbol sense and not from calculator use. Too much blame has been placed on calculators, and teachers must dispel these misconceptions.

Overcoming barriers to graphing calculator use can be difficult. Acquiring graphing calculators for all students is difficult since many schools are already restricted by tight budgets, and not all students can afford to buy a graphing calculator. But the more difficult solution will be in convincing teachers and parents of the benefits of graphing calculator use and training teachers to use graphing calculators to teach concepts (Bright, Lamphere, & Usnick, 1992). Dunham (1999) lists three recommendations that may aid in the implementation of graphing calculators in education: inform the public of research that supports calculator use, design professional development programs that prepare teachers to teach with calculators as well as reflect on their beliefs about teaching mathematics, and continually train and support teachers in the use of calculators.

Teacher Beliefs of Graphing Calculator Use

A teacher's view of mathematics and perceptions of the graphing calculator affect how a teacher utilizes technology to teach mathematics (Milou, 1999). Teachers have different reasons

for using and not using graphing technology in the classroom. Some teachers believe that students will not learn or understand the mathematics behind graphs when graphing on a calculator, that solutions found on the graphing calculator are inferior to algebraic solutions, and that students are not really doing mathematics unless they are performing algebraic manipulations.

Researchers have explored teacher beliefs of graphing calculator use. In a survey of 146 high school and middle school teachers in a large northeastern city, Milou (1999) found that teachers may not know how to teach using the graphing calculator, and they may feel uncomfortable teaching with something that they do not know well. Many teachers fear that students become too dependent on graphing calculators and do not master important algebraic manipulations. They agree that the graphing calculator allows for the de-emphasis of some topics that are currently taught, but they do not feel that high school teachers should de-emphasize topics until colleges accept the use of graphing calculators and de-emphasize the same topics. Teachers generally support the use of graphing calculators at the algebra II level, but there is disagreement whether to allow graphing calculators at the middle school and algebra I level. Szombathelyi (2001) gathered data from 96 high school mathematics teachers and 72 college instructors to determine factors that influence teachers' decisions on graphing calculator use. She found factors that do not affect use include teaching experience, educational background, and personal use of graphing calculators. Factors that affect use are professional development, mathematical and pedagogical knowledge, familiarity with graphing calculators, and the belief that graphing calculators enhance learning. The extent of a teacher's use of graphing calculators has more to do with the teacher's expertise than his/her years of experience using them. Arvanis (2003) also studied factors that influence teachers' decisions on graphing calculator use. He surveyed 879 high school teachers and found that teacher beliefs, workshops, and interaction with other teachers are factors that influence teachers' decisions to use graphing calculators. Factors that limit graphing calculator use are emphasis on basic skills, cost and availability, and lack of time, training, and materials. He also found isolation of teachers in rural schools to be a major hindrance to the implementation of graphing calculators in instruction.

Initial and continued teacher calculator training is important to convince teachers of the benefits of using graphing calculators in teaching mathematics, and training is important for teachers to use graphing calculators in ways that enhance student learning. In a study to

determine the importance of teacher training, Simonsen and Dick (1997) provided 36 high school classrooms with a classroom set of graphing calculators and a 1-day in-service. Teachers were also invited to attend an optional 1-week workshop on graphing calculators. At the end of the year 27 teachers were interviewed about their experiences in using graphing calculators in their teaching. After taking part in the workshop, teachers changed their instructional methods so that their lessons were more student-centered, involved more open-ended questions, and involved cooperative learning. The students were more active in discussions and were more enthusiastic. Similar results were found by Abuloum (1996) in his study of 43 teachers and 1697 high school and middle school students. From data gathered through surveys and an achievement test, his study showed a positive relationship between the amount of a teachers' graphing calculator training and students' achievement scores. The level of training was positively related to the level of graphing calculator use, but many teachers did not receive formal training. Most of the teachers were self-taught or taught by colleagues, and this method of training was preferred by graphing calculator users. Non-users preferred formal training, but formal training was not readily available. Poage (2002) studied the effects that an intensive graphing calculator institute had on high school and middle school mathematics teachers' content knowledge, confidence, and classroom use. Fourteen high school teachers and six middle school teachers participated in the three-week institute, and data were collected from interviews and classroom observations. Poage found that teachers are more confident in using graphing calculators in their teaching after experiencing them as students in the institute and being exposed to ideas on how to incorporate them in their teaching. Yoder (2000) surveyed 48 algebra I and algebra II teachers to determine teachers' views of learning algebra and how their views affect the ways that graphing calculators are used in their classes. She found that the teachers' views of learning algebra, the number of years taught, and the level of algebra taught were not significant factors in how calculators were used in their classes. The factor that affected calculator use the most was workshop attendance. Teachers who attended at least one workshop were more likely to use discovery methods, more likely to use graphing calculators to graph lines and quadrilaterals, and less likely to think that students would lose basic skills and become dependent on calculators.

Teachers' beliefs of graphing calculators influence ways in which they are used in mathematics classrooms. The belief that graphing calculators enhance learning is a major factor of their use in the classroom, whereas the belief that students will become dependent on them is a

major reason not to use them. Professional development and interaction with other teachers can influence a teacher's beliefs in the benefits of learning with the graphing calculator, especially if the teachers experience learning situations as students experience them.

Teacher Beliefs of Teaching Mathematics

A teacher's beliefs in teaching mathematics can affect the way in which graphing calculators are used in instruction. A teacher may view mathematics learning as memorization and application of rules, whereas another teacher may view mathematics learning as exploration and problem solving. These beliefs determine how a teacher allows graphing calculators to be used in the classroom.

Researchers have studied the effects of teachers' beliefs of mathematics on their use of graphing calculators. Herring (2000) conducted a case study of two college algebra teachers to investigate the relationship between teachers' expressed beliefs and their manifested beliefs. She found that the teachers' manifested beliefs supported their expressed beliefs, and as the teachers became aware of their beliefs, they reflected on their practices and were open to changing their teaching behaviors. The way in which they implemented graphing calculators was determined by their beliefs. Teachers who believe that students should work problems analytically first and then use the graphing calculator to check their work are teaching for instrumental understanding, whereas teachers who encourage students to learn through experimentation are teaching for relational learning. Van Cleave (1999) conducted a case study of four high school mathematics teachers who had taught for a minimum of three years with graphing calculators. She conducted interviews and classroom observations to determine the relationship between teachers' beliefs of teaching algebra and graphing calculator use. She found a high correlation between a teacher's beliefs and his/her graphing calculator use, and this correlation was strengthened by the teacher's experiences and interaction with other teachers. Teachers viewed algebra as a foundation for higher mathematics courses, and this influenced the focus of graphing calculator use to be that of a tool to do mathematics and not a tool to learn mathematics. In a study by Simmt (1997), six 11th and 12th grade teachers were interviewed and observed teaching quadratic functions. She found that teachers use graphing calculators to offer instructional variety, to save time, to generate many examples in a short amount of time, and to motivate students. Teachers used the calculator merely as an extension to their normal teaching and did not try new methods or

approaches. The only new use of graphing calculators was to demonstrate examples previously shown on the board. Each teacher's philosophy of mathematics was revealed in the way that the lessons were taught, and this philosophy guided the teacher's use of the graphing calculator. The study concluded that the use of graphing calculators in instruction is not enough to change the mathematics curriculum. Teachers must alter their philosophies and mathematical beliefs to change their practices, and calculators must be used in ways that are different from traditional practices to bring about changes in curriculum.

A major influence on graphing calculator use is the level of rule-based beliefs held by a teacher. Tharp et al. (1997) provided five 3-hour training sessions for 261 middle school and high school teachers. Each teacher responded to a questionnaire and wrote reflections in a journal during the training. The researchers found a high correlation between a teacher's view of mathematics and graphing calculator use by that teacher. Rule-based teachers are more likely to use a lecture approach to teaching, whereas non-rule-based teachers are more likely to use a discovery approach. Rule-based teachers felt that graphing calculators do not enhance and may hinder learning, and teachers who are less rule-based feel that graphing calculators are an integral part of instruction and are freer in their use of calculators. Because of involvement in the training sessions, rule-based teachers changed their beliefs about graphing calculators from a hindrance to education to an integral part of learning, but they did not change their beliefs about rule-based teaching. "As a result, rule-based teachers reported that they quickly returned to the lecture mode in order to control the use of the calculator and avoid embarrassment" (Tharp et al., 1997, p. 7). They concluded that teachers, especially rule-based teachers, need to experience procedural, conceptual, and inquiry-based learning. Being taught in this way will help them to learn how to teach in the same way. Milou (1998) surveyed 146 secondary mathematics teachers to determine how teacher beliefs in the teaching of algebra are related to graphing calculator use. He found that the majority of teachers did not allow graphing calculators to be used until the students mastered concepts and procedures. Many teachers were still unsure of the cognitive benefits that graphing calculators may provide.

Graphing calculator use is influenced by teacher beliefs. Teachers who hold non-rule-based beliefs tend to use graphing calculators in discovery situations and as a tool to learn mathematics, whereas rule-based teachers use graphing calculators for demonstration and as a

tool to do mathematics. Teachers use graphing calculators in ways consistent with their beliefs, and to change their practices, they must alter their beliefs of mathematics.

Summary

Despite the advocacy of national organizations such as NCTM and NRC and favorable achievement results from the use of graphing calculators in mathematics classes, some teachers are reluctant to use them in their classrooms. Many factors, such as access to technology, issues with implementation, and teacher beliefs affect a teacher's decision to incorporate graphing calculators in instruction. Data collected by organizations such as NCES provide an idea of the prevalence of graphing calculators in high school mathematics classrooms, and research has revealed ways in which graphing calculators are used, but there is a lack of research regarding the prevalence of graphing calculators and ways in which graphing calculators are used in rural high schools. This researcher investigated factors affecting graphing calculator use that may be unique to rural high schools such as isolation from other mathematics teachers and lack of access to professional development. Also, this researcher investigated ways in which graphing calculators are used in rural high schools in the specific region of western Kansas.

A teacher's beliefs in graphing calculator use and teaching mathematics affect the ways in which he/she uses calculators in the classroom. Opponents of graphing calculators believe that students are not learning real mathematics if they are not performing algebraic manipulations, and they believe that students easily become dependent on graphing calculators. On the other hand, proponents of graphing calculators believe that students must develop conceptual understanding so that they learn how and when to use algebraic procedures. They believe that students learn mathematics more deeply because of the graphing calculator. One way of determining teachers' beliefs is by identifying teachers by their degree of rule-based beliefs and practices. Researchers have explored the relationship between teachers' beliefs in the teaching of mathematics and graphing calculator use, however, little research has been conducted that explores this relationship at a deeper level than Likert responses.

CHAPTER 3 - Methodology

Overview

The goal of this study is to describe how graphing calculators are used in high schools of western Kansas and to determine the relationship between teachers' beliefs in teaching and learning mathematics and how graphing calculators are used in their classes. Five main questions were to be answered in this study:

1. What are characteristics of mathematics teachers in western Kansas, including their level of education, teaching experience, preparation in the use of graphing calculators, and experience with graphing calculators?
2. In what mathematics courses do students use graphing calculators, how often do they use them, what calculator functions do they use, and to what extent are graphing calculators being used to learn mathematical concepts?
3. What beliefs do teachers have regarding the teaching and learning of mathematics?
4. What beliefs do teachers have regarding the use of graphing calculators?
5. What is the relationship between teachers' beliefs about teaching and learning mathematics and how graphing calculators are used?

Research Design

This study is a mixed design of descriptive and naturalistic components (Lincoln & Guba, 1985) with quantitative and qualitative data from three main data sources. The first source is a survey, which was used to collect data about the teachers, their schools, their use of graphing calculators, obstacles to graphing calculator use, teachers' beliefs of how students should use graphing calculators, and teachers' beliefs on the effect of graphing calculators on student learning. The second source involved interviews of nine high school mathematics teachers; its purpose was to provide a deeper understanding of how graphing calculators are used. The third source involved classroom observations of six of the high school teachers interviewed; its purpose was to provide triangulation of a teacher's responses when interviewed and actions when observed.

Information collected in this study was used to determine graphing calculator use of high school mathematics teachers in western Kansas and the relationship between teaching beliefs about graphing calculator use and mathematics teaching. Survey response data were analyzed through the use of descriptive statistics as well as correlation analysis. Teachers were identified according to their level of rule-based beliefs, and comparisons were made between these beliefs and teacher beliefs of graphing calculator use.

The Setting

This study was conducted in high schools of western Kansas. This area was chosen because many students at the researcher's university graduated from high schools in this area, and many teachers from this area graduated from and continue their education at the researcher's university. Approximately 18,000 high school students attend the 90 schools in this area, and the schools range in size from 10 students to almost 2,000 students. There is one private, parochial school in this region. Over three-fourths of the schools have less than 183 students, and only four schools have more than 1,000 students. Seventy-two percent of the students are Caucasian; 23% are Hispanic; 2% are African American; and 2% are Asian. Thirty-eight percent of the students are from low socio-economic status families (KSDE, 2006).

The Participants

Survey Participants

The participants in this study were high school mathematics teachers in western Kansas. The Kansas State Department of Education website (<http://www.ksde.org>) was used to identify potential participants. There are 253 high school mathematics teachers in this region, and each teacher was sent a letter (see Appendix A) inviting him/her to participate in this study. The letter explained the study and requested the teacher's participation. A survey form (see Appendix B) was included, and by taking the survey, the teacher agreed to be a participant. A code number was assigned to each teacher and survey so that the researcher could identify the teachers who responded. The teachers were assured that individual responses would be kept confidential. To increase the participation rate, a second letter (see Appendix C) was sent two months later to those who had not responded to the survey inviting them again to take part in the study. As incentive to return the survey, two TI-83 Plus Graphing Calculators were donated by Texas

Instruments to be awarded to two teachers randomly drawn from all teachers who returned a survey. One hundred fifty-seven teachers took part in the first phase of the study. Two of the teachers did not mark responses to the Likert statements, so calculations based on the Likert responses are based on 155 teachers.

Interview Participants

In the second phase of the study the researcher interviewed nine of the teachers who responded to the survey. Stratified purposeful sampling was used to select teachers to be invited to participate in the interview phase. Statements 26–29 on the survey were used to determine the level of rule-based beliefs held by each teacher, and composite scores for these four statements had possible values in a range from 4 to 20 where 4 is at the rule-based end of the scale and 20 is at the non-rule-based end. In this study, all of the composite scores fell in a range from 10 to 20. To provide two groups for comparison of rule-based beliefs, teachers whose composite scores were below 16 were placed in the rule-based group, and teachers with scores greater than or equal to 16 were placed in the non-rule-based group. To represent a range of teacher beliefs, the investigator chose five teachers whose survey responses indicated rule-based beliefs and five teachers whose survey responses indicated non-rule-based beliefs. Two of the rule-based teachers declined to take part in the interview phase, one agreed to take part but was not able to participate because of scheduling conflicts, and two did not respond. Two of the non-rule-based teachers agreed to take part, two declined, and one did not respond. Nine more teachers, five rule-based and four non-rule-based, were chosen to be invited for an interview. Three rule-based teachers agreed to participate, and two did not respond. All four of the non-rule-based teachers agreed to participate. Each teacher selected was sent a letter (see Appendix D) advising of his/her selection for an interview and was asked to sign a consent form that explained the study and assured confidentiality (see Appendix E).

In addition to selecting teachers based on their teaching beliefs, efforts were made to choose teachers so that a variety of teaching characteristics were represented by the teachers interviewed and observed. Table 3.1 contains information about the teachers chosen for the interview and observation phases of the study. The paragraphs that follow include additional information about each participant.

Table 3.1 Interview and Observation Participants

Teacher	Age	Gender	School Enroll	Teaching Beliefs	Years of Teaching	Years of GC Use	Level of Expertise
Mr. Adams	44	Male	360	Rule-based	22	10	Advanced
Mr. Baker	25	Male	887	Rule-based	3	3	Intermediate
Ms. Clark	26	Female	81	Rule-based	3	3	Advanced
Mr. Davis	53	Female	51	Non-rule-based	30	5	Beginner
Mr. Edwards	35	Male	468	Non-rule-based	13	10	Advanced
Mr. Fort	44	Male	138	Non-rule-based	23	10	Advanced
Ms. Green	53	Female	110	Non-rule-based	32	10	Intermediate
Ms. Hays	52	Female	129	Non-rule-based	25	7	Intermediate
Ms. Irwin	46	Female	130	Non-rule-based	24	20	Intermediate

Mr. Adams, Mr. Baker, and Ms. Clark are the teachers whose survey responses indicated rule-based beliefs in teaching mathematics. Mr. Adams is a 44-year-old male who has taught for 22 years. He currently teaches calculus, trigonometry, and algebra II in a high school of 306 students, and he requires graphing calculators in these classes. Ninety-two percent of the students are Caucasian, 4% African American, 2% Hispanic, 1% Native American, and 1% Asian. Thirty percent of the students are of low socio-economic status (KSDE, 2006). He has used graphing calculators in his teaching for 10 years and considers himself at the advanced level of graphing calculator knowledge.

Mr. Baker is a 25-year-old male with 3 years of teaching experience. He rarely uses graphing calculators in his teaching, but he allows their use in the classes that he teaches, algebra I and applied geometry. His knowledge of the graphing calculator is at the intermediate level. He teaches in a high school of 887 students of which 93% are Caucasian, 2% African American, 4% Hispanic, and 1% Asian, and 24% of the students are of low socio-economic status (KSDE, 2006).

Ms. Clark is a 26-year-old female at the advanced level of graphing calculator knowledge. She has used graphing calculators during her 3 years of teaching, and she requires their use in algebra I and algebra II and allows their use in intermediate algebra. Her high school

has 81 students of which 96% are Caucasian, 3% are Hispanic, and 1% are Asian. Ten percent are of low socio-economic status (KSDE, 2006).

Ms. Davis, Mr. Edwards, Mr. Fort, Ms. Green, Ms. Hays, and Ms. Irwin are the teachers whose survey responses indicated non-rule-based beliefs in teaching mathematics. Ms. Davis is a 53-year-old female who has taught for 30 years. She has used graphing calculators in her teaching for 5 years and considers her graphing calculator knowledge to be at the beginner level. There are 51 students in her high school. Ninety-two percent are Caucasian, 2% are African American, 2% are Native American, and 4% are Asian. Forty-one percent are of low socio-economic status (KSDE, 2006). She allows graphing calculators in her applied mathematics I class, and she requires graphing calculators in her applied mathematics II and applied mathematics III classes.

Mr. Edwards is a 35-year-old male who has taught for 13 years. He identifies himself at the advanced level of graphing calculator knowledge and has used graphing calculators in his classroom for 10 years. He allows graphing calculators in his algebra II, pre-calculus, college algebra, college trigonometry, and calculus classes. There are 468 students in his high school of which 52% are Caucasian, 47% are Hispanic, and 1% are Asian. Thirty-four percent are of low-socioeconomic status (KSDE, 2006).

Mr. Fort is a 44-year-old male who has taught for 23 years. He has used graphing calculators in his teaching for 10 years, and he considers his graphing calculator knowledge to be at the advanced level. He teaches in a high school of 138 students where 89% are Caucasian, 2% are African American, 7% are Hispanic, and 1% are Asian. Thirty-five percent are of low socio-economic status (KSDE, 2006). Graphing calculators are required in his algebra II, trigonometry, and calculus classes, and they are allowed in his algebra I and geometry classes.

Ms. Green is a 53-year-old female who has taught for 32 years. She has used graphing calculators in her teaching for 10 years and considers herself to be at the intermediate level of graphing calculator knowledge. Of the 110 students in her high school, 97% are Caucasian, 1% are Hispanic, and 2% are multiethnic (KSDE, 2006). She requires graphing calculators in her algebra II class and her advanced mathematics/trigonometry class, and she allows them in her geometry class.

Ms. Hays is a 52-year-old female who has 25 years of teaching experience. She identifies her graphing calculator knowledge at the intermediate level, and she has used them in her

teaching for seven years. She teaches in a high school of 129 students of which 76% are Caucasian, 1% are African American, and 22% are Hispanic. Twenty-six percent are of low socio-economic status (KSDE, 2006). Graphing calculators are required in her college algebra class and allowed in her advanced mathematics class.

Ms. Irwin is a 46-year-old female who has taught for 24 years. She has used graphing calculators in her teaching for nearly 20 years and considers herself to be at the intermediate level of graphing calculator knowledge. She requires graphing calculators in her algebra I, advanced algebra, and AP calculus classes but does not allow their use in her fundamental mathematics class. She teaches in a high school of 130 students of which 98% are Caucasian, and 2% are multiethnic. Thirty-four percent are of low socio-economic status (KSDE, 2006).

Observation Participants

The nine teachers chosen for interviews were also asked to be observed and videotaped while teaching three lessons. All three rule-based teachers agreed to participate in the observation phase of the study, but three of the six non-rule-based teachers declined to participate in the observations. The building principal of each teacher was sent a letter (see Appendix F) explaining the observation process. Permission to observe was obtained from the teacher and the building principal. The teacher and principal were assured that the focus of the observation would be on the instructional methods of the teacher and the use of graphing calculators by the students and not on individual student behaviors. They were assured of confidentiality.

Mr. Adams, Mr. Baker, Ms. Clark, Mr. Edwards, Mr. Fort, and Ms. Irwin participated in the observation phase of the study. Mr. Adams, Mr. Baker, and Ms. Clark represented teachers with rule-based beliefs in teaching mathematics, and teachers Mr. Edwards, Mr. Fort, and Ms. Irwin represented teachers with non-rule-based beliefs in teaching mathematics.

Data Collection

The Survey Instrument

The survey instrument was developed from the Virginia Network for Technology (VANT) Attitude Questionnaire by Tharp et al. (1997), the Use of and Attitude Towards Graphing Calculators (UATGC) questionnaire by Milou (1999), and the Graphing Calculator

Survey by Yoder (2000). These surveys were chosen since they collect the type of data needed to answer the research questions in this study, and the use of previously used instruments strengthened the reliability and validity of the survey instrument. The researcher made modifications, and the modified instrument was reviewed by seven high school mathematics teachers for clarity and completeness. Modifications were made based on recommendations by the seven high school teachers, the instrument was reviewed by three experts in the field of quantitative analysis, and modifications were made based on their recommendations. The final version of the survey contained 42 questions. Seventeen items focus on teacher characteristics, teaching experience, experience with graphing calculators, and student use of graphing calculators. Short answer responses provide background information about the participants and information on how graphing calculators are used in their classes. Twenty-five statements are presented using a five-point Likert scale of Strongly Disagree, Disagree, Undecided, Agree, and Strongly Agree. These statements focus on obstacles that hinder the use of graphing calculators (statements 18–25), characteristics of rule-based and non-rule-based instruction (statements 26–29), teachers’ beliefs on how students should use graphing calculators (statements 30–36), and teachers’ beliefs about the effects of graphing calculators on student learning (statements 37–42).

Statements 18–25 form the category of obstacles that hinder graphing calculator use. Responses for statements 18–25 were assigned numerical values, and since the wording of statements 18–22 are negatively worded as compared to statements 23–25, responses were assigned values in reverse order of statements 23–25. In this way, a situation is considered to be more of an obstacle if it has a lower value, and collectively, a participant with a lower sum of response values is identified as experiencing more obstacles to graphing calculator use than a participant with a higher sum.

Statements 26–29 relate to characteristics that were used to determine whether teachers are rule-based or non-rule-based. Statements 28 and 29 are worded such that Strongly Disagree represents rule-based teaching, and statements 26 and 27 are worded such that Strongly Agree represents rule-based teaching. Responses for statements 26 and 27 were assigned values in reverse order of statements 28 and 29 so that a participant who holds rule-based teaching beliefs has a lower sum for responses to statements 26–29 than a participant who holds non-rule-based teaching beliefs.

Statements 30–36 relate to teachers’ beliefs of how students should use graphing calculators. Statements 30–34 are worded such that Strongly Agree represents a limited view on how graphing calculators should be used, and statements 35 and 36 are worded such that Strongly Disagree represents a limited view on how graphing calculators should be used. Responses for statements 30–34 were assigned values in reverse order of statements 35 and 36 so that a participant who favors a limited use of graphing calculators has a lower sum than a participant who favors unlimited graphing calculator use.

Statements 37–42 relate to teachers’ beliefs of the effects that graphing calculator use has on student learning. Statements 37–40 are worded such that Strongly Agree represents the belief that graphing calculators positively affect student learning, and statements 41 and 42 are worded such that Strongly Disagree represents the belief that graphing calculators positively affect student learning. Responses for statements 37–40 were assigned values in reverse order of statements 41 and 42 so that a participant who holds the view that graphing calculators positively affect student learning has a higher sum for responses to statements 37–42 than a participant who does not hold the view that graphing calculators positively affect student learning.

The Interview Protocol

As stated earlier, nine teachers were purposely selected to take part in the interview phase of the study. The interview protocol consisted of seven questions regarding graphing calculator use, teachers’ beliefs of how student learning is affected by graphing calculator use, and teacher level of rule-based teaching (see Appendix G). Interviews were audio taped to preserve the actual wording used by participants.

The researcher developed the interview protocol based on the survey; thus, the interview data triangulate the survey data. Questions 1 and 3 correspond to survey questions 13–17 and ask about the frequency of graphing calculator use by the students and the ways in which the students use graphing calculators. Question 2 corresponds to survey questions 18–25 and asks about obstacles to the use of graphing calculators in his/her school. In addition to corresponding to questions 13–17, Question 3 corresponds to survey questions 26–36 and pertains to the teacher’s beliefs of how mathematics should be taught and the teacher’s belief about the use of graphing calculators. Question 4 corresponds to survey questions 37–40 and pertains to ways in which the use of graphing calculators can benefit student learning. Question 5 corresponds to

survey questions 41 and 42 and pertains to student dependency on graphing calculators. Question 6 asked about ways in which the teacher has changed his/her teaching because of the graphing calculator, and its purpose is to provide additional insight into the teacher's beliefs of teaching mathematics and the use of graphing calculators. Question 7 is a statement asking for additional comments that the teacher may want to make to clarify his/her other responses in the interview. The interview protocol was piloted on two high school teachers, and modifications were made based on their responses.

The Observation Protocol

Three observations were conducted in the classrooms of six of the nine teachers who were interviewed. Observations were videotaped in all classrooms except for the classroom of a teacher in a large district that did not allow videotaping of the students. The focus of the observations was on how graphing calculators were used by the teacher and the students. A pre-lesson interview was conducted to gather preliminary information about the lesson, and a post-lesson interview was conducted to provide additional information about the lesson. Each teacher's responses from the interview phase were compared to his/her observed behavior regarding teaching practices and graphing calculator use.

The researcher developed the observation protocol to triangulate the observation data with the survey and interview data. The observation protocol (see Appendix H) was developed to focus on graphing calculator use by teachers and students, and it provided information for the second, third, and fourth main questions of this study. It was designed to focus on specific behaviors to be observed, but it allowed for field notes to be taken on behaviors not specifically mentioned in the protocol.

Data Analysis

Survey Analysis

The survey provided data about participants' length of teaching experience, length of teaching with graphing calculators, highest degree attained, gender, and age. This information was analyzed quantitatively through the calculation of means, medians, and percentages. Included were the classes taught, the classes in which graphing calculators are used, and the frequency in which graphing calculators are used.

The responses to the Likert section of the survey were interpreted in several ways. Percentages were calculated for each level of each response for each question, and percentages were calculated for each level of the categories of questions. This provided results about the teachers' beliefs for each statement and about specific characteristics defined by categories of questions. The response levels were also converted numerically so that the sum of response values for statements within a category was used to represent that category.

Since the primary focus of this study was to determine how teachers' beliefs are related to the use of graphing calculators, regression analysis was used to determine the relationship between teaching experience and graphing calculator use, between the level of rule-based teaching and graphing calculator use, and between teachers' beliefs of student learning and graphing calculator use. To reduce the risk of a Type I error, a Bonferroni adjustment was used for multiple correlations. This adjustment, however, increased the probability of a Type II error so relationships that are actually significant may not have been detected.

