DEVELOPING A MATHEMATICAL MODEL FOR PREDICTING TOTAL OPERATORS' WAGES IN PUBLIC TRANSIT

by

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DEFINITIONS

Straight Run	A regular run having no unpaid breaks.
Piece of Work	Any work assigned to an operator which pays less than eight hours.
Split Run	A regular run having unpaid breaks.
Pull Out Time	The starting time of a piece of work, straight run, or a split run.
Pull In Time	The finishing time of a piece of work, straight run, or a split run.
Travel Time	The actual running time from the garage to point of relief.
Platform Time	The time during which an operator is in charge of a bus.
Turn In	A specific amount of time which the operator receives for every straight run and each piece of work in his assignment.
Guarantee Time	The time paid for but not worked to make up a minimum guarantee for certain classes of work.
Allowance	A time equal to either the travel time, the guarantee time, or both.
Pay Time	The total pay time for a specific run.
Spread Time	The total working time plus the unpaid time of a split run.
Relief Point	Relief point is a certain location or locations on each route where operators may be relieved or assigned to the bus.

HYPOTHESIS

The task of public transit companies is becoming immeasurably greater and more complex due to the declining trend in the usage of their services. One of the urgent needs of today is that of increasing the efficiency of the transit services and to make them more attractive to the public. In view of the constant increase in the cost of labor, there is an urgent need for reducing operating costs. Proper scheduling techniques yielding efficient and effective schedules would play a major role in reducing operating costs. Hence, the aim of transit companies has been rescheduling services as a means of maintaining operations on a profitable basis in face of strong competition from automobile users.

The objective of this thesis is to develop a mathematical model for predicting the total pay time to the drivers of public transit industry in a general functional forms and parameter values. It has been found that there are various constraints and restrictions in the management agreement on which always a controversy is going on between management and trade union. Further, it is felt necessary to explore the sensitivity of the model to some general functional equation in terms of parameter values. The carrying out of computation of the model is done in 1620 IBM Computer. The total pay time functions are plotted for each combination of parameter values. Also statistically by conducting a two-way analysis of variance test it is intended to show which of these variables affect more significantly the total pay time function.

INTRODUCTION

The automobile vehicle brought a new concept to transportation, flexibility. The first twenty years of the 20th Century were devoted to perfecting the mechanical design of the vehicle. Few people recognized the potential of the motor vehicle or anticipated the revolution it would cause in American economy or in cultural and social values. It is unnecessary to enumerate the tremendous changes this brought in America and the great advance in economic progress it made possible. No longer are there isolated areas in the nation inaccessible to the amenities of life.

In many cities the bus system is one of the major means of transportation into or within the city. There are various reasons for adoption of buses in transit operations among which a few are as follows:

(1) Flexibility in traffic,

(2) Individual power supply,

(3) Ability to pass each other,

(4) Through service and off route locations,

(5) Ability to combine routes with one vehicle rendering the service,

(6) Low initial cost.

From the last few years the number of intracity bus passengers has been steadily decreasing. There are various reasons for this decrease, but the decline in the usage of the buses and the increase of automobiles on the road have not only created problems for transit companies, but also for the city planners. For reversing this trend many solutions have been suggested, but time is needed to assess the validity of these solutions. However, even those transit companies which were making tremendous profits are now facing a problem to stay in the business because of the decrease in the volume of bus passengers and increase of automobile users.

Since the transit companies are service companies, a major portion of their revenues goes as wages to operators. Thus, it has been felt that the reduction in operating cost, through rescheduling is the only way for transit companies to maintain their operation on a profitable basis. Yet, the best manual methods fall short of solving the problem, because computation cannot be made quickly enough to keep pace with the changing conditions. As a result, high speed computers have proved the best means to overcome these difficulties.

The theoretical analysis involved in scheduling has been completed in previous research[1] and computer programs to carry on the scheduling operations has been developed which have been modified here to include each company's constraints and variables.

The number of variables and restrictions is the main difficulty encountered in effectively programming a daily work schedule for each operator. Operating variables, trip times, and restrictions of the union-management agreement are a few of the factors which have to be considered in making an effective daily schedule.

One of the biggest problems faced by the transit industry is the variable nature of the daily demand of their services. Traffic is at the peak in the mornings and late afternoons and tends to fall off in the early mornings, noon, and late evenings and on week ends. To meet these peak traffic requirements, it becomes necessary to maintain a big fleet of buses in excess of that which otherwise would be needed. The graph in Figure 1, shows the requirement of the number of buses (motor coaches) needed during

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[1]Samy E. G. Elias, "A Digital Computing Solution to the Transit Operation Assignment Problems," Doctoral Thesis, Oklahoma State Univ., 1960.

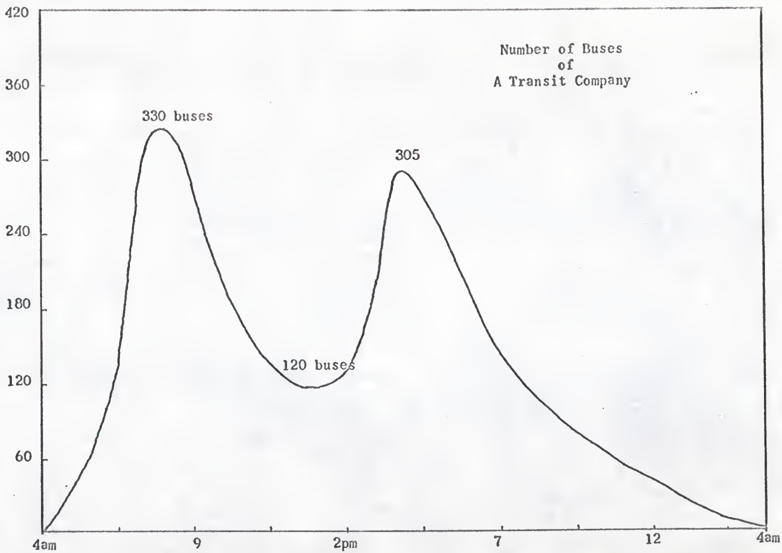

FIGURE 1

DAILY VOLUME OF TRAFFIC

the day in a hypothetical case. From the Figure, it is evident that the two peaks require almost three times as many buses as are needed during the slowest part of the day, and the additional buses in service during the peak periods will have short trips, while others may run almost twenty-four hours. As far as labor management agreement is concerned most of them require the same restrictions and basic demands.

The following is the brief list of the constraints and restrictions as founded in the labor management agreement of one of the transit companies:

> The work day of all regular trolley bus and motor coach operators shall be eight (8) hours, in that no regular run shall pay less than eight (8) hours. Time and one-half shall be paid for all work done before or after the schedule time of the regular run for all work beyond eight (8) hours daily. Forty-six percent (46%) of all runs shall be straight.
>
> Spread penalty shall be paid after a spread time of eleven (11) hours for regular operators and twelve and one-half (12½) hours for extra operations on tripper runs where the intervening time between taking out parts of the run amounts is one (1) hour or less, such intervening time shall be paid as part of run. Five (5) minutes pay shall be allowed operators making turn-in and will be considered as part of the regular run.

With the above restrictions, the schedule maker proceeds to develop different possible runs. Of course, the best possible schedule for a company would be with all straight runs, but due to the variations in trip lengths, this can never be achieved.

The use of computer techniques in solving problems of scheduling men and machine in public transit companies is feasible and would be simple were it not for the restrictions which result from the labor management agreement and the variations in the traffic situations.

The basic advantages of computer systems namely fast rate of processing data and accuracy in computation make the use of computer programming an effective and better substitute for the manual methods which involve profusive clerical work and calculations. The IBM 1620 digital computer was used in this thesis. FORTRAN (Formula translation system) which utilizes an automatic coding system resembling the language of mathematics was used in the development of straight runs.

Basically, all the program does is simulate the motion of a bus internally on the computer. Each bus is followed across its own route and decisions and calculations are made by the computer on the basis of the information fed in advance. The same work is done in the manual technique with pencil and paper but would evidently take much more time when performed by human beings. The restrictions, such as those imposed by minimum and maximum hours of work, overtime rates, spread penalties, and times passing the relief point on the route where decision must be made, are incorporated in the program and implemented automatically by the computer.

For daily scheduling a two step procedure is used which is same in both the manual and the computer approaches. The steps are:

1. Developing all possible straight runs.

2. Constructing split runs by combining pieces of work.

The main reason for following a two-step approach is that the computer storage capacity limits the use of one program to do all the steps.

