MENTAL TRANSFORMATIONS OF POSSIBLE
AND
IMPOSSIBLE 4-CORNERED TORI

by

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

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INTRODUCTION

Like any consistent system, visual space can be described by a set of constraints or rules whereby its elements exist and interact. For example, in 3-space an element of an object cannot be both figure and background to the object simultaneously, nor can an object in the mind's eye be seen from more than one perspective at a time. Two-dimensional renditions of three-dimensional objects, however, need not obey the laws of visual space, and such ungrammatical drawings are called "impossible figures." They cannot exist in the real world for, by definition, they have broken the rules or grammar of the real world. Some examples of impossible figures are shown in Figure 1.

Penrose and Penrose (1958) were the first to introduce impossible figures, and they described three: the Penrose triangle, a composite of three Penrose triangles, and an impossible staircase. Each of these figures is a perspective drawing which does not obey laws of perspective and interposition. The Penrose triangle (Figure 1a) is seen in the mind's eye from several different perspectives simultaneously, and for two surfaces of the triangle, one surface can both occlude and be occluded by the other. Such arrangements are impossible in 3-space. Equally absurd is a staircase that continually rises (from clockwise) but ends up at the starting point (Figure 1b).

Shuster (1964) described a "three-stick clevis" impossible figure which on one end is seen as clevis and on the other as three sticks (Figure 1c). The construction of such a figure breaks rules of figure ground and topology. What is background at the clevis end becomes part of the figure at the other end, and the boundary lines of the clevis arms disassociate as they approach the three sticks. It is clear that such figures cannot exist in 3-space. Though Shuster described the figure as a "new ambiguous figure,"

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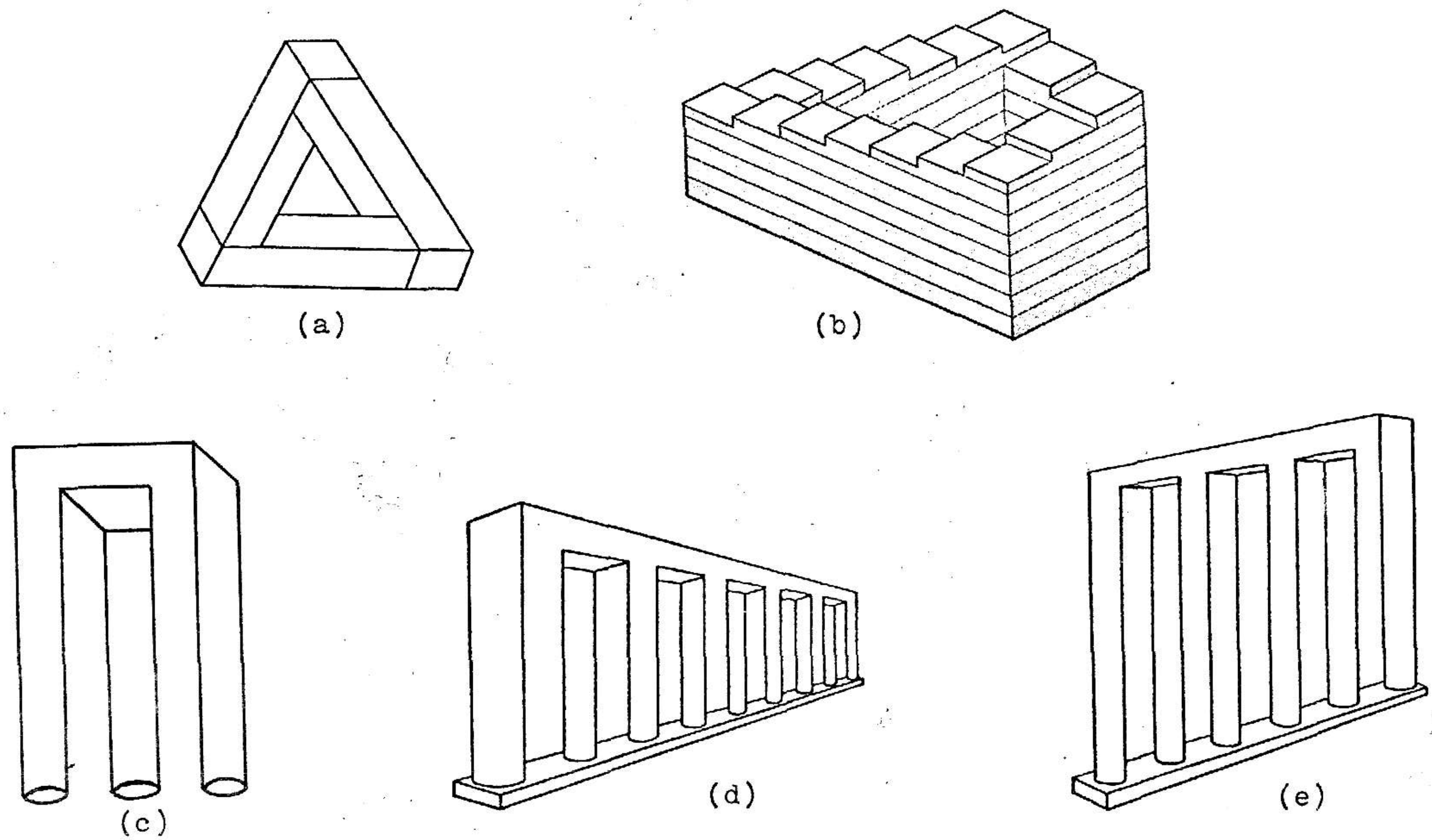


FIGURE 1. Examples of impossible figures: (a) Penrose triangle & (b) impossible staircase (Penrose & Penrose, 1958); (c) three stick clevis (Schuster, 1964); (d) impossible colonade & (e) impossible colonade with impossible perspective (Robinson & Wilson, 1973).

it is much more than simply ambiguous, for ambiguity is not a sufficient condition for absurdity. Whereas ambiguity results from insufficient information, impossibility results from the incongruity of the information with existing schemata. When the information does not conform to the laws of visual space, either the laws or the information must be discarded or modified to accomodate the inconsistency (Howard, 1975). A study by Hochberg and Brooks (1962) suggests that the confrontation may be resolved perceptually by organizing the figures two-dimensionally. When the corners of Penrose-type perspective figures are close together making a compact figure, subjects see the figure as essentially flat. When the sides are lengthened, the ratings of three-dimensionality increase. Apparently, over relatively long distances the confrontation between information and schema is less immediate and thereby mitigated.

There have been two attempts to formulate rules for the systematic generation of impossible figures. Robinson and Wilson (1973) presented interesting variations of the three-stick clevis (e.g., Figure 1d) and pointed out that the construction of such figures takes advantage of the fact that a drawing of a clevis arm requires three lines whereas a cylindrical stick requires only two. Thus any two rectilinear segments drawn in perspective can be made to terminate on five "stick-like" endings, though the figure is usually drawn terminating on three. This structural principal can, as the authors point out, be used to derive a multitide of impossibile clevises or colonades where the clevis arms are n-sided and are made to terminate on circular or alternatively, 1-sided endings where $1 \neq n$. Additional variations result if the perspective of one end of the figure is made incompatible with the perspective of the other (e.g., Figure 1e). This is a different element of impossibility, the same one involved in the figures described by Penrose and Penrose (1958) and more recently by Cowan (1974).

Cowan (1974) derived an algorithm whereby Penrose type perspective figures having n sides can be generated systematically. All such figures are constructed from various combinations of the four corners (M, N, Z, Z^{-1}) shown in Figure 2. Note that each corner is a unique perspective of a right-angled bend in an otherwise rectilinear structure. The rules of the algorithm dictate which corners can combine and in what order to form a possible figure. Thus when a figure is constructed by joining the corners according to the arrows in Figure 2, a possible figure is obtained. If one goes against the direction of the arrows at least once, then an impossible figure results. The complete set of four-cornered tori -- possible (1, 2, 7, 8) and impossible -- are shown in Figure 3.

Cowan and Pringle (1975) have begun a series of studies designed to test the extent to which psychological and formal definitions of impossibility covary. One might expect, for example, that psychological possibility would be all or none, or perhaps that it would vary indirectly with the number of times the rules of Cowan's algorithm are broken in a figure's construction. Estimations of degree of possibility were collected on all four-cornered figures drawn normally and stereoptically. The estimations were made on a scale from one to ten where torus 1 in Figure 3 was assigned a value of ten. As expected, the four possible tori were given high estimates of possibility. However, several impossible tori were also judged highly possible. When judgments of possibility are plotted as a function of the tori (Figure 4), it becomes evident that possibility is not an all or none phenomenon. Rather, some impossible figures are more possible than others. Nor is it the case that psychological impossibility corresponds to the number of times the rules of Cowan's algorithm are broken since, excluding the four possible figures, there is only a .01 correlation between the two. For the stereoptically drawn tori the correlation was .25 but was also insignificant.

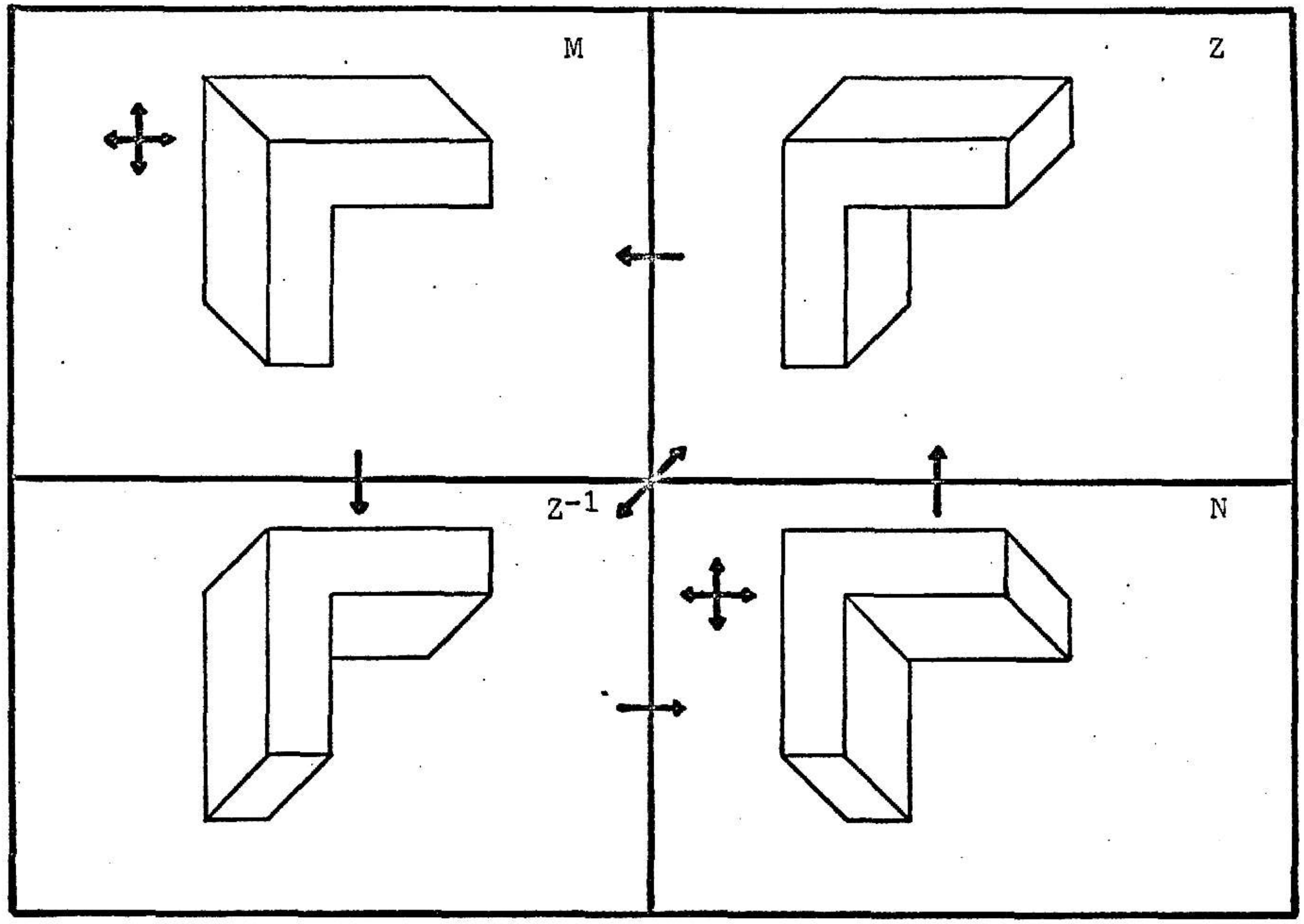


FIGURE 2. The 4 perspective corners used to construct Penrose type impossible figures. Cowan's algorithm of composition (Cowan, 1974) is indicated by the arrows. When the corners are combined in the direction of the arrows, possible figures result. When the direction of the arrows is violated at least once, impossible figures result.

