STATISTICAL AND PROBABILISTIC METHODS FOR DESIGN OF REINFORCED CONCRETE STRUCTURES

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THIS BOOK CONTAINS NUMEROUS PAGES WITH DIAGRAMS THAT ARE CROOKED COMPARED TO THE REST OF THE INFORMATION ON THE PAGE.

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INTRODUCTION

This report presents a review of simple probabilistic analysis and design techniques for reinforced concrete structures subjected to normal use loads.

The rapidly developing interest of engineers and researchers, in probabilistic procedures has resulted in a number of developments specifically aimed at applications to reinforced concrete.

The necessity to adopt methods of design of structures using probability theory has been limited for quite a long time, although many engineers see the rationality of probabilistic models of phenomena, of interest to the profession. There have been numerous papers written on probability and statistical methods for design of concrete structures by renowned authors, among the first of these being Freudenthal (9,10)*. The methods developed have not yet been widely adopted in practice, obviously. Indeed there has not been sufficient development of models to permit a unified probabilistic approach to the many aspects of civil engineering subjects where the methods could be of immense use.

A significant influence on the development of theories of diverse backgrounds is the result of a major change within the theory of applied probability and statistics. As of now, a somewhat controversial theory is developed around a framework of economic decision making. A question which has centered around probabilistic studies of natural strength

*Numbers in parentheses refer to items listed in Appendix I - References.

properties of materials is "What is the cost of excess safety margins in structural design?" The answer to this is still premature since there are many implications involved with these methods.

In place of earlier emphasis on obtaining proper objective descriptions of repetitive physical phenomena, the new concern is with making decisions involving economic gains and losses when uncertainty exists in the decision maker's mind regarding the state of nature. This new emphasis, with its new interpretations and new methods, is far more appropriate and natural for the profession of civil engineers which is more closely involved than any other profession in the economical design of structures subjected to uncertain demands of natural and man-made environmental factors.

Recent developments in probabilistic analysis have led to new concepts and tools for the assessment of safety and reliability of structures that would be of interest to every structural engineer. The ultimate objective of a structural engineer is to design structures which are both economically feasible and functionally reliable.

This report presents a discussion of the basic need to alter the existing structural safety factors in the various codes based on the numerous papers of well-known authors who strongly feel that the conservative safety factors be improved by formulating new factors of safety based on statistical and probabilistic studies.

PROBABILITY CONCEPTS

At the very outset, there is a need to know certain basic concepts of statistics viz. 1) Sample mean, 2) Sample variance, 3) Sample standard deviation, and 4) Sample coefficient of skewness.

a) The single most helpful number associated with a set of data is its average or the arithmetic mean. If the sequence of observed values is denoted as x_1, x_2, $x_3 \ldots x_n$, then the sample mean $\overline{x}$ is simply

$$\overline{x} = \frac{1}{N} \sum_{i=1}^{N} x_i \quad \text{------------------------------------} (2.1)$$

The sample mean is interpreted as a typical or central value of the data. If required to give only a single number, one would probably use this sample mean as his "best prediction" of the entire set of values. Other measures of the central tendency of a data set include mode--the most frequently occurring value in the data set, and the median--the middle value in an ordered list (middle value if N is odd or the average of two middle values if N is even).

b) Given a set of data, it is desirable to summarize in a single number something of the variability of the observations. In the past, the measure most frequently occurring in engineering reports was the range of the data. But a far more satisfactory measure of dispersion is found in sample variance, unlike the range which is more susceptible to the size of the sample and has more emphasis on the extremes.

The sample variance is analogous to the moment of inertia in that it deals with squares of distances from centre of gravity, which is simply the sample mean. The sample variance, V_x^2, is thus defined as

$$V_x^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2 \quad \text{------------------------------} \quad (2.2)$$

c) The positive square root, V_x, of the sample variance of the data is termed the sample standard deviation. The standard deviation is analogous to the radius of gyration of a structural cross-section; they are both shape rather than size-dependent parameters.

$$\text{Therefore, } V_x = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N}} \quad \text{----------------------} \quad (2.3)$$

The addition of a constant to all observed values would alter the sample mean but not the sample standard deviation. It has the same units as the original data, and next to the mean, it conveys more useful information to an engineer than any other single number that can be computed from the set of data.

When comparing relative dispersion of more than one kind of data, it is convenient to have a dimensionless ratio of the standard deviation to sample mean, called the sample coefficient of variation, V_s.

$$\text{Thus, } V_s = \frac{V_x}{\bar{x}} \quad \text{-----------------------------------} \quad (2.4)$$

d) Another numerical summary of observed data is a logical extension of the reasoning leading to the formula for sample variance. This is the sample coefficient of skewness which is related to the third moment of the probability density diagram about the mean; $m = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^3$

Therefore $a_k = \frac{m^3}{V_x{}^3}$ ---------------------------------- (2.5)

The coefficient of skewness provides a measure of the degree of asymmetry about the mean of the data. The coefficient is positive for histograms skewed to the right and negative for those skewed to the left.

Every engineering problem involves a scatter which incorporates variability. To analyze this variability, an engineer makes use of the theory of probability, a branch of mathematics dealing with uncertainty.

"To each sample point in a sample space of an experiment, is assigned a number called a probability measure. The mathematical theory of probability is not concerned with where these numbers came from or what they mean; it only tells how to use them in a consistent manner. The engineer who puts probability to work on his models of real situations must be absolutely sure what the set of numbers he assigns means, for results of probabilistic analysis of an engineering problem can be helpful only if this input is meaningful." (7)

A probability measure assigned to a sample point is that of relative frequencies. There are three axioms of probability with respect to an event E in a sample space.

<u>Axiom I</u>: The probability of an event E is a number greater than or equal to zero but less than or equal to unity.

Therefore, $0 \leq P\ [E] \leq 1$ -------------------------- (2.6)

$P\ [E]$ is a notation to denote the probability of event E.

<u>Axiom II</u>: The probability of a certain event S_e, is unity.

Therefore, $P\ [S_e] = 1$ ------------------------------ (2.7)

where S_e is the event associated with all sample points in the sample space.

<u>Axiom III</u>: The probability of an event, which is the union of two mutually exclusive events is the sum of the probabilities of these two events.

$\therefore P\ [E_1 \cup E_2] = P[E_1] + P[E_2]$ ------------------- (2.8)

If E_1 and E_2 are two events, the probability that E_2 occurs given that E_1 has already occurred, is denoted by $P\ [E_2|E_1]$ or $P\ [E_2$ given $E_1]$ and is called the conditional probability of E_2 given that E_1 has occurred.

<u>Discrete Probability Distributions</u>:

If a variable x can assume a discrete set of values x_1, x_2, $x_3 \ldots x_k$ with respective probabilities $p_1 + p_2 + p_3 \ldots + p_k = 1$, we say that a discrete probability distribution for x has been defined. The function f (x) which has the respective values $p_1, p_2, p_3, \ldots, p_k$ for $x = \{x_1, x_2, \ldots, x_k\}$ is called the probability function of x. Because x can assume certain values with given probabilities it is called a discrete random variable.

The probability distribution can be represented graphically by plotting f (x) against x, as for relative frequency distributions.

By accumulating probabilities we obtain the cumulative probability distribution, which is called the distribution function, F(x).

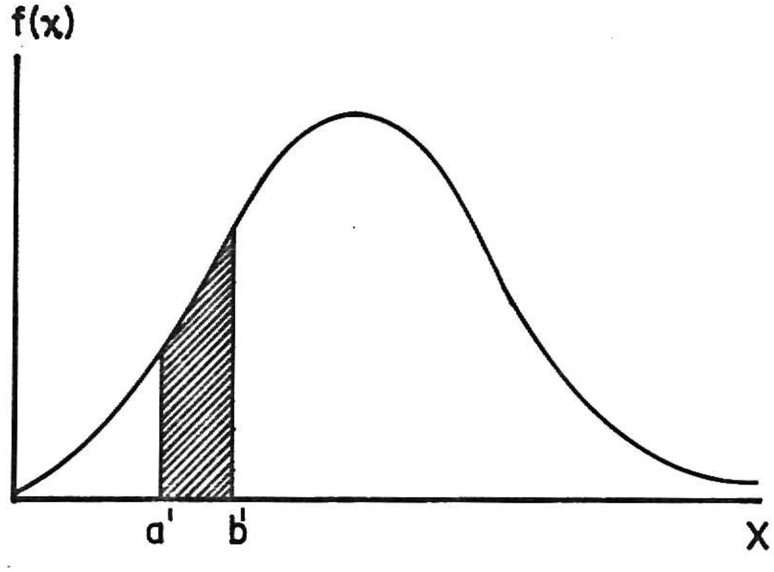

Figure. I. Probability given by area under curve.

Continuous Probability Distributions:

When a variable x assumes a continuous set of values, the relative frequency polygon of a sample, in the limiting case of the population becomes a continuous curve, as in Figure 1 whose equation is given as $Y = f(x)$. The total area under this curve bounded by x-axis is 1, and the area under the curve between $x = a'$; $x = b'$ is the probability that x lies between a' and b', i.e.,

$$P(a' < x < b')$$

Mathematical Expectation

If p is the probability that a student will score a grade of m , the mathemetical expectation is defined as pm.

In general, if x denotes a discrete random variable which can assume the values $x_1, x_2 \ldots x_k$, with respective probabilities $p_1, p_2 \ldots p_k$, where $p_1 + p_2 \ldots p_k = 1$, then the mathematical expectation of x, denoted by $E(x)$ is defined as

$$E(x) = p_1x_1 + p_2x_2 \ldots + p_kx_k = \sum_{i=1}^{K} p_ix_i = \Sigma px$$

This is generally denoted by $\mu = E(x)$, i.e., the sample mean $\bar{x}$ is analogous to the population mean μ. The sample variance, V_x^2 is analogous to σ^2 and the standard deviation, V_x is analogous to σ.

The following are the various distribution functions related to the probability distributions viz.

1) Discrete uniform distribution

2) Binomial distribution

3) Poisson distribution

4) Negative-exponential distribution

5) Normal distribution

6) Gamma distribution

7) Beta distribution

The normal distribution is the most widely used distribution and one of the most important continuous probability distributions. The equation of the normal probability density is expressed as:

$$Y = \frac{1}{\sigma\sqrt{2\pi}} \; e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

When the variable x is expressed in standard units $Z = \frac{x-\mu}{\sigma}$, the above equation is known as the standard normal distribution expressed in the form

$$Y = \frac{1}{\sqrt{2\pi}} \; e^{-\frac{z^2}{2}} ,$$

with the mean $\mu = 0$; and the variance $\sigma^2 = 1$.

EARLY DEVELOPMENT

It is imperative to analyze the safety factor in engineering structures to establish a rational method of evaluating its magnitude. The incompatibility between the refined procedures of design and the rather arbitrarily chosen safety factor has seriously hampered the development of more effective design procedures based on a best possible balance of safety and economy (9,10).

