A GOAL PROGRAMMING MODEL FOR KOREAN ECONOMIC PLANNING

by

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CHAPTER 1

INTRODUCTION

Economic planning may be described as the conscious effort of a central organization to influence, direct, and in some cases, even control changes in the principle economic variables (e.g. consumption, investment, saving, etc.) of a certain country or region over the course of time in accordance with a predetermined set of objectives. Economic plans may be either comprehensive or partial. A comprehensive plan sets its targets to cover all major aspects of the national economy. A partial plan covers only a part of the national economy, e.g., industry, agriculture, the public sector, the foreign sector, and so forth [11].

Comprehensive economic planning, which is largely adopted in the developing countries, is based on limited scarce capital and natural resources in relation to competitive targets to be achieved. Thus, in order to utilize efficiently the scarce capital and natural resources, development goals must be established in a hierarchical order of importance [10]. In other words the difficulty in the economic planning analysis is the treatment of conflicting multiple objectives or goals. Hence the conventional single objective programming technique is severely handicapped in this situation.

There are three analytical methods of solution which can handle such multiobjective problems. These are:

1. Interactive multiobjective programming [2,9]. This technique allows the decision maker to trade off one objective versus another in an interactive manner. A solution is obtained by the decision maker's cyclic involvement in a search process that attempts to locate a satisfactory course rather than a optimum course.

2. Multiobjective programming with utility function [6]. Various concepts in utility theory and preference theory can be used to solve such a problem by reducing the multiple objective function to a single objective function.
3. Linear goal programming [4,7]. In this technique, all of the decision maker's targets or goals may be incorporated into the achievement function. The objectives of linear goal programming need not be a single dimension. The physical condition of the problem must be satisfied before any goal is considered. The set of feasible solutions which satisfies the physical condition is established. The optimal solution is then selected from the feasible solution which best fulfills the decision maker's stated goals.

The first and second method of solution have some problems concerning an accurate transformation of multiple objectives into a single preference (i.e., surrogate objective function) or utility function. It is worse that interactive multiobjective programming does not have a systematic algorithm employed in the digital computer. Hence the linear goal programming technique is very appropriate tool in solving the multiple objective economic planning problems.

There have been four economic planning models for Korea. The following three models are reviewed in [8].

1. an input-output model by Adelman [1]
2. a mixed integer programming model by Westphal [1]
3. a dynamic nonlinear planning model by Kendrick and Taylor [1]

The fourth model, a multisectoral dynamic model by Eckaus and Parikh [3], has been employed by Joe [5].

The purpose of this study is to apply linear goal programming technique to the multisectoral dynamic model for Korean economic planning. Eckaus and Parikh's model is used in this study. The model was employed by Joe for Korean economic planning, but the linear programming technique was utilized. In the present study, the model is reconstructed so as to apply the linear goal programming technique. The models presented herein cover the period of 1977-1981, which corresponds to the period of Korea's Fourth Five-Year Plan.

Linear goal programming has not been widely used yet. Hence an introduction to the linear goal programming problem is presented in Chapter 2. In Chapter 3, the linear programming model of the Five-Year Korean Economic Planning is presented. The solution of this model is also presented in this chapter. In Chapter 4, this linear programming model is reorganized into a linear goal programming model and the linear goal programming solution is obtained. In the final chapter, concluding remarks and proposal for further study are presented.

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CHAPTER 2

LINEAR GOAL PROGRAMMING

1. Introduction

A tool known as the simplex method for the solution to strictly linear decision models having a single objective, has been developed by Dantzig [2] after World War II. However, it is severely limited in that it cannot solve either non-linear models or models having more than a single objective function. It is common that a system has multiple conflicting objectives to achieve. For example, in studying the feasibility of constructing a new airport, there are many conflicting objectives and interests. The study must consider the cost of construction, the capacity, accessibility of the location, traffic-flow planning, noise level for the nearby residents, conservation of natural life in the area, and so on. Obviously, a linear programming model with a single objective (goal) is not generally suitable for such decision analysis.

In the early 1960's Charnes and Cooper [1] presented an approach to the solution of linear decision models having more than a single objective. Later the work of Ijiri [5], Lee [6], and Ignizio [4] has resulted in a systematic methodology known as Goal Programming for solving linear, multiple objective problems wherein preemptive priorities and weightings are associated with the objectives.

In goal programming, instead of trying to maximize or minimize an objective directly as in linear programming, deviations between goals are to be minimized. The deviation variable is represented in two dimensions in the objective function, a positive (overattainment) and a negative (underattainment) deviation from a target value (i.e., a desired level of attainment) of each goal. Then, the achievement function entails the minimization of these deviations, based on the relative importance or priority assigned to them. Hence, goal programming is to solve problems involving multiple, conflicting goals according to the decision maker's priority structure.

Here Lee's modified simplex algorithm [6] and Ignizio's modified revised simplex algorithm [4] are presented. Both employ the same theory of simplex algorithm, but one of the advantages of Ignizio's method is that it uses the condensed tableau and the revised simplex algorithm in its computations.

2. Model Formulation

2.1 Forming the objective functions. We shall designate the goals or objectives by G_i. Thus, G_2, for example, is the second objective function. Each function must be expressed as a function of the decision variables $(\bar{x})$, that is,

$$G_i = f_i(\bar{x})$$

Every objective function will, and must, have an associated target value (b_i), that is,

$$f_i(\bar{x}) \gtreqless b_i$$

This situation allows for the intentional deviations to occur like slack variables in the linear programming. Deviations can be considered either positive, negative or zero from the goals. However, as in linear programming, all deviations in the goal programming must be structured as nonnegative variables. This might pose a problem, but it is easily handled by a simple transformation. For example, if $D_i \gtreqless 0$ is a deviation from a goal, the deviation may be replaced by the difference between two nonnegative variables, that is,

$$D_i = d_i^- - d_i^+$$

where (a) $-\infty < D_i < +\infty$

(b) $d_i^+ > 0$

(c) $d_i^- > 0$

(d) $d_i^- \cdot d_i^+ = 0$

The value of d_i^- reflects the negative deviation from $b_i(<b_i)$, while the value d_i^+ reflects positive deviation $(>b_i)$. Requirement (d) indicates that d_i^- and d_i^+ are complementary to each other. If d_i^- takes nonzero value, d_i^+ will be zero, and vice versa. Consequently, each objective will be of the following final form:

$$f_i(\bar{x}) + d_i^- - d_i^+ = b_i \qquad i = 1, \ldots, m$$

2.2 Absolute objectives. When setting up the decision model we meet two kinds of objectives. One is the decision maker's goal to attain a desired level, the other is to meet the physical demand or constraint resulting from assumptions or/and conditions of the real situation. The constraints are treated as special kind of objectives and called absolute objectives. The top priority, P_1, is assigned to these absolute objectives.

2.3 Assigning objectives to priority level. Once a set of objectives has been established, we shall rank these objectives (i.e., the nonabsolute objectives) according to preference or importance. All absolute objectives, if they exist, are assigned to the top priority level to insure that they are completely satisfied (if possible). The remaining set of nonabsolute objectives should then be grouped according to their respective priority levels. The assignment of priorities to these objectives is normally decided by the decision maker or the decision maker in conjunction with the analyst. However, the decision maker could assign a ranking that is not directly compatible with the goal programming model. The paired comparison method [7] is an approach that may be used when a single decision maker is asked to rank objectives. The essence of the procedure is to compare objectives, two at a time, until all possible pairs of objectives have been investigated. The number of such comparisons is given by $_mC_2$ where m is the total number of objectives. Consider the following typical results of such an approach:

$$G_1 > G_2 \qquad G_2 < G_3$$
$$G_1 > G_3 \qquad G_2 > G_4$$
$$G_4 < G_1 \qquad G_3 > G_4$$

where $G_1 > G_2$ is read as objective one is preferred to objective two. Rearranging these expressions so that all preference signs points to the right we have:

$$G_1 > G_2 \qquad G_3 > G_2$$
$$G_1 > G_3 \qquad G_2 > G_4$$
$$G_1 > G_4 \qquad G_3 > G_4$$

The most preferred objectives will be preferred to the three other objectives and thus it should appear on the left of our expression three times. This is true for G_1. The next most preferred goal is G_3. The final rankings are then:

Rank	Objective
1	G_1
2	G_3
3	G_2
4	G_4

Once the objectives had been ranked, we wish to group them into a minimum number of priority levels. It should be noted that only commensurable objectives may be assigned to the same priority level. Further it must be possible to assign weights to each objective within the same priority level. Weighting values can be judgement values such as: If the profit associated with the product A is four times that associated with product B then we would probably be four times more concerned with minimizing the market underrun of product A than that of product B.

It should be noted that these preemptive priority levels have the following property:

$$P_k >>> P_{k+1} \qquad k = 1,2, \ldots, K-1$$

Thus, the achievement of those objectives at any priority is immeasurably preferred to the achievement of the objective set at any lower priority. This concept is at the heart of goal programming. As a result, the solution procedure does not find a global optimum satisfying all objectives, rather the goal programming procedure finds a feasible set of optimal solutions to the priority one level, then within this set of solutions it finds another subject of optimal solutions (if possible) to the priority two level, and so forth.

2.4 Forming the achievement function. The final step in model development is the establishment of the achievement function. Let us first consider a typical objective function as shown below:

$$f_i(\bar{x}) + d_i^- - d_i^+ = b_i$$

Now we will desire to select $\bar{x}$ so as to

(a) equal or exceed the value of b_i

or (b) equal or less than the value of b_i

or (c) exactly equal to the value of b_i .

These three possibilities may be achieved by minimizing a linear function of the deviation variables as shown in Table 2-1.

Table 2-1 Procedure for Achieving an Objective

Goal	Procedure
(a) Equal or exceed b_i	Min. d_i^-
(b) Equal or be less than b_i	Min. d_i^+
(c) Equal b_i	Min. $(d_i^- + d_i^+)$

Our next step is to associate each objective with the respective preemptive priority. For example, $P_2(d_1^-)$ means that our second priority is associated with minimizing underattainment of the first objective function (i.e., min. d_1^-). The resulting achievement function will be

$$\text{Min.} \quad \bar{a} = \{P_1[g_1(\bar{d}^-, \bar{d}^+)], P_2[g_2(\bar{d}^-, \bar{d}^+)], \ldots, P_K[g_K(\bar{d}^-, \bar{d}^+)]\}$$

where $g_k(\bar{d}^-, \bar{d}^+)$ is a linear function of the deviation variables, and P_k is the kth priority level associated with $g_k(\bar{d}^-, \bar{d}^+)$. We may drop P_1, P_2, ... from the formulation of the achievement function, that is,

$$\text{Min.} \quad \bar{a} = \{g_1(\bar{d}^-, \bar{d}^+), g_2(\bar{d}^-, \bar{d}^+), \ldots, g_K(\bar{d}^-, \bar{d}^+)\}$$

2.5 The general linear goal programming model. After following the preceding steps we shall arrive at the desired model of our multiple objective decision problem. The general linear goal program model takes on the following form:

Find $\bar{x} = (x_1, x_2, \ldots, x_n)$ so as to minimize

$$\bar{a} = \{g_1(\bar{d}^-, \bar{d}^+), g_2(\bar{d}^-, \bar{d}^+), \ldots, g_K(\bar{d}^-, \bar{d}^+)\}$$

such that

$$\sum_{j=1}^{n} c_{ij}x_j + d_i^- - d_i^+ = b_i \qquad i = 1, \ldots, m$$

$$\bar{x}, \bar{d}^-, \bar{d}^+ \geq 0$$

3. Graphical Analysis

The technique for solving these models may be understood easily if we first consider a simple graphical approach. Although graphical analysis is only appropriate for a problem having no more than 3 decision variables, it does serve to aid in the understanding of the basic concept and method to be used in large problems.

Example 2-1

The Hardee toy company makes two kinds of toy dolls. Doll A is a high quality toy and Doll B is of lower quality. The respective profits are \$0.40 and \$0.30 per doll. Each doll of type A requires twice as much time as a doll of type B, and if all dolls were of type B, the company could make 500 per day. The supply of material is sufficient for only 400 dolls per day (both A and B) combined. Assuming that all the dolls for type A and type B the factory can make could be sold, how should the manager schedule production to produce the most profit?

This problem is a typical linear programming problem which is formulated as below:

$$\text{Max.} \quad Z = 0.4x_1 + 0.3x_2$$

subject to

$$x_1 + x_2 \leq 400$$

$$2x_1 + x_2 \leq 500$$

$$x_1, x_2 \geq 0$$

where x_1 and x_2 are the number of doll A and that of doll B produced respectively. The optimum solution is,

$$x_1^* = 100 \qquad x_2^* = 300$$

$$Z = \$130$$

This profit maximization problem can be formulated as a goal programming problem.

$$\text{Min.} \quad \bar{a} = \{(d_1^+ + d_2^+), (d_3^-)\}$$

such that

$$G_1: \quad x_1 + \quad x_2 + d_1^- - d_1^+ = 400$$

$$G_2: \quad 2x_1 + \quad x_2 + d_2^- - d_2^+ = 500$$

$$G_3: \quad 0.4x_1 + 0.3x_2 + d_3^- - d_3^+ = 240$$

$$\bar{x}, \bar{d}^-, \bar{d}^+ \geq 0$$

The achievement function consists of two priorities. First priority is given to the minimization of $(d_1^+ + d_2^+)$ because G_1 and G_2 are absolute objectives (i.e., they come from physical constraints). The second priority factor is assigned to the minimization of d_3^-, that is, minimize the under attainment of some arbitrarily chosen target value of \$240. This value is set arbitrarily, knowing that we will never be able to achieve a higher profit (\$0.4 x 400 = \$160 < \$240 and \$0.3 x 400 = \$120 < \$240).

In order to solve this goal programming by the graphical method, all the objective functions must be plotted on graph as shown in Figure 2-1. No feasible region is found to satisfy all three goals, i.e., G_3 is completely conflicting with G_1 and G_2. By introducing priority levels in goal programming this difficulty could be handled. The solution space satisfying the objective set of priority level 1 (P_1) is indicated by the cross-hatched area of Figure 2-2. Here both d_1^+ and d_2^+ are set to zero. Next we attempt to satisfy priority level 2(P_2) without degrading the solution of P_1. In this situation d_3^- can be minimized until $d_3^- = 110$. If d_3^- becomes smaller than this, it degrades P_1. Consequently the final solution is point A shown in Figure 2-2 and is,

$$x_1^* = 100 \qquad x_2^* = 300$$

$$\bar{a}^* = \{0, 110\}$$

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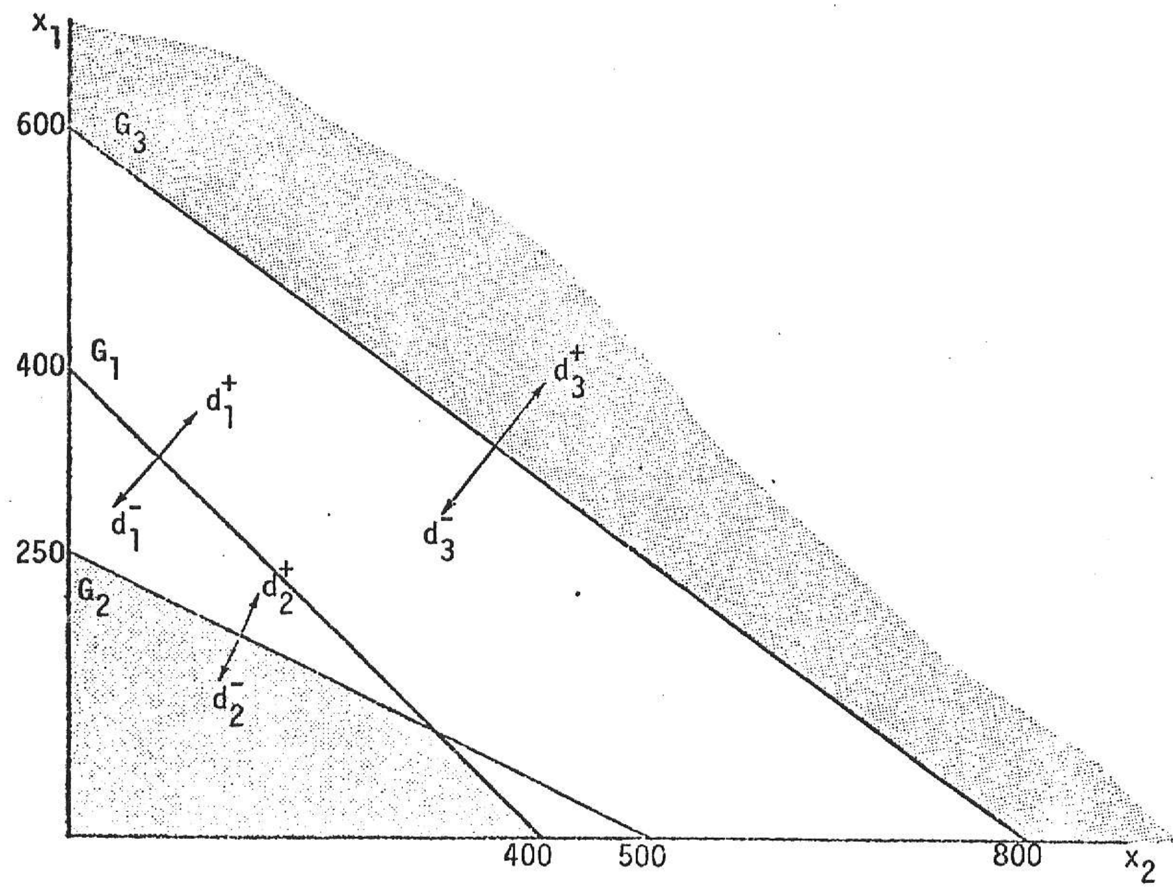

Figure 2-1 No feasible region for satisfying three goals

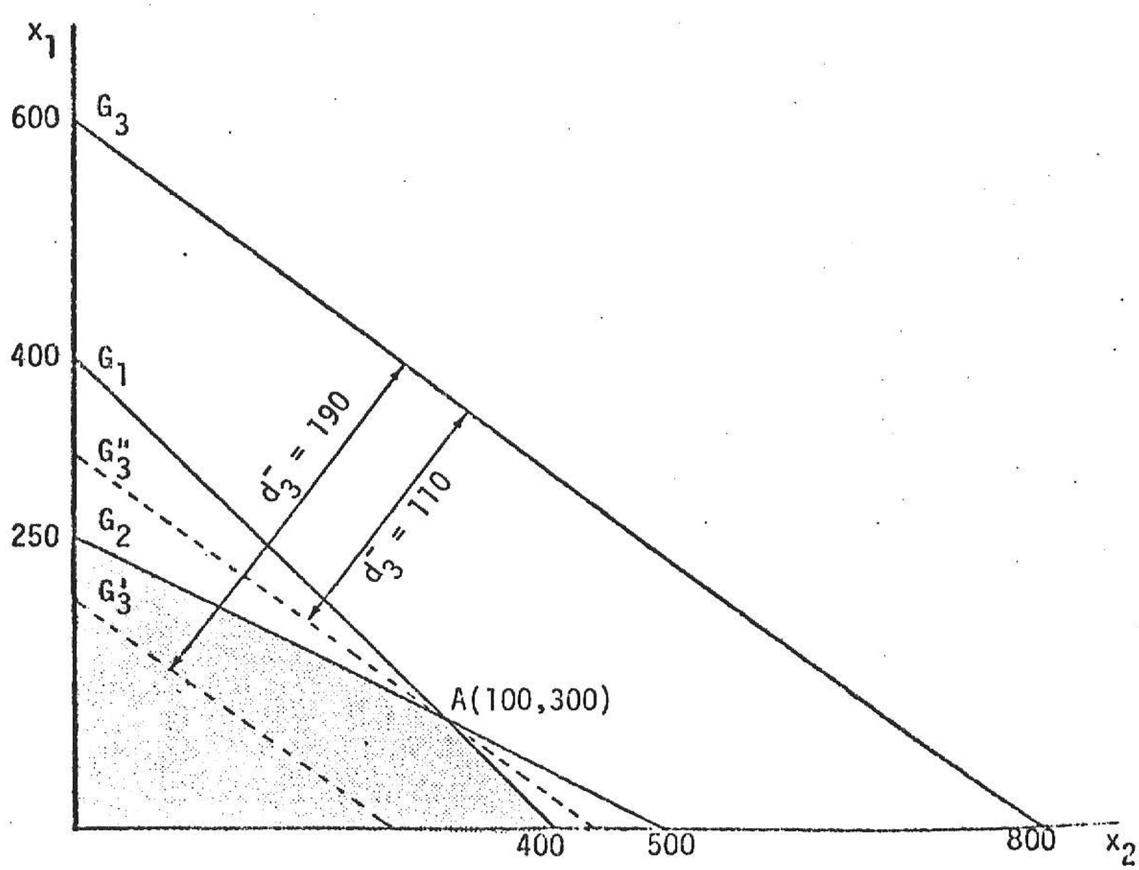

Figure 2-2 Final solution for Example 2-1

Example 2-2

Suppose the best customer of the toy company ordered 300 dolls of type A. For various reasons the manager could not decline this order. How should the manager switch the production schedule to meet the sudden demand? Assume that the other conditions are the same as Example 2-1.

If we attempt to utilize linear programming to maximize profit, the problem is formulated as below:

$$\text{Max.} \qquad Z = 0.4x_1 + 0.3x_2$$

subject to

$$x_1 + x_2 \leq 400$$

$$2x_1 + x_2 \leq 500$$

$$x_1 \geq 300$$

$$x_1, x_2 \geq 0$$

The graphical presentation of the model is shown in Fig. 2-3. Obviously, there is no area of feasible solution and consequently the above problem is unsolvable by linear programming. In simple words, the company does not have enough capacity to satisfy the customer's demand.

This difficulty can be handled, if the manager wishes to satisfy his desires as closely as possible rather than absolutely satisfy all goals. Let's assume that his priorities are:

P_1: Avoid the overtime operation of the plant (500 dolls/day)

P_2: Meet the best customer's order as closely as possible (i.e. produce at least 300 dolls of type A)

P_3: Satisfy, as much as possible, the \$240 profit.

