WHAT IS MODERN ABOUT "MODERN" MATHEMATICS IN THE INTERMEDIATE GRADES?

by

NELDA ANNE FECTEAU

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Approved by:

Harry McAnarney
Major Professor

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INTRODUCTION

Times are changing and little remains the same. Clothes change style, prices rise, discoveries in science and medicine change our lives, even the methods of teaching are changing. In a changing society, it is only natural that people want the best and the latest of everything, including a modern approach in teaching. Teaching modern concepts in science, social studies, art, and even mathematics have developed.

> To be an intelligent citizen of tomorrow, today's pupil needs not only to learn to apply mathematics to problems in his present environment but also to develop skills and procedures which will enable him to solve new problems as they arise in our complex society. What these new problems will be no one knows, but it is certain that they will arise.[1]

Statement of Problem

The purposes of the report were (1) to explain the theories of learning as related to mathematics; (2) to present principles of the modern mathematic programs; (3) to present enthusiast's and critic's viewpoints of the modern mathematic programs in current use; and (4) to present the content of modern mathematic programs by use of (a) experimental programs, (b) textbooks, and (c) programed materials.

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[1]John L. Marks and others, <u>Teaching Elementary School Mathematics for Understanding</u> (St. Louis, Missouri: McGraw-Hill Book Company, 1965), p. 5.

Importance of Study

Parents often say, "I can't help my child with his arithmetic because it is taught differently than when I went to school." Parents, teachers, and students are often confused about what should be taught and how it can best be taught. Many teachers are already teaching the "new" mathematics and many others will soon.

Revised textbooks are being used that contain "new" mathematical concepts and terms not presented before in the intermediate grades. Problems have arisen about what concepts should be presented to certain grade levels, how to present new concepts, and what preparations are needed before modern mathematics can effectively be taught. The following report was compiled to clarify what was new or modern about mathematics, what type of materials were being presented to intermediate students, and how the modern programs had been accepted in schools already teaching modern mathematics.

Teachers and administrators should examine a wide variety of textbooks before selecting the type of mathematic's program to incorporate into their field of study. The following report summarizes for the reader some of the approaches to learning that have been tried and some of the "modern" concepts in mathematics that are being presented in experimental programs, textbooks, and programed materials in the intermediate grades.

THEORIES OF LEARNING

What is modern mathematics and how does it differ from the mathematics that has been taught? To better understand the basic principles and differences of modern mathematics and traditional mathematics, one should understand some of the theories of learning that have been tried.

Drill Theory

In the 1920's the drill approach to learning was considered "the way of teaching." Learning was mechanical with the theory. A student knew each step of a process, whether he understood the arithmetic principles and relationships involved in the process or not. Repeated practice of the basic facts of arithmetic was thought to be sufficient for mastery. Therefore, a student was drilled on the mathematical facts until he knew them. From this knowledge of facts the student should be able to work the problems.

Meaning Theory

The meaning approach to learning was popular during the 1930's. The theory probably developed because of inadequacy of the drill theory. Drill alone did not help a student with the problems. Using the meaning theory, a student would achieve mastery of ideas and skills if the practice of the skills was given after the understanding was developed. The theory recognized that practice or drill was necessary for mastery, but only after a student first understood what he was doing.

Bond Psychology

Edward L. Thorndike has been given credit for developing the bond psychology. The psychology states that learning takes place through the establishing of a bond or connection between a stimulus and a response. A complicated process or problem is broken up into a number of simple steps, each of which constitutes a bond.[2] These bonds are strengthened by repeated use. It then becomes the teacher's task to divide the problem into small parts and present the parts in hope that the entire problem could be solved.

Since the bonds or parts to be established could only be strengthened by use, teachers began to use drill as the technique to establish these bonds. It was assumed that if a student made a correct response, he then derived the correct answer and understood the process.

Gestalt Psychology

The Gestalt psychology required no needed drill but

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[2]David Rappaport, Understanding and Teaching Elementary School Mathematics (New York: John Wiley and Sons, Inc., 1966), pp. 5, 6.

only an insight into the discovery of relationships. The psychology had a student begin with the whole problem and then break it into parts. The student could look at the whole problem and then find relationships and parts of the problem. The process would help the student remember facts longer and enable him to apply the knowledge to a new situation.

> Learning . . . takes place through analysis, structure, restructure, patterns, and reorganization of a situation. The child begins with the whole and breaks it up into parts. He then related the parts to the whole and the whole to the parts. Every time there is a new analysis of the situation there is a better understanding because the relationship between the whole and the parts becomes clearer. Learning is a continual recognition of knowledge.[3]

The theories of learning discussed have been used, discarded, and sometimes brought back into use when other methods failed. Educators who are still not satisfied with results try many techniques, hoping to find a "cure all" for the problems in learning. Some students discover "why" a problem is solved in a certain way, but for many the discovery never occurs. The students do their problems in a mechanical way and rarely understand why a problem is solved in such a manner.

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[3] Ibid., pp. 6, 7.

DEVELOPMENT OF MODERN MATHEMATICS

The recruitment of young men for the armed forces during World War II revealed inadequacies in mathematical achievement. By the 1950's the schools were aware of weaknesses of mathematics education. It was not until the firing of the first Earth satellite, Sputnik I in 1957, however, that the revolution in mathematics education in the United States began. Computers and automation made people realize that mathematics was of crucial importance and needed reform.[4] People began to realize that there was now a need to know not only the old knowledge but students must be prepared for a world not yet known.

Principles of the Program

The modern approach to mathematics would be a better name for the principles of the new or modern mathematics. The modern approach stresses understanding the underlying principles of mathematics; "basic elementary mathematical concepts must be gained, structural features recognized, basic properties known, relationships realized, and rationale

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[4]Kenneth E. Brown and others, "The Lively Third R," American Education (June, 1966), p. 9.

of computation understood."[5] After the understanding of the necessary concepts, the approach also recognized the need for practice or drill to reinforce some of the ideas.

Four steps have emerged from the modern approach to teaching mathematics--readiness for the concept, formal introduction of desired concepts, mastery of skills required, and finally practical application of the concepts.[6]

> Children should learn to find the answers to specific questions, to discover patterns or generalizations in the rationale of mathematics, and be able to explain the _why_ of a process. These three levels of learning may be characterized as operation, generalization, and rationalization.[7]

The operational level is the simplest level in which concrete or semiconcrete manipulative aids such as drawing circles and making marks are used. The generalization level begins only after sufficient exercises so that a student may discover generalizations--he finds a tool or rule that works. The rationalization level is the highest level in which a student is asked to prove the generalization or to

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[5]Frances Flournoy, _Elementary School Mathematics_ (Washington, D. C.: The Center for Applied Research in Education, Inc., 1964), p. 1.

[6]Albert F. Kempf and Tom E. Barnhart, _Modern Elementary Mathematics: A Parents Guide to the New Concepts_ (New York: Doubleday and Company, 1965), p. ix.