DEVELOPMENT OF STRAIGHT RUNS

Step 1:

The computer technique of developing straight runs is the same as the manual technique. The headway for each route supplies all the details about the input data. Such data for Step 1 is the route and block number, the pull out, pull in, and the relief times for each block. Cards are punched for each block. The route number, the block number, and the relief times are punched on each card. If the number of relief times is more than what could be accommodated on one card, then another card would be used for the same block. All the data received from the Company are in hours and minutes. The details of Route 26 and 3 Block are:

Pull out of garage at:	5:25 a.m.
Pull into garage at:	5:57 p.m.
Relief times:	5:25, 7:44, 8:59, 10:40, 11:10
	12:28, 12:58, 2:21, 2:51, 4:12
	4:45, 5:52, 5:57.

In order to make the data compatible for use on the computer it had to be converted onto a 24-hour clock and into hours and decimals. This requirement arises from the fact that the digital computer uses the digital system of calculation. A conversion program was used for this purpose. The converted output for the Route 26 and Block 3 was punched out in the following format:

No./Reliefs	Travel	Route	Block
13	0.0	26	3

P/Out	P/In	Reliefs
5:41	17.95	5:41, 7:73, 8:98, 12:46,
		12:96, 16.75, 17.86, 17.95.

Once the input data is prepared in the manner prescribed above, the machine is ready to start developing straight runs. The steps followed in the constructing straight runs are:

1. The machine finds the total trip time for the block and compares this with eight hours, the minimum permissible working time for a straight run. The total trip time for Route 26, Block 3 is 17.95-5.41 = 12.54 hours.

2. If the total trip time is more than 16 hours, two straight runs can be developed, one from the front of the block and another from the tail end of the block, leaving a piece of work in between. The logic behind this procedure is to avoid a late piece of work which would be difficult to use in the next step.

If the total trip time is larger than seven hours or less than sixteen hours, a straight run and a piece of work will be developed.

If the total trip time is less than seven hours, no straight run is developed but a piece of work will be punched out.

It was found that for any block having a total trip time of exactly seven hours, it would be cheaper for the company to pay guarantee time and make a straight run rather than paying spread time penalty and making a split run. Because if the company considers it as a piece of work then in order to make it a split run another piece of work of one hour will be needed. It is a restriction in the contract that the minimum pay for a piece of work has to be at least two hours. So, if we combine this seven hours piece of work in the two hours piece of work then a payment of one-half hour of overtime has to be made. Thus, total pay for this run will be 9.50 hours. There will be a possibility that the spread penalty might also be involved because of the unavailability of the pieces of work within the spread restriction. So it would be cheaper for the company to pay guarantee time and make a straight run rather than paying spread time and overtime penalties and making a split run. Hence, it was the company's policy to use a seven-hour limit in deciding whether to make a straight run or a piece of work. The total time for the example falls in the second category.

3. The platform time for the straight run is now computed - the union-management agreement states that the company has to consider the five mintues turn-in, travel time, and five minutes travel allowance, if any, as a part of the regular time. Therefore, to avoid paying overtime, the platform time is computed as eight hours minum turn-in time (five minutes), travel time and travel allowance, if any. For Route 26 and Block 3, used in the example, there is no travel time. Therefore, the platform time equals 7.91 hours (8.00 less five minutes turn-in time). The machine now checks the relief times on the block, from the pull out side, and checks for one that breaks the block into two pieces, one of them being either equal to or slightly larger or slightly smaller than the platform time. In the example, if the block is broken at relief 12.96 we get a piece having a platform time of 7.55 hours, where as the next relief time 14.35 gives a platform time of 8.94 hours.

4. The next step is to decide which of these two relief times to select as the pull in time of the straight run. This selection is chosen on the basis of cost. In the case of relief time 12.96, the company has to pay 0.36 hours as a guarantee time, but relief time 14.35 pays an overtime of 0.56 hours. Of these two alternatives,

the one most economical is chosen. Therefore, relief time 12.96 becomes the pull in time for the straight run.

5. The machine next checks the trip time of the remainder of the block. In our example this is 4.99 hours (17.95 - 12.96 = 4.99 hours). It is not enough for another straight run so a piece of work is punched out.

6. Having developed a straight run and a piece of work from the forward direction the machine now constructs another straight run from the tail end of the block. This is done to give the schedule maker the choice between selecting an early or a late straight run on the same block.

The straight run output for Route 26 and Block 3 is shown as follows:

No.	Route	Block	TF	P/Out	P/In	TB	Plat	Penlt	Turn In	Pay
1	26	3	0	5.41	12.96	0	7.55	.36	.08	8.00
	26	3	0	12.96	17.95	0	4.99			
2	26	3	0	8.98	17.95	0	8.97	0.0	. .08	9.05
	26	3	0	5.41	8,98	0	3.57			

FIGURE 2

(All times are in hours.)

CONSTRUCTING SPLIT RUNS

Step 2:

The next step in constructing the schedule is that of combining the pieces of work. To explain this step, the following two pieces of work will be used:

No.	Route	Block	TF	P/Out	P/In	TB	Platform
1	1	12	0	6.06	9.51	0	3.45
2	4	4	0	14.06	18.26	0	4.20

We have seen above that the pieces of work are punched out one per card, with the following information: Route and Block Numbers, travel time (if the piece begins away from the garage), pull out time, pull in time, travel time of the pieces of work relieved on the road, and the platform time.

The computer program for Step 2 is developed so that the machine performs the following steps:

1. The machine will read each card which contains all the above information about the piece of work and will store this information in its memory.

2. The computer selects the first piece of work from all the pieces of work fed into the machine and checks it against all the remaining pieces of work in the same sequence in which they are stored in memory. This checking will facilitiate the development of the split runs by the combination of two or more of these pieces of work within the restrictions imposed in the computer program. These are:

a. A minimum gap of five minutes plus travel, if any, must exist between the pull in time of piece (1-12) and the pull out time of piece of work (4-4), if any. The gap between the two pieces being used is 4.55; well beyond the minimum limit.

b. The spread limit is computed. There is a maximum limit of 14.0 hours on spread time. However, any time beyond 11.0 hours is paid for at one and a half times the regular rate. Spread time = (pull in time of piece (4-4) - Travel + turn in) - (pull out time of piece (1-12) - travel + turn in) or (18.26 - 0 + 8.3) - (6.06 + 0 + 8.3) = 12.20.

This figure of 12.20 hours is within the maximum limit of 14.0 hours.

As the above two pieces satisfy all the restrictions, a split run is developed. The computer output is in the following format:

No.	Route	Block	TF	P/Out	P/In	TB	Spread	Platform	Penlt	Turn In	Pay
	1	12	0	6.06	9.51			3·45			
1	4	4	0	14.06	18.26		12.36	4·20	0.19	0.16	8·00

In a similar manner, the machine picks one piece at a time and constructs all possible split runs combinations in the other remaining pieces. The schedule maker has not to make a selection from the split runs developed.

	1	3	0.0	646	983	5.0		337.			
12	68	60	0.0	1501	1971	0.0	233.3	470.	5.00	16.67	828.6
	1	3	0.0	646	983	5.0		337.			
13	68	56	0.0	1390	1876	0.0	138.3	486.	5.00	16.67	844.6
	1	3	0.0	646	983	5.0		337.			
14	4	60	0.0	1488	1915	0.0	177.3	427.	19.34	16.67	800.0
	1	3	0.0	646	983	5.0		337.			
15	7	51	0.0	1358	1850	0.0	112.3	492.	5.00	16.67	850.6
	1	4	0.0	578	958	5.0		380.			
16	1	1	5.0	1423	1843	0.0	173.3	420.	-10.00	16.67	826.6
	1	4	0.0	578	958	5.0		380.			
17	1	58	0.0	1490	1873	5.0	208.3	383.	20.34	16.67	800.0
	1	4	0.0	578	958	5.0		380.			
18	1	60	0.0	1458	1828	0.0	158.3	370.	33.34	16.67	800.0
	1	4	0.0	578	958	5.0		380.			
19	1	62	0.0	1510	1923	0.0	253.3	413.	5.00	16.67	814.6
	1	4	0.0	578	958	5.0		380.			
20	1	64	0.0	1523	1883	0.0	213.3	360.	43.34	16.67	800.0
	1	4	0.0	578	958	5.0		380.			
21	28	55	0.0	1566	1963	0.0	293.3	397.	6.34	16.67	800.0
	1	4	0.0	578	958	5.0		380.			
22	10	51	0.0	1446	1796	0.0	126.3	350.	53.34	16.67	800.0
	1	4	0.0	578	958	5.0		380.			
23	3	57	0.0	1486	1955	0.0	285.3	469.	5.00	16.67	870.6
	1	4	0.0	578	958	5.0		380.			
24	3	58	0.0	1506	1918	0.0	248.3	412.	5.00	16.67	813.6
	1	4	0.0	578	958	5.0		380.			
25	3	59	0.0	1463	1961	0.0	291.3	498.	5.00	16.67	899.6
	1	4	0.0	578	958	5.0		380.			
26	3	60	0.0	1488	1955	0.0	285.3	467.	5.00	16.67	868.6
	1	4	0.0	578	958	5.0		380.			
27	3	33	0.0	1475	1843	0.0	173.3	368.	35.34	16.67	800.0
	1	4	0.0	578	958	5.0		380.			
28	44	53	0.0	1486	1876	0.0	206.3	390.	13.34	16.67	800.0
	1	4	0.0	578	958	5.0		380.			
29	26	51	0.0	1501	1946	0.0	276.3	445.	5.00	16.67	846.6
	1	4	0.0	578	958	5.0		380.			
30	24	9	0.0	1515	1868	0.0	198.3	353.	50.34	16.67	800.0
	1	4	0.0	578	958	5.0		380.			
31	24	60	0.0	1426	1810	0.0	140.3	384.	19.34	16.67	800.0
	1	4	0.0	578	958	5.0		380.			
32	2[illegible]	54	0.0	1460	1945	0.0	275.3	485.	5.00	16.67	886.6
	[illegible]	4	0.0	578	958	5.0		380.			
33	5	55	0.0	1596	1965	0.0	295.3	369.	34.34	16.67	800.0
	1	4	0.0	578	958	5.0		380.			
34	5	57	0.0	1466	1851	0.0	181.3	385.	18.34	16.67	800.0

