SHEAR FORCE DISTRIBUTION IN BEAMS WITH ECCENTRIC RECTANGULAR WEB OPENINGS BY THE FINITE ELEMENT METHOD

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a = one-half of the opening width

A_B = area below the opening

A_{eff} = effective area of the idealized beam flange

A_f = area of the flange

$A_{R\ eff}$ = effective area of the reinforcing bars

A_T = area above the opening

A_w = web area

b_f = width of the flange

b_1 = width of top flange

b_2 = width of bottom flange

$[B]$ = element strain-displacement matrix

c = one-half of the length of the cover plate

$[C]$ = element stress-strain matrix

d = total depth of the beam section

d_B = distance from the centroid of the bottom flange to the centroid of the bottom reinforcing bar

d_T = distance from the centroid of the top flange to the centroid of the top reinforcing bar

E = elastic modulus

e = eccentricity of the web opening

ext = the extension of the reinforcing bar beyond the end of the opening

$\{\varepsilon\}$ = element strain vector

$\{\varepsilon^e\}$ = element elastic strain vector

$\{F\}$ = nodal force vector

G = shear modulus

h = one-half of the opening height

h_1 = vertical distance from upper or lower edge of opening to the centerline of the reinforcing bar

I = moment of inertia

I_B = moment of inertia of the bottom section about its centroidal axis

I_{eq} = equivalent moment of inertia

I_n = moment of inertia of the net section about its centroidal axis

I_T = moment of inertia of the top section at the opening about its centroidal axis

$[K]$ = stiffness matrix

K_B = shear shape factor for the bottom section

K_T = shear shape factor for the top section

K_I = shear shape factor for I-section

M = bending moment at a section

M_P = plastic moment of gross section

M/V = moment to shear ratio at the centerline of the opening

μ = Poisson's ratio

P = reaction force

S_B = distance from the centroid of the bottom flange to the lower edge of the opening

S_T = distance from the centroid of the top flange to the upper edge of the web opening

σ = primary bending stress at a section

σ_x = total bending stress at a section a longitudinal distance, x, from the center of the opening

σ_{x_2} = secondary bending stress at a section a longitudinal distance, x, from the center of the opening

$\{\sigma\}$ = element stress vector

t_f = thickness of the flange

t_r = thickness of the reinforcing

t_s = the thickness of the stiffener

t_w = thickness of the web

t_1 = thickness of top flange

t_2 = thickness of bottom flange

τ_{xy} = shear stress

u = distance from nearer reaction to the centerline of the opening

U_i = displacement of the i^{th} node in the x direction ($i = 1,3$)

V = shear force at a section

V_B = vertical shear force in the bottom section at the opening

V_T = vertical shear force in the top section at the opening

V_i = displacement of the i^{th} node in the y direction ($i = 1,3$)

w_s = width of the stiffener

x = perpendicular distance from the y-axis

{X} = element displacement vector

y = perpendicular distance from the x-axis

y_B = the transverse distance from the centroid of the bottom section at the opening

y_n = transverse distance from the centroid of the net section

y_T = transverse distance from the centroid of the top section at the opening

$\overline{Y}_T$ = transverse distance from the edge of the opening to the centroidal axis of the top section

$\overline{Y}_B$ = transverse distance from the edge of the opening to the centroidal axis of the bottom section

INTRODUCTION

Presently, openings are frequently cut in the webs of wide flange beams to provide access and utility passage ways for multistory buildings. By removing a portion of the web, a reduction in the shear area as well as in the moment of inertia results. This, in turn, reduces the shear capacity as well as the moment capacity of the beam. These reductions in capacity must be taken into account, or possible failure of the beam may occur.

Often different types and amounts of reinforcement have been added, almost randomly, in the vicinity of the opening in an attempt to strengthen the beam and account for any loss of capacity. This type of random reinforcing has often resulted in over or under reinforced cases, because of the uncertainty involved in the behavior of the stresses around the openings. Current methods of estimating the stresses in the vicinity of the opening are usually based on a simplified elastic analysis known as the Vierendeel Method. The Vierendeel Method makes similar assumptions to the Vierendeel girder analysis, and will be presented later.

Many of the openings cut in the webs of wide flange beams are not necessarily centered at mid-depth of the beam for practical as well as economic reasons. The Vierendeel Method cannot directly apply to beams with eccentric openings without some modifications; i.e., the shear force in the beam above and below the opening must be known.

Several equations have been presented in the current literature to determine the shear force above and below an eccentric opening, but few of the equations yield results which compare well to experimental values. The purpose of this investigation is to use a finite element method to determine the shear force distribution above and below eccentric web openings for comparison with results presented by Bower (25) and Frost (18).

LITERATURE REVIEW

Research concerning the strength and behavior of beams with openings in their webs has been conducted by many different groups and organizations. In 1963, McCutcheon, So and Gersovitz submitted a report (1) describing the progress made at McGill University. Tests were performed on beams with unreinforced circular openings and theoretical versus measured strains were indicated.

A year earlier, in 1962, Heller, Brock and Bart published a paper (2) in which a complex variable method developed by Muskhelishvili was used in conjunction with a conformal mapping technique to obtain the stresses around a rectangular opening in a beam web. Curves were presented showing the tangential stresses around the boundary of a typical family of rectangular openings.

Segner (3) studied the strength of beams with rectangular openings and various types of reinforcing. His conclusions were restricted to a comparison of the cost and performance of the several reinforcing methods.

In 1966, Bower presented two papers (4,5) concerned with the applicability of the Vierendeel Analysis. Wide flange beams with openings in their webs were tested, and it was concluded that the applicability of the analysis depended on the size of the web hole and on the magnitude of the moment to shear ratio at the hole. The analysis also seemed to apply only to flat topped holes, since the boundary conditions of a circular opening could not be satisfied.

A comparison of the stresses from the elasticity solution to the experimental tangential stress measurements, for cases of pure bending and bending with shear, was conducted by Redwood (6). Agreement was good for cases of pure bending, but, when shear was present, the stresses were underestimated.

The ultimate moment capacity of a beam was investigated by Redwood and McCutcheon (7) using several test specimens. The parameters investigated were shape, number of openings, and ratio of moment to shear at the opening. The results showed that large reductions in ultimate capacity occur and that the size of those reductions increases with the amount of shear present. Openings that were circular seemed to perform better than rectangular openings and the presence of a second opening nearby seemed to further reduce the strength of the beam.

In 1968, Bower (8) presented information intended as a first step toward providing the engineer with data supporting the need for reinforcement at an opening in the web of a beam. Criteria were provided for elastic design, plastic design and for buckling of beams with web openings. Criteria for elastic design called for the same basic factor of safety against yielding as in the AISC Specifications, except that the maximum allowable bending and shear stresses were to be computed using the actual stresses causing yielding at the hole rather than the nominal beam stresses. Plastic design called for using the AISC load factor of 1.70, except that the maximum allowable load should be computed using the actual ultimate strength of the beam at the hole rather than the gross beam strength. The buckling criteria called for the same type of buckling analysis as is used for beams without web holes so long as the edge of the hole is at least four inches from the edge of bearing.

Several papers (9,10,11) have been published concerning the plastic behavior and ultimate strength of beams with web openings. Redwood and McCutcheon (12) tested to failure beams with one or two unreinforced openings of various shapes under several different ratios of shear to moment. They found that for pure bending, the moment capacity of a beam with one or two openings could be calculated based on the plastic modulus of the net section through the opening. With the presence of shear, a reduction of moment capacity below that for pure bending resulted. The amount of this reduction was a function of the opening shape, opening dimensions, spacing of openings and the shear to moment ratio. When a single rectangular opening was present, the moment capacity of the beam was reduced to approximately 40 per cent of M_p, but when an identical opening was added at a spacing equal to the opening depth, only a small additional reduction resulted.

Redwood (13) presented a method of deriving an interaction curve relating moments and shears, as well as a simplified, slightly conservative solution to obtain an approximation to the interaction curve. Information on the plastic behavior of beams with web openings was provided by Bower (14) as well as equations predicting their ultimate strength. As the load on the beam increases, the first yielding in the vicinity of the hole occurs at the corners because of the stress concentration at that location.

An investigation concerned with estimating the strength of beams with reinforced rectangular holes, assuming perfectly plastic behavior, was conducted by Congdon and Redwood (15). The reinforcement consisted of horizontal bars welded to the web above and below the openings.

Cooper and Snell (16) performed tests on beams with reinforced web openings and confirmed the validity of the Vierendeel Analysis for the estimation of the normal stresses in the vicinity of the hole. Simple beam theory, including the effects of shear but neglecting the influence of the opening and reinforcing bars, was found to give a reasonable estimation of deflections. The measured shear stresses close to the opening did not correlate well with the horizontal shear stresses calculated from simple beam theory. The shear stresses in the regions between the reinforcing bars were higher than those predicted. Ultimate load tests were run on three beams and the results were consistent with the predictions obtained from the ultimate strength theory for reinforced web openings presented by reference (19).

An analytical method of determining the moment carrying capacity of beams with eccentric rectangular web holes was given by Richard (17). The effects of varying the opening eccentricity, opening length and opening height were investigated. By increasing the opening eccentricity, the moment carrying capacity for high shear values increased. As the opening length and opening height increased, the moment carrying capacity decreased. When the opening height became larger than the opening length, the moment carrying capacity of the beam increased. It was also found that the shear forces were unequally distributed across unequal web areas.

Frost (18) conducted an experimental investigation on eight beams to determine their ultimate strength. The web openings were tested with an eccentricity of 1 in. and 2 in. and with the moment to shear ratio at the hole at values of 0 in. and 40 in. The Vierendeel Method was used for the theoretical analysis and several different formulas for determining the shear force distribution were presented and compared.

METHOD OF ANALYSIS

Finite Element Method

The finite element method is a computer based technique for obtaining solutions to mechanics problems. The finite element method was used in this investigation to obtain the stresses of interest. The finite element method has been shown (21,22,23) to provide very good correlation with available experimental test data.

The computer program used (20) was an elastic-plastic program developed jointly by the Illinois Institute of Technology Research Institute and the Air Force Flight Dynamics Laboratory. Elastic conditions were the only ones under investigation, so the plastic portion of the program was not used. The finite element displacement method was used to perform the linear analysis in the iterative scheme. Bar and constant strain isotropic plane stress triangles were used in constructing idealizations of the beam. The two types of elements used in the analysis (the bar and the triangular plate) are shown in Fig. 1. Coordinates of the end points of the bar and the vertices of the triangle are referred to a fixed coordinate system in a plane. The ordering of the nodal components for each element is shown in Fig. 1. The geometry of the structure is determined by specifying the x and y coordinates of each node, with respect to a fixed set of coordinate axes, and the thickness (cross-sectional area in the case of bars) of the elements. The cartesian components of the nodal displacements for each of these elements comprise the element displacement

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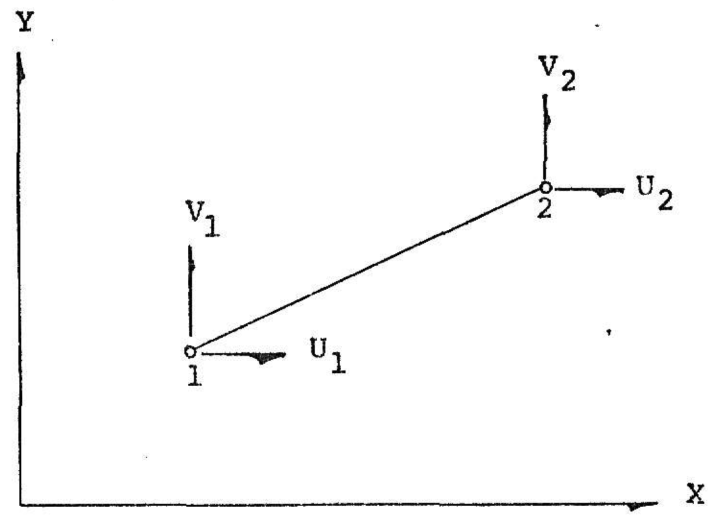
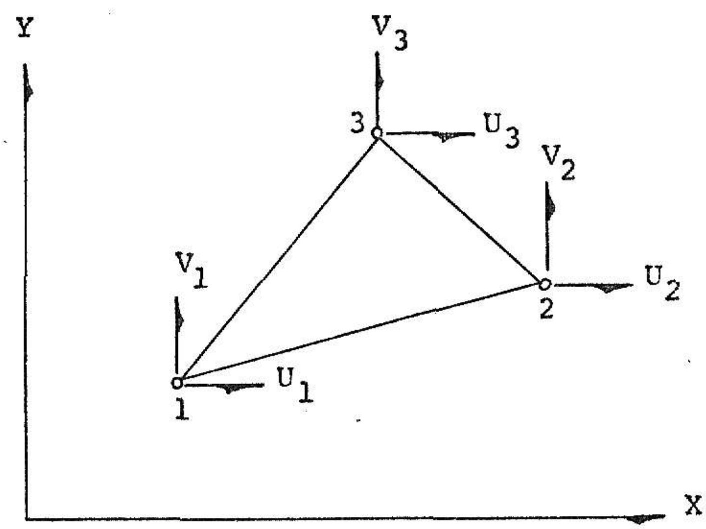


Bar Element



Triangular Plate Element

Fig. 1 - Finite Elements Used in Analysis

vector $\{X\}$. The total element strains designated by the vector $\{\varepsilon\}$ can be expressed in terms of the nodal displacements by an equation of the form

$$\{\varepsilon\} = [B]\{X\}$$

The stresses are related to the elastic strains by Hooke's Law

$$\{\sigma\} = [C]\{\varepsilon^e\}$$

The nodal forces, $\{F\}$, corresponding to given displacements, $\{X\}$, are found by the principle of virtual work.

In the elastic range of material behavior, the equilibrium equations for a structure composed of plate and bar elements of the type considered here can be written

$$\{F\} = [K]\{X\}$$

where the force and displacement vectors now have as their components the cartesian components of force and displacement at all nodes, and $[K]$ is the assembled stiffness matrix for the whole structure. The solution for the unknown displacement is given symbolically by

$$\{X\} = [K]^{-1}\{F\}$$

These displacements are used to obtain element strains and element stresses. An iterative method is applied to each element, and the decision to start a new step (when using the elastic-plastic range) is based on the largest plastic strain increment found among all the elements. (See Appendix C for program details.)

Idealization of the Beams

In order to apply a plane stress finite element method to a three-dimensional structure, certain modifications must be made. The actual flanges and web reinforcement must be replaced by equivalent flange and reinforcement elements. The equivalent flange and reinforcing elements are bars running between nodes along the paths of the flanges and the reinforcement. The equivalent elements, or bars, are pin connected to the appropriate nodes and transfer only axial forces.

The equivalent flange element was obtained as follows. The bar elements do not have an associated thickness; therefore, there is a conflict between maintaining the area of the flanges and maintaining the moment of inertia of the beam. Since maintaining the actual strength of the beam is more important than maintaining the actual area, the equivalent flanges are determined in such a way that the moment of inertia is kept a constant. (See Fig. 2a.)

The moment of inertia, I, of the beam is given as:

$$I = \frac{t_w}{12}(d - 2t_f)^3 + 2\left[\frac{b_f t_f^3}{12} + b_f t_f \left(\frac{d}{2} - \frac{t_f}{2}\right)^2\right]$$

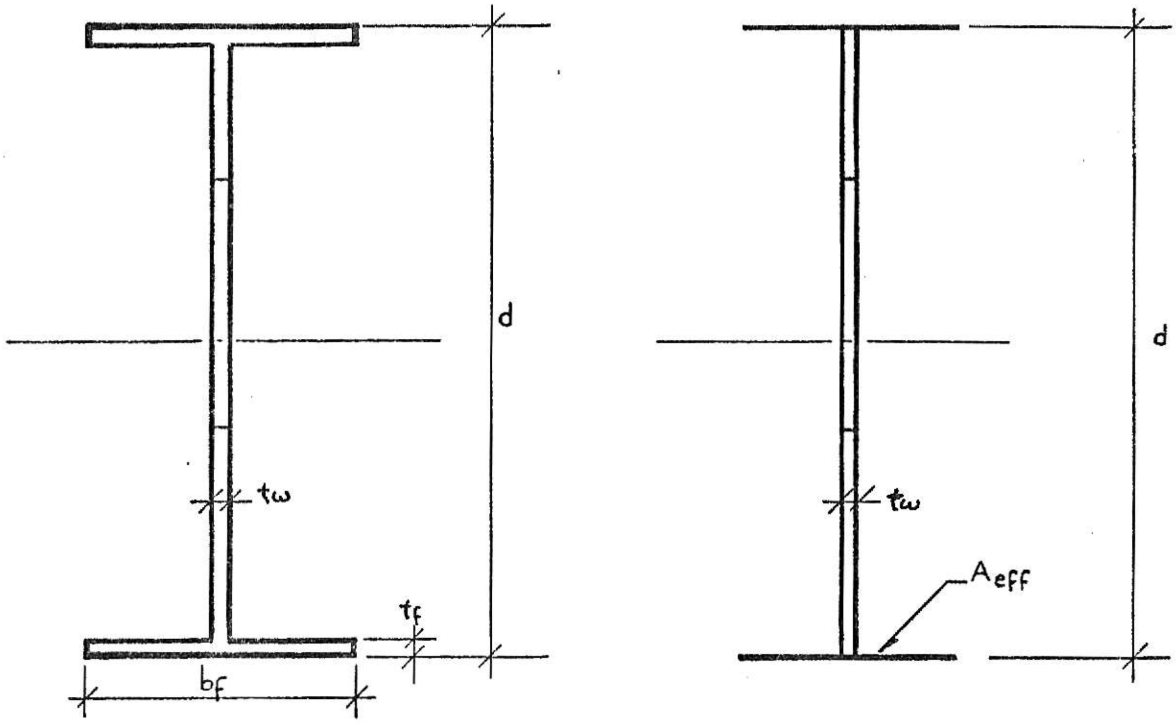
and the moment of inertia, I_{eq}, of the equivalent beam is given as:

$$I_{eq} = \frac{t_w d^3}{12} + 2\left[A_{eff}\left(\frac{d}{2}\right)^2\right]$$

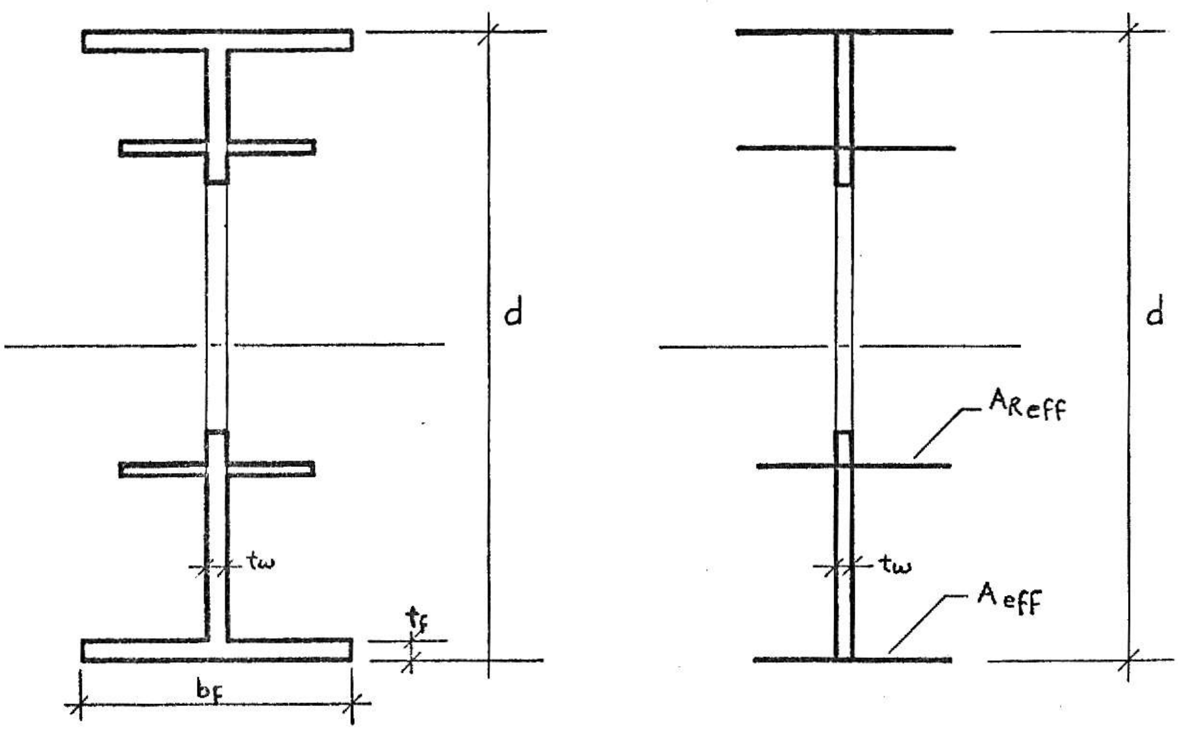
If $I_{eq} = I$, then the effective flange area, A_{eff}, of the idealized beam is given as:

$$A_{eff} = \frac{2I}{d^2} - \frac{t_w d}{6}$$

where t_w is the web thickness of the actual and idealized beam sections, d is the total depth of the beam, t_f is the thickness of the flange and b_f is the width of the flange.



a) Unreinforced



b) Reinforced

Fig. 2 - Typical, Actual and Idealized Beam Sections

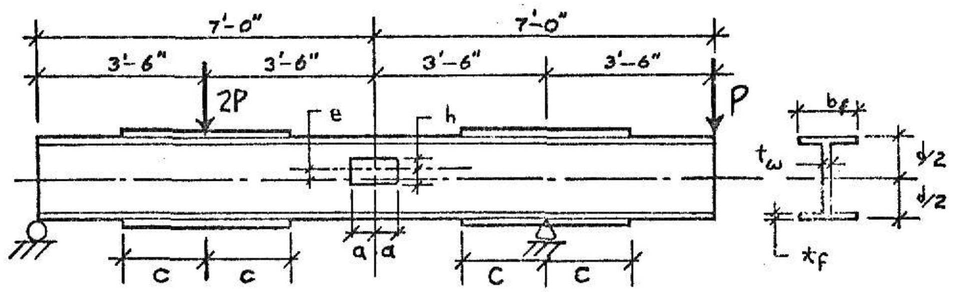
The basic actual beam sections are the same for both unreinforced and reinforced openings. It follows then that the basic idealized beam sections should be the same for both unreinforced and reinforced openings. Therefore, the only change in the idealized beam section with reinforcement is the addition of the reinforcement elements. The equivalent reinforcement elements were obtained by replacing the reinforcing by bars which have the same location and cross-sectional area as the reinforcing as shown in Fig. 2b.