[7]David Rappaport, _op. cit._, p. 9.

explain why something is true. This explanation is usually based on definitions and postulates of why a rule or method works.[8]

> Because mathematics is commonly regarded as a static science, whose truths are eternal and unchangeable, parents of school children and other thinking adults are naturally asking the question, "What is new in the new mathematics?" Unfortunately, this question does not have a simple answer, but a good beginning can be made by noting that mathematics is NOT a static science at all; it is a growing, vital subject that is expanding at an astounding rate. It has been estimated, for example, that more new mathematics has been discovered during the first half of the twentieth century than in all previous recorded history![9]

Carl O. Olson, Jr. feels that good teachers and mathematicians have known about and taught many aspects of the new mathematics for years. The content has only been redistributed within the school program with much content formerly taught in the high school being now introduced in the elementary school. "The terms 'distributive property,' 'associative property,' and 'closure' are new; the concepts they represent are not new."[10]

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[8] Ibid., pp. 9, 10.

[9] Ralph T. Heimer and Miriam S. Newman, The New Mathematics for Parents, (New York: Holt, Rinehart and Winston, 1965), p. 11.

[10] Carl O. Olson, Jr., "The New Programs Race," Education, (December, 1965), p. 222.

Whether the content of modern mathematics courses is "new" or not, the approach to teaching mathematics has changed.

> Modern mathematics curriculums called for new methods of teaching, a revised presentation of the subject, a unified approach to the various areas of mathematics, and a faster pace.
>
> One of the common elements in the new mathematics curriculum is an emphasis on the unifying theses or ideas in mathematics. This emphasis has resulted in bringing rather advanced mathematical concepts into secondary and even elementary school classes. For example, the introduction of the language and concepts of sets into the elementary grades has introduced children to symbols and words that are normally reserved for the college student.[11]

According to Heimer and Newman there are several misconceptions about the new mathematics that one must understand before fully realizing what the new mathematics is about.

> Another common misconception about the new mathematics is that the phrase "modern mathematics" is a misnomer, which has nothing to do with mathematics, *per se*, but refers rather to new methods of teaching mathematics.
>
> While it is true that methods of teaching mathematics have changed radically in recent years, there is much more to the new mathematics . . . The most widely known new technique of teaching the new mathematics is based on the principle of helping students determine mathematical truths on their own, called the discovery method, but this technique, too, is an outgrowth of the modern conception of mathematics.

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[11]Kenneth E. Brown, *op. cit.*, pp. 9, 10.

The last common misconception about the new mathematics to be considered here is that the phrase "modern mathematics" refers to nothing more than an accelerated mathematics program, for example, that in a modern mathematics program the study of algebra might be begun in the eighth rather than in the ninth grade. . . Simply examine a representative collection of the newer textbooks, many of which will probably contain subject matter that has been created in the past century, and you will see the incorrectness of this idea.

But, what then is the new mathematics? A mathematician might think of the creation of new mathematics during the past century and the certain prospect of a continued growth of mathematics in answer to this question. He might also think of the effect that this new mathematics has had on the older mathematics, or of the new applications of mathematics, especially the types of problems which could not be solved if the new mathematics had not been created. But one thought would certainly come to his mind: that of <u>structure</u> and <u>the role of deductive reasoning</u> in the construction of all mathematical systems. This is the central idea underlying the contemporary concept of the nature of mathematics.[12]

Controversies

Often when a new idea in education is developed the question arises, Is it really worthwhile trying, or is it just another fad that will come and go and be of little benefit to the students? There are mixed feelings about much of the modern mathematics taught in the intermediate grades. Large and small school systems are wondering

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[12]Ralph T. Heimer and Miriam S. Newman, *op. cit.*, pp. 107-109.

whether to teach the modern mathematical concepts, how much of the content the students can understand, and/or how to approach the teaching of modern mathematics.

> The new mathematics packs quite an emotional wallop. While proponents are debating its merits, teachers and parents are caught in the middle. That is to say, not enough teachers have been properly trained to teach it, and many parents are bewildered by the new terminology and concepts. Even the staunchest supporters of the new math deplore the fact that there is a deficiency in the learning of the fundamental operations of arithmetic. It may very well be a time for inquiring into why Johnny cannot do arithmetic.[13]

Enthusiasts

Many enthusiasts of modern mathematics point out that new problems and a mathematical revolution are taking place in research, science, military strategy, electronics, business, and everyday living that require a new and better knowledge of mathematics. There is much more to learn today in mathematics, and schools must begin earlier to teach the basic ideas of mathematics in order to have the competent doctors, engineers, scientists, and other professional people needed for the future.

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[13]Max Broder, "What About the New Mathematics?" High Points (January, 1966), p. 45.

> Experimental results tend to show that the essentials of the traditional content covered in the first eight grades can be learned by the end of grade six. This has been accomplished by starting the study of mathematics in the kindergarten and accelerating the rate of study in the first three grades. It is then possible to undertake more formal study of such topics as fractions, percentage, measures, and measurement in grades four, five, and six.[14]

Enthusiasts feel that the modern mathematics is of more interest to the student than the traditional mathematics. It is more exciting for the student to discover the fundamentals for himself and more meaningful.

Carl O. Olson, Jr. and Laidlaw Brothers have the following comments about the modern mathematics approach.

> Virtually no teacher having had a year of experience with a new math program would want to go back to a more traditional program simply because teachers are quick to see that they have known about modern math, at least intuitively, for years, that the new approach makes more sense, is more interesting to children and much more rewarding to teach.[15]
>
> The modern approach makes a determined effort to "humanize" mathematics and to make it a challenging and enjoyable subject to learn.
>
> The modern approach, rather than providing the students with ready-made problem-solving "tools" for ready-made problem situations--which may no longer exist tomorrow--gives them a set of basic "raw materials" and plans for fashioning them into all kinds of "tools", including those they may need ten years from now.

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[14]John L. Marks and others, op. cit., p. 9.

[15]Carl O. Olson, op. cit., p. 223.

> The modern approach, out to capture those students who have remained indifferent to the dictatorial JUST-SO approach, shows continuously that there are MANY DIFFERENT WAYS to reach the answer or conclusion; that there are CHOICES and, hence, decisions to be made; and that among the many choices there will be a very likely one just right for YOU! So go out, explore, and make a selection.
>
> The modern approach advocates from the very outset a clear and gradual developmental procedure from the concrete (objects, sets, numerals) to the abstract (numbers, properties of numbers, properties of the operations on numbers), using distinct terminologies for these two important levels of experience, minimizing the probability of confusion and opening the way to real understanding.[16]

Alex B. Crowder, Jr., assistant professor in the Department of Education at the Texas Technological College, felt that the key to the modern mathematics lies in the approach the teacher takes in teaching and not the subject matter.

> In traditional arithmetic, the teacher usually reverted to the lecture, or tell-all, method in which the mechanical aspects of the various operations were emphasized and explained.
>
> Modern math tends for there to be a shift of emphasis from the mechanical or <u>how</u> aspect alone to the understanding or <u>why</u> aspects as well.[17]

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[16] Laidlaw Brothers, "From Arithmetic to Elementary Mathematics," (LMS 115), pp. 4, 5.

[17] Alex B. Crowder, Jr., "Old Teaching Methods are the Pox on New Math," <u>The Texas Outlook</u> (April, 1967), p. 28.

