CREEP IN CONCRETE

by

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SYNOPSIS

There are many definitions proposed for creep. Creep of concrete is predominately a visco-elastic deformation, while indication of plastic deformation includes cracking. Theoretical consideration of the effect of change of moisture content indicates that creep due to simultaneous shrinkage is larger than would be expected if creep and shrinkage could be added.

Microstructure of cement paste has a great influence on strength as well as the deformational property of concrete. This in turn depends upon the properties of aggregates as well as water-cement ratios. It has been found that for higher water-cement ratios, the region of linearity of the relationship of sustained-stress vs. strain decreases, there-by showing plastic deformation at lower stress levels. At higher stress levels, creep no longer follows the linear relationship of sustained-stress vs. strain but it increases rapidly; this is perhaps due to the formation of minute internal cracks.

Stress relaxation, determined by Hansen, and the stress losses determined by Erzen are reviewed.

INTRODUCTION

The creep is a well discussed subject for concrete. More and more investigations have been carried out to determine the creep in concrete, especially after the time when prestressing established its importance. It was necessary to know actually what per cent loss could be specifically assigned to creep.

The creep in concrete does depend on various factors such as moisture content, composition of concrete, type of cement used, type of aggregate and water-cement ratio.

There is much confusion regarding creep and shrinkage in concrete. Davis believes both could be superimposed.[1] In the theoretical analysis by Pickett, he tries to distinguish between shrinkage strain and the deformation under load. The relation between stress and creep is firmly believed to be non-linear; linear relation has justification for only a very small per cent of ultimate strength.[2]

There is an attempt made to convert the strains in terms of stress so that a particular per cent loss may be assigned for creep. Hansen tried to establish a relation between stress and strain, if initial elastic strain and total creep are known;[4] on the other hand, Erzen developed an expression for losses with consideration that the creep in concrete is a function of instantaneous elastic strain and time.[8] The final expression gives a rapid approach to determine losses due to creep.

The purpose of this report is to investigate the behavior of material as regards creep, under various conditions of loading. So also to demonstrate the effect of other deformations such as shrinkage on it, and to determine whether it is possible to correlate creep with concrete properties such as strength, shrinkage, stress relaxation, etc.

The scope of the report is limited to a library search of the important literature. The conclusions, therefore, are those of the investigators whose work has been abstracted and are of most significance in the opinion of the author.

DEFINITION OF CREEP

The deformation of a material falls under three types. It may be elastic, viscous or plastic deformation or a combination of these types.

For real material the deformational properties of practical importance are quite complicated combinations of the three basic types of deformations. Moreover, certain factors such as strain hardening and thixotropy make it difficult to give an adequate definition.

The following equations of creep have been used commonly:

$$\epsilon_{creep} = \epsilon_{total} - \epsilon_i - S \quad (1)$$

$$\epsilon_{creep} = \epsilon_{total} - \epsilon_e - S \quad (2)$$

$$\epsilon_{creep} = \epsilon_{total} - \epsilon_{ie} - S - p \quad (3)$$

$$\epsilon_{creep} = \epsilon_{total} - \epsilon_{ie} - S \quad (4)$$

where ϵ_{creep} = creep strain of specimen some time t, after load application.

ϵ_{total} = total strain of specimen under sustained load application.

ϵ_i = initial strain of specimen when loaded at time t_o

ϵ_e = elastic strain of specimen if loaded some time t_1 after actual load application.

ϵ_{ie} = elastic strain of specimen when loaded at time t_0

S = shrinkage strain of companion specimen from t_0 to t_1

p = permanent set.

There is considerable difference between the above four definitions, but all have equal importance.

The definition of creep which gives a long-time deformation of direct practical importance should be preferred, so that the results of theoretical investigations on creep might be applicable in practice.

Definitions 2 and 4 includes the permanent set as a part of creep. The permanent set is the result of minute plastic deformations or ruptures at the first load, due to some sort of imperfections.

The difference between the definition of creep 1 and 3 is as follows:

In definition 3, creep includes the irrecoverable part of initial deformation because of continuous hydration of cement paste. Definition 1 allows even a theoretical study of the creep behavior. Definition 1 is advantageous for experimental work since during the whole work no computation regarding determination of the modulus of elasticity is needed.

Based on the sustained load considerations, again a few more definitions of deformation are available; the following are the various definitions of strains.

(i) Shrinkage (s) is that unit deformation due to any cause other than stress which would occur in an infinitesimal element if the element were unrestrained by neighboring elements.

(ii) Creep strain (c) is inelastic unit-deformation of infinitesimal element due to stress. It is not confused with the term creep, which denotes inelastic deformation of the body as a whole due to loads.

(iii) Strain due to stress (e) is the sum of the elastic-strain and creep strain in an infinitesimal element. For simple tension or compression,

$$e = \frac{\sigma}{E} + c \qquad \ldots . (1)$$

Now for a hypothetical material, after some of loading, the sustained-stress-VS-strain (σ, e) relation may be expressed as

$$e = 10^{-4}\tan \frac{\sigma}{500} \qquad \ldots . (2)$$

or solving for σ,

$$\sigma = 500 \tan^{-1} (e x 10^{4}) \qquad \ldots (2a)$$

Then from 1

$$c = 10^{-4} x \tan (\sigma/500) - \sigma/E \qquad \ldots (1a)$$

(iv) Total or Resultant, Strain (e') is the resultant unit deformation: $e' = e + s$

THEORETICAL ANALYSIS

Creep is usually obtained by a difference between total deformation of concrete and the sum of instantaneous deformation and simultaneous shrinkage, the shrinkage being measured on another specimen without load. This method of obtaining creep is alright if there is no interdependence between creep and shrinkage.

According to Pickett,[(2)] the hypothetical relation for sustained stress vs. strain could be represented by Fig. (1). For low stresses there is a straight line relationship between stress and strain as represented by line "b". For higher stresses near ultimate stress curve "a" is representative.