Parameters Investigated

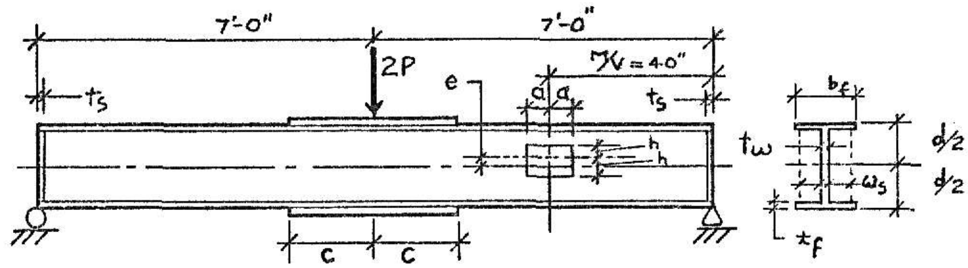
Only one beam section was investigated. A W16 x 40 was chosen since the results from the other methods, presented by both Frost and Bower, were given for that size beam. A direct comparison could then be made between the results from this investigation and those from the investigations presented by Frost and Bower.

The shear force distribution was investigated under several conditions. The eccentricity of the opening was varied from 1 in. to 3 in. by 1 in. increments, as the moment to shear ratio was varied from 0 in. to 80 in. by 40 in. increments. Each case was investigated first without reinforcing in the vicinity of the opening, then with reinforcing.

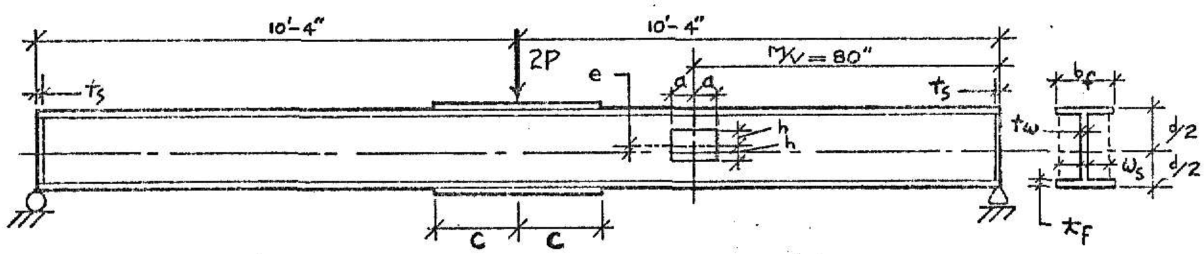
The beam sections used to obtain the moment to shear variations are shown in Fig. 3. Beam No. 1 was used to get an M/V ratio of 0 in., but was modified, as in Fig. 4, to simplify and shorten the element idealization. Beam No. 2 was used to get an M/V ratio of 40 in., and was used without modification.



Beam No. 1



Beam No. 2

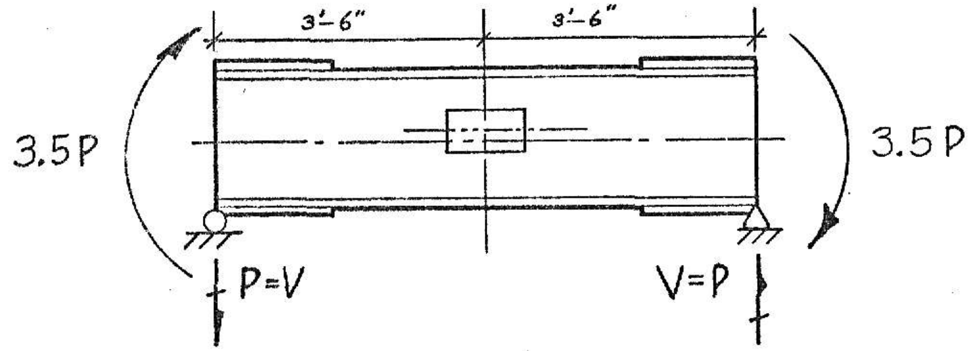


Beam No. 3

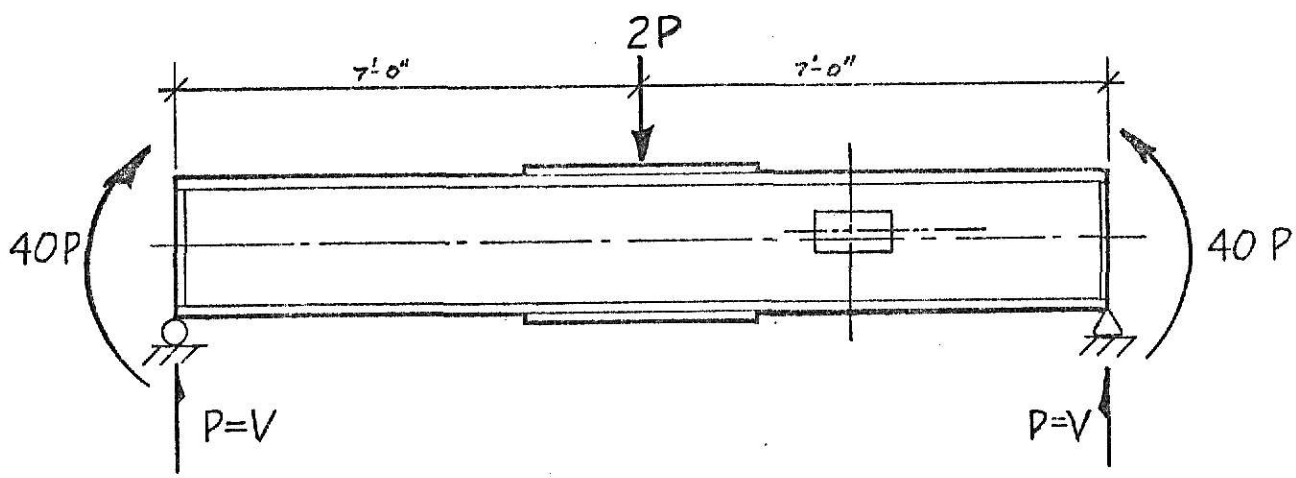
BEAM DIMENSIONS-INCHES

Beam	a	h	d	t_w	b_f	t_f	c	t_s	w_s
1	6.4	3.2	16.0	.31	6.90	.49	22	0	0
2	6.4	3.25	16.0	.31	6.94	.48	24	.25	3.25
3	6.4	3.25	16.0	.31	6.94	.48	24	.25	3.25

Fig. 3 - Beam Dimensions



Modified Beam No. 1



Modified Beam No. 3

Fig. 4 - Modified Beams Used in Calculations

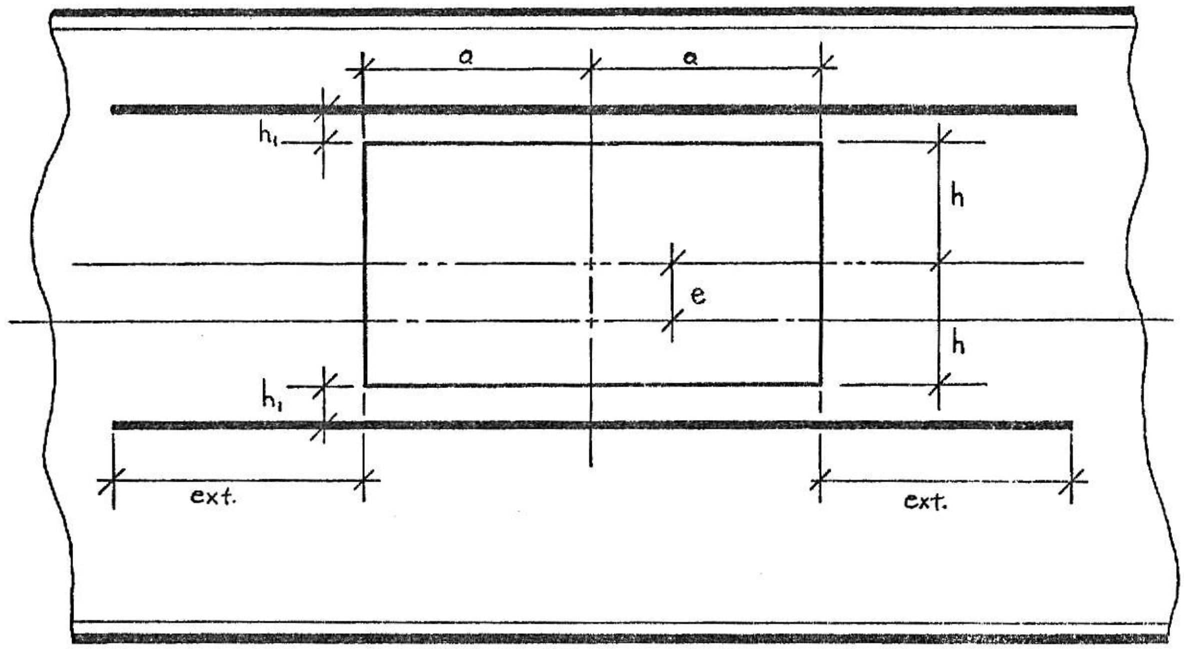
In order to obtain an M/V ratio of 80 in., Beam No. 2 was modified by adding end moments.

The modifications to the beams were designed so that the shear and moment distribution in the modified beams coincided with the shear and moment distribution in the critical portion of the actual beam.

The reinforcement details for each beam are shown in Fig. 5. All of the moment to shear ratios and the corresponding eccentricities which were under investigation for the reinforced cases are presented with their reinforcement dimensions. Some of the dimensions vary slightly in order to accommodate the particular finite element idealization being used. In most cases, the reinforcement dimensions were kept relatively constant with respect to the opening.

Discretization of the Beams

The finite element discretization for a moment to shear ratio of 0 in. is shown in Fig. 6. The finite element discretization shown is a typical example of what was used, although a different discretization was used for each change in eccentricity and moment to shear ratio. The stress value given for each element in the discretization is assumed to act at the centroid of each element. Fig. 7 shows a section of the element discretization in which the elements, numbering 252 through 285, were used to obtain the shear distribution at x = 0 in. The solid dots in each element represent the centroid of the element, as well as the point where the stress is measured. The dotted lines between centroids indicate schematically



REINFORCEMENT DIMENSIONS-INCHES

M/V	e	h	a	h_1	ext.
0	2.0	3.20	6.4	0.80	6.1
	1.0	3.25	6.4	0.75	5.6
40	2.0	3.25	6.4	0.75	5.6
	3.0	3.25	6.4	0.25	5.6
80	2.0	3.25	6.4	0.75	5.6

Fig. 5 - Reinforcement Details

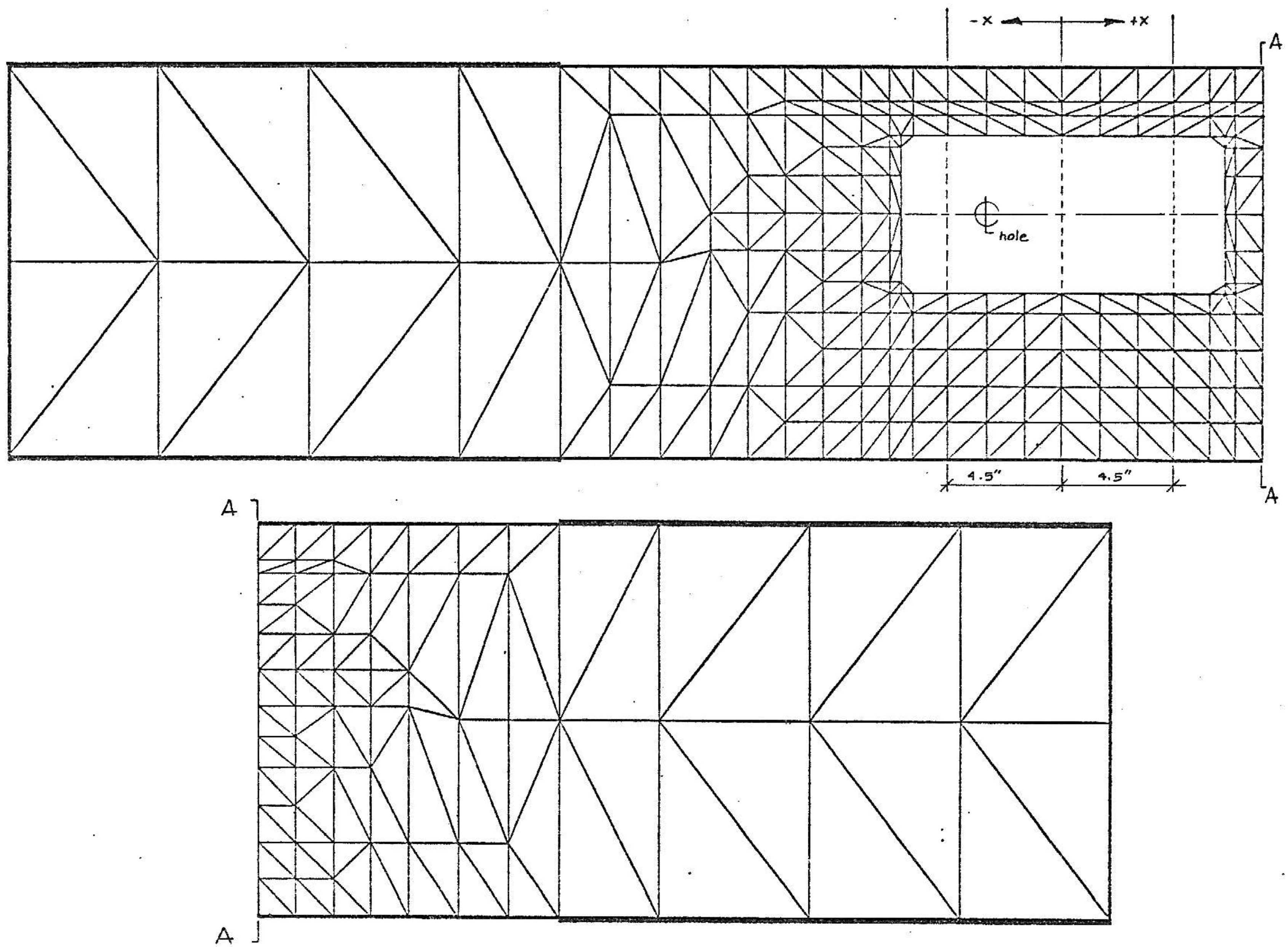


Fig. 6 - Typical Finite Element Discretization of Beam

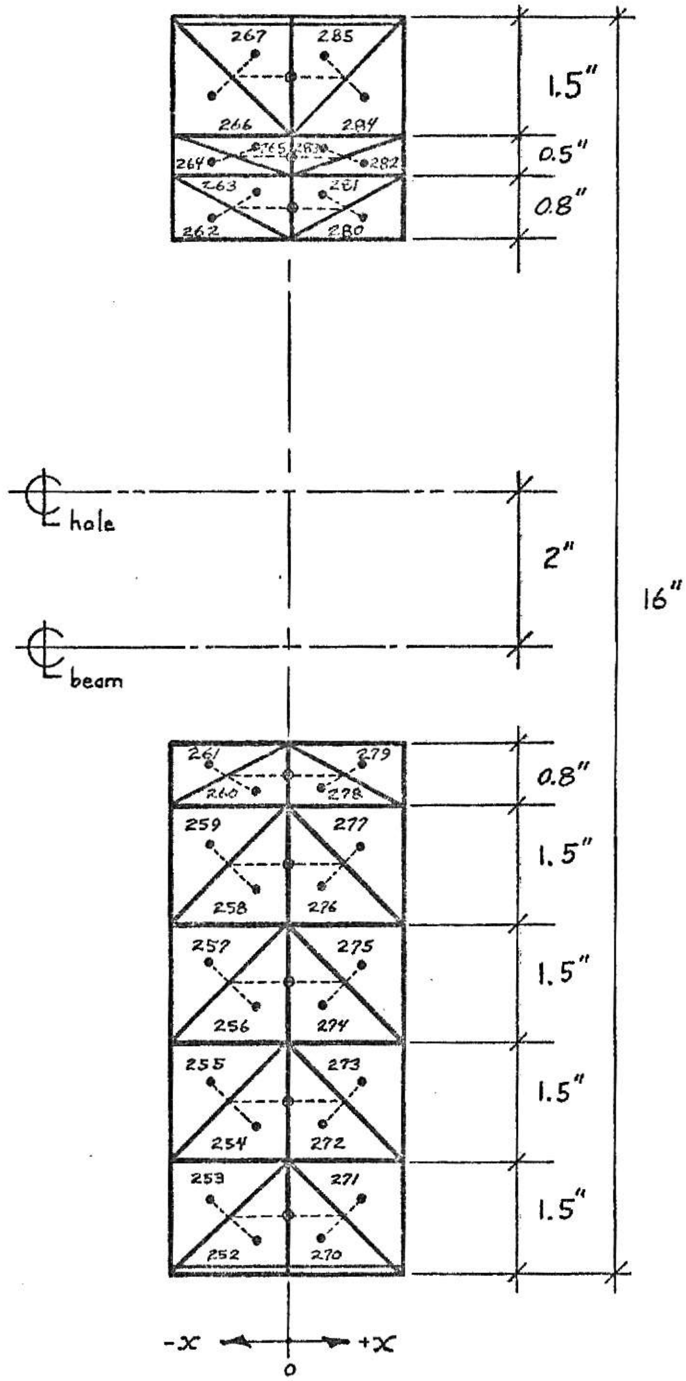


Fig. 7 - Section of Element Discretization at x = 0 in.

the weighted averages calculated. In other words, the stresses in elements numbered 252, 253, 270 and 271 are combined in a weighted average to obtain the stress at x = 0 in.

Each group of elements is averaged through the section shown, and eight stress points at x = 0 in. are obtained. These stresses at a particular section are then plotted to give the stress intensity through a section. (See Appendix B.)

The shear force is obtained by integrating the shear stress curve and multiplying that by the thickness of the elements. A numerical integration procedure was used to find the area under the curve. The area was broken down into a grid system and the number of squares and partial squares was counted.

The area above the opening multiplied by the thickness of the element gives the shear force, V_T, above the opening, and the area below the opening multiplied by the thickness of the elements gives the shear force, V_B, below the opening.

PRESENTATION AND DISCUSSION OF RESULTS

Vierendeel Analysis

The finite element method was used to obtain a shear distribution for use in the Vierendeel Analysis. The basic features of the Vierendeel Analysis are shown in Fig. 8. The portion of the beam between the reaction of the simple support and the centerline of the hole is in equilibrium with an internal bending moment caused by the reaction force, P.

The moment, M, causes basic bending stresses, σ, at the centerline of the opening

$$\sigma = \pm \frac{M\, y_n}{I_n} \qquad (1)$$

where M = Pu, I_n = moment of inertia of the net beam about its centroidal axis, and y_n = the transverse distance from the centroid of the net beam. Thus, M is assumed to be carried by the reduced section at the hole acting according to elementary beam theory.

