THE EARTH'S MAGNETIC FIELD

by

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TABLE OF CONTENTS

INTRODUCTION

Scope

This report outlines the detailed structure of the earth's magnetic field, indicates briefly the major phenomena associated with the field, and considers some possible physical models of the field. Knowledge of the field structure and the major magnetic phenomena becomes increasingly important as man ventures into the environment of space. Physical models, if simulated, would be valuable in solving such problems as the origin, spatial distribution, and future behavior of the geomagnetic field. Also, many of the processes implicit in the generation of the earth's field may be valid in the production of magnetic fields in the sun, stars, and galaxies.

As a starting point of the discussion, surface measurements of the earth's field are described, and spherical harmonic analysis is employed to obtain a magnetic potential function. From the potential function the contribution of internal and external sources to the geomagnetic field is determined. Next, a description is given of the various short term magnetic fluctuations that are superimposed upon the earth's mean field.

An examination of the results of the spherical harmonic analysis leads to several models of the earth's field; these models range from a simple, gross representation to a complex, accurate representation. The simplest two models describe the earth's field in terms of a centered dipole and an eccentric dipole. A third model adds to the internal dipole field the residual or non-dipole part of the internal field. The residual field is noteworthy since it seems directly correlated with the secular or long term variations in the earth's magnetism. The field due to external sources is added to the internal field to produce a final model; this external field appears to be associated with the transient magnetic fluctuations.

Most theories advanced in the past to explain the origin of the earth's internal field stand discredited today. The theory most nearly consistent with the present knowledge of the earth's interior is the dynamo theory. However, an exact solution for a self-regenerating internal dynamo seems impossible since so little is known about the boundary conditions, and since the equations describing the process are non-linear. Some qualitative information on the dynamo process can be gained from a dimensional analysis of the basic equations of magneto-hydrodynamics.

The concluding portion of this report deals briefly with some of the phenomena associated with the external portion of the geomagnetic field. Possible theories on the cause of magnetic storms are discussed, particularly those involving an emitted stream of particles from the sun. Some of the possible interactions between the geomagnetic field and these solar particles are considered.

Historical Background

The discovery in Asia Minor of a strange mineral having the power of attracting ferrous objects was the subject of great mystery and speculation to the ancient Greeks. Bits of this mineral, or magnets (so named after the region Magnesia where deposits of magnetic oxide of iron were found), were often exhibited by certain mystic brotherhoods as a proof of supernatural skill. The Greeks, however, had no knowledge of the poles of a magnet, or of the directive property shown by a magnet free to move in the earth's field.

In the eleventh century Chinese writings reported ships using magnets mounted upon floating straws. Some historians believe that this knowledge of the magnet's north-seeking property may have come from Arab or Persian navigators.

First mention of the compass in European literature was made in 1187 with the publication of a treatise by Alexander Neckham describing the action of the magnetic needle (Mitchell, 1932). The compass needle was soon to be described by Scandinavian sailors as the lodestone or "leading stone". Magnetic dip or inclination was first noted in 1576 when Robert Norman, a practical seaman and instrument-maker, published a discourse entitled, "A newe discovered secret and subtill propertie concernying the declinying of the Needle, touched therewith under the plaine of the Horizon".

Gilbert (1600) postulated that the earth was a magnet, and in order to demonstrate this idea he shaped a sphere of lodestone and traced the field pattern using dip needles. Next followed Gellibrand's announcement in 1634 that "variation is accompanied by a variation", i.e., the angle the compass needle made with true north varied with time. It was soon afterwards recognized by Edmund Halley (1705) that a systematic survey of magnetic declination over the oceans would greatly benefit navigators. Therefore, the first survey of this type was made in the Atlantic Ocean during 1698-1700 by the sailing vessel Paramour Pink. The data thus accumulated was published as the first isomagnetic chart.

By using the potential theory developed by Laplace and Poisson, Gauss (1839) made the first mathematical analysis of the earth's field. Since the magnetic survey data available for this analysis failed to cover extensive areas of the earth's surface, Gauss realized the importance of a complete magnetic survey. For this purpose in 1840 a world-wide system of magnetic observatories was founded.

Careful magnetic measurements taken at these observatories indicated a wide number of fluctuations, both regular and irregular, in the geomagnetic field. Particularly interesting were the irregular disturbances termed magnetic storms; these magnetic storms were soon correlated with other natural phenomena such as: (1) earth currents flowing in a telegraph system, (2) the eleven year cycle in sun spot activity, and (3) auroras in the polar regions.

Further physical investigations substantiated the correlations just described. Birkeland (1896) was able to produce auroras in his laboratory by using a magnetized steel sphere within a vacuum discharge chamber. The calculation of charged particle paths in the geomagnetic field was performed by

Störmer (1907); the results tended to support the general features of the auroras. To explain Marconi's successful transmission of radio signals across the Atlantic, Kennelly and Heaviside (1902) proposed independently the existence of an electrically conducting layer in the upper atmosphere. This layer, the ionosphere, was shown to exist, and its behavior was correlated with magnetic storm activity. A theory proposed by Chapman and Ferraro (1931) explained magnetic storm activity in terms of a shower of emitted solar particles that tended to distort the geomagnetic field upon their arrival at the earth.

Repeated mathematical analyses of the geomagnetic field have shown a general westward drift with time of the non-dipole portion of the field. Also, studies of the remanent magnetism in rocks suggested that the earth's magnetic poles were once at positions far removed from their location today, and that possibly the earth's magnetic polarity has undergone reversals in past ages. These facts give considerable support to a dynamo theory proposed by Elsasser (1946) and Bullard (1949a) as an explanation of the origin of the internal geomagnetic field.

In recent years rocket and satellite measurements taken at distances out to ten earth radii from the earth's surface have detected a belt of charged particles trapped in the geomagnetic field. This Van Allen belt has been artificially created by the detonation of a nuclear device in the earth's high atmosphere, as in the Argus experiment. Such a belt of radiation, whether due to natural or man made causes, is a definite hazard to man's operation in space and is presently the subject of intensive investigation.

DESCRIPTION OF THE EARTH'S MAGNETIC FIELD

Definition of Terms

Two standard systems may be used to specify the magnetic field at any point. The first system, of primary use to navigation and surveying, defines the total magnetic intensity vector $\bar{F}$ in terms of the magnetic elements H, D, and I. H is the magnitude of the horizontal component of $\bar{F}$ and is considered always positive. The vertical plane through $\bar{F}$ is called the local magnetic meridian. Then, at any point P, the magnetic declination D is the angle between the magnetic meridian and the geographical meridian. East declination is considered positive, and west declination is negative. The angle $\bar{F}$ makes with the horizontal plane is I, the magnetic inclination or dip; I is positive if $\bar{F}$ is inclined downwards, and negative if $\bar{F}$ points upwards.

The second system gives $\bar{F}$ in terms of its rectangular components X, Y, and Z, where X is taken along the intersection of the horizontal plane and the geographical meridian; if northward, X is positive, if southward, negative. Y is taken in the horizontal plane transverse to the geographical meridian, and is positive if eastward, and negative if westward. Z is the vertical component, and is positive if downward, negative if upward.

From Fig. 1, the following relations exist among the

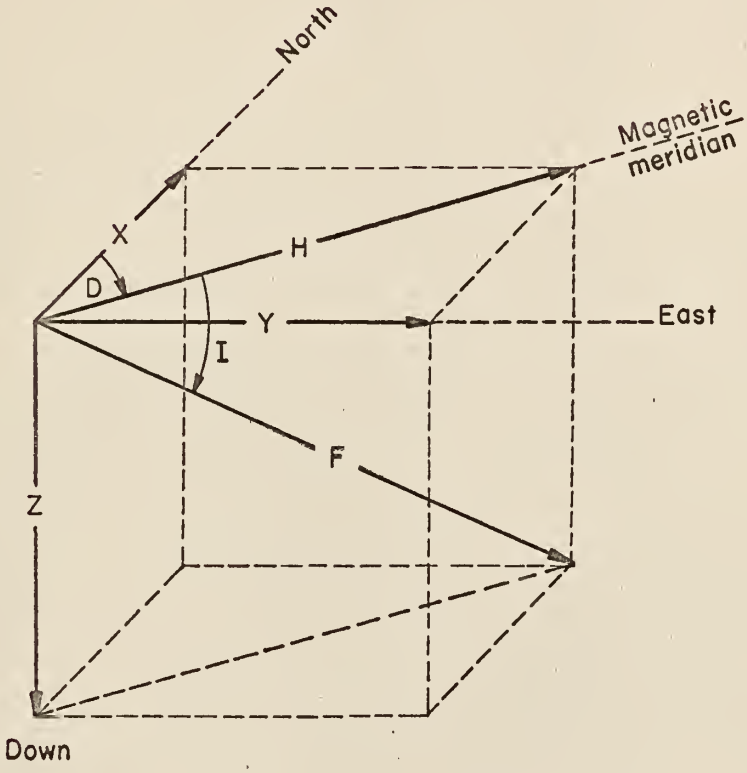

FIG. I. ELEMENTS OF THE EARTH'S MAGNETIC FIELD.

magnetic elements:

$$X = H \cos D$$

$$Y = H \sin D$$

$$Z = F \sin I$$

$$F^2 = X^2 + Y^2 + Z^2 = H^2 + Z^2$$

$$H = (X^2 + Y^2)^{\frac{1}{2}}$$

$$D = \tan^{-1} Y/X$$

$$I = \tan^{-1} Z/H \tag{1}$$

The units of magnetic force in geomagnetic measurements are the gauss (Γ) and the gamma (γ), the gamma being defined as 10^{-5} gauss. Strictly speaking, the unit of magnetic force is the oersted, and the unit of magnetic flux density is the gauss. However, the permeability of most non-ferrous materials is unity in the cgs system, and little error is introduced by this discrepancy in terminology.

