AN ANALYSIS OF AN ELECTRO-HYDRAULIC ACTUATOR

by

DESH PAUL MEHTA

B. S., Panjab University, India, 1959
M. S., Aligarh University, India, 1961

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Major Professor

PREFACE

The author of this report is a participant from India in a technical cooperation program of the United States Agency for International Development. While leaving India for the USA, the Group-Leader of the American team there gave a piece of advice. He observed that the participants should make it a part of their training to watch and feel the American way of life. This has been kept in mind during the stay and in doing so, some aspects seemed to be so attractive that they have been practically adopted by the author. One of them is, "To be equipped with a justification for your acts in all walks of life and at all times irrespective of your position or authority." Virtually it seems to be a hard demand of the democratic way of life; in fact it is a very scientific, constructive and educative approach. In writing this report, the "Preface" seems to be the most appropriate place to meet this demand.

Education often lags technological advances and it has been aptly true in the case of Fluid Power Controls. As far as technical advances are concerned, fluid power controls are being used in all the technical fields of this space age right from the automatic machine tools to the controls of the space vehicles. In the field of education, even today, as in the nineteenth century, the various demands are being met on an empirical basis. Most of the components of fluid power controls are designed, not engineered. This does not mean that the knowledge of physical phenomena are lacking. Hydrodynamics is a very old and very well developed subject and it is by no means the intention of this report to expose the theory of any aspect of this subject. Instead, one of the components of fluid power control systems, an actuator, is chosen and it is attempted in its analysis to show that a paper and pencil technique can be applied to study and show how to improve the dynamics of the component. Work using such a technique can be questioned in the presence of

computer facilities. Here the author submits that on his return to India, computer facilities are very rare, the empirical techniques are too costly and paper-pencil techniques are therefore useful for solutions to the engineering problems in developing India.

The author is thankful to the U. S. Agency for International Development for its financial support during his studies in this country. Dr. R. O. Turnquist, M. E. Department, Kansas State University, created the interest in the subject very successfully and suggested the various problems in the field. Sincere thanks go to him for his immeasurable directions and valuable guidance. The help and assistance of Mr. C. H. Cho, of Fisher Governor Company, Marshalltown, Iowa, is also gratefully acknowledged.

The author will be failing in his duties if he does not extend his sincere thanks to Dr. Ralph G. Nevins, Head of the M. E. Department, K. S. U. for his permission to the author, with a physics background, to work at the M. E. Department.

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INTRODUCTION

Human[1*] or animal muscle, and moving fluids, water or wind power, were the only sources of mechanical power before the development of the steam engine. Advances in the development of this source of mechanical power made it imperative to develop some medium for the transmission of power. Superiority of fluid power over mechanical power transmission was recognized many years ago. However the development of electrical power and its capability to transmit power over long distances much more economically and efficiently than fluid power made the use of fluid power extremely limited for some generations.

For some years back, fluid power has been undergoing rebirth and at the present time, it is a subject of great interest and importance. This is due not to any recently discovered deficiencies in the electrical techniques of power transmission, but to the rapid rise in the demand for types of performance that are difficult or impossible to obtain from electromagnetic devices alone. In general fluid power systems have the following advantages over electrical systems.

1. In fluid power systems, a material medium, unlike electricity, serves to carry off the heat produced by friction losses at the point where mechanical power is produced. This permits a great reduction in the size of the power producing component.

2. In fluid power systems, motors having torque to inertia ratios many times greater than electrical systems can be designed because electrical systems suffer from the fact that known ferromagnetic materials saturate at inconveniently low flux densities.

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* Superscript numbers refer to references.

3. Hydraulic systems as seen from the load point of view are mechanically stiff. This is highly desirable when it is required to hold the load fixed in position until further movement is desired.

4. High resolution[2] available with fluid power systems allows a smoothness of operation that permits tight control loops of which other operators are not capable.

5. The large levels of power and exceedingly rapid response of the systems being demanded for military and industrial applications are available only with fluid-power systems.

On the negative side fluid power systems are relatively expensive and suffer from other disadvantages. But in spite of these, their use has been forced upon us due to the increased demands of technology and one must learn how to use them.

Definition of Fluid Power

Fluid power can be defined[3] as the science and technology of the transfer, storage and/or control of energy by means of a pressurized fluid. The term fluid power strongly connotes the transfer of energy and differentiates energy transfer systems involving fluids from the traditional civil engineering hydraulics and hydrology; the newer technology of aerodynamics and jet propulsion; and the area of mass transfer, which is essentially a materials handling situation.

A fluid power control system consists of following principle components:

1. Pumps
2. Valves
3. Actuators; Linear or Rotary

An actuator[4] is a device which can be used to position the final control element in a control system and thereby effect a corrective change in the controlled variable. Valves and dampers are examples of mechanisms which can be controlled by actuators. There are several types of actuators, electric, pneumatic, hydraulic and combination types. Combination actuators fall in the field of high speed and high horsepower servo devices, and usually are to be considered when the horsepower requirement at the actuator exceeds one or two tenths of a horsepower (below this all-electric actuators usually are satisfactory). Pneumatic actuators are available to several horsepower but have serious limitations in frequency response, stiffness, and resolution. These limitations can be overcome by the combination actuators. The horsepower limitation of these combination actuators is primarily cost rather than feasibility.

Problem Analyzed

This report deals with the analysis of a combination actuator, a Fisher Governor Co. Type 350 electro-hydraulic actuator. The operation of the actuator under study can be described with reference to figure 1.

An increase in current to the force motor moves the coil to the left. Flapper action restricts the flow of oil from the top pump section (50 psi) at nozzle 'A'. Pressure in bellows 'A' increases while pressure in bellows 'B' decreases. The unbalance of pressure tilts a second flapper above nozzles 'N_1' and 'N_2' restricting the flow at 'N_1'. The pressure from the middle pump section builds up and is transmitted to the underside of the piston, moving it upward.

As the piston moves upward, the feedback cam causes the roller and lever assembly to move to the right, increasing the feedback spring force. Upward movement continues until feedback spring tension balances the force resulting

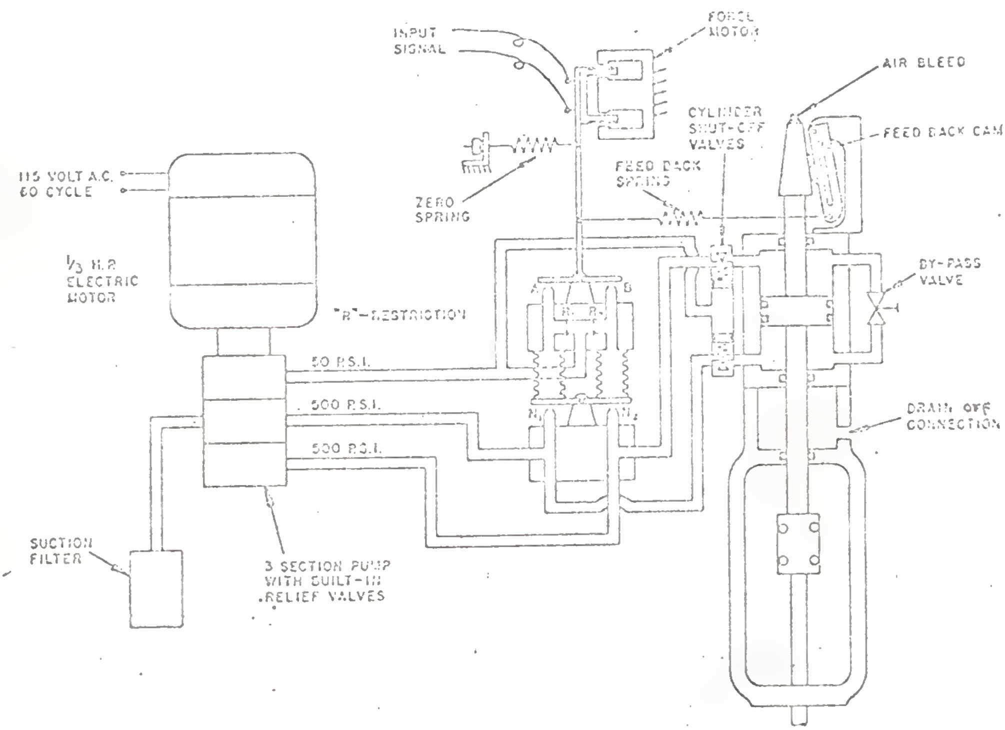


Figure (1), Principle of Operations

from the current in the force motor. When these forces are equal, the flappers assume a balanced position with the piston in the position dictated by the electrical signal.

A decrease in current to the force motor moves the coil to the right, restricting nozzles 'B' and 'N_2'; the result is downward piston motion. Spring loaded shut-off valves provide a means of locking the cylinder in position should the electric power or hydraulic pump fail. In operation, the valves are held open by pressure from the top pump section.

For the sake of analysis, the system is broken into the following sub-components:

1. Force Motor and the Beam Assembly
2. First-Stage Flapper Nozzle Amplifier
3. Second-Stage Flapper-Nozzle Amplifier and the Piston
4. Mechanical Feedback

A linear transfer function is derived for the force motor and beam assembly. Linearized transfer functions are developed for the First-Stage Flapper-Nozzle amplifier, the Second-Stage Flapper-Nozzle amplifier, and the piston assembly from the flow-pressure and velocity-force relationships. A block diagram is used to combine the transfer functions of the sub-components into an overall transfer function for the whole system.

The analytically derived transfer function has been put to experimental verification by the frequency response method. The curves are plotted for the frequency response predicted from the analytical transfer function and are compared with the experimental frequency response curves for the system. The application and the limitations of the analytical techniques are discussed, and design changes to increase the overall bandwidth of the system are indicated.

FORCE MOTOR AND BEAM ASSEMBLY

The force motor and the beam assembly of the electro-hydraulic actuator under analysis are used as an actuator for the flapper-nozzle amplifier. Such actuators require, for satisfactory operation and application, certain minimum or maximum achievements, principally with respect to force, stroke, speed of response, sensitivity, linearity and power consumption.

In addition to these, rigid specifications concerning immunity to vibration and shock and to environmental conditions of atmosphere and temperature may have to be met. As in all design problems, compromises must be made since optimization with respect to one desirable feature is generally incompatible with the best design from another point of view.

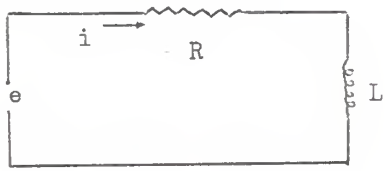
Electric force motors are of the electromagnetic type involving either moving coils or moving armatures. Action is controlled either through modulation of a polarizing field by means of a control current or through modulating an alternating control current of the same frequency as the reference field.

Electrostatic, piezoelectric or magnetostrictive devices theoretically may be considered but practically are relatively inefficient and inadequate owing to insufficient force and stroke for reasonable sizes and driving power. Crystals are rather difficult to handle, are subject to cracking and are sensitive to temperature variation.

The force motor used with the actuator analyzed is a moving coil type device and its main function is to deliver a force to the beam assembly in response to a control signal. The beam assembly converts this force into a translational motion to control the flow and pressure of the first-stage flapper-nozzle amplifier.

TRANSFER FUNCTION

The force motor windings can be considered to be a pure resistance in series with an inductance. If the resistance of the coil = R ohms, and the self-inductance of the coil = L henries, an applied voltage of e volts will cause a flow of i amperes in the circuit.



$$e = Ri + LDi \qquad [1]$$

Let us define for small changes about an operating point

$$e = e_o + \Delta e$$
$$i = i_o + \Delta i \qquad [2]$$

Substituting (2) in (1)

$$e_o + \Delta e = R(i_o + \Delta i) + LD(i_o + \Delta i) \qquad [3]$$

Also initially at a steady state operating point

$$e_o = Ri_o + LDi_o \qquad [4]$$

Subtracting (4) from (3)

$$\Delta e = R\Delta i + LD\Delta i$$

Taking the Laplace Transformation with initial conditions equal to zero

$$\Delta E = R\Delta I + SL\Delta I$$

or

$$\Delta E = (R + SL)\Delta I$$

solving for ΔI

$$\Delta I = \frac{\Delta E}{R(T_1 S + 1)} \qquad [5]$$

where $T_1 = \frac{L}{R}$ is the time constant for the force motor.

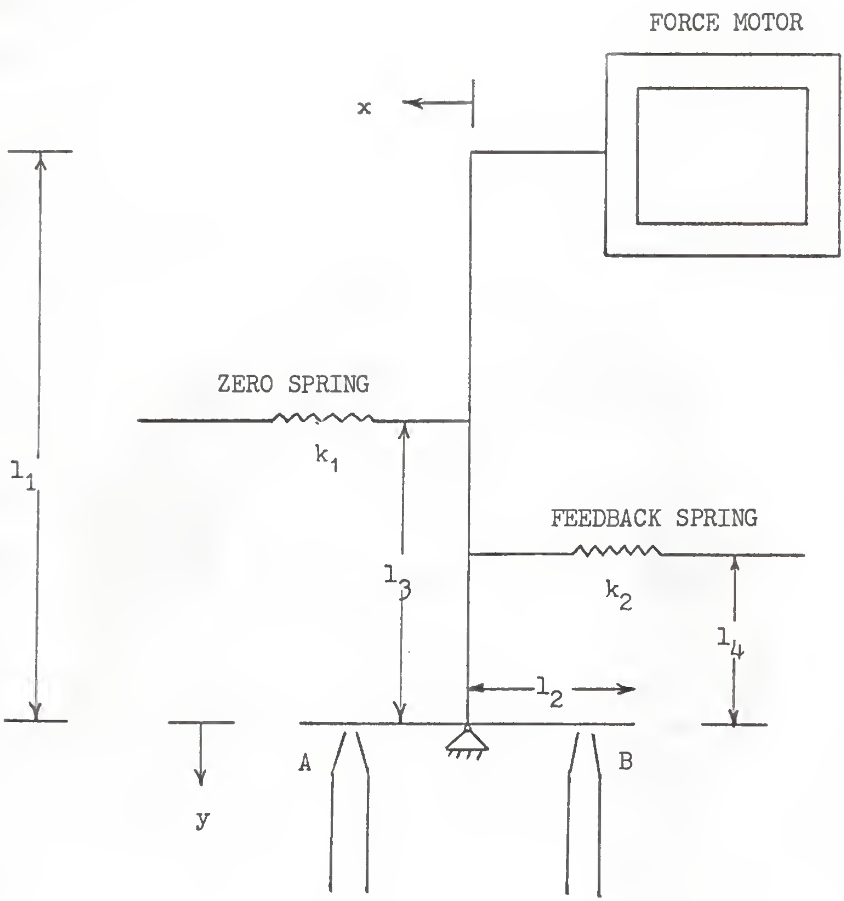


Figure (2), A View of Force Motor and Beam Assembly

This flow of current in the coil placed in the magnetic field produces a force on the coil given by

$f = B\pi dNi$; where: B = flux density produced by the permanent magnet

d = diameter of the coil

N = number of turns in the coil

For small changes in current value from an initial operating point i_o;

$$f = B\pi dN\, i$$

Taking Laplace Transformation with initial conditions equal to zero

$$\Delta F = B\pi dN\Delta I$$

Substituting for ΔI from [5], the following is obtained:

$$\Delta F = \frac{B\pi dN}{R(T_1 S + 1)}\,\Delta E \qquad [6]$$

This force applied on the coil and beam assembly rotates the beam through a small angle θ_1 resulting in a horizontal motion of the coil towards the left say ΔX units (fig. 2).