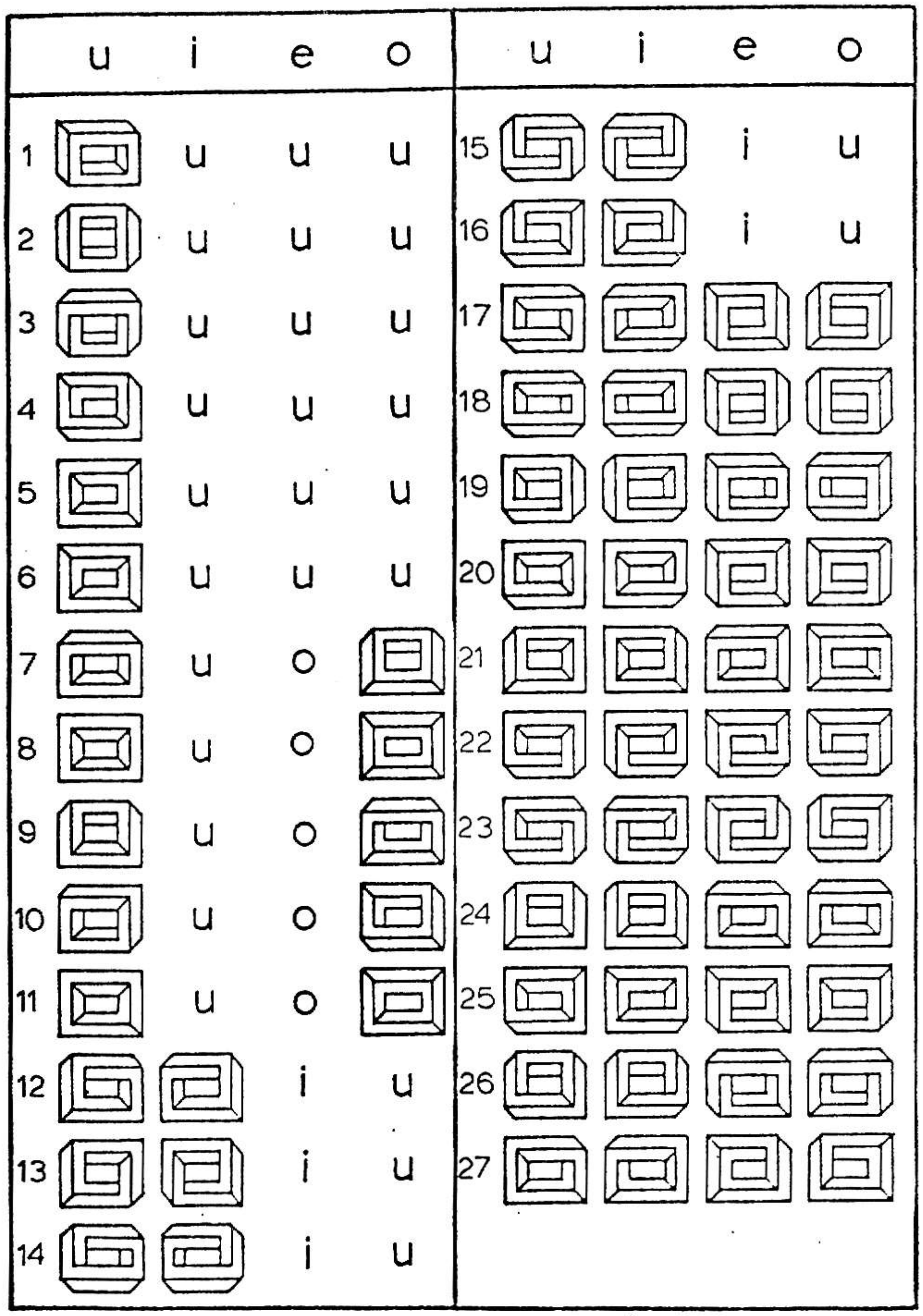
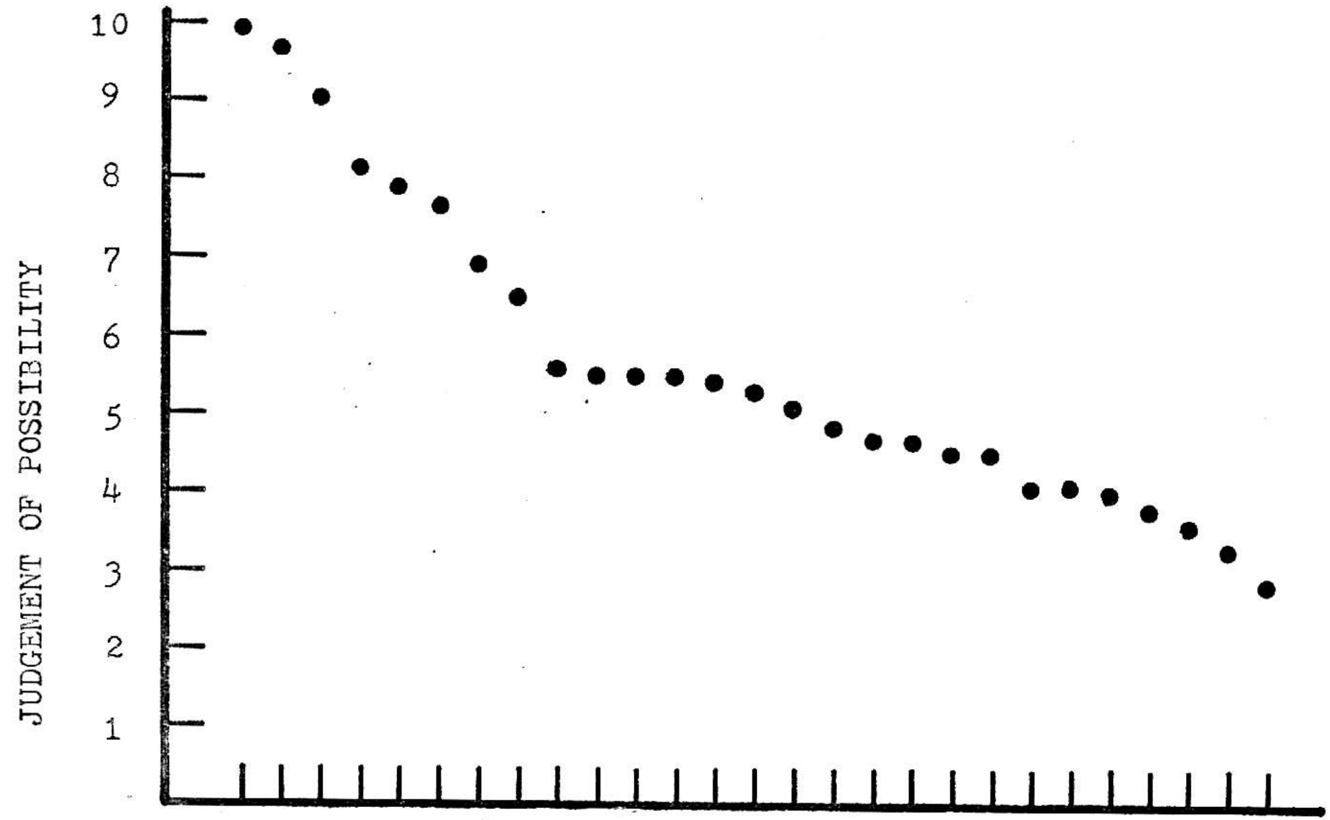


FIGURE 3. The complete set of 4-cornered tori and their vertations. u = unit figure; e = Everse; o = Obverse; i = Inverse. Note that for some tori a vertation is equivalent to the unit figure (irrespective of some rotation in the plane).



torus #---	7	1	8	2	4	10	11	6	20	27	5	19	21	3	24	26	17	22	12	25	13	18	15	9	16	14	23
# of violations--	0	0	0	0	4	2	2	4	1	3	2	1	1	2	2	3	2	2	2	3	2	1	4	2	4	2	3
net levels of depth--	0	0	0	0	0	0	0	0	1	1	0	1	1	0	0	1	2	2	2	1	2	1	4	0	2	2	3

FIGURE 4. Mean judgements of possibility (completely possible = 10) for each of the 27 tori in FIGURE 3. Number of violations of Cowan's algorithm and the net changes in levels of depth are shown for each figure.

What seemed to fit the data best were the apparent changes in "levels of depth" when one examines the corners of a torus in a consistent sequence (e.g., clockwise). Relative to a clockwise direction, a Z^{-1} corner (see Figure 2) seems to descend from the surface and a Z corner to ascend. M and N corners, on the other hand, undergo no net change. To get an intuitive understanding of these changes in depth, imagine a right-angled 3-D corner in the position which corresponds to a Z^{-1} corner. When this is done, notice that relative to a clockwise movement, the second arm of the corner is further away than the first arm. In this sense, a Z^{-1} corner can be said to descend from the surface. If four Z^{-1} corners are combined sequentially to form afour-cornered torus (torus 15), the figure will seem to spiral down in a clockwise direction. Since the consistently descending spiral ends up at the same place it started, torus 15 is the torus analogue to the impossible staircase described by Penrose and Penrose (1958). For an entire figure to undergo a net change in apparent depth of zero, every Z^{-1} corner must be balanced by the presence of a Z corner and visa versa. This type of paradoxical change in depth can account for much of the psychological dimension of impossibility as is demonstrated in Figure 4. There is a high negative correlation (-.67 for the normally drawn tori; -.78 for the tori drawn stereoptically) between the judgments of possibility and the number of net changes in depth a figure undergoes. These correlations exclude the four possible tori.

The purpose of the present study is to examine the extent to which the variables determining psychological possibility also determine the extent to which the figures can be manipulated cognitively. To rotate the figures in depth, for example, clearly has meaning for the possible figures (as formally defined), but is absurd for the impossible figures, and to the extent that formal absurdity is psychologically absurd, one would expect that possible figures are the more easily manipulated in depth. Cognitive space may not obey the same laws as real space, however, and those formally impossible figures judged highly possible may have a three-dimensional character very similar to real objects.

Cowan (1977b) has defined several transformations which can provide a basis for examining the dimensions by which the figures can be mentally manipulated. When the algorithm is used to generate all four-cornered tori, 256 figures result. When equated along various transformational dimensions, the number is reduced to 27. There are five transformations with which this paper is concerned: rotation in the plane, obversion, inversion, eversion, and identity. A <u>rotation</u> simply rotates a figure around a single point in the Euclidean plane. An <u>obversion</u> is a 180° dihedral rotation (out of the plane) around an entire axis. Technically, an obversion results from a rotation around the x-axis (Cowan, 1977b), but for the purposes of this paper an obversion will rotate around the y-axis. An <u>inversion</u> is the mirror reflection of a figure. Figures which are symmetrical about a vertical axis are unchanged by an inversion, and figures having axes of symmetry other than vertical are, when inverted, altered only by a rotation. An <u>eversion</u> brings the back surface of a figure directly up through the front surface as if turning a sock inside out or reversing a Necker cube. Note that an obverse and an everse differ by an inversion. An obversion brings the back surface forward but also inverts the components with respect to the unit figure. The final transformation is <u>identity</u>, and it does nothing to the figure. The inverse (i), obverse (o), everse (e), and identity (u) transformations comprise a group in the mathematical sense. They are interrelated in that : (i)(i) = u; (o)(o) = u; (e)(e) = u; (i)(o) = (o)(i) = e. An interesting example of this group relationship is the four surfaces of the hands. If the palm of the right hand is the unit figure, then the back of the right hand is the obverse, the palm of the left hand the inverse, and the back of the left hand the everse.

It is important to note that the formal transformations of rotation and obversion as Cowan has defined them are to be distinguished from what one might term "natural" rotation and "natural" obversion. The rotation and obversion of four-cornered objects in the real world does not change the observer's perspective. For example, a 90° natural rotation of a 3-D torus viewed from above center (e.g., torus 7) will yield the same shape -- a 3-D torus viewed from above center. The natural rotations of multiples of 90° and natural obversions always result in the same shaped figure. Cowan's treatment of the figures is more complex in that a unique perspective is "embedded" in each of the four types of corners that combine to form the figures. Formal rotations and obversions, as a consequence, are not independent of perspective. Rather, it is ultimately the perspectives themselves which are being manipulated. In the manner in which Cowan has defined the tori, torus 1, when given a natural rotation of 45°, becomes equivalent to torus 7 formally rotated 45°. Such an equality is not permissible by Cowan's treatment of the figures, but transformations of 3-D objects in the real world demand it. The terms "rotation" and "obversion" will henceforth refer to formal rotation and formal obversion respectively.

Studies which require the subject to perform mental transformations have, until recently, been confined to the rotation of two-dimensional figures in the Euclidean plane (e.g., Dearborn, 1899; Dees & Grindley, 1947; French, 1953; Arnoult, 1954; Sato, 1960). Summarizing this literature, Howard and Templeton concluded:

> ...(a) two shapes [in two dimensions] are best discriminated when they are vertically oriented (i.e., with the base horizontal); (b) regardless of whether the two shapes are vertical, they are best discriminated when both have the same orientation; and (c) the effects produced by disorienting the two shapes relative to each other depend on the type of shape used and the type of discrimination required.... [Howard and Templeton, 1966]

Shepard and Metzler (1971) and Metzler (1973) introduced a depth transformation by presenting two perspective drawings of 3-D forms simultaneously or successively and having subjects respond "same" if the figures were identical or differed only by a rotation, and "different" if they were non-superimposable regardless of rotation. The time to respond "same" was found to be a linear function of the angle of rotation between the two figures. There was no difference between reaction times for depth and picture plane rotations. The linear function increased at a rate of about 60° per second though there was some variation between subjects. Metzler (1973) demonstrated that reaction times for the "different" stimuli (different by a mirror reflection) resulted in similar linear functions when the angle to "partial congruence" was used as the independent variable. In a second experiment Metzler presented a perspective figure until the subject indicated that he could maintain the image, whereupon the figure was removed, and the subject was told to rotate the image either clockwise or counterclockwise around a prespecified axis. A second figure was then presented at an orientation corresponding to the orientation of the image if it had been rotating at the constant rate previously estimated. Under these conditions reaction times to respond "same" were constant regardless of the divergence in orientation of the second figure from the first. The linearity of the results in the first experiment and the flattening of the results in the second experiment suggest that the subject performs a transformation "that is in some sense an analog of the sort of continuous rigid rotation that might be performed on the corresponding external object," and is in that sense a mental rotation of the image (Cooper & Shepard, 1973, p. 87).