Now the linear goal programming model becomes:

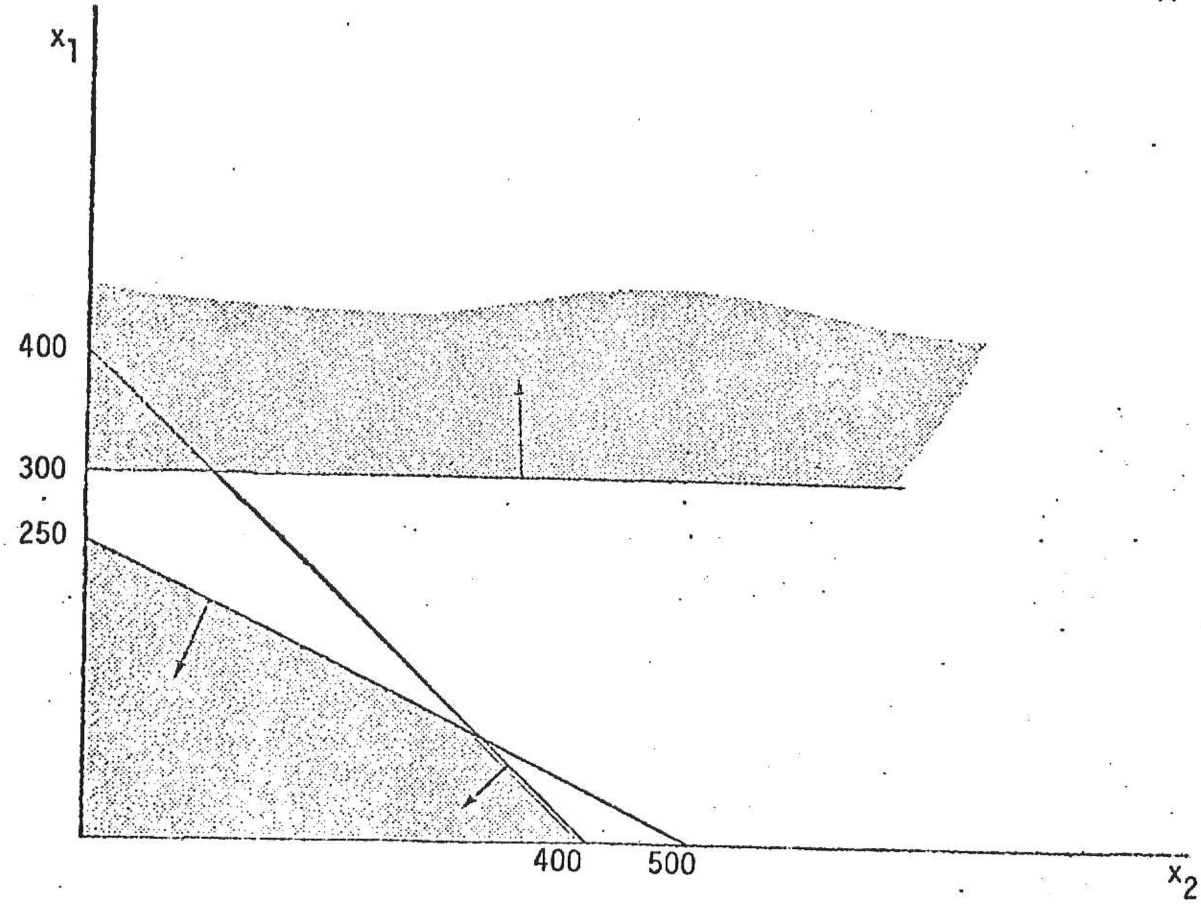

Figure 2-3 Infeasible solution for linear programming model

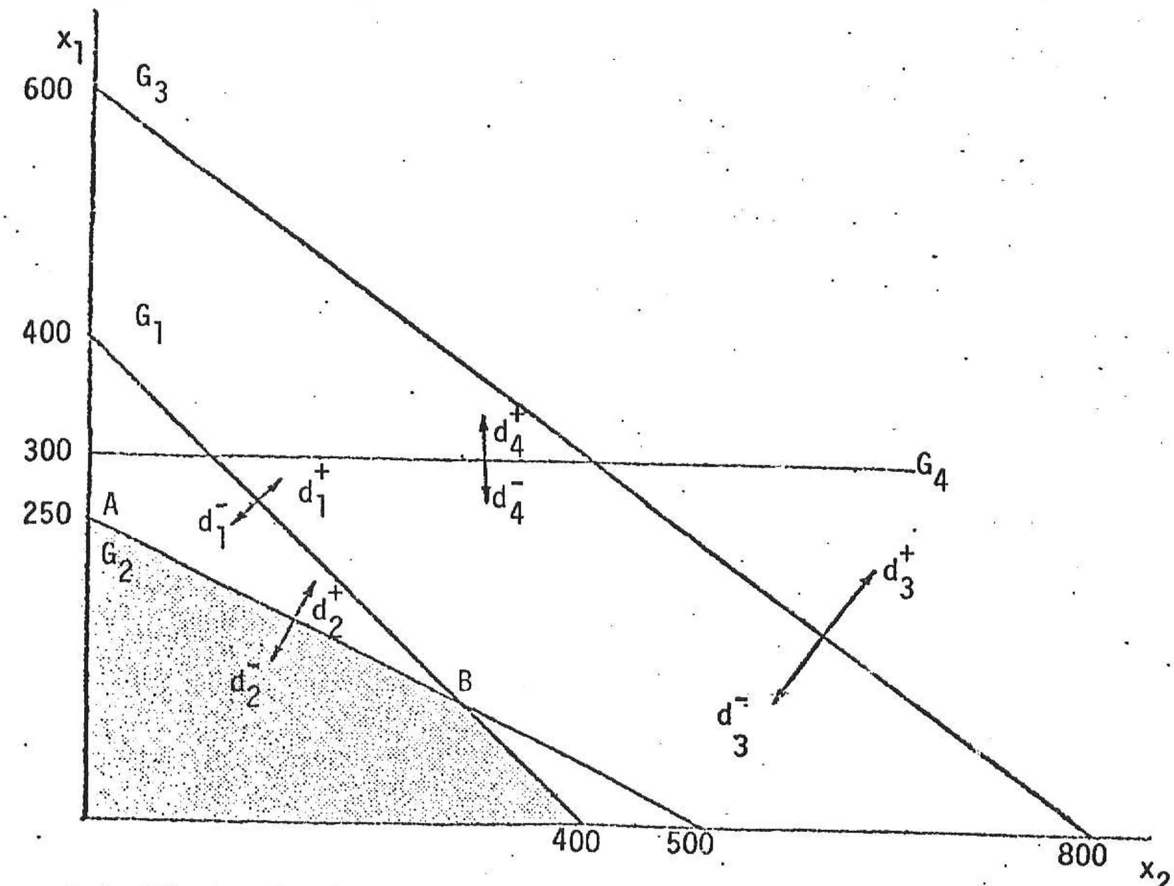

Figure 2-4 Final solution for Example 2-2

Find $\bar{x} = (x_1, x_2)$ so as to

$$\text{Min. } \bar{a} = \{(d_1^+ + d_2^+), (d_4^-), (d_3^-)\}$$

such that

$$G_1: \quad x_1 + x_2 + d_1^- - d_1^+ = 400$$

$$G_2: \quad 2x_1 + x_2 + d_2^- - d_2^+ = 500$$

$$G_3: \quad 0.4x_1 + 0.3x_2 + d_3^- - d_3^+ = 240$$

$$G_4: \quad x_1 + d_4^- - d_4^+ = 300$$

$$\bar{x}, \bar{d}^-, \bar{d}^+ \geq 0$$

The above model is shown on the graph in Figure 2-4. Now our first goal is to avoid the overtime operation of the plant by minimizing $(d_1^+ + d_2^+)$. That is $d_1^+ = d_2^+ = 0$. Hence the feasible solution must be in the cross-hatched area. Our second goal calls for production of 300 type A doll by minimizing d_4^-. We can proceed until we reach point A (that is $d_4^- = 50$) without degrading the solution to priority level 1. Finally to achieve profit maximization goal of priority level 3, we must minimize d_3^-. We can proceed from point A to point B, but this movement is against priority level 2. Hence point A has to be the final solution. That is,

$$x_1^* = 250 \qquad x_2^* = 0$$

$$\bar{a}^* = (0, 50, 140)$$

Example 2-3

Manhattan China Inc. produces the world's finest china sets. The company's production facility consists of two production lines. Production line 1 is staffed with skilled workers who can produce an average of 5 sets per hour.

Line 2 is capable of producing an average of only 3 sets per hour, as it is staffed with relatively new employees. The regular working hours for the weeks are 40 for each line. The profit from an average set is $50. It is estimated that the operating cost of the two lines are almost identical. The president of the company has suggested the following multiple goals to be achieved in the coming week in ordinal ranking of importance.

P_1: Achieve the production goal of 450 set for the week

P_2: Limit the overtime operation of line 1 to 10 hours

P_3: Avoid the underutilization of regular working hours for both lines

P_4: Limit the sum of overtime operation for both lines.

The following objective functions can be formulated.

Production capacity

$$G_1:\quad 5x_1 + 3x_2 + d_1^- - d_1^+ = 450$$

where

x_1 = number of hours line 1 is in operation

x_2 = number of hours line 2 is in operation

d_1^- = underattainment of production goal of 450 sets

d_1^+ = production in excess of 450 sets.

Regular working hours.

$$G_2:\quad x_1 + d_2^- - d_2^+ = 40$$

$$G_3:\quad x_2 + d_3^- - d_3^+ = 40$$

where

d_2^- = underutilization of regular working hours of line 1

d_2^+ = overtime operation in line 1

d_3^- = underutilization of regular working hours of line 2

d_3^+ = overtime operation in line 2.

Overtime operation for line 1.

$$G_4: \quad x_1 + d_4^- - d_4^+ = 40 + 10 = 50$$

where

d_4^- = operation hours of line 1 that total less than 50 hours

d_4^+ = overtime operation of line 1 beyond 10 hours.

The achievement function for this example is then:

Find x_1 and x_2 so as to minimize

$$\bar{a} = \{(d_1^-), (d_4^+), (5d_2^- + 3d_3^-), (3d_2^+ + 5d_3^+)\}$$

That is, our first priority is to achieve the production goal through the minimization of d_1^-. The second priority is given to limiting overtime operation of line 1 to 10 hours through minimizing of d_4^+. The third priority is assigned to the minimization of underutilization of regular working hours by minimizing d_2^- and d_3^-. Since the productivity of line 1 is 5 units per hour and that of line 2 only 3 units per hour, the president wishes to avoid the underutilization of regular working hours of line 1 more than line 2. The differential weights, then, will be 5 for d_2^- and 3 for d_3^-. The fourth and final priority is to minimize the sum of overtime operation for both lines. Again we have to assign differential weights to d_2^+ and d_3^+. The criterion to be used is the relative cost of overtime. Since ratio of the production rates for the two lines is 5 to 3, the relative cost of overtime for two lines should be 3 to 5. The final decision model is written as:

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Find x_1 and x_2 so as to minimize

$$\bar{a} = \{(d_1^-), (d_4^+), (5d_2^- + 3d_3^-), (3d_2^+ + 5d_3^+)\}$$

such that

$$G_1: \quad 5x_1 + 3x_2 + d_1^- - d_1^+ = 450$$

$$G_2: \quad x_1 \qquad + d_2^- - d_2^+ = 40$$

$$G_3: \qquad x_2 + d_3^- - d_3^+ = 40$$

$$G_4: \quad x_1 \qquad + d_4^- - d_4^+ = 50$$

$$\bar{x}, \bar{d}^-, \bar{d}^+ \geq 0$$

The above model is shown on the graph in Figure 2-5. Now our first goal is to achieve the production goal of 450 sets by minimizing d_1^-. That is $d_1^- = 0$. Hence the feasible solution must be in the cross-hatched area. Our second goal calls for the limitation of overtime operation for line 2 to 10 hours by minimizing d_4^+ or let $d_4^+ = 0$. This can be plotted as $x_1 \leq 50$, as shown in Figure 2-6. The third goal is to avoid underutilization of regular working hours for both lines. Since a greater weight is assigned to the underutilization of line 1, we should examine the working hours of line 1 first. This condition calls for $x_1 \geq 40$ (minimizing $5d_2^-$ or let $d_2^- = 0$). Plotting this on the graph we obtained the area of feasible solution shown in Figure 2-7. Now, we are also to avoid any underutilization of regular working hours for line 2, that is, to minimize $3d_3^-$ or let $d_3^- = 0$. It is evident in Figure 2-7 that the cross-hatched area already satisfies this subgoal. The last goal is to minimize the sum of overtime for both lines. However the limitation for line 2 is given a greater weight. Hence d_3^+

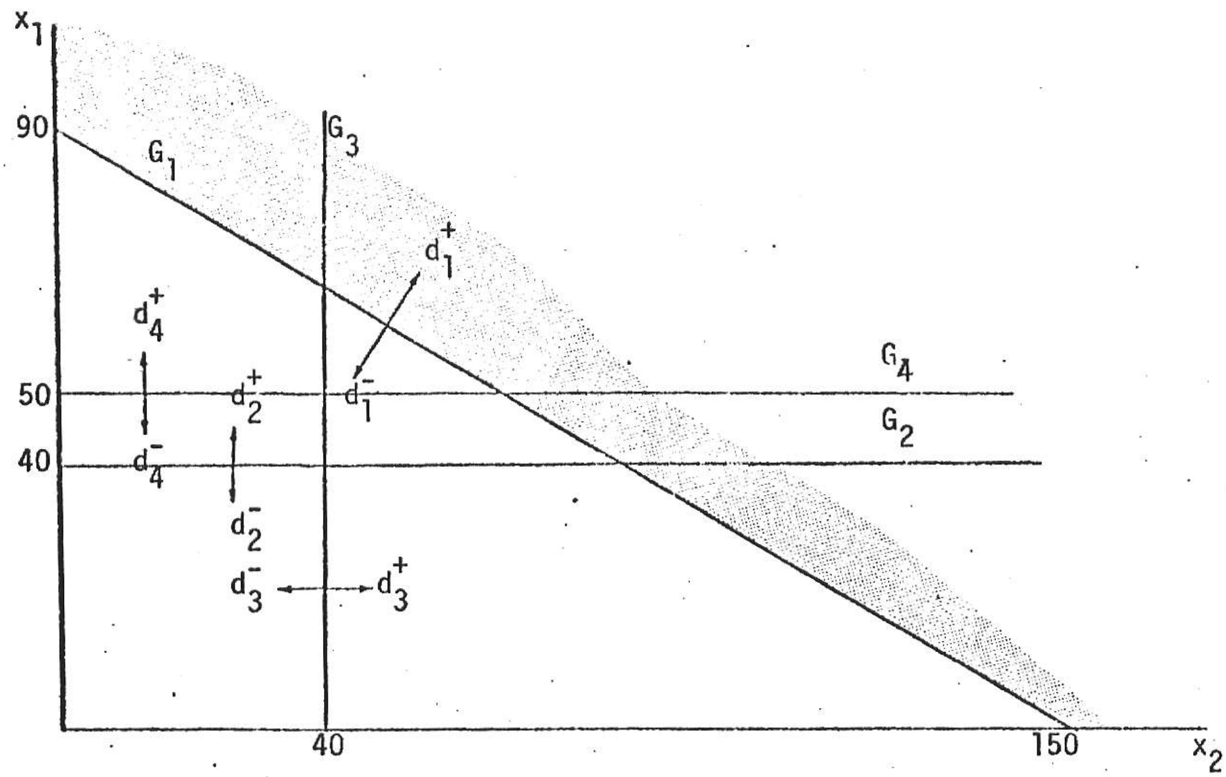

Figure 2-5 P_1 fully satisfied

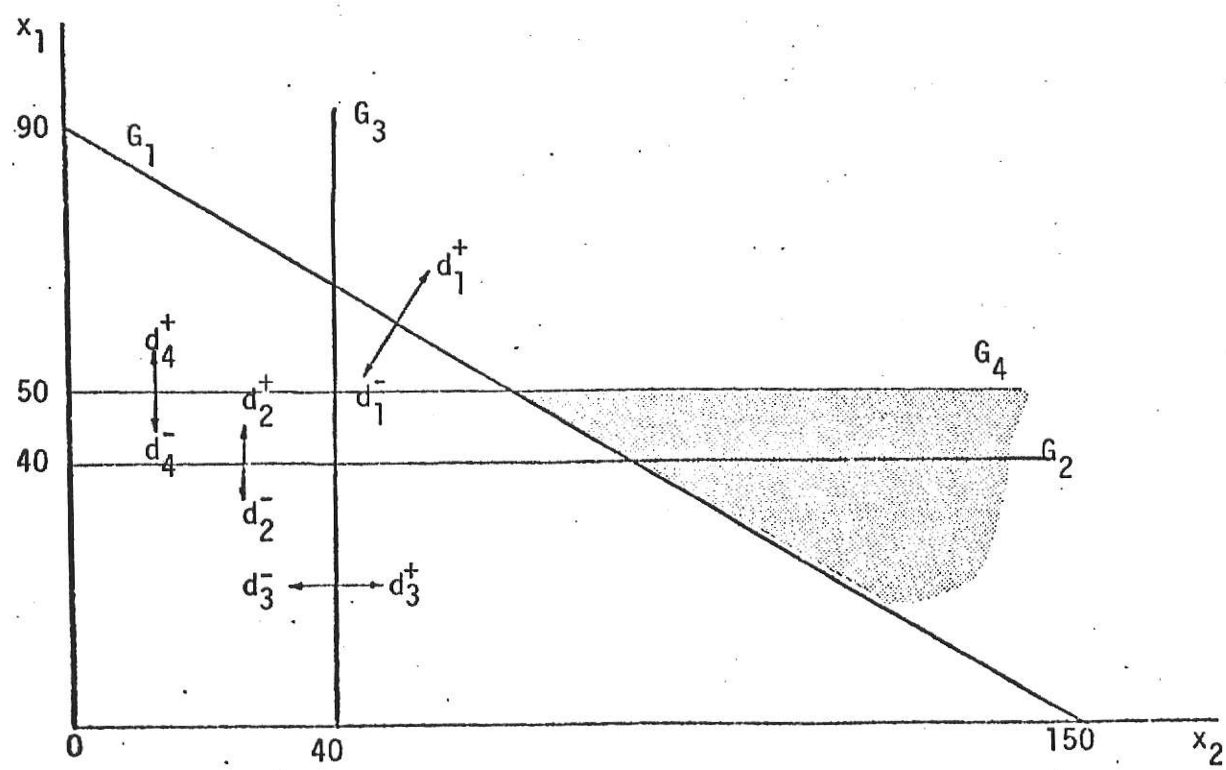

Figure 2-6 P_1 and P_2 fully satisfied.

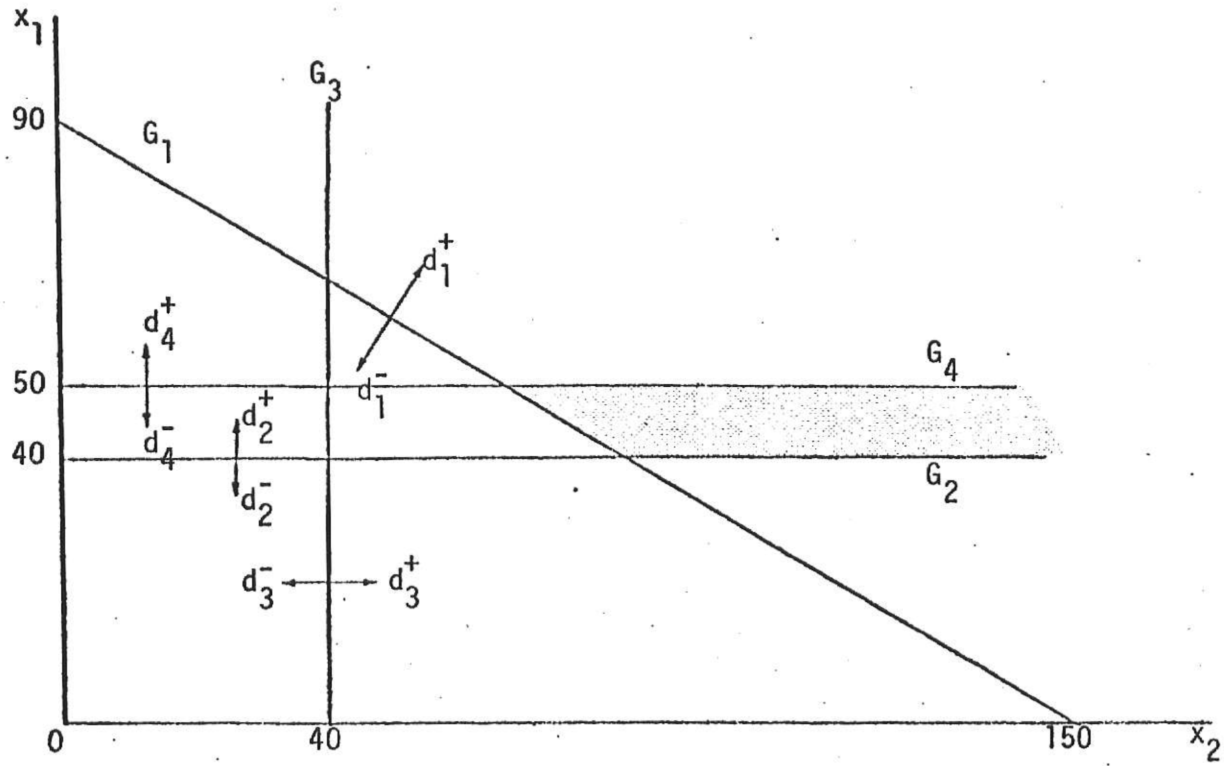

Figure 2-7 P_1, P_2 and P_3 fully satisfied.

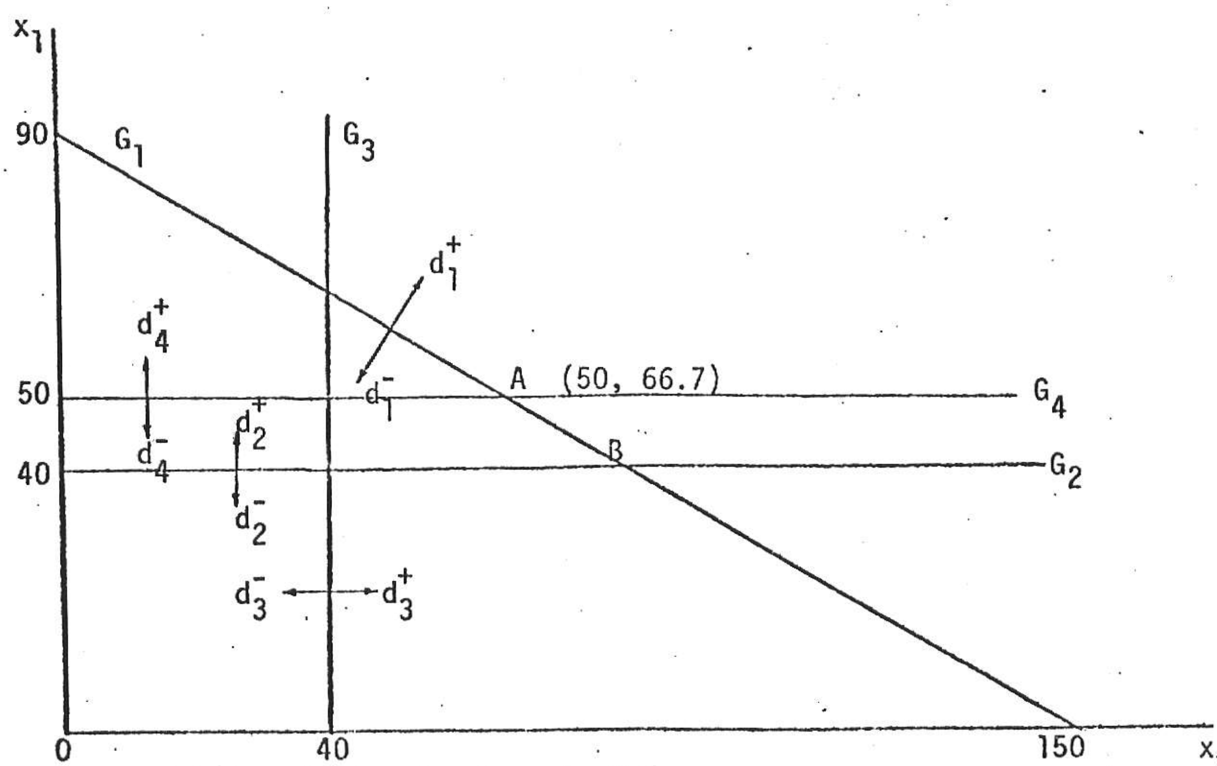

Figure 2-8 Final solution for Example 2-3

(the overtime operation of line 2) must be minimized to the fullest extent. Obviously, at point A in Figure 2-8, d_3^+ will be minimized while meeting the first three goals. Next we try to minimize d_2^+ (the overtime operation of line 1). We can proceed until it reaches point B, but this movement is against the minimizing of d_3^+. Since we cannot decrease d_2^+ any further from point A without increasing d_3^+, point A has to be the optimum point. That is:

$$x_1^* = 50 \qquad x_2^* = 66.\bar{6}$$

$$\bar{a}^* = (0,\ 0,\ 0,\ 163.\dot{3})$$

with these values ($x_1 = 50$, $x_2 = 66.7$) we could attain our first three goals completely, but could not attain the last goal. The value 163.3 comes from $g_4 = 3d_2^+ + 5d_3^+ = 3 \times 10 + 5 \times 26.7 = 163.3$.