The geomagnetic field is not constant and exhibits two distinct types of change, transient fluctuations and long-term secular changes. Transient variations, which are discussed in a later section of this report, arise largely from causes external to the earth. The secular variation extends over a period of many years or even centuries, and is due to causes within the earth.

Measurements taken of the magnetic elements H, D, I or X, Y, Z at a magnetic observation point are given in terms of mean annual values by averaging out all transient variations. The results of a magnetic survey to determine the geomagnetic field over the earth's surface are best illustrated by isomagnetic charts, i.e., maps having lines drawn through all points at which a given magnetic element has the same value. A chart giving lines of equal magnetic declination D is an isogonic chart, one giving lines of equal inclination I is an isoclinic chart, and those giving lines of equal X, Y, Z, H, or F are isodynamic charts. The annual secular variation of each magnetic element can be charted also in an isoporic chart, where points having the same annual change in a given magnetic element are connected by lines.

Spherical Harmonic Analysis

Within any region of space free of magnetic or current distributions, the following relations hold for the magnetic intensity $\bar{F}$,

$$\text{curl } \bar{F} = 0$$

$$\bar{F} = -\text{grad } V \tag{2}$$

so that V is the magnetic scalar potential satisfying Laplace's equation

$$\nabla^2 V = 0 \tag{3}$$

Let the set of spherical co-ordinates (r, ϕ, θ) be defined, where r is the length of the radius vector, ϕ is the central azimuthal angle or east longitude, and θ is the polar angle or co-latitude (Fig. 2).

For this co-ordinate system Laplace's equation has two forms of solution. If all the magnetic matter is inside a sphere of radius a, and none outside,

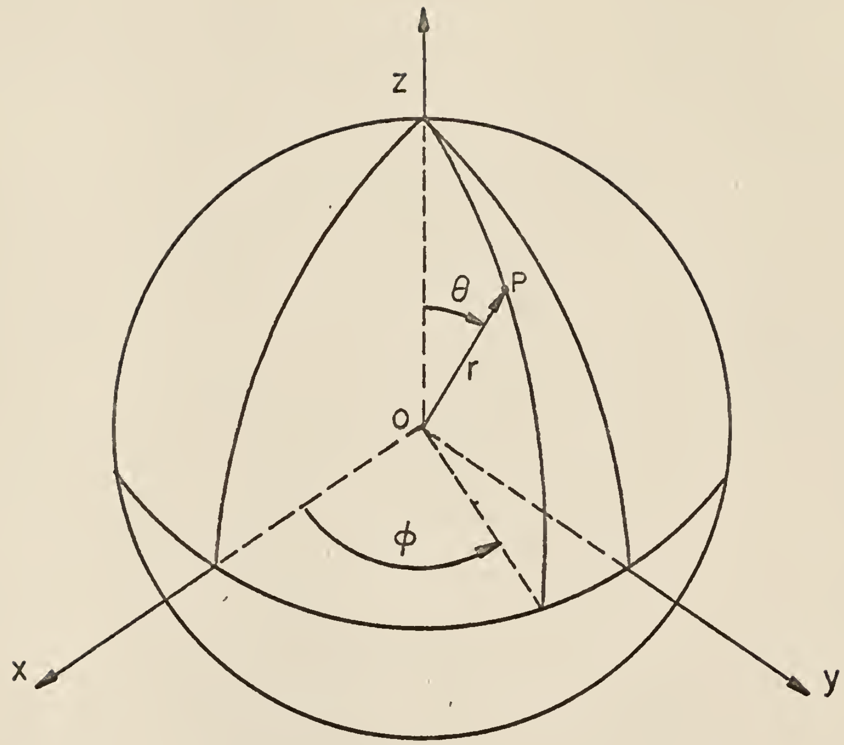

FIG. 2. CO-ORDINATE SYSTEM.

$$V_i = a \sum_{n=0}^{\infty} (a/r)^{n+1} S_n^i(\theta,\phi) \qquad r > a \tag{4}$$

If all the magnetic matter is outside the sphere, and none inside,

$$V_e = a \sum_{n=0}^{\infty} (r/a)^n S_n^e(\theta,\phi) \qquad r < a \tag{5}$$

where $S_n(\theta,\phi)$ is the spherical surface harmonic function of degree n given by

$$S_n = g_{n,0} P_n(\theta)$$

$$+ \sum_{m=1}^{n} (g_{n,m} \cos m\phi + h_{n,m} \sin m\phi)\, P_{n,m}(\theta) \tag{6a}$$

($g_{n,m}$ and $h_{n,m}$ constants)

$P_n(\theta)$ being the Legendre function of degree n, and $P_{n,m}(\theta)$ the associated Legendre function of degree n and order m.

If $\left[P_{n,m}(\theta)\right]^2$ is integrated over the region of orthagonality, then

$$\int_{-1}^{1} \left[P_{n,m}(\theta)\right]^2 d(\cos\theta) = 2/(2n+1)\,\frac{(n+m)!}{(n-m)!} \tag{7}$$

Thus for any degree n the mean square values of $P_{n,m}(\theta)$ have very diverse orders of magnitude. For example, the ratio of squares of $P_{4,1}$ and $P_{4,4}$ is 1 : 2016.

Schmidt (1935) devised a set of partially normalized Legendre functions to be used in a spherical harmonic analysis of the geomagnetic field. These functions $P_n^m(\theta)$ are defined by

$$P_n^0(\theta) = P_n(\theta)$$

$$P_n^m(\theta) = \left[2(n-m)!/(n+m)!\right]^{\frac{1}{2}} P_{n,m}(\theta) \tag{8}$$

S_n may now be rewritten

$$S_n = g_n^0 P_n^0(\theta) + \sum_{m=1}^{n} (g_n^m \cos m\phi + h_n^m \sin m\phi)\, P_n^m(\theta) \tag{6b}$$

so the average, taken over the surface of a sphere, of any term in S_n is

$$1/4\pi \int_0^{\pi} \int_0^{2\pi} \left[P_n^m(\theta) \begin{Bmatrix}\cos \\ \sin\end{Bmatrix} m\phi\right]^2 \sin\theta \, d\theta d\phi = 1/(2n+1) \tag{9}$$

Therefore, the coefficients g_n^m, and h_n^m in S_n now indicate the approximate orders of magnitude of the corresponding terms.

In the most general case magnetic material may lie both inside and outside the earth, therefore, the general expansion of the magnetic potential V may be obtained by combining equations (4), (5) and (6b).

$$V = a \sum_{n=0}^{\infty} \sum_{m=0}^{n} P_n^m(\theta) \Big[\left\{c_n^m (r/a)^n + (1-c_n^m)(a/r)^{n+1}\right\} A_n^m \cos m\phi$$

$$+ \left\{s_n^m (r/a)^n + (1-s_n^m)(a/r)^{n+1}\right\} B_n^m \sin m\phi\Big] \tag{10}$$

where c_n^m and s_n^m are numbers ranging between 0 and 1, and represent the portions of the harmonic terms, $P_n^m \cos m\phi$ or $P_n^m \sin m\phi$, in V which at, $r = a$, are due to matter outside the earth.

The observed magnetic elements X, Y, and Z are related to V by:

$$X = \partial V / r\partial\theta$$
$$Y = -\partial V/(r \sin\theta\,\partial\phi)$$
$$Z = \partial V/\partial r \qquad (11)$$

Evaluating these expressions at the earth's surface, r = a, gives

$$X_{r=a} = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \frac{dP_n^m}{d\theta}\left[A_n^m \cos m\phi + B_n^m \sin m\phi\right] \qquad (12)$$

$$Y_{r=a} = 1/\sin\theta \sum_{n=0}^{\infty} \sum_{m=0}^{n} mP_n^m\left[A_n^m \sin m\phi - B_n^m \cos m\phi\right] \qquad (13)$$

$$Z_{r=a} = \sum_{n=0}^{\infty} \sum_{m=0}^{n} P_n^m\Big[\left\{nc_n^m-(n+1)(1-c_n^m)\right\}A_n^m \cos m\phi$$
$$+ \left\{ns_n^m-(n+1)(1-s_n^m)\right\}B_n^m \sin m\phi\Big] \qquad (14)$$

The coefficients A_n^m and B_n^m of equations (12) and (13) are determined by a series expansion of Y sin θ and X, using the measured values at the earth's surface. Details of this numerical computation are given by Schmidt (1935). Since A_n^m and B_n^m are calculated by two independent means, both from X and from Y, disagreement of the two sets of values indicates a non-potential portion of the earth's field, i.e., the existence of a current flow transverse to the earth's surface. The reality of such current systems is discussed in a later section of this report.

Separation of the geomagnetic field into portions of external and internal origin proceeds by first expanding the measured values of Z into a series,

$$Z_{r=a} = \sum_{n=0}^{\infty} \sum_{m=0}^{n} P_n^m(\theta)(\alpha_n^m \cos m\phi + \beta_n^m \sin m\phi) \tag{15}$$

Next, by comparing equations (14) and (15) it follows that

$$\begin{aligned} \alpha_n^m &= \{nc_n^m - (n+1)(1-c_n^m)\} A_n^m \\ \beta_n^m &= \{ns_n^m - (n+1)(1-s_n^m)\} B_n^m \end{aligned} \tag{16}$$

As α, β, A, B are known, the values of c_n^m and s_n^m can be determined.

