COMPARISON OF POWER BY SIMULATION OF Q AND LIKELIHOOD RATIO TESTS FOR EQUALITY OF TWO NORMAL POPULATIONS IN THEIR MEANS AND VARIANCES

by

MOHAMMED ABDUL QUADEER

B.V.Sc. & A. H., Osmania University (India), 1963
M.Sc.(A. H.), Maghada University (India), 1965
Ph.D., Kansas State University, 1976

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KANSAS STATE UNIVERSITY

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Approved by:

S. K. Perng

Major Professor

CONTENTS

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PART I

COMPARISON OF SMALL SAMPLE POWER OF THE TWO TESTS FOR EQUALITY OF TWO NORMAL POPULATIONS IN THEIR MEANS AND VARIANCES

1. INTRODUCTION

Let two independent samples $X_1, \cdots, X_m$ and $Y_1, \cdots, Y_n$ be taken from normal populations $N(\mu_1, \sigma_1^2)$, $N(\mu_2, \sigma_2^2)$, respectively. There are cases of practical interest to test the equality of variances and the equality of the means at same time rather than equality of means or variances alone. For instance, it has been recognized ([5], p. 324) that the application of different treatments to otherwise homogenous experimental units often results in the treatment groups differing not only in means but also in variances. Thus, researchers are sometimes interested in the following hypothesis:

H_o: $\theta \varepsilon \Omega_o$

against

H_a: $\theta \varepsilon \Omega - \Omega_o$

on the basis of $X_1, \cdots, X_m, Y_1, \cdots, Y_n$

where $\theta = (\mu_1, \mu_2, \sigma_1^2, \sigma_2^2)$,

$$\Omega = \{\theta | -\infty < \mu_i < +\infty, o < \sigma_i^2 < \infty \text{ for } i = 1, 2\},$$

$$\Omega_o = \{\theta | \mu_1 = \mu_2 \text{ and } \sigma_1^2 = \sigma_2^2\}.$$

The need for such a comprehensive test has been recognized and various test procedures have been proposed. Neyman and Pearson [3] suggested likelihood ratio test (L-test). Sukhatme [6] derived the exact distribution of the likelihood ratio test statistic under H_o. He also tabulated the critical values of the L-test at significance level 0.05 and 0.01 for various sample sizes. Using Fisher's methods of combining two independent test statistics [2], Sukhatme [6] also considered Q-test. In [4], Perng and Littell proved that the Q-test is asymptotically optimal in the sense of Bahadur efficiency [1].

The objective of this paper is to compare the small sample power of the L-test and the Q-test by simulation.

2. TEST STATISTICS

First we define the following notation:

$$\bar{X} = \frac{1}{m} \sum_{i=1}^{m} X_i$$

$$\bar{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_i$$

$$SS_1 = \sum_{i=1}^{m} (X_i - \bar{X})^2$$

$$SS_2 = \sum_{i=1}^{n} (Y_i - \bar{Y})^2$$

$$SS = \sum_{i=1}^{m} (X_i - U)^2 + \sum_{i=1}^{n} (Y_i - U)^2$$

$$U = \left(\sum_{i=1}^{m} X_i + \sum_{i=1}^{n} Y_i\right)/N$$

$$N = (m+n)$$

$$T = (\bar{Y} - \bar{X}) \left[\frac{(m + n)(SS_1 + SS_2)}{(m + n - 2)\ m \cdot n}\right]^{-\frac{1}{2}}$$

$$F = [SS_2(m - 1] \ [SS_1(n - 1)]^{-1}$$

$$H = \begin{cases} 2[1 - G(F)] & \text{if } G(F) \geq \frac{1}{2} \\ 2\ G(F) & \text{if } G(F) < \frac{1}{2} \end{cases}$$

where G is the distribution function of the central F with (n - 1) and (m - 1) degrees of freedom.

Now we describe the two tests. First the L-test:

$$\begin{aligned} &\text{Reject } H_o && \text{if } L \leq L_o \\ &\text{accept } H_a && \text{if } L > L_o \end{aligned} \qquad (2.1)$$

where $L = \left[\frac{SS_{1/m}}{SS/N}\right]^{\frac{m}{N}} \left[\frac{SS_{2/n}}{SS/N}\right]^{\frac{n}{N}}$.

The approximate critical values L_o have been tabulated in [6] at significance level 0.05 and 0.01 for various sample sizes. Sukhatme [6] has also shown that the approximate critical values agree to the exact values to three decimal places for $m = n = 5, 12, 20, 60$ and $m = 5, n = 15$. We checked some of the approximate critical values in [6] by simulation. We found that the approximate critical values given by Sukhatme agree to three decimal places with our simulation results. Hence, Sukhatme's approximate critical values were adopted to compute the power of the L-test.

Next we describe the Q-test [4].

reject H_o if $Q \geq c_o$

accept H_a if $Q < c_o$

where $Q = -2 \log P_{H_o} [|T| > t] -2 \log P_{H_o} [H < h]$, t is the observed value of $|T|$, h is the observed value of H, and P_{H_o} [A] means the probability of event A where H_o is true. Under H_o, T has central t-distribution with $(m + n - 2)$ degrees of freedom, H has uniform distribution over (0, 1) [4]. Thus, the probabilities $P_{H_o} [|T| > t]$ and $P_{H_o} [H < h]$ can be easily calculated hence the value of Q. Since under H_o, Q has chi-squared distribution with 4 degrees of freedom, the critical value c_o can be found from chi-square table.

3. METHOD OF COMPARISON

Since the exact distributions of the test statistics L and Q under H_a are unknown, it is not possible to compare the exact power of the two tests. A Monte-Carlo study of the small sample power of these two tests was made. Normal deviates $X_1,\cdots, X_m; Y_1,\cdots, Y_n$ were generated from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ respectively. Since both these tests are designed to detect the difference between μ_1 and μ_2, and σ_1^2 and σ_2^2, hence in our Monte Carlo study, we fixed the value of μ_1 to be zero, and the value of σ_1^2 to be one, and varied the values of μ_2 and σ_2^2. Both equal and unequal sample sizes were considered: (1) $m = n = 10, 12$, and 15; (2) $(m, n) = (10, 12), (10, 15), (12, 15), (15, 30)$; and (3) with interchange of m and n in (2). In each simulation test statistics L and Q are computed based on the same generated normal deviates. Each simulated analysis was repeated a thousand times and the number of rejections of the null hypothesis H_o at five and one percent levels was counted. A computer program was written for this Monte Carlo study. A more complete description of the program may be found in Appendix B.

Since all the simulation results led to similar conclusion about the power of the two tests, we shall only present one part of our simulation results. (For more details see Appendix B). For the equal sample sizes case the simulation results, for $m = n = 15$ and $\alpha = .05$ are presented in Table 1a. Figure 1a is the graph of simulated power function based on values of Table 1a except for $\mu_2 = 0$. The results for $m = n = 15$ and $\alpha = .01$ are given in Table 1b and its corresponding graph in figure 1b (except for $\mu_2 = 0$). It is apparent that both tests have almost the same power at each alpha level for various values of μ_2 and σ_2^2. The same general pattern is observed for $m = n = 10$, 12 and 30. Similar results are obtained for $m \neq n$ case where m and n are close, for example $(m, n) = (10, 12)$, $(10, 15)$, and $(12, 15)$.

For $(m, n) = (15, 30)$ the simulation results are presented in Table 2a for $\alpha = .05$; and for $\alpha = .01$ the results are summarized in Table 2b. The values of Table 2a and 2b (except for $\mu_2 = 0$) are graphed in Figure 2a and 2b, respectively. For $\mu_2 = 0$, Q-test has greater power function of as high as 5.3% (over both alpha levels) when $\sigma_2 < 1$ and the superiority reversed in favor of L-test to the extent of 9.0% (over both alpha levels) when $\sigma_2 \geq 1$. Similar reversals of power occurred at different values of μ_2 but differences in powers of tests are considerable lower.

The simulation results with $(m, n) = (30, 15)$ are presented in Tables 3a and 3b for $\alpha = .05$ and $\alpha = .01$, respectively. Figures 3a and 3b indicate the simulated power curves and are based on the values (except $\mu_2 = 0$) from Tables

3a and 3b respectively. The difference between the powers of the two tests at various values of μ_2 and σ_2 are almost 9.3 percent over both alpha levels. However, contrary to the case (m, n) = (15, 30), now L-test is better than Q-test for $\sigma_2 \le 1$ and Q-test is better than L-test for $\sigma_2 > 1$. Though this switch in power existed for each μ_2 considered but the differences in powers decreases as μ_2 increases.

In general, the power functions are almost identical for Q and L-tests when sizes are equal or almost equal while for unequal samples neither test has an advantage -- as criss-cross in power made one test better for certain values of σ_2 and favored other test otherwise.

4. COMPARISON OF THE ACCEPTANCE REGIONS OF THE TWO TESTS

To find some insight why the power functions of the L-test and the Q-test are so close, we shall examine the acceptance regions of the two tests.

We first note that L (see (2.1)) can be written in terms of T and F of the following form.

$$L = L(T, F) = N\,(m)^{-\frac{m}{N}}(n)^{-\frac{n}{N}}\left[\frac{n-1}{m-1}F\right]^{\frac{n}{N}}\left[\left(1+\frac{n-1}{m-1}F\right)\left(1+\frac{T^2}{n-2}\right)\right]^{-1}.$$

Let $L_{\alpha,\, m,\, n}$ be the critical value of the L-test at significance level α and sample sizes (m, n). Then the boundary curve of the acceptance region of the L-test in (T, F) plane is the set of all (T, F) which satisfy the following equality

$$L(T, F) = L_{\alpha,\, m,\, n}.$$

Similarly, the set of all (T, F) which satisfy the following equality gives the boundary curve of the acceptance region of the Q-test at given significance level α and sample sizes (m, n).

$$\chi^2_{1-\alpha,\, 4} = -2\log P_{H_o}(|T_{mn}| > T) - 2\log P_{H_o}(H_{mn} < h)$$

where $\chi^2_{1-\alpha, 4}$ is the $(1 - \alpha)$ the quantile of the chi-square random variable with four degrees of freedom. In addition, we also generated two sets of 100 pairs of (T, F) values as follows: paired (T, F) values of set I were computed from samples of normal deviates generated from normal populations N(0, 1) and N(-1, 1). Set II of (T, F) values were computed from normal deviates generated from N(0, 1) and N(1, 4).

The acceptance regions of the two tests and the two sets of (T, F) values for (m, n) = (15, 15), (15, 30), (30, 15) and for $\alpha = 0.05$, are presented in Figures 4a, 5a and 6a, respectively. Corresponding graphs for $\alpha = 0.01$ are given in Figures 4b, 5b and 6b respectively. These graphs suggest that for equal sample sizes the acceptance regions of Q-test and L-test are almost the same. However, for unequal samples, there are distinct differences in the sizes of acceptance regions for two tests. Switch occurs in the sizes of acceptance regions for reversing the sample sizes. The switch in size of acceptance regions were also observed for other unequal samples sizes. Consequent to differential size in acceptance regions unequal number of (T, F) values in each set I and II were excluded from acceptance regions of Q and L -tests.

5. CONCLUSION

Simulated power functions of Q-test and L-test were compared. For equal sample sizes or approximately equal sample sizes both tests have similar power functions. However, for unequal sample sizes, the power functions of the two tests have larger difference, although no one test dominates the other test. To test the equality of two normal populations, it seems reasonable to require that the sample sizes of the two samples be close. If this is the case, together with the fact that Q-test is readily computed by using well-known tables while L-test needs special tables, we would suggest to use Q-test instead of L-test.

Table 1a. Simulated power functions of Q and L-tests for $m = 15$, $n = 15$, $\alpha = .05$, simulations = 1000, $\mu_1 = 0.0$, $\sigma_1 = 1.0$.

σ_2	Power of the Tests							
	Q	L	Q	L	Q	L	Q	L
	$\mu_2 = 0.0$		$\mu_2 = 0.25$		$\mu_2 = 0.75$		$\mu_2 = 1.50$	
0.2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.4	0.838	0.854	0.869	0.874	0.988	0.989	1.000	1.000
0.6	0.350	0.362	0.394	0.403	0.800	0.796	1.000	1.000
0.8	0.114	0.119	0.129	0.129	0.552	0.564	0.984	0.989
1.0	0.054	0.056	0.088	0.082	0.368	0.382	0.944	0.954
1.2	0.093	0.087	0.106	0.108	0.364	0.376	0.905	0.916
1.4	0.159	0.166	0.179	0.177	0.420	0.422	0.897	0.895
1.6	0.297	0.306	0.335	0.327	0.488	0.474	0.895	0.891
1.8	0.448	0.472	0.460	0.472	0.622	0.629	0.891	0.889
2.5	0.852	0.860	0.863	0.879	0.894	0.896	0.961	0.962
3.2	0.979	0.984	0.978	0.980	0.982	0.983	0.997	0.997
3.9	0.999	0.999	0.995	0.995	0.994	0.996	0.999	0.999
4.6	0.998	1.000	0.999	0.999	0.999	0.999	0.997	0.998

Table 1b. Simulated power functions of Q and L-tests for $m = 15$, $n = 15$, $\alpha = .01$, simulations = 1000, $\mu_1 = 0.0$, $\sigma_1 = 1.0$.

σ_2	Power of the Tests							
	Q	L	Q	L	Q	L	Q	L
	$\mu_2 = 0.0$		$\mu_2 = 0.25$		$\mu_2 = 0.75$		$\mu_2 = 1.50$	
0.2	.997	.996	1.000	1.000	1.000	1.000	1.000	1.000
0.4	.615	.643	0.676	0.688	0.951	0.945	1.000	1.000
0.6	.135	.147	0.166	0.158	0.577	0.562	0.982	0.984
0.8	.033	.034	0.041	0.043	0.294	0.301	0.896	0.915
1.0	.010	.011	0.020	0.019	0.153	0.166	0.785	0.819
1.2	.019	.019	0.033	0.032	0.162	0.159	0.712	0.720
1.4	.045	.045	0.045	0.048	0.184	0.179	0.712	0.723
1.6	.099	.109	0.114	0.127	0.238	0.230	0.715	0.704
1.8	.202	.217	0.217	0.234	0.356	0.354	0.737	0.728
2.5	.609	.641	0.646	0.684	0.706	0.728	0.864	0.860
3.2	.903	.915	0.909	0.922	0.912	0.925	0.959	0.968
3.9	.981	.986	0.982	0.985	0.978	0.982	0.991	0.992
4.6	.993	.994	0.997	0.998	0.992	0.993	0.994	0.994

Table 2a. Simulated power functions of Q and L-tests for m = 15, n = 30, α = .05, simulations = 1000, μ_1 = 0.0, σ_1 = 1.0.

σ_2	Power of the Tests							
	Q	L	Q	L	Q	L	Q	L
	μ_2 = 0.0		μ_2 = 0.25		μ_2 = 0.75		μ_2 = 1.50	
0.2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.4	0.950	0.945	0.969	0.966	0.999	0.995	1.000	1.000
0.6	0.558	0.505	0.632	0.587	0.919	0.908	1.000	1.000
0.8	0.141	0.114	0.219	0.194	0.724	0.703	0.995	0.994
1.0	0.047	0.050	0.087	0.080	0.517	0.535	0.986	0.988
1.2	0.064	0.073	0.092	0.110	0.461	0.483	0.969	0.970
1.4	0.136	0.173	0.187	0.227	0.492	0.507	0.947	0.950
1.6	0.332	0.396	0.359	0.422	0.617	0.633	0.954	0.958
1.8	0.481	0.571	0.531	0.594	0.723	0.749	0.958	0.962
2.5	0.909	0.945	0.906	0.947	0.957	0.973	0.994	0.997
3.2	0.989	0.993	0.997	0.999	0.998	0.999	0.999	0.999
3.9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 2b. Simulated power functions of Q and L-tests for m = 15, n = 30, α = .01, simulations = 1000, μ_1 = 0.0, σ_1 = 1.0.

σ_2	Power of the Tests							
	Q	L	Q	L	Q	L	Q	L
	μ_2 = 0.0		μ_2 = 0.25		μ_2 = 0.75		μ_2 = 1.50	
0.2	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
0.4	0.890	0.874	0.920	0.901	0.990	0.989	1.000	1.000
0.6	0.293	0.259	0.369	0.324	0.822	0.793	1.000	1.000
0.8	0.041	0.032	0.074	0.062	0.484	0.462	0.982	0.982
1.0	0.012	0.011	0.023	0.022	0.265	0.270	0.931	0.944
1.2	0.017	0.023	0.024	0.031	0.220	0.232	0.877	0.887
1.4	0.041	0.057	0.052	0.068	0.249	0.263	0.823	0.833
1.6	0.111	0.154	0.138	0.178	0.320	0.340	0.840	0.843
1.8	0.193	0.270	0.238	0.302	0.432	0.474	0.845	0.849
2.5	0.701	0.787	0.686	0.773	0.809	0.860	0.957	0.962
3.2	0.943	0.966	0.958	0.980	0.960	0.983	0.994	0.995
3.9	0.996	0.998	0.997	0.999	0.995	0.999	1.000	1.000
4.6	0.999	1.000	1.000	1.000	1.000	1.000	0.999	0.999

Table 3a. Simulated power functions of Q and L-tests for m = 30, n = 15, α = .05, simulations = 1000, μ_1 = 0.0, σ_1 = 1.0.

σ_2	Power of the Tests							
	Q	L	Q	L	Q	L	Q	L
	μ_2 = 0.0		μ_2 = 0.25		μ_2 = 0.75		μ_2 = 1.50	
0.2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.4	0.910	0.941	0.949	0.967	0.998	0.998	1.000	1.000
0.6	0.375	0.436	0.459	0.499	0.904	0.913	1.000	1.000
0.8	0.089	0.113	0.160	0.178	0.638	0.653	0.999	1.000
1.0	0.057	0.054	0.086	0.088	0.520	0.524	0.985	0.985
1.2	0.122	0.092	0.164	0.145	0.509	0.487	0.968	0.964
1.4	0.261	0.234	0.324	0.281	0.601	0.554	0.968	0.966
1.6	0.449	0.400	0.523	0.487	0.727	0.682	0.965	0.963
1.8	0.656	0.617	0.680	0.638	0.845	0.811	0.971	0.965
2.5	0.953	0.947	0.966	0.955	0.979	0.972	0.998	0.992
3.2	0.998	0.998	0.999	0.998	0.997	0.996	0.998	0.998
3.9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 3b. Simulated power functions of Q and L-tests for m = 30, n = 15, α = .01, simulations = 1000, μ_1 = 0.0, σ_1 = 1.0.

σ_2	Power of the Tests							
	Q	L	Q	L	Q	L	Q	L
	μ_2 = 0.0		μ_2 = 0.25		μ_2 = 0.75		μ_2 = 1.50	
0.2	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.4	0.705	0.798	0.790	0.858	0.978	0.982	1.000	1.000
0.6	0.115	0.166	0.192	0.232	0.686	0.700	0.998	0.999
0.8	0.014	0.025	0.044	0.054	0.359	0.370	0.978	0.982
1.0	0.007	0.007	0.018	0.019	0.242	0.248	0.933	0.935
1.2	0.034	0.028	0.044	0.037	0.262	0.248	0.883	0.888
1.4	0.109	0.087	0.136	0.111	0.360	0.324	0.905	0.892
1.6	0.223	0.192	0.292	0.260	0.501	0.448	0.911	0.895
1.8	0.418	0.376	0.439	0.402	0.642	0.592	0.918	0.899
2.5	0.887	0.868	0.893	0.880	0.929	0.918	0.978	0.974
3.2	0.990	0.989	0.986	0.985	0.991	0.990	0.995	0.995
3.9	0.998	0.998	0.998	0.996	0.999	0.997	1.000	0.999
4.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Figure 1a. POWER OF Q-TEST AND L-TEST

