CONCEPTUAL DESIGN OF A COMMERCIAL-
TOKAMAK-HYBRID-REACTOR FUELING SYSTEM

by

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SUMMARY

Possible fueling concepts were examined to determine which would provide the highest probability of success for application in the CTHR (Commercial Tokamak Hybrid Reactor). Cold fueling was chosen with this idea in mind. Thereafter, the particular scheme chosen was frozen-pellet fueling, since it provides the advantage of maximum fuel-particle density. Several methods of frozen-pellet injection were studied to determine their capabilities, their experimental verification, and projections for the future. Any characteristics which might adversely affect the normal operation of the tokamak were taken into account. To this effect, the light-gas gun was tentatively chosen to provide pellet acceleration.

The ORNL Neutral Gas Shielding Model is the basic theory used to determine the required pellet velocity. It has been modified, however, to account for operation in the commercial temperature regime (as opposed to experimental devices with temperatures around 1 keV). The required pellet velocity is a function of the depth at which the pellet has disappeared as a solid entity.

From this, the pressure level and other essential estimates to be made on the fuel injector design have been made. This leaves only the design of a fuel handling system to implement operation. The fuel handling system has been designed so that a sufficient fuel-pellet supply is produced and quality control systems may be integrated into the system at a later date.

Various ways were studied to locate the fuel injection system so as to minimize the total-system perturbation. The suggested design incorporating the fuel injector into CTHR accounts for both this aspect and that of providing the shortest possible path to the plasma center.

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NOMENCLATURE AND NOTATION

a ≡ Distance from the outer edge of the plasma to the pressure center (in m). Note that this quantity is taken as 0.9 m for CTHR.

c_p ≡ Specific heat (constant pressure) of ablatant.

c_s ≡ Sound speed (in m/s).

e ≡ Charge of the electron (in C) $\doteq 1.6021892 \times 10^{-19}$.

h ≡ Ablatant enthalpy variable.

(ℓ/a) ≡ Fraction of pressure center of tokamak to which pellet is to achieve penetration; i.e., if $(\ell/a) = 1$, pellet penetrates to the pressure center of the reactor.

m ≡ Mass of fuel molecule (in kg).

m_e ≡ Mass of electron (in kg) $\doteq 9.109537 \times 10^{-31}$.

m_{s+p} ≡ Sum of the mass of a pellet injector shell and the mass of the fuel pellet (in kg). In the present design, this is approximately 1.947×10^{-3} kg.

m_D ≡ Mass of deuteron (in kg) $\doteq 3.344549 \times 10^{-27}$.

m_H ≡ Mass of proton (in kg) $\doteq 1.673560 \times 10^{-27}$.

n_∞ ≡ Number density of electrons in the tokamak (in m^{-3}). Note: This quantity is position dependent, as in Eqn. 52b.

$\langle n_\infty \rangle$ ≡ Average electron number density in fusion plasma (in m^{-3}) = 9.8×10^{19}.

p ≡ Ablatant enthalpy variable.

$\dot{q}$ ≡ Net heat transfer rate to the ablatant, by electron collisions.

r ≡ Position variable in fusion plasma, i.e., (r/a) is dimensionless.

$\hat{r}$ ≡ Ratio of distance from pellet center to pellet radius.

r_p ≡ Initial pellet radius (in m) = 3.073×10^{-3}.

r_o ≡ Radius of pellet as a function of time (in m).

t ≡ Time variable. Used in context of dr_o/dt.

u ≡ Required pellet velocity to achieve penetration as a solid entity to (ℓ/a), (in m/sec).

u_{max} ≡ Maximum attainable velocity of light-gas-gun-projectile (in m/sec).

v ≡ Ablatant velocity variable.

v_o ≡ Speed of the ablatant (in m/sec). For the purposes of the determination of the value of ξ (see Eqn. 20), this quantity is taken to be 400 m/sec.

A_s ≡ Surface area of the pellet injector shell exposed to the propellent gas (in m^2), (numerically, $\pi/4\ (0.03)^2\ m^2$).

E_o ≡ Thermal energy (in eV) of electrons at pellet surface. Note: This quantity has been taken as 10 eV in Eqn. 51.

E_∞ ≡ Thermal energy (in eV) of electrons in fusion plasma. Note: This quantity is position dependent as illustrated in Eqn. 52a.

J ≡ Electron current, further defined in Eqn. 16.

L ≡ Distance over which the fuel injector shell and pellet are accelerated (in m), taken here to be 0.30 m.

$L(E)$ ≡ Electron energy loss function, approximated by Milora and Foster[1] as shown in Eqn. 51, (in eV m^2), where E is the electron energy (in eV).

$\langle L(E)\rangle$ ≡ Average value of L(E) in the ablatant.

M ≡ Mach number of the ablatant.

R ≡ Ideal gas constant (in J/(kg K)).

T ≡ Ablatant temperature.

$\langle T_\infty \rangle$ ≡ Average electron temperature of fusion plasma (in eV) = 1.3×10^4.

γ ≡ Ratio of specific heats.

μ_s ≡ Ratio of mass of fuel molecule to the mass of a proton.

ρ ≡ Ablatant density variable, function of r.

$\hat{\rho}$ ≡ Ratio of the mass density of the ablatant to the mass density of the ablatant at the pellet surface.

ρ_s ≡ Mass density of fuel pellet (in kg/m^3).

ρ_o ≡ Mass density of ablatant as a function of time (in kg/m^3).

ρ_D ≡ Mass density of solid deuterium (in kg/m^3) $\doteq 2.059 \times 10^2$.

NOTE: All units are assumed to be MKS unless otherwise stated.

1.0 Introduction

Westinghouse Fusion Power Systems Department is developing a design for a Commercial Tokamak Hybrid Reactor, referred to henceforth as CTHR. In the course of the design of the reactor, it became apparent that fueling system design must be given considerable attention. This is due to the fact that the fuel must cross magnetic-field lines in order to effect penetration of the reacting plasma body. In spite of this, the subject of refueling mechanisms is new to the tokamak-reactor conceptual design. The idea of crossing magnetic field lines leads, basically to two concepts of fueling. These are hot fueling, wherein the fuel consists of either ionized particles or neutral particles with a high thermal energy, and cold fueling, wherein the fuel consists of particles with very small thermal energy.

In the hot fueling concept, the particle temperatures are generally above 100 eV. Some examples of this concept are cluster beams, neutral beams, and plasma guns. The primary disadvantage to using hot ionized-particle fueling lies in the requirement that the refueling "mixture" must have at least as high a thermal energy as the reacting body; clearly, in order to allow electrically charged fuel particles to enter and penetrate the confining magnetic field, their average energy must exceed the energy which the magnetic field was designed to contain. The primary disadvantage to employing hot neutral-particle fueling is due to the rather small probability for interaction with the plasma; obviously, a wastefully excessive amount of fuel would have to be injected to maintain the necessary plasma density. There requirements also cause a secondary disadvantage. The inherent inefficiency of heating the fuel by an electromechanical device includes both the inefficiency of the device and the

Carnot/Rankine inefficiencies accompanying the electrical-power generation upon which the device is operated. Thus, it is probable that cold fueling, if feasible, will be a more efficient and a safer process.

There are, basically, three different schemes for implementation of a cold-fueling concept. These are fueling by frozen D+T pellets, squirting a jet of liquid D+T, and puffing gaseous D+T from the surface of the reactor. All three have in common the fact that the fuel particles are unaffected by the magnetic field since they are electrically neutral. The requirement for refueling here is that the fuel must penetrate the pressure "wave" of the reacting body. Obviously, the greater the density of the fuel, the greater is the probability of penetration, provided that the fuel-injection velocity is held constant. (Remember that, upon injection into the reacting body, the fuel will tend to become very quickly ionized by the surrounding plasma.) This, effectively, justifies the concentration of the study on frozen-pellet fueling while, at least temporarily, eliminating further consideration of the liquid-jet and surface-gas schemes.

There have been many methods suggested by which pellet injection may take place and, presently, some of these will be discussed briefly. However, a far more pressing question arises to the surface. How fast, i.e., with what velocity, must a pellet be injected in order to achieve penetration of the reacting body and thereby refuel the tokamak? The model used in this paper to describe the behavior of the pellet is the ORNL Neutral Gas Shielding Model of Foster and Milora.[1] The model has been modified, here, to fit more closely the realities of the CTHR temperature regime of operation. Explanation of the modification will follow in a later section.

The first method for consideration is that of linear-resonance acceleration. In this method,[2] a pellet of frozen D+T receives a metallic coating and is, subsequently, electronically charged. It is then accelerated through an electric field to achieve terminal velocity. There are several disadvantages to this system, pertaining to use in a fusion-power facility, as the following illustrates:

1. The metallic coating, received by the pellet initially, enhances bremsstrahlung losses, which, in turn, increases the value of the critical $n\tau$ required for fusion breakeven quite dramatically; for example, a 10% increase in bremsstrahlung would be realized from a pellet containing only 0.01% of iron.

2. Very high accelerating potentials are projected for use in such a reactor (~ 100 MV). To assure that there is no electric-field breakdown, such a device is expected to require a length of around 100 m.[2] The drift tube could be expected to be enormous.

In spite of these disadvantages, however, velocity achievable by this type of system is predicted to be as high as 10^4 m/sec, although no experimental data are available to confirm the theoretical projection.

Another method for consideration is that of freezing a jet of liquid into a rod which is subsequently shattered into pellets.[3] The pellets are then allowed to drift towards the target plasma without further acceleration. This method has been tested experimentally to speeds of 100 m/sec and is expected to deliver an ultimate pellet velocity of 2000 m/sec.[3]

A third possibility for accelerating the pellet lies in striking it with a laser beam on one side.[4] The resulting ablatant mass causes the pellet to recoil and to be accelerated into the tokamak with an estimated pellet velocity of 10^4 m/sec. Unfortunately, no experiments have been made to test this idea.[4]

A fourth possibility is to accelerate the pellet mechanically through the use of rotating arbor.[5,6] The primary disadvantage of this method lies in the hazard to the integrity of the vacuum caused by the rapidly moving arbor. This would necessitate a protective shield for the vacuum vessel. Terminal velocity is anticipated to exceed 1000 m/sec.[5]

The final method to be discussed here is that of using a light-gas gun to accelerate the pellet.[6,7,8] In this method, a high-pressure propellent gas strikes a disk and accelerates disk and pellet towards the reacting body. The experimental velocity achieved thus far by this method is ~330 m/sec,[7] which is higher than that for any other method studied. This method has been projected to achieve velocities of 6000 m/sec.[8]

As this is a conceptual-design paper for a possible fueling system of CTHR, it is requisite that a choice be made of methods by which the scheme of pellet-injection may be achieved. On the basis of inherent advantages and disadvantages (including lack of experimental evidence) and projected capabilities, a choice has been made for study in this paper. The system selected and recommended for use in the CTHR design is that of the light-gas gun for pellet injection.

2.0 Fuel Scheme Modeling

Several models were considered for the purpose of modeling the pellet ablation process as it pertains to CTHR.[1, 11-19, 21] These models were studied to determine the most appropriate one for determination of the pellet-injection velocity requirement. Since the ORNL Neutral Gas Shield Model[1] has a strong theoretical base of physical principles, e.g., conservation of mass, momentum, and energy laws; and since it has a record of adequacy in the lower temperature regimes (1 keV) for which it was formulated, this model was chosen to form the basis of the theory by which the ablation process of a pellet in CTHR is described. Due to the nature of the model, considerable reconstruction was required before it could be applied to the problem at hand. The ORNL Neutral Gas Shield Model was derived to form order of magnitude estimates based on discrete points at which the pellet would disappear as a solid entity. The requirement of this study is to examine the continuum of points at which the pellet will disappear for a particular reactor, namely, the Commercial Tokamak Hybrid Reactor of Westinghouse. By this, a conceptual design of a fuel injection system for CTHR may be formulated.

2.1 Laws of Conservation of Mass, Momentum, and Energy

The ORNL Neutral Gas Shield Model is based upon the laws of conservation, as mentioned previously. These may be expressed as:

$$\frac{d}{dr}\left\{ \rho v r^2 \right\} = 0; \tag{1a}$$

$$\frac{d}{dr}\left\{ \rho v^2 r^2 \right\} + r^2 \frac{dp}{dr} = 0; \tag{1b}$$

$$\left\{ \frac{1}{r} \right\}^2 \frac{d}{dr}\left\{ \rho v r^2 \left[h + \frac{v^2}{2} \right] \right\} = \dot{q}. \tag{1c}$$

These represent conservation of mass, conservation of momentum, and conservation of energy, respectively. Here, spherical geometry has been assumed and thus, the mass flow rate, w may be represented by $4\pi\rho v r^2$. First, let us manipulate equation (1a).

2.1.1 Equation of Conservation of Mass

The solution to the mass conservation equation is almost trivial, but will prove useful in the development of the model. Therefore, we give it full consideration here. The solution may be expressed in one of two ways, due to the solid-gas interface. Thus, the solution becomes, at the boundary (see Figure 1):

$$\rho v r^2 = \rho_o v_o r_o^{\ 2} .$$

But, due to the solid-gaseous interface, this may also be written as:

$$\rho v r^2 = \rho_s r_o^{\ 2} \left| \frac{dr_o}{dt} \right| .$$

Hence, one has the following relationship:

$$\hat{\rho}\hat{r}^2 v = \left(\frac{\rho_s}{\rho_o} \right) \left| \frac{dr_o}{dt} \right| = v_o . \tag{2}$$

Now, we are ready to transform the energy equation into something more easily solvable (although numerical techniques are still required).

2.1.2 Conservation of Energy

The energy equation was previously described by:

$$\left(\frac{1}{r} \right)^2 \frac{d}{dr} \left\{ \rho v r^2 \left[h + \frac{v^2}{2} \right] \right\} = \dot{q} .$$

Making use of the chain rule, we find:

$$\rho v \frac{d}{dr} \left[h + \frac{v^2}{2} \right] \quad + \quad \frac{\left[h + \frac{v^2}{2} \right]}{r^2} \frac{d}{dr} \left[\rho v r^2 \right] \quad = \quad \dot{q} .$$

However, the last term on the left-hand side of the equation is zero since mass is conserved (see equation (1a)) and thus, the energy equation is given by the following:

$$\rho v \frac{d}{dr} \left[h + \frac{v^2}{2} \right] \quad = \quad \dot{q} .$$

If we divide both sides of the above equation by the quantity, $\frac{\rho v}{dr}$, the result is that:

$$d\left[h + \frac{v^2}{2} \right] \quad = \quad \left(\frac{\dot{q}}{\rho v} \right) dr .$$

Or expressing the above equality in a slightly different manner:

$$d\left[h + \frac{v^2}{2} \right] \quad = \quad \left(\frac{\dot{q} r_o}{\rho v} \right) d\hat{r} . \tag{3}$$

Shapiro[20] gives as a general solution to the energy equation, the following, adopting Shapiro's notation temporarily (except for the ratio of specific heats):

$$\frac{dM^2}{M^2} = - \frac{2\left[1 + \frac{\gamma - 1}{2} M^2 \right]}{(1 - M^2)} \left(\frac{dA}{A} \right) + \frac{1 + \gamma M^2}{(1 - M^2)} \frac{\{ dQ - dW_x + dH \}}{c_p T}$$

$$+ \frac{\gamma M^2 \left[1 + \frac{\gamma - 1}{2} M^2 \right]}{(1 - M^2)} \left(4f \frac{dx}{D} + \frac{2dx}{\gamma p A M^2} - 2y \frac{dw}{w} \right)$$

$$+ \frac{2(1 + \gamma M^2)(1 + \frac{\gamma - 1}{2} M^2)}{(1 - M^2)} \left(\frac{dw}{w} \right) - \frac{1 + \gamma M^2}{(1 - M^2)} \left(\frac{dW}{W} \right) - \left(\frac{d\gamma}{\gamma} \right) . \tag{4}$$

The sixth bracketed term is due to a possible change in the ratio of specific heats, γ. The fifth term in brackets accounts for a change in the molecular weight, W. Both of these are zero since we neglect both disassociation of the molecule and ionization of the gas in the shield. Since we are dealing with a diatomic ideal gas, in the shield, $\gamma = 7/5$. The fourth term in brackets is zero by virtue of mass conservation. The third bracketed term accounts for frictional losses. These are negligible in the shield itself. Hence, only the first two terms are left to consider. The second term in brackets may be interpreted as[20]:

$$dQ - dW_x + dH = c_p\, dT + d(v^2/2). \qquad (5)$$

But, the differential specific enthalpy of an ideal gas is simply:

$$dh = c_p\, dT.$$

Thus equation (5) may be rewritten as:

$$dQ - dW_x + dH = d\left(h + \frac{v^2}{2} \right). \qquad (6)$$

By substituting the above relationship into equation (4), we obtain:

$$\frac{dM^2}{M^2} = - \frac{[\, 2 + (\gamma - 1)M^2 \,]}{(1 - M^2)} \left\{ \frac{dA}{A} \right\} + \frac{1 + \gamma M^2}{1 - M^2} \left[\frac{d\left(h + \frac{v^2}{2} \right)}{c_p T} \right]. \qquad (7)$$

Now let us evaluate the term, dA/A. The surface area, A, of a sphere is given by:

$$A = 4\pi r^2.$$

Upon differentiating the above equation, it is discovered:

$$dA = 8\pi r\, dr.$$

By combining the above two results, one finds that:

$$\frac{dA}{A} = \frac{8\pi r\, dr}{4\pi r^2} = 2\frac{dr}{r} = 2\frac{d\hat{r}}{\hat{r}}. \tag{8}$$

If equations (3) and (8) are substituted into equation (7), the energy equation becomes:

$$\frac{dM^2}{M^2} = -\frac{[\,2 + (\gamma - 1)M^2\,]}{(1 - M^2)}\left(\frac{2\, d\hat{r}}{\hat{r}}\right) + \frac{1 + \gamma M^2}{1 - M^2}\left(\frac{\dot{q} r_o\, d\hat{r}}{\rho v c_p T}\right). \tag{9}$$