Critics

Although many schools have some form of the modern mathematics, there are still a number of educators who feel that modern mathematics is only another fad and after a time will be substituted for yet another approach. There are many who are still quite uncertain about how much and what should be taught in the intermediate grades. Others feel that many of the concepts being developed in the modern mathematic programs are nonsense.

Often the criticism arises that new terms--sets, associative, commutative--have little meaning for the young student. It takes a certain maturity level for a student to understand such meanings. The new mathematics is unrealistic in stressing the abstract and theoretical. It goes beyond what the student needs to know and overwhelms him.

> All research on human learning indicates that, until the mind has acquired a vast reservoir of experimental knowledge and has matured to a mental age of 10 or 11 years, the ability to do two-way reflective thinking is absent, and it is impossible to understand formal logic.[18]

Many critics feel that modern mathematics is for the above average and neglects the average and below average student.

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[18] Howard F. Fehr, "Sense and Nonsense in a Modern School Mathematics Program," The Education Digest (April, 1966), p. 15.

> No one, therefore, can dispute the need for change. One can, however, question the appropriateness of certain new subject matter and the grade levels at which it is taught. For there is a tendency to think only in terms of bright youngsters, who are relatively few in number, and to forget the needs, interests, and abilities of those boys and girls who constitute the vast majority of the student body.[19]

Textbooks are poorly prepared, changes in approach have been hastily introduced, and teachers are poorly trained to teach modern mathematical concepts and vocabulary are still other criticisms of the modern programs.[20]

> Most material is of such nature that the pupil's ability to read is an important factor in his success. The new textbooks at all grade levels contain more exposition and explanation than traditional textbooks.[21]

The following is from an article written by Alice Huettig and John M. Newell. Results were given of a study that was conducted on attitudes toward a modern mathematics program in the school system, by 115 elementary school teachers near Boston.

> Despite differences in attitudes towards the modern mathematics program, the teachers in the present sample generally agreed that the new program "makes the obvious complicated": "it approaches the same concepts from too many angles," and "it is easier to go back to old methods when it becomes difficult to get a point across."[22]

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[19]Max Broder, op. cit., p. 46. [20]*Ibid*., p. 45.

[21]John L. Marks and others, *op*. *cit*., p. 8.

[22]Alice Huettig and John M. Newell, "Attitudes Toward Introduction of Modern Mathematics Program by Teachers with Large and Small Number of Years' Experiences," *Arithmetic Teacher* (February, 1966), p. 129.

Howard F. Fehr, Chairman of the Department of Mathematics Education, Teachers College, Columbia University, is among those who feel that much of the material presented in the modern mathematic programs is nonsense.

> Children are struggling to make so-called curlycue braces when they should be learning to write symbols for numbers. Set notation and symbolism, and any theory of operations on sets have no genuine significance for the learning of elementary-school mathematics. . . Recognition of collections of things is essential, but the theory of sets is nonessential in learning school mathematics.
>
> The introduction of formal logic into elementary-school mathematics is also nonsense. Truth tables, truth value, logical connectives are totally unnecessary for the acquisition of correct concepts and usages of arithmetic and physical geometry.
>
> The teaching of place systems of numeration in other bases than the decimal, and the computational algorisms in these other bases, is also nonsense. All over the world and in every type of communication--social, business, scientific, etc.,--the decimal system is the only system that at least 95 percent of the population will use, and they will probably use it every day of their lives. . . It is also nonsense to stress the distinction between numeral and number.[23]

The critics have a wide range of thinking about the mathematic programs. Good or bad, the new programs are being taught and will be used by teachers with the hope of making mathematics more meaningful for their students.

The following survey of programs points out the wide range of programs that have been written for the modern mathematics approach in the intermediate grades.

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[23]Howard F. Fehr, op. cit., pp. 14-16.

CONTENT OF SOME OF THE MODERN MATHEMATIC PROGRAMS

Experimental Programs

Some of the experimental programs include the School Mathematics Study Group (SMSG), University of Illinois; Committee on School Mathematics (UICSM); The Syracuse University-Webster College Madison Project (UMMaP); The Greater Cleveland Mathematics Program (GCMP); The Hawley and Suppes Geometry Materials Project; The Ontario Mathematics Commission (OMC); and the Boston College Mathematics Institute (BCMI). Many of the programs pertained to high school and will not be discussed in this report.

The leaders in the projects were college or university mathematicians or persons with equivalent training and experience. The primary purpose of the programs was to improve mathematics education through sound teaching procedures and content. The programs were usually sound mathematically.[24] The programs were supported by ample funds from private foundations and federal government. Funds were furnished for everything from the actual classroom tryouts to the writing of units and texts.

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[24]John L. Marks and others, op. cit., p. 7.

Most experimental programs developed a unit of study, tried the unit in a classroom situation, rewrote the unit in light of the classroom situations encountered, and then tried the unit again in the classroom. A program of teacher education accompanied some of the programs.

> Reports concerning most materials indicate they are designed to help pupils discover ideas, challenge pupils to create mathematics, and aid them to form, verify, and in some cases prove generalizations.[25]

School Mathematics Study Group (SMSG)

The SMSG was composed of university professors, college teachers, high school teachers of mathematics, and elementary teachers. The participants were divided into panels to make units on all phases of mathematics for the fourth through the twelfth grades. The units of study were developed by grade level--usually two units or books for each grade--in a series of paperbound books called *Mathematics for the Elementary School, Grade __*, and *Mathematics for the Secondary School, Grade __*. *Mathematics for the Elementary School, Grade 5*, Part I and II are presented in this report.

Part I was divided into five chapters. Chapter 1 contained information on reading large numbers, decimal

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[25] *Ibid.*, p. 7.

names for rational numbers, renaming decimals, decimals with thousandths, the base five notation with the place value of it compared with the base ten, and an introduction of other bases.

The following were the aims of Chapter 2.

> (1) to develop the technique of expressing a number as a product of prime numbers and to put this to use in (2) finding all factors of a number, and (3) finding the greatest common factor of two numbers.[26]

Chapter 3 extended multiplication and division, showed algorisms, distributive property of multiplication over addition, and showed the long and short forms for multiplication and division.

The first part of Chapter 4 was review of terms and concepts of geometric figures. Further development of geometric figures contained concepts of congruence of figures--line segments, triangles and angles. The student was also instructed in the use of a compass and straight-edge to copy and compare figures. Other exercises instructed students to draw a rectangular prism; to cut, fold, and construct a pyramid from a given pattern; and to find where planes intersected and to name angles in the

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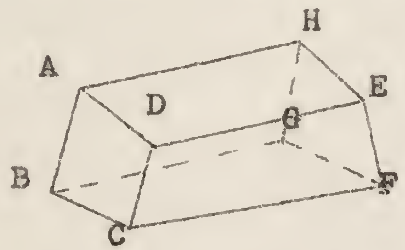
[26] Elementary School Mathematics for the School Mathematics Study Group, Mathematics for the Elementary School, Grade 5, Part I (New Haven: Yale University Press, 1963), p. 69.

given pyramid. There were also several exercises on cylinders, triangles, and half planes.

Most of the vocabulary was review from the fourth grade and included such terms as force, vertices, segment, square region, end point, intersection, pyramid, base, ray, circular region, and half plane.

Two examples of exercises included in Chapter 4 are presented here,[27] while a portion of the eight-page test can be found on page 49 of the Appendix.



