

SELECTION OF SURVEYING METHODS

by

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## INTRODUCTION

### Purpose

This investigation provides a means for judging the adequacy of the commonly available plane-surveying techniques and instruments in meeting pre-determined accuracy goals. These methods and instruments include:

1) plane table and alidade; 2) elevations by differential leveling, vertical angles, and altimetry; and 3) locations by traverse with distances by tacheometry, taping, and electronic devices, and directions by vernier transits, optical transits, and theodolites.

Because most systematic errors can be corrected or compensated for, the several sources of accidental error limit the attainable accuracy of each method or instrument. By determining, for each of the methods, the magnitude of these errors as a function of different conditions and instrument design, it should be possible to estimate the error in any future measurement. The Theory of Errors, which predicts the resultant of a series of independent accidental errors, could then be used to estimate the error for any measured point in a survey.

This is a study and tabulation of the errors in the methods; the determination of required precision rests with the user. Therefore, though the illustrations are drawn from geology, the principles and data have general application. The methods and their appropriate field procedures are well documented in existing texts and will not be described here. Similarly, though photogrammetry is the optimal method for mapping any substantial area when the measured points can be identified on photographs, there already exists a substantial body of literature on the errors of these methods and selection has been treated by others (Aguilar 1967, 1969).

One or more of the methods considered herein would be used when:

(1) available data lack the required accuracy, or (2) sketching with the aid of pace and compass, hand level, measuring wheel, range finder, and other reconnaissance methods are inadequate for the task at hand.

Meyer (1949, 1954) found errors of  $\pm 0.6$  degrees for angles measured with a compass similar in design to the geologist's Brunton. Brinker and Taylor (1961) reported errors of 1:20 for distance by pacing, and 1:50 for auto odometers.

### Applications

Although relatively imprecise values for location, direction, and elevation are sufficient for many geologic studies, some investigations do require more precise values.

The construction of contour maps with small contour intervals (less than existing USGS topographic maps of the same area) that show topography, structure, or other parameters which include a term for point elevation would require elevations of the points correct to some fraction of the chosen contour interval. The exact size of the fraction is open for debate: the USGS Topographic Instructions call for vertical map control points to be correct within one tenth of the contour interval, the National Map Accuracy Standards (Appendix 2) provide that 90 percent of the elevations of well defined points, as interpolated from contours, be correct within one-half of the contour interval. Although there are no such standards for structural and other interpretive maps, contours at intervals smaller than the errors in measurement are obviously meaningless.

For the study of ground-water flow in unconfined aquifers, differences in water level elevations of a few tenths of a foot may be significant; therefore, a corresponding accuracy in well head elevations is necessary. Most such

studies are on river flood plains where contour lines are widely spaced and for technical reasons are located less precisely than on neighboring hills, hence interpolated values would almost always be inadequate.

Attempts are sometimes made to detect and/or measure small surface displacements which may be attributed to tectonic activity, fault creep, elastic rebound, or other dynamic geologic process. In the exploitation of some earth resources there may be an accompanying subsidence or upheaval of the surface whose magnitude or areal extent may be of geologic significance. In these cases, very precise measurements are often required which necessitate the use of first order geodetic instruments and methods over the affected areas.

#### Other Investigations

Appendix 1 is a short subject index to the selected references. Values for errors reported in the literature are given in Appendices 4 and 5.

Several authors (Aquilar 1973, Veress 1973, Vreeland 1969, Wolf 1969) have discussed the distribution and propagation of surveying errors from different theoretical viewpoints. Unfortunately these authors did not include values for substitution into their error equations nor did they describe how these needed values might be obtained.

A second body of literature consists of papers on the errors of a particular instrument or a small group of related instruments. Some authors did quote values for the errors but the statistical measure of the errors was not consistent from paper to paper. Also, many reports were written by persons representing, or in the employ of the manufacturer of the particular instrument tested and thus had a vested interest in the results. Objective studies of groups of instruments include: (1) errors in precise leveling -

Lee and Karren (1964), Karren (1964), Geisler and Papo (1967); (2) errors in tacheometric measurements - Turpin (1954), Mussetter (1956a, 1956b), Colcord (1971); (3) errors in taping - Colcord and Chick (1968), Wood (1969), Golley and Sneddon (1974); (4) errors in altimetry - Greundler and others (1970, 1972); and (5) errors in solar azimuths - Berry (1958), Vanderaa (1964).

A third source of information is from texts and handbooks on surveying such as Brinker and Taylor (1961), Kissam (1966), Bomford (1971), Ewing and Mitchell (1970), and Clark (1973). These authors detailed the methods of correction and reduction of errors, but generally did not quote specific errors for particular instruments.

#### Methods of Investigation

Accidental errors in surveying result from the operator's inability to perform perfectly some task in orienting the instrument or in determining the indicated values. Common tasks include centering level bubbles, pointing at (bisecting) targets with crosshairs, matching marks, and reading instrument scales and rod intercepts.

The size of the resulting error in measurement depends upon both the operator's ability and the magnification or resolution of the instrument. Furthermore, if each task is performed independently, then each should make an independent contribution to the resultant error. Thus the resultant should be equal to a summation by the Theory of Errors of individual task errors.

Table 1 summarizes available data on measurement errors and includes error equations for those methods for which error information is partially or completely lacking. Sources and values for the available information are in Appendices 4 and 5. Several tests for measuring the error of some types

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Table 1. Status of data on errors in surveying methods.

METHOD	TASKS	AVAILABLE DATA	NEEDED DATA	ERROR EQUATION
Measurement of elevations		Errors in altimetry		
Differential leveling - precise levels	Center bubble Read rod	Test for centering error Reading error for precise invar rod		$([c \times sl]^2 + rr^2)^{\frac{1}{2}}$
Differential leveling - ordinary levels, ladders with striding levels, transit telescopes	Center bubble Read rod	Approximate equation for centering error	Rod reading error	$([c \times sl]^2 + rr^2)^{\frac{1}{2}}$
Elevations - differential leveling using vertical arc level	Center arc level bubble Set telescope with vertical arc Read rod	Approximate equation for centering error Setting error = Vertical angle Reading error	Rod reading error Setting error (mark matching error using Beaman Arc)	$([c \times sl]^2 + [m \times sl]^2 + rr^2)^{\frac{1}{2}}$
Trigonometric leveling	Center arc level Read vertical angle Read rod Determine distance	Approximate equation for centering error Angle reading error Distance error for some methods	Rod reading error Stadia distance error	$([c \times sl]^2 + [\text{var } x \text{ sl}]^2 + rr^2 + [sl \times \tan va]^2)^{\frac{1}{2}}$
Beaman method	Center arc level Match Beaman and index marks Read upper, middle and lower rod intercepts	Approximate equation for centering error	Rod reading error Mark matching error	$([c \times sl]^2 + rr^2 + [m \times sl]^2 + [\text{Beaman } x \sqrt{rr^2 + rr^2}]^2)^{\frac{1}{2}}$

Table 1. (Continued)

METHOD	TASKS	AVAILABLE DATA	NEEDED DATA	ERROR EQUATION
Self-reducing tachometers (Wild RDS, RK-1)	Center arc level or plate level	Approximate equation for centering error	Rod reading error	$([c \times sl]^2 + p^2 +$
	Point lower hair at rod target	Pointing error = $\frac{1}{2}$ reading error for rods		$[factor \times \sqrt{p^2 + rr^2}]^2)^{\frac{1}{2}}$
Measurement of Directors	Read middle hair intercept			
		Factor = 10, 20, 50, or 100		
Measurement of Distances	Pointing	Reading errors or Errors in EDM	Effect of target design on pointing error	$(s^2 + ar^2 + p^2 ar^2)^{\frac{1}{2}}$
	Instrument at target Setting initial and reading final angle or reading 2 angular values	Approximations for theodolites, optical and vernier transits Setting error = reading error	Plane table align- ment and plotting errors	or $(p^2 + ar^2 + p^2 + ar^2)^{\frac{1}{2}}$
Stadia method, transit and alidade	Read upper and lower rod intercepts	Vertical angle read- ing error for deter-	Rod reading error	
	Determine factor for inclined shots	mining factor from table	Factor reading error with Beaman arc	factor $\times (rr^2 + rr^2)^{\frac{1}{2}}$
Stadia method, self-reducing tachometers	Point lower hair at rod target	Factor constant at 100	Rod reading error	
	Read upper hair intercept	Pointing error = $\frac{1}{2}$ reading error for rods		$100 \times (p^2 + rr^2)^{\frac{1}{2}}$

s = circle setting error, ar = angle reading error, p = pointing error, rr = rod reading error, c = bubble centering error in radians, sl = sight length, var = vertical angle reading error in radians, va = vertical angle, m = mark matching error in radians, tan = tangent of angle, x = multiplication, EDM = Electronic Distance Measurement

of precise instruments were described in the literature and are referenced in Appendices 3, 4, and 5. Experiments to determine missing values and/or to evaluate existing tests for applicability to less precise instruments and methods are described in Appendix 3. Available data, new experimental values, and error equations were summarized in graphs described and used in the following sections.

## DETERMINATION OF APPROPRIATE PROCEDURES

### Accuracy Goals and Instrument Precision

The first step in the survey process is to determine the required degree of accuracy. Only the user can make this determination because only he knows the objectives. Because systematic errors, being correctable, are specifically excluded from consideration, and because replicate measurements are seldom made in surveying, the accuracy of the point location is a direct function of the precision of the measurement. As a general rule the survey measurements must be precise enough so as not to detract from the usefulness of the other observations and determinations made at the measured point. Survey measurements more precise than the observations and determinations probably mean a waste of time, money, and effort unless a more detailed study is planned for a later date. If maps are the goal then the National Map Accuracy Standards (Appendix 2) might be considered as a guide to the accuracy of planimetric detail. Contour maps have already been discussed by way of example.

### Tentative Plan

Site examination.--Assuming all available data on the project area have been assembled, the next task is to consider physical conditions of the site. If the area is large, then a control survey of some type is necessary to maintain

constant scale throughout the study area. Relief, surface texture, and vegetative cover may render some instruments and methods impossible; for example taping in badlands, or differential leveling in dense scrub or forest.

Control.--If values tied to a geodetic datum are needed, or a large area is involved, the next step is to seek out available control. Routes of control surveys by Federal agencies are shown on a series of 1:250,000 Geodetic Control Diagrams available from the USGS map distribution centers. Each diagram covers the same area as a sheet of the 1:250,000 U.S. series of topographic maps, and each bears a legend outlining the procedure for obtaining the necessary data sheets. These data sheets contain the appropriate geodetic coordinates and descriptions of the physical locations of each monument. National Ocean Survey (formerly US Coast and Geodetic Survey) control is also shown on pairs of state maps available from that agency.

If this existing control is suitably located in a large study area, it may reduce or eliminate the need for a separate control survey. Errors between points of an existing survey can be evaluated by use of agency standards (National Ocean Survey, 1974).

Tentative traverse plan.--The next step is to formulate a tentative survey route which may be outlined on available maps or on a sketch. Distances from control or arbitrary reference to the measured point should be kept to a minimum. The route should follow lines of convenient access (roads, railroads, rights-of-way) wherever possible, and lines of any significant length should be closed on some known point. It should then be possible to estimate the average and maximum distances separating measured points from control. The above information will be used in the next sections to determine which methods are adequate.

## Horizontal Measurements

Statistical parameters.--A measurement error, which is the familiar univariate statistic, differs from a location error in two dimensions (horizontal plane) which is a bivariate statistic. This difference is important because the standard deviation, the measure of error, represents different percentages of the normal population; some 68 percent for the univariate population, and 39 percent for the bivariate population. This difference is reflected in the multipliers used to arrive at other confidence limits.

The bivariate value is derived from two univariate measurements. The general case at any particular confidence level describes the measured point as lying within ellipse of certain size and shape. Rigorous solution for this type of error distribution and the summation of several such errors requires matrix algebra and terms for co-variance and is too complex for general application.

If the errors in the univariate measurements are orthogonal (at 90 degrees), then the axes of the ellipse coincide with the directions of the univariate errors. Further, if the univariate errors are equal then the ellipse becomes a circle. For this particular case, the bivariate standard deviation equals the univariate standard deviation. One can then be 68 percent sure that the true value lies within one standard deviation of each of the univariate measurements, or be 39 percent sure the true value lies within a radius of one standard deviation of the measured location. Other common confidence limits can be calculated from this value by use of Table 2.

Examination of the general method of traversing shows that the error in a measured point is due to an error in distance and an error in direction and that these errors are always orthogonal. If the precision of the traverse components are approximately equal, then the location error ellipse approaches

Table 2. Probability level conversion factors (Greenwalt and Shultz, 1962)

Linear Error Conversion Factors (univariate)					
From	To	50% PE	68.27% SD	90% MAS	99.73% NC
50%		1.0000	1.4826	2.4387	4.4475
68.27%		0.6745	1.0000	1.6449	3.0000
90%		0.4101	0.6080	1.0000	1.8239
99.73%		0.2248	0.3333	0.5483	1.0000
Circular Error Conversion Factors (bivariate)					
From	To	39.35% CSD	50% CPE	90% CMAS	99.78% CNC
39.35%		1.0000	1.1774	2.1460	3.5000
50%		0.8493	1.0000	1.8227	2.9726
90%		0.4660	0.5486	1.0000	1.6309
99.78%		0.2857	0.3364	0.6131	1.0000
Spherical Error Conversion Factors (trivariate)					
From	To	19.9% SSD	50% SPE	90% SMAS	99.89% SNC
19.9%		1.0000	1.538	2.500	4.000
50%		0.650	1.000	1.625	2.600
90%		0.400	0.615	1.000	1.600
99.98%		0.250	0.385	0.625	1.000

PE - Probable error, SD - Standard Deviation, MAS = Map Accuracy Standard,  
 NC = Near Certainty

the circular ideal.

If the error in measuring each location has a near-circular distribution, then the error in any series of locations can be determined by using the Theory of Errors and the radius of a circle substituted for the ellipse. As a conservative estimate of the radius, one could use the value of the larger of the two errors, or one could use a circular substitution equation such as that developed by Greenwalt and Shultz (1962):

$$\sigma_{\text{circular}} = (0.5222 \sigma_{\text{min.}} + 0.4778 \sigma_{\text{max.}})$$

where  $\sigma$  is the standard deviation. They cautioned against the application of this equation where  $\sigma_{\text{min.}}/\sigma_{\text{max.}}$  is smaller than 0.6.

Error charts.--Charts in this and the following discussion on vertical measurements contain plots for generalized types of instruments. Also included are actual test values for a few specific instruments that are of particular interest and which will be used in later examples. Actual test values for a number of other instruments are in Appendices 4 and 5. The user may produce similar plots for the specific instruments he wishes to consider from information in Appendices 3, 4, and 5.

Figure 1, based on the experimental derivation of rod-reading error (Appendix 3), shows the error in stadia distance measurements as a function of sight length, rod-graduation size, and telescope magnification.

Figure 2 shows the errors involved in most of the commonly available distance measurement methods or devices. Plane table errors (Table 10, Appendix 3) could not be shown because plotting error, the major source of error, causes a resultant error that is a function of the scale of the map (the smallest measured error was  $\pm 1$  min.,  $\pm 60$  sec. on the figure).