Gauss (1839) and other investigators have expressed the magnetic potential in terms of equations (4), (5), and (6b), where the coefficients g_n and h_n are termed the Gauss coefficients. Such coefficients are often employed to compare various analyses performed since Gauss's day. Table 1, due to Vestine (1947b), compares the results of the chief analyses made of the geomagnetic field. This table provides a rough picture of the secular variation of the earth's field during the past century, and shows that the major part of the field is due to a dipole.

Table 1. The first eight Gauss coefficients (external plus internal the earth's magnetic potential V expressed in units of 10 γ.

Source	Date	g_1^0	g_1^1	h_1^1	g_2^0	g_2^1	h_2^1	g_2^2	h_2^2
Gauss	1835	-3235	-311	625	51	292	12	-2	157
Erman & Peterson	1829	-3201	-284	601	-8	257	-4	-14	146
Adams	1845	-3219	-278	578	9	284	-10	4	135
Adams	1880	-3168	-243	603	-49	297	-75	61	149
Fritsche	1885	-3164	-241	591	-35	286	-75	68	142
Schmidt	1885	-3168	-222	595	-50	278	-71	65	149
Dyson & Furner	1922	-3095	-226	592	-89	299	-124	144	84
Afanasieva	1945	-3032	-229	590	-125	288	-146	150	48
Vestine	1945	-3057	-211	581	-127	296	-166	164	54

Transient Variations of the Earth's Field

A discussion of the transient variations of the geomagnetic field is necessary to complete a description of the earth's magnetism. Such variations are of interest since they yield valuable information about the behavior of the sun, the earth's high atmosphere, and outer space. Three major types of variation occur; the magnetic disturbance or D variation, the solar or S variation, and the lunar or L variation. Each magnetic element is affected in a characteristic way by each type of variation.

The continuous records of any magnetic observatory show that on some days all magnetic elements exhibit smooth and regular variations, while on other days the variations are disturbed and quite irregular. Days of the first kind are termed quiet days, and days of the second kind, disturbed days. For each observatory a figure K, ranging geometrically in scale between 0 and 9, is assigned to indicate the value of magnetic variation during each period of three Greenwich hours. To aid in world-wide analysis of magnetic variation, the K figure is modified to a K_p figure, which is as free as possible of the effects of local magnetic features, and is dependent upon the latitude of the station. For example, $K_p = 9$ would indicate a peak magnetic fluctuation of 300γ at the equator, and 2500γ at the auroral zones. A coarser measurement of the transient field is afforded by a C figure, which indicates by the digits 0, 1, 2 the mean value of the magnetic record activity for each

Greenwich day. An average of the C figure for all observatories provides the international character figure for that day. On the basis of the international character figure the five quietest days (international quiet days), and the five most disturbed days (international disturbed days) are selected for each month.

The S or solar daily variation is a function of local time, season of the year, latitude, and sunspot activity. The variation is greater and more rapid during daylight hours than during darkness hours. A spherical harmonic analysis of this variation indicates its source is largely external. When determined by averaging hourly values of variation during the five quietest days of the month, S is termed S_q, the quiet-day solar daily variation. In considering the effect of S upon each magnetic element, McNish (1939) stated that the maximum variation of X is at the equator and is of the order of 40-60γ, the maximum variation of Y is at 50° latitude and is of the order of 35-50γ, and the maximum variation of Z is at 40° latitude and is of the order of 12-18γ.

The lunar or L variation has the period of the lunar day, about 25 solar hours. Since its magnitude is only about 1/15 of the S variation, it can not be detected directly from the magnetic records, but must be calculated by statistical methods. A harmonic analysis shows that the source of the L variation is mostly external.

In overall behavior the D or magnetic disturbance variation is far more erratic than the L or S variations, although both a 27-day cycle in frequency of occurrence and a general pattern for a typical disturbance are noted. Most disturbances or magnetic storms commence at all points over the earth within $\frac{1}{2}$ minute. The initial phase is characterized by an increase in the H element for a period of 2 to 4 hours. Then H falls greatly below the undisturbed value held before the storm. Lastly, there is a slow recovery toward the normal value of H, requiring perhaps several days. Magnetic declination is but slightly affected during the storm period; the changes in vertical intensity Z are much smaller than the changes in H, and have an opposite sign. In several severe magnetic storms an H variation up to 2000γ has been noted.

Geographically, the magnitude of the D disturbance increases from low latitudes to a maximum at 65° magnetic latitude, the auroral zone, followed by a slight decrease as the poles are approached. At high latitudes few days are completely free of some magnetic disturbance. The D variation is connected with solar activity, but a time lag of one to three days occurs between peak sun-spot activity and the onset of a period of terrestrial magnetic storms.

MODELS OF THE EARTH'S FIELD

Main Dipole Field

In order to first develop a model of the earth's field, a comparison is made between the various components of the field due to internal and external sources. Table 2 lists the internal, external, and combined Gauss coefficients of two analyses made from magnetic survey data for the year 1945 (Vestine, 1960a; Fanselau and Kautzleban, 1958). As the coefficients relating to an external field source are very small, it is uncertain that the earth's long-term field has any external sources. Only the transient magnetic variations D, S, and L appear to have external sources.

Table 2. Values of $C_n^m = \{(g_n^m)^2 + (h_n^m)^2\}^{\frac{1}{2}}$ for field of origin external and internal to the earth, units of 10γ.

Source	Field	C_1^0	C_2^0	C_1^1	C_2^1	C_2^2
Fanselau-Kautzeben	Combined	3067	128	616	340	165
" "	Inner	3063	123	620	341	169
" "	Outer	4	5	8	3	9
Vestine	Combined	3057	127	618	339	173
"	Inner	3057	122	620	340	172
"	Outer	0	5	12	4	4

Line integrals of H about closed paths on the earth's surface have been shown to be non-zero with randomly distributed values. All the measurements of earth-air currents normal to the earth's surface are far too small to account for the non-vanishing of these integrals. It seems likely, therefore, that if the magnetic survey data possessed greater accuracy and more detail, these line integrals would have zero values. If the results of this line integration do have physical reality, the portion of the field attributable to a vector potential would be less than 1% of the entire field. No serious error is introduced by assuming, as in the spherical harmonic analysis, that the earth's surface field is derivable entirely from a scalar potential.

The first order terms of the magnetic potential may be written for $r = a$ as

$$V/a = g_1^0 \cos \theta + \left[g_1^1 \cos \phi + h_1^1 \sin \phi \right] \sin \theta \tag{17}$$

Now H_0, θ_0, ϕ_0 are defined as

$$H_0^{\ 2} = (g_1^0)^2 + (g_1^1)^2 + (h_1^1)^2 \tag{18}$$

$$\cos \theta_0 = g_1^0/H_0$$

$$\tan \theta_0 = h_1^1/g_1^1$$

The angle Θ between the direction (θ,ϕ) and the direction (θ_0,ϕ_0) is expressed by

$$\cos \Theta = \cos \theta \cos \theta_0 + \sin \theta \sin \theta_0 \cos(\phi - \phi_0) \tag{19}$$

The magnetic potential now becomes

$$V/a = H_0 \cos \Theta \tag{20}$$

This potential at the earth's surface could be the result of a magnetic dipole moment H_0a^3 at the center of the earth, with direction $(-\theta_0, -\phi_0)$. The axis of this dipole intersects the earth's surface at points termed the geomagnetic poles. At these poles the vertical magnetic intensity due to the dipole will be $\pm 2H_0$, and at 90° from these poles the horizontal intensity will be H_0. If the earth were uniformly magnetized throughout with a magnetization of $3H_0/4\pi$ cgs poles per cm.^{2}, a centered dipole field would also result.

A third model is a surface current system flowing east to west and having a distribution (Chapman and Bartels, 1940),

$$I = 30/4\pi\, aH_0 \cos \Theta \text{ amperes} \tag{21}$$

or a current density

$$J = -\partial I/a\partial\Theta = 0.8 \sin \Theta \text{ amperes/cm.} \tag{22}$$

An infinitude of other models can also produce the first order field observed at the earth's surface, but the three preceding models are relatively simple to visualize.

An analysis of the geomagnetic field by Finch and Leaton (1957), using data for the year 1955, gives the value of H_0 as 0.3120 gauss, and the magnetic moment of the earth's centered dipole as 8.064 X 10^{25} cgs pole cm. H_0 has been decreasing at the rate of 15 γ a year during the past century, although recent surveys indicate that this rate of decrease may now be approaching zero. The geomagnetic poles are located presently at 79°N, 290°E, and 79°S, 110°E. These poles should not be confused with the magnetic dip poles, which are points of zero vertical intensity Z, and are located asymmetrically at about 73°N, 262°E, and 68°S, 145°E. It is not certain, judging from analyses taken over the past century, that the geomagnetic poles are shifting. However, the dip poles vary rapidly in position during an otherwise slight change in the geomagnetic field pattern; their position has little significance in an analysis of the main field.

In the study of such phenomena as magnetic storms, cosmic rays, or auroras it is often convenient to adopt a geomagnetic co-ordinate system taken with reference to the axis of the centered dipole. McNish (1936) has developed a set of nomographs that give the geomagnetic co-ordinates for any point.

If the position of the main field dipole is shifted while keeping its vector moment constant, Chapman and Bartels (1940) have shown that the terms in V involving P_2^0 and P_2^1 may be made to vanish. Such a model is that of an eccentric dipole; its present position is 300 km. from the earth's center in the direction 15°N, 168°E. This eccentric dipole fits the magnetic data of the Southern hemisphere somewhat better than does the centered dipole; there is little difference between the two models over the Northern hemisphere.