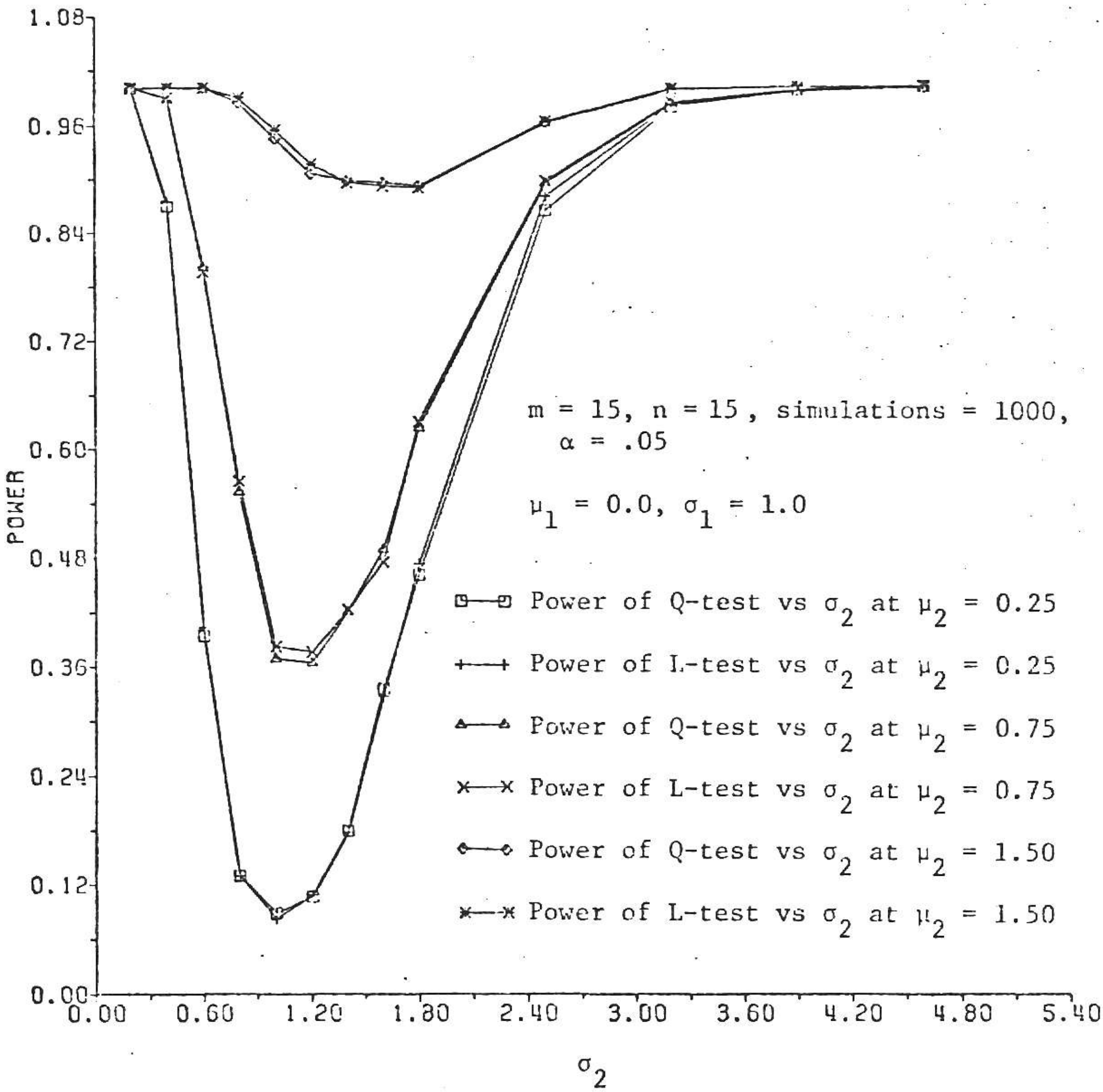


Figure 1b. POWER OF Q-TEST AND L-TEST

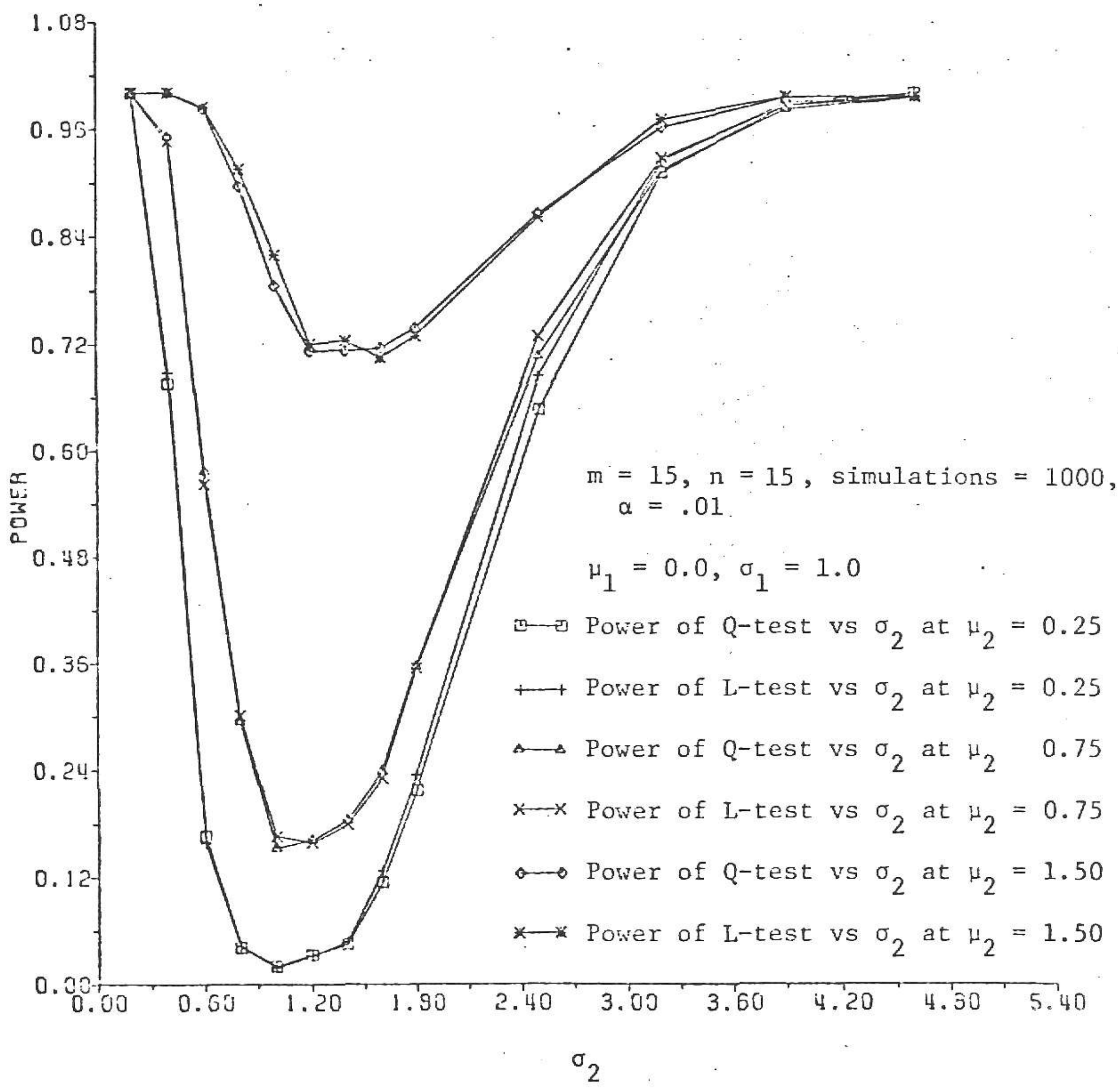


Figure 2a. POWER OF Q-TEST AND L-TEST

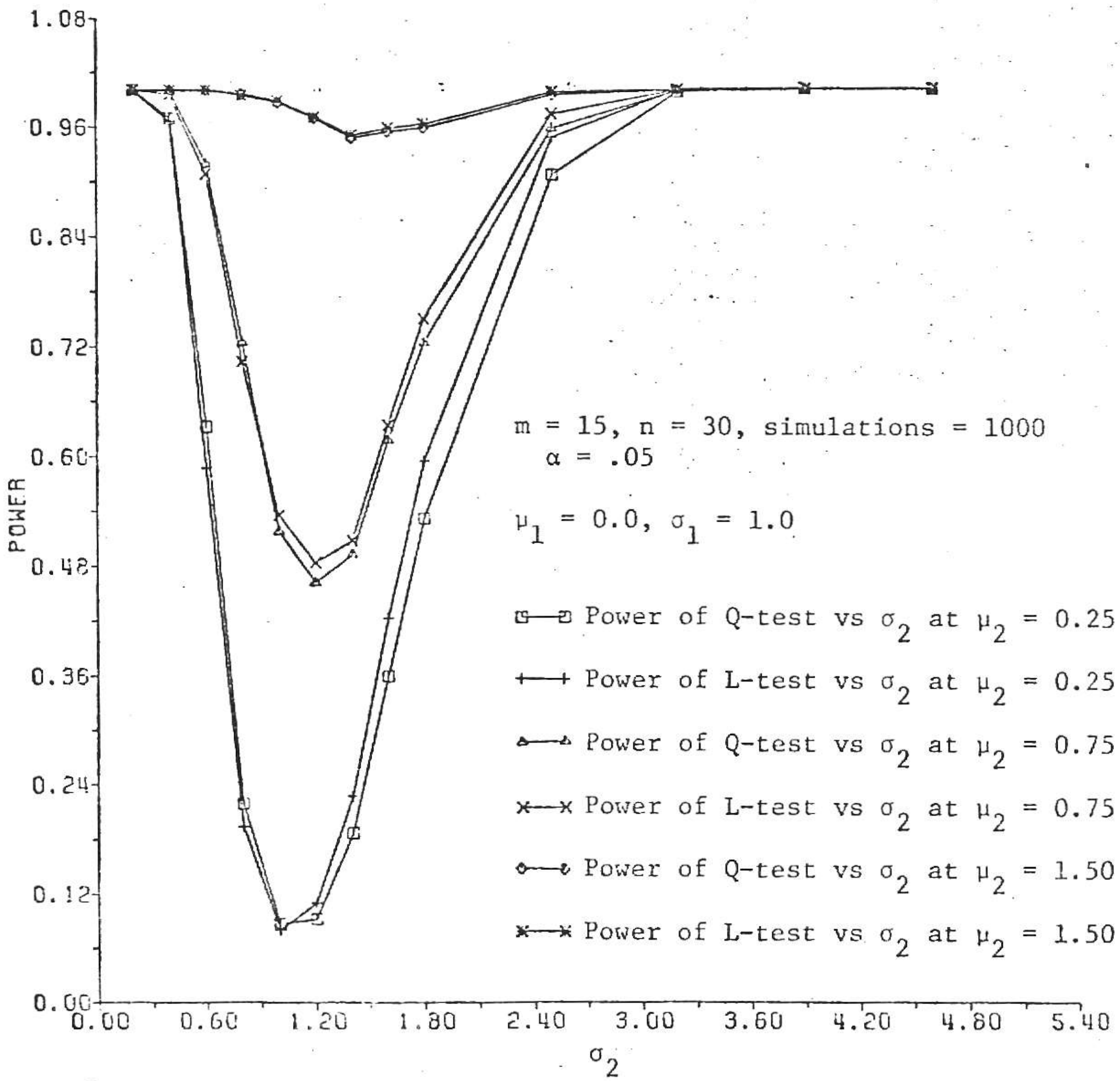


Figure 2b. POWER OF Q-TEST AND L-TEST

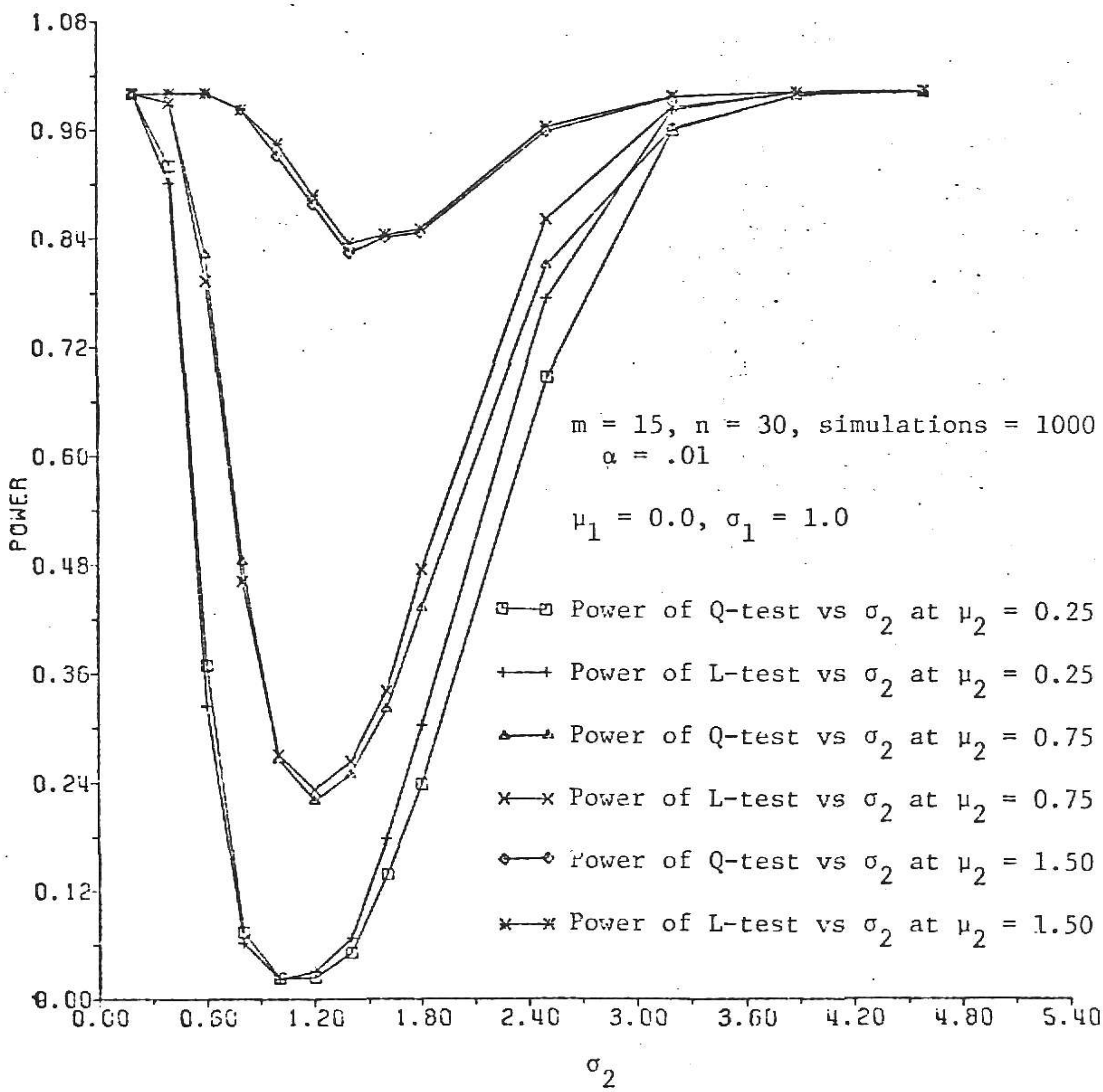


Figure 3a. POWER OF Q-TEST AND L-TEST

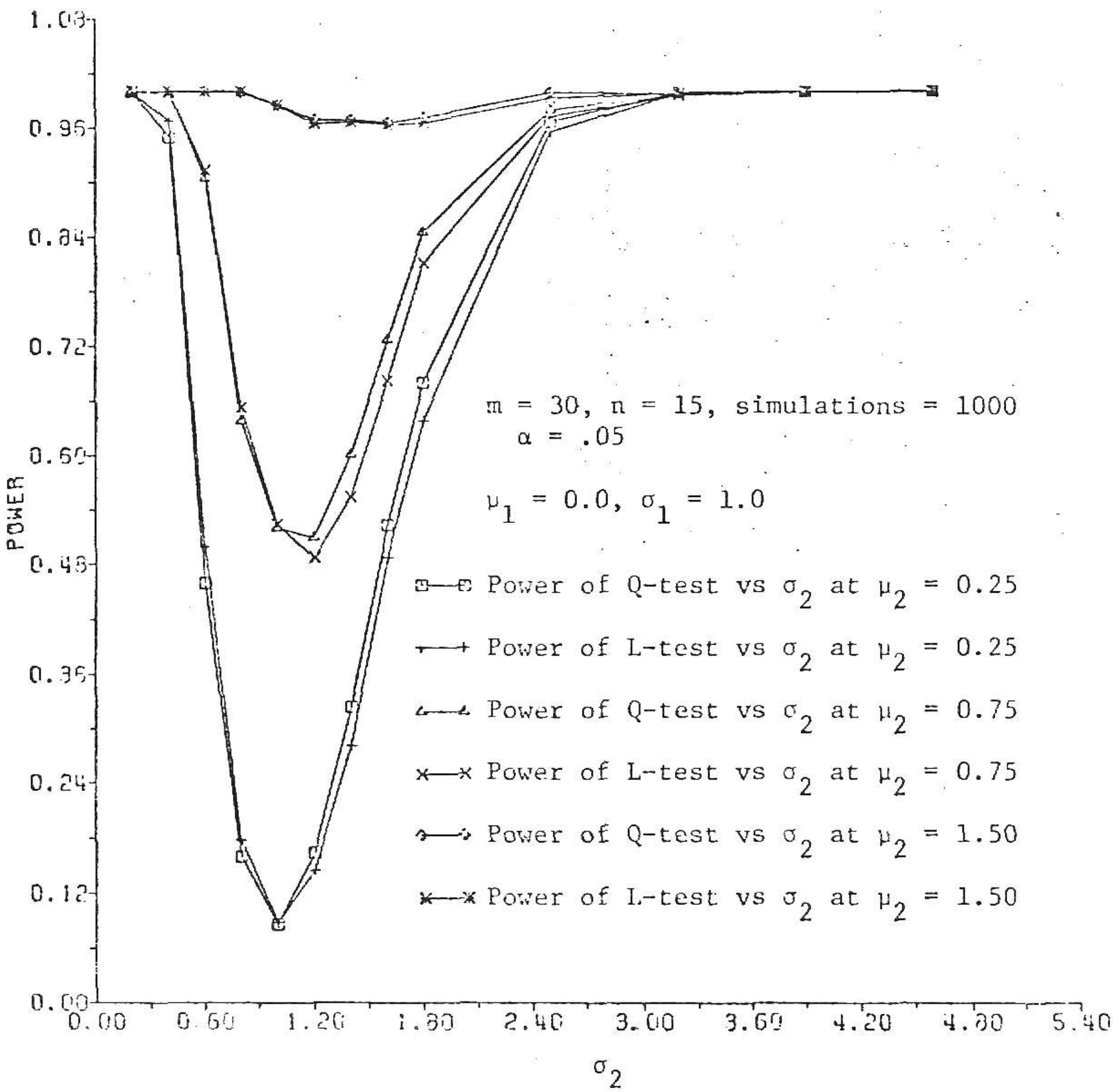


Figure 3b. POWER OF Q-TEST AND L-TEST

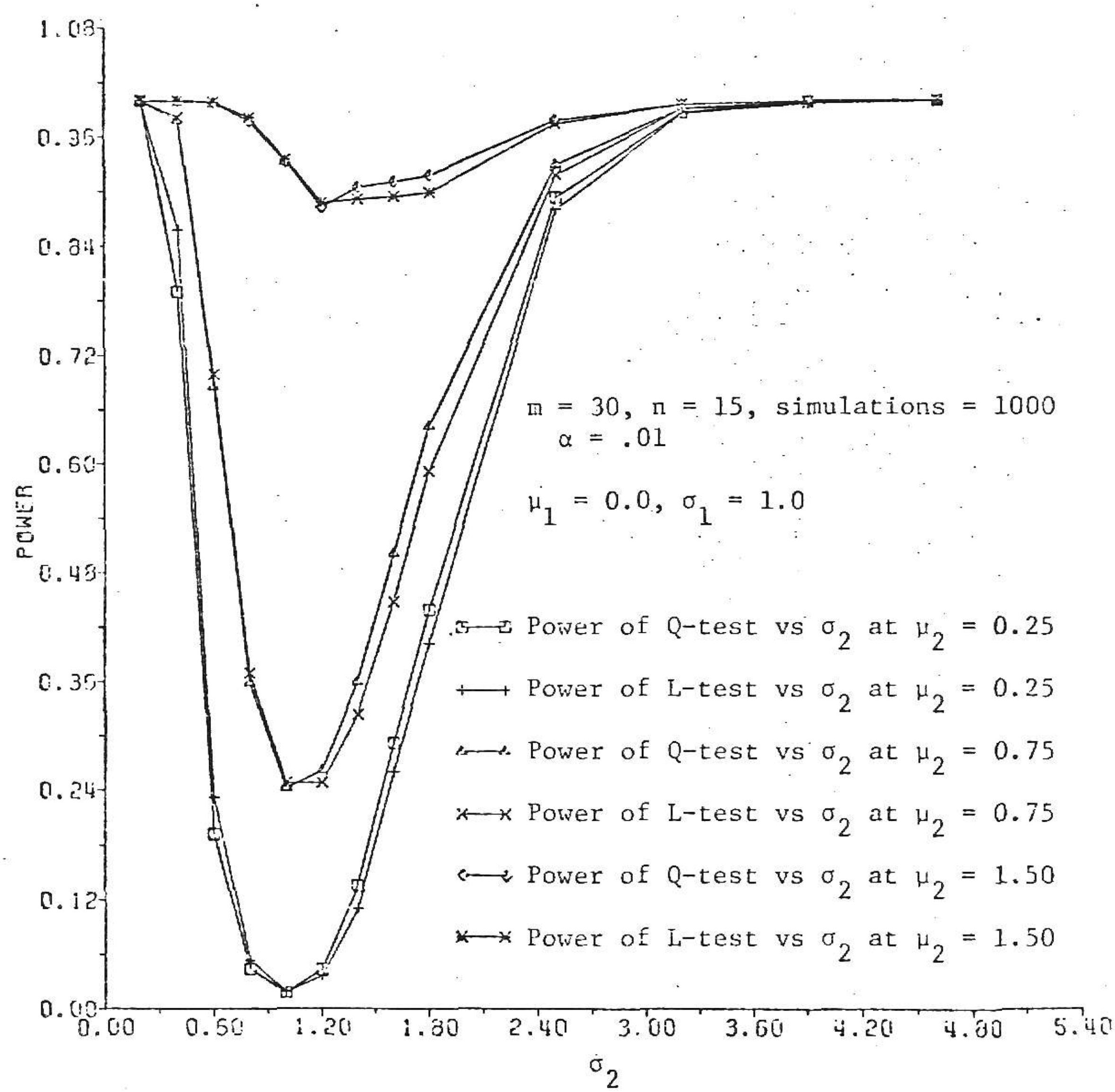


Figure 4a. ACCEPTANCE REGIONS OF Q-TEST AND L-TEST

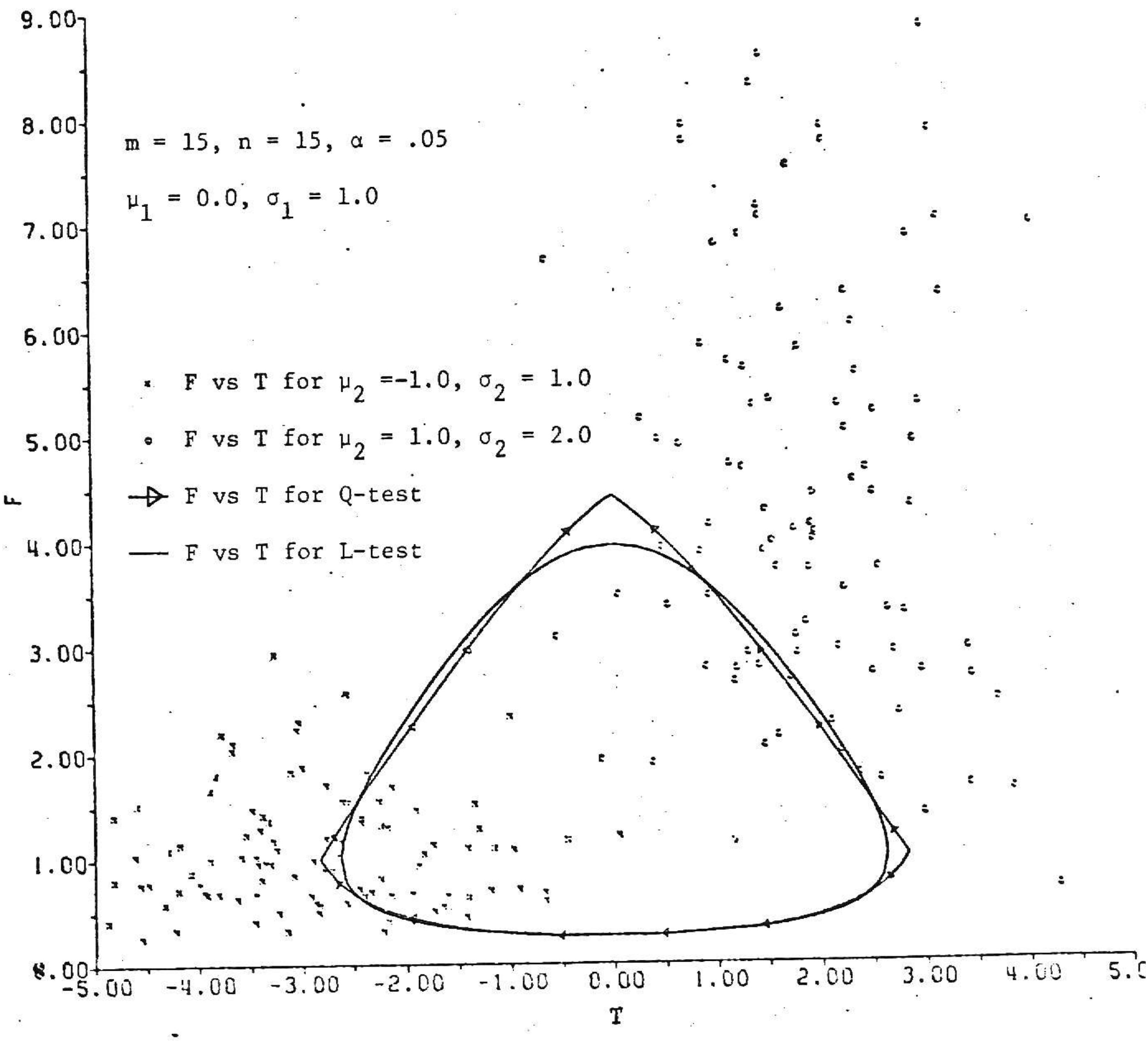


Figure 4b. ACCEPTANCE REGIONS OF Q-TEST AND L-TEST

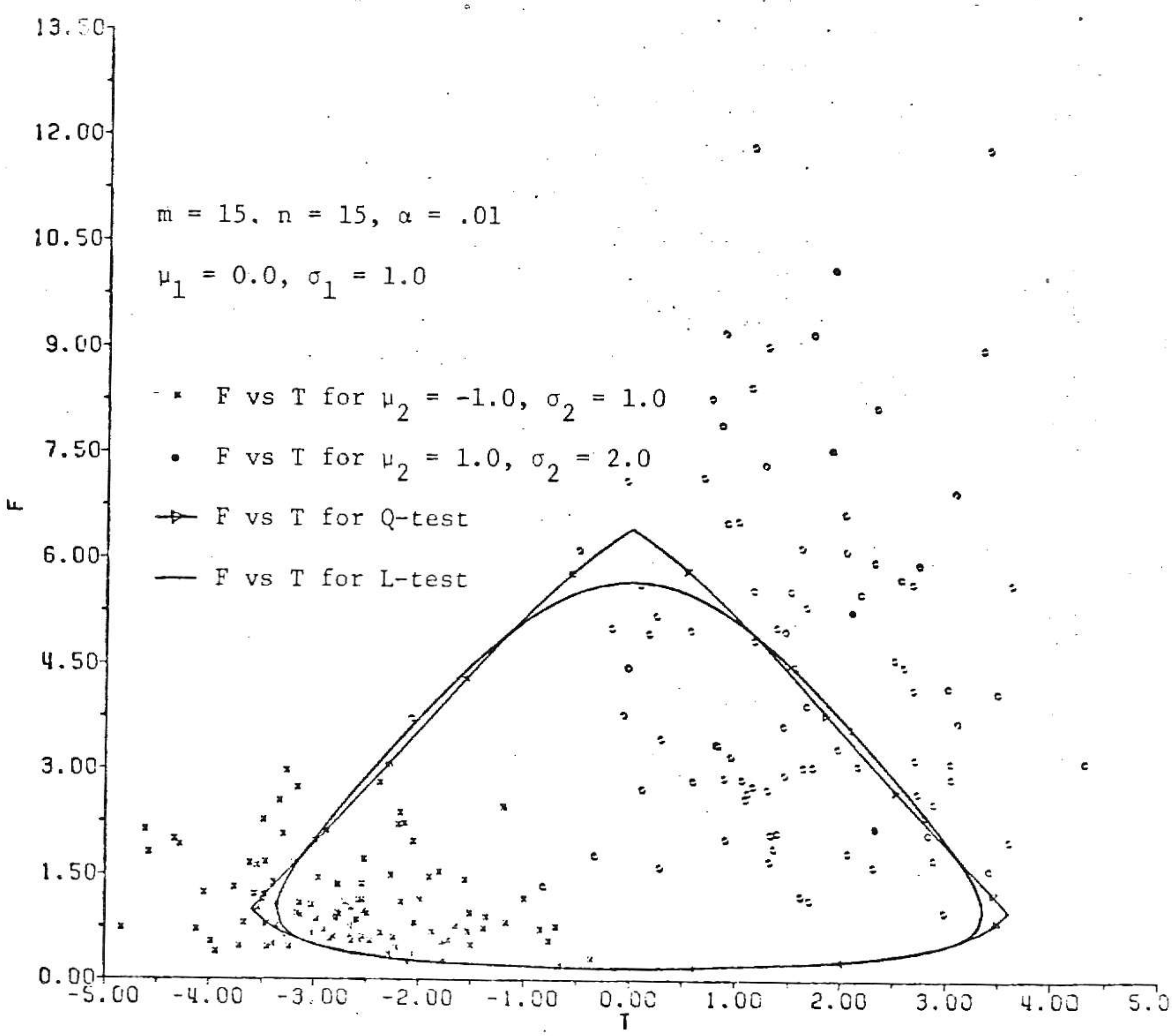


Figure 5a. ACCEPTANCE REGIONS OF Q-TEST AND L-TEST

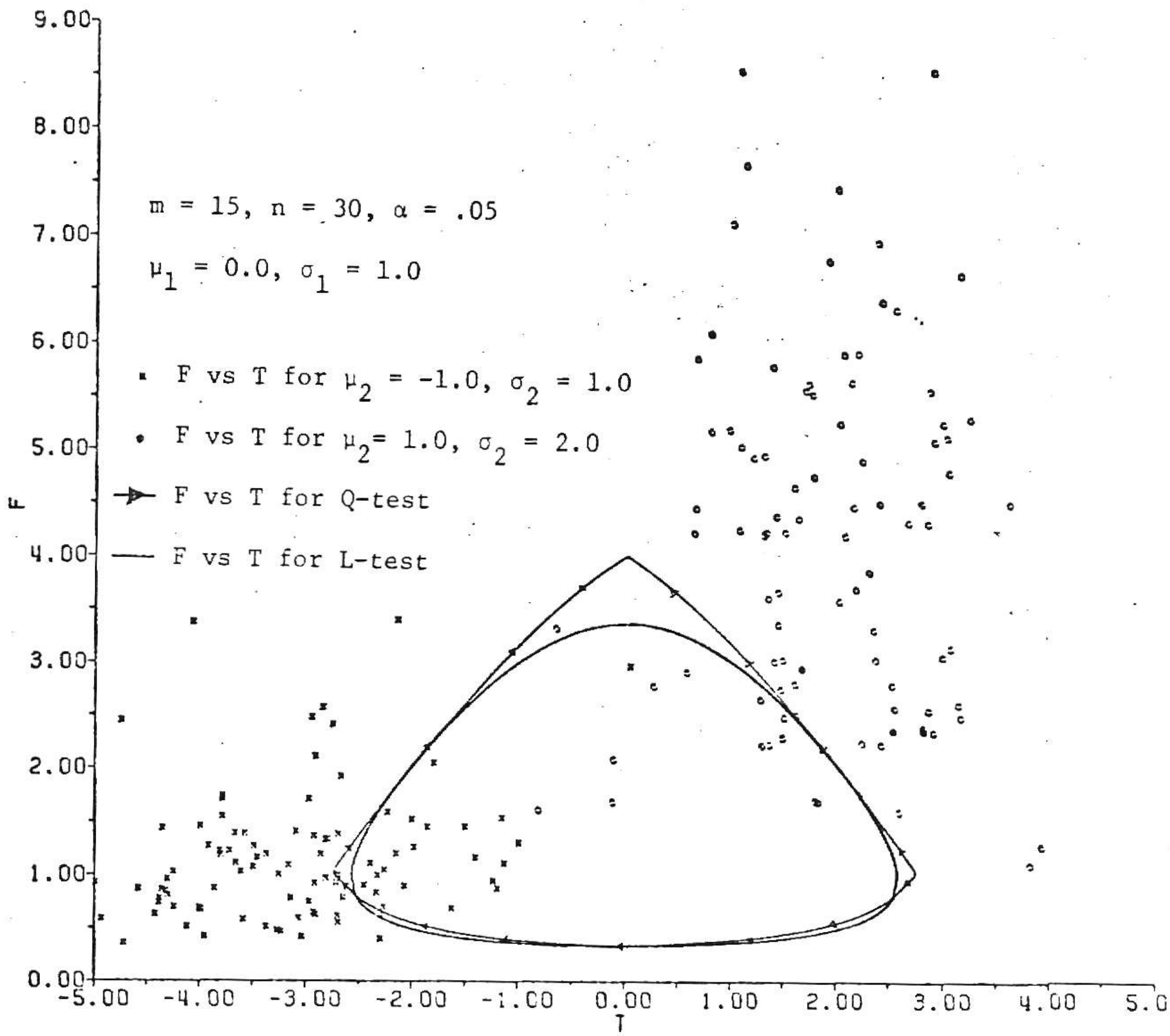


Figure 5b. ACCEPTANCE REGIONS OF Q-TEST AND L-TEST

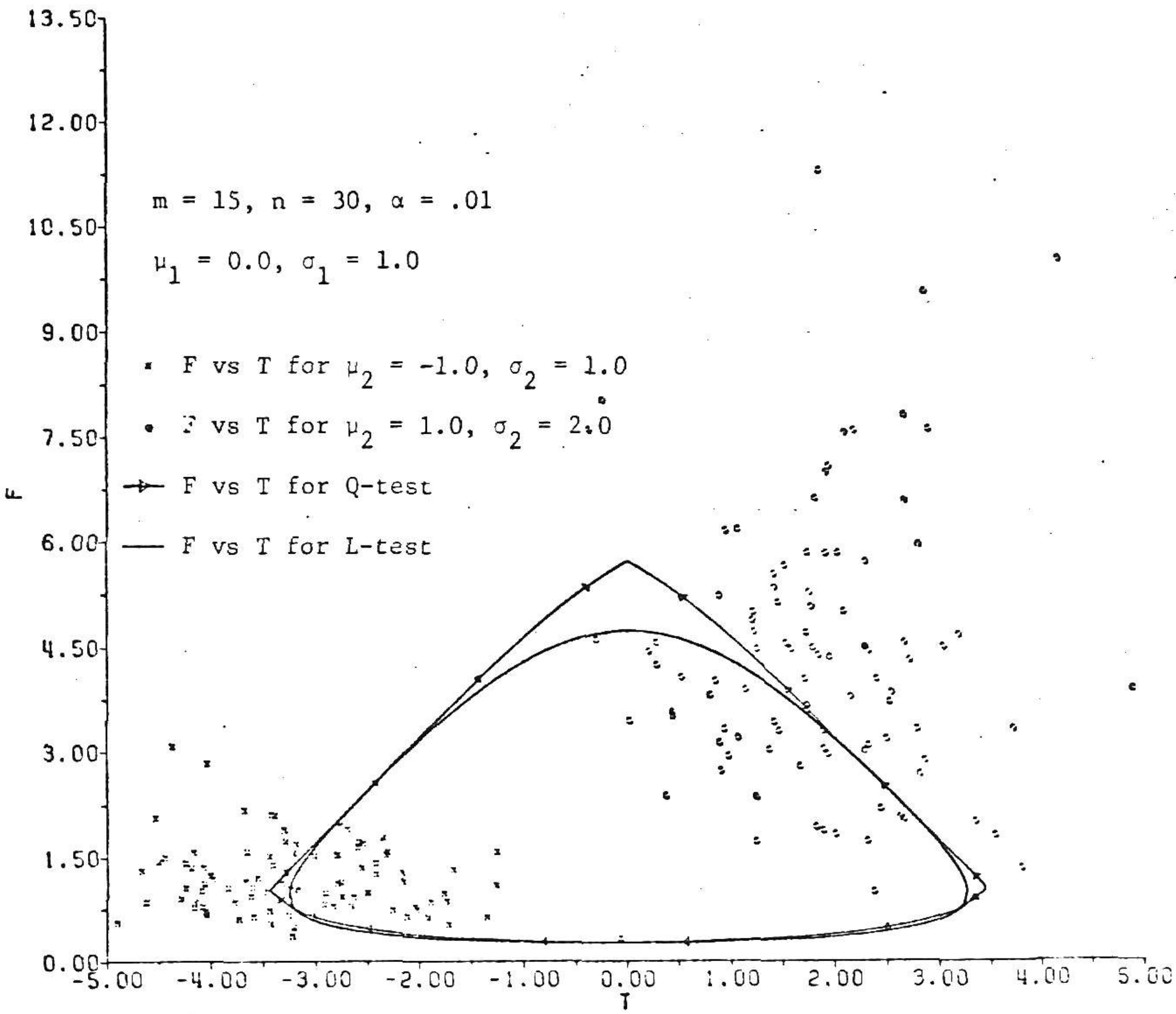


Figure 6a. ACCEPTANCE REGIONS OF Q-TEST AND L-TEST

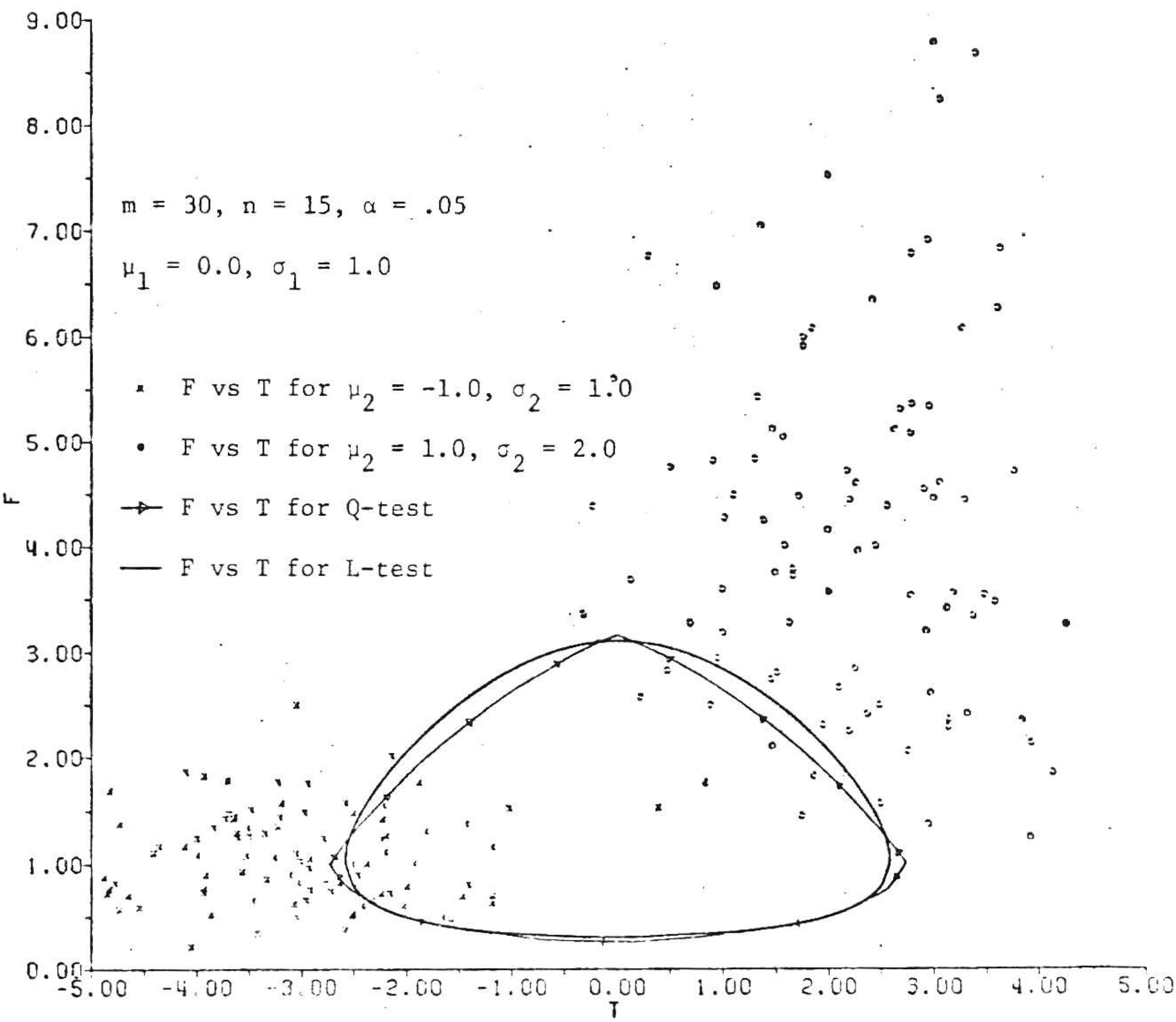
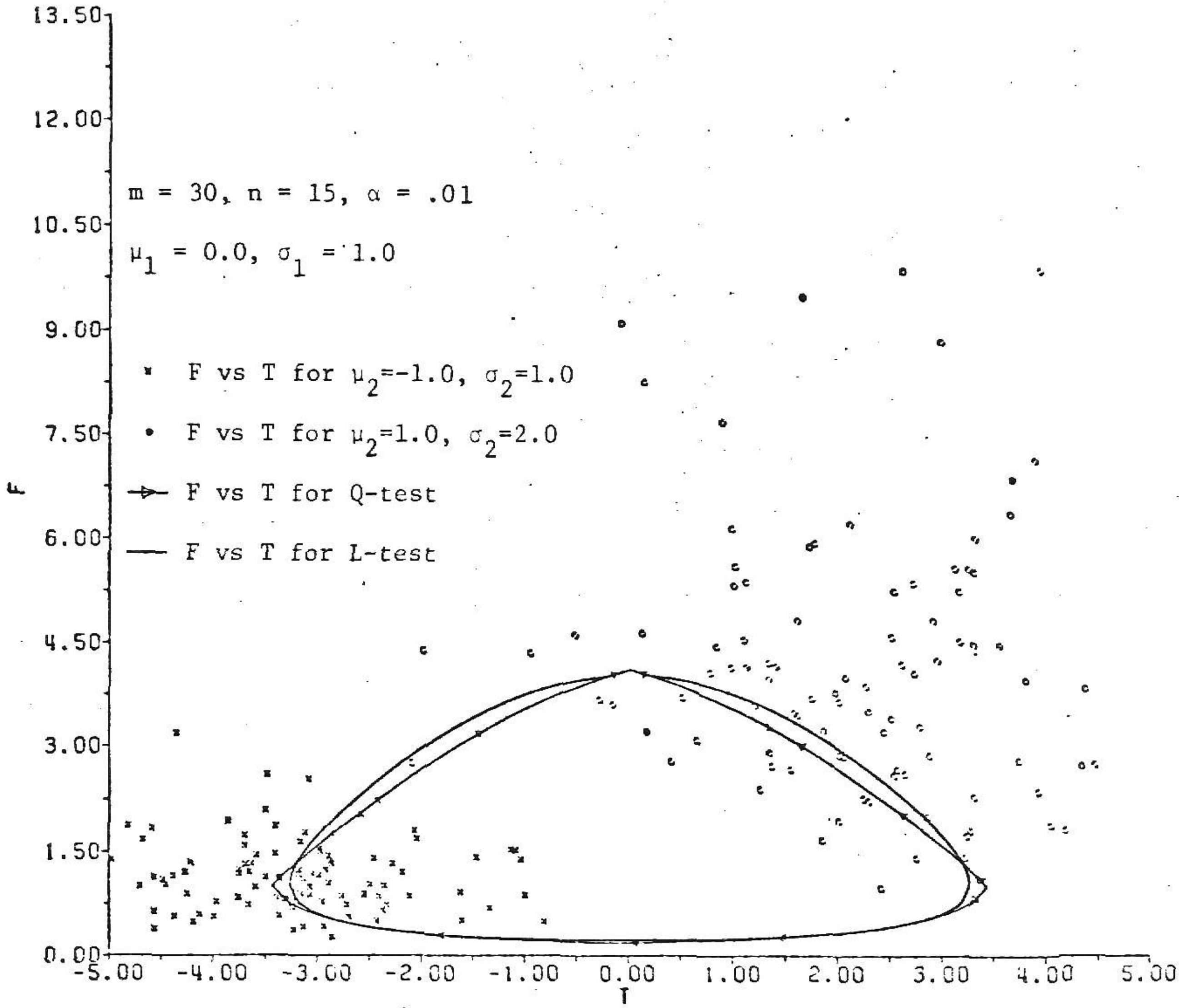


Figure 6b. ACCEPTANCE REGIONS OF Q-TEST AND L-TEST



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PART II

A P P E N D I C E S

APPENDIX A

SIMULATION OF L_o

To check a few of the $L_{.05}$ and $L_{.01}$ values from Sukhatme [6] a computer program was written. A copy of the program is at the end of this Appendix. Two independent samples $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ are generated from normal population N(0,1) and statistic L is computed in each simulation. Thus generated twenty thousand L-values are sorted and 0.5, 1.0, 2.5, 5.0, 7.5, and 10.0th percentile L-values are printed off by the program. Simulated L-values are in close agreement with those given by Sukhatme [6] for two sample sizes considered here.

(m, n)	$L_{.01}$		$L_{.05}$	
	Sukhatme's	Simulated	Sukhatme's	Simulated
(10, 10)	0.587	0.591	0.707	0.709
(5, 20)	0.632	0.627	0.742	0.743

```
      C      * THIS PROGRAM COMPUTES PERCENTILES OF LIKELIHOOD RATIO TEST STATISTIC   *
      C      * FOR TEST OF EQUALITY OF MEANS AND VARIANCES AMONG TWO NORMAL POPULA-   *
      C      * TIONS. STANDARD NORMAL RANDOM DEVIATES OF SIZES N1 AND N2 ARE GENERATED*
      C      * IN EACH SIMULATION AND L-STATISTIC COMPUTED. A MAXIMUM OF TWENTY THOU- *
      C      * SANDS SIMULATIONS ARE ALLOWED. THUS GENERATED L-VALUES ARE SORTED AND  *
      C      * 0.5, 1.0, 2.5, 5.0, 7.5 AND 10 TH PERCENTILES ARE DETERMINED.          *
             IMPLICIT REAL*8 (A-H,O-Z)
             REAL*8 M,LAMBDA(20000),LAMDA
             COMMON LAMBDA
             IX=456654333
             IJ=987654321
             M=0.D0
             S=1.D0
      C      * READS # OF SIMULATIONS AND SAMPLE SIZES.                               *
             READ(5,999) NREPS,N1,N2
         999 FORMAT(I5,2I3)
      C      * ECHO OF THE VARIABLES READ IN.                                         *
             WRITE(6,990) NREPS,N1,N2
         990 FORMAT(' ','# OF REPLICATIONS=',I5,4X,'M=',I3,4X,'N=',I3///)
             NSIZE=N1+N2
             PL=NSIZE
             PL2=PL/2.
             PN1=N1
             PK1=PN1/2.
             PN2=N2
             PK2=PN2/2.
      C      * SETS UP SPECIFIED  # OF SIMULATIONS.                                   *
             DO 10I=1,NREPS
             N=0
             DJJ=0.D0
             J=0
             CJJ=0.D0
             DO 9 K=1,NSIZE
             N=N+1
             FN=N
      C      * GENERATES STANDARD NORMAL RANDOM DEVIATES.                             *
             YFL=DEV(IX,IJ,S,M)
             IF(N-1) 1,1,2
           1 SS03  =0.D0
             SX=YFL
             GO TO 3
           2 SX=SX+YFL
      C      * POOLED SUMS OF SQUARES.                                                *
             SS03  =SS03  +((FN*YFL-SX)*(FN*YFL-SX))/DJJ
           3 DJJ=DJJ+FN+FN
             IF(K.LE.N1) GO TO 7
             J=J+1
             FJ=J
             IF(J-1)4,4,5
           4 SS02=0.D0
             SY=YFL
             GO TO 6
           5 SY=SY+YFL
      C      * SUMS OF SQUARES FOR SECOND SAMPLE.                                     *
             SS02=SS02+((FJ*YFL-SY)*(FJ*YFL-SY))/CJJ
           6 CJJ=CJJ+FJ+FJ
             GO TO 9
      C      * SUMS OF SQUARES FOR FIRST SAMPLE.                                      *
```

```
      7 SS01=SS03
      9 CONTINUE
C       * COMPUTES L-STATISTIC.                                        *
        SS01=SS01/P01
        SS01=SS01**PK1
        SS02=SS02/PM2
        SS02=SS02**PK2
        SS03=SS03/PL
        SS03=SS03**PL2
        LAMBDA(I)=SS01*SS02/SS03
     10 CONTINUE
C       * L-STATISTICS ARE SORTED.                                     *
        CALL SORT1(NREPS)
C       * DETERMINES PERCENTILES OF L-STATISTICS AND WRITES.           *
        A=.5D0
        DO 11 I=1,6
        J=NREPS*A/100.
        LAMDA=(LAMBDA(J)+LAMBDA(J+1))/2.0
        WRITE(6,980) A,LAMDA
    980 FORMAT('0',12X,F4.1,'%'///1X,'LAMBDA=',F7.4)
        K=2
        IF(I.GT.1) K=5*(I-1)
        FK=K
     11 A=.5*FK
        STOP
        END
```

```
FORTRAN IV G LEVEL  21                MAIN              DATE = 76357         10/44/48

          C      * THIS SUBROUTINE SORTS.                                                      *
                 SUBROUTINE SORT1(NREPS)
                 REAL*8 X1(20000),Y1
                 COMMON X1
                 IF (NREPS.LT.2) RETURN
                 M=NREPS
              10 I=4
                 IF (M.GT.15) I=8
                 J=M/I
                 M=2*J+1
                 NM=NREPS-M
                 DO 40 J=1,NM
                 Y1=X1(J+M)
                 J1=J+1
                 DO 20 II=1,J,M
                 I=J1-II
                 IF (X1(I).LT.Y1) GO TO 30
                 X1(I+M)=X1(I)
              20 CONTINUE
                 I=I-M
              30 X1(I+M)=Y1
              40 CONTINUE
                 IF (M.GT.1) GO TO 10
                 RETURN
                 END
```

```
FORTRAN IV G LEVEL  21                MAIN              DATE = 76357         10/44/48

          C      * THIS FUNCTION GENERATES NORMAL RANDOM DEVIATES.                             *
                 REAL FUNCTION DEV*8(IX,IJ,S,M)
                 REAL*8 S,M,YFL,DFLOAT
                 YFL=0.0D0
                 DO 1 I=1,12
                 IX=IX*65539
                 IJ=IJ*262147
               1 YFL=YFL+DFLOAT(IX+IJ)
                 DEV=(YFL*.2328306E-9)*S+M
                 RETURN
                 END
```

APPENDIX B

SIMULATION OF POWER FUNCTION OF Q-TEST AND L-TEST

A copy of the simulation program that computes powers of Q- and L-tests is at the end of this Appendix. In Part I simulation results for (m,n) = (15,15), (15,30), and (30,15) are presented in detail for the significance level .05 and .01. In those simulations the second normal population had μ_2 = 0.0, 0.25, 0.75, and 1.5 and σ_2 was varied.