The true characteristic of a safety factor is determined by introducing a statistical concept of physical qualities, composed of structural phenomena of applied stress and resistance, represented by frequency distributions rather than individual values.

The fundamental, conventional concept of "allowable stress" involves a comparison between a computed maximum stress and the strength of the material, and implies the existence of a margin between the two. The justification of this margin has never been contested. The conventional name "margin of safety" suggests, that the designer is in a dilemma, to understand the need for an adequate measure of safety as well as the consciousness of the limitations of his knowledge regarding the arbitrariness of his assumptions. Criteria for the determination of the magnitude of the measure of safety on a more rational basis have to be established rather than leave it to the experience and judgment of a designer.

The probability of structural failure can be computed on the basis of assumed distribution functions of resistance and load. In practice, the parameters of these distributions can be estimated from the samples of observed values, which are results of repeated experiments. When several distribution functions are available for the description of the same physical phenomenon, statistical tests can be used to help decide which ones may be rejected at a given level of significance. Frequently, more than one distribution cannot be rejected on this basis.

The practical size of the sample enables the middle portions of the distributions usually to fit the data equally well. Nevertheless, the tail ends might vary significantly from one distribution to another. It is well-known that the computation of failure probability depends to a great extent on the tail portions of these distributions. Therefore, it is reasonable to expect that the failure probability is sensitive to the assumed distributions.

The value of the safety factor, τ, was derived by Freudenthal, from the condition that the maximum stress, S_a, induced in the structure by actual service conditions must never cause such damage as to impede its fitness for service, even were this maximum stress to coincide with the lowest value of the structure's resistance, S_r.

$$\tau = S_r/S_a > 1 \qquad (3.1)$$

If $\pm\Delta S_a$ denotes the maximum range of fluctuation of actual applied stress about the expected value S_{aw} and $\pm\Delta S_r$ the maximum range of fluctuation of the structure's resistance about its expected value S_{rw}, the maximum stress to be expected will be (S_{aw} + ΔS_a) and the minimum resistance will not be less than (S_{rw} - ΔS_r). According to Eq. (3.1), failure is just prevented if S_r minimum = S_a maximum, or

$$S_{aw} + \Delta S_a = S_{rw} - \Delta S_r \qquad (3.2)$$

The above Eq. (3.2) leads to the condition of minimum safety based on the correlation of expected values of stress and resistance; thus,

$$S_{aw} = \frac{1 - \frac{\Delta S_r}{S_{rw}}}{1 + \frac{\Delta S_a}{S_{aw}}} \cdot S_{rw} \text{ , or}$$

$$S_{aw} = \frac{1}{\tau} \cdot S_{rw} \qquad (3.3)$$

In the above, $\tau = \dfrac{1 + \frac{\Delta S_a}{S_{aw}}}{1 - \frac{\Delta S_r}{S_{rw}}}$ $\qquad (3.4)$

denotes the safety factor. In general, the correlation of applied stress and resistance can be expressed as a correlation of load and carrying capacity.

A structure is designed by predicting its future behavior on the basis of knowledge gained by past experience and the analysis of factual data. The computation of its safety factor requires an analysis of the variability of all influences bearing upon its resistance and the applied stress.

Past experience concerning physical properties is generally recorded as a series of observations of the properties themselves, or quality characteristics which express a known relationship with the property considered. When dealing with material properties such as strength, elasticity or linear dimensions--all of which affect the resistance of a structure--it will be necessary to predict their most probable values and the probable ranges of fluctuations of individual values. Observations of a comparatively small number of samples, supplemented by relevant knowledge based on experience with the manufacture, the procedure of control and selection will be available for this purpose. The character of that process and the extent and reliability of current knowledge concerning it, will determine the character and shape of the distribution function of the property considered and therefore, the accuracy of the prediction.

The form of the distribution curve of the probabilities of occurrence of certain loading conditions may be determined by the law of large numbers. For example, the probability that a certain load will occur m times in n cases is given by the binomial function, which is the m^{th} successive term of the expression $(P_p + P_q)^n$, where P_p denotes the mathematical probability of occurrence of the particular load and $P_q =$ $(1 - P_p)$, the probability that it will not occur {is generally called the complementary of the occurrence}.

The data and statistical information, in order to be used more effectively, will have to be condensed and presented in the form of a frequency distribution. (See Fig. 3)

Subsequently, the density function should be approximated by an algebraic distribution function f $(x; \bar{x}, V_x, a_K)$

of the respective quality characteristics, x; containing the three principal statistics of the frequency distribution computed from the n observations, x_n of the characteristic x_i viz. the mean value

$$\bar{x} = \Sigma \frac{x_n}{N}, \quad \text{-----} \quad (3.5)$$

the standard deviation,

$$V_x = \sqrt{\frac{\Sigma (x_n - \bar{x})^2}{N}}, \quad \text{-----} \quad (3.6)$$

and its skewness

$$a_K = \Sigma (x_n - \bar{x})^3 / N V_x^3. \quad \text{-----} \quad (3.7)$$

If the parameters of the distribution function are chosen such that

$$\int_{-\infty}^{\infty} f\ (x;\ \bar{x},\ V_x,\ a_K)\ dx = 1$$

the integral $\int_{a'}^{b'} f\ (x;\ \bar{x},\ V_x,\ a_K)\ dx$ is a measure of the probability that an individual event will occur within the limits $x = a'$ and $x = b'$, as in Figure 1.

The shape of the distribution of a material property is an indication of the conditions under which this property has been produced. In manufacturing processes an effort is made to attain a definite value of the characteristic property. This objective is expressed either by stipulating a minimum value of the property or by imposing a limit on the fluctuations of its most probable value. The basic idea behind this is to impose controls on manufacturing processes and insure the elimination of causes of fluctuations, reducing the range of dispersion of the individual values.

The most visible effect of such control is to produce a unimodal frequency distribution of the characteristic bell-shape, which in the ideal case of maximum control, is expressed by the normal or Gaussian distribution curve; when

$$f\ (x,\ \mu,\ \sigma) = \phi(Z) = \frac{1}{\sqrt{2\pi}}\ e^{-\frac{1}{2}Z^2}\ ;\ \text{------------}\ (3.8)$$

where $Z = \frac{x - \mu}{\sigma}$.

The Gamma function, Γ, defined for non-negative arguments, is denoted by

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx;\ x > 0\ \text{---------------------}\ (3.9)$$

Also, $\Gamma(\alpha) = (\alpha-1)!$ for every positive integer α.

Now another result stems from the fact that the integral

$\int_{-\infty}^{\infty} e^{-\frac{1}{2}Z^2}\ dZ \twoheadrightarrow$ converges to $\sqrt{2\pi}$

It therefore follows that the function ϕ defined by

$$\phi(Z) = \frac{1}{\sqrt{2\pi}}\ e^{-\frac{1}{2}Z^2}\ \text{----------------------------}\ (3.10)$$

defines a probability density function on E_1 that is everywhere positive. The derivatives are

$$\phi'(Z) = \left(\frac{-Z}{\sqrt{2\pi}}\ e^{-\frac{1}{2}Z^2}\right)\ \text{-------------------------}\ (3.11)$$

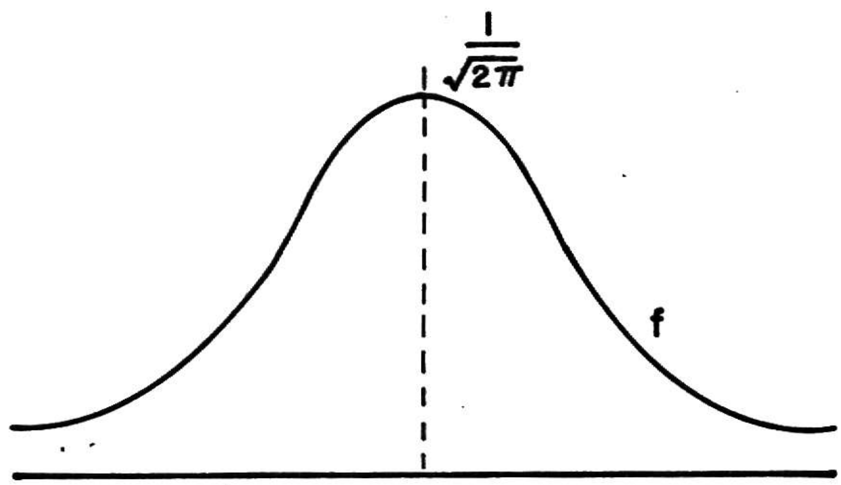

Figure. 2. NORMAL CURVE.

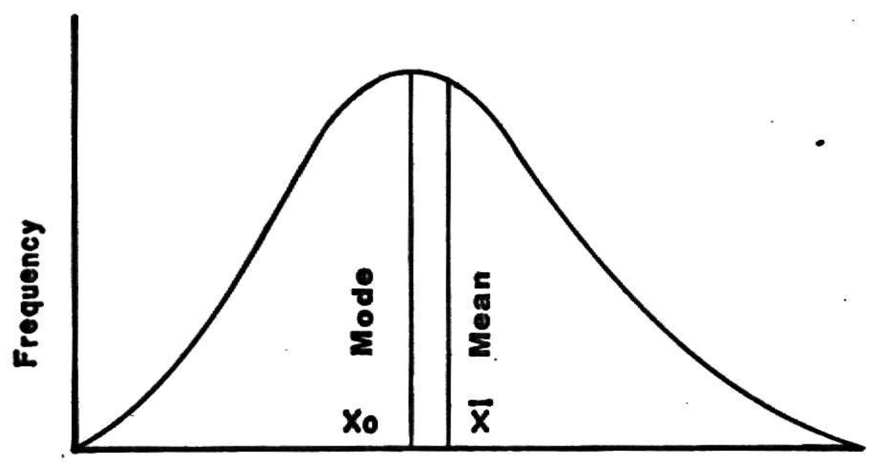

Figure. 3. Frequency distribution of characteristic X.

$$\text{and } \phi''(Z) = \frac{1}{\sqrt{2\pi}} (Z^2-1)\, e^{-\frac{1}{2} Z^2} .$$

It is seen that ϕ has a single critical value at $Z = 0$ but $\phi''(0) < 0$, so that ϕ has a single maximum at the origin. Points of inflection occur at $Z = \pm 1$, where the curve changes from concave downward to concave upward, as shown in Figure 2. Finally, $\phi(Z) \to 0$, as $Z \to \pm\infty$. These facts allow one to make a sketch for all values of ϕ, as shown in Figure 2. This bell-shaped curve is called a normal curve.

The material properties of concrete for its compression strength fit the normal curve for the frequency distribution for various 28-day cylinder strength tests. The compression strength, f'_c, obtained from such tests is the main property specified for design purposes.

To provide structural safety, continuous control is essential to ensure that the strength of the concrete as furnished is in satisfactory agreement with the value called for by the designer. The Building Code Requirements for Reinforced Concrete of the ACI (4) specify that a pair of cylinders shall be tested for each 150 yd.^{3} of concrete or for each 5,000 ft.^{2} of surface area placed but not less than once a day. The results of such strength tests of different batches mixed to identical proportions show inevitable scatter.