Residual Field

If the potential of the centered dipole is subtracted from the total potential of the geomagnetic field, the remaining potential, containing spherical harmonic terms of degree two and higher, is that of the residual field. An isodynamic chart of any residual field element has irregularly distributed regions of high and low field intensity which range in diameter from 25° to 100°. Maps of the secular variation of the earth's field also show broad regions of increasing and decreasing field with dimensions quite similar to the distribution of the residual field. It can also be demonstrated that the features of both the non-dipole field and the secular variation have drifted westward during the past half century. Thus there is every reason to believe that the two phenomena are correlated such that the secular variation is the time derivative of the residual field.

Vestine (1947a,b) published a detailed spherical analysis of the earth's field for the period 1905 to 1945. Using the data of Vestine, Bullard et al. (1950) computed the angular shift of the field for 1905 that would best fit this field to the field of 1945. As a result, the drift of the non-dipole field was given as 0.18° westward per year, and the drift of the secular variation as 0.32° westward per year. Both drifts were essentially independent of latitude. It is not known whether these drifts can be extrapolated into the past, as the older magnetic survey data was not sufficiently complete to develop an accurate map of the residual field. However, Elsasser (1950a) pointed out that the intersection of the westerly line of zero magnetic declination with the equator had shifted 90° westward in 400 years. Since other features of the older survey data also support the possibility of a westward drift of the non-dipole field, this drift is now accepted as a physical reality. Therefore, any theory about the origin of the geometric field must adequately explain this drift.

Seismic data has indicated that the earth has a core of dense fluid, probably molten iron and nickel, with a radius of about 0.45 earth radii. Thus most models of the earth's residual field have placed the sources at the surface of the molten core, since any time varying field in the interior of the core would be screened out by the high conductivity of the core. McNish (1940) represented each nest of vertical intensity isodynamic lines by a radially directed dipole placed at $r = a/2$. Vestine (1947b) assumed current functions of the form,

$$I_n = \frac{10}{4\pi} \frac{2n+1}{n} V_n \, (a/r)^{n+1} \qquad (23)$$

where V_n is the potential at the earth's surface, and I_n is the current distribution in a thin current sheet at the radius r. The total current function that would generate the residual field was then calculated and graphed for current sheet depths of 0, 1000, 2000, and 3000 km. Lowes and Runcorn (1951) used a graphical analysis on the X and Y components of the field to obtain a dipole distribution fitting the observed secular variation field to within 1% accuracy. Twelve vertical dipoles placed just beneath the surface of the earth's core were necessary; each dipole represented a horizontal circuit of current flow at the surface of the core.

External Field

A complete model of the geomagnetic field must include not only the earth's internal field for which models have now been given, but also the transient magnetic variations. Only models for the S_q variation and the D variation are considered here, as these two variations provide the major portion of the transient fluctuations. Spherical harmonic analysis indicates that the source of both the S_q and D variations is about 75% external and 25% internal to the earth. The internal portion of both variations is due to currents induced by the external field in the mantle of the earth. Just as in the case of the earth's main field, there are innumerable models that would reproduce the observed transient variations. This discussion will consider only those models that are consistent with the present knowledge of the transient variations.

The quiet day solar variation, S_q, is usually considered to have its source in the ionosphere at a height of 70 to 400 km. From observations of S_q Chapman and Bartels (1940) have developed an earth wide overhead current system that would produce this fluctuation at the earth's surface. This current system, which moves westward with the sun as the earth rotates, consists of four circuits or nests of flow lines; two circuits are in the Northern hemisphere, and two are in the Southern hemisphere. In either hemisphere one circuit covers the daylight half of the earth; the second circuit, carrying less current than the first, covers the dark half of the globe. The overall structure of the current system varies considerably with the seasons; the circuits shift southward during summer in the Northern hemisphere and northward during summer in the Southern hemisphere.

The effect of the magnetic disturbance variation D can be simulated by two superimposed current systems. One portion of D, termed the storm-time variation D_{ST}, is a function of the time elapsed since the beginning of the disturbance, and can be represented by an overhead current system that is symmetric about the earth's axis. Chapman and Bartels (1940) plotted such an idealized current system for an arbitrary height in the atmosphere. Their current system for D_{ST} has its greatest current density at the equator, with decreasing current densities as the poles are approached. Alfvèn (1955), and also Appel et al. (1962) postulated, as a more probable source of the D_{ST} variation, an equatorial ring current girdling the earth at a distance of several earth radii. Measurements made by both Russian and American satellites tend to verify that several such ring currents exist, one in the outer Van Allen belt, another well beyond the Van Allen belt at about 6 to 10 earth radii (Akasifu and Chapman, 1961; Smith et al., 1960).

The second portion of the D variation is S_D, a daily fluctuation that occurs only during periods of magnetic storm activity. The current system responsible for S_D seems to be located in the ionosphere; its position is asymmetric with respect to the earth's axis, and follows the sun as the earth rotates. At the polar regions the S_D current system has its highest current density. Over the earth its currents flow east and west along the equator, and north and south along the noon meridian. An idealized current system for D, combining the two current systems for D_{ST} and S_D, has been developed by Chapman and Bartels (1940).

THEORIES OF THE EARTH'S INTERNAL FIELD

Early Theories

In spite of the many theories devised in the past, the source of the earth's internal field is still a highly speculative topic. Any satisfactory theory must explain the field's predominantly dipolar character, the orientation and rate of change of this dipole field, the rapidly varying behavior of the non-dipole field, and the earth's past magnetic history obtained from fossil magnetism. Since the inaccessibility of the earth's interior to direct observation prohibits a conclusive test of any theory, the various theories are compared in order to select the hypothesis most closely predicting the presently known geomagnetic phenomena.

Possibly the first explanation of the origin of the internal field was the assumption of some arbitrary distribution of ferromagnetic material within the earth. However, the increase of the earth's temperature with depth seems to indicate that the Curie point of most ferromagnetic materials would be reached at a depth of 25 km. Since a Curie point increase with increasing pressure has never been verified experimentally, any distribution of magnetic material must be restricted to the crustal region. A calculation of the necessary average magnetization in the crust gave the valve of 5 to 8 cgs poles per cm.^{2}, a value more than twice the maximum remanent magnetization of iron or nickel, and over 100 times the permanent magnetism of igneous rocks. Besides requiring these impossible values of magnetization, the ferromagnetic theory could not explain the rapid variations of the non-dipole field.

Another early theory postulated that the magnetic field was produced by the rotation, with the earth, of a system of electric charges. This system could be either a uniform surface charge S, or a volume charge Q uniformly distributed throughout the earth. However, the earth's conductivity seems great enough to destroy quite rapidly any such charge distribution.

Blackett (1947), noting the existence of axial magnetic fields in various rotating cosmic bodies other than the earth, proposed a universal relationship between angular momentum and magnetic moment in large bodies; any rotating body would thus be spontaneously magnetized. For the case of the earth such a fundamental hypothesis could be directly tested; the source of the field would be distributed uniformly throughout the whole earth, and the vertical component of field intensity would not obey an inverse cube law within the earth's crust. Measurements of vertical intensity carried out in coal mines by Runcorn et al. (1951) did not support this theory, and Blackett (1951) soon announced the results of a "negative" laboratory experiment upon his theory.

As the time constant of a system of freely decaying currents within the earth is estimated to be about 2 million years, such a system might seem to offer an answer to the problem of the geomagnetic field. With the assumption that the current system was started 100 million years ago, a short interval of time compared with the probable age of the earth, the required initial current would have been about 10^{27} times the presently required value of 1000 million amperes. No process is known that could set up such enormous currents in the earth.

Nevertheless, internal current systems are a likely source of the earth's field, but some mechanism is required to maintain these systems. One proposed mechanism is the thermoelectric effect; a second mechanism is electromagnetic induction. Both mechanisms require the existence of convective flow patterns in the liquid core of the earth. The existence of such fluid motions is reasonably certain, for only a small amount of heat is necessary to initiate convection; this heat could derive from radioactive heating or slow chemical separations and crystallizations. The rapidly varying nature of the non-dipole field also points out the physical reality of convective flow patterns in the earth's core; the fluid motions interact with a current system to produce the irregular distribution of the secular variation and residual fields. There is more than a superficial resemblance between weather maps and magnetic charts; both display the phenomena of large scale fluid motion.

Elsasser (1939) suggested that thermoelectric potentials were the source of an internal current system producing the geomagnetic field. This concept, extended by Runcorn (1954), has some inherent difficulties that make it less satisfactory than the induction or dynamo theory. A thermoelectric e.m.f. is developed at the surface of the core, the core and mantle forming the two materials of the thermojunction. Convection currents in the core must maintain temperature differences between various points on the boundary of the core. The resultant current system generates a magnetic dipole moment only through the action of the Coriolis force on the convecting fluid streams. It is shown in the discussion on magneto-hydrodynamics that a magnetic line of force contained in a conducting fluid acts as if it were attached to one set of fluid particles. When the fluid moves, the line of force is deformed in following the motion of the fluid particles. Thus the Coriolis force in deflecting the fluid flow distorts the embedded magnetic field and produces a net magnetic dipole moment.

One difficulty of the thermoelectric theory is that it must include one of the following requirements: (1) large temperature differences in the core, (2) large thermoelectric coefficients, or (3) extreme geometry of convective flow. Also, the sign of the thermoelectric coefficients is unknown, and no explanation of the probable past reversals of the earth's magnetic polarity is given. There is some evidence (Vestine, 1954) that, due to high pressure, the mantle material is a semi-conductor and possesses a reasonably high Hall coefficient. Vestine then suggested that the thermoelectric and Hall effects interact in a regenerative process responsible for at least a part of the earth's internal field.