Here the results of simulated power are presented where μ_2 = 0.0, 1.0, and 2.0 for various values of σ_2. First population is N(0,1). The sample sizes considered are: (1) m = n = 10, 12, and 15; (2) (m,n) = (10,12), (10,15), (12, 15), and (15,30); and (3) with interchange of m and n in (2). Each of (m,n) listed above was considered at α = .05. Significance level of .01 was considered for (m,n) = (10,10), (12,12), (10,12), and (12,10).

For the equal sample sizes the simulation results for m = n = 10, 12, and 15 and α = 0.05 are presented in Tables B1a, B2a, and B3, respectively. Corresponding plots of simulation results are in Figures B1a, B2a, and B3, respectively. For α = .01, simulation results for m = n = 10 and 12 are presented in Tables B1b and B2b, respectively; corresponding pictures are in Figures B1b and B2b, respectively. Perusal of the tables and figures clearly indicates that both Q-test and L-test have similar powers for various values of μ_2 and σ_2.

Simulation results for α = .05 and (m,n) = (10,12), (10,15), (12,15), and (15,30) are presented in Tables B4a, B5, B6, and B7, respectively. Corresponding graphs are included in Figures B4a, B5, B6, and B7, respectively. The results for α = .01 and (m,n) = (10,12) are in Table B4b and Figure B4b. For $\mu_2 = 0.0$, Q-test has greater power function of as high as 4.2% when $\sigma_2 \leq 1$. However, when $\sigma_2 > 1$, L-test becomes better to the extent of 7.2%. Reversals of power are consistent at μ_2 = 1.0 and 2.0 but the degree of difference is quite low.

With the interchange of the sample sizes the simulation results for (m,n) = (12,10), (15,10), (15,12), and (30,15) and α = .05 are summarized in Tables B8a, B9, B10, and B11, respectively. Corresponding plots of the simulation results are presented in Figures B8a, B9, B10, and B11, respectively. The results of α = .01 for (m,n) = (12,10) are in Table B8b and its plot in Figure B8b. Contrary to above, L-test for $\mu_2 = 0.0$ has higher power to the extent of 6.2% when $\sigma_2 \leq 1$ and the power is as low as 4.2% when $\sigma_2 > 1$. Similar switch in power exists for other values of μ_2 considered but the differences are low.

In order to further investigate the switch in power for unequal sample sizes, a simulation study was carried out where the first sample in each simulation was generated from N(0,4) and the second normal population, $N(\mu_2, \sigma_2^2)$, were as discussed in Part I. A case of (m,n) = (10,12) was considered. For α = .05 the results are summarized in Table B12a. Its corresponding plot is in Figure B12a (except $\mu_2 = 0.0$). Table B12b and Figure B12b (except $\mu_2 = 0.0$) contain the results for α = .01. It is apparent that switch in power is consistent with the previous results and the point of switch has now shifted around $\sigma_2 = 2.0$.

Table B1a. Simulated power functions of Q and L-tests for m = 10, n = 10, α = .05, simulations = 1000, μ_1 = 0.0, σ_1 = 1.0.

σ_2	Q	L	Q	L	Q	L
	Power of the Tests					
	μ_2 = 0.0		μ_2 = 1.00		μ_2 = 2.00	
0.2	0.995	0.994	1.000	1.000	1.000	1.000
0.4	0.618	0.640	0.965	0.967	1.000	1.000
0.6	0.209	0.213	0.762	0.760	0.999	0.998
0.8	0.064	0.067	0.576	0.578	0.990	0.991
1.0	0.047	0.044	0.415	0.424	0.955	0.963
1.2	0.058	0.054	0.376	0.378	0.924	0.931
1.4	0.112	0.106	0.402	0.402	0.900	0.903
1.6	0.173	0.182	0.477	0.474	0.883	0.881
1.8	0.301	0.324	0.506	0.507	0.886	0.875
2.5	0.637	0.657	0.745	0.742	0.938	0.934
3.2	0.861	0.877	0.892	0.903	0.953	0.953

Table B1b. Simulated power functions of Q and L-tests for m = 10, n = 10, α = .01, simulations = 1000, μ_1 = 0.0, σ_1 = 1.0.

σ_2	Q	L	Q	L	Q	L
	Power of the Tests					
	μ_2 = 0.0		μ_2 = 1.00		μ_2 = 2.00	
0.2	0.932	0.943	1.000	1.000	1.000	1.000
0.4	0.312	0.340	0.848	0.845	1.000	1.000
0.6	0.062	0.068	0.501	0.505	0.991	0.990
0.8	0.009	0.014	0.284	0.301	0.912	0.927
1.0	0.008	0.009	0.148	0.172	0.836	0.855
1.2	0.010	0.011	0.161	0.170	0.736	0.762
1.4	0.027	0.027	0.185	0.186	0.681	0.696
1.6	0.052	0.056	0.212	0.210	0.660	0.664
1.8	0.093	0.100	0.254	0.246	0.635	0.628
2.5	0.313	0.341	0.451	0.459	0.753	0.740
3.2	0.592	0.633	0.667	0.688	0.832	0.834

Table B2a. Simulated power functions of Q and L-tests for m = 12, n = 12, α = .05, simulations = 1000, μ_1 = 0.0, σ_1 = 1.0.

σ_2	Power of the Tests					
	Q	L	Q	L	Q	L
	μ_2 = 0.0		μ_2 = 1.00		μ_2 = 2.00	
0.2	0.998	0.998	1.000	1.000	1.000	1.000
0.4	0.729	0.746	0.989	0.991	1.000	1.000
0.6	0.231	0.240	0.865	0.860	1.000	1.000
0.8	0.072	0.075	0.651	0.664	0.998	0.999
1.0	0.047	0.046	0.496	0.514	0.985	0.989
1.2	0.072	0.075	0.473	0.484	0.968	0.972
1.4	0.149	0.147	0.500	0.494	0.936	0.941
1.6	0.230	0.237	0.569	0.565	0.936	0.938
1.8	0.347	0.360	0.599	0.584	0.935	0.931
2.5	0.736	0.760	0.800	0.813	0.954	0.956
3.2	0.925	0.938	0.935	0.941	0.979	0.981

Table B2b. Simulated power functions of Q and L-tests for m = 12, n = 12, α = .01, simulations = 1000, μ_1 = 0.0, σ_1 = 1.0.

σ_2	Power of the Tests					
	Q	L	Q	L	Q	L
	μ_2 = 0.0		μ_2 = 1.00		μ_2 = 2.00	
0.2	0.981	0.988	1.000	1.000	1.000	1.000
0.4	0.430	0.452	0.942	0.936	1.000	1.000
0.6	0.081	0.088	0.637	0.626	0.996	0.997
0.8	0.014	0.017	0.361	0.373	0.967	0.977
1.0	0.008	0.009	0.266	0.286	0.909	0.926
1.2	0.010	0.009	0.245	0.252	0.863	0.884
1.4	0.034	0.036	0.251	0.249	0.861	0.866
1.6	0.061	0.072	0.274	0.263	0.818	0.828
1.8	0.130	0.139	0.339	0.328	0.810	0.802
2.5	0.436	0.461	0.593	0.599	0.862	0.859
3.2	0.739	0.768	0.803	0.821	0.927	0.925
3.9	0.891	0.911	0.934	0.947	0.966	0.968
4.6	0.966	0.977	0.980	0.981	0.982	0.984

Table B3. Simulated power functions of Q and L-tests for m = 15, n = 15, $\alpha = .05$, simulations = 1000, $\mu_1 = 0.0$, $\sigma_1 = 1.0$.

σ_2	Power of the Tests					
	Q	L	Q	L	Q	L
	$\mu_2 = 0.0$		$\mu_2 = 1.00$		$\mu_2 = 2.00$	
0.2	1.000	1.000	1.000	1.000	1.000	1.000
0.4	0.853	0.860	0.998	0.998	1.000	1.000
0.6	0.319	0.332	0.949	0.945	1.000	1.000
0.8	0.102	0.106	0.776	0.777	1.000	1.000
1.0	0.057	0.056	0.586	0.596	1.000	1.000
1.2	0.092	0.093	0.573	0.583	0.992	0.994
1.4	0.156	0.158	0.619	0.606	0.994	0.993
1.6	0.262	0.273	0.681	0.675	0.981	0.982
1.8	0.445	0.451	0.750	0.746	0.985	0.983
2.5	0.842	0.867	0.911	0.912	0.987	0.988
3.2	0.971	0.972	0.989	0.991	0.997	0.997

Table B4a. Simulated power functions of Q and L-tests for m = 10, n = 12, $\alpha = .05$, simulations = 1000, $\mu_1 = 0.0$, $\sigma_1 = 1.0$.

σ_2	Power of the Tests					
	Q	L	Q	L	Q	L
	$\mu_2 = 0.0$		$\mu_2 = 1.00$		$\mu_2 = 2.00$	
0.2	0.996	0.995	1.000	1.000	1.000	1.000
0.4	0.713	0.705	0.978	0.976	1.000	1.000
0.6	0.269	0.262	0.819	0.801	1.000	1.000
0.8	0.081	0.072	0.611	0.613	0.991	0.992
1.0	0.042	0.041	0.477	0.488	0.965	0.976
1.2	0.066	0.071	0.415	0.425	0.950	0.960
1.4	0.110	0.132	0.410	0.420	0.919	0.929
1.6	0.188	0.208	0.475	0.489	0.906	0.914
1.8	0.292	0.327	0.522	0.535	0.921	0.919
2.5	0.651	0.692	0.770	0.794	0.944	0.944
3.2	0.879	0.905	0.911	0.933	0.968	0.973

Table B4b. Simulated power functions of Q and L-tests for m = 10, n = 12, $\alpha = .01$, simulations = 1000, $\mu_1 = 0.0$, $\sigma_1 = 1.0$.