The shear forces, $V_{T \text{ or } B}$, cause secondary bending stresses,

$$\sigma_{x_2} = \pm \frac{V_{T \text{ or } B}\, x\, y_{T \text{ or } B}}{I_{T \text{ or } B}} \qquad (2)$$

that arise when the T-sections at the hole are treated as separate beams with a point of contraflexure at the midlength of the hole. $I_{T \text{ or } B}$ = the moment of inertia of the top or bottom section about its centroid, $y_{T \text{ or } B}$ = the transverse distance from the centroid of the top or bottom section, and x = the

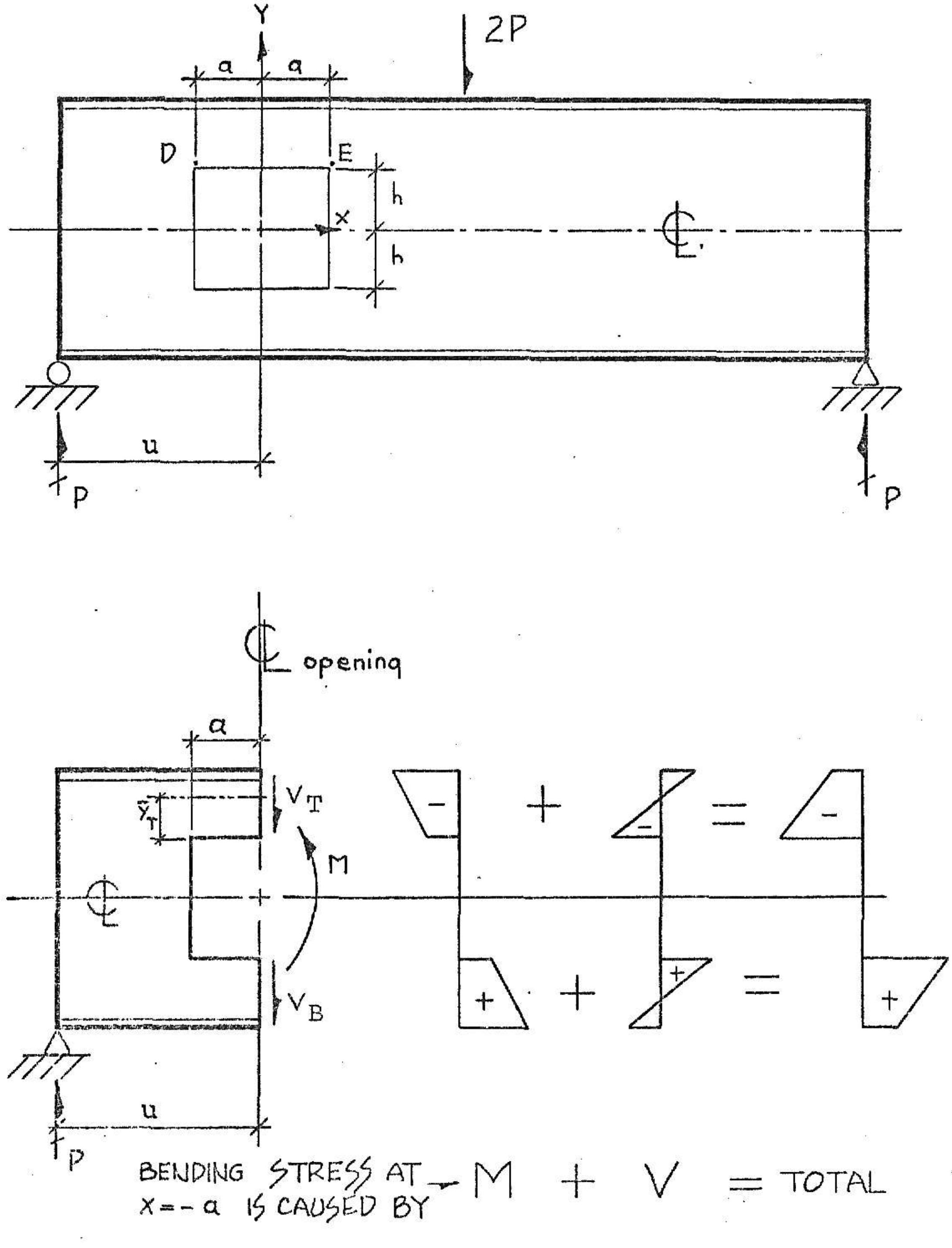


Fig. 8 - Vierendeel Analysis

longitudinal distance from the center of the hole to the section, positive to the right. For a hole symmetric with respect to the axis of the symmetric beam as shown in Fig. 8

$$V_{T \text{ or } B} = \frac{P}{2} \quad (3)$$

Equations 1 and 2 are added to obtain the total bending stress at any point through the opening. At point D in Fig. 8, the total bending stress (tensile stress being positive) is

$$\sigma_x = \pm \frac{V_T a \bar{y}_T}{I_T} \pm \frac{M y_n}{I_n} \quad (4)$$

At point E in Fig. 8, the total bending stress is

$$\sigma_x = \pm \frac{V_T a \bar{y}_T}{I_T} \pm \frac{M y_n}{I_n} \quad (5)$$

For a beam with an eccentric opening, see Fig. 9, the tee sections above and below the hole have different geometric properties, and modifications of equations 4 and 5 are required.

The total bending stress, σ_x, at any section in the tee sections is then

$$\sigma_{x \text{ top tee}} = \pm \frac{V_T \times y_T}{I_T} \pm \frac{M y_n}{I_n} \quad (6)$$

or

$$\sigma_{x \text{ bottom tee}} = \pm \frac{V_B \times y_B}{I_B} \pm \frac{M y_n}{I_n} \quad (7)$$

where V_T and V_B are the portions of the shear distributed to the top and bottom T-sections, respectively; I_T and I_B are the moments of inertia about the centroidal axes of the top and bottom tee sections, respectively; y_T and y_B are the transverse distances

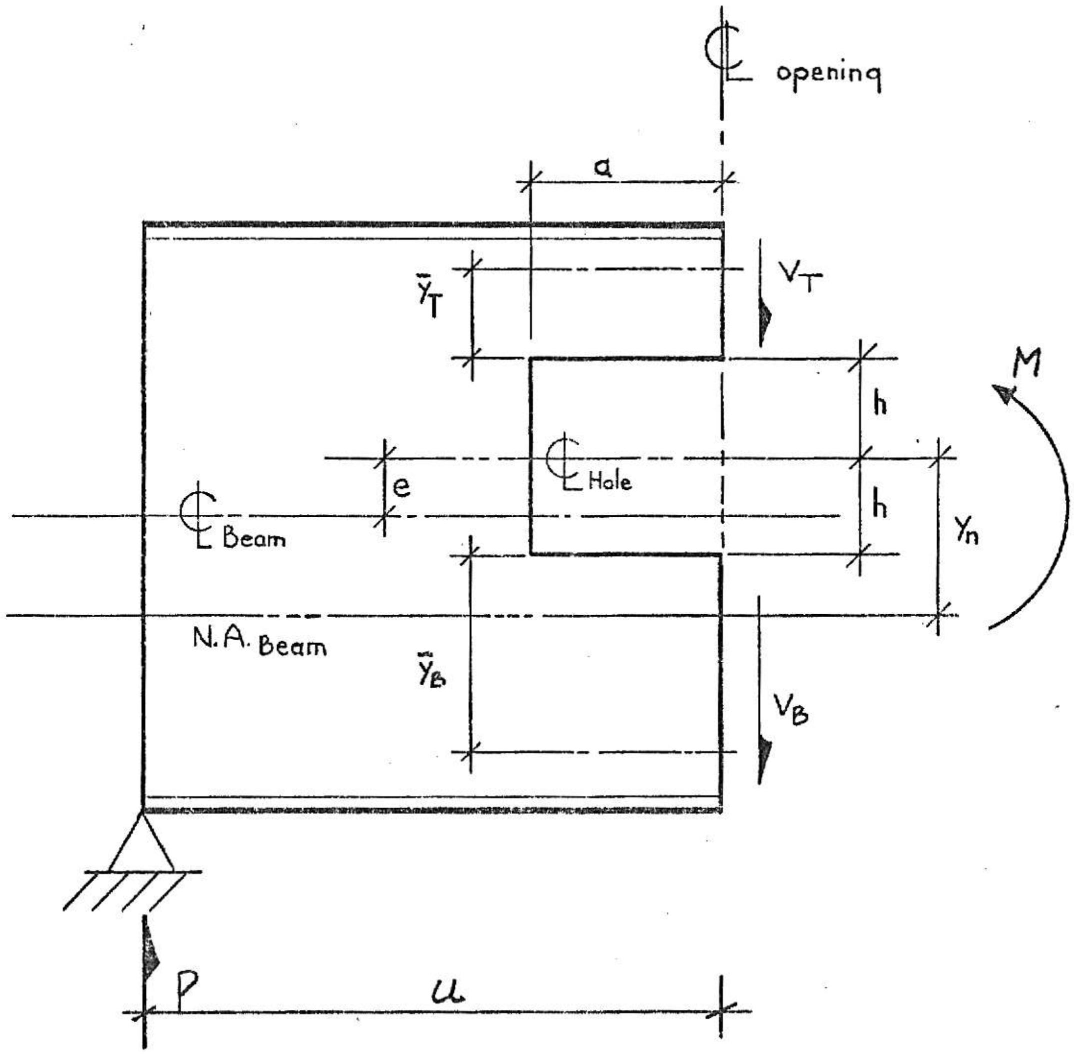


Fig. 9 - Vierendeel Analysis as Applied to an Eccentric Opening

in the top and bottom tees from the controidal axes of the T-sections; I_n is the moment of inertia about the centroidal axis of the net beam; y_n is the transverse distance from this latter axis; and x is the longitudinal distance from the center of the hole to the section, positive to the right.

Discussion of Results

Table 1 and Table 2 show the ratios of V_T to V_B which were obtained from this investigation, as well as six other ratios proposed by various authors and described in Appendix A.

The unreinforced opening cases were studied first. The values of V_T/V_B for an eccentricity of 1 in. and 2 in. were investigated for M/V ratios of 0 in., 40 in. and 80 in. The ratio of V_T/V_B was observed to be reasonably constant over the range of M/V ratios investigated; therefore, in the interest of saving time and computer costs, only one M/V ratio was investigated for an eccentricity of 3 in. To get an indication of whether or not the V_T/V_B ratio remained approximately constant for cases with reinforcing, all three M/V ratios were investigated for an eccentricity of 2 in. Based on those calculations, it again appeared that the V_T/V_B ratio was not sensitive to variations in M/V. Therefore, only one M/V ratio was investigated for eccentricities of 1 in. and 3 in. for the reinforced cases. The ratio of V_T/V_B is an indication of the amount of the total shear force that is distributed above and below the hole. If an equal amount of shear force is carried by the web tee above the opening and by the web tee below,

Without Reinforcing		Ratio of V_T/V_B						
e"	M/V"	Finite Element Method	Calculated Values (Based on Formulas From Previous Investigations*)					
			1	2	3	4	5	6
1	0	0.54	0.26	0.58	0.66	0.78	0.76	0.42
	40	0.54	0.29	0.58	0.65	0.78	0.76	0.44
	80	0.53	0.29	0.58	0.65	0.78	0.76	0.44
2	0	0.27	0.06	0.30	0.41	0.60	0.57	0.17
	40	0.26	0.07	0.30	0.41	0.60	0.57	0.17
	80	0.26	0.07	0.30	0.41	0.60	0.57	0.17
3	0							
	40	0.09	0.01	0.12	0.23	0.46	0.42	0.04
	80							

*See Appendix A

Table 1 - Ratio of V_T/V_B for a W16 x 40 Steel Beam With a 12.8" x 6.5" Eccentric Unreinforced Rectangular Opening

With Reinforcing		Ratio of V_T/V_B						
e"	M/V"	Finite Element Method	Calculated Values (Based on Formulas From Previous Investigations*)					
			1	2	3	4	5	6
1	0							
	40	0.57	0.32	0.52	0.65	0.78	0.76	0.52
	80							
2	0	0.30	0.09	0.23	0.41	0.60	0.57	0.23
	40	0.30	0.09	0.23	0.41	0.60	0.57	0.23
	80	0.30	0.09	0.23	0.41	0.60	0.57	0.23
3	0							
	40	0.14	0.03	0.11	0.23	0.46	0.42	0.09
	80							

*See Appendix A

Table 2 - Ratio of V_T/V_B for a W16 x 40 Steel Beam With a 12.8" x 6.5" Eccentric Reinforced Rectangular Opening

such as in the case of a concentric opening, the ratio V_T/V_B would be 1.0.

All of the methods used give results for the ratio of less than 1.0, therefore indicating that the smaller web tee above an eccentric opening carries less shear force than the larger web tee below the opening. The differences between the theories lie in predicting how much of a difference in shear force will result.

Method 2 is the only method that gives a reasonable comparison to the results of this study for the unreinforced opening. The comparisons of the results from Method 2 with the results of this investigation for the unreinforced case were within approximately 7 per cent for an eccentricity of 1 in., within 15 per cent for an eccentricity of 2 in., and within 33 per cent for an eccentricity of 3 in. With the addition of reinforcement, the results from Method 2 resulted in a difference of approximately 9 per cent for eccentricity of 1 in., approximately 23 per cent for an eccentricity of 2 in., and a difference of approximately 21 per cent for an eccentricity of 3 in. when compared to the results of this investigation.

This comparison in percentages may be somewhat misleading, since the values being compared for the larger eccentricities are relatively small. As can be seen from Table 1, the actual differences between the results from Method 2 and the experimental investigation for the unreinforced opening were small and relatively constant. For the reinforced opening the shear ratio increased by the finite element method and decreased by Method 2. Method 2, as given in Appendix A, is a relatively cumbersome method for determining the shear force ratio, and is particularly so for the reinforced case.

While Method 6 doesn't yield results as close to the results of this investigation as Method 2 for unreinforced openings it does show an increase in the shear ratio when the opening is reinforced and thus compares favorably in the reinforced case. Method 6 is a reasonably straightforward approach to obtaining the shear force ratio; however, the results from that method for unreinforced cases vary from those obtained in this investigation by 19 per cent for an eccentricity of 1 in., by 42 per cent for an eccentricity of 2 in., and by 55 per cent for an eccentricity of 3 in. For the reinforced cases, the variations were from 9 per cent for an eccentricity of 1 in., from 23 per cent for an eccentricity of 2 in., and from 36 per cent for an eccentricity of 3 in. Again, these percentages may be somewhat misleading for the larger eccentricities because of the relatively small values being compared. (See Table 1.)

All the other methods varied from extremes of 92 per cent below to 300 per cent above the values from this investigation.

When reinforcement was added, the V_T/V_B ratio, calculated by the finite element method, increased in each case. This indicated that when reinforcement was present, the per cent shear force carried by the top web tee was increased, while the per cent shear force carried by the bottom web tee was decreased. When reinforcement is added above and below the hole, the web tee's become essentially I shapes, and the shear stress distribution changes from the shape associated with a tee section to that associated with an I section. This redistribution would not necessarily change the total shear force distribution however. The change in shear force distribution indicated by the finite element method was possibly caused by the added stiffness of the top tee relative to the bottom tee which resulted when reinforcement was added. The stiffness of the section above and below the opening is measured by its moment of inertia. An indication of the relative stiffness of the section above the hole to the stiffness of the section below the hole is given by the ratio of the moment of inertia of the section above and below the hole. This ratio of moment of inertias increased each time reinforcement was added to the opening. For an eccentricity of 1 in., the ratio increased from .287 to .323, for an eccentricity of 2 in. the increase was from .059 to .089, and for an eccentricity of 3 in., the increase was from .0071 to .0253. This increase indicates that when reinforcement is added to the section, the stiffness of the top section increases in a greater proportion than the stiffness of the bottom section.

CONCLUSIONS

From this experimental investigation, the following conclusions can be reached:

1. The distribution of shear force above and below the hole did not appear to be very sensitive to the variation of the moment to shear ratio at the center of the hole.
2. The greatest effect on the distribution of shear force above and below the hole resulted from varying the eccentricity of the hole. The eccentricity as investigated was always above the beam centerline and, as the eccentricity increased, the shear force in the web above the hole decreased and the shear force in the web below the hole increased.
3. When reinforcement was added above and below the hole, the shear force became more concentrated between the reinforcement and the flanges and the percentage of the shear force carried by the section above the hole, as determined by the finite element method, increased.
4. For the unreinforced case Method 2 gave results which compared reasonably well with the finite element results. For the reinforced case Method 2 and Method 6 gave nearly identical results. However, Method 6 was relatively easy to apply while Method 2 was quite cumbersome and tedius to apply.

SUGGESTIONS FOR FURTHER RESEARCH

Additional research should be carried out using a larger range of parameters. Only one beam size and three moment to shear ratios were used in this investigation. However, in order to cover the practical range of the parameters involved, several different beam sizes need to be investigated as well as several different moment to shear ratios. Three eccentricities were considered in this investigation and those were all located above the beam centerline. Other eccentricities should be looked at both above and below the centerline of the beam.

One size and length of reinforcement was used for each case in this investigation. A more elaborate investigation into the reinforcement details and their effects on the shear force also needs to be considered.

REFERENCES

1. McCutcheon, J. O., So, W-C. and Gersovitz, B., "A Study of the Effects of Large Circular Openings in the Webs of Wide Flange Beams," McGill University, Applied Mechanics Series No. 2, November, 1963.

2. Heller, S. R. Jr., Brock, J. S. and Bart, R., "The Stresses Around a Rectangular Opening with Rounded Corners in a Beam Subjected to Bending with Shear," Proceedings, Vol. 1, Fourth United States National Congress on Applied Mechanics, Berkeley, California, 1962.

3. Segner, E. P. Jr., "Reinforcement Requirements for Girder Web Openings," Journal of the Structural Division, American Society of Civil Engineers, Vol. 90, No. ST3, Proc. Paper 3919, June, 1964.

4. Bower, J. E., "Experimental Stresses in Wide Flange Beams with Holes," Journal of the Structural Division, American Society of Civil Engineers, Vol. 92, No. ST5, Proc. Paper 4945, October, 1966.

5. Bower, J. E., "Elastic Stresses Around Holes in Wide-Flange Beams," Journal of the Structural Division, American Society of Civil Engineers, Vol. 92, No. ST2, April, 1966.

6. Redwood, R. G., "Experimental Stresses in Wide-Flange Beams With Holes," Journal of the Structural Division, American Society of Civil Engineers, ASCE, Vol. 93, No. ST1, Discussion, February, 1967.

7. Redwood, R. G. and McCutcheon, J. O., "Experimental Tests of Wide-Flange Beams with Large Unreinforced Web Openings," McGill University, Structural Mechanics Series No. 1, April, 1967.

8. Bower, J. E., "Design of Beams With Web Openings," Journal of the Structural Division, American Society of Civil Engineers, Vol. 94, No. ST3, Proc. Paper 5869, March, 1968.

9. Redwood, R. G., "Ultimate Strength Design of Beams with Multiple Openings," American Society of Civil Engineers, Annual Meeting and National Meeting on Structural Engineering, Pittsburgh, Pennsylvania, September 30-October 4, 1968.

10. Delesques, R., "Stability of the Web Posts of Castellated Beams," Translated from, Construction Metallique, No. 3, September, 1968.

11. Redwood, R. G., "The Strength of Steel Beams with Unreinforced Web Holes," Civil Engineering and Public Works Review, Vol. 64, No. 755, London, June, 1969.

12. Redwood, R. G. and McCutcheon, J. O., "Beam Tests with Unreinforced Web Openings," Journal of the Structural Division, American Society of Civil Engineers, Vol. 94, No. ST1, Proc. Paper 5706, January, 1968.

13. Redwood, R. G., "Plastic Behavior and Design of Beams with Web Openings," Proceedings of the Canadian Structural Engineering Conference, Canadian Institute of Steel Construction, Toronto, February, 1968.

14. Bower, J. E., "Ultimate Strength of Beams with Rectangular Holes," Journal of the Structural Division, American Society of Civil Engineers, Vol. 94, No. ST6, Proc. Paper 5982, June, 1968.

15. Congdon, J. G. and Redwood, R. G., "Plastic Behavior of Beams with Reinforced Holes," Journal of the Structural Division, American Society of Civil Engineers, Vol. 96, No. ST9, Proc. Paper 7561, September, 1970.

16. Cooper, P. B. and Snell, R. R., "Tests on Beams with Reinforced Web Openings," An unpublished report on research supported by AISC, March, 1972.

17. Richard, M. W., "Ultimate Strength Analysis of Beams With Eccentric Rectangular Web Openings," A Master's Thesis, Department of Civil Engineering, Kansas State University, 1971.

18. Frost, R. W., "Behavior of Steel Beams with Eccentric Web Holes," Technical Report 46.019-400 (1) Research Laboratory, United States Steel Corporation, February, 1973.

19. Congdon, J. G., "Ultimate Strength of Beams with Reinforced Rectangular Openings," A Master's Thesis, McGill University, Montreal, Quebec, Canada, July, 1969.

20. Salmon, M., Berke, L. and Sandhu, R., "An Application of the Finite Element Method to Elastic-Plastic Problems of Plane Stress," Illinois Institute of Technology Research Institute and the Air Force Flight Dynamics Laboratory, Project No. 1467, "Structural Analysis Methods," January, 1967, through December, 1968.

21. Lee, Hsian-Huan, "Plate Bending Finite Element Analysis of Beams with Web Holes," A Master's Report, Department of Civil Engineering, Kansas State University, 1972.

22. Hsu, Young-Ih, "Plane Stress Finite Element Analysis of Beams with Web Openings," A Master's Report, Department of Civil Engineering, Kansas State University, 1972.

23. Liov, Farn-Shinn, "Finite Element Method Analysis of Wide-Flange Beam with Reinforced Opening," A Master's Report, Department of Civil Engineering, Kansas State University, 1973.

24. Cowper, G. R., "The Shear Coefficient in Timoshenko's Beam Theory," Transactions, American Society of Mechanical Engineers, Series E, Vol. 33, June, 1966.

25. Bower, J. E., "Analysis and Experimental Verification of Steel Beams with Unreinforced Web Openings," Notes prepared for the University of Wisconsin Institute, "Design of Beams with Web Openings," Milwaukee, Wisconsin, January, 1970.

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APPENDIX A

REVIEW OF METHODS OF CALCULATING SHEAR DISTRIBUTION

An eccentric opening in the web of a beam creates a cross section as shown in Fig. 10a. Unequal Tee Sections cause unequal shear force distributions in the web section above and below the opening. Several methods of calculating unequal shear force distributions were presented by Bower (25).

The first method assumes that the shear forces are proportional to the bending stiffness of the tees.

$$\frac{V_T}{V_B} \stackrel{?}{=} \frac{I_T}{I_B} \quad \text{--} \quad (1)$$

where I_T is the moment of inertia of the top tee section with respect to its centroid, and I_B is the moment of inertia of the bottom tee section with respect to its centroid.

The second method relates the shear forces to the shearing stiffness of the tees.

$$\frac{V_T}{V_B} \stackrel{?}{=} \frac{A_T K_T}{A_B K_B} \quad \text{--} \quad (2)$$

where A_T is the area of the top tee section, A_B is the area of the bottom tee section, K_T is the shear shape factor for the top tee section, and K_B is the shear shape factor for the bottom tee section.

The shear shape factors, K_T and K_B, in method 2 are given (24) by the equation:

$$K_B \text{ or } K_T = \frac{13\ (1 + 4m)^2}{(15.3 + 122m + 350m^2 + 257m^3) + 3n^2m\ (11 + 15m + m^2)}$$

where $m = \dfrac{A_f}{t_w\ S_{B \text{ or } T}}$, $n = \dfrac{b_f}{S_{B \text{ or } T}}$

and A_f is the area of the flange, t_w, S_B and S_T are shown in Fig. 10a and listed in Table 3.