<u>Example 2-4</u>

To demonstrate the significance of priority level in goal programming, Example 2-3 is examined again, changing only the ordinal priority of the president's decision.

New priority levels are as follows:

P_1 = Old P_4

P_2 = Old P_3

P_3 = Old P_2

P_4 = Old P_1

then the achievement function will be

$$\bar{a} = \{(3d_2^+ + 5d_3^+),\ (5d_2^- + 3d_3^-),\ (d_4^+),\ (d_1^-)\}$$

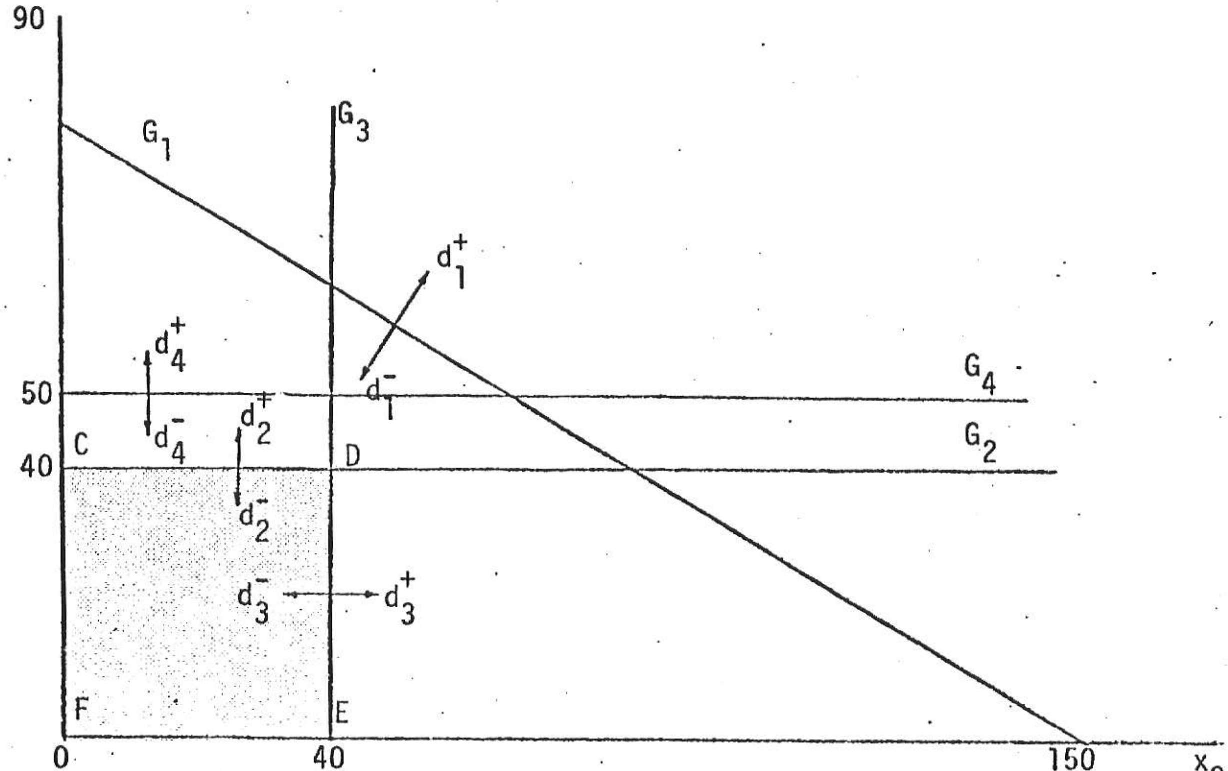

Figure 2-9 Solution for Example 2-4

The graphical solution to Example 2-4 is demonstrated in Figure 2-9. Priority level 1 is to minimize $(3d_2^+ + 5d_3^+)$, and the feasible region CDEF satisfies this goal. In priority level 2, d_2^- has larger weight, hence d_2^- is to be minimized first, and line segment CD is the solution. Line DE is the solution to minimize d_3^-. However, to satisfy the minimization of d_2^-, point D is the solution. At point D, d_4^+ has already become zero (i.e., P_3 fully satisfied). Now we attempt to achieve priority level 4 by minimizing d_1^- or $d_1^- = 0$, but this is against all higher priorities, level 1 through level 3. The final solution is point D, that is,

$$x_1^* = 40 \qquad x_2^* = 40$$

$$\bar{a} = (0,\ 0,\ 0,\ 130)\ .$$

The above two examples show that there is no feasible solution which satisfies all four goals as long as these goals are conflicting with each other. However, through priority level ranking of the goals, we can reach an optimal solution. Further, it is worth noticing that changes of priority levels give a distinctive optimal solution.

4. The Computational Algorithm

Two kinds of algorithms have been presented to solve linear goal programming problems. The generalized inverse technique has been introduced by Ijiri [5], while Lee [6] and Ignizio [4] use the modified simplex method. The modified simplex method is presented here.

4.1 The modified simplex method.

Establishing the Initial Tableau

The extended tableau for this method is shown in Table 2-2. After a brief explanation of the tableau, some examples will be used to demonstrate the procedure. The elements within this tableau may be defined as follows:

Table 2-2 The Extended Tableau for the Modified Simplex Method

				P_K	$w_{K,1}$	$\cdots$	$w_{K,n}$	$w_{K,n+1}$	$\cdots$	$w_{K,n+m}$	$w_{K,n+m+1}$	$\cdots$	$w_{K,n+2m}$	
		Left Stub		$\vdots$		$\vdots$			$\vdots$			$\vdots$		Top Stub
				P_1	$w_{1,1}$	$\cdots$	$w_{1,n}$	$w_{1,n+1}$	$\cdots$	$w_{1,n+m}$	$w_{1,n+m+1}$	$\cdots$	$w_{1,n+2m}$	
	P_K	$\cdots$	P_1	V	x_1	$\cdots$	x_n	d_1^-	$\cdots$	d_m^-	d_1^+	$\cdots$	d_m^+	$\bar{b}$
	$u_{1,K}$	$\cdots$	$u_{1,1}$	d_1^-	$e_{1,1}$	$\cdots$	$e_{1,n}$	$e_{1,n+1}$	$\cdots$	$e_{1,n+m}$	$e_{1,n+m+1}$	$\cdots$	$e_{1,n+2m}$	b_1
		$\vdots$		$\vdots$		$\vdots$			$\vdots$			$\vdots$		$\vdots$
	$u_{m,K}$	$\cdots$	$u_{m,1}$	d_m^-	$e_{m,1}$	$\cdots$	$e_{m,n}$	$e_{m,n+1}$	$\cdots$	$e_{m,n+m}$	$e_{m,n+m+1}$	$\cdots$	$e_{m,n+2m}$	b_m
				P_1	$I_{1,1}$	$\cdots$	$I_{1,n}$	$I_{1,n+1}$	$\cdots$	$I_{1,n+m}$	$I_{1,n+m+1}$	$\cdots$	$I_{1,n+2m}$	a_1
		Index Rows		$\vdots$		$\vdots$			$\vdots$			$\vdots$		$\vdots$
				P_K	$I_{K,1}$	$\cdots$	$I_{K,n}$	$I_{K,n+1}$	$\cdots$	$I_{K,n+m}$	$I_{K,n+m+1}$	$\cdots$	$I_{K,n+2m}$	a_K

Headings:

P_k = the kth priority level, k = 1, ..., K

V = decision and deviational variables. The variables below V (d_i^-) are the initial set of basic variables.

$\bar{b}$ = the elements below $\bar{b}$ are the b_is, i.e., the right-hand-side values of each objective.

Elements:

j = 1, ..., n; i = 1, ..., m; s = 1, ..., S; k = 1, ..., K .

$e_{i,s}$ = coefficient of the sth variable in the ith objective.

$w_{i,s}$ = weighting factor of priority k (P_k) associated with the sth variable.

$u_{i,k}$ = weighting factor of priority k (P_k) associated with ith basic variable.

$I_{k,s}$ = index number for priority k under the sth variable.

a_k = level of achievement of priority k.

All elements except $I_{k,s}$ and a_k are from the initial mathematical decision model. However, $I_{k,s}$ and a_k must be computed as follows:

$$I_{k,s} = \sum_{i=1}^{m} (e_{i,s} \cdot u_{i,k}) - w_{k,s}$$

$$a_k = \sum_{i=1}^{m} (b_i \cdot u_{i,k})$$

Referring again to the extended tableau of Table 2-2, it should be noted that if we remove the rows and columns associated with the "Top Stub", "Left Stub" and "Index Rows", the remaining matrix is simply the matrix of objective function coefficients, and the right hand side values of each objective. In the initial tableau the basic variables will always be the set of negative deviation variables (d_i^-). That is, with m objectives, only m variables may be basic and all others are set to a value of zero and denoted as nonbasic. The iterations of the simplex algorithm consist of exchanging a present non-basic variable for a present basic variable if such an exchange improves the present solution. If only a single priority level had been established, this tableau would be similar to the traditional "single objective" simplex tableau of linear programming. The solution to the linear goal programming model, at any stage, is given by a_1, a_2, ..., a_K, where a_k represent the level of achievement for priority k. Since the achievement function in linear goal programming model is always of the minimization form, the lower the value of a_k, the better the level of achievement. A zero value means that this particular priority level has been completely achieved. The set of index rows ($I_{k,s}$) in the tableau serve to indicate whether or not the present solution is optimal and, if not, the proper exchange between a basic and non-basic variable for a better solution.

<u>The Algorithm</u>

The steps of the modified simplex algorithm are:

Step 1: <u>Initialization</u>. Establish the initial tableau and the index row. Set k = 1 and proceed to Step 2.

Step 2: <u>Check for Optimality</u>. Examine a_k. If a_k is zero go to Step 6. Otherwise, examine each positive valued index number ($I_{k,s}$) in the kth index row. Select the largest, positive $I_{k,s}$ for which there are no negative valued index numbers at a higher priority in the same column. Designate this column as optimum column (s'). Ties in the selection of $I_{k,s}$ may be broken arbitrarily. If no such $I_{k,s}$ may be found, go to Step 6. Otherwise, go to Step 3.

Step 3: Determining the New Entering Variable. The nonbasic variable in the optimum column (s') is the new entering variable.

Step 4: Determining the Departing Variable. Determine the row associated with the minimum nonnegative value of:

$$b_i \,/\, e_{i,s'}, \qquad i = 1,2, \ldots, m$$

In the event of ties, select that row having the basic variable with the higher priority level. Designate this row as key row (i'). The basic variable associated with key row (i') is the departing variable.

Step 5: Establishment of the New Tableau.

(a) Set up a new tableau with all $e_{i,s}$, b_i, $I_{k,s}$ and a_k elements empty. Replace the position of the basic variable heading in optimum row with the nonbasic variable heading in the key column.

(b) Make each objective have one basic variable with a coefficient of +1, and make sure that this basic variable does not appear in any other objective (by the Gaussian elimination method). This can be done [3] by performing the elementary row operations (c) and (d).

(c) Multiplying an objective by a nonzero constant.

(d) Adding a multiple of one objective to another objective.

(e) Establish the new values for $I_{k,s}$ and a_k.

(f) Return to Step 2.

Step 6: Evaluate Next Lower Priority Level. Set k = k+1. If k exceeds K (the total number of priority levels), then stop as the solution is optimal. If $k \leq K$ go to Step 2.

Example 2-5

Example 2-1 illustrated by graphical solution is again considered. Find $\bar{x} = (x_1, x_2)$ so as to

$$\text{Min.}\quad \bar{a} = \{(d_1^+ + d_2^+),\ (d_3^-)\}$$

such that

$$x_1 + \quad x_2 + d_1^- - d_1^+ = 400$$

$$2x_1 + \quad x_2 + d_2^- - d_2^+ = 500$$

$$0.4x_1 + 0.3x_2 + d_3^- - d_3^+ = 240$$

$$\bar{x},\ \bar{d}^-,\ \bar{d}^+ \geq 0$$

Step 1: The initial tableau for this example is given in Table 2-3. For clarity, the zero elements of the tableau have been omitted. The basic solution of this tableau is,

$$d_1^- = 400$$

$$d_2^- = 500$$

$$d_3^- = 240\ .$$

All other variables (those nonbasic) are zero. P_1 is completely satisfied since $a_1 = 0$. P_2 is not completely satisfied since $a_2 = 240$.

Step 2: Since $a_1 = 0$, go to Step 6.

Step 6: $k = k+1 = 2$ $k \leq K$ since $k = 2$, $K = 2$ go to Step 2.

Step 2: $a_2 = 240$. Hence, examine all positive valued indicators in index row 2 ($I_{2,s}$). $I_{2,1}$ is the largest value (+0.4) and there are no negative valued index numbers above $I_{2,1}$ ($I_{1,1} = 0$). Thus $s' = 1$ go to Step 3.

Step 3: x_1 is the new entering variable.

Table 2-3 The Initial Tableau for Example 2-1

		P_2					1				
		P_1						1	1		
P_2	P_1	V	x_1	x_2	d_1^-	d_2^-	d_3^-	d_1^+	d_2^+	d_3^+	$\bar{b}$
		d_1^-	1	1	1			-1			400
		d_2^-	2	1		1			-1		500
1		d_3^-	.4	.3			1			-1	240
		P_1						-1	-1		0
		P_2	$\boxed{.4}$	.3						-1	240

Table 2-4 Second Tableau for Example 2-1

		P_2					1				
		P_1						1	1		
P_2	P_1	V	x_1	x_2	d_1^-	d_2^-	d_3^-	d_1^+	d_2^+	d_3^+	$\bar{b}$
		d_1^-		.5	1	-.5		-1	.5		150
		x_1	1	.5		.5			-.5		250
1		d_3^-		.1		-.2	1		.2	-1	140
		P_1						-1	-1		0
		P_2		$\boxed{.1}$		-.2			.2	-1	140

Table 2-5 Third Tableau for Example 2-1

		P_2					1				
		P_1						1	1		
P_2	P_1	V	x_1	x_2	d_1^-	d_2^-	d_3^-	d_1^+	d_2^+	d_3^+	$\bar{b}$
		x_2		1	2	-1		-2	1		300
		x_1	1		-1	1		1	-1		100
1		d_3^-			-.2	-.1	1	.2	.1	-1	110
		P_1						-1	-1		0
		P_2			-.2	-.1		.2	.1	-1	110

Step 4: Computing nonnegative entering $b_i/e_{i,s'}$.

We obtain

$$b_1/e_{1,1} = 400/1 = 400$$

$$b_2/e_{2,1} = 500/2 = 250 \text{ (minimum value)}$$

$$b_3/e_{3,1} = 240/0.4 = 600 \ .$$

Hence i' = 2 and departing variable is d_2^-.

Step 5: Replace basic variable of d_2^- with x_1.

New $e_{i,s'}$ values can be calculated by the following way,

$$\text{New } e_{2,s} = \text{Old } e_{2,s}/2 \text{ (since } e_{i',s'} = 2)$$

$$\text{New } e_{1,s} = \text{Old } e_{1,s} - \text{New } e_{2,s}$$

$$\text{New } e_{3,s} = \text{Old } e_{3,s} - 0.4 \text{ New } e_{2,s}$$

New tableau is set up in Table 2-4.

Go to Step 2.

Step 2: $a_2 = 140$, so priority level 2 is not satisfied. Hence examine all nonnegative indicator elements in row 2 and find that $i_{2,7} = 0.2$ is the largest. However, since there is a negative index number above this element ($I_{1,7} = -1$), an exchange cannot be made here without degrading the achievement of priority level 1. Hence, go to $I_{2,2} = 0.1$, thus s' = 2.

Go to Step 3.

Step 3: x_2 is the new entering variable.

Step 4: Computing nonnegative $b_i/e_{i,s'}$, the following is obtained:

$$b_1/e_{1,2} = 150/0.5 = 300 \text{ (minimum value)}$$

$$b_2/e_{2,2} = 250/0.5 = 500$$

$$b_3/e_{3,2} = 140/0.1 = 1400$$

Thus $i' = 1$ and departing variable is d_1^- .

Step 5: Third tableau is shown in Table 2-5.

Go to Step 2.

Step 2: $a_2 = 110$, so priority 2 is not completely satisfied. However, positive $I_{2,6}$ and $I_{2,7}$ values have negative elements above them at a higher priority (P_1), Hence, go to Step 6.

Step 6: $k = 2+1 = 3$, and $k > K$, thus the solution is optimal.

The solution to this example is then:

$$x_1^* = 100 \qquad x_2^* = 300$$

$$\bar{a}^* = \{0,\ 110\}$$

Example 2-6

In this example we shall solve the model of Example 2-3 again. Only the tableaus will be shown since by now the reader should be able to follow the steps resulting in each tableau with little difficulty. Tables 2-6 through 2-11 illustrate the progress of the algorithm.

The optimal solution is read from the last tableau and is:

$$x_1^* = 50 \qquad x_2^* = 200/3$$

$$\bar{a}^* = (0,\ 0,\ 0,\ 490/3)$$

Example 2-7

By the same token, Table 2-12 through 2-14 illustrate the progress of the algorithm applying to solve Example 2-4.

The optimal solution is read from the final tableau and is:

$$x_1^* = 40 \qquad x_2^* = 40$$

$$\bar{a}^* = (0,\ 0,\ 0,\ 130)$$

- Table 2-6 Initial Tableau for Example 2-3

P_4	P_3	P_2	P_1	V	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+	b
				P_4								3	5		
				P_3				5	3						
				P_2										1	
				P_1			1								
			1	d_1^-	5	3	1				-1				450
	5			d_2^-	1			1				-1			40
	3			d_3^-		1			1				-1		40
				d_4^-	1					1				-1	50
				P_1	$\boxed{5}$	3					-1				450
				P_2										-1	0
				P_3	5	3						-5	-3		320
				P_4								-3	-5		0

Table 2-7 Second Tableau for Example 2-3

P_4	P_3	P_2	P_1	V	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+	b
				P_4								3	5		
				P_3				5	3						
				P_2										1	
				P_1			1								
			1	d_1^-		3	1	-5			-1	5			250
				x_1	1			1				-1			40
	3			d_3^-		1			1				-1		40
				d_4^-				-1		1		1		-1	10
				P_1		3		-5			-1	$\boxed{5}$			250
				P_2										-1	0
				P_3		3		-5					-3		120
				P_4								-3	-5		0

Table 2-8 Third Tableau for Example 2-3

				P_4								3	5		
				P_3				5	3						
				P_2										1	
				P_1			1								
P_4	P_3	P_2	P_1	V	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+	b
			1	d_1^-		3	1			-5	-1			5	200
				x_1	1					1				-1	50
	3			d_3^-		1			1				-1		40
3				d_2^+				-1		1		1		-1	10
				P_1		3				-5	-1			$\boxed{5}$	200
				P_2										-1	0
				P_3		3		-5					-3		120
				P_4				-3		3			-5	-3	30

Table 2-9 Fourth Tableau for Example 2-3

				P_4								3	5		
				P_3				5	3						
				P_2										1	
				P_1			1								
P_4	P_3	P_2	P_1	V	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+	b
		1		d_4^+		3/5	1/5			-1	-1/5			1	40
				x_1	1	3/5	1/5				-1/5				90
	3			d_3^-		1			1				-1		40
3				d_2^+		3/5	1/5	-1			-1/5	1			50
				P_1			-1								0
				P_2		$\boxed{3/5}$	1/5			-1	-1/5				40
				P_3		3		-5					-3		120
				P_4		9/5	3/5	-3			-3/5		-5		150

Table 2-10 Fifth Tableau for Example 2-3

P_4	P_3	P_2	P_1	V	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+	b
				P_4								3	5		
				P_3				5	3						
				P_2										1	
				P_1			1								
		1		d_4^+			1/5		-3/5	-1	-1/5		3/5	1	16
				x_1	1		1/5		-3/5		-1/5		3/5		66
				x_2		1			1				-1		40
3				d_2^+			1/5	-1	-3/5		-1/5	1	3/5		26
				P_1			-1								0
				P_2			1/5		-3/5	-1	-1/5		$\boxed{3/5}$		16
				P_3				-5	-3						0
				P_4			3/5	-3	-9/5		-3/5		-16/5		78

Table 2-11 Final Tableau for Example 2-3

P_4	P_3	P_2	P_1	V	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+	b
				P_4								3	5		
				P_3				5	3						
				P_2										1	
				P_1			1								
5				d_3^+			1/3		-1	-5/3	-1/3		1	5/3	80/3
				x_1	1					1				-1	50
				x_2		1	1/3			-5/3	-1/3			5/3	200/3
3				d_2^+				-1		1		1		-1	10
				P_1			-1								0
				P_2										-1	0
				P_3				-5	-3						0
				P_4			5/3	-3	-5	-16/3	-5/3			16/3	490/3

Table 2-12 Initial Tableau for Example 2-4

P_4	P_3	P_2	P_1	V	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+	$\bar{b}$
				P_4			1								
				P_3										1	
				P_2				5	3						
				P_1								3	5		
1				d_1^-	5	3	1				-1				450
		5		d_2^-	1			1				-1			40
		3		d_3^-		1			1				-1		40
				d_4^-	1					1				-1	50
				P_1								-3	-5		0
				P_2	$\boxed{5}$	3						-5	-3		320
				P_3										-1	0
				P_4	5	3					-1				450

Table 2-13 Second Tableau for Example 2-4

P_4	P_3	P_2	P_1	V	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+	$\bar{b}$
				P_4			1								
				P_3										1	
				P_2				5	3						
				P_1								3	5		
1				d_1^-		3	1	-5			-1	5			250
				x_1	1			1				-1			40
		3		d_3^-		1			1				-1		40
				d_4^-				-1		1		1		-1	10
				P_1								-3	-5		0
				P_2		$\boxed{3}$		-5					-3		120
				P_3										-1	0
				P_4		3		-5			-1	5			250

Table 2-14 Final Tableau for Example 2-4

				P_4			1								
				P_3										1	
				P_2				5	3						
				P_1								3	5		
P_4	P_3	P_2	P_1	V	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+	b
1				d_1^-			1	-5	-3		-1	5	3		130
				x_1	1			1				-1			40
				x_2		1			1				-1		40
				d_4^-				-1		1		1		-1	10
				P_1								-3	-5		0
				P_2				-5	-3						0
				P_3										-1	0
				P_4				-5	-3		-1	5	3		130

4.2 The modified revised simplex method. A condensed tableau is introduced by Ignizio [4]. Table 2-2 is condensed to Table 2-15. This new tableau eliminates d_i^- from the heading. The columns of d_i^- in the heading simply reflect the set of basic variables associated with the identity matrix of the tableau. Now, it is unnecessary to include the columns associated with d_i^-, since, if these variables are basic, there will always be an identity matrix associated with their columns. Removal of these columns of the extended tableau results in a condensed tableau shown in Table 2-15. Explanation of headings and elements are identical with the extended tableau.

Another savings in computation may be achieved by computing only those index rows necessary for the iteration under consideration. That is, if we are presently interested in achieving priority level k then only the index rows associated with P_1, ..., P_k need to be computed (i.e., P_{k+1}, ..., P_K is not needed).