Assume that the magnitude of the flow forces on the beam assembly is negligible as compared with other forces (because fluid pressure is 50 psig and the diameter of the nozzle = .0465 inches). Also assume that the mass of the beam is small compared to the mass of the coil.

Summing forces on the coil-beam assembly

$$f = M_1\ddot{x} + B_1\dot{x} + \left(k_1 \frac{l_3}{l_1} + k_2 \frac{l_4}{l_1}\right) x$$

For small changes in x about an initial operating point x_o

$$\Delta f = M_1\Delta\ddot{x} + B_1\Delta\dot{x} + \left(k_1 \frac{l_3}{l_1} + k_2 \frac{l_4}{l_1}\right) \Delta x$$

where M_1 = The mass of the coil

B_1 = Damping coefficient for coil-beam assembly

k_1 = spring stiffness for zero spring

k_2 = spring stiffness for feedback spring

l_1, l_2, l_3, l_4 are the lengths on the beam assembly as shown in fig. 2.

Taking Laplace Transformation with initial conditions equal to zero

$$\Delta F = \left[M_1 S^2 + B_1 S + \left(\frac{k_1 l_3}{l_1} + \frac{k_2 l_4}{l_1} \right) \right] \Delta X$$

Substituting for ΔF

$$\frac{B\pi dN}{R(T_1 S + 1)} \Delta E = \left[M_1 S^2 + B_1 S + \left(\frac{k_1 l_3}{l_1} + \frac{k_2 l_4}{l_1} \right) \right] \Delta X$$

or $$\frac{\Delta X}{\Delta E} = \frac{B\pi dN}{R(T_1 S + 1) \left[M_1 S^2 + B_1 S + \left(\frac{k_1 l_3}{l_1} + \frac{k_2 l_4}{l_1} \right) \right]} \qquad [7]$$

which is the required transfer function for the coil and beam assembly.

The displacement ΔX in the horizontal direction is transmitted to the flappers of the first-stage flapper-nozzle amplifier. If l_1 is the length of the beam and l_2 is the length of the flapper from the fulcrum,

$$\frac{\Delta X}{l_1} = \frac{\Delta Y}{l_2}$$

substituting in [7] the following is obtained:

$$\Delta X = \frac{l_1}{l_2} \Delta Y$$

$$\frac{\Delta Y}{\Delta E} = \frac{l_2 B\pi dN}{l_1 R(T_1 S + 1) \left[M_1 S^2 + B_1 S + \left(\frac{k_1 l_3}{l_1} + \frac{k_2 l_4}{l_1} \right) \right]} \qquad [8]$$

FIRST-STAGE FLAPPER-NOZZLE AMPLIFIER

This component of the electro-hydraulic actuator is a power-amplifying device. This is needed because the power required to effect a corrective change in the controlled variable, piston position, is large compared with the power available in the electrical signal used as an input.

DERIVATION OF TRANSFER FUNCTION

Reference is made to fig. 3. The following assumptions are made:

1. During all operation, the supply pressure p_1 is constant.
2. The tube connecting the bellows and the nozzle is rigid and does not undergo expansion during operation.

Considering the flow through the restriction R_1, nozzle A and into the bellows A,

$$q_2 = q_4 + q_5 \qquad [9]$$

Also $$q_2 = K_d \; A_1 \sqrt{\frac{2(p_1 - p_2)}{\rho}} \qquad [10]$$

$$q_4 = K_d \pi \, 2 \, r_1 \, (y_c - y) \sqrt{\frac{2p_2}{\rho}} \qquad [11]$$

$$q_5 = (V_o + A_2 z) \frac{\dot{p}_2}{B_s} + A_2 \dot{z} \qquad [12]$$

From [9]

$$q_5 = q_2 - q_4$$

Substituting for q_2 and q_4

$$q_5 = K_d \; A_1 \sqrt{\frac{2(p_1 - p_2)}{\rho}} - K_d \; 2\pi r_1 (y_c - y) \sqrt{\frac{2p_2}{\rho}};$$

$$q_5 = f(p_2, y)$$

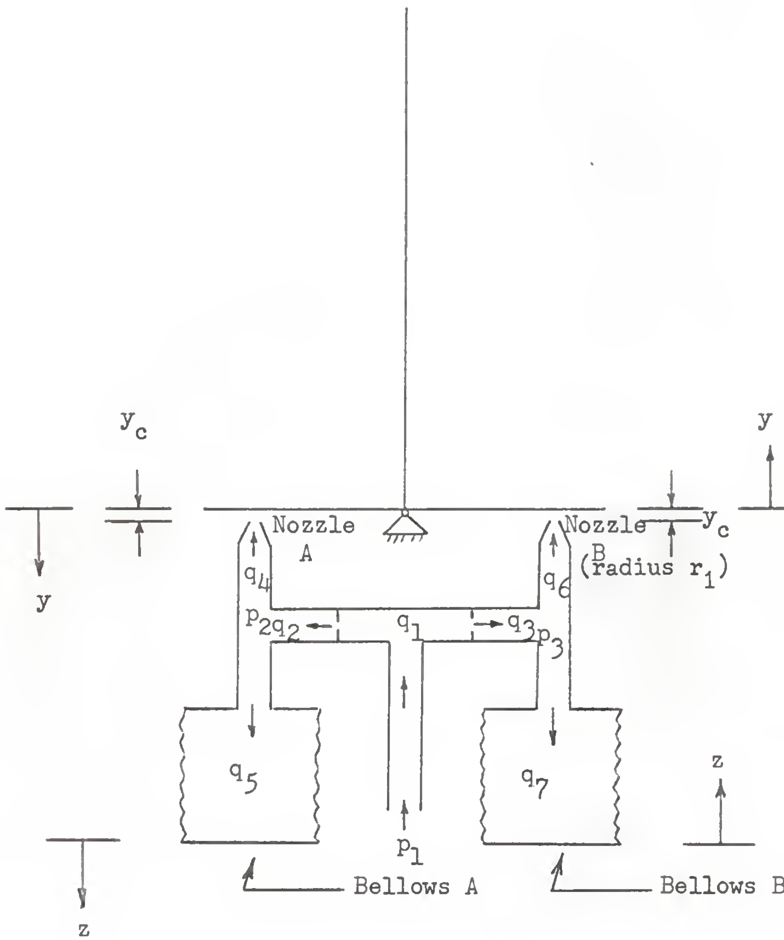


Figure (3), First-Stage Flapper-Nozzle Amplifier

Using a Taylor's series expansion and neglecting terms of order higher than one under the assumption that changes in p_2 and y are small

$$\Delta q_5 = \left.\frac{\partial q_5}{\partial p_2}\right|_{\substack{p_{2_o} \\ y_o}} \Delta p_2 + \left.\frac{\partial q_5}{\partial y}\right|_{\substack{p_{2_o} \\ y_o}} \Delta y$$

where $\Delta q_5 = q_5 - q_{5_o}$

$$\Delta p_2 = p_2 - p_{2_o} \qquad [13]$$

$$\Delta y = y - y_o$$

Let $\left.\frac{\partial q_5}{\partial p_2}\right|_{\substack{p_{2_o} \\ y_o}} = K_1$ and $\left.\frac{\partial q_5}{\partial y}\right|_{\substack{p_{2_o} \\ y_o}} = K_2$

then $\Delta q_5 = K_1 \Delta p_2 + K_2 \Delta y$ [14]

From equation [12]

$$q_5 = \frac{V_o \dot{p}_2}{B_s} + \frac{A_2 z \dot{p}_2}{B_s} + A_2 \dot{z} \qquad [15]$$

Let $a_1 = \frac{A_2}{B_s} z \dot{p}_2$

$= f(z, \dot{p}_2)$

Define $\Delta z = z - z_o$ [16]

$$\Delta p_2 = \dot{p}_2 - \dot{p}_{o2}$$

$$\Delta a_1 = a_1 - a_{1_o}$$

Using Taylor's expansion and neglecting terms of order higher than one under the assumption that changes in z and $\dot{p}_2$ are small.

$$a_1 = \left.\frac{\partial a_1}{\partial z}\right|_{\substack{z_o \\ \dot{p}_{2_o}}} \Delta z + \left.\frac{\partial a_1}{\partial \dot{p}_2}\right|_{\substack{z_o \\ \dot{p}_{2_o}}} \Delta \dot{p}_2$$

The term $\left.\frac{\partial a_1}{\partial z}\right|_{\substack{z_o \\ \dot{p}_{2_o}}}$ will be zero because $\dot{p}_{2_o}$ is zero

when p_{2_o} is a constant.

$$\Delta a_1 = \left.\frac{\partial a_1}{\partial \dot{p}_2}\right|_{\substack{z_o \\ \dot{p}_{2_o}}} \Delta \dot{p}_2 = \frac{A_2 z_o \Delta \dot{p}_2}{B_s} \qquad [17]$$

Substituting [14], [16] and [17] in [5], the following is obtained

$$q_{5_o} + \Delta q_5 = \frac{V_o}{B_s} (\dot{p}_{2_o} + \Delta \dot{p}_2) + a_{1_o} + \Delta a_1 + A_2 (\dot{z}_o + \Delta \dot{z}) \qquad [18]$$

Also initially

$$q_{5_o} = \frac{V_o \dot{p}_{2_o}}{B_s} + a_{1_o} + A_2 \dot{z}_o \qquad [19]$$

Subtracting [19] from [18]

$$\Delta q_5 = \frac{V_o \Delta \dot{p}_2}{B_s} + \Delta a_1 + A_2 \Delta \dot{z}$$

Substituting for Δa_1 from [17]

$$\Delta q_5 = \frac{V_o \Delta \dot{p}_2}{B_s} + \frac{A_2 z_o \Delta \dot{p}_2}{B_s} + A_2 \Delta \dot{z} \qquad [20]$$

Equating [14] and [20]

$$K_1 \Delta p_2 + K_2 \Delta y = \frac{V_o \Delta \dot{p}_2}{B_s} + \frac{A_2 z_o \Delta \dot{p}_2}{B_s} + A_2 \Delta \dot{z}$$

$$K_2 \Delta y = \frac{(V_o + A_2 z_o)}{B_s} \Delta \dot{p}_2 - K_1 \Delta p_2 + A_2 \Delta \dot{z}$$

Taking Laplace Transformation with initial conditions equal to zero

$$K_2 \Delta Y = \left[\frac{(V_o + A_2 z_o)}{B_s} S - K_1 \right] \Delta P_2 + A_2 S \Delta Z \qquad [21]$$

Summing forces on the bellows A and making the following assumptions:

1. The mass of the bellows is very small.
2. The bellows has a spring effect having a stiffness constant k_3.
3. The magnitude of the damping and friction forces is negligible as compared with other forces.

$$A_2 p_2 - k_3 z = 0$$

or $A_2 p_2 = k_3 z$

$$p_2 = \frac{k_3 z}{A_2}$$

For small changes about an operating point

$$\Delta p_2 = \frac{k_3}{A_2} \Delta z$$

Taking Laplace Transformation with initial conditions equal to zero

$$\Delta P_2 = \frac{k_3}{A_2} \Delta Z$$

Substituting this value of ΔP_2 in [21]

$$K_2 \Delta Y = \left[\frac{(V_o + A_2 z_o)}{B_s} S - K_1 \right] \frac{k_3}{A_2} \Delta Z + A_2 S \Delta Z$$

or $$\frac{\Delta Z}{\Delta Y} = \frac{A_2 K_2}{\left[\frac{k_3 (V_o + A_2 z_o)}{B_s} + A_2^2 \right] S - K_1 k_3} \qquad [22]$$

<u>Treatment for bellows B</u>

$$q_7 = q_3 - q_6 \qquad [23]$$

$$q_3 = K_d A_1 \sqrt{\frac{2(p_1 - p_3)}{\rho}} \qquad [24]$$

$$q_6 = K_d 2\pi r_1 (y_c + y) \sqrt{\frac{2p3}{\rho}} \qquad [25]$$

$$q_7 = (V_o - A_2 z) \frac{\dot{p}_3}{B_s} - A_2 \dot{z} \qquad [26]$$

Substituting for q_3 and q_6 the following is obtained

$$q_7 = K_d A_1 \sqrt{\frac{2(p_1 - p_3)}{\rho}} - K_d\, 2\pi r_1 (y_c + y) \sqrt{\frac{2p_3}{\rho}}$$

$$q_7 = f(p_3, y)$$

Using Taylor's expansion and neglecting terms of the order higher than the first under the assumption that changes in p_3 and y are small.

$$\Delta q_7 = \left.\frac{\partial q_7}{\partial p_3}\right|_{\substack{p_{3_o} \\ y_o}} \Delta p_3 + \left.\frac{\partial q_7}{\partial y}\right|_{\substack{p_{3_o} \\ y_o}} \Delta y$$

Let $\left.\frac{\partial q_7}{\partial p_3}\right|_{\substack{p_{3_o} \\ y_o}} = K_3$ and $\left.\frac{\partial q_7}{\partial y}\right|_{\substack{p_{3_o} \\ y_o}} = K_4$

$$\therefore q_7 = K_3 \Delta p_3 + K_4 \Delta Y \qquad [27]$$

From equation [26]

$$q_7 = \frac{V_o\dot{p}_3}{B_s} - \frac{A_2 z\,\dot{p}_3}{B_s} - A_2\dot{z} \qquad [28]$$

The non-linear term $\frac{A_2 z\dot{p}_3}{B_s}$ makes the equation non-linear.