Shepard and Klun (1973), Cooper (1973) and Cooper and Shepard (1973) have adjusted the above paradigm to include 2-D objects which have a preferred orientation. One might expect that in order to determine if a stimulus matches some internal representation stored in long-term memory, a subject might first rotate the stimulus to test the congruence with the stored representation. Such a process is probably not necessary for stimuli which differ grossly in structural features. Consequently, in these studies subjects were presented stimuli which were either the correct or backward (mirror reflection) version of a familiar figure. Subjects were required to respond "correct" or "backwards" by pressing one of two keys. It is probably not necessary to mentally rotate an upside down "R" to identify it as an "R;" the structural features are probably sufficient. To identify it as a forward or backward version of an "R" is more difficult and, as these studies demonstrate, involves mental rotation of the "R" back to its preferred orientation presumable to match it against an "R" template. Rotations of letters (Cooper & Shepard, 1973; Cooper, 1973; Shepard & Klun, 1973), of hands (Cooper & Shepard, 1975), and of random shapes (Cooper, 1973) follow a slightly non-linear function when reaction times to identify the shapes as forward or backward are plotted against the degree of angular divergence of the test stimulus from the preferred orientation. This non-linearity seems to be a product of stimuli with preferred orientations and can be eliminated by having subjects mentally rotate an image of the shape away from the preferred orientation before the test stimulus is presented.

When both stimulus identity and orientation are known in advance of presentation of the test stimulus, and the information is accurate, the functions for forward-backward identification are flat. When either orientation or letter identity (but not both) are known, there is a slight decrement in overall reaction time from a no advance information condition, but there is no flattening of the function. Such results are further support for the hypothesis that subjects are mentally rotating an image of the stimulus from a preferred orientation in order to match it against the test stimulus. The same process seems to be involved whether the two stimuli are both physically present or if one is present and the other is in long-term memory.

Unlike the perspective figures used by Shepard and Metzler (1971), these 2-D stimuli were rotated rapidly at about 450° per second. One might argue that the letters used in some of these studies were simpler and more familiar than the perspective drawings used in the Shepard and Metzler studies, and that as a consequence, were more easily rotated. Cooper (1973), however, obtained the same type results using random 2-D shapes, and found no differences in rotation velocity for stimuli differing in complexity. She defined complexity in terms of the number of vertices of the random shapes. In addition, these stimuli were rotated at rates comparable to common letters. These results make it difficult to account for the differences in rotation rates found in the Shepard and Metzler studies and the Cooper and Shepard studies on the basis of complexity and/or familiarity. The random shapes, though unfamiliar by definition, were, however, preexposed to the subjects in order to create an orientation bias similar to that found for letters. Consequently, one cannot completely exclude familiarity as an explanation for the discrepant rates of rotation. In support of this conclusion it is interesting to note that subjects showed a tendency to give the stimuli familiar descriptions (e.g., a crab) which further confounds any attempt to assess the effects of familiarity.

This paper attempts to apply Cowan's perspective figures to the mental rotation paradigm developed by Shepard and colleagues. Rather than focusing simply on rotations in depth and in the Euclidean plane, this study will in addition examine the more complex transformations of inversion, eversion, and obversion. Such an analysis will serve both to delineate the critical variables in the perception of impossible figures and to extend the Shepard studies to more complex mental operations.

EXPERIMENT 1

In an attempt to examine the effects of stimulus complexity on the rate of mental rotation, Cooper (1973) had subjects rotate random shapes differing in number of vertices. Regardless of the degree of complexity measured in this manner, all shapes were rotated at comparable rates. The present study replicates the Cooper experiment using torus figures varying along two orthogonal dimensions of complexity: perimeter shape and possibility type. Nine tori were chosen from Figure 3 so as to conform with the 3 x 3 matrix shown in Figure 5. The three types of perimeter shape vary complexity in a manner similar to that used by Cooper. Perimeters 1 and 2 have six vertices; perimeter 3 has eight. Though P1 and P2 both have six vertices, they differ in the organization of those vertices. Within a particular perimeter shape level the outside torus structure is constant across the three possibility types. Only the internal organization is allowed to vary.

Unlike perimeter shape, variations along a "possibility" dimension would seem to manipulate complexity on a higher cognitive level than the number of vertices used by Cooper, and accordingly, complexity on a possible-impossible dimension might affect the rate of mental rotation where Cooper's shapes did not. One would expect impossible tori to be rotated at slower rates than the possible tori. However, since there are two interpretations of possibility, formal and psychological (Cowan & Pringle, 1975), it is an empirical question as to which is more applicable to rates of rotation. The three types of possibility used in this study were selected so as to test the relative influence of these two interpretations of possibility. The T1 tori are formally possible with 0-net-levels of depth; T2, formally impossible with 0-net-levels of depth; T3 formally impossible with 2-net-levels of depth. Both the formal and the psychological definitions of

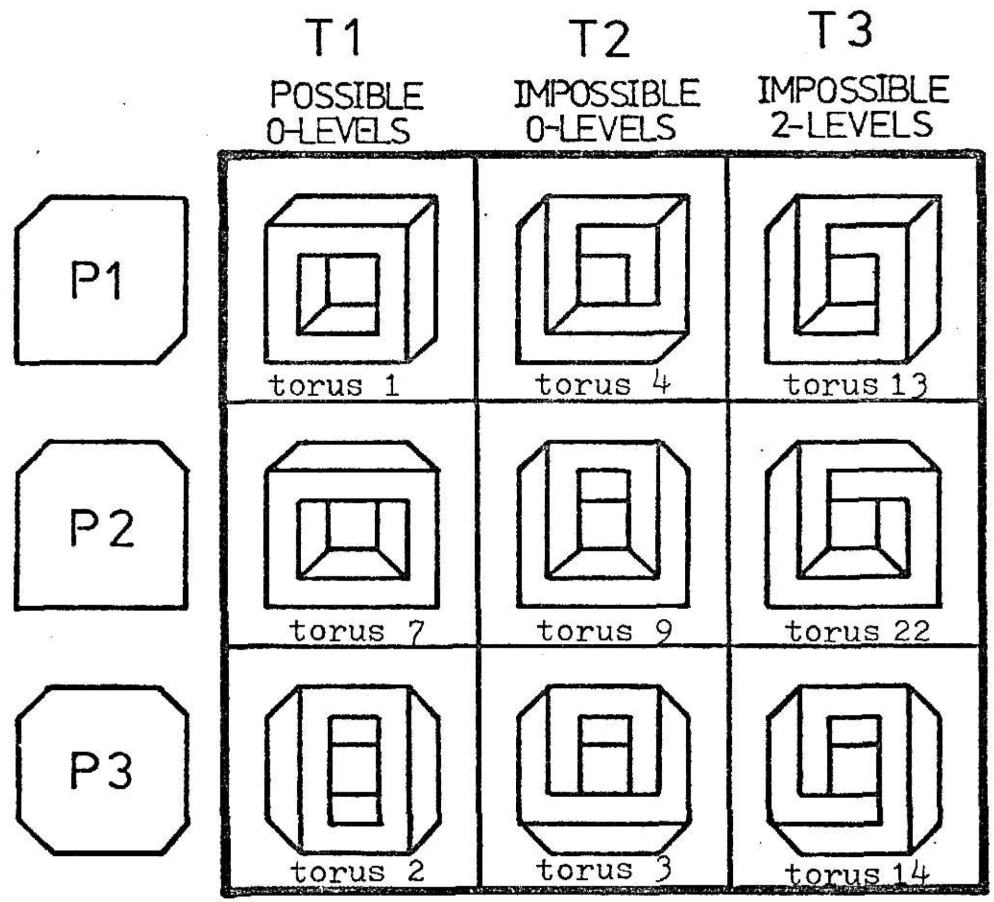


FIGURE 5. Matrix of stimuli used in Experiment 1 and 2. T = possibility type; P = perimeter shape.

possibility agree that the T1 tori are possible and that the T3 tori are impossible. However, they disagree on how to classify the T2 tori. Formally they are impossible -- they cannot exist in 3-space -- but they have 0-net-levels of depth, indicating that psychologically they are relatively high in possibility. Any differences between the T1 and T3 tori will indicate an influence of possibility; the T2 tori will help determine if the continuum of possibility is formally or psychologically defined. If the continuum is a psychological one, then the T2 figures should behave more like the T1 figures. If the continuum is formal, then they should behave more like the T3 figures.

In summary, the present experiment attempts to ascertain (a) if four-cornered tori are mentally rotated in a manner consistent with previous studies on mental rotation; (b) if torus complexity affects the rate of rotation and if such effects are specific to the possible-impossible dimension of complexity; and (c) if the effects of the possible-impossible complexity follow formal or psychological definitions of possibility.

METHOD

Subjects. Four subjects were used. Three were undergraduates (female) recruited from psychology courses at Kansas State University and received partial course credit for their participation. One subject (male) was a graduate student volunteer who has some previous experience with Cowan's set of tori. All subjects had normal or corrected vision.

Stimuli. Each torus subtended a 3.7° visual angle in the center of a white circular background subtending 7.6°. Tori were always presented in pairs in a two channel tachistoscope. The "standard" torus was presented 3.7° to the right of midline. Each "same" torus pair consisted of a standard torus chosen from Figure 5 and a comparison that was a 0°, 45°, 90°, 135°, or 180° clockwise rotation of the standard.

The traditional procedure for constructing "different" stimuli by pairing standards with their mirror reflections (e.g., Shepard & Metzler, 1971; Cooper, 1973) was not followed here. Six of the nine tori of Figure 5 are symmetrical, and a mirror reflection of a symmetrical figure is always a simple planar rotation of the original figure. Thus if the correct response is to be "different," then symmetrical tori must differ by more than mirror reflection -- there must be structural differences. Such structurally different stimuli were constructed from the tori of Figure 5. So that these "structurally different" stimuli retained a measure of similarity, each standard and comparison pair had identical perimeter shapes.

If structural differences are too great, subjects would be able to perform a match without mental rotation. To minimize the use of such a strategy, an additional class of "different" stimuli was included. For these stimuli the comparison was always a mirror reflected version of the standard, but neither the standard or comparison were taken from Figure 5. They were taken from the assymmetrical tori in Figure 3 that were not otherwise included in Figure 5 (i.e., tori 12, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, & 27). Overall, 40% of the "different" stimuli were of the structurally different class, and 60% were of the mirror reflection class. It is important to note that the test of the adaquacy of this procedure is simply whether or not subjects mentally rotate the tori. To the extent that the tori are mentally rotated to perform a match, the procedure can be said to have been a sufficient inducement of mental rotation.

Apparatus. The standard and comparison of each torus pair were presented simultaneously via a two channel tachistoscope under an illumination of 67 lm. To initiate a trial the experimenter closed a switch which sounded a 500 msec warning tone. The torus pair was presented 1 sec after the termination of the tone. The subject pressed a key to indicate when he or she had decided if the tori were "same" or "different." Immediately upon pressing the key the subject responded "same" or "different" verbally. A Standard 60 second clock began timing the response interval when the tori were presented and was terminated by the key press.

Procedure. The complete experimental design included four factors: three possibility types, three perimeter shapes, five extents of rotation, and four replications. The five extents of rotation were 0°, 45°, 90°, 135°, and 180°. Due to torus 2 the experimental design is actually incomplete across all five rotations. Torus 2 has two axes of symmetry, and consequently, its 135° and 180° rotations are redundant with its 45° and 0° rotations respectively. There are no unique 135° and 180° rotations for torus 2. Thus the experimental design is complete only across the 0°, 45°, and 90° rotations.

Each subject participated in five one-hour sessions of 117 trials each. The first session was considered practice, and the first 27 trials of each session were warm-up trials involving tori of Figure 3 that were not otherwise included in the experiment. Nine warm-up stimuli pairs (5 "same" and 4 "different") were presented three times in random orders with appropriate counterbalancing of extent of rotation and the orientation of the standard torus. Ninety experimental trials followed the 27 warm-up trials and consisted of 45 "same" and 45 "different" pairs presented in one of four random orders. Each of these random orders was presented once in the final four sessions of each subject according to 4 x 4 Latin square designs.

The 45 "same" stimuli consisted of the nine tori of Figure 4 presented in each of five rotation conditions (0°, 45°, 90°, 135°, & 180° from a partial congruence of the standard and comparison tori). The orientation of the standard tori of these 45 "different" pairs was random within the confines of always being horizontal, vertical, or left or right oblique.

Only correct responses to "same" stimuli were included in the analysis, and all incorrect responses to the "same" stimuli were repeated (with filler trials) at a later point in the session.

RESULTS

Mean reaction times to correctly respond to the "same" stimuli are shown in Tables 1, 2, 3, and 4. Reaction times pooled across all four subjects are plotted as a function of extent of rotation in Figure 6. Since the 135^{o} and 180^{o} rotations of torus 2 are undefined, only the first three extents of rotation are included. However, the data across all five extents of rotation will be discussed at a later point. It is apparent from Figure 6 that the reaction times increased in a linear manner. A regression analysis attributed 98.9% of the variance to the linear component. This linearity is in agreement with other rotation studies (e.g., Shepard & Metzler, 1971) and suggests that in order to make an identity match of four-cornered tori, subjects mentally rotate (at a constant rate) one torus into congruence with the other.

Analyses of variance were performed on each subject individually and were based on the first three extents of rotation. The results are reported in Table 5. There was a significant main effect of degree of rotation for three subjects (the fourth approached significance: $p < .07$), and a trend analysis revealed significant linear components for all subjects. All residual components were insignificant. Mean reaction times to the three extents of rotation are shown graphically for each subject in Figure 7. The inverse of the slopes of the functions in Figure 7 estimate the rates of mental rotation. They were 115, 97, 113, and 30 degrees per second for S1, S2, S3, and S4 respectively. The average rate across all subjects was 67 degrees per second which is similar to the 60 degrees per second obtained by Shepard and Metzler (1971) for their perspective figures.