	Power of the Tests					
σ_2	Q	L	Q	L	Q	L
	$\mu_2 = 0.0$		$\mu_2 = 1.00$		$\mu_2 = 2.00$	
0.2	0.980	0.982	1.000	1.000	1.000	1.000
0.4	0.449	0.452	0.923	0.912	1.000	1.000
0.6	0.074	0.076	0.571	0.571	0.998	0.998
0.8	0.021	0.017	0.324	0.335	0.959	0.963
1.0	0.010	0.011	0.220	0.227	0.853	0.872
1.2	0.016	0.019	0.174	0.188	0.772	0.801
1.4	0.028	0.031	0.183	0.188	0.727	0.741
1.6	0.041	0.051	0.223	0.230	0.720	0.724
1.8	0.095	0.114	0.253	0.261	0.702	0.709
2.5	0.316	0.361	0.504	0.534	0.787	0.782
3.2	0.651	0.702	0.684	0.729	0.854	0.866
3.9	0.824	0.868	0.854	0.888	0.921	0.936
4.6	0.910	0.938	0.948	0.960	0.959	0.972

Table B5. Simulated power functions of Q and L-tests for m = 10, n = 15, $\alpha = .05$, simulations = 1000, $\mu_1 = 0.0$, $\sigma_1 = 1.0$.

	Power of the Tests					
σ_2	Q	L	Q	L	Q	L
	$\mu_2 = 0.0$		$\mu_2 = 1.00$		$\mu_2 = 2.00$	
0.2	0.999	0.999	1.000	1.000	1.000	1.000
0.4	0.785	0.763	0.984	0.980	1.000	1.000
0.6	0.310	0.282	0.877	0.867	1.000	1.000
0.8	0.104	0.094	0.674	0.661	0.995	0.995
1.0	0.047	0.044	0.516	0.523	0.986	0.988
1.2	0.066	0.078	0.466	0.474	0.961	0.970
1.4	0.102	0.119	0.452	0.465	0.948	0.955
1.6	0.174	0.204	0.515	0.538	0.920	0.928
1.8	0.289	0.349	0.561	0.577	0.924	0.927
2.5	0.655	0.726	0.774	0.804	0.952	0.956
3.2	0.868	0.918	0.930	0.950	0.979	0.981

Table B6. Simulated power functions of Q and L-tests for m = 12, n = 15, $\alpha = .05$, simulations = 1000, $\mu_1 = 0.0$, $\sigma_1 = 1.0$.

σ_2	Power of the Tests					
	Q	L	Q	L	Q	L
	$\mu_2 = 0.0$		$\mu_2 = 1.00$		$\mu_2 = 2.00$	
0.2	1.000	1.000	1.000	1.000	1.000	1.000
0.4	0.835	0.837	0.995	0.992	1.000	1.000
0.6	0.327	0.321	0.904	0.900	1.000	1.000
0.8	0.105	0.098	0.732	0.733	0.999	1.000
1.0	0.044	0.041	0.573	0.586	0.996	0.996
1.2	0.079	0.084	0.520	0.530	0.983	0.988
1.4	0.118	0.135	0.524	0.529	0.968	0.968
1.6	0.227	0.263	0.576	0.572	0.961	0.963
1.8	0.358	0.395	0.681	0.686	0.966	0.968
2.5	0.761	0.798	0.878	0.890	0.981	0.980
3.2	0.928	0.945	0.967	0.970	0.998	0.998

Table B7. Simulated power functions of Q and L-tests for m = 15, n = 30, $\alpha = .05$, simulations = 1000, $\mu_1 = 0.0$, $\sigma_1 = 1.0$.

σ_2	Power of the Tests					
	Q	L	Q	L	Q	L
	$\mu_2 = 0.0$		$\mu_2 = 1.00$		$\mu_2 = 2.00$	
0.2	1.000	1.000	1.000	1.000	1.000	1.000
0.4	0.948	0.938	1.000	1.000	1.000	1.000
0.6	0.536	0.494	0.990	0.987	1.000	1.000
0.8	0.144	0.123	0.904	0.899	1.000	1.000
1.0	0.053	0.057	0.790	0.791	1.000	1.000
1.2	0.073	0.081	0.698	0.710	1.000	1.000
1.4	0.180	0.218	0.717	0.731	0.996	0.996
1.6	0.282	0.354	0.752	0.761	0.997	0.997
1.8	0.515	0.584	0.816	0.834	0.996	0.996
2.5	0.922	0.949	0.971	0.981	0.998	0.999
3.2	0.993	0.998	0.999	0.990	1.000	1.000

Table B8a. Simulated power functions of Q and L-tests for m = 12, n = 10, α = .05, simulations = 1000, μ_1 = 0.0, σ_1 = 1.0.

σ_2	Power of the Tests					
	Q	L	Q	L	Q	L
	μ_2 = 0.0		μ_2 = 1.00		μ_2 = 2.00	
0.2	0.993	0.995	1.000	1.000	1.000	1.000
0.4	0.616	0.660	0.977	0.975	1.000	1.000
0.6	0.224	0.243	0.782	0.785	1.000	1.000
0.8	0.067	0.075	0.594	0.603	0.991	0.994
1.0	0.046	0.043	0.443	0.463	0.963	0.967
1.2	0.065	0.064	0.422	0.418	0.955	0.962
1.4	0.127	0.120	0.456	0.441	0.938	0.938
1.6	0.226	0.219	0.528	0.506	0.926	0.916
1.8	0.351	0.347	0.588	0.566	0.922	0.912
2.5	0.704	0.697	0.820	0.812	0.950	0.946
3.2	0.896	0.897	0.940	0.942	0.976	0.974

Table B8b. Simulated power functions of Q and L-tests for m = 12, n = 10, α = .01, simulations = 1000, μ_1 = 0.0, σ_1 = 1.0.

σ_2	Power of the Tests					
	Q	L	Q	L	Q	L
	μ_2 = 0.0		μ_2 = 1.00		μ_2 = 2.00	
0.2	0.958	0.972	0.999	0.999	1.000	1.000
0.4	0.329	0.374	0.882	0.884	1.000	1.000
0.6	0.060	0.072	0.528	0.535	0.997	0.997
0.8	0.013	0.017	0.298	0.316	0.948	0.962
1.0	0.008	0.006	0.235	0.250	0.864	0.887
1.2	0.013	0.011	0.192	0.204	0.801	0.824
1.4	0.032	0.032	0.224	0.214	0.770	0.777
1.6	0.056	0.054	0.274	0.263	0.754	0.749
1.8	0.121	0.125	0.319	0.291	0.755	0.735
2.5	0.424	0.427	0.574	0.569	0.826	0.815
3.2	0.745	0.751	0.782	0.780	0.904	0.893
3.9	0.898	0.906	0.910	0.912	0.938	0.938
4.6	0.951	0.954	0.966	0.964	0.975	0.976

Table B9. Simulated power functions of Q and L-tests for m = 15, n = 10, α = .05, simulations = 1000, μ_1 = 0.0, σ_1 = 1.0.

σ_2	Q	L	Q	L	Q	L
	Power of the Tests					
	μ_2 = 0.0		μ_2 = 1.00		μ_2 = 2.00	
0.2	0.996	0.998	1.000	1.000	1.000	1.000
0.4	0.659	0.738	0.985	0.984	1.000	1.000
0.6	0.207	0.257	0.832	0.840	1.000	1.000
0.8	0.078	0.087	0.618	0.641	0.995	0.997
1.0	0.043	0.047	0.522	0.530	0.983	0.986
1.2	0.069	0.066	0.521	0.520	0.971	0.973
1.4	0.154	0.137	0.527	0.503	0.956	0.953
1.6	0.264	0.231	0.608	0.577	0.942	0.937
1.8	0.413	0.376	0.658	0.628	0.946	0.931
2.5	0.777	0.761	0.872	0.861	0.975	0.965
3.2	0.953	0.949	0.965	0.959	0.983	0.979

Table B10. Simulated power functions of Q and L-tests for m = 15, n = 12, α = .05, simulations = 1000, μ_1 = 0.0, σ_1 = 1.0.

σ_2	Q	L	Q	L	Q	L
	Power of the Tests					
	μ_2 = 0.0		μ_2 = 1.00		μ_2 = 2.00	
0.2	1.000	1.000	1.000	1.000	1.000	1.000
0.4	0.766	0.806	0.995	0.995	1.000	1.000
0.6	0.275	0.303	0.879	0.879	1.000	1.000
0.8	0.095	0.106	0.700	0.723	0.998	0.999
1.0	0.054	0.056	0.559	0.572	0.994	0.996
1.2	0.089	0.089	0.536	0.537	0.987	0.989
1.4	0.149	0.137	0.554	0.546	0.971	0.971
1.6	0.287	0.279	0.632	0.612	0.972	0.968
1.8	0.429	0.425	0.720	0.704	0.969	0.965
2.5	0.842	0.839	0.912	0.904	0.986	0.987
3.2	0.952	0.951	0.970	0.967	0.994	0.992

Table B11. Simulated power functions of Q and L-tests for m = 30, n = 15, α = .05, simulations = 1000, μ_1 = 0.0, σ_1 = 1.0.

σ_2	Q	L	Q	L	Q	L
	μ_2 = 0.0		μ_2 = 1.00		μ_2 = 2.00	
0.2	1.000	1.000	1.000	1.000	1.000	1.000
0.4	0.907	0.933	1.000	1.000	1.000	1.000
0.6	0.393	0.455	0.987	0.989	1.000	1.000
0.8	0.090	0.104	0.872	0.889	1.000	1.000
1.0	0.045	0.046	0.783	0.788	1.000	1.000
1.2	0.099	0.081	0.752	0.739	0.998	0.998
1.4	0.295	0.264	0.786	0.764	0.999	1.000
1.6	0.476	0.434	0.851	0.819	0.999	0.999
1.8	0.667	0.627	0.878	0.855	0.998	0.997
2.5	0.960	0.951	0.986	0.979	0.998	0.998
3.2	0.997	0.995	0.996	0.996	0.999	0.999

Table B12a. Simulated power functions of Q and L-tests for m = 10, n = 12, α = .05, simulations = 1000, μ_1 = 0.0, σ_1 = 2.0.

σ_2	Q	L	Q	L	Q	L	Q	L
	μ_2 = 0.0		μ_2 = 0.25		μ_2 = 0.75		μ_2 = 1.50	
0.2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.4	0.997	0.997	0.991	0.991	0.997	0.997	1.000	1.000
0.6	0.924	0.918	0.924	0.924	0.944	0.943	0.990	0.986
0.8	0.733	0.716	0.721	0.711	0.811	0.808	0.936	0.924
1.0	0.440	0.435	0.468	0.460	0.589	0.574	0.837	0.821
1.2	0.241	0.233	0.261	0.249	0.355	0.352	0.669	0.647
1.4	0.136	0.129	0.160	0.143	0.210	0.202	0.520	0.508
1.6	0.082	0.078	0.075	0.071	0.149	0.147	0.399	0.394
1.8	0.057	0.052	0.068	0.067	0.119	0.115	0.320	0.323
2.5	0.076	0.084	0.074	0.084	0.105	0.111	0.274	0.279
3.2	0.200	0.221	0.170	0.194	0.208	0.227	0.346	0.349
3.9	0.343	0.387	0.343	0.385	0.422	0.465	0.483	0.516
4.6	0.555	0.604	0.552	0.607	0.578	0.625	0.627	0.662

Table B12b. Simulated power functions of Q and L-tests for m = 10, n = 12, α = .01, simulations = 1000, μ_1 = 0.0, σ_1 = 2.0.

	Power of the Tests							
σ_2	Q	L	Q	L	Q	L	Q	L
	μ_2 = 0.0		μ_2 = 0.25		μ_2 = 0.75		μ_2 = 1.50	
0.2	1.000	1.000	0.999	0.999	0.999	0.999	1.000	1.000
0.4	0.974	0.974	0.973	0.971	0.986	0.987	0.999	0.999
0.6	0.767	0.771	0.767	0.778	0.838	0.831	0.958	0.952
0.8	0.450	0.456	0.431	0.441	0.548	0.538	0.810	0.788
1.0	0.202	0.201	0.205	0.206	0.323	0.307	0.597	0.570
1.2	0.090	0.086	0.093	0.090	0.150	0.133	0.404	0.380
1.4	0.034	0.034	0.054	0.051	0.070	0.064	0.243	0.239
1.6	0.021	0.018	0.016	0.014	0.046	0.041	0.173	0.171
1.8	0.013	0.014	0.008	0.008	0.030	0.029	0.115	0.114
2.5	0.020	0.023	0.018	0.017	0.027	0.028	0.100	0.102
3.2	0.059	0.068	0.042	0.054	0.065	0.070	0.135	0.139
3.9	0.114	0.145	0.116	0.140	0.170	0.197	0.218	0.228
4.6	0.247	0.291	0.244	0.300	0.262	0.317	0.320	0.347