FIG.3. COMPUTER OUTPUT OF SPLIT RUN DEVELOPMENT
(TIMES IN HUNDRED OF HOURS)

SPLIT RUN SELECTION

Since the computer program in previous sections is designed in order to construct all possible split runs, a selection operation is required by the schedule maker. Such a selection would depend on the objective of the particular company, which sometimes might be governed or restricted by the labor management. As an example, some companies aim at reducing the number of trippers (very small pieces of work are called as trippers), others might have the objective of combining the large pieces of work first and then combining as many as possible of the left over pieces. Various methods have been found for selecting the split runs from the output of Section 2. For this experiment, the computer approach for the selection of split runs has been adopted. The computer approach of selection is based on the total pay time of each split run. The total pay time of a split run includes all the allowances, spread penalty, overtime penalty, and the trip time of the constituting pieces of work.

The input data prepared for this selection consists of cards punched out for each split run from the previous section having the information as the run number, route numbers, block numbers, of both the pieces of work making the run and the total pay time of the split run. First, the machine is instructed to arrange the cards in the ascending order of the total pay time (first card for minimum pay time, second card for little more time and subsequently, they are arranged in the increasing time order). Then the machine picks up the first card and compares the route numbers and block numbers of the pieces of work for this run with the route numbers and block numbers of pieces of work of other runs. If the same route number and block number are on any of the cards it will reject that card and read the next one. It thus goes on eliminating other runs and finally makes a total selection of split runs in which no piece of work is used more than once. Then it punches out the selection output according to a prescribed format giving route numbers and block numbers and total pay time for each split run selected.

Finally a complete list of the various split runs is then given when the minimum total pay time is selected.

SELECTION OF PIECES OF WORK NOT USED FOR SPLIT RUNS

After making the selection of split runs it becomes necessary to eliminate the pieces of work used for making the split runs from the total pieces of work used for Section 2. The main idea behind this process of elimination of pieces of work used is to allocate the penalty for the pieces which could not be used in Section 2. This is done in the following steps by making use of the computer.

(1) The output of "Split Run Selection," is stored in the machine.

(2) Then all the pieces of work from Section 2 are read by the machine one by one, and each piece of work read by the machine is compared with the pieces of work which are used for selected split runs stored in the machine's memory. If the piece of work read by the machine is not used for the split runs, then it is punched out. Otherwise, it reads another piece of work and thus again compares it with the pieces of work used for split runs.

(3) The Steps 1 and 2 are repeated until all the pieces of work are read and compared with pieces of work of split runs.

The output is punched in the same format as of the piece of work of Section 2.

ALLOCATING PENALTIES TO THE UNUSED PIECES OF WORK

The pieces of work which are not used for the development of straight runs are then allotted a penalty for the purpose of calculating total pay time of the company for a particular division. The system of assigning a penalty for the pieces of work not used differs from one company to another. An usual system of assigning a penalty adopted for carrying out this research is given below:

PLATFORM TIME OF PIECE OF WORK		PENALTY
BETWEEN	190. Min - 299 Min	4 hours
	300. Min - 499 Min.	6 hours
	500. Min - Above	8 hours

By adopting the above penalty system, the total time for the pieces of work not used is then calculated.

Generally, the trippers (pieces of work having platform time less than 190. Min) are not assigned any penalty by the transit companies. So tripper times are just added together which will give total tripper time for the complete division.

The total time for which the company makes payment to the operators daily will consist of:

1. Straight run pay time.
2. Split run pay time (including spread and overtime penalties).
3. Penalty for pieces of work not used for split runs.
4. Total tripper time.

DESIGN OF EXPERIMENT

Here we are concerned with the establishment of the mathematical or statistical relationship existing between a number of economic variables. A model or hypothesis as to the assumed relationships between the economic variables is to be constructed. Then, economic measurements are applied to each variable and the degree of relationship is determined. There are various constraints and restrictions in labor management agreements of public transit companies. There are always a few constraints like platform time and spread time in the agreement on which controversy is always going on between the trade union and the management. While conducting this experiment for predicting total pay time of operators in public transit companies it was felt that platform and spread time are the two main deterministic variables. Here we are establishing the likelihood that these two variables have a relationship to the total pay time. So, the model building for the prediction of total pay time seeks the basic pay time determinants such as platform time and spread time.

In Step 1 of straight run development there is a restriction that time and one-half is paid as overtime for all work beyond eight hours. Also, in Step 2 of split runs development there is a restriction that the spread time must not exceed fourteen hours. So the two constraints which are varied for this experiment are spread time and platform time. The platform times are varied from 700 minutes to 900 minutes and the spread time is varied from 1150 mintues to 1450 minutes. The input data used for the various sets of combination of spread time and platform time is obtained from one of the public transit companies. The sequence of steps is as follows:

(1) Development of straight runs as done in Step 1 of Scheduling, introducing various platform time limits say 700 minutes, 725 minutes, 850 minutes and on.

(2) Selection of straight runs, trippers (pieces having platform time less than 190 minutes) and pieces of work.

(3) Development of split runs from the pieces of work as done in Step 2 of Scheduling by introducing various spread time limits say 1150 minutes, 1200 minutes, 1250 minutes, and so on.

(4) Selection of split runs on the basis of total pay time which mainly includes spread penalty, overtime penalty and the trip time of pieces of work.

(5) Making a list of the pieces of work not used for the development of split runs and assigning penalties to them.

(6) Making a list of trippers and adding their trip times.

Figure 4 gives the tabulated results for total pay time in detail for the various platform times and spread times. The total pay time for operators of a complete division consists of time of straight runs, time of split runs, penalty time for pieces not used for split runs and total tripper time.

Platform Time	Time of Sec. 1 (St.Runs)	(Time of Sec. 2) + (Penalty for Pieces Unused for Sec.2) For Various Spread Times					Tripper Time	Total Time of Schedule 1 & Schedule 2 Plus Tripper and Penalty of Pieces Unused For Various Spread Times				
Spread / Plat.		1150	1200	1300	1400	1450		1150	1200	1300	1400	1450
700	45375	62224	59792	56594	51762	52658	5090	112689	110257	107059	102227	103123
750	44084	60217	57867	54938	50519	50807	5683	109984	107634	104705	100286	100574
800	47276	60995	60644	44618	53713	52905	3971	112242	111891	105865	104906	104152
825	45789	61652	58857	55398	52053	51042	4704	112145	109350	105891	102546	101535
850	47521	62245	60403	56592	51425	51676	4253	114019	112117	108366	103199	103450
875	43441	63600	63575	57519	53856	53147	4957	111998	111973	105917	102254	101545
900	45541	62400	59521	57292	54034	52778	4754	112696	109817	107588	104330	103074

FIGURE 4

ACTUAL TOTAL PAY TIME FOR OPERATORS

(All times are expressed in hundreds of hours.)

MULTIVARIATE AND BIRARIATE CORRELATIONAL ANALYSIS

While conducting an experiment for predicting total pay time of operators in public transit companies it is felt that platform time and the spread time are the two deterministic variables. Here we are establishing the likelihood that these two variables have a relationship to the total pay time. The hypothesis that $Y = f(x_1, x_2)$, namely that Y is a function of x_1 and x_2 where,

Y = total pay time

x_1= platform time

x_2= spread time.

The data presented in Figure 4 for the total pay time (Y) together with two dependent variables, platform time (x_1) and spread time (x_2) is used for establishing correlational equations. Both spread time and platform time are expressed in minutes and fraction of minutes. The total pay time is expressed in hours and fraction of hours.