	T1			T2			T3			
	Possible, 0-Net-Levels			Impossible, 0-Net-Levels			Impossible, 2-Net-Levels			
D	P1	P2	P3	P1	P2	P3	P1	P2	P3	$\overline{D}$
0°	1.53	1.56	1.47	1.75	1.56	1.56	1.69	1.59	1.87	1.62
45°	2.00	1.69	1.65	2.33	2.02	1.64	2.52	1.90	2.28	2.00
90°	2.38	1.64	1.94	2.52	2.10	2.07	2.91	2.89	3.15	2.40
135°	3.02	2.15	2.16	2.66	2.02	1.96	3.40	2.63	3.14	2.57
180°	2.76	1.79	2.40	3.12	2.14	2.03	3.71	3.73	3.97	2.85
$\overline{PT}$	2.34	1.77	1.92	2.48	1.97	1.85	2.85	2.55	2.88	
$\overline{T}$		2.01			2.10			2.76		

$\overline{P1} = 2.55$ $\overline{P2} = 2.09$ $\overline{P3} = 2.21$

TABLE 1. Mean reaction times (seconds) for Subject 1. T = possibility type; P = perimeter shape (see Figure 5); D = degree of rotation. The 135° & 180° rotation times of torus 2 (T1, P3) were extrapolated from its 0°, 45°, & 90° rotation times (see text).

D	T1 Possible, 0-Net-Levels P1	P2	P3	T2 Impossible, 0-Net-Levels P1	P2	P3	T3 Impossible, 2-Net-Levels P1	P2	P3	$\overline{D}$
0°	1.55	1.55	1.44	1.59	1.68	1.57	1.71	2.38	2.24	1.74
45°	1.68	1.95	1.55	2.05	2.16	1.95	2.67	2.77	2.26	2.11
90°	2.53	2.04	1.55	2.45	1.94	2.70	4.45	2.71	3.61	2.66
135°	2.50	1.92	1.63	2.44	2.41	2.55	3.81	4.22	5.25	2.97
180°	2.11	1.94	1.68	3.35	1.78	1.93	3.18	3.39	4.49	2.65
$\overline{PT}$	2.07	1.88	1.57	2.38	2.00	2.14	3.16	3.09	3.56	
$\overline{T}$		1.84			2.17			3.27		
		$\overline{P1}$ = 2.54			$\overline{P2}$ = 2.32			$\overline{P3}$ = 2.43		

TABLE 2. Mean reaction times (seconds) for Subject 2. T = possibility type; P = perimeter shape (see Figure 5); D = degree of rotation. The 135° & 180° rotation times of torus 2 (T1, P3) were extrapolated from its 0°, 45°, & 90° rotation times (see text).

	T1			T2			T3			
	Possible, 0-Net-Levels			Impossible, 0-Net-Levels			Impossible, 2-Net-Levels			
D	P1	P2	P3	P1	P2	P3	P1	P2	P3	$\overline{D}$
0°	1.43	1.08	0.78	1.99	1.44	1.26	2.63	2.58	1.73	1.66
45°	2.06	2.17	0.78	2.57	1.65	1.44	4.06	2.61	2.90	2.25
90°	1.87	1.76	0.89	2.77	1.49	1.45	3.52	2.08	4.34	2.24
135°	2.88	1.58	0.95	3.55	3.33	1.61	3.70	3.13	3.02	2.64
180°	2.43	2.33	1.02	3.62	3.29	2.70	3.75	5.05	3.63	3.09
$\overline{PT}$	2.13	1.78	0.88	2.90	2.24	1.69	3.53	3.09	3.12	
$\overline{T}$		1.60			2.28			3.25		
	$\overline{P1}$ = 2.85			$\overline{P2}$ = 2.37			$\overline{P3}$ = 1.90			

TABLE 3. Mean reaction times (seconds) for Subject 3. T = possibility type; P = perimeter shape (see Figure 5); D = degree of rotation. The 135° & 180° rotation times of torus 2 (T1, P3) were extrapolated from its 0°, 45°, & 90° rotation times.

	T1			T2			T3			
	Possible, 0-Net-Levels			Impossible, 0-Net-Levels			Impossible, 2-Net-Levels			
D	P1	P2	P3	P1	P2	P3	P1	P2	P3	
0°	2.23	1.34	1.29	3.51	1.81	2.00	3.87	2.81	3.37	2.47
45°	4.72	1.77	1.56	6.01	1.73	2.22	8.65	3.66	7.92	4.25
90°	6.60	2.34	2.18	4.71	2.04	3.58	8.53	8.47	10.78	5.47
135°	5.71	2.91	2.57	4.78	2.17	2.95	10.10	4.71	9.01	4.99
180°	6.31	2.08	3.03	7.29	1.92	2.38	7.64	7.93	11.05	5.51
$\overline{PT}$	5.12	2.09	2.12	5.26	1.93	2.62	7.76	5.51	8.43	
$\overline{T}$		3.11			3.27			7.23		
	$\overline{P1}$ = 6.05			$\overline{P2}$ = 3.18			$\overline{P3}$ = 4.39			

TABLE 4. Mean reaction times (seconds) for Subject 4. T = possibility type; P = perimeter shape (see Figure 5); D = degree of rotation. The 135° & 180° rotation times of torus 2 (T1, P3) were extrapolated from its 0°, 45°, & 90° rotation times (see text).

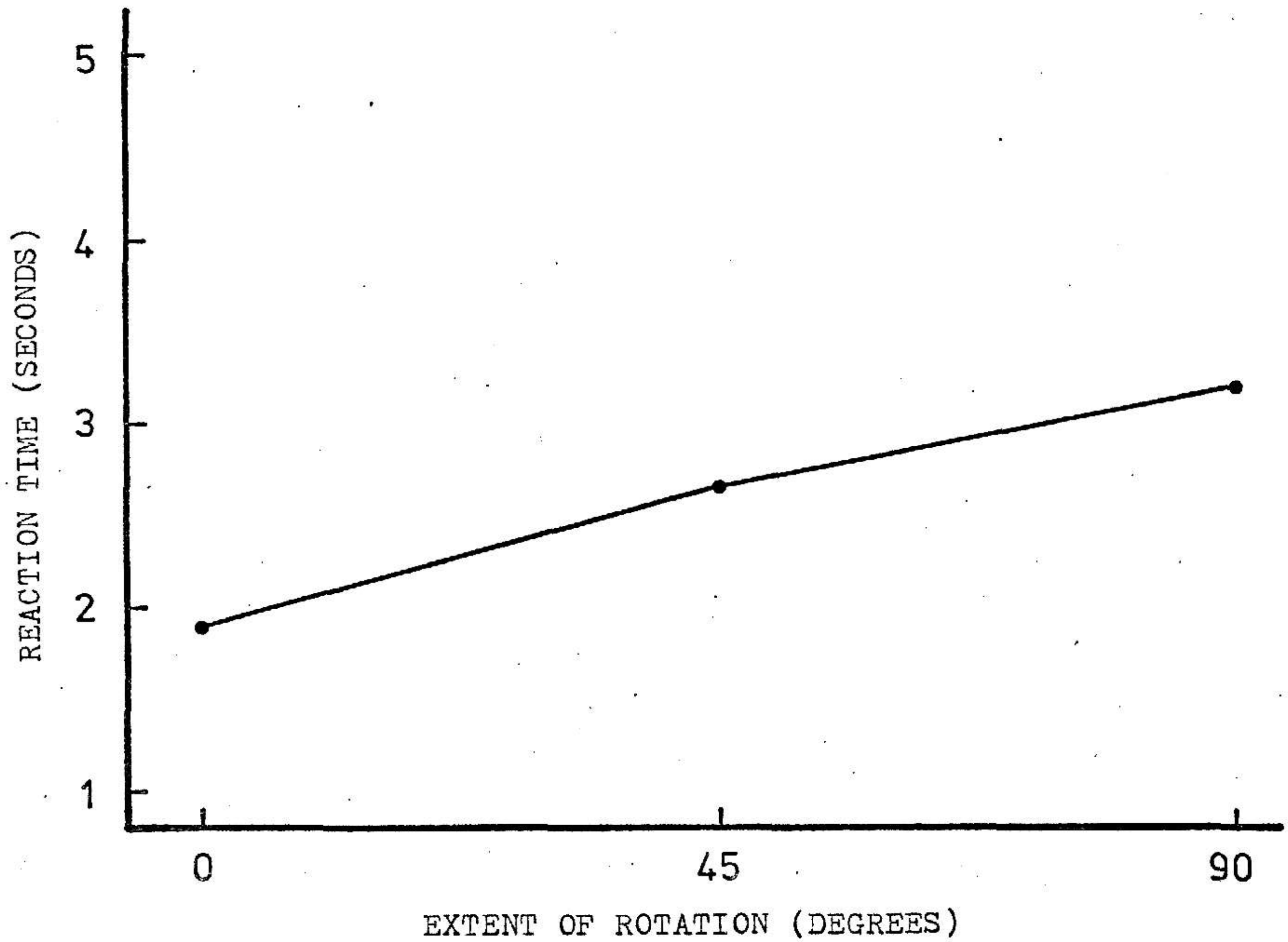


FIGURE 6. Mean time to respond "same" (pooled across subjects) as a function of the planar rotation of the comparison torus from the standard.

SOURCE	df	S1	S2	S3	S4
Mean	1	3345.**	1215.**	309.6**	300.5**
Possibility Type (T)	2	22.51**	23.05**	15.35**	26.22**
T2-T1	1	5.13*	2.69	1.60	<1.0
T3-T2	1	18.82**	23.87**	16.32**	34.80**
Perimeter Shape (P)	2	7.00**	1.00	4.65*	9.95**
P2-P3	1	<1.0	---	<1.0	2.98
P1-(P2+P3)/2	1	9.93**	---	9.05**	12.69**
Degree Rotation (D)	2	43.83**	18.38**	2.83	13.82**
linear	1	87.64**	36.31**	4.20*	27.32**
residual	1	<1.0	<1.0	1.48	<1.0
T x P	4	1.88	<1.0	1.17	1.56
T x D	4	4.68**	2.06	0.32	3.40**
(T1-T2)D	2	<1.0	<1.0	---	<1.0
[T3-(T1+T2)/2] D	2	5.81**	3.89*	---	6.47**
P x D	4	1.34	3.23*	0.67	1.11
(P1-P3)D	2	---	1.54	---	---
[P2-(P1+P3)/2] D	2	---	4.92*	---	---
T x P x D	8	<1.0	1.17	<1.0	<1.0
R(TPD)	81				

TABLE 5. Abbreviated analysis of variance table (F ratios) for each subject. The analysis is based on three extents of rotation (0°, 45°, & 90°). $*p<.05$; $**p<.01$.

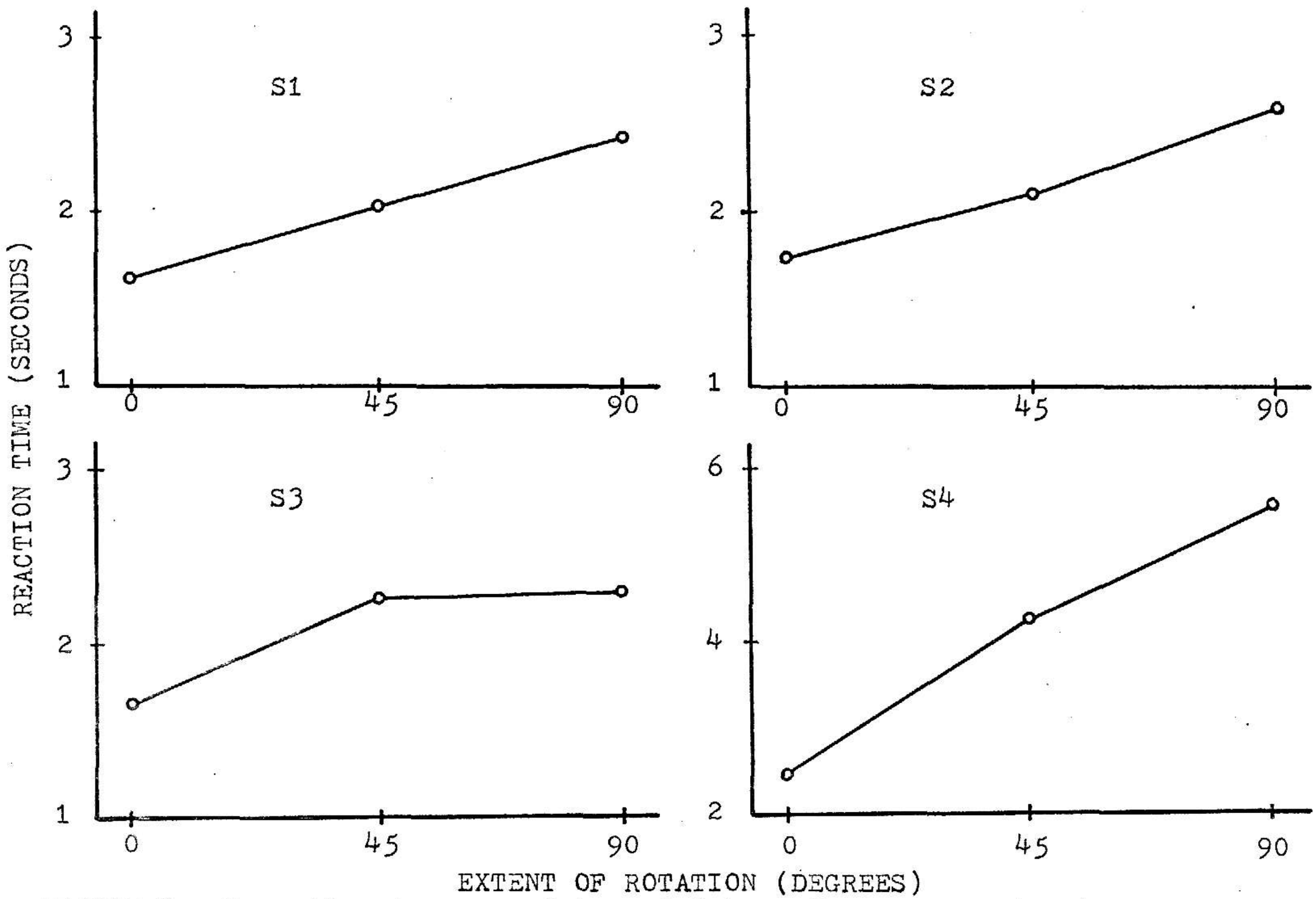


FIGURE 7. Mean time to respond "same" (shown for each subject) as a function of the planar rotation of the comparison torus from the standard.