In the tests carried out by Pickett, rectangular prisms of concrete are subjected to various loading conditions. To simplify mathematical analysis, a concrete prism drying from two opposite surfaces is considered. Pickett assumes a parabolic distribution of the type.

$$S = A\,(y/a)^2 + B$$

for drying from top and bottom surfaces. A and B are functions of time and 2a is the height of the prism. S is the shrinkage strain and y is the distance from center towards drying surface S in inches.

Another assumption is also made. It is assumed that the prism is long and narrow so that only longitudinal stresses and strains need be considered.

The cases for non-linear distribution have been analyzed in the following discussion.

Shrinkage alone (Case I).

In the first case a hypothetical distribution of shrinkage strain over a prism drying from two opposite surfaces is dealt with and is shown in Fig. 2a. The distribution is given by equation:

$$S = -\left[3\,(y/a)^2 + 2\right]\ \text{x}10^{-4} \qquad \ldots\ldots (3)$$

The relation is shown in Fig. 2b.

This type of distribution means that of the shrinkage of each infinitesimal element of the prism, if the element were unrestrained by neighboring elements. However, with further assumptions that plane sections remain plane and shrinkage is symmetric with respect to a horizontal plane through the longitudinal axis, then the resultant strain (e') is constant and equal to s + e. . . . . . (4)

Now the equation of hypothetical sustained stress-vs-strain curve is assumed to be of the form

$$= 500\ \tan^{-1}(e\text{x}10^4) \qquad \ldots\ldots (5)$$

Substituting $e = e' - s$ in this, we have,

$$\sigma = 500\ \tan^{-1}(e' - s)\ 10^4 \qquad \ldots\ldots (6)$$

Now for equilibrium

$$\int_{-a}^{+a} \sigma\,dy = 0 \qquad \ldots\ldots (7)$$

From equations (3), (6) and (7), the result is

$$500\int_{-a}^{+a}\left[\tan^{-1}\left\{e'\ \text{x}10^4 + 3(y/a)^2 + 2\right\}\right] dy = 0 \qquad \ldots\ldots (8)$$

Equation 8, solved by trial, gives a value for e' of approximately -2.929×10^{-4} in. Under the conditions assumed this is the resultant strain. Unfortunately, it is commonly called shrinkage and incorrectly used as if it were something apart from elastic or plastic deformation.

The shrinkage-stress is now found by substituting for e' and s in Equation 6.

$$\sigma = 500 \tan^{-1}\left[3(y/a)^2 - 0.929\right] \quad \ldots\ldots \quad (9)$$

This distribution is shown in Fig. 3.

Tensile load and shrinkage (Case II).

In this case, in addition to the shrinkage-strain, a load is assumed to be acting, which would give a uniform tensile stress of 500 psi.

Under this load equilibrium requires that

$$\int_{-a}^{+a} \sigma \, dy = 500 \times 2a \quad \ldots\ldots \quad (7a)$$

Combining Equations 4, 6 and 7a the result becomes

$$500 \int_{-a}^{+a} \tan^{-1}\left[e' \times 10^4 - 3(y/a)^2 - 2\right] dy = 1000a \quad \ldots \quad (8a)$$

Equation (8a) is approximately satisfied by a value of

$$e' = -1.163 \times 10^{-4} \text{ in.} \quad \ldots\ldots$$

The equation for resultant stress becomes

$$\sigma = 500 \tan^{-1} \; 3(y/a)^2 + 0.837 \quad \ldots\ldots \quad (9a)$$

This distribution is shown in Fig. 4.

Tension with shrinkage (Case III).

Here $e' = e$. Since shrinkage is not present, a uniform tension applied to a prism produces uniform stress across the section. In this case the strain may be found directly from the stress-strain relation. For $\sigma = 500$psi, this strain is given by the equation $e' = 10^{-4}\tan \sigma/500$. It is found to be equal to 1.557×10^{-4} in/in. It is shown in Fig. 5.

Moment load without shrinkage (Case IV).

In this case, prism is loaded to produce flexural stresses.

The expression for strain in this case is,

$$e' = \frac{y}{p} \qquad \ldots (10)$$

for equilibrium

$$\int_{-a}^{+a} \sigma y\,dy = M$$

or

$$500\int_{-a}^{+a} y\tan^{-1}(e\times10^{4})\,dy = 333a^{2} \qquad \ldots (11)$$

as $s = 0$, $e' = e$

from (10) and (11)

$$500\int_{-a}^{+a} y\tan^{-1}\left[y/p \times10^{4}\right] dy = 333a^{2} \qquad \ldots (12)$$

The curvature $1/p$ is found by trial to be $\frac{1.236\times10^{-4}}{a}\,\text{in}^{-1}$

The resultant stress $\sigma = 500\tan^{-1}(1.236y/a)$

This distribution is shown in Fig. 7.

Moment load and shrinkage (Case V)

This is a combination of case (1) and (IV).

In this case $e' = y/p + k$ . . . (13)

where k is the average unit change in length.

The expression for e becomes,

$$e = \frac{y}{p} + k + \left[3(y/a)^2 + 2\right] \times 10^{-4} \quad \ldots (14)$$

Again for equilibrium,

$$\int_{-a}^{+a} \sigma \, dy = 0 \quad \text{and also} \quad \int_{-a}^{+a} \sigma y \, dy = M$$

with these expressions and equation 2.

$$500 \int_{-a}^{a} y \tan^{-1} \left[3(y/a)^2 + 2 + (y/p + k) \times 10^4\right] dy = 333a^2 \quad \ldots (16)$$

By trial

$$\frac{1}{p} = \frac{1.881}{a} \times 10^{-4} \text{ in}^{-1} \quad \text{and} \quad k = -2.74 \times 10^{-4} \text{ in/in}$$

Substituting these values into equation (2a),

$$= 500 \tan^{-1} \left[3(y/a)^2 + 1.881 y/a - 0.742 \right]$$

This distribution is shown in Fig. 8.