Let $a_2 = \frac{A_2 z\dot{p}_3}{B_s}$

$= f(z,\dot{p}_3)$

Define $\Delta z = z - z_o$

$$\Delta q_7 = q_7 - q_{7_o}$$

$$\Delta\dot{p}_3 = \dot{p}_3 - \dot{p}_{3_o} \qquad [29]$$

$$\Delta a_2 = a_2 - a_{2_o}$$

Using Taylor's expansion and neglecting terms of order higher than one under the assumption that changes in z and $\dot{p}_3$ are small.

$$\Delta a_2 = \left.\frac{\partial a_2}{\partial z}\right|_{\substack{z_o\\ \dot{p}_{3_o}}}\Delta z + \left.\frac{\partial a_2}{\partial \dot{p}_3}\right|_{\substack{z_o\\ \dot{p}_{3_o}}}\Delta\dot{p}_3$$

The term $\left.\frac{\partial a_2}{\partial z}\right|_{\substack{z_o\\ \dot{p}_{3_o}}}$ will be zero because $\dot{p}_{3_o}$ is zero when $\dot{p}_{3_o}$ is a constant.

$$\Delta a_2 = \left.\frac{\partial a_2}{\partial \dot{p}_3}\right|_{\substack{z_o\\ \dot{p}_{3_o}}}\Delta\dot{p}_3 = \frac{Az_o\Delta\dot{p}_3}{B_s} \qquad [30]$$

Substituting [29] and [30] in [28], the following is obtained

$$q_{7_o} + \Delta q_7 = \frac{V_o}{B_s}(\dot{p}_{3_o} + \Delta\dot{p}_3) - (a_{2_o} + \Delta a_2) - A_2(\dot{z}_o + \Delta\dot{z}) \quad [31]$$

Also initially

$$q_{7_o} = \frac{V_o}{B_s}\dot{p}_{3_o} - a_{2_o} - A_2 z_o \quad [32]$$

Subtracting [32] from [31], the following is obtained

$$\Delta q_7 = \frac{V_o}{B_s}\Delta\dot{p}_3 - \Delta a_2 - A_2\Delta\dot{z}$$

Substituting for Δa_2

$$\Delta q_7 = \frac{V_o}{B_s}\Delta\dot{p}_3 - \frac{A_2 z_o \Delta p_3}{B_s} - A_2\Delta\dot{z} \quad [33]$$

Equating [27] and [33]

$$K_3\Delta p_3 + K_4\Delta y = \frac{V_o}{B_s}\Delta\dot{p}_3 - \frac{A_2 z_o \Delta\dot{p}_3}{B_s} - A_2\Delta\dot{z}$$

or $$K_4\Delta y = \frac{V_o}{B_s}\Delta\dot{p}_3 - \frac{A_2 z_o}{B_s}\Delta\dot{p}_3 - K_3\Delta p_3 - A_z\Delta\dot{z}$$

Taking Laplace Transformation with initial conditions equal to zero

$$K_4\Delta Y = \left[\frac{(V_o - A_2 z_o)}{B_s} S - K_3\right]\Delta P_3 - A_2 S\,\Delta Z \quad [34]$$

Summing forces on the bellows B and making the following assumptions:

1. The mass of the bellows is very small.
2. The bellows has a spring effect having a stiffness constant k_3.
3. The magnitude of the damping and friction forces is negligible as compared with other forces.

$$A_2 p_3 = k_3 z$$

Define

$$p_3 - p_{3_o} = \Delta p_3$$

$$z - z_o = \Delta z$$

$$A_2(p_{3_o} + \Delta p_3) = k_3(z_o + \Delta z)$$

Also initially

$$A_2 p_{3_o} = k_3 z_o$$

∴ for small changes about an operating point

$$\Delta p_3 = \frac{k_3}{A_2} \Delta z$$

Taking Laplace Transformation with initial conditions equal to zero.

$$\Delta P_3 = \frac{k_3}{A_2} \Delta Z$$

Substituting this value of ΔP_3 in [34]

$$K_4 \Delta Y = \left[\frac{(V_o - A_2 z_o)}{B_s} S - K_3\right] \frac{k_3}{A_2} \Delta Z - A_2 S \Delta Z$$

$$\frac{\Delta Z}{\Delta Y} = \frac{A_2 K_4}{\left[\frac{k_3(V_o - A_2 z_o)}{B_s} - A_2^2\right] S - K_3 k_3} \qquad [35]$$

Expressions [22] and [35] are the required transfer functions for the first stage of the flapper-nozzle amplifier.

SECOND STAGE FLAPPER NOZZLE AMPLIFIER (PILOT ASSEMBLY) AND ACTUATOR

The pilot assembly used for the actuator under study consists of a flapper-nozzle valve. A servovalve is a fluid valve that varies output flow or pressure in response to an input control signal. The signal can be electrical, mechanical, fluid pressure, force-reaction of a gyroscope, or even thermal expansion of a trapped liquid. In the present case, the control signal to the pilot assembly is the mechanical movement of the flapper over nozzles N_1 and N_2 caused by the pressure variations in the bellows.

There are two main types of servovalves--pressure control and flow control. The pressure control type provides a pressure proportional to input displacement, has low impedance, is highly load sensitive. The flapper-nozzle valve being used here is a flow-control type and it provides an output flow proportional to input displacement at constant load pressure.

An "ideal" valve has high static stiffness. It will accelerate inertia loads quickly, resist disturbances and has low static error. Undesirable valve characteristics are poor linearity and zero pressure-flow slope at neutral, which has a destablizing effect. The actuator itself is a cylinder-piston assembly. Oil enters one side of the piston and flows out the other side.

The actuator receives energy from the pilot assembly and converts it into mechanical power delivered to the load. In general overall system performance will depend on a good choice of actuator type and size. An increase in the desired range of load displacement can make the dynamic performance poor. Selection of an actuator is a trial and error process involving the following factors.

1. Load characteristics.
2. Dynamic performance desired.
3. Control valve type.
4. Allowable pressure or flow.
5. Gear ratio in the case of rotary actuators.
6. Size of the actuator.

A study of the various forces acting on an actuator is also an interesting field. The actuator must be capable of overcoming its own friction forces or torques as well as those of the load. It must also be capable of accelerating the load and itself at the rates necessary to execute the desired load motion. In addition if the load has other forces or torques associated with it, the actuator must be able to deliver the force or torque necessary to overcome them. The actuator must be able to deliver all these forces or torques at the velocities necessary to execute the desired load motion. Therefore to select an actuator one must know the following.

1. Load velocities and damping.
2. Load acceleration and mass.
3. Load friction.
4. Load forces or torques.

In addition to these forces there may be a "stiction" force due to the formation of tiny welds between sliding metal surfaces in either the load or cylinder when the system has been in a fixed position for some period of time.

Derivation of Transfer Function

With reference to fig. 4, the following assumptions are made:

1. The pressure under the piston is uniform throughout the cylinder, line and valve and is equal to p_4.

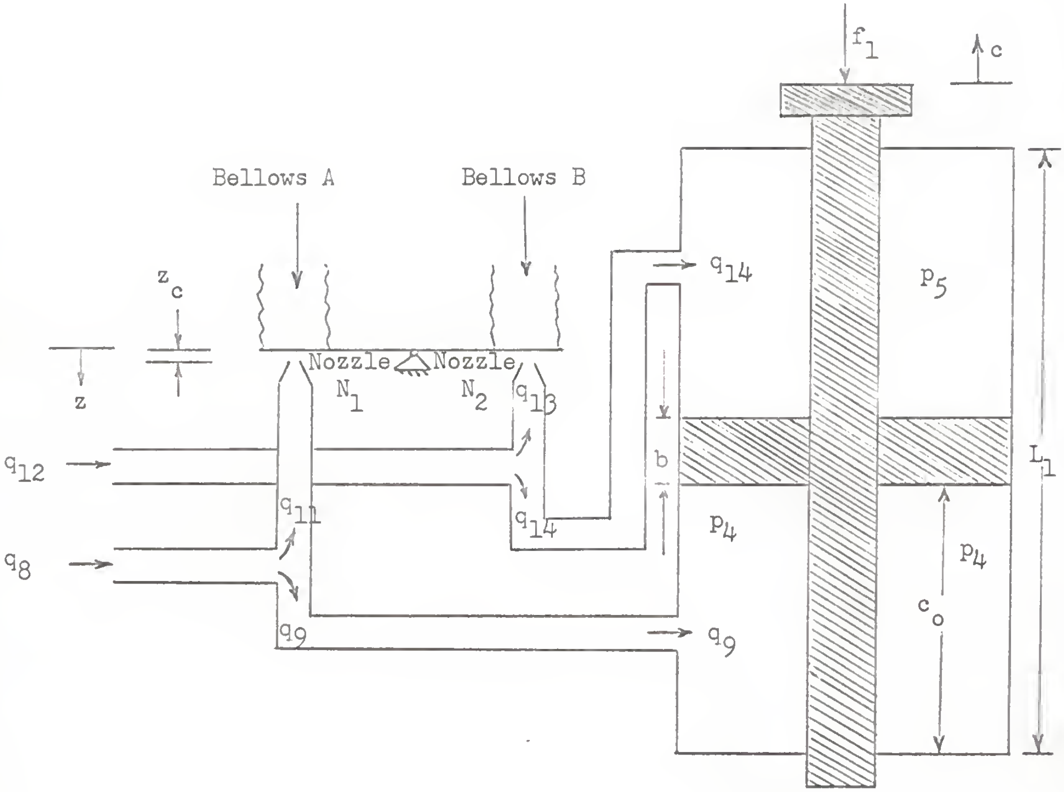


Figure (4), Second-Stage Flapper-Nozzle Amplifier and the Actuator

2. The pressure above the piston is uniform throughout the cylinder, line and valve and is equal to p_5.
3. No leakage across the piston because o-rings used.
4. The flow from the pump is constant; that is, it is a constant delivery pump.
5. K_d for the nozzles is constant and equal.
6. Lines connecting the valve and cylinder are infinitely rigid.
7. Piston and cylinder are infinitely rigid.

Considering the flow from the pump, at the nozzle N_1 and into the cylinder under the piston.

$$q_9 = q_8 - q_{11} \qquad [36]$$

Also $q_{11} = K_d \, 2\pi r_2 \, (z_c - z) \sqrt{\frac{2p_4}{\rho}}$

Substituting in [36]

$$q_9 = q_8 - K_d \, 2\pi r_2 \, (z_c - z) \sqrt{\frac{2p_4}{\rho}} \qquad [37]$$

$q_9 = f(p_4, z)$; where q_8 is constant.

Using Taylor's expansion and neglecting terms of order higher than one under the assumption that changes in z and p_4 are small.

$$\Delta q_9 = \left.\frac{\partial q_9}{\partial p_4}\right|_{\substack{p_{4_o} \\ z_o}} \Delta p_4 + \left.\frac{\partial q_9}{\partial z}\right|_{\substack{p_{4_o} \\ z_o}} \Delta z$$

Let $\left.\frac{\partial q_9}{\partial p_4}\right|_{\substack{p_{4_o} \\ z_o}} = K_5$ and $\left.\frac{\partial q_9}{\partial z}\right|_{\substack{p_{4_o} \\ z_o}} = K_6$

$$\Delta q_9 = K_5 \Delta p_4 + K_6 \Delta z$$

Taking Laplace Transformation with initial conditions equal to zero.

$$\Delta Q_9 = K_5 \Delta P_4 + K_6 \Delta Z \qquad [38]$$

Also considering the flow into the chamber below the piston

$$q_9 = (V_1 + A_3 c) \frac{1}{B_s} \dot{p}_4 + A_3 \dot{c}$$

or $$q_9 = \frac{V_1 \dot{p}_4}{B_s} + \frac{A_3 c\, \dot{p}_4}{B_s} + A_3 \dot{c} \qquad [39]$$

This is non-linear equation having the non-linear term $\frac{A_3 c\, \dot{p}_4}{B_s}$.

Let $$a_3 = \frac{A_3 c\, \dot{p}_4}{B_s} \qquad [40]$$

Also define

$$\begin{aligned} c - c_o &= \Delta c \\ \dot{p}_4 - \dot{p}_{4_o} &= \Delta \dot{p}_4 \\ q_9 - q_{9_o} &= \Delta q_9 \\ a_3 - a_{3_o} &= \Delta a_3 \end{aligned} \qquad [41]$$

From [40]

$$a_3 = f(c, \dot{p}_4)$$

Using Taylor's expansion and neglecting the terms of order higher than one under the assumption that the changes in c and $\dot{p}_4$ are small

$$\Delta a_3 = \left.\frac{\partial a_3}{\partial c}\right|_{\substack{c_o \\ \dot{p}_{4_o}}} \cdot \Delta c + \left.\frac{\partial a_2}{\partial \dot{p}_4}\right|_{\substack{c_o \\ \dot{p}_{4_o}}} \Delta \dot{p}_4.$$

If p_{4_o} is a constant, $\dot{p}_{4_o}$ is zero, hence $\left.\frac{\partial a_3}{\partial c}\right|_{c_o, \dot{p}_{4_o}}$ is zero

$$\Delta a_3 = \left.\frac{\partial a_3}{\partial \dot{p}_4}\right|_{c_o, \dot{p}_{4_o}} \Delta \dot{p}_4 = \frac{A_3 c_o}{B_s} \Delta \dot{p}_4 \qquad [42]$$

Substituting [41] in [39]

$$q_9 = \frac{V_1}{B_s} (\dot{p}_{4_o} + \Delta \dot{p}_4) + a_{3_o} + \Delta a_3 + A_3 c_o + A_3 \Delta c \qquad [43]$$

Also initially

$$q_{9_o} = \frac{V_1}{B_s} \dot{p}_{4_o} + a_{3_o} + A_3 \dot{c}_o \qquad [44]$$

Subtracting [44] from [43]

$$\Delta q_9 = \frac{V_1}{B_s} \Delta \dot{p}_4 + \Delta a_3 + A_3 \Delta \dot{c}$$

Substituting Δa_3 from [42]

$$\Delta q_9 = \frac{V_1}{B_s} \Delta \dot{p}_4 + \frac{A_3 c_o}{B_s} \Delta \dot{p}_4 + A_3 \Delta \dot{c}$$

Taking Laplace Transformation with initial conditions equal to zero

$$\Delta Q_9 = \frac{(V_1 + A_3 c_o)}{B_s} S \Delta P_4 + A_3 S \Delta C \qquad [45]$$

Equating [38] and [45]

$$K_5 \Delta P_4 + K_6 \Delta_Z = \frac{V_1 + A_3 c_o}{B_s} S \Delta P_4 + A_3 S \Delta C$$

Rearranging

$$\Delta P_4 = \frac{A_3 S \Delta C - K_6 \Delta Z}{K_5 - \frac{(V_1 + A_3 c_o) S}{B_s}}$$

Let $\frac{V_1 + A_3 c_o}{B_s} = K_9$

$$\Delta P_4 = \frac{A_3 S \Delta C - K_6 \Delta Z}{K_5 - K_9 S} \qquad [46]$$

Now considering the flow from the pump, at the nozzle N_2 and into the cylinder above the piston.

$$q_{14} = q_{12} - q_{13} \qquad [47]$$

Also $q_{13} = K_d \, 2 \pi r_2 (z_c + z) \sqrt{\frac{2p_5}{\rho}}$

$$q_{14} = q_{12} - K_d \, 2\pi r_2 (z_c + z) \sqrt{\frac{2p_5}{\rho}}$$

$q_{14} = f\,(z, p_5)$; where q_{12} is constant.