1. a) Place your finger on the top face.
      Place your finger on the bottom face.
      How many faces has a chalkbox?

   b) Trace any edge of the box with your finger tip.
      How many edges has the box?

   c) Point to a vertex of the box.
      How many vertices has the box?

2. Suppose we name each corner (vertex) of the box with the letter given in the above sketch.

   a) Name 3 edges of the rectangular prism.

   b) Name 4 faces of this rectangular prism.

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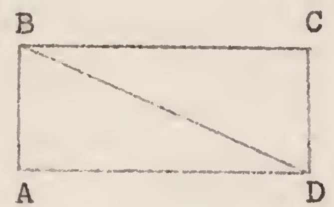
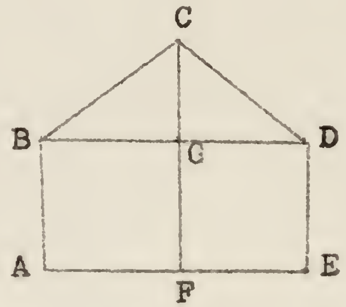
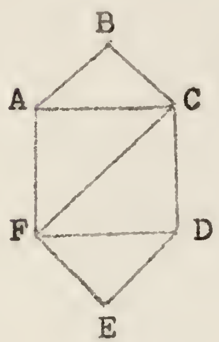
[27] *Ibid.*, pp. 308, 331.

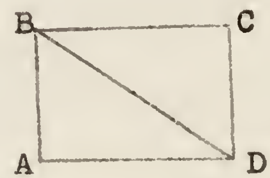
Exercise Set 2

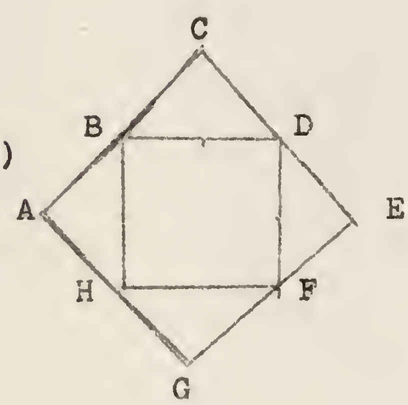
By tracing one triangle on a sheet of this paper find the triangles which are congruent to each other. Be sure to name corresponding vertices in order. In Exercise 1, state your answer like this: $\cong \triangle$ DCB. In Exercises 3, 5, and 6 you may have to trace more than one triangle.

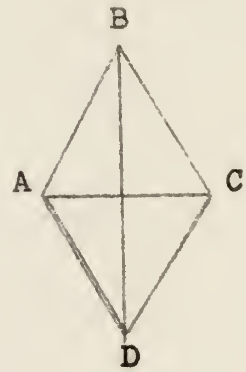
1) 

2) 

3) 

4) 

5) 

6) 

Chapter 5 contained multiplication of larger numbers, a short way of multiplying, and division by numbers greater than ten and less than 100.

Mathematics for the Elementary School, Grade 5, Part II, was a continuation of Part I--Chapters 6-10. Chapter 6 contained addition and subtraction of rational numbers; addition, subtraction, multiplication, and division of fractions; expressing fractions in simplest terms, finding other names for certain fractions; estimating sums of fractions; comparing fractions with decimals; and adding and subtracting decimals.

In Chapter 7 a compass and protractor were used to measure angles and compare right angles. How to estimate areas with odd shapes; finding definite areas of rectangles; finding the area of a right triangle; and finding the area of general triangles were included in Chapter 8. In Chapter 9 work with objects and pictures was introduced to help find ratio of different objects. Chapter 10 was a review for the book with a portion presented in the Appendix, page 52.

The following two examples (page 23) show work presented for fractions and decimals.[28]

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[28] Ibid., pp. 605, 629.

1) $\frac{34}{5} = \frac{(5 \times 6) + 4}{5}$

$$\frac{5 \times 6}{5 \times 1} + \frac{4}{5}$$

$$6 + \frac{4}{5}$$

$$\frac{34}{5} = 6\,\frac{4}{5}$$

2)
$$\begin{array}{r} 6.37 \\ +\ 3.24 \\ \hline \end{array}$$

$$\begin{array}{l} 6 + .3 + .07 \\ 3 + .2 + .04 \\ \hline 9 + .5 + .11 = \\ 9 + .5 + .1 + .01 = \\ 9 + .6 + .01 = 9.61 \end{array}$$

(or)

$$\begin{array}{r} 14.56 \\ +\ 27.25 \\ \hline .11 \\ .7 \\ 11. \\ 30. \\ \hline 41.81 \end{array}$$

The Madison Project

The Madison Project started in 1957 as an experimental mathematics curriculum for some of the weaker students at Madison Junior High School in Syracuse, New York. It later developed into a more extensive five-year program with the following objectives.

> 1. To make "probes" and "thrusts" into the unknown world of possible mathematical experiences of children . . . and to report the results via motion picture films showing actual classroom lessons. . .
> 2. To move new topics into the working repetoire of teachers, schools, and textbook authors.
> 3. To devise "informal exploratory experiences" which are fully designed (but flexible) classroom experiences for children . . .

4. To formulate explicitly the working hypotheses and value judgments upon which the proceding activities are based, in order to contribute to the development of a "theory of instruction.[29]

The Project Director was Professor Robert B. Davis of Syracuse, New York. The Project had a number of specialist teachers who went into classrooms, by invitation, and taught a unit. The regular teacher would gradually take over the teaching after becoming acquainted with the materials and procedures. The specialist teachers developed "informal exploratory experience" units for the participating schools, helped develop new units, worked with classroom teachers individually, and helped operate in-service and summer courses for the teachers. In the "exploratory experiences" units the material was usually regarded as an enrichment--largely separate from the school's pre-existing arithmetic program. In the fourth, fifth, and sixth grades, algebra or geometry was introduced in the "exploratory experiences" units.

Other topics were number lines; introduction of signed numbers; concepts of open sentences and true and

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[29]Robert B. Davis, "A Modern Mathematics Program as it Pertains to the Interrelationship of Mathematical Content, Teaching Methods and Classroom Atmosphere, (Cooperative Research Project No. D-044, Syracuse University and Webster College, 1963), p. 5.

false statements; practice of variables using quadratic equations; linear graphs; identities; role of axioms and theorems; uses of monotonicity; concepts of angles, area, volume, and length; and word problems.[30]

According to Bowin, "The Madison Project attempts to remove the emphasis on the memorization of steps and a mathematical vocabulary and place it on the creative thinking of the student."[31]

Greater Cleveland Mathematics Program (GCMP)

The Educational Research Council of Greater Cleveland was created in 1959 by Dr. George H. Baird, the Council's Executive Director. The Council was an independent, non-profit organization and worked on developing a modern school curricula. As one project the Council undertook a research and implementation study of mathematics education. It developed a program for students in grades kindergarten through twelve. The Council made a study of existing arithmetic and mathematic programs, materials,

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[30] *Ibid.*, pp. 17-23.

[31] William F. Bowin, "An Evaluation of the Madison Project of Teaching Arithmetic in Grades 4, 5, 6." (New York: Cooperative Research Project No. 1193, New York State Department of Education and Board of Education, 1963), p. 1.

learning theories, and needs of the students and professional staff of the Council schools.