In the first case, shrinkage produces a non-uniform internal stress over the section. This must be added to stresses due to external loads. This brings non-uniformity of stress distribution. Some parts will be over-stressed above proportional limit and some parts will bestressed lower than that given by proportionality limit.

From the stress pattern for other cases, the following observations are available. The creep under simultaneous shrinkage is larger than would be expected if creep and shrinkage could be added. This is also found in case of beams loaded with moments.

Pickett assumed the hypothetical relation of Fig. (1) but usually the tensile strength of concrete is not more than about one-tenth of compressive strength and so the relation shown in Fig. (9) is perhaps more appropriate. However, in such a condition, material will behave differently in flexure, compression or tension. This pattern is suggested by Hansen. (4)

Usually, the compressive working load is much less than compressive strength. Fig. (10) shows the stress distribution due to non-uniform shrinkage in concrete prism drying from the two sides. The sum of the stresses due to usual working load and additional stress due to non-uniform shrinkage will rarely exceed one-half of the compressive strength. This would lead to the decision that there would be no increase in creep due to non-uniform shrinkage, under compressive loads.

In the case of flexure, however, stresses may be produced so that the tensile strength is exceeded in some part of the member, producing cracks and subsequent increase in creep.

Visco-elastic Property of Creep

The modulus of viscosity, and the delayed elastic modulus of concrete increase with the amount of cement gel formed. They also increase with decrease in water cement ratio. (4)

According to Nevile (9), the creep rate at any time after load application is inversely proportional to the strength of cement mortar, and independent of the composition of the mortar and the type of cement used, indicating relation between creep rate and gel density. However, this suggestion by Neville would be valid only for cement gel and not for concrete, since inclusion of aggregate imbolizes part of the total volume, hence decreasing creep. The creep is, however, actually proportional to the volume of cement paste, and aggregates have little effect on strength properties of gel so long as the water cement ratio is the same.

On the basis of the logical model shown in Fig. (11), Hansen has established the general rheological creep-equation of all cement paste, mortars and concretes cured and stored in water or under conditions not allowing any drying or wetting and subjected to a constant compressive, tensile or flexural load.

It is expressed by Equation (17).

$$\frac{\epsilon_{creep}}{\sigma} = \beta \frac{\left\{0.31g(t_0) - \frac{w_0}{c}\right\} V_1}{(Nk_1 - 0.31)g(t_0)} (1-e^{-m(t_1 - t_0)})$$

$$+ \alpha_1 w_0/C \, . \, V_1 \ln (t_1/t_0) \qquad \ldots . (17)$$

Specific creep = Delayed elasticity + Viscosity

where t_1 = age of concrete at time t_1 in days

t_0 = age of concrete when loaded, in days

w_0/c = water-cement ratio by weight, corrected for bleeding.

V_1 = volume concentration of cement paste in mortar or concrete

$g(t_0)$ = degree of hydration of cement at the time of load application.

and m = coefficients to be determined experimentally.

k_1 = weight ratio of non-evaporable water to cement, when all cement is hydrated. k_1 is a function of the proportions of cement components.

$= 0.178\ (C_3S) - 0.158\ (C_2S) - .665\ (C_3A)$

$-.213\ (C_4AF)$

and $N = 0.75\ (1-4k)$

where $k = 0.230\ (C_3S) - 0.320\ (C_2S) - 0.317\ (C_3A)$

$- 0.368\ (C_4AF)$

Various tests by Davis (10), Hansen (4), Neville (9), and others provide excellent proofs for the above equation and their results are shown in Fig. (12,13,14,15).

But according to Davis and Troxell, linear relation between specific creep per unit stress and time in a semilogarithmic graph is valid only up to about 1000 days sustained loading. Then creep rate is reduced considerably. This is shown in Fig. (16).

At high overloads plastic deformation of concrete takes place. When plastic deformation takes place, a kind of internal rupture may be observed in the concrete. This would happen when the proportional limit is exceeded.

In the tests carried out by Hansen, concrete cubes were tested under a compressive load, at constant rate, and longitudinal wave velocity was measured by an ultrasonic pulse method. Fig. (17) shows stress-strain relationship for these tests. It is to be noted that the stress at the proportionality limit, and the stress at which the first microcracks occur, are almost the same. Pulse signals indicated by sonoscope at three different stages during the loading to rupture of concrete cubes were observed. The decreasing of signal amplitude indicates internal minute cracks, then external cracks appear at any time after that. And according to these tests, plastic deformation is due to internal crack formation in the concrete.

When the concrete is in destruction stage at stresses above the proportional limit, it is difficult to determine what part of the total deformation is due to c reep and how much is due to plasticity at this instant. Therefore, enough information about the creep at high overload is not available. But at this stage if the proportional limit could be determined, it would be possible to distinguish between creep and plastic deformation up to that extent. Composition of concrete has much influence on this. Microcracks are formed due to failure of bond that exists in cement crystals and aggregates. The type of aggregate has great influence upon the adhesion between the cement gel and aggregates. Another important factor affecting the adhesion is water-cement ratio.

INTERNAL MECHANISM OF PERMANENT SET AND CONCRETE CREEP

The instantaneous permanent set,is probably the result of innumerable minute plastic deformations or ruptures at first loading of concrete, due to stress concentrations at imperfections or inhomogeneities in the material, such as notches, holes, pores and inclusions. (4) A collapse of gel structures, when sustained to loading, may also contribute somewhat to the permanent set. It is possible that part of the permanent set due to collapse of the gel structure only is released after some time of sustained loading. This may be the cause of the rapid creep of concrete, which is always observed during the very first days of loading.