Using Taylor's expansion and neglecting terms of order higher than one under the assumption that changes in z and p_5 are small.

$$\Delta q_{14} = \frac{\partial q_{14}}{\partial p_5}\bigg|_{\substack{z_o \\ p_{5_o}}} \Delta p_5 + \frac{\partial q_{14}}{\partial z}\bigg|_{\substack{z_o \\ p_{5_o}}} \Delta z$$

Let $\frac{\partial q_{14}}{\partial p_5}\bigg|_{\substack{z_o \\ p_{5_o}}} = K_7$ and $\frac{\partial q_{14}}{\partial z}\bigg|_{\substack{z_o \\ p_{5_o}}} = K_8$

$$\Delta q_{14} = K_7 \Delta p_5 + K_8 \Delta z$$

Taking Laplace Transformation with initial conditions equal to zero.

$$\Delta Q_{14} = K_7 \Delta P_5 + K_8 \Delta Z \qquad [48]$$

Now considering the flow into the chamber above the piston

$$q_{14} = \left[\frac{V_2 + A_3 \left[L_1 - (b + c_o)\right]}{B_s} . \right] \dot{p}_5 - A_3 \dot{c}$$

$$\text{or } q_{14} = \frac{V_2}{B_s} \dot{p}_5 + \frac{A_3 L_1 \dot{p}_5}{B_s} - \frac{A_3 b\, \dot{p}_5}{B_s} - \frac{A_3 c_o \dot{p}_5}{B_s} - A_3 \dot{c} \qquad [49]$$

This is a non-linear equation having the non-linear term $\frac{A_3 c\, \dot{p}_5}{B_s}$.

Let $a_4 = \frac{A_3 c\, \dot{p}_5}{B_s}$

$$a_4 = f\,(c,\, \dot{p}_5)$$

Using Taylor's expansion and neglecting terms of order higher than one under the assumption that changes in c and $\dot{p}_5$ are small.

$$\Delta a_4 = \left.\frac{\partial a_4}{\partial c}\right|_{c_o, \dot{p}_{5_o}} \Delta c + \left.\frac{\partial a_4}{\partial \dot{p}_5}\right|_{c_o, \dot{p}_{5_o}} \Delta \dot{p}_5$$

If p_{5_o} is constant, $\dot{p}_{5_o}$ is zero and $\left.\frac{\partial a_4}{\partial c}\right|_{c_o, \dot{p}_{5_o}}$ is zero.

$$\Delta a_4 = \left.\frac{\partial a_4}{\partial \dot{p}_5}\right|_{c_o, \dot{p}_{5_o}} \Delta \dot{p}_5 = \frac{A_3}{B_s} . c_o \;\Delta \dot{p}_5 \qquad [50]$$

Define the following

$$c - c_o = \Delta c$$

$$\dot{p}_5 - \dot{p}_{5_o} = \Delta \dot{p}_5$$

$$p_5 - p_{5_o} = \Delta p_5 \qquad [51]$$

$$q_{14} - q_{14_o} = \Delta q_{14}$$

$$a_3 - a_{3_o} = \Delta a_3$$

Using [49] [50] and [51] the following is obtained

$$\Delta q_{14} = \frac{V_2 \Delta \dot{p}_5}{B_s} + \frac{A_3 L_1 \Delta \dot{p}_5}{B_s} - \frac{A_3 b \Delta \dot{p}_5}{B_s} - \frac{A_3 c_o \Delta \dot{p}_5}{B_s} - A_3 \Delta \dot{c}$$

$$\text{or } \Delta q_{14} = \left[\frac{V_2 + A_3(L_1 - b - c_o)}{B_s} \right] \Delta \dot{p}_5 - A_3 \Delta \dot{c}$$

Taking Laplace Transformation with initial conditions equal to zero

$$\Delta Q_{14} = \left[\frac{V_2 + A_3(L_1 - b - c_o)}{B_s} \right] S \Delta P_5 - A_3 S \Delta C \qquad [52]$$

Equating [48] and [52] and rearranging, the following is obtained

$$\Delta P_5 = \frac{A_3 S \Delta C + K_8 \Delta Z}{\left[\frac{V_2 + A_3(L_1 - b - c_o)}{B_s} \right] S - K_7}$$

Let $\dfrac{V_2 + A_3(L_1 - b - c_o)}{B_s} = K_{10}$,

So that

$$\Delta P_5 = \frac{A_3 S \Delta C + K_8 \Delta Z}{K_{10} S - K_7} \qquad [53]$$

Next taking a force balance on the piston

$$p_4A_3 - p_5A_3 - f_1 - B_2\dot{c} - \left[K_s(p_4 + p_5) + f_2\right]\frac{\dot{c}}{|\dot{c}|} - M_2\ddot{c} = 0$$

where f_1 = load force

B_2 = cylinder viscous damping

K_s = pressure dependent coulomb friction

f_2 = pressure independent coulomb friction

A_3 = effective area of the piston

M_2 = mass of piston and rod and load

Assumption: friction forces represented by the term $\left[K_5(p_4 + p_5) + f_2\right]\frac{\dot{c}}{|\dot{c}|}$ have small magnitude when compared with the magnitude of the other forces acting on the actuator. This term will be neglected for the present analysis in which it is desired to build up a linear model so that the usual techniques available for linear systems can be applied.

Therefore

$$p_4A_3 - p_5A_3 - f_1 - B_2\dot{c} = M_2\ddot{c} \qquad [54]$$

Let us define

$$\begin{aligned} \ddot{c} - \ddot{c}_o &= \Delta\ddot{c} \\ \dot{c} - \dot{c}_o &= \Delta\dot{c} \\ c - c_o &= \Delta c \\ p_4 - p_{4_o} &= \Delta p_4 \\ p_5 - p_{5_o} &= \Delta p_5 \\ f_1 - f_{1_o} &= \Delta f_1 \end{aligned} \qquad [55]$$

Substituting [55] in [54]

$$(p_{4_o} + \Delta p_4)A_3 - (p_{5_o} + \Delta p_5)A_3 + (f_{1_o} + \Delta f_1) - B_2(\dot{c}_o + \Delta\dot{c}) = M_2(\ddot{c}_o + \Delta\ddot{c}) \qquad [56]$$

Also initially

$$p_{4_o} A_3 - p_{5_o} A_3 + f_{1_o} - B_2 \dot{c}_o = M_2 \ddot{c}_o \qquad [57]$$

Subtracting [57] from [56]

$$\Delta p_4 A_3 - \Delta p_5 A_3 - \Delta f_1 - B_2 \Delta \dot{c} = M_2 \Delta \ddot{c}$$

Taking Laplace Transformation with initial conditions zero.

$$(\Delta P_4 - \Delta P_5) A_3 - \Delta F_1 - B_2 S \Delta C = M_2 S^2 \Delta C$$

Substituting for ΔP_4 from 46 and for ΔP_5 from 53 ;

$$\left[\frac{A_3 S \Delta C - K_6 \Delta Z}{K_5 - K_9 S} - \frac{A_3 S \Delta C + K_8 \Delta Z}{K_{10} S - K_7} \right] A_3 = M_2 S^2 \Delta C + B_2 S \Delta C + \Delta F_1 \qquad [58]$$

Simplifying left hand side, the following is obtained

$$\left[\frac{(A_3^2 K_{10} + A_3^2 K_9) S^2 - (A_3^2 K_5 + A_3^2 K_7) S}{(K_5 - K_9 S)(K_{10} S - K_7)} \right] \Delta C + \left[\frac{A_3 (K_8 K_9 - K_6 K_{10}) S + (K_6 K_7 - K_5 K_8) A_3}{(K_5 - K_9 S)(K_{10} S - K_7)} \right] \Delta Z$$

Let $A_3^2 K_{10} + A_3^2 K_9 = K_{11}$

$A_3^2 K_5 + A_3^2 K_7 = K_{12}$

$A_3 (K_8 K_9 - K_6 K_{10}) = K_{13}$

$A_3 (K_6 K_7 - K_5 K_8) = K_{14}$

So equation [58] becomes

$$\frac{K_{11} S^2 - K_{12} S}{(K_5 - K_9 S)(K_{10} S - K_7)} \Delta C + \frac{K_{13} S + S_{14}}{(K_5 - K_9 S)(K_{10} S - K_7)} \Delta Z$$

$$= M_2 S^2 \Delta C + B_2 S \Delta C + \Delta F_1$$

Solving for ΔC

$$\left[\frac{K_{11} S^2 - K_{12} S}{(K_5 - K_9 S)(K_{10} S - K_7)} - M_2 S^2 - B_2 S \right] \Delta C = \Delta F_1 - \frac{K_{13} S + K_{14}}{(K_5 - K_9 S)(K_{10} S - K_7)} \Delta Z$$

Simplifying, the left hand side is equal to

$$\frac{M_2K_9K_{10}S^4 + (K_9K_{10}B_2 - M_2K_5K_{10} - K_7K_9M_2)\,S^3 + (K_{11} + K_5K_7M_2 - K_5K_{10}B_2 - K_7K_9B_2)S^2 + (K_5K_7B_2 - K_{12})S}{(K_5 - K_9S)(K_{10}S - K_7)}\,\Delta C$$

And the right hand side is equal to

$$\Delta F_1 - \frac{K_{13}S + K_{14}}{(K_5 - K_9S)(K_{10}S - K_7)}\,\Delta Z$$

for $F_1 = 0$

$$\frac{\Delta C}{\Delta Z} = \frac{-K_{13}S - K_{14}}{M_2K_9K_{10}S^4 + (K_9K_{10}B_2 - M_2K_5K_{10} - K_7K_9M_2)\,S^3 + (K_{11} + K_5K_7M_2 - K_5K_{10}B_2 - K_7K_9B_2)\,S^2 + (K_5K_7B_2 - K_{12})S} \qquad [59]$$

Equation [59] represents the required transfer function for the pilot assembly and the actuator.

MECHANICAL FEEDBACK

The actuator under analysis has mechanical feedback. This type of feedback improves the system's reliability and this increase in reliability has made the use of mechanical feedback actuators very popular; for example, for engine positioning on several space boosters[6].

The improvement in reliability is achieved by eliminating the use of an electrical position transducer (potentiometer, LVDT, or other) together with its associated wiring and power. Potentiometers place a serious limitation on the life and environmental capabilities of servoactuators. Substitution of an LVDT imposes additional electrical complexity and still leaves the feedback dependent upon an electrical power supply. Elimination of an electrical transducer altogether is highly desirable.

Even[7] more significant is the drastic loss of control resulting from electrical failure in a conventional electrical feedback actuator. If the potentiometer or LVDT opens, or if one or more wires in the connecting cable are severed, or if part or all of the electrical supply is lost, the actuator will drive hardover, usually at high velocity. Also with an electrical feedback actuator, the servoamplifier is located within the servoloop and loss of the amplifier or its connecting cables causes an open loop failure. The actuator analyzed, using mechanical feedback, will "fail neutral" with loss of electrical power.

However the change in role of the servoamplifier in a mechanical feedback actuator may impose additional design complexity. With the servoamplifier outside the servoloop, its gain, stability, linearity and dynamic response directly affect the system. These requirements are compounded by the electrical input characteristics of the force motor. Force motors respond to the current input from the amplifier, so ideally the amplifier should be a current source (high impedance source). The coil resistance may vary by ±30%, over the normal operating temperature range of the actuator and the amplifier should be insensitive to these changes in its ability to supply current in response to electrical commands. If not, the basic sensitivity of the actuator will vary.

Development of a mechanism needed to relate piston displacement to force motor force is also a critical problem. If a simple spring is used, the spring rate required is prohibitively low. Such a spring is too weak and too subject to vibrations. The alternative is to use a precision motion - reduction device between the piston and the force motor feedback spring. This device must be free from backlash, have good wear characteristics, be linear, have high acceleration and vibrational capability, and not be subject to inaccuracy with temperature variations, actuator loading, and working or field handling. The motion reduction device between the piston and the torque motor feedback spring used in the actuator selected is shown in fig. 5.

This device is a cone-shaped linear cam. The follower is supported in a close fitting support with bearing areas at each end.