Figure B1a. POWER OF Q-TEST AND L-TEST

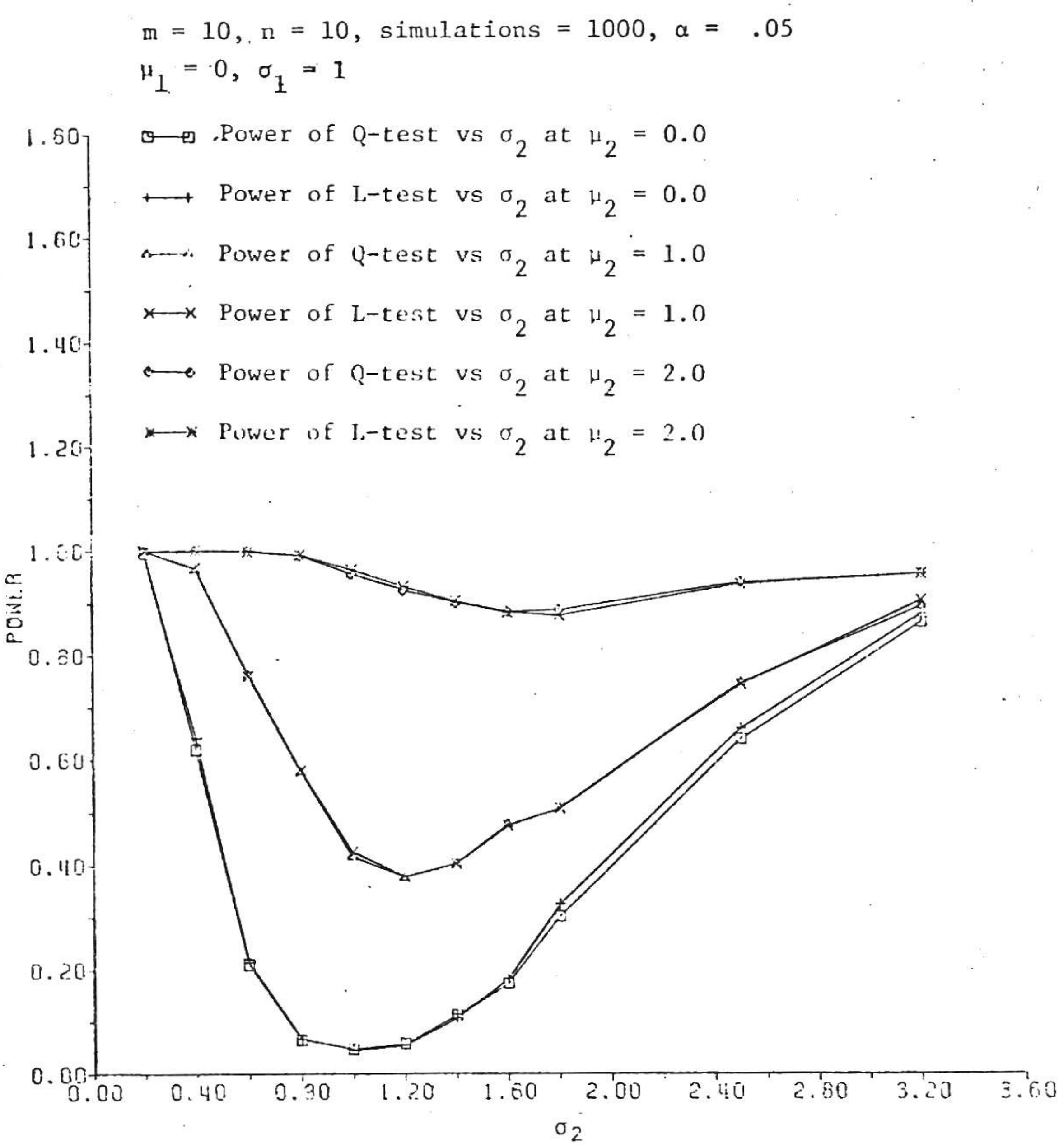


Figure B1b. POWER OF Q-TEST AND L-TEST

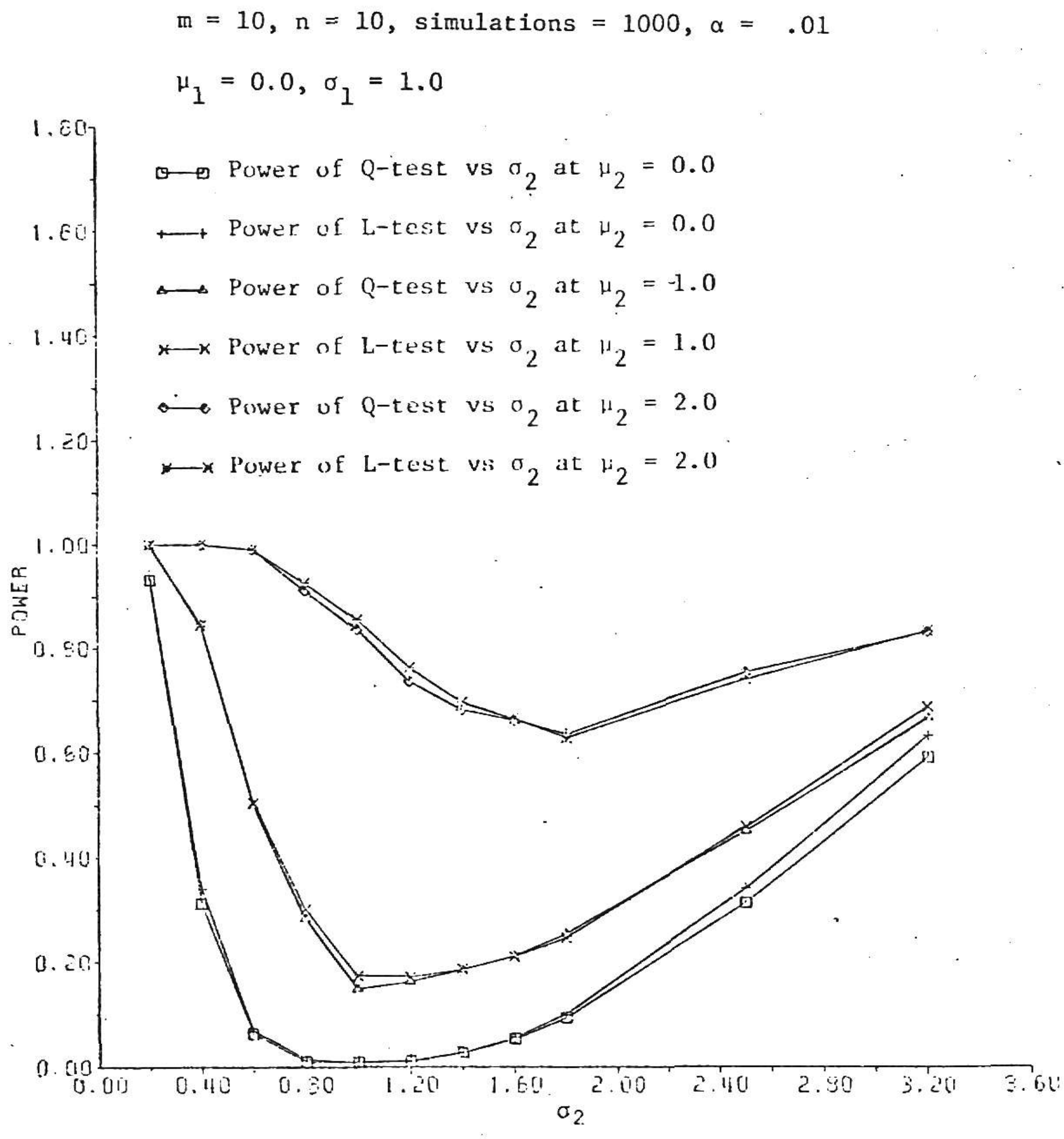


Figure B2a. POWER OF Q-TEST AND L-TEST

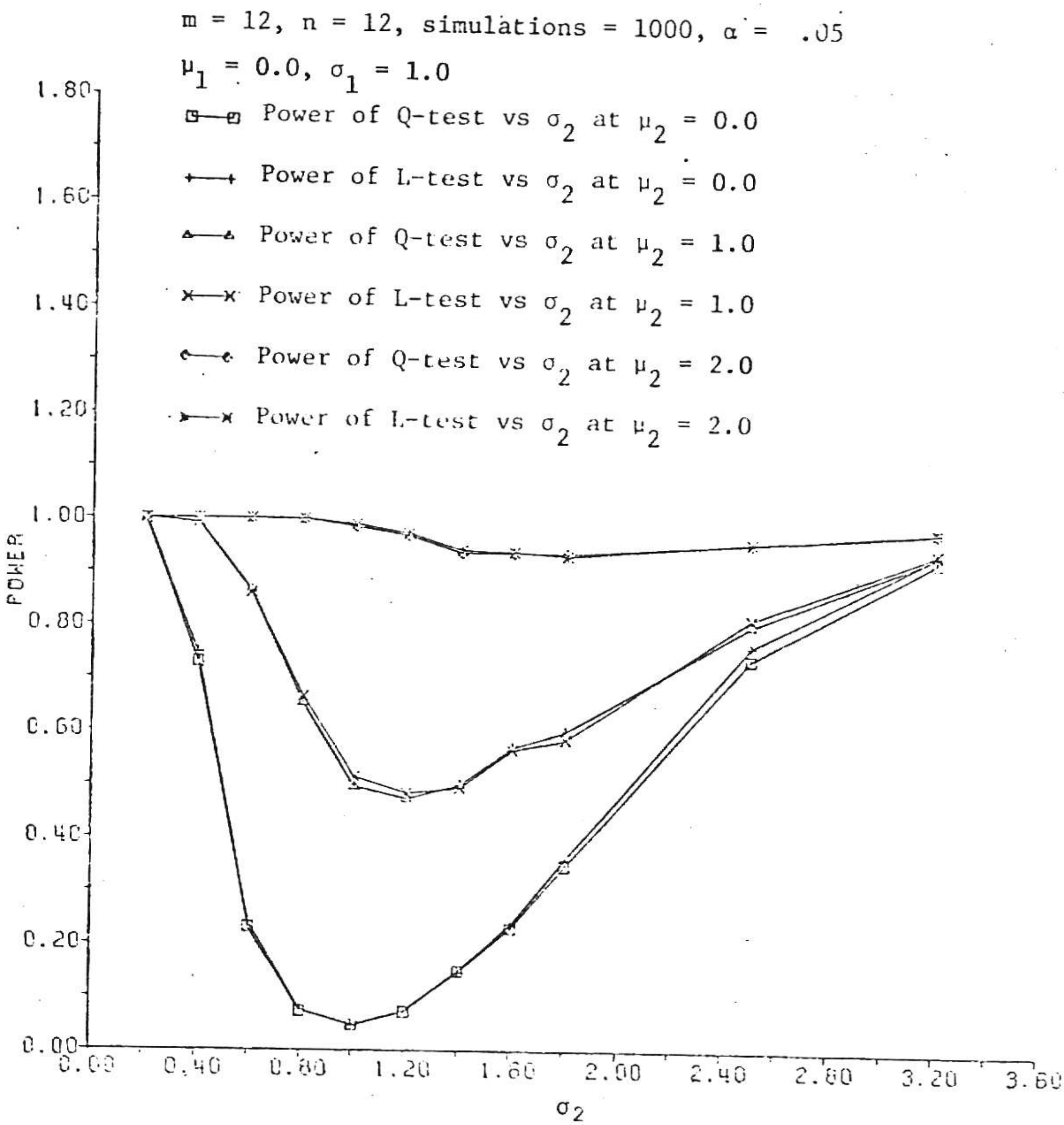


Figure B2b. POWER OF Q-TEST AND L-TEST

$m = 12$, $n = 12$, simulations = 1000, $\alpha = .01$

$\mu_1 = 0.0$, $\sigma_1 = 1.0$

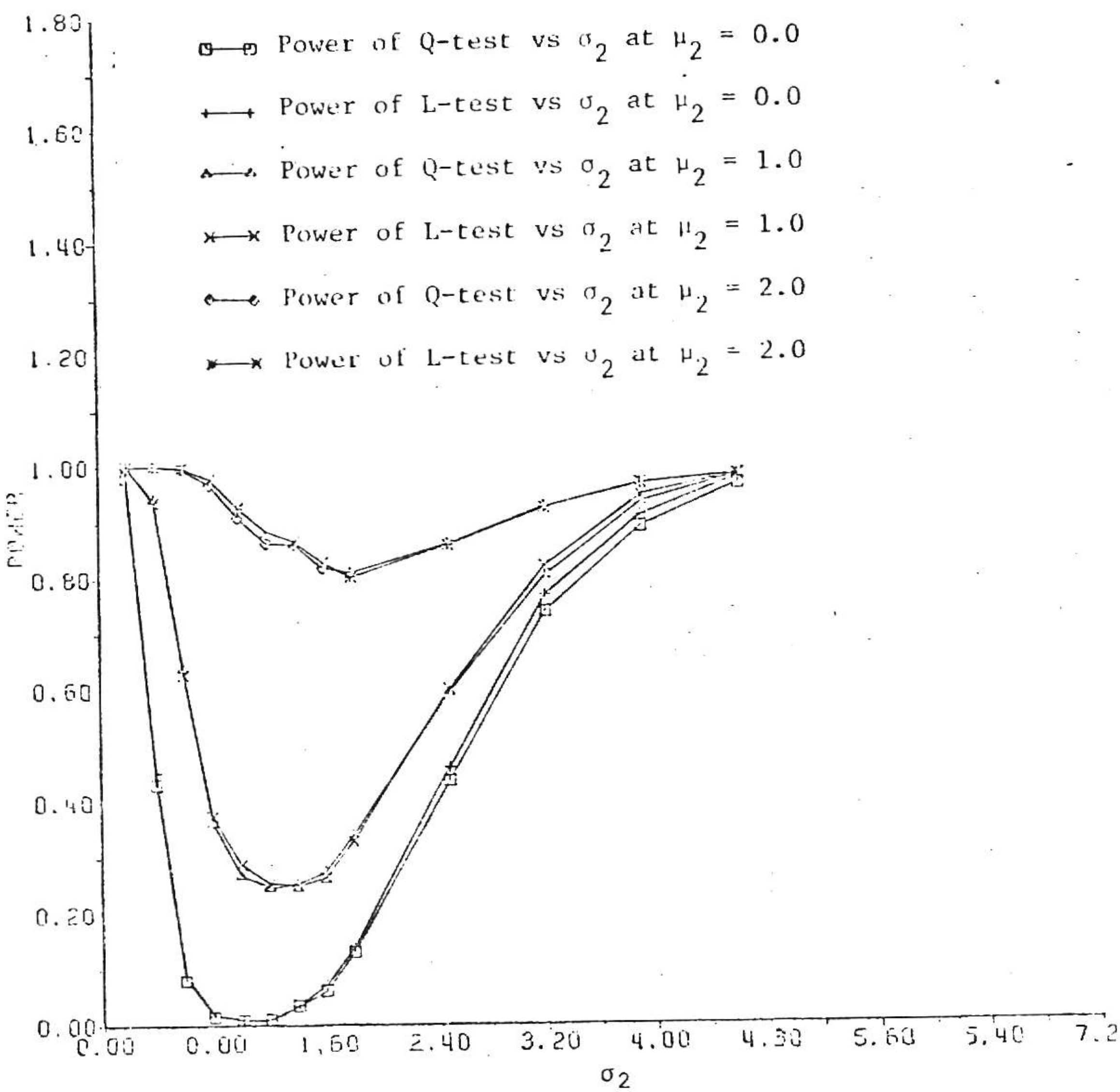


Figure B3. POWER OF Q-TEST AND L-TEST

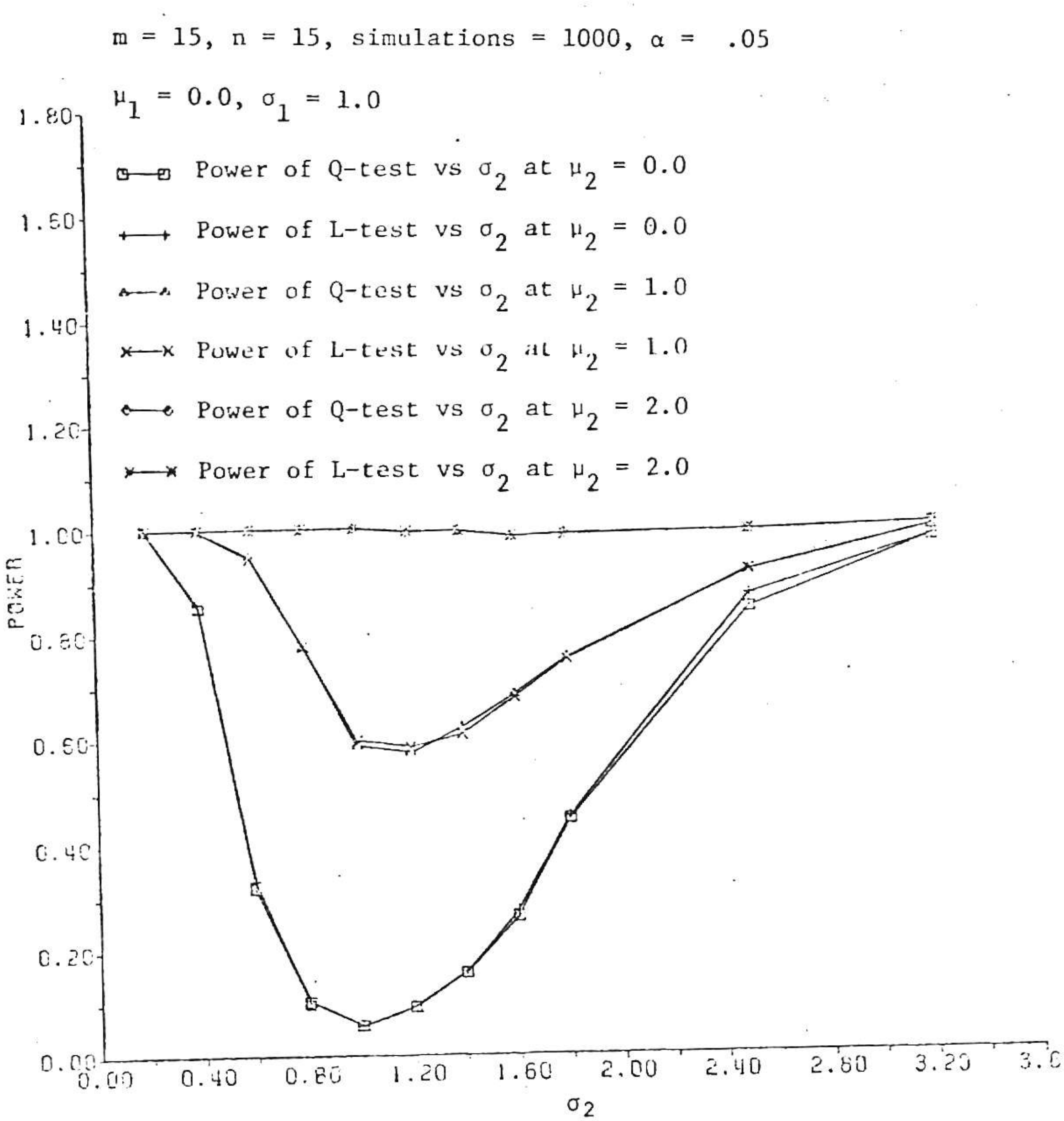


Figure B4a. POWER OF Q-TEST AND L-TEST

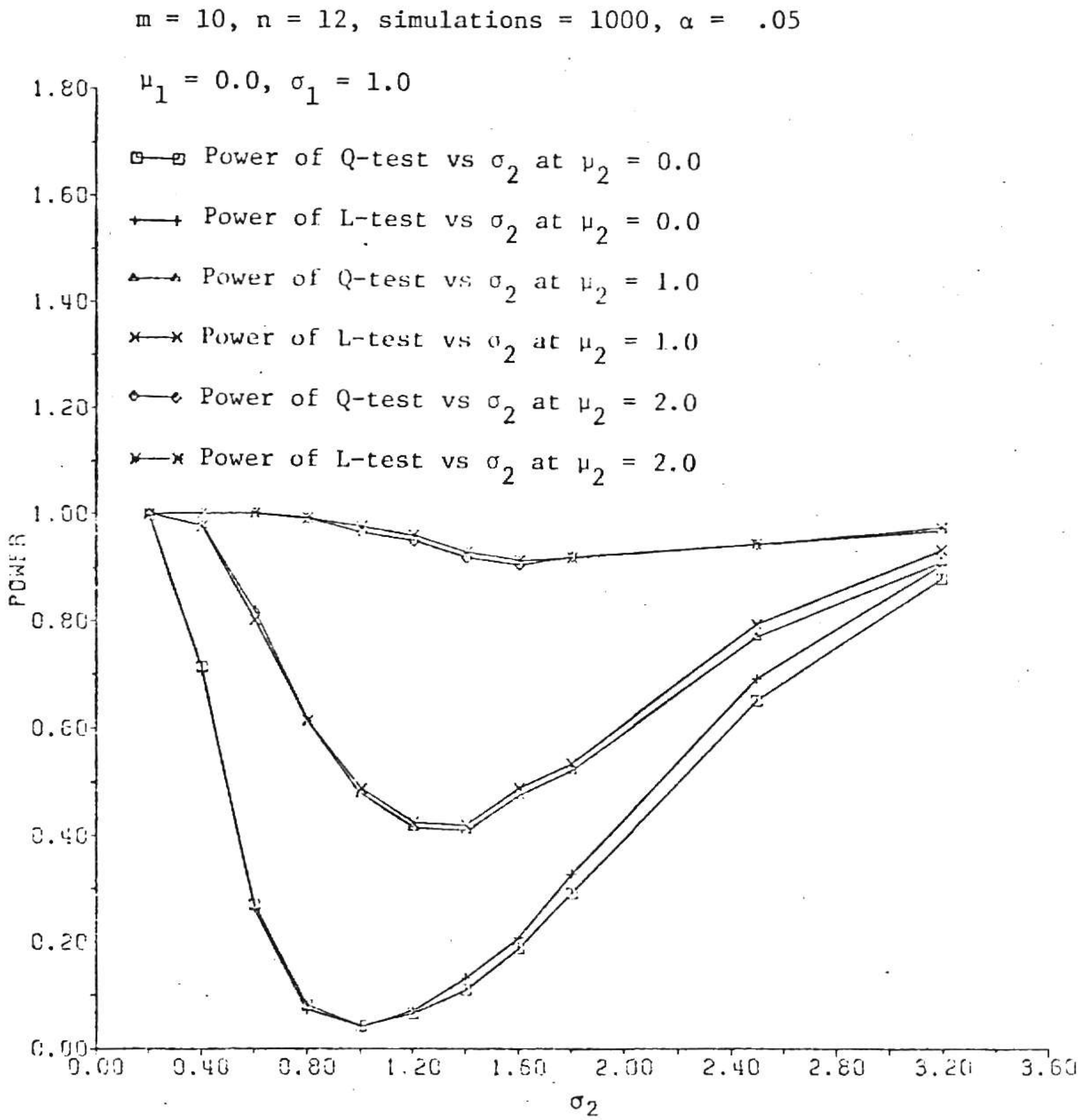


Figure B4b. POWER OF Q-TEST AND L-TEST

$m = 10$, $n = 12$, simulations = 1000, $\alpha = .01$

$\mu_1 = 0.0$, $\sigma_1 = 1.0$

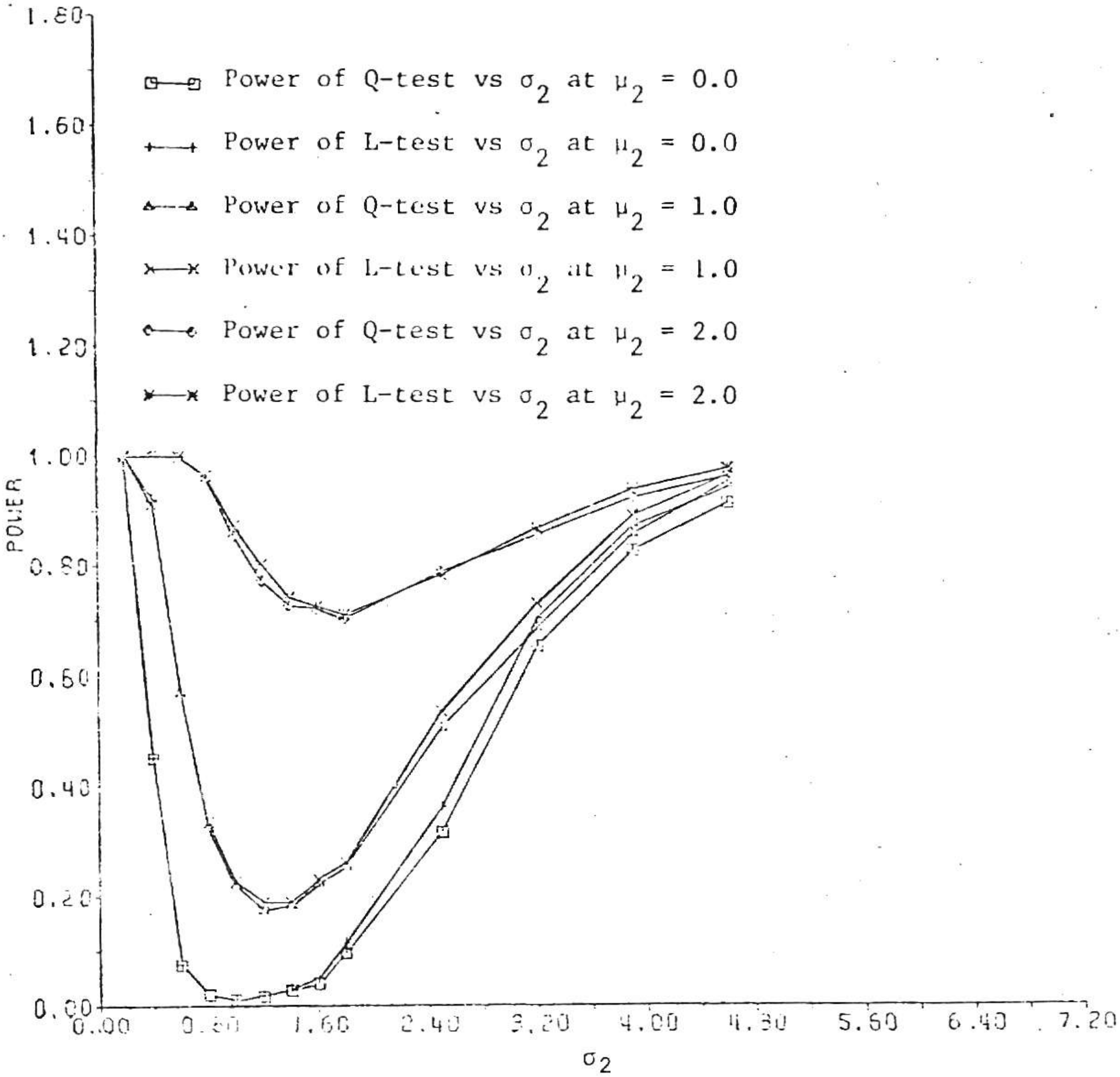


Figure B5. POWER OF Q-TEST AND L-TEST

$m = 10$, $n = 15$, simulations = 1000, $\alpha =$ .05

$\mu_1 = 0.0$, $\sigma_1 = 1.0$