The Algorithm.

Throughout 6 steps of the algorithm, only Step 5 is changed.

Step 5: Establishment of the New Tableau.

(a) Set up a new tableau with all $e_{i,s}$, b_i, $I_{k,s}$ and a_k elements empty. Exchange the positions of the basic variable heading in optimum row (i') with the nonbasic variable heading in key column (s'). Calculations of the $e_{i,s}$ and b_i elements is given in (b), (c) and (d) below.

(b) Row i' of the new tableau (except for $e_{i',s'}$) is obtained by dividing row i' of the previous tableau by $e_{i',s'}$.

(c) Column s' of the new tableau (except for $e_{i',s'}$) is obtained by dividing column s' of the previous tableau by $(-e_{i',s'})$.

Table 2-15 The Condensed Tableau for the Modified Revised Simplex Method

			P_K	$w_{K,1}$	...	$w_{K,n}$	$w_{K,n+1}$	...	$w_{K,n+m}$		Top Stub
			⋮		⋮			⋮			
	Left Stub		P_1	$w_{1,1}$	...	$w_{1,n}$	$w_{1,n+1}$	...	$w_{1,n+m}$		
P_K	...	P_1	V	x_1	...	x_n	d_1^+	...	d_m^+	$\bar{b}$	
$u_{1,K}$	...	$u_{1,1}$	d_1^-	$e_{1,1}$	...	$e_{1,n}$	$e_{1,n+1}$	...	$e_{1,n+m}$	b_1	
	⋮		⋮		⋮			⋮		⋮	
$u_{m,K}$	...	$u_{m,1}$	d_m^-	$e_{m,1}$	...	$e_{m,n}$	$e_{m,n+1}$	...	$e_{m,n+m}$	b_m	
			P_1	$I_{1,1}$	...	$I_{1,n}$	$I_{1,n+1}$	...	$I_{1,n+m}$	a_1	
Index Rows			⋮		⋮			⋮		⋮	
			P_K	$I_{K,1}$	...	$I_{K,n}$	$I_{K,n+1}$	...	$I_{K,n+m}$	a_K	

(d) The new element at position $e_{i',s'}$ is the reciprocal of $e_{i',s'}$.
The remaining elements are computed as follows.
Let $\hat{b}_i$ and $\hat{e}_{i,s}$ represent the new set of elements to be computed and b_i and $e_{i,s}$ represent the previous value for these elements (from the previous tableau). Then for those elements not in row i' or column s':

$$\hat{e}_{i,s} = e_{i,s} - \frac{(e_{i's})(e_{i,s'})}{e_{i',s'}}$$

$$\hat{b}_i = b_i - \frac{(b_{i'})(e_{i,s'})}{e_{i',s'}}$$

(e) Establish the new values for $I_{k,s}$ and a_k

(f) Return to Step 2.

To illustrate the procedure of the modified revised simplex method, we shall use the same Examples of 2-1, 2-3, and 2-4. The reader will realize that both methods give exactly the same tableau and solutions except for the simpler heading.

Example 2-8

Tables 2-16 through 2-18 illustrate the progress of applying the algorithm to Example 2-1. The optimal solution is read from the last tableau and is:

$$x_1^* = 100 \qquad x_2^* = 300$$

$$\bar{a}^* = (0, 110)$$

Example 2-9

Tables 2-19 through 2-24 illustrate the progress of applying the algorithm to Example 2-3. Until the third tableau we don't have to calculate index row of $k>1$ because we are presently interested in achieving priority level 1. Index rows in the fourth and fifth tableau are calculated up to the 2nd row and all index row calculations are performed in the final tableau. The optimal solution is:

$$x_1^* = 50 \qquad x_2^* = 200/3$$

$$\bar{a}^* = (0,\ 0,\ 0,\ 490/3)$$

Example 2-10

Table 2-25 through 2-27 illustrate the progress of applying the algorithm to Example 2-4. The optimal solution is

$$x_1^* = 40 \qquad x_2^* = 40$$

$$\bar{a}^* = (0,\ 0,\ 0,\ 130)$$

5. Complications and Their Resolutions

5.1 Right-hand side value. In an objective such as maximizing profit or minimizing cost, there is no target value specified. This causes no problem as long as a reasonable value (an aspiration level) is given. Alternatively, one could use a right-hand side value that determines either an upper bound in maximization or a lower bound in minimization. Nevertheless, it is still important that such upper or lower bound values be reasonable. Unreasonable levels can serve to degrade any resulting solution. For example, if we set the profit of an objective to some unreasonably high value, the achievement of this value can result in less likelihood of the achievement of lower priority objectives.

5.2 Negative right-hand side. Consider the objective below:

$$G_1 = -2x_1 - 3x_2 + d_1^- - d_1^+ = -30$$

Table 2-16 Initial Condensed Tableau for Example 2-1.

		P_2						
		P_1			1	1		
P_2	P_1	V	x_1	x_2	d_1^+	d_2^+	d_3^+	$\bar{b}$
		d_1^-	1	1	-1			400
		d_2^-	2	1		-1		500
1		d_3^-	0.4	0.3			-1	240
		P_1			-1	-1		0
		P_2	$\boxed{0.4}$	0.3			-1	240

Table 2-17 Second Condensed Tableau for Example 2-1.

		P_2						
		P_1			1	1		
P_2	P_1	V	d_2^-	x_2	d_1^+	d_2^+	d_3^+	$\bar{b}$
		d_1^-	-1/2	1/2	-1	1/2		150
		x_1	1/2	1/2		-1/2		250
1		d_3^-	-0.2	0.1		0.2	-1	140
		P_1			-1	-1		0
		P_2	-0.2	$\boxed{0.1}$		0.2	-1	140

Table 2-18 Final Condensed Tableau for Example 2-1.

		P_2						
		P_1			1	1		
P_2	P_1	V	d_2^-	d_1^-	d_1^+	d_2^+	d_3^+	Б
		x_2	-1	2	-2	1		300
		x_1	1	-1	1	-1		100
1		d_3^-	-0.1	-0.2	0.2	0.1	-1	110
		P_1			-1	-1		0
		P_2	-0.1	-0.2	0.2	0.1	-1	110

Table 2-19 Initial Condensed Tableau for Example 2-3.

				P_4				3	5		
				P_3							
				P_2						1	
				P_1							
P_4	P_3	P_2	P_1	V	x_1	x_2	d_1^+	d_2^+	d_3^+	d_4^+	Б
			1	d_1^-	5	3	-1				450
	5			d_2^-	1			-1			40
	3			d_3^-		1			-1		40
				d_4^-	1					-1	50
				P_1	$\boxed{5}$	3	-1				450

Table 2-20 Second Condensed Tableau for Example 2-3.

				P_4				3	5		
				P_3	5						
				P_2						1	
				P_1							
P_4	P_3	P_2	P_1	V	d_2^-	x_2	d_1^+	d_2^+	d_3^+	d_4^+	$\bar{b}$
			1	d_1^-	-5	3	-1	5			250
				x_1	1			-1			40
	3			d_3^-		1			-1		40
				d_4^-	-1			1		-1	10
				P_1	-5	3	-1	$\boxed{5}$			250

Table 2-21 Third Condensed Tableau for Example 2-3.

				P_4					5		
				P_3	5						
				P_2						1	
				P_1							
P_4	P_3	P_2	P_1	V	d_2^-	x_2	d_1^+	d_4^-	d_3^+	d_4^+	$\bar{b}$
			1	d_1^-		3	-1	-5		5	200
				x_1				1		-1	50
	3			d_3^-		1			-1		40
3				d_2^+	-1			1		-1	10
				P_1		3	-1	-5		$\boxed{5}$	200

Table 2-22 Fourth Condensed Tableau for Example 2-3.

P_4	P_3	P_2	P_1	V	d_2^-	x_2	d_1^+	d_4^-	d_3^+	d_1^-	$\bar{b}$
				P_4					5		
				P_3	5						
				P_2							
				P_1						1	
		1		d_4^+		3/5	-1/5	-1		1/5	40
				x_1		3/5	-1/5			1/5	90
	3			d_3^-		1			-1		40
3				d_2^+	-1	3/5	-1/5			1/5	50
				P_1						-1	0
				P_2		$\boxed{3/5}$	-1/5	-1		1/5	40

Table 2-23 Fifth Condensed Tableau for Example 2-3.

P_4	P_3	P_2	P_1	V	d_2^-	d_3^-	d_1^+	d_4^-	d_3^+	d_1^-	$\bar{b}$
				P_4					5		
				P_3	5	3					
				P_2							
				P_1							
		1		d_4^+		-3/5	-1/5	-1	3/5	1/5	16
				x_1		-3/5	-1/5		3/5	1/5	66
				x_2		1			-1		40
				d_2^+	-1	-3/5	-1/5		3/5	1/5	26
				P_1						-1	0
				P_2		-3/5	-1/5	-1	$\boxed{3/5}$	1/5	16

Table 2-24 Final Condensed Tableau for Example 2-3.

				P_1							
				P_2	5	3					
				P_3					1		
				P_4						1	
P_4	P_3	P_2	P_1	V	d_2^-	d_3^-	d_1^+	d_4^-	d_4^+	d_1^-	$\bar{b}$
5				d_3^+		-1	-1/3	-5/3	5/3	1/3	80/3
				x_1				1	-1		50
				x_2			-1/3	-5/3	5/3	1/3	200/3
3				d_2^+	-1			1	-1		10
				P_1						-1	0
				P_2					-1		0
				P_3	-5	-3					0
				P_4	-3	-5	-5/3	-16/3	16/3	5/3	490/3

Table 2-25 Initial Condensed Tableau for Example 2-4.

				P_4							
				P_3						1	
				P_2							
				P_1				3	5		
P_4	P_3	P_2	P_1	V	x_1	x_2	d_1^+	d_2^+	d_3^+	d_4^+	$\bar{b}$
1				d_1^-	5	3	-1				450
		5		d_2^-	1			-1			40
		3		d_3^-		1			-1		40
				d_4^-	1					-1	50
				P_1				-3	-5		0
				P_2	$\boxed{5}$	3		-5	-3		320

Table 2-26 Second Condensed Tableau for Example 2-4.

P_4	P_3	P_2	P_1	V	d_2^-	x_2	d_1^+	d_2^+	d_3^+	d_4^+	$\bar{b}$
				P_4							
				P_3						1	
				P_2	5						
				P_1				3	5		
				d_1^-	-5	3	-1	5			250
				x_1	1			-1			40
		3		d_3^-		1			-1		40
				d_4^-	-1			1		-1	10
				P_1				-3	-5		0
				P_2	-5	$\boxed{3}$			-3		120

Table 2-27 Final Condensed Tableau for Example 2-4.

P_4	P_3	P_2	P_1	V	d_2^-	d_3^-	d_1^+	d_2^+	d_3^+	d_4^+	$\bar{b}$
				P_4							
				P_3						1	
				P_2	5	3					
				P_1				3	5		
1				d_1^-	-5	-3	-1	5	3		130
				x_1	1			-1			40
				x_2		1			-1		40
				d_4^-	-1			1		-1	10
				P_1				-3	-5		0
				P_2	-5	-3					0
				P_3						-1	0
				P_4	-5	-3	-1	5	3		130

There is basically nothing wrong with this objective from a mathematical or physical point of view. However, the modified simplex algorithm requires that all right-hand side values be nonnegative. This requirement is easily satisfied by simply multiplying G_1 by (-1). The new objective becomes:

$$-G_1 = 2x_1 + 3x_2 - d_1^- + d_1^+ = 30$$

If we desire to achieve exactly -30 in G_1, this is accomplished by minimizing both d_1^- and d_1^+ at the same priority level. However, if we wish to make G_1 equal to or greater than -30, then d_1^- must be minimized in $-G_1$ Similarly, to satisfy the desire to have G_1 equal to or less than -30, d_1^+ must be minimized in $-G_1$.

5.3 Tie for entering variable. It may occur, during the progress of the modified simplex algorithm, that a tie occurs in the selection of the new entering variable, i.e., two or more columns with the same $I_{k,s}$ value at the highest unattained priority level. If the tie cannot be broken, selection between the contending variables may be made arbitrarily. The other variable will generally be introduced into the solution base in the subsequent iterations.

5.4 Tie for the departing variable. The departing variable is selected through the choice of the smallest, nonnegative $b_i/e_{i,s'}$. It may happen that two or more rows have the same $b_i/e_{i,s'}$ value. The tie may be broken by selecting the row that has the basic variable with the higher priority level. In some cases the basic variables will not have an associated priority level and ties must be broken arbitrarily.

5.5 Alternative optimal solutions. It is possible that two or more points provide optimal solutions that attain exactly the same level of goals. Such a situation never occurs as long as there is a single deviation variable at each priority level and differential weights are assigned in the same priority level when there is more than a single goal at each priority level. To illustrate the point, let us examine Example 2-3. Suppose the president of the company altered the P_4 level and made each weight same value. The achievement function is then:

$$\bar{a} = \{(d_1^-), (d_4^+), (5d_2^- + 3d_3^-), (d_2^+ + d_3^+)\}$$

Optimal solution is at any point on the line segment AB in Figure 2-8.

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CHAPTER 3

LINEAR PROGRAMMING MODEL

1. Introduction

Throughout the less developed nations of the world the quest for rapid economic progress has been predicated largely upon the formulation and implementation of comprehensive development plans, which cover all major aspects of the national economy [12]. Thus, after World War II, many developing countries started their economic planning in order to establish new patterns of welfare statism. Korea is one of the first nations to recognize this urgent need. Soon after the military coup in 1961, the Korean government made clear its intention to manage the national economy in accordance with a fully articulated, comprehensive plan. The First Five-Year Development plan was drawn up in 1961. The second development plan lasted from 1967 to 1971, and the third development plan began in 1972 and ended in 1976. The model presented herein covers the period 1977-1981, which corresponds to the period of Korea's Fourth Five-Year Development plan.

Korea's development planning methodology involved a three-pronged approach, the plan consisting of an aggregate model, a sectoral model, and an investment program. The aggregate model was to be used to select a growth rate consistent with the supply of foreign and domestic savings and foreign exchange. Given the growth rate, the sectoral annual input-output model was to be employed to assure the balance of demand and supply in each sector and to set minimal levels of investment. The sectoral model was to rely on some macroeconomically derived parameters, such as private consumption and imports, for the exogenous estimates of final demand. Thus, the Korean sectoral model

Symbols Used in the Model

Variables and Parameters*		Dimensions for n sectors, k activities T periods
A(t)	net foreign capital inflow in period t	T
a(t)	matrix of interindustry current flow coefficients appropriate to period t	nxk
b(t)	diagonal matrix of capital-output ratio	kxk
c(t)	column vector, each term of which indicates the proportion of the sector's output in total consumption	n
C(t)	aggregate consumption in each period	T
D(t)	vector of the amount of fixed capital (components) in each sector that is completely depreciated in period t	k
d	diagonal matrix of depreciation rate to capital stock	kxk
E(t)	column vector of exports by each sector	n
F(t)	column vector of deliveries by each sector for private consumption purposes	n
G(t)	column vector of deliveries by each sector for government consumption	n
H(t)	column vector of deliveries by each sector for inventory accumulation	n
I	identity matrix	nxn or kxk
I(t)	column vector of investment by each sector	n
J(t)	column vector of deliveries of intermediate inputs by each sector	n
K(t)	column vector of fixed-capital capacity in each sector	k
M(t)	column vector of total imports	n
M'(t)	column vector of noncompetitive imports	k
m'(t)	diagonal matrix of import coefficients relating noncompetitive imports to sectoral output	kxk

Variables and Parameters*		Dimensions for n sectors, k activities T periods
$M''(t)$	column vector of competitive imports	k
$m''(t)$	column vector of coefficients indicating in each sector the maximum use of the foreign exchange available after noncompetitive imports requirements have been satisfied	n
$p(t)$	capital composition matrix in period t	nxk
r	social discount rate	1
$S^D(t)$	sum of domestic savings in period t	T
$S^F(t)$	savings by foreign sectors in period t	T
$s(t)$	matrix of inventory coefficient in period t	nxk
u	unit row vector	nxn
W	social welfare indicator	1
$X(t)$	column vector of gross domestic outputs	k
$Z(t)$	column vector of new additions to fixed-capital capacity in each sector	k
β_0	savings constant	1
β_1	marginal propensity to save	1
$\rho(t)$	minimum rate of growth of aggregate consumption C(t) over C(t-1)	1
δ	marginal contribution of capital stock to the aggregate consumption	1

*Variables in capital letters; parameters in small letters.

was designed to test the consistency of a development program based on targets derived from the macro-model and to indicate total investment requirements and the appropriate sectoral allocation of investment [1].

This linear programming model was adopted from Eckaus and Parikh's multi-sectoral dynamic model [4]. Joe [7] embodied the terminal condition of capital stock into the objective function, thus deleting terminal conditions of capital from the set of constraints. Lack of reliable labor coefficients impedes the addition of labor supply constraints to this model.

The primary purpose of this chapter is to suggest the up-to-date linear economic model of Korea as a first step in the development of the linear goal programming model, hence the postoptimal sensitivity analysis is not considered.

2. Model Formulation

2.1 Industry classification. The 1970 input-out tables, recently published by the Bank of Korea, are in the form of a 56x56 matrix. But, for the purpose of this study they are further aggregated into 16 sectors so that the program becomes manageable. The sixteen sectors are:

1. Agriculture, forestry, and fisheries
2. Mining
3. Processed foods
4. Textiles
5. Lumber, plywood, wood product, and furniture
6. Chemicals
7. Petroleum products and coal products (fuel)
8. Non-metallic mineral products
9. Metallurgical products
10. Machinery

11. Other manufacturing
12. Construction
13. Electricity and water
14. Transportation and communication
15. Banking, insurance, and real estate
16. Commerce, services, and others

2.2 Objective function. Ideally, the objective function should be a social welfare function. Since this function is unknown and aggregate private consumption is the most important determinant of welfare, Eckaus and Parikh [4] has used this exclusively as the objective function. To this definition of the objective function Joe [7] has embodied the terminal condition of capital stock into the objective function, and calls it social welfare indicator (W). Hence,

$$W = f(C,K) \tag{3.1}$$

The derivative of eq. (3.1) is

$$dW = \frac{\partial f}{\partial C} dC + \frac{\partial f}{\partial K} dK \tag{3.2}$$

It is assumed that W is proportional to C at a given value of K, because K employs only in the final planning year. The mathematical expression of this will be $\frac{\partial f}{\partial C} = 1$. Equation (3.2) is divided by $\frac{\partial f}{\partial C}$, it becomes

$$dW = dC + \frac{\partial C}{\partial K} dK \tag{3.3}$$

Integrating both sides yields

$$\int dW = \int dC + \frac{\partial C}{\partial K} \int dK \tag{3.4}$$

or $$W = C + \frac{\partial C}{\partial K} \cdot K \tag{3.4)'$$

A one-year investment lag makes the investment in year t matured into capital stock in the following year. Assuming that the capital stock at the end of (t+1) year continues to produce the aggregate consumption at the constant marginal productivity of $\delta = \frac{\partial C}{\partial K}$ and that the social discount rate is constant after (t+1) years, the sum of a geometric progression gives $(\frac{1+r}{r})\delta$. First, starting with a single year planning horizon and measuring the variables in total amounts;

$$W = C(t) + \left(\frac{1+r}{r}\right) \delta K(t+1) \qquad (3.5)$$

A multi-year planning horizon requires discounting as follows;

$$W = \sum_{t=1}^{T} \frac{C(t)}{(1+r)^t} + \frac{\left(\frac{1+r}{r}\right) \delta K(T+1)}{(1+r)^{T+1}} \qquad (3.6)$$

where T is the terminal year. A simplification of eq. (3.6) leads to;

$$W = \sum_{t=1}^{T} \frac{C(t)}{(1+r)^t} + \frac{\delta K(T+1)}{(1+r)^T \cdot r} \qquad (3.6)'$$

If the depreciation of the capital stock is considered, the final form of the objective function becomes

$$W = \sum_{t=1}^{T} \frac{C(t)}{(1+r)^t} + \frac{\delta^*}{(1+r)^T \cdot r} K(T+1) \qquad (3.7)$$

where $\delta^* = \delta(1-d/b)$

2.3 Constraints. Four types of constraints are involved in the model, namely, physical balance, capacity, foreign exchange, and savings constraints. Brief explanations of these follow:

1) Consumption growth constraints (CG)

$$C(t+1) \geq [1 + \rho(t)]C(t) \qquad t = 0, \ldots, T-1 \tag{3.8}$$

$$C(0) = \overline{C(0)}$$

where $\overline{C(0)}$ = the aggregate private consumption in the pre-plan period. From now on "bar" indicates preassigned value.

These constraints represent the monotonic growth of aggregate consumption between successive periods.

2) Capacity constraints (CC)

$$b(t)\ X(t) \leq K(t) \qquad t = 1, \ldots, T \tag{3.9}$$

$$K(1) = \overline{K(1)}$$

This insures that output in each sector in each period can not exceed the amount producible with the fixed capital in that sector available at the beginning of that period.

3) Capital accounting relationships (CAR)

$$K(t+1) = K(t) + I(t) - D(t) \qquad t = 1, \ldots, T \tag{3.10}$$

$$K(1) = \overline{K(1)}$$

This states that the capital available at the beginning of period (t+1) is equal to the capital available at the beginning of the previous period (t) plus the new completed additions to capacity less the depreciated capacity. Investment and depreciation variables have the following relationships.

3-1) Investment requirement

$$I(t) = p(t)Z(t+1) \qquad t = 1, \ldots, T \tag{3.11}$$

3-2) Depreciation of capital

$$D(t) = dX(t) \qquad t = 1, \ldots, T \tag{3.12}$$

With these subrelationship, eq (3.10) becomes

$$K(t+1) - K(t) - p(t)Z(t+1) + dX(t) = 0 \qquad t = 1, \ldots, T \qquad (3.10)'$$

4) Balance of payment constraints (BOP)

$$uM(t) \leq \overline{A(t)} + u\overline{E(t)} \qquad t = 1, \ldots, T \tag{3.13}$$

The total amount of imports in each period is limited by the available foreign exchange. Imports are divided into non-competitive imports and competitive imports. Import subrelationships follow:

$$M(t) = M'(t) + M''(t) \qquad t = 1, \ldots, T \tag{3.14}$$

$$M'(t) = m'(t)\, X(t) \qquad t = 1, \ldots, T \tag{3.15}$$

with the subrelationships above, eq (3.13) becomes

$$um'(t)\, X(t) + uM''(t) \leq \overline{A(t)} + u\overline{E(t)} \qquad t = 1, \ldots, T \qquad (3.13)'$$

5) Competitive import ceilings (CIC)

$$uM''(t) \leq m''(t)\, [\overline{A(t)} + u\overline{E(t)} - uM'(t)] \qquad t = 1, \ldots, T \tag{3.16}$$

This indicates that competitive imports make use of whatever foreign exchange is left after non-competitive imports are satisfied. With import subrelations of eq. (3.14) and (3.15), eq. (3.16) becomes

$$[m''(t)]^{-1}\ M''(t) + um'(t)\ X(t) \leq \overline{A(t)} + u\overline{E(t)} \qquad t = 1, \ldots, T \tag{3.16)'$$

6) Distributional relationships (DR)

$$J(t) + H(t) + I(t) + F(t) + G(t) + E(t) \leq M(t) + X(t) \qquad t = 1, \ldots, T \tag{3.17}$$

The first six terms represent uses of the output of each sector, and two terms in RHS are the sources of availability of the outputs. Hence, total demand for each sector in each year can not exceed availability in that year. Left-hand-side terms of eq. (3.17) have the following subrelationships with the key decision variables.