The scatter can be reduced by closer control, but occasional tests below the cylinder strength specified in the design cannot be avoided (18). To assure adequate concrete strength, the ACI Code (4) stipulates that concrete quality is satisfactory if

1) No individual strength test result (average of a pair of cylinders) falls below the required compression strength, f_c', by more than 500 psi.

2) The average of any three consecutive strength tests is equal to or exceeds the required f_c'.

It is evident that if concrete were proportioned so that its mean strength were just equal to the required f_c', it would not pass these quality requirements, because half of its strength-test results would fall below the required f_c'. Therefore it is essential to proportion concrete such that its mean strength exceeds the required design strength, f_c', by an amount sufficient to meet the two quoted requirements. This minimum amount by which the mean strength must exceed f_c' is determined only by statistical methods which take into account the variability of the material. The requirements for the execution of the above tests have been derived to limit the probability of strength deficiencies to the following specifications.

According to ACI (4) Specifications, Art. 4.2.2.1, Concrete Quality, where the concrete production facility has a record based on at least 30 consecutive strength tests representing similar materials and conditions to those expected, the strength used as the basis for selecting proportions shall exceed the required f_c' by at least

i) 400 psi if the standard deviation is less than 300 psi.

ii) 550 psi if the standard deviation is 300-400 psi.

iii) 700 psi if the standard deviation is 400-500 psi.

iv) 900 psi if the standard deviation is 500-600 psi.

Example: The table given below is an example of 92 tests performed on concrete cylinders for different strength properties with the respective frequencies.

Cell No.	Cell Boundaries in Ksi	Cell Midpoint in Ksi	Frequency
1	2.5 - 2.6	2.55	1
2	2.6 - 2.7	2.65	1
3	2.7 - 2.8	2.75	2
4	2.8 - 2.9	2.85	3
5	2.9 - 3.0	2.95	6
6	3.0 - 3.1	3.05	7
7	3.1 - 3.2	3.15	7
8	3.2 - 3.3	3.25	7
9	3.3 - 3.4	3.35	8
10	3.4 - 3.5	3.45	8
11	3.5 - 3.6	3.55	9
12	3.6 - 3.7	3.65	7
13	3.7 - 3.8	3.75	6
14	3.8 - 3.9	3.85	8
15	3.9 - 4.0	3.95	4
16	4.0 - 4.1	4.05	3
17	4.1 - 4.2	4.15	1
18	4.2 - 4.3	4.25	0
19	4.3 - 4.4	4.35	2
20	4.4 - 4.5	4.45	1
21	4.5 - 4.6	4.55	1

If the specified compressive strength, f_c', is given as 3000 psi, find the allowable, f_c', and also how many standard deviations the specified f_c' (3000 psi) falls below the allowable f_c'. Determine the percent of cylinders that fail the required f_c' based on theoretical probability population and check to see if these cylinder tests comply with the ACI Code Specifications (4).

Solution:

a) For the given data, compute the mean from

$$\bar{x} = \Sigma \frac{x_i f_i}{N}, \text{ and standard deviation } \sigma = \sqrt{\Sigma \frac{(x_i - \bar{x})^2 f_i}{N}}$$

computing the mean $\bar{x}$ and standard deviation σ for the above, from the given data,

$\bar{x}$ = 3.456 Ksi = 3456 psi

σ = 0.292 Ksi = 292 psi

For the data sample it has been found that the normal distribution curve, F_n (x) is a very good fit, and that the 92 test data sample is a good representation of the population. The shapes of the normal distribution curve and the cumulative polygon agree well and therefore coincide at the 50% point. (See Figures 4 & 5)

b) According to ACI Code Specifications (4), the strength used as the basis for selecting proportions shall exceed the required f_c' by at least 400 psi if the standard deviation is less than 300 psi.

Now, for the data given, the actual f_c', or the mean has been computed as 3456 psi and the standard deviation, 292 psi, which truly complies with the specification.

c) To find the percent of cylinders that fail the specified f_c' (3000 psi), we shall compute the probability of a random variable falling below the required f_c'.

For a standard normal distribution with the mean μ and standard deviation σ, the equation

$$Z = \frac{x-\mu}{\sigma} ,$$

gives the area under the required interval [where $\mu = \bar{x}$].

$$\therefore\ Z = \frac{3000 - 3456}{292} = \frac{-456}{292} = -1.56.$$

The value of $Z = -1.56$ can be found from the standard normal distribution tables. These give $P(-1.56 < Z < 0) = 0.4406$ or $P(Z < -1.56) = .5000 - 0.4406 = 0.059$. The percent of cylinders that fail the required $f_c' = 5.9 = 6\%$.

The ACI Code accepts $\leq 10\%$ of cylinder failures, for strength properties. Thus we see that the specified level is -1.56 standard deviation below the mean.

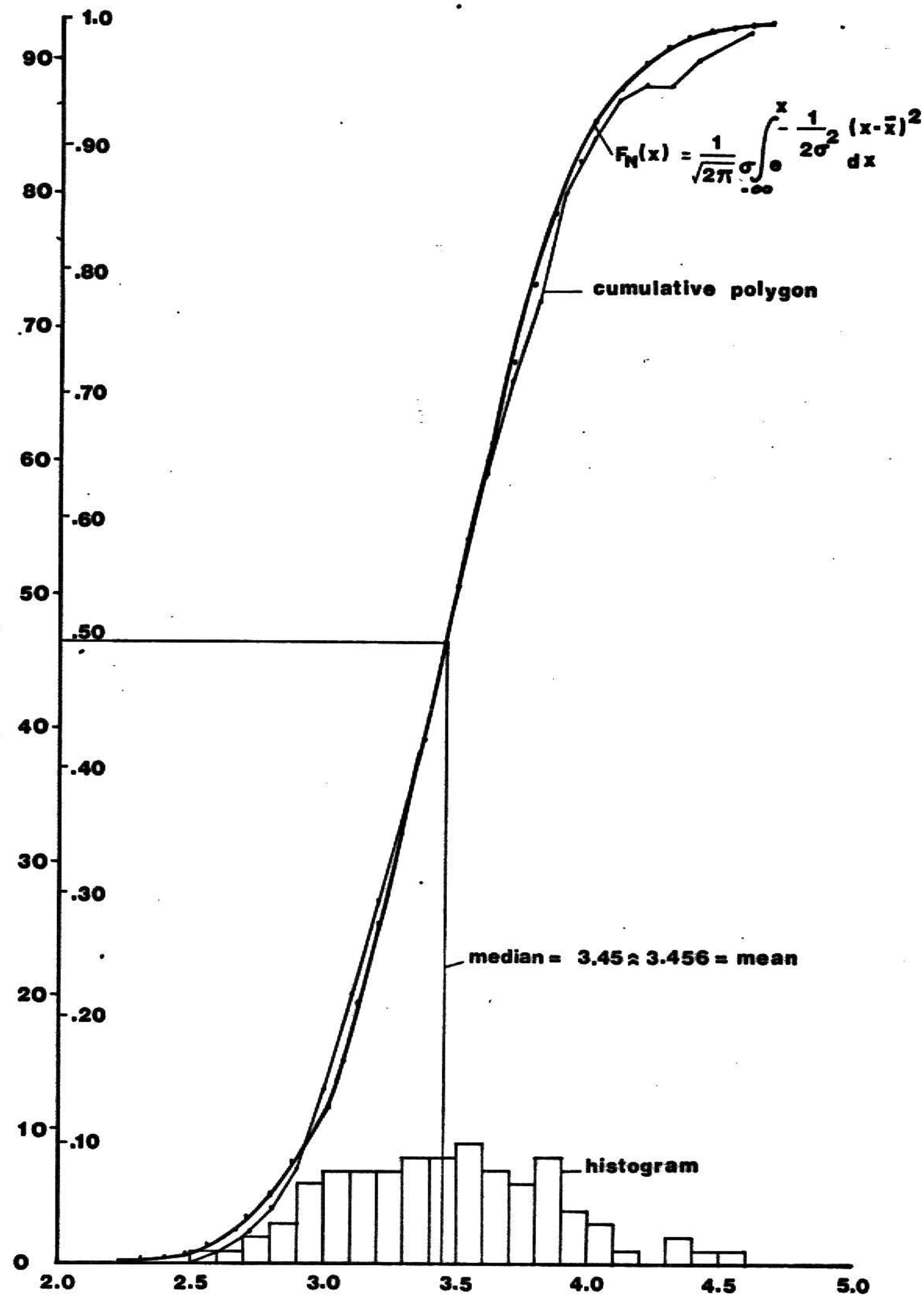

Fig.4. Normal Distrbution Curve and Cumulative Polygon.

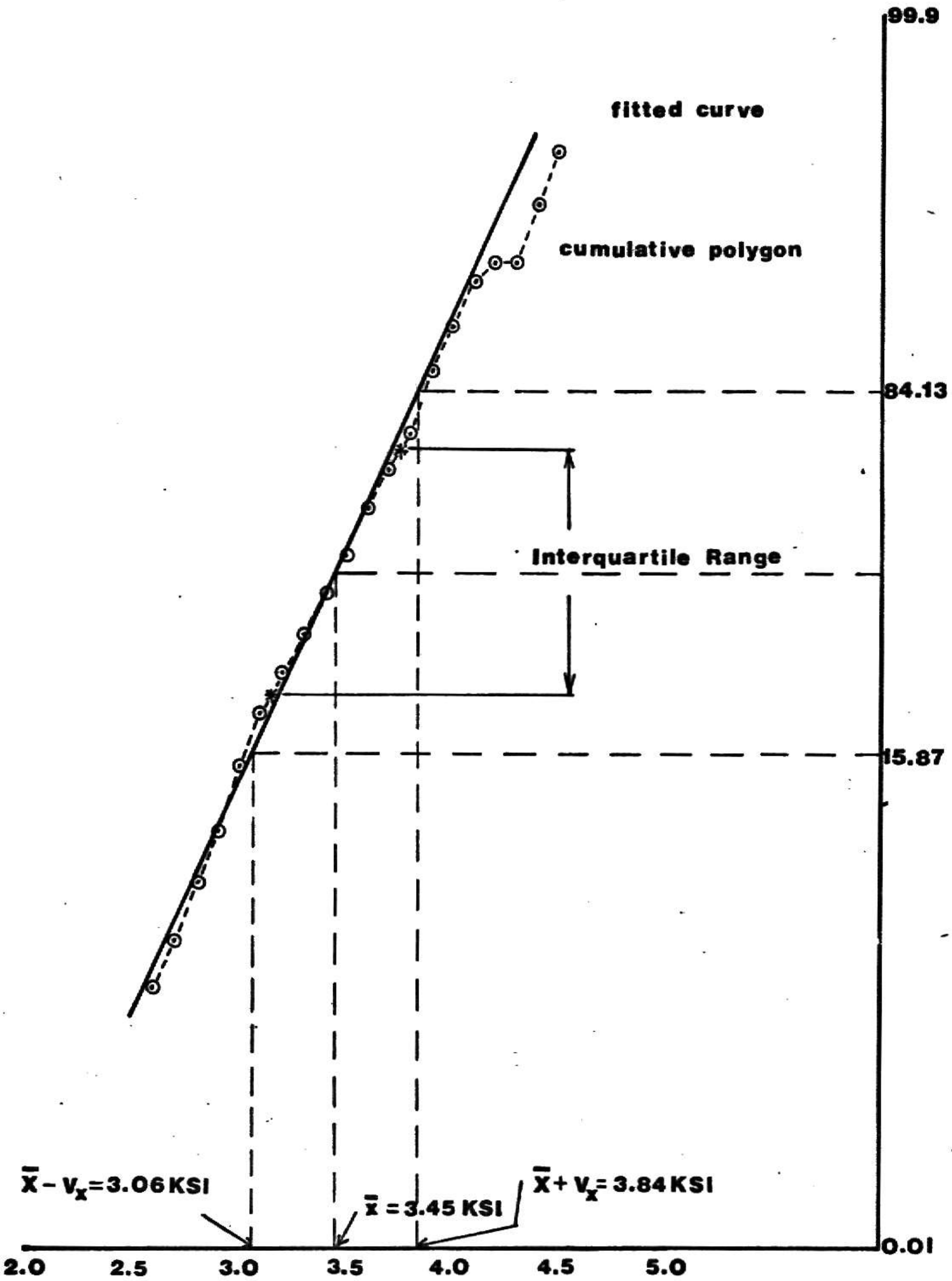

Fig. 5. Fitted Curve.