When reinforcement is added to the beam, the tee sections change to unsymmetrical I-sections, as shown in Fig. 10b, and the shear shape factor above does not apply. A new shear shape factor for an unsymmetrical I-shape was derived, based on unpublished calculations provided by G. R. Cowper, the author of reference (24).

$$K_I = \frac{20(1+\mu)\Delta}{5\mu[(\alpha n_1^2 + \beta n_2^2)\rho\Delta-\Delta^2] + 3\rho\gamma}$$

where

$$\gamma = \frac{1}{2\rho^3}\{(3+4\mu)(\nu^5+\omega^5) + 5(1+\mu)\ \nu^2\omega^2(\nu+\omega)$$

$$+ 20(1+\mu)(\alpha\omega+\beta\nu)(\nu^3+\omega^3)\rho\}$$

$$+ 5\alpha\omega^2[(4+5\mu)n_1^2 - \frac{(2+\mu)\omega^2}{\rho^2} + \frac{3(1+\mu)}{\rho}\{\frac{\omega^2+\nu^2}{\rho} + 4(\alpha\omega+\beta\nu)\}]$$

$$+ 5\beta\nu^2[(4+5\mu)n_2^2 - \frac{(2+\mu)\nu^2}{\rho^2} + \frac{3(1+\mu)}{\rho}\{\frac{\omega^2+\nu^2}{\rho} + 4(\alpha\omega+\beta\nu)\}]$$

and

$$\Delta = (12\alpha\beta+4\alpha+4\beta+1)$$

$$\alpha = \frac{b_1 t_1}{d_{T\text{ or }B}\, t_w},\quad \beta = \frac{b_2 t_2}{d_{T\text{ or }B}\, t_w},\quad \nu = 2\alpha+1,\quad \omega = 2\beta+1,$$

$$\rho = \alpha+\beta+1,\quad n_1 = \frac{b_1}{d_{T\text{ or }B}},\quad n_2 = \frac{b_2}{d_{T\text{ or }B}},\quad \mu = \text{Poisson's ratio}$$

and where b_1, t_1, b_2, t_2, $d_{T\text{ or }B}$ and t_w are shown in Fig. 10b-1 and listed in Table 4.

This shear shape factor was then used in Method 2 for the reinforced openings.

The third method differs from methods 1 and 2 in that it is exclusively a function of the beam web, instead of the tee section. The shear force is related to the ratio of the total depth of the tee webs.

$$\frac{V_T}{V_B} \stackrel{?}{=} \frac{(d - 2h - 2e)}{(d - 2h + 2e)} \qquad (3)$$

where d is the total depth of the beam, 2h is the depth of the opening, and e is the eccentricity of the centerline of the opening.

The fourth method is similar to method 3 and is also a function of the beam web. It relates the shear forces to the ratio of the beam depths above and below the centerline of the hole.

$$\frac{V_T}{V_B} \stackrel{?}{=} \frac{(d - 2e)}{(d + 2e)} \qquad (4)$$

where d is the total depth of the beam, and e is the eccentricity of the centerline of the opening.

Method 5 is a refined version of method 4 that takes into account the effect of the beam flanges in altering the shear stress distribution in a normal wide-flange beam.

$$\frac{V_T}{V_B} \stackrel{?}{=} \frac{12\ C^{*2}\ (d - 2e) - [d^3 - (2e)^3]}{12\ C^{*2}\ (d - 2e) - [d^3 + (2e)^3]} \qquad (5)$$

where $C^{*2} = \left(\frac{d}{2}\right)^2 [1 + \frac{4\ b_f t_f}{t_w\ d}]$

and d, t_f, b_f, t_w, and e are shown in Fig. 10a.

Method 6 presented by Frost (18) indicates that the shear is distributed between the two section according to their relative stiffnesses.

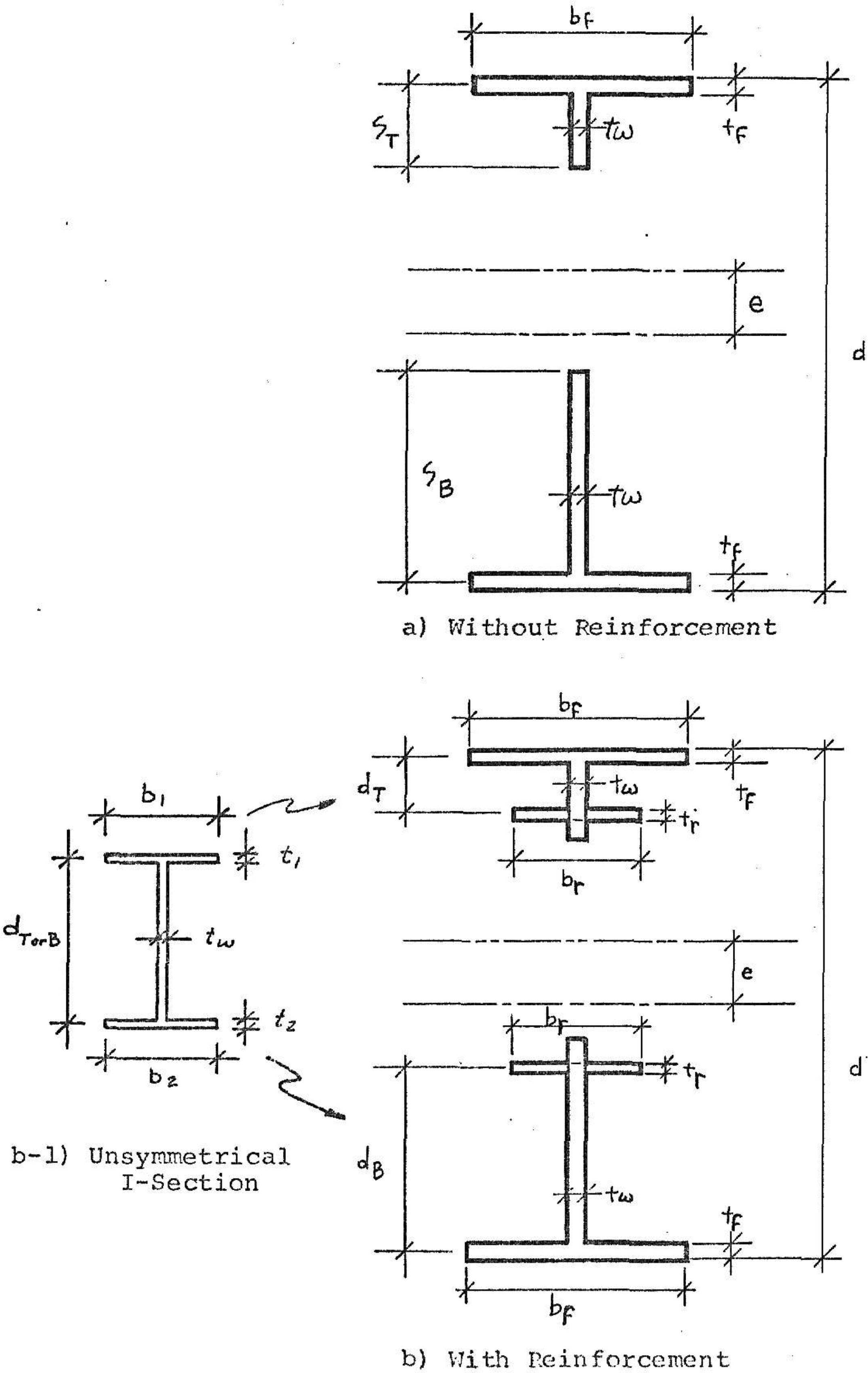


Fig. 10 - Beam Sections With Eccentric Openings

$$\frac{V_T}{V_B} \stackrel{?}{=} \frac{\dfrac{a^2}{3\ E\ I_B} + \dfrac{1.2}{A_{wB}G}}{\dfrac{a^2}{3\ E\ I_T} + \dfrac{1.2}{A_{wT}G}} \quad \text{------------------------------} \quad (6)$$

where a is the half length of the hole, A_w is the web area, I is the moment of inertia of the tee with respect to its centroid, E is the elastic modulus, and G is the shear modulus. The subscripts T and B refer to the top and bottom tee sections, respectively.

e	M/V	b_f	t_f	t_w	S_T	S_B	d
1	0	6.90	0.49	0.31	3.55	5.55	16.0
	40	6.94	0.48	0.31	3.51	5.51	16.0
	80	6.94	0.48	0.31	3.51	5.51	16.0
2	0	6.90	0.49	0.31	2.55	6.55	16.0
	40	6.94	0.48	0.31	2.51	6.51	16.0
	80	6.94	0.48	0.31	2.51	6.51	16.0
3	0						
	40	6.94	0.48	0.31	1.51	7.51	16.0
	80						

Table 3 - Dimensions Used in the Calculation of the Values of the Shear Shape Factor of the Unreinforced Beam (inches)

e	M/V	b_f	t_f	t_w	d_T	d_B	b_r	t_r	d
1	0								
	40	6.94	0.48	0.31	2.76	4.76	4.31	0.25	16.0
	80								
2	0	6.90	0.49	0.31	1.76	5.76	4.31	0.25	16.0
	40	6.94	0.48	0.31	1.76	5.76	4.31	0.25	16.0
	80	6.94	0.48	0.31	1.76	5.76	4.31	0.25	16.0
3	0								
	40	6.94	0.48	0.31	1.26	7.26	4.31	0.25	16.0
	80								

Table 4 - Dimensions Used in the Calculation of the Values of the Shear Shape Factor of the Reinforced Beam (inches)

APPENDIX B

PLOTS OF THE SHEAR DISTRIBUTION FOR THE VARIOUS CASES INVESTIGATED

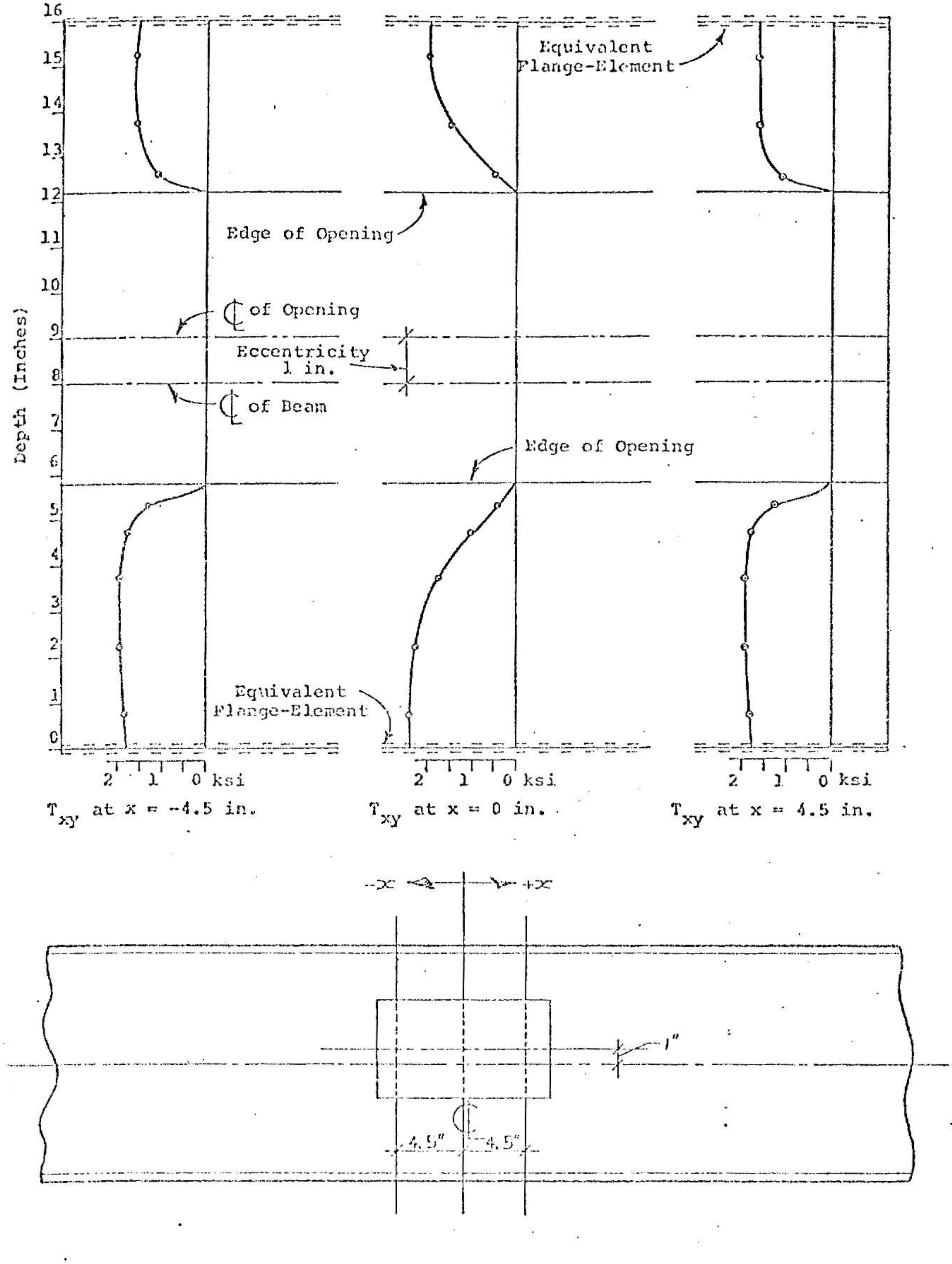


Fig. 11 - Shear Distribution for Beam With Unreinforced Hole Where e = 1 in. and M/V = 0 in.

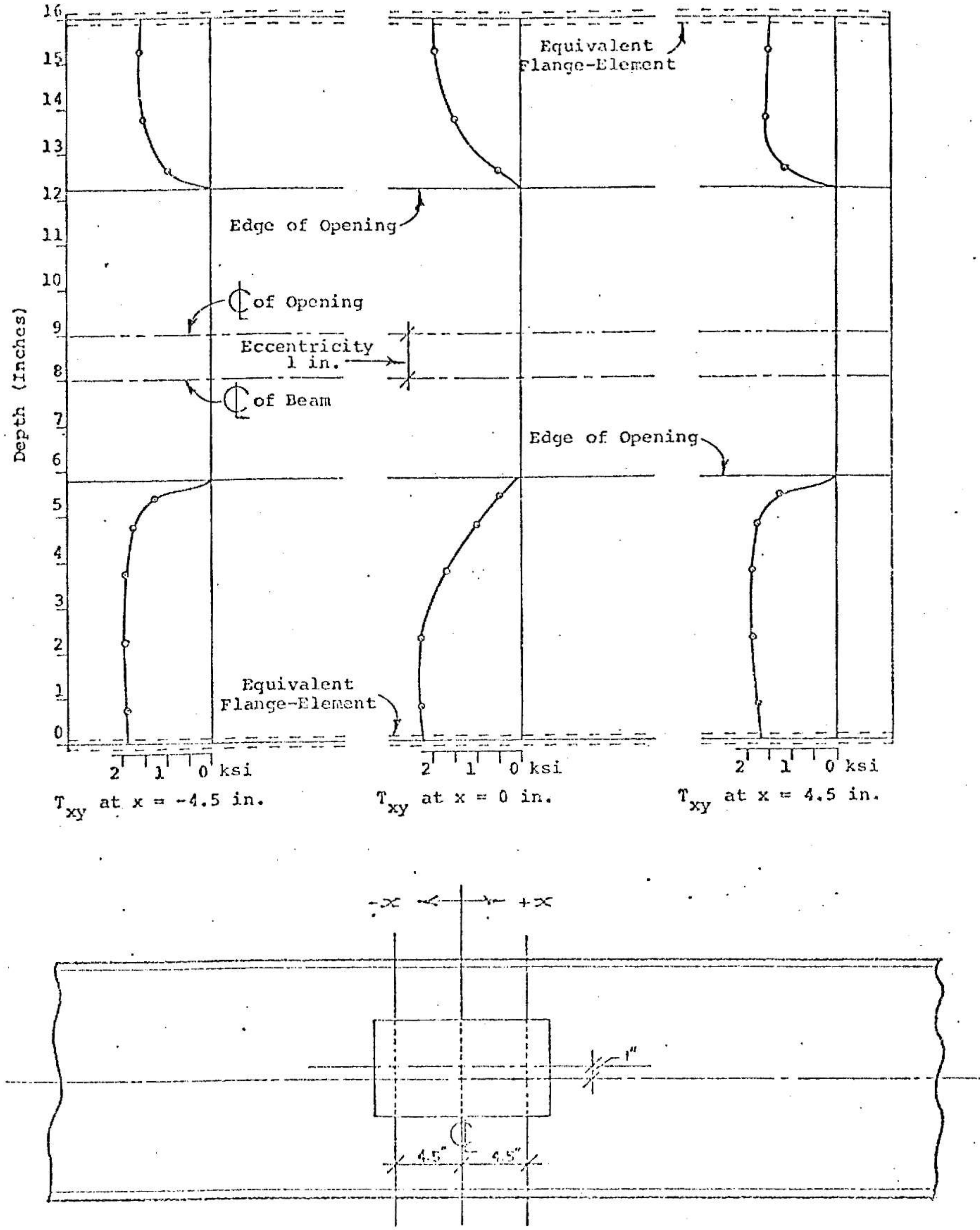
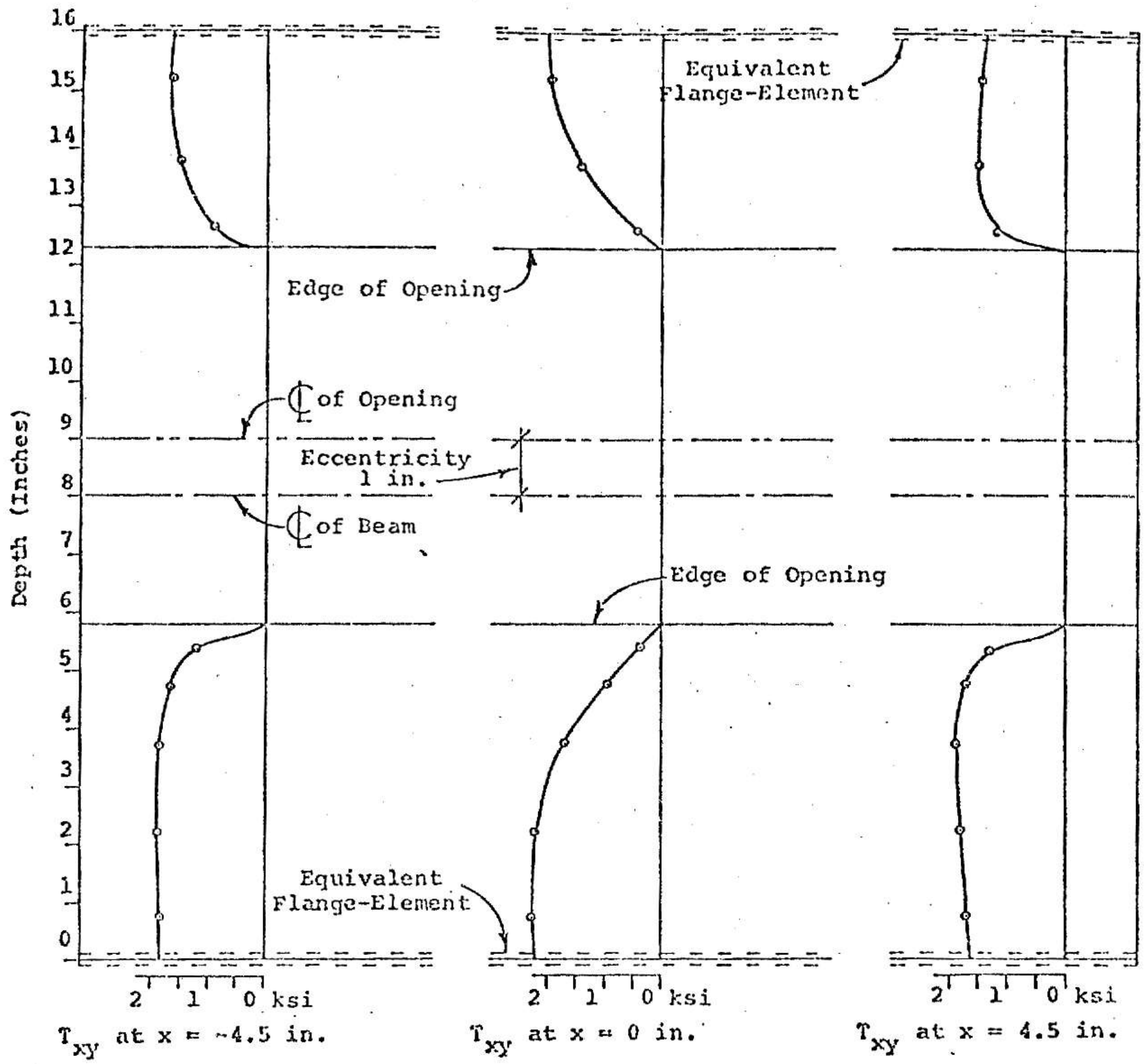


Fig. 12 - Shear Distribution for Beam With Unreinforced Hole Where e = 1 in. and M/V = 40 in.



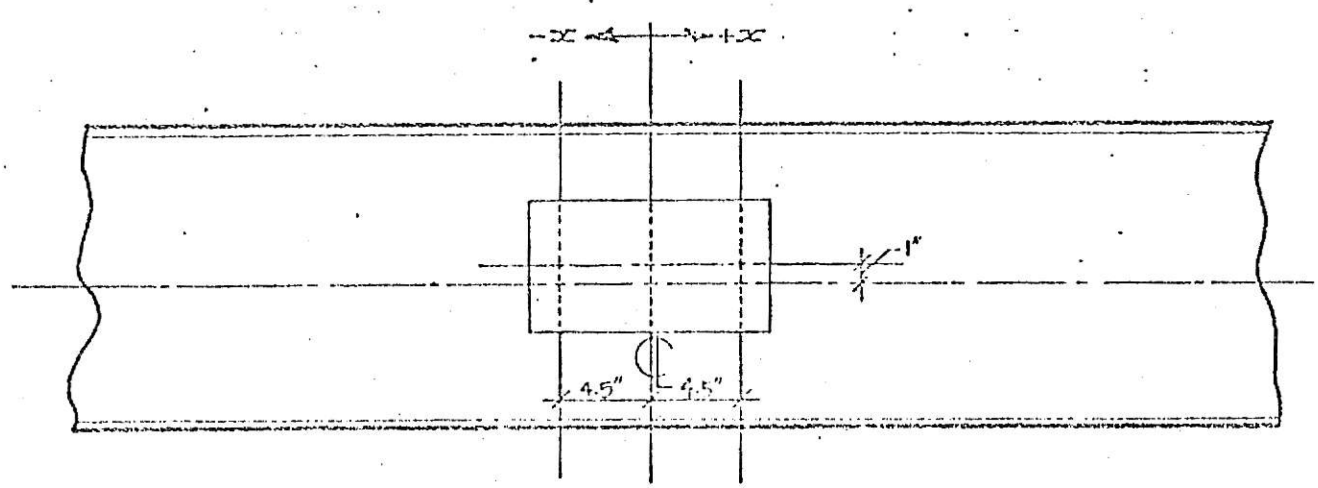


Fig. 13 - Shear Distribution for Beam With Unreinforced Hole Where e = 1 in. and M/V = 80 in.