The visco-elastic behavior of concrete can be roughly described by means of a rheological model, a Burgers body as shown in Fig. (11) where the four rheological constants vary with the age of concrete.

A rheological model is a combination of ideal elastic and viscous elements in series or parallel, which represents the behavior of actual materials under load or deformation.

The element E_M describes the instantaneous elastic deformation of concrete, λM the viscous part of creep, E_k and λ_k in series describe the delayed elastic part of creep.

The use of this model is to give a phenomeno-logical description of the visco-elastic response of concrete, but it does not necessarily imply anything about the molecular mechanisms responsible for the observed behavior. Several attempts have been made to develop a molecular theory of the visco-elastic behavior of concrete, but no theory is commonly accepted.

STRESS RELAXATION OBTAINED FROM CONCRETE CREEP (4)

Many attempts have been made to transform creep functions into stress relaxation functions for concrete. The problem is of great importance in structural engineering when designing prestressed concrete structures or when calculating the internal stresses in mass concrete structures, due to shrinkage or temperature changes.

If it is assumed that the rheological model shown in Fig.(11) represents the deformational behavior of concrete, it is then possible to obtain an exact solution of the relaxation problem.

The following is the general equation for the model.

$$\frac{\lambda_K}{E_K}\frac{d^2\epsilon}{dt^2} + \frac{d\epsilon}{dt} = \frac{\lambda_K}{E_M E_K}\frac{d^2\sigma}{dt^2} + \frac{\lambda_M E_K + \lambda_M E_M + \lambda_K E_M}{\lambda_M E_K E_M}\frac{d\sigma}{dt} + \frac{\sigma}{\lambda_M} \quad . \; . \; (18)$$

where ϵ = strain

σ = strain, lb. per sq. in.

t = time, sec.

E = modulus of elasticity, lb. per sq. in.

λ = modulus of viscosity lb. sec. per sq. in.

If a unit stress σ_{t_0}, then the total instantaneous strain E_{t_0} is elastic and due to the deformation of the elastic element E_M. According to Hook's Law we get:

$$\sigma_{t_0} = E_M \cdot \epsilon_{t_0} = 1$$

or

$$\epsilon_{t_0} = 1/E_M$$

If, however, the total strain ϵ_{t_0} is sustained for some time, then the other elements of the model λ_M, E_k and λ_k are slowly compressed by a certain amount, thus causing an elongation of the spring element E_M by the same amount. This results in relaxation of the stress in element E_M and therefore in the model as a whole.

With $\epsilon_{t_0} = 1/E_M$ we obtain $\frac{d\epsilon}{dt} = 0$ and $\frac{d^2\epsilon}{dt^2} = 0$

and therefore, from Eq. (18)

$$\frac{d^2\sigma}{dt^2} = \frac{\lambda_M E_k + \lambda_M E_k + \lambda_k E_M}{\lambda_M \lambda_k} \cdot \frac{d\sigma}{dt} + \frac{E_M E_k}{\lambda_M \lambda_k} \sigma = 0$$

From this differential equation of second order, σ has been derived:

$$\sigma = \epsilon_{t_0} E_M \left(\frac{a - \frac{E_k}{\lambda_k}}{a-b} e^{-at} - \frac{E_k/\lambda_k - b}{a-b} e^{-bt} \right)$$

$$= \frac{a - E_k/\lambda_k}{a-b} e^{-at} + \frac{E_k/\lambda_k - b}{a-b} e^{-bt}$$

where

$$a,b = 1/2 \left(\frac{E_k + E_M}{\lambda_k} + \frac{E_M}{\lambda_M} \right) \pm 1/2 \sqrt{\left(\frac{E_k + E_M}{k} - \frac{E_M}{\lambda_M} \right)^2 - 4\frac{E_k E_M}{\lambda_k \cdot \lambda_M}}$$

The above equation is mainly of theoretical interest because it is generally very difficult to obtain the exact value of all rheological constants. The following approximate method is suggested for practical purposes. The method is based on the mechanical behavior of the rheological model.

The stress σ_{t_1}, necessary to maintain the constant strain of the model at any time t_1 after load application, is the same as the stress in the element E_M. It can be calculated as follows:

$$\sigma_{t_1} = E_M : \epsilon_{t_1}$$

For all possible values of t_1 we get the stress relaxation function $\sigma(t)$.

ϵ_{t_1} is the strain remaining in the element E_M at time t_1. ϵ_{t_1} can be written as the total instantaneous deformation ϵ_{t_0} of the model at time t_0, diminished by the deformation $\epsilon_{creep, \sigma(t)}$ of the element λ_M, E_k and λ_k during the period of time $t = t_0$ to $t = t$ under the stress $\sigma(t)$ in the model, or

$$\epsilon_{t_1} = \epsilon_{t_0} - \epsilon_{creep,\ \sigma(t)} \qquad \ldots\ldots\ldots (19)$$

As an approximation we can write,

$$\epsilon_{creep,\ \sigma(t)} = \frac{\sigma_{t_0} + \sigma_{t_1}}{2}\ \epsilon_{creep,\ t_1} = \frac{1 + \sigma_t}{2}\ \epsilon_{creep,\ t}, \qquad \ldots\ldots\ldots (20)$$

where $\epsilon_{creep,\ t_1}$ = total creep obtained by the model (the concrete) from time of load application $t = t_0$ to $t = t_1$, under constant unit load $\sigma(t) = 1$.