Interview Analysis

Each teacher chosen was interviewed once regarding his/her use of graphing calculators. Interviews were recorded on cassette tape, and the researcher took notes during and after the interview. Recordings were transcribed and categories of types and frequency of use, obstacles to the use of graphing calculators, teachers' beliefs of teaching mathematics, and teachers' beliefs of graphing calculators were coded. Passages were sorted according to the categories, and themes within each category were identified within individual interviews and among all interviews.

Comparisons were made between the survey responses and the interview responses. Access codes were used to match interview responses from each participant to his/her survey responses. Of particular interest was how well the interview responses match the survey responses. Comparisons of themes were made between interviewees that had similar survey responses.

Observation Analysis

An observation protocol was followed and field notes taken on graphing calculator availability and amount of use, teaching methods used, graphing calculator use by the teacher and the students, and the degree to which graphing calculators were used in the teaching and

learning of mathematics. Information gathered from the observations provided insight to the responses given by the teachers in the interviews. The observation provided evidence of the teaching methods and graphing calculator use claimed by the participant in the survey and interview.

Observation information was sorted according to categories of types and frequency of use, obstacles to the use of graphing calculators, practices that reveal teachers' beliefs of teaching mathematics, and practices that reveal teachers' beliefs of graphing calculators. Themes within each category were identified within individual observations and among all observations.

Instrument Validity

Survey instruments from previous studies (Milou, 1999; Tharp et al., 1997; Yoder, 2000) were used in the design of the survey instrument for this study. Specifically, definitions for rule-based teaching and non-rule-based teaching, questions in the survey, and the scoring system used in this study to determine the level of rule-based teaching were similar to the measure of perspective on mathematics learning, the View of Learning Math as a Rule-Based Subject (VLMRBS), by Tharp et al. This instrument was found to have concurrent validity with Perry's scheme with correlation in the expected direction ($r = -.43$, $p < .05$). Fifteen of the twenty-five questions used in the Likert response section of the survey were similar to questions from the study by Tharp et al.; seven questions related to obstacles, three questions related to teacher beliefs of teaching mathematics, one question related to graphing calculator beliefs, and three questions related to beliefs of student learning.

Three additional questions from the Graphing Calculator Survey by Yoder (2000) were included in this study; one question related to obstacles, one related to graphing calculator beliefs, and one related to beliefs of student learning. This instrument was reviewed by four of Yoder's colleagues to ensure the clarity of each question, and modifications were made accordingly.

Six questions from the Use of and Attitude Towards Graphing Calculators by Milou (1999) were included in this study; five related to graphing calculator beliefs, and one related to beliefs of student learning. Milou's survey was reviewed by three professors of mathematics and two high school mathematics teachers, and a pilot study was conducted. The researcher included

an additional question related to teaching beliefs and an additional question related to student learning.

The questions were sorted according to categories of obstacles to the use of graphing calculators, teachers' beliefs of teaching mathematics, teachers' beliefs of using graphing calculators, and teachers' beliefs of the impact graphing calculators have on student learning. The researcher included an additional question related to teaching beliefs and an additional question related to student learning, and the researcher modified some questions to be negatively worded in relationship to similar questions. The modified instrument was reviewed by seven high school mathematics teachers for clarity and completeness, and revisions were made based on their recommendations. A panel of three experts in the area of survey research reviewed the survey, and additional revisions were made based on their recommendations.

Verification

Creswell (1998) relates the need for verification of naturalistic research to the need for validity in quasi-experimental/experimental research. He views verification as one of the strengths of naturalistic research and as a way of identifying qualitative research as a legitimate approach to research. He recommends that the researcher strive for verification by employing at least two of eight recommended procedures in naturalistic research. The procedures chosen for this study were triangulation and clarifying researcher bias.

Triangulation

One of Creswell's recommended procedures for verification is triangulation. Information was gathered by three methods: a survey, an interview, and observations of teaching practices. In order to gather comparable information from the three parts of the study, survey questions were grouped into categories so that responses to interview questions and observations of teaching practices could be compared to the category responses. In addition to providing a richer description of the survey findings in this study, the interview and observation provided evidence to responses in the survey.

Clarifying Researcher Bias

Another procedure recommended by Creswell is to clarify researcher bias so that the reader understands how the researcher's biases and assumptions may have affected decisions in designing the study. The paragraphs that follow describe the researcher's biases.

The researcher has taught with graphing calculators for 13 years and is a proponent of graphing calculator use by students. Personal experience has led the researcher to believe that students can gain a deeper understanding of mathematics through multiple-representations of problem situations. They can analyze graphs of functions more quickly when graphed on a calculator than by hand, and this makes it easier to see patterns in families of functions so that generalizations can be made. Teachers should teach more than how to perform the calculator functions; they should teach so that students learn mathematical concepts through the use of the graphing calculator.

The researcher also recognizes that obstacles prevent the use of graphing calculators in high schools. Obstacles include a lack of access to technology, insufficient teacher knowledge of and comfort with the graphing calculator, and teachers' fears of student dependency on technology and loss of basic skills. Moreover, it can be difficult to plan lessons and activities that utilize the graphing calculator as a learning tool instead of merely a tool for computation. Many teachers and students view mathematics as a subject that is learned through memorization of rules and procedures, and for teachers who have learned mathematics in this way it can be difficult to teach differently. Student understanding has often been gauged by how well the procedures have been executed regardless of the understanding of when to use the procedure and why the procedure works.

The researcher believes that a student constructs knowledge from previous knowledge, and the student learns concepts more deeply when provided with activities that lead him/her to the discovery of the concepts. The student constructs knowledge in a way that is unique to him/her, and interaction with other students allows for the social construction of knowledge.

The researcher's teaching experience at the university is the source of his opinion that students have had a wide variety of graphing calculator experience at the high school level, and he is particularly interested to understand why this variety of experience exists. These researcher biases, interests, and beliefs have influenced his decision to conduct this particular study.

Trustworthiness

Steps were taken to strengthen the trustworthiness of this study by reducing the influence of researcher's biases on the results of the study. One step was to adapt the survey instrument from three surveys developed and used by other researchers. It was reviewed by seven high school mathematics teachers for clarity and completeness, and modifications were made accordingly. It was then reviewed by a panel of three experts in the area of survey research, and modifications were made according to their recommendations.

Another step to strengthen trustworthiness was to take precautions so that the researcher's biases did not affect subject responses in the interviewing stage of the study. The wording of the questions, the order of the questions, and the researcher comments and reactions may have affected subject responses. To reduce the effect of researcher bias on the interpretation of interview responses, the interview underwent member checks so that the subjects of the interviews could judge the accuracy of the findings in the study. This is another of Creswell's (1998) recommended procedures for qualitative research.

Pre- and post-interviews were conducted for the observations to give the teachers the opportunity to explain their reasoning for approaching the lesson in the way chosen and to provide their perspective on the observed session. This helped to diminish the effect that researcher bias has in interpreting the teacher's actions.

Summary

In order to determine how teachers and students use graphing calculators in high schools of western Kansas and how teachers' beliefs in the teaching and learning of mathematics are related to the way in which graphing calculators are used, a three-phase study was conducted. The first component was a survey of high school mathematics teachers of western Kansas, the second component involved interviews of nine of the high school mathematics teachers who responded to the survey, and the third component involved classroom observations of six of the high school teachers interviewed. The survey gathered information about the teachers, their schools, the use of graphing calculators at the schools, obstacles to graphing calculator use, teachers' beliefs of how students should use graphing calculators, and teachers' beliefs on the effect that graphing calculators have on student learning. The interviews provided a deeper understanding of the survey responses and provided evidence of the survey responses, and the

observations provided evidence of the interview responses and examples of how graphing calculators are being used.

CHAPTER 4 - Results

Overview

The purpose of this study is to describe how graphing calculators are used in high schools of western Kansas and to determine the relationship between teachers' beliefs in teaching and learning mathematics and how graphing calculators are used in their classes. Five main questions were addressed in this study:

1. What are characteristics of mathematics teachers in western Kansas, including their level of education, teaching experience, preparation in the use of graphing calculators, and experience with graphing calculators?
2. In what mathematics courses do students use graphing calculators, how often do they use them, what calculator functions do they use, and to what extent are graphing calculators being used to learn mathematical concepts?
3. What beliefs do teachers have regarding the teaching and learning of mathematics?
4. What beliefs do teachers have regarding the use of graphing calculators?
5. What is the relationship between teachers' beliefs about teaching and learning mathematics and how graphing calculators are used?

Data were collected through surveys, interviews, and observations, and these data sources were used to answer these questions. Numerical survey data were analyzed quantitatively through the calculation of means, medians, and percentages. Correlation analyses were conducted on various teacher and school characteristics, and independent sample t-tests were conducted to determine mean differences on survey responses between selected groups. Interview information was collected to provide a deeper understanding of the survey responses, and observation information was collected to support the interview responses and provide examples of how graphing calculators are used. This chapter discusses the results of the data and information collected as related to these questions.

This chapter is divided into four main sections. The first section discusses information gathered in the survey as it relates to the five main questions of this study. The second section discusses information gathered from the interviews of nine high school mathematics teachers as it relates to questions 2, 3, 4, and 5 of this study. The third section contains descriptions of the

observations of six high school mathematics teachers, and this section discusses information from the observations as it relates to questions 2, 3, 4, and 5 of this study. The fourth section of this chapter triangulates the information from the surveys, interviews, and observations.

Survey Information

Characteristics of Mathematics Teachers of Western Kansas

Teacher responses to the short answer questions on the survey provided information to answer Question 1: What are characteristics of mathematics teachers in western Kansas, including their level of education, teaching experience, preparation in the use of graphing calculators, and experience with graphing calculators? With a 62% return rate of the surveys it is assumed that the survey responses provide an accurate representation of the population of western Kansas mathematics teachers, and with 88% of the schools represented by teachers who responded to the survey, it is assumed that the responses provide an accurate representation of graphing calculator use in western Kansas high schools.

Table 4.1 Highest Degree Obtained

Degree/Hours	BS	BS+15	BS+30	MA	MA+15	MA+30
Frequency/	25	22	61	11	13	24
Percent of Teachers	16%	14.1%	39.1%	7.1%	8.3%	15.4%

Table 4.2 Age Distribution

Age Group	20-29	30-39	40-49	50-59	60-69
Frequency/	29	34	36	47	10
Percent of Teachers	18.6%	21.8%	23.1%	30.1%	6.4%

Table 4.3 Years of Teaching Experience

Years Teaching	1-9	10-19	20-29	30-39	40-49
Frequency/	54	43	29	29	1
Percent of Teachers	34.6%	27.6%	18.6%	18.6%	0.6%

Table 4.4 Level of Graphing Calculator Expertise

Level	No Experience	Beginner	Intermediate	Advanced
Frequency/	9	26	86	32
Percent of Teachers	5.9%	17.0%	56.2%	20.9%

Table 4.5 Years of Teaching with a Graphing Calculator

Years of Teaching	0	1-5	6-10	11-15	16-20
Frequency/	20	51	50	17	3
Percent of Teachers	14.2%	36.2%	35.4%	12.1%	2.1%

Thirty-one percent of mathematics teachers in western Kansas who responded to the survey earned a master’s degree. The mean age of these teachers is 43 years, and the average length of teaching experience is over 16 years. They have used graphing calculators in their teaching for an average of six years, and one-fourth of the teachers have used graphing calculators for ten or more years. Approximately 6% report no experience with graphing calculators, 17% are at the beginner level, 56% are at the intermediate level, and 21% are at the advanced level of graphing calculator use. Sixty-four percent attended classes, workshops, or conferences to learn or increase their knowledge about graphing calculators, and 25% claimed that all of their graphing calculator knowledge was self-taught. Thirty-six percent attribute part or all of their graphing calculator knowledge to their colleagues, and 82% attribute part or all of their graphing calculator knowledge to self-teaching. Distributions for survey data regarding teacher characteristics are provided in Tables 4.1 – 4.5.

Table 4.6 Correlation Analysis of Teacher Characteristics

		Age	Years	Degree	Years GC Use	Expertise	Gender
Age	Pearson Correlation	1	.784***	.569***	.380***	-.178	-.013
	Sig. (2-tailed)	.	.000	.000	.000	.028	.876
	N	156	156	156	141	153	156
Years	Pearson Correlation	.784***	1	.546***	.520***	.081	-.119
	Sig. (2-tailed)	.000	.	.000	.000	.319	.138
	N	156	156	156	141	153	156
Degree	Pearson Correlation	.569***	.546***	1	.246*	.007	.054
	Sig. (2-tailed)	.000	.000	.	.003	.933	.499
	N	156	156	156	141	153	156
Years GC Use	Pearson Correlation	.380***	.520***	.246*	1	.404***	.010
	Sig. (2-tailed)	.000	.000	.003	.	.000	.907
	N	141	141	141	141	138	141
Expertise	Pearson Correlation	-.178	.081	.007	.404***	1	-.152
	Sig. (2-tailed)	.028	.319	.933	.000	.	.062
	N	153	153	153	138	153	153
Gender	Pearson Correlation	-.013	-.119	.054	.010	-.152	1
	Sig. (2-tailed)	.876	.138	.499	.907	.062	.
	N	156	156	156	141	153	157
*p<.003 (with Bonferroni adjustment .05/15)							
**p<.0007 (with Bonferroni adjustment .01/15)							
***p<.00007 (with Bonferroni adjustment .001/15)							

Various teacher characteristics were compared using correlation analysis to determine relationships. The results of this analysis are displayed in Table 4.6. Because 15 correlations were conducted, a Bonferroni adjustment (.05/15) was used to reduce the risk of a Type I error. As can be expected, the number of years of teaching experience was significantly related to the age of the teacher [$r(156) = .784, p < .003$], the highest degree obtained by the teacher [$r(156) = .546, p < .003$], and years of graphing calculator use [$r(141) = .520, p < .003$]. Older teachers

have taught longer than younger teachers, they have earned advanced degrees, and they have had the opportunity to use graphing calculators for a longer period of time. A significant positive relationship was also found between graphing calculator expertise and years of graphing calculator use [$r(138) = .404, p < .003$] indicating that teachers who have used graphing calculators for a longer period of time have a higher level of graphing calculator expertise than teachers who have not used them as long. There was also a positive significant relationship between the highest degree earned and years of graphing calculator use [$r(141) = .246, p < .003$] indicating that teachers with advanced degrees have used graphing calculators for a longer period of time than teachers who have not earned advanced degrees. No significant relationship was found between the number of years of teaching experience and expertise in graphing calculator use, between age and expertise in graphing calculator use, or between the highest degree obtained and expertise in graphing calculator use. There was no significant relationship between gender and any of the other factors.

Table 4.7 Correlation Analysis of School Characteristics

		Hispanic	Enroll	SE Status	Provide GC
Hispanic	Pearson Correlation	1	.733***	.392***	.032
	Sig. (2-tailed)	.	.000	.000	.692
	N	156	155	156	154
Enrollment	Pearson Correlation	.733***	1	.471***	.001
	Sig. (2-tailed)	.000	.	.000	.988
	N	155	155	155	153
SE Status	Pearson Correlation	.392***	.471***	1	.091
	Sig. (2-tailed)	.000	.000	.	.263
	N	156	155	156	154
Provide GC	Pearson Correlation	-.032	-.001	-.001	1
	Sig. (2-tailed)	.692	.988	.988	.
	N	154	153	153	155
* $p < .008$ (with Bonferroni adjustment .05/6)					
** $p < .002$ (with Bonferroni adjustment .01/6)					
*** $p < .0002$ (with Bonferroni adjustment .001/6)					

Various school characteristics were compared using correlation analysis to determine relationships. Hispanic enrollment was chosen as a characteristic for comparison because of the growing Hispanic population in western Kansas. The results of this analysis are displayed in Table 4.7. A Bonferroni adjustment (.05/6) was used to reduce the risk of a Type I error. Significant positive relationships were found between the enrollment of the school and the Hispanic population of the school [$r(155) = .733, p < .008$], between the enrollment of the school and the socio-economic status of the school [$r(155) = .471, p < .008$], and between the Hispanic population of the school and the socio-economic status of the school [$r(156) = .392, p < .008$]. Larger schools tend to have a higher percentage of Hispanic students and a higher percentage of low socio-economic students. No significant relationships were found between the ability of a school to provide graphing calculators and the school's enrollment, the school's Hispanic enrollment, and the school's socio-economic status.

Table 4.8 Correlation Analysis of Teacher Characteristics and School Characteristics

		Enroll	Hispanic	SE Status	Provide GC
Years	Pearson Correlation	-.150	-.175	-.081	-.040
	Sig. (2-tailed)	.063	.029	.315	.619
	N	155	156	156	154
Age	Pearson Correlation	-.077	-.045	.013	-.018
	Sig. (2-tailed)	.339	.575	.871	.822
	N	155	156	156	154
Degree	Pearson Correlation	-.042	-.025	-.063	-.006
	Sig. (2-tailed)	.606	.759	.432	.945
	N	155	156	156	154
Gender	Pearson Correlation	.031	.091	-.018	.198
	Sig. (2-tailed)	.698	.261	.827	.014
	N	155	156	156	154
Expertise	Pearson Correlation	.037	-.070	-.059	.212
	Sig. (2-tailed)	.651	.391	.466	.009
	N	152	153	153	151
Years GC Use	Pearson Correlation	-.182	-.204	-.025	.115
	Sig. (2-tailed)	.031	.015	.770	.175
	N	140	141	141	140
*p<.002 (with Bonferroni adjustment .05/24)					
**p<.0004 (with Bonferroni adjustment .01/24)					
***p<.00004 (with Bonferroni adjustment .001/24)					

Various teacher characteristics were compared with various school characteristics to determine relationships. The results of this analysis are displayed in Table 4.8. Because 24 correlations were conducted, a Bonferroni adjustment (.05/24) was used to reduce the risk of a Type I error. With the adjustment of the significance level at .002, no significant relationships were found between any of the teacher characteristics of teaching experience, age, degree, gender, graphing calculator expertise, and years of graphing calculator use and school

characteristics of enrollment, Hispanic enrollment, socio-economic status, and ability to provide graphing calculators.

Graphing Calculator Use in the Classroom

Survey responses provided information to answer Question 2: In what mathematics courses do students use graphing calculators, how often do they use them, what calculator functions do they use, and to what extent are graphing calculators being used to learn mathematical concepts? To answer this question, teachers were asked about the brands and models of graphing calculators being used, the ways in which graphing calculators are used by their students, the extent at which schools provide graphing calculators for students, the course levels at which graphing calculators are required or not allowed, and obstacles to graphing calculator use. The following sections discuss the information gathered for this question.

Calculator Brands and Models

Three brands of graphing calculators are used by students in high schools of western Kansas: Texas Instruments, Casio, and Hewlett-Packard. The most widely used brand of graphing calculator is Texas Instruments with 98% of the teachers indicating that Texas Instruments graphing calculators are used in their schools. Twelve percent of the teachers reported the use of Casio graphing calculators in their schools, and less than 2% reported the use of Hewlett-Packard graphing calculators. The most widely used Texas Instrument models are various types of TI-83/84 calculators with 87% of the teachers who indicated specific calculator models reporting their use. Twelve percent of the teachers who reported specific calculator models reported students using TI-89 and TI-92 graphing calculators that have computer algebraic system capabilities. Table 4.9 shows the number and percent of teachers who indicated the various brands and models of graphing calculators used by their students. Eight teachers indicated that their students use Texas Instruments graphing calculators but did not indicate a particular model.

Table 4.9 Graphing Calculator Brands and Models Used by Students

Brand/Model	Up to TI 82	TI 83	TI 84	TI 85/86	TI 89/92	Casio	HP
Frequency/	36	100	43	22	15	15	2
Percent of Teachers	29.8%	82.6%	35.5%	18.2%	12.4%	12.4%	1.7%

Student Access to Graphing Calculators

Schools vary in their approach to providing graphing calculators for students. Factors that determine how graphing calculators are provided include the level of the mathematics class, the supply of calculators available, and security of the calculators. Ways in which the policies vary are discussed throughout the next few paragraphs.

Seventy-nine percent of the teachers surveyed reported that their school provides graphing calculators for students, but the level at which they provide them varies from in-class use only to possession of the calculator for the entire school year. Table 4.10 contains information about the ways in which teachers allow students access to graphing calculators provided by their school.

Table 4.10 Access to School-Issued Graphing Calculators

Student Access	In-class Only – Higher-level Only	Take Home – All Students	In-class Only – All Students	Take Home – Higher-level Only
Frequency/	17	22	31	49
Percent of Teachers	11.3%	14.6%	20.5%	32.5%

Twenty-one percent of the teachers surveyed reported that their schools provide graphing calculators for students to be used only in the classroom. An additional 11% of teachers surveyed reported that only higher-level mathematics students are allowed to use graphing calculators, and they may use them only in the classroom. However, the distinction of classes that are considered higher-level varies between geometry, algebra II, pre-calculus, trigonometry, college algebra, and calculus. Fifteen percent of the teachers reported that all of their students were allowed to take them home, and 33% reported that only higher-level students were allowed to take them home. Of the teachers who allow students to take graphing calculators home, 14% issue graphing calculators for an entire semester or year. Eleven percent of these teachers issue graphing

calculators to higher-level mathematics students, whereas 3% of these teachers issue graphing calculators to students at all levels of high school mathematics.

Twenty-one percent of the teachers surveyed allow graphing calculators to be taken home by students in higher-level classes, but students in lower-level classes are allowed to use graphing calculators in the classroom only. The distinction of classes that are considered higher-level varies between geometry, algebra II, pre-calculus, trigonometry, college algebra, and calculus. In general, permission to take a calculator home increases as the level of mathematics class increases.

Requiring, Allowing, and Not Allowing Graphing Calculators

Teachers vary in their approach to requiring, allowing, and not allowing graphing calculators in their classes. Approximately one-half of the teachers who responded to the survey require graphing calculators in at least one of their classes, 75% allow but do not require the use of graphing calculators in at least one of their classes, and 22% do not allow graphing calculators in at least one of their classes. Table 4.11 shows that graphing calculators are required more often in higher-level courses than in lower-level classes, and that graphing calculators are allowed or required in all mathematics courses above Algebra II (n = number of teachers who responded that they teach that particular class).

Table 4.11 Level of Allowed Use of Graphing Calculators by Class

Course	Require	Allow	Do Not Allow
Pre-Algebra, Consumer Math, Applied Math (n = 47)	9%	51%	40%
Algebra I (n = 114)	31%	59%	10%
Algebra II (n = 75)	40%	56%	4%
Geometry (n = 79)	5%	79%	16%
Trigonometry (n = 31)	71%	29%	0%
Calculus (n = 47)	72%	28%	0%
College Algebra (n = 35)	80%	20%	0%
Statistics (n = 12)	83%	17%	0%

Responses from teacher surveys indicate that teachers and schools vary in the way that they require, allow, and do not allow graphing calculators. Generally, the requirement to use graphing calculators increases as the level of the class increases, and teachers of all classes above the level of algebra II allow or require graphing calculators. Graphing calculators are not allowed in some lower-level classes, but they are allowed in most classes to some extent. The extent to which they are used will be discussed in the next section.

Frequency of Graphing Calculator Use

The frequency at which graphing calculators are used by students varies from daily use to rare use. On the survey teachers were asked to indicate the frequency of student use of graphing calculators in their classrooms. Table 4.12 shows the frequency of graphing calculator use in the classrooms of the teachers surveyed whose students use graphing calculators. Percentages are calculated for 135 teachers who responded to this question on the survey.

Table 4.12 Frequency of Graphing Calculator Use by Students

Frequency of GC Use by Students	Every Day	Several Times Per Week	Several Times Per Month	Less Than Once Per Month
Frequency/ Percent of Teachers	50 37.0%	41 30.4%	19 14.1%	25 18.5%

Students in about two-thirds of the classes of teachers who responded to the survey use graphing calculators no less than several times per week. Thirty-seven percent of the teachers use graphing calculators daily in their teaching, and 30% use them several times per week. Nineteen percent use them less than once per month. The amount of use varies within a teacher’s classes with more frequent use in higher-level courses than lower-level courses.

Allowable Use on Course Requirements

There is considerable variation in the ways in which teachers allow graphing calculators to be used by students. On the survey teachers were asked to indicate all of the course requirements in which they allow graphing calculators to be used by students from choices such as completing in-class activities and assignments, working on homework, taking quizzes, and taking tests. Table 4.13 displays the number and percent of teachers who responded to the survey

who allow their students to use graphing calculators for specific requirements. Percentages are calculated for 134 teachers who responded to this question on the survey.

Table 4.13 Allowable Use on Course Requirements

Requirement	Activities	Homework	Quizzes	Tests
Frequency/	129	106	114	119
Percent of Teachers	96.3%	79.1%	85.1%	88.8%

Ninety-six percent of the teachers surveyed allow their students to use graphing calculators on in-class activities and assignments, 79% allow graphing calculators to be used on homework, 85% allow them on quizzes, and 89% allow them on tests.

Student Uses of Graphing Calculator Capabilities

Survey data provided information about the ways in which students use graphing calculators. Teachers were asked to indicate all of the ways in which their students use graphing calculators from choices such as checking answers, performing calculations, participating in discovery exercises, and analyzing graphs. Table 4.14 displays the number and percent of teachers who responded to the survey who reported ways in which their students use graphing calculators. Percentages are calculated for 133 teachers who responded to this question on the survey.

Table 4.14 Student Uses of Graphing Calculators

Use	Check Answers	Perform Calculations	Discovery Exercises	Analyze Graphs
Frequency/	105	122	84	111
Percent of Teachers	78.9%	91.7%	63.2%	83.5%

Seventy-nine percent of the teachers surveyed indicated that their students use graphing calculators to check answers, 92% of the teachers indicated that their students perform calculations, 63% indicated that their students use them for discovery exercises, and 84% indicated that their students use them to analyze graphs.

Graphing calculators have a variety of functions and capabilities utilized by high school students. Survey data provided information about the capabilities of the graphing calculator used

by high school students. On the survey teachers were asked to indicate all of the graphing calculator capabilities used by their students from choices such as graphing functions, programming, calculating statistics, working with matrices, using symbolic algebra capabilities, and executing applications (APPS). Table 4.15 displays the number and percent of teachers who responded to the survey who reported the use of specific graphing calculator capabilities used by their students. Percentages are calculated for 132 teachers who responded to this question on the survey.

Table 4.15 Graphing Calculator Capabilities Used by Students

Capability	Graphing Functions	Programming	Stats	Matrices	Symbolic Algebra	APPS
Frequency/	114	23	68	66	15	29
Percent of Teachers	86.4%	17.4%	51.5%	50.0%	11.4%	22.0%

Eighty-six percent of the teachers surveyed indicated that their students use graphing calculators to graph functions, 17% indicated that their students write programs on their graphing calculators, 52% indicated that their students utilize statistical functions, 50% indicated that their students manipulate matrices, 11% indicated that their students perform calculations using symbolic algebra capabilities, and 22% indicated that their students execute applications (APPS).

Obstacles to Graphing Calculator Use

Obstacles prevent schools from implementing graphing calculators as fully as they would like. Survey statements 18–25 were used to determine how obstacles such as lack of calculator access, inadequate funding, lack of personal knowledge and confidence, lack of teaching time, inadequate student ability, inadequate supply of calculators, lack of administrative support, and lack of departmental support hindered them from using graphing calculators. Table 4.16 shows the relative frequency of each response.