A Symbolic Programming System program has been used for finding the regression equation. The equation is developed in the multiplicative form and the coefficient of multiple determination (R^2) is also determined which enables one to know how close the estimated values are to the actual values.

The standard formula for R^2, taken from Nemmers (1962) is,

$$R^2 = 1 \left[\frac{\sum (d^2)}{\sum (Y^2) - (\sum Y/N)^2 . N} \right] \left[\frac{N-1}{N-M} \right]$$

$\sum$ = The sum of.

d = The deviation of the actual values from the estimated.

N = Number of observations.

Y = The actual values.

M = The number of constants in the multiple regression equation.

The first predicting equation is:

$$\log_e (Y) = 9.58359 + 0.05147 \log_e (x_1) + 0.41229 \log_e (x_2)$$

or

$$Y = e^{9.58359} \cdot (x_1)^{0.05147} \cdot (x_2)^{-0.41229} \quad (1)$$

where x_1 = Platform time in minutes.

x_2 = Spread time in minutes.

Y = Total pay time in hours and fraction of hours.

For the above equation R^2 is 0.87659 or $R = 0.94$ which shows that there is not much explained and unexplained variation. By putting the different values of x_1 and x_2 in the above equation Y's are computed which are tabulated in Figure 5, and graphs are plotted for Y against x_1 and Y against x_2, which shows that the total pay time decreases by about 10% for the spread time change from 1125 minutes to 1450 minutes. Also, it is seen that for change in platform time from 700 minutes to 900 minutes, the total pay time increases by about 1·5%.

PLAT / SPREAD	TOTAL PAY TIME FOR DIFFERENT PLAT TIME								
	700.	725.	750.	775.	800.	825.	850.	875.	900.
1125.	1123.	1125.	1127.	1129.	1131.	1133.	1134.	1136.	1138.
1150.	1113.	1115.	1117.	1119.	1121.	1123.	1124.	1126.	1128.
1175.	1103.	1105.	1107.	1109.	1111.	1113.	1115.	1116.	1118.
1200.	1094.	1096.	1098.	1100.	1101.	1103.	1105.	1107.	1108.
1225.	1084.	1087.	1089.	1090.	1092.	1094.	1096.	1097.	1099.
1250.	1076.	1078.	1079.	1081.	1083.	1085.	1086.	1088.	1090.
1275.	1067.	1069.	1071.	1073.	1074.	1076.	1078.	1079.	1081.
1300.	1058.	1060.	1062.	1064.	1066.	1067.	1069.	1071.	1072.
1325.	1050.	1052.	1054.	1056.	1057.	1059.	1061.	1062.	1064.
1350.	1042.	1044.	1046.	1047.	1049.	1051.	1053.	1054.	1056.
1375.	1034.	1036.	1038.	1040.	1041.	1043.	1044.	1046.	1048.
1400.	1027.	1028.	1030.	1032.	1034.	1036.	1037.	1038.	1040.
1425.	1019.	1021.	1023.	1024.	1026.	1028.	1029.	1031.	1032.
1450.	1012.	1014.	1015.	1017.	1019.	1020.	1022.	1024.	1025.

FIG. 5. PREDICTED TOTAL PAY TIME FROM EQ. 1.

(TOTAL PAY IN HRS.,PLAT AND SPREAD IN HUNDRED OF HRS.)

PLAT / SPREAD	TOTAL PAY TIME FOR DIFFERENT PLAT TIME								
	700.	725.	750.	775.	800.	825.	850.	875.	900.
1125.	1124.	1126.	1128.	1129.	1131.	1132.	1134.	1136.	1137.
1150.	1114.	1116.	1118.	1119.	1121.	1123.	1124.	1126.	1128.
1175.	1105.	1107.	1108.	1110.	1111.	1113.	1115.	1116.	1118.
1200.	1095.	1097.	1099.	1100.	1102.	1104.	1105.	1107.	1109.
1225.	1086.	1088.	1090.	1091.	1093.	1095.	1096.	1098.	1099.
1250.	1077.	1079.	1081.	1082.	1084.	1086.	1087.	1089.	1090.
1275.	1068.	1070.	1072.	1073.	1075.	1077.	1078.	1080.	1082.
1300.	1060.	1062.	1063.	1065.	1066.	1068.	1070.	1071.	1073.
1325.	1051.	1053.	1055.	1056.	1058.	1060.	1061.	11063.	1064.
1350.	1043.	1045.	1046.	1048.	1050.	1051.	1053.	1055.	1056.
1375.	1035.	1037.	1038.	1040.	1041.	1043.	1045.	1046.	1048.
1400.	1027.	1029.	1030.	1032.	1033.	1035.	1037.	1038.	1040.
1425.	1019.	1021.	1022.	1024.	1026.	1027.	1029.	1030.	1032.
1450.	1011.	1013.	1015.	1016.	1018.	1019.	1021.	1022.	1024.

FIG. 6. PREDICTED TOTAL PAY TIME FROM EQ. 2.

(TOTAL PAY IN HRS.,PLAT AND SPREAD IN HUNDRED OF HRS.)

The previous equation for the total pay times expressed as a function of spread time and platform time is in the multiplicative form and it gave a coefficient of multiplicative determination of 0.877. Then an attempt was made to change the form of equation expressing it as a function of the same variables so that the value of R^2 may become greater than the previous value. The following is the second form of the predicting equation for the total pay time in terms of the two determinants platform time and spread time.

$$Y = 4207.75112 - 445\text{-}39419 \log_e (x_2) + 0.06529 (x_1) \qquad (2)$$

where Y = Total pay time in hours and fraction of hours.

x_1= Platform time in minutes.

x_2= Spread time in minutes.

For the above equation (2) the coefficient of multiplicative determination (R^2) is 0.903 or $R = 0.95$. It can thus be concluded that a better correlation of spread, plat and total pay time is given by this equation. From this above equation, Y's are calculated for different values of x_1 and x_2 and then they are tabulated in Figure 6. The graphs are also plotted for Y against x_1 and Y against x_2. It is seen that total pay decreases by about 9·8% for change in spread time from 1125 minutes to 1450 minutes. Also, total pay increases by about 12% for a change in platform time from 700 minutes to 900 minutes.

On analyzing both the equations which give total pay time as a function of platform time and spread time it is noticed that total pay time is not as significantly affected by platform time as it is by spread time. So for each platform time the total pay time is expressed as a linear function of spread time and the coefficient of multiplicative determination is determined. The results are summarized as follows:

Platform Time (Minutes)	Equation Y = Total Pay Time (Hr.s) x_2= Spread Time (Mins.)	R^2
700	$Y = 1519 - 0.346\ x_2$	0.983000
750	$Y = 1476 - 0.331\ x_2$	0.997030
800	$Y = 1458 - 0.293\ x_2$	0.996660
825	$Y = 1517 - 0.349\ x_2$	0.995060
850	$Y = 1580 - 0.383\ x_2$	0.999631
875	$Y = 1533 - 0.358\ x_2$	0.999495
900	$Y = 1473 - 0.306\ x_2$	0.995820

To confirm statistically that platform time does not effect total pay time as significantly as spread time, two-way analysis of variance is tested out in the next section.

TWO-WAY ANALYSIS OF VARIANCE

What is here to investigate is the effect of K, the different platform time restrictions, and n, the different spread restrictions on the total pay time. Using notation introduced in Freund (1963), let the "Plats" be A_i for $i = 1, 2, \ldots, k$ the spreads be B_j for $j = 1, 2, \ldots n$. Suppose that total pay time is found for each possible combination of plats and spread, that the (n, k) different pay times are randomized, and that x_{ij} is the pay time obtained with the i^{th} plat and j^{th} spread. A possible model for this kind of problem is to look upon the x_{ij} as values assumed by independent random variables having normal distributions with the means μ_{ij} and variance σ^2, where

$$\mu_{ij} = \mu + \alpha_i + \beta_j \qquad (3)$$

and $$\sum_{i=1}^{k} \alpha_i = 0 \quad \text{and} \sum_{j=1}^{n} \beta_j = 0$$

where α_i is the effect of i^{th} plat and β_j is the effect of j^{th} we could also specify these assumptions by writing spread,

$$x_{ij} = \mu + \alpha_i + \beta_j + e_{ij} \text{ for } \begin{matrix} i=1,2,\ldots k \\ j=1,2,\ldots n \end{matrix} \qquad (4)$$

where e_{ij} are values assumed by independent random variables having normal distributions with 0 means and the common variance σ^2. It is to be seen that in this model the effects of the two variables, that is, the α_i and β_j, are added to μ.