Substituting Eq. (20) in Eq. (19) gives,

$$\epsilon_{t_1} = \epsilon_{t_0} - \frac{1 + \sigma_{t_1}}{2}\ \epsilon_{creep,\ t_1} \qquad \ldots\ldots\ldots (21)$$

From equations $\sigma_{t_0} = E_M . \epsilon_{t_0} = 1$ and $\sigma_{t_1} = E_M . \epsilon_{t_1}$

$$\epsilon_{t_1} = \frac{\epsilon_{t_0}\ \sigma_{t_1}}{\sigma_{t_0}} = \epsilon_{t_0} . \sigma_{t_1} \qquad \ldots\ldots\ldots (22)$$

Inserting Eq. (22) in Eq. (21)

$$\sigma_{t_1}\epsilon_{t_0} = \epsilon_{t_0} - \frac{1-\sigma_{t_1}}{2}\epsilon_{creep,\,t_1} \quad \ldots\ldots \quad (23)$$

or

$$\sigma_{t_1} = \frac{2\epsilon_{t_0} - \epsilon_{creep,\,t_1}}{2\epsilon_{t_0} + \epsilon_{creep,\,t_1}} \quad \ldots\ldots\ldots\ldots \quad (24)$$

where

σ_{t_1} = remaining stress in concrete at time t_1

ϵ_{t_0} = initial elastic strain under stress $\sigma_{t_0} = 1$

$\epsilon_{creep,\,t_1}$ = total creep obtained by the model (the concrete) from time of load application $t = t_0$ to $t = t_1$ under constant unit load $\sigma(t) = 1$.

Equation (24) is the fundamental equation from which the remaining stress in a concrete specimen at any time t_1, or the relaxation function for a constant sustained strain, can be calculated, when the initial elastic strain ϵ_{t_0} under the initial stress $\sigma_{t_0} = 1$, and the creep function or the total creep obtained by the concrete from time of load application $t = t_0$ to $t = t_1$ under constant unit stress $\sigma_t = 1$, is known.

When determining the stress relaxation function of concrete, exposed simultaneously to other volume changes, it must be remembered that not only the internal creep of the elements E_M, λ_k and E_k, but also other volume changes, contributes to the relaxation.

If the volume changes on concrete, other than those due to load, during the period of restrain $t = t_0$ to $t = t_1$ are S, then Equation (19a) applies instead of Equation (19).

$$\epsilon_{t_1} = \epsilon_{t_0} - \epsilon_{creep}^{\sigma(t)-S} \quad . \; . \; . \; . \; (19a)$$

the following relation is obtained

$$\sigma_{t_1} = \frac{2\,\epsilon_{t_0} - \epsilon_{creep,\,t_1} - S}{2\,\epsilon_{t_0} + \epsilon_{creep,t_1}} \quad . \; . \; . \; . \; (24a)$$

where S = total shrinkage during time of restraint $t = t_0$ to $t = t_1$, positive for shrinkage, negative for swelling, when concerned with the relaxation of compressive stress. Vice-versa when dealing with the relaxation of the tensile stress.

Equation (24) has been tested on several experimentally determined relaxation curves for different concretes. The calculated relaxation curves are always in good agreement with the experimentally determined curves. For instance, the relaxation curves have been computed according to this approximate method from creep data reported by Hansen (1953), Fig. 18. From Fig.19 it will be seen that there is a good correlation between computed and experimentally determined relaxation curves.

Due to the approximation made, the computed values always overestimate the relaxation to some degree. But as long as we are concerned with concretes having normal creep properties, that is as long as the total creep does not exceed twice the elastic deformation under the same load, Hansen observed that the computed relaxation curves will be well within the experimental accuracy and acceptable for most practical purposes. When concerned with concrete with extreme creep properties, the approximate method should not be use.

Based on the same principles, it is possible to calculate the loss in stress in the steel of prestressed concrete elements, due to creep and shrinkage of concrete.

According to Erzen total strain ϵ_{total} in concrete is affected by the following factors:

- σ = unit stress
- E_{ie} = modulus of elasticity of concrete at time of loading.
- k = time of loading measured from the day of casting of concrete.
- t = time at which total strain is measured. origin of time taken at time of casting of concrete.

The total strain is the sum of the instantaneous elastic strain ϵ_{ie} and the creep strain ϵ_{creep} taking place during the time t-k.

Now according to Erzen, assuming creep strain varies linearly with the initial unit stress and taking $\sigma = 1$ psi, the equation for the total strain becomes

$$\frac{\epsilon_{total}}{\epsilon_{ie}} = e^{\alpha\left[1 - (k/t)^{B}\right]} \quad . \; . \; . \; . \; . (25)$$

in which α, and B are constants, to be determined experimentally. This is plotted in Fig. (20).

To determine the total strain from Equation (25), ϵ_{ie}, or instantaneous modulus of elasticity, should be known. It could be found from the formula

$$\frac{E_t}{E_{28}} = \frac{4}{3 + 28/t} \qquad \ldots\ldots (26)$$

This formula is applicable to ordinary portland cement.

With this much information about total strain, it is possible to determine loss of stress in prestressed beam due to creep.

In the wire of a prestressed beam, the loss of prestress occurs in two stages. In the first stage, initial loss occurs at the time of immediate transfer of prestress to the concrete.

The unit strain in the wires due to elastic loss on transfer is

$$\frac{\Delta F_o}{A_s \; E_s}$$

where F_o is the prestress after transfer, A_s, area of steel and E_s, is the modulus of elasticity of steel wires.

Then $$\frac{\Delta F_o}{A_s \; E_s} = \frac{F_i - \Delta F_o}{A_c \; E_k} \; (1 + c^2/r^2) - \frac{M_G \cdot C}{E_k \cdot I}$$

$$\Delta F_o = \frac{\left(F_i - \frac{M_G \cdot C}{r^2 + c^2}\right)(1 + c^2/r^2)\; n_{k.p}}{1 + (1 + c^2/r^2)\; n_{k.p}} \qquad \ldots (27)$$

where

M_G = dead load moment

c = eccentricity of steel wires from the centroidal axis of beam.

r = radius of gyration of beam section.