Basic Magneto-hydrodynamics

Before proceeding to a detailed discussion of the dynamo theory, it is desirable to consider the basic equations of magneto-hydrodynamics. Any internal dynamo action must satisfy these equations.

If the displacement current, all purely electrostatic effects, and relativistic effects of order higher than v/c are ignored, Maxwell's equations in the rationalized mks system are:

$$\mathrm{curl}\ \bar{E} = -\partial\bar{B}/\partial t \tag{24}$$

$$\mathrm{curl}\ \bar{B} = \mu\bar{J} \tag{25}$$

$$\mathrm{div}\ \bar{B} = 0 \tag{26}$$

where $\bar{E}$, $\bar{B}$, and $\bar{J}$ are the vectors of the electric field, magnetic flux density, and current density, respectively. The magnetic permeability is μ.

For an electrically conducting fluid having relative motion with respect to the magnetic field, the current density is

$$\bar{J} = \sigma(\bar{E} + \bar{v} \times \bar{B}) \qquad (27)$$

with σ as the electrical conductivity, and $\bar{v}$ as the velocity vector; $\sigma\bar{v} \times \bar{B}$ being the motional induction term.

Taking the curl of equation (25), and using equations (24) and (27) to eliminate $\bar{E}$ gives

$$\text{curl curl } \bar{B} = \mu\sigma\left[-\partial\bar{B}/\partial t + \text{curl } (\bar{v} \times \bar{B})\right] \qquad (28)$$

Now,

$$\text{curl curl } \bar{B} = \text{grad div } \bar{B} - ✡\bar{B} \qquad (29)$$

where $✡\bar{B}$ is the vector Laplacian of $\bar{B}$. Combining equations (26), (28) and (29) yields

$$\partial\bar{B}/\partial t = \text{curl } (\bar{v} \times \bar{B}) + \nu_m ✡\bar{B} \qquad (30)$$

The quanity $1/\mu\sigma$ has been written as ν_m and is called the magnetic viscosity.

The Napier-Stokes equation of hydrodynamics for a moving, electrically conducting fluid is

$$\partial\bar{v}/\partial t + (\bar{v} \cdot \nabla)\bar{v} = -1/\rho \nabla p - \nabla U + \nu ✡\bar{v} + 1/\rho \; \bar{J} \times \bar{B} \qquad (31)$$

with ρ the fluid density, U the gravitational potential, ν the kinematic viscosity, and p the pressure. The term, $1/\rho \; \bar{J} \times \bar{B}$, is the pondermotive force which the electromagnetic field exerts on the fluid, and represents the coupling between the field and the fluid. A second necessary equation of hydrodynamics is the continuity equation which, for an incompressible fluid, is

$$\text{div } \bar{v} = 0 \qquad (32)$$

Using equation (25) in equation (31) results in

$$\partial\bar{v}/\partial t + (\bar{v}\cdot\nabla)\bar{v} = -1/\rho\,\nabla p - \nabla U + \nu\nabla^2\bar{v}$$
$$+\ 1/\rho\mu\ \mathrm{curl}\ \bar{B}\times\bar{B} \qquad (33)$$

Equations (30, (32), and (33) contain only the vectors $\bar{v}$ and $\bar{B}$, and are the basic equations of field motion. The equations include non-linear terms of three kinds: the term curl $(\bar{v}\times\bar{B})$, the term $1/\rho\mu$ curl $\bar{B}\times\bar{B}$, and the inertia term $(\bar{v}\cdot\nabla)\bar{v}$. These non-linearities make a mathematical solution almost impossible. However, by a dimensional analysis Elsasser (1954a) obtained some important qualitative results.

This analysis begins by combining equations (25) and (27) into

$$\bar{E} + \bar{v}\times\bar{B} = \nu_m\ \mathrm{curl}\ \bar{B} \qquad (34)$$

If L, T, and V represent the orders of magnitude of length, time, and velocity, respectively, then, by equation (24), $\bar{E}$ has the order of magnitude VB, as has the $\bar{v}\times\bar{B}$ term, while the term ν_m curl $\bar{B}$ is of the order $\nu_m L^{-1}B$.

In hydrodynamics the nondimensional Reynolds number is defined as $R = LV/\nu$. Thus the viscous term $\nu\nabla^2\bar{v}$ in equation (33) is of the order $1/R$ compared to the inertia term $(\bar{v}\cdot\nabla)\bar{v}$. A magnetic Reynolds number may be defined as

$$R_m = LV/\nu_m \qquad (35)$$

The terms magnetic viscosity and magnetic Reynolds number can be justified by noting that the dissipative term $\nu_m\nabla^2\bar{B}$ is of order $1/R_m$ compared to the other terms in equation (30) a relation analogous to the purely hydrodynamic case.

The right-hand side of equation (34) is of the order $1/R_m$ when compared to the left-hand side. When R_m takes on large values due to L becoming large, as V, B, and ν_m remain constant, equation (34) gives

$$\bar{E} = -\bar{v} \times \bar{B} \tag{36}$$

Therefore, the actual electric currents corresponding to a given magnetic field can become very small in systems of cosmic dimensions.

A simple physical interpretation of R_m is gained by considering an electric conductor in which a system of currents flows. If electromotive forces are absent, these currents will decay exponentially. The decay time τ can be estimated from equation (30) by setting $\bar{v} = 0$, and is of the order

$$\tau = L^2/\nu_m \tag{37}$$

so that

$$R_m = \tau V/L = \tau/T \tag{38}$$

where T is the time for a fluid particle to travel a distance equivalent to the extension L of the conductor. If, as in the case of the earth's interior, R_m has a large value, then the decay time τ must be very large compared to a period of mechanical motion. Fluid motions may be considerable during a time in which the decay of the electromagnetic field is small.

One more general result of magneto-hydrodynamics is found by combining equations (24) and (30), and writing them in integral form as

$$\iint \partial\bar{B}/\partial t \cdot d\bar{A} = \int \bar{v} \times \bar{B} \cdot d\bar{l} - \nu_m \int \operatorname{curl} \bar{B} \cdot d\bar{l} \tag{39}$$

Next let a surface S and its contour move bodily with the fluid, sweeping out a disk-shaped volume during time dt. The flux passing through the strip that forms the edge of the disk will be, by Fig. 3,

$$d\Phi = \iint (d\bar{r} \times d\bar{l}) \cdot B = - \left[\int \bar{v} \times \bar{B} \cdot d\bar{l} \right] dt \tag{40}$$

Thus the total time derivative of flux crossing S as it moves with the fluid will be

$$d/dt \iint \bar{B} \cdot d\bar{A} = \partial/\partial t \iint \bar{B} \cdot d\bar{A} - \int \bar{v} \times \bar{B} \cdot d\bar{l} \tag{41}$$

The first term on the right is the flux changing with time for a surface fixed in space; the second integral on the right represents the change in flux for a fixed $\bar{B}$ as the surface is displaced. Substituting equation (41) into equation (39) provides

$$d/dt \iint \bar{B} \cdot d\bar{A} = -\nu_m \int \mathrm{curl}\ \bar{B} \cdot d\bar{l} \tag{42}$$

For ideal conductors $\nu_m = 0$ and

$$d/dt \iint \bar{B} \cdot d\bar{A} = 0 \tag{43}$$

Equations (42) and (43) hold equally well for both incompressible and compressible fluids, and indicate that the magnetic lines of force in a conducting fluid are carried bodily along with the fluid motion. For a fluid of finite conductivity the right-hand side of equation (42) represents a "slippage" of the lines of force when following the fluid motion.

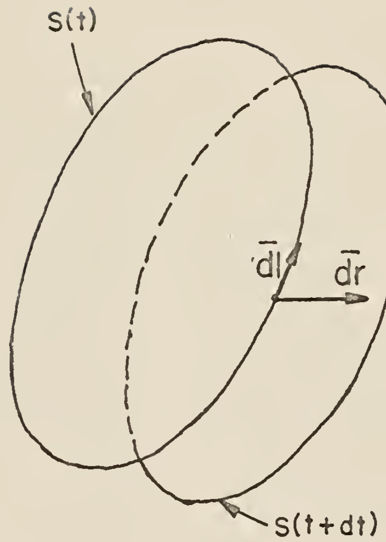

FIG. 3.

The Dynamo Theory

The dynamo theory has the widest acceptance of any theory of the earth's internal field since it presupposes the least in the way of natural phenomena and can easily predict much of the behavior of the field. The equations of magneto-hydrodynamics suggest that the magnetic field is produced by the mechanism of electromagnetic induction; magnetic energy is drawn from the kinetic energy of the fluid in the core. Thus, $\bar{B}$ and $\bar{v}$ form a single dynamic system; energy is not separately conserved for each of these fields.

Since the basic equations involving $\bar{B}$ and $\bar{v}$ are non-linear, it is not possible to obtain analytically a steady-state dynamo solution satisfying the necessary boundary conditions. All approaches taken so far have separated the hydrodynamic and the electromagnetic problems and attempted to solve only the latter problem. Some velocity distribution is assumed, and interactions between the magnetic field and the fluid motion, satisfying Maxwell's equations and capable of maintaining the field, are sought.

An important characteristic of any mechanical dynamo is its low order of symmetry; the armature has the symmetry of a clock face in which the two directions of rotation are not equivalent. Depending upon the rotational direction, such a dynamo process will be either degenerative or regenerative. A simple body like a sphere has a high order of structural symmetry. Thus to act as a dynamo the earth's core must possess some asymmetry in fluid motion, for Cowling (1934) has shown that highly symmetrical fluid motions in a sphere cannot produce any dynamo action. The necessary asymmetry in fluid flow is most likely provided by the Coriolis force. As in the mechanical dynamo, the resultant induction reaction between the fluid and the field must have the proper sign in order that the field be maintained.