PROBABILISTIC METHODS FOR DESIGN OF CONCRETE STRUCTURES

LOADS

The influences that affect the factor of safety are basically divided into two groups bearing upon load and resistance (carrying capacity). In addition to these two groups, an intermediate group embodying influences of the method and procedure of computation of load are introduced (9,10).

The load classifications are mentioned below.

Group A: Causes of fluctuation in applied stress(load).

1) Uncertainty and variability of loading conditions
    - a) Dead Load
    - b) Live Load

2) Uncertainty and variability of external conditions that are independent of the load
    - a) Change of temperature
    - b) Wind forces
    - c) Behavior of the subsoil

Intermediate Group: Causes of uncertainty of applied load computations.

3) Variation of rigidity

4) Imperfection of methods and shortcomings of assumptions.

Group B: Causes of fluctuations of resistance (strength).

5) Uncertainty and inaccuracy of the assumed mechanism of resistance.

a) Inaccuracy or inadequacy of conceived mechanism.

b) Variability of resistance limits of materials.

6) Variation of structural dimensions.

An examination and analysis of these aforementioned causes of uncertainty will give a rational insight into the methods that can be adopted for better conditions of safety in engineering structures.

Analysis of Influences:

a) Dead Load: The dead load fluctuations are caused due to the variability of the specific weight of materials concerned and of the dimensions of the structure. These two influences will, together, give a reasonably accurate estimate of the variation of the dead load, given a careful consideration.

It is a frequent experience that computations made for control purposes yield dead loads considerably in excess of the design load after the structure has been completed, due to the varying unit weights of concrete at a given site and the variation in the dimensions of the structural members.

b) Live Load: The most conspicuous and important service condition that is subject to controversy is the design live load of any structure.

Basically, the design live load should represent the most probable, actual service conditions of reasonably high frequency and provide for and anticipate an increase of the service load during the assumed life of the structure. Allowance should be made for possible, yet comparatively infrequent, adverse conditions of service in the ranges of fluctuations of the specified design value and this allowance should be embodied in the factor of safety. In general, the conventional specifications do not fulfill any of the foregoing requirements; the design loads stipulated represent, usually, highly unfavorable conditions, the occurrence of which is infrequent and improbable as well. Safety factors currently adopted are not concerned with the probability that such design live loads may occur, attributing to the unsafety of the structure in actuality.

The properties by which failure and unserviceability of structure are determined are dependent on the time factor. The time dependence of a loading has a critical effect on the operating conditions of a structure. The major aspects of this time dependence are a) the rate of load application, b) the duration of the loading, and c) the number, frequency and sequence of load repetitions.

The rate of load application for long span bridges of heavy mass has a narrow range of variation, as also for prestressed concrete structures and in cases of floor loads of buildings, the possible range of variation in the rate of load application is negligibly small.

Analysis of Bridge Loading Frequencies

The selection of the proper live load to be used for the design of various parts and types of highway bridges represents one of the most important and difficult problems encountered by the designers (16). By and large, the choice of a design live load not only depends upon the maximum sizes and weights of vehicles but also their speeds, spacings and other operating conditions necessary to ensure that a given bridge will perform the functions for which it is intended, safely and economically.

Successful planning of any particular highway bridge requires adequate information about the site and foundation conditions; a thorough knowledge of bridge design procedures, their relation to physical properties of the materials of construction, and last and most important, perhaps, for the designer, to arrive at a design live load commensurate with present and anticipated traffic conditions.

Mathematical Basis for Study of Vehicle Group Frequencies

The proper design live load for highway bridges is a function of the sizes, gross weights and frequencies of individual vehicles that might be expected to occur on a given length of a bridge, as a result of the probability grouping of two or more vehicles at the same time. The application of probability theory for chance grouping of vehicles and the frequency of specified vehicle groups needs a few simple assumptions, viz.

1) That vehicles, both individually and by types, are of random distribution in ordinary highway traffic.

2) That the average composition, volume and speed of traffic remain constant during the time period under consideration.

The first assumption means the time and distance spacings of vehicles occur entirely by chance. The second assumption implies that the time period under consideration should be of small duration for the average composition, volume and speed of the traffic remain constant during that particular time. These assumptions have been found by numerous studies to be compatible to the actual traffic behavior on an ordinary highway.

Moreover, these studies have shown that the time and distance spacings of vehicles--both individually and by groups--in ordinary traffic agree with the distributions given by the Poisson Frequency Distribution Formula. The Poisson distribution law determines mathematically the probability of vehicle groups occurring within specified lengths of time and the time interval can be estimated, too.

This Poisson's law las been found to provide an excellent estimate of the frequency distribution of various intensities of vehicle loads measured in terms of their H-truck loading equivalencies on a given span.

The Poisson's law is an approximation of the binomial distribution law when the sample size of the binomial distribution tend to ∞. In other words, the binomial distribution tends to approach the Poisson's distribution as a limit as the number of trials or sample size becomes very large.

$$P(n;K) = \frac{K^n e^{-K}}{n!} \text{ ---------------------------------- } (4.1)$$

where n is the number of arrivals; K = average number of successes.

$$K = \frac{\text{(Average No. of Veh./hr.)(x)}}{\text{(Average Speed of Veh./hr.)(5280)}}$$

The successive terms of a binomial expansion have as their limits the corresponding terms in the Poisson distribution as

$$\sum_{n=0}^{\infty} P(n;K) = e^{-K} + Ke^{-K} + \frac{K^2e^{-K}}{2!} + \frac{K^3e^{-K}}{3!} + \ldots + \frac{K^ne^{-K}}{n!} \ldots$$

$$\therefore \Sigma\, P(n;K) = 1, \text{ for } n = 0, 1, 2, 3, \text{ ----------- } (4.2)$$

Assuming the average composition, volume and speed of traffic remains constant during a particular time period, the problem of estimating the frequency of specified vehicle groups occurring within specified lengths of time or distance can be computed by various methods.

a) Occurrence of n unspecified vehicles in either or both directions.

At any particular highway, for any given average composition, volume and speed of traffic, the probability of n vehicles, unspecified as to type, occurring in any manner in either or both directions of travel within a time interval of t seconds or a distance of x feet is given by Poisson's formula

$$P(n;\ x;\ \frac{\ell'}{2}) = \frac{K^ne^{-K}}{n!} \text{ --------------------------- } (4.3)$$

where K is the average number of vehicles expected within the distance based on the total number of vehicles per hour (both directions) at the given location, and ℓ' denotes the number of lanes on the highway.

If time, t, instead of distance is used to measure the interval, then the probability that n vehicles will occur within t seconds is given by

$$P(n;t,\frac{\ell'}{2}) = \frac{K^n e^{-K}}{n!} \text{ ------------------------------ } (4.4)$$

The vehicle interval is calculated as follows:

$$V'(n;x,\frac{\ell'}{2}) = \frac{K}{P(n,x;\ell'/2} \text{ veh./given distance x -- } (4.5)$$

$$V'(n;t,\frac{\ell'}{2}) = \frac{K}{P(n,t;\ell'/2)} \text{ veh./given time ------- } (4.6)$$

depending on whether the length is measured in time or distance.

The time interval between vehicles is given by

$$T(n;t,\frac{\ell'}{2}) = \frac{V'(n;x,\frac{\ell'}{2})}{R'} \text{ ------------------------- } (4.7)$$

where R' is the rate of interval of vehicles/unit time.

b) The occurrence of n unspecified vehicles in each direction;

$$P(n;x,\ell') = (\frac{K^n e^{-K}}{n!})^2 \text{ ---------------------------- } (4.8)$$

where K is the average number of vehicles in each direction ($K_1 = K_2 = K$).

The vehicle interval; $V'(n;x,\ell') = \dfrac{2K}{\dfrac{(K^n e^{-K})^2}{n!}}$ ------ (4.9)

The time interval; $T(n;x,\ell') = \dfrac{V'(n;x,\ell')}{R'}$ --------- (4.10)

Example: Assume that the vehicular traffic on a single-lane county highway bridge is distributed in a Poisson process with a traffic composition of 75% M, 20% L, 5% H, vehicles.

Let the number of vehicle arrivals be approximated by the Poisson process with a parameter intensity α = 12 vehicles/hour. Now K is the total number of arrivals = αt, where t is the time period.

$$\therefore\ f(t) = \alpha e^{-\alpha t};\ t \geq 0. \qquad (4.11)$$

a) What is the probability that, in the next 45 minutes, there will not be more than 8 arrivals?

$$\alpha = 12 \text{ veh/hr.}$$

$$\alpha' = \frac{12}{60} = \frac{1}{5} \text{ veh/min.}$$

$$\therefore\ K = \alpha' t = \frac{1}{5} \times 45 = 9 \text{ veh.}$$

$$\therefore\ P(n \leq 8) = \sum_{K=0}^{8} \frac{e^{-9}(9)^8}{8!}$$

$$= e^{-9}[1 + \frac{9}{1!} + \frac{9^2}{2!} + \frac{9^3}{3!} + \frac{9^4}{4!} + \frac{9^5}{5!}$$

$$+ \frac{9^6}{6!} + \frac{9^7}{7!} + \frac{9^8}{8!}]$$

$$\therefore\ P[n \leq 8] = 0.456.$$

b) In the first 10 minutes, what is the concurrent probability that there will be 1 vehicle in each direction of the bridge at the same time?

$$\therefore\ P[n;K;\ell'] = \left(\frac{K^n e^{-K}}{n!}\right)^2,$$

where ℓ' corresponds to both the lanes on the bridge.

$$\alpha = 12 \text{ veh/hr.}$$

$$\alpha' = \frac{12}{60} = \frac{1}{5} \text{ veh/min.}$$

$$\therefore\ \alpha' t = \frac{1}{5} \times 10 = 2 \text{ vehicles} = K$$

$$\therefore\ P[n = 1, \text{ in each direction}] = \left(\frac{2^1 \ e^{-2}}{1!}\right)^2$$

$$= \{e^{-2}(1 + \frac{2}{1!} + \frac{2^2}{2!})\}^2 = (.676)^2$$

$$\therefore\ P[n = 1, \text{ in each direction}] = 0.458.$$

Live Floor Loads

Statistical Model for Live Floor Loads:

Hasofer (12), in his paper has proposed a statistical model for live floor loads correlating them with actual results from a live-load survey conducted at Melbourne University.