$p = A_s/A_c$

$n_k = E_s/E_k$

In the second stage, creep losses occur due to change of strain with time. The initial strain in concrete at transfer would produce a total strain after time given by the following expression.

$$\epsilon_1 = \frac{1 + c^2/r^2}{A_c\ E_k} \left(F_i - \Delta F_o - \frac{M_G \cdot C}{r^2 + c^2}\right) e^{\alpha\left[1 - (k/t)^B\right]} \quad \ldots (a)$$

and because of the change of strain, a continuous loss of stress occurs as shown in Fig. (3a) and denoted by $\Delta\bar{F}(t)$. The total loss at any time is then given by

$$\Delta F = \Delta F_o + \Delta\bar{F}(t)$$

Thus as the strain, due to initial stress, occurs, a strain corresponding to $\Delta\bar{F}(t)$ occurs and is given by

$$\epsilon_2 = \int_k^t e^{\alpha\left[1 - (p/t)^B\right]} \frac{d\ \epsilon(p)}{dp}\ dp \quad \ldots (b)$$

in which $\frac{d\ \epsilon(p)}{dp}\ dp$ is the increment of strain due to $\Delta F(p)$ during dp.

The net unit strain in the concrete is the difference of a and b.

Hence

$$\frac{F}{A_s\ E_s} = \frac{1 + c^2/r^2}{A_c\ E_k}\left(F_i - F_o - \frac{M_G}{r^2 + c^2}\right) e^{1-(k/t)^B}$$

$$= e^{1-(k/t)^B} \frac{d\ (p)}{dp}\ dp \qquad \text{. . . . . (28)}$$

The net unit strain in the concrete is the difference of a and b. Hence

$$\frac{\Delta F}{A_s\ E_s} = \frac{1 + \frac{c^2}{r^2}}{A_c\ E_k}\left(F_i - \Delta F_o - \frac{M_G}{r^2+c^2}\right) e^{\alpha\left[1-(k/t)^B\right]}$$

$$- \int_k^t e^{\alpha\left[1-(k/t)^B\right]} \frac{d\ \sigma(p)}{dp}\ dp$$

on simplification it becomes,

$$\frac{\Delta F}{A_s\ E_s} = \frac{1 + c^2/r^2}{A_c\ E_k}\left(F_i - \Delta F_o - \frac{M_G}{r^2+c^2}\right) e^{\alpha\left[1-(k/t)^B\right]}$$

$$- \frac{\Delta F(t)}{A_c\ E_t}\left(1 + \frac{c^2}{r^2}\right) - \frac{\alpha B}{t^B} \cdot \frac{1+\frac{c^2}{r^2}}{A_c} \int_k^t e^{\alpha\left[1-(p/t)^B\right]} . p^{B-1}$$

$$\frac{\Delta F(p)}{Ep} .\ dp \qquad \text{. . . . .(28)}$$

The integral is solved by approximation. And for the numerical solution the last term of Equation (28) is transformed from an integral to a series as,

$$- \frac{\alpha B}{t^B} \cdot \frac{1+c^2/r^2}{A_c} \leq e^{\alpha\left[1-(p/t)^B\right]}\ p^{B-1}\ \frac{\Delta F(p)}{E\ p} .\ p$$

The method of solution is illustrated in the following examples.

Given a beam that possesses the following properties:

$A_c = 200$ in.^{2}	$I = 18,000$
$A_s = 4.52$ sq.in.	$r^2 = 90$ sq. in.
$p = 0.0226$	$E_s = 29\text{x}10^6$ psi
$c = 11$ in.	$F_i = 19,200$ lb.

Taking $E_7 = 2.11\text{x}10^6$ psi, $n_7 = 13.7$

Therefore, from Equation (4)

$$\Delta F_o = \frac{0.0226\text{x}13.7\ (1+\frac{121}{90})\ F_i}{1+0.0226\text{x}13.7\ (1+\frac{121}{90})} = 0.42\ F_i = 80,500\ \text{lb.}$$

To find the loss on the succeeding days Equation (28) is used. Taking B = 1, there is found

$$\frac{F_o + \Delta F(t)}{p(1+\frac{c^2}{r^2})\ E_s} = \frac{F_i - \Delta F_o}{E_k}\ e^{\alpha(1-\frac{k}{t})} - \frac{\Delta F(t)}{E_t} - \frac{\alpha}{t} \leq e^{(1-\frac{p}{t})}$$

$$\frac{\Delta F(p)}{Ep} \cdot \Delta p$$

in which F(p). p is the differential area as shown in Fig. (22a). To find the additional loss on the eighth day, in the above equation t and p are taken as and 7.5, respecitively, and

$$F(7.5)\ \ p = \tfrac{1}{2}\ \ F(8)\ \text{x}1$$

Since

$$\frac{E_t}{E_{28}} = \frac{12}{11+\frac{28}{t}}$$

$$E_t = \frac{12}{11+\frac{28}{t}} \cdot E_{28}$$

$$E_7 = \frac{12}{11+4} \cdot E_{28} \qquad \text{where } E_{28} = 3.56\text{x}10^6 \text{ psi}$$

$$= 2.11\text{x}10^6 \text{ psi}$$

$$E_8 = 2.17\text{x}10^6 \text{ psi}$$

$$E_{7.5} = 2.15\text{x}10^6 \text{ psi}$$

$$1/E_8 = 0.46\text{x}10^{-6} \text{ psi}$$

and $1/E_{7.5} = 0.466\text{x}10^{-6}$ psi

the equation becomes with $\alpha = 0.74$

$$0.65(\Delta F(8) + 80500) = 52{,}800\ e^{.74(1-7/8)} - 0.46\Delta F(8)$$

$$-0.0925\text{x}1.0477\text{x}\tfrac{1}{2}\Delta F(8)\text{x}0.466$$

$$0.65\Delta F(8) + 52{,}400 = 57{,}900 - 0.46\Delta F(8) - 0.2255\Delta F(8)$$

$$1.1325\Delta F(8) = 5{,}400$$

$$\Delta F(8) = 4{,}760 \text{ lb.}$$

Similarly for t= 9

$$\frac{1}{E_9} = 0.448\text{x}10^{-6} \qquad \frac{1}{E_{8.5}} = 0.454\text{x}10^{-6}$$