The null hypotheses we shall be testing are:

(1) That the α_i are all equal to 0, and

(2) That the β_j are all equal to 0, the corresponding alternative hypotheses are that the respective parameters are not all equal to 0. The tests of these hypotheses are based on the following analysis of the total variability of the data, decomposing it into terms attributed to differences among the A's, differences among the B's, and chance (experimental error):

$$\sum_{i=1}^{k} \sum_{j=1}^{n} (x_{ij} - \bar{x})^2 = n \sum_{i=1}^{k} (\bar{x}_{i\cdot} - \bar{x})^2 + k \sum_{j=1}^{n} (\bar{x}_{\cdot j} - \bar{x})^2$$
$$\sum_{i=1}^{k} \sum_{j=1}^{n} (x_{ij} - \bar{x}_{i\cdot} - \bar{x}_{\cdot j} + \bar{x})^2 \tag{5}$$

where

$$\bar{x}_{\cdot j} = \frac{1}{k} \left(\sum_{i=1}^{k} x_{ij} \right)$$

$$\bar{x}_{i\cdot} = \frac{1}{n} \left(\sum_{j=1}^{n} x_{ij} \right)$$

$$\bar{x} = \frac{1}{k.n} \left(\sum_{i=1}^{k} \sum_{j=1}^{n} x_{ij} \right)$$

Equation (5) may be written as,

SST = SSA + SSB + SSE

where

SST = Total Sum of squares.

SSA = Sample sum of squares for variable A.

SSB = Sample sum of squares for variable B.

SSE = Error sum of squares.

It is seen that if the null hypothesis concerning the α_i is true, then SSA/σ^2 and SSE/σ^2 are values assumed by independent random variables having chi-square distributions with k-1 and (n-1) and (k-1) degrees of freedom; if this null hypothesis is not true, then SSA can be attributed, at least in part, to differences among the A's, that is differences among the platform time. Similarly, if the null hypothesis concerning the β_j is true, it can be seen that SSB/σ^2 and SSE/σ^2 are values assumed by independent random variables having chi-square distributions with (n-1) and (n-1)(k-1) degrees of freedom; if this null hypothesis is not true, then SSB can be attributed, at least in part, to differences among the B's, that is, differences among the spread. If both of the null hypothesis are true, it is seen, furthermore, that SST/σ^2 is a value assumed by a random variable having a chi-square distribution with (nk-1) degrees of freedom:

In accordance with the following theorem:

"If x_1 and x_2 are independent random variables having chi-square distributions with v_1 and v_2 degrees of freedom, then;

$$Y = \frac{x_1/v_1}{x_2/v_2}$$

has an F distribution with v_1 and v_2 degrees of freedom.

The test of the null hypothesis concerning the α_i can be based on the static:

$$F_A = \frac{SSA/(k-1)}{SSE/(n-1)(k-1)} = \frac{(n-1)(SSA)}{SSE} \quad \ldots, \qquad (6)$$

which, under the null hypothesis that the α_i and all equal to 0, is a value assumed by a random variable having the F distribution with k-1 and (n-1)(k-1) degrees of freedom. We reject this null hypothesis if F_A is greater than or equal to the critical value given in Table VI b of Freund (1963).

Similarly, the test of the null hypothesis that the β_j are all equal to 0 can be based on the static,

$$F_B = F_B = \frac{SSB/(n-1)}{SSE/(n-1)(k-1)} = \frac{(k-1)\ SSB}{SSE} \tag{7}$$

which, under the null hypothesis that the β_j are all to 0, is a value assumed by a random variable having the F distribution with n-1 and (n-1)(k-1) degrees of freedom. We reject this null hypothesis if F_B is greater than or equal to the critical value given in Table VI b of Freund (1963).

The analysis of variance table for this kind of a two-way analysis is usually presented in the following fashion:

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F
Between A's	k-1	SSA	$MSA=\frac{SSA}{k-1}$	$\frac{MSA}{MSE}$
Between B's	n-1	SSB	$MSB=\frac{SSB}{n-1}$	$\frac{MSB}{MSE}$
Error	(n-1)(k-1)	SSE	$MSE=\frac{SSE}{(n-1)(k-1)}$	
Total	nk-1	SST		

FIGURE 7

ANALYSIS OF VARIANCE TABLE

Here the mean squares are again the sums of squares divided by the respective degrees of freedom.

The experiment designed to test seven different spreads and five different plats yielded the results of total pay time shown in the following table: (Total plat is in hours.)

Spread / Plat	1150	1200	1300	1400	1425	$\sum_{j=1}^{5} x_{ij}$	
700	1127	1102	1070	1022	1031	5352	
750	1100	1076	1047	1003	1006	5232	
800	1128	1114	1074	1047	1037	5400	
825	1121	1093	1059	1025	1015	5313	
850	1140	1122	1083	1032	1034	5417	
875	1120	1120	1059	1022	1015	5336	
$\sum_{i=j}^{7} x_{ij}$	7863	7725	7468	7194	7159	37425	$\sum_{j=1}^{5}\sum_{i=1}^{7} x_{ij}$

FIGURE 8

ACTUAL TOTAL PAY TIME

Using the modified form of formula (5) we get,

$$SST = \sum_{i=1}^{k} \sum_{j=1}^{n} x_{ij}^2 - \frac{1}{k \cdot n} \left[\sum_{i=1}^{k} \sum_{j=1}^{n} x_{ij} \right]^2$$

$$= \sum_{i=1}^{7} \sum_{j=1}^{5} x_{ij}^2 - \frac{1}{(7)(5)} \left[\sum_{i=1}^{7} \sum_{j=1}^{5} x_{ij} \right]^2$$

$$= 4066645 - \frac{1}{35} \cdot (1400630625)$$

$$= 4066645 - 40018018$$

$$= 48627$$

$$SSA = \frac{1}{n} \sum_{i=1}^{k} \left[\sum_{j=1}^{n} x_{ij} \right]^2 - \frac{1}{k \cdot n} \left[\sum_{i=1}^{k} \sum_{j=1}^{n} \right]^2$$

$$= \frac{1}{5} \sum_{i=1}^{7} \left[\sum_{j=1}^{5} x_{ij} \right]^2 - \frac{1}{35} \left[\sum_{i=1}^{7} \sum_{j=1}^{5} x_{ij} \right]^2$$

$$= \frac{1}{5} (200113107) - \frac{1}{35} (1400630625)$$

$$= 40022621 - 40018018$$

$$= 4603.$$

$$SSB = \frac{1}{k} \sum_{j=1}^{n} \left[\sum_{i=1}^{k} x_{ij} \right]^2 - \frac{1}{k \cdot n} \left[\sum_{i=1}^{k} \sum_{j=1}^{n} x_{ij} \right]^2$$

$$= \frac{1}{7} \sum_{j=1}^{5} \left[\sum_{i=1}^{7} x_{ij} \right]^2 - \frac{1}{35} \left[\sum_{i=1}^{7} \sum_{j=1}^{5} x_{ij} \right]^2$$

$$= 40039762 - 40018018$$

$$= 21744$$

and $SSE = SST - SSA - SSB$

$$= 48627 - 4603 - 21744$$

$$= 22280.$$

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F
Between A's	6	4603	767.1	0.827
Between B's	4	21744	5436	5.85
Error	24	22280	928	
Total	34	48267		

FIGURE 9

COMPUTATION FOR ANALYSIS OF VARIANCE

Critical values of the static F for Table VI b of Freund (1963) are:

$F_{.01,6,24} = 3.67$

$F_{.0.,4,24} = 4.22$

Since $F_A = 0{\cdot}827$ which is less than $F_{.01,6,24}$ and $F_B = 5{\cdot}85$, much greater than $F_{.01,4,24}$, the null hypothesis for A's cannot be rejected, but the null hypothesis for the B's is rejected at the .01 level of significance. We may conclude that the total pay time is not much affected by platform restrictions and is significantly affected by different spread restrictions.