In his model, Hasofer points out that there should be specific rules formulated to reduce the live loads on the basis of areas supported by structural members. It is suggested that statistical analysis recommends reduction of loads for different types of occupancy.

The model is developed on the basis of studying fluctuations of live loads on a building for a given type of occupancy. The type of occupancy has been taken as a random element for a hypothetical number of buildings, the live load thus becoming a random variable.

The two following assumptions made for the purpose of analysis are:

1) The probability distribution of the live load on two equal areas in different parts of a building having the same occupancy is the same. This is called, in probabilistic terms, a stationarity assumption.

2) The live loads on two non-overlapping areas of the building are distributed independently of each other.

The conceptual model for the total load over an area corresponding to the two assumptions is known as a stochastic process with independent stationary increments which follow a compound Poisson process.

It has been shown by Hasofer (12), that the mean value and variance of total load on the floor area is directly proportional to the floor area itself. The mean value is based on the stationarity assumption while the variance is a consequence of independence. The results obtained from independence assumptions for variance are in accordance with the actual results obtained from a survey of live loads in a building, carried by the University of Melbourne, Australia.

<u>Temperature changes</u>: in a structure involve chance fluctuations. The probability of extreme temperature

conditions to occur is comparatively small. However, temperature changes will occur frequently within a definite range and therefore are to be included in any study concerning the service conditions of a structure. It may not influence greatly the safety of the structure, but investigations on the thermal coefficient of expansion of concrete will be of immense value in the construction of highways. The thermal coefficient of expansion of concrete varies with the type of aggregate and mix. Some of the coarse aggregate concretes are found to vary by about $\pm$ 50% from the acceptable constant of 0.000006, which would lead to undue cracking.

Wind Forces: The magnitude and distribution of wind forces are chance events. Within a certain range of wind pressures, the peak frequency could be high and consequently, the structure built to withstand these pressures can be expected to conform to the designed intensities. But extreme wind forces such as storms and tornadoes occur very seldom. Hence, the actual design can only be restricted to moderate wind forces. Nevertheless, data of the wind velocities of the specific area would greatly enhance the designer to prepare distribution curves from which design values and fluctuations can be derived.

Behavior of Subsoil: The foundation of a structure is one of its integral parts. The behavior of the foundation plays an important role in a structure and furthermore, is directly dependent upon the behavior of the subsoil itself. It is assumed that the supports remain fixed or settle in small increments due to strain as a result of movement in the subsoil. Fluctuations varying about the design load stresses induced in the subsoil will increase the differential settlements and hence increase the required action of safety. The subsoil has a unique characteristic of having a high degree of rigidity and almost a perfect elasticity under the action of moving loads. Therefore, the effect of subsoil to structure will diminish to only dead load stresses.

Variation of Structural Rigidity: The rigidity of a structure is expressed as the product of a sectional value, such as moment of inertia and modulus of elasticity divided by the length of the member. Therefore, the amount of variation in the rigidity will depend on the change in either the modulus of elasticity or the sectional value or both. The rigidity of all parts or members of one and the same structure may not have uniform variation. In concrete and reinforced concrete structures the creep that occurs under action of sustained loads makes the initial modulus of elasticity appear to decrease. This property apparently affects the state of strain and thus the safety of the structure. The effect of such variation upon the strain of redundant structures may be determined by the method "analytical experiment" by Hardy Cross (19).

Imperfection of Methods & Shortcomings of Assumption: Methods of strain computations are generally based upon the perfect elasticity assumption. This assumption is justified for concrete within the range of conventional working stresses for specified boundary conditions. But for short cut methods there is a range of error which is compensated by an increase in the safety factor. The boundary conditions, chosen arbitrarily, are expressed in terms of displacement, angular deflection, movement of the subsoil and foundations.

STRENGTH

The resistance of engineering structures is affected by the character of load, by external conditions and by the characteristic properties of the structural materials. Structural resistance is generally defined with reference to the state of strain which delimits the fitness for service of a structure. Basically it is a function of two variables; sectional characteristic and the strength of the material. For practically all structural materials this resistance is influenced by the rate of which load is applied.

The real resistance mechanism is usually complex and it is hardly possible to conceive such a mechanism which will effectively reproduce the actual phenomenon, and be suitable for practical design pruposes at the same time. The efficiency of the resistance of any structure can be ascertained by statistically interpretable experiments, with the range of dispersion of individual values about the average being established.

The range of uncertainty characterizing a certain resistance mechanism is affected by the state defined as undesirable with regard to the safety of the structure. The perfect correlation of structural resistance limits with the material strength limits has been achieved so far for only simple conditions.

Uncertainty and Variation of Resistance Limits of Structural Materials

The field of applied load, the rate and duration of load, the amplitude of load cycles, and the number of cycles, are the major factors that govern the mechanical resistance of engineering materials. The influence of loading is generally expressed by the mechanism of resistance. Since the resistance of a material relevant for purposes of design, is its resistance in the area of the heaviest concentration of internal forces, the resistance of the material should therefore be determined experimentally for an idealization of the structure around the specified critical area.

The resistance-reducing effects of structural connections subject to load cycles of different amplitudes have been noted for steel structures. Different limits have also been obtained for different numbers of load cycles. Statistical information regarding the dynamic resistance range of fluctuations is not available as yet. The concept of damage due to overstraining has considerable bearing on the safety of structures. The capacity of a structural material to sustain occasional overstrain without damage to its ultimate bearing limit is important. In fact, the design live load should accommodate service conditions of reasonably high frequency of occurrence and at the same time a less frequent and less probable occurrence of extreme loading has to be taken care of by the factor of safety.

The resistance values of structural materials are assumed constants in a statistical sense; the level of control enforced in all stages of manufacturing processes determining the character of the distribution function. Concrete has been found to have a range of chance fluctuations for individual values of compressive strength, around 35% about the mean, under normal control on the site.

CONSEQUENCES OF FAILURE

The term "Factor of Safety" (13) is without real meaning unless it is correlated with the corresponding probabilities. The resisting properties of materials and the maximum load effects to which structures are subjected vary. It is imperative to realize, however, that regardless of how conservative the design and how well a structure is built, there is always some probability of its failure. Therefore, the correlation of factors of safety and factors of serviceability with probabilities of survival and serviceability for each individual structure is commonly impracticable. However, the framing of design rules and regulations can play a significant role for the practicability of the correlation factors.

The question "What is the probability of loss?" has been of immense importance in the discretion of a designer and perhaps been the utmost hurdle any designer had to cross before contemplating the design. Statistical and probability studies can be of importance as guides supplemented by the application of common sense and engineering judgment.

Considerable work on the application of statistical and probabilistic methods has been done following the lead of A. M. Freudenthal (9, 10). This work is evidently of little avail until such time as structural engineers have acquired

a) A statistical background regarding resistance of materials and structures, including time-yield, dynamic and fatigue effects;

b) A similar background regarding load effects; and,

c) The necessary competence in the calculus of probability, which includes the elements of statistical analysis.

The term "Factor of Safety" has been outlined in two major categories as given below.

1) Minimum required factor of safety: to assure that a given probability of failure, P_F, of the structure is not exceeded, is defined as the ratio (greater than unity) of R, the mean estimated resistance to collapse during the anticipated life of a large number of structures to be identical with the subject structure, and W, the mean load effect for which the subject structure is designed, i.e., $P_F = \frac{R}{W} > 1$.

2) Minimum required factor of serviceability: to assure that a given probability of the structure becoming unserviceable, for the purpose and during the anticipated life for which it is designed, is not exceeded, is defined as a similar ratio of resistance and load, but with respect to serviceability rather than collapse.

The above definitions basically have to do with structures rather than individual members of the structure. This is because the resistance of a structure is not necessarily governed by the strength of individual members.

The main difference in the two definitions involving "Factor of Safety" in terms of collapse and serviceability can be understood by an example--assume a high office building so constructed that the probability of collapse is negligible. However, during gusty wind storms which are prevalent in that area, it vibrates to such an extent as to cause alarm and chaos in the occupants leading to vacating the building. This shows that although the building is "safe", it is "unserviceable" in that it does not fulfill the purpose for which it is designed.

Ordinarily, one is expected to choose a considerably greater probability for the structure becoming unserviceable than for failure by collapse. Therefore, the factor of serviceability is smaller than the factor of safety. In cases where the principle of superposition applies, an adequate design for serviceability will usually result in an adequate design for safety. But where the principle of superposition does not apply in the case for members carrying compressive loads, an adequate design either for serviceability or safety will not assure adequate design for the other. Hence, a safe rule is to calculate both factors independently.

The above definitions involve terms that are associated with given probabilities. When selecting these probabilities, considerations should be given to at least the following:

1) The type of failure--will it be without warning as in the case of concrete failure due to tension? Or on the

other hand, will increases in deformation give warning of impending danger?

2) The value of human lives which may be lost, in case of failure.

3) The importance of the structure and its cost, including costs other than of the structure itself incurred on account of it becoming unserviceable.

4) The capitalized cost of maintaining the structure in serviceable conditions.

5) The replacement cost in case of failure.

6) A charge against the structure equal to total cost of failure multiplied by the probability of its occurrence.

The factors of safety and serviceability can be considered as "load factors" by which the mean of the design-load effect is multiplied to equal the mean calculated resistance which may in some cases correspond to and be limited to a comparatively small deformation required to render the structure unserviceable, while in other cases, it may be almost as high as the ultimate strength. In order that the resistance of a structure in the design stage can be estimated, it is necessary to have statistical data pertaining to the in-place strength, lower yield point, and fatigue strength of materials similar to those which will be used. These data should preferably include the results of a large number of tests and should be in such form that the following quantities and graphs can be determined and prepared for each material involved; i.e., median and average strengths, coefficients of variation, values of lowest and highest tests, histograms and cumulative frequency graphs indicating the statistical distribution of test results.

ECONOMIC ASPECTS

From an engineering point of view, if it were possible to build a structure exactly as designed, with all the materials of definite strength, only subjected to loads within those used in the design, and with the analysis and all assumptions of design correct, then the structure would not fail and there would be no need to consider providing additional strength for the purpose of safety. In fact, all these factors are variables and few can be accurately predicted. To provide for this uncertainty the strength of the structure is increased, which provides a certain level of safety against extreme conditions of the variables (8).