6-1) Intermediate product

$$J(t) = a(t)\ X(t) \qquad t = 1, \ldots, T \tag{3.18}$$

6-2) Inventory accumulation

$$H(t) = s(t)\ [X(t) - X(t-1)] \qquad X(0) = \overline{X(0)} \qquad t = 1, \ldots, T \tag{3.19}$$

6-3) Private consumption

$$F(t) = c(t)C(t) \qquad t = 1, \ldots, T \tag{3.20}$$

6-4) Government Consumption

$$G(t) = \overline{G(t)} \qquad t = 1, \ldots, T \tag{3.21}$$

6-5) Export

$$E(t) = \overline{E(t)} \qquad t = 1, \ldots, T \tag{3.22}$$

By substituting eq. (3.11), (3.14), (3.15), (3.18), (3.19), (3.20), (3.21) and (3.22) into eq. (3.17), the following equation is obtained.

$$[a(t) - I - m'(t) + s(t)]\ X(t) - s(t)\ X(t-1) + p(t)\ Z(t+1)$$

$$+ c(t)C(t) - M''(t) \leq - \overline{G(t)} - \overline{E(t)} \quad (3.17)'$$

$$t = 1, \ldots, T$$

7) Savings constraints (SC)

$$uI(t) + uH(t) \leq S^D(t) + S^F(t) \qquad t = 1, \ldots, T \quad (3.23)$$

This states that investment and increase in inventory should be limited by the amount of available investment, which is the sum of domestic savings and savings in the foreign sectors. Two kinds of savings have the following subrelationships.

$$S^D(t) = \beta_0 + \beta_1\{[I - a(t)]\ X(t)\} \qquad t = 1, \ldots, T \quad (3.24)$$

$$S^F(t) = \overline{A(t)} + u[\overline{E(t)} - M(t)] \qquad t = 1, \ldots, T \quad (3.25)$$

By substituting eqs. (3.11), (3.19), (3.24) and (3.25) into eq. (3.23), it becomes

$$up(t)\ Z(t+1) + u\{s(t) + m'(t) - \beta_1[I-a(t)]\}\ X(t) - us(t)\ X(t-1)$$

$$+ uM''(t) \leq \beta_0 + \overline{A(t)} + u\overline{E(t)} \qquad t = 1, \ldots, T \quad (3.23)'$$

<u>2.4 The basic model</u>. The model has the following final form of the linear programming problem.

$$\text{maximize } W = \sum_{t=1}^{T} \frac{C(t)}{(1+r)^t} + \frac{\delta(1-d/b)}{(1+r)^T \cdot r} K(T+1) \quad (3.7)$$

subject to

$$[1+\rho(t)]C(t) - C(t+1) \leq 0 \qquad t = 0, \ldots, T-1 \quad (3.8)$$

$$-K(t) + b(t)\ X(t) \leq 0 \qquad t = 1, \ldots, T \quad (3.9)$$

$$-K(t) + K(t+1) + dX(t) - p(t)\ Z(t+1) = 0 \qquad t = 1, \ldots, T \quad (3.10)'$$

$$uM''(t) + um'(t)\ X(t) \leq \overline{A(t)} + \overline{uE(t)} \qquad t = 1, \ldots, T \qquad (3.13)'$$

$$[m''(t)]^{-1}M''(t) + um'(t)\ X(t) \leq \overline{A(t)} + u\overline{E(t)} \qquad t = 1, \ldots, T \qquad (3.16)'$$

$$c(t)C(t) - M''(t) - s(t)\ X(t-1) + [a(t) - I - m'(t) + s(t)]\ X(t)$$

$$+ p(t)\ Z(t+1) \leq - \overline{E(t)} - \overline{G(t)} \qquad t = 1, \ldots, T \qquad (3.17)'$$

$$uM''(t) - us(t)\ X(t-1) + u\{s(t) + m'(t) - \beta_1[I-a(t)]\}\ X(t) + up(t)\ Z(t+1)$$

$$\leq \beta_0 + \overline{A(t)} + \overline{uE(t)} \qquad t = 1, \ldots, T \qquad (3.23)'$$

$$C(t),\ K(t),\ M''(t),\ X(t),\ Z(t) \geq 0$$

where $C(o) = \overline{C(o)}$

$K(1) = \overline{K(1)}$

$X(o) = \overline{X(o)}$

3. Estimation of Data

3.1 Estimation of parameters

1) Intermediate Input-Output Coefficients (a_{ij}): This model assumes that input coefficients are constant over the planning horizon, due to the difficulty of adjusting those coefficients to future changes in industrial structure and in order to narrow the scope of this study to the programming technique itself. Thus, the 1970 Input-Output Tables, the latest version available, published by the Bank of Korea [2], are used to estimate the input coefficients. The 16 sectors' input coefficients for this model are calculated simply by a further aggregation of the 52 sectors' transaction tables, as shown in Table 3-1.

2) Capital coefficients (b_{ij}): The Bank of Korea's estimation shows that the aggregate capital coefficients by industries are not in a matrix form of (b_{ij}), hence Song's [11]estimation by 16 sectors is used. The 16 sectors' capital coefficients are shown in Table 3-2.

Table 3-1 Input Coefficient Matrix for 1970 by 16 Sectors (a_{ij}) [2].

Sectors	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1. Agriculture, forestry, & fisheries	.111347	.021698	.410849	.087655	.498706	.002544	.000062	.001314	.000271	.000453	.081144	.018195	.000046	.000043	.000179	.020892
2. Mining	.000671	.001675	.002820	.000280	.000325	.021355	.471228	.137231	.043070	.002072	.001002	.020177	.035385	.000063		.001029
3. Processed foods	.046015		.115709	.000952	.004238	.025450		.000046	.000142	.000096	.034298	.000021	.000092			.007038
4. Textiles	.006303	.002163	.004646	.369410	.001833	.004656	.001429	.001113	.001623	.003345	.016990	.000724	.000902	.003989	.000766	.008041
5. Lumber, plywood, wood products & furniture	.002424	.001057	.002245	.000360	.054554	.002299	.000217	.003379	.002036	.010588	.005720	.066080	.000795	.000510	.000138	.001811
6. Chemicals	.041653	.034108	.019419	.117818	.037561	.283103	.012105	.035882	.015498	.026316	.070010	.017759	.005669	.002925	.000288	.016139
7. Petroleum products, & coal products	.006444	.013403	.005737	.007003	.008321	.038862	.013565	.046481	.016559	.010668	.007862	.014463	.124183	.095453	.002851	.011433
8. Non-metallic mineral products	.000437	.001122	.002152	.000169	.004303	.008785	.000318	.052686	.011846	.010790	.013469	.120352	.001406	.001222	.000075	.002190
9. Metallurgical products	.003022	.020153	.006061	.002555	.006917	.009756	.005560	.027616	.548427	.174378	.015694	.136981	.001604	.002826	[illegible]	.002150
10. Machinery	.003156	.018559	.003761	.005776	.006475	.004697	.007664	.008046	.006094	.237628	.005303	.064434	.024830	.077597	[illegible]	.006762
11. Other Manufacturing	.003164	.010133	.015845	.011268	.005591	.032661	.002547	.039392	.004642	.026843	.169585	.007620	.004324	.026930	.012571	.048102
12. Construction	.000766	.002781	.000503	.000476	.00032	.000790	.000738	.000572	.000342	.000554	.000758	.001050	.015555	.003280	.088422	.009241
13. Electricity & water	.000113	.035946	.018216	.014071	.008503	.038844	.005257	.061047	.037823	.009933	.010782	.001035	.023027	.002842	.002079	.005852
14. Transportation & communication	.006816	.011321	.018190	.009504	.021894	.033837	.064921	.088229	.026041	.017027	.020999	.040299	.025869	.033649	.013372	.024630
15. Banking, insurance, & real estate	.006286	.021112	.007287	.014368	.017032	.021466	.008495	.014927	.014452	.016718	.012311	.008300	.035847	.010058	.012180	.008537
16. Commerce, service, and others	.035680	.055546	.102843	.092206	.083248	.119416	.074612	.077158	.076465	.110184	.094255	.102349	.043900	.100595	.031512	.108954
Total intermediate inputs	.274297	.250793	.734276	.733882	.759800	.648521	.669718	.595120	.805331	.677593	.560209	.619839	.343407	.361982	.167096	.282801
Consumption of fixed capital	.011428	.084496	.011059	.016024	.022906	.059973	.030552	.066196	[illegible]	.023625	.013251	.010473	.114908	.116908	.068516	.005896
d(I-a)	.725703	.749207	.265724	.266118	.240200	.351479	.331282	.404880	.194669	.322407	.439791	.380161	.656593	.638018	.832904	.717199

Table 3-2 Capital Coefficients Matrix (b_{ij}) and Capital Composition Matrix (p_{ij}) [8].

Sectors	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	.124620 (.256)															
2																
3																
4																
5	.003410 (.007)	.004560 (.009)	.003260 (.013)	.003090 (.007)	.005930 (.026)	.007930 (.017)	.010660 (.020)	.004490 (.005)	.002800 (.007)	.007160 (.023)	.007700 (.033)					
6																
7																
8																
9	.012410 (.026)	.008650 (.017)	.004270 (.016)	.003500 (.008)	.007360 (.033)	.014440 (.030)	.006130 (.011)	.012190 (.014)	.008050 (.021)	.011990 (.038)	.004730 (.020)					
10	.101360 (.20[illegible])	.275990 (.550)	.145420 (.557)	.285330 (.664)	.141180 (.628)	.303280 (.638)	.354560 (.659)	.572090 (.657)	.254900 (.654)	.149120 (.473)	.129200 (.546)		.506700 (.144)	2.520690 (.546)	.258640 (.042)	.279580 (.283)
11	.001480 (.003)	.000590 (.001)	.000500 (.002)	.000940 (.002)	.000120 (.001)	.005970 (.013)	.000240 (.001)	.006690 (.008)	.001110 (.003)	.002160 (.007)	.000920 (.004)					
12	.242980 (.500)	.211900 (.423)	.107460 (.412)	.137060 (.319)	.070070 (.312)	.143990 (.302)	.166080 (.309)	.274560 (.316)	.122570 (.315)	.144580 (.459)	.093950 (.397)	.206600 (1.0)	3.021840 (.856)	2.096790 (.454)	5.899500 (.958)	.706820 (.717)
13																
14																
15																
16																
Total	.486260 (1.0)	.501690 (1.0)	.260910 (1.0)	.429920 (1.0)	.224660 (1.0)	.475610 (1.0)	.537670 (1.0)	.870020 (1.0)	.389430 (1.0)	.315010 (1.0)	.236500 (1.0)	.206600 (1.0)	3.528540 (1.0)	4.617480 (1.0)	6.158140 (1.0)	.986400 (1.0)

The capital composition matrix (P_{ij}) is shown in parenthesis in the same Table 3-2.

3) Composition of Private Consumption coefficients ($c_i(t)$): Estimation of private consumption by sectors is obtained from Song's [11] table. Table 3-3 shows the proportions of each sector based on Song's estimation of private consumption by 16 sectors through 1981.

4) Import coefficients (m', m''): Non-competitive import coefficients (m') are calculated directly from the inter-industry complementary import transaction table for 1970, 1970 I-O Tables, and Framework of the 1970 Input-Output Tables. To estimate ratio of competitive imports to uncommitted foreign exchange (m''), competitive imports by sectors are first calculated by subtracting non-competitive imports from corresponding total imports. Uncommitted foreign exchange is defined as the sum of exports of commodities and services and net foreign capital inflow less noncompetitive imports. The values of m' and m'' are estimated as shown in Table 3-4.

5) Inventory-Output Coefficients (s_{ij}): The Korea Development Institute estimated the inventory coefficients matrix by using Han's [6] aggregate inventory-output coefficient by sectors for 1968. The inventory-output coefficient matrix for 52 sectors is further aggregated into 16 sectors for this model, as shown in Table 3-5.

6) Marginal Contribution of Capital to Aggregate Consumption ($\delta = \partial C/\partial K$): The estimation of marginal contribution of capital by way of $\Delta C(t+1)/\Delta K(t)$ is to reflect the assumed one-year investment lag. Here 10 years average of 0.356 is used to represent the value of $\partial C/\partial K$, as shown in Table 3-6.

Table 3-3 Composition of Private Consumption by Sectors [8].

Sectors	M(j)	1977	1978	1979	1980	1981
1	.506595	.2099	.2021	.1950	.1885	.1826
2	.870212	.0010	.0010	.0010	.0010	.0010
3	1.856269	.1492	.1521	.1547	.1571	.1593
4	.724539	.0596	.0589	.0583	.0577	.0572
5	.883089	.0030	.0030	.0030	.0030	.0030
6	1.878018	.0030	.0306	.0311	.0316	.0320
7	1.564144	.0241	.0244	.0247	.0250	.0253
8	.78888	.0016	.0016	.0016	.0016	.0016
9	.972799	.0027	.0027	.0027	.0027	.0027
10	3.111818	.0382	.0394	.0405	.0415	.0424
11	1.588226	.1094	.1111	.1126	.1139	.1152
12						
13	1.930684	.0118	.0120	.0123	.0125	.0126
14	1.853652	.0890	.0907	.0923	.0937	.0950
15	.760211	.0425	.0421	.0417	.0414	.0410
16	1.039909	.2280	.2283	.2285	.2288	.2291

Table 3-4 Imports Coefficients m', m" and $(m")^{-1}$ [2] .

Sectors	m'	m"	$(m")^{-1}$
1	.001051	.036457	.304165
2	.002685	.003216	.026831
3	.033822	.130452	1.088379
4	.067614	.175680	1.465723
5	.465055	.012705	.105999
6	.209636	.040104	.334593
7	.336877	.002073	.017295
8	.007835	.012367	.103180
9	.085699	.124716	1.040523
10	.087107	.110523	.922109
11	.031087	.119768	.999242
12	.007320	.152592	1.273096
13	.007752	.001806	.015068
14	.016420	.022546	.188104
15	.000181	.002108	.017587
16	.003213	.052888	.441252

Note that $(m")^{-1}$ is obtained by using a generalized inverse method; that is, $A^{-} = \frac{A'}{A' \cdot A}$

Table 3-5 Inventory-output Coefficient Matrix (s_{ij}) [9].

Sectors	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	.0318	.0151	.0590	.0306	.1231	.0007		.0005	.0002	.0001	.0452	.0010				.0194
2		.0018			.0002	.0097	.1581	.0810	.0094	.0003	.0002	.0009	.0018			.0011
3	.0131		.0170	.0002	.0009	.0084					.0187					.0125
4	.0004	.0021	.0010	.0849	.0005	.0012	.0007		.0003	.0007	.0084		.0001	.0001		.0101
5	.0001	.0009	.0010	.0011	.0222	.0109		.0310	.0010	.0058	.0257	.0020				.0144
6	.0121	.0356	.0040	.0452	.0099	.0503	.0044	.0185	.0048	.0068	.0358	.0004	.0004			.0272
7	.0004	.0109	.0010	.0023	.0022	.0222	.0054	.0339	.0068	.0025	.0034	.0013	.0073	.0052		.0211
8		.0010			.0012	.0026		.0253	.0056	.0027	.0076	.0052		.0001		.0039
9	.0009	.0182	.0010	.0005	.0030	.0026	.0015	.0131	.1545	.0471	.0078	.0054				.0021
10	.0001	.0171		.0016	.0009	.0013	.0027	.0031	.0024	.0605	.0021	.0047	.0007	.0047		.0089
11	.0009	.0071	.0010	.0007	.0003	.0013		.0015	.0004	.0030	.0319		.0002	.0011		.0818
12																
13																
14																
15																
16																
Total	.0598	.1098	.0850	.1671	.1644	.1112	.1728	.2079	.1854	.1295	.1868	.0209	.0105	.0112		.2124

Table 3-6 Marginal Contribution of Capital ($\frac{\Delta C}{\Delta K}$) [3].

Year	Consumption (C)	ΔC	ΔK	$\Delta C_{(t+1)}/\Delta K_{(t)}$
1962	1,017.73		133.38	.283
1963	1,055.51	37.78	167.79	.443
1964	1,124.20	68.69	155.12	.496
1965	1,201.12	76.92	195.40	.416
1966	1,282.37	81.25	294.28	.389
1967	1,396.87	114.50	358.63	.415
1968	1,545.55	148.68	498.30	.321
1969	1,705.63	160.08	639.23	.279
1970	1,884.25	178.62	650.20	.301
1971	2,080.12	195.87	680.14	.214
Average (1962-1971)				.356

7) The other parameters (ρ, d, r, β_1): From the historical data we assumed that the minimum growth rate of private consumption (ρ) is 10%, the social discount rate is 10%, the depreciation rate is 10%, and marginal propensity data (β_1) is 0.2629.

3.2 Estimation of exogenous variables.

1) Exports and Net Capital Inflow: The Korean Government has set up an export plan (1971-1980), as shown in Table 3-7, aiming to reach 10 billion U.S. dollars in 1980 in current U.S. dollars. Exports for the target year are estimated by using an average growth rate of export during 1977-1980. Since the export plan is in current U.S. dollars, it is deflated to 1970 constant prices.

Projections of the other source of foreign exchange earnings are obtained from "Long-term projections of the Korean economy" as shown in Table 3-8.

2) Government Consumption (G(t)): Lack of fundamental data in the U.S.A. makes it necessary to use Song's estimation of government consumption by 16 sectors, his projections are based on 1968 constant prices and are converted to 1970 constant prices, as shown in Table 3-9.

3) Initial Capital Stocks and Inventories: The estimation of initial capital stock by sectors is made simply by exogenously estimating X(1) directly and then multiplying the estimated X(1) by the aggregated capital output ratio, b(t). A simple estimation of X(1) exogenously is to interpolate the past trends of outputs by sectors. The estimation of initial capital stock and inventory levels using Song's [11] output estimation by sectors is shown in Table 3-10.

Table 3-7 Export Projections (At 1970 Prices) [10].
(Unit: $ in Million US Dollars, W in Million Won)*

Sectors	1977 $	1977 W	1978 $	1978 W	1979 $	1979 W	1980 $	1980 W	1981 $	1981 W
1	372	115,566	415	128,924	448	139,176	487	151,291	531	164,960
2	41	12,737	41	12,737	41	12,737	41	12,737	41	12,737
3	39	12,116	39	12,116	41	12,737	42	13,048	43	13,048
4	1,333	414,110	1,452	451,078	1,571	488,047	1,696	526,879	1,838	570,993
5	193	59,957	202	62,753	206	63,996	211	65,549	216	67,103
6	141	43,803	169	52,502	205	63,385	249	77,354	301	93,509
7	63	19,572	71	22,057	84	26,095	104	32,309	123	38,211
8	63	19,572	70	21,746	75	23,300	82	25,474	90	27,959
9	354	109,974	450	139,797	556	172,727	692	214,977	866	269,032
10	1,442	447,972	1,893	588,079	2,461	764,534	3,166	983,550	4,116	1,278,077
11	511	158,747	565	175,523	635	197,269	672	208,764	736	228,646
12										
13										
14										
15										
16										
Total	4,551	1,413,813	5,367	1,667,312	6,323	1,964,303	7,438	2,310,689	8,901	2,765,184

* The 1970-averaged foreign exchange rate, 310.66, is applied to calculate won equivalents from U.S. dollars.

Table 3-8 Availability of Foreign Exchanges on Current Account (In Million Dollars) [5].

	1977	1978	1979	1980	1981
Exports	4,551	5,367	6,323	7.438	8,901
Invisible Trade Balance	-380	-430	-476	-534	-591
Transfer (net)	109	85	83	76	76
Total Availability	4,280 (1,329,625)	5,022 (1,560,135)	5,930 (1,842,214)	6,980 (2,168,407)	8,386 (2,605,195)

*Figures in parenthesis are won equivalents in millions of 1970 won.

Table 3-9 Projection of Government Consumption (1977-1981) [8]
(At 1970 Prices).

Sectors	1977	1978	1979	1980	1981
1	2,488.9	2.693.0	2,914.1	3,152.8	3,452.9
2	27.4	30.6	34.3	38.4	43.2
3	246.2	275.4	274.2	307.6	345.3
4	328.2	336.6	377.1	384.5	431.6
5	1,039.3	1,101.6	1,199.9	1,307.2	1,424.3
6	3,309.5	3,580.4	3,874.0	4,191.0	4,575.2
7	2,488.9	2,662.4	2,879.8	3,114.4	3,409.8
8	519.7	612.1	685.7	769.0	863.3
9	27.4	30.6	34.1	38.4	43.2
10	8,287.5	9,464.2	10,593.6	11,957.8	13,509.8
11	83,365.0	95,875.7	109,809.3	125,191.0	142,565.4
12	49,724.4	57,133.7	65,515.3	74,783.9	85,245.8
13	3,883.9	4,192.5	4,559.7	4,960.0	5,438.5
14	10,119.9	11,016.7	11,999.2	13,111.3	14,373.1
15	1,996.7	1,224.1	342.9	346.0	345.3
16	105,493.4	115,889.3	127,739.5	140,801.4	155,600.5
Total	273,346.3	306,118.9	342,832.7	384,454.7	431,667.2

(Unit: in million won)

Table 3-10 Estimation of Initial Capital Stocks and Inventory Levels [8] (Unit: Million Won).

Sectors	Initial Capital Stocks	Initial Inventory Levels
1	776,475	199,221
2	80,872	63,602
3	182,230	69,750
4	342,001	86,781
5	40,439	67,140
6	138,403	169,016
7	140,601	65,898
8	179,485	27,018
9	162,665	100,749
10	237,549	66,361
11	250,737	153,751
12	182,015	
13	576,211	
14	2,668,903	
15	1,308,605	
16	1,549,832	
Total	8,817,023	1,069,287

4. Analysis of Solution

After all the possible substitutions have been made, there remain five key variables which are the gross domestic output X(t), the level of aggregate consumption C(t), competitive imports M"(t), capital stocks K(t), and new capital Z(t). Hence the linear programming model has five key variables and seven key constraints covering the five year planning period. These key variables can be divided into 16 industrial sectors except for C(t). And these distribution relationship (DR), capital constraint (CC) and capital accounting relationship (CAR) have 16 rows each through (16x16) matrix coefficient multiplication. From the computational point of view, this linear model has 325, i.e., (4 x 16 x 5 + 1 x 1 x 5) technological variables and 260, i.e., (3 x 16 x 5 + 4 x 1 x 5) rows.

This linear programming problem required 3 minutes 15 seconds solved by the MPS/360 computer programming using an IBM 370-158 computer. Row and column notations used in computer printout are shown in Table 3-11. For example row 211 indicates the first industrial sector in the third year's distributional relationship (DR) constraints. The computer printout of the optimal solution is presented in Table 3-12.

From the optimal solution the following two important characteristics can be shown. First, total domestic output grows at an annual rate of around 0.24% $\left(= \sqrt[3]{\frac{9011}{8946}}\right)$, but declined a small amount in the final year, as shown in Table 3-13. It shows rapid growth of manufacturing sector, especially in the heavy and chemical industry where the growth rate is 11.9% $\left(= \sqrt[3]{\frac{2500}{1782}}\right)$, while some sectors like mining and social overhead capital achieved negative growth. Secondly, turning to the national income accounts the gross national product

Table 3-11 Notations for Computer Output

1. Row (Constraints)

Row \ Year (t)	1	2	3	4	5
Object	101				
CG(t)	102	156	210	264	318
DR(t)	103-118	157-172	211-226	265-280	319-334
CC(t)	119-134	173-188	227-242	281-296	335-350
CAR(t)	136-151	190-205	244-259	298-313	352-367
BOP(t)	152	206	260	314	368
CIC(t)	153	207	261	315	369
SC(t)	154	208	262	316	370

2. Column (technological variables)

Col. \ Year (t)	1	2	3	4	5
X(t)	101-116	166-181	231-246	296-311	361-376
C(t)	117	182	247	312	377
M(t)	118-133	183-198	248-263	313-328	378-393
Z(t+1)	134-149	199-214	264-279	329-344	394-409
K(t+1)	150-165	215-230	280-295	345-360	410-425

ILLEGIBLE DOCUMENT

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Table 3-12 Computer Printout of Optimal Solution.