Table 4.16 Responses to Survey Statements on Obstacles to Graphing Calculator Use

Statement	SD	D	U	A	SA
18. I have difficulty gaining access to graphing calculators for use in my classroom.	39%	26%	9%	21%	5%
19. It is difficult to get funds to buy graphing calculators.	17%	25%	15%	32%	11%
20. I lack the knowledge and confidence to teach using graphing calculators.	34%	35%	13%	12%	6%
21. I often do not have time to teach both the required algebra curriculum and graphing calculator technology.	14%	31%	14%	32%	9%
22. Most students lack the ability to work with a calculator as complex as a graphing calculator.	29%	44%	12%	12%	3%
23. I have enough calculators for individual student use.	19%	30%	6%	25%	20%
24. My administration encourages the use of graphing calculators.	1%	7%	40%	32%	20%
25. Teaching with a graphing calculator is a high priority in my department.	8%	24%	20%	31%	17%

While only 43% of the teachers surveyed reported that it is difficult to get the funds to buy graphing calculators, 65% disagreed or strongly disagreed that they have difficulty in acquiring graphing calculators for use in their classrooms. Sixty-nine percent of the teachers surveyed reported that they are knowledgeable and confident in using graphing calculators, and 73% reported that most of their students have the ability to work with a graphing calculator. Forty-five percent reported that they have enough time to teach the curriculum as well as how to use the graphing calculator, but almost just as many, 41%, reported that they do not have enough time to do both. Fifty-two percent reported that the administration encourages the use of graphing calculators, but 40% are not sure how the administration feels about graphing calculator use. Forty-eight percent indicated that graphing calculator use is a high priority in their department whereas 32% do not think that graphing calculator use is a high priority in their department.

Table 4.17 Correlation Analysis of Obstacles and School Characteristics

	Hispanic	Enroll	SE Status	Provide GC
Obstacles Pearson Correlation	.226*	.255**	.122	-.367***
Sig. (2-tailed)	.005	.001	.129	.000
N	155	154	155	153
*p<.0125 (with Bonferroni adjustment .05/4)				
**p<.0025 (with Bonferroni adjustment .01/4)				
***p<.00025 (with Bonferroni adjustment .001/4)				

Information from the surveys provided data used to identify relationships between obstacles to graphing calculator use and various school and teacher characteristics. A bivariate correlation was conducted between obstacles to graphing calculator use and school characteristics, and the results of this analysis are displayed in Table 4.17. A Bonferroni adjustment (.05/4) was used to reduce the risk of a Type I error. A significant positive relationship was found between obstacles to using graphing calculators and school enrollment [$r(154) = .255, p < .0125$]. This implies that teachers in schools with higher enrollment perceive more obstacles to using graphing calculators than smaller schools. The relationship between obstacles to using graphing calculators and the Hispanic enrollment of the school was found to be significant [$r(155) = .266, p < .0125$]. Teachers of schools with a higher Hispanic enrollment perceive more obstacles to graphing calculator use than schools with a lower Hispanic enrollment. A significant negative relationship exists between obstacles and schools that provide graphing calculators [$r(153) = -.367, p < .0125$] indicating that teachers of schools that provide graphing calculators for students perceive fewer obstacles to graphing calculator use. No significant relationship was found between obstacles and the school's socio-economic status.

Table 4.18 Correlation Analysis of Obstacles and Teacher Characteristics

	Years	Age	Gender	Degree	Yrs GC Use	Expertise
Obstacles Pearson Correlation	-.191	-.054	-.045	-.165	-.437***	-.500***
Sig. (2-tailed)	.017	.506	.582	.040	.000	.000
N	155	155	155	155	140	152
*p<.008 (with Bonferroni adjustment .05/6)						
**p<.002 (with Bonferroni adjustment .01/6)						
***p<.0002 (with Bonferroni adjustment .001/6)						

A bivariate correlation was conducted between obstacles to graphing calculator use and teacher characteristics, and the results of this analysis are displayed in Table 4.18. Because six correlations were conducted, a Bonferroni adjustment (.05/6) was used to reduce the risk of a Type I error. Significant negative relationships were found between obstacles and the teacher’s years of graphing calculator use [$r(140) = -.437, p < .008$] and between obstacles and the teacher’s expertise with graphing calculators [$r(152) = -.500, p < .008$]. Fewer obstacles to graphing calculator use are perceived by teachers have used graphing calculators for a longer period of time and possess a higher level of expertise with graphing calculators. No significant relationship was found between obstacles to graphing calculator use and degree, gender, or age.

Rule-Based versus Non-Rule-Based Beliefs

The level of a teacher’s rule-based beliefs was determined from responses to survey statements 26–29. Teachers were asked to respond to statements that compare the importance of knowing why a procedure works to knowing how to do the procedure; compare mathematics to memorizing facts and rules; compare algebra to exploring problems, discovering patterns, and making generalizations; and question how well students learn through discovery activities. Table 4.19 shows the relative frequency of each response.

Table 4.19 Responses to Survey Statements on Teacher Beliefs

Statement	SD	D	U	A	SA
26. When learning algebra, knowing why a procedure works is not as important as knowing how to do the procedure.	26%	48%	8%	14%	4%
27. Learning mathematics is mostly memorizing a set of facts and rules.	36%	52%	5%	7%	0%
28. Learning algebra means exploring problems to discover patterns and make generalizations.	1%	6%	8%	71%	14%
29. Students learn a concept better when they discover the concept in an activity.	0%	6%	20%	62%	12%

Seventy-four percent of the teachers surveyed agreed or strongly agreed that knowing why a procedure works is as important as knowing how to do the procedure. Seven percent indicated that learning mathematics is mostly memorizing a set of facts and rules, and of the 7%, none of the teachers strongly agree with this statement. Eighty-five percent agreed or strongly agreed that learning algebra means exploring problems, discovering patterns, and making generalizations, and 84% indicated that students learn better through discovery activities. The percentages of these responses indicate that teachers generally report having non-rule-based beliefs.

The statements were assigned values from 1 to 5 in such a way that a participant who holds rule-based teaching beliefs has a lower sum for responses to statements 26–29 than a participant who holds non-rule-based teaching beliefs. Even though it is possible for the sum of the scores to range from 4 to 20, teacher sums for this survey ranged from 10 to 20 indicating that teachers generally report non-rule-based beliefs. However, the sum of scores can be used to identify the strength at which a teacher’s beliefs are rule-based or non-rule based. In order to divide the teachers into a rule-based group and a non-rule-based group for comparisons, a sum of 16 was used as the dividing value for the groups. This value was chosen since the average teacher sum was 15.7, and the median sum was 16, and this choice provided groups that were the

Table 4.21 Correlation Analysis Between Teaching Beliefs and Teacher Characteristics

	Age	Years	Years GC Use	Gender	Degree	Expertise
Teaching Pearson Correlation	.096	.032	.166	.208	.220*	.158
Beliefs Sig. (2-tailed)	.237	.693	.050	.010	.006	.053
N	155	155	140	155	155	152
*p<.008 (with Bonferroni adjustment .05/6)						
**p<.002 (with Bonferroni adjustment .01/6)						
***p<.0002 (with Bonferroni adjustment .001/6)						

Survey information was used to determine relationships between teaching beliefs and various teacher characteristics. A bivariate correlation was conducted between teaching beliefs and the variables of a teacher’s age, years of teaching, years of graphing calculator use, gender, highest degree earned, and expertise with graphing calculators. The results of this analysis are in Table 4.21. A Bonferroni adjustment (.05/6) was used to reduce the risk of a Type I error. A significant relationship was found between teaching beliefs and the highest degree earned by the teacher [$r(153) = .220, p < .008$]. Teachers with higher degrees tend to hold beliefs that are more non-rule-based than teachers who have not earned higher degrees. No significant relationship was found to exist between teaching beliefs and a teacher’s age, years of teaching, years of graphing calculator use, gender, or graphing calculator expertise.

Teacher Beliefs of Graphing Calculator Use

Survey statements 30–36 were used to determine teacher beliefs in how graphing calculators should be used in instruction. Teachers responded to statements regarding when graphing calculators should be introduced to students, how soon they should be used in learning a concept, to what degree they should be used to learn a concept, whether or not they should be used in testing, and whether or not graphing calculators with algebraic symbolic capabilities should be used. Table 4.22 shows the relative frequency of each response.

Table 4.22 Responses to Survey Statements on Graphing Calculator Use

Statement	SD	D	U	A	SA
30. When graphing calculators are used in instruction, students should first solve algebraically and support graphically.	1%	13%	20%	60%	6%
31. When graphing calculators are used in instruction, students should solve graphically only when algebraic methods are too difficult.	6%	62%	18%	12%	2%
32. Students should use graphing calculators only after they have mastered a concept or procedure.	3%	36%	18%	35%	8%
33. Students should not be introduced to graphing calculators before they are in higher-level algebra, trigonometry, or calculus classes.	12%	43%	14%	24%	7%
34. Calculators should only be used to check work.	22%	61%	9%	8%	0%
35. Students should be permitted to use graphing calculators on ALL tests.	10%	39%	16%	31%	4%
36. Students should be permitted to use graphing calculators that have algebraic symbolic manipulator capabilities (like the TI-89 or TI-92) in algebra classes.	16%	34%	35%	15%	0%

Sixty-six percent of the teachers surveyed agreed or strongly agreed that students should solve problems first and then support the answer graphically. Sixty-eight percent of the teachers disagreed or strongly disagreed with the statement that graphing calculators should be used only when algebraic methods are too difficult. Teacher responses are somewhat mixed on whether students should be allowed to use graphing calculators before they have mastered a concept with 43% agreeing or strongly agreeing that the students should master the concept before using the calculator and 39% disagreeing or strongly disagreeing with that statement. Fifty-five percent of the teachers indicated that students should be allowed to use graphing calculators prior to taking high level mathematics classes, but 31% indicated that the students should not be allowed to use them until they are in high level mathematics classes. Only eight percent indicated that graphing

calculators should be used only to check work. Thirty-five percent of the teachers agreed or strongly agreed that students should be allowed to use graphing calculators on all tests while 49% disagreed or strongly disagreed with that statement. Fifty percent of the teachers disagreed or strongly disagreed that students should be allowed to use calculators with symbolic algebra capabilities in algebra classes, and 35% were unsure if students should be allowed to use such calculators.

In summary, responses to statements 30 and 36 indicate ways in which teachers believe that graphing calculator use should be limited, and overall the results indicate that the teachers surveyed favor limiting graphing calculator use. Responses to statements 31 and 34 indicate ways in which teachers believe that graphing calculator use should be unlimited, and from the responses to these statements, it seems that the teachers surveyed do not favor limiting graphing calculator use. This difference as well as the spread of responses to statements 32, 33, and 35 indicates a wide range of beliefs about limiting graphing calculator use.

Effects on Student Learning

Teacher beliefs about the effects that graphing calculators have on student learning were investigated using survey statements 37–42. Teachers were asked to respond to statements regarding graphing calculator influence on the amount of detail and level of difficulty of mathematics topics, student ability to solve problems that they could not solve previously, student learning, multiple representations, student dependency on calculators, and student ability to think. Teacher beliefs of the effects that graphing calculators have on student learning are generally positive. Table 4.23 shows the relative frequency of each response.

Table 4.23 Responses to Survey Statements on Student Learning

Statement	SD	D	U	A	SA
37. Graphing calculators allow for greater detail and/or difficulty of algebraic topics than in classes that are not using graphing calculators.	1%	8%	19%	63%	9%
38. A graphing calculator can be used to solve problems that the students could not solve before.	2%	12%	15%	59%	12%
39. Using a graphing calculator to teach mathematics enhances student learning or understanding of concepts.	1%	3%	17%	59%	20%
40. Using a graphing calculator allows the student to develop multiple representations of a problem.	0%	3%	9%	68%	20%
41. When students use graphing calculators on a regular basis, they become dependent on them and are unable to master basic algebraic manipulations.	2%	36%	31%	28%	3%
42. If students are taught to use the graphing calculator, they will rely on it and lose their ability to think.	14%	53%	20%	12%	1%

Seventy-two percent agreed or strongly agreed that they can teach with greater detail and difficulty than if they were not using the calculator, and 71% agreed or strongly agreed that students are able to solve problems using the graphing calculator that they were not able to solve previously. Seventy-nine percent agreed or strongly agreed that student learning is enhanced by using graphing calculators, and only 4% disagreed or strongly disagreed with this statement. Eighty-eight percent of the survey respondents reported that they believe that graphing calculators allow students to develop multiple representations of problems.

Teacher beliefs were somewhat mixed when it comes to student dependency on graphing calculators. Thirty-one percent agreed or strongly agreed that students become dependent on them whereas 38% disagreed or strongly disagreed that they will become dependent on them. However, 67% of the teachers indicated that students will not lose their ability to think.

Relationship Between Teaching Beliefs and Graphing Calculator Beliefs

The relationship between teacher beliefs of how mathematics should be taught and teacher beliefs of graphing calculator use was investigated by using composite scores for survey statements pertaining to teacher beliefs (statements 26–29) and composite scores for survey statements pertaining to graphing calculator beliefs (statements 30–36). A bivariate correlation was conducted between teacher beliefs of how mathematics should be taught and teacher beliefs of how graphing calculators should be used, and the results of this analysis are in Table 4.24. The relationship was found to be significant [$r(155) = .213, p < .05$] indicating that rule-based teachers favor a more limited use of calculators than non-rule-based teachers.

Table 4.24 Correlation Analysis Between Teaching Beliefs and GC Beliefs

	GC Beliefs
Teaching Beliefs Pearson Correlation	.213**
Sig. (2-tailed)	.008
N	155
* $p < .05$	
** $p < .01$	
*** $p < .001$	

A comparison of mean composite scores was made between rule-based teachers and non-rule-based teachers using an independent samples t-test on their responses to statements 30–36. A Bonferroni adjustment ($.05/7$) was used to reduce the risk of a Type I error. Table 4.25 shows statements that had significant results at the significance level of .007.

Table 4.25 Comparison of Means for Graphing Calculator Use

Statement	Non-Rule-Based	Rule-Based	t	Sig.
32. Students should use graphing calculators only after they have mastered a concept or procedure.	N=89 \bar{x} =2.89 SD=1.08	N=66 \bar{x} =3.35 SD=.984	-2.76	.006
34. Calculators should only be used to check work.	N=89 \bar{x} =1.85 SD=.716	N=66 \bar{x} =2.26 SD=.829	-3.18	.002

Table 4.25 suggests that there is a significant difference in how teachers responded to these two statements depending upon their beliefs in how mathematics should be taught. The mean composite scores for rule-based teachers were significantly higher than the mean composite scores for non-rule-based teachers for each statement indicating that, on average, rule-based teachers agreed more strongly than non-rule-based teachers that students should master a concept or procedure before using graphing calculators and calculators should only be used to check work.

Relationship Between Teaching Beliefs and Beliefs of Student Learning

The relationship between teacher beliefs of how mathematics should be taught and teacher beliefs of student learning was investigated using the composite scores for survey statements pertaining to teacher beliefs (statements 26–29) and the composite scores of statements pertaining to teacher beliefs of the effects that graphing calculators have on student learning (statements 37–42). A bivariate correlation was conducted between teacher beliefs of how mathematics should be taught and teacher beliefs of the effects that graphing calculators have on student learning, and the results of this analysis are in Table 4.26. The relationship was significant [$r(155) = .357, p < .05$] indicating that non-rule-based teachers believe more strongly that graphing calculators positively affect student learning.

Table 4.26 Correlation Analysis Between Teaching Beliefs and Student Learning Beliefs

	Student Learning
Teaching Beliefs	Pearson Correlation Sig. (2-tailed) N
	.357*** .000 155
*p<.05 **p<.01 ***p<.001	

A comparison of mean composite scores was made between rule-based teachers and non-rule-based teachers using an independent samples t-test on their responses to statements 37-42. A Bonferroni adjustment (.05/6) was used to reduce the risk of a Type I error. Table 4.27 shows the statement that had significant results at the significance level of .008.

Table 4.27 Comparison of Means for Student Learning

Statement	Non-Rule-Based	Rule-Based	t	Sig.
42. If students are taught to use the graphing calculator, they will rely on it and lose their ability to think.	N=89 \bar{x} =2.15 SD=.806	N=66 \bar{x} =2.62 SD=.973	-3.23	.002

Table 4.27 suggests that there is a significant difference in how teachers responded to this statement depending upon their beliefs in how mathematics should be taught. The mean composite score for non-rule-based teachers was significantly higher than the mean composite score for rule-based teachers on statement 42 indicating that, on average, rule-based teachers agreed more strongly than non-rule-based teachers that students will lose their ability to think if they use graphing calculators.

Relationship Between Graphing Calculator Beliefs and Beliefs of Student Learning

The relationship between graphing calculator beliefs and teacher beliefs in student learning was investigated using composite scores for survey statements pertaining to graphing

calculator beliefs (statements 30–36) and the composite scores of statements pertaining to teacher beliefs of the effects that graphing calculators have on student learning (statements 37–42). A bivariate correlation was conducted between teacher beliefs of how graphing calculators should be used and teacher beliefs of the effects that graphing calculators have on student learning, and the results of this analysis are in Table 4.28. The relationship was significant [$r(155) = .421, p < .05$] indicating that teachers who favor unlimited graphing calculator use believe more strongly that graphing calculators positively affect student learning.

Table 4.28 Correlation Analysis Between GC Beliefs and Student Learning Beliefs

		Student Learning
GC Beliefs	Pearson Correlation	.421***
	Sig. (2-tailed)	.000
	N	155
* $p < .05$		
** $p < .01$		
*** $p < .001$		

Summary of Survey Information

Survey data provided information related to the five main questions of this study. This data provided information about the characteristics of high school mathematics teachers of western Kansas, ways in which graphing calculators are used in their classrooms, their beliefs in teaching mathematics, and their beliefs about the use of graphing calculators and student learning. The data related to teaching beliefs, graphing calculator beliefs, and beliefs about student learning were used to determine relationships between teacher beliefs of teaching mathematics and their beliefs of graphing calculator use. This information is compared later in this chapter with information gathered from interviews and observations.

Teacher Interviews

Teacher interviews were used to collect information regarding questions 2, 3, 4, and 5 of this study. Stratified purposeful sampling was used to select nine teachers to be invited to

participate in the interview phase. Statements 26–29 on the survey were used to determine the level of rule-based beliefs held by each teacher. The statements were assigned values from 1 to 5 in such a way that a participant who holds rule-based teaching beliefs has a lower sum for responses to statements 26–29 than a participant who holds non-rule-based teaching beliefs. Even though it is possible for the sum of the scores to range from 4 to 20, teacher sums for this survey ranged from 10 to 20 indicating that teachers generally hold non-rule-based beliefs. To provide two groups for comparison of rule-based beliefs, teachers whose composite scores were below 16 were placed in the rule-based group, and teachers with scores greater than or equal to 16 were placed in the non-rule-based group. Five teachers whose survey responses indicated rule-based beliefs and five teachers whose survey responses indicated non-rule-based beliefs were chosen to participate in the interviews. Two of the rule-based teachers declined to participate, one agreed to participate but was not able because of scheduling conflicts, and two did not respond. Two of the non-rule-based teachers agreed to participate, two declined, and one did not respond. Nine more teachers, five rule-based and four non-rule-based, were chosen to be invited for an interview. Three rule-based teachers agreed to participate, and two did not respond. All four of the non-rule-based teachers agreed to participate. This selection process resulted in three teachers from the rule-based group and six teachers from the non-rule-based group participating in the interview phase of the study. The scores of the nine teachers interviewed are shown in Table 4.29.

Table 4.29 Response Sum of Teacher Belief Statements for Teachers Interviewed

Teacher	Response Sum for Rule-Based Statements
Mr. Adams	13
Mr. Baker	13
Ms. Clark	15
Ms. Davis	18
Mr. Edwards	18
Mr. Fort	18
Ms. Green	19
Ms. Hays	19
Ms. Irwin	20

Graphing Calculator Use in the Classroom

Interview responses provided information to answer Question 2: In what mathematics courses do students use graphing calculators, how often do they use them, what calculator functions do they use, and to what extent are graphing calculators being used to learn mathematical concepts? To answer this question, teachers were asked to describe student uses of the graphing calculator that are required, permitted, or encouraged in their classrooms. They were to include ways in which students use graphing calculators in learning the lesson, working on problems in class, working on problems outside the classroom and at home, and on taking tests. They were asked to describe their school's, department's, or personal policy on the use of graphing calculators, and they were asked to identify factors that hinder the students from using them according to the policy. The following sections discuss the information gathered for this question.

Student Access to Graphing Calculators

The teachers interviewed reported various ways in which graphing calculators are provided for students that ranged from not providing graphing calculators to checking them out to the students for the semester or year. Survey data were used to organize the ways in which graphing calculators are provided for students, and interview information was used to describe the ways in which graphing calculators are provided. Table 4.30 contains survey and interview information regarding each teacher's policy on providing graphing calculators for students.

Table 4.30 Access to School-Issued Graphing Calculators

Teacher	How Schools Provide Graphing Calculators for Students
Mr. Adams	Provided for students in algebra I for in-class use only
Mr. Baker	Does not provide them for his algebra I students, but his school does provide graphing calculators for higher-level classes for in-class use only
Ms. Clark	Provided for all students for use in-class and at home
Ms. Davis	Provided for all students for several days for certain topics
Mr. Edwards	Provided for in-class use, but students may check one out for special situations
Mr. Fort	Does not provide graphing calculators for his students
Ms. Green	Provided for students who cannot afford one for long-term use
Ms. Hays	Provided for in-class use for lower-level classes; higher-level students may rent one for the entire year for \$10
Ms. Irwin	Provided for in-class use for algebra I, geometry, and advanced algebra; checked out for the entire year in calculus and trigonometry

Information from interviews yielded a variety of ways in which students are provided (or not provided) access to graphing calculators. Some teachers, such as Mr. Baker and Mr. Fort, do not provide graphing calculators for their students. Mr. Baker's school provides graphing calculators for students in higher-level classes for in-class use only, but because of the limited number of graphing calculators owned by the school, not all teachers have a classroom set. Therefore, teachers who teach lower-level classes such as algebra I and applied mathematics courses may not have graphing calculators available for students. Mr. Fort's school does not provide graphing calculators for students, so he encourages students to buy their own. He would like the school to adopt a policy for the students to be required to buy their own graphing calculator for algebra II courses and higher since many of them may need them in college mathematics courses.

Mr. Adams, Mr. Edwards, Ms. Hays, and Ms. Irwin provide graphing calculators for in-class use, and some of these teachers allow graphing calculators to be taken home in certain situations or by students in higher-level classes. Mr. Adams' school has graphing calculators that can be used by students in algebra I class, but students in courses at the algebra II level and

higher are required to buy their own. Mr. Edwards’ school prefers that calculators remain in the classroom, but calculators may be taken home for use on the ACT test or homework if the teacher carefully documents each occurrence. Each teacher has a classroom set and is responsible for that particular set of calculators so teachers are especially careful when issuing calculators to students. Ms. Hays issues graphing calculators to advanced mathematics and college algebra classes for a \$10 fee, but students in other classes may use the school’s calculators only in the classroom. Ms. Irwin issues graphing calculators to her trigonometry and calculus students for the year, but her students in lower-level classes may use them in the classroom only.

Ms. Clark, Ms. Davis, and Ms. Green allow all students to take graphing calculators home. Ms. Clark allows students to take graphing calculators home when needed for homework. Ms. Davis allows students to take graphing calculators home for several days when working on certain topics. Ms. Green’s school issues graphing calculators for extended periods of time to students who cannot afford to buy their own in all levels of mathematics classes.

Requiring, Allowing, and Not Allowing Graphing Calculators

Teachers vary in the degree to which they require, allow, and not allow graphing calculators in their classrooms. Policy on how graphing calculators are to be used varies from no known school policy to policies that are established by the teacher. Table 4.31 shows the variety of policies reported in the interviews.

Table 4.31 Policy for the Use of Graphing Calculators

Teacher	Policy
Mr. Adams	Require graphing calculators for algebra II and higher courses
Mr. Baker	Use of graphing calculators in class only
Ms. Clark	No school policy, teacher’s discretion
Ms. Davis	No school policy, teacher’s discretion
Mr. Edwards	Use graphing calculators in class except for special instances
Mr. Fort	No school policy, teacher’s discretion
Ms. Green	No school policy, teacher’s discretion
Ms. Hays	No school policy, teacher’s discretion
Ms. Irwin	Graphing calculators issued to calculus and trig students for the year

Most of the schools of the teachers interviewed do not have a school policy on how graphing calculators were to be used, so decisions related to graphing calculator use are made by each teacher or by the mathematics department. Mr. Adams, Mr. Baker, Mr. Edwards, and Ms. Irwin reported the policies followed by their respective schools. Mr. Adams indicated that his school requires each student in a class at the level of algebra II and higher to have his/her own graphing calculator, preferably a TI-83 or TI-84. Mr. Baker's school does not have enough graphing calculators for all of the students in mathematics classes, so classroom sets of calculators are used by students in higher-level classes. The calculators are to be used only in the classroom so that students in every class that meets in that classroom during the day have access to them. Mr. Edwards' school policy is that the calculators are to be used in the classroom. Teachers may allow students to take them home, but this is to be done in rare situations such as an occasional homework assignment or for ACT testing. The teachers must keep accurate records of when a calculator is checked out including the student's signature. Ms. Irwin's school checks out graphing calculators to trigonometry and calculus students for the entire year. Students in lower-level classes may use the classroom set of calculators in the classroom only.

Ms. Clark, Ms. Davis, Mr. Fort, Ms. Green, and Ms. Hays indicated that their school did not have a policy on graphing calculator use, and decisions on how graphing calculators are to be used are made by the instructor. Also, decisions not covered by the policies described in the previous paragraph are made by the respective teachers. Survey data were used to organize the ways in which the teachers interviewed require, allow, and do not allow graphing calculators in their classes, and interview information was used to describe the ways in which they are allowed or required. Table 4.32 displays information from the surveys and interviews of classes in which each teacher interviewed requires, allows, and does not allow graphing calculators in the classroom.

Table 4.32 Level of Allowed Use of Graphing Calculators

Teacher	Require	Allow (but not require)	Do Not Allow
Mr. Adams	Algebra II, Trigonometry, Calculus		
Mr. Baker		Algebra I, Applied Geometry	
Ms. Clark	Algebra I, Algebra II	Integrated Math	
Ms. Davis	Applied Math II, Applied Math III	Applied Math I	
Mr. Edwards		Algebra II, College Algebra, Trigonometry, Pre-Calculus, Calculus	
Mr. Fort	Algebra II, Trigonometry, Calculus	Algebra I, Geometry	
Ms. Green	Algebra II, Advanced Math/Trig	Geometry	
Ms. Hays	College Algebra	Advanced Math	
Ms. Irwin	Algebra, Advanced Algebra, Calculus		Fundamental Math

Only one teacher, Ms. Irwin, reported a class in which graphing calculators have not been allowed. She has struggled for ways in which she can incorporate their use in her fundamental mathematics class since the course involves concepts at a basic skill level, but she plans to use them in preparation for the state mathematics assessments for concepts such as box-and-whisker plots and stem-and-leaf plots. She thinks that the use of graphing calculators can be motivational to these students who currently see other students in higher-level mathematics classes using them.

Several teachers allow but do not require graphing calculators to be used in lower-level mathematics classes, and one teacher, Mr. Edwards, allows but does not require their use in his higher-level classes of algebra II, college algebra, trigonometry, pre-calculus, and calculus. However, Mr. Edwards uses graphing calculators often in all of these classes. Mr. Baker allows

graphing calculators in his algebra I and applied geometry class, but he rarely uses them in these classes since not all students have access to graphing calculators. He has found it difficult to teach concepts when some students have access to the technology and others do not, and he has found it difficult for students to share calculators when too few students own one.

In general, the teachers interviewed required graphing calculators to be used in higher-level classes and allowed them to be used in lower-level classes. Graphing calculators are required by most of the teachers interviewed in classes at the algebra II level and higher. Mr. Adams requires graphing calculators for all of his classes at the algebra II level and above. He may teach algebra I next year, and he plans to allow the use of graphing calculators in that class so that the students become familiar with them. Ms. Clark requires graphing calculators in her algebra I and algebra II class, and she allows them to be used in her integrated mathematics class. Ms. Davis allows graphing calculators in her applied mathematics I class, and she requires their use in applied mathematics II and applied mathematics III. Mr. Fort requires graphing calculators in his algebra II, trigonometry, and calculus classes, and he allows them in his Algebra I and Geometry classes. Ms. Green requires their use in algebra II and advanced mathematics/trigonometry, and she allows them to be used in geometry. Ms. Hays allows graphing calculators to be used in her advanced mathematics class, and their use is required by the community college that offers the college algebra class that she teaches.