The final level of safety is composed of that provided for both engineering purposes as well as other consequences of failure. The engineering purposes being extreme conditions of loadings, etc., and the other consequences being the financial cost, inconvenience and risk of death and injury associated with failure.

Structures of different levels of safety can be built, but the level provided depends largely on the amount of money available. As the safety increases, the cost of the structure also goes up, because to ensure additional strength, either large sections are required with magnification of the foundation, or stronger or more expensive materials are required.

A structure in which failure can be minimized to the extent of impossibility can be constructed, but the cost would be prohibitive. Hence, in practice, a lower standard of safety is prevalent. The methods of specifying safety have, therefore, to provide a solution to the engineering and other problems.

Engineering structures fail when they collapse or show functional deficiency when applied loads exceed the capacity of the structure to resist them at the critical time. This situation results from the inability to determine the applied loads, the strength capacity of the structure, or both in the design, or some error in construction. The solution to this problem requires reasonable methods of specification which limit the chance of structural failure.

The solution to other problems require provision of additional safety, above that required by engineering considerations. This additional level of safety protects the owner, the user and the general public from the consequences of failure. The owner has a financial interest in the structure; the user has a risk of injury or death associated with collapse; and the general public may lose confidence in a particular type of structure.

<u>Design and Construction</u>

The work of design involves the provision of a structure with a decided level of safety at a minimum cost. The three basic steps involved are:

1) Determination of design loads.

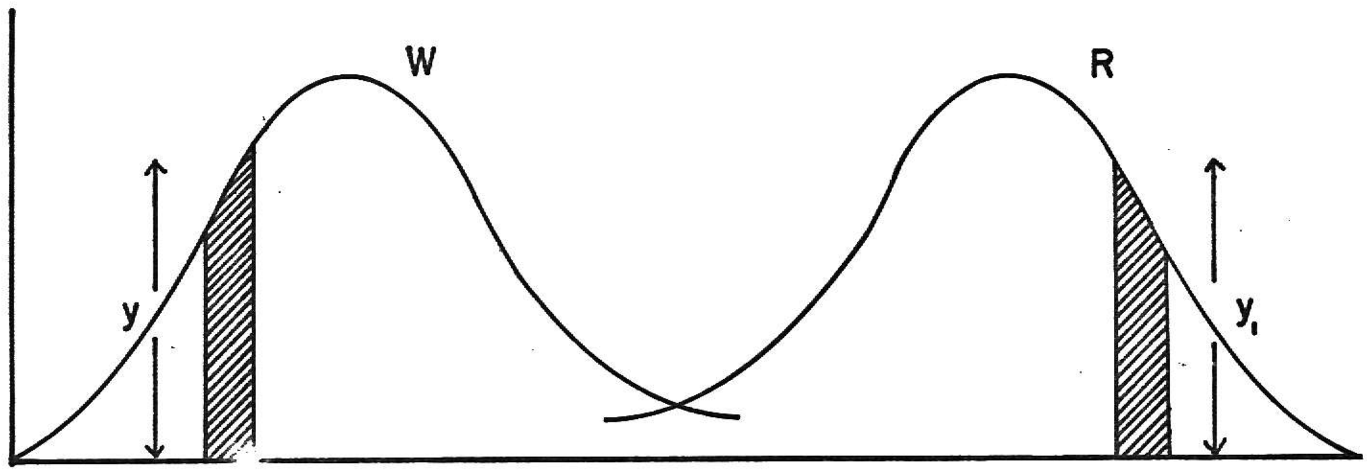

Figure. 6. Magnitude of Load and Resistance.

2) Determination of the strength of the structure.

3) Determination of a quantitative value for safety.

The determination of the value of safety provides for the inadequacy of knowledge in Steps (1) and (2) and also any social safety level required.

The determination of strength of the structure includes the consideration of material strength; selection of sections and structural arrangement; analysis of resistance of the structure with regard to material properties and sectional arrangements.

Generally, the determination of design loading is considered to be the weakest part in the design procedure. The design loading consisting of load patterns, intensities, frequencies of occurrence and durations require extensive data in order to deal with the problem on a statistical basis and they are difficult and expensive to obtain.

A method of specifying safety from the engineering aspect must attempt to avoid the excess of applied loads over the resistance of the structure.

In Figure 6, the curve W shows the different magnitude of loads to which an idealized structure may be subjected, plotted against the relative frequency of occurrence. Also, curve R indicates the variability of strength magnitude of a large number of structures designed and constructed to the same requirements. Failure occurs where the two curves intersect, and where any value of W is in excess of R for a particular structure. Under the existing conditions there is always a range of loads and strengths and whatever arrangements of W and R curves exist, the tails will intersect, implying a possibility of failure. Under these circumstances no structure can be completely safe and, hence, a chance of failure.

Attempts at providing safety for these curves can be made by:

1) Specifying as a minimum the ratio $\frac{R_1}{W_1}$, known as the factor of safety, W_1 being the specified maximun design load anticipated under working conditions and R_1 the resistance of the structure arrived at by specifying the variables in design.

2) Specifying as a maximum the probability of failure.

3) Specifying as a minimum a factor of safety coupled with a maximum probability of failure.

The application of the above safety factors for engineering purposes can suitably be changed to give an additional level of safety for social consequences of failure.

The social consequences which include financial loss to the owner, loss of life, injury, damage to human society, interference with use, repairs and reconstruction costs resulting from failure can be included as a capitalized cost, along with the initial cost of the structure, according to Asplund (5). He also suggests that in the decision of safety level, consideration of the structures not built but which could be built with the money saved if the safety level is lowered, should be made. This involves cost of lives and inconvenience caused by the absence of structures, leading to implications that are social, administrative and political, and yet, if considered in the manner suggested, affect the cost of structures. It appears that the consideration of all these social consequences cannot be achieved in the decision of safety level to be provided in a single structure.

A simple example would be safety levels for bridges. Here the effects of safety level on the number of bridges being built in a country could be considered. The level of safety decided being that which provides the maximum economic gain to the community. For instance, the value of 1,000 bridges built to a greater level of safety compared to 1,100 cheaper bridges built to a lower level of safety may have to be decided, taking into account the cost of inconvenience, risk of injury or death associated with 100 unbuilt bridges in the first case or the justification of the 1,100 bridges built to a lower safety level. Baker (6), in his paper seems to discuss that it is illogical to have different factors of safety for failure of steel and concrete in reinforced concrete; it is pointed out that only one level of safety should be kept constant for all parts of a structure for all practical purposes. However, for social reasons, different levels of safety can be specified.

Basically, since there is a greater variance in the strength of concrete than in steel, the specification of the safety level should depend upon the method of selecting material strength to be used in design. The design strengths used in a structure should be such that the percentage of test results below the selected design value are the same for all materials. The decision on the social level of safety to be provided needs discretion on the part of the designer as it is impossible to determine the numerical effect of social consequences of failure on the probability of risk or failure. Much of the social consequences of failure can be expressed as a cost and if some of this cost is included as a capitalized cost, the decision on the social safety level can become an engineering consideration.

DESIGN APPLICATIONS FOR CONCRETE STRUCTURES

DESIGN OF A T-BEAM BRIDGE

A highway bridge consisting of a concrete slab and concrete girders is to be designed by AASHTO Specifications (2,3), for these conditions; loading - HS20-44; clear width - 28 ft.; effective span - 54 ft.; concrete strength - 3000 psi.; reinforcement - intermediate grade.

The slab and girders will be poured monolithically, and the slab will include a 3/4 in. wearing surface. The design is, in addition, to make an allowance of 15 psf. for future paving.

Design the slab and the cross section of the girder.

For bridge members it is necessary to modify the wheel loads to allow for the effects of dynamic loading and lateral distribution of loads resulting from the rigidity of the slab.

Figure 7 shows the spacing of the girders and the dimensions of the members obtained on a trial basis.

The values from the specification are n = 10, in stress calculations; $f_c = 0.4 f_c' = 1200$ psi; for beams with web reinforcement, $v_u = 0.075 f_c' = 225$ psi; $f_s = 20{,}000$ psi; bond stress $u = 0.10 f_c' = 300$ psi; where n is modular ratio.

a) Computation of design coefficients for balanced design;

$$k = f_c/(f_c + f_{s/n}) = \frac{1200}{\left(1200+\frac{20,000}{10}\right)} = \frac{1200}{3200} = 0.375$$

$$\therefore\ j = 1 - \frac{K}{3} = 1 - \frac{0.375}{3} = 1 - 0.125 = 0.875$$

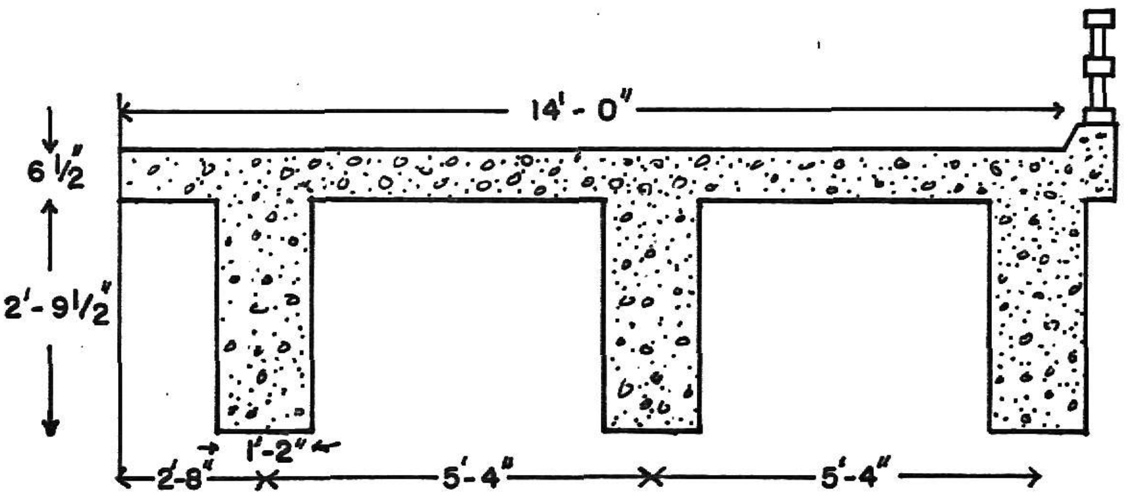

Fig. 7. Transverse Section of T-beam bridge.

$$K = \frac{1}{2} f_c k_j = \frac{1}{2} \times 1200 \times 0.375 \times 0.875 = 197 \text{ psi.}$$

b) As shown in the AASHTO Specification, the wheel-load system of HS20-44 truck comprises two loads of 16 kips each and one load of 4 kips. Since the girders are simply supported, an axle spacing of 14 ft. will induce the maximum shear and bending moment in these members.