> The Greater Cleveland Mathematics Program is a concept-oriented modern mathematics program in which the primary emphasis has been placed upon thinking, reasoning, and understanding, rather than on purely mechanical responses to standard situations. The child is continuously encouraged to investigate how and why things happen in mathematics. He is led to make generalizations, to test these generalizations, and to find new applications for them.
>
> Problem situations and experiences are presented in such a manner that discovery has a good chance of taking place spontaneously. Then students are led to the established symbolism. The logical structure of mathematics stimulates the imagination of children and leads to an appreciation of mathematics as a dynamic and meaningful study. Continuity and creativity are stressed throughout the program.
>
> Many topics are placed anywhere from 3 months to 16 months earlier than is usually the case for these grade levels.
>
> In kindergarten and the first three grades the children are expected to learn and use 250 terms with special meaning for the mathematics program.[32]

The Science Research Associates took many of the GCMP ideas and planned their own programs for the primary and intermediate grades. The following information was taken from the Science Research Associates programs using the GCMP materials.

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[32]National Council of Teachers of Mathematics, An Analysis of New Mathematics Programs (Washington, D. C., 1963), pp. 10, 11, 13, 14.

The Science Research Associates programs contained twelve sets or units of books for the intermediate level. Units 1-4 were used for the fourth grade level, units 5-8 for the fifth grade level, and units 9-12 for the sixth grade level. The units contained material about sets and numbers; mathematical sentences; multiplication and division of whole numbers; set of integers; set of fractional numbers; geometry, measurement, and statistics; problem solving; percentage; and set of rational numbers.

In Unit 6 the following method was shown for adding fractions.[33] $\frac{2}{9}+\frac{4}{3}=\frac{(2 \times 3)+(9 \times 4)}{9 \times 3}=\frac{6+36}{27}=\frac{42}{27}$. The explanation was followed by forty-eight problems for the student to work. Then subtraction--using the same process--was introduced, also followed by forty-eight practice problems. The work was reinforced by sixty-four more practice problems of addition and subtraction of fractions. Next, seventy-two problems of multiplication and division of fractions appeared with no explanation of how to solve such problems. $11/7 \times 6/11$ $13/19 \div 7/10$.

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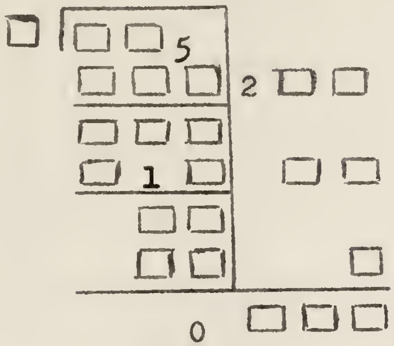
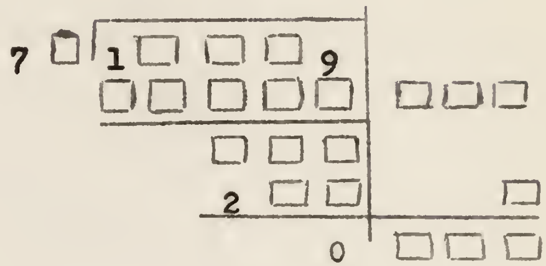
[33]Educational Research Council of Greater Cleveland, Greater Cleveland Mathematics Program, Intermediate Unit 6, (Chicago: Science Research Associates, 1965), p. 105.

Students were to construct exercises like the following in Unit 8.[34]





Textbooks

There have been several publishing companies that have published modern mathematics series: <u>Modern Arithmetic Through Discovery</u> by Silver Burdett; <u>Seeing Through Arithmetic</u> by Scott, Foresman and Company; <u>Discovering Mathematics</u> by Charles E. Merrill Books; <u>Teaching Mathematics We Need</u> by Ginn and Company; <u>Elementary Mathematics</u> by Harcourt, Brace and World; and <u>Modern School Mathematics</u> by Houghton Mifflin Company.

Most of the companies have tried to introduce the new mathematics in a variety of ways so that it could be easily taught and be of use to teachers and students. In order to show the type of content in textbooks, three texts have been chosen to be presented.

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[34]<u>Ibid</u>., Unit 8, p. 307.

Modern Arithmetic Through Discovery

In the series of books by Silver Burdett, the chapters are called learning stages. Each book contained nine or ten learning stages.

> The term learning stage is used because the pupil does not go abruptly from one unit to the next higher one, but rather progresses from one stage of understanding and skill to the succeeding stage. In this learning plan, growth in every topic is maintained and extended to a higher level, once the topic is introduced.[35]

Number sentences, place value numeration systems other than ten, and inverse operations were introduced in the intermediate grades. Most of the content in the series was similar to that introduced in traditional textbooks, except for the addition of geometry, line segments, rays, and measuring angles.

Specific topics presented in the fifth grade included numbers and numerals--reteaching and extension by comparing decimal numeration system with base five numeration system; addition and subtraction--reteaching and extension; multiplication and division--commutative, associative and distributive principles of multiplication; fractions--commutative and associative principles; decimals; measure-

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[35]Robert Lee Marton and others, Modern Arithmetic Through Discovery, Grade 5 (Chicago: Silver Burdett Company, 1963), p. 361.

ment; geometry--point, line, ray, rectangular prisms, cubes, and spheres; graphs; and problem solving.

Modern School Mathematics

The teacher's edition for the series was in a spiral notebook form. The edition contained two major aims: (1) to explain the mathematical background of the program, and (2) to provide suggestions for teaching materials, procedures, and activities. There were several pages of introduction for each chapter with key mathematical ideas, concept development, and procedure suggestions for each page of content.

The series stressed that practice was essential for reinforcing the basic skills. The following abilities might be taken as evidence that a student had mastery of a fact.

1. He can describe the fact in words.
2. He can demonstrate the fact with objects or diagrams.
3. He can write the fact in symbols.
4. He can relate the fact to other facts he knows.
5. He can use the fact in computation.[36]

Chapters in the fourth grade book contained: sets, numbers, and numerals; addition and subtraction; geometry;

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[36]Ernest R. Duncan and others, Modern School Mathematics: Structure and Use (Boston: Houghton Mifflin, 1967), Grade 4, p. vii.

multiplication and division; and fractions and fractional numbers. The chapter names in the fifth grade were very similar to those in the fourth grade, with the addition of chapters on statements; number theory; and addition, subtraction, multiplication, and division of fractional numbers.

Patterns like the following were used to help the student think through and practice the process of addition and subtraction, or multiplication and division of one number less than ten.[37]

+	2	3	7	4	8
32	a	b	c	d	e

+	2	b	a	c	e
34	36	40	39	38	37

Examples of extended and short forms of mathematical processes were shown throughout the series.

$$\begin{array}{r} 4527 \\ +3898 \\ \hline 15 \\ 110 \\ 1300 \\ 7000 \\ \hline 8425 \end{array} \qquad \begin{array}{r} 4527 \\ +3898 \\ \hline 8425 \end{array}$$

Other concepts included separating sets; set diagrams --subsets and intersection of subsets; factor trees; and base eight numeration system compared with the base ten. Models were given for construction of a pyramid, cube, cone, and a cylinder. The student was asked how many planes,

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[37] _Ibid_., pp. 44, 45.

vertices, edges, and faces there were for each figure constructed.