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SOLUTION (OPTIMAL)

TIME = 3.25 MINS. ITERATION NUMBER = 364

...NAME...	...ACTIVITY...	DEFINED AS
FUNCTIONAL RESTRAINTS	33951.20580	ROW101 LIMITS

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SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	ROW101	BS	33951.20580	33951.20580-	NONE	NONE	1.00000
2	ROW102	LL	3306.00000	.	3306.00000	NONE	27.18008
3	ROW103	UL	82.00000	.	NONE	82.00000	7.83796-
4	ROW104	UL	51.00000	.	NONE	51.00000	4.39505-
5	ROW105	UL	58.00000	.	NONE	58.00000	5.92305-
6	ROW106	LL	327.00000	.	327.00000	NONE	7.83796
7	ROW107	UL	7.00000	.	NONE	7.00000	6.60614-
8	ROW108	UL	122.00000	.	NONE	122.00000	7.83796-
9	ROW109	UL	44.00000	.	NONE	44.00000	4.49058-
10	ROW110	UL	7.00000	.	NONE	7.00000	3.09227-
11	ROW111	LL	9.00000	.	9.00000	NONE	7.83796
12	ROW112	LL	389.00000	.	389.00000	NONE	7.83796
13	ROW113	LL	88.00000	.	88.00000	NONE	4.31541
14	ROW114	LL	49.00000	.	49.00000	NONE	3.94013
15	ROW115	LL	3.00000	.	3.00000	NONE	2.32594
16	ROW116	LL	10.00000	.	10.00000	NONE	2.86241
17	ROW117	LL	1.00000	.	1.00000	NONE	7.83795
18	ROW118	LL	105.00000	.	105.00000	NONE	7.83796
19	ROW119	UL	777.00000	.	NONE	777.00000	11.47201-
20	ROW120	BS	77.76569	3.23431	NONE	81.00000	.
21	ROW121	BS	179.39837	3.60163	NONE	183.00000	.
22	ROW122	UL	342.00000	.	NONE	342.00000	4.23032-
23	ROW123	BS	20.68532	20.31468	NONE	41.00000	.
24	ROW124	UL	139.00000	.	NONE	139.00000	5.06110-
25	ROW125	BS	101.64027	39.35973	NONE	141.00000	.
26	ROW126	BS	109.25360	70.74640	NONE	180.00000	.
27	ROW127	UL	163.00000	.	NONE	163.00000	4.66379-
28	ROW128	UL	238.00000	.	NONE	238.00000	7.61492-
29	ROW129	BS	213.55719	37.44281	NONE	251.00000	.
30	ROW130	BS	108.09794	73.90206	NONE	182.00000	.
31	ROW131	BS	488.83970	88.16030	NONE	577.00000	.
32	ROW132	BS	2263.65769	405.34231	NONE	2669.00000	.
33	ROW133	UL	1309.00000	.	NONE	1309.00000	1.07666-
34	ROW134	UL	1550.00000	.	NONE	1550.00000	5.69148-
35	ROW136	EQ	506.00000	.	506.00000	506.00000	15.13401-
36	ROW137	EQ	80.80000	.	80.80000	80.80000	29.70586-
37	ROW138	EQ	182.30000	.	182.30000	182.30000	41.17836-
38	ROW139	EQ	342.00000	.	342.00000	342.00000	12.37783-
39	ROW140	EQ	40.40000	.	40.40000	40.40000	1.98500-
40	ROW141	EQ	138.40000	.	138.40000	138.40000	16.74163-
41	ROW142	EQ	140.60000	.	140.60000	140.60000	5.66630-
42	ROW143	EQ	179.50000	.	179.50000	179.50000	2.04200-
43	ROW144	EQ	149.40000	.	149.40000	149.40000	13.21193-
44	ROW145	EQ	237.50000	.	237.50000	237.50000	2.33015-
45	ROW146	EQ	250.70000	.	250.70000	250.70000	34.82347-
46	ROW147	EQ	182.00000	.	182.00000	182.00000	2.09800-
47	ROW148	EQ	576.20000	.	576.20000	576.20000	3.91396-
48	ROW149	EQ	2668.90000	.	2668.90000	2668.90000	4.55119-
49	ROW150	EQ	1308.60000	.	1308.60000	1308.60000	5.41328-

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NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
50	ROW151	EQ	1549.80000	.	1549.80000	1549.80000	19.07493-
51	ROW152	UL	1330.00000	.	NONE	1330.00000	7.83796-
52	ROW153	BS	1105.15757	224.84243	NONE	1330.00000	.
53	ROW154	BS	2112.71818	326.28182	NONE	2439.00000	.
54	ROW156	LL	.	.	.	NONE	19.56227
55	ROW157	LL	131.00000	.	131.00000	NONE	5.69498
56	ROW158	LL	12.00000	.	12.00000	NONE	5.69498
57	ROW159	LL	12.00000	.	12.00000	NONE	5.69498
58	ROW160	LL	451.00000	.	451.00000	NONE	5.69498
59	ROW161	LL	63.00000	.	63.00000	NONE	4.98939
60	ROW162	LL	56.00000	.	56.00000	NONE	5.69498
61	ROW163	LL	24.00000	.	24.00000	NONE	4.05362
62	ROW164	LL	22.00000	.	22.00000	NONE	2.28559
63	ROW165	LL	139.00000	.	139.00000	NONE	5.69498
64	ROW166	LL	597.00000	.	597.00000	NONE	2.98255
65	ROW167	LL	271.00000	.	271.00000	NONE	2.95407
66	ROW168	LL	57.00000	.	57.00000	NONE	2.71603
67	ROW169	LL	4.00000	.	4.00000	NONE	1.90728
68	ROW170	LL	11.00000	.	11.00000	NONE	2.04483
69	ROW171	LL	1.00000	.	1.00000	NONE	5.69498
70	ROW172	LL	115.00000	.	115.00000	NONE	5.69498
71	ROW173	UL	.	.	NONE	.	8.28394-
72	ROW174	UL	.	.	NONE	.	5.17931-
73	ROW175	UL	.	.	NONE	.	4.23958-
74	ROW176	UL	.	.	NONE	.	4.02330-
75	ROW177	BS	23.83228-	23.83228	NONE	.	.
76	ROW178	UL	.	.	NONE	.	3.96628-
77	ROW179	BS	20.73305-	20.73305	NONE	.	.
78	ROW180	BS	77.39688-	77.39688	NONE	.	.
79	ROW181	UL	.	.	NONE	.	4.65419-
80	ROW182	BS	25.22401-	25.22401	NONE	.	.
81	ROW183	BS	8.57958-	8.57958	NONE	.	.
82	ROW184	BS	549.91540-	549.91540	NONE	.	.
83	ROW185	BS	46.40924-	46.40924	NONE	.	.
84	ROW186	BS	185.61929-	185.61929	NONE	.	.
85	ROW187	UL	.	.	NONE	.	.77784-
86	ROW188	UL	.	.	NONE	.	4.83189-
87	ROW190	EQ	.	.	.	.	6.85007-
88	ROW191	EQ	.	.	.	.	24.52655-
89	ROW192	EQ	.	.	.	.	36.93878-
90	ROW193	EQ	.	.	.	.	8.35453-
91	ROW194	EQ	.	.	.	.	1.98500-
92	ROW195	EQ	.	.	.	.	12.77536-
93	ROW196	EQ	.	.	.	.	5.66630-
94	ROW197	EQ	.	.	.	.	2.04200-
95	ROW198	EQ	.	.	.	.	8.55774-
96	ROW199	EQ	.	.	.	.	2.33015-
97	ROW200	EQ	.	.	.	.	34.82347-
98	ROW201	EQ	.	.	.	.	2.09800-
99	ROW202	EQ	.	.	.	.	3.91396-
100	ROW203	EQ	.	.	.	.	4.55119-

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
101	ROW204	EQ	.	.	.	.	4.63544-
102	ROW205	EQ	.	.	.	.	14.24304-
103	ROW206	UL	1561.00000	.	NONE	1561.00000	5.69498-
104	ROW207	BS	1215.13184	345.86816	NONE	1561.00000	.
105	ROW208	BS	856.76476	743.23524	NONE	1600.00000	.
106	ROW210	LL	.	.	.	NONE	14.11751
107	ROW211	LL	142.00000	.	142.00000	NONE	1.49281
108	ROW212	LL	12.00000	.	12.00000	NONE	7.59916
109	ROW213	LL	13.00000	.	13.00000	NONE	7.59916
110	ROW214	LL	488.00000	.	488.00000	NONE	7.59916
111	ROW215	LL	65.00000	.	65.00000	NONE	4.14700
112	ROW216	LL	67.00000	.	67.00000	NONE	7.59916
113	ROW217	LL	28.00000	.	28.00000	NONE	6.00418
114	ROW218	LL	23.00000	.	23.00000	NONE	4.13356
115	ROW219	LL	172.00000	.	172.00000	NONE	7.59916
116	ROW220	LL	775.00000	.	775.00000	NONE	4.57037
117	ROW221	LL	307.00000	.	307.00000	NONE	7.59916
118	ROW222	LL	65.00000	.	65.00000	NONE	3.77819
119	ROW223	LL	4.00000	.	4.00000	NONE	2.65190
120	ROW224	LL	11.00000	.	11.00000	NONE	7.59916
121	ROW225	LL	.	.	.	NONE	7.59916
122	ROW226	LL	127.00000	.	127.00000	NONE	7.59916
123	ROW227	BS	2.74661-	2.74661	NONE	.	.
124	ROW228	UL	.	.	NONE	.	9.51125-
125	ROW229	UL	.	.	NONE	.	16.81835-
126	ROW230	UL	.	.	NONE	.	5.39920-
127	ROW231	BS	27.62986-	27.62986	NONE	.	.
128	ROW232	UL	.	.	NONE	.	5.24524-
129	ROW233	BS	2.49116-	2.49116	NONE	.	.
130	ROW234	BS	85.28111-	85.28111	NONE	.	.
131	ROW235	UL	.	.	NONE	.	4.25957-
132	ROW236	BS	95.56288-	95.56288	NONE	.	.
133	ROW237	UL	.	.	NONE	.	15.33634-
134	ROW238	BS	777.77703-	777.77703	NONE	.	.
135	ROW239	BS	22.21772-	22.21772	NONE	.	.
136	ROW240	UL	.	.	NONE	.	1.05307-
137	ROW241	UL	.	.	NONE	.	1.04619-
138	ROW242	UL	.	.	NONE	.	5.58472-
139	ROW244	EQ	.	.	.	.	6.85007-
140	ROW245	EQ	.	.	.	.	15.01530-
141	ROW246	EQ	.	.	.	.	20.12043-
142	ROW247	EQ	.	.	.	.	2.95533-
143	ROW248	EQ	.	.	.	.	1.98500-
144	ROW249	EQ	.	.	.	.	7.53012-
145	ROW250	EQ	.	.	.	.	5.66630-
146	ROW251	EQ	.	.	.	.	2.04200-
147	ROW252	EQ	.	.	.	.	4.29817-
148	ROW253	EQ	.	.	.	.	2.33015-
149	ROW254	EQ	.	.	.	.	19.48713-
150	ROW255	EQ	.	.	.	.	2.09800-
151	ROW256	EQ	.	.	.	.	3.91396-

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
152	ROW257	EQ	.	.	.	.	3.49812-
153	ROW258	EQ	.	.	.	.	3.58924-
154	ROW259	EQ	.	.	.	.	8.65833-
155	ROW260	UL	1843.00000	.	NONE	1843.00000	7.59916-
156	ROW261	BS	1519.78541	323.21459	NONE	1843.00000	.
157	ROW262	BS	744.78362	1138.21638	NONE	1883.00000	.
158	ROW264	LL	.	.	.	NONE	7.69598
159	ROW265	LL	154.00000	.	154.00000	NONE	1.33449
160	ROW266	LL	12.00000	.	12.00000	NONE	6.81588
161	ROW267	LL	13.00000	.	13.00000	NONE	6.81588
162	ROW268	LL	142.00000	.	142.00000	NONE	3.62856
163	ROW269	LL	66.00000	.	66.00000	NONE	3.59384
164	ROW270	LL	81.00000	.	81.00000	NONE	6.81588
165	ROW271	LL	35.00000	.	35.00000	NONE	6.81588
166	ROW272	LL	26.00000	.	26.00000	NONE	4.59445
167	ROW273	LL	215.00000	.	215.00000	NONE	6.81588
168	ROW274	LL	995.00000	.	995.00000	NONE	4.62439
169	ROW275	LL	333.00000	.	333.00000	NONE	6.81588
170	ROW276	LL	74.00000	.	74.00000	NONE	3.58331
171	ROW277	LL	4.00000	.	4.00000	NONE	6.81588
172	ROW278	LL	13.00000	.	13.00000	NONE	6.81588
173	ROW279	LL	.	.	.	NONE	6.81588
174	ROW280	LL	140.00000	.	140.00000	NONE	6.81588
175	ROW281	BS	5.25686-	5.25686	NONE	.	.
176	ROW282	UL	.	.	NONE	.	9.01613-
177	ROW283	UL	.	.	NONE	.	15.68326-
178	ROW284	BS	34.21515-	34.21515	NONE	.	.
179	ROW285	BS	26.03012-	26.03012	NONE	.	.
180	ROW286	UL	.	.	NONE	.	4.05901-
181	ROW287	UL	.	.	NONE	.	2.87422-
182	ROW288	BS	77.39850-	77.39850	NONE	.	.
183	ROW289	UL	.	.	NONE	.	2.04182-
184	ROW290	UL	.	.	NONE	.	.28515-
185	ROW291	UL	.	.	NONE	.	13.67404-
186	ROW292	BS	831.57123-	831.57123	NONE	.	.
187	ROW293	UL	.	.	NONE	.	1.20013-
188	ROW294	UL	.	.	NONE	.	.91240-
189	ROW295	UL	.	.	NONE	.	.94009-
190	ROW296	UL	.	.	NONE	.	4.63628-
191	ROW298	EQ	.	.	.	.	6.85007-
192	ROW299	EQ	.	.	.	.	5.99917-
193	ROW300	EQ	.	.	.	.	4.43716-
194	ROW301	EQ	.	.	.	.	2.95533-
195	ROW302	EQ	.	.	.	.	1.98500-
196	ROW303	EQ	.	.	.	.	3.47111-
197	ROW304	EQ	.	.	.	.	2.79209-
198	ROW305	EQ	.	.	.	.	2.04200-
199	ROW306	EQ	.	.	.	.	2.25635-
200	ROW307	EQ	.	.	.	.	2.04500-
201	ROW308	EQ	.	.	.	.	5.81309-
202	ROW309	EQ	.	.	.	.	2.09800-

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
203	ROW310	EQ	.	.	.	.	2.71383-
204	ROW311	EQ	.	.	.	.	2.58572-
205	ROW312	EQ	.	.	.	.	2.64916-
206	ROW313	EQ	.	.	.	.	4.02204-
207	ROW314	UL	2169.00000	.	NONE	2169.00000	6.81588-
208	ROW315	BS	1533.22341	635.77659	NONE	2169.00000	.
209	ROW316	BS	577.18264	1231.81736	NONE	2209.00000	.
210	ROW318	LL	.	.	.	NONE	2.80099
211	ROW319	LL	168.00000	.	168.00000	NONE	3.45830
212	ROW320	LL	12.00000	.	12.00000	NONE	3.45830
213	ROW321	LL	13.00000	.	13.00000	NONE	3.45830
214	ROW322	LL	571.00000	.	571.00000	NONE	3.45830
215	ROW323	LL	68.00000	.	68.00000	NONE	3.29754
216	ROW324	LL	98.00000	.	58.00000	NONE	3.45830
217	ROW325	LL	41.00000	.	41.00000	NONE	3.45830
218	ROW326	LL	28.00000	.	28.00000	NONE	2.84985
219	ROW327	LL	269.00000	.	269.00000	NONE	3.45830
220	ROW328	LL	118.00000	.	118.00000	NONE	2.63428
221	ROW329	LL	371.00000	.	371.00000	NONE	3.45830
222	ROW330	LL	85.00000	.	85.00000	NONE	2.09800
223	ROW331	LL	5.00000	.	5.00000	NONE	3.45830
224	ROW332	LL	14.00000	.	14.00000	NONE	3.45830
225	ROW333	LL	.	.	.	NONE	3.45830
226	ROW334	LL	155.00000	.	155.00000	NONE	3.45830
227	ROW335	UL	.	.	NONE	.	4.69207-
228	ROW336	UL	.	.	NONE	.	4.16117-
229	ROW337	UL	.	.	NONE	.	2.32016-
230	ROW338	UL	.	.	NONE	.	.82733-
231	ROW339	BS	26.77825-	26.77825	NONE	.	.
232	ROW340	UL	.	.	NONE	.	1.53911-
233	ROW341	UL	.	.	NONE	.	.82409-
234	ROW342	BS	82.76840-	82.76840	NONE	.	.
235	ROW343	UL	.	.	NONE	.	.16035-
236	ROW344	BS	320.27434-	320.27434	NONE	.	.
237	ROW345	UL	.	.	NONE	.	3.72609-
238	ROW346	BS	828.74491-	828.74491	NONE	.	.
239	ROW347	UL	.	.	NONE	.	.57483-
240	ROW348	UL	.	.	NONE	.	.43072-
241	ROW349	UL	.	.	NONE	.	.46316-
242	ROW350	UL	.	.	NONE	.	1.82504-
243	ROW352	EQ	.	.	.	.	2.15800-
244	ROW353	EQ	.	.	.	.	1.83800-
245	ROW354	EQ	.	.	.	.	2.11700-
246	ROW355	EQ	.	.	.	.	2.12800-
247	ROW356	EQ	.	.	.	.	1.98500-
248	ROW357	EQ	.	.	.	.	1.93200-
249	ROW358	EQ	.	.	.	.	1.96800-
250	ROW359	EQ	.	.	.	.	2.04200-
251	ROW360	EQ	.	.	.	.	2.09600-
252	ROW361	EQ	.	.	.	.	2.04500-
253	ROW362	EQ	.	.	.	.	2.08700-

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NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
254	ROW363	EQ	.	.	.	.	2.09800-
255	ROW364	EQ	.	.	.	.	2.13900-
256	ROW365	EQ	.	.	.	.	2.15500-
257	ROW366	EQ	.	.	.	.	2.18600-
258	ROW367	EQ	.	.	.	.	2.19700-
259	ROW368	UL	2606.00000	.	NONE	2606.00000	3.45830-
260	ROW369	BS	2049.90215	556.05785	NONE	2606.00000	.
261	ROW370	BS	1371.51746	1274.48254	NONE	2646.00000	.

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
262	VAR101	BS	1598.76543	.	.	NONE	.
263	VAR102	BS	154.91172	.	.	NONE	.
264	VAR103	BS	687.35009	.	.	NONE	.
265	VAR104	BS	795.34884	.	.	NONE	.
266	VAR105	BS	91.93477	.	.	NONE	.
267	VAR106	BS	292.01681	.	.	NONE	.
268	VAR107	BS	188.92243	.	.	NONE	.
269	VAR108	BS	125.57885	.	.	NONE	.
270	VAR109	BS	419.02314	.	.	NONE	.
271	VAR110	BS	755.55556	.	.	NONE	.
272	VAR111	BS	901.08519	.	.	NONE	.
273	VAR112	BS	522.21229	.	.	NONE	.
274	VAR113	BS	138.52074	.	.	NONE	.
275	VAR114	BS	490.28756	.	.	NONE	.
276	VAR115	BS	212.56902	.	.	NONE	.
277	VAR116	BS	1572.00811	.	.	NONE	.
278	VAR117	BS	3306.00000	.90900	.	NONE	.
279	VAR118	BS	131.17457	.	.	NONE	.
280	VAR119	LL	.	.	.	NONE	3.44291-
281	VAR120	LL	.	.	.	NONE	1.91491-
282	VAR121	BS	113.70366	.	.	NONE	.
283	VAR122	LL	.	.	.	NONE	1.23182-
284	VAR123	BS	178.48871	.	.	NONE	.
285	VAR124	LL	.	.	.	NONE	3.34737-
286	VAR125	LL	.	.	.	NONE	4.74569-
287	VAR126	BS	181.30799	.	.	NONE	.
288	VAR127	BS	254.12209	.	.	NONE	.
289	VAR128	LL	.	.	.	NONE	3.52254-
290	VAR129	LL	.	.	.	NONE	3.89783-
291	VAR130	LL	.	.	.	NONE	5.51202-
292	VAR131	LL	.	.	.	NONE	4.97555-
293	VAR132	BS	30.76138	.	.	NONE	.
294	VAR133	BS	45.15255	.	.	NONE	.
295	VAR134	BS	869.58043	.	.	NONE	.
296	VAR135	LL	.	.	.	NONE	3.72824-
297	VAR136	LL	.	.	.	NONE	3.73988-
298	VAR137	LL	.	.	.	NONE	4.17316-
299	VAR138	LL	.	.	.	NONE	3.94595-
300	VAR139	LL	.	.	.	NONE	3.59104-
301	VAR140	LL	.	.	.	NONE	4.20166-
302	VAR141	LL	.	.	.	NONE	3.90455-
303	VAR142	LL	.	.	.	NONE	4.01035-
304	VAR143	LL	.	.	.	NONE	3.13925-
305	VAR144	LL	.	.	.	NONE	3.66157-
306	VAR145	LL	.	.	.	NONE	1.84213-
307	VAR146	LL	.	.	.	NONE	2.36999-
308	VAR147	LL	.	.	.	NONE	3.84359-
309	VAR148	LL	.	.	.	NONE	1.99609-
310	VAR149	LL	.	.	.	NONE	2.87952-