The specification does not present moment coefficients for the design of continuous members. The positive and negative reinforcement will be made identical using straight bars for both. A coefficient of 0.10 is assumed for positive and negative dead load moments. The span length, S, of a slab continuous over more than two supports can be taken as the clear distance between supports.

c) Verify the adequacy of slab size and design reinforcement. In computing the effective depth, disregarding the wearing surface, assume the use of No. 6 bars and 1" for insulation, as required by AASHTO;

$$\therefore t = 6.5 - 0.75 - 1 - 0.38 = 4.37 \text{ in.}$$

$$w_{DL} = \frac{6.5}{12} \times 150 + 15 = 81.25 + 15 = 96 \text{ psf., say.}$$

$$M_{DL} = 0.10 W_{DL} S^2 = 0.10 \times 96 \times (4.17)^2 = 167 \text{ ft-lb./ft.}$$

where, $S = 5'4'' - 7'' - 7'' = 64'' - 14'' = 50'' = 4.17$ ft.

$$M_{LL} = 0.8 \frac{(S+2)}{32} P_{20} \text{ by AASHTO - 1.3.2 (c)}$$

$$= \frac{0.8\ (4.17+2)}{32} \times 16{,}000 = 2467 \text{ ft-lb./ft.}$$

$$\therefore M_{LL} = 2467 \text{ ft-lb./ft.}$$

The impact factor I.F. by AASHTO = 30%.

$\therefore\ M_{total} = \{167 + 1.3\ (2467)\} \times 12 = 40{,}500$ in-lb.

The moment corresponding to balanced design is $M_b = Kbd^2$

$\therefore\ M_b = 197 \times 12 \times (4.37)^2 = 45{,}100$ in-lb.

Therefore the concrete section is excessive, but a 6 in. slab would be inadequate.

The steel is stressed to capacity at design load.

$$\therefore\ A_s = \frac{M}{f_s jd} = \frac{40{,}500}{20{,}000 \times 0.875 \times 4.37}$$

$\therefore\ A_s = 0.53$ in.$^{-2}$

Use #6 bars, 10 in c-c, top and bottom.

The transverse reinforcement resists the tension caused by thermal effects and by load distribution.

By AASHTO; $A_t = 0.67\ (A_s)$

$A_t = 0.67\ (0.53) = 0.36$ in.^{2}

Use #5 bars in each panel.

ii) Interior Girder Moments:

The required stem dimensions are governed by either the maximum moment or maximum shear.

a) Dead-load moments:

The weight of the slab per foot of beam is $96 \times 5.33 = 512$ plf.

The stem section below the slab is assumed as 14×33.5 in., which adds a weight of 488 plf.

$\therefore$ Total weight of the slab = 1000 plf.

$\therefore$ Total dead load moment at the center of the span is $M_{DL} = \frac{1}{8} \times (1) \times (54)^2 = 364.5$ K-ft.

b) Live-load moments:

The maximum live-load bending moment occurs under the center load shown in Figure 8 for an HS20 truck loading.

The resultant P_R of the load group has the location

$$d = \frac{16(14) + 4(28)}{16 + 16 + 4} = 9.33 \text{ ft.}$$

The AASHTO prescribes a distribution factor $\frac{S}{6}$, but we will use $\frac{S}{5}$ for this problem.

$$\therefore \ DF = \frac{5.33}{5} = 1.066$$

$\therefore$ The load from the rear wheel = 16 x 1.066

= 17.06 kips.

Load from the front wheel = 4 x 1.066 = 4.26 Kips.

$\therefore$ P = 17.06 x 2 + 4.26 = 38.38 Kips.

$$\therefore \ R_L = \frac{38.38 \times 29.33}{54} = 20.85 \text{ Kips - from Figure 9.}$$

$\therefore$ Maximum live-load moment

$$M_{LL} = 20.85\ (29.34) - 17.06\ (14)$$

$$M_{LL} = 372.9 \text{ K-ft.}$$

Impact Moments:

$$\text{Impact coefficient} = \frac{50}{\ell + 125} = \frac{50}{54 + 125}$$

$$= \frac{50}{179} = 0.28$$

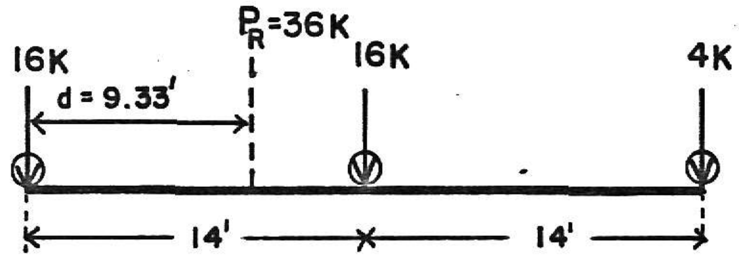

Fig. 8. Load group and its resultant.

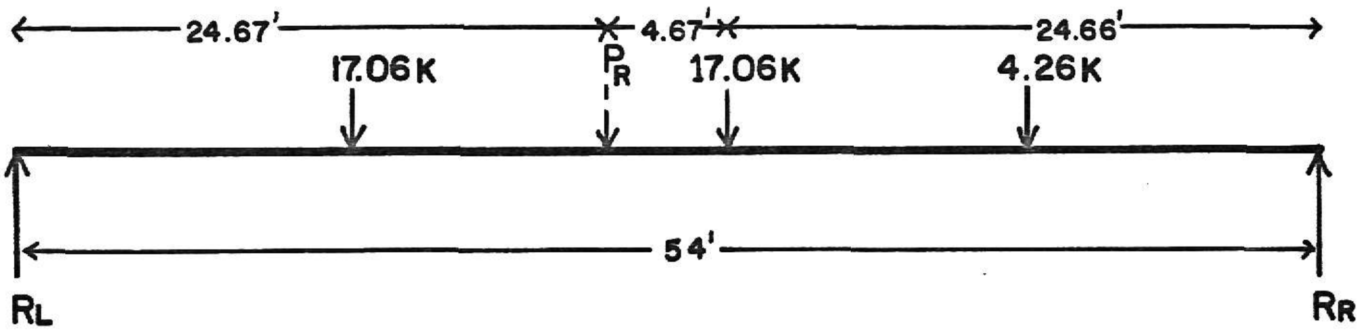

Fig. 9. Loading for maximum moment.

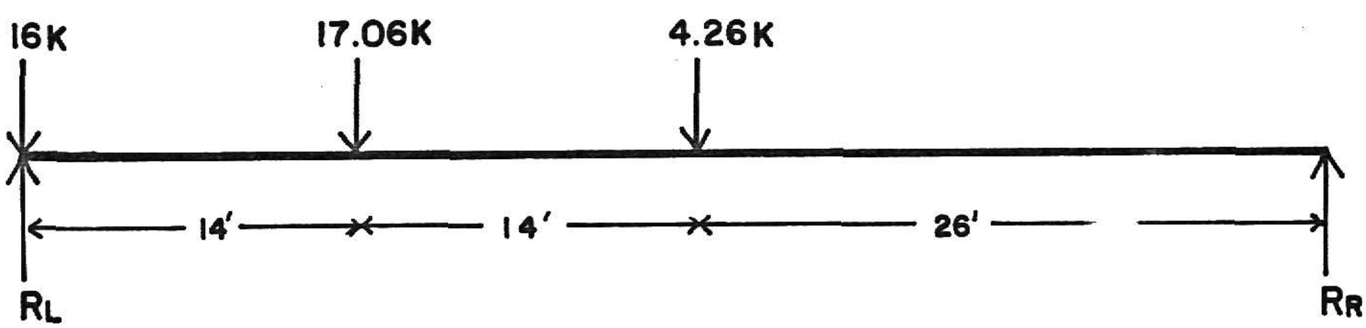

Fig. 10. Loading for maximum shear.

$\therefore$ Impact Moment = 0.28 x 372.9 = 104.4 K-ft.

$\therefore$ M_{max} = Maximum total moments = 364.5 + 372.9 + 104.4 = 841.8 K-ft.
(sum of max. D.L., max. L.L., Impact moments)

iii) Shears:

a) Dead-load shears:

The maximum dead-load shear at the end of the beam is 1000 x 27 = 27000 lb. = 27 Kips.

$$\therefore M_{DL} = \frac{1}{8} \times 1 \times (54)^2 = 364.5 \text{ K-ft.}$$

b) Live-load shears:

The absolute maximum shear occurs with the truck on the span in the position shown in Figure 10.

$\therefore$ Max. shear due to live load

$$V_{LL} = 16 + \frac{17.06(40)}{54} + \frac{4.26(26)}{54}$$

$$V_{LL} = 30.69 \text{ Kips.}$$

c) Impact shears:

End shear = 0.28 x 30.69 = 8.6 Kips.

$\therefore$ Total shear = 27 + 30.69 + 8.6 = 66.29 Kips.

$$\therefore M_{total} = 12\ (364.5 + 1.28 \times 372.8)$$

$$M_{total} = 10{,}100 \text{ in-Kips.}$$

d) In establishing the effective depth of the girder, assume #4 stirrups will be supplied and that the main reinforcement will consist of three rows of #11 bars.

AASHTO requires 1 1/2" insulation for stirrups and a clear distance of 1" between rows of bars. But we will use 2" of insulation and the center-to-center spacing of rows will be taken as 2.5 times the bar diameter.

$$\therefore\ d = 5.75 + 33.5 - 2 - 1 - 1.41\ (2.5) = 32.72\ \text{in.}$$

$$\therefore\ v = \frac{V}{b_w jd} = \frac{66290}{14 \times 0.875 \times 32.72} = 166\ \text{psi.}$$

$$\therefore\ v = 166\ \text{psi.} < 225\ \text{psi.}\ \therefore\ \text{acceptable.}$$

Since the concrete is poured monolithically the girder and beam act as a T-beam.

Moment capacity of the girder at balanced design is now computed.

$$kd = 0.375(32.72) = 12.27\ \text{in};\ 12.27\text{-}5.75 = 6.52\ \text{in.}$$

At balanced design, $f_{c_1} = \dfrac{1200 \times 6.52}{12.27} = 638$ psi.

Effective flange width governed by AASHTO, $b = 64$ in.

$$\therefore\ C_{b_1} = 5.75(64) \times \frac{1}{2}\ (1.2 + 0.638) = 338\ \text{Kips.}$$

$$C_{b_2} = 638 \times \frac{1}{2} \times 14 \times 6.52 = 29.12\ \text{Kips.}$$

$$C_b = 338 + 29.12 = 367.12\ \text{Kips}$$

The action line of this resultant force lies at the centroidal axis of the stress trapezoid.

$$Z = \frac{5.75}{3}\ \frac{(1200 + 2 \times 638)}{(1200 + 638)} = 2.58\ \text{in.}$$

$$jd = 32.72 - 2.58 = 30.14$$

$$\therefore\ M_{bal} = 367.12\ (30.14) = 11{,}065\ \text{in-Kips.}$$

The concrete section is therefore slightly excessive and the steel is stressed to capacity.

$$\therefore\ A_s = \frac{10{,}100}{20(30.14)} = 16.76\ \text{in.}^2$$

Use 11 #11 bars, arranged in three rows.