Perimeter Shape. The reaction times pooled across subjects are plotted against extent of rotation for the three perimeter shapes in Figure 8. There were significant main effects of perimeter shape for three subjects, and all three were due primarily to the longer RT's associated with P1 than P2 or P3. A perimeter shape effect due to the number of vertices would predict longer RT's to P3 than to P1 or P2, and in this respect these results are in agreement with Cooper (1973) where no main effects of number of vertices was found. Perimeter shape does have an effect, however, and it might be due to the type of symmetries associated with the three perimeters. P1 has diagonal symmetry whereas P2 and P3 both have perpendicular symmetry. It is important to note that these symmetries are not defined relative to gravity but relative to the perimeter structure itself.

The "savings" associated with the perpendicularly symmetrical perimeters did not give rise to faster mental rotation. There was, for example, only one subject (S2) who showed a perimeter shape by extent of rotation interaction, and it was inconsistent with the symmetry dichotomy (e.g., there was no difference between the rotation rates of the P1 and P3 tori). Nor was it consistent with an effect of number of vertices since the P1 and P3 tori have 6 and 8 vertices respectively. Thus as in Cooper's random shape study, number of vertices (and perimeter shape in general) did not effect the rate of mental rotation.

Possibility Type. The mean reaction times across all subjects are plotted against extent of rotation for the three possibility types in Figure 9. The main effect of possibility type was significant for all subjects and was primarily due to the slower RT's associated with the impossible, 2-net-level tori (T3). For all subjects the impossible, 0-net-level tori (T2) elicited slower RT's than the possible, 0-net-level tori (T1), but this difference reached significance for only one subject (S1).

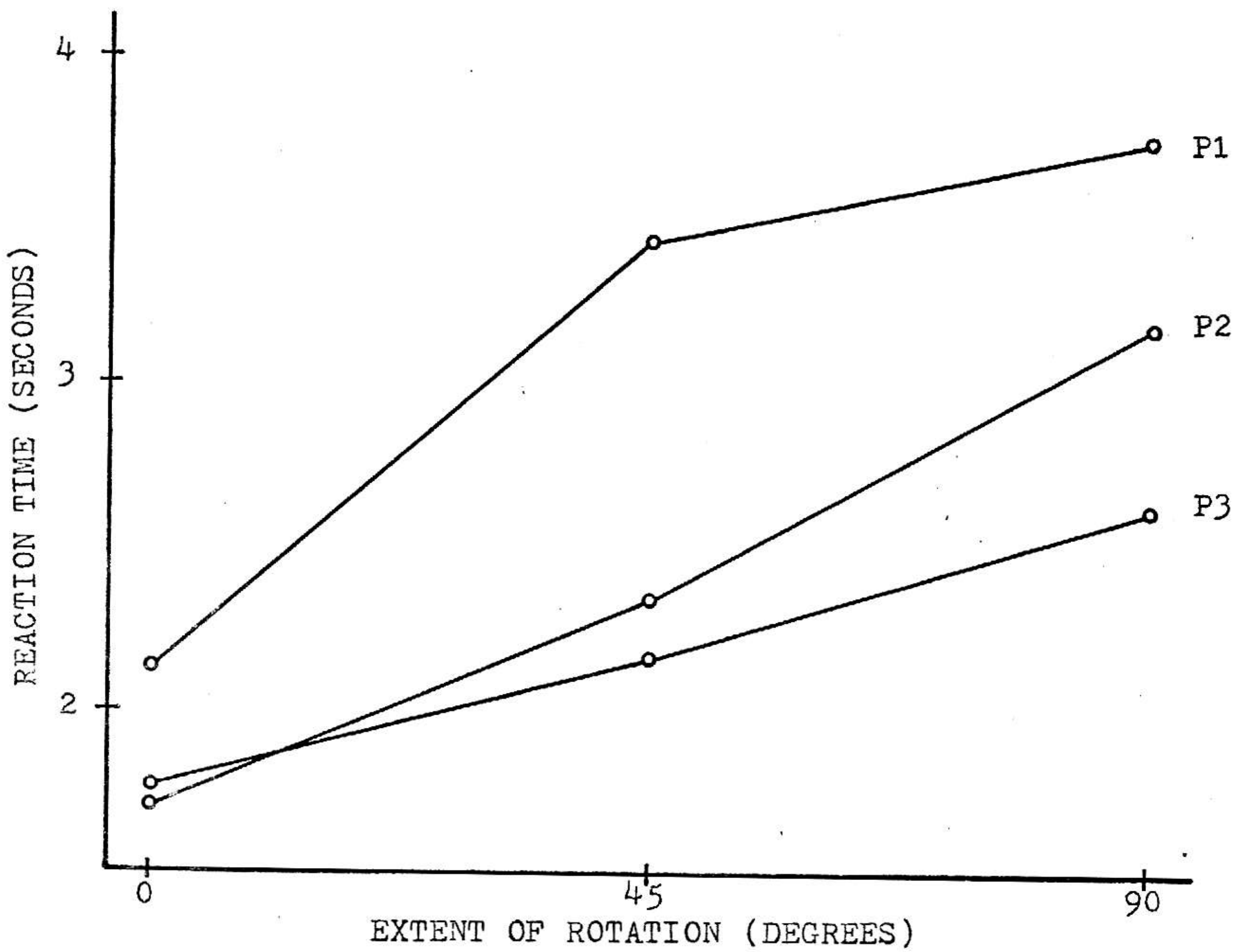


FIGURE 8. Mean time to respond "same" (pooled across subjects) as a function of planar rotation for each Perimeter Shape.

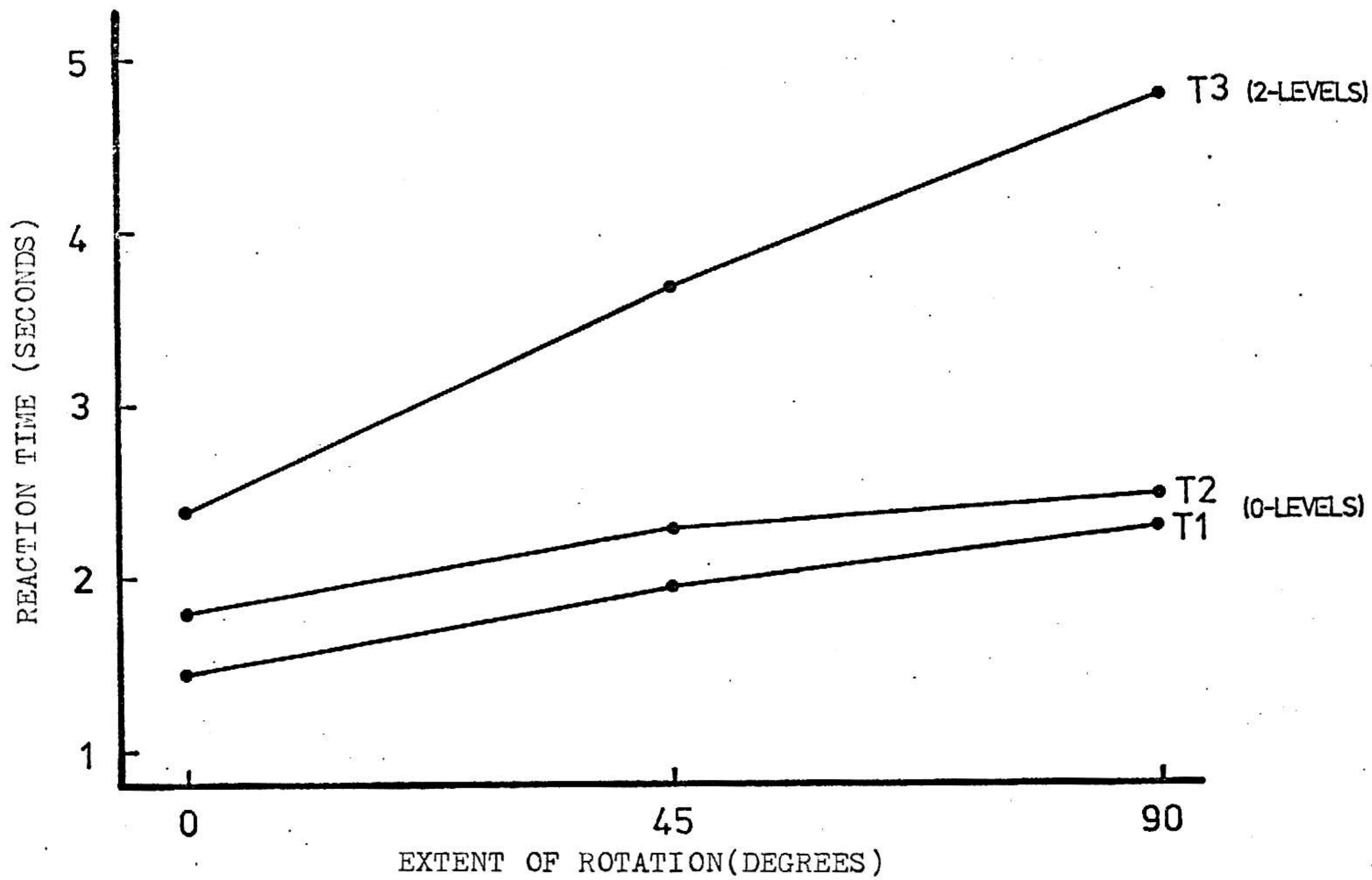


FIGURE 9. Mean time to respond "same" (pooled across subjects) as a function of planar rotation shown for each Possibility Type.

In contrast, the difference between the 0- and 2-net-level impossible tori was significant for all subjects. The directions of these differences were consistent: $T1 < T2 < T3$.

In addition to the main effect of possibility type there were significant possibility type by extent of rotation interactions for three of the four subjects. These interactions are shown graphically in Figure 10. In each case the interaction was due to the greater slope (slower rotation rate) of the impossible, 2-net-level tori (T3) relative to the 0-net-level tori (T1 & T2). There were no slope differences between the possible and impossible, 0-net-level tori. On average, the possible and impossible, 0-net-level tori were rotated at 113 degrees per second. The 2-net-level tori, on the other hand, were rotated at 37 degrees per second. These rotation rates are broken down according to individual subjects in Table 6, and in each case the 2-net-level tori are associated with slower rates than the 0-net-level tori.

The reaction time and rotation rate differences attributable to possibility type seem to follow changes in the psychological (net levels of depth) rather than the formal dimension of possibility. The fact that tori are impossible makes little difference in rate of mental rotation unless they are considered highly impossible on a psychological dimension. This psychological dimension can be defined by the number of net levels of depth a torus undergoes when examined in a consistent (e.g., clockwise) direction (Cowan & Pringle, 1975), and thus it is not surprising that these net levels of depth determine the ease of mental manipulation of the tori. In contrast to Cooper's (1973) study, the present study demonstrates that complexity can influence the rate of mental rotation when complexity is of a high order.

Five-Rotation Analyses. An attempt was made to generalize these results to all five extents of rotation used in this study. Two analyses

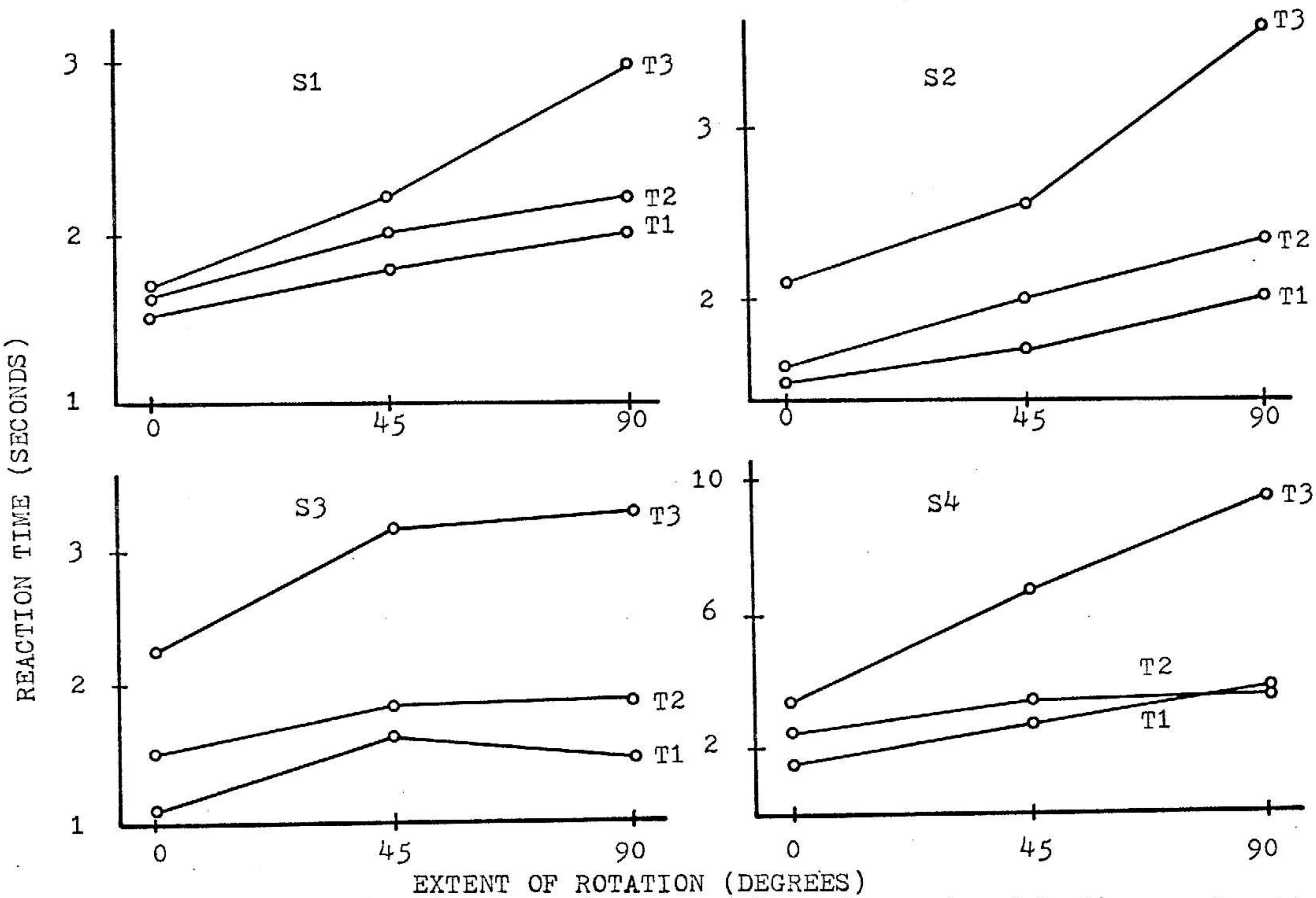


FIGURE 10. Mean time to respond "same" (shown for each subject) as a function of planar rotation for each Possibility Type.