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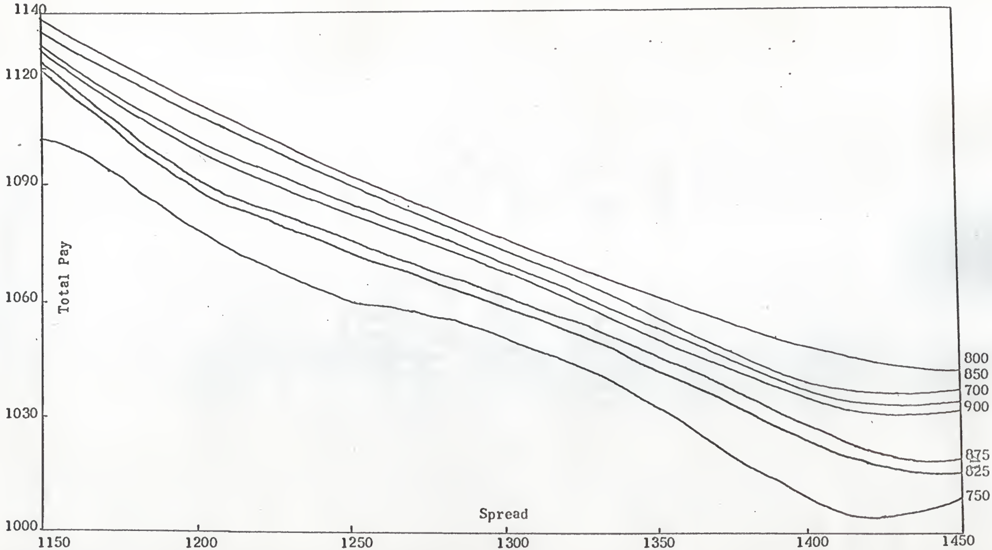
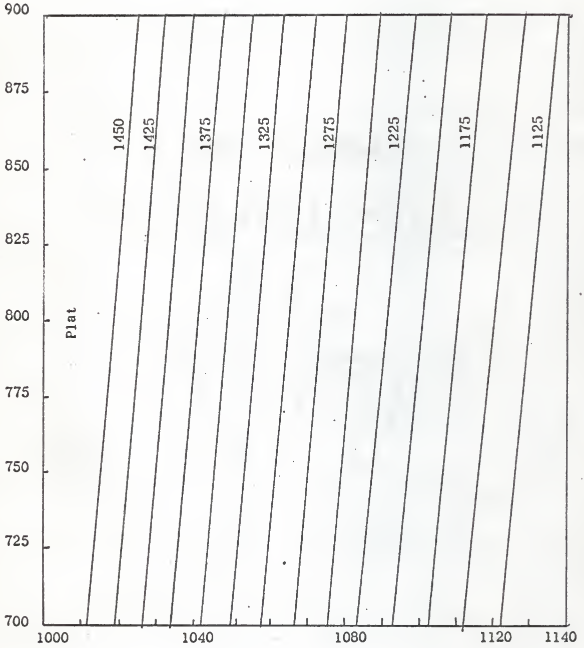
APPENDIX



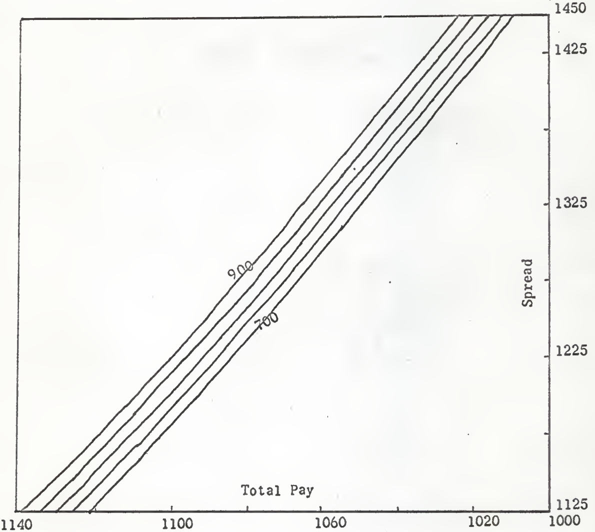
GRAPH 1

ACTUAL PAY AND SPREAD FOR VARIOUS PLATS



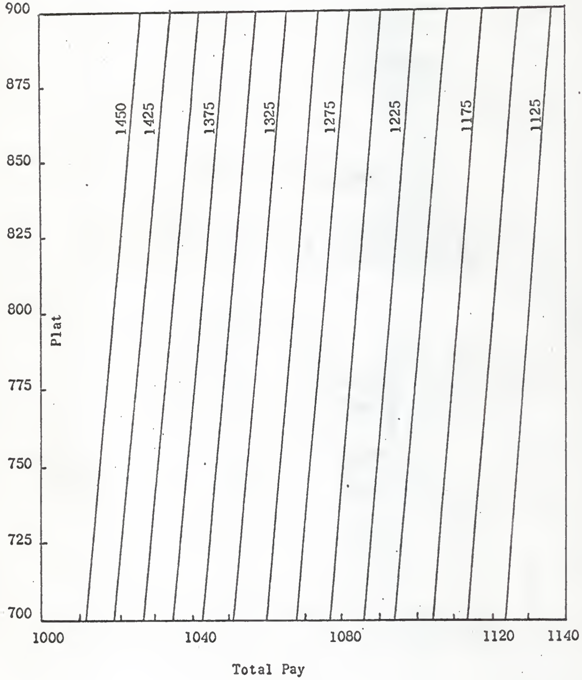
GRAPH 2

BETWEEN TOTAL PAY FROM EQUATION 1 AND PLAT



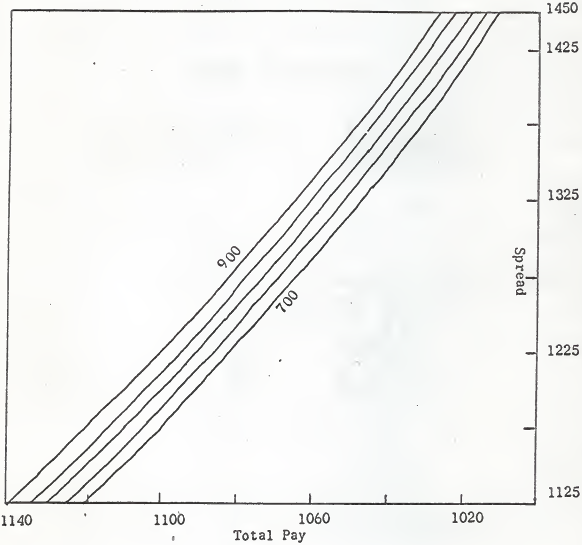
GRAPH 3

BETWEEN TOTAL PAY FROM EQUATION 1 AND SPREAD FOR VARIOUS PLATS



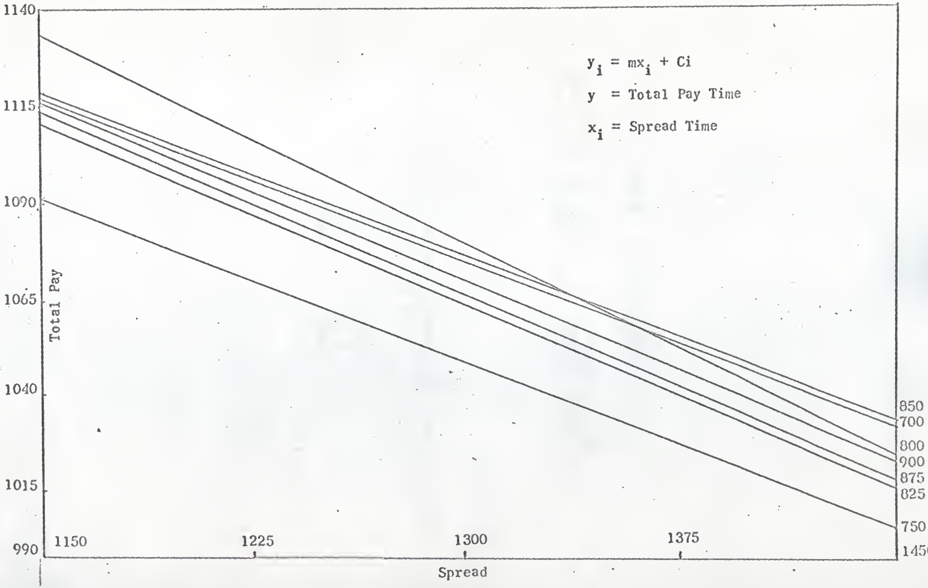
GRAPH 4

TOTAL PAY FROM EQUATION 2 versus PLAT



GRAPH 5

TOTAL PAY FROM EQUATION 2 AND SPREAD FOR VARIOUS PLATS



GRAPH 6

TOTAL PAY versus SPREAD FOR VARIOUS PLATS

```
      PROGRAM NO. 1

      CONVERSION PROGRAM FOR HOURS MINUTES TO HOURS DECIMALS
      DIMENSION REL(100)
 1111 READ1112,TRAVEL
 1112 FORMAT(F2.0)
      TRAVEL=TIME(TRAVEL,1.667)
  111 N1=1
      N2=13
      READ1,LINE,NOTRIP,POUT,PIN,(REL(I),I=N1,N2)
    1 FORMAT(2I2,15F5.0)
      IF(LINE)100,100,2
    2 IF(REL(N2))3,4,3
    3 IF(REL(N2)-PIN)5,6,5
    5 N1=1+N2
      N2=N2+15
      READ1,LINE,NOTRIP,(REL(I),I=N1,N2)
      GO TO 2
    4 DO 7 I=1,N2
      IF(REL(I))7,8,7
    7 CONTINUE
    8 I=I-1
      GO TO 9
    6 I=N2
    9 DO 10 J=1,I
   10 REL(J)=TIME(REL(J),1.667)
      POUTC=TIME(POUT,1.667)
      POINC=TIME(PIN, 1.667)
   17 PUNCH18,I,TRAVEL,LINE,NOTRIP,POUTC,POINC,(REL(J),J=1,I)
       GO TO 111
  100 PAUSE
      GO TO 1111
   18 FORMAT(I3,F5.1,2I3,11F6.0/26X,9F6.0/26X,9F6.0/26X,9F6.0/26X,9F6.0/
     126X,9F6.0/26X,9F6.0)
      END
** TIME FUNCTION
      FUNCTION TIME(X,Y)
      IF(X)1,2,2
    1 X=-X+1200.
    2 IX=X/100.
      HR=IX*100
      TIME=(X-HR)*Y+HR
      RETURN
      END
```

PROGRAM NO. 2.