At this point, consider an ideal gas for which the following holds[20]:

$$c_s^{\,2} = \left.\partial p/\partial \rho\,\right|_s = \gamma R\hat{T}\ .$$

Then, we have the following relationships:

$$M^2 \equiv v^2/c_s^{\,2} = v^2/(\gamma RT); \tag{10a}$$

$$c_p \equiv \frac{\gamma}{\gamma - 1} R. \tag{10b}$$

Dividing (10a) by $v^2/(\gamma-1)$, we find that:

$$(\gamma - 1)\frac{M^2}{v^2} = \frac{1}{\frac{\gamma}{\gamma - 1} RT}\ . \tag{11}$$

But, the term in the denominator of the right-hand side of equation (11) is given by equation (10b) as simply, $c_p T$. Thus, it is seen that:

$$\frac{1}{c_p T} = (\gamma - 1)\frac{M^2}{v^2}\ . \tag{12}$$

Substitution of this relationship into equation (9) yields:

$$\frac{dM^2}{M^2} = -\frac{[\,2 + (\gamma - 1)M^2\,]}{(1 - M^2)}\left(\frac{2\, d\hat{r}}{\hat{r}}\right) + \frac{1 + \gamma M^2}{1 - M^2}\left(\frac{\dot{q}}{\rho} r_o \frac{1}{v^3}(\gamma - 1)M^2\, d\hat{r}\right). \tag{13}$$

Upon rearranging equation (2), one obtains:

$$\frac{1}{v} = \beta \hat{r}^2 \left(\frac{\rho_o}{\rho_s} \right) \left| \frac{dr_o}{dt} \right|^{-1} .$$

and thus, equation (13) becomes:

$$\frac{dM^2}{M^2} = - \frac{[\, 2 + (\gamma - 1)M^2 \,]}{(1 - M^2)} \left(\frac{2\, d\hat{r}}{\hat{r}} \right) + \frac{1 + \gamma M^2}{1 - M^2} \left\{ \left(\frac{\dot{q}}{\rho} \right) r_o \, (\beta \hat{r}^2)^3 \left| \frac{dr_o}{dt} \right|^{-3} \times (\gamma - 1)M^2 \, d\hat{r} \right\} . \quad (14)$$

Using the chain rule on the right-hand side, and multiplying through by the quantity $M(1-M^2)/(2\, d\hat{r})$, one discovers, upon rearranging terms:

$$(1 - M^2) \frac{dM}{d\hat{r}} = \frac{(\gamma - 1)}{2} \left(\frac{\rho_o}{\rho_s} \right)^3 \left(\frac{\dot{q}}{\rho} \right) r_o \left| \frac{dr_o}{dt} \right|^{-3} (\beta \hat{r}^2)^3 M^3 (1 + \gamma M^2) - \frac{2M}{\hat{r}} \left\{ 1 + \frac{\gamma - 1}{2} M^2 \right\} . \quad (15)$$

At this time, it is necessary to define further the heat deposition rate, $\dot{q}$. The ORNL Neutral Gas Shield Model assumes that the primary mechanism by which heat is deposited in the shield is electron collision. This is given by[1]:

$$\dot{q} = eJ \frac{dE}{dr} = eJ \frac{\rho}{m} L(E), \text{ where } J \equiv n_\infty \sqrt{\frac{e\, E_\infty}{3\pi\, m_e}} . \quad (16)*$$

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*L(E) may be approximated by[1]:

$$L(E) \doteq (2.35 \times 10^{18} + 4 \times 10^{15}\, E + 2 \times 10^{21}/E^2)^{-1} .$$

Thus equation (15) becomes:

$$(1 - M^2) \frac{dM}{d\hat{r}} = \frac{\gamma - 1}{2} \left(\frac{\rho_o}{\rho_s}\right)^3 \left(\frac{e\, L(E)}{m} J\right) r_o \left| \frac{dr_o}{dt} \right|^{-3} (\beta \hat{r}^2)^3 M^3 (1 + \gamma M^2)$$

$$- \frac{2M}{\hat{r}} \left\{ 1 + \frac{\gamma - 1}{2} M^2 \right\}. \tag{17}$$

At this point it becomes useful to define a dimensionless parameter, ξ:

$$\xi \equiv \frac{\gamma - 1}{2} \left(\frac{\rho_o}{\rho_s}\right)^3 \left\{ \frac{e\, L(E)}{m} J \right\} r_o \left| \frac{dr_o}{dt} \right|^{-3}. \tag{18}$$

Hence, equation (17) becomes, upon substitution of the definition of ξ:

$$(1 - M^2) \frac{dM}{d\hat{r}} = \xi\, (\beta \hat{r}^2)^3 M^3 (1 + \gamma M^2) - \frac{2M}{\hat{r}} \left\{ 1 + \frac{\gamma - 1}{2} M^2 \right\}. \tag{19a}$$

Or, by performing an algebraic manipulation, one finds:

$$\frac{dM}{d\hat{r}} = \frac{M}{1 - M^2} \left\{ \xi\, (\beta \hat{r}^2)^3 M^2 (1 + \gamma M^2) - \frac{[2 + (\gamma - 1) M^2]}{\hat{r}} \right\}. \tag{19b}$$

As it turns out, equation (1b) will yield a differential equation in terms of β and M as a function of $\hat{r}$; so, momentarily at least, let us turn our attention to the definition of ξ since we shall have two equations of β and M as a function of $\hat{r}$ (and thus, though the solution must be numerical, we can solve for both variables).

Recalling equation (2), we have the relationship:

$$v_o = \left(\frac{\rho_s}{\rho_o}\right) \left|\frac{dr_o}{dt}\right|.$$

If this result is further applied to equation (18), one discovers:

$$\xi = \frac{\gamma - 1}{2} \frac{e\,L(E)}{m} J \frac{r_o}{v_o^{3}} . \tag{20}$$

The practical application of the ORNL Neutral Gas Shield Model demands that ξ be given a numerical value. The resultant set of coupled differential equations is then solved. Thus r_o is taken to be the initial pellet radius, r_p; v_o is taken to be the sound speed of a low temperature hydrogen gas (about 400 m/sec); and J is taken to be the average electron current of the plasma.

2.1.3 Equation of Conservation of Momentum

The momentum equation is recalled to be:

$$\frac{d}{dr}\left\{ \rho v^2 r^2 \right\} + r^2 \frac{dp}{dr} = 0 .$$

Using the ideal gas law, one finds:

$$\frac{d}{dr}\left\{ \rho v^2 r^2 \right\} + r^2 \frac{d(\ \rho RT\ )}{dr} = 0.$$

Dividing the above equation by $\rho_o r_o$ yields:

$$\frac{d}{d\hat{r}}\left\{ \hat{\rho} \hat{r}^2 v^2 \right\} + \hat{r}^2 \frac{d(\ \hat{\rho} RT\ )}{d\hat{r}} = 0. \tag{21}$$

Now, recall the definition of the Mach Number, equation (10a):

$$M^2 \equiv \frac{v^2}{\gamma RT} .$$

By algebraically rearranging the above equation, one obtains:

$$RT = \frac{v^2}{\gamma M^2} .$$

Upon substitution of the above relationship into equation (21), one finds:

$$\frac{d}{d\hat{r}}\left\{\beta\hat{r}^2v^2\right\} + \hat{r}^2\frac{d}{d\hat{r}}\left\{\frac{\beta v^2}{\gamma M^2}\right\} = 0. \tag{22}$$

Equation (2) may be rearranged to provide the following relationship:

$$v = \frac{v_o}{\beta\hat{r}^2}. \tag{23}$$

The substitution of equation (23) into equation (22) produces the following:

$$\frac{d}{d\hat{r}}\left\{\frac{{v_o}^2}{\beta\hat{r}^2}\right\} + \hat{r}^2\frac{d}{d\hat{r}}\left\{\frac{{v_o}^2}{\gamma M^2\beta\hat{r}^4}\right\} = 0.$$

If the above equation is divided through by ${v_o}^2$, one has:

$$\frac{d}{d\hat{r}}\left\{\frac{1}{\beta\hat{r}^2}\right\} + \frac{\hat{r}^2}{\gamma}\frac{d}{d\hat{r}}\left\{\frac{1}{M^2\beta\hat{r}^4}\right\} = 0. \tag{24}$$

The chain rule may be applied to equation (24) to yield:

$$\frac{d}{d\hat{r}}\left\{\frac{1}{\beta\hat{r}^2}\right\} + \frac{\hat{r}^2}{\gamma}\left[\frac{1}{M^2\hat{r}^2}\frac{d}{d\hat{r}}\left\{\frac{1}{\beta\hat{r}^2}\right\} + \frac{1}{\beta\hat{r}^2}\frac{d}{d\hat{r}}\left\{\frac{1}{M^2\hat{r}^2}\right\}\right] = 0. \tag{24b}$$

Upon rearranging, one finds:

$$\left\{1 + \frac{1}{\gamma M^2}\right\}\frac{d}{d\hat{r}}\left\{\frac{1}{\beta\hat{r}^2}\right\} + \frac{1}{\gamma\beta}\frac{d}{d\hat{r}}\left\{\frac{1}{M^2\hat{r}^2}\right\} = 0.$$

Again using the chain rule yields:

$$\frac{d}{d\hat{r}}\left(\frac{1}{\hat{\rho}\hat{r}^2}\right)\left[\frac{\gamma M^2+1}{\gamma M^2}\right] + \frac{1}{\gamma\hat{\rho}M^2}\frac{d}{d\hat{r}}\left(\frac{1}{\hat{r}^2}\right) + \frac{1}{\gamma\hat{\rho}\hat{r}^2}\frac{d}{d\hat{r}}\left(\frac{1}{M^2}\right) = 0.$$

This may be reexpressed as follows by differentiating and then multiplying the equation by $-\gamma M^2\hat{\rho}\hat{r}^2$:

$$\frac{1}{(\hat{\rho}\hat{r}^2)}\frac{d}{d\hat{r}}(\hat{\rho}\hat{r}^2)\left[1+\gamma M^2\right] + \frac{2}{\hat{r}} + \frac{2}{M}\frac{dM}{d\hat{r}} = 0.$$

The following results from using the chain rule of differentiation and combining terms:

$$\frac{d\hat{\rho}}{d\hat{r}}\left[\frac{1+\gamma M^2}{\hat{\rho}}\right] + \frac{2(2+\gamma M^2)}{\hat{r}} + \frac{2}{M}\frac{dM}{d\hat{r}} = 0.$$

If equation (19b) is substituted into the above relationship, we have:

$$\frac{d\hat{\rho}}{d\hat{r}}\left[\frac{1+\gamma M^2}{\hat{\rho}}\right] = -\frac{2(2+\gamma M^2)}{\hat{r}} - \frac{2}{M}\left[\frac{M}{1-M^2}\left\{\xi(\hat{\rho}\hat{r}^2)^3 M^2(\gamma M^2+1) - \frac{2+(\gamma-1)M^2}{\hat{r}}\right\}\right].$$

Upon rearranging terms further, the following is found:

$$\left(\frac{1+\gamma M^2}{\hat{\rho}}\right)\frac{d\hat{\rho}}{d\hat{r}} = -\frac{2}{1-M^2}\left[\xi(\hat{\rho}\hat{r}^2)^3 M^2(1+\gamma M^2) - \frac{2}{\hat{r}} + \frac{2}{\hat{r}}(1-M^2) - \frac{(\gamma-1)M^2}{\hat{r}} + \frac{\gamma M^2}{\hat{r}}(1-M^2)\right].$$

This may be reexpressed in the following form by combining terms:

$$\left(\frac{1+\gamma M^2}{\hat{\rho}}\right)\frac{d\hat{\rho}}{d\hat{r}} = -\frac{2M^2}{1-M^2}\left[\xi(\hat{\rho}\hat{r}^2)^3(1+\gamma M^2) - \frac{2}{\hat{r}} + \frac{1}{\hat{r}} - \frac{\gamma M^2}{\hat{r}}\right].$$

Combining terms again in the above equation yields:

$$\left(\frac{1+\gamma M^2}{\hat{\rho}}\right)\frac{d\hat{\rho}}{d\hat{r}} = -\frac{2M^2}{1-M^2}\left[\xi(\hat{\rho}\hat{r}^2)^3(1+\gamma M^2) - \frac{(\gamma M^2+1)}{\hat{r}}\right].$$

Finally, we have:

$$\frac{d\hat{\beta}}{d\hat{r}} = -\frac{2M^2\hat{\beta}}{1 - M^2}\left\{\xi\,(\hat{\beta}\hat{r}^2)^3 - \frac{1}{\hat{r}}\right\}. \tag{25}$$

2.2 Boundary Conditions of the Model

First, let us recall the coupled differential equations which must be solved in order to evaluate the pellet velocity requirement. These are equations (19b) and (25).

$$\frac{dM}{d\hat{r}} = \frac{M}{1 - M^2}\left[\xi\,(\hat{\beta}\hat{r}^2)^3\,M^2\,(\gamma M^2 + 1) - \frac{[2 + (\gamma - 1)M^2]}{\hat{r}}\right]; \tag{19b}$$

$$\frac{d\hat{\beta}}{d\hat{r}} = -\frac{2M^2\hat{\beta}}{1 - M^2}\left\{\xi\,(\hat{\beta}\hat{r}^2)^3 - \frac{1}{\hat{r}}\right\}. \tag{25}$$

The difficulty in solving the above set of differential equations stems from the fact that the known boundary conditions are insufficient. Specifically, these boundary conditions are:

(i) $\hat{\beta}(\hat{r} = 1) = 1$,

(ii) $\lim_{\hat{r}\to\infty} \frac{dM}{d\hat{r}} = 0$.

It can be shown that through the use of equations (19b) and (24b) that the second boundary condition is equivalent to the following:

$$\lim_{\hat{r}\to\infty} M = \sqrt{(5/\gamma)}.$$

Equation (24b) yields a power law solution for $\hat{\beta}$ at large values of $\hat{r}$ but, due to the nature of the equations, integrating from large values of $\hat{r}$ to the pellet surface, numerically, is unsuccessful in attaining the first boundary condition. Thus, there are only two possibilities left. The first is to guess the boundary value. While this possibility works in principle, it can become very expensive in terms of computer time usage and thus, the second possibility was explored and found to be fruitful. This possibility was to attempt to provide a solution for $M(\hat{r} = 1) = M_o$ and will now be explained here, in detail.

First, however, let us examine the defining equations in more detail to determine further boundary conditions. We note that for all finite $\hat{r}$:

$$\frac{d\beta}{d\hat{r}} < 0 .$$

Any other condition might bring about recoalescence, which is physically unrealistic for a scheme to effect fuel injection into a fusion plasma. It may also be noted that, based on physical grounds, for all finite $\hat{r}$, the following is true:

$$\frac{dM}{d\hat{r}} > 0 .$$

Examination of equation (19b) will reveal that for cases where the Mach Number is subsonic, the quantity in brackets must be positive and that when the Mach Number exceeds unity, this same quantity must be negative. In order to avoid a singularity at $M = M_1 = 1$, the bracketed quantity must be zero. Thus, we have the following:

$$\left\{ \xi \, (\beta \hat{r}^2)^3 \, (1 + \gamma) \; - \; \frac{(1 + \gamma)}{\hat{r}} \right\}_{M = 1} = 0 .$$

The above equality reduces to the following relationship upon dividing through the equation written above by the quantity, $(1 + \gamma)$:

$$\left\{ \xi \ (\beta \hat{r}^2)^3 \ - \ \frac{1}{\hat{r}} \right\}_{M = 1} = 0. \tag{26}$$

This provided a motivation to introduce the following definition:

$$\beta \ \equiv \ \eta \ \xi^{-1/3} \ \hat{r}^{-7/3} . \tag{27}$$

Upon substitution of equation (27) into equation (26), one finds:

$$\eta_1 \ = \ \eta\Big|_{M = 1} \ = 1 \ .$$

Next, we substitute the definition of η into equation (19b) to obtain:

$$\frac{dM}{d\hat{r}} \ = \ - \frac{M}{1 - M^2} \left\{ \frac{\eta^3 \ M^2 \ (1 + \gamma M^2)}{\hat{r}} \ - \ \frac{2 + (\gamma - 1)M^2}{\hat{r}} \right\} .$$

Upon combining terms in the above equation, one discovers:

$$\frac{dM}{d\hat{r}} \ = - \frac{M}{\hat{r}} \left\{ \frac{2 + \gamma M^2 - M^2 - \eta^3 M^2 - \gamma \eta^3 M^4}{1 - M^2} \right\} .$$

By rearranging the terms in the above equality, the energy conservation equation becomes:

$$\frac{dM}{d\hat{r}} \ = \ - \frac{M}{\hat{r}} \left\{ \frac{2 + M^2[(1 + \eta^3) - \gamma(1 - M^2 \eta^3)]}{1 - M^2} \right\} \ \equiv \ - \frac{U}{\hat{r}(1 - M^2)} \ . \tag{28}$$

Again, the definition of η is substituted into one of the conservation equations. Thus, equation (25) becomes, upon carrying out the differentiation of the definition of β:

$$\left\{ \frac{d\eta}{d\hat{r}} \ - \ \frac{7\eta}{3\hat{r}} \right\} \hat{r}^{-7/3} \ \xi^{-1/3} \ = \ - \frac{2M^2 \eta \ \xi^{-1/3} \ \hat{r}^{-7/3}}{1 - M^2} \left\{ \frac{\eta^3 - 1}{\hat{r}} \right\} .$$

The quantity $\xi^{-1/3}\,\hat{r}^{-7/3}$ may be eliminated from the above equation immediately to yield:

$$\frac{d\eta}{d\hat{r}} - \frac{7\eta}{3\hat{r}} = -\frac{2M^2\eta}{1 - M^2}\left\{\frac{\eta^3 - 1}{\hat{r}}\right\}.$$

If the above equation is solved for $d\eta/d\hat{r}$, directly (placing terms on a common denominator):

$$\frac{d\eta}{d\hat{r}} = \left\{\frac{-6M^2(\eta^3 - 1) + 7(1 - M^2)}{1 - M^2}\right\}\frac{\eta}{\hat{r}}.$$

Rearrangement of terms in the above equation yields the following simplification:

$$\frac{d\eta}{d\hat{r}} = \left\{\frac{7 - M^2(1 + 6\eta^3)}{3(1 - M^2)}\right\}\frac{\eta}{\hat{r}} \equiv \frac{V}{\hat{r}(1 - M^2)}. \tag{29}$$

Consider, now, the following transformation:

$$s = \ln \hat{r}.$$

Therefore, by taking the derivative with respect to $\hat{r}$ of both sides of the above equation, one finds:

$$\frac{ds}{d\hat{r}} = \frac{1}{\hat{r}}.$$

Upon dividing both sides of equations (28) and (29) by the above, one obtains:

$$\frac{dM}{ds} = -\frac{M\{2 - M^2[(1 + \eta^3) - \gamma(1 - M^2\eta^3)]\}}{1 - M^2} \equiv -\frac{U}{1 - M^2}; \tag{30a}$$

$$\frac{d\eta}{ds} = \frac{\eta[7 - M^2(1 + 6\eta^3)]}{3(1 - M^2)} \equiv \frac{V}{1 - M^2}. \tag{30b}$$

Thus an autonomous system of differential equations has been developed. Upon the elimination of the parametric variable, s, from equations (30a) and (30b), we discover:

$$\frac{dM}{d\eta} = \frac{-3M\{2 - M^2[(1 + \eta^3) - \gamma(1 - M^2\eta^3)]\}}{\eta[7 - M^2(1 + 6\eta^3)]} = -\frac{U}{V}. \tag{31}$$

From equation (27), we can see that when $\hat{r} = 1$, i.e., when $M = M_o$, then $\eta(\hat{r} = 1) = \sqrt[3]{\xi}$. This means that if equation (31) can be integrated from any given point on the physically realistic solution, M_o may be determined. There are two points where the boundary values are known. These are at $M = M_1 = 1$ with $\eta = \eta_1 = 1$; and at $M = \sqrt{(5/\gamma)}$ with $\eta = \sqrt[3]{(7\gamma - 5)/30}$ (see equation (28)). However, upon inspection, it will be discovered that both of these points result in an indeterminate derivative and thus equation (31) cannot yet be integrated, since it is obvious that equation (31) very probably has no simple analytical solution and for this reason will require a numerical integration.