	S1	S2	S3	S4	$\bar{X}$
T1	191.5	167.5	105.4	43.1	101.5
T2	146.4	119.0	207.2	75.6	123.5
T3	70.2	58.1	75.6	15.2	37.2
$\bar{X}$	115.3	96.7	113.9	29.7	67.4

(degrees/second)

TABLE 6. Mean rotation rates for each subject based on three extents of rotation (0°, 45°, & 90°). Each rate was derived from a least squares estimate of the best fitting line.

of variance, which made different assumptions about the 135° and 180° rotations of torus 2, were performed for each subject across all rotations. In the first the 135° and 180° rotation times of torus 2 were assumed equivalent to its 45° and 0° rotation times respectively. In the second the 135° and 180° rotation times of torus 2 were estimated by a linear extrapolation from its 0°, 45°, and 90° rotation times. The degree to which the second analysis differs from the first can be considered a measure of the amount of influence the anomolous nature of torus 2 had on the five-rotation analyses. The results of these two analyses are shown in Tables 7 and 8. There were essentially no differences in the two ANOVA's. Thus torus 2 had a minimal impact on the analyses, and the results cannot be attributed to the nature of its 135° and 180° rotations.

The results of the five-rotation analyses are in basic agreement with the results of the analysis based on three rotations. There were main effects of perimeter shape due primarily to the slower times associated with P1. There were no perimeter shape by extent of rotation interactions. Main effects of possibility type were due principally to the slower RT's to the 2-net-level tori. There were three possibility type by degree of rotation interactions, and all were due to the slower rate of rotation of the 2-net-level tori than the 0-net-level tori. Unlike the three rotation analysis, however, two subjects showed non-linear rotation functions. The rotation functions across all five rotations are shown for each subject in Figure 11. Whatever the cause of the non-linearity, it seems to have diminished substantially by the final replication (final session) as demonstrated by the dotted line functions in Figure 11.

In addition, the five-rotation analyses revealed significant possibility type by perimeter shape interactions for three subjects. The significance of these interactions seems to result from the increased power of the five-

SOURCE	df	S1	S2	S3	S4
Mean	1	4030.**	1765.**	492.0**	583.3**
Possibility Type (T)	2	52.23**	58.04**	20.70**	54.59**
T2-T1	1	4.57*	6.24*	7.33*	<1.0
T3-T2	1	57.54**	61.28**	13.85**	76.14**
Perimeter Shape (P)	2	17.50**	1.24	7.08**	20.51**
P2-P3	1	<1.0	---	3.65	5.06*
P1-(P2+P3)/2	1	35.01**	---	10.48**	16.43**
Degree Rotation (D)	4	31.55**	14.21**	4.71**	8.56**
linear	1	121.0**	40.89**	17.82**	23.33**
residual	3	1.70	5.31**	<1.0	3.74*
T x P	4	3.66**	2.55*	<1.0	4.42**
(P2-P1)T	2	<1.0	<1.0	---	<1.0
[P3-(P2+P1)/2] T	2	6.61**	4.68*	---	8.45 **
P x D	8	1.50	1.36	<1.0	<1.0
T x D	8	5.91**	3.22**	<1.0	2.15*
(T1-T2)D	4	<1.0	<1.0	---	<1.0
[T3-(T1+T2)/2] D	4	11.04**	6.19**	---	7.84**
T x P x D	16	1.47	1.50	<1.0	1.03
R(TPD)	135				

TABLE 7. Abbreviated analysis of variance table (F ratios) for each subject. The analysis is bases on five extents of rotation (0°, 45°, 90°, 135°, & 180°). $*p<.05$; $**p<.01$.

SOURCE	df	S1	S2	S3	S4
Mean	1	3870.**	1777.**	489.4**	584.7**
Possibility Type (T)	2	41.51**	56.78**	19.80**	51.63**
T2-T1	1	1.04	5.53*	6.62*	<1.0
T3-T2	1	53.79**	61.44**	13.57**	74.24**
Perimeter Shape (P)	2	14.03**	1.15	6.60**	19.56**
P2-P3	1	1.94	---	3.20	7.01**
P1-(P2+P3)/2	1	26.13**	---	10.03**	167.1**
Degree Rotation (D)	4	34.57**	14.49**	4.93**	9.10**
linear	1	135.6**	42.94**	18.37**	26.53**
residual	3	<1.0	5.1.**	<1.0	3.31*
T x P	4	2.51*	2.24	<1.0	3.52**
(P2-P1)T	2	<1.0	---	---	<1.0
[P3-(P2+P1)/2]T	2	4.28*	---	---	6.54**
P x D	8	1.03	1.49	<1.0	<1.0
T x D	8	4.63**	3.12**	<1.0	1.93
(T1-T2)D	4	<1.0	<1.0	---	<1.0
[T3-(T1+T2)/2]D	4	8.34**	3.91**	---	3.60**
T x P x D	16	<1.0	1.54	<1.0	<1.0
R(TPD)	135				

TABLE 8. Abbreviated analysis of variance table (F ratios) for each subject. The analysis is based on five extents of rotation (0°, 45°, 90°, 135°, & 180°). The 135° & 180° rotation times of torus 2 were extrapolated from its 0°, 45°, & 90° rotation times. $*p<.05$; $**p<.01$.

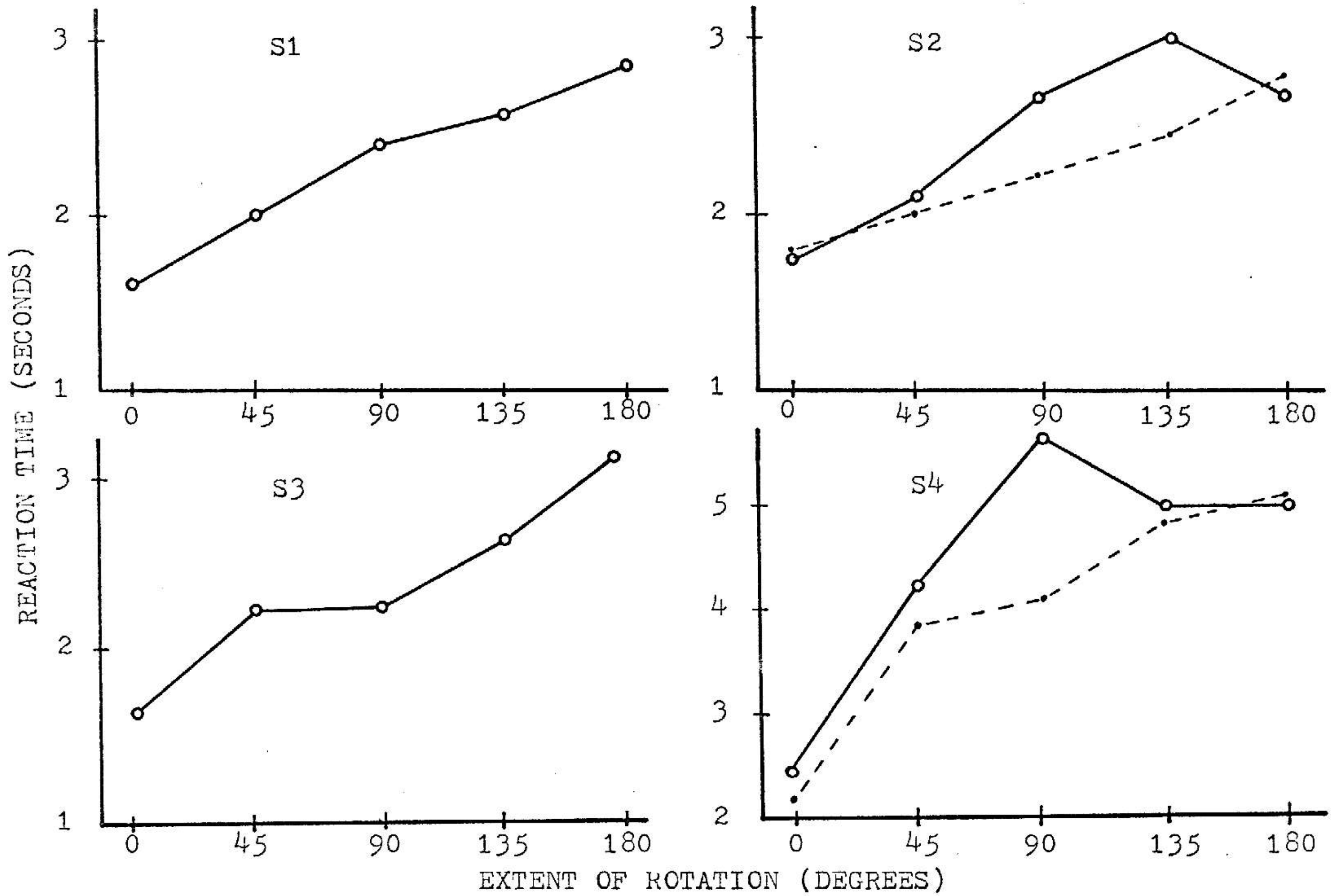


FIGURE 11. Mean time to respond "same" (shown for each subject) as a function of all five extents of rotation. Dotted line functions are data from final replications.

rotation analyses. Partitioning of the interactions indicates they were due to the greater effect of possibility type on the P3 tori than on the P1 and P2 tori. Effects of possibility type were in the same direction for all perimeter shapes, however.

CONCLUSIONS

Subjects seem to mentally rotate one torus into congruence with another when required to match two tori differing only in orientation. Rates of rotation differ according to figure complexity if the complexity is defined along a possible-impossible dimension. Complexity of perimeter shape, on the other hand, does not seem to affect rotation rate. Net levels of depth is more critical to rotation rate than formal distinctions of possibility, a result that suggests psychological possibility is the critical factor.

An alternative interpretation is available, however. The 0-net-level tori of this study are symmetrical whereas the 2-net-level tori are not. It could be the case that symmetrical figures are rotated faster than assymmetrical figures. That would be an interesting result in its own right. However, it is extremely difficult to separate net levels of depth from overall figure symmetry. There is, for example, only one figure (torus 24) which has 0-net-levels of depth and is assymmetrical, and a more profound problem is that ultimately levels of depth can be defined in terms of a symmetry between corners. Thus the two explanations are perhaps forever confounded.

A recent study by Corballis and Roldan (1975), however, strongly suggests that symmetry is not the critical factor. In order to decide if dot patterns were symmetrical or assymmetrical around a predesignated axis, subjects mentally rotated the axis to vertical. The important point for the present study is that the rates of rotation for the symmetrical and assymmetrical patterns did not differ. To the extent that these findings can be generalized to the rotation of four-cornered tori, they indicate that net levels of depth -- not symmetry -- is the determinant of rate of mental rotation.

EXPERIMENT 2

The purpose of Experiment 2 was twofold: (1) to examine the effects of possibility type and perimeter shape on the ability (reaction time) of subjects to perform the complex transformations of inversion, eversion, and obversion; and (2) to replicate the mental rotation results of the first experiment in the context of these more complex transformations. Two subjects in Experiment 1 reported regarding the tori as flat figures -- a strategy that would tend to mitigate any effects of the dimension of possibility. To the extent that subjects could ignore the possibility dimension in this manner, the differential rates of rotation found in the first experiment would seem to be due to torus symmetry. Since eversions and obversions transform a torus out of the plane, such transformations demand a three-dimensional interpretation of the tori.

METHOD

Subjects. Subjects were drawn from undergraduate psychology courses at Kansas State University and received partial course credit for their participation. Subjects were selected for the experiment on the basis of their ability to identify six types of torus transformations: inverse, everse, obverse, and 0°, 90°, 180° planar rotations. The description of this procedure is given in detail below. Subjects who identified 95% or more of the transformations were included in the experiment. Twelve subjects participated in this pretest; eight met the above criteria. Five were female, three male.

Stimuli. As in Experiment 1, the tori were presented in pairs. In each case the standard torus was presented 3.7° to the left of visual midline and the comparison 3.7° to the right. One of six transformations (everse, obverse, inverse, and 0°, 90°, 180° rotations) was named by the experimenter prior to each trial, and when the torus pair was presented, the subject's task was to decide if the comparison torus differed from the standard torus by the transformation named. The nine tori of Figure 5 served as standard tori. Each comparison for the "same" trials was simply that transformation of the standard torus named by the experimenter prior to the trial. 54 "same" trials were constructed by combining factorially the nine tori of Figure 5 with the six types of transformations listed above. The comparison for a "different" trial differed from the standard by a transformation other than the one named. Two types of "different" transformations were used: (1) structural transformations (the comparison was an altogether different torus from the standard but nevertheless had the same perimeter shape) and (2) vertation and/or rotation transformations (the comparison was a rotation, a vertation, or a rotation and vertation of the standard). One third (18) of the "different" trials were of the structurally different class, and two thirds (36) were of the vertation/rotation class for a total of 54 "different" trials. The type of "different" transformations was counterbalanced with the nine standard toruses. Within these confines the selection of the comparisons for the "different" trials was randomized for each subject.