$$0.65(\Delta F(9) + 80{,}500) = 52800\text{x}1.178 - 0.448\Delta F(9)$$

$$-0.0823(\tfrac{1}{2}\text{x}4760\text{x}1.131\text{x}0.466$$

$$+\frac{4760 + \Delta F(9)}{2} \text{x } 1.0856\text{x}0.454$$

where $\frac{1}{2}$x4800x1 and $.\frac{4760 + \Delta F(9)}{2}$ x 1 are the two consecutive differential areas,

$$0.65\,\Delta F(9) + 52,400 = 62,300 - .448\,\Delta F(9)$$
$$-103.5 - 96.5 - .02035\,\Delta F(9)$$
$$1.118\,\Delta F(9) = 62,300 - 52,600 = 9,700 \text{ lb.}$$
$$\Delta F(9) = 8,670 \text{ lb.}$$

Similar losses $\Delta F(t)$ on succeeding days could be found.

It is found that the above losses can be expressed by equation

$$F = \left\{\frac{1.74}{(1+7/t)} - 0.8\right\} \Delta F_o$$

CONCLUSION

Under sustained load, concrete deforms rapidly in the beginning and then the rate of deformation decreases with time. The stress-creep relation is not linear and deformation of concrete is different at higher loads from that at working load. The linear relation holds only for a small per cent of ultimate strength. Again, it is affected by properties of cement gel, aggregate, and water-cement ratio. As the water-cement ratio is increased, creep is increased.

Under normal working load, there is a proportionality between stress and strain in short-time tests, and between stress and rate of strain in long-time tests. But the different behavior of concrete at ultimate strength, leads to the conclusion that the plastic deformation of concrete is due to internal cracking, which is perhaps a sign of destruction of material.

From the tests carried out by Pickett[2] on the concrete prisms under various conditions, it was found that the creep deformation under simultaneous shrinkage is larger than would be expected if creep and shrinkage could be added; it is true even under flexural load.

Losses due to creep in concrete may be evaluated knowing the instantaneous and total creep in concrete. The problem is of great importance to come up with some definite value of losses due to creep, especially in structural engineering when designing prestressed concrete structures.

ACKNOWLEDGEMENT

The author wishes to thank his major adviser Dr. R.F. Morse, Dr. Blackburn, Head, of Civil Engineering Department, and Dr. Cecil Best of Applied Mechanics Department for their help and guidance.

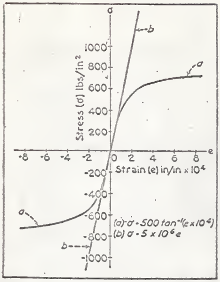

Fig.1

Fig.1. Hypothetical sustained-strss vs.strain relation some time t after load is applied.
(Pickett)

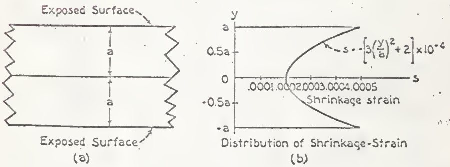

Fig. 2

Fig.2. Hypothetical distribution of shrinkage-strain over a plate of concrete, drying from both sides.

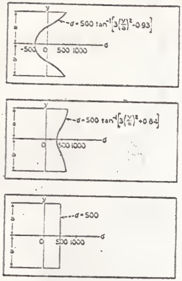

Fig. 3,4,5

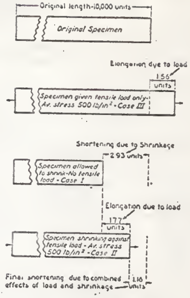

Fig. 6.

Fig.3- Distribution of shrinkage stress (case I)

Fig.4-Distribition of stress for tension and shrinkage (caseII)

Fig.5- Distribution of stress for tension(caseIII)

Fig.6-Diagrammatic summary of the analuses of cases I, II, III

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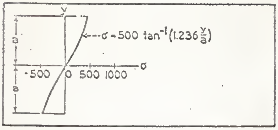

Fig.7

ig.7-Distribution of stress for bending (case IV)

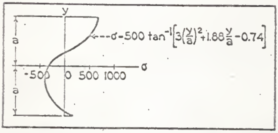

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Fig.8

rig.8-Distribution of stress for bending and shrinkage(case V)

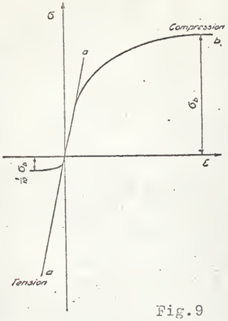


Fig.9

Fig.9 Hypothetical sustained-stress vs.-strain relatinn some time t after load is applied.

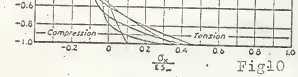


Fig10

.ig.10-Theoretical distribution of stresses due to non-uniform shrinkage in a plate of concrete drying from two sides.(T is a nondimensional parameter which is directly proportional to the time t after drying starts. E and S are constants.)

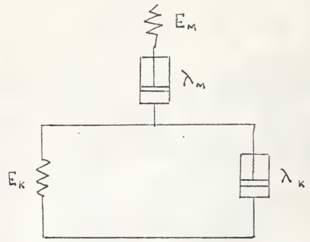


Fig. 11. Suggested rheological model for concrete

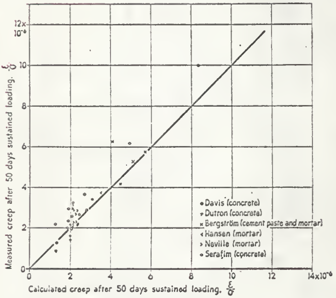


Fig.12. Graph showing agreement between calculated and experimentally determined creep after 50 days' sustained loading.