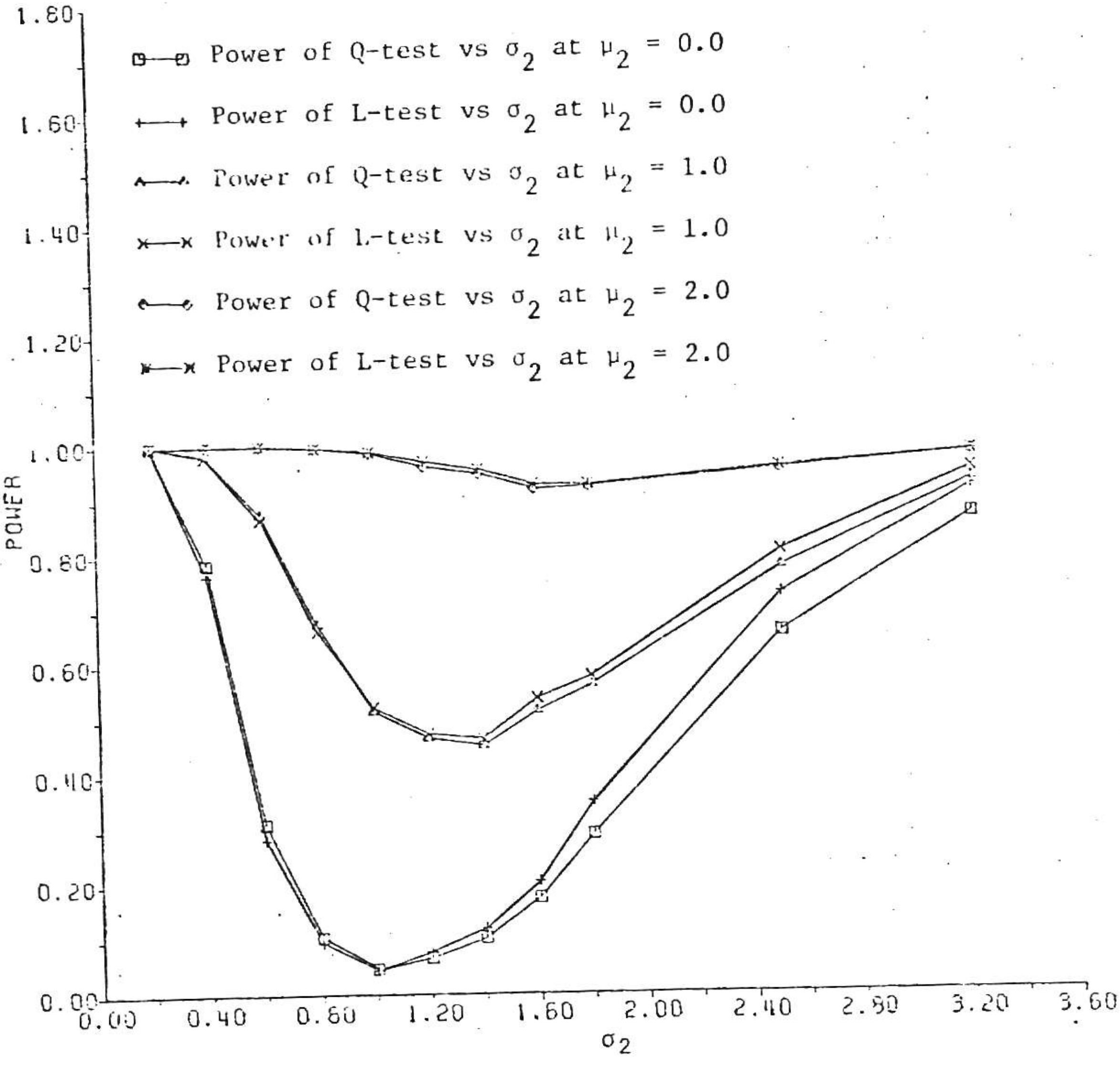


Figure B6. POWER OF Q-TEST AND L-TEST

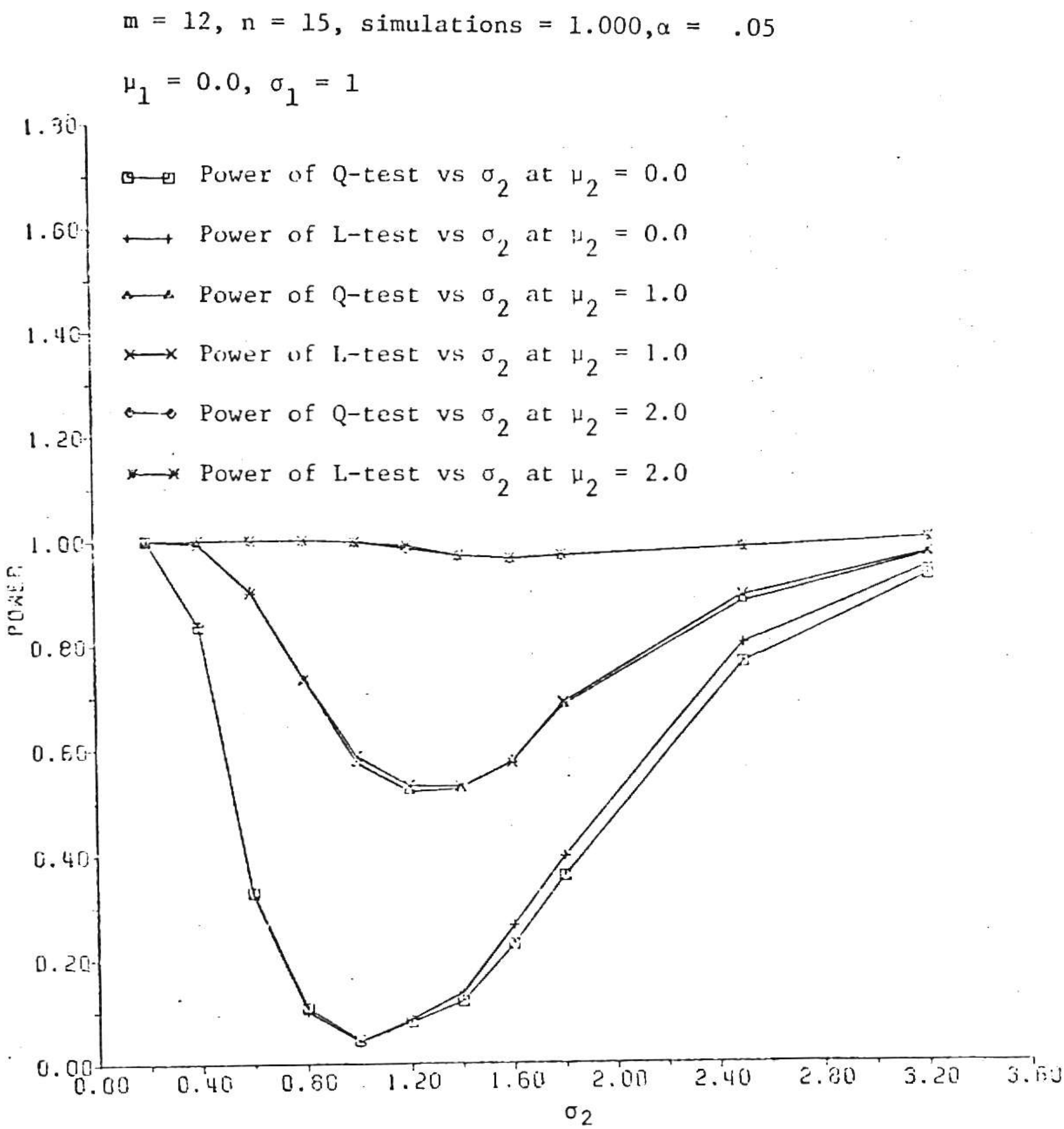


Figure B7. POWER OF Q-TEST AND L-TEST

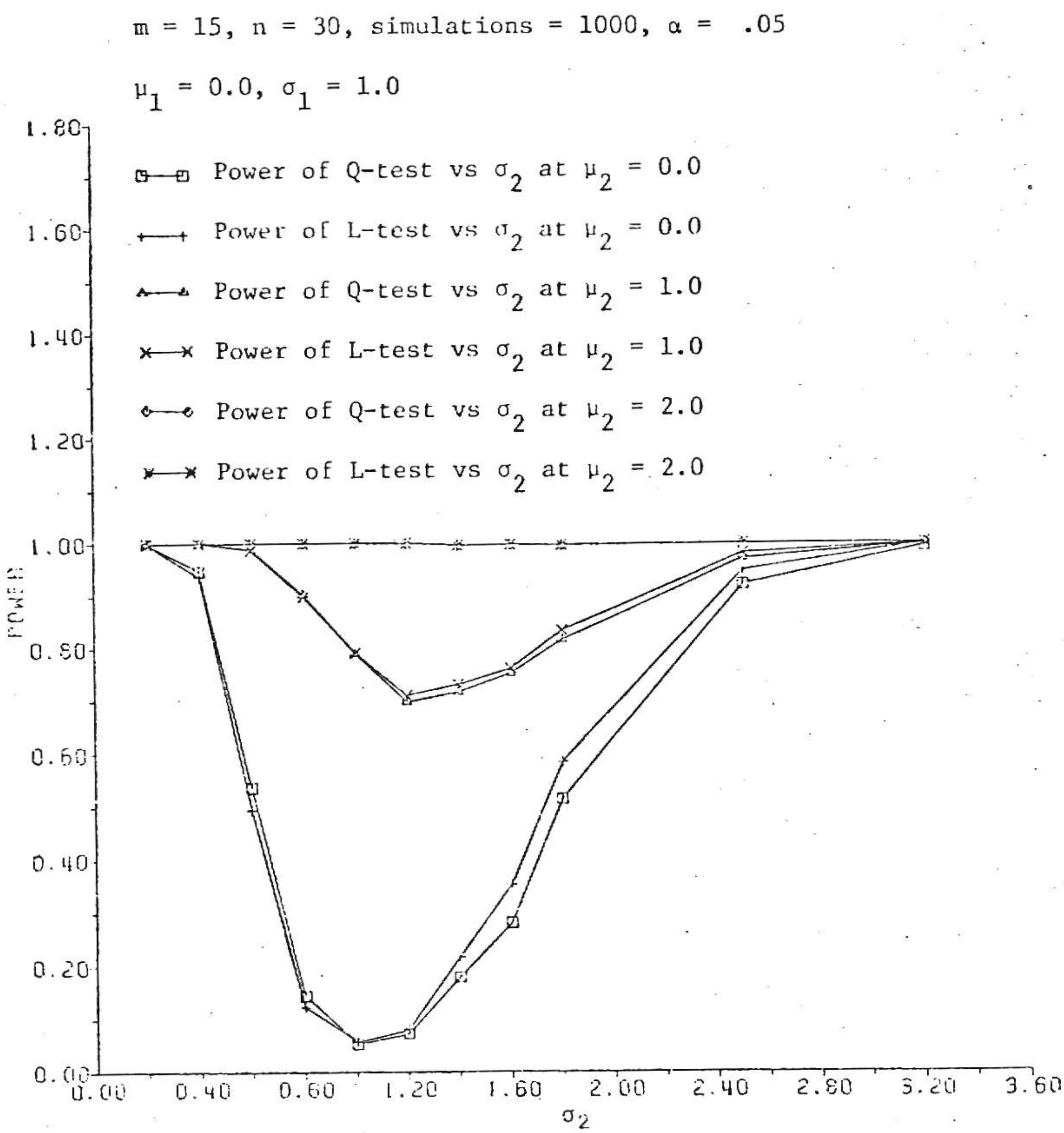


Figure B8a. POWER OF Q-TEST AND L-TEST

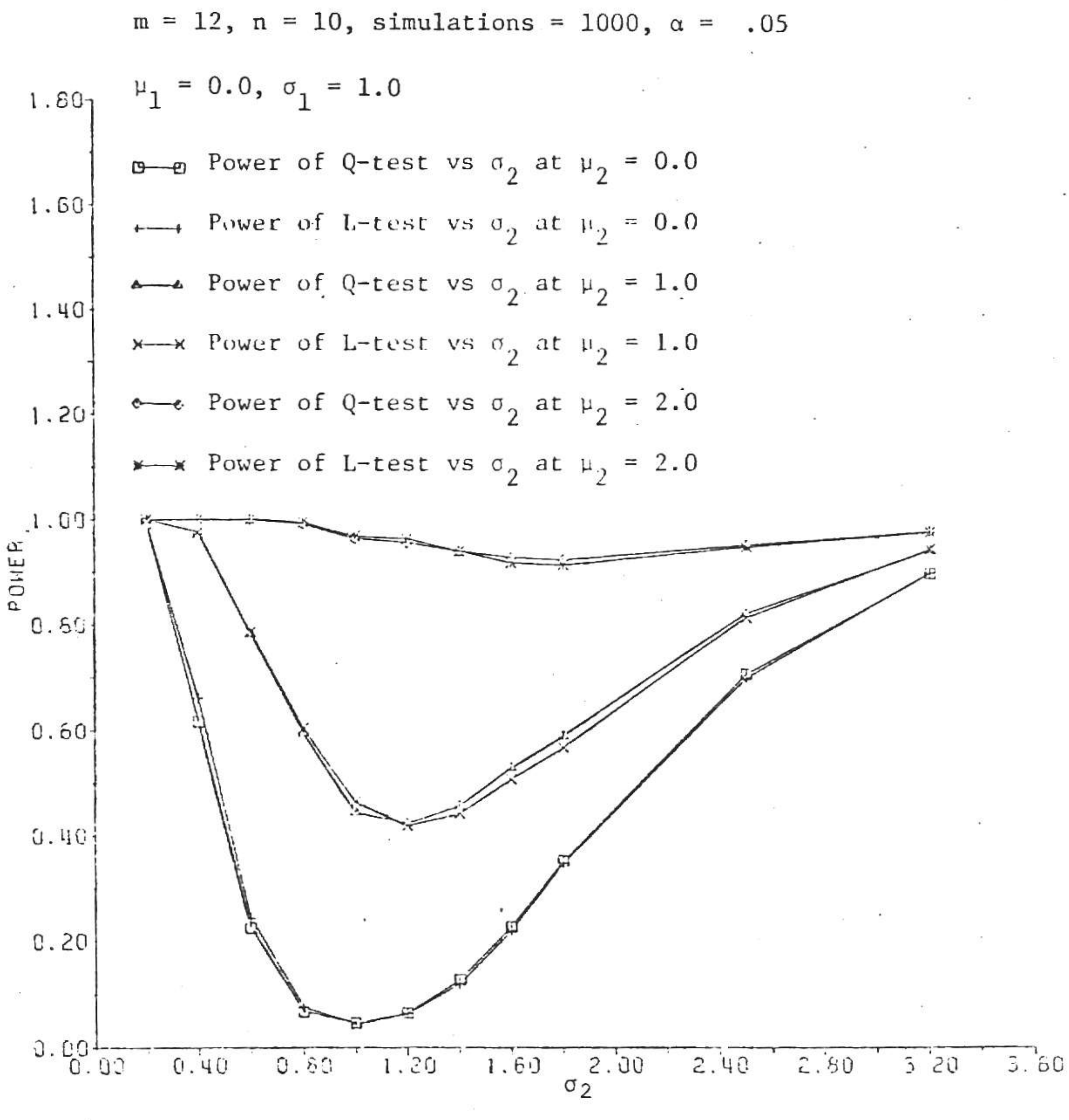


Figure B8b. POWER OF Q-TEST AND L-TEST

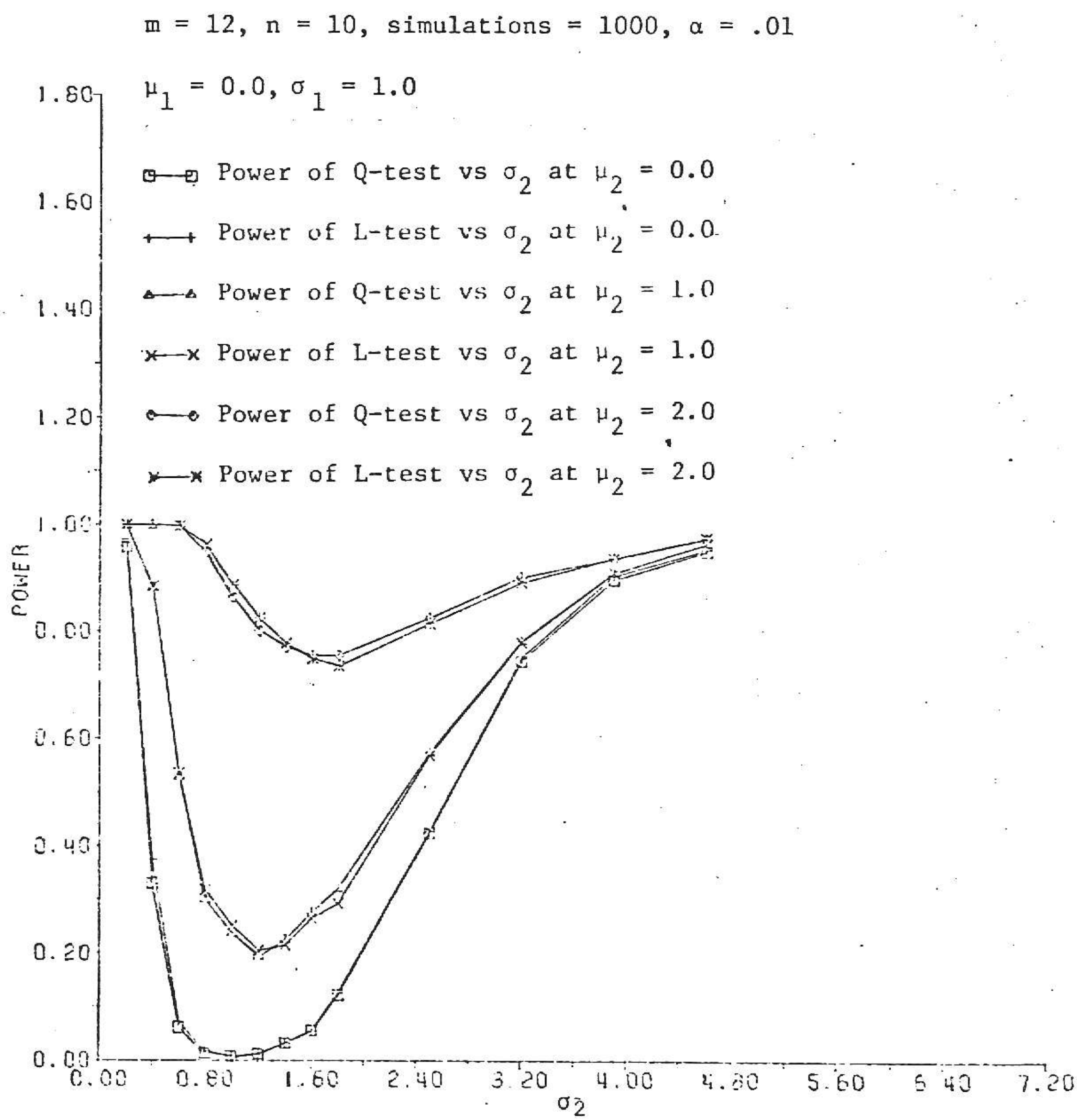


Figure B9. POWER OF Q-TEST AND L-TEST

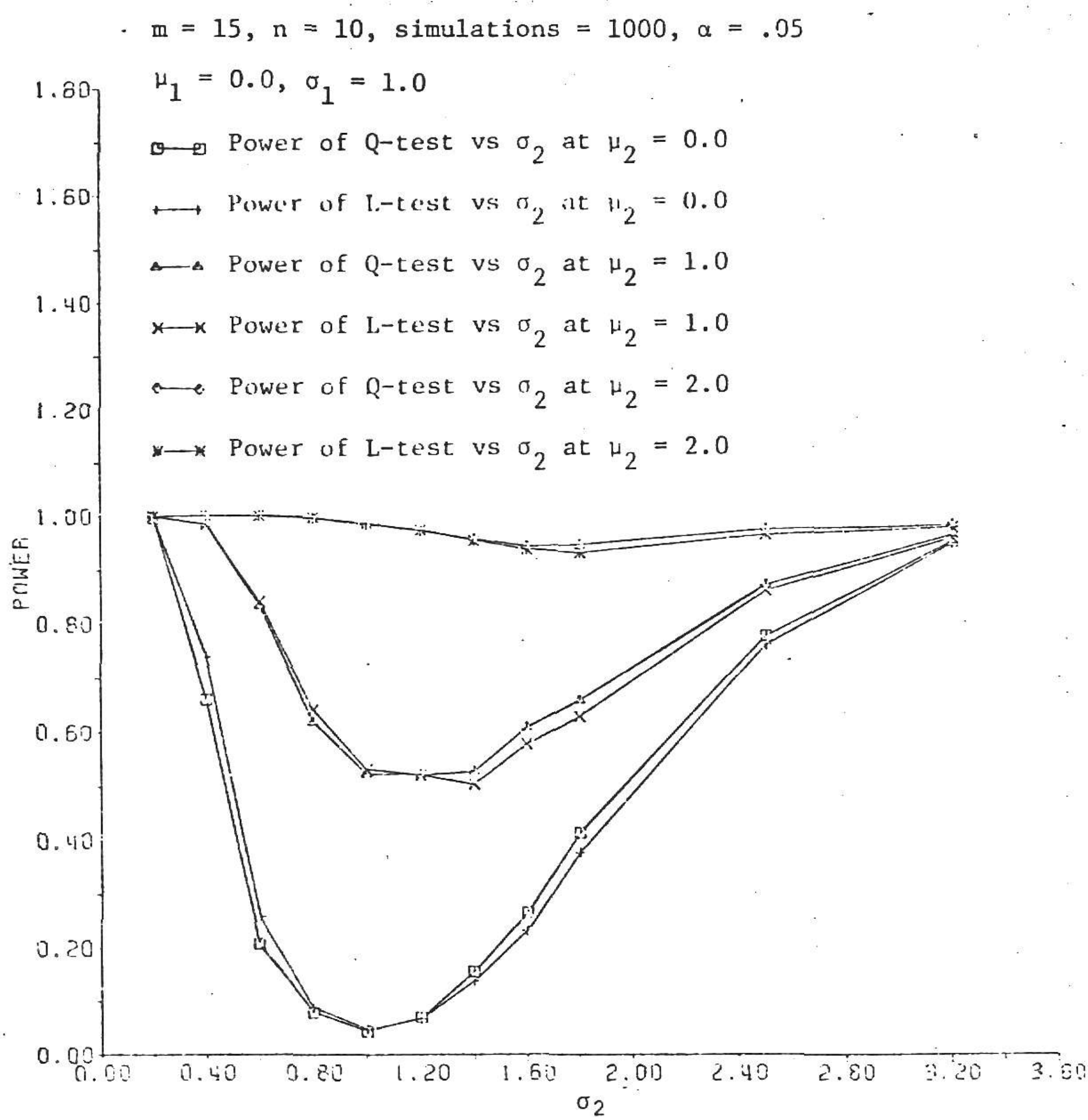


Figure B10. POWER OF Q-TEST AND L-TEST

$m = 15$, $n = 12$, simulations = 1000, $\alpha = .05$

$\mu_1 = 0.0$, $\sigma_1 = 1.0$

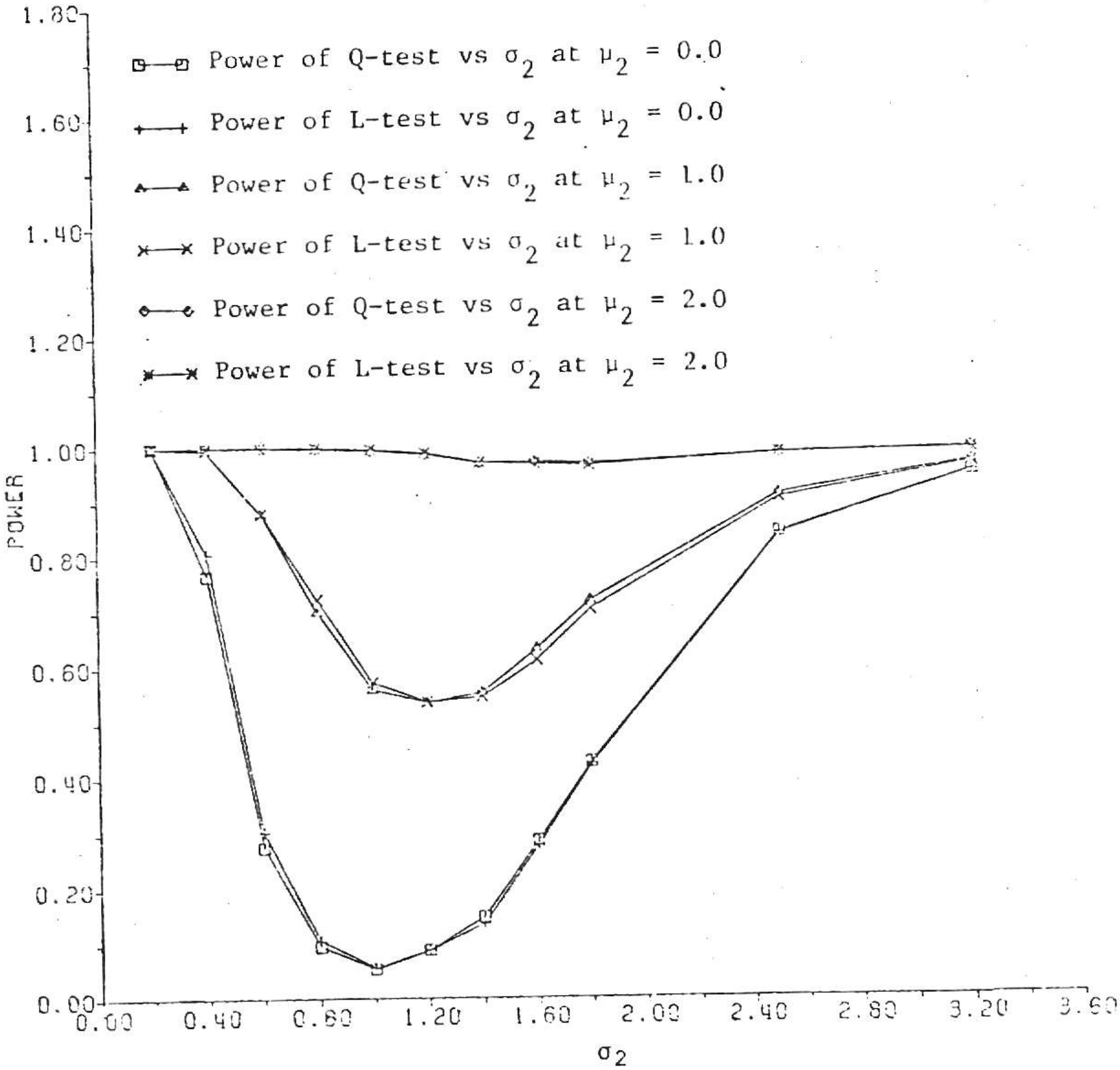


Figure B11. POWER OF Q-TEST AND L-TEST

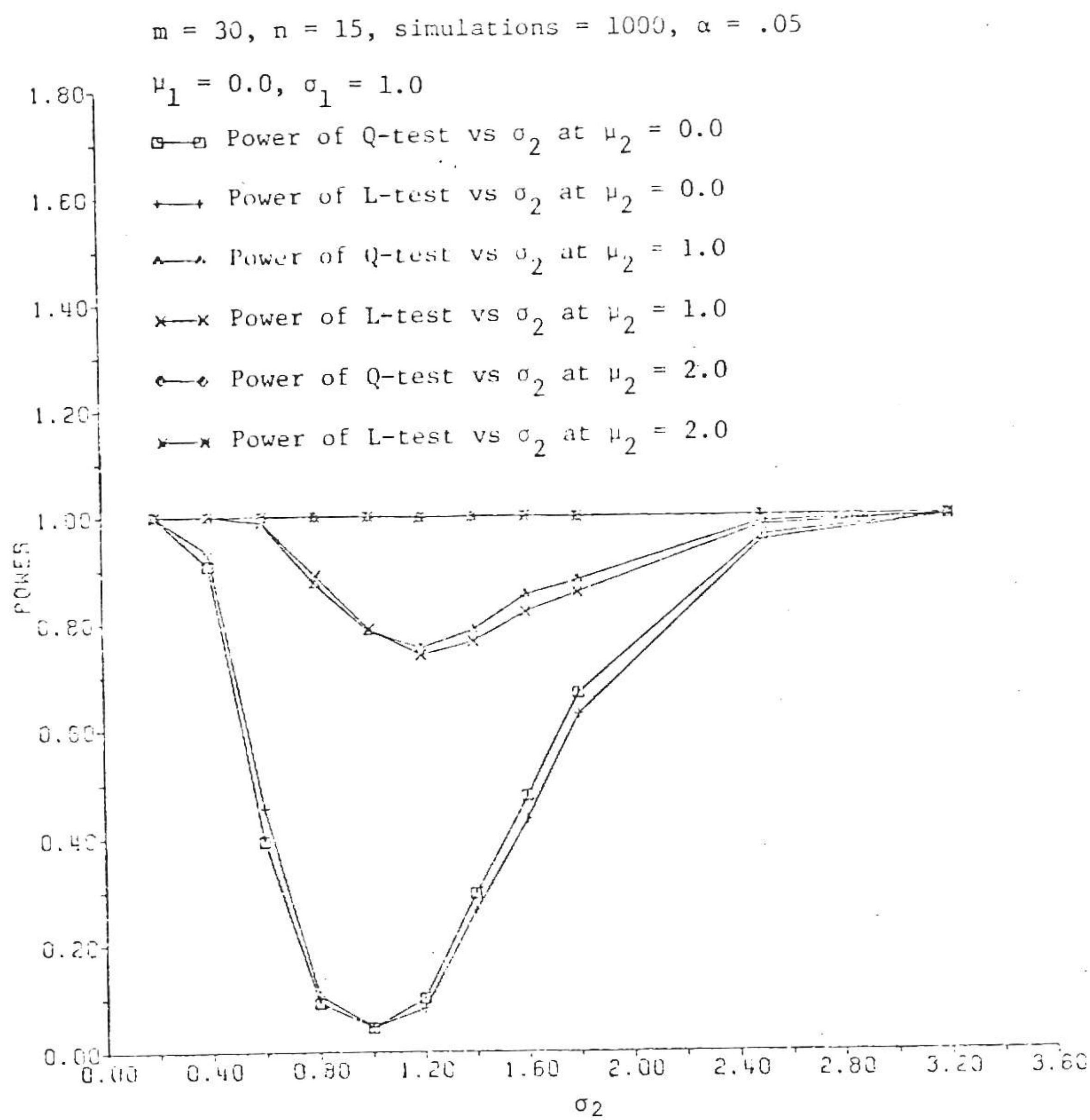


Figure B12a. POWER OF Q-TEST AND L-TEST

m = 10, n = 12, simulations = 1000, α = .05

μ_1 = 0.0, σ_1 = 2.0

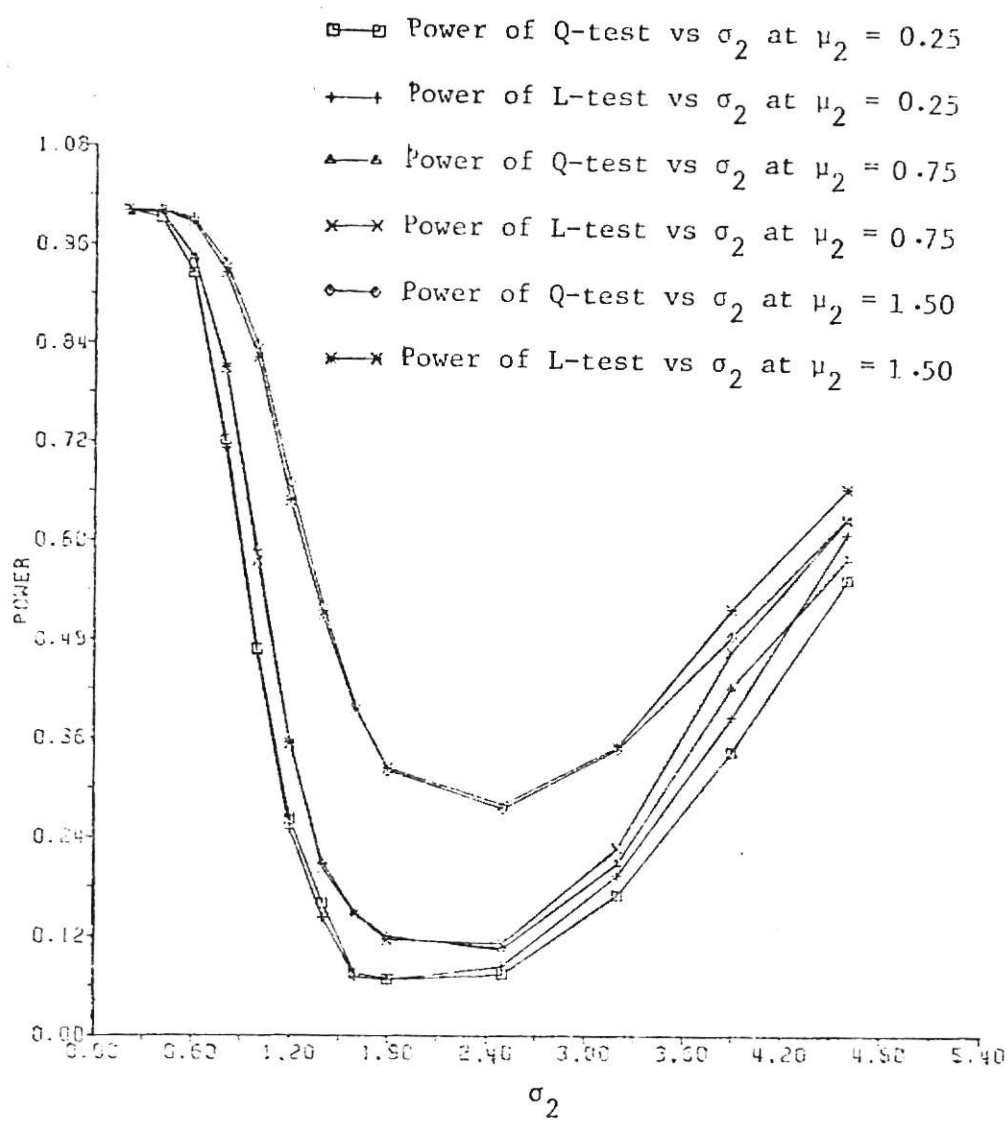
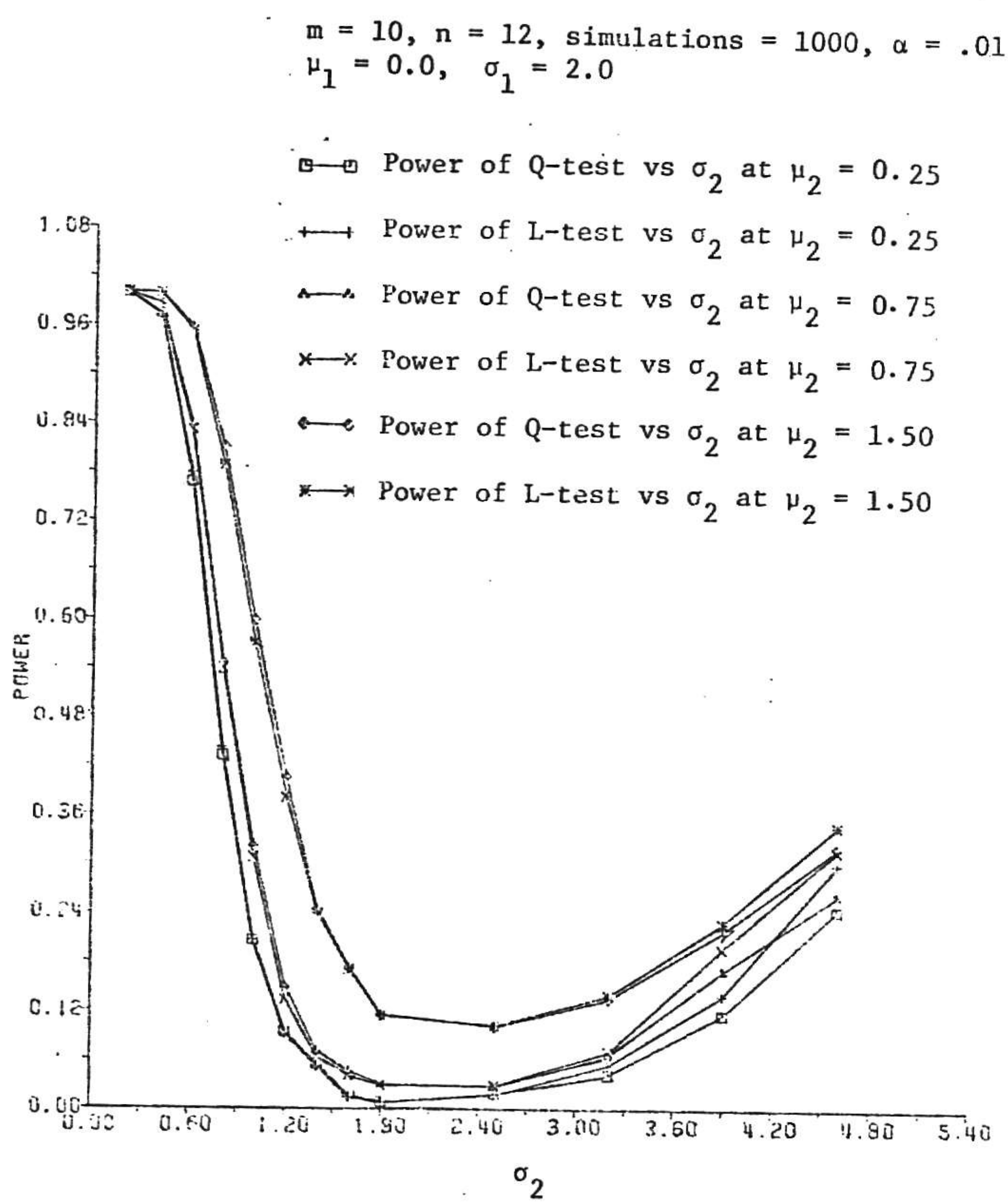