Transfer Function:

Referring to fig. 6, $\frac{\Delta z_1}{\Delta c_1} = \tan \theta_1$,

$$\frac{\Delta z_1}{l_5} = \varphi \quad \left[\frac{\text{Arc length}}{\text{Radius}} = \text{Angle enclosed}\right]$$

Similarly

$$\frac{\Delta z_2}{l_6}$$

$$\therefore \Delta z_2 = \frac{l_6}{l_5} \Delta z_1$$

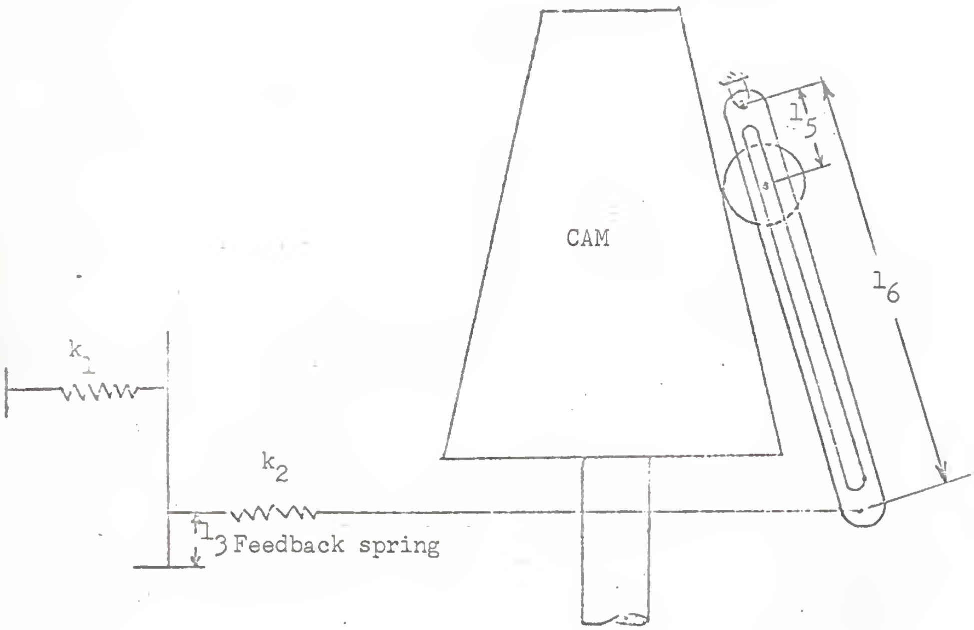


Figure (5), A . view of the Mechanical Feedback Device

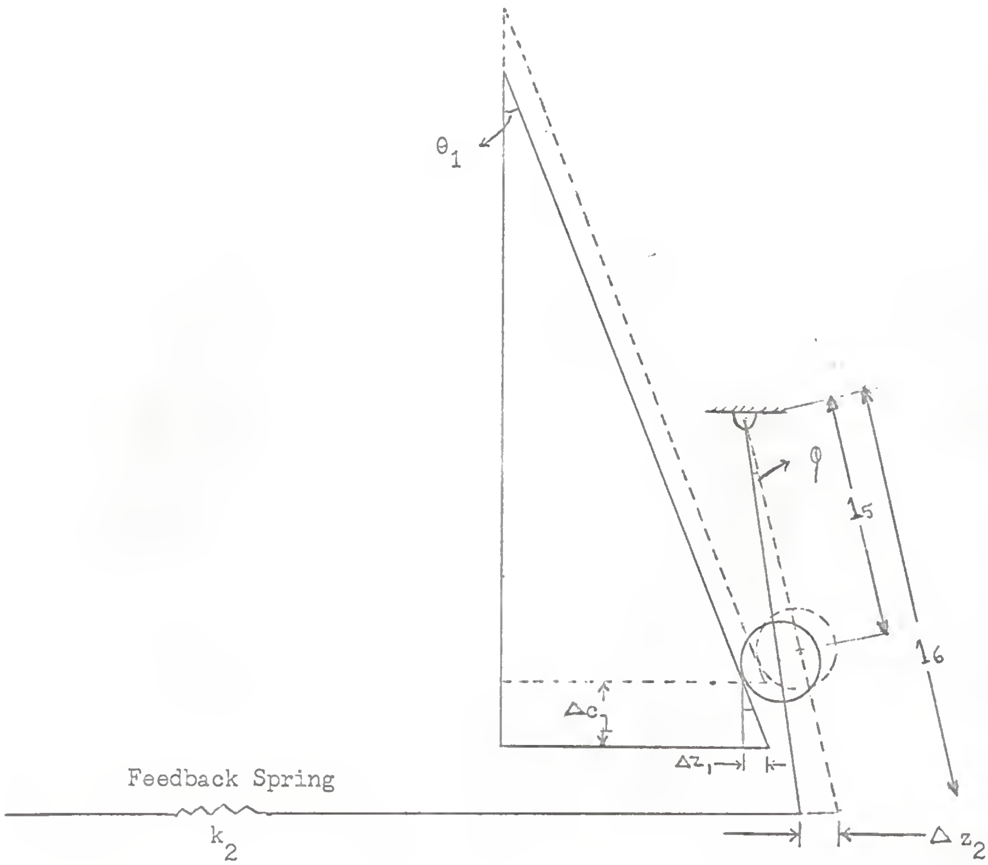


Figure (6) Extension of the Feedback Spring

Substituting for Δz_1

$$\Delta z_2 = \frac{l_6}{l_5} \Delta c_1 \operatorname{Tan} \theta_1;$$

$$\therefore \text{feedback force} = k_2 \frac{l_6}{l_5} \operatorname{Tan} \theta_1. \qquad [60]$$

where Δz_1 = Displacement of the center of the wheel

Δz_2 = Displacement of the feedback spring

l_5 = Distance between the center of the wheel and the fulcrum

l_6 = Distance between the pivot fulcrum and the point of feedback spring connection.

BLOCK DIAGRAM AND OVERALL TRANSFER FUNCTION

Let the transfer functions be identified as follows:

1. Force Motor = G_1
2. Beam Assembly = G_2
3. First-Stage Flapper-Nozzle Amplifier = G_3
4. Second-Stage Flapper-Nozzle Amplifier and the Actuator = G_4
5. Mechanical Feedback Device = H
6. Load Force Conversion = G_5

Then the system can be represented by a block diagram as shown in fig. 7. From fig. 7, assuming $\Delta F_1 = 0$

$$\Delta C = G_2 G_3 G_4 (G_1 \Delta E - H \Delta C)$$

$$\frac{\Delta C}{\Delta E} = \frac{G_1 G_2 G_3 G_4}{1 + G_2 G_3 G_4 H} \qquad [61]$$

This is the closed loop transfer function for the electro-hydraulic actuator. The open loop transfer function for the electro-hydraulic actuator analyzed is given by

$$G_2 G_3 G_4 H \qquad [62]$$

In the present analysis

$$G_1 = \frac{B \pi d N}{R(T_1 S + 1)}$$

$$G_2 = \frac{l_2}{l_1 \left(M_1 S^2 + B_1 S + \frac{k_1 l_3}{l_1} + \frac{k_2 l_4}{l_1}\right)}$$

$$G_3 = \frac{A_2 K_2}{\left[\frac{k_3 (V_o + A_2 z_o)}{B_s}\right] S - K_1 k_3}$$

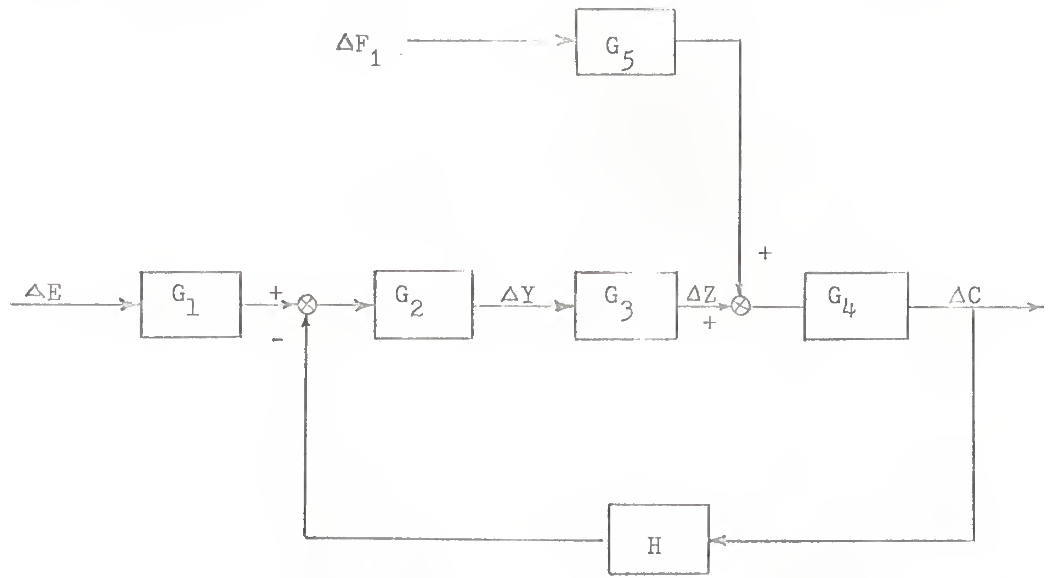


Figure (7), Block Diagram for the System

$$G_4 = \frac{-K_{13}S - K_{14}}{[M_2K_9K_{10}S^4 + (K_9K_{10}B_2 - M_2K_5K_{10} - K_7K_9M_2)S^3 + (K_{11} + K_5K_7M_2 - K_5K_{10}B_2 - K_7K_9B_2)S^2 + (K_5K_7B_2 - K_{12})S]}$$

$$H = \frac{k_2 l_6 \operatorname{Tan} \theta}{l_5}$$

$$G_5 = \frac{(K_5 - K_9S)(K_{10}S - K_7)}{(-K_{13}S - K_{14})}$$

From Appendices A, B, C and D

$$G_1 = \frac{1.092}{(.00287S + 1)}$$

$$G_2 = \frac{.169}{(.00077S^2 + 1)}$$

$$G_3 = \frac{12.82}{(.043S + 1)}$$

$$G_4 = \frac{.207}{(.00062S + 1)(.00695S^2 + .11S + 1)}$$

$$H = 1.98$$

Substituting these values, the closed loop transfer function for the electro-hydraulic actuator becomes

$$\frac{1}{[43.8 \times 10^{-14}S^7 + 1.55 \times 10^{-8}S^6 + 2.7 \times 10^{-5}S^5 + 1.04 \times 10^{-5}S^4 + 4.3 \times 10^{-4}S^3 + 2.06 \times 10^{-3}S^2 + 1.61 \times 10^{-1}S + 1.88]} \qquad [63]$$

Substituting the values of G_2, G_3, G_4, and H in [62], the open loop transfer function is

$$\frac{.887}{(.00077S^2 + 1)(.043S + 1)(.00062S + 1)(.00695S^2 + .11S + 1)} \qquad [64]$$

FREQUENCY RESPONSE OF THE THEORETICAL TRANSFER FUNCTION

The open loop transfer function has been derived to be

$$\frac{.887}{(.043S + 1)(.00062S + 1)(.00077S^2 + 1)(.00695S^2 + .11S + 1)}$$

This transfer function is analyzed for frequency response in Table 1. Bode's plots for the frequency response are represented in figures 8 and 9.

Table 1. Characteristics of log magnitude and angle diagram for various factors of the theoretical open loop transfer function.

	Factor	Corner Frequency r.p.s.*1	Corner Frequency c.p.s.*2	Log Magnitude	Angle Characteristics
1.	.887	None	None	Constant magnitude −1.04 db.	Constant 0 degree.
2.	$(.043S + 1)^{-1}$	23.25	3.69	0 slope below the corner frequency. -20 db/decade slope above the corner frequency.	Varies from 0 degree to -90 degrees. (-45 degrees at corner frequency).
3.	$(.00062S + 1)^{-1}$	1613	257	0 slope below the corner frequency. -20 db/decade slope above the corner frequency.	Varies from 0 degree to -90 degrees (-45 degrees at corner frequency.)
4.	$(.00077S^2 + 1)^{-1}$	36	5.72	0 slope below the corner frequency -40 db/decade slope above the corner frequency.	Varies from 0 degree -180 degrees (-90 degrees at corner frequency.)
5.	$(.00695S^2 + .11S + 1)^{-1}$	12.00	1.91	0 slope below the frequency. -40 db/decade slope above the corner frequency.	Varies from 0 degree to -180 degrees. (-90 degrees at corner frequency.)

*1 radians per second.

*2 cycles per second.

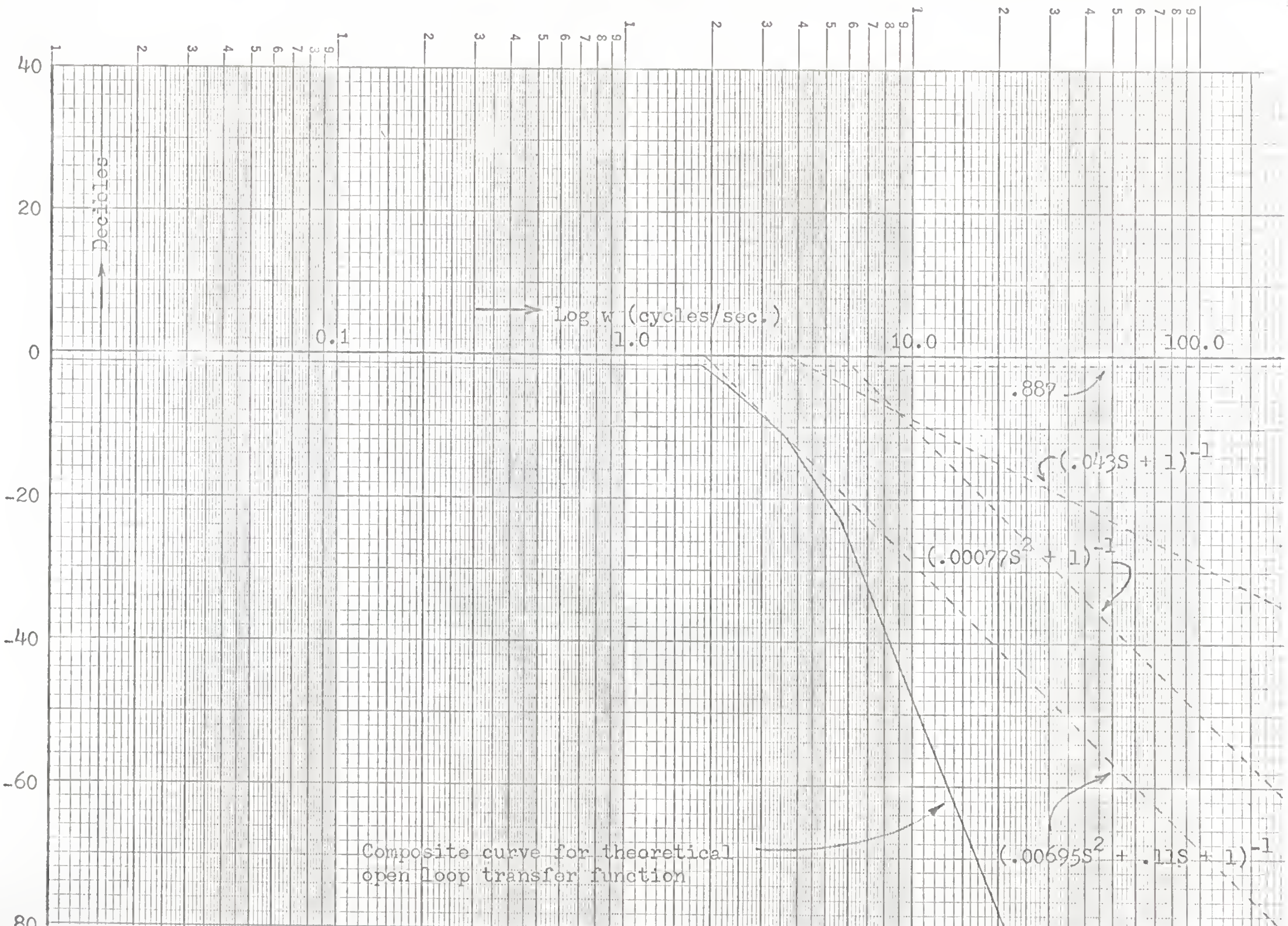


Figure (8), Log Magnitude vs. Frequency Curve (Theoretical open loop transfer function)