Frequency of Graphing Calculator Use

Teacher interviews provided information about the frequency of graphing calculator use in the mathematics classroom. The frequency of use varies among the teachers and the level of the classes. Survey data were used to organize the frequency of graphing calculator use as reported by each teacher interviewed, and interview information was used to describe how frequency of use varies among the teachers. Table 4.33 displays information collected in the survey and interviews about the frequency of graphing calculator use in the classrooms of the teachers interviewed.

Table 4.33 Frequency of Graphing Calculator Use by Students

Teacher	Frequency
Mr. Adams	Every day
Mr. Baker	Less than once per month
Ms. Clark	Several times per week
Ms. Davis	Several times per month
Mr. Edwards	Several times per week
Mr. Fort	Several times per week
Ms. Green	Several times per week
Ms. Hays	Every day or several times per month depending upon the class
Ms. Irwin	Every day or several times per month depending upon the class

Several teachers indicated that they use graphing calculators in their teaching daily in at least some of their classes. Mr. Adams, Ms. Hays, and Ms. Irwin reported daily use of graphing calculators in higher-level classes and less frequent use in lower-level classes. Mr. Adams uses graphing calculators daily in his algebra II, trigonometry, and calculus classes, but he plans to use them less frequently in the algebra I class that he may teach next year. Ms. Hays uses the graphing calculator daily in her college algebra class, and she uses them several times per month in her advanced mathematics class. Ms. Irwin uses the graphing calculator daily in her AP calculus class and several times per month in her algebra I class.

Ms. Clark, Mr. Edwards, Mr. Fort, and Ms. Green reported that they use graphing calculators several times per week in some or all of their classes. Ms. Clark uses graphing calculators several times per week, but she does not use them as often when starting a new chapter. Her policy is that students should learn the basics well before using the calculator, but once the basics are learned, then students use the calculators often for the rest of the chapter. Mr. Edwards has increased his use of graphing calculators since the school purchased graphing calculators for each classroom. He uses them several times per week in all of his classes. Mr. Fort uses them several times per week in all of his classes except algebra I. He rarely uses them in that class because he wants them to learn the basic processes of algebra. Ms. Green uses them often in her algebra II and advanced mathematics/trigonometry classes, but rarely uses them in geometry since there is not much need for her geometry students to use graphing calculators.

Mr. Baker and Ms. Davis rarely use graphing calculators in their teaching. Mr. Baker rarely uses graphing calculators in his algebra I class since he does not have a classroom set. He illustrates some concepts on the graphing calculator, but he does not have his students work with the calculator since there are not enough for everyone in the class. Ms. Davis uses graphing calculators several times per month in her applied mathematics classes. Her students are ones who have traditionally struggled with mathematics, and she allows them to use the graphing calculators so that they are familiar with their capabilities when they use them on assessments. She also believes that these students feel better about themselves since they are doing some of the same things that the advanced students are doing.

Allowable Use on Course Requirements

Teacher interviews provided information about ways in which the teachers allow graphing calculators to be used on course requirements. All of the teachers interviewed reported that their students are allowed to use graphing calculators on course requirements such as in-class activities and assignments, homework, quizzes, and tests, but at times the use of graphing calculators on tests is restricted.

Mr. Adams, Ms. Clark, and Ms. Irwin reported allowing unlimited use of graphing calculators on all course requirements except for quizzes and tests. Students are occasionally given quizzes and tests in which graphing calculators are not allowed. Mr. Adams allows unlimited use of graphing calculators on in-class activities, assignments, and homework in his higher-level classes, but he plans to restrict the use of them in the algebra I class that he may teach next year. He generally allows unlimited use on quizzes and tests, but he occasionally gives tests in which the graphing calculator is not allowed to make sure that the students do not become dependent on the calculator. Ms. Clark allows unlimited use of graphing calculators in all of her classes, but she occasionally restricts their use on certain parts of quizzes and tests to test students on basic skills. Ms. Irwin has a similar way of allowing graphing calculators on class requirements. She often separates tests into graphing calculator and non-graphing calculator sections. "When I allow them to use the calculator, it is that they need the calculator. I don't let them have a calculator so they can flounder around with it and maybe come up an answer that they otherwise wouldn't be able to by hand." Instead of giving tests or parts of tests in which graphing calculators are not allowed, Ms. Davis, Mr. Edwards, Mr. Fort, Ms. Green, and Ms. Hays reported that they require students to show work on homework and tests.

Some students may not have access to graphing calculators at home, so Mr. Edwards and Mr. Baker adjust their lessons and class time so that these students are not at a disadvantage. To limit the number of calculators checked out to students, Mr. Edwards allows class time for students to work on homework problems that require a graphing calculator. He alerts them as to which problems require graphing calculators so that they can complete them before the end of the class. If there is not enough time for them to complete all of the problems that require a graphing calculator, then he provides time at the beginning of the next day's class. Mr. Baker does not assign homework problems that require the use of a graphing calculator since many of his students do not have access to graphing calculators at home. He allows their use on quizzes and tests but mainly for calculations.

Student Uses of Graphing Calculator Capabilities

Teacher interviews provided information about the ways in which their students use graphing calculators. Survey data were used to organize student uses of graphing calculators and graphing calculator capabilities used by the students, and interview information was used to describe these uses and capabilities. Table 4.34 displays information gathered from the surveys about the ways in which each teacher's students use graphing calculators, and Table 4.35 displays information gathered from surveys about the graphing calculator capabilities used by students.

Table 4.34 Student Uses of Graphing Calculators

Teacher	Graphing Calculator Uses
Mr. Adams	Check answers, Perform calculations, Discovery exercises, Analyze graphs
Mr. Baker	Perform calculations
Ms. Clark	Check answers, Perform calculations, Discovery exercises, Analyze graphs
Ms. Davis	Perform calculations, Analyze graphs
Mr. Edwards	Check answers, Perform calculations, Analyze graphs
Mr. Fort	Check answers, Perform calculations, Discovery exercises, Analyze graphs
Ms. Green	Check answers, Perform calculations, Discovery exercises, Analyze graphs
Ms. Hays	Check answers, Perform calculations, Discovery exercises, Analyze graphs
Ms. Irwin	Perform calculations, Discovery exercises, Analyze graphs

Survey data reveal that all of the teachers mentioned that their students perform calculations, and all but Mr. Baker mentioned that their students analyze graphs. Six of them indicated that their students use graphing calculators to check answers, and six of them indicated that students use them for discovery exercises.

Table 4.35 Graphing Calculator Capabilities Used by Students

Teacher	Graphing Calculator Capabilities
Mr. Adams	Graphing functions, Statistics, Matrices
Mr. Baker	Did not respond to this survey question.
Ms. Clark	Graphing functions, Statistics, Matrices, Applications, Symbolic algebra capabilities
Ms. Davis	Graphing functions, Statistics
Mr. Edwards	Graphing functions, Statistics, Matrices
Mr. Fort	Graphing functions, Programming, Statistics, Matrices
Ms. Green	Graphing functions, Statistics, Matrices, Applications
Ms. Hays	Graphing functions, Statistics, Matrices
Ms. Irwin	Graphing functions, Programming (pre-programmed for motion detector, light sensor, etc. for CBL), Statistics, Matrices, integral and derivative commands in calculus, parametric equations

Information from the surveys and interviews reveal that students of all but one teacher interviewed graph functions on the graphing calculator, and students of seven of the teachers use the calculators for the statistical and matrix capabilities.

Obstacles to Graphing Calculator Use

In the interviews, teachers were asked to describe factors that hinder them from using graphing calculators. Of the teachers interviewed, only Mr. Baker, Mr. Edwards, and Mr. Fort identified obstacles that they have in using graphing calculators. Mr. Baker reported that the lack of a classroom set of graphing calculators hinders him from using them in his classes. Mr. Edwards reported that even though each classroom in his school has a set of graphing calculators for in-class use, students who do not have their own calculator are not able to do homework problems outside of class that require a graphing calculator. He allows time in class for these

problems to be done, but his students' use of graphing calculators is hindered by not having use of the calculators at all times. Mr. Fort reported that he is not able to incorporate graphing calculators as fully as he would like because not all students have their own graphing calculator.

The other teachers interviewed did not identify obstacles to using graphing calculators. Ms. Clark, Ms. Davis, and Ms. Green provide graphing calculators for their students. Ms. Hays and Ms. Irwin provide graphing calculators for students in higher-level classes, but even though their students in lower-level classes may use the classroom set of calculators in the classroom only, they did not identify the lack of access to graphing calculators outside the classroom as an obstacle. Mr. Adams provides graphing calculators for in-class use only, but he did not identify the lack of access to graphing calculators outside the classroom as an obstacle.

Typical Lessons

In the interviews, teachers were asked to describe a typical lesson in which the use of graphing calculators is incorporated in the teaching of the lesson. They were also asked to describe the way in which they had taught the lesson before they used graphing calculators in the classroom. Because of the focus of the question on a specific use of graphing calculators, the teachers did not elaborate on all of their survey responses listed in Table 4.34 and Table 4.35. Their responses did, however, provide the researcher with specific graphing calculator uses and capabilities. Several teachers described more than one typical lesson, and all of these lessons are included in the discussion of typical lessons. Lessons were coded according to two criteria. They were coded as rule-based or non-rule-based, and they were coded according to the way in which graphing calculators were used.

The teaching style for the lesson was coded as rule-based if the content of the lesson was taught as a set of rules, facts, or procedures. The teaching style for the lesson was coded as non-rule-based if the lesson involved exploring problems, discovering patterns, and making generalizations.

The type of graphing calculator use was coded as visualization if the calculator was used to present a visual representation of a concept. Visualization, along with data tables, is one of the multiple representations provided by the graphing calculator. The type of graphing calculator use was coded as efficiency if the calculator was used to perform a task more efficiently than by hand. This type of use was evident in lessons in which the students used graphing calculators to perform tasks on the graphing calculator to save time that would have been spent on numerous

calculations. The type of graphing calculator use was coded as motivation if the calculator was used as a motivational factor for students who may not ordinarily use graphing calculators. This was most commonly identified in lower-level classes in which graphing calculators may not ordinarily be used. The type of graphing calculator use was coded as a tool aiding in the solution of problems that are too difficult to solve by hand. In some cases, the solution may be possible without the graphing calculator, but the calculator allows the students to explore concepts that were previously beyond the scope of the course.

A typical lesson described by Mr. Adams was coded as non-rule-based. The lesson involved the students determining the amplitude, phase shift, and vertical shift of a sinusoidal function in trigonometry. Mr. Adams described how he presents the graph of the function to the students, and they use their graphing calculators to make adjustments to the parent function until they match his function. They are encouraged to use their analytical skills to determine the graph's function.

Mr. Adams' use of graphing calculators in this lesson was coded as visualization and efficiency. The students are able to see the changes that take place in the graph of the function when changes are made to the function, and they are able to graph functions quickly and receive instant feedback that they did not receive when graphing by hand.

Mr. Baker does not use graphing calculators often in his classroom since all of his students do not have access to them, so the students that have graphing calculators use them mainly for calculations. However, he did mention in his interview that, in the previous year, he had his students graph several linear functions to determine the effect that the slope and y-intercept have on the graph of the function. Prior to using the graphing calculator he used a similar lesson but the students graphed several functions by hand.

Mr. Baker's lesson was coded as non-rule-based since it involved exploring problems and making generalizations. Graphing calculator use was coded as visualization and efficiency since the students are able to see the changes in the graph of the function when changes are made to the function, and the students are able to graph more functions on the calculator than they would be able to graph by hand.

Ms. Clark described a lesson that was coded as non-rule-based, and her use of graphing calculators was coded as visualization and efficiency. Her typical lesson involved transformations of absolute value and quadratic functions. The students were asked to determine

how changes to a parent absolute value function or quadratic function affect the graph of the function, and because of the graphing calculator they were able to graph several functions quickly, observe the changes, and generalize the effect of certain changes. Prior to using the graphing calculator for this lesson, Ms. Clark's approach was rule-based. She gave the students the parent absolute value or quadratic function and told them about the changes that take place as values in the function are changed.

Ms. Clark also described a lesson that was coded as rule-based. The students use matrices to determine the equation of a parabola when given three points. Like she had done prior to using the graphing calculator for this lesson, she teaches her students the matrix fundamentals of solving a system of equations with two unknowns, but now she allows the use of the calculator for systems with three unknowns. Since graphing calculators allow her students to solve problems that had previously been beyond the scope of the class, her graphing calculator use was coded as tool.

Two typical lessons described by Ms. Davis were coded as rule-based. One lesson involved graphing linear functions, and the other lesson utilized the statistical capabilities of the calculator. She teaches the paper and pencil techniques before teaching the concepts with technology, and her approach is similar to how she taught the lessons prior to using the graphing calculator.

Ms. Davis' graphing calculator use for both lessons was coded as motivation. She teaches lower-level mathematics classes, and allowing her students to use graphing calculators motivates them since they are using the same technology as the advanced classes. The use of calculators in the statistics lesson was also coded as efficiency. The calculators were used to remove much of the calculation involved in the standard deviation so that the students are able to focus on the meaning of the standard deviation and use it to solve other problems.

One of the typical lessons described by Mr. Edwards was coded as non-rule-based. The lesson involved the regression capabilities of the calculator to determine the line or curve of best fit for a set of data. The students collect actual data to be used in this lesson, and they make predictions from the regression equation. Since the calculator provides the correlation coefficient and the coefficient of variation, the students determine how well the equation models the data.

Calculator use for Mr. Edwards' lesson was coded as a tool to allow his students to explore concepts that were previously beyond the scope of the course. Prior to using the graphing

calculator to teach this lesson, Mr. Edwards taught only linear regression by estimating the location of the line of best fit and choosing two points on the line to determine the equation of the line. The calculator has enabled his students to determine the regression equation for data that are not linear.

Mr. Edwards also described a lesson coded as rule-based, and the graphing calculator use was coded as visualization. In this lesson the students learn the concepts of quadratic equations and functions using the graphing calculator. The calculator is used to provide a visual representation of the curve, and students locate extreme values and x-intercepts graphically and algebraically. He continues to teach the algebraic methods that he used before using the graphing calculator to teach this lesson, but now he incorporates the graphing calculator for the visual representation that it provides.

One of Mr. Fort's typical lessons involves sinusoidal functions, and it was coded as non-rule-based. His students create a situation in which an object travels along the rim of a wheel that is partially submerged in water. With the help of a graphing calculator, the students model this situation and are able to determine how long the object is submerged during each revolution.

Mr. Fort's graphing calculator use for this lesson was coded as visualization. This project reinforced concepts such as phase shift and amplitude, and it allowed the students to see a visual interpretation of the situation. Prior to using graphing calculators, Mr. Fort taught this lesson similar to the way he does now except the students had to test points to see if they met certain conditions. Now the students are provided with a visual representation of the situation as well as a table of values that can help them to determine the behavior of the function.

Another of Mr. Fort's typical lessons was coded as rule-based, and the graphing calculator use was coded as efficiency and visualization. The lesson involves determining the maximum volume of a box or cylinder. The calculator provides a visual representation of the volume function, and calculator commands provide an efficient way of determining the maximum value. His approach to this lesson has not changed from how he taught the lesson prior to the use of graphing calculators in that the students must be able to write the function for the situation, but the method of solution has changed. Without graphing calculators the students tested values in the function to determine the maximum value.

A third lesson described by Mr. Fort was coded as rule-based, and his graphing calculator use was coded as tool and efficiency. In this lesson students utilize the matrix capabilities of the

calculator to quickly determine the equation of a parabola when given three points. A system of equations such as this may not have “nice” solutions, but the graphing calculator is able to solve for them. Prior to using the graphing calculator for matrix problems Mr. Fort used points that resulted in “nice” coefficients so that the solutions calculated by hand were not too difficult. Graphing calculators enabled him to assign problems that are more realistic.

Three typical lessons described by Ms. Green were coded as non-rule-based. The first lesson involves analyzing a quadratic function for roots through exploration on the graphing calculator. Graphing calculator use for this lesson was coded as visualization. After discussing how the roots can be determined on the graphing calculator, they discuss how these values can be calculated by factoring, completing the square, or using the quadratic formula. When this concept is extended to inequalities, the shade feature on the graphing calculator is used to show the solution to the inequality. Prior to using the graphing calculator in this lesson, Ms. Green had the students graph the parabolas by hand. She used a lecture approach in teaching this lesson, but now her approach involves some discovery and experimentation.

Graphing calculator use in the second typical lesson described by Ms. Green was coded as visualization. In this lesson Ms. Green has students graph a quadratic function, make changes to a parameter of the function, and explain what happens to the graph of the function. After changing that parameter several times, the students describe the effect that the parameter has on the function. She then changes a different parameter and repeats the process. In this way the students can “discover” the effect that the parameters have on the function without being told about the relationship. Ms. Green uses a similar discovery lesson with absolute value functions. She has her students graph the parent function $y = |x|$ and notice changes to the graph when a value is added inside the absolute value or outside the absolute value. They continue with subtraction, multiplication, and division of values inside or outside the absolute value. They generalize the effect that these changes have on the function.

Graphing calculator use in Ms. Green’s third lesson was coded as visualization and tool. Students use Skittles, a pizza box, and the regression capabilities of the graphing calculator to simulate population growth. After shaking the box, the students count the number of Skittles in which the “S” side is up. This signifies that each of these Skittles has reproduced, so another Skittle is added for each one. This process is repeated 15 times, and the total number of Skittles in the box is counted each time. The students enter this information in the calculator and

calculate the exponential function that best fits the growth of the population. They compare the model with what they actually produce with the Skittles. Prior to using the graphing calculator, Ms. Green's students explored situations that were linear, but now they are able to explore situations that are not linear.

Ms. Hays described a lesson on the transformation of functions that was coded as non-rule-based. She has the students determine how the function changes from the parent function as well as determine the changes to the equation needed to perform a desired transformation. The ability to graph quickly and receive instant visual feedback are reasons why she feels this lesson is valuable to the students in learning about transformations of functions, and therefore, graphing calculator use for this lesson was coded as visualization and efficiency. Prior to using the graphing calculator in this lesson, Ms. Hays had her students create t-tables and sketch the graphs by hand. This process was much slower than using the graphing calculator.

Three lessons described by Ms. Irwin were coded as non-rule-based, and the graphing calculator use for these lessons was coded as visualization. One of Ms. Irwin's lessons is a discovery lesson on linear equations. Students graph several linear equations that are in slope-intercept form, decide which value is the slope and which is the y-intercept, and describe the effect that the slope and the y-intercept have on the graph of the line.

A second lesson described by Ms. Irwin incorporates the use of a motion detector. The students walk in front of the motion detector, graph their movement on the graphing calculator, and determine the meaning of the slope and y-intercept as they relate to their movement. Prior to using the graphing calculator, Ms. Irwin taught this lesson through telling the students about the meaning of the slope and the y-intercept.

The third of Ms. Irwin's lessons is designed to teach her students to factor by using the x-intercepts from the graph of the function. The students use what they know about the graph of the function to determine how many factors the function has and where the roots are located. She feels that this lesson connects everything that they have done previously with factoring, and it provides the students with a way to approach factoring that is different than the procedures often taught. Prior to using the graphing calculator, Ms. Irwin taught this lesson by showing them the process for factoring.

Rule-Based versus Non-Rule-Based Practices

Of the 18 lessons described in the interviews, 12 were coded as rule-based and 6 were coded as non-rule-based. Table 4.36 displays the lessons described in teacher interviews as rule-based or non-rule-based.

Table 4.36 Rule-Based and Non-Rule-Based Practices Reported in Interviews

Teacher	Rule-Based Lessons	Non-Rule-Based Lessons
Mr. Adams		Transformation of Functions
Mr. Baker		Linear Functions
Ms. Clark	Matrices	Transformation of Functions
Ms. Davis	Graphing Linear Functions Calculating Statistics	
Mr. Edwards	Graphing Quadratic Functions	Regression
Mr. Fort	Maximum Volume Matrices	Sinusoidal Functions
Ms. Green		Roots of a Quadratic Function Transformation of Functions Skittle Lesson
Ms. Hays		Transformation of Functions
Ms. Irwin		Linear Functions Motion Detector Factoring

Six lessons described by the teachers interviewed were coded as rule-based. These lessons were coded this way because they involved teaching a set of rules, facts, and procedures. Rule-based lessons included Ms. Clark’s lesson on matrices, Ms. Davis’ lesson on graphing linear functions and her lesson on calculating statistics, Mr. Edwards’ lesson on graphing quadratic functions, and Mr. Fort’s lesson on maximum volume and his lesson on matrices.

Twelve lessons described by the teachers interviewed were coded as non-rule-based. In order for a lesson to be considered non-rule-based, it must have included one or more of the following features as described by the teacher: involve exploration of problems, discovery of patterns or concepts, and/or making generalizations. One type of non-rule-based lesson is on the

transformation of functions. Mr. Adams, Ms. Clark, Ms. Green, and Ms. Hays described lessons in which the students experiment with the graphing calculator to determine the effects that values in the function have on the graph of the function, and then they generalize their findings. Lessons by Mr. Baker and Ms. Irwin on linear equations are non-rule-based in that they are discovery lessons where the students are to determine the effects that the slope and y-intercept have on the graph of the line. This is particularly evident in Ms. Irwin's lesson that uses the motion detector. Students graph their movements and determine the effects that their movements have on the graph.

Mr. Fort's lesson on sinusoidal functions is another example of a non-rule-based lesson. In this lesson the students develop their own problem and devise their own solution to the problem. There is no predetermined procedure for the students to follow.

Another example of a non-rule-based lesson is Ms. Green's Skittle lesson. By modeling the situation first, she helps the students to develop an understanding of exponential functions without telling them what an exponential function is.

Ms. Irwin's lesson on factoring also involves discovery. Students combine their knowledge of x-intercepts with their knowledge of factoring to discover the connection between factoring and x-intercepts. Students generalize their findings after studying several examples.

Interviews provided information about typical graphing calculator lessons used by the teachers interviewed. These lessons were coded as rule-based or non-rule-based depending upon how the lesson was approached. Rule-based lessons were taught as a set of rules, facts, or procedures; and non-rule-based lessons were taught by exploring problems, discovering patterns, and making generalizations. Six teachers described more than one typical lesson, and three of these teachers described at least one rule-based lesson and at least one non-rule-based lesson.

Types of Graphing Calculator Use

The graphing calculator uses for the lessons described in the interviews were coded using categories of visualization, efficiency, tool, and motivation. In some cases the graphing calculator use had two codes assigned to it. Table 4.37 represents the types of graphing calculator uses coded for each teacher in the interviews. The numeral in each box represents the number of lessons in which the use was coded.

Table 4.37 Types of Graphing Calculator Use Coded in the Interviews

Teacher	Visualization	Efficiency	Tool	Motivation
Mr. Adams	1	1		
Mr. Baker	1	1		
Ms. Clark	1	1	1	
Ms. Davis		1		2
Mr. Edwards	1		1	
Mr. Fort	2	2	1	
Ms Green	3		1	
Ms. Hays	1	1		
Ms. Irwin	3			

Thirteen of the 18 lessons were coded as visualization. Calculators were used to provide a visual representation of an equation, inequality, or function. Seven were coded as efficiency since the calculator was used to create graphs or perform calculations more quickly than could be done by hand. Four were coded as a tool since the content of the lesson was extended beyond what had been the normal scope of the class. Two lessons were coded as motivation since students who ordinarily did not use graphing calculators were allowed to use them for motivational purposes.

Lessons that were coded as visualization involved analyzing graphs and graphing functions, and these lessons were designed to teach concepts such as transformations of functions, analyzing functions for x-intercepts and extreme values, and identifying x-intercepts as solutions to equations and values in factoring polynomials. All of the teachers who were interviewed reported using the graphing calculator for lessons of this type. The reasons given for using graphing calculators for this type of lesson included the visual representation and instant feedback provided by the calculator. The teachers did not change their teaching style much when they began using graphing calculators in these lessons, but calculators were used to replace paper-and-pencil graphing.

Lessons that were coded as efficiency involved statistical capabilities of the calculator and determining a curve of best fit for a set of data. Ms. Davis, Mr. Edwards, and Ms. Green reported using graphing calculators for these purposes, and the reason for using calculators was to easily perform repeated calculations. Graphing calculators changed the way in which the teachers had previously taught these lessons by providing an efficient way to model nonlinear situations and to determine values, such as standard deviation, that would otherwise be cumbersome.

Lessons that were coded as tool involved matrix capabilities of the graphing calculator and non-linear regression. Ms. Clark and Mr. Fort described using the matrix capabilities of the graphing calculator to solve systems of equations. Their teaching did not change much from how they taught prior to using calculators, but their students were able to solve larger and more complicated systems than before. Mr. Edwards and Ms. Green described how they had taught linear regression in the past, but with graphing calculators their students are able to model non-linear situations.

Ms. Davis described lessons that were coded as motivation. She uses the graphing calculator for her lessons on linear functions and statistics so that her students can experience using technology that normally only the students in higher-level classes use.

In the interviews teachers discussed ways in which they have changed their teaching because of the graphing calculator. The teachers reported that they teach the concepts of their typical lessons in much the same way that they did before incorporating graphing calculators in the lessons. Their use of graphing calculators has provided a visual representation of the concept, provided students with multiple examples of the concept quickly, provided students with a tool that enables them to work with more difficult problems than before, or provided them with motivation to learn mathematics with a tool commonly allowed for higher-level classes.

Summary of Graphing Calculator Use

Interviews provided information about the ways in which graphing calculators are used in nine high school mathematics classrooms. Teachers reported how they provide graphing calculators for students, the frequency of their use, obstacles to their use, ways in which they are allowed, and ways in which they are used. This section summarizes interview information provided by each teacher.

Mr. Adams described one non-rule-based lesson, and he uses graphing calculators for visualization and efficiency. He reported that his school provides graphing calculators for algebra I students to be used in the classroom only, and his school requires graphing calculators for students in courses at the algebra II level and higher. He did not report obstacles to using graphing calculators at his school. Mr. Adams' students use graphing calculators daily for tasks such as checking answers, performing calculations, discovering concepts, and analyzing graphs, and they utilize the graphing functions, statistics commands, and matrix capabilities of the calculator. He incorporates the use of graphing calculators in lessons on transformations of functions because of the visual representation and instant feedback provided by the graphing calculator. Prior to using graphing calculators his students graphed the transformations of functions by hand, and they were not able to produce as many examples with which to make generalizations.

Mr. Baker described one non-rule-based lesson, and he uses graphing calculators for the visualization and efficiency that graphing calculators provide. He reported the lack of graphing calculators as an obstacle to their use in his algebra I classroom since his school provides graphing calculators for students in upper-level classes for in-class use only. Because not all of his students have graphing calculators he does not use graphing calculators often in his teaching. He taught a lesson on graphing linear functions in a previous year, and the use of graphing calculators allowed his students to generate more examples than could be generated by hand. The students in his class who own a graphing calculator use it primarily for performing calculations.

Ms. Clark described one non-rule-based lesson and one rule-based lesson. The graphing calculator is used for visualization and efficiency, and it is a tool that allows her students to solve problems that they could not solve before using graphing calculators. Her students use graphing calculators several times per week. She requires her algebra I and algebra II students to use graphing calculators, and she allows her integrated mathematics students to use them. Ms. Clark has a classroom set of calculators, and she allows students to take them home when needed. Her students use graphing calculators to graph functions, calculate statistics, manipulate matrices, execute applications, and utilize symbolic algebra capabilities. They check answers, perform calculations, participate in discovery lessons, and analyze graphs. Graphing calculators allow her students to graph several functions quickly, helping them to make generalizations. The

calculator's matrix capabilities allow her students to solve problems that may otherwise be too difficult or impossible to solve.

Ms. Davis described two rule-based lessons, and she uses graphing calculators for efficiency and motivation. She reported that she allows the students in her applied mathematics I course to use graphing calculators, and she requires the students in her applied mathematics II and applied mathematics III classes to use graphing calculators. She uses graphing calculators several times per month in these classes to motivate the students and to provide them with graphing calculator experience since these students may never take a higher-level mathematics class in which graphing calculators are used. Her students use graphing calculators to graph and analyze functions, calculate statistics, and to perform calculations. She teaches the paper-and-pencil techniques before showing the graphing calculator procedures, but the graphing calculator relieves her students from cumbersome calculations and allows them to focus on the meaning of the result.