A pool of practice stimuli was constructed from the tori (and vertations) of Figure 3 which were not also included in Figure 5. This pool allowed the selection of 30 practice trials consisting of 5 trials for each transformation. 15 of the 30 trials were "same," and 15 were "different."

Apparatus. The stimuli were presented in the same manner as employed in Experiment 1. The subject pressed a key to indicate that he or she had decided if the tori were "same" or "different." Immediately upon pressing the key, the subject responded "same" or "different" verbally. A 60 second clock timed the response interval.

Pretesting Procedure. All subjects participated in 1 to 2 one-hour sessions of training and pretesting. During these sessions the transformations were defined and the subjects practiced identifying the transformations. Ability to correctly identify the transformations served as a basis of admission into the experiment proper.

Tori 6 and 7 of Figure 3 were used as models with which to demonstrate the transformations. The 0°, 90°, and 180° rotations were defined as clockwise and confined to the plane. Inversion was always referred to as mirror image and was defined as a left-right reversal. Examples of the mirror reversals of various letters were given. An eversion was referred to as the back side brought forward and was defined as conceptually bringing the back surfacr of the torus up through the front surface "as if you were turning a sock inside out." An obversion was referred to as the back side turned over and was defined as rotating the torus out of the plane around the vertical axis. Using torus 7 the differences between the back side turned over and the back side brought forward were pointed out. It was noted that the side brought forward (everse) and the back side turned over (obverse) always differed by a mirror image. All eight subjects seemed to understand this relationship.

At this point the subject was given one of two practice booklets. Each booklet contained six standard tori not included in Figure 5. Below the standard torus were four to eight comparisons. The subject was instructed to pick out each of the six transformations for each standard -- a total of 36 transformations. Subjects were allowed as much time as necessary to complete the booklet. This required about 15-20 minutes for those subjects who were eventually run in the experiment proper, and 40-50 minutes for those subjects discarded. The latter required two sessions to complete pretesting. Answers to the first booklet were checked and corrections pointed out to the subjects. The second booklet was then administered, and subjects who incorrectly identified more than two transformations or more than one transformation of a given type were not run in the experiment proper. The subjects meeting these criteria missed an average of 3.1 in the first booklet and an average of 0.2 in the second booklet. Those subjects discarded missed an average of 16 in the first booklet and 9 in the second booklet. Incorrect eversions and obversions accounted for most of the errors (98%).

Testing Procedure. The 54 "same" and 54 "different" transformation trials previously described were presented to those 8 subjects who met criterion. Each subject was run in two sessions of 84 trials each, where the first 30 trials of each session were practice. The remaining 54 trials of each session consisted of 27 "same" and 37 "different" transformations. Each of the nine tori of Figure 5 was presented as a standard six times per session (three "same" and three" different"). Stimuli were presented to each subject according to one of four random orders of presentation.

The orientation of the standard torus was essentially random within subjects but was counterbalanced across subjects such that, for subsets of four subjects, a given perpendicular axis of each torus appeared once in each of four orientations (horizontal, vertical, and left and right oblique) for each of the six transformations.

The complete experimental design included three possibility types, three perimeter shapes, and six transformations.

RESULTS

Three analyses of variance were performed on the data. One was across all six transformations, one on the three rotations, and one on the three vertations. The results of all three analyses are shown in Table 9 with mean reaction times shown in Table 10. The overall analysis indicated significant effects of possibility type and transformation. The mean reaction

	COMPLETE DESIGN		PARTIAL DESIGNS			
	Vertations & Rotations		Vertations		Rotations	
SOURCE	df	F	df	F	df	F
Mean	1	106.4**	1	84.12**	1	108.9**
Subjects (S)	7		7		7	
Possibility Type (T)	2	19.83**	2	12.52**	2	12.87**
Perimeter Shape (P)	2	1.19	2	1.74	2	1.62
Transformation (V)	5	20.35**	2	19.91**	2	41.85**
ST	14		14		14	
SP	14		14		14	
TP	4	1.40	4	1.03	4	1.03
SV	35		14		14	
TV	10	1.33	4	0.48	4	4.91**
PV	10	0.96	4	0.84	4	0.49
STP	28		28		28	
STV	70		28		28	
SPV	70		28		28	
TPV	20	0.51	8	0.54	8	0.54
STPV	140		56		56	

TABLE 9. Abbreviated analysis of variance tables for the complete design in Experiment 2, including both vertations (inverse, obverse, & everse) and rotations (0°, 90°, 180°); and for the partial designs which include vertations or rotations.

	POSSIBLE, 0-LEVELS (T1)			IMPOSSIBLE, 0-LEVELS (T2)			IMPOSSIBLE, 2-LEVELS (T3)			
	P1	P2	P3	P1	P2	P3	P1	P2	P3	$\overline{V}$
0°	2.44	2.84	2.22	3.25	3.18	2.87	3.26	3.50	3.18	2.97
90°	4.91	3.60	3.65	4.92	4.50	4.67	8.57	7.74	8.06	5.62
180°	5.69	4.18	2.93	5.56	5.38	5.98	8.70	6.72	9.94	6.12
$\overline{TP}_{rotation}$	4.35	3.54	2.94	4.58	4.35	4.51	6.84	5.98	7.06	
$\overline{T}_{rotation}$		3.61			4.48			6.63		
Inverse	3.55	3.52	3.33	4.33	3.74	4.00	5.41	5.51	5.84	4.36
Everse	4.79	5.62	5.52	5.08	6.90	6.17	9.98	7.37	10.67	6.90
Obverse	7.32	7.40	6.54	7.60	7.42	9.85	9.49	8.77	13.32	8.64
$\overline{TP}_{vertation}$	4.43	4.26	4.39	6.61	6.63	7.46	8.14	7.86	9.90	
$\overline{T}_{vertation}$		4.36			6.90			8.64		

TABLE 10. Mean reaction times (seconds) based on all 8 subjects in Experiment 2. T=Possibility Type; P=Perimeter Shape; V=Transformation.

times to the six transformations are shown graphically in Figure 12. There were no effects of perimeter shape; nor were any interactions significant. In the rotation analysis, however, there was a significant possibility type by extent of rotation interaction.

Rotation Analysis. It is apparent from the graph in Figure 12 (rotation points only) that the rotation function is non-linear. Trend analyses revealed both significant linear ($F = 45.0$, $df = 1, 7$, $p < .001$) and significant residual ($F = 28.8$, $df = 1, 7$, $p < .01$) components. This non-linearity is in marked contrast with the findings of Experiment 1. However, subjects in Experiment 2 had considerably less experience in rotating tori than the subjects in Experiment 1 due to the presence of the vertation transformations and the lack of replications. Thus a plausable explanation of the non-linearity is that the subjects were simply not well practiced. To test this notion, the 0°, 90°, and 180° points from the first session of Experiment 1 are shown (dotted line) in Figure 12. The curves are essentially identical. It is evident that lack of practice gives rise to non-linearity, and it seems reasonable that lack of practice was responsible for the non-linearity in the present experiment.

Partitioning of the main effect of possibility type revealed significant effects between the 0- and 2-net-level impossible tori ($F = 35.1$, $df = 1, 7$, $p < .001$) and between the possible and impossible 0-net-level tori ($F = 8.6$, $df = 1, 7$, $p < .05$). The larger effect was clearly due to the effect of net levels of depth (see mean reaction times in Table 10). The extent of rotation by possibility type interaction is shown graphically in Figure 13 and was due to the relatively greater slope of the 2-net-level tori versus the 0-net-level tori ($F = 12.6$, $df = 2, 14$, $p < .001$). There was no difference in slope between the possible and impossible 0-net-level tori ($F = 1.6$, $df = 2, 16$). The average rate of rotation for the possible

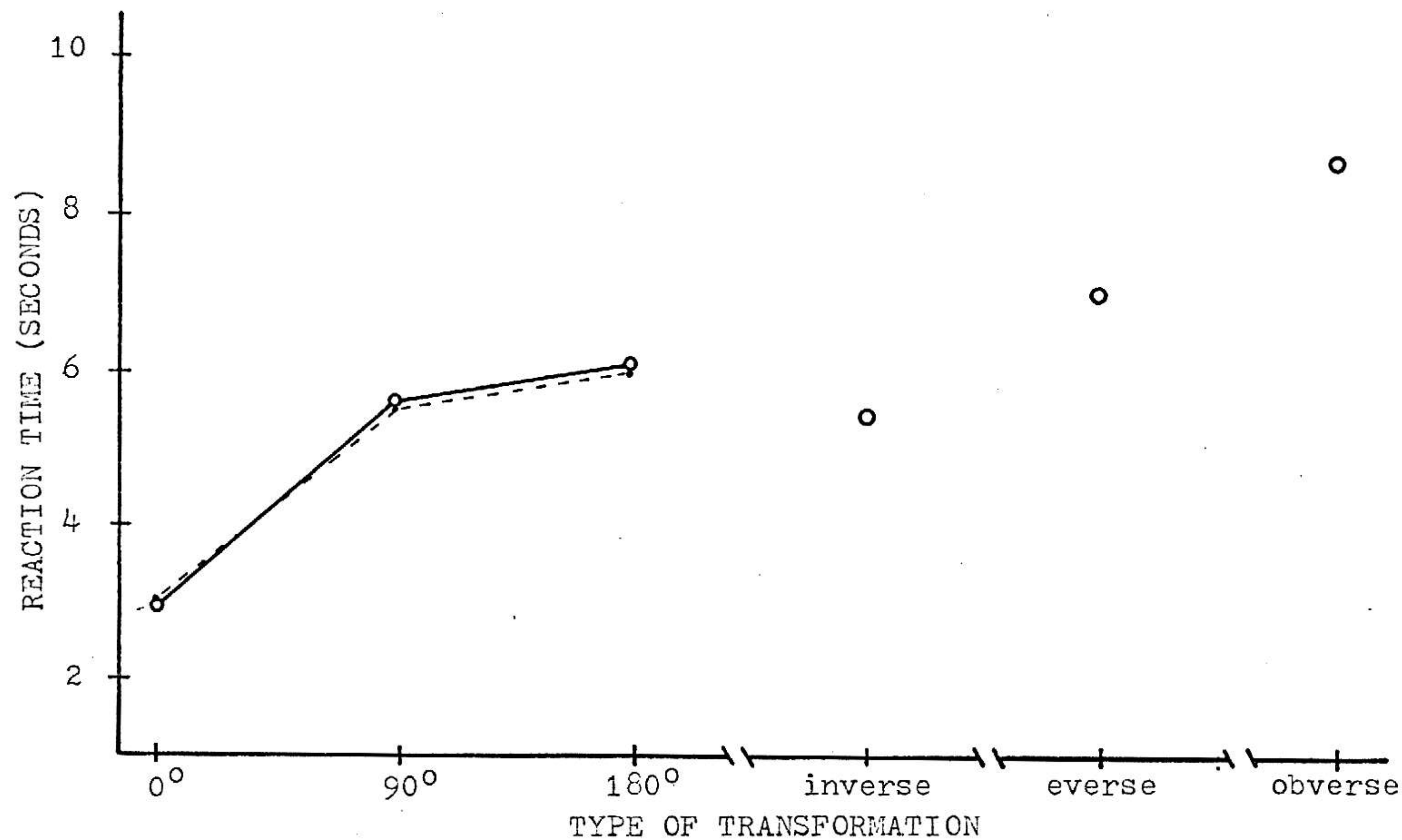


FIGURE 12. Mean time to respond "same" as a function of transformation type (Experiment 2). Dotted line function is data from the first replication in Experiment 1 pooled across subjects.