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NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
311	VAR150	BS	711.02617	.	.	NONE	.
312	VAR151	BS	67.63250	.	.	NONE	.
313	VAR152	BS	174.73915	.	.	NONE	.
314	VAR153	BS	329.27442	.	.	NONE	.
315	VAR154	BS	44.37256	.	.	NONE	.
316	VAR155	BS	120.87899	.	.	NONE	.
317	VAR156	BS	129.45358	.	.	NONE	.
318	VAR157	BS	171.21180	.	.	NONE	.
319	VAR158	BS	172.00909	.	.	NONE	.
320	VAR159	BS	400.23940	.	.	NONE	.
321	VAR160	BS	241.59463	.	.	NONE	.
322	VAR161	BS	611.56809	.	.	NONE	.
323	VAR162	BS	560.27011	.	.	NONE	.
324	VAR163	BS	2611.53635	.	.	NONE	.
325	VAR164	BS	1293.53274	.	.	NONE	.
326	VAR165	BS	1540.36795	.	.	NONE	.
327	VAR166	BS	1463.01681	.	.	NONE	.
328	VAR167	BS	134.72610	.	.	NONE	.
329	VAR168	BS	669.49866	.	.	NONE	.
330	VAR169	BS	765.75446	.	.	NONE	.
331	VAR170	BS	91.29015	.	.	NONE	.
332	VAR171	BS	253.94746	.	.	NONE	.
333	VAR172	BS	202.08276	.	.	NONE	.
334	VAR173	BS	107.83323	.	.	NONE	.
335	VAR174	BS	442.18275	.	.	NONE	.
336	VAR175	BS	1190.52505	.	.	NONE	.
337	VAR176	BS	983.18588	.	.	NONE	.
338	VAR177	BS	297.83912	.	.	NONE	.
339	VAR178	BS	145.61090	.	.	NONE	.
340	VAR179	BS	525.43146	.	.	NONE	.
341	VAR180	BS	210.12224	.	.	NONE	.
342	VAR181	BS	1562.23930	.	.	NONE	.
343	VAR182	BS	3636.60000	.82600	.	NONE	.
344	VAR183	BS	160.55223	.	.	NONE	.
345	VAR184	BS	34.82845	.	.	NONE	.
346	VAR185	BS	68.61926	.	.	NONE	.
347	VAR186	BS	181.40989	.	.	NONE	.
348	VAR187	LL	.	.	.	NONE	.70559-
349	VAR188	BS	243.52307	.	.	NONE	.
350	VAR189	LL	.	.	.	NONE	1.64136-
351	VAR190	LL	.	.	.	NONE	3.40939-
352	VAR191	BS	231.41778	.	.	NONE	.
353	VAR192	LL	.	.	.	NONE	2.71243-
354	VAR193	LL	.	.	.	NONE	2.74091-
355	VAR194	LL	.	.	.	NONE	2.97895-
356	VAR195	LL	.	.	.	NONE	3.78770-
357	VAR196	LL	.	.	.	NONE	3.65015-
358	VAR197	BS	48.43671	.	.	NONE	.
359	VAR198	BS	161.94218	.	.	NONE	.
360	VAR199	BS	353.89516	.	.	NONE	.
361	VAR200	LL	.	.	.	NONE	.56675-

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
362	VAR201	LL	.	.	.	NONE	.54753-
363	VAR202	LL	.	.	.	NONE	.55473-
364	VAR203	LL	.	.	.	NONE	.55430-
365	VAR204	BS	79.29581	.	.	NONE	.
366	VAR205	LL	.	.	.	NONE	.61763-
367	VAR206	LL	.	.	.	NONE	.34391-
368	VAR207	LL	.	.	.	NONE	.48665-
369	VAR208	LL	.	.	.	NONE	.32949-
370	VAR209	LL	.	.	.	NONE	.51598-
371	VAR210	LL	.	.	.	NONE	.61803-
372	VAR211	LL	.	.	.	NONE	.62298-
373	VAR212	LL	.	.	.	NONE	.63679-
374	VAR213	LL	.	.	.	NONE	.61947-
375	VAR214	LL	.	.	.	NONE	.62776-
376	VAR215	BS	785.53015	.	.	NONE	.
377	VAR216	BS	56.18078	.	.	NONE	.
378	VAR217	BS	167.37456	.	.	NONE	.
379	VAR218	BS	317.02235	.	.	NONE	.
380	VAR219	BS	46.09818	.	.	NONE	.
381	VAR220	BS	105.64214	.	.	NONE	.
382	VAR221	BS	117.53069	.	.	NONE	.
383	VAR222	BS	164.09480	.	.	NONE	.
384	VAR223	BS	183.58924	.	.	NONE	.
385	VAR224	BS	495.86771	.	.	NONE	.
386	VAR225	BS	230.90575	.	.	NONE	.
387	VAR226	BS	809.48461	.	.	NONE	.
388	VAR227	BS	543.52486	.	.	NONE	.
389	VAR228	BS	2550.06087	.	.	NONE	.
390	VAR229	BS	1279.43430	.	.	NONE	.
391	VAR230	BS	1530.95452	.	.	NONE	.
392	VAR231	BS	1610.66570	.	.	NONE	.
393	VAR232	BS	111.91391	.	.	NONE	.
394	VAR233	BS	641.28224	.	.	NONE	.
395	VAR234	BS	737.26127	.	.	NONE	.
396	VAR235	BS	82.08143	.	.	NONE	.
397	VAR236	BS	221.93728	.	.	NONE	.
398	VAR237	BS	213.82812	.	.	NONE	.
399	VAR238	BS	90.59045	.	.	NONE	.
400	VAR239	BS	471.95177	.	.	NONE	.
401	VAR240	BS	1270.80898	.	.	NONE	.
402	VAR241	BS	974.28586	.	.	NONE	.
403	VAR242	BS	153.17674	.	.	NONE	.
404	VAR243	BS	147.72092	.	.	NONE	.
405	VAR244	BS	552.31988	.	.	NONE	.
406	VAR245	BS	207.76783	.	.	NONE	.
407	VAR246	BS	1552.73277	.	.	NONE	.
408	VAR247	BS	4000.26000	.75100	.	NONE	.
409	VAR248	LL	.	.	.	NONE	6.10635-
410	VAR249	BS	59.18332	.	.	NONE	.
411	VAR250	BS	170.14641	.	.	NONE	.
412	VAR251	BS	255.45558	.	.	NONE	.

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT CCST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
413	VAR252	LL	.	.	.	NONE	3.45216-
414	VAR253	BS	293.92373	.	.	NONE	.
415	VAR254	LL	.	.	.	NONE	1.59498-
416	VAR255	LL	.	.	.	NONE	3.46560-
417	VAR256	BS	219.83686	.	.	NONE	.
418	VAR257	LL	.	.	.	NONE	3.02629-
419	VAR258	BS	85.64075	.	.	NONE	.
420	VAR259	LL	.	.	.	NONE	3.82097-
421	VAR260	LL	.	.	.	NONE	4.94726-
422	VAR261	BS	4.55895	.	.	NONE	.
423	VAR262	BS	64.63034	.	.	NONE	.
424	VAR263	BS	261.08038	.	.	NONE	.
425	VAR264	BS	94.91601	.	.	NONE	.
426	VAR265	LL	.	.	.	NONE	2.00680-
427	VAR266	LL	.	.	.	NONE	1.99746-
428	VAR267	LL	.	.	.	NONE	2.04158-
429	VAR268	LL	.	.	.	NONE	2.08465-
430	VAR269	LL	.	.	.	NONE	1.71306-
431	VAR270	LL	.	.	.	NONE	2.06348-
432	VAR271	LL	.	.	.	NONE	1.96501-
433	VAR272	LL	.	.	.	NONE	2.04348-
434	VAR273	LL	.	.	.	NONE	1.92301-
435	VAR274	LL	.	.	.	NONE	1.98028-
436	VAR275	LL	.	.	.	NONE	1.68019-
437	VAR276	LL	.	.	.	NONE	1.76090-
438	VAR277	LL	.	.	.	NONE	1.98624-
439	VAR278	LL	.	.	.	NONE	1.70373-
440	VAR279	LL	.	.	.	NONE	1.83882-
441	VAR280	BS	792.11132	.	.	NONE	.
442	VAR281	BS	46.66810	.	.	NONE	.
443	VAR282	BS	160.32056	.	.	NONE	.
444	VAR283	BS	305.22617	.	.	NONE	.
445	VAR284	BS	44.87472	.	.	NONE	.
446	VAR285	BS	92.32591	.	.	NONE	.
447	VAR286	BS	104.91483	.	.	NONE	.
448	VAR287	BS	158.11583	.	.	NONE	.
449	VAR288	BS	186.05706	.	.	NONE	.
450	VAR289	BS	485.11083	.	.	NONE	.
451	VAR290	BS	218.52478	.	.	NONE	.
452	VAR291	BS	855.41085	.	.	NONE	.
453	VAR292	BS	526.53695	.	.	NONE	.
454	VAR293	BS	2485.43945	.	.	NONE	.
455	VAR294	BS	1265.09832	.	.	NONE	.
456	VAR295	BS	1521.67812	.	.	NONE	.
457	VAR296	BS	1518.95581	.	.	NONE	.
458	VAR297	BS	92.96435	.	.	NONE	.
459	VAR298	BS	614.25502	.	.	NONE	.
460	VAR299	BS	630.25818	.	.	NONE	.
461	VAR300	BS	83.75381	.	.	NONE	.
462	VAR301	BS	193.56199	.	.	NONE	.
463	VAR302	BS	195.00899	.	.	NONE	.

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NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
464	VAR303	BS	92.77854	.	.	NONE	.
465	VAR304	BS	478.25577	.	.	NONE	.
466	VAR305	BS	1540.03437	.	.	NONE	.
467	VAR306	BS	922.04549	.	.	NONE	.
468	VAR307	BS	115.16725	.	.	NONE	.
469	VAR308	BS	149.20288	.	.	NONE	.
470	VAR309	BS	538.32347	.	.	NONE	.
471	VAR310	BS	205.43981	.	.	NONE	.
472	VAR311	BS	1543.28410	.	.	NONE	.
473	VAR312	BS	4400.28600	.68300	.	NONE	.
474	VAR313	LL	.	.	.	NONE	5.48139-
475	VAR314	BS	66.05787	.	.	NONE	.
476	VAR315	BS	260.50260	.	.	NONE	.
477	VAR316	LL	.	.	.	NONE	3.18733-
478	VAR317	LL	.	.	.	NONE	3.22205-
479	VAR318	BS	335.67058	.	.	NONE	.
480	VAR319	BS	41.02585	.	.	NONE	.
481	VAR320	LL	.	.	.	NONE	2.22143-
482	VAR321	BS	303.49682	.	.	NONE	.
483	VAR322	LL	.	.	.	NONE	2.19149-
484	VAR323	BS	208.47415	.	.	NONE	.
485	VAR324	LL	.	.	.	NONE	3.23257-
486	VAR325	BS	1.50865	.	.	NONE	.
487	VAR326	BS	64.16482	.	.	NONE	.
488	VAR327	BS	80.18208	.	.	NONE	.
489	VAR328	BS	376.19602	.	.	NONE	.
490	VAR329	BS	.86369	.	.	NONE	.
491	VAR330	LL	.	.	.	NONE	2.13995-
492	VAR331	LL	.	.	.	NONE	2.14455-
493	VAR332	LL	.	.	.	NONE	2.23628-
494	VAR333	LL	.	.	.	NONE	2.27658-
495	VAR334	LL	.	.	.	NONE	2.08736-
496	VAR335	LL	.	.	.	NONE	2.24212-
497	VAR336	LL	.	.	.	NONE	2.24392-
498	VAR337	LL	.	.	.	NONE	2.26482-
499	VAR338	LL	.	.	.	NONE	2.11910-
500	VAR339	LL	.	.	.	NONE	2.14631-
501	VAR340	LL	.	.	.	NONE	1.48531-
502	VAR341	LL	.	.	.	NONE	1.64286-
503	VAR342	LL	.	.	.	NONE	2.03768-
504	VAR343	LL	.	.	.	NONE	1.53126-
505	VAR344	LL	.	.	.	NONE	1.79494-
506	VAR345	BS	774.52387	.	.	NONE	.
507	VAR346	BS	38.76613	.	.	NONE	.
508	VAR347	BS	153.56375	.	.	NONE	.
509	VAR348	BS	295.14204	.	.	NONE	.
510	VAR349	BS	42.95443	.	.	NONE	.
511	VAR350	BS	80.68819	.	.	NONE	.
512	VAR351	BS	93.40930	.	.	NONE	.
513	VAR352	BS	151.95245	.	.	NONE	.
514	VAR353	BS	186.07951	.	.	NONE	.

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NUMBER	COLUMN	AT	ACTIVITY	INPUT COST	LOWER LIMIT	UPPER LIMIT	REDUCED COST
515	VAR354	BS	448.32965	.	.	NONE	.
516	VAR355	BS	206.54078	.	.	NONE	.
517	VAR356	BS	854.69102	.	.	NONE	.
518	VAR357	BS	509.37862	.	.	NONE	.
519	VAR358	BS	2422.45560	.	.	NONE	.
520	VAR359	BS	1250.92298	.	.	NONE	.
521	VAR360	BS	1512.41841	.	.	NONE	.
522	VAR361	BS	1593.67051	.	.	NONE	.
523	VAR362	BS	77.22337	.	.	NONE	.
524	VAR363	BS	588.36687	.	.	NONE	.
525	VAR364	BS	686.37683	.	.	NONE	.
526	VAR365	BS	71.89416	.	.	NONE	.
527	VAR366	BS	169.51300	.	.	NONE	.
528	VAR367	BS	173.62324	.	.	NONE	.
529	VAR368	BS	79.56787	.	.	NONE	.
530	VAR369	BS	478.35350	.	.	NONE	.
531	VAR370	BS	406.52479	.	.	NONE	.
532	VAR371	BS	871.48008	.	.	NONE	.
533	VAR372	BS	125.34356	.	.	NONE	.
534	VAR373	BS	144.34078	.	.	NONE	.
535	VAR374	BS	524.68174	.	.	NONE	.
536	VAR375	BS	203.13787	.	.	NONE	.
537	VAR376	BS	1533.89292	.	.	NONE	.
538	VAR377	BS	4840.31460	.62100	.	NONE	.
539	VAR378	BS	75.62824	.	.	NONE	.
540	VAR379	BS	65.82052	.	.	NONE	.
541	VAR380	BS	357.95391	.	.	NONE	.
542	VAR381	BS	418.48974	.	.	NONE	.
543	VAR382	LL	.	.	.	NONE	.16076-
544	VAR383	BS	356.34348	.	.	NONE	.
545	VAR384	BS	66.52607	.	.	NONE	.
546	VAR385	LL	.	.	.	NONE	.60845-
547	VAR386	BS	94.18992	.	.	NONE	.
548	VAR387	LL	.	.	.	NONE	.82402-
549	VAR388	BS	308.15491	.	.	NONE	.
550	VAR389	LL	.	.	.	NONE	1.36030-
551	VAR390	BS	3.66940	.	.	NONE	.
552	VAR391	BS	100.56377	.	.	NONE	.
553	VAR392	BS	79.34522	.	.	NONE	.
554	VAR393	BS	364.48564	.	.	NONE	.
555	VAR394	LL	.	.	.	NONE	.50417-
556	VAR395	LL	.	.	.	NONE	.36045-
557	VAR396	LL	.	.	.	NONE	.36983-
558	VAR397	LL	.	.	.	NONE	.41411-
559	VAR398	LL	.	.	.	NONE	.45052-
560	VAR399	LL	.	.	.	NONE	.36360-
561	VAR400	LL	.	.	.	NONE	.43094-
562	VAR401	LL	.	.	.	NONE	.42376-
563	VAR402	LL	.	.	.	NONE	.42730-
564	VAR403	LL	.	.	.	NONE	.37028-
565	VAR404	LL	.	.	.	NONE	.39779-

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
566	VAR405	BS	1.41653	.	.	NONE	.
567	VAR406	LL	.	.	.	NONE	.08486-
568	VAR407	LL	.	.	.	NONE	.32175-
569	VAR408	LL	.	.	.	NONE	.02475-
570	VAR409	LL	.	.	.	NONE	.16677-
571	VAR410	BS	756.59349	2.15800	.	NONE	.
572	VAR411	BS	32.20215	1.83800	.	NONE	.
573	VAR412	BS	147.09172	2.11700	.	NONE	.
574	VAR413	BS	284.16001	2.12800	.	NONE	.
575	VAR414	BS	41.30087	1.98500	.	NONE	.
576	VAR415	BS	70.51741	1.93200	.	NONE	.
577	VAR416	BS	83.16553	1.96800	.	NONE	.
578	VAR417	BS	146.74097	2.04200	.	NONE	.
579	VAR418	BS	186.07551	2.09600	.	NONE	.
580	VAR419	BS	438.57305	2.04500	.	NONE	.
581	VAR420	BS	195.21154	2.08700	.	NONE	.
582	VAR421	BS	854.85412	2.09800	.	NONE	.
583	VAR422	BS	492.77943	2.13900	.	NONE	.
584	VAR423	BS	2361.06784	2.15500	.	NONE	.
585	VAR424	BS	1236.90646	2.18600	.	NONE	.
586	VAR425	BS	1503.21506	2.19700	.	NONE	.

Table 3-13 Output Level by Industry.

(Unit: Billions of 1970 Won)

Industry	1977	1978	1979	1980	1981
Agriculture, Forestry & Fishery	1599	1463	1610	1618	1594
Mining	155	134	111	93	77
Manufacturing	4256	4705	4701	4750	4525
Heavy & Chemical Industry	1782	2197	2267	2500	2308
Other Manu.	2474	2508	2434	2250	2217
Social Overhead Capital (SOC)	1151	969	852	802	793
Trade, Services to Others. (TOS)	1785	1772	1759	1748	1736
Total	8946	9043	9033	9011	8725

Source: Table 3-12

Note: Sectors 6 to 10 are classified to heavy and chemical industry; sectors 3,4,5, and 11 to other manufacturing; SOC covers sectors 12, 13, and 14; and trade, services, and other sectors 15 and 16.

as calculated from the expenditure side is the sum of private consumption (C), government consumption (G), business investment (I+H), and the sale of goods and services abroad minus purchases from abroad (E-M). GNP, calculated this way, grows at the annual growth rate of 4.1% $\left(= \sqrt[4]{\frac{5274}{4486}}\right)$ from 4,486 billion Won to 5,274 billion Won as shown in Table 3-14.

Table 3-14 National Income Accounts.

(Unit: Billions of 1970 Won)

Classification	1977	1978	1979	1980	1981
Consumption	3579	3943	4343	4784	5272
Private	3306	3637	4000	4400	4840
Government	273	306	343	384	432
Investment	907	480	207	0	2
Fixed Inv.	870	433	95	0	2
Inventory Inc.	37	47	112	0	0
Foreign Exchange	0	0	0	0	0
Receipts	1330	1560	1842	2168	2605
Payments	1330	1560	1842	2168	2605
GNP	4486	4423	4550	4784	5274

Source: Table 3-12

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Chapter 4

LINEAR GOAL PROGRAMMING MODEL

1. Introduction

Economic planning, especially in developing countries, is the selection of the optimum allocation of resources among many competing development goals. To employ this comprehensive economic planning effectively, economic development goals are desired to be established in a hierarchical order of importance. Since economic planning problems involve multiple goals, it has long been desired to have efficient analytical techniques to handle this problem. Goal programming is very appropriate for solving the economic planning problem because it can handle multiple economic goals in such a way that goal satisfaction is based on a hierarchial structure of priorities [2].

The linear model solved by the linear programming technique in Chapter 3 is again considered from the viewpoint of the linear goal programming technique. The decision of priority level in the linear goal programming model is very hard to make at this time, hence the author's option here is to present two kinds of priority structure. The results obtained by the linear goal programming model are then compared with the results obtained by the linear programming model.

2. Model Formulation

There are two kinds of objective functions in goal programming model, and these are the absolute and nonabsolute objective functions. Among seven types of constraints in the linear programming model, the private consumption

Additional Symbols Used in this Chapter

Variables and Parameters		Dimensions for n sectors, k activities T periods, P priorities
a_i^*	goal achievement in the ith priority level	P
$D_i^-(t)$	column vector of underachievement of the ith objective function group in period t, i.e., $[D_i^-(t)]^T = [d_1^-(t)\ d_2^-(t)\ \ldots\ d_k^-(t)]$	k
$D_i^+(t)$	column vector of overachievement of the ith objective function group in period t.	k
$d_k^-(t)$	underachievement of the kth objective function in period t	T
$d_k^+(t)$	overachievement of the kth objective function in period t	T
d_i^{-*}	underachievement of the ith objective function in computer printout	1
d_i^{+*}	overachievement of the ith objective function in computer printout	1
$\overline{GNP(t)}$	specified GNP goal in period t excluding government consumption and foreign exchange available	T
$\alpha(t)$	specified rate of growth of GNP(t)	1
$\gamma(t)$	specified rate of growth of aggregate capital stock K(t+1) over K(t)	1

constraint and the balance of payment constraint are selected as candidates to be nonabsolute objectives, because they are assumed to be more flexible constraints and be more meaningful objectives in the linear goal programming. Gross national product (GNP) growth and capital stock growth are introduced as nonabsolute objectives. Hence this model is composed of five kinds of absolute objectives and four kinds of nonabsolute objectives.

2.1 Forming the absolute objective functions.

1) Distributional relationships (DR: Ref. eq. (3.17)')

$$c(t)C(t) - M''(t) - s(t)X(t-1) + [a(t) - I - m'(t) + s(t)]\, X(t) + p(t)\, Z(t+1) + D_1^-(t) - D_1^+(t) = -\overline{E(t)} - \overline{G(t)}$$

$$t = 1, \ldots, T \qquad (4.1)$$

2) Capacity relationships (CC: Ref. eq. (3.9))

$$-K(t) + b(t)\, X(t) + D_2^-(t) - D_2^+(t) = 0$$

$$t = 1, \ldots, T \qquad (4.2)$$

3) Capital accounting relationships (CAR: Ref. eq. (3.10'))

$$-K(t) + K(t+1) + dX(t) - p(t)\, Z(t+1) + D_3^-(t) - D_3^-(t) = 0$$

$$t = 1, \ldots, T \qquad (4.3)$$

4) Competitive import ceilings (CIC: Ref. eq. (3.16)')

$$[m''(t)]^{-1}\, M''(t) + um'(t)\, X(t) + d_{3k+1}^-(t) - d_{3k+1}^+(t) = \overline{A(t)} + u\, \overline{E(t)}$$

$$t = 1, .., T \qquad (4.4)$$

5) Savings relationships (SC: Ref. eq. (3.23)')

$$uM''(t) - us(t)\ X(t-1) + u\{s(t) + m'(t) - \beta_1[I-a(t)]\}\ X(t)$$

$$+ up(t)\ Z(t+1) + d^-_{3k+2}(t) - d^+_{3k+2}(t)$$

$$= \beta_0 + \overline{A(t)} + u\ \overline{E(t)} \qquad t = 1, \ldots, T \qquad (4.5)$$

2.2 Forming the nonabsolute objective functions.

1) Consumption growth (CG: Ref. eq. (3.8))

$$-[1 + \rho(t)]\ C(t) + C(t+1) + d^-_{3k+3}(t) - d^+_{3k+3}(t) = 0$$

$$t = 0, \ldots, T-1 \quad (4.6)$$

where $d^-_{3k+3}(t)$ = underachievement on grass private consumption growth goal in period t

$d^+_{3k+3}(t)$ = overachievement on gross private consumption growth goal in period t.

2) Capital stock growth (CSG)

$$-u[1+\gamma(t)]\ K(t) + uK(t+1) + d^-_{3k+4}(t) - d^+_{3k+4}(t) = 0$$

$$t = 1, \ldots, T \qquad (4.7)$$

where $d^-_{3k+4}(t)$ = underachievement on gross capital stock growth goal in period t

$d^+_{3k+4}(t)$ = overachievement on gross capital stock growth goal in period t.

3) Balance of payment (BOP: Ref. eq. (3.13)')

$$uM''(t) + um'(t)\ X(t) + d^-_{3k+5}(t) - d^+_{3k+5}(t) = \overline{A(t)} + u\overline{E(t)}$$

$$t = 1, \ldots, T \qquad (4.8)$$

where $d^-_{3k+5}(t)$ = amount of foreign exchange surplus in period t

$d^+_{3k+5}(t)$ = amount of foreign exchange deficit in period t.

4) Gross national product (GNP)

The GNP can be expressed by an expenditure concept as follows;

GNP = Consumption + Business investment + Net export

where Consumption = Private consumption + Government consumption

Business investment = New investment + Inventory increase

Net export = Foreign exchange available - Imports.