STRAIGHT RUN MAKING PROGRAM

```
      DIMENSION  REL(100)
    1 K=0
    2 TURN=8.33
      TF=0
      TB=0
  201 REL(I)=0
   10 DO 201 I=1,100
      PUNCH 202
  202 FORMAT(//)
      READ11,M,TRAVEL,ROUTE,BLOCK,TIMON,TIMOFF,(REL(L),L=1,M)
   14 BACK=0
      FRONT=0
      POUT=TIMON
      POIN=TIMOFF
   18 FIGHT=600.-TURN
   20 ALLOW=0
   21 IF(TIMOFF-TIMON-600.)24,24,22
   22 BACK=1
   24 IF(TIMOFF-TIMON)10,10,25
   25 TRIPT=TIMOFF-TIMON
   26 IF(TIMON-POUT)27,28,27
   27 TF=TRAVEL
   28 IF(TIMOFF-POIN)29,30,29
   29 TB=TRAVEL
   30 IF(TRIPT-600.)31,136,136
  136 IF(TRIPT-600.)36,36,150
  150 IF(FRONT)9000,50,200
   31 IF(TRIPT-190.)331,331,431
  331 PUNCH 332,ROUTE,BLOCK,TF,TIMON,TIMOFF,TB,TRIPT
  332 FORMAT(9X,2I4,F5.1,2I6,F5.1,5X,I6,29X,1H-)
      GO TO 33
  431 PUNCH32,ROUTE,BLOCK,TF,TIMON,TIMOFF,TB,TRIPT
   33 TF=0
      TB=0
      IF(BACK)9000,10,100
   36 IF(TIMON-POUT)37,38,37
   37 TF=TRAVEL
   38 IF(TIMOFF-POIN)39,40,39
   39 TB=TRAVEL
   40 ALLOW=TF+TB
      TOTAL=TRIPT+TURN+TF+TB
   42 PUNCH43,K,ROUTE,BLOCK,TF,TIMON,TIMOFF,TB,TRIPT,ALLOW,TURN,TOTAL
      TF=0
      TURN2=16.67
      K=K+1
      TB=0
      IF(BACK)100,10,100
      FRONT=1
   51 DO 53 L=1,M
   50 PLAT=TIMON+600.-TRAVEL-TURN
```

```
52 IF(PLAT-REL(L))65,54,53
53 CONTINUE
   IF(L-M)53,31,31
54 TB=TRAVEL
55 TOTAL=REL(L)-TIMON+TURN+TRAVEL
56 ALLOW=TF+TB
   TRIPT=REL(L)-TIMON

   K=K+1
59 PUNCH43,K,ROUTE,BLOCK,TF,TIMON,REL(L),TB,TRIPT,ALLOW,TURN,TOTAL
   FRONT=1
   TF=0
   TB=0
   TIMON=REL(L)
63 EIGHT=600.-TURN-TRAVEL
   GO TO 24
65 PWO=(REL(L)-PLAT)*.5
66 PWP=PLAT-REL(L-1)
67 IF(PWO-PWP)68,68,74
68 K=K+1
   TB=TRAVEL
   ALLOW=TF+TB
   TOTAL=REL(L)-TIMON+TURN+TF+TB
   TRIPT=REL(L)-TIMON
   GO TO 59
74 K=K+1
   TB=TRAVEL
   ALLOW=TF+TB+PWP
   TOTAL=REL(L-1)-TIMON+TURN+ALLOW
   TRIPT=REL(L-1)-TIMON
   PUNCH43,K,ROUTE,BLOCK,TF,TIMON,REL(L-1),TB,TRIPT,ALLOW,TURN,TOTAL
   TIMON=REL(L-1)
   TB=0
   FRONT=1
   GO TO 63
99 TE=TRAVEL
100 TIMON=POUT
   TIMOFF=POIN
200 PLAT=TIMOFF-600.+TRAVEL+TURN
101 DO 103 L=1,M
   IF(PLAT-REL(L))121,104,103
103 CONTINUE
   IF(L-M)103,31,31
104 TF=TRAVEL
   ALLOW=TF+TB
   TOTAL=600.
   K=K+1
   TRIPT=TIMOFF-REL(L)
109 PUNCH 43,K,ROUTE,BLOCK,TF,REL(L),TIMOFF,TB,TRIPT,ALLOW,TURN,TOTAL
   FRONT=0
111 TF=0
113 TIMOFF=REL(L)
```

```
114 TIME=TIMOFF-TIMON
115 IF(TIME)9000,10,116
116 IF(TIME-(600.-TRAVEL-TRAVEL))117,119,199
199  TB=TRAVEL
    GO TO 50
117 BACK=0
    GO TO 25
119 BACK=0
120 GO TO 36
121 PWO=(PLAT-REL(L-1))*.5
122 PWP=REL(L)-PLAT
123 IF(PWO-PWP)124,124,134
124 TF=TRAVEL
    ALLOW=TF+TB
    TOTAL=TIMOFF-REL(L-1)+TURN+ALLOW
    K=K+1
    TRIPT=TIMOFF-REL(L-1)
    PUNCH 43,K,ROUTE,BLOCK,TF,REL(L-1),TIMOFF,TB,TRIPT,ALLOW,TURN,TOTA
   1L
    FRONT=0
    TF=0
    TIMOFF=REL(L-1)
    GO TO 114
134 TF=TRAVEL
    ALLOW=TF+TB+PWP
    TOTAL=TIMOFF-REL(L)+TURN+ALLOW
    K=K+1
    TRIPT=TIMOFF-REL(L)
    GO TO 109
9000 STOP
 32  FORMAT(9X,2I4,F5.1,2I6,F5.1,5X,I6)
 43 FORMAT(I3,6X,2I4,F5.1,2I6,F5.1,5X,I6,2F6.1,F7.0)
 11 FORMAT(I3,F5.1,2F3.0,11F6.0/26X,9F6.0/26X,9F6.0/26X,9F6.0/26X,9F6.
   10/26X,9F6.0)
    END
```