From numerical considerations, either point would allow for solution of equation (31), but the point $\eta(M = M_1) = \eta_1$ is chosen as the most appropriate since this will preclude numerical discrepancies due to integration through a critical point. Therefore, in a sufficiently small reqion of the critical point, linearization will be valid and we find, for $M \doteq 1 + \mu$ and $\eta \doteq 1 + \varepsilon$, by substitution into equation (31):

$$\frac{d\mu}{d\varepsilon} = \frac{-3(1 + \mu)\{2 - (1 + \mu)^2[(1 + (1 + \varepsilon)^3) - \gamma(1 - (1 + \mu)^2(1 + \varepsilon)^3)]\}}{(1 + \varepsilon)[7 - (1 + \mu)^2(1 + 6(1 + \varepsilon)^3)]}.$$

Upon linearizing M^y and η^x (where x and y are arbitrary integer powers) it is found that:

$$\frac{d\mu}{d\varepsilon} \approx \frac{-3(1 + \mu)\{2 - (1 + 2\mu)[(2 + 3\varepsilon) - \gamma(1 - (1 + 2\mu)(1 + 3\varepsilon))]\}}{(1 + \varepsilon)[7 - (1 + 2\mu)(7 + 18\varepsilon)]}.$$

Linearizing the above equation further, one obtains:

$$\frac{d\mu}{d\varepsilon} \simeq \frac{-3(1+\mu)\{2-(1+2\mu)[(2+3\varepsilon)+\gamma(2\mu+3\varepsilon)]\}}{(1+\varepsilon)[7-(7+14\mu+18\varepsilon)]} .$$

Upon rearranging terms, one discovers:

$$\frac{d\mu}{d\varepsilon} \simeq \frac{-3(1+\mu)\{2-(1+2\mu)[2+3(\gamma+1)\varepsilon+2\gamma\mu]\}}{(1+\varepsilon)[-14\mu-18\varepsilon]} .$$

If we linearize equation (31) once again, the following relationship will be provided:

$$\frac{d\mu}{d\varepsilon} \simeq \frac{-3(1+\mu)\{2-[2+3(\gamma+1)\varepsilon+2\gamma\mu+4\mu]\}}{-14\mu-18\varepsilon} .$$

By rearranging terms in the above equation, we see that:

$$\frac{d\mu}{d\varepsilon} \simeq \frac{-3(1+\mu)\{-3(\gamma+1)\varepsilon-2(\gamma+2)\mu\}}{-14\mu-18\varepsilon} .$$

A final round of linearization yields:

$$\frac{d\mu}{d\varepsilon} \simeq -\frac{9(\gamma+1)\varepsilon+6(\gamma+2)\mu}{18\varepsilon+14\mu} . \tag{32}$$

Thus, if η and M are very close to the critical point $\eta(M = 1)$, the above equation will prove to be mathematically equivalent to equation (31). If we now consider that for an independent variable τ:

$$\frac{d\mu}{d\tau} \simeq [9(\gamma+1)\varepsilon+6(\gamma+2)\mu]; \text{ and} \tag{33a}$$

$$\frac{d\varepsilon}{d\tau} \simeq -(18\varepsilon+14\mu). \tag{33b}$$

We again have an autonomous system of differential equations which are not only the mathematical equivalent of equation (31) but are also linear. They may be rewritten as a matrix equation, as shown below:

$$\begin{bmatrix} \frac{d\mu}{d\tau} \\ \frac{d\varepsilon}{d\tau} \end{bmatrix} \simeq \begin{bmatrix} 6(\gamma + 2) & 9(\gamma + 1) \\ -14 & -18 \end{bmatrix} \times \begin{bmatrix} \mu \\ \varepsilon \end{bmatrix}. \tag{34}$$

This is a system of equations of the form:

$$\overline{X'} = \vec{\vec{A}} \times \vec{X}. \tag{35}$$

If the following is hypothesized:

$\vec{\vec{A}} \times \vec{X} = \vec{\vec{\lambda}} \times \vec{X}$, where $\vec{\vec{\lambda}}$ is a diagonal matrix, equation (35) may be solved in terms of the elements of $\vec{\vec{\lambda}}$. Thus, we have:

$$\begin{bmatrix} 6(\gamma + 2) - \lambda & 9(\gamma + 1) \\ -14 & -18 - \lambda \end{bmatrix} = 0. \tag{36}$$

The characteristic equation, obtained by taking the determinant of both sides of equation (36), becomes, after algebraic simplification:

$$\lambda^2 - 6(\gamma - 1)\lambda + 18(\gamma - 5) = 0; \ \lambda = \lambda_1, \lambda_2. \tag{37}$$

It is important to note here that since $1 < \gamma \leq 5/3$, by virtue of the fact that $\gamma = (\nu + 2)/\nu$ where the integer $\nu \geq 3$ is the number of degrees of freedom, and since $\lambda_1\lambda_2 = 18(\gamma - 5)$, the product $\lambda_1\lambda_2$ must always be negative and therefore, λ_1 and λ_2 are of opposite sign. The net effect of this is that the critical point $M = 1$, $\eta = 1$ (or $\mu = 0$, $\varepsilon = 0$) is a saddle point.

Now, if the expression $\vec{\vec{A}} \times \vec{X} = \vec{\vec{\lambda}} \times \vec{X}$ is written in matrix form we have (for $\lambda_i = \lambda_1, \lambda_2$):

$$\begin{bmatrix} 6(\gamma + 2) & 9(\gamma + 1) \\ -14 & -18 \end{bmatrix} \times \begin{bmatrix} \mu \\ \varepsilon \end{bmatrix} = \begin{bmatrix} \lambda_i & \\ & \lambda_i \end{bmatrix} \times \begin{bmatrix} \mu \\ \varepsilon \end{bmatrix}. \qquad (38)$$

Rewriting the above expression as a set of coupled algebraic equations, one obtains:

$$6(\gamma + 2)\mu + 9(\gamma + 1)\varepsilon = \lambda_i \mu; \qquad (39a)$$

$$-14\mu - 18\varepsilon = \lambda_i \varepsilon. \qquad (39b)$$

If equation (39b) is solved for the ratio (μ/ε), one finds:

$$\frac{\mu}{\varepsilon} = -\frac{\lambda_i + 18}{14}; \quad i = 1, 2. \qquad (40)$$

Equation (40) is true for both of the roots of equation (37). At the point $\mu = 0$, $\varepsilon = 0$ (or $M = 1$, $\eta = 1$), the above equation becomes, through the application of L'Hospital's Rule:

$$\frac{d\mu}{d\varepsilon} = -\frac{\lambda_i + 18}{14}; \quad i = 1, 2. \qquad (41)$$

But, $d\mu/d\varepsilon = dM/d\eta$ and ergo:

$$M'_{si} \equiv \left.\frac{dM}{d\eta}\right|_{M = 1,\ \eta = 1} = -\frac{\lambda_i + 18}{14}; \quad i = 1, 2. \qquad (42)$$

Thus, upon simultaneous solution of equations (42) and (37), we discover:

$$M'_{s1} = -3[(\gamma + 5) - \sqrt{(\gamma - 2)^2 + 7}\,]/14; \qquad (43a)$$

$$M'_{s2} = -3[(\gamma + 5) + \sqrt{(\gamma - 2)^2 - 7}\,]/14. \qquad (43b)$$

At this point, we have both the location of the saddle point and two possibilities for the slope at the saddle. One of these slopes must be physically correct and the other must be physically impossible.

Consider now Figure 2, serving as an illustrative example for the specific case of our interest where $\gamma = 7/5$. However, the general conclusions remain valid for any physically permissible value of the ratio of specific heats, i.e., $1 < \gamma \leq 5/3$. The isocline of $U = 0$ (see equation (28)) is intersected by the line $M = 1$. Since values of M that are greater than $\sqrt{(5/\gamma)}$ are not physically possible, another line is drawn at $M = \sqrt{(5/\gamma)}$. This forms five regions. The uppermost region is Region V. The four remaining areas are defined in the following manner. Starting at the upper left-hand corner of the as yet undefined areas and numbering clockwise, Region I is where $\sqrt{(5/\gamma)} > M > 1$ and $U > 0$; Region II is where $\sqrt{(5/\gamma)} > M > 1$ and $U < 0$; Region III is where $M < 1$ and $U < 0$; and Region IV is where $M < 1$ and $U > 0$.

It may be seen from equation (28) (and in light of the fact that $dM/d\hat{r}$ must be positive) that for $M < 1$, U must be negative <u>and</u> that for $M > 1$, U must be positive. This clearly excludes Regions II and IV from the physically realistic solution set. Region V is excluded as well since M must be less than $\sqrt{(5/\gamma)}$. Upon further examination of Figure 2, it will be discovered that the solution to equation (31) based on M'_{s2} is not valid on either side of the sonic point since it occupies Region II above the sonic point and Region IV below the sonic point. However, the other solution (based on M'_{s1}) never enters either Region II or IV. This results in the conclusion that the solution to equation (31) based on M'_{s1} must be the only physically possible solution. **This solution is presented graphically in Figure 3.** The computer program that generated these solutions (Figures 2 and 3) is in Appendix I.

Let us now consider equation (27). We know that at the point $\hat{r} = 1$, $M = M_o$ and $\beta = 1$. If equation (27) is evaluated at this point, we have the seemingly trivial solution of $\eta(\hat{r} = 1) = \xi^{1.3}$. However, if equation (31) is integrated to this value of η, the corresponding Mach Number will be M_o. Hence, M_o may be readily determined by performing the aforementioned integration. Finally, having determined M_o, the equations of conservation may be solved and the pellet velocity requirement may, in principle, be determined.

The equations of conservation may be solved in one of two ways. The first is to integrate directly equations (19b) and (25), utilizing the value of M_o that has just been determined. The second method is to apply a simple quadrature to the solution set of $M(\eta)$. The method chosen for application here is the former. The equations of conservation were solved for two separate cases. The reference case ($\xi = 1000$) is shown in Figure 4. The second case solved was for CTHR and is shown in Figure 5. The computer program used to generate these results and those of the next section are contained in Appendix II.

2.3 Pellet Velocity Requirement

As stated previously, the pellet velocity requirement can now be determined, at least in principle, since the conservation laws may be readily solved. Actually, only the point at which the pellet disappears as a solid entity may be determined. The actual pellet velocity requirement will depend upon how far into the tokamak the pellet must go as a solid entity for the ablated pellet to travel the rest of the way to the plasma center. This would require a transport study and is well beyond the capabilities of the KSU computer. Thus, as a matter of practical compromise, the problem is parameterized so that when the penetration requirement on the solid entity is determined, the mechanical requirements of the design will be known.

Recall the definition of $\dot{q}$ from equation (16). This may be solved for $\rho(r)$ as follows:

$$\frac{dE}{dr} = \frac{\rho \, L(E)}{m} . \tag{16}$$

Upon rearranging the above relationship, one finds:

$$\rho \, dr = \frac{m \, dE}{L(E)} . \tag{44}$$

Integrating from the pellet surface, one has:

$$\int_{r_o}^{\infty} \rho \, dr = \int_{E_o}^{E_\infty} \frac{m \, dE}{L(E)} ; \tag{45}$$

where E_o is the electron energy at the pellet surface and E_∞ is the electron energy of the reacting body. We may approximate E_o as 10 eV or less and thus, for all practical purposes in very hot plasmas, E_o is roughly zero. Upon dividing both sides of equation (45) by $\rho_o r_o$, we may define a new parameter which may be readily evaluated from the solution of the conservation equations:

$$f(\xi) \equiv \int_{1}^{\infty} \hat{\rho} \, d\hat{r} = \frac{m}{\rho_o r_o} \int_{E_o}^{E_\infty} \frac{dE}{L(E)} . \tag{46}$$

Solving the above equation for ρ_o, one discovers:

$$\rho_o = \frac{m}{r_o} \int_{E_o}^{E_\infty} \frac{dE}{L(E)} \; f(\xi)^{-1} . \tag{47}$$

If we solve the working form of equation (18) for dr_o/dt [1], we find:

$$\frac{dr_o}{dt} = - \left(\frac{\rho_o}{\rho_s} \right) \xi^{-1/3} \left(\frac{\gamma - 1}{2} \; \frac{e \, \langle L(E) \rangle}{m} \; J \; r_o \right)^{1/3} .$$

Upon substitution of equation (47) into the above equality, one may observe:

$$\frac{dr_o}{dt} = -\left\{\frac{m}{\rho_s}\right\} [\xi^{1/3} f(\xi)]^{-1} \frac{1}{r_o} \left\{\frac{\gamma - 1}{2} \frac{e \langle L(E)\rangle}{m} J r_o\right\}^{1/3} \int_{E_o}^{E_\infty} \frac{dE}{L(E)} . \tag{48}$$

The above relationship may be rearranged to produce the following equality:

$$r_o^{2/3} \frac{dr_o}{dr} \frac{dr}{dt} = -\left\{\frac{m}{\rho_s}\right\} [\xi^{1/3} f(\xi)]^{-1} \left\{\frac{\gamma - 1}{2} \frac{e}{m}\right\}^{1/3}$$

$$\times \left\{\frac{J}{E_\infty - E_o} \int_{E_o}^{E_\infty} L(E)\, dE\right\}^{1/3} \int_{E_o}^{E_\infty} \frac{dE}{L(E)} . \tag{49}$$

Here, a new relationship is introduced:

$$\frac{m}{\rho_s} = 2 \frac{m_D}{\rho_D} .$$

Upon substitution of the above relationship into equation (49) and rearranging (and noting that dr/dt is just the pellet velocity, u), we discover:

$$u\, r_o^{2/3} \frac{dr_o}{dr} = -2 \left\{\frac{m_D}{\rho_D}\right\} [\xi^{1/3} f(\xi)]^{-1} \left\{\frac{\gamma - 1}{2} \frac{e}{m_H}\right\}^{1/3} \frac{1}{\mu_s^{1/3}}$$

$$\times \left\{\frac{J}{E_\infty - E_o} \int_{E_o}^{E_\infty} L(E)\, dE\right\}^{1/3} \int_{E_o}^{E_\infty} \frac{dE}{L(E)} . \tag{50}$$

If equation (50) is integrated, one obtains:

$$\frac{3}{5} u\, r_p^{5/3} = 2 \left\{\frac{m_D}{\rho_D}\right\} [\xi^{1/3} f(\xi)]^{-1} \left\{\frac{\gamma - 1}{2} \frac{e}{m_H}\right\}^{1/3} \frac{a}{\mu_s^{1/3}}$$

$$\times \int_{(\ell/a)}^{1} \left\{\frac{J}{E_\infty - E_o} \int_{E_o}^{E_\infty} L(E)\, dE\right\}^{1/3} \int_{E_o}^{E_\infty} \frac{dE}{L(E)}\, d(r/a) .$$

Solving the above relationship for u, the pellet velocity requirement, we discover:

$$u = \frac{10}{3} \frac{m_D}{\rho_D} [f(\xi)\ \xi^{1/3}]^{-1} \left\{ \frac{\gamma - 1}{2} \frac{e}{m_H} \right\}^{1/3} \frac{a}{\mu_s^{1/3}}$$

$$\times \int_{(\ell/a)}^{1} \left\{ \frac{J}{E_\infty - E_o} \int_{E_o}^{E_\infty} L(E)\ dE \right\}^{1/3} \int_{E_o}^{E_\infty} \frac{dE}{L(E)}\ d(r/a); \qquad (51)$$

where: J is as given by equation (16);

$L(E) \doteq (2.35 \times 10^{18} + 4 \times 10^{15}\ E + 2 \times 10^{15}/E^2)^{-1}$ eV m^2;

E is in eV;

$f(\xi)$ is given by equation (46); and

ξ is given by equation (20).