A unit on clock equations was also introduced in the fourth grade. The statement, 1 o'clock is five hours after 8 o'clock was written: $8 + 5 \overset{12}{=} 1$. The $\overset{12}{=}$ meant that the student used a clock with only twelve numbers. Pictures of clocks were also given to help the student figure out such problems as: $3 + 4 \overset{7}{=} a$; $3 \times 15 \overset{12}{=} m$; and $3 - 1 \overset{7}{=} t$.[38]

The fifth grade text presented ideas of base five compared with the base ten, exponents, the metric system of measure, use of formulas in addition and subtraction, measurement of angles, number patterns, area and volume extended to use of formulas, multiplication of fractions by fractions, and extended use of clock equations.

An introduction of exponents gave students equations --$25 = 5^c$; $91 = 3^m$; $27 = 3^g$--to study and solve. Problem 1 below was an example of a solution for all the students to study; while Problem 2 was meant for students more advanced in mathematical thinking.[39]

1. $$\begin{aligned} 2^3 &= 2 \times 2 \times 2 \\ &= (2 \times 2) \times 2 \\ &= 4 \times 2 \\ &= 8 \end{aligned}$$

2. $(7 \times 10^4) + (6 \times 10^3) + (5 \times 10^2) + (3 \times 10) + 4 = i$

---

[38] *Ibid.*, pp. 331-332. [39] *Ibid.*, Grade 5, pp. 20-21.

Formulas for addition and subtraction problems and the use of number patterns were also presented in the fifth grade.

> Study the equations with 2 placeholders, then copy and complete the number charts.[40]
>
> 5. $x + 1 = y$
>
> | x | 2 | 4 | 7 | 9 | 10 | 1 |
> |---|---|---|---|---|---|---|
> | y | 3 | 5 | n | r | e | s |
>
> 6. $x - 6 = y$
>
> | x | 8 | 9 | 13 | 6 | 15 | 12 |
> |---|---|---|---|---|---|---|
> | y | 2 | x | a | e | n | i |
>
> Complete the number patterns. Replace each □ by a numeral, each △ by + or −.[41]
>
> $$\begin{array}{r} 43 \\ + \square \\ \hline \end{array} \qquad \begin{array}{r} 102 \\ -\ 22 \\ \hline \end{array} \qquad \begin{array}{r} 36 \\ + \square \\ \hline \end{array} \qquad \begin{array}{r} 64 \\ - \square \\ \hline \end{array}$$
>
> $$66 + 14 \qquad \square = 87 \;\triangle\; 7 = 80$$
>
> $$\begin{array}{r} +\ 27 \\ \hline \end{array} \qquad\qquad\qquad \begin{array}{r} -68 \\ \hline \end{array}$$
>
> $$\square = 16 \;\triangle\; 25 = 29 + \square$$

__Elementary Mathematics__

The teacher's edition for the series was divided into two sections. The first contained an overall view of contemporary material concepts and explanation of the vocabulary; a detailed explanation of the content for the particular grade; and a suggested testing program for that grade. The second, and largest, part of the book contained the actual pages of the student's book with detailed suggestions on how to teach each page--purpose of page,

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[40] __Ibid.__, p. 38. [41] __Ibid.__, p. 81.

mathematical content, pre-book teaching, procedure for the page, and supplementary material.

The intermediate grade books contained material about numeration systems, whole numbers, fractional numbers, geometry and measurement, problem solving, and estimation and mental computation.

A more detailed look at the material revealed that the fourth grade presented ideas on Roman numerals; sets of numbers; addition and subtraction of whole numbers; decimal system to thousands; planes and paths; line symmetry; fractional numbers; multiplication and division of whole numbers; rays and angles; parallel and perpendicular lines; thought problems; units of measure, length, and time; area; square and triangular numbers; cones and pyramids; decimal system to millions; graphing number pairs; and an introduction to the metric system.

The fifth grade contained further analysis of the topics with several additional topics. Exercises on paper-folding of a pentagon and hexagon; finding perimeters of polygons using the metric units; rectangular and triangular prisms; cubes, pyramids, cylinders, cones, and spheres; and volume were all included in the geometry section.

Additional units included addition, subtraction, and multiplication of fractions and an explanation of reciprocals; extensive use of graphing number pairs; a study of ancient numeration systems; ratio; scale drawings; base-six numerals; and an extension of multiplication and division of whole numbers.

A sample of problems using the metric system of measure and use of fractions follow.[42]

Copy and complete.

20. 100 m make ___hm
21. 1,000 m make ___hm
22. ___hm make 1 km
23. ___dkm make 1 km

36. 18 cm make 1 dm ___cm
37. 43 m make 4 dkm ___m
38. 76 hm make 7 km ___hm
39. 54 dm make ___m 4 dm

Copy and complete each of the following number sentences to make it true. Then tell what two numerals are shown to be equivalent by each true sentence.

3. $$\frac{\frac{3}{4}}{\frac{5}{7}} = \frac{\frac{1}{2} \times \frac{3}{4}}{\frac{1}{2} \times \frac{5}{7}} = \frac{?}{\frac{5}{14}}$$

13. $$\frac{4}{\frac{5}{7}} = \frac{\frac{7}{5} \times 4}{\frac{7}{5} \times \frac{5}{7}} = \frac{?}{?}, \text{ or } \_\_$$

Programed Materials

Most programed materials are used by only a few students who are capable of going ahead on their own, not the whole class.

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[42] Joseph N. Payne and others, Elementary Mathematics, Grade 5, (New York: Harcourt, Brace and World, Inc., 1966), pp. 161, 316.

> These books are, of course, intended as enrichment material and are not to be considered as a substitute for the regular arithmetic curriculum. They are written primarily for pupils who I.Q.'s are over 100 and for pupils who are working above their grade level in arithmetic. . . Such pupils should also possess a degree of motivation or interest that enables them to work alone with selfpacing materials.[43]

Most programed materials have a series of learning steps organized sequentially. If a student would rush through the material, just in order to get done, he might find that he no longer understood the concepts and then must repeat parts in order to understand the later material presented. Most programed materials have individual answer sheets, so nothing is written in the book. Perodically there are tests for the student.

The main purpose of programed materials is that a student can find out immediately whether his response was correct. It would give the student an immediate awareness of his strong and weak points.

Mathematics Enrichment

The Discovery Edition of the book Growth in Arithmetic by Harcourt, Brace and World had a group of three books--Program A, B, and C--that were used in the fourth,

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[43]George Spooner, Mathematics Enrichment Programs A, B, C. (Teacher's Manual), (New York: Harcourt, Brace and World, Inc., 1962), pp. 11, 12.

fifth, and sixth grades. Each book contained material on sets, geometry, and numerations.

The books are much thinner than normal textbooks and are constructed quite different. A series of questions appear on each page, with the answers in the right-hand column. A skil-slide is used to cover all answers on the page. The student uncovers the answers as he completes each question in order to check the answer immediately. Only the right-hand page is used to read the information and answer questions. When the student reaches the end of the book, the book is turned over and the student must begin with the last page and work forward.