Mr. Edwards described one rule-based lesson and one non-rule-based lesson. He uses graphing calculators for visualization and as a tool. He reported that his school provides graphing calculators for in-class use by all students, and the calculators may be checked out for special purposes. Mr. Edwards provides class time for graphing calculator problems on homework so that there is no need for the students to take the calculators home. He allows the use of graphing calculators in his algebra II, college algebra, trigonometry, pre-calculus, and calculus classes, and his students use them several times per week. His students use the calculators to graph and analyze functions, calculate statistics, manipulate matrices, perform calculations, and check answers. His typical lessons include regression analysis and analyzing functions, and graphing calculators provide visual representations of functions and have allowed his students to model situations that are not linear.

Mr. Fort described two rule-based lessons and one non-rule-based lesson. His uses of the graphing calculator include visualization, efficiency, and a tool to extend concepts beyond what had normally been taught in the course. Because his school does not provide graphing calculators for his students and not all students own a graphing calculator, Mr. Fort reported that he is not able to incorporate them in his teaching as much as he would like. He allows them to be used in algebra I and geometry, and he requires them to be used in algebra II, trigonometry, and calculus. His students use graphing calculators several times per week for graphing and analyzing

functions, programming, calculating statistics, manipulating matrices, and participating in discovery exercises. Much of his teaching has remained the same as when he taught prior to using graphing calculators, but he now stresses the importance of visualizing situations in graphical form. Graphing calculators enable his students to solve realistic problems that may be too difficult to solve otherwise.

Ms. Green described three non-rule-based lessons, and she uses graphing calculators for visualization and as a tool. She reported that she allows graphing calculators to be used in geometry, and she requires them to be used in algebra II and advanced mathematics/trigonometry. Her school provides long-term use of graphing calculators for students who cannot afford them. Her students use them several times per week to graph and analyze functions, calculate statistics, manipulate matrices, execute applications, and participate in discovery activities. Ms. Green's typical lessons include analyzing functions, generalizing transformations of functions, and modeling situations through regression analysis. The graphing calculator has allowed her to incorporate more discovery learning in her teaching and to model situations that are not linear.

Ms. Hays described one non-rule-based lesson, and her graphing calculator uses include visualization and efficiency. She reported that she allows graphing calculators to be used in advanced mathematics and requires them to be used in college algebra. Her school provides graphing calculators for students in lower-level classes for in-class use only, and students in higher-level classes may rent one for the year for \$10. Her advanced mathematics class uses graphing calculators several times per month, and her college algebra class uses them daily. Her students use graphing calculators to graph and analyze functions, calculate statistics, manipulate matrices, and participate in discovery exercises. Her typical lesson involves the transformation of functions, and she uses graphing calculators for this lesson to allow her students to graph several transformations quickly and receive instant feedback.

Ms. Irwin described three non-rule-based lessons, and she uses graphing calculators for visualization. She reported that her school provides a classroom set of graphing calculators for in-class use only for algebra I, geometry, and advanced algebra classes, and her school checks out graphing calculators for the entire year to students in calculus and trigonometry. She requires graphing calculators to be used in algebra, advanced algebra, and calculus, but she does not allow them to be used in fundamental mathematics class since this class involves concepts at a

basic skill level. However, she may eventually allow students in this class to use them for motivational purposes and preparation for certain concepts that will be tested on the state mathematics assessment. The frequency of graphing calculator use depends upon the level of the class such that lower-level classes use them several times per month and higher-level classes use them daily. Her students utilize the graphing, programming, statistical, and matrix capabilities of the graphing calculator, and they typically use the calculator to participate in discovery exercises, to analyze graphs, and to perform calculations.

Teacher Beliefs of Graphing Calculator Use

Teacher interviews provided information about their beliefs of how graphing calculators should be used. In the interviews the teachers were asked to describe ways in which their students learn better because of graphing calculators, ways in which their students are dependent on graphing calculators, and ways in which graphing calculators have influenced their teaching practices. The following sections describe the teachers' beliefs of the effects that graphing calculators have on student learning, student dependency on graphing calculators, and ways in which their teaching has changed because of graphing calculators.

Effects on Student Learning

All of the teachers interviewed reported that they believe that graphing calculators enhance student learning by allowing them to learn more information at a deeper level than before. Mr. Adams commented, "I do think that I have been able to cover more material, more in depth than I ever did before." This is most evident in his lesson on transformations of sinusoidal functions. He believes that his students gain a deeper understanding of these functions since they can experiment with them on the graphing calculator instead of depending solely on algebraic methods.

Another aspect of graphing calculator use that was mentioned by eight of the teachers in the interviews was the visual representation provided by the calculator. Even though the teachers require students to determine a function's characteristics and important points by algebraic methods, they also encourage students to graph the function on a graphing calculator to help them visualize the function. As stated by Mr. Edwards, "I always try to stress to them they need to look at things more than one way. They can solve a problem algebraically, or numerically, and graphically, and so the calculator helps with the graphical perspective. It helps them make that

connection to what they are doing algebraically.” Ms. Clark also stated that visual representations contribute to student understanding. “If you show them, I think that helps them absorb and helps them understand better maybe where that is all coming from.” Ms. Green stated, “It seems like a lot of times they remember so much better when they have actually seen the process and see the physical picture of it.”

Mr. Edwards and Mr. Fort reported that the regression capabilities of the graphing calculator have allowed them to teach students about modeling situations that are not linear. Ms. Irwin uses a motion detector to collect data that are fit to a curve through regression analysis, providing her students with a realistic situation. Ms. Green uses the graphing calculator to simulate population growth in her Skittles lesson. By reducing the number of tedious calculations she feels that her students have a better understanding of harder problems. “Sometimes the grind of finding the answer isn’t necessarily what you want. You want to be able to use that result for some other reason.” She mentioned that she is able to assign more word problems that are more realistic than before.

Two teachers also mentioned that student motivation to learn was increased when using graphing calculators. Ms. Davis, who teaches lower-level mathematics classes for students who have struggled with mathematics in the past, allows her students to use the same technology as the students in higher-level mathematics classes. She commented, “I want them to have the same technology that is available to them, and I think that most of the kids feel better about themselves that they can do something the ‘smart kids’ are doing.” Ms. Green reported that her students explore the graphing calculator and learn mathematics without realizing it. They are aware that the calculator has many capabilities that she will not cover in class, so they explore various calculator functions for “things that they think are going to get them an answer.”

Student Dependency

Survey information indicated that 31% of the teachers surveyed believe that students become dependent on graphing calculators and 38% do not believe that students become dependent on them. Teacher interviews also reveal a variety of beliefs of student dependency on graphing calculators. Some teachers indicated that student dependency is not a problem at all, and others indicated that students occasionally depend on the graphing calculator to graph a function when mental methods should have been sufficient. The next few paragraphs include descriptions of ways that the teachers reported student dependency in their classrooms.

Five of the nine teachers interviewed, Mr. Baker, Ms. Clark, Ms. Davis, Ms. Green, and Ms. Hays, reported that the main problem with student dependency is with calculators in general. Of these five teachers, two of them, Mr. Baker and Ms. Clark, were identified as rule-based, and the other three were identified as non-rule-based. Ms. Clark reported that students in the middle school are allowed to use calculators for all calculations and have forgotten how to calculate with fractions. She periodically reviews calculations such as these with her high school students so that they become less dependent on the calculator. Ms. Green stated that her students are weak at calculating with fractions. Ms. Davis has her students use paper-and-pencil mathematics one day of each week to sharpen her students' computational skills.

Two teachers, Mr. Adams and Ms. Irwin reported a type of dependency specific to graphing calculators. They reported that their students occasionally use graphing calculators to graph functions such as lines and parabolas that they know well. They relate this type of graphing calculator dependency to basic calculator dependency of performing a simple calculation instead of using mental mathematics. They both stress to their students the importance of using the graphing calculator for a visual representation of a problem, but they think that the graphs of some functions are so basic that graphing calculators are not needed to visualize the function.

Another example of student dependency on graphing calculators is that students often trust the calculator's graph of a function without a clear idea if it is correct or not. Mr. Adams reported that students forget that the calculator has limitations and that certain characteristics, such as holes found in graphs of rational functions, do not graph properly. Because of the high level of confidence that students place on technology, they may not challenge the calculator's graph.

The final issue identified by the teachers as a student dependency issue was that students may begin to think that algebraic methods are not important since the calculator can do so much for them. A situation such as this was reported for calculators with symbolic algebra capabilities. Mr. Fort mentioned one student who did not believe that algebraic paper-and-pencil work was important since his TI-89 calculator could provide answers for him, but he was eventually convinced that algebraic methods were still important after the teacher gave him some problems that the calculator could not handle properly.

Eight of the nine teachers interviewed, Mr. Adams, Ms. Clark, Ms. Davis, Mr. Edwards, Mr. Fort, Ms. Green, Ms. Hays, and Ms. Irwin, reported measures that they take to ensure that their students do not become dependent on graphing calculators. This information is organized in Table 4.38.

Table 4.38 Measures to Reduce Student Dependency

Measure Taken	Teachers Implementing This Measure
Teach paper-and-pencil methods before teaching graphing calculator methods	Ms. Clark, Ms. Hays
Test students without the use of the graphing calculator	Mr. Adams, Ms. Clark, Ms. Irwin
Require work to be shown on tests for full credit	Ms. Davis, Mr. Edwards, Mr. Fort, Ms. Green, Ms. Hays

One measure taken is to teach the paper-and-pencil method before the graphing calculator method. Ms. Clark teaches early sections of each chapter without the use of the graphing calculator so that the students learn the paper-and-pencil techniques before using the calculator. After she is convinced that the students can properly work problems without technology, then she teaches the students how to use the graphing calculator. Ms. Hays also teaches her students the paper-and-pencil techniques before teaching with the graphing calculator, so she does not think that her students' basic skills suffer because of the graphing calculator.

Another measure used by teachers is to test students without the use of the graphing calculator. Mr. Adams, Ms. Clark, and Ms. Irwin give two-part tests in which the students are allowed graphing calculators on one part of the test but not the other. Mr. Adams gives quizzes or parts of tests in which students are not allowed to use graphing calculators so that they are reminded of the importance of the basics. On tests in which Ms. Clark allows the graphing calculator to be used, she requires that they show their work for full credit even though some problems can be solved with the graphing calculator.

Ms. Davis, Mr. Edwards, Mr. Fort, Ms. Green, and Ms. Hays allow use on all tests, but they require work to be shown to earn credit on the problems. Mr. Edwards commented, "They

are allowed to use them, but they have to justify the solutions that they get with their paper-and-pencil work.”

The teachers who were interviewed reported that they took adequate measures, such as teaching paper-and-pencil methods before graphing calculator methods, testing without calculators, and requiring work to be shown on tests, to reduce student dependency on graphing calculators. The main concern was student dependency on calculators being used for simple calculations.

Lesson Observations

Six of the nine teachers interviewed agreed to participate in the observation phase of this study. Mr. Adams, Mr. Baker, and Ms. Clark from the rule-based group participated, and Mr. Edwards, Mr. Fort, and Ms. Irwin from the non-rule-based group participated. Each teacher was observed teaching three lessons, and they provided additional information about their lessons through pre-lesson and post-lesson interviews. Scheduling issues determined whether teachers were observed in one day or in separate visits and if they were observed teaching the same class or different classes. Teachers were asked to teach as they normally would teach the lessons and not prepare anything special to be observed. A description of each teacher’s lessons is provided in the following section.

Lessons were videotaped and field notes were taken. Pre-observation interviews were conducted to provide information about the lesson, and post-observation interviews were conducted to further explain the teaching and graphing calculator use of the lesson. Lessons were coded according to two criteria. They were coded as rule-based or non-rule-based, and they were coded according to the way in which graphing calculators were used.

The teaching style for the lesson was coded as rule-based if the content of the lesson was taught as a set of rules, facts, or procedures. The teaching style for the lesson was coded as non-rule-based if the lesson involved exploring problems, discovering patterns, and making generalizations. In addition to coding the overall teaching style of the lesson, episodes within rule-based lessons that involved non-rule-based teaching and episodes within non-rule-based lessons that involved rule-based teaching were identified and coded. Episodes were coded as rule-based if the content of episode was taught as a set of rules, facts, or procedures. The

teaching style for the episode was coded as non-rule-based if the episode involved exploring problems, discovering patterns, and making generalizations.

The type of graphing calculator use was coded as visualization if the calculator was used to present a visual representation of a concept. Visualization, along with data tables, is one of the multiple representations provided by the graphing calculator. The type of graphing calculator use was coded as efficiency if the calculator was used to perform a task more efficiently than by hand. This type of use was evident in lessons in which the students used graphing calculators to perform tasks on the graphing calculator to save time that would have been spent on numerous calculations. The type of graphing calculator use was coded as motivation if the calculator was used as a motivational factor for students who may not ordinarily use graphing calculators. This was most commonly identified in lower-level classes. The type of graphing calculator use was coded as a tool aiding in the solution of problems that are too difficult to solve by hand. In some cases, the solution may be possible without the graphing calculator, but the calculator allows the students to explore concepts that were previously beyond the scope of the course.

Descriptions of Observations

Observations of Mr. Adams

Mr. Adams' classroom was arranged with six tables in three rows. An aisle separated the tables in each row, and the tables were slightly slanted toward the aisle. Three chairs were situated at each table so that the classroom could comfortably seat 18 students. The chairs faced the whiteboard at the front of the classroom. There was also a screen at the front of the classroom that was lowered over the whiteboard, and a projector was mounted to the ceiling in the middle of the room. Another whiteboard was located on the wall to the right of the students. Large windows occupied much of the space on the wall behind the students. The teacher's desk was to the left of the students, and a television was located above the door in the front, left corner of the room. A computer and a TI-Presenter were on the teacher's desk, and both were connected to the projector. The TI-Presenter allowed the view screen of the teacher's calculator to be projected on the screen at the front of the class.

Mr. Adams was observed in two visits. On the first visit he was observed teaching advanced placement calculus and college algebra, and on the second visit he was observed

teaching algebra II. Mr. Adams' school followed block scheduling, and the periods were 75 minutes long.

There were 10 seniors, 2 males and 8 females, in the advanced placement calculus class. The students provided their own graphing calculators. The topic of the lesson was the Trapezoid Rule and Simpson's Rule of estimating the area under a curve. Mr. Adams projected the image of his textbook's computer software illustration of the Trapezoid Rule and Simpson's Rule on the screen. The illustration provided students with a visual representation of the way in which each rule divided the area under a curve into a sum of areas that approximate the area under the curve. He changed parameters in the program to show how the approximate area approached the actual area as the number of subintervals increased. The graphing calculator was used mainly as a computational tool by the students, and because of the complexity of the calculations, Mr. Adams illustrated keystrokes on the screen using a graphing calculator and the TI-Presenter. Students also used the graphing calculators to graph functions as visual representations of the areas. After the teacher finished his presentation of the lesson, the students began working on their homework, and the teacher answered questions as needed.

Even though Mr. Adams' lesson involved learning the formulas for the Trapezoid Rule and Simpson's Rule, his lesson was coded as non-rule-based in that he guided the students to the discovery of the concepts and the development of the rules. The students explored the graphs of curves that were divided into smaller areas to develop the formulas. One episode of this lesson, coded as rule-based, involved the part of the lesson in which Mr. Adams showed the procedure of entering the formula for the Trapezoid Rule and Simpson's Rule on the calculator.

Mr. Adams' graphing calculator use for this lesson was coded as visualization and efficiency. In his post-observation interview, Mr. Adams reported that he did not feel that graphing calculators were necessary for this lesson, but they were helpful in providing the students with a visual representation of the curve that make problems more meaningful. Values could have been calculated on a scientific calculator or by hand, but calculating by hand would have taken much longer. The development of the concepts could have been achieved by drawing the curves on the board, but interaction with the textbook software allowed the discovery of the rules to be accomplished more quickly with more accurate graphs.

Mr. Adams' college algebra class consisted of 13 seniors, 4 females and 9 males. They brought their own graphing calculators to class. The topic of the lesson was on transformations

of functions. The class started with Mr. Adams explaining a question from a recent test. He used his graphing calculator and the TI-Presenter to illustrate on the screen the concept of vertical asymptote. He also explained homework questions on increasing and decreasing intervals of a function by graphing the function and analyzing a table of values for the function. He then began the lesson on transformation of functions. The students graphed the functions along with the teacher, and they were asked to describe changes to the function when values in the function were changed. After the presentation of the lesson, the students began working on their homework, and the teacher answered questions as needed.

Mr. Adams' lesson was coded as non-rule-based in that the students explored the graphs of functions, discovered the effects that changes to a function have on the graph of the function, and generalized their findings. His graphing calculator use was coded as visualization and efficiency. In his post-observation interview, Mr. Adams described how the graphing calculator shortened the time that it had taken to teach this lesson prior to his use of graphing calculators from three or four days to one day. He was able to present numerous examples of eight basic graphs within the class period, and the students were able to generalize the effects of the transformations more easily when shown the same transformation of various functions within a short amount of time.

Mr. Adams' algebra II class consisted of 10 junior and sophomore students of which 5 were male and 5 were female. The students provided their own graphing calculators. The topic of the lesson was solving polynomial equations by using the graphing calculator. The students had already learned how to solve polynomial equations by factoring, but this lesson presented equations that could not be solved by factoring. Mr. Adams explained the relationship between the x-intercepts of a function and the solution to the equation. He provided examples on the screen in which students could have solved by factoring to help them to recognize this relationship. He also provided an example in which every value in the function was multiplied by negative one, and he showed how the function has the same x-intercepts since it is simply a reflection across the x-axis. He used his graphing calculator and the TI-Presenter to show them how to use the calculator's zero function to locate the x-intercepts. After the presentation of the lesson, the students began working on their homework, and the teacher answered questions as needed.

Mr. Adams' lesson was coded as rule-based since he taught the procedure of locating x-intercepts with the graphing calculator. The lesson did not involve exploration or discovery. His graphing calculator use was coded as tool. When asked why he used the graphing calculator for this lesson, Mr. Adams commented, "The graphing calculator is a necessity. Since I have also taught physics, some of the equations used in that discipline are not able to be solved by means of factoring. Without the use of the graphing calculator, this lesson would become very repetitive in trying to estimate zeros."

Two of Mr. Adams' lessons were coded as non-rule-based, and one lesson was coded as rule-based. One episode within a non-rule-based lesson was coded as rule-based since Mr. Adams taught a procedure on the graphing calculator. The uses of graphing calculators in two lessons were coded as visualization and efficiency, and the use of graphing calculators in one lesson was coded as tool.

Observations of Mr. Baker

The desks in Mr. Baker's classroom were arranged in four closely situated columns of seven desks per column with little room between the desks. The desks faced a chalkboard at the front of the room, and two teachers' desks were located to the right of the students. One teacher desk was the desk of the teacher whom I observed, and it had a computer on it. The other desk was used by another teacher who also taught in that room. A chalkboard was also located at the back of the room, but the back row of student desks were pushed against the wall, so it did not seem likely that the teacher would use that chalkboard. A screen was lowered over the chalkboard at the front of the room, and an overhead projector was used to project onto the screen. A television was located near the door at the front right corner of the room. A large bulletin board was located to the left of the students. Since the room was on the interior of the building, there were no windows. The school provided a laptop computer for each student, and the students carried these with them throughout the day. Most of the students brought calculators with them, and the types of calculators varied among various types of scientific and graphing calculators.

Mr. Baker was observed in three visits. His algebra I class was observed each time. His school followed block scheduling, so each class period was 75 minutes long. Following school policy for the safety and privacy of students, Mr. Baker's administration did not allow students to be videotaped.

There were 26 freshmen in Mr. Baker's algebra I class, and since almost all of the desks were occupied, the room seemed crowded. Nine of them were male, and 17 of them were female. The topic of the first lesson observed was solving systems of equations. The students had already learned how to solve systems of equations a few days before, and this class period was spent reviewing how to solve them and preparing for a quiz that was given at the end of the period. The methods of solution included substitution and elimination, but since all of his students did not have access to graphing calculators, he did not teach them the graphing method on the calculator. He reminded them of the graphing method, but this method only involved paper-and-pencil techniques. He answered their questions by solving the problems on the overhead projector. After reviewing with them, he gave the students the quiz and they worked on it until the end of the period.

The topic of the second lesson was a continuation of the teacher's lesson from the previous period, solving quadratic equations by factoring. The students were familiar with solving equations using this method, but they had several questions for the teacher. He answered their questions by writing the solutions to the problems on the overhead projector. After their questions had been answered, the students worked on their homework problems, and Mr. Baker answered individual student questions as needed.

The third lesson observed was a review of concepts in preparation for the final exam, but most of the time was spent on concepts from the first two lessons. Graphing calculators were not used in the review, but graphing of linear equations by hand was reviewed. Mr. Baker worked examples for the students on the overhead projector. After their questions had been answered, the students worked on their review problems, and Mr. Baker answered individual student questions as needed.

Mr. Baker's lessons were coded as rule-based in that he taught the procedures needed to solve systems of equations and quadratic equations, and the students practiced skills instead of exploring and experimenting. In his post-observation interviews he explained that he did not incorporate graphing calculators in his lessons since not all of his students had access to one, and he wanted make sure that his students knew how to graph linear equations by hand. The use of calculators was limited to performing calculations. Graphing calculator use was not coded since graphing calculators were not used in these lessons.

Observations of Ms. Clark

The 12 desks in Ms. Clark's large room were rearranged by the students for each class that I observed so that no real row/column pattern was observed. The desks faced the whiteboard at the front of the room, and an overhead projector was positioned in front of the left side of the whiteboard. A TI-Viewscreen was used to project the output of a graphing calculator on the whiteboard. Chalkboards were located on the walls to the left and right of the students. The teacher's desk and table were located to the right of the students, and a computer was on the teacher's desk. The door was located in the front, right corner of the room, and large windows were located on the wall behind the students. A cabinet in which graphing calculators are stored was located in the right, rear corner of the room. Students were allowed to borrow a calculator at any time during class if needed.

Ms. Clark was observed in one visit. The classes observed included an integrated mathematics class and two algebra II classes. Class periods were 50 minutes long.

Ms. Clark's integrated mathematics class consisted of 3 males and 1 female. One of the male students was a foreign exchange student from Saudi Arabia. The students grouped their desks to the left side of the classroom near the overhead projector, and one student moved his desk near the Saudi Arabian student to help him when communication became difficult. The students were juniors who had already taken algebra and geometry but were not prepared to take higher-level mathematics classes. The topic of the lesson was graphing scatter plots on the graphing calculator. This was the first time that these students used the graphing calculator to plot data, so Ms. Clark taught the steps involved with this process. She showed the students the process with the TI-Viewscreen projected on the whiteboard, and the students copied her steps on their own calculators. After the presentation of the lesson, the students worked on homework, and the teacher answered questions when needed.

Ms. Clark's approach was coded as rule-based in that she taught the students the procedure needed to graph a scatter plot on the graphing calculator. One episode in this lesson was coded as non-rule-based since the students were asked to discuss why her graph appeared different than their graphs, to experiment with window values, and to determine what they could do to make the graphs look the same.

Ms. Clark's use of the graphing calculator was coded as visualization and motivation. In her post-observation interview Ms. Clark reported that she used the graphing calculator in this

lesson as motivation for the students to become more interested in mathematics and have them do something in the class other than paper-and-pencil work. This class did not use technology often in this class, and the only use of the graphing calculator prior to this class was for calculations. This lesson enabled the students to create an accurate graph that could be analyzed for various characteristics such as local minima, local maxima, increasing intervals, and decreasing intervals.

The other two classes observed were algebra II classes. The first algebra II class consisted of 7 males and 3 females, and all of them were juniors. The second class was an advanced class of sophomores that consisted of 5 males and 2 females. The students in both of these classes organized the desks into columns of unequal desks per row, and they moved the desks so that they were not all clustered in the front, left corner of the room. All students had access to graphing calculators since there was a classroom set available to them.

The topic of the lesson for the algebra II classes was complex numbers. The presentation of the lesson was similar between the two classes. Ms. Clark listed the definition of the imaginary unit and the definition of complex number, and then she developed the rules for calculating with complex numbers. She showed the students how to graph complex numbers on paper and how to determine the distance between two points on a complex plane. She had originally planned to teach the students how to use the capabilities of the graphing calculator to calculate with complex numbers so that they could check their answers, but she decided during the class to wait to teach that until the next day. She wanted to make sure that they knew what they were doing on paper before calculating on the calculator. After the presentation of the lesson, the students worked on their homework problems, and Ms. Clark answered individual student questions as needed.

Ms. Clark's approach to these lessons were coded as rule-based in that she listed the properties of imaginary numbers, described the form and parts of complex numbers, and showed the students the procedures needed to graph complex numbers and to determine the distance between two points on a complex plane. Her lesson included an episode coded as non-rule-based in which she allowed the students to help her develop the rules for calculating with complex numbers. Graphing calculator use was not coded since graphing calculators were not used in this lesson.

All three of Ms. Clark's lessons were coded as rule-based in that she taught the content of the lessons as a set of rules and procedures. One episode was coded as non-rule-based since Ms. Clark asked the students to explore various windows on the graphing calculator to match her window. The uses of graphing calculators in one of her lessons were coded as visualization and motivation. Graphing calculator use for the other two lessons was not coded since calculators were not used in the lessons.

Observations of Mr. Edwards

Mr. Edwards' classroom was arranged with the students' desks in five columns with four desks in each column. The desks faced a whiteboard at the front of the room, and a screen was positioned above the whiteboard. The teacher's desk and podium were in the front, left corner of the room, and a television was mounted high on the wall in the front, right corner of the room. A computer on a wheeled cart was also located in the front, right corner of the room. Bookshelves, cabinets, and a counter were to the left of the students, and a whiteboard was to their right. To the left of the whiteboard was as a coordinate grid whiteboard, and to the right was a small window. A bulletin board was located on the wall behind the students. The door was located on the wall to the left of the students in the left, rear corner of the room.

All three of Mr. Edwards' lessons were observed on the same day. The classes observed were pre-calculus, calculus, and algebra II. The class periods were 50 minutes long.

Mr. Edwards' pre-calculus class consisted of 6 males and 8 females. They were mostly juniors and seniors with two sophomores. There were three Hispanic students in the class. All students had access to graphing calculators since Mr. Edwards provided them for in-class use for anyone who did not own one. The topic of the lesson was logarithmic functions. The students learned how to evaluate expressions involving logarithms, solve logarithmic equations and inequalities, and graph logarithmic functions. The procedure used by the teacher to teach this lesson was to solve an equation or inequality, then graph the equation or inequality on the calculator to verify the answer and to provide the students with a visual interpretation of the problem. Students were shown the answer graphically by the intersections of the graphs from the two sides of the equation. Mr. Edwards had the students analyze the table of ordered pairs generated on the calculator to identify the intersection or solution set to an inequality for three problems. He also showed the students how to graph a logarithmic function on paper and the graphing calculator. Then he reviewed concepts such as domain, range, and asymptotes with the

students. He stressed the need for algebraic methods in addition to calculator methods when analyzing graphs because of limitations that the calculator has in properly graphing a function. After the presentation of the lesson, the students worked on their homework problems, and Mr. Edwards answered individual student questions as needed.

Mr. Edwards' lesson was coded as rule-based in that he presented the solution of logarithmic equations and inequalities. His graphing calculator use was coded as visualization. In his post-observation interview he explained that he used the graphing calculator in this lesson because he believes that students learn concepts better when they can visualize them. He represented every problem solved in class algebraically and geometrically, and he represented three of them in table form. In his post-lesson interview he commented, "I used the graphing calculator in the lesson on logarithmic functions in order to show the students that there can be different ways of solving equations and problems which may include analytical, algebraic, and graphing techniques, perhaps even a combination of these techniques. It was also my intention to use the graphing calculator in order to help give the students a visual perspective on the logarithmic properties and concepts that we are currently studying in this lesson."