Thus, the only things that are now required are the temperature and density distributions of the plasma. These are taken to be[1]:

$$E_\infty = \frac{45}{16} \langle T_\infty \rangle \ [1 - (r/a)^2]^2; \qquad (52a)$$

$$n_\infty = \frac{3}{2} \langle n_\infty \rangle \ [1 - (r/a)^2]. \qquad (52b)$$

2.4 Results of the Theoretical Treatment

As stated previously, the equations of conservation were solved for two cases, a reference case (ξ = 1000) and the CTHR case. As a means of comparison, for the reference case, the value of $[f(\xi)\ \xi^{1/3}]^{-1}$ obtained by this study is 1.156 whereas Foster and Milora[1] obtained a value of 1.23. The value of this parameter for the CTHR case was found to be 1.0978. The penetration dependent pellet velocity requirement is plotted in Figure 6.

Earlier, we decided to neglect disassociation and ionization processes. At a distance of 10 pellet radii from the pellet center, we may observe that the temperature increase over the pellet surface temperature is a factor of 26.7 for the reference case and 63.5 for the CTHR case. Assuming that the temperature at the pellet surface is 20 K, we find that for the reference case, the temperature is ~535 K, and for CTHR, the temperature is ~1270 K. This is sufficient justification for neglecting the dissociation and ionization processes.

3.0 Fuel Injector Design

3.1 Gas Gun Design

The light gas gun is the type of design chosen for the fuel-injection system of CTHR, whose pertinent plasma dimensions are given by a minor radius of 1.40 m, an aspect ratio of 4.33, and an elongation of 1.60. The conceptual design is shown in Figs. 7 and 8. It is a system which is easily used in rapid-fire mode, say, one to three pellets per second per gas gun system. Three such systems will be utilized in CTHR. The pellet radius is given by the following:

$$r_p = [\,(3/(4\pi))\,(m_D/\rho_D)\,(n_\infty V_{pl})/(\dot{p}\,\tau)]^{1/3}\,,$$

where: V_{pl} ≡ volume of the reacting body,

$\dot{p}$ ≡ rate at which pellets are injected into CTHR, and

τ ≡ confinement time for CTHR, equat to 1.64 sec.

By substituting appropriate values into the above equation, we find that for a spherical pellet, $r_p \doteq 3.073$ mm.

The final parameter required for this design is the pressure used by the light gas gun. Since the required pellet velocity is a function of the penetration depth and since the required gas pressure depends on the velocity needed, the gas pressure may be expressed as a function of penetration depth.

The gas pressure as a function of injection velocity is given by the following expression:[9]

$$P = \frac{m_{s+p}}{A_s}\,\frac{u\,u_{max}}{2L}\quad\left(\frac{\gamma-1}{\gamma+1}\right)\,\left[\left(1-\frac{u}{u_{max}}\right)^{\frac{\gamma+1}{\gamma-1}}-1\right]$$

(assuming that the propellent is an ideal gas, expansion of the propellent is to zero pressure and that the injection system is frictionless). Numerically, with assumed parameters where P is the gas pressure in (Pa) and u is the required pellet velocity (in m/sec), one has for hydrogen gas:

$$P = 5033.7\ u\ [(1 - \frac{u}{6578.4})^{-6} - 1],$$

and for D+T gas:

$$P = 3187.7\ u\ [(1 - \frac{u}{4166.0})^{-6} - 1].$$

The required gas pressure is plotted as a function of the penetration fraction in Figures 9 and 10 for natural hydrogen and a deuterium-tritium mixture, respectively.

This brings up one final question. What type of performance can we expect from a light-gas-gun such as that which is conceptualized here? If the performance of the gun were limited to that already achieved by others,[7] i.e., a pellet-injection speed of 330 m/sec, the pellet would disappear as a solid entity after penetrating about 25% of the distance to the pressure center (see Fig. 4). The maximum performance to be expected of a light-gas gun employed in the refueling of CTHR is determined by assuming that the maximum applicable propellent pressure is 100 GPa. Thus, the performance limit for the D+T propellent is found from Figs. 10 and 6 to result in a penetration of about 45% of the pressure-center distance, achieved with an injection velocity of approximately 3600 m/sec. Based on Figs. 9 and 6, the limit for the H_2 propellent turns out to be somewhat higher, say a maximum penetration of about 50%, corresponding to an injection velocity of approximately 5500 m/sec. The extremely high pressure assumed above is justified when one notes that this value may be decreased by a factor of 5 to 10 were this design modified such that each shell could be used only for a single shot.

3.2 Fuel Handling System

The fuel handling system prepares the D+T pellets and subsequently, makes them available to the fuel injection system. To make fuel handling simpler, the pellets have been given a cylindrical shape in place of the spherical one assumed earlier in this report. The net result of this is a cylinder diameter $\sqrt[3]{16/3}$ times the sphere's radius and a height equal to the diameter. The effects of this geometry shift on ablation are unknown but, obviously, expected to be small. [Note that the ratio of surface area to volume is increased only by about 15 percent.]

The fuel handling system works on the following principles (see Figure 11). Initially, the D+T mixture is cooled by a liquid nitrogen stage. The next stage, which liquifies the fuel, is a liquid helium stage. These discrete steps are suggested since the pellets will be mass produced and liquid helium is more expensive than liquid nitrogen. The very cold D+T mixture (5 to 10 K) is injected into a mold whose temperature is maintained at 4.2 K by the liquid helium until the pellets are frozen. The newly formed pellets are subsequently ejected from the mold by the liquid D+T for the next cycle of pellets. Several of these pellet makers could be in operation simultaneously and quality control may be instituted if and where necessary.

Now that the fuel pellets have been formed, they may be loaded into the fuel injector mechanically. The remaining problem is to implement the system into CTHR.

3.3 Implementation into CTHR

For implementation into CTHR, several criteria must be taken into account:

1. The pellet must take the shortest path possible to the fusion plasma.
2. The amount of radiation shielding used to isolate the system from the environment should be minimized within good safety practices.
3. The fuel injection system must be located at a position in the system to make maintenance feasible.
4. Surrounding systems (and subsystems) must be affected as little as possible by the introduction of the fuel injection system.

These goals are attained as shown in Figures 12 and 13. Figure 12 illustrates the overall system and how each of 3 injector systems may be put into CTHR. In Figure 13, a closeup view is provided, to illustrate maintenance possibilities and to illustrate better radiation-shield requirement and system implementation. As can be seen from Fig. 12, interaction of the fuel injection systems with CTHR is very limited, indeed.

4.0 Conclusions

After rejection of several hot-fueling schemes because of inherent difficulties anticipated in practical implementation, the three possibilities of tokamak refueling with cold fuel were considered: liquid-fuel-jet injection, gas puffing, and frozen-pellet injection. It was concluded that pellet fueling appeared to be the most likely of these three alternatives to succeed as a practical scheme. Several methods by which pellet fueling could be implemented into CTHR were then investigated. This led to the further conclusion that the light-gas-gun approach entailed probably the fewest number of real drawbacks. The ORNL Neutral-Gas-Shielding-Model was used to model pellet ablation in CTHR and a required pellet injection velocity was determined as a function of required penetration-depth in the CTHR plasma. On this basis, a conceptual design for a light-gas-gun fuel-injection system for CTHR was developed and the gas-pressure requirement for the device was determined. Finally, suggested means for implementation into the overall design of CTHR have been discussed.

It must be recalled, however, that the penetration-depth requirement for CTHR has not been firmly established. This penetration-depth requirement will ultimately determine the potential feasibility of the design presented in this report. To this end, such a study is highly recommended.

In conclusion, the findings of this study are as follows:

1. If it is determined that the required penetration depth must be 50% of the pressure center distance or more, the light-gas gun will most probably not succeed in refueling CTHR.
2. If the required penetration is between 45% and 50% of the pressure center distance, the D+T propellent mixture will most likely not

succeed in achieving the required injection velocity, but the light-gas-gun design might be practically feasible with H_2 as the propellent.

3. If the required penetration depth is between 25% and 45% of the pressure center depth, the proposed light-gas-gun injection system should be able to refuel the CTHR, with either propellent.
4. If the required penetration depth is less than 25% of the pressure center distance, the proposed scheme of light-gas gun injection will be able to achieve the necessary penetration velocity based on currently tested and proven state-of-the-art technology.

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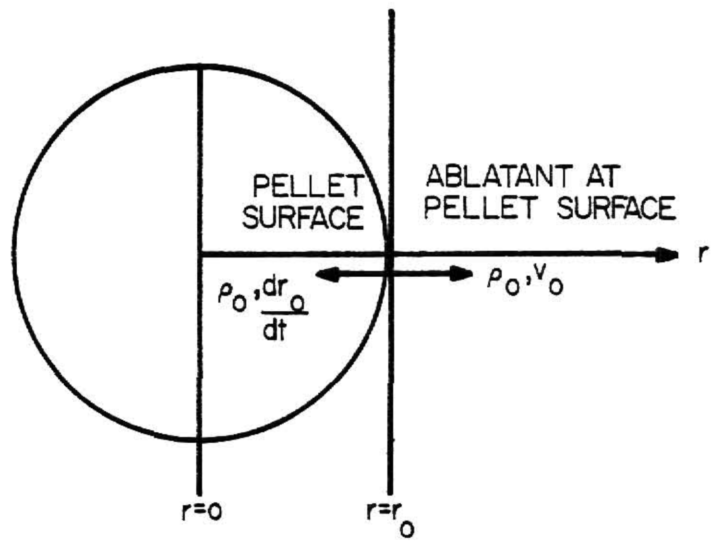

Figure 1. Explanatory diagram of spatial coordinate system and velocity and density boundary conditions at pellet surface. (The origin of the coordinate system moves with the pellet center at the pellet speed, u.)

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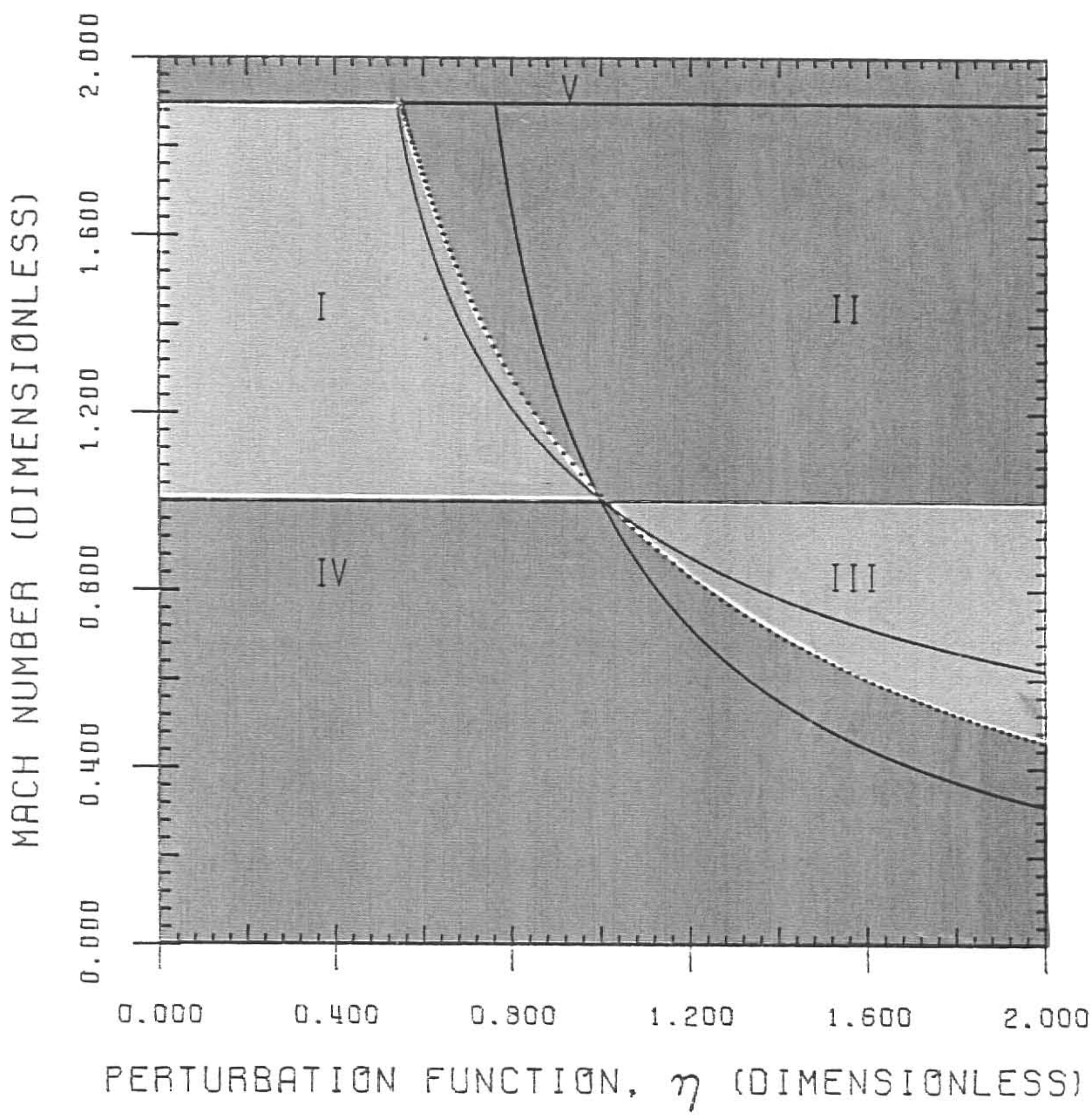


Figure 2. The M, η domain is divided into five regions. In Region V, $M > \sqrt{(5/\gamma)}$. In Regions II and III, $U < 0$. In Regions I and IV, $U > 0$. In Regions III and IV, $M < 1$. In Regions I and II, $\sqrt{(5/\gamma)} > M > 1$. Hence, the physically realistic solution must lie in Regions I or III. Furthermore, any solution in Regions II, IV, or V must be physically impossible. The dotted line represents the solution to $U = 0$.

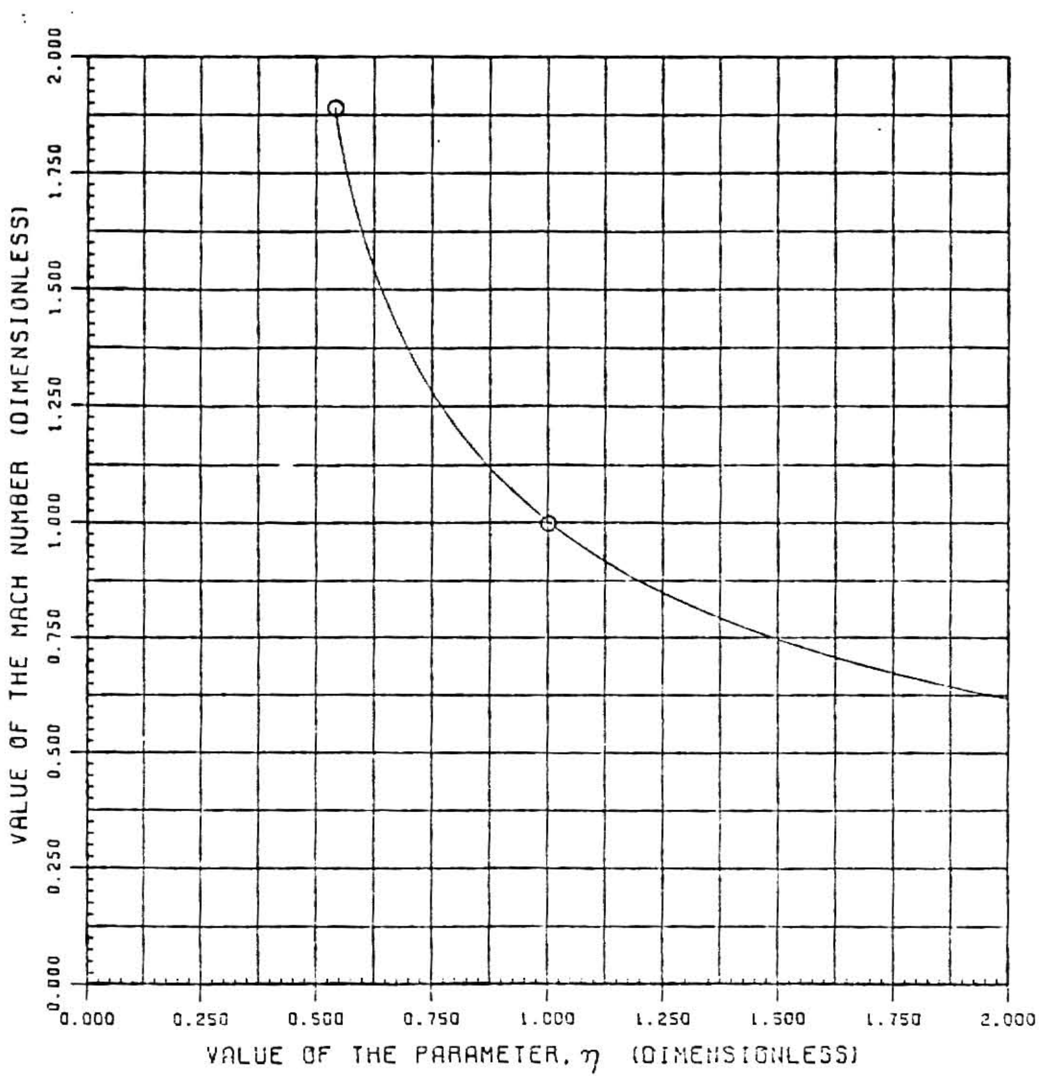


Figure 3. Solution of differential equation (31) for the physically realistic sound-barrier conditions.