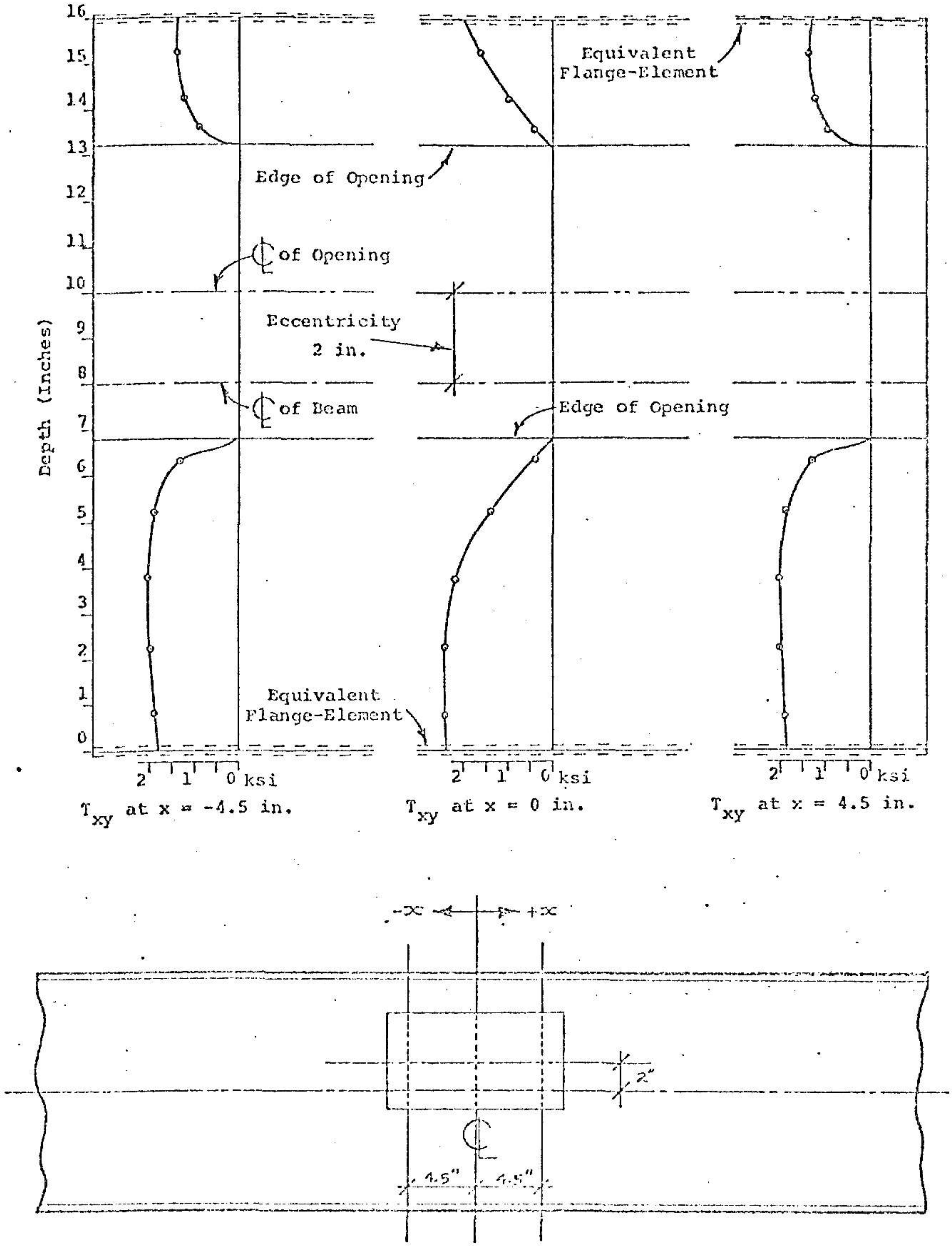


Fig. 14 - Shear Distribution for Beam With Unreinforced Hole Where e = 2 in. and M/V = 0 in.

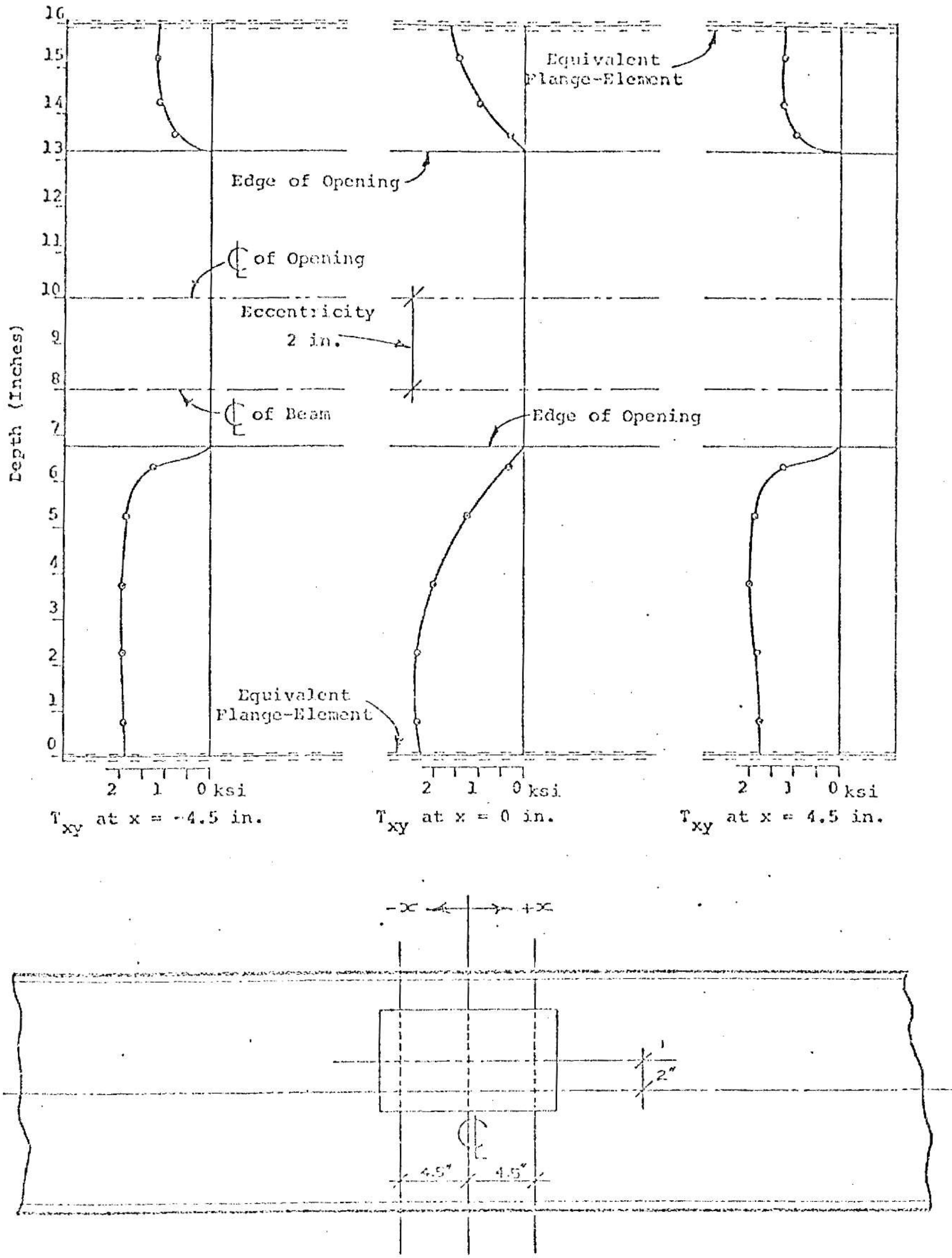
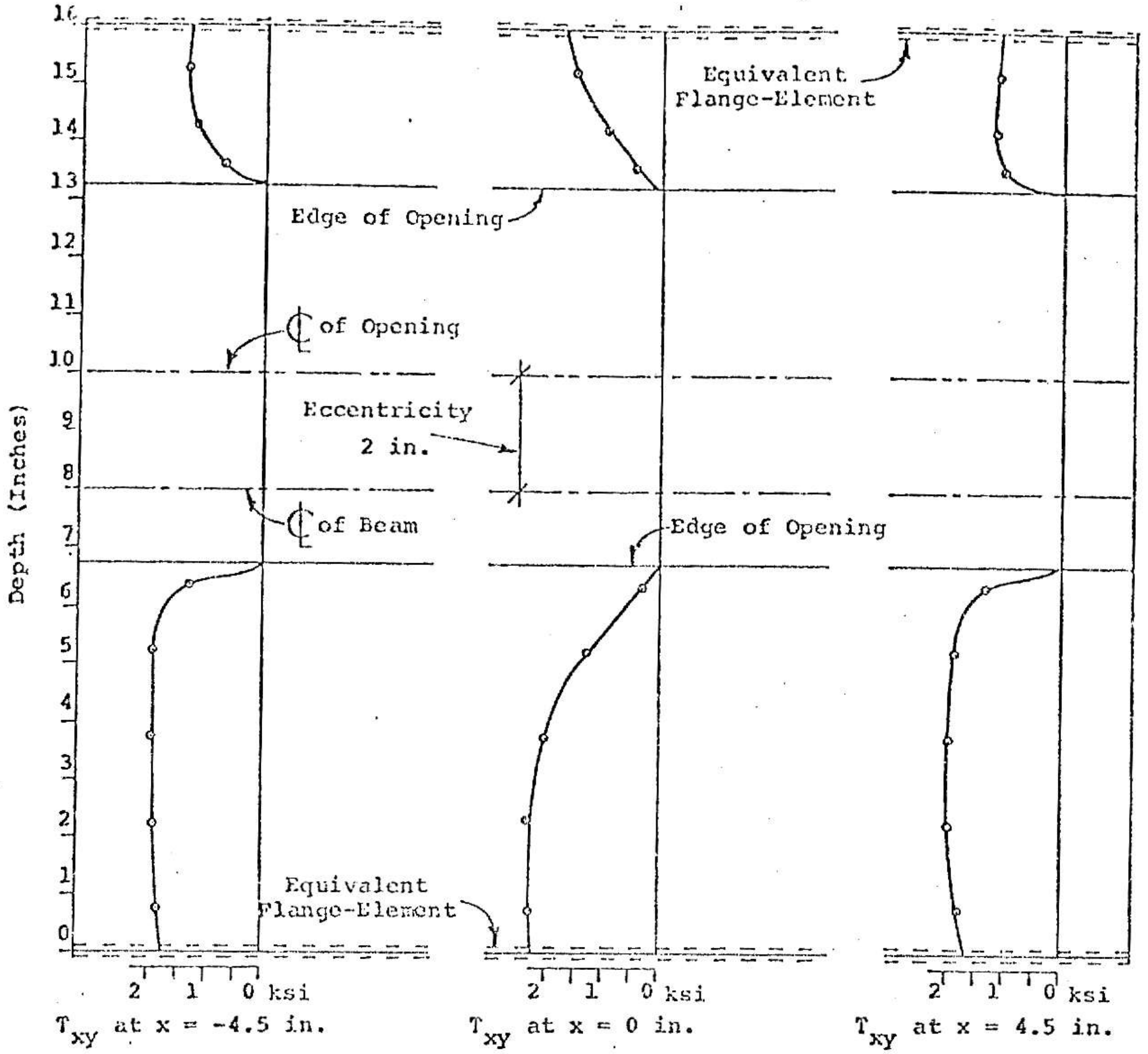


Fig. 15 - Shear Distribution for Beam With Unreinforced Hole Where e = 2 in. and M/V = 40 in.



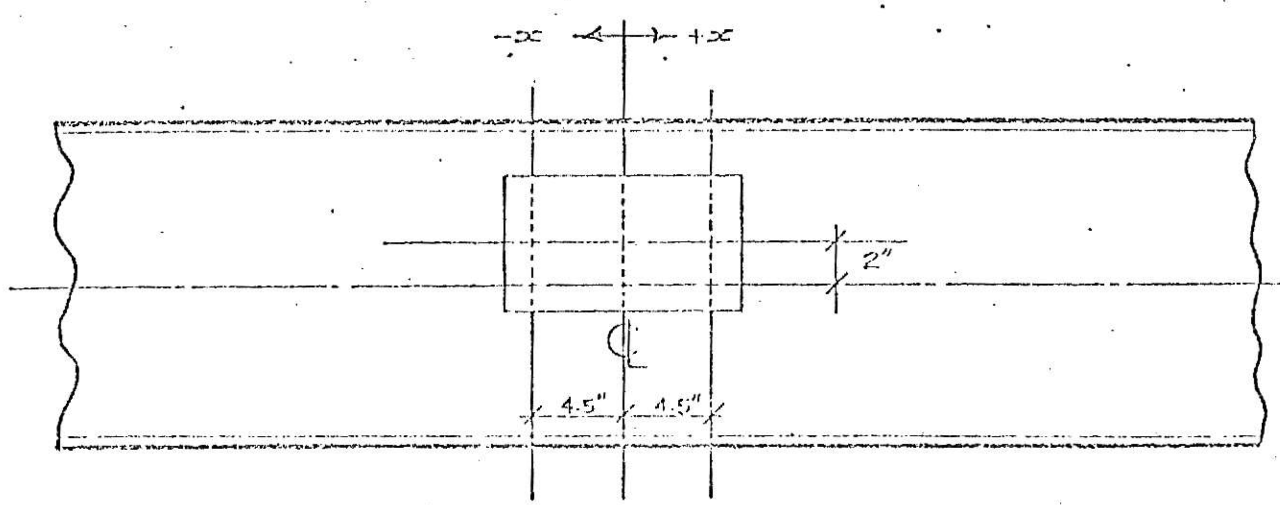


Fig. 16 - Shear Distribution for Beam With Unreinforced Hole Where e = 2 in. and M/V = 80 in.

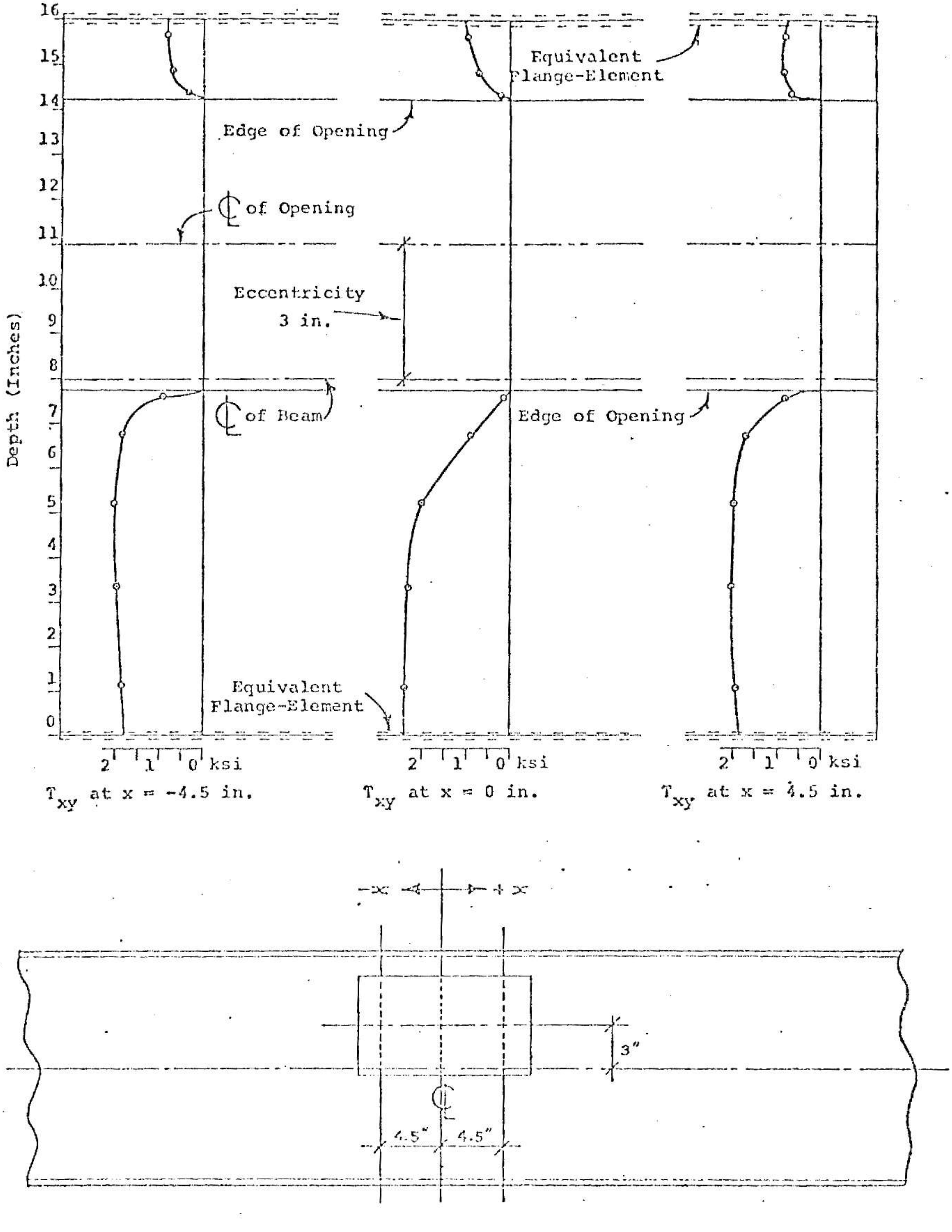


Fig. 17 - Shear Distribution for Beam With Unreinforced Hole Where e = 3 in. and M/V = 40 in.