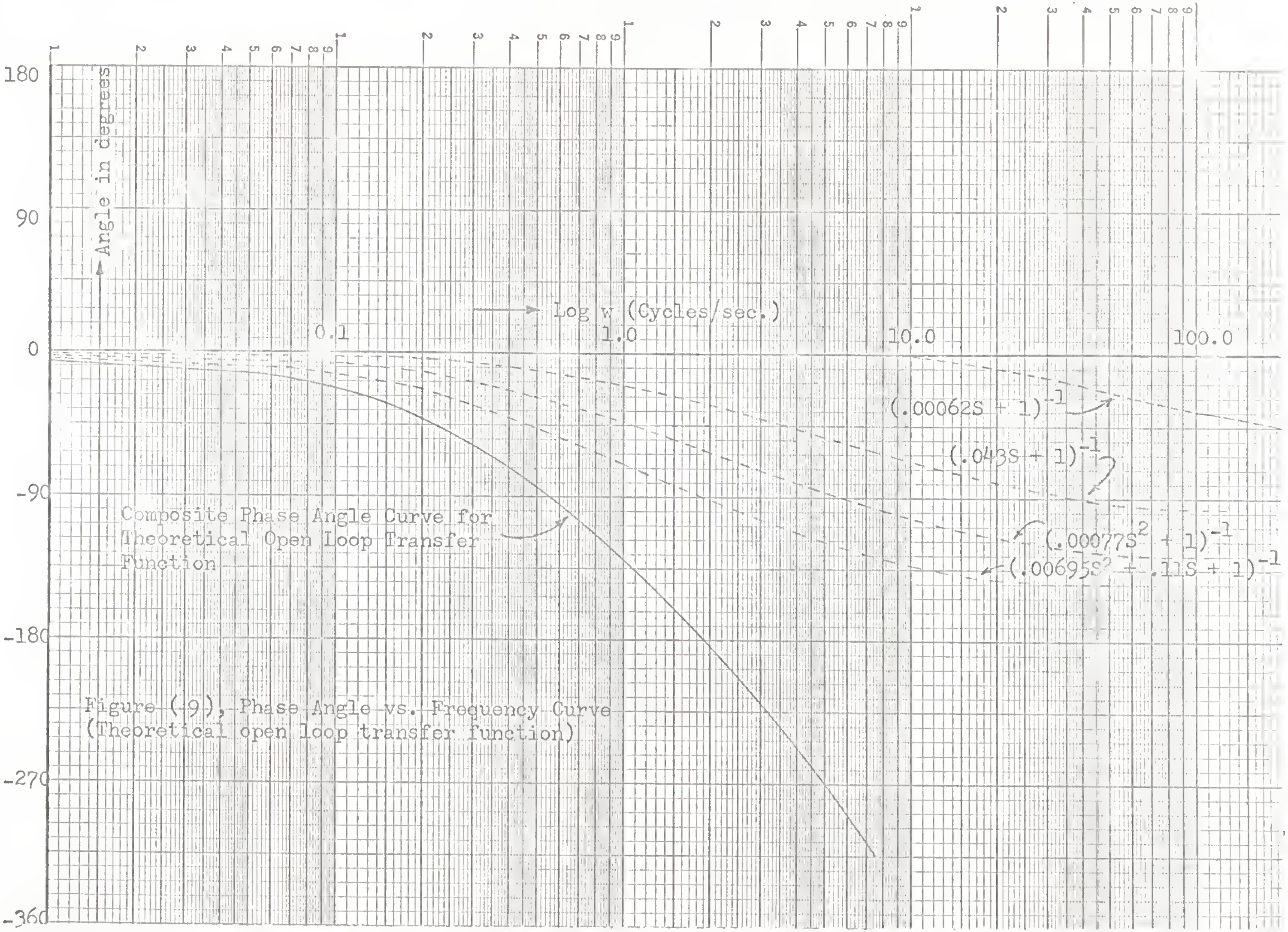


Figure (9), Phase Angle vs. Frequency Curve
(Theoretical open loop transfer function)

The closed loop transfer function has been derived to be

$$\frac{1}{\left[43.8 \times 10^{-14} S^7 + 1.55 \times 10^{-8} S^6 + 2.7 \times 10^{-5} S^5 + 1.04 \times 10^{-5} S^4 + 4.3 \times 10^{-4} S^3 + 2.06 \times 10^{-3} S^2 + 1.61 \times 10^{-1} S + 1.88\right]}$$

In order to plot the frequency response plot for the closed loop transfer function to be compared with the experimental plot, S = jw is substituted. After separating the complex and real parts, the closed loop transfer function becomes

$$\frac{1}{\left[(-1.55 \times 10^{-8} w^6 - 1.04 \times 10^{-5} w^4 - 2.06 \times 10^{-3} w^2 + 1.88) + j(-43.8 \times 10^{-14} w^7 + 2.7 \times 10^{-5} w^5 - 4.3 \times 10^{-4} w^3 + 1.61 \times 10^{-1} w)\right]}$$

Let $(-1.55 \times 10^{-8} w^6 - 1.04 \times 10^{-5} w^4 - 2.06 \times 10^{-3} w^2 + 1.88) = A$

and $(-43.8 \times 10^{-14} w^7 + 2.7 \times 10^{-5} w^5 - 4.3 \times 10^{-4} w^3 + 1.61 \times 10^{-1} w) = B$

So that the closed loop transfer function becomes

$$\frac{1}{(A + jB)}$$

The log magnitude is given by $-20 \log \sqrt{(A^2 + B^2)}$

The angle is given by $-\tan^{-1} \frac{B}{A}$.

Curves are shown in figs. 11 and 12.

Table 2. Characteristics of log magnitude and angle diagram for the theoretical closed loop transfer function.

Frequency r.p.s.	Frequency c.p.s.	$-20 \log A^2 + B^2$	$-\tan^{-1} \frac{B}{A}$
.0628	.01	-5.413 db	-18°
.628	.1	-6.022 db	-23°
6.28	1.0	-6.431 db.	-126°
12.56	2.0	-9.510 db.	-181°
31.40	5.0	-23.531 db.	-272°
62.80	10.0	-47.120 db.	--

EXPERIMENTAL VERIFICATION

Description of Apparatus and Procedure

A schematic view of the test setup is shown in fig. 10. The input to the force motor was a sinusoidally varying voltage from a low-frequency function generator. The frequency and the amplitude of the voltage from the function generator could be controlled accurately. One channel of a Sanborn Recorder was connected in parallel to the input voltage to record it.

The motion of the actuator was sensed with a linear potentiometer and was fed to the second channel of the Sanborn Recorder. This arrangement made it possible to record the input and the output traces simultaneously on a strip chart.

Before running the experiment for the test data, the Sanborn Recorder was calibrated to measure 50 millivolts per cm. deflection of the pen on the strip chart. This provided for the measurement of the voltages for the input as well as for the output directly from their traces on the strip chart.

The D.C. voltage applied to the linear potentiometer was 6.3 volts. The total travel available for the sliding contact was 1.328 inches. Calibration of the potentiometer gave .211 inch motion per volt. The linearity and the calibration of the potentiometer was confirmed by measuring its motion with a Micrometer and recording its output on the Sanborn Recorder.

Test data was obtained for two different levels of input voltage and at various frequencies. The results for the first input voltage (.475 volts) are shown in figures 11 and 12.

The results for the second input voltage (.3125 volts) are also shown in figs. 11 and 12, respectively.