These or similar charts enable the user to select balanced traverse components and to estimate the error in any measurement. The optical methods of distance measurement are severely limited in range, whereas direction

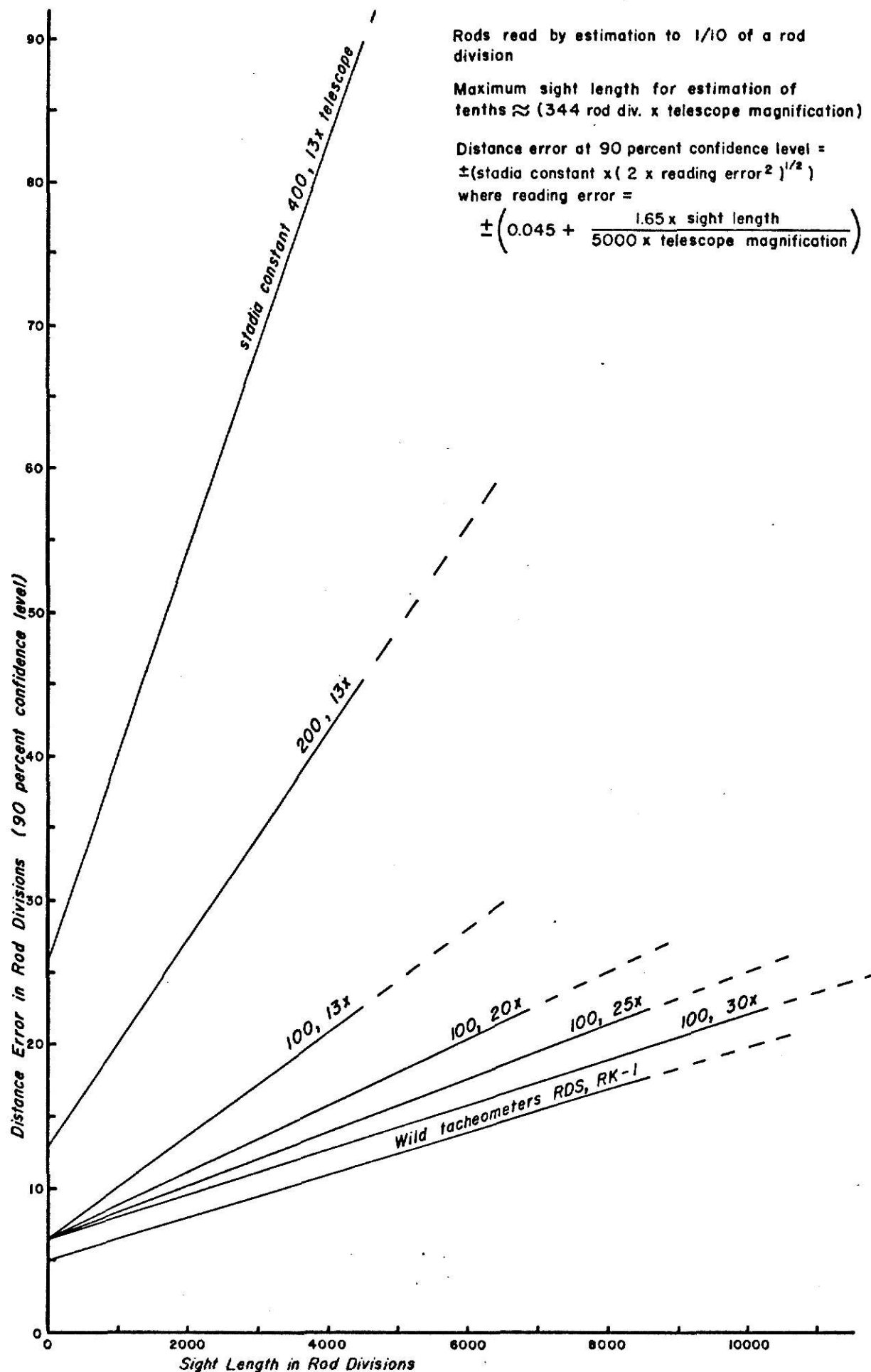


Figure 1 Errors in Stadia Distance Measurements

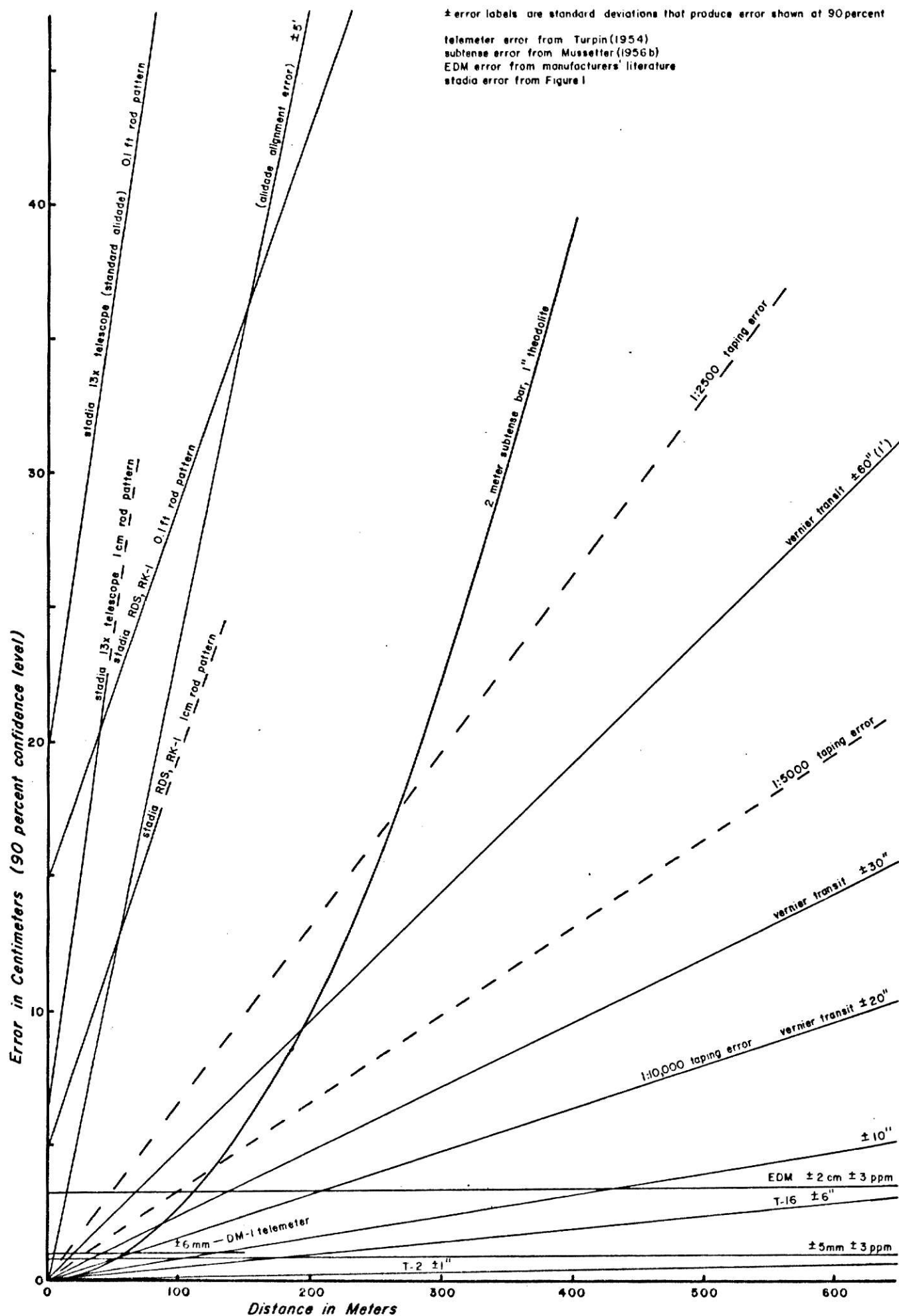


Figure 2 Errors in Horizontal Measurements

measurements are limited only by the need for intervisibility and the need for a suitable target.

Estimates of the individual errors of the tentative traverse can be derived from the graph and combined to give a bivariate error radius for each single location. This value must then be multiplied by the appropriate constant to convert it to the desired probability level. The individual error radii can then be combined using the Theory of Errors to give the location error of any point on the traverse. Practically, the spacing between traverse stations will probably depend upon the methods of measuring elevation, most of which have a short range. Therefore, this calculation must be postponed until the method of elevation measurement has been selected if both are to be done in one operation.

#### Vertical Measurements

Statistical parameters.--For true geometric location in three dimensional space there is a corresponding trivariate statistic and error ellipsoid. However, because elevations almost always are used, and often determined independently of exact horizontal location, the univariate statistic is appropriate for most uses. For spherical substitutions and confidence limits see Greenwalt and Shultz (1962).

Error charts.--Figure 3 shows the expected error in one sight for the standard exploration model alidade. Figure 4 shows the expected elevation error for the Wild RDS and RK-1 tacheometers. Both figures are based on the equations in Table 1 and experimental values in Appendix 3.

Errors for most optical methods of elevation measurement are shown in Figure 5. Trigonometric leveling depends upon the accuracy of both the vertical angle measurement and the distance measurement. Values can be derived using the equation in Table 1 and data in Appendix 4, but there are too many

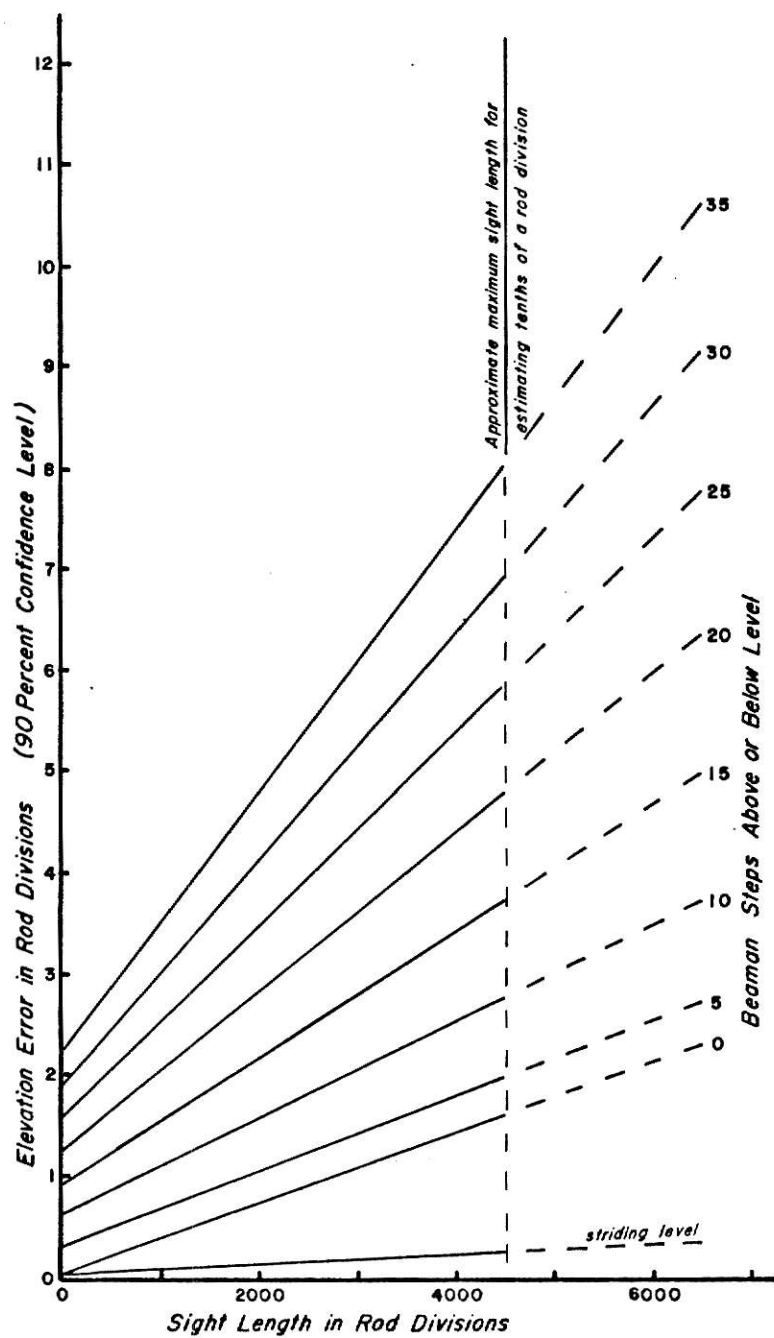


Figure 3 Errors in elevations, Standard alidade (13x telescope) and Beaman Method

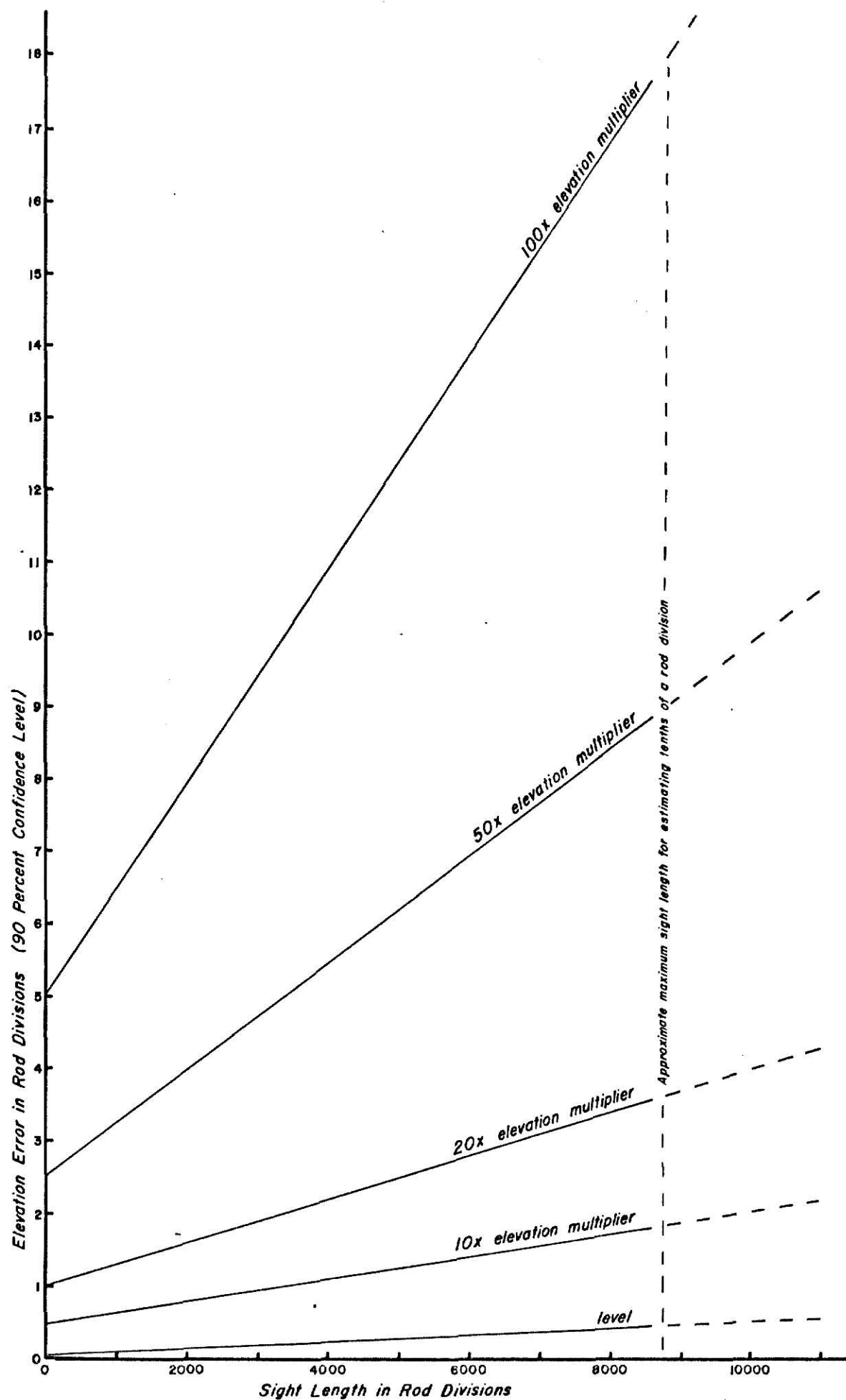


Figure 4 Errors in elevations, Wild self-reducing tachometers RDS and RK-1

## ALTIMETRY

Method	Error (90% C.L.)
Single base	$\pm 10.7$ feet
Double base	$\pm 9.3$ feet
Triple base	$\pm 8.8$ feet
Leap frog	$\pm 12.7$ feet