Since the beam is designed under the AASHTO Specification, the nominal bond stress, u, is to be checked at critical locations.

$$u = \frac{V}{\Sigma_o jd}$$, where Σ_o is the sum of bar perimeters at the section.

Checking the bond at the center line of supports, where the shear is greatest and the total bar perimeter is least, four #11 bars provide a perimeter of 17.72 in.

$$\therefore\ u = \frac{V}{\Sigma_o jd} = \frac{66290}{17.72 \times 0.875 \times 32.72} = 130 \text{ psi.}$$

$$\therefore\ u = 130 \text{ psi.} < 300 \text{ psi.} \ \therefore \text{ Acceptable.}$$

Web reinforcement:

The portion of the beam through which web reinforcement is required is determined by computing unit shears at various points on the beam.

The concrete resists a unit shear of $0.03\ f_c' = 90$ psi. and the remainder is taken up by stirrups.

Use #5 stirrups.

AASHTO Specifications call for maximum spacing of 0.5d = 0.5 x 30 = 15"

$$\therefore \text{ Spacing } S = \frac{A_v f_s}{(v-v_c)b_w}$$

$$\therefore\ v-v_c = \frac{A_v f_s}{b_w S}$$

$$v_c = 0.95 \sqrt{f_c'}$$

$$= 0.95 \sqrt{3000}$$

$$= 52 \text{ psi.}$$

$$\therefore A_v = \frac{(v-v_c)b_w S}{f_s}$$

$$A_v = \frac{(90-52)\ 14 \times 15}{20{,}000} = 0.40 \text{ in.}^2$$

$\therefore$ Total shear stress = 90 + 52 = 142 psi.

The stirrups are then placed by calculating from the expression

$$S = \frac{A_v f_s}{(v-v_c)b_w}$$

AASHTO specifies that where maximum stresses are produced in any member by loading any number of traffic lanes simultaneously, the following percentage of the resultant live load stresses shall be used in view of improbable maximum loading:

for one or two lanes - % age to be used - 100

Since the T-beam designed in the above example is for two lanes, the resultant live load stresses need not be reduced.

The HS20-44 truck loadings have been standardized for the provision of overloading, viz., a beam designed for an HS loading has provision for overloads of infrequent type.

The character and distribution of traffic vary with local conditions. But this cannot be a basis for differentiations in specifications of traffic loads, for changes and development of industries around those local conditions. However, a degree of differentiation may be justified with regard to the volume of traffic, which will affect the number of load repetitions, but not the load intensity.

In order to estimate the number of load repetitions as well as the number of consecutive load impulses on highway bridges, a statistical data and evidence concerning volume and distribution of traffic is necessary.

The value of the safety factor, τ, may be derived from the condition that the maximum applied working moment, S_a, induced in the structure by actual service conditions must never cause such damage as to impair its fitness for service, even were this maximum working moment to coincide with the lowest value of the structure's resistance, S_r.

Therefore, $\tau = \frac{S_r}{S_a} > 1$; as in Eq. (3.1).

For the T-beam designed for a highway bridge on page 49 the maximum total working moment computed was 841.8 K-ft., i.e., maximum moment due to dead load, live load and impact = S_a = 841.8 K-ft.

AASHTO (3,4) Specifications for flexure, Art. 1.5.32 (c) of I-&T- sections with tension reinforcement only,

(1) When the compression flange thickness is equal to or greater than the depth of the equal rectangular stress block, a, the design moment strength may be computed by the following equations:

$$S_r = M_u = \phi[A_s f_y d\ (1 - 0.6\ \rho\ \frac{f_y}{f'_c}\ )] \text{ ----------- } (5.1)$$

$$= \phi[A_s f_y\ (d - \frac{a}{2})]$$

where $a = \dfrac{A_s f_y}{0.85 \times f'_c \times b}$

First let us check for condition (1) of Art. 1.5.32 (c).

Compression flange thickness of the T-beam is t = 5.75 in.

$$a = \frac{A_s f_y}{0.85 \times f'_c\ b}$$

$$= \frac{16.76 \times 40{,}000}{0.85 \times 3000 \times 64} = 4.11$$

a = 4.11 in.

Therefore, t = 5.75 in. > a = 4.11 in.

Hence, the design moment strength may be computed by equation (5.1).

a = 4.11 in.

d = 32.72 in.

A_s = 16.76 sq. in.

f_y = 40,000 psi.

ϕ = capacity reduction factor = 0.90 for flexure.

Design Moment Strength, $M_u = \phi[A_s f_y\ (d - \frac{a}{2})]$

$$M_u = 0.90\ [16.76 \times 40{,}000\ (32.72 - \frac{4.11}{2})]$$

M_u = 1713.15 K-ft.

Therefore, M_u = 1713.15 K-ft. is the ultimate resistance of the structure and will fail when subjected to a working moment equal to its value.

Factor of safety, $\tau = \frac{S_r}{S_q} > 1.$

$S_r = M_u = 1713.15$ K-ft.

S_a = total working moment due to bending = 841.8 K-ft.

Therefore, $\tau = \frac{1713.15}{841.8} = 2.04 > 1.$

Hence, the structure (highway bridge) is safe from failure due to actual service conditions.

SUMMARY AND CONCLUSIONS

The safety factor is an objective value of correlation between applied load and potential strength that can be ascertained for all practical purposes of design. Loads, external conditions, physical properties of materials have to be carefully analyzed in relation to the magnitude of the safety factor. This factor of safety is intended to provide for relatively infrequent loading, and other conditions whereas the design itself considers the conditions of relatively frequent occurrence. The computation of the minimum safety factor and the consequent permissible stresses are directly responsible for a balanced design.

In the analysis of a structure, the evaluation of loads, as well as of the probability of failure or unserviceability is as important as the analysis of the stresses. The information required for the analysis, producing a rational value for the safety of the structure related to acceptable probabilities of failure, are the distribution functions of the applied loads leading to structural fatigue and unserviceability. The effect of the rate of applied loads and the capacity of the resisting mechanism to sustain occasional overloads has to be carefully considered and should not be neglected.

Probability theory formalizes operations involving chance elements. It does not allow the making of forecasts with certainty--even of chance results. There is a dearth of proper statistical data for the profession of Civil Engineering to move in the direction of design procedures based on probabilistic and statistical concepts. Once this hurdle is crossed, the design approach based on probabilistic methods will greatly enhance the future of structural engineering in terms of safety, serviceability and economy of structures.

With increasing perfection of design methods the element of "ignorance" can be largely eliminated; but the element of "uncertainty" is caused by circumstances that can be altered to a certain extent, but cannot be removed. Hence, a rational "safety factor" can minimize the measure of uncertainty.

The scope of research carried out to compile the necessary statistical data for the application of design procedures based on probabilistic methods has been tragically limited. It has been seen by the writer in his review of a large number of papers published by well-known authors on probabilistic and statistical methods for design of structures, that there have been only a few tests conducted for the purposes of obtaining data, and the data from these tests have been the basis for the majority of the papers. It would greatly increase the validity of the existing literature if more tests were conducted for an elaborate compilation of data, thus justifying the basis for design procedures to be based on probabilistic methods.

In private conversation with Dr. Swartz, it has been pointed out that the ACI Code is thinking of implementing the concept of probabilistic and statistical methods for design of reinforced concrete members in its new code.

Although a great deal of money is involved, there is a need for greater research and more statistical data to be compiled, for the implementation of probabilistic methods for design purposes. The significance of a general code format based on concepts of probability is on the threshold at this juncture.

LIST OF FIGURES

APPENDIX I - REFERENCES

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APPENDIX II - NOTATION

a = depth of equivalent rectangular stress block

a_K = sample coefficient of skewness

a' = ordinate point on x-axis of a normal curve

A_s = area of tension reinforcement

A_t = area of transverse reinforcement

b = width of compression face of member

b' = ordinate point on x-axis of a normal curve

b_w = web width in I-and T-sections

c = distance of extreme compression fiber to neutral axis

d = distance of extreme compression fiber to centroid of tension reinforcement

e = eccentricity of design load parallel to axis measured from the centroid of the section

f_c = extreme fiber stress (compressive) in concrete at service loads

f'_c = specified compressive strength of concrete, psi.

f_s = tensile stress in reinforcement at service loads, psi.

$f(x)$ = probability function of x

f_y = specified yield strength of steel, psi.

$F(x)$ = probability distribution function of x

jd = distance between action lines of C and T forces, inches

k = effective length factor for compression members

K = total number of arrivals

K_b = constant of proportionality

ℓ' = number of lanes in a highway bridge

M = computed moment capacity

M_b = moment capacity at balanced conditions

M_u = ultimate design moment strength of a section

m = third moment about the mean

n = number of arrivals of vehicles

N = sample size

p = probability value

$P[\cdot]$ = measure of probability

P_F = probability of failure

P_p = mathematical probability of occurrence of a particular load

P_q = the probability that P_p will not occur

P_R = resultant of live loads

R = resistance or strength of the structural member

R' = rate of interval of vehicles per second

S = shear reinforcement spacing

S = span length

S_a = maximum range of fluctuations of applied stress

S_{aw} = expected value of S_a

S_r = maximum range of fluctuation of structure's resistance

S_{rw} = expected value of S_r

t = time period

T = time interval of vehicles

u = nominal bond stress

v = unit shear stress of concrete

v_u = nominal ultimate shear stress for beams with web reinforcement

V = total applied design shear force at section

V' = vehicle interval

V^2 = sample variance

V_x = sample standard deviation

W = mean load effect for which the structure is subjected

x = variable associated with probabilities

$\bar{x}$ = mean of the sample

Y = normal or Gaussian probability distribution function

z = distance from extreme compression fiber to action of lime C-force

Z = standard normal distribution parameter $\frac{x-\mu}{\sigma}$

μ = mathematical expectation of a random variable x about the first moment = E(x)

σ = standard deviation (population)

σ^2 = variance (population)

$\phi(Z)$ = function of standard normal distribution curve

Γ = gamma distribution function

τ = factor of safety

ρ = steel ratio

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STATISTICAL AND PROBABILISTIC METHODS FOR DESIGN OF REINFORCED CONCRETE STRUCTURES

by

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ABSTRACT

The objective of this report is to discuss the factor of safety based on simple statistical and probabilistic analysis for the design of reinforced concrete structures.

The factor of safety underlines the principle of computed structural loads that cannot be equated to the measurable physical strength properties of materials implying the existence of a margin between the two. The factors of safety can only be obtained from a complete structural analysis based on the expected variations of loadings and of strength properties of the structural materials, together with the economic aspects involved.

The report presents a review of the early development of the probabilistic methods and the analyses of loadings and strength properties. The consequences of failure or unserviceability and economic aspects of building structures based on the optimum value for the factor of safety is discussed later on. A design example of a T-beam highway bridge is worked out and correlated to the existing factor of safety for its compatibility.

Consequently, with the future trend of building codes based on probabilistic methods, the main emphasis is laid upon the rational insight obtained into the true conditions of safety in structural engineering.