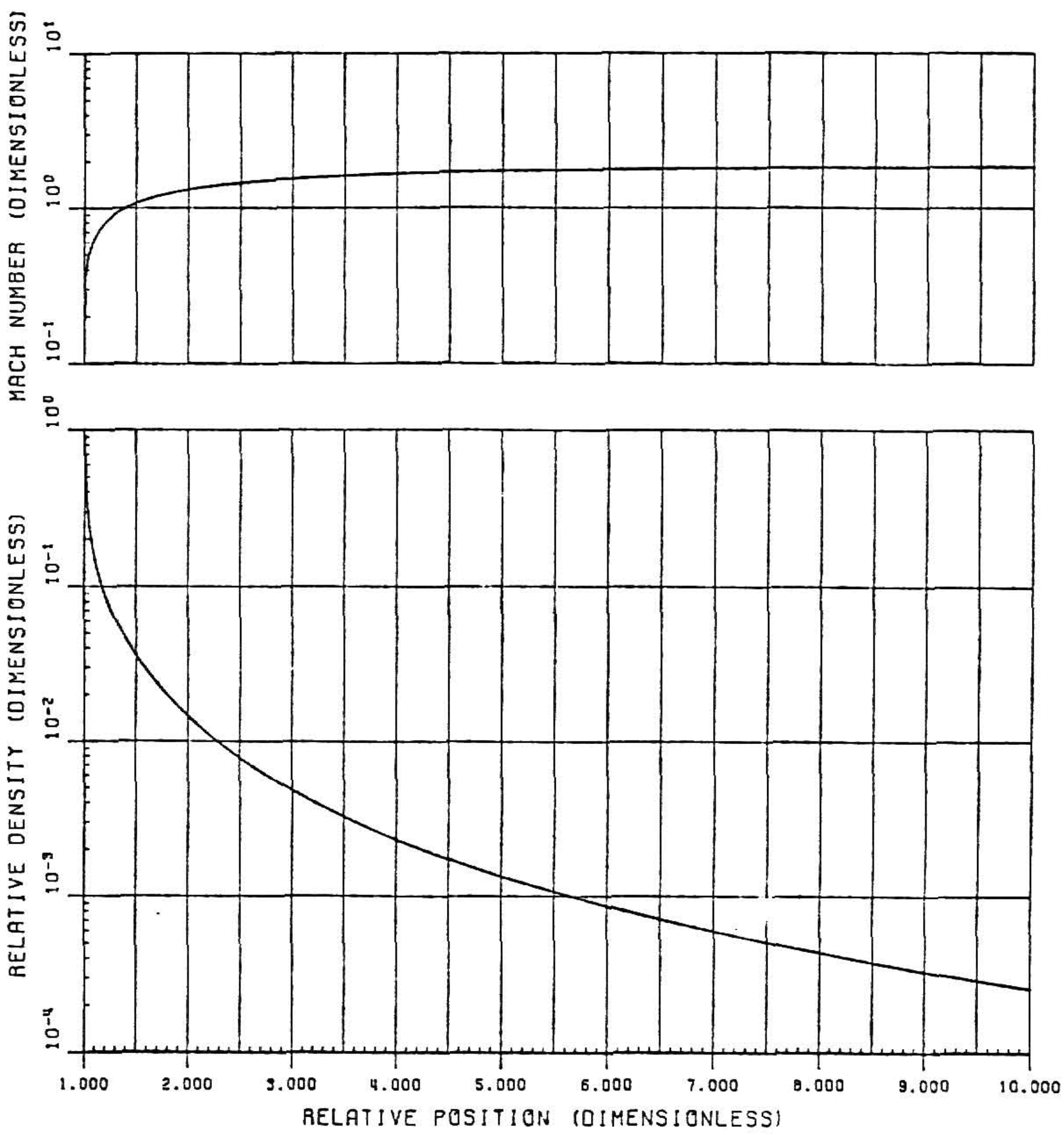


Figure 4. Solution to system of coupled differential equations for case of ξ = 1000 (reference case - Milora and Foster[1]).

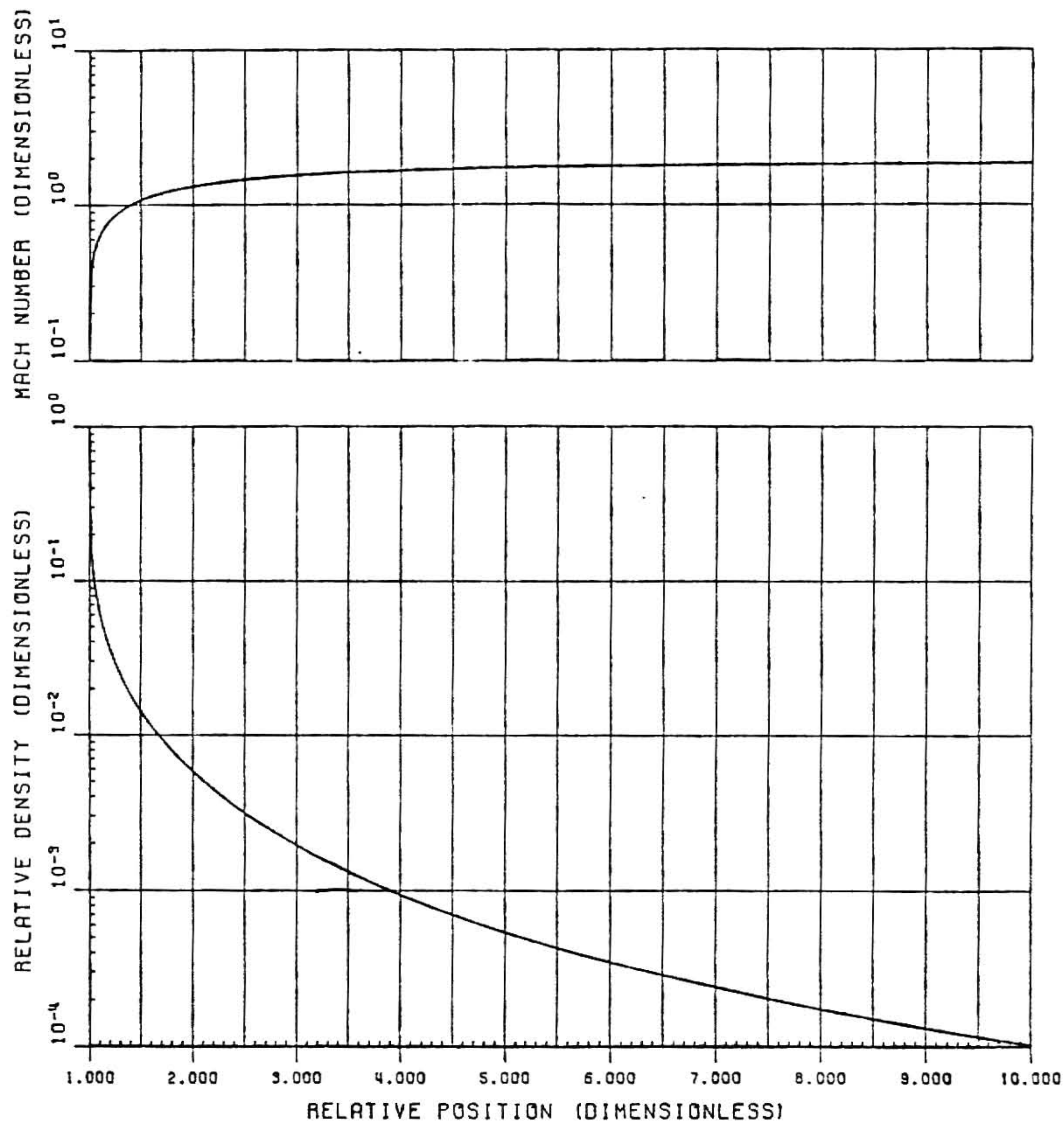


Figure 5. Solution to system of coupled differential equations for CTHR case (ξ = 15,301).

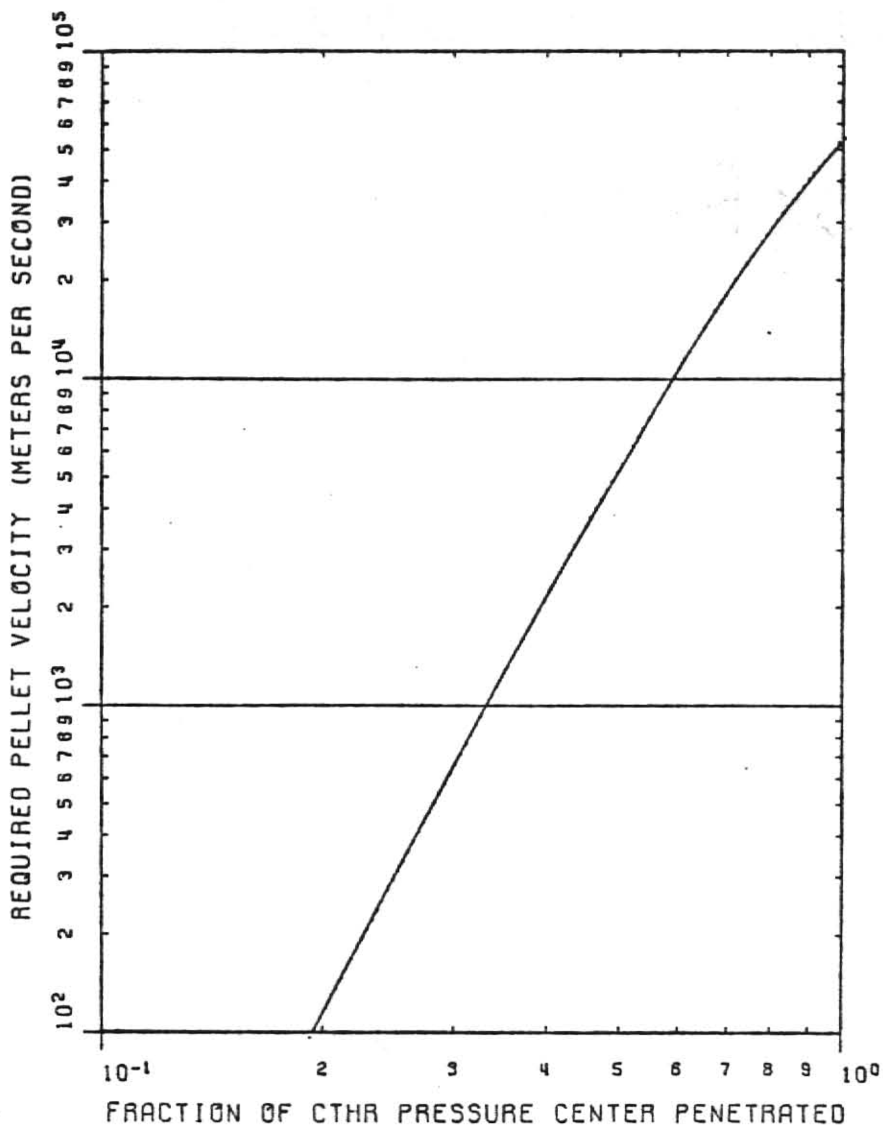


Figure 6. Required pellet velocity is plotted as a function of the point at which the pellet will be totally ablated, which is known as the penetration fraction or depth.

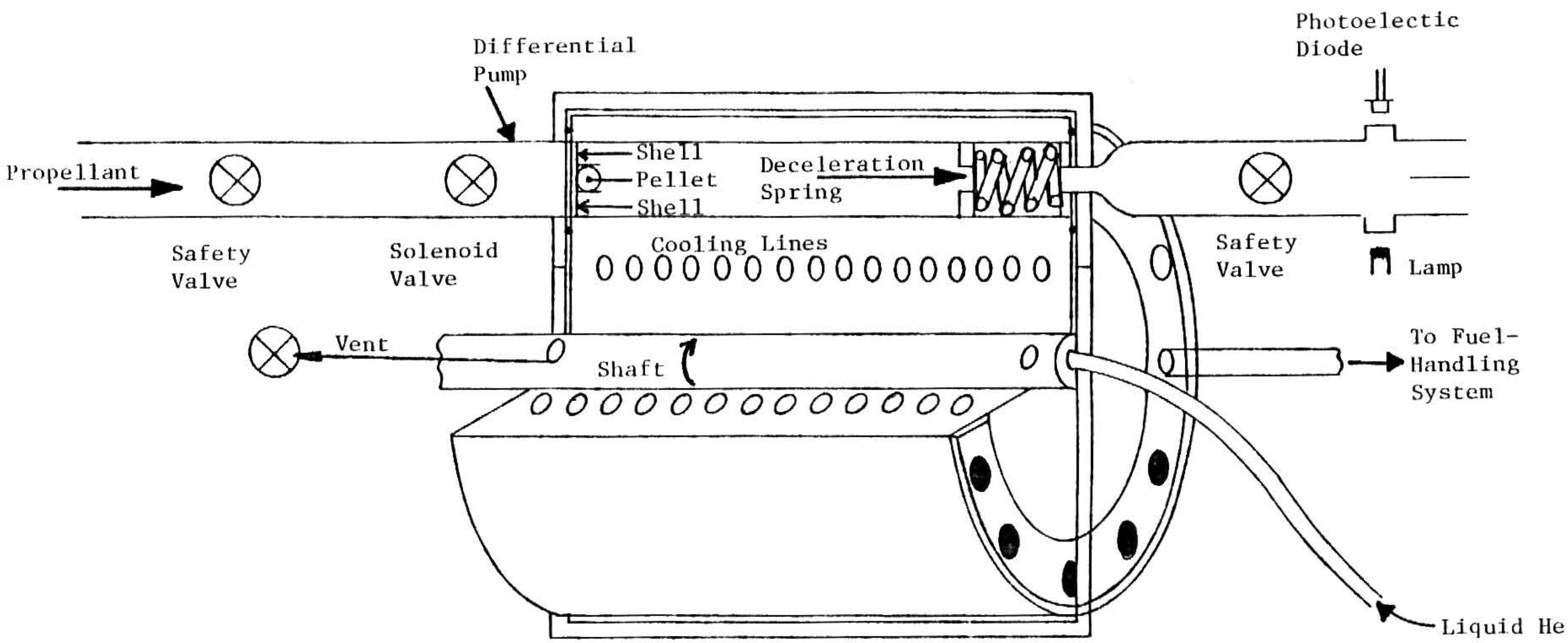


Figure 7. Fuel injection system for CTHR

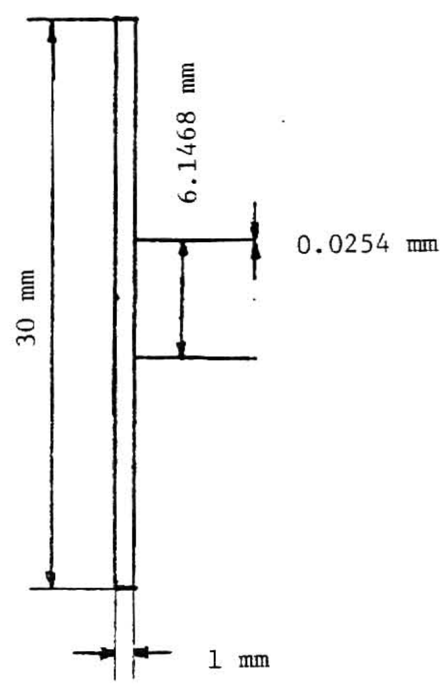


Figure 8. The shell for the fuel injector is a 1mm by 30mm dia disk intersected by a cylindrical shell whose thickness is 0.0254 mm and whose inner diameter is 6.1648 mm.

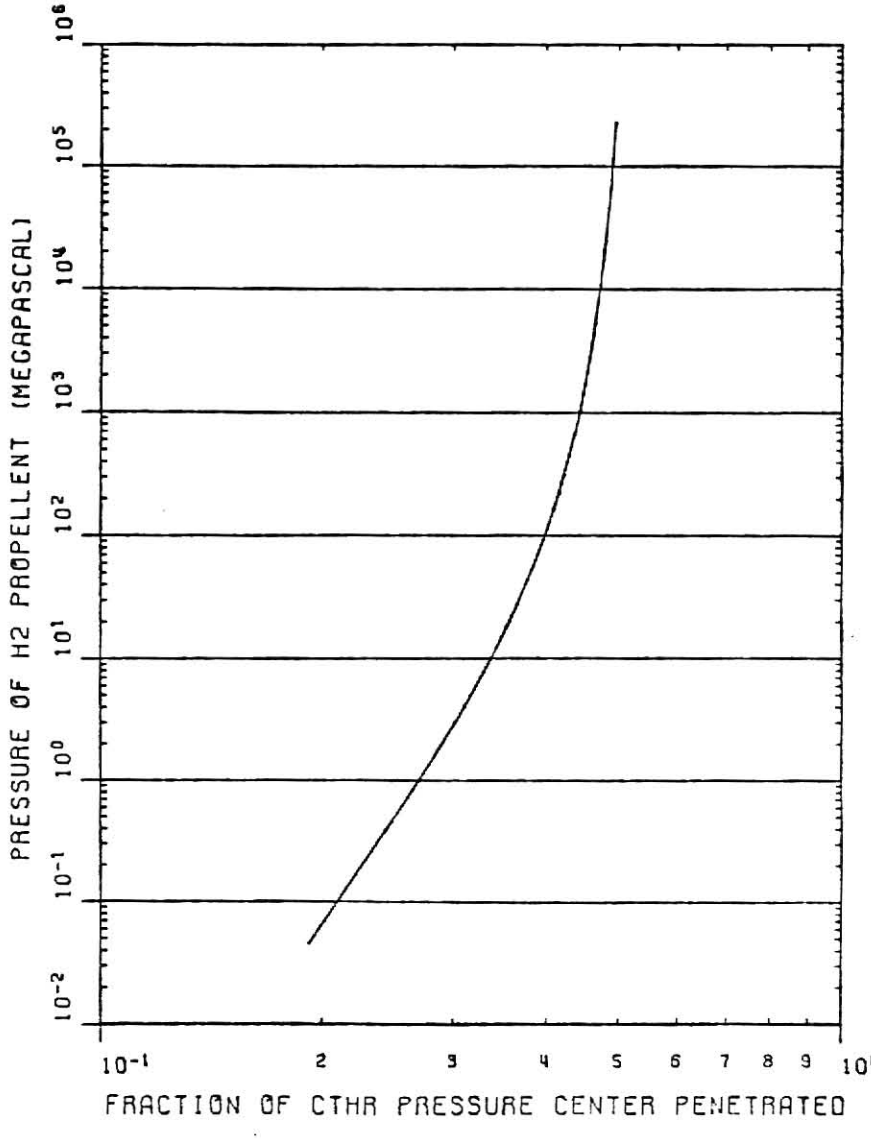


Figure 9. The required gas pressure, utilizing H_2 as propellent, is plotted as a function of the penetration fraction.