Derived from Table 14, Appendix 5

NOTE: label (  $\pm A''$  Bx )

A = error in leveling instrument, see Appendix 5

B = telescope magnification

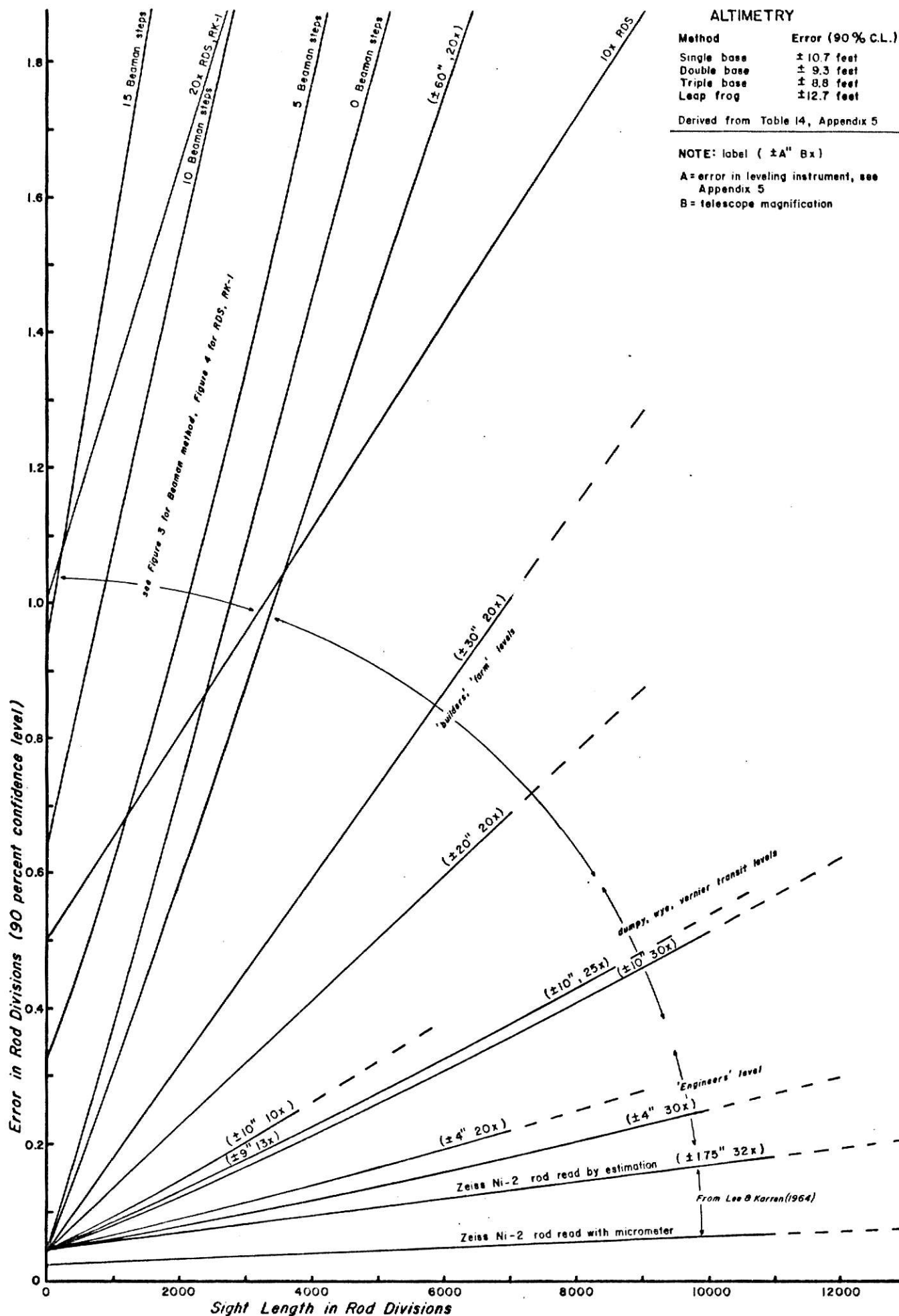


Figure 5 Errors in Vertical Measurements

variables to plot meaningful values on the general chart. Trigonometric measurements based upon a vernier transit and stadia distances fall in the same range as Beaman values.

With these or similar charts the error in each measurement can be estimated and using the Theory of Errors an estimate for the error in any measured point can be derived. For purposes of planning and selection, one selects a convenient average sight-length and divides this into the distance from control to give the number of sight-lengths necessary to arrive at the distant points. Because sight-lengths are equal, the error in each are all equal and the equation from the Theory of Errors simplifies to:

$$\text{resultant error} = \text{error in one measurement} \times (\text{number of measurements})^{\frac{1}{2}}.$$

From a theoretical viewpoint, one will find that for any error line on the chart and any given traverse distance there exists an optimal sight length (i.e. one that will give a minimum resultant error). This optimal value increases with traverse length. Practically, shorter sight lengths mean more measurements and hence more time and field expense. Therefore, the sight lengths should be kept in the maximum third of the instrument range if scintillation and systematic errors do not interfere.

These charts and principles apply to accidental errors only, predicted results cannot be achieved unless systematic errors in the measurements are corrected or eliminated.

#### SELECTION FROM AMONG APPROPRIATE PROCEDURES

By using the methods and data in the preceeding sections the required precision can be used to divide the methods into adequate and inadequate groups. The next step is to select a method from the group of adequate methods.

### Immediate Availability and Costs

The user may have available only one of the adequate methods and the problem is solved. If several methods are available or rental or purchase is being considered, then overall cost determines the final selection. In addition to capital investment, the cost equation includes terms for the efficiency of the instrument, volume of present work, and probability of future work in which the instrument could be used.

Present experiments were terminated before any data on relative efficiency of different instruments could be gathered, and considerations of present and future workload are too complex and abstract to be discussed here. Some general observations include: (1) more precise instruments, although requiring larger capital investments have lower operating costs because they are also generally easier and faster to use; (2) the probability is larger that a more precise instrument will be adequate for future tasks than the probability for a less precise instrument..

### Systematic Errors

Some methods or instruments require ancillary measurements for the correction of systematic errors which must be made with each primary measurement if the results are to be corrected. For example, precision taping requires temperature, tension, and sag data for each measurement. Several other constants which vary from tape to tape must also be determined in the laboratory. Seven separate corrections (Table 12, Appendix 4) must be calculated and applied to each measurement. Thus, if EDM (Electronic Distance Measurement) devices are available, they would be preferred because similar corrections need be applied only to the most precise measurements.

Other methods or instruments require use of certain field procedures to eliminate systematic errors. Balanced sight lengths are desirable for

all types of differential leveling, but are particularly important when using an automatic level (Karren 1964). The automatic level is probably the fastest and easiest method of determining elevations if terrain conditions permit the balancing of sight lengths without substantially reducing the range of the instrument. If balanced sight lengths are not possible, then a well adjusted spirit level would be needed for gentle slopes, or a method employing vertical angles could be used on steeper slopes if accuracy requirements permit.

#### Operator Convenience and Speed

Some instruments, generally the more expensive ones, are designed so as to be less demanding upon the skills of the operator. As a result these instruments are faster and easier to use and chances of blunders are reduced.

Karren (1964) found the accuracy of a good automatic level, such as the Zeiss Ni-2, comparable to the best of the generally available spirit instruments, such as the Wild N-3, under normal operating conditions. Because the automatic level releases the operator from the task of precisely centering the bubble, it is faster and easier to use. However, he found also that accuracy deteriorated much more rapidly for automatic instruments when sight lengths were not balanced.

Elevation measurements with the Wild RDS tachometer are comparable to values obtained by trigonometric leveling with transit and stadia. RK-1 elevations are comparable to values obtained with a standard alidade and the Beaman method. In both examples the first instrument requires fewer and simpler operator tasks and simpler mathematical computations than the second instrument, thus the probability of blunders is reduced and measurements are made more rapidly. The RK-1 also has approximately twice the range of the standard alidade using any given rod pattern because of a more powerful telescope.

Elevations determined with the Wild RDS and RK-1 are also comparable, but the RK-1 must be transported between stations in a bulky case whereas the RDS can be left on the tripod, or placed in a container which is smaller and lighter than that of the RK-1. The RK-1 plane table and tripod are also awkward to carry. Therefore, the RDS would be preferred over the RK-1 if the plane table was not needed to draw a map.

The Wild T-16 and other optical transits are not only more accurate than the common vernier transit, but they are easier to read which reduces the probability of blunders. These instruments are also lighter and smaller than their vernier counterparts so they are more convenient to transport and use in the field.

#### Practical Examples

Yarrow and Schmidt.--Yarrow (1974) and Schmidt (1974) made a structure contour map of the top of a sedimentary rock unit over an area of 800 square miles (fig. 6). The area was completely covered on the 1:24,000 (7½ min.) series of USGS topographic maps. Also available were 1:62,500 geologic maps that aided in locating desired outcrops. As the structure map was ultimately reduced to a scale of 1:250,000, horizontal location by inspection on the 1:24,000 maps was more than sufficient. The selected contour interval was 20 feet; adopting the USGS topographic standards, the elevations of the measured points should be determined within  $\pm 2$  feet. The northern half of the area was covered on recent (1964) topographic maps at a 10-foot contour interval which contained checked spot elevations for vertical control every one or two miles along section roads. In the southern area control consisted of values for benchmarks, bridge abutments, and other highway structures supplied by state and county engineers.

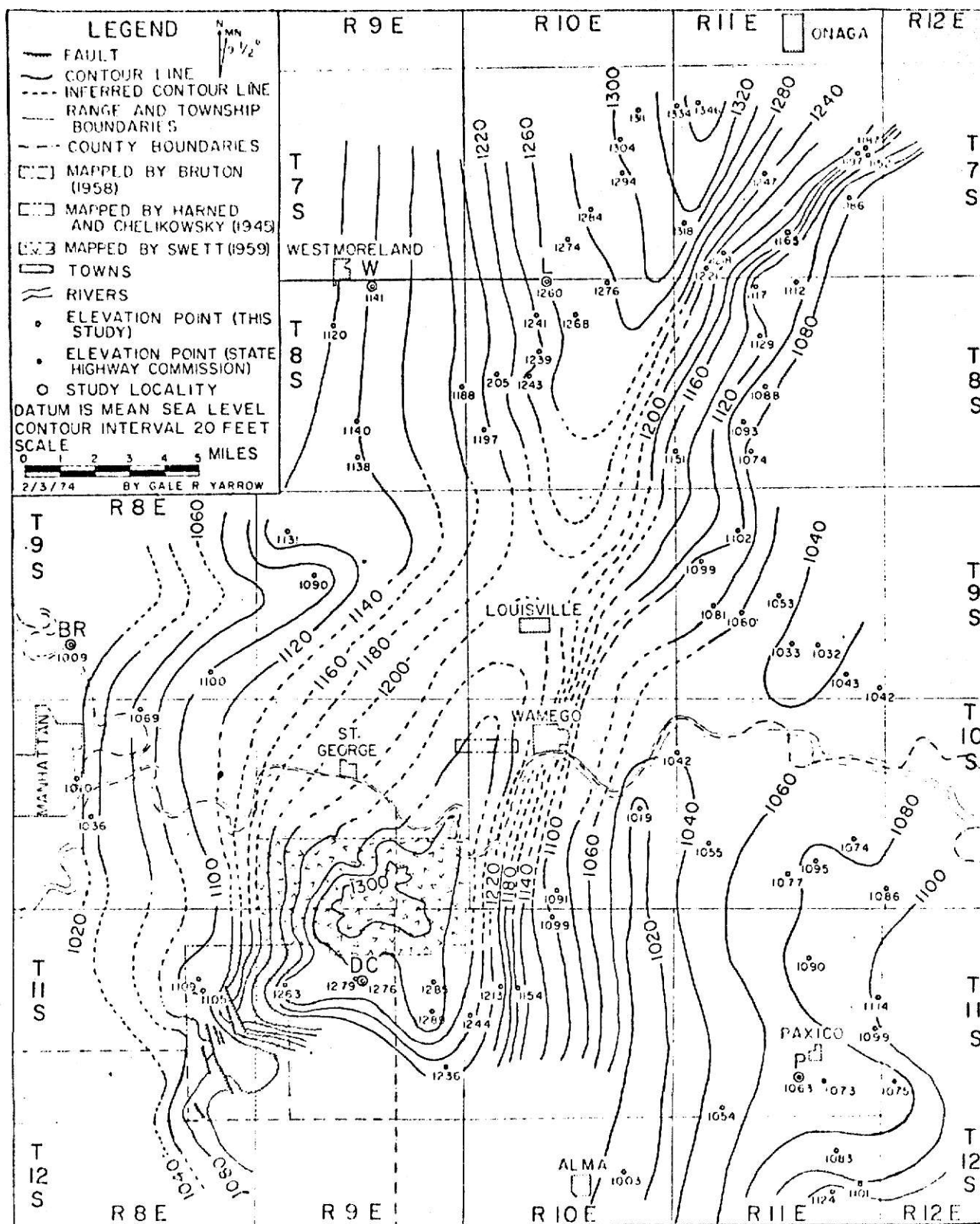


FIGURE 6 STRUCTURE CONTOURS ON TOP OF AMERICUS LIMESTONE IN POTTAWATOMIE, RILEY, AND WABAUNSEE COUNTIES, KANSAS (Yarrow, 1974)

The terrain is rolling with several hundred feet of total relief, thus sight lengths for an automatic level would have been restricted by terrain and maintenance of the required balanced sight lengths would have increased the number of measurements. Both the plane table and the RDS would produce suitable results but the RDS required less effort and time. The average slope of the line of sight was about  $2\frac{1}{2}$  degrees, which is in the 10x multiplier range of the RDS, or about 5 Beaman steps. Most points are within  $1\frac{1}{2}$  miles of a starting elevation. Instrument errors from Figures 3 and 4 and the equation from the Theory of Errors yield the results in Table 3. In this example the limiting uncertainty was that of the starting elevation of  $\pm 1$  foot.