For a sphere the vector fields of $\bar{E}$, $\bar{B}$, $\bar{J}$, and $\bar{v}$ can be derived from the scalar functions:

$$\Psi_n^{ms} = R(r)\ P_{n,m}(\cos\theta)\ \sin m\phi$$

$$\Psi_n^{mc} = R(r)\ P_{n,m}(\cos\theta)\ \cos m\phi \tag{44}$$

The fields derived from these functions are of two kinds: a toroidal field $\bar{T}$ = curl $(\bar{r}\Psi)$ = grad $\Psi \times \bar{r}$, and a poloidal field $\bar{S}$ = curl $\bar{T}$.

To produce the poloidal field $\bar{S}_1$ (corresponding to the main dipole field), Bullard and Gellman (1954a) postulated that a much more powerful toroidal magnetic field must exist in the earth's core. This toroidal field would not appear in any measurements made at the earth's surface. Bullard also showed that no single type of velocity field could provide dynamo action. Of the combinations of two velocity fields possibly allowing dynamo action, the process requiring T_1 and S_2^{2c} fluid motions was the simplest. The details of this $T_1S_2^{2c}$ interaction are given in Fig. 4 and Table 3. After extended numerical computation, it was concluded that the $T_1S_2^{2c}$ interaction could produce dynamo action. This particular solution is by no means unique, and some doubt has been cast upon the convergence

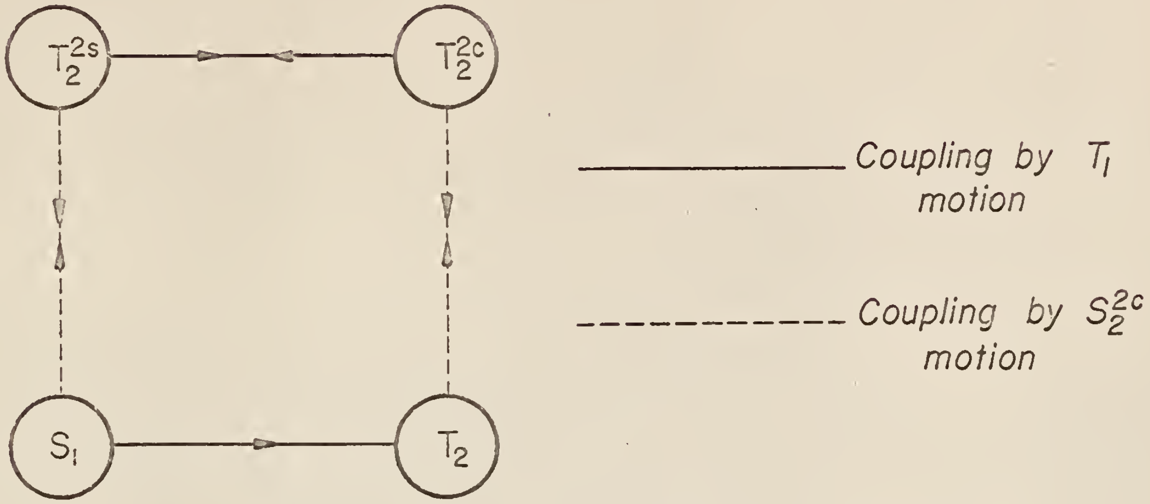

FIG. 4. RELATIONS BETWEEN INDUCED AND INDUCING FIELDS FOR A $T_1S_2^{2c}$ DYNAMO PROCESS.

Table 3. Relations between induced and inducing fields for T_1 and S_2^{2c} fluid motions.

Velocity	Inducing field	Current	Induced field
T_1	S_1	S_2	T_2
S_2^{2c}	S_1	S_2^{2s}	T_2^{2s}
S_2^{2c}	T_2	S_2^{2c}	T_2^{2c}
S_2^{2c}	T_2^{2c}	S_2	T_2
S_2^{2c}	T_2^{2s}	T_1	S_1
T_1	T_2^{2c}	S_2^{2s}	T_2^{2s}
T_1	T_2^{2s}	S_2^{2c}	T_2^{2c}

of the solution. Nevertheless, Bullard's results indicate that the internal dynamo theory is quite feasible and can provide a self-consistent account of the earth's magnetic field.

In order to show that a magnetic field could be deformed in such a way so as to produce a self-sustaining dynamo, Parker (1955) and Elsasser (1955) considered two effects of the earth's rotation upon fluid motions in the core. First, a simple $\bar{T}_1$ toroidal magnetic field with lines of force circling about the axis of the earth, parallel to lines of latitude, is assumed. This toroidal field results from the deformation of the poloidal $\bar{S}_1$ (dipole) field by the non-uniform rotation of fluid particles in the earth's core (Fig. 5). The toroidal field can be generated directly from the poloidal field, but not vice versa.

The production of a poloidal field from the toroidal field is obtained by considering the action of convective fluid motions as they deform the toroidal field. The convective process is pictured to be an arbitrary distribution of radially directed rising and sinking columns of fluid. The earth's rotation enters into the process a second time when the Coriolis force imparts a cyclonic motion to each convective column. For the case of an outwardly directed column passing through a small region of the toroidal field, each line of force is lifted and then twisted 90° counterclockwise to form an elemental loop of magnetic flux lying in a meridional plane.(Fig. 6). Parker further showed, that for any arbitrary distribution of convective columns, the elemental loops of flux will coalesce and regenerate the poloidal field.

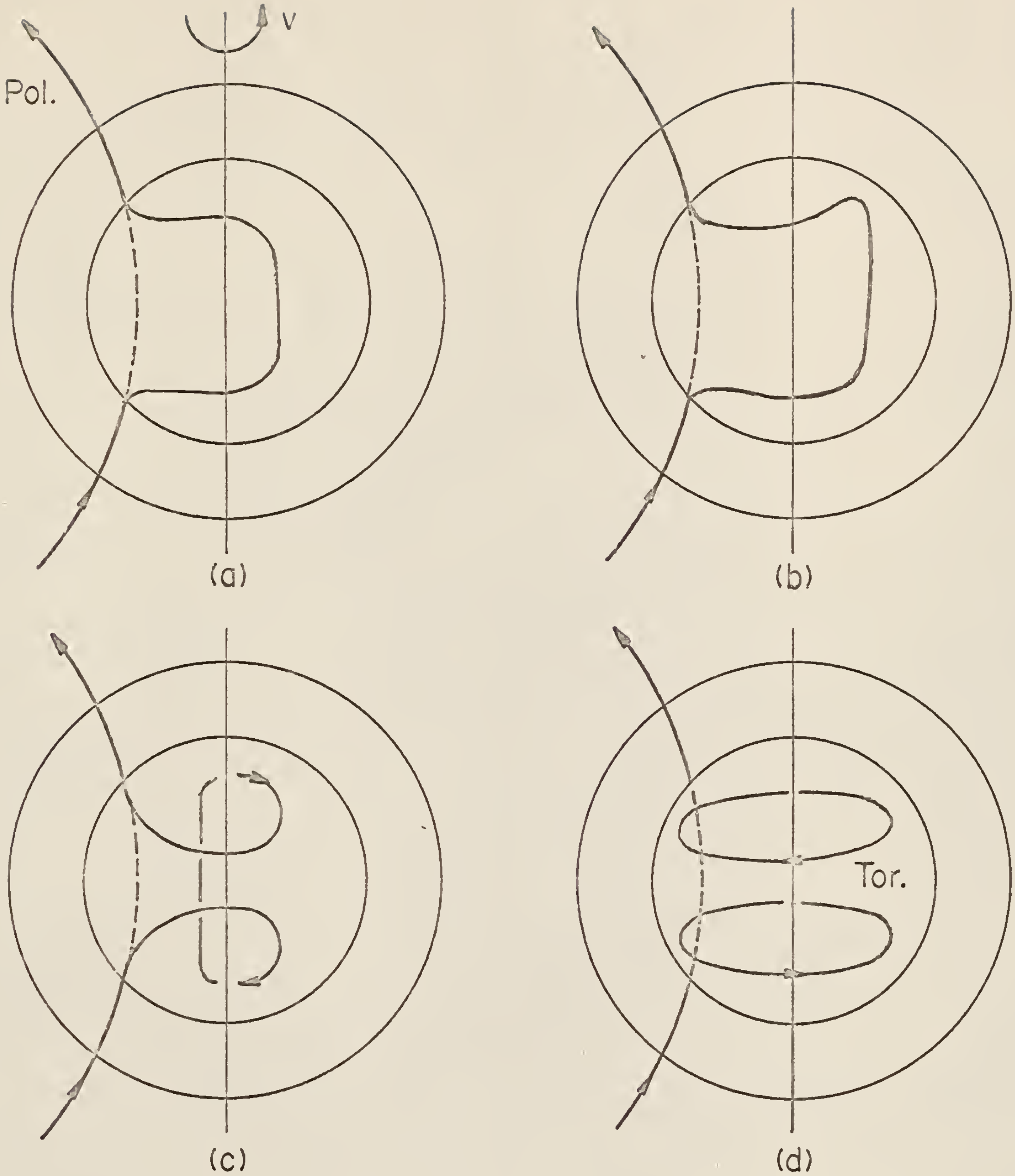

FIG. 5. GENERATION BY NON-UNIFORM ROTATION OF A TOROIDAL FIELD FROM A POLOIDAL FIELD.