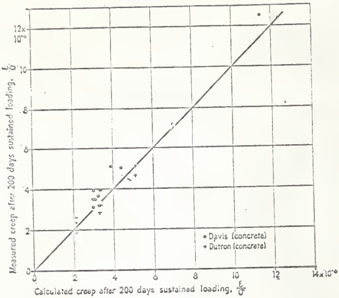


Fig.13. Graph showing agreement between calculated and experimentally determined creep after 100 days' sustained loading.

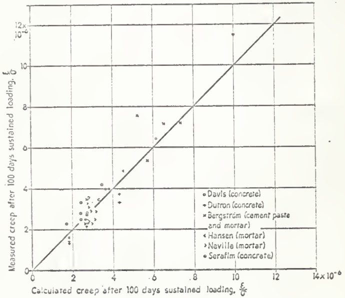


Fig. 14. Graph showing agreement between calculated and experimentally determined creep after 200 days' sustained loading.

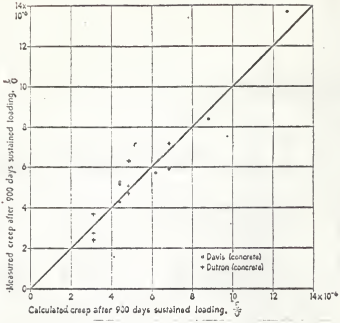


Fig. 15. Graph showing agreement between calculated and experimentally determined creep after 900 days'

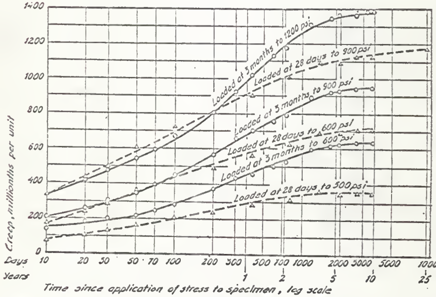


Fig. 1 .. Creep of concrete continues after 25 years sustained loading.

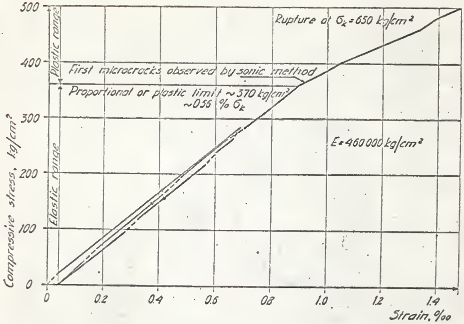
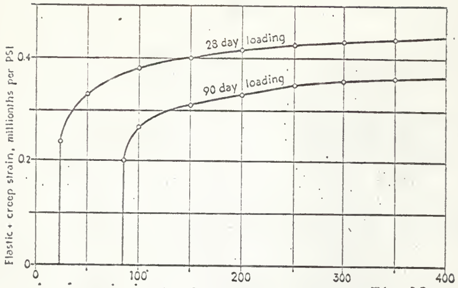


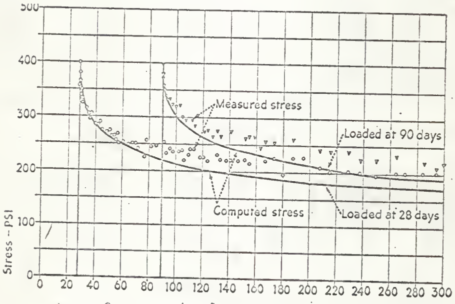
Fig. 17

Fig. 17- Stress-strain relationship for concrete showing accordance between the plastic limit and the formation of the first microcracks.



Age of concrete-days Fig.18

Fig.18-Creep curves for Ross dam concrete.



Age of concrete-days Fig.19

Fig. 19- Graph showing accordance between measured and calculated stress relaxation.

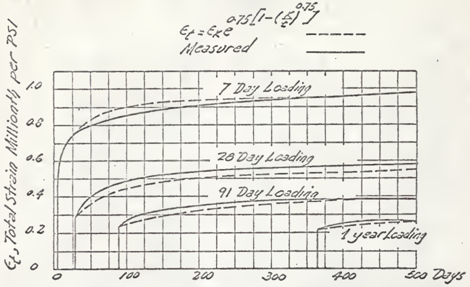


Fig.20

Fig.20. Elastic plus creep strain-Shasta Dam concrete

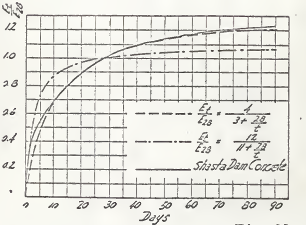


Fig.21

Fig.21- Variation of modulus of elasticity with time

APPENDIX B - SELECTED READING REFERENCES

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CREEP IN CONCRETE

by

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B.E.(Civil), Bombay University, 1961

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

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Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1964

Many articles have been published on creep in concrete, and it has been interpreted in many different ways.

Creep in concrete explained in the report is a result of reviewing several references containing informations on creep.

A brief discussion of various definitions of creep is followed by a theoretical analysis of relations between stress and strain under sustained loading. A consideration to visco-elastic properties of creep is also given. Then stress relaxation determined by Hansen and stress losses determined by Erzen are reviewed.

From important literatures on creep in concrete,the author arrives at the conclusion that creep in conncrete is dependent upon many factors such as type of loading,moisture content, composition of concrete,type of cement used, type of aggregates used and water-cement ratio.It is also concluded that it is possible to correlate creep with stresses under sustained loading.