Table 3. Expected elevation error in  $1\frac{1}{2}$  miles of traverse.

Instrument	Rod Pattern	Sight Length	Single Error	Shots/ $1\frac{1}{2}$ mi	Resultant
K&E alidade	0.1 ft	450 ft	$\pm 0.2$ ft	18	$\pm 0.9$ ft
RDS	0.1 ft	800 ft	$\pm 0.17$ ft	10	$\pm 0.5$ ft
RDS	1 cm	80 meters	$\pm 1.7$ cm	30	$\pm 0.4$ ft

Bell.--Bell (1974) was engaged in a groundwater study involving wells on a river flood plain that were about one mile apart. Location by inspection on existing 1:24,000 maps was sufficient for horizontal location. The groundwater surface was ultimately contoured at a five-foot interval, adopting the USGS topographic standards, well head elevations within  $\pm 0.5$  ft would be required (fig. 7). Spot elevations on the maps were unsuitable for vertical control because they were given only to the nearest foot. The railroad right-of-way through the valley had been used for a first order geodetic level line. The specifications for such surveys call for permanent bench marks to be established every one to two km along the route. Leveling using the striding level of the alidade was one possibility, but a Zeiss Ni-2 level was available and was much faster and easier to use. Table 4 gives the results of using a line length of three miles, instrument errors from Figure 5, and the Theory of



Errors. These calculations are for accidental errors only; care is needed to eliminate systematic error if these results are to be achieved.

Table 4. Expected elevation error in 3 miles of leveling.

Level Inst.	Rod Pattern	Sight Length	Single Error	Shots/3 mi.	Resultant
$\pm$ 4 sec 30x	0.1 ft.	500 ft.	$\pm 0.014$ ft.	34	$\pm 0.06$ ft.
$\pm$ 4 sec 30x	0.1 ft.	900 ft.	$\pm 0.022$ ft.	16	$\pm 0.09$ ft.
$\pm 10$ sec 30x	0.1 ft.	500 ft.	$\pm 0.022$ ft.	34	$\pm 0.16$ ft.
$\pm$ 9 sec 13x	0.1 ft.	450 ft.	$\pm 0.025$ ft.	36	$\pm 0.15$ ft.
(RDS)	0.1 ft.	800 ft.	$\pm 0.17$ ft.	20	$\pm 0.8$ ft.
(alidade)	0.1 ft.	450 ft.	$\pm 0.16$ ft.	36	$\pm 1.0$ ft.

Hypothetical Example 1.--Yarrow's project area is used for two of the three hypothetical examples. Assume that an accuracy of  $\pm 1$  foot was desired for the construction of the structural contour map. The map spot elevations no longer would be suitable for control and measurements would have to be made from monumented points. Figure 8 shows the routes of USGS and USC & GS level and traverse lines through the area. Specifications for this work call for monuments at about one-mile intervals along these lines. The exact locations of the monuments are described on separate data sheets that are available from the appropriate agency. Examination of Figure 8 shows most measured points should be within three miles of a monument. Table 5 shows the results of using a line length of three miles, instrument errors from Figures 3 and 4, and the Theory of Errors. In this example the alidade would be a marginal failure, but the RDS would be acceptable.

Table 5. Expected elevation error in 3 miles of traverse.

Instrument	Rod Pattern	Sight Length	Single Error	Shots/3 mi.	Resultant
K & E alidade	0.1 ft.	450 ft.	$\pm 0.2$ ft.	36	$\pm 1.2$ ft.
RDS	0.1 ft.	800 ft.	$\pm 0.17$ ft.	20	$\pm 0.8$ ft.
RDS	1 cm	80 meters	$\pm 1.7$ cm	60	$\pm 0.5$ ft.

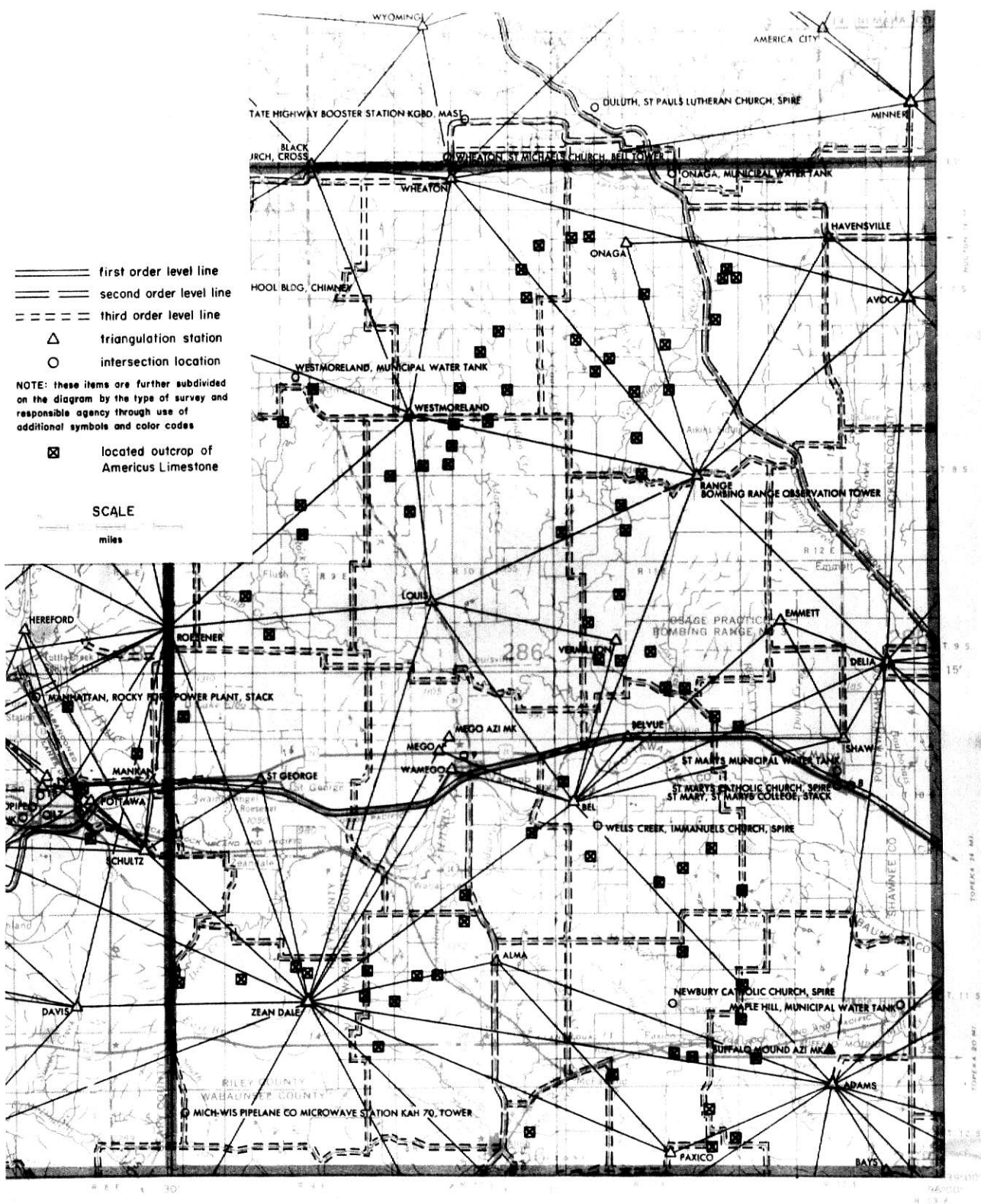


Figure 8 Section of Geodetic Control Diagram Manhattan, Kansas (NJ 14-3) 1967

Hypothetical Example 2.--This area contains more than the usual number of control survey lines. As a second example, consider the same area without the 1:24,000 maps and only the USC & GS primary level lines available. Horizontal locations could still be made with adequate precision on the 1:62,500 geologic maps. Because some data points would be ten or more miles from a benchmark, the avoidance of blunders becomes an important consideration. Most or all measurements in any one map area would be affected by a blunder in the survey line and this could seriously affect the structure shown on the map.

The project would have to be executed in two steps: (1) a control survey to establish level lines, and (2) ties between the level line and the outcrops. For the level line a fast and convenient instrument such as the Wild or Zeiss automatic level should be chosen. The level line should be laid out along roads wherever possible. As a check for blunders, the lines should be closed on second benchmarks. Ideally the line would be run twice in one or two-mile sections to isolate any blunders and avoid later re-runs.

Figure 9 shows routes for four possible level lines to carry elevations to the vicinity of the outcrops. The two longer lines can be closed to existing control and adjusted. The other two lines could be closed on bridge values supplied by state engineers as a check for blunders, but adjustment probably would not be warranted. Table 6 gives the expected accidental error of several instruments for running the level lines in Figure 9. Also included are the standards for third-order leveling. After adjustment, the expected error and the center of the adjusted line would be half the value shown in the table.

An effort of this magnitude should include permanent marks for use by later investigators. Details of field operations are discussed else where

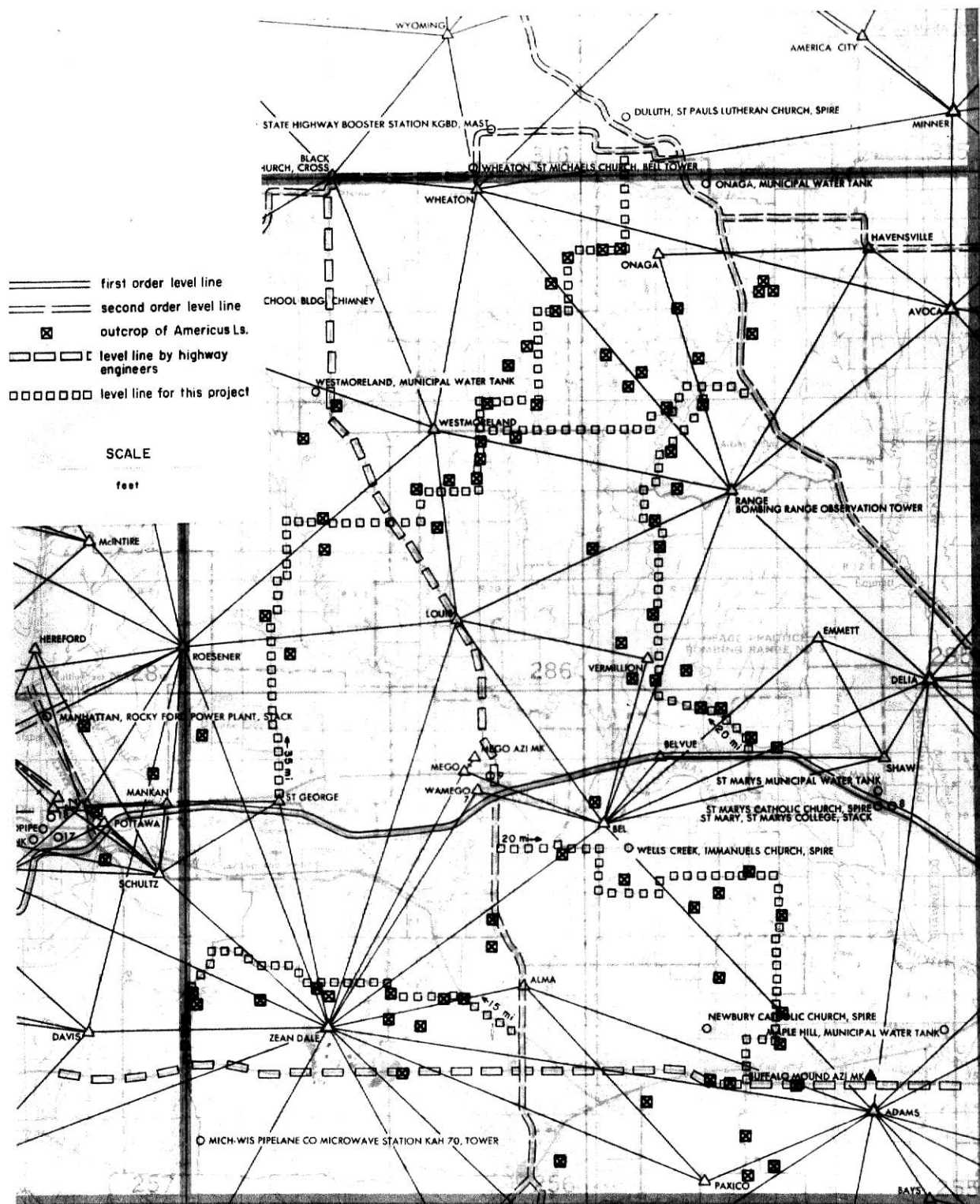


Figure 9 Tentative routes of control level survey for Hypothetical example 2

Table 6. Expected error for level lines of 15, 20, and 35 miles

Rod Pattern and Sight Length	Instrument	Error in millimeters for line length		
		15 miles	20 miles	35 miles
1 cm rod 30 m sight	Ni-2 micrometer	10	12	15
	Ni-2 estimation	23	26	35
	$\pm 4$ sec 30x*	27	31	41
	$\pm 10$ sec 30x	45	52	69
	$\pm 20$ sec 20x	91	105	139
	$\pm 30$ sec 20x	130	151	199
1 cm rod 50m sight	Ni-2 micrometer	9	11	14
	Ni-2 estimation	24	28	37
	$\pm 4$ sec 30x	30	34	45
	$\pm 10$ sec 30x	57	66	87
	$\pm 20$ sec 20x	112	129	171
	$\pm 30$ sec 20x	160	185	245
1 cm rod 70 m sight	Ni-2 micrometer	9	11	14
	Ni-2 estimation	25	29	38
	$\pm 4$ sec 30x	33	39	51
	$\pm 10$ sec 30x	67	77	102
	$\pm 20$ sec 20x	128	148	196
	$\pm 30$ sec 20x	195	225	298
1 cm rod 90 m sight	Ni-2 micrometer	10	11	15
	Ni-2 estimation	26	30	40
	$\pm 4$ sec 30x	36	42	55
	$\pm 10$ sec 30x	75	87	115
	$\pm 20$ sec 20x	144	167	220
	$\pm 30$ sec 20x	210	242	320
Third Order Standards		59	68	80

\*  $\pm A$  sec Bx, where A = error in leveling the instrument (standard dev.)  
B = magnification of the level telescope

such as in the USGS Topographic Instructions 2E-Leveling (1966).

Hypothetical Example 3.--Assume one is interested in monitoring the strike-slip fault shown in Figure 10 for relative movements of the blocks which might indicate an increase in strain leading to failure and an earthquake. Assume also that previous investigations have shown that the effects of past faulting do not extend beyond one km either side of the fault trace. Three monuments, A, B, and C are erected as shown in the figure. Relative movement of the blocks could be detected by repeating measurements of the distance A-C, or the angle ABC at suitable time intervals. In this example a measure in only one direction, parallel to the fault, is of interest and not the location of the points; therefore, the univariate statistic is appropriate.

Several surveying instruments are available and some idea of the effectiveness of each is needed. Table 7 gives the minimum lateral displacements detectable by a one-tail T-test for the instruments and number of repetitions listed. Advance knowledge of the accidental error of each instrument permits calculation of a T-test by substituting the known standard deviation into the equation. This permits one to judge the suitability of the instruments before any measurements are made.