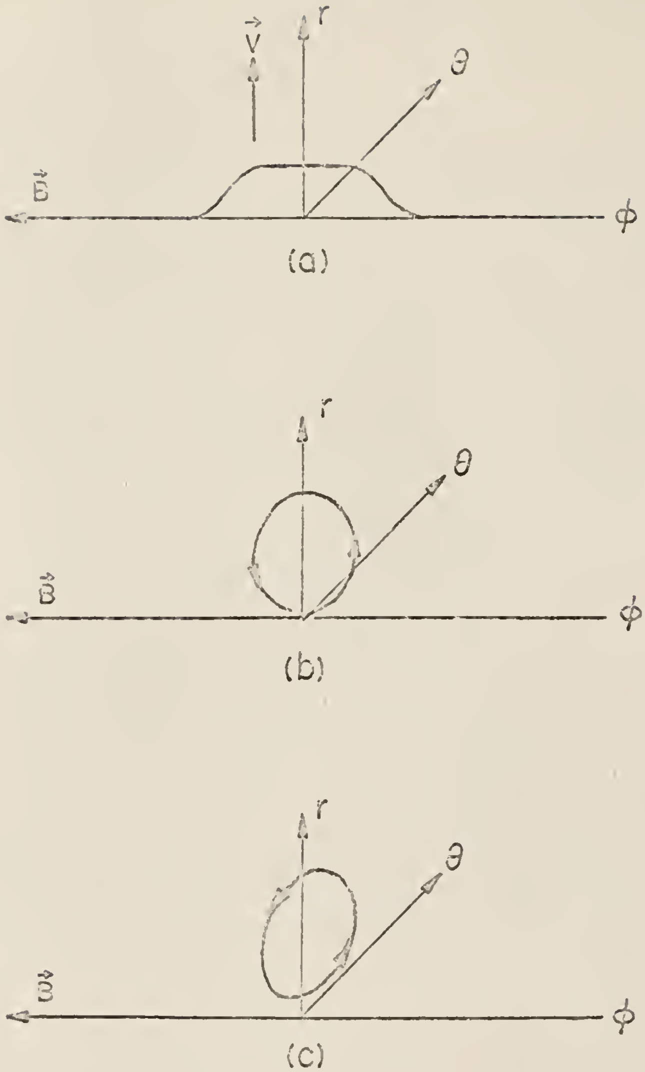

FIG. 6. GENERATION OF MERIDIONAL LOOPS OF MAGNETIC FLUX.

THEORIES OF THE EARTH'S EXTERNAL FIELD

Theory of the Solar Daily Variation

As the primary source of the S or solar daily variation, Stewart proposed, "that convective currents established by the sun's heating influence in the upper regions of the atmosphere are to be regarded as conductors moving across lines of magnetic force, and thus the vehicle of electric currents which act upon the magnet". This atmospheric dynamo theory for the S variation was developed into a quantitative form by Schuster (1908), and remains quite well accepted today. Other theories involving atmospheric thermoelectric currents or the change of paramagnetic behavior of oxygen when heated do not have any basis in physical fact.

The dynamo action given by Schuster takes place in a thin spherical shell of the ionosphere. The dynamo field is given by

$$\bar{E} = \bar{v} \times \bar{H} \qquad \text{(emu system)} \qquad (44)$$

where $\bar{v}$ is the velocity vector in the presence of a magnetic field $\bar{H}$. If the motion of the air of the shell is supposed horizontal and irrotational, it is associated with a velocity potential Ψ, such that,

$$\bar{v} = - \text{grad}\,\Psi \qquad (45)$$

Let the x direction be southward, the y direction be eastward, and the z direction be upward, and let a be the radius of the shell. Since $v_z = 0$, Ψ is independent of z, and equations (44) and (45) can be combined to give

$$E_x = -H_z \frac{\partial \Psi}{a \sin \theta \, \partial \phi}$$

$$E_y = H_z \frac{\partial \Psi}{a \, \partial \theta} \tag{46}$$

where θ and ϕ are the angles of co-latitude and of east longitude. Thus the dynamo field depends only on the vertical component of the geomagnetic field.

The existence of a steady current system, without a growing accumulation of charge, implies that the southward and eastward components (U, V) of the current density are associated with a current function R. Therefore,

$$U = \frac{\partial R}{a \sin \theta \, \partial \phi}$$

$$V = - \frac{\partial R}{a \, \partial \theta} \tag{47}$$

The electric force required to drive this current must have the components U/K, and V/K, where K is the conductivity of the spherical shell. However, the dynamo components E_x and E_y are not, in general, related to any function R in the manner of U/K and V/K. The dynamo field will set up a charge distribution whose field, together with the dynamo field and any additional field due to self or mutual induction, is such that a current system having the current function R is produced. If self or mutual induction is ignored, the equation satisfying R is

$$K\left[\frac{\partial^2 R}{\sin \theta \, \partial \phi^2} + \frac{\partial}{\partial \theta}\left(\sin \theta \frac{\partial R}{\partial \theta}\right)\right] - \left[\frac{\partial K}{\partial \phi} \frac{\partial R}{\sin \theta \, \partial \phi} + \sin \theta \frac{\partial K}{\partial \theta} \frac{\partial R}{\partial \phi}\right]$$

$$= aK^2\left[\frac{\partial (v_x H_z)}{\partial \theta} + \frac{\partial (v_y H_z \sin \theta)}{\partial \theta}\right] \tag{48}$$

Thus R is determined if H_z, K, and $\overline{v}$ (or the velocity potential

Ψ) are known.

Theories of Magnetic Storms

Any theory of magnetic storms must explain not only the magnetic disturbance variation D, but also such allied phenomena as the auroras and the Van Allen belts. These phenomena will be briefly considered before the discussion of magnetic storm theories. Auroral displays occur in two primary zones 20° to 25° from the geomagnetic poles; Vestine (1960) has shown that these zones are linked by geomagnetic lines of force. The displays always take place at times when the magnetic field is disturbed, but not all storms are accompanied by auroras. During more intense disturbances, the auroral zones tend to move toward lower latitudes. Most of the displays are at a height of approximately 100 km. Since the spectra of the displays have many strong hydrogen lines, proton bombardment of the upper atmosphere must certainly play a part in aurora generation. The incidence of auroras is at a maximum one or two years after the peak of the eleven year sunspot cycle. For any year auroras are more prevalent during the equinoctial season.

Regions of intense radiation in the space surrounding the earth were first discovered by Van Allen et al. (1958) through analysis of data obtained by earth satellites Explorers I and III. This data and the data from later satellites showed the existence of two distinct, widely separated, high intensity zones of trapped particles encircling the earth. The trapped radiation is largely protons and electrons that are constrained to trochoidal orbits by the geomagnetic field; the particles eventually leak out of the radiation zones by virtue of collisional energy loss and scattering in the thin outer atmosphere. The inner Van Allen belt lies 1.5 earth radii from the geocenter; its particles seem to be the products of radioactive decay of neutrons, these neutrons being released from cosmic ray induced nuclear disintegrations in the top of the atmosphere. The outer Van Allen belt is centered at 3 to 4 earth radii from the geocenter; the source of its particles appears to be solar plasma that has reached the earth. This outer belt may play a central role in auroras, geomagnetic storms, and atmospheric heating. An artificial Van Allen belt was created by the Argus experiment in 1958; three small nuclear devices were detonated at high altitudes to inject electrons into trapped orbits in the earth's magnetic field (Christofilos, 1959).

Birkeland (1896) gave the first promising suggestion on the correlation of auroras and magnetic disturbances. He showed that when an electron stream was projected towards a magnetized sphere in an evacuated chamber, the electrons were deflected towards the poles by the magnetic field. A behavior quite similar to the earth's aurora took place; two circular discharge zones, one about each pole, appeared upon the sphere.

Störmer (1907, 1911, 1912) developed mathematically the results of Birkeland's experiments by calculating the possible paths of particles of one sign moving under the influence of the geomagnetic field. With the assumption that these particles were emitted from the sun, Störmer was able to show that such particles could reach the earth's upper atmosphere only in the auroral zones. A further consequence of these calculations was the prediction of a westward flowing ring current about the earth. The calculations showed that, in a bundle of solar particles reaching the earth, many particles would be deflected around the earth in the equatorial plane. The resultant ring current could account for the main phase decrease in H during a magnetic storm and could also explain the tendency of the auroral zones to shift equatorwards during a severe magnetic disturbance.

One fatal objection to the Birkeland-Störmer theory is that if the emitted corpuscles from the sun were of one sign only, they would disperse through mutual electrostatic repulsion long before reaching the earth. Nevertheless, the Störmer motion of charged particles plays a predominant role in the generation of the auroras and the Van Allen belts.

Chapman and Ferraro (1931) hypothesized that the effects of a magnetic storm were due to an electrically neutral stream of solar particles. The details of a solar process that could produce such a neutral ionized stream are not yet completely known. The time interval of about a day between the occurrence of a solar flare and the onset of a magnetic storm requires that the stream have a velocity of 600 to 1400 miles per second. It was estimated that the width of the stream front was at least 10^7 km., so that the surface of the front appears as a plane as it nears the earth. The composition of the stream was considered to be largely electrons and protons, with negligible recombination taking place during the passage of the stream from the sun to the earth.

As this plane front of ionized, highly conducting gas approaches the earth, currents are induced in the stream's surface by the geomagnetic field. The resultant magnetic effect is equivalent to an image dipole situated inside the stream at the same distance from the stream surface as the earth dipole is outside the stream. Such an image dipole compresses the lines of force of the earth dipole on the side nearest the stream, and causes an increase in magnetic intensity typical of the first phase of a magnetic storm.

In sweeping by the earth, the solar stream is repelled from the earth's surface by the compressed geomagnetic field lines, and a hollow space forms in the stream (Fig. 7). Surface charges on the walls of the hollow produce an electric field that gives rise to a discharge across the intervening gap. This discharge leads to a net westward flow of current around the earth. When the stream has passed beyond the earth, a ring current will be left behind. This current produces the decrease in H characteristic of the main phase of a magnetic storm.