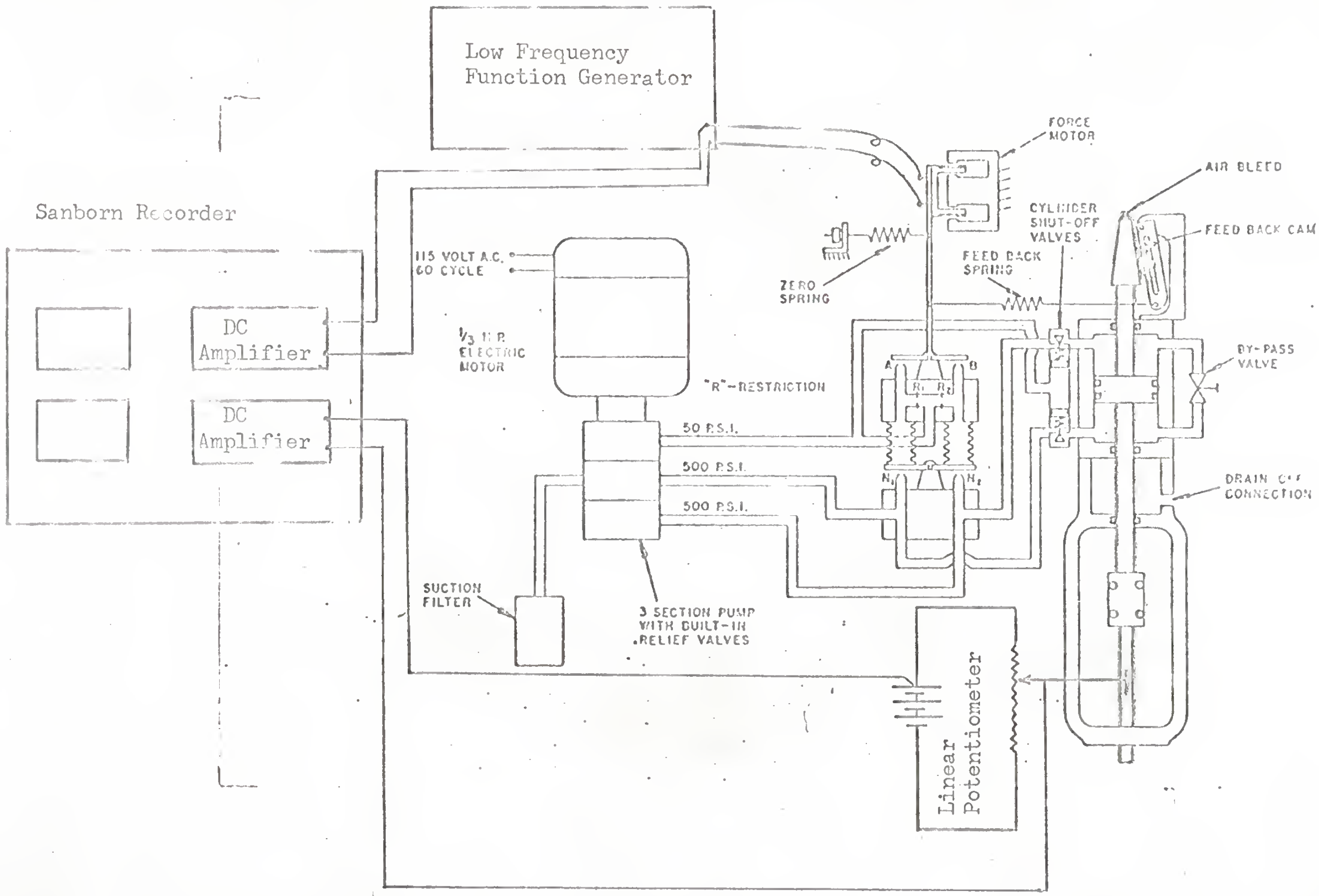


Figure (10), Experimental Setup

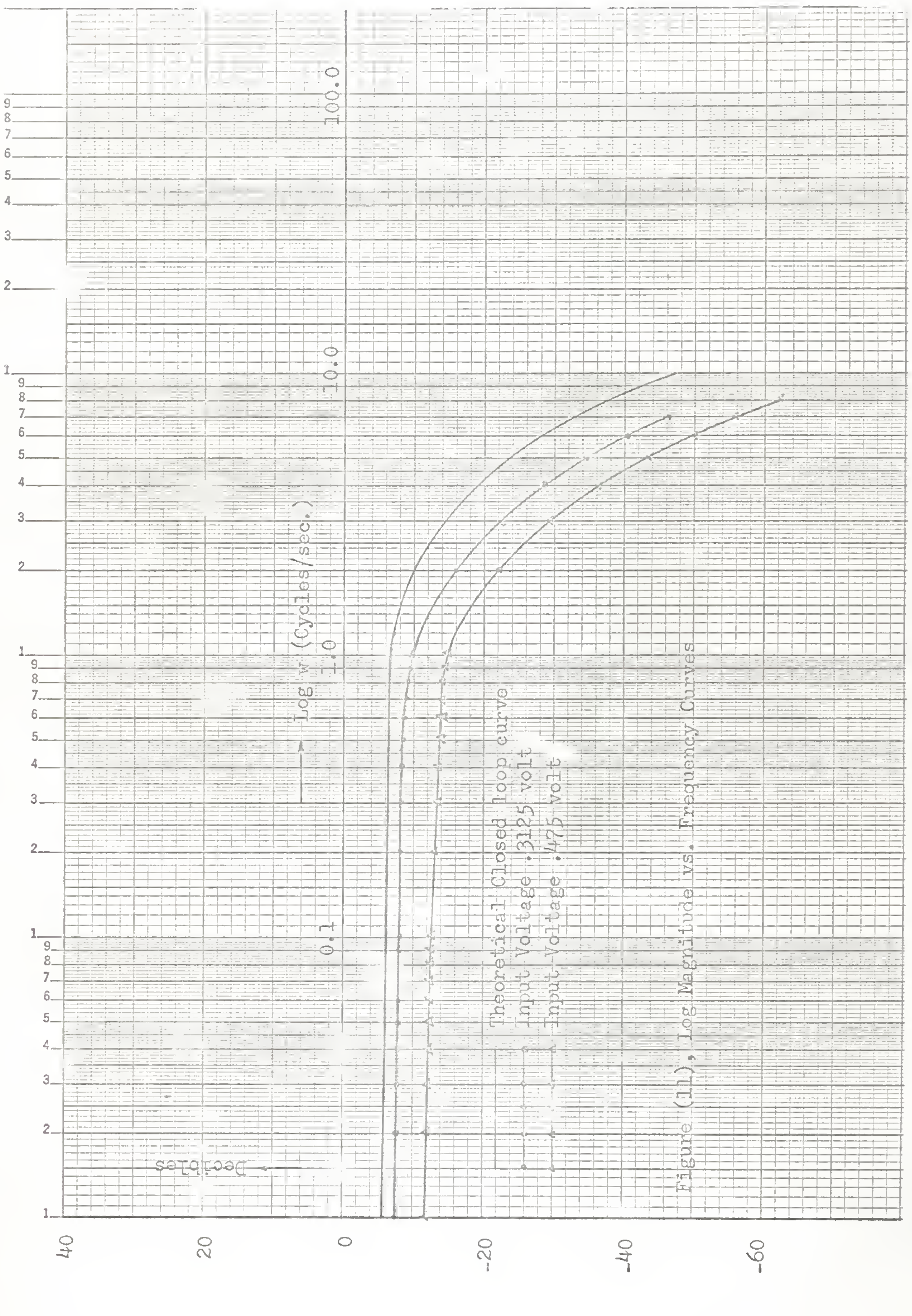


Figure (11), Log Magnitude vs. Frequency Curves

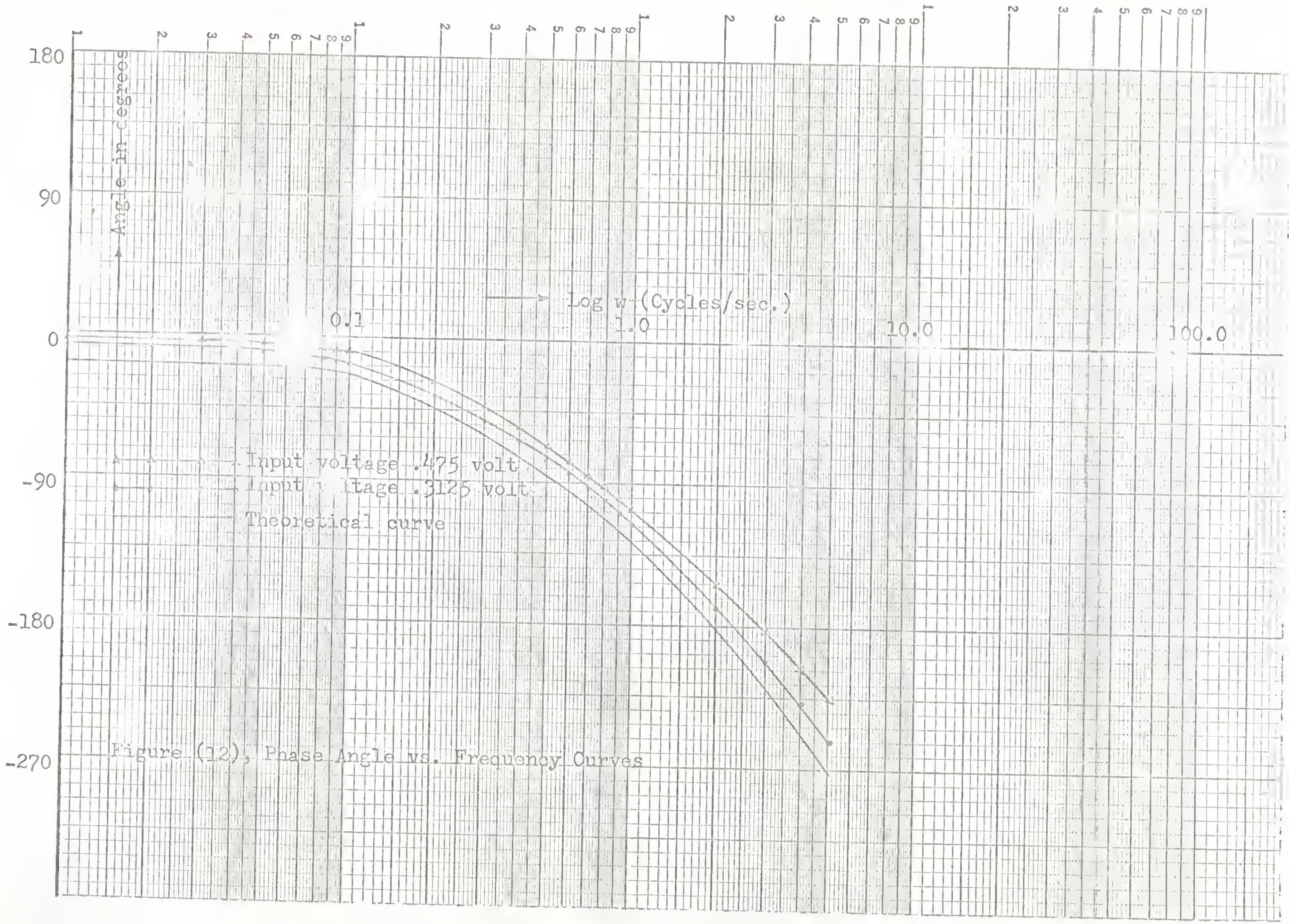


Figure (12), Phase Angle vs. Frequency Curves

SUMMARY AND CONCLUSIONS

1. The experimental magnitude ratio vs. frequency plots of fig. 11 show a drop of approximately 100 db/decade which is the same as predicted from the plot of the linearized transfer function. However the experimental plot for an input voltage amplitude equal to .475 volts shows a deviation of -6 db. in its log magnitude at very low frequencies. This deviation can be explained on the basis of the fact that in the derivation of the linearized transfer function, it was assumed that there would be small changes of all variables about some initial operating point. This assumption does not apparently hold true when the level of input is .475 volts because the change in variables is too large. The assumption proves to be practical when the level of the input is .3125 volts. This is fully supported by the plots of fig. 11.

2. The phase angle plot for the linearized open loop transfer function shows a phase crossover at a frequency of 1.9 cycles/sec. The corresponding gain margin from fig. 8 is 2 db. Changing the gain raises or lowers the log magnitude curve without changing the angle characteristics. Increasing the gain raises the curve, thereby decreasing the gain margin, with the result that stability is decreased. Decreasing the gain lowers the curve and increases stability. Examining the factors influencing the gain term in the linearized open loop transfer function, it is concluded that in order to increase the stability of the actuator, it should be redesigned with a decrease in the following factors:

   l_2, the length of the flapper

   A_2, effective area of the bellows

   A_3, effective area of the piston

However there will be a limit to the decrease in these factors because reduction in steady-state errors demands increase in gain.

3. The linearized open loop transfer function of the actuator can be analyzed to seek for an alternative to increase stability. The open loop transfer function for the actuator may be written qualitatively as

$$\frac{K}{(S + b_1)(S + b_2)(S^2 + 2\xi_1 w_1 S + W_1^2)(S^2 + 2\xi_2 W_2 S + W_2^2)};$$

The root locus plot for the poles of this transfer function is shown in fig. 13. An increase in the gain makes the complex poles move toward the right-half plane. This decreases system stability. Stability can be increased, with increasing gain, by introducing two pairs of complex zeros in the neighborhood of the complex poles. This can be achieved by redesigning and compensating the actuator. The compensator required to introduce complex zeros may consist of an electric network or mechanical devices consisting of springs, levers, dashpots, etc. The compensator may be placed in cascade with the forward transfer function or in the feedback path. However this will increase the cost of the actuator.

4. The linear analysis shows a natural frequency of about 2 cycles/sec. In order to improve dynamic performance, a higher natural frequency is desirable. The linearized open loop transfer function can be analyzed to indicate changes in the parameters which can increase the natural frequency of the actuator. Characteristics of log magnitude and angle diagram for various factors of the open loop transfer function are given in Table 1. From Table 1, it can be seen that the factors limiting the natural frequency of the actuator are

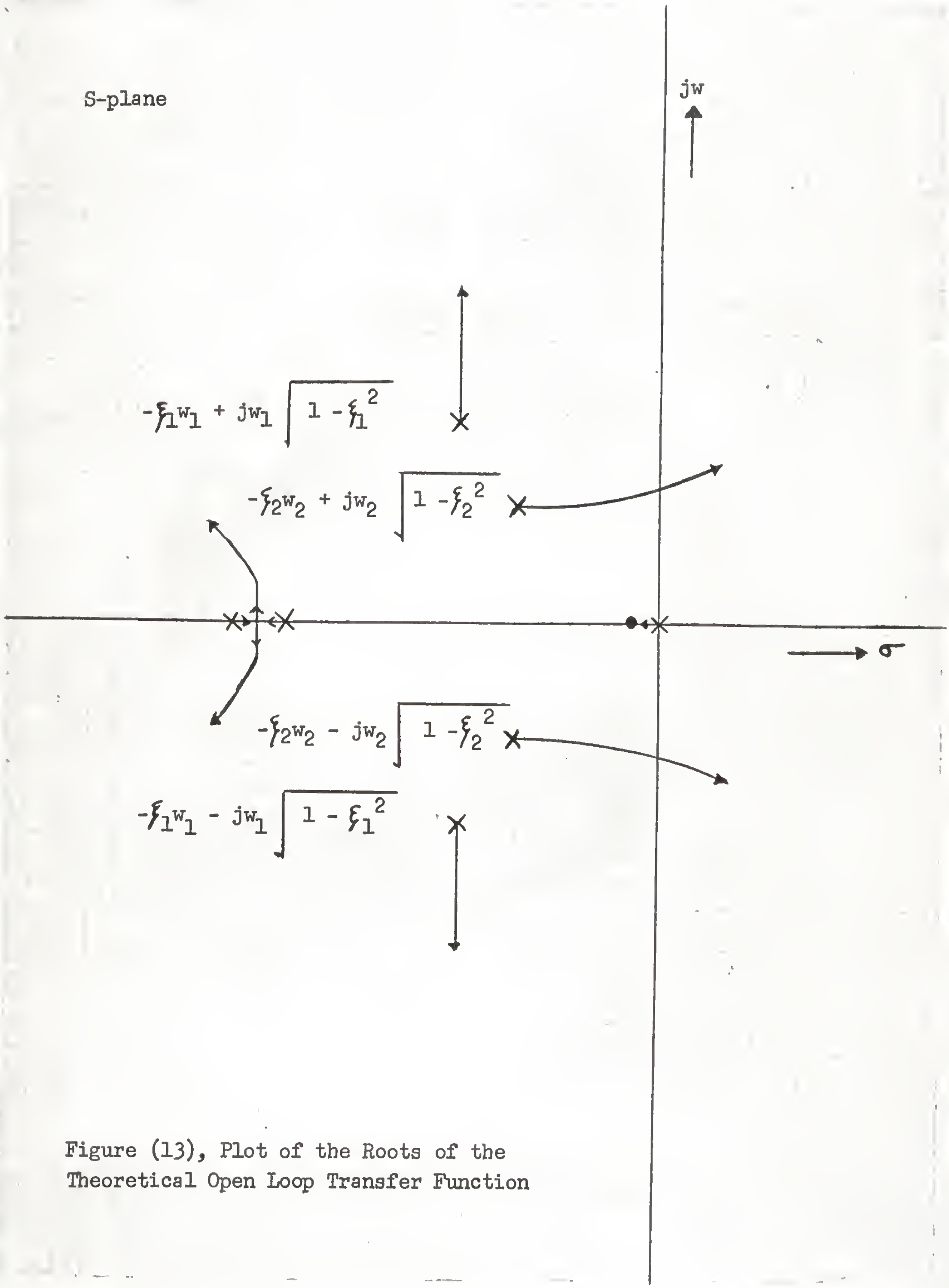


Figure (13), Plot of the Roots of the Theoretical Open Loop Transfer Function a, $(.00695S^2 + .11S + 1)^{-1}$

b, $(.043S + 1)^{-1}$

c, $(.00077S^2 +1)^{-1}$

In order to increase the overall bandwidth of the system, the coefficients of S^2 and S should be decreased. Parameters going into the coefficients of these factors should be decreased for an improved design to increase the overall bandwidth. Going back to the linearized transfer functions for individual components, the following parameters should be decreased:

A_2; effective area of the bellows

k_3; spring stiffness of the bellows

K_5; K_7; K_9; K_{10}.

M_1; Mass of the coil

M_2; Mass of the piston and rod assembly

V_o; Initial volume of the bellows.

The analytical techniques applied made use of some assumptions which may be only crude approximations due to a lack of knowledge of their actual effects. Most significant of these are coulomb friction and stiction forces. In fact, some lack of correlation in the experimental results may be attributed to the neglect of these.

In spite of all this, it goes without saying that the analytical technique used proves to be a useful tool if it should be desired to redesign the actuator to improve its performance. Not only this, the technique can be used to design an actuator with particular specifications, or some other similar type system. The result will be a saving in time and cost as compared to that required for an empirical approach.

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APPENDIX A

CALCULATIONS FOR FORCE MOTOR AND BEAM ASSEMBLY

1. Calculation for the self-inductance of the coil.

   Self-Inductance = $\frac{1.257\ N^2 A}{1 \times 10^8}$ henry

   where N = number of turns in the coil

   A = Area of the coil in sq. cm.

   l = length of windings in cm.

   Taking N = 2160

   Internal diameter of the coil = 2"

   External diameter of the coil = 2.38"

   Length of the windings = .75"

   Self-Inductance L = .747 Henry

2. Calculations for the transfer function of the force motor

   $$G_1 = \frac{B\pi dN/R}{\left(\frac{L}{R}S + 1\right)} \qquad [A1]$$

   Substituting $B\pi dN$ = 28.55 lb_f/ampere.