Mr. Edwards had 4 males and 5 females in his calculus class. All of the students were seniors. All students had access to graphing calculators since Mr. Edwards provided them for in-class use for anyone who did not own one. The topic of the lesson was integration by substitution. Mr. Edwards taught the process of integration by substitution, and he used the graphing calculator to illustrate that the value of the integral was equal to the area under the curve. When evaluating the integral he stressed the importance of changing the limits of the integral to match the substitution. He used the graphing calculator to illustrate this by graphing the original function and having the calculator evaluate the integral, and then he graphed the function from the substitution and had the calculator evaluate the integral for the new limits to show that the values are the same. The students followed the presentation and graphed on their calculators as shown by the teacher. After the presentation of the lesson, the students worked on their homework problems, and Mr. Edwards answered individual student questions as needed.

The lesson used by Mr. Edwards was coded as rule-based. He presented the lesson as a set of procedures, and the students worked along with him on paper and their graphing calculators. His graphing calculator use was coded as visualization. In his post-observation interview he explained that he used the graphing calculator in this lesson to show how

substitution changed the function, but when evaluating the new integral at the new limits, the areas under that curve and the original curve were the same.

Mr. Edwards' algebra II class consisted of freshmen, sophomores, and juniors of which 8 were males and 7 were females. Six of the students were Hispanic. All of the students had access to a graphing calculator in class whether it was their own or it belonged to the school. The topic of the lesson was solving polynomial equations, and this lesson involved using a combination of algebraic methods and calculator methods to solve each problem. The graphing calculator was used to identify possible rational roots from the x-intercepts, and then synthetic division was used to determine which of the x-intercepts were actually roots of the function. The quadratic formula was used to determine irrational and imaginary solutions. Mr. Edwards encouraged the students to look at tables of values generated by the graphing calculator to identify x-intercepts. The students followed along with the teacher and worked the problems on paper with the use of their graphing calculators. After the presentation of the lesson, the students worked on their homework problems, and Mr. Edwards answered individual student questions as needed.

The lesson used by Mr. Edwards was coded as rule-based in that he presented the procedures needed to solve the polynomial equations. An episode that occurred during guided practice of the lesson was coded as non-rule-based. Because of the experimental nature of locating and testing x-intercepts, students explored the graphs of the polynomials to determine possible roots. Algebraic techniques were used to verify that their conjectured roots were actually solutions to the equation.

Graphing calculator use in this lesson was coded as visualization and efficiency. In his post-observation interview Mr. Edwards commented on the usefulness of incorporating the graphing calculator in this lesson by saying, "My primary objective was to help reinforce to the students that by using the graphing calculator when solving polynomial equations, it not only saves time, but it also gives a visual perspective on the functions being analyzed and a nice connection to the traditional equation solving techniques such as synthetic division."

All three of Mr. Edwards' lessons were coded as rule-based. One episode within a rule-based lesson was coded as non-rule-based since the students were exploring the graphs for possible roots of a polynomial. The uses of graphing calculators in three lessons were coded as visualization, and the use of graphing calculators in one lesson was coded as efficiency.

Observations of Mr. Fort

The desks in Mr. Fort's classroom were arranged in four columns with four desks in each column. The desks faced a chalkboard at the front of the room. A whiteboard was located on the wall to the left of the students, and a chalkboard was located on the wall to the right of the students. A window was located on each side of the whiteboard, and the door was located on the wall to the right of the students in the front, right corner of the classroom. There was a teacher's desk at the front of the room and a teacher's desk and tables at the back of the room. A computer was located on the teacher's desk at the back of the room. There was no overhead projector, but there was a screen above the whiteboard.

Mr. Fort's advanced mathematics/calculus class was observed in three visits. Mr. Fort's school uses block scheduling so each class period was 75 minutes long. Seven females and no males were in the advanced mathematics/calculus class. They brought their own graphing calculators. Before each class, the teacher had written problems on the whiteboard and two chalkboards so that the students could see the problems for the day as they entered the room. This seemed to save some time since he did not have to write the problems out during class; he had to write only the steps for solution of the problems.

The first class observed was a continuation of a lesson on logarithms from the day before. He explained the solution to each problem with the help of the students. He graphed equations on his graphing calculator and had them graph equations on their graphing calculators to verify solutions, but since he did not have a TI-Viewscreen or overhead projector, he sketched the solutions on the whiteboard and chalkboards by hand. Students compared their calculator graphs with his hand-drawn graphs. He showed the students how to graph a logarithmic function with a base other than 10 or e , and he had them use the trace function to locate specific points on the curve. The last problems discussed were application problems in which he urged them to think for a few moments about the problem before attempting to solve them. The students provided some ideas for solution and then followed along as the teacher solved the problems. After the presentation of the lesson, the students began working on their homework while the teacher answered individual student questions as needed.

The second lesson by Mr. Fort was on exponential growth and decay. Graphing calculators were used mainly for calculations, but students were also encouraged to generate tables of values to compare to graphs of the functions. The teacher led the students through the

procedures of solving problems involving exponential growth and decay. After the presentation of the lesson, the students began working on homework problems while the teacher answered individual student questions.

The third lesson was on evaluating infinity limits, limits at infinity, and one-sided limits. Mr. Fort had the students try to determine the limits of functions mentally, and then he had them graph the functions so that they could visualize the limits of the functions. They also traced the functions to see the value that the function approached. After the presentation of the lesson, the students began working on homework problems while the teacher answered individual student questions.

All three of Mr. Fort's lessons were coded as rule-based. He presented the procedures while students worked along with him on paper and their graphing calculators. Also, he often used the statements "by definition" and "by rule" when evaluating the limits algebraically. His third lesson involved an episode coded as non-rule-based since students were encouraged to explore the graphs of functions for limit values and behavior.

Mr. Fort's use the graphing calculator in these three lessons was coded as visualization. Students used them to verify solutions and represent problems graphically. In his post-observations interview he commented several times on the importance of visualization. After the lesson on logarithms he said, "Graphing a function lets students actually 'see' the concept." After the lesson on exponential growth and decay, Mr. Fort commented "The graphic calculator provides positive visual feedback to the solution of exponential equations." After the lesson on infinity limits he commented, "They [graphing calculators] provide an excellent visual feedback to students learning limits."

Observations of Ms. Irwin

Ms. Irwin's classroom was arranged in four groups with four desks in each group. The four desks were situated in a rectangle so that group members were facing each other. Whiteboards were located at the front and back of the classroom. Windows were located on the left wall, and the door was located at the right, rear corner of the room on the right wall. Part of the whiteboard at the back of the classroom was covered with tessellations. The teacher's desk was located near the front, left corner of the classroom. Cabinets were located in the front, right corner of the classroom along the right wall, and near the cabinets was a graphing calculator

holder with a classroom set of graphing calculators. There was no screen or overhead projector in the room.

Ms. Irwin was observed in one visit. The classes observed were advanced algebra and two algebra I classes. The classes were 50 minutes long.

Thirteen juniors and one senior were in Ms. Irwin's advanced algebra class. Three were males and eleven were females. Some students had their own graphing or scientific calculator, and graphing calculators were available for any student that needed one for in-class use. The topic of the lesson was evaluating radicals with negative radicands, and graphing calculators were used only for a few calculations in this lesson. Ms. Irwin gave four different sets of problems to each group. Each set was on a differently colored sheet of paper than the other sets of problems. She then told the students to find other students in the class who had the same colored sheet of paper. Students with the same colored paper were to become "experts" for the type of problems on their sheet for their original group. After all of the "expert" groups solved their problems and were comfortable that all of their members knew the solution to their problems well, the students formed their original groups and worked each set of colored problems with guidance from the "expert" for each set of problems.

Ms. Irwin's lesson was coded as non-rule-based. Her students used cooperative learning groups to explore problems, and they shared what they knew with others in their original groups. The students presented solutions to homework problems at the board, and they were asked to "pick a buddy" to compare answers and explain their solutions. The teacher's role was to guide the students to discovery of the solution and to correct incorrect student explanations. In her post-observation interview Ms. Irwin described why she used activities in her teaching. "Once a topic is presented, I like to let the students do a little more investigating on their own. With me walking around, the questions still come, just more individually or within the groups."

Ms. Irwin's use of the graphing calculator was coded as tool. The calculator enabled them to explore inconsistencies in the way in which the textbook produced answers and the way in which the graphing calculator produced answers. On the use of graphing calculators to perform computation instead of graphing in this lesson, she commented, "I need to continue to use the calculator more in advanced algebra so the students are comfortable using the computation part of it and not just the graphing calculator part."

Ms. Irwin's first algebra I class consisted of four freshmen males and 2 freshmen females. Her second algebra I class consisted of six freshmen, one male and five females. Within each class the students divided into two groups of three students per group, and one person from each group distributed graphing calculators to the other members in the group. The class began with students presenting homework problems on the board and explaining their solutions. Then Ms. Irwin handed three differently color sheets of paper to each group so that each student had a different problem. As a student completed his/her problems, the teacher gave him/her another colored sheet of paper with other problems to solve. The lesson was on linear regression and the students used the graphing calculators to determine the equation for the line of best fit.

Ms. Irwin's lesson was coded as non-rule-based in that the students explored various problems. She guided the students individually as they encountered troubles, and her students communicated with each other throughout the class. One episode was coded as rule-based when Ms. Irwin taught the procedure of entering data in the calculator.

Her graphing calculator use was coded as visualization. In her post-observation interview Ms. Irwin explained that the students had used graphing calculators for plotting data only once before this lesson and revisiting the concept at this time was valuable in helping them to remember how to plot data, in introducing them to linear regression, and in providing them with a visual representation of a situation. Referring to her use of graphing calculators with algebra I students, she explained, "The graphing calculator provides a more exploratory approach to algebra that a lot of first year algebra students do not get."

All three of Ms. Irwin's lessons were coded as non-rule-based, and one episode was coded as rule-based. The uses of graphing calculators in two lessons were coded as visualization, and the use of graphing calculators in one lesson was coded as tool.

Summary of Observations

The classroom observations that were described in the previous sections provided information as to the ways in which teachers actually teach. In order to observe the teachers' normal use of the graphing calculator, the researcher asked the teachers to teach lessons without any special preparation for the use of graphing calculators. They were to teach the lessons as they have in the past so that their normal graphing calculator use (or non-use) could be observed.

Because of this graphing calculator use was observed at varying levels from non-use to extensive use. Of special interest to the researcher were ways in which teachers used the calculators in non-rule-based ways.

Thirteen of the 18 observed lessons were coded as rule-based. Lessons were taught as a set of facts, rules, and procedures. Table 4.39 displays the lessons and episodes observed as rule-based or non-rule-based.

Table 4.39 Rule-Based and Non-Rule-Based Practices Observed

	Lessons		Episodes	
	Rule-Based	Non-Rule-Based	Rule-Based	Non-Rule-Based
Mr. Adams	Locating x-intercepts	Trapezoid and Simpson's Rule, Transformations of functions	Formula for Trapezoid and Simpson's Rule	
Mr. Baker	Systems of equations, Quadratic equations, Linear equations			
Ms. Clark	Scatter plots, Complex numbers, Complex numbers			Explored windows
Mr. Edwards	Logarithms, Integration by substitution, Polynomial equations			Explored polynomial functions for roots
Mr. Fort	Logarithms, Exponential growth and decay, Limits			Explored limits graphically
Ms. Irwin		Radicals, Linear regression, Linear regression	Entering data	

One type of rule-based lesson observed included calculator procedures that the teacher taught to the students. Mr. Adams taught the procedure of locating x-intercepts on the graphing calculator, and Ms. Clark taught her students how to graph a scatter plot on the calculator. A

second type of rule-based lesson included algebraic procedures that the teacher explained to the students. Mr. Baker taught the procedures needed to solve systems of equations and quadratic equations. Ms. Clark taught the procedure of graphing complex numbers on paper. Mr. Edwards taught how to algebraically solve logarithmic equations and inequalities, he taught the process of integration by substitution, and he taught the procedures needed to solve polynomial equations. Mr. Fort showed the process of solving logarithmic equations and growth and decay problems, and he taught them the algebraic processes of evaluating limits.

Five of the lessons were coded as non-rule-based. Mr. Adams guided his students to the development of the Trapezoid Rule and Simpson's Rule, and he had his students explore transformations of functions on the calculator. Ms. Irwin had her students explore differences in values calculated by hand and values calculated on the calculator for radicals, and she had her students explore problems involving linear regression and the line of best fit.

Two episodes of rule-based teaching were observed in non-rule-based lessons. In Mr. Adams' non-rule-based lesson on the Trapezoid Rule and Simpson's Rule he taught his students the process of entering the formulas for each rule on the calculator. In Ms. Irwin's non-rule-based lesson on linear regression she taught her students the process of entering data in the graphing calculator.

Three episodes of non-rule-based teaching were observed in a rule-based lesson. In Ms. Clark's rule-based lesson on graphing scatter plots on the graphing calculator she asked her students to experiment with the window of the calculator until it matched the window on her calculator. In Mr. Edwards' rule-based lesson on solving polynomial equations his students explored the graphs of polynomials to determine possible solutions when working on practice problems. Mr. Fort had his students explore limit behavior graphically on the calculator before evaluating the limits algebraically.

The graphing calculator uses for the observed lessons were coded using categories of visualization, efficiency, tool, and motivation. In some cases the graphing calculator use had two codes assigned to it, and no graphing calculator use was evident in five lessons. Table 4.40 represents the types of graphing calculator uses coded for each teacher in the observations. The numeral in each box represents the number of lessons in which the use was coded.

Table 4.40 Types of Graphing Calculator Use Coded in the Observations

Teacher	Visualization	Efficiency	Tool	Motivation
Mr. Adams	2	2	1	
Mr. Baker				
Ms. Clark	1			1
Ms. Davis				
Mr. Edwards	3	1		
Mr. Fort	3			
Ms Green				
Ms. Hays				
Ms. Irwin	1		1	

Ten of the 18 uses of the graphing calculator were coded as visualization. Calculators were used to provide a visual representation of an equation, inequality, or function. Mr. Adams' students and Mr. Edwards' students graphed functions so that they could visualize areas under curves. Mr. Adams' students explored transformations of functions on the graphing calculator. Calculators were used to verify algebraic solutions graphically. Mr. Edwards' students solved logarithmic equations algebraically and verified solutions graphically on the graphing calculator. They identified solutions in tables generated by the graphing calculator for three problems. Mr. Edwards' students also used the graphing calculator to identify rational zeros of a polynomial. Mr. Fort's students solved exponential equations algebraically and graphically, and they used the graphing calculator to visualize infinity limits and limits at infinity. Graphing calculators were also used to display graphs of data. Ms. Clark used the graphing calculator for graphing scatter plots, and Ms. Irwin's students used graphing calculators to provide them with visual representations of the least squares regression line for a set of data.

Three uses of the graphing calculator were coded as efficiency since the calculator was used to create graphs or perform calculations more quickly than could be done by hand. Mr. Adams' students used graphing calculators to determine the area under a curve using the lengthy formulas for the Trapezoid Rule and Simpson's Rule. Mr. Adams had his students explore transformations of functions on the graphing calculator because of the time saved by not

graphing the functions by hand. Mr. Edwards incorporated graphing calculators into the solution of polynomial equations providing more efficient methods of solving the equations.

Two were coded as a tool since the content of the lesson was extended beyond what had been the normal scope of the class. Mr. Adams taught the graphing calculator solution of polynomial equations that cannot be solved easily by algebraic methods since these equations are often encountered in the physics class that he teaches. Ms. Irwin included the computational aspect of the graphing calculator with her students so that they would become comfortable with them in situations where calculators are needed for complicated calculations.

One lesson was coded as motivation since students who ordinarily did not use graphing calculators were allowed to use them for motivational purposes. Ms. Clark taught the graphing of scatter plots on graphing calculators to students in a lower-level class that ordinarily does not use graphing calculators.

The teachers were observed using techniques to prevent the students from becoming dependent on graphing calculators. Mr. Baker did not use graphing calculators in his lesson on solving systems of equations, solving quadratic equations, and graphing lines. One of his reasons for not using graphing calculators was because of the lack of graphing calculators in his room, but another reason was that he wanted to make sure that the students could accomplish these tasks using paper-and-pencil methods. Ms. Clark taught operations of complex numbers without the calculator, and she planned on teaching them how to perform the operations on the calculator in the next day's lesson. She mentioned that if she introduced the graphing calculator too soon in the lesson, then the students would have worked the problems on the graphing calculator before understanding the processes of combining and simplifying complex numbers. Mr. Edward's students and Mr. Fort's students solved equations algebraically before verifying their solutions graphically. Occasionally they were asked to graph a problem before solving algebraically so that they could have a better idea of what they were expected to find, but the students were still required to show their algebraic work.

Triangulation of Data

Data were collected in surveys, interviews, and observations to answer the five main questions of this study. Information for Question 1 was collected in the surveys, and information

for Questions 2, 3, 4, and 5 was collected in surveys, interviews, and observations. The following sections include triangulation of the data collected from the three phases of the study.

Question 1

Data for Question 1 were collected in the survey and were not collected in the interviews and observations. Because data were collected from only one source, triangulation was not performed on Question 1 data.

Question 2

Question 2 asked, In what mathematics courses do students use graphing calculators, how often do they use them, what calculator functions do they use, and to what extent are graphing calculators used to learn mathematical concepts? Survey information revealed that graphing calculators are required or allowed in all mathematics courses higher than the level of algebra II, and graphing calculators are not allowed in some lower-level mathematics courses. Interview data revealed that the only course in which graphing calculators were not allowed is Ms. Irwin's Fundamental Mathematics class. Mr. Baker allows their use in his algebra I class, but he does not teach the students to use them. Similar to information gathered in the surveys, the interview information indicates that the requirement to use graphing calculators increases with higher-level classes. Observations support the data from the surveys and interviews. The only class in which graphing calculators were not used in the teaching of the lesson was an algebra I class. Graphing calculators were used in all higher-level mathematics classes.

Teacher interviews supported survey responses on the frequency of graphing calculator use, and even though only three observations were conducted for each teacher, the information gathered from the observations supported teacher survey and interview responses. Mr. Baker did not use them in his classes on the days he was observed, and Ms. Clark changed her algebra II lessons so that graphing calculators were to be used the next day, but otherwise graphing calculators were used in all of the other classes.

Survey information and interview information were consistent in regard to the limitations of graphing calculator use on course requirements, but because the observations were conducted only during class time and students were not observed during testing, only limitations during class time could be compared. Students were allowed unlimited access to graphing calculators during class time if they had access to calculators. Mr. Baker's students were allowed to use

graphing calculators in class if they had them, but they were using them mainly for calculations since he did not teach how to use graphing calculators for the content of the lesson.

Survey and interview responses supported each other in regard to student uses of graphing calculators, but with the exception of lessons by Ms. Irwin and Mr. Adams, the use of graphing calculators for discovery exercises was not observed. Almost three-fourths of the teachers surveyed reported using discovery lessons, and several teachers who were interviewed reported using them, but few of the lessons observed included discovery activities. The discovery lessons in Ms. Irwin's classes involved students working in cooperative groups to learn the concepts. Mr. Adams guided his students through the discovery lesson. In reference to the ways in which students used graphing calculators, students were observed checking answers, performing calculations, graphing functions, and analyzing graphs, but they were not observed working with matrices, calculating statistics, or running applications.

Interviews and observations show that teachers allow their students to use graphing calculators for problems of all difficulty levels, not only for those that are too difficult to solve algebraically. An example of this is analyzing functions. Students were often encouraged to combine algebraic methods with graphical methods to analyze functions. However, some problems were impossible to solve algebraically, and teachers allowed graphing calculators to be used for those problems. Mr. Fort reported that his students solve realistic problems with solutions that cannot be found with algebraic methods. Students are able to see that problems in life do not always have "nice" solutions. Ms. Davis reported that she allows her applied mathematics students to use graphing calculators to calculate the standard deviation of a data set. The calculation is not necessarily difficult, but it can be cumbersome. The calculator allows the student to find the standard deviation easily, and the student can then use the standard deviation for other purposes.

Information regarding obstacles to graphing calculator use was consistent between the surveys, interviews, and observations. Except for the lack of graphing calculators in Mr. Baker's classroom, obstacles were not obvious in the observations. Students provided their own or had access to a classroom set of calculators. Since observations were conducted only during class time, the obstacle of some students not being able to use graphing calculators on homework was not observed.

Question 3

Question 3 asked, What beliefs do teachers have regarding the teaching and learning of mathematics? Information was collected in surveys, interviews, and observations to provide insight into the beliefs held by high school mathematics teachers in western Kansas. Survey responses to specific statements were used to identify each teacher along a continuum of rule-based to non-rule-based beliefs. All of the teachers' scores were within a range from the middle of the scale to the non-rule-based end of the scale. Even though the range of scores suggests that teachers generally hold non-rule-based beliefs, the teachers were divided into two groups for comparisons. Teachers with scores greater than or equal to 16 were placed in the non-rule based group, and teachers with scores below 16 were placed in the rule-based group. Six teachers from the non-rule-based group and three teachers from the rule-based group were interviewed, and three teachers from each group were observed teaching three lessons each. Of the teachers observed, Mr. Adams, Mr. Baker, and Ms. Clark were identified as rule-based, and Mr. Edwards, Mr. Fort, and Ms. Irwin were identified as non-rule-based. Ms. Davis, Ms. Green, and Ms. Hays, who were interviewed but not observed, were identified as non-rule-based.

Each teacher interviewed was asked to describe a typical lesson in which graphing calculators are used, and several teachers described more than one lesson. Six of the teachers interviewed were observed teaching three lessons. The lessons described in the interviews and the observed lessons were coded as rule-based or non-rule based, and this information is presented in Table 4.41.

Table 4.41 Comparison of Beliefs and Practices for Observed Teachers

Teacher	Survey	Interview	Observation
Mr. Adams	Rule-based	One non-rule-based lesson	Two non-rule-based lessons, one rule-based lesson, one rule-based episode
Mr. Baker	Rule-based	One non-rule-based lesson	Three rule-based lessons
Ms. Clark	Rule-based	One non-rule-based lesson, one rule-based lesson	Three rule-based lessons, one non-rule-based episode
Ms. Davis	Non-rule-based	Two rule-based lessons	Not observed
Mr. Edwards	Non-rule-based	One non-rule-based lesson, one rule-based lesson	Three rule-based lessons, one non-rule-based episode
Mr. Fort	Non-rule-based	One non-rule-based lesson, two rule-based lessons	Three rule-based lessons
Ms Green	Non-rule-based	Three non-rule-based lessons	Not observed
Ms. Hays	Non-rule-based	One non-rule-based lesson	Not observed
Ms. Irwin	Non-rule-based	Three non-rule-based lessons	Three non-rule-based lessons, one rule-based episode

As illustrated in the table, there is little consistency among information gathered in the surveys, interviews, and observations. Of the teachers who were not interviewed, Ms. Green and Ms. Hays were identified as non-rule-based teachers, and their descriptions of typical graphing calculator lessons were consistent with this; however, Ms. Davis was identified as a non-rule-based teacher from her survey responses, but she described two rule-based lessons.

Of the teachers who were observed, only Ms. Irwin's survey responses, typical graphing calculator lessons, and observed lessons were consistent in indicating non-rule-based beliefs. The other teachers' survey responses, described lessons, and observed lessons provided information that was inconsistent across data sources. Mr. Adams, Mr. Baker, and Ms. Clark, all of whom scored in the rule-based range, identified non-rule-based lessons. Ms. Clark also described a rule-based lesson. Two of Mr. Clark's observed lessons were non-rule-based, and one was rule-based.

Mr. Baker's observed lessons and Ms. Clark's observed lessons were rule based. Mr. Edwards, who scored in the non-rule-based range on the survey, described one non-rule-based lesson and one rule-based lesson and was observed teaching rule-based lessons. Mr. Fort, who scored in the non-rule-based range, described rule-based and non-rule-based lessons and was observed teaching rule-based lessons.

Of the 18 typical lessons described, 12 were coded as non-rule based and 6 were identified as rule-based. Of the 18 lessons observed, 5 were identified as non-rule-based and 13 were identified as rule-based. This discrepancy may be due to the teachers describing lessons specifically for the graphing calculator, whereas their observed lessons were to be typical of their everyday teaching without any special preparation for graphing calculator use.

In summary, the teachers involved in the interview and observation phases of the study generally reported teaching non-rule-based lessons that are specifically designed for graphing calculators but were observed teaching mainly rule-based lessons. The primary use of graphing calculators in the rule-based lessons was to provide visual representations of algebraic solutions. The teaching of these lessons has not changed much because of graphing calculators, but rather the graphing calculator has been added to the teaching of the lesson to provide deeper understanding of the concepts. Graphing calculators changed the way in which some concepts were taught, specifically to enable students to solve more difficult problems than they were able to solve previously.

Question 4

Question 4 asked, What beliefs do teachers have regarding the use of graphing calculators? Almost 80% of the teachers surveyed reported that they agree that students understand mathematical concepts better because of the graphing calculator. All nine of the teachers interviewed expressed this belief in both their surveys and interviews. Teachers use graphing calculators for visualization, efficiency, as a tool to solve problems too difficult to solve by hand, and as a way to motivate students.

Interview and observation information provided a variety of beliefs in how soon students should be introduced to graphing calculators. Most of the teachers interviewed indicated that students should be exposed to graphing calculators before taking high level mathematics classes, but some believe that their use should be restricted. Mr. Baker uses them for demonstration, but

since all of his students do not have access to them, the students do not use them in lessons. Ms. Davis uses them in her applied mathematics class so that the students become familiar with them before they take higher-level classes. She also feels it is important that they are familiar with them for assessment tests. Mr. Fort does not use them in algebra I since he wants the students to learn the basics of algebra.

Survey data indicated that about one-third of the teachers surveyed agreed that students will become dependent on graphing calculators, about one-third disagreed, and about one-third were uncertain. Interview responses indicated varying beliefs of student dependency, but there did not appear to be significant differences between rule-based and non-rule-based teachers in their beliefs of student dependency. Five teachers indicated that dependency on calculators for simple calculations is the main problem of dependency. Two teachers reported a minor problem of students graphing simple functions when they should have been able to mentally visualize the graph. One teacher reported that students often trust the calculator's graph without realizing the limitations of the calculator, and another teacher mentioned that one of his students thought that algebraic processes were not important since his calculator had symbolic algebra capabilities. Teachers reported that they protect against student dependency by requiring work to be shown on tests or by giving two-part tests in which students may not use graphing calculators on one of the parts. The observations indicated that teachers stressed the algebraic solutions while incorporating graphical solutions and representations of problem.

In summary, the teachers who were interviewed and observed believe that graphing calculators enhance student learning. The teachers believe that the visual representation provided by the graphing calculator is invaluable to student understanding of concepts. They do not think that student dependency on graphing calculators is much of a problem because of measures that they have implemented to protect against dependency such as teaching paper-and-pencil methods before graphing calculator methods, requiring algebraic work to be shown on tests, and giving tests in which graphing calculators are not allowed.

Question 5

Question 5 asked, What is the relationship between teachers' beliefs about teaching and learning mathematics and how graphing calculators are used? Survey data indicated that non-rule-based teachers believed more strongly that graphing calculators positively affect student

learning. Interview information indicated that all teachers interviewed believed that graphing calculators positively affect student learning, and there was no apparent difference in how strongly either group believed this.

Survey information indicated that rule-based teachers believe that students should use graphing calculators to solve equations only when algebraic methods are too difficult, that students should master a concept or procedure before using a graphing calculator, and that calculators should only be used to check work. None of the teachers who were interviewed indicated that calculators should be used only to check work or should be used only when algebraic methods are too difficult. As for mastering a concept before using the graphing calculator, interview information indicated that rule-based teachers, Mr. Adams and Mr. Baker, did not believe that graphing calculators are necessary at the algebra I level in which students are learning algebraic processes. Of the non-rule-based teachers, Mr. Fort did not use graphing calculators in his algebra I class, and Ms. Irwin did not use graphing calculators in her fundamental mathematics class since the students were learning basic computational processes.