Alfvèn (1955) accepted the idea that the solar stream consists of a neutral ionized gas, but he proposed also that the stream, because of its high electrical conductivity, may possess an embedded magnetic field. Thus, as the stream moves by a point, an electrical field, $\bar{E} = \bar{v} \times \bar{H}$, will appear; in effect

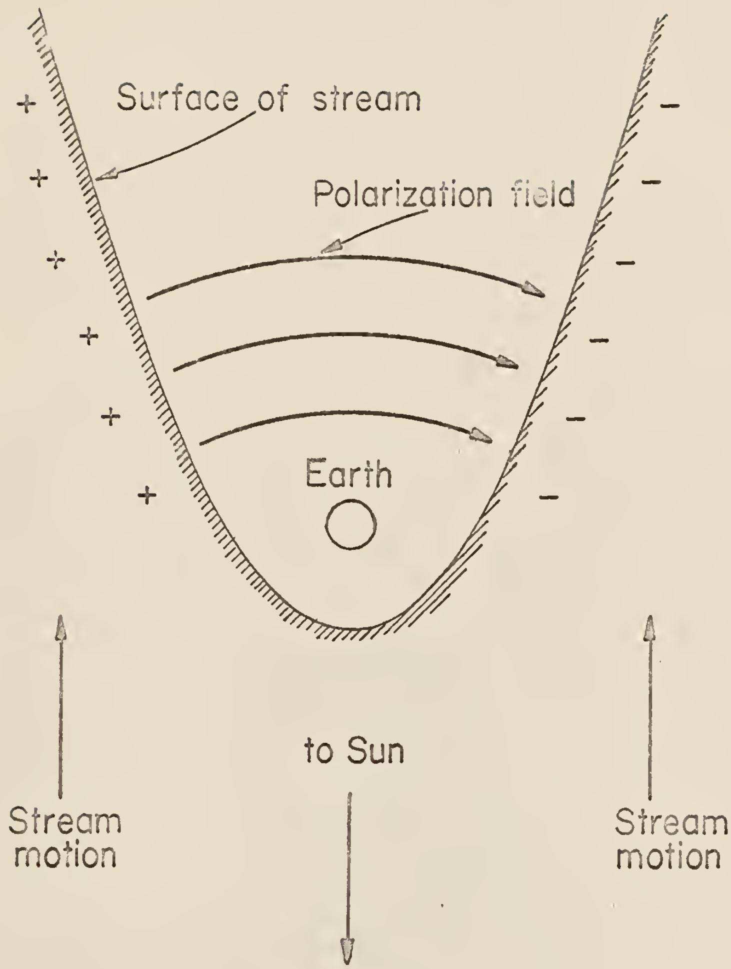
Surface of stream
Polarization field
+
+
+
+
+
+
–
–
–
–
–
–
Earth
to Sun
Stream motion
Stream motion

FIG. 7

the stream is electrically polarized. By methods developed by Störmer, Alfvèn calculated the motion of electrons under the influence of this electric field and the earth's dipole field. These electrons drift in trochoidal orbits about the earth; the motion is equivalent to a ring current in the equatorial plane. During the onset of a magnetic storm Alfvèn's ring current flows west to east; the current reverses in direction during the main phase of the storm. In comparing his ring current with the ring current of the Chapman-Ferraro theory, Alfvèn pointed out that an electron moving in a trochoidal path which has an overall circular drift pattern represents a stable orbit, whereas it had never been proved that a simple ring current was stable.

Still another theory of the magnetic storm was developed by Singer (1957). This theory considers that the stream from a solar flare gives some of its energy to the production of a shock wave in the interplanetary gas. This shock wave propogates at a higher velocity than the solar flare itself, and arrives at the earth up to nine hours earlier than the solar stream. When the shock wave reaches the earth, it is retarded in the equatorial plane by the geomagnetic field. Above and below this plane the shock wave is bent and guided by the lines of force, and becomes compressed by the field as it travels into the atmosphere of the auroral zone. There auroral displays are produced and a current is initiated which is responsible for the sudden commencement of a magnetic storm.

The main phase of the magnetic storm results when the solar stream particles reach the earth. The solar gas penetrates into the geomagnetic field, which has been perturbed by the shock wave. After being scattered, the charged particles are attached to lines of magnetic force and execute spiraling or trochoidal motions about these lines. At the same time these trapped particles drift in azimuth around the earth and generate a large westward flowing current of the right magnitude to cause the main phase effect of the storm.

Singer's concept of a trapped region of solar particles seems amply supported by the discovery of the Van Allen belts, and by the results of Project Argus. The particles oscillate rapidly to and fro between magnetic mirror points at fairly high northern and southern latitudes. Simultaneously, they circle about the magnetic field lines, and also drift around the earth, protons westward, and electrons eastward.

Another recent magnetic storm theory was presented by Dessler and Parker (1959). In this theory the storm is caused by the collision of the earth's magnetic field with relatively dense plasma clouds ejected from the sun. The initial phase of the storm occurs when a solar plasma cloud pushes against the earth's field and compresses it. The rapid increase in magnetic intensity at the beginning of the storm is propogated rapidly around the earth by a hydromagnetic wave.

Upon diffusion of the plasma into the geomagnetic field, the main phase begins. The trapped particles, mostly protons, exert three stresses upon the geomagnetic field. The chief stress, due to centrifugal force of the particles, accounts for most of the main phase changes. The recovery from the main phase is attributed to the relief of the geomagnetic field stress by a transfer of the energy of the trapped protons to atoms of neutral hydrogen through an ion-atom charge exchange.

CONCLUSION

At the earth's surface the geomagnetic field is derivable from a scalar potential. By the use of spherical harmonic analysis it is possible to determine the components of the field due to external and internal sources. The internal field, much larger than the external field, has a large component that can be produced by a simple dipole, and a smaller non-dipole or residual component. With respect to the earth's dipole, a system of geomagnetic co-ordinates useful in the study of magnetic storms can be derived.

The earth's magnetism exhibits two distinct types of variation, secular and transient. The secular variation, of internal origin, is clearly associated with the non-dipole field, and has displayed a westward drift during the past half century. Three major forms of transient variation exist: (1) the solar daily variation, (2) the lunar daily variation, and (3) the magnetic disturbance variation. All three forms are largely of external origin.

For a simple model of the earth's magnetism, the internal field dipole may be placed at the center of the earth. A somewhat better representation is possible by displacing the dipole 250 km. from the geocenter. The residual field may be represented by a distribution of dipoles or current functions at the surface of the earth's liquid core. A combination of an overhead ionospheric current system and an equatorial ring current produces an adequate model of the transient magnetic variations. An accurate simulation of the geomagnetic field is obtained by combining the models for the dipole field, the residual field, and the external field.

A dynamo process seems the most likely source of the internal field. The non-linearity of the basic magneto-hydrodynamic equations allows only a partial solution of the dynamo action. For certain assumed fluid motions, a solution satisfying Maxwell's equations and the postulated boundary conditions does exist, but the solution is not necessarily unique. This solution requires an internal toroidal field that is not detectable at the earth's surface and is much stronger than the earth's dipole field.

Tides in the upper atmosphere appear responsible for the solar daily variation in the magnetic field. Emitted solar particles reaching the earth lead to magnetic storms and auroral displays. These particles also enter into the formation of the Van Allen belts. The development of a comprehensive magnetic storm theory must certainly await a more complete measurement of the external geomagnetic field and the solar wind of emitted plasma particles.

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THE EARTH'S MAGNETIC FIELD

by

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This report gives a detailed description of the geomagnetic field, presents some physical models of the field, and discusses a number of theories relating to the field's origin.

The magnetic elements used in geomagnetic surface measurements are defined. With the use of spherical harmonic analysis a scalar magnetic potential function may be obtained from world wide magnetic data. This potential function is separable into two component potentials, one due to internal sources, and the other due to external sources.

The internal sources produce a much larger field than the external sources; this internal field has a large component that could be produced by a simple dipole, and a smaller non-dipole or residual component. The residual internal field is associated with the secular variation of the earth's field.

External sources superimpose upon the earth's mean field three major forms of transient magnetic variation: the solar daily variation, the lunar daily variation, and the magnetic disturbance variation.

The simplest model of the earth's magnetism would represent the main field by either a centered dipole or by a slightly eccentric dipole. More detailed models must also include the internal non-dipole field and the external field. A distribution of dipoles or current functions at the surface of the earth's core could generate the non-dipole field. A combination of an ionospheric current system and an external ring current in the equatorial plane produces an adequate model of the external field.

Various theories on the origin of the internal field, based on ferromagnetism, spontaneous magnetization of rotating bodies, systems of rotating electric charge, or slowly decaying current systems, are shown to fail in many aspects. The internal field is probably due to internal current systems maintained either by the thermoelectric effect or by a dynamo action; the latter process seems more likely, as it requires fewer assumptions.

The basic equations governing dynamo action in the earth's fluid core are non-linear and, therefore, not easily amenable to analysis. Some qualitative results may be obtained by a dimensional analysis of these equations. Another approach to the problem assumed a particular fluid motion, and then showed that a self regenerating dynamo action, consistent with Maxwell's equations, is possible.

The solar daily variation in the magnetic field is most likely a result of upper atmospheric tides. The magnetic disturbance variation and its associated phenomena apparently result from the interaction of emitted solar particles with the earth's field. Any comprehensive magnetic storm theory must await a more complete determination of the external geomagnetic field and the nature of the solar wind.