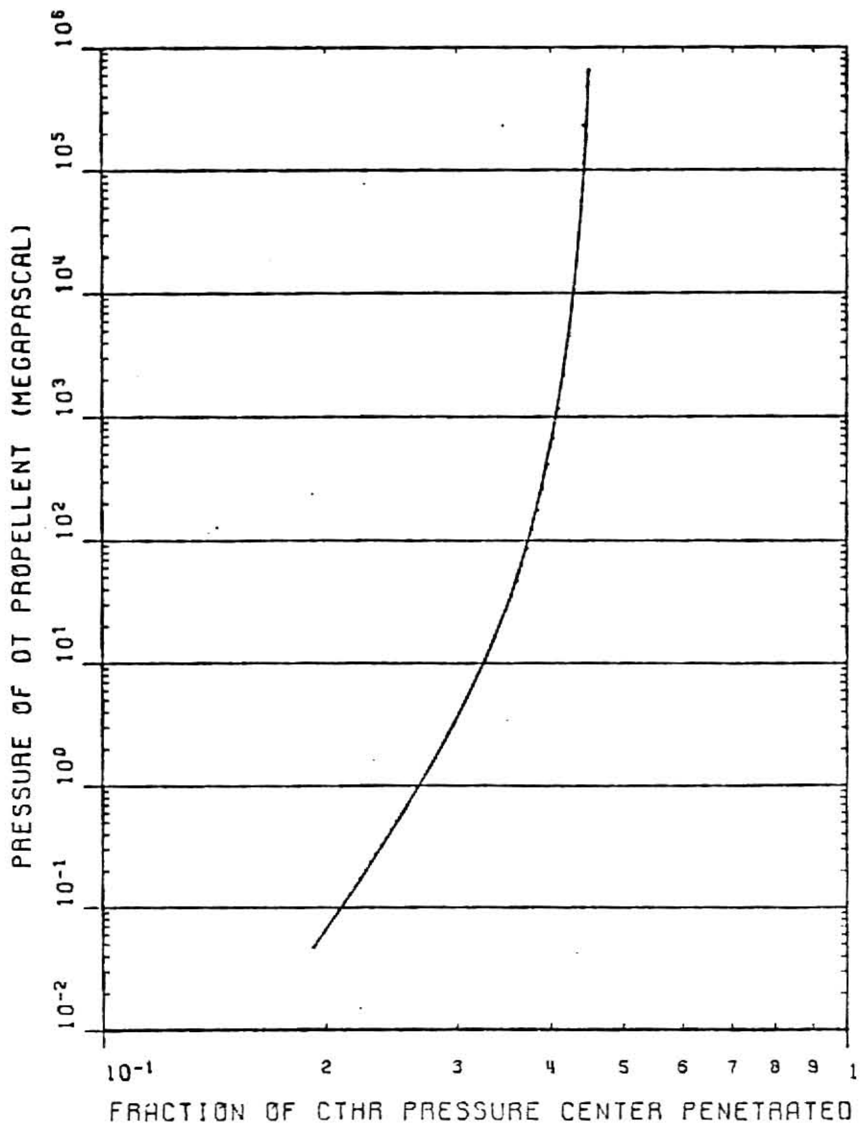


Figure 10. The required gas pressure, utilizing DT as propellant, is plotted as a function of the penetration fraction.

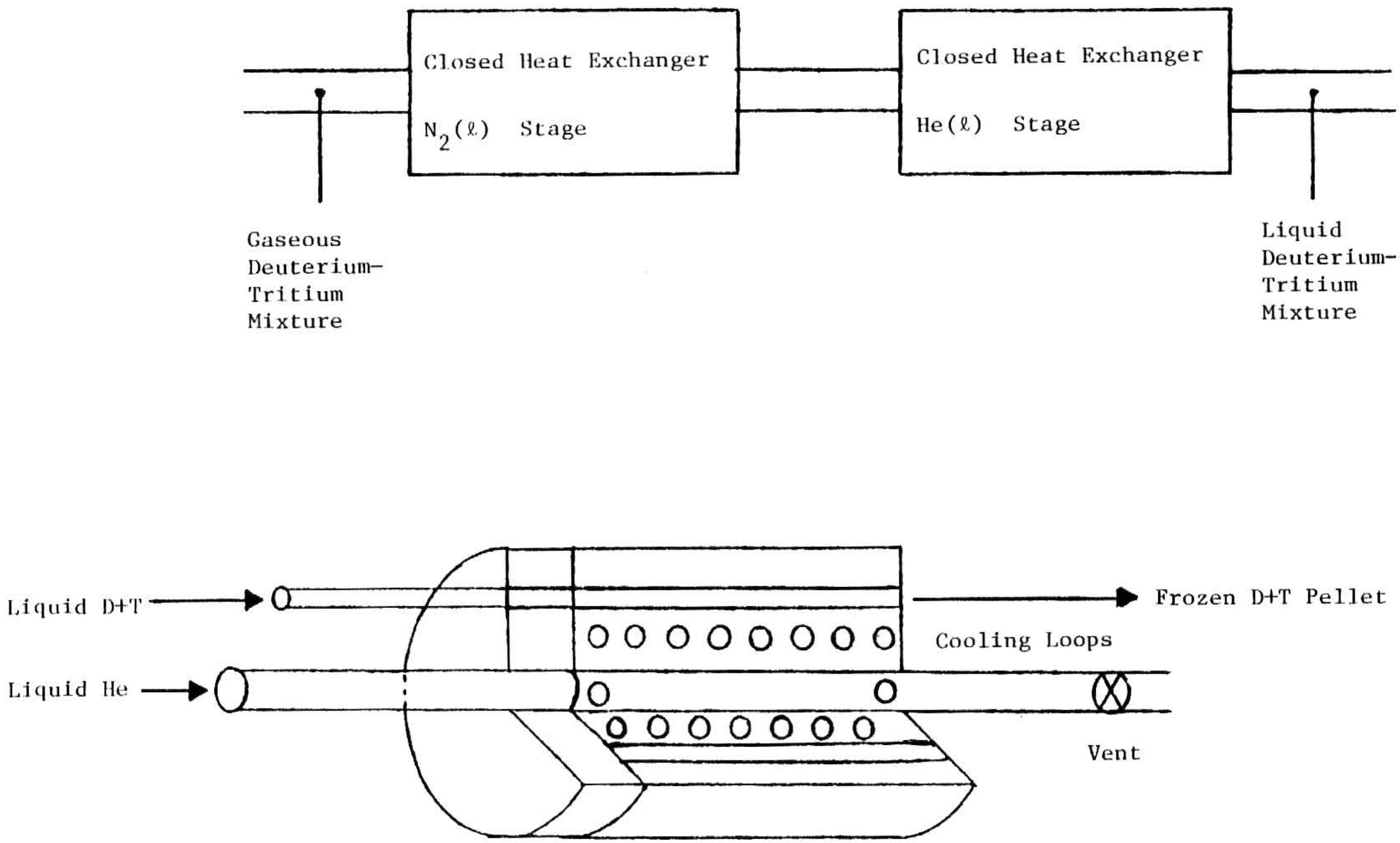
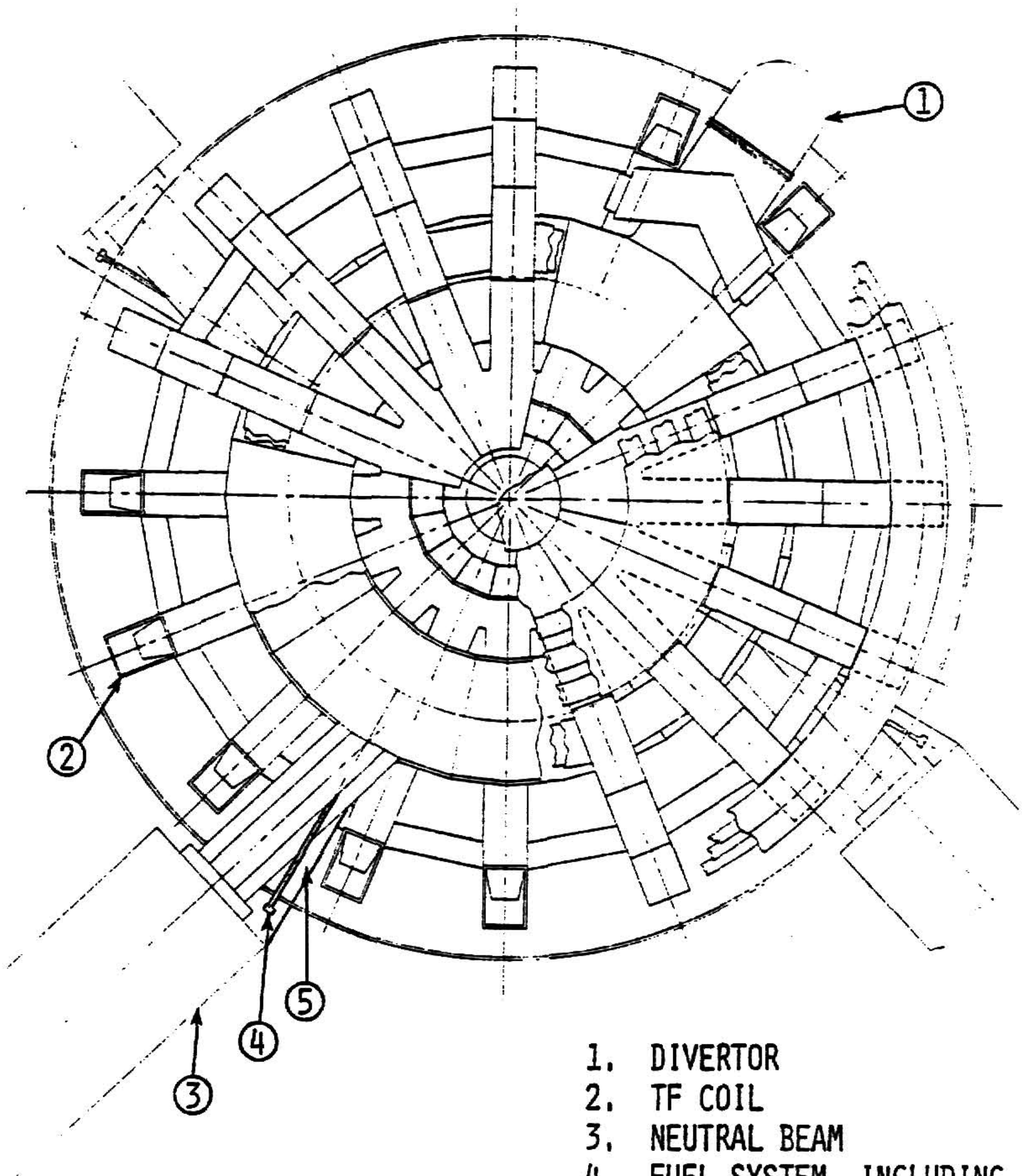


Figure 11. The fuel handling system is as diagrammed above. The liquid DT mixture enters the cylindrical chamber (above), is frozen by the liquid He, and on the next cycle is forced out as a pellet by the entering DT(ℓ).



1. DIVERTOR
2. TF COIL
3. NEUTRAL BEAM
4. FUEL SYSTEM, INCLUDING DRIFT TUBE
5. SHIELDING

Figure 12. Incorporation of fueling systems into CTHR.

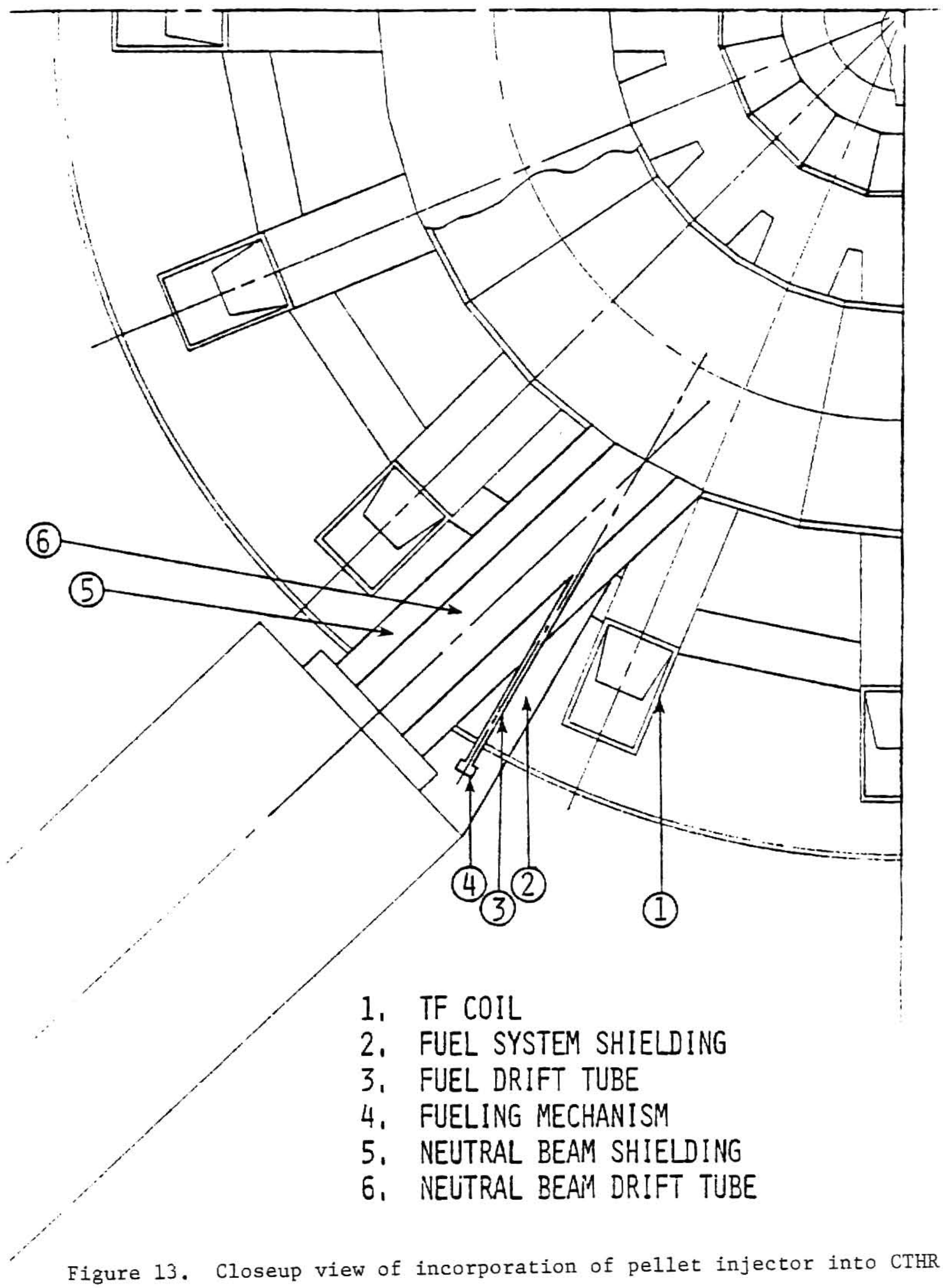


Figure 13. Closeup view of incorporation of pellet injector into CTHR.

Appendix I. Computer Program to Determine M_o.