## CONCLUSION

Use of these concepts and data permits division of available methods into adequate and inadequate groups on the basis of required accuracy. Further research could introduce more refinement into the determination of the error elements upon which the graphs are based. However, because most of the accidental error elements include a human operator making decisions, a truly definitive value for the error inherent in any instrument can never be derived. All that is possible are successively closer approximations to

Table 7. Minimum detectable displacements for several measurement devices,  
Hypothetical example 3.

Angular instrument error in one reading	Error in one angle measurement	Angle repeated 10 times at each test degrees freedom = 18 minimum displacement	Angle repeated 16 times at each test degrees freedom = 30 minimum displacement
$\pm 1$ sec.(T-2)	$\pm 1.4$ sec.	.9 cm	.7 cm
$\pm 6$ sec.(T-16)	$\pm 8.5$ sec.	5.1 cm	4.0 cm
$\pm 20$ sec.(transit)	$\pm 28.3$ sec.	16.9 cm	13.2 cm
$\pm 30$ sec.(transit)	$\pm 42.4$ sec.	25.4 cm	19.8 cm
$\pm 60$ sec.(transit)	$\pm 84.9$ sec.	50.8 cm	39.6 cm

EDM devices	Error in one distance measurement	Dist. repeated 10 times at each test	Distance repeated 16 times at each test
$\pm 3\text{mm} \pm 3\text{ppm}$	$\pm 1.5$ cm	0.9 cm	0.7 cm
$\pm 1.5$ cm $\pm 3$ ppm	$\pm 2.1$ cm	1.3 cm	1.0 cm
$\pm 1.0$ cm $\pm 3$ ppm	$\pm 1.8$ cm	0.9 cm	0.8 cm

for names of EDM devices see table 13, appendix 4.

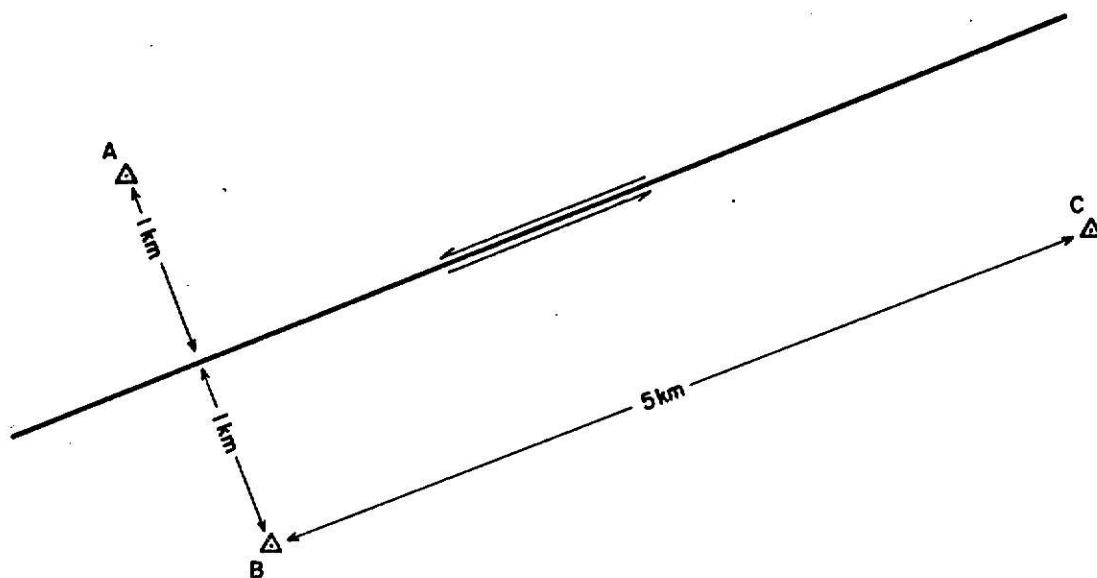


Figure 10 Plan view of fault study, Hypothetical example 3

the truth. A first approximation are the included general charts. As a second approximation the user could make similar charts for his particular instruments. A closer approximation would be charts for the performance of each operator with each instrument, but even these values would be affected by the working environment of the operator. Closer approximations probably are not practical and efforts beyond the second approximation would be worthwhile only in special circumstances.

This study is important to the geologist or other occasional user of surveying techniques because hopefully it will acquaint him with the existence and capabilities of the available surveying instruments. This information is not readily available in any single source and persons having true acquaintance with more than a few instruments are unusual. When confronted with a measurement task for which reconnaissance methods are inadequate, this text should enable selection of an adequate method thus avoiding possible experimental failure or inadvertent violation of the principle of significant figures.

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## Appendix 1- A short subject index to the selected references

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## Appendix 2 - National Map Accuracy Standards

Formulated for use by all federal mapping agencies by U. S. Bureau of the Budget, adopted June 10, 1941, as revised June 17, 1947.

### UNITED STATES NATIONAL MAP ACCURACY STANDARDS

With a view to the utmost economy and expedition in producing maps which fulfill not only the broad needs for standard or principal maps, but also the reasonable particular needs of individual agencies, standards of accuracy for published maps are defined as follows:

1. Horizontal accuracy. For maps on publication scales larger than 10 percent of the points tested shall be in error by more than  $1/30$  inch, measured on the publication scale; for maps on publication scales of 1:20,000 or smaller,  $1/50$  inch. These limits of accuracy shall apply in all cases to positions of well defined points only. "Well defined" points are those that are easily visible or recoverable on the ground, such as the following: monuments or markers, such as bench marks, property boundary monuments; intersections of roads, railroads, etc.; corners of large buildings or structures (or center points of small buildings); etc. In general what is "well defined" will also be determined by what is plottable on the scale of the map within  $1/100$  inch. Thus while the intersection of two road or property lines meeting at right angles would come within a sensible interpretation, identification of the intersection of such lines meeting at an acute angle would obviously not be practicable within  $1/100$  inch. Similarly, features not identifiable upon the ground within close limits are not considered as test points within the limits quoted, even though their positions may be scaled closely upon the map. In this class would come timber lines, soil boundaries, etc., etc.

2. Vertical accuracy, as applied to contour maps on all publication scales, shall be such that not more than 10 percent of the elevations tested shall be in error more than one-half the contour interval. In checking elevations taken from the map, the apparent vertical error may be decreased by assuming a horizontal displacement within the permissible horizontal error for a map of that scale.

3. The accuracy of any map may be tested by comparing the positions of points whose locations or elevations are shown upon it with corresponding positions as determined by surveys of a higher accuracy. Tests shall be made by the producing agency, which shall also determine which of its maps are to be tested, and the extent of such testing.

4. Published maps meeting these accuracy requirements shall note this fact in their legends, as follows: "This map complies with the National Map Accuracy Standards."

5. Published maps whose errors exceed those aforestated shall omit from their legends all mention of standard accuracy.

6. When a published map is a considerable enlargement of a map drawing ("manuscript") or of published map, that fact shall be stated in the legend. For example, "This map is an enlargement of a 1:20,000 scale map drawing," or "This map is an enlargement of a 1:24,000 scale published map."

### Appendix 3 - Description of Experiments Conducted by the Author

Experiment 1 - Direct Measurement of Reading Error.--Aguilar (class notes 1969) postulated a reading-error distribution with a minimum at other than zero sight length, thus there would be an optimal sight length for measurements. To develop the actual error curves, he proposed making repeated estimates of the same rod image; the mean of the set would be the best estimate of the true value, and the standard deviation would be a measure of the reading error. Several rod patterns would be observed over a range of distances to produce a family of curves, one for each pattern. A long, level test site was needed and a 3,000 foot line at the base of Tuttle Creek Dam was selected.

Several days of observations failed to produce the expected results and the effort was abandoned. The failure can be attributed to faulty experimental logic. As Backstrom (1930, 1932, 1933) and Turnbull and Ellis (1952) noted, an observer subjectively subdivides an interval into a series of regions and the placement of these regions remains constant with time. Thus repeated estimates of the same image will all be the same, the mean will not provide a better estimate of the true value, and there will be no standard deviation even though an error actually exists.

Experiment 2 - Effect of Target on Reading and Pointing.--Kissam and Irish (1951) proposed a comparative field test for evaluating levels, the "Princeton Standard Test". Kissam (1963) further standardized the test by providing a fixed course length (309 ft.), 10 intermediate fixed instrument stations, and 4 trips over the course (80 observations - 40 calculated differences) to make one data set for computation. Test results are reported as an angular standard deviation of the line of sight in the vertical plane. Relative errors of a series of rod patterns might be determined by comparing the resultant Princeton Test values, provided all patterns were read from the

same instrument station before the instrument was moved.

The Wild Na-2 automatic level was chosen as the test instrument because it was not included in the Princeton Test values of Lee and Karren (1964). Also, a parallel plate micrometer was available as an accessory for this instrument. This accessory replaces the operator task of estimating tenths of a division with the task of pointing at (bisecting) the appropriate graduation boundary with the crosshair. Use of this accessory might suggest some conclusions as to the effect of target design on pointing error as well as the desirability of different rod patterns a micrometer reading system.

The test course was laid out on the old intramural field south of the KSU President's residence according to the directions of Kissam (1963). Segments of four rod patterns were placed on a specially built target which was mounted on a tripod set up over each of the course end points.

The patterns tested were: (1) a 1 cm checkerboard pattern (Wild and Kern tacheometric rods, USGS leveling rods, (2) the Philadelphia rod pattern (very common in engineering practice), (3) a 1 cm pennant rod pattern (USGS standard stadia pattern was [Birdseye 1928] a 0.1 ft. pennant pattern and is fairly common in that application), and (4) a 0.5 cm checkerboard pattern. Because the micrometer was graduated in metric units, the size of the Philadelphia pattern was modified slightly so that one rod division corresponded to 3 mm rather than 0.1 ft.(= 3.048 mm). The 0.5 cm and 0.3 cm patterns were estimated to the tenth of a rod unit when estimation was used, values converted to metric units, and micrometer readings added where applicable. Calculations of test results were made using these metric values.

Kuznetsov (1969), in another study of rod patterns, found smaller reading errors (estimation errors) when the crosshair was on a light-colored graduation than on a dark one (i.e. the rod should have a continuous series of light-colored graduations bounded by black).

The expected ranking of patterns in this test, in order of decreasing precision, was:

A. for readings by estimation

- 1) 1 cm pennant - meets Kuznetsov's parameter, provides midpoint reference, is large enough to be well-defined at all test distances.
- 2) 1 cm checkerboard - meets Kuznetsov's parameter, large enough to be a well-defined image at all test distances.
- 3) 0.5 cm checkerboard - meets Kuznetsov's parameter, but smaller size should make estimation more difficult.
- 4) 0.3 cm Philadelphia - only half of the graduations meet Kuznetsov's parameter, small size.

B. for readings with micrometer

- 1) 1 cm pennant - notch in pattern provides definite target for pointing.
- 2) 1 cm checkerboard - boundary for pointing not as well-defined because it is obscured crosshair, pointing requires several up and down passes past the boundary and centering by intuition.
- 3) 0.5 cm checkerboard - because of smaller size and consequent poorer definition of boundary.
- 4) 0.3 cm Philadelphia - because pattern does not even supply the clues for pointing by intuition.

Three sets of tests using estimation of tenths, and three sets using the micrometer (Table 8) suggest no conclusions on the effect of rod pattern on reading error. The failure may be attributable to any of several factors:

- 1) masking effect of other larger errors - accidental errors accumulate as the square root of the sum of their squares

Table 8. Princeton Test results, Wild Na-2 automatic level

Test no.	Inst. no.	3mm Phila.	Rod Patterns		
			5mm Block	1 cm pennant	1 cm Block
without optical micrometer					
1	69	$\pm 1.99''$	$\pm 1.56''$	$\pm 1.33''$	$\pm 2.16''$
2	67	$\pm 1.63''$	$\pm 2.01''$	$\pm 1.61''$	$\pm 2.01''$
3	69	$\pm 1.83''$	$\pm 1.63''$	$\pm 2.06''$	$\pm 1.65''$
with optical micrometer					
4	71	$\pm 1.72''$	$\pm 1.67''$	$\pm 1.96''$	$\pm 1.66''$
5	73	$\pm 1.88''$	$\pm 1.23''$	$\pm 1.22''$	$\pm 1.54''$
6	73	$\pm 2.05''$	$\pm 1.85''$	$\pm 2.01''$	$\pm 1.81''$

(Theory of Errors), thus changes in a small error would have little effect on the resultant if other larger errors were present.

- 2) difference between absolute and relative error - for those sets read by estimation, Backstrom 1930, 1932, 1933, and Carr and Garner 1952, found an increase in relative error with smaller scales but a decrease in absolute error. The Princeton Test is a measure of absolute error. Also, only a few of the 80 observations in each set involved viewing distances where the visual size of the smaller graduations fell below 1 min. of arc, the point at which the psychologists (Murrell and others 1958, Grenther and Williams 1949, Leyzorek 1949, Kappauf and Smith 1951) observed a marked increase in estimation error.
- 3) graduation errors - for those sets read with a micrometer (The micrometer is intended for use with a carefully calibrated invar rod but in these experiments the graduations were simply drawn in ink on millimeter cross-section paper. Any error in a graduation boundary location would introduce a systematic error into the readings.)

Experiment 3 - Direct measurement of total error in tacheometric elevations.--Lee and Karren (1964) produced plots of leveling error versus sight length for the Zeiss Ni-2 and Wild N-3 by making repeated observations on a pair of level rods. Stations were at 30 foot intervals from 30 to 360 feet. The error was calculated from the variance of the observed differences of each pair of rod readings. The rods were side by side at one end of the test course so that sight lengths were balanced and systematic errors due to curvature, refraction, and collimation were self-canceling. Because errors were calculated from observed differences, changes in instrument height between

pairs of observations would not affect the results. In this experiment the same technique was used in an attempt to determine the error in elevation measurements made by the Beaman method and with the Wild tachometers RDS and RK-1. A test course was laid out on a long slope (about 4 degrees) adjacent to the south side of the KSU Auditorium. Two rods, supported by tripods, were placed side by side at the top of the hill. A set of data contained 25 pairs of observations from one instrument station. One test used the RK-1 alidade, 1 cm pennant rods, and stations at 50 foot intervals from 50 to 350 feet downslope from the rods. A second test used the RK-1, 0.1 ft. pennant rods, and stations at 100-foot intervals from 200 to 1000 feet. A third test used a standard alidade, 0.1 ft. rods and stations at 100 foot intervals from 100 to 500 feet. A fourth test was made on a level site (the old intramural field); this test used 1 cm rods, the RK-1 alidade, and stations at 50-foot intervals from 50 to 500 feet. In this last test the index level was centered, then the telescope leveled by matching the index and vertical arc graduations; the difference between the rods was calculated using the differential leveling procedure. The standard deviation for each set of test data was plotted yielding a series of error versus sight length curves, each showed the expected increase in error with sight length.

The RDS was then set up on the same test course as the first three tests above. The rods had a 0.1 ft. pattern and the difference in height was 0.47 ft. (0.47 ft. by happenstance, the higher rod was set on a pipe driven to refusal and the difference was measured later). The RK-1 under the same conditions had yielded an appropriate scattering of 0.4's and 0.6's for the measured difference because the minimum RK-1 elevation multiplier is 20, hence all elevation products and the differences were in even units. The RDS, however, has a minimum multiplier of 10; all observed differences were 0.5 ft. and the standard deviation was zero. The earlier RK-1 test values seemed to

include an artifact that was due to experimental structure; if the actual difference had not fallen accidentally about midway between two possible RK-1 values, the calculated results might have been different. Further RDS tests were suspended until this difficulty could be resolved.