   R = 260 ohms.

   l_1 = 4.25 inches.

   l_2 = .688 inches

   L = .747 Henry

   $$G_1 = \frac{1.092}{(.00287S + 1)} \qquad [A2]$$

3. Calculations for the transfer function of the coil and beam assembly

   $$G_2 = \frac{l_2}{l_1\left(M_1 S^2 + B_1 S + \frac{k_1 l_3}{l_1} + \frac{k_2 l_4}{l_1}\right)}$$

Substituting

$$M_1 = .008 \quad \frac{lb_f - Sec^2}{in.}$$

$$B_1 = 0 \text{ lb - sec./in}$$

$$k_1 = .53 \text{ lb}_f\text{/inch}$$

$$k_2 = 1.5 \text{ lb}_f\text{/inch}$$

$$l_3 = 2.125 \text{ inches}$$

$$l_4 = 2.125 \text{ inches}$$

$$G_2 = \frac{.169}{(.00077S^2 + 1)} \qquad [A3]$$

APPENDIX B

CALCULATIONS FOR THE FIRST-STAGE FLAPPER-NOZZLE AMPLIFIER

The transfer function is given by

$$G_3 = \frac{A_2 K_2}{\left[\frac{k_3 (V_o + A_2 z_o)}{B_s}\right] S - K_1 k_3} \qquad [B1]$$

1. Calculations for K_1

$$K_1 = \left.\frac{\partial q_5}{\partial p_2}\right|_{\substack{p_{2_o} \\ y_o}}$$

Substituting $q_5 = K_d A_1 \sqrt{\frac{2(p_1 - p_2)}{\rho}} - K_d 2\pi \gamma_1 (y_c - y) \sqrt{\frac{2 p_2}{\rho}}$ [B2]

Partial differentiation yields

$$K_1 = \frac{-K_d A_1}{\sqrt{2\rho (p_1 - p_{2_o})}} - K_d \pi \gamma_1 (y_c - y_o) \sqrt{\frac{2}{\rho\, p_{2_o}}}$$

Substituting

$K_d = .6$

$A_1 = .00061$ sq. inches

$y_c = .004$ inches

$y_o = .002$ inches

$\gamma_1 = .028$ inches

$\rho = 8.0 \times 10^{-5} \quad \frac{lb_f - Sec^2}{In.^4}$

$p_{2_o} = 37.5$ psig

$K_1 = -.073$ [B3]

2. Calculations for K_2

$$K_2 = \left. \frac{\partial q_5}{\partial y} \right|_{\substack{p_{2_o} \\ y_o}}$$

Substituting the value of q_5 from B2 and differentiating partially

$$K_2 = K_d \, 2 \pi r_1 \sqrt{\frac{2 \, p_{2_o}}{\rho}}$$

which gives $K_2 = 102.5$ [B4]

3. Inserting the following values

A_2 = .3545 sq. inches

K_1 = -.073; K_2 = 102.5

k_3 = 39 lb_f/inch

V_o = .338 inch^3.

z_o = .004 inch.

Bs = 220,000 psi

$$G_3 = \frac{12.82}{(.043S + 1)}$$ [B5]

APPENDIX C

CALCULATIONS FOR SECOND-STAGE FLAPPER-NOZZLE AMPLIFIER (PILOT ASSEMBLY) AND THE ACTUATOR

The transfer function is given by

$$G_4 = \frac{-K_{13}\,S - K_{14}}{\left[M_2K_9K_{10}S^4 + (K_9K_{10}B_2 - M_2K_5K_{10} - K_7K_9M_2)\,S^3 + (K_{11} + K_5K_7M_2 - K_5K_{10}B_2 - K_7K_9B_2)\,S^2 + (K_5K_7B_2 - K_{12})\,S\right]} \qquad [C1]$$

1. Calculations for K_5

$$K_5 = \left.\frac{\partial q_9}{\partial p_4}\right|_{\substack{p_{4_o} \\ z_o}}$$

Substituting $q_9 = q_8 - K_d\, 2\pi\gamma_2(z_c - z)\sqrt{\dfrac{2\,p_4}{\rho}}$ and differentiating

$$K_5 = \frac{-K_d\pi\gamma_2\,(z_c - z_o)}{\quad}\sqrt{\frac{2}{\rho\, p_{4_o}}}$$

Inserting $K_d = .6$

$$\begin{aligned} \gamma_2 &= .0507 \text{ inch} \\ z_c &= .009 \text{ inch} \\ z_o &= .005 \text{ inch} \\ p_{4_o} &= 257.8 \text{ psig} \\ p_{5_o} &= 256.6 \text{ psig} \end{aligned} \qquad [C2]$$

$$K_5 = -.0457 \qquad [C3]$$

2. Calculations for K_6

$$K_6 = \left.\frac{\partial q_9}{\partial z}\right|_{\substack{p_{4_o} \\ z_o}}$$

Substituting the value of q_9 and differentiating

$$K_6 = K_d \, 2\pi r_2 \sqrt{\frac{2 p_{4_o}}{\rho}} \quad \text{which gives}$$

$$K_6 = 50$$ [C4]

3. Calculations for K_7

$$K_7 = \left.\frac{\partial q_{14}}{\partial p_5}\right|_{\substack{p_{5_o} \\ z_o}}$$

Substituting $q_{14} = q_{12} - K_d \, 2\pi r_2 \, (z_c + z) \sqrt{\frac{2 p_5}{\rho}}$

Partial differentiation yields

$$K_7 = -K_d \pi r_2 \, (z_c + z_o) \sqrt{\frac{2}{\rho \, p_{5_o}}}$$

Numerical values from C2 yield

$$K_7 = -.0121$$ [C5]

4. Calculations for K_8

$$K_8 = \left.\frac{\partial q_{14}}{\partial z}\right|_{\substack{p_{5_o} \\ z_o}}$$

Proceeding as for K_7, the following is obtained

$$K_8 = 49$$ [C6]

5. Calculations for K_9

$$K_9 = \frac{V_1 + A_3 c_o}{B_s}$$

Substituting $V_1 = 4.58$ inch^3

$c_o = 1$ inch

$A_3 = 4.58$ inch^2

$B_s = 220{,}000$ psi

$K_9 = 416 \times 10^{-7}$ [C7]

6. Calculations for K_{10}

$$K_{10} = \frac{V_2 + A_3(L_1 - b - c_o)}{B_s}$$

Putting $L_1 = 3.125$ inches

$K_{10} = 415 \times 10^{-7}$

7. Using the values of K_5, K_6, K_7, K_8, K_9 and K_{10}

$K_{11} = 17.45 \times 10^{-4}$

$K_{12} = -1.21$

$K_{13} = 4.16 \times 10^{-6}$

$K_{14} = -.127$

8. Substituting the values of K's plus

$M_2 = .0181 \ \dfrac{lb_f - Sec^2}{Inch}$, and

$B_2 = 0$, under the assumption that the damping force is very small as compared with other forces acting on the piston

$$G_4 = \frac{.207}{(.00062S + 1)(.00695S^2 + .11S + 1)}$$ [C8]

APPENDIX D

CALCULATIONS FOR THE MECHANICAL FEEDBACK DEVICE

The transfer function for the mechanical feedback is given by

$$H = \frac{l_6}{l_5} \cdot k_2 \cdot \mathrm{Tan}\,\theta$$

Substituting $l_6 = 3.8$ inches

$l_5 = .75$ inches

$k_2 = 1.5\ lb_f/$ inches

$\theta = 15^{\circ}$

$$H = 1.98 \qquad [D1]$$

NOMENCLATURE

The variables in the lower case letters are in the time domain while the Laplace Transforms of these same variables are represented by capital letters.

A_1 Area of the restriction R_1 and equal to the area of the restriction R_2, $in.^2$

A_2 Effective area of the bellows, $in.^2$

A_3 Effective area of the piston, $in.^2$

B Flux density produced by the permanent magnet, $lines/in.^2$

B_1 Damping coefficient for the coil and beam assembly, lb - sec/inch.

B_2 Damping coefficient for the piston and cylinder assembly, lb - sec./in.

B_s Adiabatic tangent bulk modulus for the oil, psi.

b Thickness of the piston, in.

c Distance between the cylinder wall and the piston face, in.

d Diameter of the coil, in.

e. Voltage applied to the coil, volts.

f Force produced by passing current through the coil, lb_f/amp.

f_1 Load force on the piston, lb_f.

i Current flowing in the coil, amps

k_1 Spring stiffness for zero-adjustment spring, lb_f/in.

k_2 Spring stiffness for feed-back spring, lb_f/in.

k_3 Spring stiffness for the bellows, lb_f/in.

$$K_1 = \left.\frac{\partial q_5}{\partial p_2}\right|_{\substack{p_{2_o} \\ y_o}} , \; in.^5/lb_f - sec.$$

NOMENCLATURE--Continued

$$K_2 = \left.\frac{\partial q_5}{\partial y}\right|_{p_{2_o},\, y_o} \text{, in.}^2\text{/sec.}$$

$$K_3 = \left.\frac{\partial q_7}{\partial p_3}\right|_{p_{3_o},\, y_o} \text{, } \frac{\text{in.}^5}{\text{lb}_f - \text{sec}}$$

$$K_4 = \left.\frac{\partial q_7}{\partial y}\right|_{p_{3_o},\, y_o} \text{, in.}^2\text{/sec.}$$

$$K_5 = \left.\frac{\partial q_9}{\partial p_4}\right|_{p_{4_o},\, z_o} \text{, in.}^5\text{/lb}_f - \text{sec.}$$

$$K_6 = \left.\frac{\partial q_9}{\partial z}\right|_{p_{4_o},\, z_o} \text{, in.}^2\text{/sec.}$$

$$K_7 = \left.\frac{\partial q_{14}}{\partial p_5}\right|_{p_{5_o},\, z_o} \text{, in.}^5\text{/lb}_f - \text{sec.}$$

$$K_8 = \left.\frac{\partial q}{\partial z}\right|_{p_{5_o},\, z_o} \text{, in.}^2\text{/sec.}$$

NOMENCLATURE--Continued

$K_9 = \dfrac{V_1 + A_3 c_o}{B_s}$, in^3/psi

$K_{10} = \dfrac{V_2 + A_3(L_1 - b - c_o)}{B_s}$, $in.^3/psi$

$K_{11} = A_3^2 (K_9 + K_{10})$, $in.^7/psi$

$K_{12} = A_3^2 (K_5 + K_7)$, $in.^9/lb_f$ - sec.

$K_{13} = A_3(K_8K_9 - K_6K_{10})$, $in.^7/$ psi - sec.

$K_{14} = A_3(K_6K_7 - K_5K_8)$, $in.^9/lb_f - sec.^2$

K_d = Discharge coefficient, .6

l_1 Length of the beam, in.

l_2 Length of the flapper, in.

l_3 Distance between the flapper and the feedback spring, in.

l_4 Distance between the flapper and the zero-adjustment spring, in.

l_5 Distance between the center of the travel adjustment wheel and the fulcrum of the follower, in.

l_6 Distance between the feedback spring and the fulcrum of the follower, in.

L Self-Inductance of the coil, Henry

L_1 Effective length of the cylinder, in.

M_1 Mass of the coil, $\dfrac{lb - Sec^2}{in.}$

M_2 Mass of the piston and the cam, $\dfrac{lb - sec^2}{in.}$

p_1 Supply pressure to the amplifier, psig

p_2 Pressure in the bellows A, psig.

NOMENCLATURE--Continued

p_3	Pressure in the bellows B, psig
p_4	Pressure in the cylinder under the piston, psig
p_5	Pressure in the cylinder above the piston, psig
q_1	Total flow into the amplifier, $in.^3/sec.$
q_2	Flow through the restriction R_1, $in.^3/sec.$
q_3	Flow through the restriction R_2, $in.^3/sec.$
q_4	Flow through the nozzle A, $in.^3/sec.$
q_5	Flow into the bellows A, $in.^3/sec.$
q_6	Flow through the nozzle B, $in.^3/sec.$
q_7	Flow into the bellows B, $in.^3/sec.$
q_8	Flow from the pump in the middle section, $in.^3/sec.$
q_9	Flow into the cylinder under the piston, $in.^3/sec.$
q_{11}	Flow at the nozzle N_1, $in.^3/sec.$
q_{12}	Flow from the pump in the bottom section, $in.^3/sec.$
q_{13}	Flow at the nozzle N_2, $in.^3/sec.$
q_{14}	Flow into the cylinder above the piston, $in.^3/sec.$
r_1	Radius of the nozzle A or nozzle B, in.
r_2	Radius of the nozzle N_1 or nozzle N_2, in.
R	Resistance of the coil, ohms
T_1	L/R, Henry/ohm
V_o	Initial volume of the bellows, $in.^3$
V_{1_o}	Initial volume of the cylinder below the piston, $in.^3$
V_{2_o}	Initial volume of the cylinder above the piston, $in.^3$
x	Distance traveled by the coil, in.

NOMENCLATURE--Concluded

y Distance traveled by the flapper, in.

z Distance traveled by the bellows, in.

ρ Density of the oil, $\frac{lb_f - sec^2}{in}$

ζ Damping ratio

w_n Natural frequency, rad./sec.

VITA

Desh Paul Mehta

Candidate for the Degree of

Master of Science

Report: AN ANALYSIS OF AN ELECTRO-HYDRAULIC ACTUATOR

Major Field: Mechanical Engineering

Biographical:

Personal Data: Born in India, September 5, 1940, the son of Mr. M. L. Mehta and Mrs. K. Mehta.

Education: Attended high school in Panjab; graduated from Panjab University in 1955; received the Bachelor of Science degree from Panjab University, with a major in Physics, in May, 1959; received the Master of Science degree from the Aligarh University, with a major in Physics, in June, 1961; completed requirements for the Master of Science degree in January, 1967.

Professional experience: Joined U.P. Agricultural University, India, as an instructor in Physics in 1962. Came to U.S.A. under U.S. Agency for International Development. Was elected to Phi Kappa Phi in 1966. Student member of ASME.

AN ANALYSIS OF AN ELECTRO-HYDRAULIC ACTUATOR

by

DESH PAUL MEHTA

B. S., Panjab University, India, 1959
M. S., Aligarh University, India, 1961

AN ABSTRACT OF A MASTER'S REPORT

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requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

Kansas State University
Manhattan, Kansas

1967

ABSTRACT

For some years, fluid power control systems have been undergoing a "re-birth" and at the present time they are the subject of great interest and importance. However, education lags technological advances in this field. The various demands of the field are being met, in many cases, using an empirical approach.

In this report, a fluid power control system was selected for analysis. It was attempted, in its analysis, to show that an analytical technique can be applied to the study and improvement of the dynamics of such systems. The system analyzed was a combination type, electro-hydraulic actuator.

For the sake of analysis, the system was broken into its subcomponents. The analysis of these components involved equations which were non-linear in nature. These equations were linearized about an operating point and linearized transfer functions were developed from flow-pressure and velocity-force relationships. A block diagram was developed for the system and used to combine the transfer functions of the individual components into an overall open loop as well as closed loop transfer function for the system.

Log magnitude vs. frequency and phase angle vs. frequency plots have been drawn for the analytically derived overall transfer function. The actuator was subjected to an experimental frequency response test and results have been plotted to be compared with the theoretical predictions. The results were compared and have been found to agree favorably with each other.

The applications and the limitations of the analytical techniques have been discussed. The analytical transfer function has been analyzed from a point of view to improve stability and to increase the overall band width of the system.