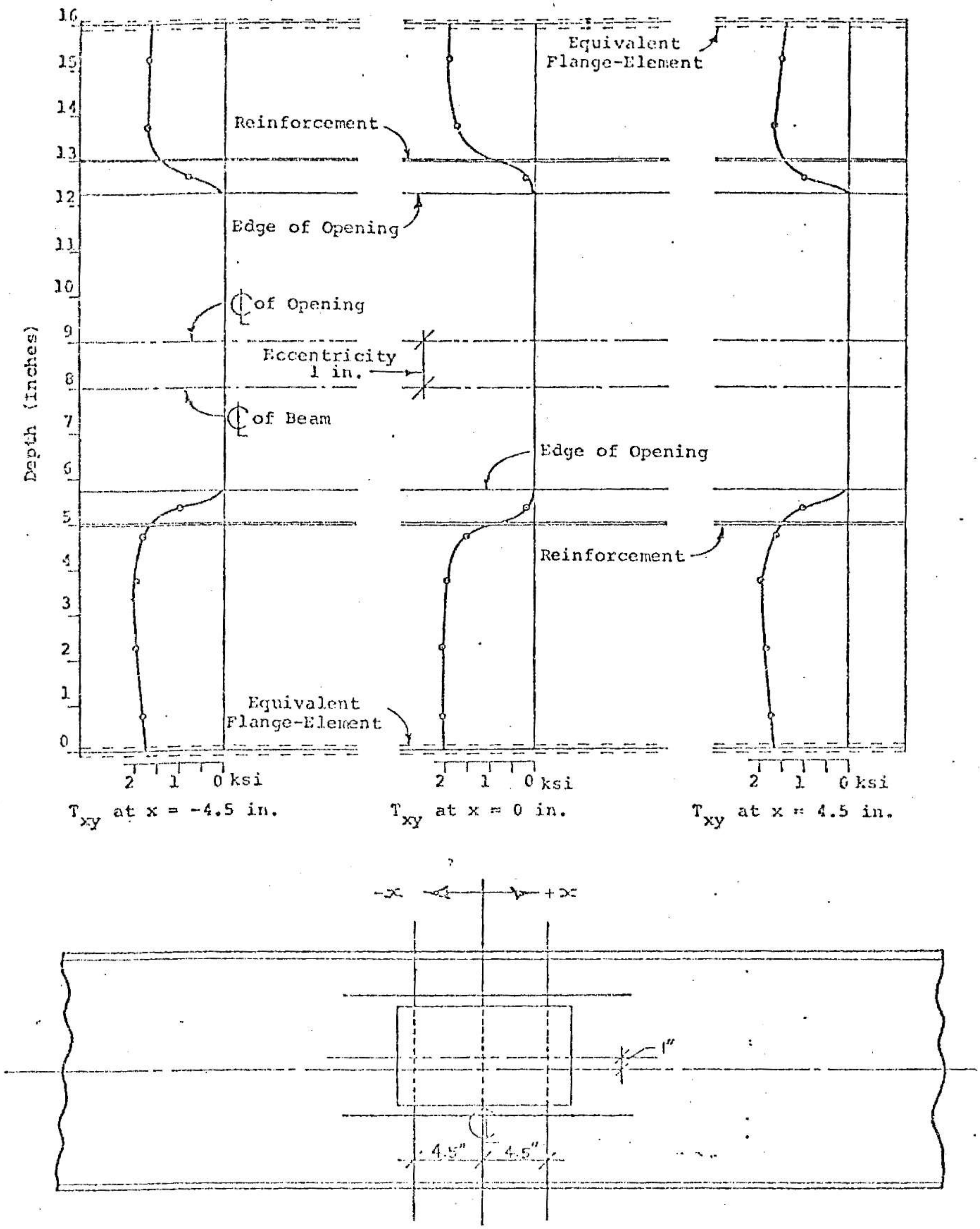
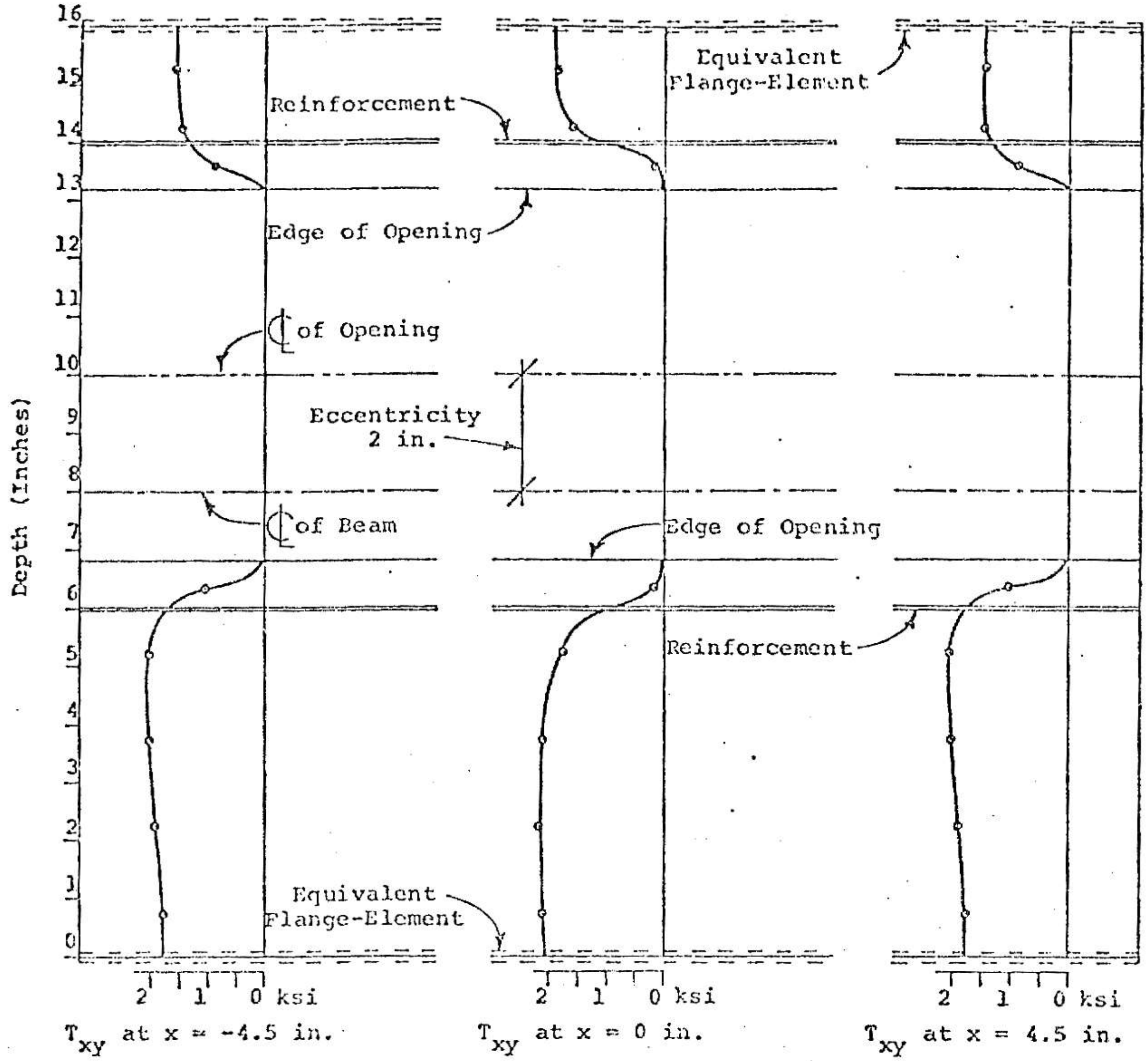


Fig. 18 - Shear Distribution for Beam With Reinforced Hole Where e = 1 in. and M/V = 40 in.



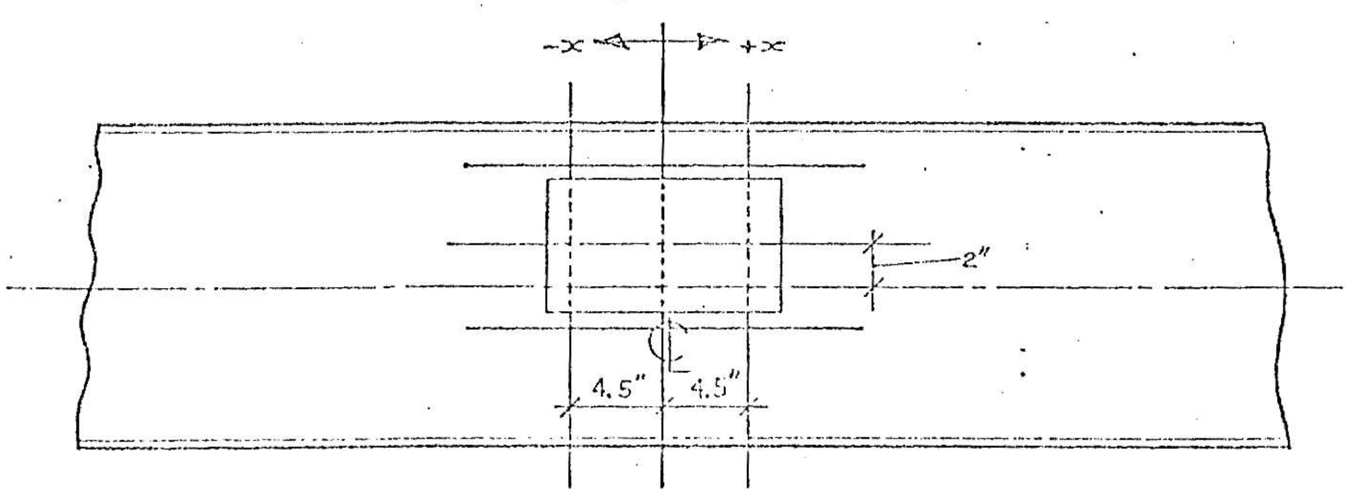


Fig. 19 - Shear Distribution for Beam With Reinforced Hole Where e = 2 in. and M/V = 0 in.

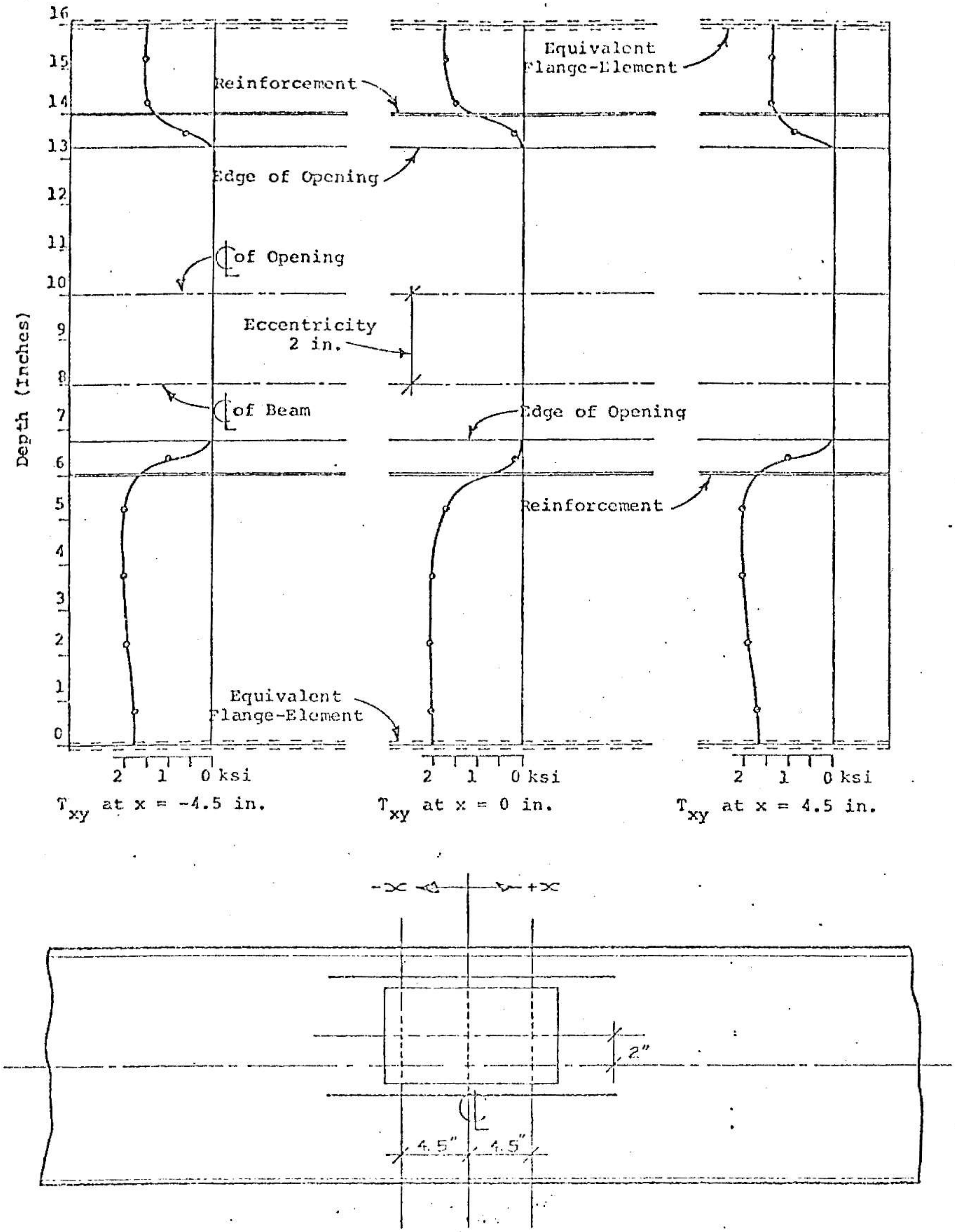


Fig. 20 - Shear Distribution for Beam With Reinforced Hole Where e = 2 in. and M/V = 40 in.

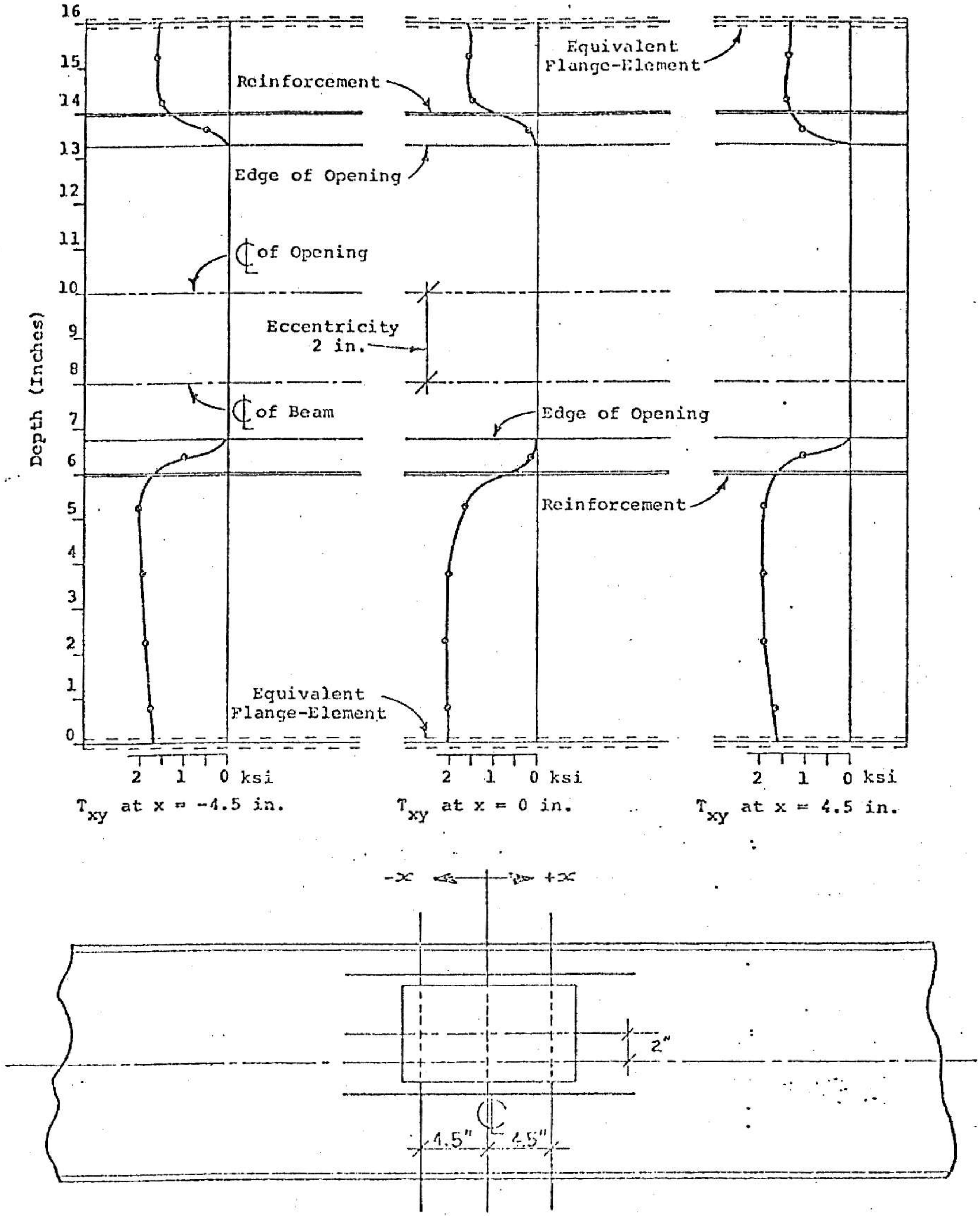


Fig. 21 - Shear Distribution for Beam With Reinforced Hole Where e = 2 in. and M/V = 80 in.

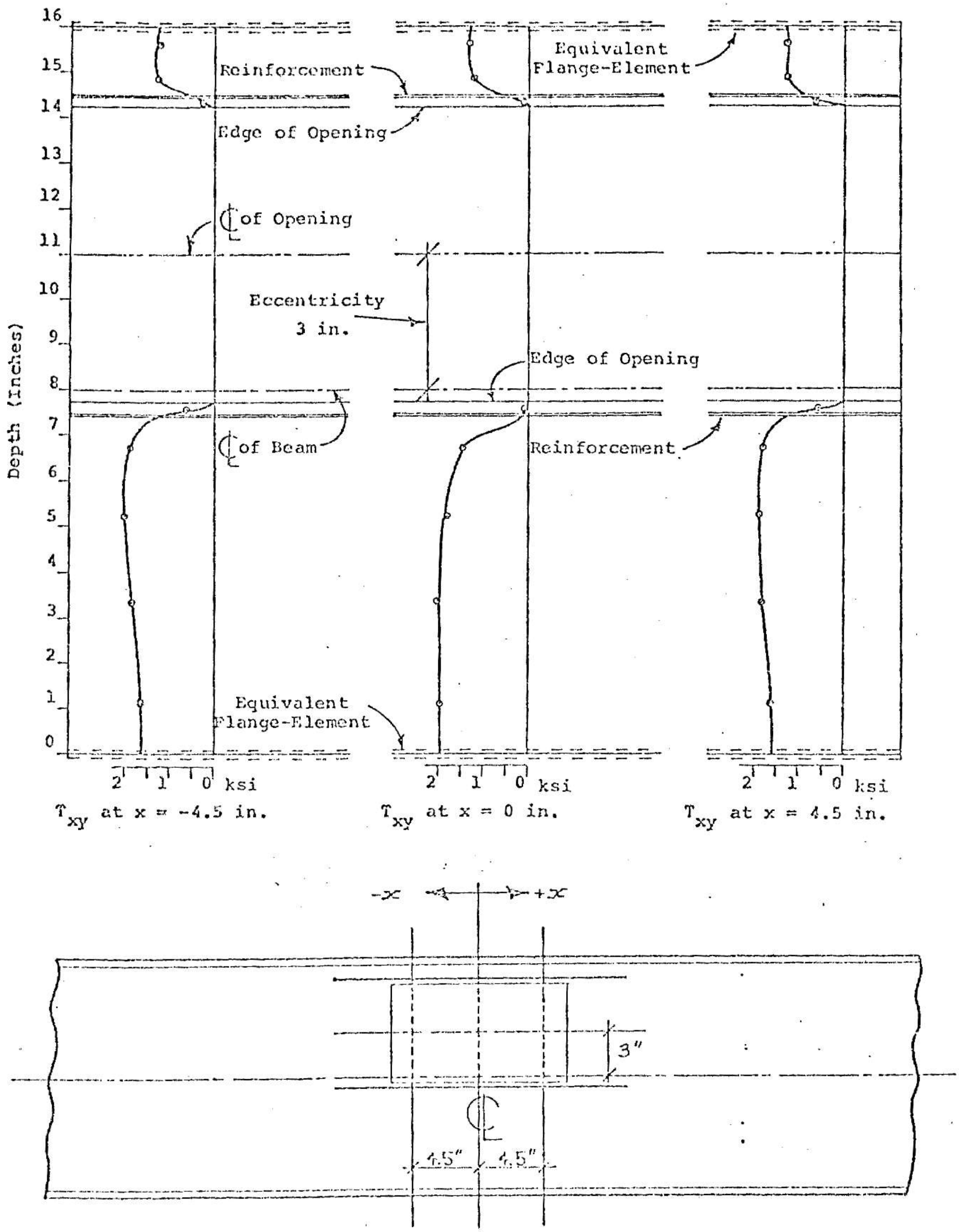


Fig. 22 - Shear Distribution for Beam With Reinforced Hole Where e = 3 in. and M/V = 40 in.

APPENDIX C

TYPICAL COMPUTER INPUT DATA
(W16 x 40 Beam, 12.8" x 6.5" unreinforced opening, M/V = 0", e = 2")

FST BEAM WITH CUTOUT - M/V = 0 INCHES

BILINEAR LAW
MATERIAL---------------------- 1
MODULUS OF ELASTICITY-------- 0.2900E 05
SECANT YIELD STRESS---------- 0.3600E 02
SHAPE PARAMETER-------------- 0.0
POISSON'S RATIO-------------- 0.3000
ERROR TOLERANCE-------------- 0.0200

BILINEAR LAW
MATERIAL---------------------- 2
MODULUS OF ELASTICITY-------- 0.2900E 05
SECANT YIELD STRESS---------- 0.3600E 02
SHAPE PARAMETER-------------- 0.0
POISSON'S RATIO-------------- 0.3000
ERROR TOLERANCE-------------- 0.0200

BILINEAR LAW
MATERIAL---------------------- 3
MODULUS OF ELASTICITY-------- 0.2900E 05
SECANT YIELD STRESS---------- 0.3600E 02
SHAPE PARAMETER-------------- 0.0
POISSON'S RATIO-------------- 0.3000
ERROR TOLERANCE-------------- 0.0200

NO. OF NODES NNODE = 284
NO. OF ELEMENTS NELEM = 536
NO. OF STEPS NDIV = 1
NO. OF ITERATIONS/STEP NIT = 30

BOUNDARY CONDITION ARRAY

NODAL PT	X CODE	X VALUE	Y CODE	Y VALUE	SLIDING CODE	SLIDING VALUE
1	0	0.0	1	0.0	0	0.0
147	1	0.0	0	0.0	0	0.0
282	0	0.0	1	0.0	0	0.0

NODAL PT	X-COORD	X-FORCE	Y-COORD	Y-FORCE	CODE
1	0.0	-26.2500	0.0	0.0	10
2	0.0	0.0	8.0000	0.0	0
3	0.0	26.2500	16.0000	0.0	0
4	6.0000	0.0	0.0	0.0	0
5	6.0000	0.0	8.0000	0.0	0
6	6.0000	0.0	16.0000	0.0	0
7	12.0000	0.0	0.0	0.0	0
8	12.0000	0.0	8.0000	0.0	0
9	12.0000	0.0	16.0000	0.0	0
10	18.0000	0.0	0.0	0.0	0
11	18.0000	0.0	8.0000	0.0	0
12	18.0000	0.0	16.0000	0.0	0
13	22.0000	0.0	0.0	0.0	0
14	22.0000	0.0	8.0000	0.0	0
15	22.0000	0.0	16.0000	0.0	0
16	24.0000	0.0	0.0	0.0	0
17	24.0000	0.0	3.0000	0.0	0
18	24.0000	0.0	8.0000	0.0	0