During earlier observations with the alidades the plane table was left in place between pairs of observations. The consecutive pairs of observations with the RK-1 were made at increasing 5 rod unit (5 cm or 0.5 ft.) intervals on the rod (standard alidade interval was one Beaman step). If the plane table and rods remained at the same height during the set of observations, then all 50 observations can be treated as backsights on a known benchmark. Elimination of the fixed difference from the calculation should also remove the artifact. The standard deviation of the resulting instrument elevations was plotted yielding another set of error versus sight length curves (fig. 11 and 12).

Experiment 4 - Princeton Test as a measure of instrument errors.--Plots from the previous experiment and those of Lee and Karren (1964) show a resultant due to several sources of error. These sources can be divided into those caused by errors in making instrument settings and those arising in reading the rod. Perhaps the reading (estimation) error could be derived indirectly by measuring the errors associated with setting the instrument and factoring them out of the total using the Theory of Errors. Knowledge of each of the individual error elements would also permit construction of error plots for instrument settings which were not actually tested, if the propagation pattern were known.

Comparison of Lee and Karren's plots of error versus distance and Princeton Test values suggested that this test might be an appropriate measure of instrument-related errors (fig. 13). A second Princeton Test course was laid out on the same slope as used in the last experiment to test this

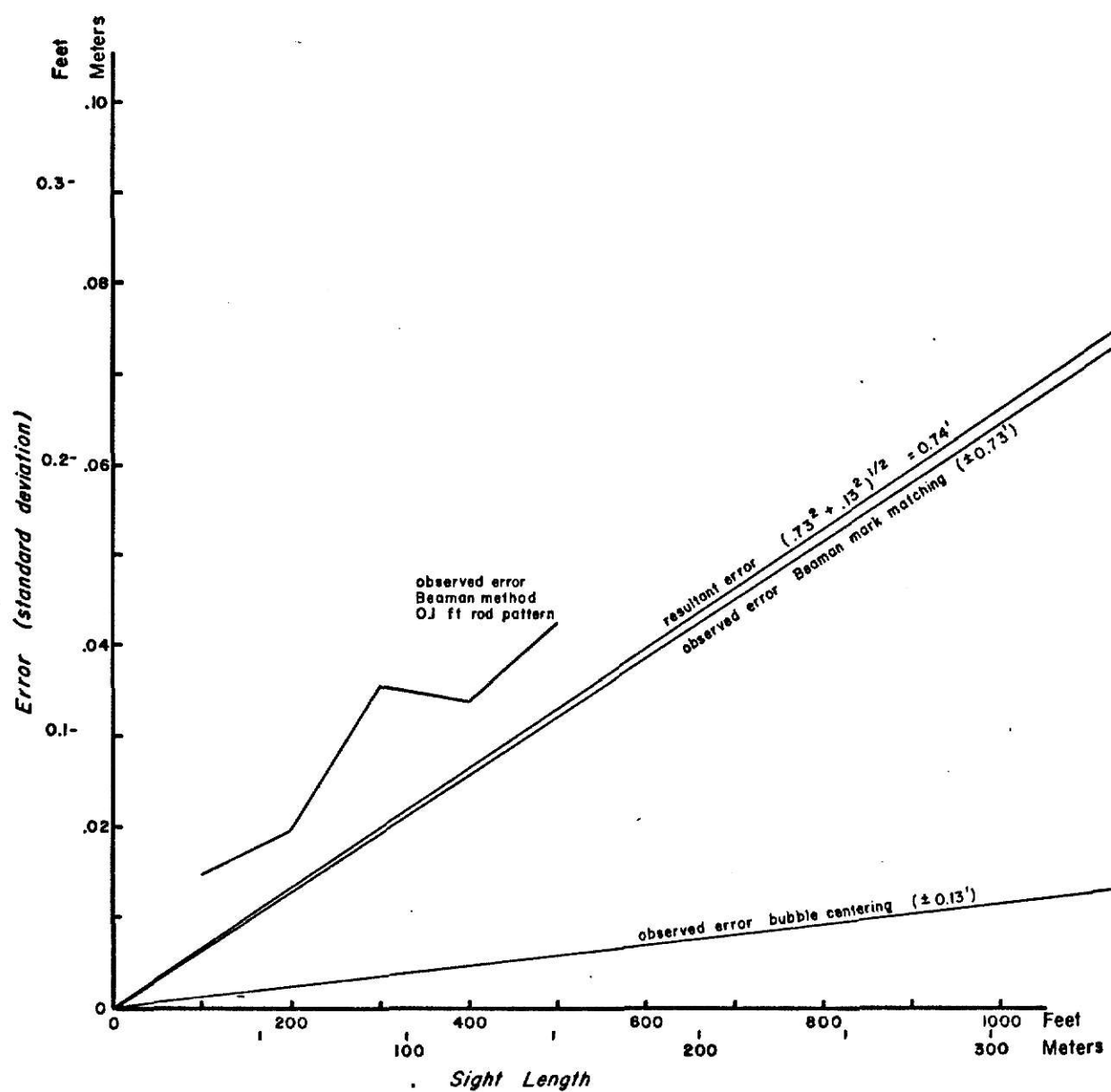


Figure II Observed errors, K&E alidade

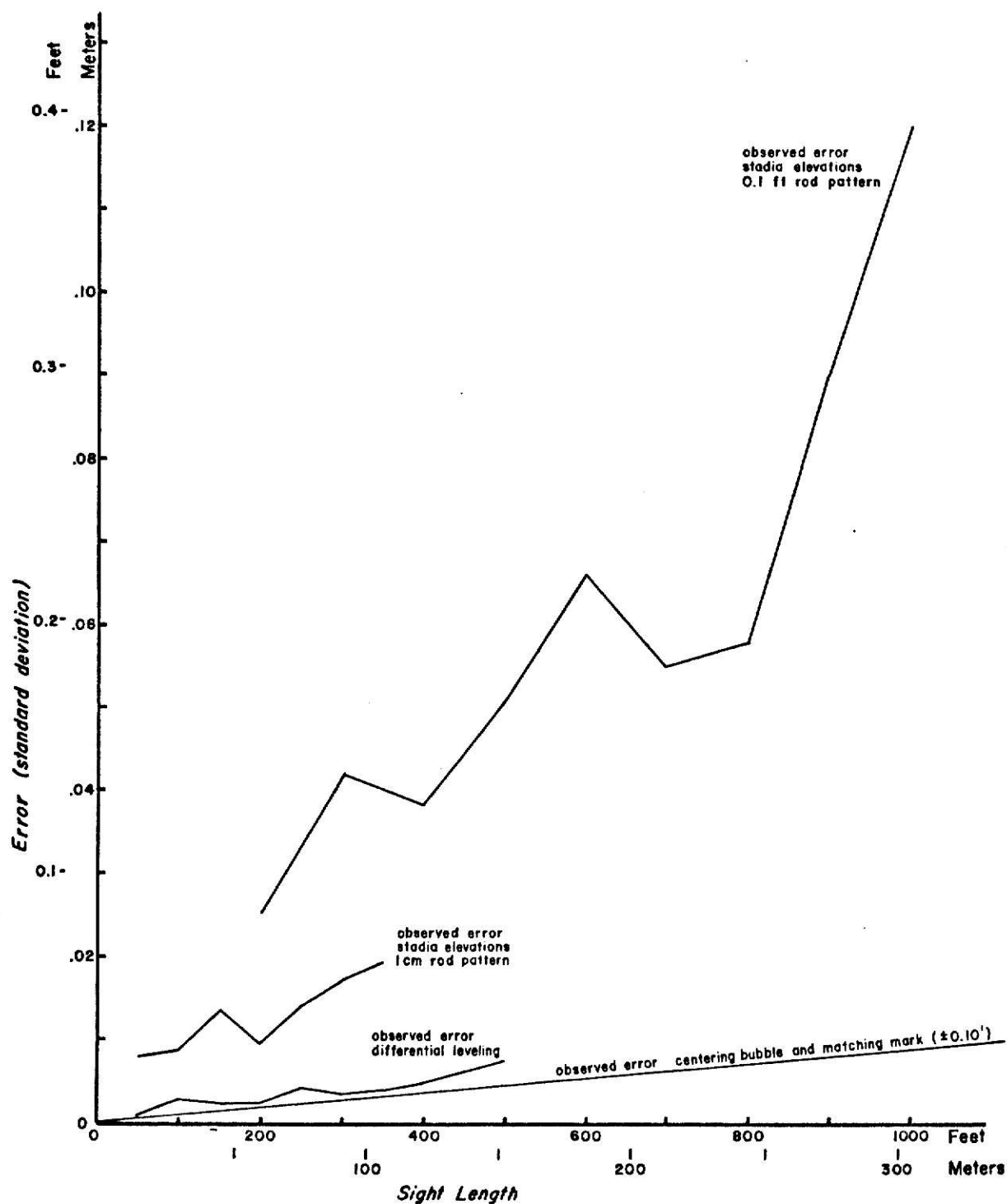


Figure 12 Observed errors, Wild RK-1 alidade

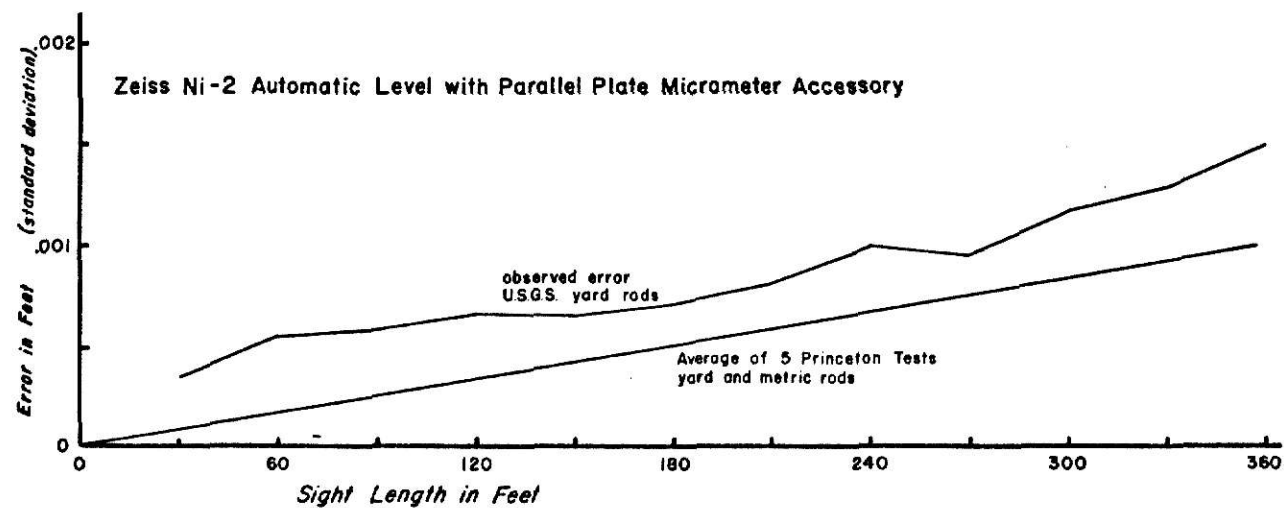
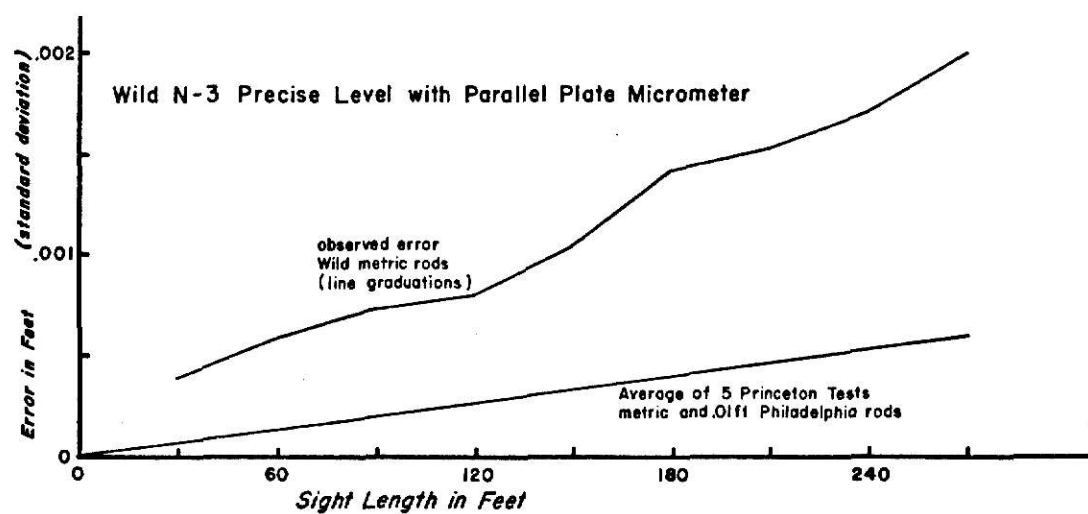


Figure 13 Results of USGS level tests, Wild N-3 and Zeiss Ni-2 levels (Lee and Karren 1964)

possibility. The RK-1 and 1 cm rods were used for two Princeton Tests with the alidade in the 20x multiplier range to give values of  $\pm 60$  sec. and  $\pm 76$  sec. A third test was made on the test site of experiment 2 with the RK-1, 1 cm rods, and the differential leveling procedure to give a value of  $\pm 20$  sec.

The resultant of these angular errors (angle in radians x distance), when plotted on the graphs of the previous experiment, seem to be too large by at least a factor of two (fig. 14), being larger than the total observed error for most of the usable viewing distances. The Princeton Test was not a suitable measure of instrument related errors for procedures which are less refined than precision leveling, so efforts with this test were terminated.

Experiment 5 - Direct measurement of instrument errors.--If one had access to an optical equipment shop, it might be possible to measure each of the several instrument errors directly against some more precise standard or tool. In this experiment, however, it was necessary to improvise.

At a distance of 34.37 meters, 1 cm subtends an arc of 1 min. A 1-cm pennant rod was set up vertically, and a heavy stand was placed 34.37 meters away. This was done in the hallway of Thompson Hall to minimize atmospheric effects and so the stand and rod could remain in place between tests. If the relative height of the stand and rod remain constant, and the instrument on the stand is stationary with only a pivoting motion of the telescope on the transit axis, then rod readings made through the telescope should be directly translatable into angular units of telescope rotation in the vertical plane, provided the range of movement is small.