Government consumption and foreign exchange available can be excluded from the GNP formulation due to their exogenous characteristics of this model (i.e., preassigned values). Hence GNP objective function becomes,

$$C(t) + up(t)\, Z(t+1) + us(t)\, [X(t) - X(t-1)] - um'(t)\, X(t)$$

$$- uM''(t) + d^-_{3k+6}(t) - d^+_{3k+6}(t) = \overline{GNP(t)} \qquad t = 1, \ldots, T \qquad (4.9)$$

where $d^-_{3k+6}(t)$ = underachievement on GNP goal in period t

$d^+_{3k+6}(t)$ = overachievement on GNP goal in period t.

$\overline{GNP(t)}$ = projected GNP goals (set at 10% annual growth rate ($\alpha(t) = 0.1$) are shown in Table 4-1.)

2.3 Forming the achievement functions. Since at the beginning of the simulation the decision maker's priority structure of his goals would not be fixed, it might be desirable to simulate the model for different priority structures. In this study two priority structures will be used.

Table 4-1 GNP Target

(Unit: billion Won)

	1977	1978	1979	1980	1981
$\overline{G(t)}$	273	306	343	384	432
$\overline{A(t)} + u\overline{E(t)}$	1330	1560	1842	2168	2605
$\overline{GNP(t)}$*	3443	3684	3920	4164	4350
GNP**	5045	5550	6105	6715	7387

*Right hand side value of eq. (4.9)

**10% annual growth rate of GNP target

Priority Structure 1

P_1: Satisfy all absolute objectives

P_2: Achieve $\rho(t) \times 100\%$ annual growth of gross private consumption

P_3: Achieve $\gamma(t) \times 100\%$ annual growth of gross capital stock

P_4: Achieve balance of payment in foreign trade

P_5: Achieve $\alpha(t) \times 100\%$ annual growth of GNP

then the achievement function is;

$$\text{Min. } \bar{a} = \Big\{ \sum_{t=1}^{T} [uD_1^+(t) + uD_2^+(t) + uD_3^+(t) + uD_3^-(t) + d_{3k+1}^+(t) + d_{3k+2}^+(t)], \\ [\sum_{t=1}^{T} d_{3k+3}^-(t)], [\sum_{t=1}^{T} d_{3k+4}^-(t)], [\sum_{t=1}^{T} d_{3k+5}^+(t)], [\sum_{t=1}^{T} d_{3k+6}^-(t)] \Big\} \tag{4.10}$$

Priority Structure 2

P_1: Satisfy all absolute objectives

P_2: Achieve $\alpha(t) \times 100\%$ annual growth of GNP

P_3: Achieve balance of payment in foreign trade

P_4: Achieve $\gamma(t) \times 100\%$ annual growth of gross capital stock

P_5: Achieve $\rho(t) \times 100\%$ annual growth of gross private consumption

the achievement function of this structure is;

$$\text{Min. } \bar{a} = \Big\{ \sum_{t=1}^{T} [uD_1^+(t) + uD_2^+(t) + uD_3^+(t) + uD_3^-(t) + d_{3k+1}^+(t) + d_{3k+2}^+(t)], \\ [\sum_{t=1}^{T} d_{3k+6}^-(t)], [\sum_{t=1}^{T} d_{3k+5}^+(t)], [\sum_{t=1}^{T} d_{3k+4}^-(t)], [\sum_{t=1}^{T} d_{3k+3}^-(t)] \Big\} \tag{4.11}$$

In the above two achievement functions each year's weight is assumed to be the same within each priority level, but it is optional. A heavier weight could be assigned to the latter years if the last year's success was considered to be of higher merit.

3. Analysis of Solutions

The planning span is 5 years actually and the model formulation is also planned for the entire 5 year period, but its computer solution is made up to the 2nd year due to its excessive computer execution time required. The simplified model consists of 130, i.e., (4x16x2 + 1x1x2) technological variables; 108, i.e., (3x16x2 + 6x1x2) positive (d_i^{+*}) and negative (d_i^{-*}) deviational variables and 5 priorities. The computer linear goal programming code used herein was constructed by Bershader [1]. This model is solved on an IBM 370/158 computer using FORTRAN G level with 256K region size required. The solution is obtained after approximately 11 minutes of execution time. (For one-year planning, it took about 1.5 minutes; for three-years planning it required more than 40 minutes.) Here the results of 2 year economic planning with 2 kinds of priority structure with 10% annual growth rate of private consumption, capital stock and GNP respectively (i.e., $\alpha(t) = \gamma(t) = \rho(t) = 0.1$) are presented. Notations used in the computer printout are shown in Table 4-2. Notice that, for the column identification, ZBAR is a_k^*, XSTAR is technological variable, PSTAR is d_i^{+*}, and NSTAR is d_i^{-*}. For example, GNP objective function has deviational variable of (d_1^{-*} and d_1^{+*}) for the first year, and of (d_{55}^{-*} and d_{55}^{+*}) for the second year. For technological variable, optimal amount of C(1) is found in the 17th row, and C(2) in the 82th row in column name of XSTAR.

3.1 Run-1. The result, based on the priority structure 1 with twice the weight for the first year than that of the second year (i.e., $a_k = 2d_i(1) + d_i(2)$) within the same priority level, are shown in Table 4-3. Its goal achievements are as follows;

Table 4-2 Notations for Computer Output

1. Deviational variables (d_i^{-*} and d_i^{+*})

Object \ Year (t)	1	2
GNP(t)	1	55
CG(t)	2	56
DR(t)	3 - 18	57 - 72
CC(t)	19 - 34	73 - 88
CSG(t)	35	89
CAR(t)	36 - 51	90 - 105
BOP(t)	52	106
CIC(t)	53	107
SC(t)	54	108

2. Technological variables

Var. \ Year (t)	1	2
X(t)	1 - 16	66 - 81
C(t)	17	82
M(t)	18 - 33	83 - 98
Z(t+1)	34 - 49	99 - 114
K(t+1)	50 - 65	115 - 130

Table 4-3 Computer Printout of Optimal Solution (Run-1).

PROBLEM 1 READ IN SUCCESSFULLY

SUBSCRIPT	5 TERMS ZBAR	130 TERMS XSTAR	108 TERMS PSTAR	108 TERMS NSTAR
1	0.0	1596.6531	0.0	432.3337
2	0.0	162.3253	0.0	0.0
3	146.4320	686.7476	0.0	0.0
4	589.4719	795.4541	0.0	0.0
5	1734.4224	86.2086	0.0	0.0
6		289.9158	0.0	0.0
7		274.2078	0.0	8.7011
8		205.7468	0.0	0.0
9		419.0229	0.0	112.3786
10		755.5557	0.0	61.4366
11		1061.1238	0.0	0.0
12		749.4929	0.0	0.0
13		126.2331	145.9685	0.0
14		533.0271	0.0	0.0
15		203.7463	0.0	0.0
16		1572.1121	0.0	0.0
17		3305.9941	0.0	0.0
18		66.2261	0.0	0.0
19		45.8578	0.0	0.0
20		0.0	0.0	0.0
21		98.2216	0.0	13.0488
22		0.0	0.0	0.0
23		189.8657	0.0	20.9395
24		0.0	0.0	0.0
25		0.0	0.0	0.0
26		200.0142	0.0	0.0
27		292.0486	0.0	0.0
28		0.0	0.0	0.0
29		0.0	0.0	0.0
30		0.0	0.0	26.1963
31		0.0	0.0	79.6697
32		31.9924	0.0	293.0310
33		98.2973	0.0	0.0
34		545.2532	0.0	0.0
35		0.0	0.0	0.0
36		0.0	0.0	0.0
37		0.0	0.0	0.0
38		0.0	0.0	0.0
39		0.0	0.0	0.0
40		0.0	0.0	0.0
41		0.0	0.0	0.0
42		0.0	0.0	0.0
43		0.0	0.0	0.0
44		0.0	0.0	0.0
45		211.9616	0.0	0.0
46		0.0	0.0	0.0
47		0.0	0.0	0.0
48		0.0	0.0	0.0
49		305.9824	0.0	0.0
50		397.7612	0.0	0.0
51		68.2987	0.0	0.0
52		179.8605	139.8954	0.0

53	328.6357	0.0	130.7047
54	41.3312	0.0	0.0
55	120.6050	0.0	869.7549
56	139.4356	0.0	0.0
57	165.7947	0.0	0.0
58	176.6630	0.0	0.0
59	417.9451	0.0	0.0
60	229.6133	0.0	0.0
61	876.1379	0.0	0.0
62	574.0129	0.0	0.0
63	2625.8399	0.0	0.0
64	1308.8762	0.0	0.0
65	1549.0259	0.0	0.0
66	1846.3821	0.0	0.0
67	122.3280	0.0	0.0
68	552.4365	0.0	0.0
69	764.1968	0.0	0.0
70	93.2538	0.0	0.0
71	253.3721	0.0	0.0
72	184.5712	0.0	0.0
73	123.0399	0.0	0.0
74	148.3447	0.0	0.0
75	1335.2952	0.0	0.0
76	961.6321	0.0	0.0
77	627.1226	0.0	32.1433
78	102.3780	0.0	0.0
79	541.3091	0.0	57.2634
80	227.0620	0.0	63.0426
81	1571.2522	0.0	120.2588
82	3634.2771	0.0	0.0
83	0.0	0.0	0.0
84	0.0	0.0	745.6465
85	207.8875	0.0	146.8871
86	180.6349	0.0	215.7045
87	0.0	0.0	0.0
88	292.7983	0.0	0.0
89	91.3199	0.0	146.4320
90	0.0	0.0	0.0
91	449.7903	0.0	0.0
92	0.0	0.0	0.0
93	29.0443	0.0	0.0
94	0.0	0.0	0.0
95	0.0	0.0	0.0
96	0.0	0.0	0.0
97	31.9776	0.0	0.0
98	193.2694	0.0	0.0
99	1058.7817	0.0	0.0
100	0.0	0.0	0.0
101	0.0	0.0	0.0
102	0.0	0.0	0.0
103	0.0	0.0	0.0
104	0.0	0.0	0.0
105	0.0	0.0	0.0
106	0.0	309.6814	0.0
107	0.0	0.0	0.0
108	0.0	0.0	0.0
109	0.0	0.0	0.0
110	0.0	0.0	0.0
111	0.0	0.0	0.0
112	0.0	0.0	0.0

113	0.0	0.0	0.0
114	0.0	0.0	0.0
115	1142.4526	0.0	0.0
116	74.5722	0.0	0.0
117	180.7644	0.0	0.0
118	319.0454	0.0	0.0
119	42.0643	0.0	0.0
120	105.4026	0.0	0.0
121	124.9066	0.0	0.0
122	166.8400	0.0	0.0
123	203.6764	0.0	0.0
124	609.9502	0.0	0.0
125	219.7820	0.0	0.0
126	1403.1145	0.0	0.0
127	573.1824	0.0	0.0
128	2572.4199	0.0	0.0
129	1301.6521	0.0	0.0
130	1548.9031	0.0	0.0

P_1 is achieved ($a_1^* = 0$). All of the economic constraints are satisfied.

P_2 is satisfied ($a_2^* = 0$). 10% annual growth of gross private consumption is exactly achieved.

P_3 is partially achieved ($a_3^* = 2d_{35}^{-*} + d_{89}^{-*} = 0 + 146.4 = 146.4$). The first year's growth of capital stock is 10% and second year's is 8.49% which comes from (9699 x 1.1 - 146.4)/9699 = 1.0849.

P_4 is not achieved ($a_4^* = 2d_{52}^{+*} + d_{106}^{+*} = 2 \times 139.9 + 309.7 = 589.5$). The first year's import is exceed by 10.53% (139.9/1330) and the second year's by 19.87% (309.7/1560) than that allocated.

P_5 is not achieved ($a_5^* = 2d_1^{-*} + d_{55}^{-*} = 2 \times 432.3 + 869.8 = 1734.4$). The first year's GNP is 4164 billion Won (5046-432) and the second is 4680 billion Won (5550-870). These are 0.58% and 1.44% of the annual growth rate respectively.

<u>3.2 Run-2.</u> The result, based on the priority structure 2 with the same weight for both planning years, are shown in Table 4-4. Its goal achievements are as follows;

P_1 is achieved ($a_1^* = 0$). All of the economic constraints are satisfied.

P_2 is not achieved ($a_2^* = d_1^{-*} + d_{55}^{-*} = 119.4 + 389.0 = 508.4$). The first year's GNP is 4927 billion Won (5046-119) and the second is 5161 billion Won (5550-389), which are 7.41% and 4.76% of the growth rate respectively.

P_3 is partially achieved ($a_3^* = d_{52}^{+*} + d_{106}^{+*} = 134.0 + 0 = 134.0$). The first year inport volume is exceed by 10.08% (134/1330) but the second year import volume is only 83.91% (1-251/1560) of the allocated fund because of $d_{106}^{-*} = 251$.

Table 4-4 Computer Printout of Optimal Solution (Run-2).

PROBLEM 1 READ IN SUCCESSFULLY

SUBSCRIPT	5 TERMS ZBAR	130 TERMS XSTAR	108 TERMS PSTAR	108 TERMS NSTAR
1	0.0	1601.9719	0.0	119.4108
2	503.3533	156.3475	249.2009	0.0
3	134.0671	713.0210	0.0	0.0
4	0.0	796.4158	0.0	0.0
5	1294.6646	58.7337	0.0	0.0
6		289.9158	0.0	0.0
7		215.9848	0.0	0.0
8		134.3853	0.0	0.0
9		419.0227	0.0	0.0
10		755.5559	0.0	0.0
11		931.1431	0.0	0.0
12		866.4338	0.0	0.0
13		121.3263	0.0	0.0
14		546.1755	0.0	0.0
15		212.7372	0.0	0.0
16		1571.1228	0.0	0.0
17		3555.1909	0.0	0.0
18		36.3338	0.0	0.0
19		0.0	0.0	0.0
20		0.0	0.0	10.6237
21		135.9182	0.0	4.9509
22		0.0	0.0	0.0
23		204.1054	0.0	28.9179
24		0.0	0.0	0.0
25		0.0	0.0	31.8470
26		174.6114	0.0	64.6263
27		288.5459	0.0	0.0
28		0.0	0.0	0.0
29		0.0	0.0	22.1183
30		0.0	0.0	0.0
31		0.0	0.0	75.1691
32		57.9434	0.0	222.6918
33		163.1592	0.0	0.0
34		326.6736	0.0	0.0
35		0.0	38.0079	0.0
36		0.0	0.0	0.0
37		0.0	0.0	0.0
38		0.0	0.0	0.0
39		0.0	0.0	0.0
40		0.0	0.0	0.0
41		0.0	0.0	0.0
42		0.0	0.0	0.0
43		0.0	0.0	0.0
44		0.0	0.0	0.0
45		0.0	0.0	0.0
46		738.0215	0.0	0.0
47		41.6018	0.0	0.0
48		0.0	0.0	0.0
49		0.0	0.0	0.0
50		839.2991	0.0	0.0
51		78.4476	0.0	0.0
52		169.9759	134.0671	0.0

53	329.2644	0.0	106.4259
54	41.8342	0.0	0.0
55	120.6051	0.0	388.9426
56	178.2258	0.0	1294.6646
57	172.8000	0.0	0.0
58	169.7333	0.0	0.0
59	413.7112	0.0	0.0
60	249.3178	0.0	0.0
61	990.8979	0.0	0.0
62	575.4485	0.0	0.0
63	2607.0864	0.0	0.0
64	1311.9255	0.0	0.0
65	1549.3184	0.0	0.0
66	1674.0000	0.0	0.0
67	174.8885	0.0	0.0
68	612.5229	0.0	0.0
69	769.1279	0.0	0.0
70	127.6394	0.0	0.0
71	201.3345	18.8662	0.0
72	222.7191	0.0	0.0
73	200.0584	0.0	0.0
74	440.1558	0.0	0.0
75	1307.8391	0.0	17.0195
76	877.4229	0.0	0.0
77	1126.3923	0.0	18.7831
78	116.8235	0.0	0.0
79	466.4678	0.0	38.0636
80	237.8890	0.0	0.0
81	1577.2065	0.0	0.0
82	2618.5974	0.0	0.0
83	0.0	0.0	48.1176
84	16.6010	0.0	776.1729
85	0.0	0.0	142.4508
86	113.6833	0.0	485.6982
87	0.0	0.0	0.0
88	341.0784	0.0	0.0
89	0.0	318.4673	0.0
90	0.0	0.0	0.0
91	372.3289	0.0	0.0
92	0.0	0.0	0.0
93	0.0	0.0	0.0
94	0.0	0.0	0.0
95	0.0	0.0	0.0
96	0.0	0.0	0.0
97	0.0	0.0	0.0
98	0.0	0.0	0.0
99	968.7336	0.0	0.0
100	0.0	0.0	0.0
101	0.0	0.0	0.0
102	0.0	0.0	0.0
103	0.0	0.0	0.0
104	0.0	0.0	0.0
105	0.0	0.0	0.0
106	0.0	0.0	251.1645
107	0.0	0.0	433.3484
108	0.0	0.0	0.0
109	0.0	0.0	0.0
110	558.1323	0.0	0.0
111	0.0	0.0	0.0
112	0.0	0.0	0.0

113	0.0	0.0	0.0
114	0.0	0.0	0.0
115	1084.0107	0.0	0.0
116	74.7755	0.0	0.0
117	170.0735	0.0	0.0
118	325.0000	0.0	0.0
119	20.9415	0.0	0.0
120	108.2660	0.0	0.0
121	117.8131	0.0	0.0
122	167.8243	0.0	0.0
123	167.6931	0.0	0.0
124	581.1802	0.0	0.0
125	232.5183	0.0	0.0
126	2023.4778	0.0	0.0
127	574.8132	0.0	0.0
128	7569.9675	0.0	0.0
129	1310.1362	0.0	0.0
130	1548.5032	0.0	0.0

P_4 is achieved ($a_4^* = 0$). Due to $d_{35}^{+*} = 38$, $d_{89}^{+*} = 318$, the first year's growth rate of gross capital stock is 10.43% ((9699 + 38)/ 8817) and the second is 12.83% ((9699 x 1.1 + 318)/(9699+38)).

P_5 is not achieved ($a_5^* = d_2^{-*} + d_{56}^{-*} = 0 + 1294.7 = 1294.7$). Due to $d_2^{+*} = 249.2$, the first year's gross private consumption growth rate is 18.3% ((3306 + 249)/3005) and the second is -34.43% ((3636-1295)/(3306+249)).

3.3 Comparison with linear programming model. Aggregate values of five key variables from two runs in linear goal programming and the linear programming model of two year planning are shown in Table 4-5. Domestic output does not vary much, but total volume from run-2 increased due to its top priority goal of GNP. Private consumption from linear programming model and from run-1 has the same value because maximization of private consumption is one part of the objective function in the linear programming model and also private consumption growth is top priority goal in run-1. In competitive import side, the volume in run-2 is sharply decreased to boost the GNP achievement as top priority level. In run-2 private consumption reduced much, hence this amount is left for increasing new additions to fixed capital capacity. Fixed capital stock is relatively consistent because its priority level among goals does not change much.

A real contribution of the linear goal programming is found in GNP achievement. In the linear programming model GNP grows at the average rate of 4.1%, whereas it grows 0.58% and 1.44% during the first and the second year respectively by run-1, and 7.4% and 4.8% respectively by run-2 in the linear goal programming model.

Table 4-5 Aggregate Values of Endogenous Variables

(Unit: Billion Won)

Var.	Method		1st year	2nd year	Total
$\sum_{i=1}^{16} X_i$	LP		9,404	9,635	19,039
	G/P	RUN-1	9,517	9,450	18,967
		RUN-2	9,389	10,129	19,518
C	LP		3,306	3,637	6,934
	G/P	RUN-1	3,306	3,634	6,940
		RUN-2	3,555	2,618	6,173
$\sum_{i=1}^{16} M_i''$	LP		922	1,120	2,042
	G/P	RUN-1	1,021	1,477	2,498
		RUN-2	1,061	844	1,905
$\sum_{i=1}^{16} Z_i$	LP		1,026	729	1,755
	G/P	RUN-1	1,063	1,058	2,121
		RUN-2	1,107	1,527	2,634
$\sum_{i=1}^{16} K_i$	LP		9,610	10,098	19,708
	G/P	RUN-1	9,700	10,589	20,289
		RUN-2	9,749	10,910	20,659

Source: Table 4-3 and 4-4.

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CHAPTER 5

CONCLUDING REMARKS AND PROPOSAL FOR FURTHER STUDY

The results obtained in this study show that linear goal programming is a successful analytical method for solving a multiobjective economic planning problem. To make the linear goal programming technique a more reliable planning tool for Korea, the following suggestions are made.

1. Extended Computer Run for 5 Year Planning

The planning span for Korea economic planning is 5 years and the model is also formulated for the entire 5 year period, but its computer solution is made up to the 2nd year only due to the excessive computer execution time required. It is expected that around 3 hours (1.5 min. X 5^3) computer execution time (using IBM 370/158 computer) is needed to solve the 5 year planning by linear goal programming.

This amount of computer execution time is probably acceptable as a national project. It is, however, too much for this project, so therefore an alternative approach must be considered. One way is to reduce the number of variables in the model. This linear goal programming model has 5 key variables which are composed of 16 industrial sectors with a 5 year planning span, hence the model has 325 (i.e., 4 x 16 x 5 + 1 x 1 x 5) technological variables and 270 (i.e., 3 x 16 x 5 + 6 x 1 x 5) positive and negative deviational variables. If we aggregate 16 industrial sectors into 8 sectors, this model will have 165 (i.e., 4 x 8 x 5 + 1 x 1 x 5) technological variables and 150 (i.e., 3 x 8 x 5 + 6 x 1 x 5) positive and negative variables, and it may be solved within 25 minutes (1.5 min x $5^3/2^3$). But if the number of variables used in the model is too small, the model will not reflect a sharp image of the real economic situation.

Another way considered is to use what is called here "Sequential Linear Programming" technique. This is the maximization/minimization of priority one nonabsolute objective first within the feasible solution set formed by all the other objectives. Then maximize/minimize priority two nonabsolute objective within the feasible solution set formed by low/upper bound of the attained priority one nonabsolute objective as well as the other objectives, and so on. As long as goals are conflicting, it may not form any feasible region which is the necessary condition in the linear programming. Hence this rationale can not be utilized. The solution of this model run by the linear programming technique using this algorithm turns out to be infeasible. It indicates that the selected goals in the linear goal programming model are conflicting in one way or the other.

2. Modification of Priority Level

After initial solutions with the different priority structure have been made, the linear goal programming technique gives the planner a chance to review hierarchical structure of priority among goals. Regrouping or change of priority level can be considered. The best attainable solution of this economic model can be obtained with this modification procedure.

A GOAL PROGRAMMING MODEL FOR
KOREAN ECONOMIC PLANNING

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Many economic planning problems are composed of multiple objectives or goals which the planner wishes to achieve. Furthermore, the degree of importance of these goals are not equal. Because of these characteristics of planning problems, analytical techniques which can efficiently solve this type of problem are desired. Linear goal proprogramming, which is a relatively new analytical technique, is introduced to handle multiple objective planning problems.

Eckaus and Parikh's multisectoral dynamic model is reorganized for the Fourth Five-Year Plan of Korea so as to apply the linear goal programming technique. Chosen economic goals in the model are GNP growth, private consumption growth, capital stock growth and balance of payment in foreign trade. Two kinds of priority structures are tried. The solutions from both linear programming model and linear goal programming model are compared.

While certain improvements are needed to make it a reliable planning tool for Korea, linear goal programming is shown to be a suitable technique for the economic planning problems.