19	24.0000	0.0	14.0000	0.0	0
20	24.0000	0.0	16.0000	0.0	0
21	26.0000	0.0	0.0	0.0	0
22	26.0000	0.0	3.0000	0.0	0
23	26.0000	0.0	8.0000	0.0	0
24	26.0000	0.0	14.0000	0.0	0
25	26.0000	0.0	16.0000	0.0	0
26	28.0000	0.0	0.0	0.0	0
27	28.0000	0.0	3.0000	0.0	0
28	28.0000	0.0	8.5000	0.0	0
29	28.0000	0.0	10.0000	0.0	0
30	28.0000	0.0	14.0000	0.0	0
31	28.0000	0.0	16.0000	0.0	0
32	29.5000	0.0	0.0	0.0	0
33	29.5000	0.0	3.0000	0.0	0
34	29.5000	0.0	6.0000	0.0	0
35	29.5000	0.0	8.5000	0.0	0
36	29.5000	0.0	10.0000	0.0	0
37	29.5000	0.0	11.5000	0.0	0
38	29.5000	0.0	14.0000	0.0	0
39	29.5000	0.0	16.0000	0.0	0
40	31.0000	0.0	0.0	0.0	0
41	31.0000	0.0	1.5000	0.0	0
42	31.0000	0.0	3.0000	0.0	0
43	31.0000	0.0	6.0000	0.0	0
44	31.0000	0.0	8.5000	0.0	0
45	31.0000	0.0	10.0000	0.0	0
46	31.0000	0.0	11.5000	0.0	0
47	31.0000	0.0	14.0000	0.0	0
48	31.0000	0.0	14.5000	0.0	0
49	31.0000	0.0	16.0000	0.0	0
50	32.5000	0.0	0.0	0.0	0
51	32.5000	0.0	1.5000	0.0	0
52	32.5000	0.0	3.0000	0.0	0
53	32.5000	0.0	4.5000	0.0	0
54	32.5000	0.0	6.0000	0.0	0
55	32.5000	0.0	7.3000	0.0	0
56	32.5000	0.0	8.5000	0.0	0
57	32.5000	0.0	10.0000	0.0	0
58	32.5000	0.0	11.5000	0.0	0
59	32.5000	0.0	12.7000	0.0	0
60	32.5000	0.0	14.0000	0.0	0
61	32.5000	0.0	14.5000	0.0	0
62	32.5000	0.0	16.0000	0.0	0
63	34.0000	0.0	0.0	0.0	0
64	34.0000	0.0	1.5000	0.0	0
65	34.0000	0.0	3.0000	0.0	0
66	34.0000	0.0	4.5000	0.0	0
67	34.0000	0.0	6.0000	0.0	0
68	34.0000	0.0	7.3000	0.0	0
69	34.0000	0.0	8.5000	0.0	0
70	34.0000	0.0	10.0000	0.0	0
71	34.0000	0.0	11.5000	0.0	0
72	34.0000	0.0	12.7000	0.0	0
73	34.0000	0.0	14.0000	0.0	0
74	34.0000	0.0	14.5000	0.0	0
75	34.0000	0.0	16.0000	0.0	0
76	35.1000	0.0	0.0	0.0	0
77	35.1000	0.0	1.5000	0.0	0
78	35.1000	0.0	3.0000	0.0	0

79	35.1000	0.0	4.5000	0.0	0
80	35.1000	0.0	6.0000	0.0	0
81	35.1000	0.0	6.8000	0.0	0
82	35.1000	0.0	7.3000	0.0	0
83	35.1000	0.0	8.5000	0.0	0
84	35.1000	0.0	10.0000	0.0	0
85	35.1000	0.0	11.5000	0.0	0
86	35.1000	0.0	12.7000	0.0	0
87	35.1000	0.0	13.2000	0.0	0
88	35.1000	0.0	14.0000	0.0	0
89	35.1000	0.0	14.5000	0.0	0
90	35.1000	0.0	16.0000	0.0	0
91	35.6000	0.0	6.8000	0.0	0
92	35.6000	0.0	7.3000	0.0	0
93	35.6000	0.0	8.5000	0.0	0
94	35.6000	0.0	10.0000	0.0	0
95	35.6000	0.0	11.5000	0.0	0
96	35.6000	0.0	12.7000	0.0	0
97	35.6000	0.0	13.2000	0.0	0
98	36.1000	0.0	0.0	0.0	0
99	36.1000	0.0	1.5000	0.0	0
100	36.1000	0.0	3.0000	0.0	0
101	36.1000	0.0	4.5000	0.0	0
102	36.1000	0.0	6.0000	0.0	0
103	36.1000	0.0	6.8000	0.0	0
104	36.1000	0.0	13.2000	0.0	0
105	36.1000	0.0	14.0000	0.0	0
106	36.1000	0.0	14.5000	0.0	0
107	36.1000	0.0	16.0000	0.0	0
108	37.5000	0.0	0.0	0.0	0
109	37.5000	0.0	1.5000	0.0	0
110	37.5000	0.0	3.0000	0.0	0
111	37.5000	0.0	4.5000	0.0	0
112	37.5000	0.0	6.0000	0.0	0
113	37.5000	0.0	6.8000	0.0	0
114	37.5000	0.0	13.2000	0.0	0
115	37.5000	0.0	14.0000	0.0	0
116	37.5000	0.0	14.5000	0.0	0
117	37.5000	0.0	16.0000	0.0	0
118	39.0000	0.0	0.0	0.0	0
119	39.0000	0.0	1.5000	0.0	0
120	39.0000	0.0	3.0000	0.0	0
121	39.0000	0.0	4.5000	0.0	0
122	39.0000	0.0	6.0000	0.0	0
123	39.0000	0.0	6.8000	0.0	0
124	39.0000	0.0	13.2000	0.0	0
125	39.0000	0.0	14.0000	0.0	0
126	39.0000	0.0	14.5000	0.0	0
127	39.0000	0.0	16.0000	0.0	0
128	40.5000	0.0	0.0	0.0	0
129	40.5000	0.0	1.5000	0.0	0
130	40.5000	0.0	3.0000	0.0	0
131	40.5000	0.0	4.5000	0.0	0
132	40.5000	0.0	6.0000	0.0	0
133	40.5000	0.0	6.8000	0.0	0
134	40.5000	0.0	13.2000	0.0	0
135	40.5000	0.0	14.0000	0.0	0
136	40.5000	0.0	14.5000	0.0	0
137	40.5000	0.0	16.0000	0.0	0
138	42.0000	0.0	0.0	0.0	0

139	42.0000	0.0	1.5000	0.0	0
140	42.0000	0.0	3.0000	0.0	0
141	42.0000	0.0	4.5000	0.0	0
142	42.0000	0.0	6.0000	0.0	0
143	42.0000	0.0	6.8000	0.0	0
144	42.0000	0.0	13.2000	0.0	0
145	42.0000	0.0	14.0000	0.0	0
146	42.0000	0.0	14.5000	0.0	0
147	42.0000	0.0	16.0000	0.0	100
148	43.5000	0.0	0.0	0.0	0
149	43.5000	0.0	1.5000	0.0	0
150	43.5000	0.0	3.0000	0.0	0
151	43.5000	0.0	4.5000	0.0	0
152	43.5000	0.0	6.0000	0.0	0
153	43.5000	0.0	6.8000	0.0	0
154	43.5000	0.0	13.2000	0.0	0
155	43.5000	0.0	14.0000	0.0	0
156	43.5000	0.0	14.5000	0.0	0
157	43.5000	0.0	16.0000	0.0	0
158	45.0000	0.0	0.0	0.0	0
159	45.0000	0.0	1.5000	0.0	0
160	45.0000	0.0	3.0000	0.0	0
161	45.0000	0.0	4.5000	0.0	0
162	45.0000	0.0	6.0000	0.0	0
163	45.0000	0.0	6.8000	0.0	0
164	45.0000	0.0	13.2000	0.0	0
165	45.0000	0.0	14.0000	0.0	0
166	45.0000	0.0	14.5000	0.0	0
167	45.0000	0.0	16.0000	0.0	0
168	46.5000	0.0	0.0	0.0	0
169	46.5000	0.0	1.5000	0.0	0
170	46.5000	0.0	3.0000	0.0	0
171	46.5000	0.0	4.5000	0.0	0
172	46.5000	0.0	6.0000	0.0	0
173	46.5000	0.0	6.8000	0.0	0
174	46.5000	0.0	13.2000	0.0	0
175	46.5000	0.0	14.0000	0.0	0
176	46.5000	0.0	14.5000	0.0	0
177	46.5000	0.0	16.0000	0.0	0
178	47.9000	0.0	0.0	0.0	0
179	47.9000	0.0	1.5000	0.0	0
180	47.9000	0.0	3.0000	0.0	0
181	47.9000	0.0	4.5000	0.0	0
182	47.9000	0.0	6.0000	0.0	0
183	47.9000	0.0	6.8000	0.0	0
184	47.9000	0.0	13.2000	0.0	0
185	47.9000	0.0	14.0000	0.0	0
186	47.9000	0.0	14.5000	0.0	0
187	47.9000	0.0	16.0000	0.0	0
188	48.4000	0.0	6.8000	0.0	0
189	48.4000	0.0	7.3000	0.0	0
190	48.4000	0.0	8.5000	0.0	0
191	48.4000	0.0	10.0000	0.0	0
192	48.4000	0.0	11.5000	0.0	0
193	48.4000	0.0	12.7000	0.0	0
194	48.4000	0.0	13.2000	0.0	0
195	48.9000	0.0	0.0	0.0	0
196	48.9000	0.0	1.5000	0.0	0
197	48.9000	0.0	3.0000	0.0	0
198	48.9000	0.0	4.5000	0.0	0

199	48.9000	0.0	6.0000	0.0	0
200	48.9000	0.0	6.8000	0.0	0
201	48.9000	0.0	7.3000	0.0	0
202	48.9000	0.0	8.5000	0.0	0
203	48.9000	0.0	10.0000	0.0	0
204	48.9000	0.0	11.5000	0.0	0
205	48.9000	0.0	12.7000	0.0	0
206	48.9000	0.0	13.2000	0.0	0
207	48.9000	0.0	14.0000	0.0	0
208	48.9000	0.0	14.5000	0.0	0
209	48.9000	0.0	16.0000	0.0	0
210	50.0000	0.0	0.0	0.0	0
211	50.0000	0.0	1.5000	0.0	0
212	50.0000	0.0	3.0000	0.0	0
213	50.0000	0.0	4.5000	0.0	0
214	50.0000	0.0	6.0000	0.0	0
215	50.0000	0.0	7.3000	0.0	0
216	50.0000	0.0	8.5000	0.0	0
217	50.0000	0.0	10.0000	0.0	0
218	50.0000	0.0	11.5000	0.0	0
219	50.0000	0.0	12.7000	0.0	0
220	50.0000	0.0	14.0000	0.0	0
221	50.0000	0.0	14.5000	0.0	0
222	50.0000	0.0	16.0000	0.0	0
223	51.5000	0.0	0.0	0.0	0
224	51.5000	0.0	1.5000	0.0	0
225	51.5000	0.0	3.0000	0.0	0
226	51.5000	0.0	4.5000	0.0	0
227	51.5000	0.0	6.0000	0.0	0
228	51.5000	0.0	7.3000	0.0	0
229	51.5000	0.0	8.5000	0.0	0
230	51.5000	0.0	10.0000	0.0	0
231	51.5000	0.0	11.5000	0.0	0
232	51.5000	0.0	12.7000	0.0	0
233	51.5000	0.0	14.0000	0.0	0
234	51.5000	0.0	14.5000	0.0	0
235	51.5000	0.0	16.0000	0.0	0
236	53.0000	0.0	0.0	0.0	0
237	53.0000	0.0	1.5000	0.0	0
238	53.0000	0.0	3.0000	0.0	0
239	53.0000	0.0	6.0000	0.0	0
240	53.0000	0.0	8.5000	0.0	0
241	53.0000	0.0	10.0000	0.0	0
242	53.0000	0.0	11.5000	0.0	0
243	53.0000	0.0	14.0000	0.0	0
244	53.0000	0.0	14.5000	0.0	0
245	53.0000	0.0	16.0000	0.0	0
246	54.5000	0.0	0.0	0.0	0
247	54.5000	0.0	3.0000	0.0	0
248	54.5000	0.0	6.0000	0.0	0
249	54.5000	0.0	8.5000	0.0	0
250	54.5000	0.0	10.0000	0.0	0
251	54.5000	0.0	11.5000	0.0	0
252	54.5000	0.0	14.0000	0.0	0
253	54.5000	0.0	16.0000	0.0	0
254	56.0000	0.0	0.0	0.0	0
255	56.0000	0.0	3.0000	0.0	0
256	56.0000	0.0	8.5000	0.0	0
257	56.0000	0.0	10.0000	0.0	0
258	56.0000	0.0	14.0000	0.0	0

259	56.0000	0.0	16.0000	0.0	0
260	58.0000	0.0	0.0	0.0	0
261	58.0000	0.0	3.0000	0.0	0
262	58.0000	0.0	8.0000	0.0	0
263	58.0000	0.0	14.0000	0.0	0
264	58.0000	0.0	16.0000	0.0	0
265	60.0000	0.0	0.0	0.0	0
266	60.0000	0.0	3.0000	0.0	0
267	60.0000	0.0	8.0000	0.0	0
268	60.0000	0.0	14.0000	0.0	0
269	60.0000	0.0	16.0000	0.0	0
270	62.0000	0.0	0.0	0.0	0
271	62.0000	0.0	8.0000	0.0	0
272	62.0000	0.0	16.0000	0.0	0
273	66.0000	0.0	0.0	0.0	0
274	66.0000	0.0	8.0000	0.0	0
275	66.0000	0.0	16.0000	0.0	0
276	72.0000	0.0	0.0	0.0	0
277	72.0000	0.0	8.0000	0.0	0
278	72.0000	0.0	16.0000	0.0	0
279	78.0000	0.0	0.0	0.0	0
280	78.0000	0.0	8.0000	0.0	0
281	78.0000	0.0	16.0000	0.0	0
282	84.0000	-26.2500	0.0	0.0	10
283	84.0000	0.0	8.0000	0.0	0
284	84.0000	26.2500	16.0000	0.0	0

ELEMENT	NODE 1	NODE 2	NODE 3	ELEMENT TYPE	AREA OR THICK.	MATERIA
1	1	4	0	BAR	0.56580E 01	1
2	1	4	5	PLATE	0.31000E 00	2
3	1	5	2	PLATE	0.31000E 00	2
4	2	5	3	PLATE	0.31000E 00	2
5	3	5	6	PLATE	0.31000E 00	2
6	3	6	0	BAR	0.56580E 01	1
7	4	7	0	BAR	0.56580E 01	1
8	4	7	8	PLATE	0.31000E 00	2
9	4	8	5	PLATE	0.31000E 00	2
10	5	8	6	PLATE	0.31000E 00	2
11	6	8	9	PLATE	0.31000E 00	2
12	6	9	0	BAR	0.56580E 01	1
13	7	10	0	BAR	0.56580E 01	1
14	7	10	11	PLATE	0.31000E 00	2
15	7	11	8	PLATE	0.31000E 00	2
16	8	11	9	PLATE	0.31000E 00	2
17	9	11	12	PLATE	0.31000E 00	2
18	9	12	0	BAR	0.56580E 01	1
19	10	13	0	BAR	0.56580E 01	1
20	10	13	14	PLATE	0.31000E 00	2
21	10	14	11	PLATE	0.31000E 00	2
22	11	14	12	PLATE	0.31000E 00	2
23	12	14	15	PLATE	0.31000E 00	2
24	12	15	0	BAR	0.56580E 01	1
25	13	16	0	BAR	0.32200E 01	1
26	13	16	17	PLATE	0.31000E 00	2
27	13	17	14	PLATE	0.31000E 00	2

28	14	17	18	PLATE	0.31000E 00	2
29	14	18	19	PLATE	0.31000E 00	2
30	14	19	15	PLATE	0.31000E 00	2
31	15	19	20	PLATE	0.31000E 00	2
32	15	20	0	BAR	0.32200E 01	1
33	16	21	0	BAR	0.32200E 01	1
34	16	21	22	PLATE	0.31000E 00	2
35	16	22	17	PLATE	0.31000E 00	2
36	17	22	23	PLATE	0.31000E 00	2
37	17	23	18	PLATE	0.31000E 00	2
38	18	23	19	PLATE	0.31000E 00	2
39	19	23	24	PLATE	0.31000E 00	2
40	19	24	20	PLATE	0.31000E 00	2
41	20	24	25	PLATE	0.31000E 00	2
42	20	25	0	BAR	0.32200E 01	1
43	21	26	0	BAR	0.32200E 01	1
44	21	26	27	PLATE	0.31000E 00	2
45	21	27	22	PLATE	0.31000E 00	2
46	22	27	28	PLATE	0.31000E 00	2
47	22	28	23	PLATE	0.31000E 00	2
48	23	28	29	PLATE	0.31000E 00	2
49	23	29	24	PLATE	0.31000E 00	2
50	24	29	30	PLATE	0.31000E 00	2
51	24	30	25	PLATE	0.31000E 00	2
52	25	30	31	PLATE	0.31000E 00	2
53	25	31	0	BAR	0.32200E 01	1
54	26	32	0	BAR	0.32200E 01	1
55	26	32	33	PLATE	0.31000E 00	2
56	26	33	27	PLATE	0.31000E 00	2
57	27	33	34	PLATE	0.31000E 00	2
58	27	34	28	PLATE	0.31000E 00	2
59	28	34	35	PLATE	0.31000E 00	2
60	28	35	29	PLATE	0.31000E 00	2
61	29	35	36	PLATE	0.31000E 00	2
62	29	36	37	PLATE	0.31000E 00	2
63	29	37	30	PLATE	0.31000E 00	2
64	30	37	38	PLATE	0.31000E 00	2
65	30	38	31	PLATE	0.31000E 00	2
66	31	38	39	PLATE	0.31000E 00	2
67	31	39	0	BAR	0.32200E 01	1
68	32	40	0	BAR	0.32200E 01	1
69	32	40	41	PLATE	0.31000E 00	2
70	32	41	33	PLATE	0.31000E 00	2
71	33	41	42	PLATE	0.31000E 00	2
72	33	42	43	PLATE	0.31000E 00	2
73	33	43	34	PLATE	0.31000E 00	2
74	34	43	44	PLATE	0.31000E 00	2
75	34	44	35	PLATE	0.31000E 00	2
76	35	44	45	PLATE	0.31000E 00	2
77	35	45	36	PLATE	0.31000E 00	2
78	36	45	37	PLATE	0.31000E 00	2
79	37	45	46	PLATE	0.31000E 00	2
80	37	46	38	PLATE	0.31000E 00	2
81	38	46	47	PLATE	0.31000E 00	2
82	38	47	48	PLATE	0.31000E 00	2
83	38	48	39	PLATE	0.31000E 00	2
84	39	48	49	PLATE	0.31000E 00	2
85	39	49	0	BAR	0.32200E 01	1
86	40	50	0	BAR	0.32200E 01	1
87	40	50	51	PLATE	0.31000E 00	2

88	40	51	41	PLATE	0.31000E 00	2
89	41	51	52	PLATE	0.31000E 00	2
90	41	52	42	PLATE	0.31000E 00	2
91	42	52	53	PLATE	0.31000E 00	2
92	42	53	43	PLATE	0.31000E 00	2
93	43	53	54	PLATE	0.31000E 00	2
94	43	54	55	PLATE	0.31000E 00	2
95	43	55	44	PLATE	0.31000E 00	2
96	44	55	56	PLATE	0.31000E 00	2
97	44	56	57	PLATE	0.31000E 00	2
98	44	57	45	PLATE	0.31000E 00	2
99	45	57	46	PLATE	0.31000E 00	2
100	46	57	58	PLATE	0.31000E 00	2
101	46	58	59	PLATE	0.31000E 00	2
102	46	59	47	PLATE	0.31000E 00	2
103	47	59	60	PLATE	0.31000E 00	2
104	47	60	48	PLATE	0.31000E 00	2
105	48	60	61	PLATE	0.31000E 00	2
106	48	61	49	PLATE	0.31000E 00	2
107	49	61	62	PLATE	0.31000E 00	2
108	49	62	0	BAR	0.32200E 01	1
109	50	63	0	BAR	0.32200E 01	1
110	50	63	64	PLATE	0.31000E 00	2
111	50	64	51	PLATE	0.31000E 00	2
112	51	64	65	PLATE	0.31000E 00	2
113	51	65	52	PLATE	0.31000E 00	2
114	52	65	66	PLATE	0.31000E 00	2
115	52	66	53	PLATE	0.31000E 00	2
116	53	66	67	PLATE	0.31000E 00	2
117	53	67	54	PLATE	0.31000E 00	2
118	54	67	68	PLATE	0.31000E 00	2
119	54	68	55	PLATE	0.31000E 00	2
120	55	68	69	PLATE	0.31000E 00	2
121	55	69	56	PLATE	0.31000E 00	2
122	56	69	70	PLATE	0.31000E 00	2
123	56	70	57	PLATE	0.31000E 00	2
124	57	70	58	PLATE	0.31000E 00	2
125	58	70	71	PLATE	0.31000E 00	2
126	58	71	59	PLATE	0.31000E 00	2
127	59	71	72	PLATE	0.31000E 00	2
128	59	72	60	PLATE	0.31000E 00	2
129	60	72	73	PLATE	0.31000E 00	2
130	60	73	61	PLATE	0.31000E 00	2
131	61	73	74	PLATE	0.31000E 00	2
132	61	74	62	PLATE	0.31000E 00	2
133	62	74	75	PLATE	0.31000E 00	2
134	62	75	0	BAR	0.32200E 01	1
135	63	76	0	BAR	0.32200E 01	1
136	63	76	77	PLATE	0.31000E 00	2
137	63	77	64	PLATE	0.31000E 00	2
138	64	77	78	PLATE	0.31000E 00	2
139	64	78	65	PLATE	0.31000E 00	2
140	65	78	79	PLATE	0.31000E 00	2
141	65	79	66	PLATE	0.31000E 00	2
142	66	79	80	PLATE	0.31000E 00	2
143	66	80	67	PLATE	0.31000E 00	2
144	67	80	81	PLATE	0.31000E 00	2
145	67	81	68	PLATE	0.31000E 00	2
146	68	81	82	PLATE	0.31000E 00	2
147	68	82	83	PLATE	0.31000E 00	2