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```
         C****************************************************************************
         C****                                                                    ****
         C****                                                                    ****
         C**** PROGRAM AND SUBROUTINES BY KENNETH DALE MATNEY.                    ****
         C****                                                                    ****
         C**** THE PURPOSE OF THIS PROGRAM IS TO CALCULATE THE DEPENDENCE OF THE****
         C**** MACH NUMBER ON THE FUNCTION, ETA, ASSUMING THAT LAMBDA-1 IS THE  ****
         C**** APPROPRIATE ROOT OF THE QUADRATIC EQUATION.                       ****
         C****                                                                    ****
         C****                                                                    ****
         C****************************************************************************
ISN 0002       IMPLICIT REAL*8(A-H,O-Z)
ISN 0003       REAL ERR,E,ER
ISN 0004       REAL*8 K1,K2,K3,K4
ISN 0005       DIMENSION X(1005),Y(1005,1),K1(1),K2(1),K3(1),K4(1),E(2),
              <DMDF(1005),YO(2),YP(8),YC(8),ER(8)
ISN 0006       COMMON H
ISN 0007       COMMON/HANDAT/CRIT,PSI,GAMMA,IFRKG
ISN 0008     1 FORMAT(D9.2,1X,D23.16,7X,I1)
ISN 0009     2 FORMAT(1PD22.15,8X,D22.15,12X,'L1 N,M DATA ',I4)
ISN 0010    10 FORMAT(//11X,'**THE VALUE OF THE LAST INCREMENT IS:',1PD12.2,'**')
ISN 0011    11 FORMAT(1H1,112X,'PAGE ',I5)
ISN 0012    12 FORMAT(11X,'THE MACH NUMBER AND DM/DN AS A FUNCTION OF F FOR H = '
              <,1PD9.2,'.')
ISN 0013    13 FORMAT(11X,'THE MACH NUMBER AND DM/DN AS A FUNCTION OF F FOR H = '
              <,1PD9.2,' -- CONTINUED.')
ISN 0014    14 FORMAT(//22X,'N',27X,'M',25X,'DM/DN'/6X,3(5X,23('-')))
ISN 0015    15 FORMAT(/11X,1PD23.16,5X,D23.16,5X,D23.16)
ISN 0016    22 FORMAT(11X,'N AND DN/DM AS A FUNCTION OF THE MACH NUMBER FOR H = '
              <,1PD9.2,'.')
ISN 0017    23 FORMAT(11X,'N AND DN/DM AS A FUNCTION OF THE MACH NUMBER FOR H = '
              <,1PD9.2,' -- CONTINUED.')
ISN 0018    24 FORMAT(//22X,'M',27X,'N',25X,'DN/DM'/6X,3(5X,23('-')))
ISN 0019       ND=1005
ISN 0020       GAMMA=1.400
ISN 0021       IPAGE=1
ISN 0022    99 CONTINUE
ISN 0023       IFRKG=-1
ISN 0024       WRITE(6,11)IPAGE
ISN 0025       IPAGE=IPAGE+1
ISN 0026       X(1)=1.D0
ISN 0027       Y(1,1)=1.D0
ISN 0028       N=1
         C****                                                                    ****
         C****                                                                    ****
         C**** READ THE VALUE OF THE INCREMENT TO BE TAKEN, THE VALUE OF THE     ****
         C**** HEATING PARAMETER, AND IPUNCH.  IF IPUNCH IS SET EQUAL TO ZERO,   ****
         C**** THE PUNCHING OF THE OUTPUT WILL BE SUPPRESSED.                     ****
         C****                                                                    ****
         C****                                                                    ****
ISN 0029       READ(5,1,END=9999)H,PSI,IPUNCH
ISN 0030       D=DABS(H)
ISN 0031       ITAG=H/9.9D-1/DABS(H)
ISN 0032       HSAV=H
ISN 0033       IF(H.LT.0.)WRITE(6,22)HSAV
ISN 0035       IF(H.GT.0.)WRITE(6,12)HSAV
```

```
ISN 0037          IF(H.LT.0.)WRITE(6,24)                                                    ML057
ISN 0039          IF(H.GT.0.)WRITE(6,14)                                                    ML058
ISN 0041          NINC=5.D-02/DABS(H)+0.1D0                                                 ML059
ISN 0042          IF(NINC.LT.1)NINC=1                                                       ML060
ISN 0044          NINC1=1                                                                   ML061
ISN 0045          KO=0                                                                      ML062
ISN 0046          XSAV=PSI**(1.D0/3.D0)                                                     ML063
ISN 0047          IF(H.LT.0.D0)XSAV=DSQRT(5.D0/GAMMA)                                       ML064
ISN 0049          NTOT=(XSAV-1.D0)/DABS(H)+1.1D0                                            ML065
ISN 0050          NSAV=NTOT                                                                 ML066
ISN 0051      100 CONTINUE                                                                  ML067
ISN 0052          NINTER=NTOT                                                               ML068
ISN 0053          IF(NINTER.GT.1000)NINTER=1003                                             ML069
ISN 0055          IF(NTOT.LE.1000)NTOT=0                                                    ML070
ISN 0057          IF(NTOT.GT.1000)NTOT=NTOT-1000                                            ML071
ISN 0059          X0=1.D0                                                                   ML072
ISN 0060          Y0(1)=1.D0                                                                ML073
ISN 0061          NC=1                                                                      ML074
ISN 0062          A1=0.D0                                                                   ML075
ISN 0063          A2=0.D0                                                                   ML076
ISN 0064          ERR=1.E-14                                                                ML077
ISN 0065          CALL HAMMIN(X0,Y0,NC,NINTER,D,A1,A2,ERR,X,Y,IER,YP,YC,K1,K2,K3,K4,        ML078
                 <E,ER,ND)                                                                  ML079
ISN 0066          IFRKG=IFRKG+1                                                             ML080
ISN 0067          IF(NTOT.EQ.0)D=XSAV-X(NINTER-1)                                           ML081
ISN 0069          IF(NTOT.EQ.0)CALL RKGSDQ(NINTER,X,Y,NC,D,E,K1,K2,K3,K4,ND)                ML082
ISN 0071          DO 200 J=1,NINTER,1                                                       ML083
ISN 0072          IF(H.GT.0.)DMDF(J)=F(L,X,Y,J)                                             ML084
ISN 0074          IF(H.LT.0.)DMDF(J)=F(L,X,Y,J)                                             ML085
ISN 0076      200 CONTINUE                                                                  ML086
ISN 0077          IF(IPUNCH.EQ.0)GO TO 251                                                  ML087
ISN 0079          DO 250 J=1,1000,1                                                         ML088
ISN 0080          IF(J.GT.NINTER)GO TO 251                                                  ML089
ISN 0082          IF(H.LT.0.D0)PUNCH 2,Y(J,1),X(J),ITAG                                     ML090
ISN 0084          IF(H.GT.0.D0)PUNCH 2,X(J),Y(J,1),ITAG                                     ML091
ISN 0086          ITAG=ITAG+H/9.9D-1/DABS(H)                                                ML092
ISN 0087      250 CONTINUE                                                                  ML093
ISN 0088      251 CONTINUE                                                                  ML094
ISN 0089          DO 300 J=NINC1,NINTER,NINC                                                ML095
ISN 0090          IF(NINTER.LT.1000.AND.NINTER.LT.NINC1)GO TO 299                           ML096
ISN 0092          WRITE(6,15)X(J),Y(J,1),DMDF(J)                                            ML097
ISN 0093          JINTER=J                                                                  ML098
ISN 0094          KO=KO+1                                                                   ML099
ISN 0095          IF(KO.NE.25)GO TO 300                                                     ML100
ISN 0097          KO=0                                                                      ML101
ISN 0098          WRITE(6,11)IPAGE                                                          ML102
ISN 0099          IPAGE=IPAGE+1                                                             ML103
ISN 0100          IF(H.LT.0.)WRITE(6,23)HSAV                                                ML104
ISN 0102          IF(H.GT.0.)WRITE(6,13)HSAV                                                ML105
ISN 0104          IF(H.LT.0.)WRITE(6,24)                                                    ML106
ISN 0106          IF(H.GT.0.)WRITE(6,14)                                                    ML107
ISN 0108          GO TO 300                                                                 ML108
ISN 0109      299 CONTINUE                                                                  ML109
ISN 0110          WRITE(6,15)X(NINTER),Y(NINTER,1),DMDF(NINTER)                             ML110
ISN 0111          GO TO 301                                                                 ML111
ISN 0112      300 CONTINUE                                                                  ML112
ISN 0113      301 CONTINUE                                                                  ML113
ISN 0114          IF(NTOT.NE.0)GO TO 350                                                    ML114
```

```
      WRITE(6,10)D
      GO TO 99
  350 CONTINUE
      DO 400 J=1,4,1
      X(J)=X(NINTER-4+J)
      Y(J,1)=Y(NINTER-4+J,1)
  400 CONTINUE
      NINC1=NINC+1
      GO TO 100
 9999 CONTINUE
      STOP
      END
```

OS/360 FORTRAN H

```
          C*************************************************************************
          C***                                                                   ***
          C***                                                                   ***
          C***    THE PURPOSE OF HAMMIN IS TO SOLVE A SET OF SIMULTANEOUS FIRST  ***
          C***    ORDER FUNCTIONALS OF Y WITH RESPECT TO X USING HAMMING'S FIFTH ***
          C***    ORDER PREDICTOR-CORRECTOR.  FOURTH ORDER RUNGA-KUTTA-GILL      ***
          C***    PROVIDES STARTING VALUES WHICH ARE ITERATED UPON BY THREE OTHER ***
          C***    INTEGRATION FORMULAES UNTIL A CONSISTENT STARTER IS OBTAINED.  ***
          C***                                                                   ***
          C***                                                                   ***
          C***    X0 = INITIAL VALUE OF X.                                       ***
          C***                                                                   ***
          C***    Y0 = ARRAY OF INITIAL Y-VALUES.                                ***
          C***                                                                   ***
          C***    NC = NUMBER OF COUPLED ECUATIONS.                              ***
          C***                                                                   ***
          C***    H = INTEGRATION INCREMENT                                      ***
          C***                                                                   ***
          C***    N = NUMBER OF INTEGRATIONS TO BE PERFORMED.                    ***
          C***                                                                   ***
          C***    A1,A2 = PREDICTOR-CORRECTOR PARAMETERS TO BE CHOSEN BY USER.   ***
          C***                                                                   ***
          C***    ERR = CONVERGENGE PARAMETER FOR STARTER.                       ***
          C***                                                                   ***
          C***    X = ARRAY OF X-VALUES                                          ***
          C***                                                                   ***
          C***    Y = 2-DIMENSIONAL ARRAY OF Y-VALUES.                           ***
          C***                                                                   ***
          C***    IER = ERROR PARAMETER.  IF IER=0, INTEGRAL INSIDE OF ASSYMPTOTES. ***
          C***                            IF IER=1, STARTER FAILED TO CONVERGE.  ***
          C***                            IF IER=2, INTEGRAL DIVERGES.           ***
          C***                                                                   ***
          C***    YC,W1,W2,W3,W4,E = WORK ARRAYS OF LENGTH NC.                   ***
          C***                                                                   ***
          C***    YP,ER  = WORK ARRAY OF MINIMUM SIZE (4,NC).                    ***
          C***                                                                   ***
          C***    ND = DIMENSION SIZE OF Y IN MAIN PROGRAM.                      ***
          C***                                                                   ***
          C***    TO GET DOUBLE PRECISION VERSION, REMOVE C'S FROM PC240 AND PC245. ***
          C***                                                                   ***
          C***                                                                   ***
          C*************************************************************************
ISN 0002        SUBROUTINE HAMMIN(X0,Y0,NC,N,H,A1,A2,ERR,X,Y,IER,YP,YC,W1,W2,W3,W4
               $,E,ER,ND)
ISN 0003        REAL*8 X0,Y0,H,A1,A2,X,Y,YP,YC,W1,W2,W3,W4,C1,C2,C3,C4,C5,C6,C7,EP **
               $,EC,A0,B0,B1,B2,B3,BCN1,BC0,BC1,BC2,CRIT,F,PSI,GAMMA              **
ISN 0004        COMMON/HAMDAT/CRIT,PSI,GAMMA,IFRKG                                 *
ISN 0005        DIMENSION Y0(1),X(1),Y(ND,1),YP( 4,1),YC(1),E(1),W1(1),W2(1),W3(1)
               $,W4(1),ER( 4,1)
ISN 0006        IER=0
ISN 0007        C1=9.D+00
ISN 0008        C2=19.D+00
ISN 0009        C3=5.D+00
ISN 0010        C4=24.D+00
ISN 0011        C5=4.D+00
ISN 0012        C6=3.D+00
```

```
ISN 0013          C7=8.D+00
ISN 0014          EP=(2.51D+02-C2*A1-C7*A2)/6.D+00
ISN 0015          EC=(-C2+11.D+00*A1-C7*A2)/6.D+00
ISN 0016          A0=1.D+00-A1-A2
ISN 0017          B0=(55.D+00+C1*A1+C7*A2)/C4
ISN 0018          B1=(-59.D+00+C2*A1+32.D+00*A2)/C4
ISN 0019          B2=(37.D+00-C3*A1+C7*A2)/C4
ISN 0020          B3=(A1-C1)/C4
ISN 0021          BCN1=(C1-A1)/C4
ISN 0022          BC0=(C2+13.D+00*A1+C7*A2)/C4
ISN 0023          BC1=(-C3+13.D+00*A1+32.D+00*A2)/C4
ISN 0024          BC2=(1.D+00-A1+C7*A2)/C4
ISN 0025          IF(IFRKG.GE.0)GO TO 155
ISN 0027          X(1)=X0
ISN 0028          DO 100 L=1,NC,1
ISN 0029          Y(1,L) =Y0(L)
ISN 0030      100 CONTINUE
ISN 0031          DO 115 J=2,4,1
ISN 0032          CALL RKGSDQ(J,X,Y,NC,H,E,W1,W2,W3,W4,ND)
ISN 0033      115 CONTINUE
ISN 0034          JCHECK=0
ISN 0035      120 CONTINUE
ISN 0036          JCHECK=JCHECK+1
ISN 0037          DO 130 L=1,NC,1
ISN 0038          YP(2,L)=Y(1,L) +H*(C1*F(L,X,Y,1)+C2*F(L,X,Y,2)-C3*F(L,X,Y,3)+F(L,
                 $X,Y,4))/C4
ISN 0039          YP(3,L)=Y(1,L) +H*(F(L,X,Y,1)+C5*F(L,X,Y,2)+F(L,X,Y,3))/C6
ISN 0040          YP(4,L)=Y(1,L) +H*(F(L,X,Y,1)+C6*(F(L,X,Y,2)+F(L,X,Y,3))+F(L,X,Y
                 $,4))/C7*C6
ISN 0041          DO 125 J=2,4,1
ISN 0042          ER(J,L)=(Y(J,L)-YP(J,L))/Y(J,L)
ISN 0043          ER(J,L)=ABS(ER(J,L))
ISN 0044      125 CONTINUE
ISN 0045      130 CONTINUE
ISN 0046          ITEST=0
ISN 0047          DO 140 L=1,NC,1
ISN 0048          DO 135 J=2,4,1
ISN 0049          IF(ER(J,L).GT.ERR)ITEST=1
ISN 0051          Y(J,L)=YP(J,L)
ISN 0052      135 CONTINUE
ISN 0053      140 CONTINUE
ISN 0054          IF(JCHECK.GE.500)GO TO 145
ISN 0056          IF(ITEST.EQ.1)GO TO 120
ISN 0058      145 DO 150 L=1,NC,1
ISN 0059          YP(1,L)=0.D+00
ISN 0060          YC(L)=0.D+00
ISN 0061      150 CONTINUE
ISN 0062      155 CONTINUE
ISN 0063          DO 300 J=4,N,1
ISN 0064          X(J+1)=X(J)+H
ISN 0065          DO 175 L=1,NC,1
ISN 0066          YP(2,L)=YP(1,L)
ISN 0067      175 CONTINUE
ISN 0068          DO 200 L=1,NC,1
ISN 0069          YP(1,L)=A0*Y(J,L)+A1*Y(J-1,L)+A2*Y(J-2,L)+H*(B0*F(L,X,Y,J)+B1*F(L,
                 $X,Y,J-1)+B2*F(L,X,Y,J-2)+B3*F(L,X,Y,J-3))
ISN 0070      200 CONTINUE
ISN 0071          DO 225 L=1,NC,1
```

```
ISN 0072          Y(J+1,L)=YP(1,L)-(EP/(EP-EC))*(YP(2,L)-YC(L))                        PC575
ISN 0073      225 CONTINUE                                                              PC580
ISN 0074          DO 250 L=1,NC,1                                                       PC585
ISN 0075          YC(L)=A0*Y(J,L)+A1*Y(J-1,L)+A2*Y(J-2,L)+H*(BCN1*F(L,X,Y,J+1)+BC0*F   PC590
                 $(L,X,Y,J)+BC1*F(L,X,Y,J-1)+BC2*F(L,X,Y,J-2))                          PC595
ISN 0076      250 CONTINUE                                                              PC600
ISN 0077          DO 275 L=1,NC,1                                                       PC605
ISN 0078          Y(J+1,L)=YC(L)-(EC/(EP-EC))*(YP(1,L)-YC(L))                           PC610
ISN 0079      275 CONTINUE                                                              PC615
ISN 0080          IF(J-N)285,300,400                                                    PC620
ISN 0081      285 CONTINUE                                                              PC625
            C     IF(Y(J+1,1).GT.1.D+00.OR.Y(J+1,1).LT.Y(J,1))GO TO 500                 PC630
ISN 0082      300 CONTINUE                                                              PC635
ISN 0083          IER=ITEST                                                             PC640
ISN 0084      400 N=N+1                                                                 PC645
ISN 0085          RETURN                                                                PC650
ISN 0086          END                                                                   PC670
```

```
         C*****************************************************************************
         C***                                                                       ***
         C***                                                                       ***
         C***   THE PURPOSE OF RKGSDQ IS TO SOLVE, USING RUNGA-KUTTA-GILL, A SET    ***
         C***   OF SIMULTANEOUS DIFFERENTIAL EQUATIONS, WHICH ARE FIRST ORDER IN    ***
         C***   A COMMON (DUMMY) VARIABLE.  THE QUADRATURE IS FOURTH ORDER.         ***
         C***                                                                       ***
         C***                                                                       ***
         C***   J IS THE ITERATE OF Y BEING SOLVED.                                 ***
         C***                                                                       ***
         C***   X(J) IS THE DUMMY VARIABLE.                                         ***
         C***                                                                       ***
         C***   Y(L,J) IS THE SOLUTION TO THE L'TH DIFFERENTIAL EQUATION.           ***
         C***                                                                       ***
         C***   N IS THE NUMBER OF SIMULTANEOUS DIFFERENTIAL EQUATIONS.             ***
         C***                                                                       ***
         C***   H IS THE INCREMENT TO BE ADDED TO X(J-1) TO GET X(J).               ***
         C***                                                                       ***
         C***   E IS A PARAMETER USED IN THE RULE OF COLLATZ, OF LENGTH,N.          ***
         C***                                                                       ***
         C***   K1 IS A REAL WORK ARRAY OF LENGTH, N.                               ***
         C***                                                                       ***
         C***   K2 IS A REAL WORK ARRAY OF LENGTH, N.                               ***
         C***                                                                       ***
         C***   K3 IS A REAL WORK ARRAY OF LENGTH, N.                               ***
         C***                                                                       ***
         C***   K4 IS A REAL WORK ARRAY OF LENGTH, N.                               ***
         C***                                                                       ***
         C***                                                                       ***
         C*****************************************************************************
ISN 0002          SUBROUTINE RKGSDQ(J,X,Y,N,H,E,K1,K2,K3,K4,M)
ISN 0003          REAL*8  X,Y,H,K1,K2,K3,K4,AMD,ADD,AMS,AM,AD,AX,F
         C        REAL K1,K2,K3,K4
ISN 0004          DIMENSION X(1),Y(M,1),E(1),K1(1),K2(1),K3(1),K4(1)
ISN 0005          AMD=2.D+00
ISN 0006          ADD=1.D+00+DSQRT(5.D-01)
ISN 0007          AMS=5.D-01-DSQRT(5.D-01)
ISN 0008          AM=2.D+00-DSQRT(2.D+00)
ISN 0009          AD=2.D+00+DSQRT(2.D+00)
ISN 0010          AX=6.D+00
ISN 0011          X(J)=X(J-1)
ISN 0012          DO 100 L=1,N,1
ISN 0013          Y(J,L)=Y(J-1,L)
ISN 0014      100  CONTINUE
ISN 0015          DO 110 L=1,N,1
ISN 0016          K1(L)=F(L,X,Y,J)*H
ISN 0017      110 CONTINUE
ISN 0018          DO 120 L=1,N,1
ISN 0019          Y(J,L)=Y(J,L)+(K1(L)/AMD)
ISN 0020      120 CONTINUE
ISN 0021          X(J)=X(J)+(H/AMD)
ISN 0022          DO 130 L=1,N,1
ISN 0023          K2(L)=F(L,X,Y,J)*H
ISN 0024      130 CONTINUE
ISN 0025          DO 140 L=1,N,1
ISN 0026          Y(J,L)=Y(J,L)+(K2(L)-K1(L))*(AMD-ADD)
```