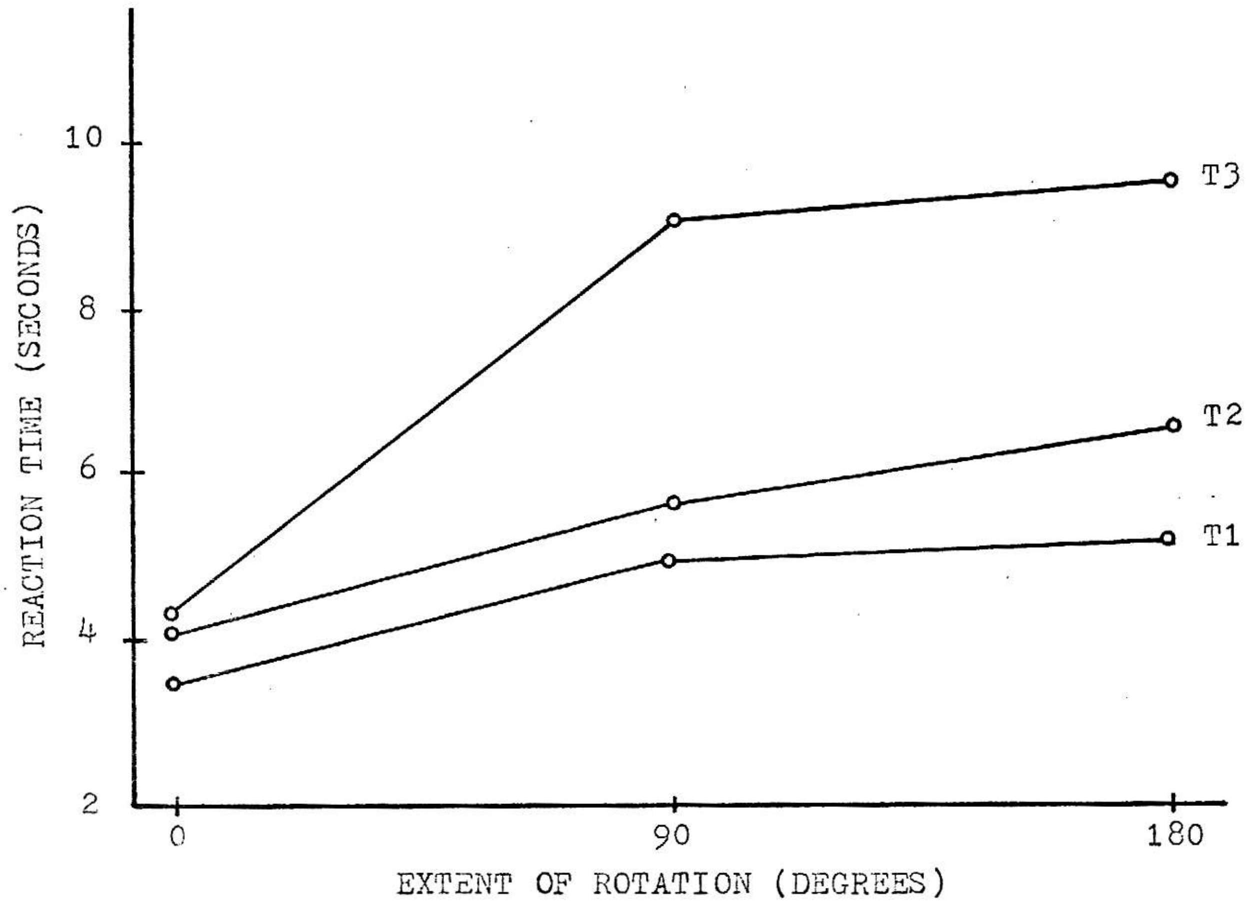


FIGURE 13. Mean time to respond "same" as a function of extent of rotation for each Possibility Type (Experiment 2).

and impossible 0-net-level tori was 85 and 61 degrees per second respectively. The 2-net-level tori, on the other hand, were rotated at 28 degrees per second.

Vertation Analysis. Comparisons between possibility types indicated a significant difference between the 0- and 2-net-level impossible tori ($F = 11.1$, $df = 1, 7$, $p < .05$) and no difference between the possible and impossible 0-net-level tori ($F = 1.4$, $df = 1, 7$). As can be seen in Figure 14, however, the numerical differences between the 0-net-level tori were in the direction consistent with all previous analyses involving the two possibility types (i.e., faster reaction times for the possible tori).

Inversion took reliably less time than eversion ($F = 19.6$, $df = 1, 7$, $p < .01$) which in turn took less time than obversion ($F = 9.5$, $df = 1, 7$, $p < .05$), and these relationships were consistent across perimeter shapes, possibility types, and subjects, and held up for all but one of the tori used (torus 13: $i < o < e$). Because of the psychological similarity of a figure and its mirror image (e.g., Bradshaw, Bradley, & Patterson, 1976), it was to be expected that mirror reversals would be faster than the more complex transformations (inversions also took less time than the 90° rotation: $F = 20.8$, $df = 1, 7$, $p < .01$). However, that obversion would take more time than eversion was unexpected. An eversion (bringing the back side up through the front surface) might be considered to be a more complex transformation than simply turning a figure over (obversion). However, several subjects commented during the training and pretesting sessions that it was easier to find the mirror image (inverse) of the back side brought forward (everse) to get to the back side turned over (obverse) than to turn the back side over directly. An inversion does not require a three-dimensional interpretation of the torus, whereas an everse requires at least a conceptual recognition that there is another plane of depth in the torus

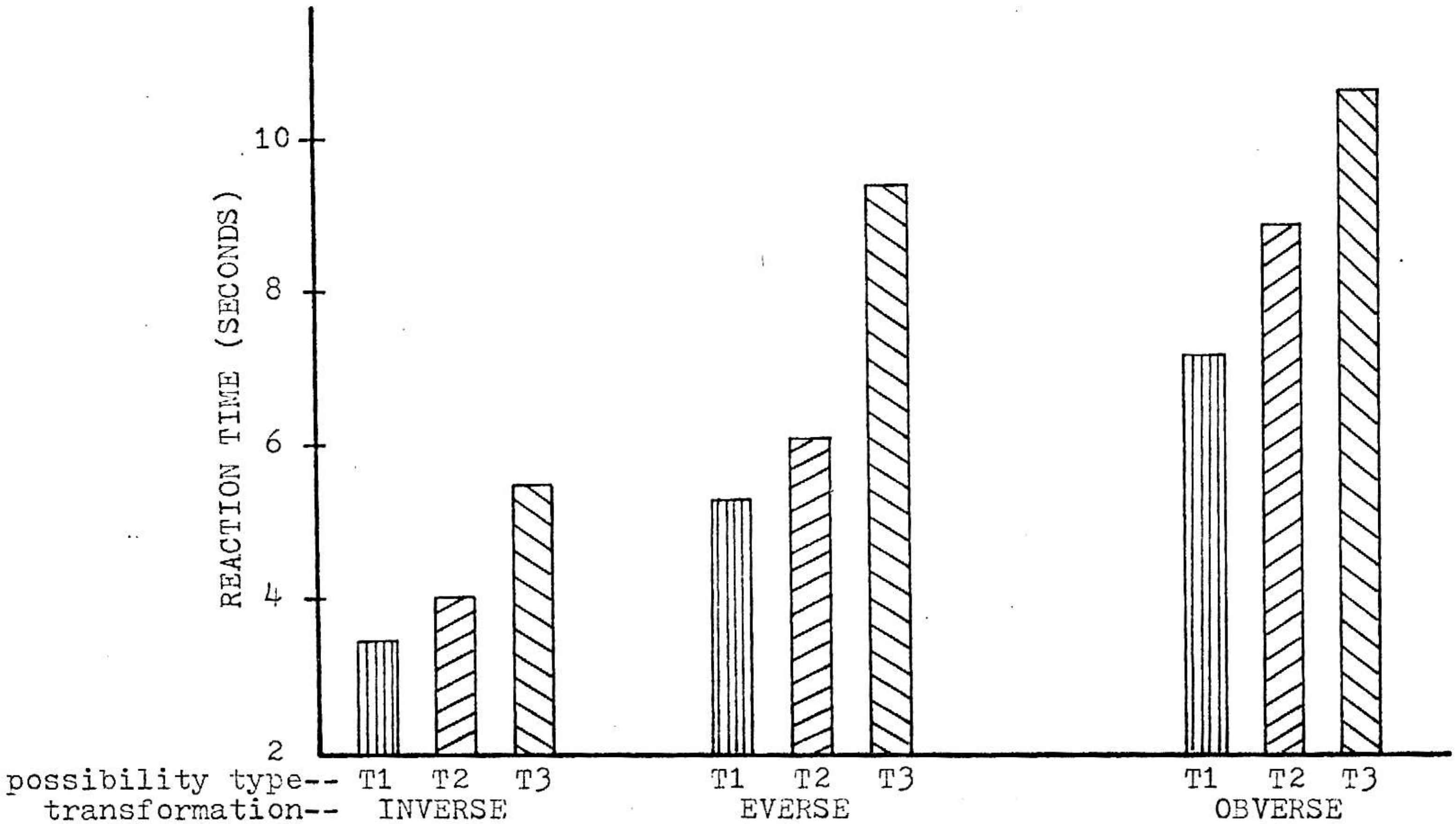


FIGURE 14. Mean time to respond "same" as a function of vertation type shown for each Possibility Type (Experiment 2).

(i.e., the back side). Obversion, on the other hand, requires both the recognition of the back surface and a rotation of the torus out of the plane. It may be the case that such rotations are resisted by the subjects if an alternative route is available. An alternative was available in this experiment; it was pointed out to the subjects that eversions and obversions differed by a mirror image. If subjects selected this route, then we can infer that eversion plus inversion is a simpler operation than a rotation out of the plane. To examine the extent to which this route might have been taken, reaction times to obversion were compared to the sum of the reaction times to inversion and eversion. Presumably, the reaction time to the 0° rotation represents matching and response times independent of a mental transformation. By subtracting the 0° rotation times from each of the complex transformation times, the cognitive times for the inverse, everse, and obverse transformations can be estimated and the appropriate comparisons made (i.e., [o-s] versus [i-s] + [e-s] where "s" is the reaction time to the 0° rotation). These transformation time estimates are shown in Table 11 along with the appropriate F ratios testing the differences between (e+i) and (o). The test was made on each possibility type as well as across all possibility types. None of the differences approached significance, and thus, within the reliability of the test, there is no evidence that subjects did not perform an eversion and inversion sequentially to get to an obversion. One might expect that the time to turn a torus 180° out of the plane around the y-axis would equal the time to turn a tours 180° in the plane. Shepard and Metzler (1971) found such a relation for their perspective drawings, and it is not unreasonable to expect similar findings for the torus figures used here. However, such was not the case; obversion time was reliably greater than 180° rotation time ($F = 8.6$, $df = 1, 7$, $p < .05$) -- a relation the direction of which held for all possibility types

	INVERSE	EVERSE	E + I	OBVERSE	F(1,7)
T1	1.16	2.80	3.96	4.59	1.75
T2	0.93	2.95	3.88	4.19	1.21
T3	2.28	6.03	8.31	7.22	0.59
$\bar{X}$	1.39	3.93	5.42	5.67	0.65

TABLE 11. Mean transformation times corrected for response time without transformation (i.e., 0° rotation), and F ratios testing differences between (Everse + Inverse) and (Obverse). T=Possibility Type; E= Everse; I= Inverse.

and perimeter shapes. This finding seems to suggest that subjects were not, in fact, rotating the tori out of the plane in spite of the fact that such rotations might have taken less time. One could argue that impossible tori are simply more difficult to rotate in depth than in the plane, but there is no a priori reason that the possible tori should not rotate one way as fast as the other. That they do not again suggests that the subjects opted for the alternative route of everting the figure and inverting the result (or visa versa) to get to an obversion.

DISCUSSION AND CONCLUSIONS

Results of Experiment 2 confirmed that (1) variations in torus possibility affected the rate of mental rotation in a manner consistent with a net levels of depth description of possibility, and (2) rate of rotation was unaffected by variations in formal possibility or in perimeter shape.

The main effect of perimeter shape was not significant in this experiment. However, the reaction times were in the same direction as those of Experiment 1 ($P1 < P2$ & $P3$), and a comparison between P1 and (P2 & P3) approached significance ($F = 4.5$, $df = 1, 7$, $p < .10$). In Experiment 1 this relation was interpreted as a "savings" for perpendicularly symmetrical tori relative to tori having diagonal symmetry, and that hypothesis is at best only weakly supported in Experiment 2. In neither experiment was there an effect of the number of vertices -- a result in agreement with Cooper (1973).

Unlike the findings of Experiment 1, reaction time was a somewhat nonlinear function of the extent of rotation. This discrepancy was interpreted as a result of a relative lack of experience of the Experiment 2 subjects. In this regard, S3 of Experiment 1 also participated in this experiment, and it is interesting to note that only she, out of the eight subjects showed linear data in the second experiment.

There are three findings of importance in regard to the complex transformations used in Experiment 2. First, most subjects were capable of making such transformations with great accuracy even on the unusually complex figures used in this study. Even those subjects who were discarded on the basis of their performance in the pretest showed a substantial improvement between the first and second booklets. Second, the time to perform the vertation transformations was affected by possibility in a manner consistent with that found for the rotation studies: possible, 0-net-level impossible, 0-net-level impossible, 2-net-level. Only the latter difference was reliable, however, suggesting that the differences between the 0- and 2-net-levels was a more important determinant of complex transformation times than a difference in formal possibility. Third, inversion times were shorter than eversion times which in turn were shorter than obversion times. The distinction between eversion and obversion times was interpreted as arising from an apparent resistance by subjects to rotating the tori out of the plane in spite of the fact that such a rotation consumes less time. They preferred instead to reach the same ends by successively everting the figure and inverting the result (or visa versa).

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MENTAL TRANSFORMATIONS OF POSSIBLE AND IMPOSSIBLE 4-CORNERED TORI

by

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When subjects give magnitude estimations of "degree of possibility" of 4-cornered tori, the ratings fall along a smooth continuum from very impossible to very possible (Cowan & Pringle, 1975). Thus some impossible figures are considered more impossible than others. A parameter closely correlated with this psychological continuum is "net levels of depth"--the algebraic sum of the changes in depth a figure seems to undergo when examined in a consistent direction (e.g., clockwise). This study examined the effect of this parameter on the time required for subjects to perform various types of mental transformations. Four types of transformations were used: (1) rotations in the plane, (2) 180° dihedral rotations (obversions), (3) mirror reversals (inversions) and (4) reversals of front and back surfaces (eversions).

A related study (Cooper, 1973) demonstrated that figure complexity (defined in terms of the number of vertices) did not effect the time required to mentally rotate random shapes. Since degree of possibility is a higher order of complexity than variations of perimeter structure, it was predicted that variations of possibility would affect transformation times where variations in number of vertices would not. To examine this notion, degree of possibility was varied orthogonally with perimeter shapes.

The time required for subjects to determine if two tori were "same" irrespective of rotations in the plane (Experiment 1) or irrespective of a predesignated complex transformation (Experiment 2) indicated no effects of number of vertices, but a substantial effect of degree of possibility. The effect of possibility was along a psychological rather than a formal continuum of possibility, and was thereby consistent with a "net levels of depth" description of the torus figures.

Reaction times to the tori differing by planar rotations increased linearly with increases in the angle of rotation between the figures, indicating that subjects were mentally rotating one torus into congruence with the other in order to make the match. The slope of this function was greater for the psychologically impossible tori than for the psychologically possible tori. Since slope reflects rate of rotation, the impossible tori were rotated slower than the possible tori.