The first test was for the error in centering the striding level bubble of the alidade. The standard alidade was placed on the stand and the striding level affixed to the telescope. The bubble was centered using the telescope tangent screw and rod intercept values recorded for 80 trials. Recorded values

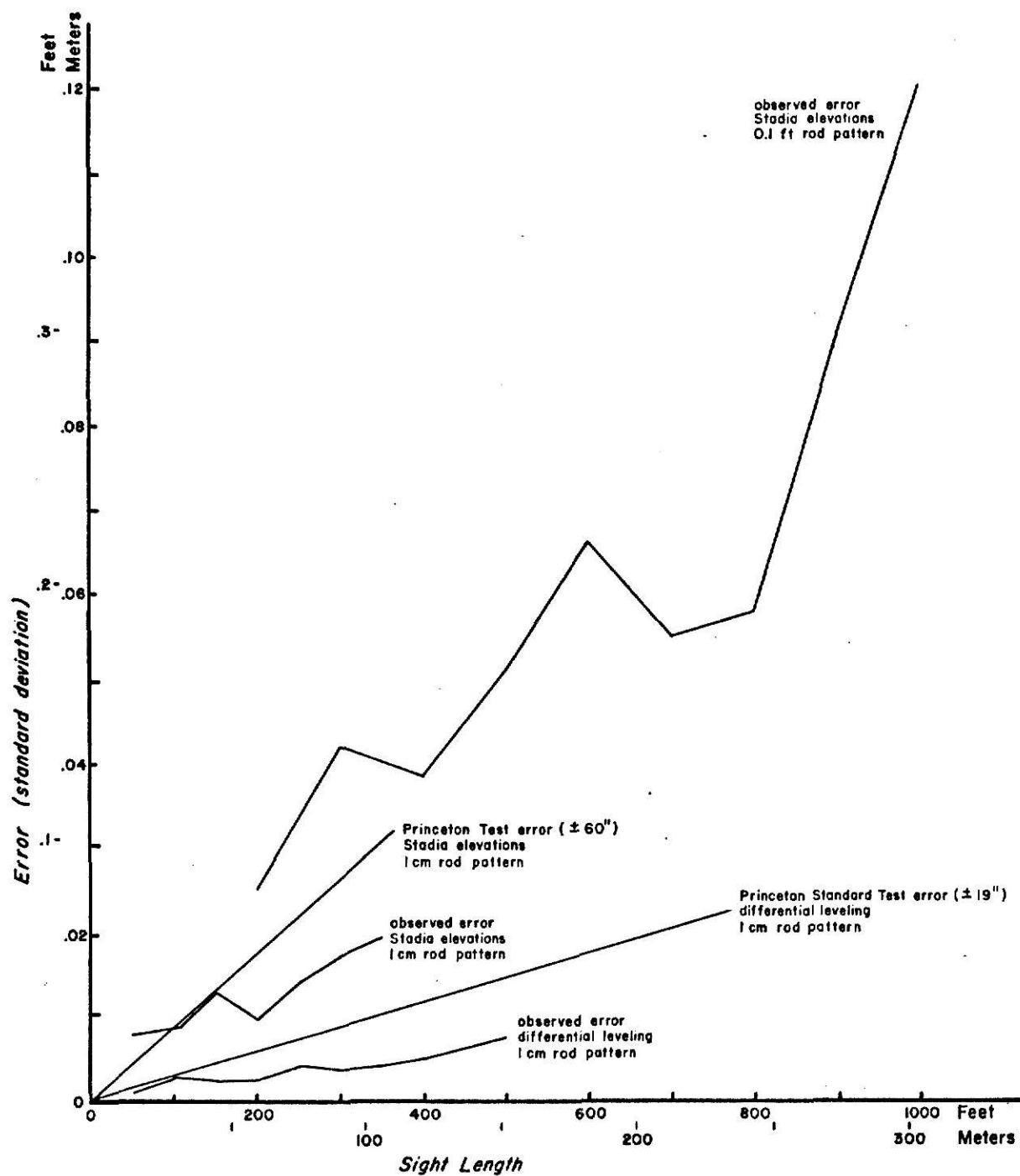


Figure 14 Observed error and Princeton Test values, Wild RK-1 alidade

were the upper, middle, and lower wire intercepts as per Kissam's (1954) three-wire method to reduce the effects of estimation error. These data were reduced to give an angular standard deviation in centering the striding level bubble ( $\pm 5$  sec.).

For the second test, the telescope tangent clamp was released and the telescope index arm was clamped to the vertical arc with a C-clamp, thus the telescope and arc moved as one unit when the arc tangent screw was turned. The vertical arc level was repeatedly centered and data recorded as before for 80 trials to give an angular standard deviation in centering the arc bubble ( $\pm 8$  sec.).

In the third test the arc was leveled and left undisturbed. The telescope was then leveled by turning the telescope tangent screw until the index mark coincided with a Beaman value of 50. These two marks were matched 80 times and data recorded as before to give an angular error in matching Beaman marks ( $\pm 0.73$  min.).

The fourth test used the RK-1 alidade. The arc level was centered and the telescope leveled by matching the vertical arc marks as one operation. This was repeated 80 times as before to give an angular standard deviation in leveling the RK-1 telescope with the vertical arc ( $\pm 6$  sec.).

An attempt was made to measure the mark matching error alone by the same method as was used with the Beaman scale, but the error was too small to produce any detectable difference. The difference between the two instruments is in the design of the graduations. The graduations of the standard alidade are scribed lines in metal, filled with paint, and viewed directly or with the aid of a small magnifying glass. The RK-1 graduations are sharp, black, photographically produced images on glass; the images overlap and are illuminated from behind. They are viewed in the telescope eyepiece and because of the design, mismatches are easier to detect.

Experiment 6 - Calculation of estimation error.--The data from experiments 3 and 5, the Theory of Errors, and a set of equations for the error elements in each method (Table 9) were used to produce a series of calculated-estimation-error versus visual-size-of-the-gradation plots (fig. 15). Most plots fell together in one group. The plots from experiment 3 and those of other workers did not trend to zero at zero distance but to some finite value, which would have to be due to some minimum value for estimation error because the instrument-related errors would have a zero resultant at the instrument.

Excluding observer bias, the population of all possible values is uniformly and continuously distributed. A correct estimate of the true value to the nearest tenth of a scale division could still be in error by as much as .05 scale divisions. If this distribution of true values were incorrectly treated as a normal population, it would yield a standard deviation of about  $\pm .03$  scale divisions. For the purposes of error calculations  $\pm .034$  scale divisions would contain the appropriate 68 percent of the population, with  $\pm .045$  divisions encompassing 90 percent.

A convenient straight line was drawn through the plotted values of estimation error corresponding to the equation:

$$\pm (.034 \text{ scale divisions} + \text{sight length}/[5000 \times \text{telescope magnification}])$$

The method of least squares could have been used to obtain a best fitting line through the data points; this would result in changing the coefficient of 5000 to 4849, but the data quality does not warrant this refinement.

The equation value for estimation error was substituted back into the equations to produce a series of plots for stadia distance errors, Beaman elevation errors, and RDS and RK-1 elevation errors (figs. 1, 3, 4). These were combined with other published data on instrument error to produce a pair of comprehensive error charts (figs. 2 and 5).

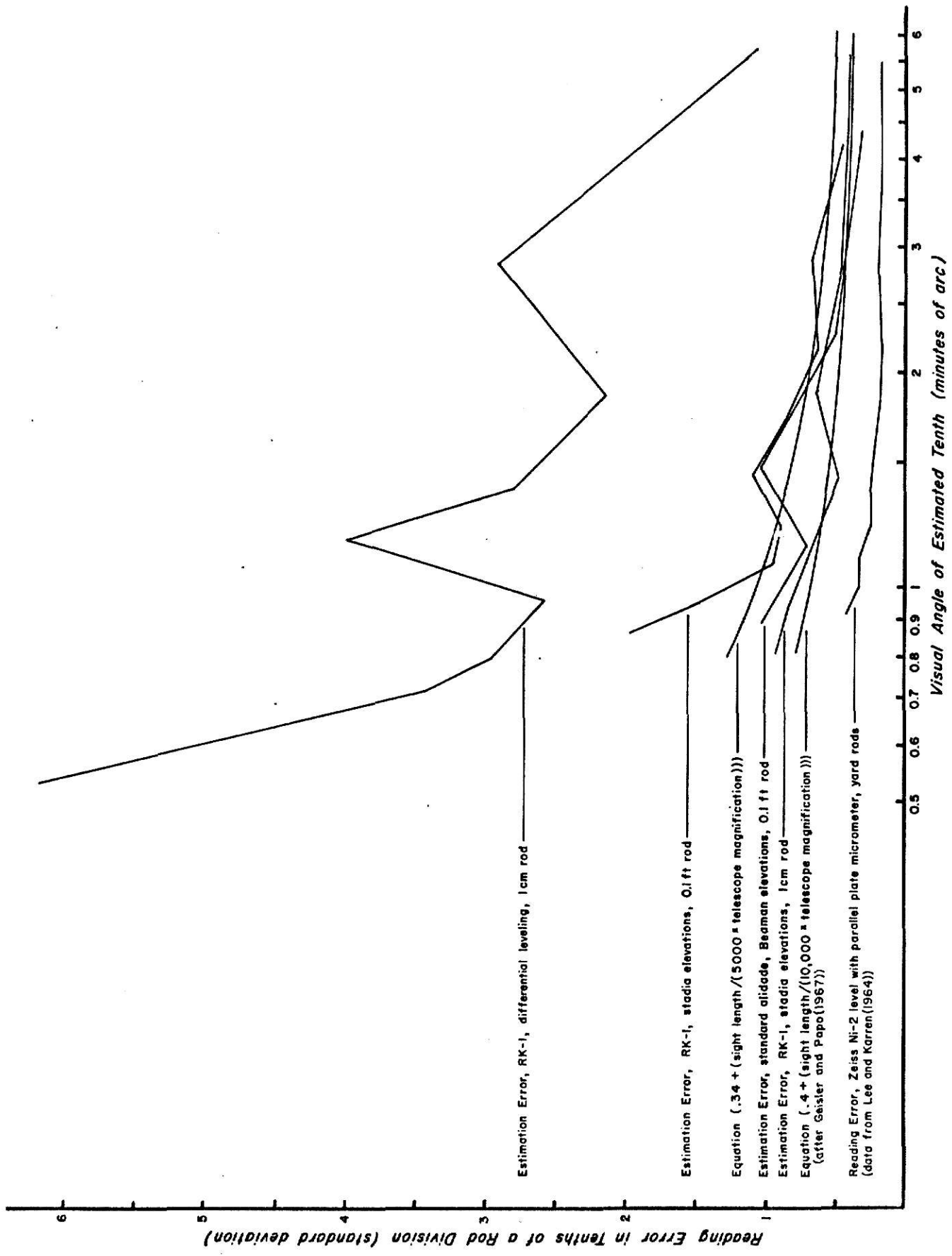


Figure 15 Calculated estimation errors

Table 9. Error equations for the K &amp; E and Wild alidades

Method	Equation using Theory of Errors	Solved for estimation error
Differential leveling	$TE = (L^2 + E_m^2)^{\frac{1}{2}}$	$E = (TE^2 - L^2)^{\frac{1}{2}}$
Differential leveling with arc level	$TE = (L^2 + E_m^2 + M^2)^{\frac{1}{2}}$	$E = (TE^2 - L^2 - M^2)^{\frac{1}{2}}$
Beaman method	$TE = (L^2 + E_m^2 + M^2 + [B E_u^2 + E_l^2]^2)^{\frac{1}{2}}$	$E = \left( \frac{TE^2 - M^2 - L^2}{1 + 2 B^2} \right)^{\frac{1}{2}}$
Self-reducing tachometer	$TE = (L^2 + P_1^2 + [C P_1^2 + E_m^2]^2)^{\frac{1}{2}}$	$E = \left( \frac{TE^2 - L^2}{.25 + 1.25C^2} \right)^{\frac{1}{2}}$

TE = total observed error  
 L = leveling error  
 E = estimation error  
 C = tachometer constant

M = mark matching error  
 P = pointing error  $\approx 0.5 E$   
 B = Beaman Value  
 Subscripts u, m, l = upper, middle, lower wire

Experiment 7 - Plane Table orientation errors.--Location errors in the plane table method are due largely to graphic plotting and orientation errors. Determination of plotting error requires use of a precision instrument such as a monocomparator to measure the x-y coordinates of the plotted point for comparison with knowledge of where the point is truly located. Because such a device was unavailable, the commonly quoted value of  $\pm 0.5$  mm for plotting error was used (Low 1952, 1957, Duran 1950, Denault 1944).

An angular orientation error occurs each time the plane table is set up. The orientation may be made by compass or by backsight. A plane table was set up 34.37 meters from a 1-cm rod supported horizontally on a second tripod. A telescope was fastened to the center of the board and aimed at the rod so that small angular rotations of the board could be measured. Conventional practice would use a theodolite mounted on the board, and a collimator in place of the rod, but the T-3 and T-2 theodolites have steel parts which might affect the compass, and the T-16 offers no advantage in precision.

The K & E alidade and Wild RK-1 box compass were placed on the board in turn and the board was rotated until the compass was correctly oriented. This procedure was repeated 50 times with each compass to give an angular standard deviation of  $\pm 5.8$  min. for the K & E compass and  $\pm 6.5$  min. for the Wild compass. A similar test was made of the combined tasks of aligning the alidade with a compass line drawn on the plane table surface and orienting the board. The error for the K & E alidade was  $\pm 7.2$  min. The RK-1 compass is a separate accessory and was not tested this way.

In orientation by backsight there is an error in aligning the alidade blade with the previously plotted points, and an error in pointing the telescope at the old instrument station. The size of the pointing error depends on how the station was marked. The alignment error depends upon the spacing of the

plotted points on the map.

Tests were made to measure the relationship between point spacing and angular alignment error. Point spacings ranged from 1 to 11 inches at 2-inch intervals, the sample size at each spacing was 50. The plane table and rod were arranged as in the previous tests and the plotted points oriented at right angles to the rod. The instrument was aligned with the particular pair of points, the rod intercept recorded, and the process repeated. The standard deviation of each set was calculated and listed in Table 10.

Experiment 8.--The collimation error of the arc and striding levels of six standard alidades were determined using a standard two-peg test. Each test was repeated ten times and averaged to eliminate the effects of bubble centering error. The results are reported in Table 11.

Table 10. Errors in aligning alidade with previously plotted points.

Graphic station separation (inches)	K & E standard alidade (angular std. dev.)	Wild RD-1 alidade	Angular equivalent of .5 mm plotting error
1	7.0'	3.8'	67.7'
3	2.8'	1.5'	22.6'
5	1.9'	0.9'	13.5'
7	1.0'	0.9'	9.7'
9	1.1'	1.0'	7.5'
11	1.2'		6.2'

Table 11. Examples of collimation errors, six K &amp; E alidades

Alidade serial no.	Striding level angular deviation (min)	Rod units/100 units distance	Index level angular deviation (min)	Rod units/100 units distance
76 734	0.0	0.0	1.0	0.3
110 497	-1.0	-0.3	-1.3	-0.4
182 366	-40.7	-11.8	8.7	2.5
182 370	-6.2	-1.8	28.7	8.3
183 098	5.2	1.5	8.7	2.5
183 388	1.6	0.5	18.7	5.4

## Appendix 4 - Summary of errors in horizontal measurements.

### DIRECTION MEASUREMENTS

#### Operator Tasks

In the measurement of directions the operator must perform the following tasks:

- 1) orient the instrument by pointing at some known azimuth mark,
- 2) read the initial angle (or set the initial angle prior to pointing),
- 3) point the instrument in the sought direction,
- 4) read the indicated angular value and calculate the difference.

#### Resultant Error

From the Theory of Errors the resultant error equation would be:

Resultant error =

$$\left( \text{pointing error}^2 + \text{reading error}^2 + \text{pointing error}^2 + \text{reading error}^2 \right)^{\frac{1}{2}} \text{ or}$$

$$\left( \text{setting error}^2 + \text{pointing error}^2 + \text{pointing error}^2 + \text{reading error}^2 \right)^{\frac{1}{2}}$$

Pointing Error.--Pointing error is affected by the size and design of the target and the magnification and resolution of the telescope.