PROGRAM NO. 3

A SPLIT RUN DEVELOPMENT PROGRAM

```
      DIMENSION ROUTE(200),BLOCK(200),TF(200),TIMON(200),TIMOFF(200),TB(
     1200),TRIPT(200)
    1 READ2,IPW
    2 FORMAT(3XI3)
      N=0
    3 TURN=8.33
    4 FIVE=8.33
      M=0
    6 DO 9 I=1,IPW
    9 CONTINUE
   10  L=1
   11 DO 1000 J=1,IPW
    7  READ8,ROUTE(I),BLOCK(I),TF(I),TIMON(I),TIMOFF(I),TB(I),TRIPT(I)
   12  PENLT1=0
      PFNLT2=0
      PENLT3=0
   16 IF(TIMON(J)-TIMON(L))1000,1000,17
   17 IF(TIMOFF(J)-TIMOFF(L))1000,1000,18
   18 IF(TIMON(J)-TIMOFF(L))1000,1000,19
   19 GAP=TIMON(J)-TIMOFF(L)
   20 BREAK=TB(L)+TF(J)+FIVE
   21 IF(GAP-BREAK)1000,22,22
   22 IF(GAP-100.)500,23,23
   23 SPREAD=TIMOFF(J)-TIMON(L)+TF(L)+TB(J)+TURN
   24 IF(1400.-SPREAD)1000,25,124
  124 IF(SPREAD-1100.)26,26,25
   25 PENLT3=(SPREAD-1100.)
   26 PW1=TRIPT(L)+TF(L)+TB(L)
   27 PW2=TRIPT(J)+TF(J)+TB(J)
   28 IF(PW1-100.)29,31,31
   29 PENLT1=100.-PW1
   30 PW1=100.
   31 IF(PW2-100.)32,34,34
   32 PENLT2=100.-PW2
   33 PW2=100.
   34 TOTAL=PW1+PW2+2.*TURN
   35 IF(TOTAL-750.)1000,36,45
   36 PENLTY=50.
   37 PAYTIM=TOTAL+PENLTY
      ALLOW=PENLTY+PENLT1+PENLT2+ TF(L)+TB(J) +TB(L)+TF(J)
      M=M+1
   40 PUNCH41,ROUTE(L),BLOCK(L), TF(L),TIMON(L),TIMOFF(L),TB(L),TRIPT(L)
   42 PUNCH43,M,ROUTE(J),BLOCK(J),TF(J),TIMON(J),TIMOFF(J),TB(J),PENLT3,
     1TRIPT(J),ALLOW,TURN2,PAYTIM
        GO TO 1000
   45 IF(800.-TOTAL)50,46,46
   46 PENLTY=800.-TOTAL
   47 PAYTIM=PENLTY+TOTAL
   48 ALLOW=PENLTY+PENLT1+PENLT2+TF(L)+TB(L)+TF(J)+TB(J)
   49 M=M+1
```

```
      GO TO 40
   50 IF(900.-TOTAL)1000,51,51
   51 PENLTY=0
      GO TO 47
  500 SUM=TRIPT(L)+TRIPT(J)+TF(L)+TB(J)+TURN+GAP
  501 IF(SUM-750.)1000,502,511
  502 ALLOW=GAP-TB(L)-TF(J)+800.-SUM
  503 N=N+1
      TOTAL=SUM-TURN
      PAYTIM=800.
  505 BLOCKX=BLOCK(L)*100.+BLOCK(J)
      ROUTEX=ROUTE(L)*100.+ROUTE(J)
      PUNCH509,N,ROUTEX,BLOCKX,TF(L),TIMON(L),TIMOFF(J),TB(J),TOTAL,ALLO
     1W,TURN,PAYTIM
       GO TO 1000
  511 IF(SUM-800.)502,502,512
  512 IF(SUM-900.)513,513,1000
  513 ALLOW=GAP-TB(L)-TF(J)
      N=N+1
      TOTAL=SUM-TURN
      PAYTIM=SUM
      GO TO 505
 1000 CONTINUE
      L=L+1
      IF(L-IPW)11,11,1005
 1005 STOP
    8 FORMAT(9X,2I4,F5.1,2F6.0,F5.1,5X,F6.0)
   41 FORMAT(3X,2I5,F5.1,2I5,F5.1,7X,F6.0)
  509 FORMAT(I3,2I5,F5.1, 2I5,F5.1,7X,  F6.0,2F7.2,F7.1)
   43 FORMAT(I3,2I5,F5.1, 2I5,F5.1,F7.1,F6.0,2F7.2,F7.1)
      END
```

PROGRAM NO. 4

TOTAL PAY PROGRAM

```
101 OVER=0.
    PENLTY=0
103 READ1,ROUT1,BLOCK1
    IF(ROUT1-100.)102,103,103
102 READ2,N,ROUT2,BLOCK2,PENLT,PAY
    BLOX=ROUT1*100.+BLOCK1
    ROUX=ROUT2*100.+BLOCK2
    IF(PAY-800.)15,9,10
 10 OVER=(PAY-800.)*.5
  9 IF(PENLT)15,14,11
 14 PENLTY=0
    GO TO 12
 11 PENLTY=PENLT*.5
 12 PAYX=OVER+PENLTY+PAY
    PUNCH3,N,BLOX,ROUX,PAYX
    GO TO 101
  1 FORMAT (3X,2I5)
  2 FORMAT(I3,2I5,20X,I5,22X,F7.1)
  3 FORMAT(I4,2I6,F7.1)
 15 STOP
    END
```

PROGRAM NO. 5

A SELECTION PROGRAM

```
      DIMENSION BLOK1(1000),ROUT1(1000),PAY(1000),BLOKX(75),ROUTX(75),P
     1YX(75)
      READ100,NL
  100 FORMAT(I5)
      DO22I=1,NL
      READ1,BLOK1(I),ROUT1(I),PAY(I)
    1 FORMAT(3I6)
   22 CONTINUE
  444 READ14,N
   14 FORMAT(I3)
      DO333 L=1,N
      MX=0
      CHECK=0.0
      DO4 J=1,NL
      IF(BLOK1(L)-BLOK1(J))2,4,2
    2 IF(ROUT1(L)-ROUT1(J))44,4,44
   44 IF(CHECK)15,55,6
    6 DO5 M=1,JK
      IF(ROUT1(J)-ROUTX(M))77,4,77
   77 IF(ROUT1(J)-BLOKX(M))5,4,5
    5 CONTINUE
      DO7 M=1,JK
      IF(BLOK1(J)-BLOKX(M))88,4,88
   88 IF(BLOK1(J)-ROUTX(M))7,4,7
    7 CONTINUE
   55 MX=MX+1
   66 BLOKX(MX)=BLOK1(J)
      ROUTX(MX)=ROUT1(J)
      PAYX(MX)=PAY(J)
      JK=MX
      CHECK=1.0
    4 CONTINUE
      PUNCH12,BLOK1(L),ROUT1(L),PAY(L)
      IF(MX)15,15,34
   34 PUNCH12,(BLOKX(MX),ROUTX(MX),PAYX(MX), MX=1,JK)
   12 FORMAT(20X,3I5,45X)
      PUNCH 16
   16 FORMAT(///)
  333 CONTINUE
      GO TO 444
   15 STOP
      END
```

PROGRAM NO. 6

A SELECTION OF UNUSED PIECES PROGRAM

```
      DIMENSION BLOK(100),ROUT(100)
      READ 3,N
    3 FORMAT(I3)
      DO 5 I=1,N
      READ1,BLOK(I),ROUT(I)
    1 FORMAT(20X,2I5)
    5 CONTINUE
    7 READ8,  ROUTE,BLOCK,TF,TIMON,TIMOFF,TB,TRIPT
    8 FORMAT(9X,2I4,F5.1,2F6.0,F5.1,5X,F6.0)
      BLOK1=ROUTE*100.+BLOCK
      DO 6 J=1,N
      IF(BLOK1-BLOK(J))6,7,6
    6 CONTINUE
   20 DO9 K=1,N
      IF(BLOK1-ROUT(K))9,7,9
    9 CONTINUE
   21 PUNCH8, ROUTE,BLOCK,TF,TIMON,TIMOFF,TB,TRIPT
      GO TO 7
      END
```

PROGRAM NO. 7

A COMPUTATION OF VALUES PROGRAM FOR EQ. 2

```
   DIMENSION SPREAD(20)
 3 FORMAT(I3)
33 FORMAT(F4.0)
 2 FORMAT(F3.0)
 4 FORMAT(3F20.10)
   READ3,N
   DO1J=1,N
   READ33,SPREAD(J)
 1 CONTINUE
22 READ2,PLAT
   DO 44 J=1,N
   Y=4207.75112-445.39419*LOGF(SPREAD(J))+.06529*PLAT
   A=EXPF(Y)
   PUNCH4,Y,SPREAD(J),PLAT
44 CONTINUE
   GO TO 22
   END
```

PROGRAM NO. 8.

A COMPUTATION OF VALUES PROGRAM FOR EQ. 1

```
   DIMENSION SPREAD(20)
 3 FORMAT(I3)
33 FORMAT(F4.0)
 2 FORMAT(F3.0)
 4 FORMAT(3F20.10)
   READ3,N
   DO1J=1,N
   READ33,SPREAD(J)
 1 CONTINUE
22 READ2,PLAT
   DO 44 J=1,N
   Y=9.58359+.05147*LOGF(PLAT)-.41229*LOGF(SPREAD(J))
   A=EXPF(Y)
   PUNCH4,A,SPREAD(J),PLAT
44 CONTINUE
   GO TO 22
   END
```

DEVELOPING A MATHEMATICAL MODEL FOR PREDICTING TOTAL OPERATORS' WAGES IN PUBLIC TRANSIT

by

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AN ABSTRACT OF A MASTER'S THESIS

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The objective of this thesis is to develop a mathematical model for predicting the total pay to the operators of the public transit industry in general functional forms and parameter values. Economic measurements were applied to each variable and the degree of relationship was determined. There are usually several constraints and restrictions in labor management agreements of public transit companies. Platform time and spread time are the two main constraints on which controversy is always going on between management and trade unions. It is felt that platform time and spread time are the two main deterministic variables. These variables are used to establish a likelihood that they have a relationship to the total pay time.

By conducting a two-way analysis of variance test it is concluded that on the total pay time the affect of spread time is more significant than the affect of platform time. The computation was carried on IBM 1620 computer and the total pay time functions were plotted for each combination of parameter values.