Figure B12b. POWER OF Q-TEST AND L-TEST



```
C      * THIS PROGRAM COMPUTES POWER OF  Q- AND LR-TEST FOR EQUALITY OF  MEANS   *
C      * AND VARIANCES AMONG TWO NORMAL POPULATIONS. TWO SETS OF POWER FUCTIONS *
C      * FOR ALPHA .05 AND .01 ARE GENERATED. MEAN AND VARIANCE OF THE FIRST    *
C      * POPULATION IS SPECIFIED.  SECOND POPULATION COULD BE GENERATED WITH 4  *
C      * MEANS AND 20 DIFFERENT VARIANCES. SAMPLES OF SIZES N1 AND N2 ARE GENE- *
C      * RATED  FROM POPULATION  ONE AND TWO RESPECTIVELY FOR SPECIFIED NUMBER  *
C      * OF TIMES. IN EACH REPLICATION Q- AND LR- STATSTICS ARE COMPUTED  AND   *
C      * COMPARED AGAINST CORRESPONDING CRITICAL VALUES .                      *
       IMPLICIT REAL*8 (A-H,O-Z)
       REAL*8 M,IM,IS,LAMDA,LAMBDA,M2,M2I,LAMDA2
       REAL*8 MEAN1(20,4),MEAN2(20,4),VAR1(20,4),VAR2(20,4)
       REAL*4 PLOT(20,9),MU(4),PLOT1(20,9)
C      * READS SAMPLE SIZES,  # OF REPLICATIONS, CRITICAL VALUES, POPULATION   *
C      * PARAMETERS ETC.                                                      *
       READ(5,999) N1,N2,LAMDA, LPHA,NREPS,IX,IJ,M,S,M2,S2,M2I,S2I,S2GT,
      1S2II,L,LAMDA2,LPHA2
   999 FORMAT(2I3,F3.3,  I2,I4,1X,I9,1X,I9,1X,8F4.2,I2,1X,F3.3,1X,I2)
       CISQR1=9.4877
       CISQR2=13.277
       ALPHA=LPHA*.01
       ALPHA2=LPHA2*.01
C      * ECHO OF THE VARIABLES READ IN.                                       *
       WRITE(6,100) IX,IJ,M2,S2,M2I,S2I,S2GT,S2II,L,N1,N2,ALPHA,NREPS,
      1LAMDA,CISQR1
   100 FORMAT(' ',20X,'IX=',
      2I9,4X,'IJ=',I9
      3        /1X,20X,'INITIAL MU 2=',F6.2,2X,'INITIAL SIGMA 2=',F6.2,
      42X,'INCREMENT OF MU 2=',F6.2,2X,'INCREMENT OF SIGMA 2=',F6.2/1X,20
      5X,  'SIGMA 2 GR. TH.=',F6.2,2X,'SIGMA 2 INCREASE=',F6.2,2X,'# OF
      6 SIGMA 2=',I2///1X,'M=',I3,4X,'N=',I3,4X,'ALPHA=',F4.2,4X,'REPLICA
      7TIONS=',I4,4X,'CRITICAL LAMDA=',F6.3,4X,'CHISQUARE=',F8.4////)
C      * WRITES  HEADINGS  OF POWER FUNCTION TABLE.                           *
       WRITE(6,101) M,S
   101 FORMAT(' ','MU 1=',F6.2,10X,'SIGMA 1=',F6.2//1X,35X,'POWER
      1   OF          THE'//1X,11X,4(3X,'Q-TEST',4X,'LR-TEST'))
       DO 50 III=1,4
       DO 50 II=1,L
       MEAN1(II,III)=0.D0
       MEAN2(II,III)=0.D0
       VAR1(II,III)=0.D0
    50 VAR2(II,III)=0.D0
       NSIZE=N1+N2
       PL=NSIZE
       PL2=PL/2.
       PN1=N1
       PK1=PN1/2.
       PN2=N2
       PK2=PN2/2.
       LAMDA=LAMDA**PL2
       PLAMDA=LAMDA2**PL2
       NDF1=N1-1
       NDF2=N2-1
       NDF3=NSIZE-2
       DF1=NDF1
       DF2=NDF2
       DF3=NDF3
       PREPS=NREPS
       IM=M2
```

```
      C      * COMPUTES POWER FOR Q AND LR- TESTS FOR VARIOUS COMBINATIONS OF 4 MEANS *
      C      * AND SPECIFIED VARIANCES IN SECOND POPULATION AT ALPHA = .05 AND .01.   *
             DO 23 III=1,4
             IS=S2
             DO 22 II=1,L
             ICONT1=0
             ICONT2=0
             ICONT3=0
             ICONT4=0
      C      * COMPUTES POWERS IN A COMBINATION ON THE BASIS OF SPECIFIED # OF REPS.. *
             DO 10 I=1,NREPS
             N=0
             DJJ=0.D0
             J=0
             CJJ=0.D0
      C      * GENERATES NORMAL RANDOM DEVIATES AND COMPUTES SUM OF SQUARES IN FIRST  *
      C      * SAMPLE.                                                                *
             DO 9 K=1,N1
             N=N+1
             FN=N
             YFL=DEV(IX,IJ,S,M)
             IF(N-1) 1,1,2
           1 SSQ3  =0.D0
             SX=YFL
             GO TO 3
           2 SX=SX+YFL
             SSQ3  =SSQ3  +((FN*YFL-SX)*(FN*YFL-SX))/DJJ
           3 DJJ=DJJ+FN+FN
           9 CONTINUE
             SSQ1=SSQ3
             XBAR=SX
      C      * GENERATES NORMAL RANDOM DEVIATES AND COMPUTES SUM OF SQUARES IN SECOND *
      C      * SAMPLE AND POOLED SUM OF SQUARES.                                      *
             DO 11 K=1,N2
             N=N+1
             FN=N
             YFL=DEV(IX,IJ,IS,IM)
             SX=SX+YFL
             SSQ3  =SSQ3  +((FN*YFL-SX)*(FN*YFL-SX))/DJJ
             DJJ=DJJ+FN+FN
             J=J+1
             FJ=J
             IF(J-1)4,4,5
           4 SSQ2=0.D0
             SY=YFL
             GO TO 6
           5 SY=SY+YFL
             SSQ2=SSQ2+((FJ*YFL-SY)*(FJ*YFL-SY))/CJJ
           6 CJJ=CJJ+FJ+FJ
          11 CONTINUE
             XBAR=XBAR/FN1
             YBAR=SY/FN2
             MEAN1(II,III)=MEAN1(II,III)+XBAR
             MEAN2(II,III)=MEAN2(II,III)+YBAR
             VAR1(II,III)=VAR1(II,III)+SSQ1
             VAR2(II,III)=VAR2(II,III)+SSQ2
      C      * COMPUTES Q STATISTIC.                                                  *
             FMN=(SSQ2*DF1)/(SSQ1*DF2)
```

```
                CALL FSUB(FMN,NDF2,NDF1,FPROB)
                GMN=1.-FPROB
                HMN=2.*GMN
                IF(GMN.GE..5) HMN=2.*FPROB
                TMN=(YBAR-XBAR)/DSQRT(PL*(SSQ1+SSQ2)/(DF3*PN1*PN2))
                PROB=TPROB(TMN,NDF3)
                IF(TMN.GT.0.)  PROB=1.-PROB
                PROB=2.*PROB
      C         * CHECKS FOR ARGUMENTS OF LOG VALUES.                                 *
                IF(HMN.LE.0.0)  HMN=.1E-20
                IF(PROB.LE.0.0) PROB=.1E-20
                QMN=-2.*DLOG(PROB)-2.*DLOG(HMN)
      C         * COMPUTES # OF REJECTIONS OF Q STATISTIC  FOR ALPHA .05 AND .01.    *
                IF(QMN.GE.CISQR1) ICONT1=ICONT1+1
                IF(QMN.GE.CISQR2) ICONT3=ICONT3+1
      C         * COMPUTES  LR- STATISTIC.                                           *
                SSQ1=SSQ1/PN1
                SSQ2=SSQ2/PN2
                SSQ1=SSQ1**PK1
                SSQ2=SSQ2**PK2
                SSQ3=SSQ3/PL
                SSQ3=SSQ3**PL2
                LAMBDA    =SSQ1*SSQ2/SSQ3
      C         * COMPUTES # OF REJECTIONS OF LR- STATISTIC FOR ALPHA .05 AND .01.   *
                IF(LAMBDA.LE.LAMDA) ICONT2=ICONT2+1
                IF(LAMBDA.LE.PLAMDA) ICONT4=ICONT4+1
             10 CONTINUE
                PCONT1=ICONT1
                PCONT2=ICONT2
                PCONT3=ICONT3
                PCONT4=ICONT4
      C         * COMPUTES RELATIVE FREQUENCY OF REJECTIONS OF Q AND LR- STATISTICS. *
                PQ=PCONT1/PREPS
                PLR=PCONT2/PREPS
                PQ2=PCONT3/PREPS
                PLR2=PCONT4/PREPS
                PLOT(II,1)=IS
                PLOT(II,2*III)=PQ
                PLOT(II,2*III+1)=PLR
                PLOT1(II,1)=IS
                PLOT1(II,2*III)=PQ2
                PLOT1(II,2*III+1)=PLR2
                IS=IS+S2I
                IF(IS.GT.S2GT) IS=IS+S2II
             22 CONTINUE
                MU(III)=IM
                IM=IM+(III*M2I)
             23 CONTINUE
      C         * PRINTS POWER OF Q AND LR- TESTS FOR ALPHA .05.                     *
                WRITE(6,102) MU
            102 FORMAT(' ',5X,'SIGMA 2',8X,4('MU 2=',F5.2,10X))
                DO 24 I=1,L
                WRITE(6,200)(PLOT(I,J),J=1,9)
            200 FORMAT(' ',9F10.4)
      C         * WRITES POWER POINTS ON DISK  FOR PLOTTING PURPOSES.                *
                WRITE(11) (PLOT(I,J),J=1,9),(PLOT1(I,J),J=1,9)
             24 CONTINUE
                WRITE(6,104)
```

```
      104 FORMAT(////)
C         * WRITES  HEADINGS  OF POWER FUNCTION TABLE.                          *
          WRITE(6,103)M1,M2,ALPHA2,NREPS,LAMDA2,CISQ2
      103 FORMAT(' ','M=',I3,4X,'N=',I3,4X,'ALPHA=',F4.2,4X,'REPLICATIONS=',
         1I4,4X,'CRITICAL LAMBDA=',F6.3,4X,'CHISQUARE=',F8.4)
          WRITE(6,105)
      105 FORMAT(//)
          WRITE(6,101) M,S
          WRITE(6,102) MU
C         * PRINTS POWER OF Q AND LR- TESTS FOR ALPHA .01.                      *
          DO 25 I=1,L
       25 WRITE(6,200) (PLOT1(I,J),J=1,9)
          DO 51 III=1,4
          DO 51 II=1,L
          MEAN1(II,III)=MEAN1(II,III)/PREPS
          MEAN2(II,III)=MEAN2(II,III)/PREPS
          VAR1(II,III)=DSQRT(VAR1(II,III)/(DF1*PREPS))
          VAR2(II,III)=DSQRT(VAR2(II,III)/(DF2*PREPS))
       51 CONTINUE
C         * PRINTS MEAN AND STD. DEVIATIONS OF NORMAL RANDOM DEVIATES FOR VARIOUS *
C         * COMBINATIONS OF MEAN AND VARIANCES OF TWO NORMAL POPULATIONS.         *
          WRITE(6,201)
      201 FORMAT('1',  'SAMPLE  #1',40X,'SAMPLE    #2')
          DO 52 I=1,L
          WRITE(6,202) (MEAN1(I,J),J=1,4),(MEAN2(I,J),J=1,4)
      202 FORMAT(//1X,'MEAN',4F10.4,10X,4F10.4)
          WRITE(6,203)(VAR1(I,J),J=1,4),(VAR2(I,J),J=1,4)
      203 FORMAT(' ',' STD',4F10.4,10X,4F10.4)
       52 CONTINUE
          STOP
          END
```

FORTRAN IV G LEVEL 21 MAIN DATE = 76357 23/27/25

```
C         * THIS FUNCTION GENERATES NORMAL RANDOM DEVIATES.                     *
          REAL FUNCTION DEV*8(IX,IJ,S,M)
          REAL*8 S,M,YFL,DFLOAT
          YFL=0.0D0
          DO 1 I=1,12
          IX=IX*65539
          IJ=IJ*262147
        1 YFL=YFL+DFLOAT(IX+IJ)
          DEV=(YFL*.2328306E-9)*S+M
          RETURN
          END
```

```
FORTRAN IV G LEVEL  21                MAIN               DATE = 76357          23/27/26

      C     * THIS SUBROUTINE COMPUTES UPPER TAIL AREAS OF F- RANDOM VARIABLE.          *
            SUBROUTINE FSUB(F,J,K,FPROB)
            IMPLICIT REAL*8(A-H,O-Z)
            IF(F.GT.1000.)GO TO 11199
            XA=.5*J
            XB=.5*K
            TEMP=XB+XA*F
            XX=XA*F/TEMP
            FPROB=1.0
            IF(F.LE.0.0.OR.XX.LE.0.0) GO TO 1986
            XC=XB/TEMP
            AB=XA+XB
            CON=0.
            SGN=1.
            IF(F.GE.1.0) GO TO 11120
            TEMP=XA
            XA=XB
            XB=TEMP
            TEMP=XC
            XC=XX
            XX=TEMP
            CON=1.
            SGN=-1.
      11120 TOP=AB
            BOT=XB+1.
            XSUM=1.
            TERM=1.
      11130 TEMP=XSUM
            TERM=TERM*(TOP/BOT)*XC
            XSUM=XSUM+TERM
             TOP=TOP+1.
            BOT=BOT+1.
            IF(XSUM.GT.TEMP) GO TO 11130
            FPROB=CON+SGN*DEXP(XA*DLOG(XX)+XB*DLOG(XC)+DLGAMA(AB)-DLGAMA(XA)
           1-DLGAMA(XB))*XSUM/XB
            GO TO 1986
      11199 FPROB=0.0
            IF(K.EQ.1 ) FPROB=1.E50
       1986 RETURN
            END
```

```
FORTRAN IV G LEVEL  21                MAIN               DATE = 76357          23/27/26

      C     * THIS SUBROUTINE COMPUTES LOWER TAIL AREAS OF T- RANDOM VARIABLE.          *
            REAL FUNCTION TPROB*8(T,DF)
            IMPLICIT REAL*8(A-H,O-Z)
            INTEGER DF
            B=DABS(T)
            X=B*(1.-.25/DF)/DSQRT(1.+B*B/(2.*DF))
            TPROB=1.0D0-5.0D-1/(1.0D0+4.9867347D-2*X+2.1141006D-2*X*X
           1+3.2776263D-3*X**3+3.80036D-5*X**4+4.88906D-5*X**5+5.383D-6*X**6)
           2**16
            IF(T.LT.0) TPROB=1.-TPROB
            RETURN
            END
```

APPENDIX C

COMPARISON OF THE ACCEPTANCE REGIONS OF THE TWO TESTS

A copy of the computer simulation program that generates pairs of (T,F) values determining the boundary curves of the acceptance regions of the Q- and the L-tests is given at the end of this Appendix.

The acceptance regions of the two tests and two sets of (T,F) values ($\leq$ 100 pairs in each set) for α = .05 and (m,n) = (10,10), (10,12), and (12,10) are plotted in Figures C13a, C14a, and C15a, respectively. Corresponding pictures for α = .01 are in Figures C13b, C14b, and C15b, respectively. These graphs indicate that for equal sample sizes the acceptance regions of both the tests are almost similar. Switch occurs in the sizes of acceptance regions upon interchanging the sample sizes.