Optimal target.--Kissam (1961) reported the optimal target for pointing consists of a pair of parallel lines so spaced that the crosshair can be centered between them with a white space about one to two times the width of a crosshair separating the crosshair from each of the lines. He reported the probable error (i.e. 50 percent confidence level) in pointing at such a target is  $\pm 0.15$  seconds. This target is not practical for general application because the proper physical spacing of the lines would change with sight length. It is used for special purposes where the viewing distance is fixed.

Common targets.--The present experiments (Experiment 2, Appendix 3) produced inconclusive results as to the effect of target design on pointing. No published numerical values for pointing error with ordinary targets was found.

Sun as a target.--Vanderaa (1964) made extensive studies of solar pointing error using different instruments and methods. Because of the specialized and technical nature of the errors in solar azimuths see Vanderaa (1964) and Berry (1958) for details.

Setting Error.--Unless the instrument has some special provision for setting the initial circle reading at zero, the setting error presumably would be equal to the circle reading error.

Reading Error.--The size of the reading error depends upon the method used to subdivide the main circle graduations.

Theodolites.--Theodolites (Wild T-2, T-3, Kern DKM-1, DKM-2, DKM-3, Askania K & E-2) use an optical micrometer and the task of matching or centering marks to subdivide the scale. Vanderaa (1964) found errors of  $\pm 0.1$  sec. for the T-3,  $\pm 0.8$  sec. for the T-2, and  $\pm 0.5$  sec. for the K & E-2 by making repeated matchings of the micrometer marks while other instrument settings were left undisturbed.

Optical Transits.--Optical transits (Wild T-16, Askania K & E-6) use a scale in the reading microscope eyepiece to subdivide the scale. Estimation of tenths of a microscope division provides the least reading. Studies of estimation errors on similar scales (Backstrom 1930, 1932, 1933) indicate an error in the range of  $\pm 0.09$  to  $\pm 0.11$  scale divisions ( $= \pm 6$  sec. for the T-16).

Vernier instruments.--Vernier instruments employ the selection of matching marks on an auxiliary vernier scale. Perfect selection of the matching mark would leave a residual error of  $\pm 0.34$  vernier units at a 68 percent confidence

level. Studies of systematic errors in vernier construction by Kissam (1961) suggest that a value of  $\pm 1$  vernier unit would be more appropriate because these errors are commonly unmeasured and uncorrected.

## DISTANCE MEASUREMENTS

### Steel Tape

Kissam (1966) reported errors of 1:2500 for experienced tapemen using the method of breaking tape. Use of a tension handle improved the results to 1:3000. Use of temperature corrections resulted in further improvement to 1:5000. For the method of slope taping, he reported a practical upper limit of 1:10,000.

Colcord and Chick (1968) gave the derivations of a series of tape correction equations (Table 12) for the systematic errors in taping.

Wood (1969) compared the true catenary value with the more common parabolic sag corrections. He found the parabolic correction was too large by as much as the initial sag error for long tapes (300 ft.) and steep slopes (45 deg).

### Electronic Distance Measurement EDM

These devices are subject to an accidental error of constant size due to residual errors in the timing and phase comparison electronics, and a second accidental error that is proportional to the measured distance caused by uncertainty in the propagation velocity of the electromagnetic signal. Reported errors of the available devices are in Table 13.

### Subtense Bar

Mussetter (1956b) derived an equation for distance error with a subtense bar and demonstrated its appropriateness by comparison with actual field

Table 12. Tape correction equations (Colcord and Chick 1968)

Correction	Equation
Standardization	$C_{std} = (e \times p)/L$
Temperature	$C_{temp} = a \times T \times p$
Tension (Pull)	$C_{pull} = (\Delta P \times p)/(A \times E)$
Alignment	$C_{align} = d^2/2p$
Wind	$C_{wind} = 8X^2/3p$
Sag (Tape level)	$C_{sag} = (w^2 p^3)/(24p^2)$
(Tape sloping)	$C_{sag} = k \times p \approx (w^2 p^3)/(24p^2) \cos^2 \theta$
Slope	$C_{slope} = S \text{ vers } \theta \approx V^2/25$

where A = tape cross-sectional area, in square inches

d = eccentric distance from the theodolite center to the side chaining dot (about 0.2 ft. for the Wild T-1 theodolite)

E = Young's modulus 28,000,000 pounds per square inch for steel

k = slope factor =  $(wp/P)^2 \cos^2 \theta (1 - [wp/P]^2 \sin \theta)/24$

p = partial unsupported tape length

L = full standardized tape length

P = tension read from tension handle

S = corrected slope distance

V = vertical difference in elevation from the theodolite to the chaining point

w = weight of the tape in pounds per foot

a = coefficient of thermal expansion (0.00000645 feet per foot per degree fahrenheit, for steel)

$\Delta P$  = difference from standard pull

T = difference from standard temperature (68 deg F)

8 = total standardization correction for full length L

$\theta$  = vertical angle (slope angle)

X = side sag of the tape due to wind (usually estimated)

Table 13. Errors for Electronic Distance Measurement (from manufacturers literature).

Instrument	Carrier	Range	Accuracy
Tellurometer			
MRA 4	microwave	25m - 50km	$\pm 3\text{mm} \pm 3\text{ppm}$
MRA 101	microwave	50m - 50km	$\pm 1.5\text{cm} \pm 3\text{ppm}$
MRA 3	microwave	50m - 50km	$\pm 1.5\text{cm} \pm 3\text{ppm}$
CA 1000	microwave	50m - 30km	$\pm 1.5\text{cm} \pm 3\text{ppm}$
MA 100	infrared	10m - 3km	$\pm 1.5\text{mm}$
Cubic Electrotape			
DM 20	microwave	50m - 50km	$\pm 1.5\text{cm} \pm 3\text{ppm}$
Wild			
Distomat 150	microwave	50m - 50km	$\pm 1.5\text{cm} \pm 3\text{ppm}$
DI 160	microwave	20m - 50km	$\pm 1\text{cm} \pm 3\text{ppm}$
DI 10	infrared	10m - 1km	$\pm 2\text{cm}$
DI 3	infrared	10m - 1km	$\pm 5\text{mm}$
Kern			
DM 1000	infrared	10m - 1km	$\pm 4\text{mm}$
MDM 500	infrared	10m - .5km	$\pm 5\text{mm} \pm 2\text{ppm}$
AGA			
Geodimeter	6 light	10m - 1.5km - 15km	$\pm 1.5\text{cm} \pm 3\text{ppm}$ (day) (night)
Geodimeter	8 laser	50m - 50km	$\pm 3\text{mm} \pm 3\text{ppm}$
Hewlett-Packard			
3800	infrared	10m - 3km	$\pm 3\text{mm} \pm 10\text{ppm}$

### Optical Wedge Telemeter

Turpin (1954) tested the Wild DM-1 telemeter on a traverse with legs of from 30 to 110 meters. Comparisons with taped distances indicated a constant error of  $\pm 6$  mm over the range of distances tested. The manufacturers (Kern, Wild) claim an accuracy of  $\pm 1$  to 2 cm.

### Stadia Measurements

Figure 1 in the text shows stadia distance errors based upon the current experiments. For the self-reducing tachometers, Wild RDS and Kern DK-RV, the manufactures claim accuracies of  $\pm 1:2000$  to  $\pm 1:3300$  when the instruments are used with a special short range rod (maximum distance 100 meters).

## Appendix 5 - Summary of errors in elevation measurements.

### ALTIMETRY

Greundler and others (1970, 1972) studied four methods of barometric leveling by comparing altimeter values with known elevations. Their results are in Table 14. These results may not be applicable to other areas where atmospheric conditions are different.

### OPTICAL METHODS

All of the optical methods use a telescope and some means of reference to the local level surface, usually a level bubble. Elevations are determined by differential comparisons between known and unknown points.

#### Direct Differential Leveling

Tasks.--Direct differential leveling uses the following operator tasks:

- 1) leveling the instrument, usually by centering a bubble,
- 2) reading rod intercept values.

Resultant error.--From the Theory of Errors the resultant error equation would be:

Resultant error =

$$([\text{leveling error in radians} \times \text{sight length}]^2 + \text{rod reading error}^2)^{\frac{1}{2}}$$

Leveling error.--Brinker and Taylor (1961) reported  $\pm 1/10$  of the stated sensitivity (angular rotation per 2 mm run of bubble) as the error in centering an ordinary bubble, and  $\pm 1/40$  of the stated sensitivity for centering bubbles using the split image system of prism levels and some European theodolites.

Kissam and Irish (1951) and Kissam (1963) proposed the Princeton Standard Test as a method for comparing the performance of precise levels. The test

results are reported as an angular standard deviation of the line of sight in the vertical plane. Lee and Karren (1964) used the Princeton Test to evaluate the levels listed in Table 15. Comparison of their Princeton Test results with other error plots in their text seems to indicate that the Princeton Test is a good measure of the leveling error.

Rod reading error.--Geisler and Papo (1967) proposed:

$$\pm (\text{scale interval}/25 + \text{sight length}/400,000)$$

as an estimate of the reading error for precise invar leveling rods and high magnification precision levels.

The present experiments suggest:

$$\pm (0.034 \text{ scale divisions} + \text{sight length}/[\text{telescope magnif.} \times 5000])$$

as the reading error for ordinary instruments and rods.

Kissam (1954) described the three-wire method as a means of reducing rod reading error.

Lee and Karren (1964) reported a 50 percent reduction in the resultant error when a parallel plate micrometer was used in place of estimation of rod readings.

### Trigonometric Leveling

Tasks.--Trigonometric leveling involves the following tasks:

- 1) leveling the instrument,
- 2) reading a rod intercept or pointing at a target of known height,
- 3) reading the vertical angle,
- 4) determining the horizontal or slope distance from the instrument to the rod or target.

Resultant error.--From the Theory of Errors the resultant error equation would be:

$$\text{resultant error} = ([\text{leveling error in radians} \times \text{sight length}]^2 + [\text{angle}$$

reading error in radians x sight length]<sup>2</sup> + [rod reading error<sup>2</sup> or pointing error<sup>2</sup>] + [distance error x tangent of vertical angle]<sup>2</sup>)<sup>1/2</sup>

where:

Distance error, pointing error, and angle reading error are as in Appendix 4, leveling error as per Brinker and Taylor (1961) above, and rod reading error as above.

#### Beaman Method

See Figure 3 in text.

#### Self Reducing Tacheometers

See Figure 4 in text.

Table 14. Observed errors in barometric leveling (Greundler and others 1970, 1972).

Altimetry method employed	Average error feet	Percentage of errors greater than:			
		6 feet	8 feet	10 feet	12 feet
Single base	4.4	28	16	6	3
Double base	3.8	20	9	2.5	1
Triple base	3.6	17	5	2	1
Leap frog	5.2	26	18	12	9

based on 187 measurements corrected by each of the four methods.

Table 15. Princeton Test results, USGS level tests (Lee and Karren, 1964)

Instrument	Type	Tele- scope Magnif.	Course Length	Number Set up	Micro- meter Used?	Rod Type	Angular Std Dev	Colli- mation Error
Ertel INA	auto.	30x	100m	40	no	metric	2.21"	14.3"
			100m	40	no	metric	2.95"	21.1"
			210ft	56	yes	metric	1.65"	
Kern GK 1A	auto.	25x	210ft	56	no	metric	5.71"	5.3"
			100m	240	no	metric	3.80"	
Hilger & Watts	auto.	32x	210ft	28	yes	metric	1.39"	30.1"
Zeiss Ni-2	auto.	32x	360ft	22	yes	yard	1.68"	16.0"
			360ft	44	yes	yard	1.58"	9.6"
			360ft	66	yes	yard	2.35"	2.7"
			210ft	42	yes	yard	1.80"	3.6"
			100m	40	yes	metric	1.40"	4.0"
			360ft	44	yes	yard	.30"	
Vickers S-700	auto.	32x	360ft	44	yes	yard	1.74"	5.8"
			100m	40	no	yard	2.31"	
			100m	40	no	yard	1.75"	
Filotechnica 5127	auto.	24x	210ft	28	no	yard	2.79"	16.0"
			210ft	28	yes	yard	2.16"	13.0"
			360ft	22	yes	yard	1.56"	50.0"
			360ft	22	yes	yard	1.97"	8.0"
Kern GK 23	spirit	30x	210ft	14	yes	metric	2.58"	3.5"
			210ft	56	yes	metric	4.05"	3.0"
Path L-11	spirit	28x	360ft	22	no	yard	2.15"	2.3"
			210ft	14	yes	metric	3.10"	1.5"
Wild N 3	spirit	42x	210ft	28	yes	Phila.	0.42"	8.7"
			210ft	28	yes	Phila.	2.82"	105. "
			210ft	33	yes	Wild	0.44"	
			210ft	42	yes	Wild	0.91"	8.0"
			100m	40	yes	metric	1.33"	10.4"
			100m	40	yes	metric	0.78"	2.7"
Hilger & Watts	spirit	38x	210ft	28	yes	Phila.	3.57"	7.0"
			360ft	44	no	metric	2.80"	7.0"
			210ft	44	yes	Phila.	1.22"	
			210ft	42	yes	Phila.	2.45"	6.0"
			360ft	44	no	metric	2.71"	7.0"
			210ft	28	yes	Phila.	3.32"	7.0"
(Wild Na-2	auto.	30x	309ft	40	no/yes	see table 8)		

SELECTION OF SURVEYING METHODS

by

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## ABSTRACT

This study was directed toward the tabulation of experimental values for the resultant accidental errors of the surveying methods and instruments commonly used in engineering practice. The instruments include: plane table and alidade; vernier transits, optical transits, and theodolites; altimeters, spirit and automatic levels; and steel tape, optical and electronic devices for measuring distance.

Accidental errors are postulated as limiting the accuracy of the several surveying methods because most systematic errors can be corrected and blunders can be avoided by careful work. By combining the Theory of Errors, which predicts the resultant of several independent accidental errors, with the tabulated values, an objective evaluation can be made of the expected performance of each surveying method in any given measurement task. Thus the adequacy of any method in meeting specified accuracy goals can be judged without dependence on intuition or previous experience, neither of which is transferable by the printed page.

The tabulations were reduced to charts that show the error in a single measurement as a function of distance or sight length. These charts are for generalized types of instruments, selection among specific instruments would require preparation of similar charts for those particular instruments using information in the appendix.

Selection begins with the determination of the required accuracy for the particular project. The next steps are an examination of the site and formulation of a tentative survey plan. The error charts and the Theory of Errors are then used to determine the adequacy of each method in meeting the accuracy goals. Once the adequate methods have been identified, selection can proceed on the basis of availability, costs, and operator convenience.

Published data, included in the appendix, were concerned primarily with the errors of the more precise methods of measurement. The present experiments supplement these data by deriving values for the errors in tacheometric methods: stadia, Beaman method, Wild RDS and RK-1. The equation:  $\pm (.034 \text{ scale divisions} + \text{sight length} / [\text{magnification} \times 5000])$  was found to be a useful approximation for rod-reading error (standard deviation). Experiments to measure the effect of rod and target pattern on reading and pointing errors were inconclusive.