The information from the interviews and observations supported the idea that non-rule-based teachers favor unlimited graphing calculator use more strongly than rule-based teachers. Ms. Clark did not allow her students to use graphing calculators until they learned paper-and-pencil methods first. Ms. Davis was the only non-rule-based teacher that taught paper-and-pencil methods before allowing her students to use graphing calculators. The other non-rule-based teachers allowed their students to use graphing calculators from the beginning of the lesson.

Survey information indicated that teachers who hold non-rule-based beliefs perceived fewer obstacles to graphing calculator use than teachers who hold rule-based beliefs. Interview information did not support this. One rule-based teacher, Mr. Baker, and two non-rule-based teachers, Mr. Edwards and Mr. Fort, indicated the lack of a classroom set of calculators as an obstacle to graphing calculator use.

Survey information indicated that teachers who hold rule-based beliefs feel more strongly than teachers who hold non-rule-based beliefs that students will lose their ability to think if they use graphing calculators. There was no difference indicated between rule-based teachers and non-rule-based teachers in the interviews and observations.

In summary, even though significant differences were found between rule-based teachers and non-rule-based teachers in reference to the relationship between their beliefs in teaching

mathematics and using graphing calculators, few differences were indicated in the interviews and noticed in the observations between rule-based and non-rule based teachers. Two of the three rule-based teachers restricted the use of graphing calculators for some classes whereas only two non-rule-based teachers restricted their use. One of the three rule-based teachers and one of the six non-rule-based teachers did not allow her students to use graphing calculators until they learned the algebraic processes.

Summary

The goal of this study was to describe how graphing calculators are used in high schools of western Kansas and to determine the relationship between teachers' beliefs in teaching and learning mathematics and how graphing calculators are used in their classes. Data were collected through surveys, interviews, and observations to help achieve this goal. Surveys were sent to 253 mathematics teachers in western Kansas, and 62% were returned. The returned surveys represented 88% of the schools in the region. Means and percentages were calculated for survey data, and the data were analyzed for correlations and significant differences. Nine teachers were purposefully chosen to be interviewed, and six of them were chosen to be observed teaching three lessons. Information from the interviews and observations were used to explain and verify information from the surveys.

The surveys provided information about characteristics of the mathematics teachers of western Kansas who returned the survey. They have taught for an average of 16 years and have used graphing calculators in their classrooms for an average of six years. About one-third of them have earned a master's degree in mathematics. Approximately three-fourths of them claim to be at the intermediate or advanced level of expertise in using the calculators, and about two-thirds had attended conferences, workshops, or courses to learn how to use them. About half received training from their colleagues, and one-fourth of them taught themselves how to use graphing calculators.

Schools of teachers who returned the survey vary in the way in which graphing calculators are provided and the way in which they require and allow graphing calculators. Seventy-nine percent of the schools provide graphing calculators for student use, but the level at which they provide them varies. Thirty percent provide them for in-class use only, 42% provide them for students to take home, and 21% allow them to be taken home by higher-level students

only. Of the schools that allow them to be taken home, some schools issue calculators for the school year, and others issue them for overnight use.

Most of the schools of teachers who returned the survey have no official policy on requiring and allowing graphing calculators, and the decisions on how to use graphing calculators are generally made by teachers for their own classrooms. Approximately one-half of the teachers require graphing calculators for at least one of their classes, and approximately three-fourths allow graphing calculators in at least one of their classes. Graphing calculators are required more often in higher-level classes, and they are required in all mathematics courses above algebra II. They are not allowed in some lower-level classes, and there are often restrictions to their use in lower-level classes in which they are allowed.

Survey, interview, and observation data indicate that the frequency of graphing calculator use increases as the level of the class increases. Thirty-seven percent of the teachers use them daily, 31% use them several times per week, and 19% use them less than once per month. Teachers also vary in how they allow graphing calculators to be used in their classes. Ninety-six percent allow them to be used on in-class activities and assignments, 79% allow them to be used on homework, 85% allow them to be used on quizzes, and 89% allow them on tests. Teachers differ in the way in which they allow graphing calculators on tests. Some teachers allow the calculator on all parts of the test but require work to be shown, and others test the students with a two-part test. Graphing calculators are allowed on one part but not the other.

Teachers reported a variety of ways in which students use graphing calculators. Eighty-six percent use them for graphing functions, 52% use them for statistics, 50% for matrices, and 22% for APPS. Seventeen percent write their own programs, and 11% use the symbolic algebra capabilities. Teacher responses to the interviews indicated that they believe that students learn better from the visualization that graphing calculators provide, and the ability to quickly graph functions works well with discovery learning and making generalizations about functions.

Obstacles such as an insufficient number of calculators available, lack of funds to buy calculators, and lack of time to teach content and technology prevent teachers from using calculators as fully as they would like. Teachers of large schools and schools with a large Hispanic population perceived more obstacles to using graphing calculators than teachers of small schools, and teachers in schools that provide graphing calculators perceived fewer

obstacles. Teachers who have taught with graphing calculators for a long period of time, earned advanced degrees, and have a high level of expertise perceived fewer obstacles.

Most of the teachers who were surveyed indicated that knowing why a procedure works is as important as knowing how to do the procedure, and few indicated that mathematics is mostly memorizing facts. Most indicated that learning algebra means exploring problems, discovering patterns, and making generalizations. Teachers with higher degrees tended to be less rule-based than teachers who had not earned higher degrees.

Most of the teachers who were surveyed indicated that students should solve problems first and then support the answer graphically, and most did not think that graphing calculators should be used only when algebraic methods were too difficult. Very few teachers indicated that calculators should be used only to check work. Teacher responses were mixed as to the extent at which graphing calculators should be used on tests.

Survey responses were used to identify teachers who hold mathematics teaching beliefs that are relatively more rule-based than teachers who hold non-rule-based beliefs. Rule-based teachers favored a more limited use of graphing calculators than non-rule-based teachers, non-rule-based teachers held stronger beliefs than rule-based teachers that graphing calculators positively affect student learning, and teachers who favor unlimited graphing calculator use believe that graphing calculators positively affect student learning. Interview information, however, revealed little difference between rule-based teachers and non-rule-based teachers in regard to the positive effects of graphing calculators on student learning.

Interviews and observations provided additional information about ways in which graphing calculators are used in lessons. Interviews revealed that teachers use graphing calculators in lessons that tend to be non-rule-based, but observations revealed that teachers primarily taught rule-based lessons. Graphing calculators provided visual representations of algebraic problems and processes and allowed students to solve problems that they could not solve previously.

CHAPTER 5 - Conclusions, Discussion, Recommendations

Background

Graphing calculators have been used in education for over 20 years, but no consensus has been reached on how they should be used. The National Council of Teachers of Mathematics (NCTM) strongly recommends that all students should have access to technology including the graphing calculator, and the National Research Council supports full implementation of calculators and computers in education, but this view is not shared by all teachers. Decisions on whether to use graphing calculators, at what levels they should be used, and to what extent they should be used center around beliefs of how graphing calculators affect learning.

Ample research supports the positive benefits of graphing calculator use. Meta-analyses by Hembree and Dessart (1986, 1992) found that graphing calculators enhance student learning and increase understanding of mathematical concepts. A meta-analysis by Smith (1996) found that students who used graphing calculators showed higher achievement on problem solving, computation, and conceptual understanding than students who did not use graphing calculators. Results from TIMSS indicated that students who used graphing calculators daily performed better than students who rarely used calculators (The International Study Center, 1998), and there is a positive correlation for most countries that report high calculator use and achievement (NCES, 2005). NAEP data showed that students who used graphing calculators daily scored higher than students who rarely used calculators (NCES, 2005). Research also exists that does not support positive benefits of graphing calculator use. Studies by Hunter (2005), Autin (2001), Lesmeister (1996), and Ritz (1999) found no significant differences in understanding and achievement between students who used graphing calculators and students who did not use them.

Much research has been conducted to determine the effects that graphing calculators have on student learning and achievement, but researchers have called for more research to be conducted. Dick (1994) suggested that research be conducted on why graphing calculators are used, Dunham (1999) suggested that research be conducted on how, how often, and when calculators are used, Simmt (1997) suggested that research be conducted on teachers' views of technology and mathematics and how those views influence teaching, and Tharp, Fitzsimmons,

and Ayers (1997) suggested research regarding teacher beliefs and teacher change toward a discovery approach.

In addition to addressing these suggestions, the researcher intended to identify professional development needs of pre-service and in-service teachers in the service area of the researcher's university. Information will also be valuable in determining factors that contribute to disparity in the knowledge of and the experience with graphing calculators by the students from western Kansas who attend the researcher's university.

Methodology

Surveys, interviews, and observations were used to collect information regarding teacher characteristics and their graphing calculator use. Surveys were sent to 253 mathematics teachers in western Kansas, and 62% were returned. Survey statements were used to identify the level of rule-based teaching beliefs held by the teachers, and responses to these statements revealed that all of the teacher's scores were located from the middle of the teaching belief scale to the non-rule-based end of the teaching belief scale. Nine teachers were purposefully chosen to be interviewed based on their responses to these statements so that teachers at both ends of the range of scores were represented. The teaching beliefs of six of the teachers were at the non-rule-based end, and the beliefs of three of the teachers were closer to the rule-based end. Six of the nine teachers were chosen for observations with each person teaching three lessons. Three were teachers identified as non-rule-based, and three were teachers identified as rule-based. Information from the interviews and observations were used to explain and verify information from the surveys and to generate themes from each case and across cases.

This chapter discusses conclusions based on the results of the study, contributions of this study to the research community, contributions of this study to inform local professional development, and recommendations for future research. The five main questions of this study are used to guide the discussion of conclusions.

Findings for Research Questions

The goal of this study was to describe how graphing calculators are used in high schools of western Kansas and to determine the relationship between teachers' beliefs in teaching and

learning mathematics and how graphing calculators are used in their classes. Five main questions were to be answered in this study:

1. What are characteristics of mathematics teachers in western Kansas, including their level of education, teaching experience, preparation in the use of graphing calculators, and experience with graphing calculators?
2. In what mathematics courses do students use graphing calculators, how often do they use them, what calculator functions do they use, and to what extent are graphing calculators being used to learn mathematical concepts?
3. What beliefs do teachers have regarding the teaching and learning of mathematics?
4. What beliefs do teachers have regarding the use of graphing calculators?
5. What is the relationship between teachers' beliefs about teaching and learning mathematics and how graphing calculators are used?

Conclusions of each question are discussed in the following sections.

Question 1

Question 1 asked, What are characteristics of mathematics teachers in western Kansas, including their level of education, teaching experience, preparation in the use of graphing calculators, and experience with graphing calculators? Information about mathematics teachers of western Kansas was collected in the surveys. The data suggest that mathematics teachers in this geographic region are experienced, well-educated teachers who are generally experienced users of graphing calculators. The average length of teaching experience is 16 years, and almost one-third of the teachers have earned a master's degree. They have used graphing calculators in their teaching for an average of six years, and one-fourth of them have used graphing calculators for ten or more years. Less than one-fourth of the teachers identify themselves at or below the beginner level of expertise in graphing calculator use with 6% reporting no graphing calculator experience. Teacher graphing calculator knowledge is generally acquired through a combination of sources such as workshops, classes, colleagues, and self-teaching. Twenty-five percent of the teachers attribute all of their graphing calculator knowledge to self-teaching, and 82% attribute part or all of their graphing calculator knowledge to self-teaching.

Question 2

Question 2 asked, In what mathematics courses do students use graphing calculators, how often do they use them, what calculator functions do they use, and to what extent are graphing calculators being used to learn mathematical concepts? Information regarding graphing calculator use was collected in the surveys, interviews, and observations. The most widely used graphing calculator brand is Texas Instruments, and the most widely used models are the TI-83/84 calculators. Teachers reported student use of Casio and Hewlett-Packard graphing calculators, but these brands were much less common than Texas Instruments. Graphing calculators with computer algebraic system capabilities are rarely used, but the model most commonly used is the TI-89 calculator.

Most of the schools provide graphing calculators for their students, but the level in which they are provided depends upon the level of the class. Since students in higher-level classes are allowed, and often required, to use graphing calculators, schools provide students with graphing calculators, often for an extended period of time such as the entire school year. Students in lower-level classes in schools that provide graphing calculators are generally allowed to use them in the classroom only, and in some cases they are not allowed to use graphing calculators.

Teachers and schools vary in the way they require, allow, and do not allow graphing calculators. Generally, the requirement to use graphing calculators increases as the level of the class increases, and teachers of all classes above the level of algebra II allow or require graphing calculators. Graphing calculators are not allowed in some lower-level classes, but they are allowed to be used in most classes to some extent.

The frequency of graphing calculator use varies widely, but generally they are used more frequently in higher-level classes than in lower-level classes. It is common for teachers to allow graphing calculators on all class activities, assignments, and quizzes, but certain restrictions may be placed on how graphing calculators may be used on tests. Some teachers administer two-part tests in which students may not use graphing calculators on the first part. Others allow students to use graphing calculators on all tests, but the students must show their work for full credit. Teachers who do not allow graphing calculators to be taken home provide class time for students to complete problems that require use of the graphing calculator.

In addition to performing calculations, the most common use of graphing calculators is analyzing functions. This use involves the graphing and table-generating capabilities of the

calculator. It is common for students to check their answers graphically and to use graphing calculators in discovery exercises whether the discovery exercises are formal exercises that last the entire period or short exercises within a lecture lesson. Other common graphing calculator capabilities that students use include statistical functions and matrices. Few students execute applications (APPS), write programs, or perform calculations using symbolic algebra capabilities.

Teachers face several obstacles that prevent them from using graphing calculators as fully as they would like. The lack of school funding prevents some classes from having access to graphing calculators and limits other classes to using graphing calculators only in the classroom. Some schools require that students purchase their own calculator, but this practice is common only for higher-level mathematics students.

Another factor that may be an obstacle to graphing calculator use is time. Most teachers stated that they possess adequate knowledge to incorporate graphing calculators in their teaching, but many said that they do not have time to teach students how to use the graphing calculator in addition to the mathematics content that must be taught. Most teachers believe that their students possess the knowledge and ability to use graphing calculators effectively, and if students are allowed to explore the calculators, they often become more knowledgeable about the calculators than the teacher.

Relationships were found between perceived obstacles to graphing calculator use and teacher and school characteristics. Teachers of larger schools and schools with higher Hispanic enrollment perceive more obstacles to graphing calculator use than teachers of smaller school or schools with lower Hispanic enrollment, but the socioeconomic status of the school does not seem to affect the way in which teachers perceive obstacles to graphing calculator use. Teachers of schools that provide graphing calculators perceive fewer obstacles to graphing calculator use. Fewer obstacles to graphing calculator use are perceived by teachers who hold non-rule-based beliefs, have used graphing calculators for a longer period of time, and possess a higher level of expertise with graphing calculators.

Most high schools provide graphing calculators for students, but the level at which they provide them varies. There are exceptions, but in general schools that provide graphing calculators allow them to be taken home by students in advanced classes for longer periods of time than by students in lower-level classes, and many schools allow them to be used for

classroom use only for students in lower-level classes. The willingness to provide graphing calculators does not depend on the number of years that the teacher has taught with graphing calculators, the teacher's number of years of experience, or the teacher's beliefs of teaching mathematics; and the ability of a school to provide graphing calculators does not depend on the school's enrollment, the school's Hispanic enrollment, or the school's socio-economic status.

In addition to teaching students how to perform various functions on the graphing calculator, teachers also teach students various mathematical concepts through the use of the calculator. This seemed to be an important factor in the way in which teachers believed that graphing calculators enhanced learning. Students use graphing calculators to learn mathematical concepts instead of merely simplifying calculations and graphing. Teachers believe that students learn certain concepts better through the use of graphing calculators because they are able to visualize situations on the calculator that would otherwise be difficult to visualize. Teachers stressed the importance of combining algebraic methods with graphing calculator methods so that the students are provided with multiple representations of a problem and multiple ways of solving a problem. Even though the graphing calculator methods are important, the teachers still believe that the algebraic methods are important and must not be ignored.

Question 3

Question 3 asked, What beliefs do teachers have regarding the teaching and learning of mathematics? Information regarding teacher beliefs of teaching mathematics was collected in the surveys, interviews, and observations. Four survey questions were used to determine the level of rule-based beliefs held by each teacher. Their survey responses and answers to interview questions indicate that mathematics teachers generally hold non-rule-based beliefs, but the observations revealed mainly rule-based practices. This discrepancy may be due to the way in which teachers were asked for a typical graphing calculator lesson in the interview but asked not to prepare anything special for their observed lessons. In describing the typical lesson, the teachers may have discussed lessons that were special to them, and the first lessons that they recalled were discovery lessons and lessons in which they taught differently than their normal teaching style. Since they were asked not to prepare special lessons for the observations, the lessons observed may have accurately portrayed their normal teaching style. Had they been

asked to prepare specifically for graphing calculator use, more discovery lessons may have been observed.

A non-rule-based practice discussed in several interviews involves discovery learning. Since students can quickly and easily create graphs of functions on the graphing calculator, they can observe patterns and generalize their findings more easily than if they were to graph each function by hand. A common discovery lesson involves the transformation of functions. Students create several graphs from a family of functions and notice the changes among the graphs as they relate to changes in the functions.

Even though the observed practices were mainly rule-based, teachers incorporated certain aspects of non-rule-based teaching such as allowing students to guide the teacher through the problem, developing a rule with the students before stating the rule, and representing a situation in multiple ways. The use of multiple representations such as algebraic, graphical, and table of values supports the teachers' belief that students learn better when able to visualize the problem.

Differences were found between rule-based teachers and non-rule-based teachers. Teachers with higher degrees tend to hold beliefs that are less rule-based than teachers who have not earned higher degrees, and teachers who hold non-rule-based beliefs perceive fewer obstacles to graphing calculator use than rule-based teachers. Teaching beliefs did not seem to be related to the teacher's age, years of teaching experience, years of graphing calculator use, gender, or expertise with the graphing calculator. Also, teaching beliefs did not seem to be related to a school's enrollment, Hispanic enrollment, socio-economic status, or ability to provide graphing calculators for students.

Question 4

Question 4 asked, What beliefs do teachers have regarding the use of graphing calculators? Information regarding teacher beliefs of graphing calculator use was collected in the surveys, interviews, and observations. Seven survey questions were used to determine the graphing calculator beliefs held by each teacher. Most teachers believe that students should solve problems algebraically before supporting the answer graphically, but very few teachers believe that calculators should only be used to check answers. Occasionally teachers encourage their students to graph a problem on the calculator before solving the problem algebraically so that they have a better idea of the meaning of the problem.

Even though most teachers believe that students should be able to solve problems algebraically before learning how to solve them graphically, this does not mean that they believe that the students must master a concept before the introduction of the concept on the graphing calculator. Most teachers allow graphing calculators to be used while learning the paper-and-pencil methods, but some teachers teach the concept with the graphing calculator after the students have shown that they can work the problem without it. They may not expect the students to have mastered the concept, but they want the students to be exposed to the paper-and-paper methods before relying too quickly on the graphing calculator.

Most teachers allow the use of graphing calculators on problems of all difficulty levels and not only those that are too difficult for algebraic methods, and a common practice is for students to graph problems to check answers and to visualize situations. Teachers also allow their students to use graphing calculators on problems that are difficult or impossible to solve without technology. These problems do not always have “nice” solutions and help students to see how real-life problems can often be solved.

Most teachers believe that students should be introduced to graphing calculators before taking higher-level classes, but many believe that students must learn the basic properties of algebra first to avoid becoming dependent on graphing calculators. One fear is that students may begin to think that algebraic methods are not important since the calculator can replace many of the methods. Another fear involves trust that students have in technology in which they often feel that a calculator result must be correct, and they may not challenge calculator results. Another dependency feared by teachers is that students may be quick to graph a simple function whose graph they should know mentally, yet the teachers admit this may be due to their emphasis on the use of multiple representations. However, interview responses revealed that the teachers are not as concerned with the problem of student dependency on functions exclusive to graphing calculators as they are with the problem of student dependency on all types of calculators for simple calculations. Many teachers feel that students do not adequately know math facts and often rely on the calculator to provide them with an answer that they should know mentally.

There is much uncertainty on the part of teachers in regard to student use of calculators with symbolic algebra capabilities since these calculators can replace many algebraic processes. Students can easily develop the impression that algebraic processes are unimportant, and while

that may be true for some processes, teachers believe that many algebraic processes remain to be important and must be learned for success in future mathematics classes.

Teachers feel that teaching algebraic methods in addition to graphical methods and requiring the students to show their work on homework and tests decreases the chance that the students will develop dependency on graphing calculators. This may partially explain why teachers who basically hold non-rule-based beliefs teach with rule-based practices. They feel that they must place emphasis on the procedures so that the students do not become dependent on the calculator. Requiring algebraic work on tests and not allowing graphing calculators to be used on parts of tests are ways in which teachers attempt to determine if a student is dependent on the calculator.

Most teachers believe that the use of graphing calculators enhances student learning of mathematics. They are able to teach with more detail and difficulty than if calculators were not used. Students are able to solve problems with graphing calculators that they were not able to solve without graphing calculators, and they are able to view situations in multiple representations that are not easily generated without graphing calculators. Students are exposed to more realistic mathematical situations when allowed to use graphing calculators. Graphing calculators and Calculator-Based Laboratory Systems (CBLs) are used to allow for the collection of real-life data that can be analyzed, and the ability of the calculator to quickly provide statistical data can aid in simulations. Graphing calculators are used to analyze several sets of data so that decisions can be made about the data without performing multiple calculations that may cause the students to focus too much on the calculations and not enough on the overall problem.

Question 5

Question 5 asked, What is the relationship between teachers' beliefs about teaching and learning mathematics and how graphing calculators are used? Survey responses to statements involving teacher beliefs of teaching mathematics and teacher beliefs of graphing calculator use were compared to determine the relationship between teachers' beliefs in how mathematics should be taught and their beliefs in graphing calculator use. Interview and observation information was used to provide evidence of the relationship.

Rule-based teachers favor more limited use of graphing calculators than non-rule-based teachers. They believe more strongly in student mastery of algebraic processes before learning the calculator processes and in the use of calculators only as a checking tool. Non-rule-based teachers believe more strongly that the use of graphing calculators enhances student learning and does not cause students to lose their ability to think.

Even though teachers' beliefs in teaching mathematics relate to their beliefs in how graphing calculators should be used, the extent of their use seems to be determined more by the course level at which they teach. Also, there did not seem to be a difference in the frequency of discovery lessons based only on teaching beliefs as related to the graphing calculator. The greatest difference in the use of discovery lessons observed in this study was due to a particular teacher's extensive use of organized cooperative learning strategies.

Conclusions

Answers to the five questions provided information specific to each question. This section reports findings according to specific themes of teacher characteristics, calculator access, graphing calculator uses, teacher beliefs of graphing calculators on student learning, and the effect of graphing calculators on curriculum.

Teacher Characteristics

High school mathematics teachers in western Kansas have taught for an average of 16 years, and approximately one-third of them have earned a master's degree. The average length of time in which they have used graphing calculators is six years, and one-fourth of them have used graphing calculators for ten years or more. Almost four-fifths of them are at the intermediate or advanced level of graphing calculator knowledge, and only 6% have had no experience with graphing calculators.

Teachers generally report non-rule-based beliefs, and teachers who have earned advanced degrees tend to be less rule-based than teachers who have not earned advanced degrees. This may be due to the exposure that teachers who have earned advanced degrees may have had to non-rule-based teaching techniques in their coursework. Also, newer teachers who have not had the opportunity to work on an advanced degree may utilize teaching methods in which they are familiar and allow them to maintain control of the direction of the lesson.

Three-fourths of teachers believe that having their students know why a procedure works is as important as knowing how to do the procedure, and less than 10% of them believe that mathematics is mostly memorizing facts and rules. Approximately 85% of the teachers believe that learning algebra means exploring problems, discovering patterns, and making generalizations, and they believe that students learn better through discovery activities. Similar findings by Yoder (2000) were that 96% of the teachers believed that learning algebra means discovering patterns and making generalizations, and 88% of the teachers believed that graphing calculators allowed their students to experiment with algebraic concepts.

Calculator Access

Student access to graphing calculators depends more on the level of the mathematics course taken than on the school attended. Almost all high schools in western Kansas allow or require the use of graphing calculators at some level, and four out of five schools provide graphing calculators for students. The frequency of use increases as the level of the class increases, and the length of time that students are allowed to possess a school-issued calculator often depends upon the level of the mathematics class. Students in higher-level mathematics classes are issued graphing calculators for greater lengths of time than students in lower-level classes, and in some cases students in lower-level classes may use graphing calculators only in the classroom. The extensive use of graphing calculators in western Kansas high schools does not agree with findings by Arvanis (2003) who found that the isolation of teachers in rural schools limits graphing calculator use.

Graphing calculators are allowed or required in all mathematics courses above the level of algebra II, and in those classes, graphing calculators are required in three-fourths of the trigonometry, calculus, college algebra, and statistics classes. Teachers in almost all algebra I and algebra II classes require or allow the use of graphing calculators, but teachers in almost half of the pre-algebra, consumer mathematics, and applied mathematics classes do not allow graphing calculators to be used. Milou (1999) also found that teachers generally support the use of graphing calculators at the algebra II level and above; however, Arvanis (2003) found that less than three-fourths of the algebra I teachers in rural Illinois used graphing calculators.

Most teachers have little difficulty in acquiring graphing calculators for their classrooms, but almost half of them find it difficult to buy graphing calculators. Teachers who hold non-rule-

based beliefs perceive fewer obstacles to using graphing calculators than teachers who hold rule-based beliefs. Teachers who possess a higher level of graphing calculator expertise and have used graphing calculators for a longer period of time perceive fewer obstacles to using graphing calculators than teachers who are not as familiar with graphing calculators. Teachers in larger schools and schools with higher Hispanic enrollment perceive more obstacles to using graphing calculators than smaller schools and schools with smaller Hispanic enrollment, but the ability of the school to provide graphing calculators for students was not related to the school's size, Hispanic enrollment, or socio-economic status. One possible explanation for teachers of smaller schools perceiving fewer obstacles is that a teacher in a small school may be the only mathematics teacher and can make decisions regarding graphing calculators without reaching a consensus with other mathematics teachers.

Graphing Calculator Uses

The frequency of graphing calculator use ranges from daily use to rare use with students in higher-level courses generally using them more frequently than students in lower-level courses. About one-third of the teachers use them in their classes daily, and one-third of the teachers use them in their classes several times per week. Almost all of the teachers who allow graphing calculators to be used allow students to use them on all in-class activities and assignments, and most teachers allow students to use them on homework, quizzes, and tests.

The most common student uses of graphing calculators are performing calculations and analyzing graphs. About half of the teachers have their students utilize the statistical functions and matrix capabilities of the calculator. Less than one-fifth of the teachers have their students write programs, and about one-tenth allow their students to perform calculations using the symbolic algebra capabilities of the calculator.

The most common lessons involving the use of a graphing calculator are transformations of functions and the analysis of functions, and the reasons for using graphing calculators include the ability to graph functions more quickly and easily so that generalizations can be made. Another common use of graphing calculators is to provide a visual representation of equations and inequalities that are solved algebraically.

Teachers teach concepts with the graphing calculator that they have not taught prior to their use of graphing calculators. These new concepts include regression analysis for functions

that are not linear, matrix methods on systems that include decimals, and statistical processes that were previously cumbersome. Even though the calculator makes some of the processes easier, teachers believe that the calculator should be used mainly for teaching mathematical concepts. This finding does not agree with Van Cleave (1999) who found that teachers primarily used the graphing calculator as a tool to do mathematics and not to learn mathematics.

Teacher Beliefs of Graphing Calculators on Student Learning

Most teachers believe that the visual representation provided by the graphing calculator is valuable to student understanding. They encourage their students to graph equations and inequalities so that they can “see” the solutions, and they incorporate graphing calculator methods of calculating important function values with algebraic processes to reinforce student understanding of the properties of the function. This finding supports the finding by Smith and Shotsberger (1997) that students were able to visualize concepts more easily with the graphing calculator.