Approximately two tests are administered for each of the units in a book. Before each test, the student should talk with the teacher about material covered, ask questions about information not clearly understood, or the teacher may ask for information from the student that might have been misunderstood. The teacher grades the test and, if the student needs no further help, permits him to continue at his own rate until the next test.

George Spooner emphasized the following seven points in using the books of the series. These points are important for teachers as well as the students to understand before beginning the series.

1. A programed book is a teaching, _not_ a testing device.

2. An error _on_ a step is in no way comparable to an error on a test question.

3. _No_ scoring or grading will be done of response pads.

4. There is no such thing as "cheating" because there is no grading of responses to Programed Instruction.

5. When pupils move the cover slide down to reveal the correct answer to a step and discover their response is wrong, they are always to cross out their incorrect answer and write in the correct one on _the same line_.

6. There is no pressure for speed in Programed Instruction. Pupils are to set their own rate of progress, and no two pupils are expected to work at the same rate of speed.

7. Programed Instruction is scientifically constructed to enable pupils to learn faster and more easily.[44]

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[44] _Ibid._, p. 22.

SUMMARY

Change is a key word in modern society. Change is inevitable and can be seen all around--in modern dress, attitudes, science, technology--so textbooks and methods of teaching should change with the times. Whether people refer to mathematics as new, traditional, or modern, it is changing as is the rest of society. What is being taught now will be obsolete to some degree for the students of tomorrow.

There is a wide variety of materials published for modern mathematics on *how* it should be taught and *what* should be taught. Experimental programs are mathematically sound, but were often compiled by people too long away from the actual classroom situation and who had forgotten the average or slow learner. The programed materials are fine for some students, but are not meant for all. Students need to reach a certain maturity before they can understand many concepts that the programs provide.

Many testbook publishing companies have recognized the maturity level, needs, and interests of the students. The companies have introduced new concepts--geometry, sets, and algebra--into the intermediate programs, and have provided many experiences for the primary student so that he will be ready for the "new" concepts by the intermediate grades. By repeated and expanded use of concepts--in fourth, fifth, and sixth grades--the student should develop an understanding of the why as well as the how for mathematical procedures.

The term "modern" or "new" mathematics means a modern approach to teaching so that a student can develop an understanding of why as well as how something is done. The why is accomplished by an earlier development of terms and concepts in the experiences of the students. Terms and concepts formerly introduced in high school and college are now introduced in the primary and intermediate grades through conversation, pictures, use of objects, and other experiences of the student. Many instructional objects and devices have been developed to show the student that there are several ways to develop mathematical concepts or to solve certain mathematical problems.

A program--new or traditional--in any subject is only as successful as the teacher, how he views the materials, his interests and aspirations, and the interests and attitudes of the students.

Each student is different and each group. One method of teaching might work with one child or group and be a complete failure for another. Teachers need to examine a wide variety of textbooks, be alert to new trends and concepts, understand the needs and interests of the students, and supplement the text when a need arises in order to build a modern mathematics program for the students of tomorrow.

Critics and enthusiasts have expressed opinions of the modern mathematic programs. For some students the modern mathematics is a new and exciting experience, for others it has become a difficult and puzzling procedure, and yet for others it has already become the commonplace. Already educators are trying to discover yet another approach to make mathematics meaningful. Modern mathematics in use today will be called old or traditional five or ten years from now. Change is inevitable and for mathematics to be "modern" it must change with the needs, interests, and aspirations of society.

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REFERENCES

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APPENDIX

APPENDIX

The following are examples of test items of an eight-page test from <u>Mathematics for the Elementary School</u>, <u>Grade 5</u>, Part I, of the SMSG Program (Pages 389-391, 393, and 396).

1. Choose the item from Column 2 that matches each item in Column 1. Write the Word in the space.

A. Matching Symbols

Column 1		Column 2
______	$\triangle$	A. ray
______	$\cong$	B. line
______	$\overrightarrow{AB}$	C. segment
______	$\angle$	D. angle
______	$\overline{DE}$	E. triangle
______	$a > b$	F. a is greater than b
______	$\overleftrightarrow{GH}$	G. a is less than b
______	$a < b$	H. congruent