148	68	83	69	PLATE	0.31000E 00	2
149	69	83	84	PLATE	0.31000E 00	2
150	69	84	70	PLATE	0.31000E 00	2
151	70	84	71	PLATE	0.31000E 00	2
152	71	84	85	PLATE	0.31000E 00	2
153	71	85	72	PLATE	0.31000E 00	2
154	72	85	86	PLATE	0.31000E 00	2
155	72	86	87	PLATE	0.31000E 00	2
156	72	87	73	PLATE	0.31000E 00	2
157	73	87	88	PLATE	0.31000E 00	2
158	73	88	74	PLATE	0.31000E 00	2
159	74	88	89	PLATE	0.31000E 00	2
160	74	89	75	PLATE	0.31000E 00	2
161	75	89	90	PLATE	0.31000E 00	2
162	75	90	0	BAR	0.32200E 01	1
163	76	98	0	BAR	0.32200E 01	1
164	76	98	99	PLATE	0.31000E 00	2
165	76	99	77	PLATE	0.31000E 00	2
166	77	99	100	PLATE	0.31000E 00	2
167	77	100	78	PLATE	0.31000E 00	2
168	78	100	101	PLATE	0.31000E 00	2
169	78	101	79	PLATE	0.31000E 00	2
170	79	101	102	PLATE	0.31000E 00	2
171	79	102	80	PLATE	0.31000E 00	2
172	80	91	81	PLATE	0.31000E 00	2
173	80	102	91	PLATE	0.31000E 00	2
174	91	102	103	PLATE	0.31000E 00	2
175	81	91	92	PLATE	0.31000E 00	2
176	91	103	92	PLATE	0.31000E 00	2
177	81	92	82	PLATE	0.31000E 00	2
178	82	92	93	PLATE	0.31000E 00	2
179	82	93	83	PLATE	0.31000E 00	2
180	83	93	94	PLATE	0.31000E 00	2
181	83	94	84	PLATE	0.31000E 00	2
182	84	94	85	PLATE	0.31000E 00	2
183	85	94	95	PLATE	0.31000E 00	2
184	85	95	86	PLATE	0.31000E 00	2
185	86	95	96	PLATE	0.31000E 00	2
186	86	96	87	PLATE	0.31000E 00	2
187	87	96	97	PLATE	0.31000E 00	2
188	96	104	97	PLATE	0.31000E 00	2
189	87	97	88	PLATE	0.31000E 00	2
190	88	97	105	PLATE	0.31000E 00	2
191	97	104	105	PLATE	0.31000E 00	2
192	88	105	89	PLATE	0.31000E 00	2
193	89	105	106	PLATE	0.31000E 00	2
194	89	106	90	PLATE	0.31000E 00	2
195	90	106	107	PLATE	0.31000E 00	2
196	90	107	0	BAR	0.32200E 01	1
197	98	108	0	BAR	0.32200E 01	1
198	98	108	109	PLATE	0.31000E 00	2
199	98	109	99	PLATE	0.31000E 00	2
200	99	109	110	PLATE	0.31000E 00	2
201	99	110	100	PLATE	0.31000E 00	2
202	100	110	111	PLATE	0.31000E 00	2
203	100	111	101	PLATE	0.31000E 00	2
204	101	111	112	PLATE	0.31000E 00	2
205	101	112	102	PLATE	0.31000E 00	2
206	102	112	113	PLATE	0.31000E 00	2
207	102	113	103	PLATE	0.31000E 00	2

208	104	114	105	PLATE	0.31000E 00	2
209	105	114	115	PLATE	0.31000E 00	2
210	105	115	106	PLATE	0.31000E 00	2
211	106	115	116	PLATE	0.31000E 00	2
212	106	116	107	PLATE	0.31000E 00	2
213	107	116	117	PLATE	0.31000E 00	2
214	107	117	0	BAR	0.32200E 01	1
215	108	118	0	BAR	0.32200E 01	1
216	108	118	119	PLATE	0.31000E 00	2
217	108	119	109	PLATE	0.31000E 00	2
218	109	119	120	PLATE	0.31000E 00	2
219	109	120	110	PLATE	0.31000E 00	2
220	110	120	121	PLATE	0.31000E 00	2
221	110	121	111	PLATE	0.31000E 00	2
222	111	121	122	PLATE	0.31000E 00	2
223	111	122	112	PLATE	0.31000E 00	2
224	112	122	123	PLATE	0.31000E 00	2
225	112	123	113	PLATE	0.31000E 00	2
226	114	124	115	PLATE	0.31000E 00	2
227	115	124	125	PLATE	0.31000E 00	2
228	115	125	116	PLATE	0.31000E 00	2
229	116	125	126	PLATE	0.31000E 00	2
230	116	126	117	PLATE	0.31000E 00	2
231	117	126	127	PLATE	0.31000E 00	2
232	117	127	0	BAR	0.32200E 01	1
233	118	128	0	BAR	0.32200E 01	1
234	118	128	129	PLATE	0.31000E 00	2
235	118	129	119	PLATE	0.31000E 00	2
236	119	129	130	PLATE	0.31000E 00	2
237	119	130	120	PLATE	0.31000E 00	2
238	120	130	131	PLATE	0.31000E 00	2
239	120	131	121	PLATE	0.31000E 00	2
240	121	131	132	PLATE	0.31000E 00	2
241	121	132	122	PLATE	0.31000E 00	2
242	122	132	133	PLATE	0.31000E 00	2
243	122	133	123	PLATE	0.31000E 00	2
244	124	134	125	PLATE	0.31000E 00	2
245	125	134	135	PLATE	0.31000E 00	2
246	125	135	126	PLATE	0.31000E 00	2
247	126	135	136	PLATE	0.31000E 00	2
248	126	136	127	PLATE	0.31000E 00	2
249	127	136	137	PLATE	0.31000E 00	2
250	127	137	0	BAR	0.32200E 01	1
251	128	138	0	BAR	0.32200E 01	1
252	128	138	139	PLATE	0.31000E 00	2
253	128	139	129	PLATE	0.31000E 00	2
254	129	139	140	PLATE	0.31000E 00	2
255	129	140	130	PLATE	0.31000E 00	2
256	130	140	141	PLATE	0.31000E 00	2
257	130	141	131	PLATE	0.31000E 00	2
258	131	141	142	PLATE	0.31000E 00	2
259	131	142	132	PLATE	0.31000E 00	2
260	132	142	143	PLATE	0.31000E 00	2
261	132	143	133	PLATE	0.31000E 00	2
262	134	144	135	PLATE	0.31000E 00	2
263	135	144	145	PLATE	0.31000E 00	2
264	135	145	136	PLATE	0.31000E 00	2
265	136	145	146	PLATE	0.31000E 00	2
266	136	146	137	PLATE	0.31000E 00	2
267	137	146	147	PLATE	0.31000E 00	2

268	137	147	0	BAR	0.32200E 01	1
269	138	148	0	BAR	0.32200E 01	1
270	138	148	139	PLATE	0.31000E 00	2
271	139	148	149	PLATE	0.31000E 00	2
272	139	149	140	PLATE	0.31000E 00	2
273	140	149	150	PLATE	0.31000E 00	2
274	140	150	141	PLATE	0.31000E 00	2
275	141	150	151	PLATE	0.31000E 00	2
276	141	151	142	PLATE	0.31000E 00	2
277	142	151	152	PLATE	0.31000E 00	2
278	142	152	143	PLATE	0.31000E 00	2
279	143	152	153	PLATE	0.31000E 00	2
280	144	154	155	PLATE	0.31000E 00	2
281	144	155	145	PLATE	0.31000E 00	2
282	145	155	156	PLATE	0.31000E 00	2
283	145	156	146	PLATE	0.31000E 00	2
284	146	156	157	PLATE	0.31000E 00	2
285	146	157	147	PLATE	0.31000E 00	2
286	147	157	0	BAR	0.32200E 01	1
287	148	158	0	BAR	0.32200E 01	1
288	148	158	149	PLATE	0.31000E 00	2
289	149	158	159	PLATE	0.31000E 00	2
290	149	159	150	PLATE	0.31000E 00	2
291	150	159	160	PLATE	0.31000E 00	2
292	150	160	151	PLATE	0.31000E 00	2
293	151	160	161	PLATE	0.31000E 00	2
294	151	161	152	PLATE	0.31000E 00	2
295	152	161	162	PLATE	0.31000E 00	2
296	152	162	153	PLATE	0.31000E 00	2
297	153	162	163	PLATE	0.31000E 00	2
298	154	164	165	PLATE	0.31000E 00	2
299	154	165	155	PLATE	0.31000E 00	2
300	155	165	166	PLATE	0.31000E 00	2
301	155	166	156	PLATE	0.31000E 00	2
302	156	166	167	PLATE	0.31000E 00	2
303	156	167	157	PLATE	0.31000E 00	2
304	157	167	0	BAR	0.32200E 01	1
305	158	168	0	BAR	0.32200E 01	1
306	158	168	159	PLATE	0.31000E 00	2
307	159	168	169	PLATE	0.31000E 00	2
308	159	169	160	PLATE	0.31000E 00	2
309	160	169	170	PLATE	0.31000E 00	2
310	160	170	161	PLATE	0.31000E 00	2
311	161	170	171	PLATE	0.31000E 00	2
312	161	171	162	PLATE	0.31000E 00	2
313	162	171	172	PLATE	0.31000E 00	2
314	162	172	163	PLATE	0.31000E 00	2
315	163	172	173	PLATE	0.31000E 00	2
316	164	174	175	PLATE	0.31000E 00	2
317	164	175	165	PLATE	0.31000E 00	2
318	165	175	176	PLATE	0.31000E 00	2
319	165	176	166	PLATE	0.31000E 00	2
320	166	176	177	PLATE	0.31000E 00	2
321	166	177	167	PLATE	0.31000E 00	2
322	167	177	0	BAR	0.32200E 01	1
323	168	178	0	BAR	0.32200E 01	1
324	168	178	169	PLATE	0.31000E 00	2
325	169	178	179	PLATE	0.31000E 00	2
326	169	179	170	PLATE	0.31000E 00	2
327	170	179	180	PLATE	0.31000E 00	2

328	170	180	171	PLATE	0.31000E 00	2
329	171	180	181	PLATE	0.31000E 00	2
330	171	181	172	PLATE	0.31000E 00	2
331	172	181	182	PLATE	0.31000E 00	2
332	172	182	173	PLATE	0.31000E 00	2
333	173	182	183	PLATE	0.31000E 00	2
334	174	184	185	PLATE	0.31000E 00	2
335	174	185	175	PLATE	0.31000E 00	2
336	175	185	186	PLATE	0.31000E 00	2
337	175	186	176	PLATE	0.31000E 00	2
338	176	186	187	PLATE	0.31000E 00	2
339	176	187	177	PLATE	0.31000E 00	2
340	177	187	0	BAR	0.32200E 01	1
341	178	195	0	BAR	0.32200E 01	1
342	178	195	179	PLATE	0.31000E 00	2
343	179	195	196	PLATE	0.31000E 00	2
344	179	196	180	PLATE	0.31000E 00	2
345	180	196	197	PLATE	0.31000E 00	2
346	180	197	181	PLATE	0.31000E 00	2
347	181	197	198	PLATE	0.31000E 00	2
348	181	198	182	PLATE	0.31000E 00	2
349	182	198	199	PLATE	0.31000E 00	2
350	182	188	183	PLATE	0.31000E 00	2
351	182	199	188	PLATE	0.31000E 00	2
352	188	199	200	PLATE	0.31000E 00	2
353	188	200	189	PLATE	0.31000E 00	2
354	183	188	189	PLATE	0.31000E 00	2
355	189	200	201	PLATE	0.31000E 00	2
356	189	201	190	PLATE	0.31000E 00	2
357	190	201	202	PLATE	0.31000E 00	2
358	190	202	191	PLATE	0.31000E 00	2
359	191	202	203	PLATE	0.31000E 00	2
360	191	203	204	PLATE	0.31000E 00	2
361	191	204	192	PLATE	0.31000E 00	2
362	192	204	205	PLATE	0.31000E 00	2
363	192	205	193	PLATE	0.31000E 00	2
364	193	205	206	PLATE	0.31000E 00	2
365	193	206	194	PLATE	0.31000E 00	2
366	184	193	194	PLATE	0.31000E 00	2
367	184	194	185	PLATE	0.31000E 00	2
368	185	194	207	PLATE	0.31000E 00	2
369	194	206	207	PLATE	0.31000E 00	2
370	185	207	208	PLATE	0.31000E 00	2
371	185	208	186	PLATE	0.31000E 00	2
372	186	208	209	PLATE	0.31000E 00	2
373	186	209	187	PLATE	0.31000E 00	2
374	187	209	0	BAR	0.32200E 01	1
375	195	210	0	BAR	0.32200E 01	1
376	195	210	196	PLATE	0.31000E 00	2
377	196	210	211	PLATE	0.31000E 00	2
378	196	211	197	PLATE	0.31000E 00	2
379	197	211	212	PLATE	0.31000E 00	2
380	197	212	198	PLATE	0.31000E 00	2
381	198	212	213	PLATE	0.31000E 00	2
382	198	213	199	PLATE	0.31000E 00	2
383	199	213	214	PLATE	0.31000E 00	2
384	199	214	200	PLATE	0.31000E 00	2
385	200	214	215	PLATE	0.31000E 00	2
386	200	215	201	PLATE	0.31000E 00	2
387	201	215	202	PLATE	0.31000E 00	2

388	202	215	216	PLATE	0.31000E 00	2
389	202	216	203	PLATE	0.31000E 00	2
390	203	216	217	PLATE	0.31000E 00	2
391	203	217	218	PLATE	0.31000E 00	2
392	203	218	204	PLATE	0.31000E 00	2
393	204	218	219	PLATE	0.31000E 00	2
394	204	219	205	PLATE	0.31000E 00	2
395	205	219	206	PLATE	0.31000E 00	2
396	206	219	220	PLATE	0.31000E 00	2
397	206	220	207	PLATE	0.31000E 00	2
398	207	220	221	PLATE	0.31000E 00	2
399	207	221	208	PLATE	0.31000E 00	2
400	208	221	222	PLATE	0.31000E 00	2
401	208	222	209	PLATE	0.31000E 00	2
402	209	222	0	BAR	0.32200E 01	1
403	210	223	0	BAR	0.32200E 01	1
404	210	223	211	PLATE	0.31000E 00	2
405	211	223	224	PLATE	0.31000E 00	2
406	211	224	212	PLATE	0.31000E 00	2
407	212	224	225	PLATE	0.31000E 00	2
408	212	225	213	PLATE	0.31000E 00	2
409	213	225	226	PLATE	0.31000E 00	2
410	213	226	214	PLATE	0.31000E 00	2
411	214	226	227	PLATE	0.31000E 00	2
412	214	227	215	PLATE	0.31000E 00	2
413	215	227	228	PLATE	0.31000E 00	2
414	215	228	216	PLATE	0.31000E 00	2
415	216	228	229	PLATE	0.31000E 00	2
416	216	229	217	PLATE	0.31000E 00	2
417	217	229	230	PLATE	0.31000E 00	2
418	217	230	231	PLATE	0.31000E 00	2
419	217	231	218	PLATE	0.31000E 00	2
420	218	231	232	PLATE	0.31000E 00	2
421	218	232	219	PLATE	0.31000E 00	2
422	219	232	233	PLATE	0.31000E 00	2
423	219	233	220	PLATE	0.31000E 00	2
424	220	233	234	PLATE	0.31000E 00	2
425	220	234	221	PLATE	0.31000E 00	2
426	221	234	235	PLATE	0.31000E 00	2
427	221	235	222	PLATE	0.31000E 00	2
428	222	235	0	BAR	0.32200E 01	1
429	223	236	0	BAR	0.32200E 01	1
430	223	236	224	PLATE	0.31000E 00	2
431	224	236	237	PLATE	0.31000E 00	2
432	224	237	225	PLATE	0.31000E 00	2
433	225	237	238	PLATE	0.31000E 00	2
434	225	238	226	PLATE	0.31000E 00	2
435	226	238	239	PLATE	0.31000E 00	2
436	226	239	227	PLATE	0.31000E 00	2
437	227	239	228	PLATE	0.31000E 00	2
438	228	239	240	PLATE	0.31000E 00	2
439	228	240	229	PLATE	0.31000E 00	2
440	229	240	230	PLATE	0.31000E 00	2
441	230	240	241	PLATE	0.31000E 00	2
442	230	241	242	PLATE	0.31000E 00	2
443	230	242	231	PLATE	0.31000E 00	2
444	231	242	232	PLATE	0.31000E 00	2
445	232	242	243	PLATE	0.31000E 00	2
446	232	243	233	PLATE	0.31000E 00	2
447	233	243	244	PLATE	0.31000E 00	2

448	233	244	234	PLATE	0.31000E 00	2
449	234	244	245	PLATE	0.31000E 00	2
450	234	245	235	PLATE	0.31000E 00	2
451	235	245	0	BAR	0.32200E 01	1
452	236	246	0	BAR	0.32200E 01	1
453	236	246	237	PLATE	0.31000E 00	2
454	237	246	247	PLATE	0.31000E 00	2
455	237	247	238	PLATE	0.31000E 00	2
456	238	247	239	PLATE	0.31000E 00	2
457	239	247	248	PLATE	0.31000E 00	2
458	239	248	240	PLATE	0.31000E 00	2
459	240	248	249	PLATE	0.31000E 00	2
460	240	249	241	PLATE	0.31000E 00	2
461	241	249	250	PLATE	0.31000E 00	2
462	241	250	251	PLATE	0.31000E 00	2
463	241	251	242	PLATE	0.31000E 00	2
464	242	251	252	PLATE	0.31000E 00	2
465	242	252	243	PLATE	0.31000E 00	2
466	243	252	244	PLATE	0.31000E 00	2
467	244	252	253	PLATE	0.31000E 00	2
468	244	253	245	PLATE	0.31000E 00	2
469	245	253	0	BAR	0.32200E 01	1
470	246	254	0	BAR	0.32200E 01	1
471	246	254	247	PLATE	0.31000E 00	2
472	247	254	255	PLATE	0.31000E 00	2
473	247	255	248	PLATE	0.31000E 00	2
474	248	255	256	PLATE	0.31000E 00	2
475	248	256	249	PLATE	0.31000E 00	2
476	249	256	257	PLATE	0.31000E 00	2
477	249	257	250	PLATE	0.31000E 00	2
478	250	257	251	PLATE	0.31000E 00	2
479	251	257	258	PLATE	0.31000E 00	2
480	251	258	252	PLATE	0.31000E 00	2
481	252	258	259	PLATE	0.31000E 00	2
482	252	259	253	PLATE	0.31000E 00	2
483	253	259	0	BAR	0.32200E 01	1
484	254	260	0	BAR	0.32200E 01	1
485	254	260	255	PLATE	0.31000E 00	2
486	255	260	261	PLATE	0.31000E 00	2
487	255	261	256	PLATE	0.31000E 00	2
488	256	261	262	PLATE	0.31000E 00	2
489	256	262	257	PLATE	0.31000E 00	2
490	257	262	263	PLATE	0.31000E 00	2
491	257	263	258	PLATE	0.31000E 00	2
492	258	263	264	PLATE	0.31000E 00	2
493	258	264	259	PLATE	0.31000E 00	2
494	259	264	0	BAR	0.32200E 01	1
495	260	265	0	BAR	0.32200E 01	1
496	260	265	261	PLATE	0.31000E 00	2
497	261	265	266	PLATE	0.31000E 00	2
498	261	266	262	PLATE	0.31000E 00	2
499	262	266	267	PLATE	0.31000E 00	2
500	262	267	268	PLATE	0.31000E 00	2
501	262	268	263	PLATE	0.31000E 00	2
502	263	268	269	PLATE	0.31000E 00	2
503	263	269	264	PLATE	0.31000E 00	2
504	264	269	0	BAR	0.32200E 01	1
505	265	270	0	BAR	0.32200E 01	1
506	265	270	266	PLATE	0.31000E 00	2
507	266	270	271	PLATE	0.31000E 00	2

508	266	271	267	PLATE	0.31000E 00	2
509	267	271	268	PLATE	0.31000E 00	2
510	268	271	272	PLATE	0.31000E 00	2
511	269	272	269	PLATE	0.31000E 00	2
512	269	272	0	BAR	0.32200E 01	1
513	270	273	0	BAR	0.56580E 01	1
514	270	273	271	PLATE	0.31000E 00	2
515	271	273	274	PLATE	0.31000E 00	2
516	271	274	275	PLATE	0.31000E 00	2
517	271	275	272	PLATE	0.31000E 00	2
518	272	275	0	BAR	0.56580E 01	1
519	273	276	0	BAR	0.56580E 01	1
520	273	276	274	PLATE	0.31000E 00	2
521	274	276	277	PLATE	0.31000E 00	2
522	274	277	278	PLATE	0.31000E 00	2
523	274	278	275	PLATE	0.31000E 00	2
524	275	278	0	BAR	0.56580E 01	1
525	276	279	0	BAR	0.56580E 01	1
526	276	279	277	PLATE	0.31000E 00	2
527	277	279	280	PLATE	0.31000E 00	2
528	277	280	281	PLATE	0.31000E 00	2
529	277	281	278	PLATE	0.31000E 00	2
530	278	281	0	BAR	0.56580E 01	1
531	279	282	0	BAR	0.56580E 01	1
532	279	282	280	PLATE	0.31000E 00	2
533	280	282	283	PLATE	0.31000E 00	2
534	280	283	284	PLATE	0.31000E 00	2
535	280	284	281	PLATE	0.31000E 00	2
536	281	284	0	BAR	0.56580E 01	1

SHEAR FORCE DISTRIBUTION IN BEAMS WITH ECCENTRIC RECTANGULAR WEB OPENINGS BY THE FINITE ELEMENT METHOD

by

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ABSTRACT

In some instances it has been desirable to permit the passage of utility components through the webs of wide flange beams. Since web holes do not always lie on the centerline of the beam, it becomes necessary to be able to analyze a beam with an eccentric hole. The Vierendeel Analysis provides a way of analyzing the stresses in a beam with an eccentric hole, provided that the shear force above and below the opening is known.

This investigation uses a finite element computer program to determine the shear force above and below the hole. Several parameters were investigated while determining the shear force distribution, the moment to shear ratio was varied from 0 in. to 80 in. by 40 in. increments, the eccentricity of the hole was varied from 1 in. to 3 in. by 1 in. increments, and reinforcement was added to determine the effect on the shear force with and without reinforcement.

In general, the moment to shear ratio was not found to cause a significant change in the shear force distribution. Changes in the eccentricity and the addition of the reinforcement seemed to cause the most substantial effects.

Several other methods of calculating the shear force distribution were compared with the results of this investigation, and none of these methods compared gave results which were entirely consistent with the finite element results.