Figure C13a. ACCEPTANCE REGIONS OF Q-TEST AND L-TEST

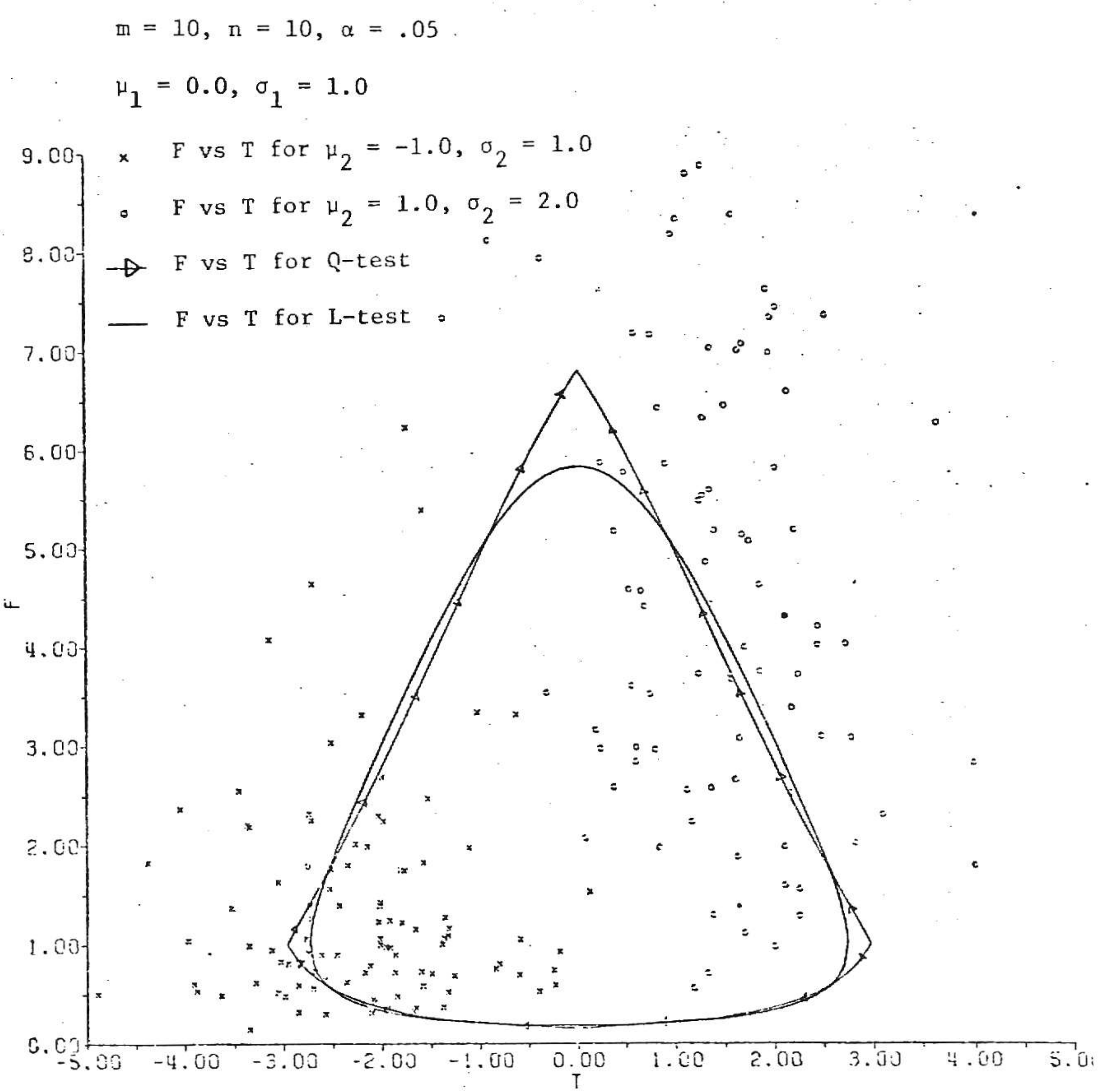


Figure C13b. ACCEPTANCE REGIONS OF Q-TEST AND L-TEST

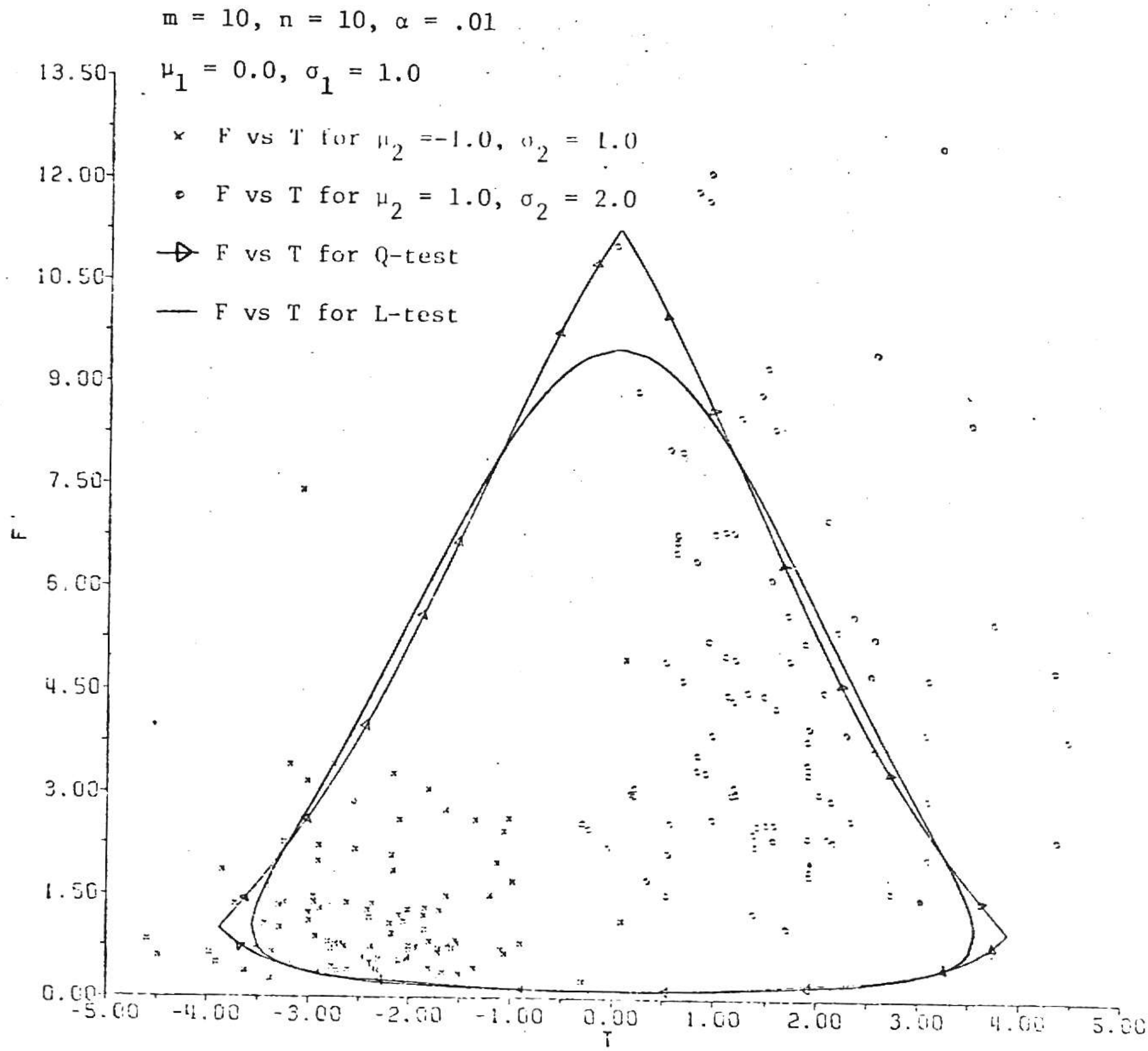


Figure C14a. ACCEPTANCE REGIONS OF Q-TEST AND L-TEST

$m = 10$, $n = 12$, $\alpha = .05$

$\mu_1 = 0.0$, $\sigma_1 = 1.0$

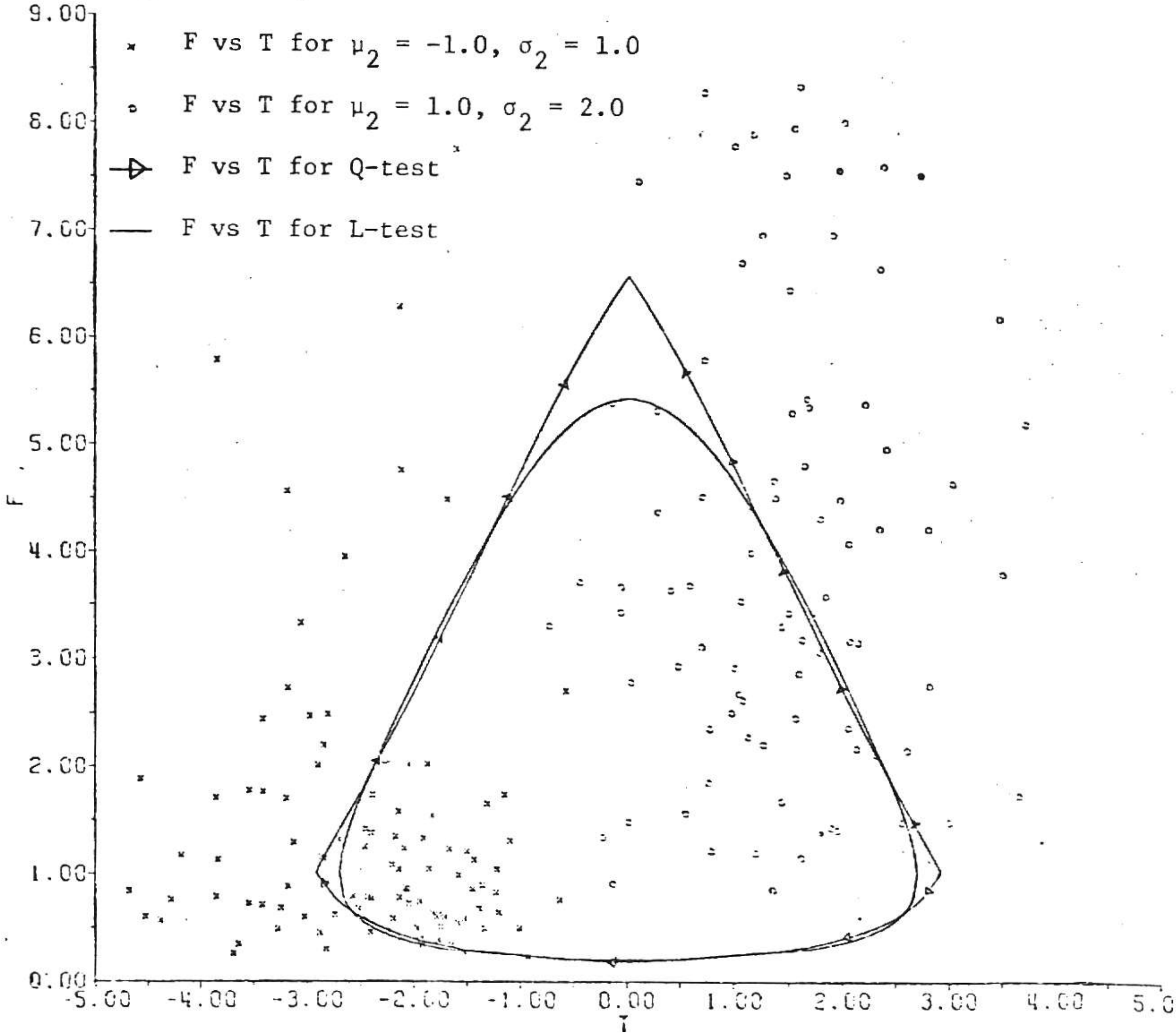


Figure C14b. ACCEPTANCE REGIONS OF Q-TEST AND L-TEST

$m = 10$, $n = 12$, $\alpha = .01$

$\mu_1 = 0.0$, $\sigma_1 = 1.0$

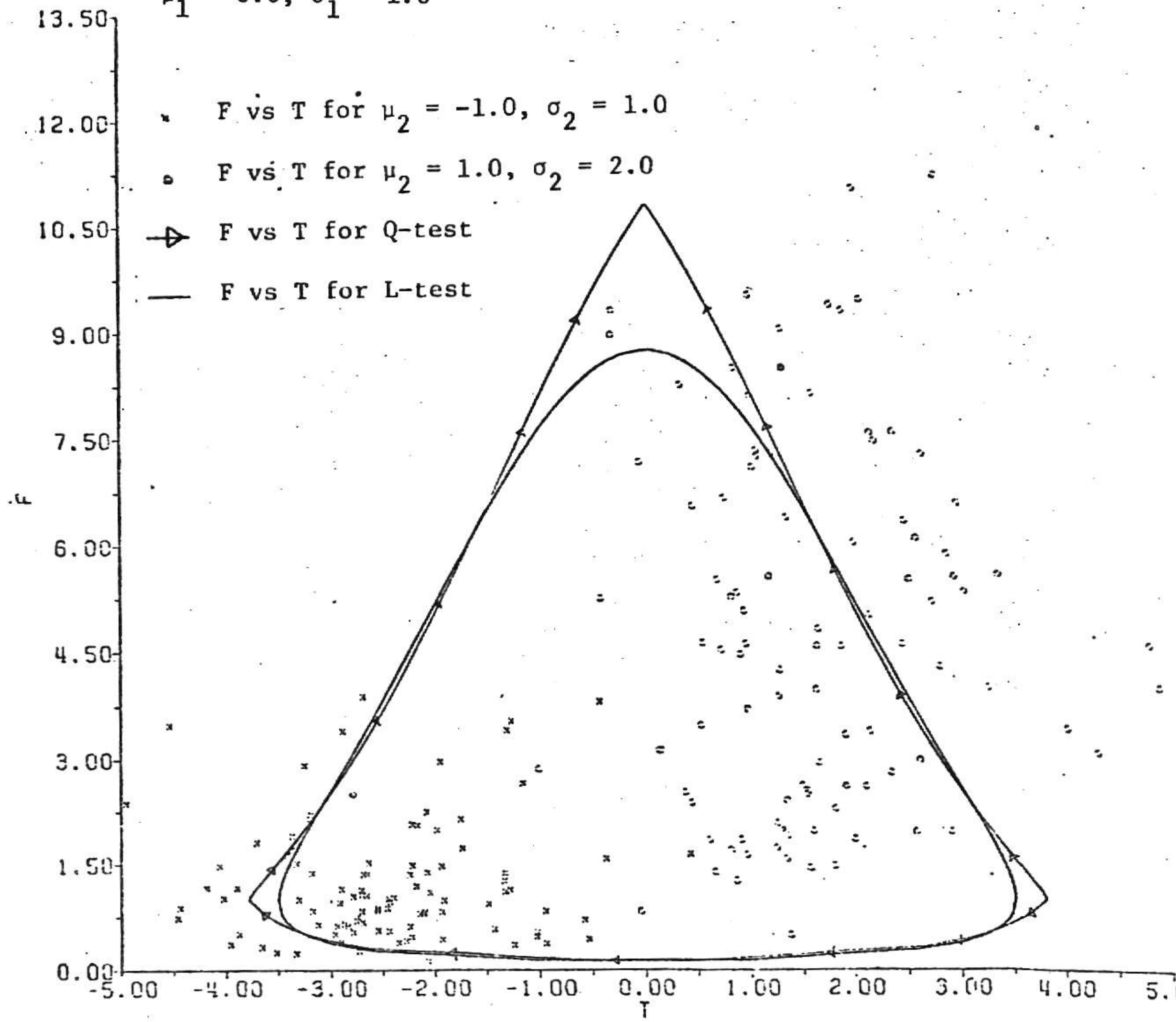


Figure C15a. ACCEPTANCE REGIONS OF Q-TEST AND L-TEST

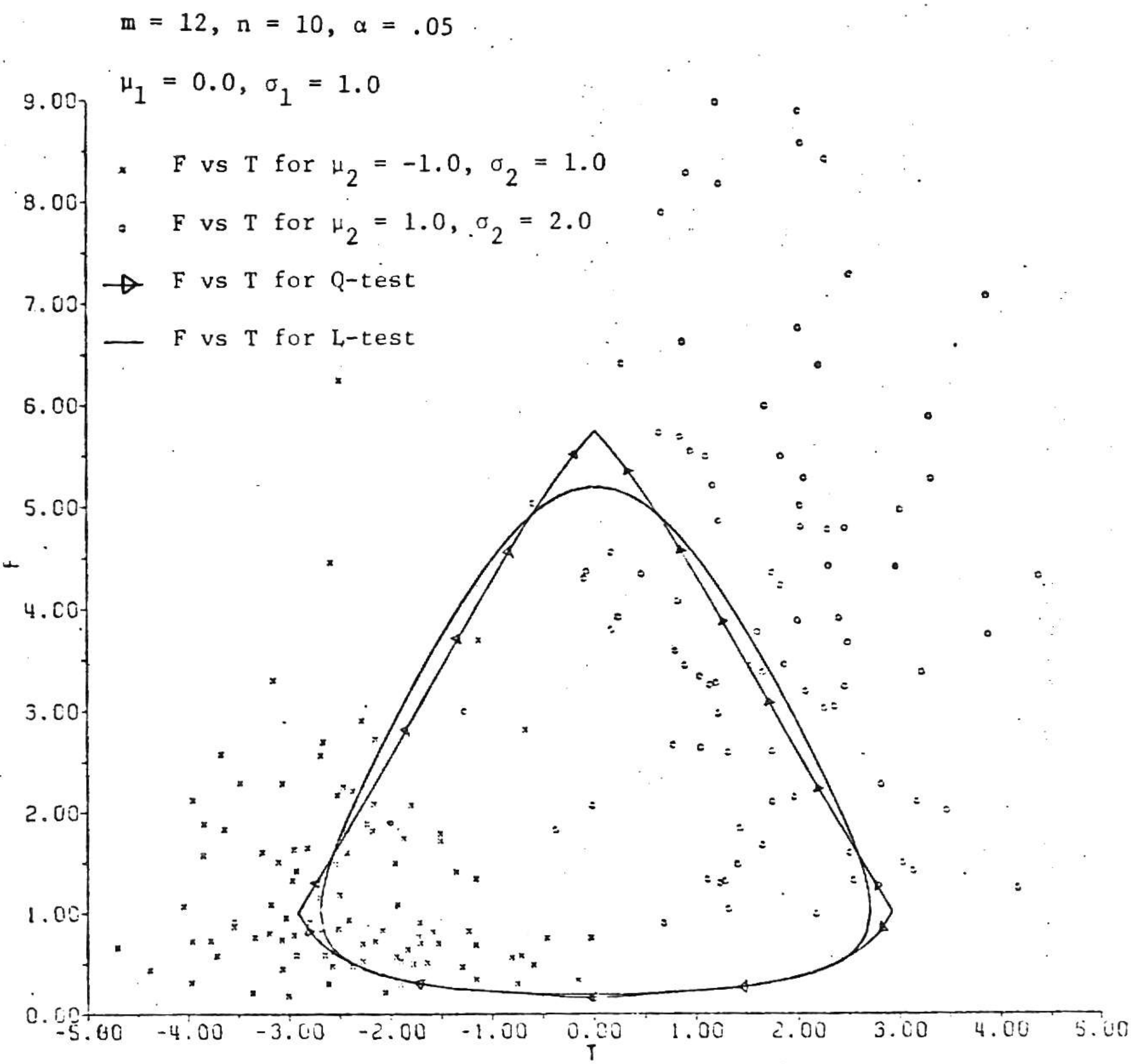
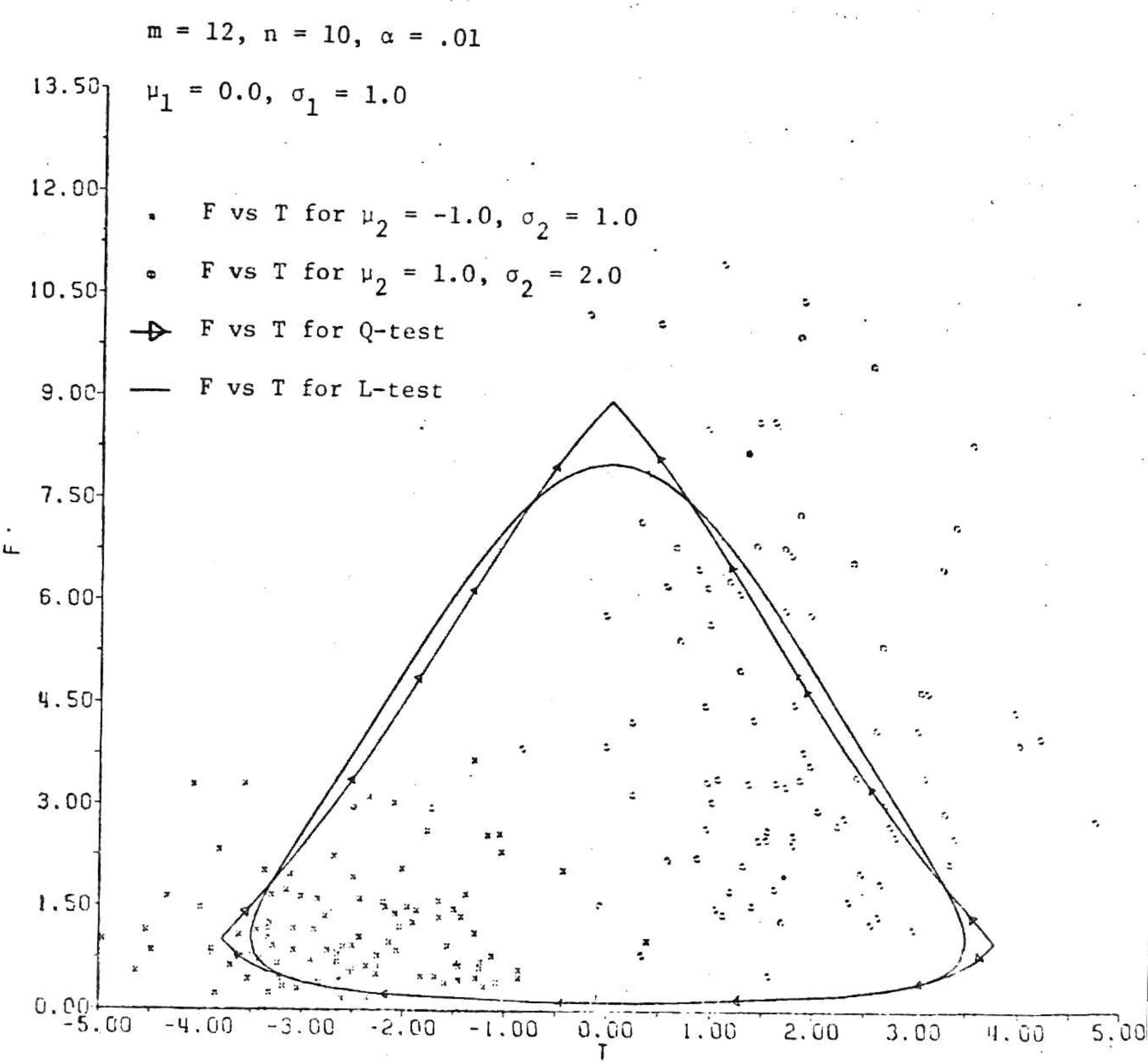


Figure C15b. ACCEPTANCE REGIONS OF Q-TEST AND L-TEST



```
C       * THIS PROGRAM GENERATES TWO SETS OF PAIRS OF (T,F) VALUES THAT DETERMINE*
C       * THE  BOUNDARY CURVES OF THE ACCEPTANCE REGIONS OF THE Q-TEST AND L-TEST*
C       * (FOR TESTING EQUALITY OF MEANS AND VARIANCES OF TWO NORMAL POPULATIONS)*
C       * A MAXIMUM OF THOUSAND (T,F) VALUES COULD BE GENERATED IN EACH SET.    *
C       * SIGNIFICANCE LEVEL AND SAMPLE SIZES N1 AND N2 ARE SPECIFIED. IN ADDI-  *
C       * TION, TWO SETS OF HUNDRED PAIRS OF (T,F) VALUES ARE GENERATED: PAIRED  *
C       * (T,F) VALUES IN EACH SET ARE COMPUTED FROM SAMPLES OF NORMAL DEVIATES  *
C       * GENERATED FROM NORMAL POPULATION N(0,1) AND THE MEAN AND VARIANCE OF   *
C       * THE SECOND NORMAL POPULATION COULD BE SPECIFIED.                       *
        IMPLICIT REAL*8 (A-H,O-Z)
        REAL*8 LAMDA,AF(1000),AT(1000)
        REAL*8 M1,IM1,IS1,IM2,IS2
C       * READS SAMPLE SIZES, CRITICAL VALUE OF L-STATISTIC, ALPHA LEVEL, MEAN   *
C       * AND VARIANCE SPECIFICATIONS FOR NORMAL POPULATIONS ETC.....            *
        READ(5,999) N1,N2,LAMDA, LPHA,IX,IJ,M1,S1,IM1,IS1,IM2,IS2,FINC,FC,
       1TC
  999 FORMAT(2I3,F3.3,  I2,1X,I9,1X,I9,1X,6F4.2,F2.2,2F2.0)
        CHISQR=9.4877
        IF(LPHA.EQ.1) CHISQR=13.277
        ALPHA=LPHA*.01
C       * ECHO OF THE VARIABLES READ IN.                                         *
        WRITE(6,10) N1,N2,ALPHA,LAMDA,CHISQR,IX,IJ,FINC,FC,TC
   10 FORMAT(' ','M=',I3,4X,'N=',I3,4X,'ALPHA=',F4.2,4X,'LAMBDA=',F6.3,
       14X,'CHISQUARE=',F6.3,2X,'IX=',I9,2X,'IJ=',I9/1X,20X,'F INCREMENT='
       2,F6.2,2X,'F TRUNCATE=',F6.2,2X,'T TRUNCATE=',F6.2)
        NSIZE=N1+N2
        PL=NSIZE
        PL2=PL/2.
        PN1=N1
        PK1=PN1/2.
        PN2=N2
        PK2=PN2/2.
        LAMDA=LAMDA**PL2
        NDF1=N1-1
        NDF2=N2-1
        NDF3=NSIZE-2
        DF1=NDF1
        DF2=NDF2
        DF3=NDF3
        PL=PL**PL2
        PN1=PN1**PK1
        PN2=PN2**PK2
        C=PN1*PN2*LAMDA
        FINC2=2.*FINC
C       * NEWTON-RAPHSON TECHNIQUE OF FINDING ROOTS FOLLOWS TO DETERMINE CORRES- *
C       * PONDING VALUES OF T FOR EACH VALUE OF F THAT DETERMINE THE BOUNDARY    *
C       * CURVE OF THE ACCEPTANCE REGION OF Q-TEST.                              *
        WRITE(6,1)
    1 FORMAT(//1X,'Q-TEST'//1X,5X,'F',9X,'T')
        ICOUNT=0
C       * F-VALUE IS INITIALIZED.                                                *
        FMN=1.0D0
        DO 70 I=1,2
        IF (I.EQ.2) TMAX=AT(ICOUNT)
        IF (I.EQ.2) FMAX=AF(ICOUNT)
        IF(I.EQ.2) FMN=.8
        CALL FSUB(FMN,NDF2,NDF1,FPROB)
        GMN=1.-FPROB
```

```
            HMN=2.*GMN
            IF(GMN.GE..5) HMN=2.*FPROB
            T=.01
         30 PROB=TPROB(T,NDF3)
            PROB=2.*(1.-PROB)
            IF(HMN.LE.0.0) HMN=.1E-20
            IF(PROB.LE.0.0) PROB=.1E-20
            FT=CHISQR+2.*DLOG(HMN)+2.*DLOG(PROB)
            IF(FT) 50,50,40
         40 T=T+.5
            GO TO 30
         20 CALL FSUB(FMN,NDF2,NDF1,FPROB)
            GMN=1.-FPROB
            HMN=2.*GMN
            IF(GMN.GE..5) HMN=2.*FPROB
            IF(HMN.LE.0.0) HMN=.1E-20
            IF(PROB.LE.0.0) PROB=.1E-20
            FT=CHISQR+2.*DLOG(HMN)+2.*DLOG(PROB)
         50 DFT=2./PROB
            TMN=T+FT/DFT
            IF(TMN) 70,51,51
         51 PROB=TPROB(TMN,NDF3)
            PROB=2.*(1.-PROB)
            IF(HMN.LE.0.0) HMN=.1E-20
            IF(PROB.LE.0.0) PROB=.1E-20
            FT=CHISQR+2.*DLOG(HMN)+2.*DLOG(PROB)
            IF(DABS(TMN-T)-.1E-10)63,63,62
         62 T=TMN
            GO TO 50
         63 ICOUNT=ICOUNT+1
            AF(ICOUNT)=FMN
            AT(ICOUNT)=TMN
            FMN=FMN+FINC
            IF(I.EQ.2) FMN=FMN-FINC2
            GO TO 20
         70 CONTINUE
            FMIN=AF(ICOUNT)
            TMIN=AT(ICOUNT)
            NLOG=23
      C     * SUBROUTINE DEL IS CALLED TO SORT AND CHOOSE A MAXIMUM OF TWO HUNDRED OF*
      C     * PAIRED (T,F) VALUES.                                                   *
            CALL DEL(AF,AT,ICOUNT,NLOG,FMAX,TMAX,FMIN,TMIN)
      C     * NEWTON-RAPHSON TECHNIQUE OF FINDING ROOTS FOLLOWS TO DETERMINE CORRES- *
      C     * PONDING VALUES OF T FOR EACH VALUE OF F THAT DETERMINE THE BOUNDARY    *
      C     * CURVE OF THE ACCEPTANCE REGION OF L-TEST.                              *
            WRITE(6,2)
          2 FORMAT(///1X,'LR-TEST'//1X,5X,'F',9X,'T')
            ICOUNT=0
      C     * F-VALUE IS INITIALIZED.                                                *
            FMN=1.
            DO 120 I=1,2
            IF (I.EQ.2) TMAX=AT(ICOUNT)
            IF (I.EQ.2) FMAX=AF(ICOUNT)
            IF(I.EQ.2) FMN=.8
            T=.01
         80 FT=C*((1.+DF2*FMN/DF1)*(1.+T  *T  /DF3))**PL2-PL*(DF2*FMN/DF1)**PK
                                        12
            IF(FT)90,90,100
```

```
      90 T=T+.5
         GO TO 80
     100 DFT=C*PL2*((1.+DF2*FMN/DF1)**PL2)*((1.+T   *T   /DF3)**(PL2-1.))*2.*
        1T   /DF3
         TMN=T-FT/DFT
         IF(TMN) 120, 99, 99
      99 FT=C*((1.+DF2*FMN/DF1)*(1.+TMN*TMN/DF3))**PL2-PL*(DF2*FMN/DF1)**PK
        12
         IF(DABS(TMN-T)-.1E-10) 119,119,118
     118 T=TMN
         GO TO 100
     119 ICOUNT=ICOUNT+1
         AF(ICOUNT)=FMN
         AT(ICOUNT)=TMN
         FMN=FMN+FINC
         IF(I.EQ.2) FMN=FMN-FINC2
         GO TO 80
     120 CONTINUE
         FMIN=AF(ICOUNT)
         TMIN=AT(ICOUNT)
         NLOG=24
C        * SUBROUTINE DEL IS CALLED TO SORT AND CHOOSE A MAXIMUM OF TWO HUNDRED OF*
C        * PAIRED (T,F) VALUES.                                                    *
         CALL DEL(AF,AT,ICOUNT,NLOG,FMAX,TMAX,FMIN,TMIN)
C        * SUBROUTINE COMP IS CALLED TO GENERATE HUNDRED PAIRS OF (T,F) VALUES IN *
C        * THE FIRST SET.                                                          *
         NLOG=11
         WRITE(6,3) M1,S1,IM1,IS1
       3 FORMAT(//1X,'MU 1=',F6.2,4X,'SIGMA 1=',F6.2/1X,'MU 2=',F6.2,4X,'SI
        1GMA 2=',F6.2//1X,5X,'F',9X,'T')
         CALL COMP(IX,IJ,N1,N2,S1,M1,IS1,IM1,NLOG,FC,TC)
C        * SUBROUTINE COMP IS CALLED TO GENERATE HUNDRED PAIRS OF (T,F) VALUES IN *
C        * THE SECOND SET.                                                         *
         NLOG=18
         WRITE(6,3) M1,S1,IM2,IS2
         CALL COMP(IX,IJ,N1,N2,S1,M1,IS2,IM2,NLOG,FC,TC)
         STOP
         END
```

```
      C     * THIS SUBROUTINE CHOOSES A MAXIMUM OF TWO HUNDRED OF PAIRED (T,F) VALUES*
      C     * THAT DETERMINE THE BOUNDARY CURVE OF THE ACCEPTANCE REGIONS OF Q-TEST  *
      C     * AND L-TEST.                                                            *
            SUBROUTINE GEL(AF,AT,ICOUNT,NLOG,FMAX,TMAX,FMIN,TMIN)
            IMPLICIT REAL*8(A-H,O-Z)
            REAL*8 AF(ICOUNT),AT(ICOUNT)
            REAL*4 X1,X2
            INC=ICOUNT/100.+.99
            IF(ICOUNT.LE.100) INC=1
            K=0
            DO 1 I=1,ICOUNT,INC
            K=K+1
            AF(K)=AF(I)
            AT(K)=AT(I)
          1 CONTINUE
            I=K/2
            AF(I)=FMAX
            AT(I)=TMAX
            AF(K)=FMIN
            AT(K)=TMIN
            CALL SORT(AF,AT,K)
            DO  2 I=1,K
            X1=AF(I)
            X2=AT(I)
            WRITE(6,60) X1,X2
         60 FORMAT(' ',2F10.4)
      C     * WRITES PAIRED (T,F) VALUES ON DISK FOR PLOTTING PURPOSES.              *
            WRITE(NLOG) X1,X2
          2 CONTINUE
            DO 3 I=1,K
            J=I-1
            X1=AF(K-J)
            X2=-1.*AT(K-J)
            WRITE(6,60) X1,X2
      C     * WRITES PAIRED (T,F) VALUES ON DISK FOR PLOTTING PURPOSES.              *
            WRITE(NLOG) X1,X2
          3 CONTINUE
            RETURN
            END
```

```
FORTRAN IV G LEVEL  21                 MAIN                DATE = 76357          15/21/13

            C      * THIS SUBROUTINE COMPUTES UPPER TAIL AREAS OF F RANDOM VARIABLE.     *
                   SUBROUTINE FSUB(F,J,K,FPROB)
                   IMPLICIT REAL*8(A-H,O-Z)
                   IF(F.GT.1000.)GO TO 11199
                   XA=.5*J
                   XB=.5*K
                   TEMP=XB+XA*F
                   XX=XA*F/TEMP
                   FPROB=1.0
                   IF(F.LE.0.0.OR.XX.LE.0.0) GO TO 1986
                   XC=XB/TEMP
                   AB=XA+XB
                   CON=0.
                   SGN=1.
                   IF(F.GE.1.0) GO TO 11120
                   TEMP=XA
                   XA=XB
                   XB=TEMP
                   TEMP=XC
                   XC=XX
                   XX=TEMP
                   CON=1.
                   SGN=-1.
            11120  TOP=AB
                   BOT=XB+1.
                   XSUM=1.
                   TERM=1.
            11130  TEMP=XSUM
                   TERM=TERM*(TOP/BOT)*XC
                   XSUM=XSUM+TERM
                    TOP=TOP+1.
                   BOT=BOT+1.
                   IF(XSUM.GT.TEMP) GO TO 11130
                   FPROB=CON+SGN*DEXP(XA*DLOG(XX)+XB*DLOG(XC)+DLGAMA(AB)-DLGAMA(XA)
                  1-DLGAMA(XB))*XSUM/XB
                   GO TO 1986
            11199  FPROB=0.0
                   IF(K.EQ.1 ) FPROB=1.E50
             1986  RETURN
                   END


FORTRAN IV G LEVEL  21                 MAIN                DATE = 76357          15/21/13

            C      * THIS SUBROUTINE COMPUTES LOWER TAIL AREAS OF T RANDOM VARIABLE.     *
                   REAL FUNCTION TPROB*8(T,DF)
                   IMPLICIT REAL*8(A-H,O-Z)
                   INTEGER DF
                   B=DABS(T)
                   X=B*(1.-.25/DF)/DSQRT(1.+B*B/(2.*DF))
                   TPROB=1.0D0-5.0D-1/(1.0D0+4.9867347D-2*X+2.11410061D-2*X*X
                  1+3.2776263D-3*X**3+3.80036D-5*X**4+4.88906D-5*X**5+5.383D-6*X**6)
                  2**16
                   IF(T.LT.0) TPROB=1.-TPROB
                   RETURN
                   END
```

```
FORTRAN IV G LEVEL  21                 MAIN              DATE = 76357         15/21/13

      C       * THIS SUBROUTINE SORTS PAIRED (T,F) VALUES.                                 *
              SUBROUTINE SORT(A,B,NREPS)
              REAL*8 A(NREPS),B(NREPS),Y1,Y2
              IF (NREPS.LT.2) RETURN
              M=NREPS
           10 I=4
              IF (M.GT.15) I=8
              J=M/I
              M=2*J+1
              NM=NREPS-M
              DO 40 J=1,NM
              Y1=A(J+M)
              Y2=B(J+M)
              J1=J+1
              DO 20 II=1,J,M
              I=J1-II
              IF(A(I).GT.Y1) GO TO 30
              A(I+M)=A(I)
              B(I+M)=B(I)
           20 CONTINUE
              I=I-M
           30 A(I+M)=Y1
              B(I+M)=Y2
           40 CONTINUE
              IF (M.GT.1) GO TO 10
              RETURN
              END
```

```
FORTRAN IV G LEVEL  21                 MAIN              DATE = 76357         15/21/13

      C       * THIS FUNCTION GENERATES NORMAL RANDOM DEVIATES.                            *
              REAL FUNCTION DEV*8(IX,IJ,S,M)
              REAL*8 S,M,YFL,DFLOAT
              YFL=0.0D0
              DO 1 I=1,12
              IX=IX*65539
              IJ=IJ*262147
            1 YFL=YFL+DFLOAT(IX+IJ)
              DEV=(YFL*.2328306E-9)*S+M
              RETURN
              END
```

```
C      * THIS SUBROUTINE COMPUTES HUNDRED PAIRS OF (T,F) VALUES. PAIRED (T,F)   *
C      * VALUES ARE COMPUTED FROM SAMPLES OF NORMAL DEVIATES GENERATED FROM     *
C      * NORMAL POPULATION N(0,1) AND THE MEAN AND VARIANCE OF THE SECOND NORMAL*
C      * POPULATION COULD BE SPECIFIED.                                         *
       SUBROUTINE COMP(IX,IJ,N1,N2,S1,M1,IS,IM,NLOG,FC,TC)
       IMPLICIT REAL*8 (A-H,O-Z)
       REAL*8  M,IM,IS,M1
       REAL*4 FMN,TMN,F,T,ABS
       F=FC
       T=TC
       PL=N1+N2
       PN1=N1
       PN2=N2
       DF1=N1-1
       DF2=N2-1
       DF3=N1+N2-2
       DO 200 J=1,100
       M=M1
       S=S1
       L=N1
       DO 201 I=1,2
       N=0
       DJJ=0.D0
       DO 9 K=1,L
       N=N+1
       FN=N
C      * GENERATES NORMAL RANDOM DEVIATES.                                      *
       YFL=DEV(IX,IJ,S,M)
       IF(N-1) 1,1,2
     1 SSQ   =0.D0
       SX=YFL
       GO TO 3
     2 SX=SX+YFL
       SSQ   =SSQ   +((FN*YFL-SX)*(FN*YFL-SX))/DJJ
     3 DJJ=DJJ+FN+FN
     9 CONTINUE
       IF(I.EQ.2) GO TO 201
C      * COMPUTES SUMS OF SQUARES IN THE FIRST SAMPLE.                          *
       SSQ1=SSQ
       XBAR=SX/PN1
       L=N2
       M=IM
       S=IS
   201 CONTINUE
C      * COMPUTES SUMS OF SQUARES IN THE SECOND SAMPLE.                         *
       SSQ2=SSQ
       YBAR=SX/PN2
C      * COMPUTES T AND F VALUES.                                               *
       FMN=(SSQ2*DF1)/(SSQ1*DF2)
       TMN=(YBAR-XBAR)/DSQRT(PL*(SSQ1+SSQ2)/(DF3*PN1*PN2))
       WRITE(6,202) FMN,TMN
   202 FORMAT(' ',2F10.4)
       IF(FMN.GT.F ) GO TO 200
       IF(ABS(TMN).GT.T ) GO TO 200
C      * WRITES  (T,F) ON DISK FOR PLOTTING PURPOSES.                           *
       WRITE(NLOG) FMN,TMN
   200 CONTINUE
       RETURN
       END
```

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COMPARISON OF POWER BY SIMULATION OF Q AND LIKELIHOOD RATIO TESTS FOR EQUALITY OF TWO NORMAL POPULATIONS IN THEIR MEANS AND VARIANCES

by

MOHAMMED ABDUL QUADEER

B.V.Sc. & A. H., Osmania University (India), 1963
M.Sc.(A. H.), Maghada University (India), 1965
Ph.D., Kansas State University, 1976

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Simulated power functions of Q-test and L-test for testing equality of $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ were compared. Generally, the parameters of the first normal population were fixed at $\mu_1 = 0.0$ and $\sigma_1^2 = 1.0$ and various values of μ_2 and σ_2^2 were considered. Both equal and unequal sample sizes were used: (1) $m = n = 10$, 12, and 15; (2) $(m,n) = (10,12)$, (10,15), (12,15), and (15,30); and (3) with interchange of m and n in (2). Significance levels of .05 and .01 were used. Acceptance regions of the two tests were compared for the sample sizes and α levels.

For equal sample sizes both tests have almost similar power functions and acceptance regions. However, for unequal sample sizes, the powers of the two tests differ and switch occurs in their power functions around $\sigma_1^2/\sigma_2^2 = 1.0$, so that no test appears dominating. Similarly, acceptance regions differ with a switch in their sizes when sample sizes are unequal.