B. Matching the word with the sentence that describes it.

___ A triangle with only two sides that are congruent.

___ A connected part of a circle.

___ A set of points of two rays which have a common endpoint and which do not lie in the same straight line.

___ A triangle which has at least two sides which are congruent to each other.

___ A part of a line which includes two endpoints and all points of the line between them.

___ The intersection of two sides of a triangle.

___ A triangle which has three angles, each congruent to the other two.

___ The set of points in a plane all of which are equidistant from a given point.

I. angle

J. segment

K. isosceles

L. vertex

M. equilateral

N. arc

O. circle

D. Suppose you have two trangles $\triangle$ MNO and $\triangle$ PQR. All you know about them is that

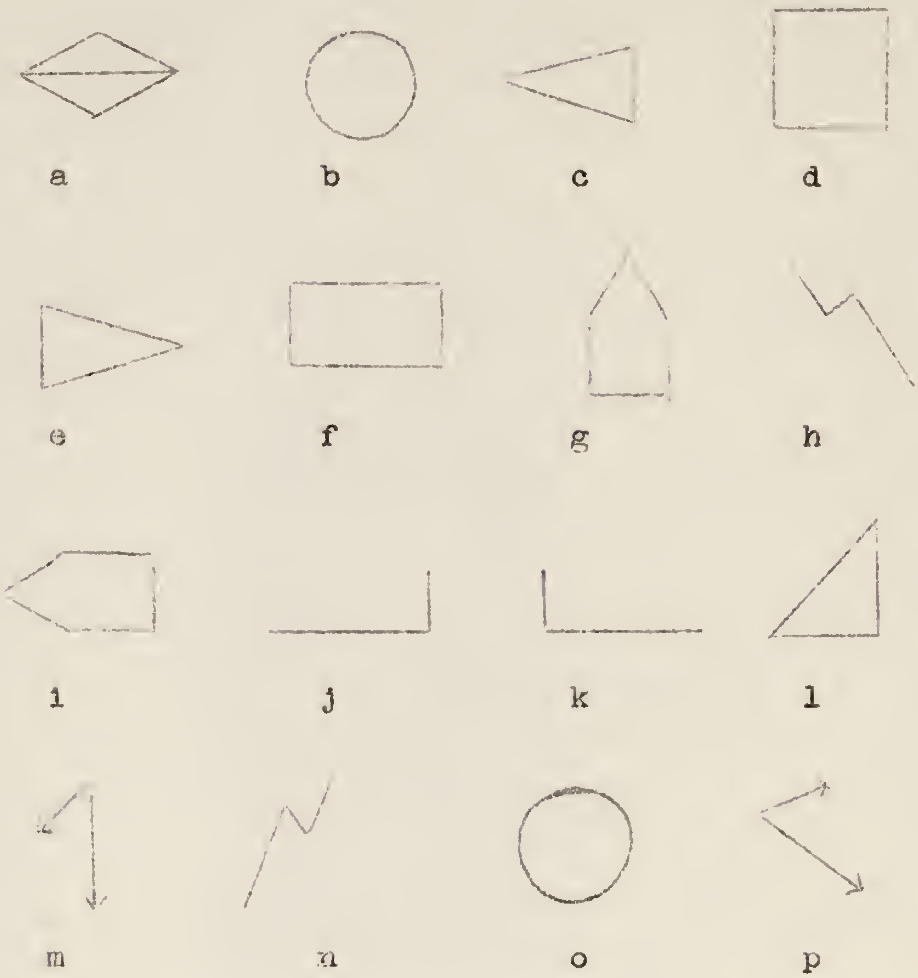
$\angle M \cong \angle P$,

$\angle N \cong \angle Q$, and

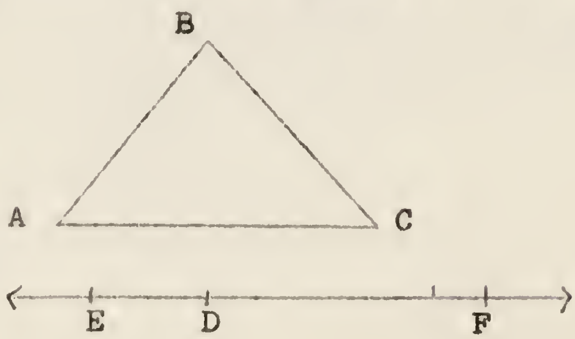
$\angle O \cong \angle R$

Can you be certain that the two triangles are congruent?

2. Choose the pairs of figures which appear to be congruent.

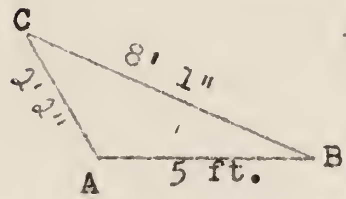


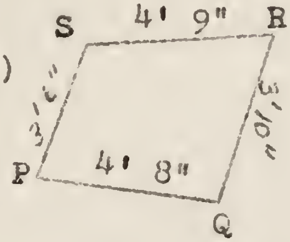
17. Use your compass and straightedge to copy $\angle a$ so that the copy has point D as its vertex; one side shall be $\overrightarrow{DE}$ and the interior of the angle shall be below $\overleftrightarrow{EF}$.



The following are samples of review items that appeared in the Mathematics for the Elementary School, Grade 5, Part II, of the SMSG program (pages 911, 928).

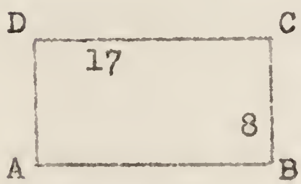
1. Find the perimeter of each polygon. On your paper write the answer beside the letter which names each polygon.

a) 

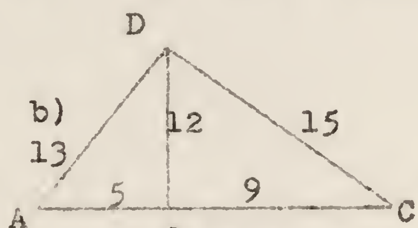
b) 

c) 

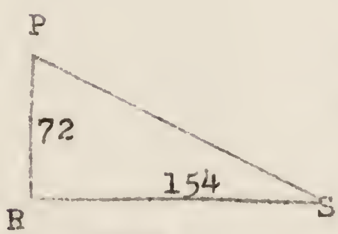
2. The polygons shown below are either rectangles or triangles. The numbers are the measures. Find the measure in square units of the area of each.

a) 

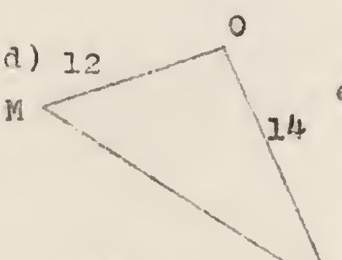
ABCD =

b) 

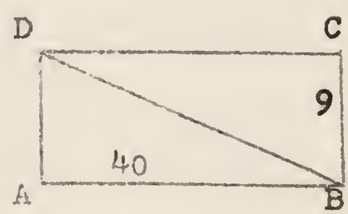
Area of $\triangle$ ABD =
Area of $\triangle$ CBD =
Area of $\triangle$ ACD =

c) 

$\triangle$RSP =

d) 

$\triangle$MNO =

e) 

Area of $\triangle$ABD =
Area of $\triangle$DBC =
Area of ABCD =

WHAT IS MODERN ABOUT "MODERN"
MATHEMATICS IN THE INTERMEDIATE GRADES?

by

NELDA ANNE FECTEAU

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STATEMENT OF PROBLEM

The purposes of the report were (1) to explain the theories of learning as related to mathematics; (2) to present principles of the modern mathematics program; (3) to present enthusiast's and critic's viewpoints of the modern mathematic programs in current use; and (4) to present the content of modern mathematic programs by use of (a) experimental programs, (b) textbooks, and (c) programed materials.

METHOD USED TO COLLECT DATA

The report was compiled after reading and analyzing magazine articles, books, textbooks, and pamphlets of current trends, controversies, and content of the modern mathematic programs for the intermediate grades.

SUMMARY

Whether people refer to mathematics as new, modern, or traditional, it is changing as is the rest of society. What is being taught now will be obsolete to some degree for the students of tomorrow. To emphasize the fact of change a section on theories and psychologies of learning was included to summarize the methods used in teaching. For example, after dissatisfactions developed over the Drill Theory the Meaning Theory was incorporated into teaching methods. As years

passed other psychologies or theories were tried until the present term "modern" or "new" has emerged in the teaching of mathematics.

The term "modern" or "new" mathematics means a modern approach to teaching so that a student can develop an understanding of <u>why</u> as well as the <u>how</u> something is done. The <u>why</u> is accomplished by an earlier development of terms and concepts during the experiences of the student. Terms and concepts formerly introduced in high school and college are now introduced in the primary and intermediate grades through conversation, pictures, and use of objects. Many instructional aids and devices have been developed to show the student that there are several ways to develop mathematical concepts or to solve certain mathematical problems.

Controversies develop about most phases of life, and critics and enthusiasts have expressed opinions of the modern mathematic programs. For some students and teachers the modern mathematics is a new and exciting experience, for others it has become a difficult and puzzling procedure, and yet some have already found mathematics commonplace.

There is a wide variety of materials published for modern mathematics on <u>how</u> it should be taught and <u>what</u> should be taught. Experimental programs seemed mathematically sound but often neglected the average or slow learner. The programed materials are appropriate for some students, but are not meant for all. Students need to reach a certain maturity before they can understand many concepts that the programs provide.

Many textbook authors have attempted to recognize the maturity level, needs, and interests of the students. New concepts--geometry, sets, and algebra--have been introduced into the intermediate programs while the basic mathematical facts are still stressed. Many experiences for the primary student have been provided in the textbook series so the students will be ready for the "new" concepts by the intermediate grades. By repeated and expanded use of concepts--in fourth, fifth, and sixth grades--the student should develop an understanding of the <u>why</u> as well as the <u>how</u> of mathematical procedures.

No one program of study or method of teaching would be appropriate for all school systems or all students. The mathematical programs must provide a variety of teaching procedures, student activities, and content in order to be of benefit to students and teachers.