Teachers’ beliefs of student dependency on graphing calculators are mixed. About one-third of teachers believe that students will become dependent on them, but more than one-third do not believe that students will become dependent on them. Two-thirds of the teachers believe that students will not lose their ability to think. The main concern of dependency, however, was not directly related to the graphing calculator but to calculators in general. Teachers reported that many students are dependent on calculators to perform simple calculations. Rule-based teachers believe more strongly than non-rule-based teachers that students should master a concept or procedure before using graphing calculators and calculators should be used only to check work. Most of the teachers in this study have their students use graphing calculators while learning concepts instead of using them after a concept has been mastered, and this disagrees with a finding by Milou (1998) that most teachers did not allow students to use graphing calculators until a skill or concept was mastered.

Teachers who hold non-rule-based beliefs believe more strongly than teachers who hold rule-based beliefs that graphing calculators positively affect student learning, and they do not believe that students will lose their ability to think. Teachers who believe that graphing calculators positively affect student learning favor unlimited graphing calculator use to a greater extent than teachers who do not believe that graphing calculators positively affect student

learning. Tharp et al. (1997) also found that non-rule-based teachers believed more strongly that graphing calculators enhance learning, and they favored unlimited graphing calculator use to a greater extent than rule-based teachers.

The Effect of Graphing Calculators on Curriculum

Even though most teachers believe that graphing calculators enhance student learning, teachers have not changed their teaching style much. Similar to a finding by Simmt (1997), this researcher found that most teachers use graphing calculators as an extension of their previous style of teaching. This may be due to teachers teaching the way in which they were taught and with which they are familiar. They are able to incorporate graphing calculators into their lessons without major changes to their lessons. The calculator is used for demonstrations that were previously drawn on the board or for visual, graphical representations of equations and inequalities. Teachers who used discovery for lessons such as the transformation of functions now have the students use the graphing calculator to generate functions more quickly than they could have graphed by hand, and the time needed to teach the lesson has decreased. Similar to this study, Simmt (1997) found that graphing calculators were used to save time by generating many examples quickly.

Teachers made minor changes to the way in which they had previously taught concepts. Students are presented with more realistic problems with answers that are not “nice” since the calculator can easily handle such situations. Also, teachers who had previously taught linear regression now have an efficient way of teaching regression analysis for functions that are not linear. Calculator matrix capabilities allow students to solve systems of equations that are difficult to solve without the calculator.

Contributions of the Study

This study was conducted to determine the graphing calculator use of mathematics teachers in western Kansas and the relationship between teachers’ belief of teaching mathematics and graphing calculator use. The contributions of the study will be discussed in two ways, contributions to the body of research of graphing calculators and contributions to inform local professional development.

Contributions to Research

This study extends the knowledge of graphing calculator use by identifying mathematics teacher characteristics and high school characteristics for the geographic region of western Kansas. Specifically, this study provides information on the level, frequency, and types of graphing calculator use by high school mathematics teachers. In addition to gathering survey data, this study included the qualitative components of interviews and observations, a characteristic that is uncommon in previous graphing calculator research. These qualitative components provided insight into the ways in which teachers utilize graphing calculators in their teaching, the reasons for using or not using graphing calculators, and information about the types of graphing calculator lessons employed by high school mathematics teachers.

Graphing calculators are widely used in high school mathematics classes of western Kansas, and they are used in higher-level mathematics classes more frequently than in lower-level classes. In addition to using graphing calculators to make calculations easier and to solve problems that they could not solve before, students are using graphing calculators to learn mathematical concepts. The most common student uses of graphing calculators are performing calculations and analyzing graphs, and the most common lessons involve the transformations of functions, the analysis of functions, and the visual representation of equations and inequalities solved algebraically.

Mathematics teachers believe that graphing calculators enhance student learning because they provide students with visual representations of algebraic concepts. They identify student dependency on calculators for simple calculations as more of a problem than student dependency on graphing calculators. To prevent students from becoming dependent on graphing calculators teachers teach algebraic processes before teaching graphing calculator methods, test students without graphing calculators for parts of tests, and/or require algebraic work to be shown on tests.

Contributions to Inform Local Professional Development

This study is beneficial to the researcher in that it has provided valuable information about mathematics teachers and their graphing calculator needs in the service area of the researcher's university. As the institution that provided and continues to provide pre-service teacher education for many of the teachers in this region, it is important for the researcher to be

aware of issues that must be addressed in pre-service courses to adequately prepare candidates for teaching. Also, faculty members in the researcher's mathematics department provide in-service training for teachers in this region, and it is important for them to know what needs must be met for teachers already in the field.

Even though many teachers indicated that they are at or above the intermediate level of expertise in graphing calculator use, almost one-fourth of the teachers identify themselves at the beginner level or no experience level. This signifies that there is a need to continue to offer workshops and/or conference sessions that concentrate on basic graphing calculator functions and capabilities. Teachers who are unfamiliar with graphing calculators will naturally be reluctant to use them, and in order to increase the use of technology in the classroom, teachers must become familiar with the technology. Fewer beginning teachers should be unfamiliar with graphing calculators since they are used extensively in the researcher's university in the undergraduate program.

In addition to the need for in-service training on basic graphing calculator functions and capabilities, advanced topics should also be addressed to provide teachers with ideas of calculator uses for their classrooms. Even though many teachers claim that they have taught themselves at least some of what they know about graphing calculators, there continue to be capabilities and functions unknown to them. However, a greater need for teachers with adequate graphing calculator knowledge is information on how to use graphing calculators to teach mathematical concepts. Teachers addressed the importance of using graphing calculators to learn mathematical concepts, but these concepts are mainly in graphical analysis and transformations. Not much was discussed in the area of matrices, statistics, and programming. Also, few teachers discussed the use of applications (APPS), and these programs are useful in providing probability simulations, explanations of a variety of mathematical concepts, and interactive programs.

This study has also provided information to why students from western Kansas enter university mathematics classes with varied graphing calculator experiences and knowledge. The findings of this study indicate that graphing calculators are widely used by western Kansas mathematics teachers and that students are not necessarily exposed to or denied the use of graphing calculators because of the school they attend. Rather, a student's experience is mostly dependent upon the level of mathematics classes taken. All students who take mathematics classes at the algebra II level or higher should have had experience using graphing calculators.

Students who have taken only lower-level classes from certain high schools will not be exposed to graphing calculators. Generally a student's expertise level increases as he/she takes more mathematics classes. This indicates that there is a need for workshops and/or conference sessions that focus on ways to incorporate graphing calculators in lower-level mathematics classes.

Recommendations for Future Research

Results of this study have led to questions that require additional research. One such study involves schools in urban settings. It would be interesting to see not only similarities and differences in how graphing calculators are used in urban schools, but also how urban teacher characteristics compare with characteristics of rural mathematics teachers.

Second, more in-depth case studies of teachers are needed to understand more clearly how they typically engage in the use of calculators. Three observations provided valuable evidence as to how a teacher teaches, but more observations would provide a more accurate picture of what the teacher does throughout the year. The three observations did not provide much of an opportunity to observe discovery lessons, and though this may indicate that not much discovery learning is utilized, more observations may reveal that more discovery learning takes place than this researcher was able to observe.

Since the amount of graphing calculator use varies with the level of the class, another possible research idea is to study graphing calculator use specific to a particular course. This study collected information about the classes that they teach and the use of graphing calculators in general, but teachers were not asked to specifically match graphing calculator use to each class that they teach. It may be interesting to investigate the type and frequency of graphing calculator use for specific classes.

In this study teachers were asked to identify ways in which they acquired knowledge of graphing calculators, but they were not asked how much of their knowledge was acquired from each source or how many and what type of classes or workshops they attended. This could have provided insight into differences that there may be between schools of different size. Even though no significant difference was found to exist between school size and teacher expertise of using graphing calculators, a study such as the one proposed may provide information about experiences that may be limited to teachers in smaller schools (no colleagues in which to share ideas, lack of course or workshop opportunities, etc.).

A study could be conducted to further investigate equity issues related to graphing calculator use. One area that could be investigated is the level of administration support of graphing calculators at schools compared across various levels of enrollment, Hispanic enrollment, and socio-economic status. Another area of study could involve graphing calculator access by students compared across various levels of ethnicity, socio-economic status, and level of courses taken.

Summary

One goal of this study was to determine the level of graphing calculator use in high school mathematics classes of western Kansas. Another goal of this study was to determine the relationship between teachers' beliefs in teaching and learning mathematics and the ways in which graphing calculators are used in their classrooms. Information from surveys, interviews, and observations provided evidence of the level of graphing calculator use, reasons for using them, and factors that affect their use. The information also provided insight into the beliefs held by teachers regarding the teaching of mathematics and the use of graphing calculators. This information has led to three main findings in this study.

Graphing calculators are widely accepted and used in high school mathematics classrooms of western Kansas. Very few teachers have no experience using them, and most teachers are comfortable in using them. Graphing calculators are used more frequently in higher-level mathematics classes than in lower-level classes, they are allowed or required in all classes above the level of algebra II, and they are allowed or required in almost all of the classes at or above the level of algebra I. A student's familiarity with the graphing calculator depends more upon the level of classes taken than on the high school attended.

Mathematics teachers believe that graphing calculators enhance student learning. They can teach concepts that they could not teach without the graphing calculator, and they believe that they can teach concepts with deeper understanding than they could previously. In addition to teaching students how to perform calculator functions, the teachers teach mathematical concepts with the graphing calculator. The teachers identify visualization provided by the graphing calculator as the main reason for increased student understanding. They do not think that students will become dependent on graphing calculators, but they identify the use of calculators in general to perform simple calculations as a common dependency problem. They protect

against dependency on graphing calculators by teaching algebraic methods before graphing calculator methods, requiring work to be shown on tests, and/or providing tests in which students are not allowed to use graphing calculators.

Teachers generally hold non-rule-based beliefs in teaching mathematics. Most teachers believe that students should know why a procedure works as well as knowing how to do the procedure, and very few teachers believe that learning mathematics is mostly memorizing facts and rules. Most teachers believe that students should explore problems, discover patterns, and make generalizations, and most teachers believe that students learn better through discovery lessons.

The graphing calculator is used as an extension of a teacher's previous style of teaching and is mainly used for visualization of an equation, inequality, or function. Graphing calculators are also used to solve problems that could not be solved previously and to generate graphs more quickly and easily than can be graphed by hand so that generalizations can be made without tedious calculations. The mathematics curriculum has not changed significantly, but teachers have been able to include topics, such as nonlinear regression analysis, that they were not able to teach previously.

This study has contributed to the body of research on graphing calculators by identifying how graphing calculators are used in high school mathematics classrooms of western Kansas, why they are used, and at what levels they are used. Information was gathered on teacher beliefs of teaching mathematics and the use of graphing calculators, and insight is provided for the relationship between teacher beliefs of teaching mathematics and the use of graphing calculators.

This study has contributed information that can be used for professional development. There is a need for more in-service offerings at the beginner level of graphing calculator use, and there is a need for in-service offerings that focus on advanced topics. Especially beneficial are sessions that provide teachers with ways of teaching mathematical concepts through the use of graphing calculators.

This study has also opened the door for future studies. A similar study could be conducted in an urban setting to provide a comparison of graphing calculator use between urban and rural settings. More in-depth case studies could be conducted to provide a more accurate picture of how teachers use graphing calculators in their lessons. A similar study could be conducted to identify graphing calculator use specific to a particular course level or to further

explore equity issues in schools of western Kansas. Another area of research could focus specifically on ways in which teachers acquired graphing calculator knowledge.

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Appendix A - Teacher Survey Letter

Dear High School Mathematics Teacher:

I am a mathematics instructor at Fort Hays State University, and I am currently working on my doctoral dissertation at Kansas State University. For my dissertation I have chosen to conduct a study on the use of graphing calculators in high school mathematics classes of western Kansas.

My study consists of three parts. The first part is a 42-question survey in which I am inviting you to participate. The survey and an addressed, postage-paid envelope are included with this letter. The second and third parts of the study involve interviews and observations with at least six of the teachers who respond to the survey. By taking the survey, you are agreeing only to the survey part of the study, and if asked to participate in the interview and observation, you may decline. Code numbers at the top of each survey will be used to identify each teacher with his/her survey and aid in the selection process for interviews and observations.

I will be greatly appreciative if you complete the survey and return it to me in the envelope provided within the next three weeks. All data obtained from the survey will be kept confidential and no identifying factors will be included in the dissertation report. To encourage participation in the survey, Texas Instruments has donated two TI-83 Plus graphing calculators to be awarded to two randomly selected teachers from all teachers who return the survey. If you would like a copy of the final report and/or a list of teachers who were selected to receive the TI-83 Plus calculators, please respond accordingly in the survey or contact me at 785-628-5280 or kdreilin@fhsu.edu. Thank you for your time and consideration.

Sincerely,

Keith Dreiling
RH 396, FHSU
600 Park Street
Hays, KS 67601

Appendix B - Graphing Calculator Survey

Please complete the survey as it relates to your high school mathematics classes. If you teach junior high classes as well as high school classes, then answer the questions as they relate only to your high school mathematics classes.

1. How many years have you taught mathematics? _____
2. What is your gender? Male Female
3. What is your age? _____
4. Circle your highest degree and the number of hours beyond the degree?
 BS BS + 15 BS + 30 MA MA + 15 MA + 30
5. Circle your level of expertise with graphing calculators.
 No experience Beginner Intermediate Advanced
6. Circle the sources that contributed to your graphing calculator knowledge.
 Courses Workshops Conferences Colleagues Self-taught
7. How many students attend your high school? _____
8. Circle the range that most closely represents the ethnicity of students in your classes.

White	<10%	10-20%	20-30%	30-40%	40-50%	>50%
Hispanic	<10%	10-20%	20-30%	30-40%	40-50%	>50%
Black	<10%	10-20%	20-30%	30-40%	40-50%	>50%
Asian	<10%	10-20%	20-30%	30-40%	40-50%	>50%
Other	<10%	10-20%	20-30%	30-40%	40-50%	>50%
9. Circle the range that most closely represents the percentage of economically disadvantaged students in your classes (free and reduced lunches).
 <20% 20-40% 40-60% 60-80% >80% unsure
10. Does your school provide graphing calculators for students? Yes No
 If yes, please explain the extent to which graphing calculators are provided (in-class only or allowed to be taken home, all students or only higher-level students, etc.)

11. Please list the mathematics classes that you teach and indicate classes in which you allow or require students to use graphing calculators. If you do not allow or require your students to use graphing calculators, then skip to #18.

Class	Allow use of GCs		Require use of GCs	
	Yes	No	Yes	No
_____	Yes	No	Yes	No
_____	Yes	No	Yes	No
_____	Yes	No	Yes	No
_____	Yes	No	Yes	No
_____	Yes	No	Yes	No

12. How many years have you used graphing calculators to teach mathematics? _____

13. What brand(s)/models(s) of graphing calculator(s) do your students use? _____

14. Approximately how often do your mathematics students use graphing calculators?

- _____ Every day
- _____ Several times per week
- _____ Several times per month
- _____ Less than once per month

15. In what ways do you allow your students to use graphing calculators?

- _____ To do in-class activities and assignments
- _____ To do homework
- _____ To take quizzes
- _____ To take tests
- _____ Other (Please describe) _____

16. In what ways do your mathematics students use graphing calculators?

- _____ Check answers
- _____ Perform calculations
- _____ Discovery exercises
- _____ Analyze graphs
- _____ Other (Please describe) _____

17. What calculator capabilities do your students use?

- _____ Computations
- _____ Graphing functions
- _____ Programming
- _____ Statistics
- _____ Matrices
- _____ Applications (APPS)
- _____ Symbolic algebra capabilities
- _____ Other (Please describe) _____

For statements 18 – 42, please circle the response that best fits each question (SD = Strongly Disagree, D = Disagree, U = Undecided, A = Agree, SA = Strongly Agree).

- | | | | | | |
|--|----|---|---|---|----|
| 18. I have difficulty gaining access to graphing calculators for use in my classroom. | SD | D | U | A | SA |
| 19. It is difficult to get funds to buy graphing calculators. | SD | D | U | A | SA |
| 20. I lack the knowledge and confidence to teach using graphing calculators. | SD | D | U | A | SA |
| 21. I often do not have time to teach both the required algebra curriculum and graphing calculator technology. | SD | D | U | A | SA |
| 22. Most students lack the ability to work with a calculator as complex as a graphing calculator. | SD | D | U | A | SA |
| 23. I have enough calculators for individual student use. | SD | D | U | A | SA |
| 24. My administration encourages the use of graphing calculators. | SD | D | U | A | SA |
| 25. Teaching with a graphing calculator is a high priority in my department. | SD | D | U | A | SA |
| 26. When learning algebra, knowing why a procedure works is not as important as knowing how to do the procedure. | SD | D | U | A | SA |

27. Learning mathematics is mostly memorizing a set of facts and rules.	SD	D	U	A	SA
28. Learning algebra means exploring problems to discover patterns and make generalizations.	SD	D	U	A	SA
29. Students learn a concept better when they discover the concept in an activity.	SD	D	U	A	SA
30. When graphing calculators are used in instruction, students should first solve algebraically and support graphically.	SD	D	U	A	SA
31. When graphing calculators are used in instruction, students should solve graphically only when algebraic methods are too difficult.	SD	D	U	A	SA
32. Students should use graphing calculators only after they have mastered a concept or procedure.	SD	D	U	A	SA
33. Students should not be introduced to graphing calculators before they are in higher-level algebra, trigonometry, or calculus classes.	SD	D	U	A	SA
34. Calculators should only be used to check work.	SD	D	U	A	SA
35. Students should be permitted to use graphing calculators on ALL tests.	SD	D	U	A	SA
36. Students should be permitted to use graphing calculators that have algebraic symbolic manipulator capabilities (like the TI-89 or TI-92) in algebra classes.	SD	D	U	A	SA
37. Graphing calculators allow for greater detail and/or difficulty of algebra topics than in classes that are not using graphing calculators.	SD	D	U	A	SA
38. A graphing calculator can be used to solve problems that the students could not solve before.	SD	D	U	A	SA

39. Using a graphing calculator to teach mathematics enhances student learning or understanding of concepts. SD D U A SA
40. Using a graphing calculator allows the student to develop multiple representations of a problem. SD D U A SA
41. When students use graphing calculators on a regular basis, they become dependent on them and are unable to master basic algebraic manipulations. SD D U A SA
42. If students are taught to use the graphing calculator, they will come to rely on it and lose their ability to think. SD D U A SA

If you would like to know the results from this survey and/or the winners of the TI-84 graphing calculators, please include your e-mail address _____

Appendix C - Second Teacher Survey Letter

Dear High School Mathematics Teacher:

You should have received a letter in early November inviting you to participate in a graphing calculator survey. My records show that I have not yet received your survey, so I am sending this letter as another invitation to participate. I apologize if I am sending this letter in error. In case you did not receive the first letter, I am including the contents of that letter below.

I am a mathematics instructor at Fort Hays State University, and I am currently working on my doctoral dissertation at Kansas State University. For my dissertation I have chosen to conduct a study on the use of graphing calculators in high school mathematics classes of western Kansas.

My study consists of three parts. The first part is a 42-question survey in which I am inviting you to participate. The survey and an addressed, postage-paid envelope are included with this letter. The second and third parts of the study involve interviews and observations with at least six of the teachers who respond to the survey. By taking the survey, you are agreeing only to the survey part of the study, and if asked to participate in the interview and observation, you may decline. Code numbers at the top of each survey will be used to identify each teacher with his/her survey and aid in the selection process for interviews and observations.

I will be greatly appreciative if you complete the survey and return it to me in the envelope provided within the next three weeks. All data obtained from the survey will be kept confidential and no identifying factors will be included in the dissertation report. To encourage participation in the survey, Texas Instruments has donated two TI-83 Plus graphing calculators to be awarded to two randomly selected teachers from all teachers who return the survey. If you would like a copy of the final report and/or a list of teachers who were selected to receive the TI-83 Plus calculators, please respond accordingly in the survey or contact me at 785-628-5280 or kdreilin@fhsu.edu. Thank you for your time and consideration.

Sincerely,

Keith Dreiling
RH 396, FHSU
600 Park Street
Hays, KS 67601

Appendix D - Teacher Interview Letter

Dear High School Mathematics Teacher:

Thank you for taking part in the graphing calculator survey. I appreciate your time and thought in completing the survey. As stated in my previous letter, the second part to this study is to interview several teachers who responded to the survey. The third part of the study is to observe the teaching of three lessons by 6 of the teachers who were interviewed. I have chosen 10 participants who have a variety of teaching experiences and beliefs about teaching mathematics and the use of graphing calculators. I am writing to inform you that you are one of the teachers whom I have chosen to be interviewed.

By participating in the survey you did not commit to the interview, so I need your consent to participate in the interview. In addition to the interview, I am also asking you to allow me to observe you teach three classes if you are chosen for the third phase of this study. Would you please contact me at 785-628-5280 or kdreilin@fhsu.edu to let me know if you are willing to take part in the interview and the observation? I will send the consent form if you indicate that you are willing to be interviewed and observed teaching.

All data obtained from the survey will be kept confidential and no identifying factors will be included in the dissertation report. If you would like a copy of the final report, please contact me at 785-628-5280 or kdreilin@fhsu.edu. Thank you for your time and consideration.

Sincerely,

Keith Dreiling
RH 396, FHSU
600 Park Street
Hays, KS 67601

Appendix E - Teacher Informed Consent Form

KANSAS STATE UNIVERSITY

INFORMED CONSENT TEMPLATE

PROJECT TITLE: Graphing Calculator Use by High School Mathematics Teachers of Western Kansas

APPROVAL DATE OF PROJECT: 9/29/05 EXPIRATION DATE OF PROJECT: _____

PRINCIPAL INVESTIGATOR: CO-INVESTIGATOR(S): Dr. Jennifer Bay-Williams

CONTACT AND PHONE FOR ANY PROBLEMS/QUESTIONS: Dr. Jennifer Bay-Williams
785-532-5808

IRB CHAIR CONTACT/PHONE INFORMATION: Rick Scheidt/785-532-3224

SPONSOR OF PROJECT: _____

PURPOSE OF THE RESEARCH: The study is designed to determine graphing calculator use of high school mathematics teachers of western Kansas. Reasons for use/non-use of graphing calculators will be explored. Specifically, the relationship between teachers' beliefs of teaching mathematics and graphing calculator use will be explored.

PROCEDURES OR METHODS TO BE USED: Interview and observation of three lessons

ALTERNATIVE PROCEDURES OR TREATMENTS, IF ANY, THAT MIGHT BE ADVANTAGEOUS TO SUBJECT:

LENGTH OF STUDY: 1 semester

RISKS ANTICIPATED: none

BENEFITS ANTICIPATED: The information gathered by the researcher may be used to determine professional development needs of western Kansas high school mathematics teachers.

EXTENT OF CONFIDENTIALITY: All data from the survey will be reported as group data in order to protect the confidentiality of individual participants. Identities of the teachers interviewed and observed will be protected through the use of generic names (Teacher A, Teacher B,...). Records will be stored in a locked file cabinet during and after the study.

IS COMPENSATION OR MEDICAL TREATMENT AVAILABLE IF INJURY OCCURS: _____

PARENTAL APPROVAL FOR MINORS: _____

TERMS OF PARTICIPATION: I understand this project is research, and that my participation is completely voluntary. I also understand that if I decide to participate in this study, I may withdraw my consent at any time, and stop participating at any time without explanation, penalty, or loss of benefits, or academic standing to which I may otherwise be entitled.

I verify that my signature below indicates that I have read and understand this consent form, and willingly agree to participate in this study under the terms described, and that my signature acknowledges that I have received a signed and dated copy of this consent form.

Last revised on May 20, 2004

(Remember that it is a requirement for the P.I. to maintain a signed and dated copy of the same consent form signed and kept by the participant

Participant Name: _____

Participant Signature: _____ Date: _____

Witness to Signature: (project staff) _____ Date: _____

Appendix F - Principal Observation Letter

Dear High School Principal:

I am a mathematics instructor at Fort Hays State University, and I am currently working on my doctoral dissertation at Kansas State University. For my dissertation I have chosen to conduct a study on the use of graphing calculators in high school mathematics classes of western Kansas.

My study consists of three parts. The first part is a 42-question survey in which one of your teachers has participated. The second part is an interview in which the teacher participated. The third part is a series of three observations that I would like to conduct in the teacher's classroom. I am asking you for permission to conduct the observations. All data obtained from the survey will be kept confidential and no identifying factors will be included in the dissertation report. The focus of the observations will be on the teacher's method of teaching and the students' use of the graphing calculator. Students will not be individually identified.

If you would like a copy of the final report, please contact me at 785-628-5280 or kdreilin@fhsu.edu. Thank you for your time and consideration.

Sincerely,

Keith Dreiling
RH 396, FHSU
600 Park Street
Hays, KS 67601

I agree to allow Keith Dreiling to observe the teaching of _____ for three class sessions. It is understood that the teacher or I can withdraw from the study at any time, and no information from the teacher will then be used in the data analysis or final report.

Teacher's signature _____ Date _____

Principal's signature _____ Date _____

Researcher's signature _____ Date _____

Appendix G - Graphing Calculator Interview Protocol

Beginning time of interview: _____ Ending time of interview: _____

Date: _____ Place: _____

Interviewer: Keith Dreiling

Interviewee: _____ Position: _____

The purpose of this study is to describe how calculators are being used in high schools of western Kansas and to determine how teachers' beliefs in the teaching and learning of mathematics affect the way in which graphing calculators are used in their classes. This study will attempt to answer the following question: How are graphing calculators being utilized in mathematics instruction of high schools in western Kansas?

1. Describe student uses of the graphing calculator that you require, permit, or encourage. Include the ways in which students use calculators in learning the lesson, working on problems in class, working on problems outside the classroom and at home, and on tests.
2. Describe your school's, department's, or personal policy on the use of graphing calculators, and describe factors that hinder your students from using them according to the policy.
3. Describe a typical lesson in which you incorporate graphing calculators in the teaching of the lesson, and explain why graphing calculators are used in the lesson.

4. Describe ways in which your students learn concepts better through the use of graphing calculators.

5. Describe tasks that your students cannot perform without the use of a graphing calculator that they should be able to do. What do you do in your teaching to prevent the students in becoming dependent on graphing calculators?

6. Describe the way in which you had taught the lesson from Question 3 before you taught the lesson with graphing calculators. If you do not use graphing calculators, explain why you choose not to use graphing calculators in the lesson.

7. Are there any additional comments that you would like to make regarding the other questions or any other aspects of graphing calculator use that were not addressed in the other questions?

Thank you for participating in this interview. Your responses will be kept confidential.

Appendix H - Graphing Calculator Observational Protocol

Beginning time of observation: _____ Ending time of observation: _____

Date: _____ Place: _____

Observer: Keith Dreiling

Class Observed: _____ Teacher: _____

Number of students: _____ Males: _____ Females: _____

Pre-Observation Questionnaire (to be completed by the teacher prior to observation)

1. What is the topic of the lesson?
2. What are the goals/objectives of the lesson?
3. How will graphing calculators be used in the lesson by the teacher?
4. How will graphing calculators be used in the lesson by the students?
5. Why are graphing calculators being (or not being) used?

Classroom Observation (to be completed by researcher during lesson)

1. Describe the classroom setting (seating arrangement, chalkboard/overhead placement, atmosphere, bulletin boards).

2. Describe the calculator equipment that is available to the teacher and the students (brand, quantity, how provided).

3. Describe ways in which the teacher used graphing calculators in the teaching of the lesson.

4. Describe ways in which the students used graphing calculators in the learning of the lesson.

5. Describe ways in which the teacher's approach to teaching the lesson was rule-based and non-rule based.

6. Compare ways in which the students were learning how to use the graphing calculator as compared to learning mathematical concepts through the use of graphing calculators?

7. Describe any episodes that made an impression, or comment on other observations.

Post-Observation Debriefing

1. Why did you use the graphing calculator in this lesson?
2. How did your use of the graphing calculator in teaching this lesson match the way in which you planned to use the graphing calculator?
3. How did the use of graphing calculators by the students match the way in which you planned for them to use graphing calculators?
4. What impact do you feel that graphing calculators had on the lesson?
5. Do you have any additional comments about this lesson?

General Impressions of the Observation (to be completed soon after the post-observation debriefing)