```
  140 CONTINUE                                                          RK 57
      DO 150 L=1,N,1                                                    RK 58
      K3(L)=F(L,X,Y,J)*H                                                RK 59
  150 CONTINUE                                                          RK 60
      DO 160 L=1,N,1                                                    RK 61
      Y(J,L)=Y(J,L)+AMS*K1(L)-K2(L)+ADD*K3(L)                           RK 62
  160 CONTINUE                                                          RK 63
      X(J)=X(J-1)+H                                                     RK 64
      DO 170 L=1,N,1                                                    RK 65
      K4(L)=F(L,X,Y,J)*H                                                RK 66
  170 CONTINUE                                                          RK 67
      DO 180 L=1,N,1                                                    RK 68
      Y(J,L)=Y(J-1,L)+(K1(L)+AM*K2(L)+AD*K3(L)+K4(L))/AX                RK 69
  180 CONTINUE                                                          RK 72
      RETURN                                                            RK 73
      END                                                               RK 74
```

```
               C*************************************************************************M.N01
               C****                                                                 ****M.N02
               C****                                                                 ****M.N03
               C**** THE PURPOSE OF THIS FUNCTION IS TO PROVIDE DERIVATIVE FUNCTIONS ****M.N04
               C**** FOR SUBROUTINES RKGSDO AND HAMMIN.                              ****M.N05
               C****                                                                 ****M.N06
               C****                                                                 ****M.N07
               C*************************************************************************M.N08
ISN 0002             FUNCTION F(L,X,Y,J)                                                  M.N09
ISN 0003             REAL*8 X,Y,S,G,M,NUM,DEN,F3,L1,L2,F,CRIT,FNEW,H                      M.N10
ISN 0004             COMMON H                                                             M.N11
ISN 0005             COMMON/HAMDAT/CRIT,S,G,IFRKG                                         M.N12
ISN 0006             DIMENSION X(1),Y(1005,1)                                             M.N13
ISN 0007             M=Y(J,1)                                                             M.N14
ISN 0008             IF(H.LT.0.D0)M=X(J)                                                  M.N15
ISN 0010             F3=X(J)*X(J)*X(J)                                                    M.N16
ISN 0011             IF(H.LT.0.D0)F3=Y(J,1)*Y(J,1)*Y(J,1)                                 M.N17
ISN 0013             FNEW=X(J)                                                            M.N18
ISN 0014             IF(H.LT.0.D0)FNEW=Y(J,1)                                             M.N19
ISN 0016             NUM=-3.D0*(2.D0-M*M*((1.D0+F3)-G*(1.D0-M*M*F3)))*M                   M.N20
ISN 0017             DEN=(7.D0-M*M*(1.D0+6.D0*F3))*FNEW                                   M.N21
ISN 0018             IF(M.EQ.1.D0.AND.FNEW.EQ.1.D0)GO TO  00                              M.N22
ISN 0020             IF(H.LT.0.D0)GO TO 100                                               M.N23
ISN 0022             F=NUM/DEN                                                            M.N24
ISN 0023             RETURN                                                               M.N25
ISN 0024         100 CONTINUE                                                             M.N26
ISN 0025             F=DEN/NUM                                                            M.N27
ISN 0026             RETURN                                                               M.N28
ISN 0027         200 CONTINUE                                                             M.N29
ISN 0028             L1=3.D0*((G-1.D0)-DSQRT((2.D0-G)*(2.D0-G)+7.D0))                     M.N31
ISN 0029             L2=3.D0*((G-1.D0)+DSQRT((2.D0-G)*(2.D0-G)+7.D0))                     M.N30
ISN 0030             IF(H.LT.0.D0)GO TO 300                                               M.N32
ISN 0032             F=-(L1+1.8D1)/1.4D1                                                  M.N33
ISN 0033             RETURN                                                               M.N34
ISN 0034         300 CONTINUE                                                             M.N35
ISN 0035             F=-1.4D1/(L1+1.8D1)                                                  M.N36
ISN 0036             RETURN                                                               M.N37
ISN 0037             END                                                                  M.N38
```

Appendix II. Computer Program to Solve Equations of Conservation and Determine the Pellet Injection Velocity Requirement

```
         C*******************************************************************************
         C***                                                                         ***
         C***                                                                         ***
         C***   PROGRAM BY KENNETH D. MATNEY.                                         ***
         C***                                                                         ***
         C***                                                                         ***
         C***   THE ULTIMATE PURPOSE OF THIS PROGRAM IS TO DETERMINE FUEL PELLET      ***
         C***   VELOCITY REQUIREMENTS FOR TOKAMAK TYPE REACTORS USING AN EQUAL        ***
         C***   PARTS MIXTURE OF DEUTERIUM AND TRITIUM.                               ***
         C***                                                                         ***
         C***                                                                         ***
         C*******************************************************************************
ISN 0002          IMPLICIT REAL*8(A-H,O-Z)
ISN 0003          REAL EW(8),D,ER,MACH(2005),DENS(2005),RPCS(2005)
ISN 0004          COMMON/FDATA/PSI,GAMMA,T,A,B,C,DARG
ISN 0005          COMMON/HAMDAT/CRIT,IFRKG,LG
ISN 0006          COMMON R(1005),Y(1005,2),D(2),XINIT
ISN 0007          DIMENSION W1(2),W2(2),W3(2),W4(2),W(82),X(82),Y0(2),Y1(2),Y2(8)
ISN 0008        2 FORMAT(2(D34.27,6X))
ISN 0009        3 FORMAT(1PD22.15,8X,D22.15,13X,'MACH DATA ',I5)
ISN 0010        4 FORMAT(1PD22.15,8X,D22.15,13X,'DENS DATA ',I5)
ISN 0011        5 FORMAT(4(D16.9,4X))
ISN 0012        6 FORMAT(I6,4X,D21.14)
ISN 0013       10 FORMAT(///)
ISN 0014       11 FORMAT(1H1,115X,'PAGE ',I5)
ISN 0015       12 FORMAT(1H0,5X,'THE VALUE OF THE LOSS FUNCTION IS ',1PD25.18)
ISN 0016       13 FORMAT(1H0,5X,'THE VALUE OF THE PELLET RADIUS IS ',1PD25.18/6X,
                 $'THE VALUE OF THE ABLATANT VELOCITY AT THE BOUNDARY IS ',D25.18)
ISN 0017       14 FORMAT(1H0,5X,'THE VALUE OF THE HEATING PARAMETER IS ',1PD25.18)
ISN 0018       15 FORMAT(1H0,5X,'THE STABILITY POINTS ARE ',1PD25.18,' AND ',D25.18)
ISN 0019       17 FORMAT(1H0,5X,'THE VALUE OF THE DESIRED CONSTANT IS ',1PD25.18)
ISN 0020       18 FORMAT(1H0,5X,'THE BOUNDARY MACH NUMBER IS ',1PD25.18/
                 $6X,'THE CONVERGENCE EFFICIENCY IS ',D25.18)
ISN 0021       19 FORMAT(1H0,5X,I4,7X,1PD25.18,2(10X,D25.18))
ISN 0022       20 FORMAT( 6X,'TABLE',I4,'.  INTEGRATION DATA.'/6X,29(1H-))
ISN 0023       21 FORMAT( 6X,'TABLE',I4,'.  INTEGRATION DATA -- CONTINUED.'/6X,42(1H
                 $-))
ISN 0024       22 FORMAT(1H0,5X,'R(',I6,') = ',1PD25.18,', WHEN M = ',D25.18,', AND'
                 $,/6X,'RHO = ',D25.18,', WHERE DM/DR'' = ',D25.18)
ISN 0025       23 FORMAT(/1H0/1H0,5X,'*F(PSI) = ',1PD25.18)
ISN 0026       24 FORMAT(/1H0,5X,'STEP',5X,'VALUE OF THE RELATIVE POSITION',5X,'VALU
                 $E OF THE RELATIVE DENSITY ',5X,'VALUE OF ABLATANT MACH NUMBER '/
                 $6X,4('-'),3(5X,30('-')))
ISN 0027       25 FORMAT(/6X,'THE NUMBER OF INTEGRATIONS PERFORMED THIS STEP = ',I6)
ISN 0028       26 FORMAT(6X,'FOR THE PARAMETER L/A = ',1PD22.15,':'/)
ISN 0029       27 FORMAT(1H0,5X,'THE VALUE OF THE PELLET VELOCITY MUST BE ',1PD22.15
                 $,' TIMES THE PRESSURE CENTER DISTANCE.')
ISN 0030       28 FORMAT(1PD22.15,8X,D22.15,11X,'PELLET SPEED',I5)
ISN 0031          FL(E)=1.D+00/(2.35D+18+4.D+15*E+2.D+21/E/E)
ISN 0032          ND=1005
ISN 0033          LG=0
ISN 0034          IPAGE=1
ISN 0035          ITABLE=1
ISN 0036          K=0
ISN 0037          NSTART=1
ISN 0038          WRITE(6,11)IPAGE
```

```
ISN 0039            IPAGE=IPAGE+1                                                  A0057
ISN 0040            READ(5,6)N                                                     A0058
ISN 0041            DO 100 J=1,N,1                                                 A0059
ISN 0042            READ(5,2)X(J),W(J)                                             A0060
ISN 0043            X(J+N)=(1.D+00+X(J))/2.D+00                                    A0061
ISN 0044            X(J  )=(1.D+00-X(J))/2.D+00                                    A0062
ISN 0045            W(J  )=W(J)/2.D+00                                             A0063
ISN 0046            W(J+N)=W(J)                                                    A0064
ISN 0047        100 CONTINUE                                                       A0065
ISN 0048            NSAVE=N+N                                                      A0066
ISN 0049            N=NSAVE                                                        A0067
ISN 0050            READ(5,5)EMAX,DN,V0,RP                                         A0068
ISN 0051            E=1.6021892D-19                                                A0069
ISN 0052            XM=(2.01410222D+00+3.016049720+00)*1.660571D-27                A0070
ISN 0053            PI=3.141592653589793D+00                                       A0071
ISN 0054            GAMMA=1.4D+00                                                  A0072
ISN 0055            PSI=0.D+00                                                     A0073
ISN 0056            DO 105 J=1,N,1                                                 A0074
ISN 0057            PSI=PSI+W(J)*FL(EMAX*X(J))                                     A0075
ISN 0058        105 CONTINUE                                                       A0076
ISN 0059            WRITE(6,12)PSI                                                 A0077
ISN 0060            WRITE(6,13)RP,V0                                               A0078
ISN 0061            PSI=PSI*(GAMMA-1.D+00)/2.D+00*E/XM*RP/V0**3*DSQRT(E*EMAX/3.D+00/PI A0079
                   $/1.660571D-27/548.579D-06)*DN                                  A0080
ISN 0062            T= MAX*2.D+00/3.D+00                                           A0081
ISN 0063            READ(5,2)A1SAVE,A2SAVE                                         A0082
ISN 0064            S=-1.D+00/3.D+00                                               A0083
ISN 0065            CRIT=DSQRT(5.D+00/GAMMA)                                       A0084
ISN 0066            READ(5,6)NSAV,HSAV                                             A0085
ISN 0067            READ(5,6)NTABLE                                                A0086
ISN 0068        110 CONTINUE                                                       A0087
ISN 0069            READ(5,2,END=999)XMACH,DELTA                                   A0088
              C****                                                            ****A0N01
              C****                                                            ****A0N02
              C**** THE PURPOSE OF THIS, AN OBVIOUS ADDITION TO THE MAIN PROGRAM, IS ****A0N03
              C**** TO REDUCE THE CPU BUT MAINTAIN THE INTEGRITY IN COMPUTATION.   ****A0N04
              C**** SINCE THE FOLLOWING SECTION ONLY COMPUTES THE VALUE OF THE     ****A0N05
              C**** VARIABLE, CONST; IT IS SUBMITTED HERE AS COMPUTED IN EARLIER RUNS****A0N06
              C**** IN THE FORM OF A DATA STATEMENT.  THE SECTION OF PROGRAM WHICH ****A0N07
              C**** FOLLOWS IS THEREFORE BYPASSED, YIELDING A CPU SAVINGS OF ABOUT ****A0N08
              C**** 90%.                                                       ****A0N09
              C****                                                            ****A0N10
              C****                                                            ****A0N11
ISN 0070            DATA CONST/1.0977634558861560D0/                               A0N12
ISN 0071            IF(IPAGE.GT.0)GO TO 337                                        A0N13
ISN 0073            ROSAV=1.D+00                                                   A0089
ISN 0074            RHOSAV=1.D+00                                                  A0090
ISN 0075            NINC=5.D-3/HSAV+5.D-3                                          A0091
ISN 0076            IF(NINC.LT.1)NINC=1                                            A0092
ISN 0078            NEWTAB=0                                                       A0093
ISN 0079            INITIL=1                                                       A0094
ISN 0080            H=HSAV                                                         A0095
ISN 0081            RO=ROSAV                                                       A0096
ISN 0082            YO(1)=XMACH                                                    A0097
ISN 0083            YO(2)=RHOSAV                                                   A0098
ISN 0084            FPSI=0.D+00                                                    A0099
ISN 0085            NC=2                                                           A0100
ISN 0086            A1=A1SAVE                                                      A0101
```

```
ISN 0087          A2=A2SAVE                                                         A0102
ISN 0088          ER=1.E-14                                                         A0103
ISN 0089          IFRKG=-1                                                          A0104
ISN 0090          WRITE(6,14)PSI                                                    A0105
ISN 0091          WRITE(6,15)A1SAVE,A2SAVE                                          A0106
ISN 0092          WRITE(6,19)INITIL,ROSAV,RHCSAV,XMACH                              A0107
ISN 0093          WRITE(6,23)FPSI                                                   A0108
ISN 0094          WRITE(6,11)IPAGE                                                  A0109
ISN 0095          IPAGE=IPAGE+1                                                     A0110
ISN 0096          NSTORE=0                                                          A0111
ISN 0097          IF(NEWTAB.EQ.0)WRITE(6,20) ITABLE                                 A0112
ISN 0099          IF(NEWTAB.EQ.1)WRITE(6,21) ITABLE                                 A0113
ISN 0101          WRITE(6,24)                                                       A0114
ISN 0102          NEWTAB=1                                                          A0115
ISN 0103          LINES=0                                                           A0116
ISN 0104      120 CONTINUE                                                          A0117
ISN 0105          N=1003                                                            A0118
ISN 0106          CALL HAMMIN(RO,YO,NC,N,H,A1,A2,ER,R,Y,I,Y2,Y1,W1,W2,W3,W4,D,EW,ND) A0119
ISN 0107          NSTORE=NSTORE+1000                                                A0120
ISN 0108          N=1001                                                            A0121
ISN 0109          DO 200 J=3,N,2                                                    A0122
ISN 0110          FPSI=FPSI+(Y(J-2,2)+4.D+00*Y(J-1,2)+Y(J,2))*CABS(H)/3.D+00        A0123
ISN 0111      200 CONTINUE                                                          A0124
ISN 0112          DO 300 J=NSTART,N,NINC                                            A0125
ISN 0113          WRITE(6,19)J,R(J),Y(J,2),Y(J,1)                                   A0126
ISN 0114          K=K+1                                                             A0127
ISN 0115          RPOS(K)=R(J)                                                      A0128
ISN 0116          MACH(K)=Y(J,1)                                                    A0129
ISN 0117          DENS(K)=Y(J,2)                                                    A0130
ISN 0118          LINES=LINES+1                                                     A0131
ISN 0119          IF(LINES-20)300,250,250                                           A0132
ISN 0120      250 CONTINUE                                                          A0133
ISN 0121          IF(J+125 -N)275,300,300                                           A0134
ISN 0122      275 CONTINUE                                                          A0135
ISN 0123          WRITE(6,11)IPAGE                                                  A0136
ISN 0124          IPAGE=IPAGE+1                                                     A0137
ISN 0125          WRITE(6,21)ITABLE                                                 A0138
ISN 0126          WRITE(6,24)                                                       A0139
ISN 0127          LINES=0                                                           A0140
ISN 0128      300 CONTINUE                                                          A0141
ISN 0129          N=N-1                                                             A0142
ISN 0130          IFRKG=IFRKG+1                                                     A0143
ISN 0131          DO 320 J=1,4,1                                                    A0144
ISN 0132          DO 310 L=1,NC,1                                                   A0145
ISN 0133          Y(J,L)=Y(J+N,L)                                                   A0146
ISN 0134      310 CONTINUE                                                          A0147
ISN 0135          R(J)=R(J+N)                                                       A0148
ISN 0136      320 CONTINUE                                                          A0149
ISN 0137          NSTART=NINC+1                                                     A0150
ISN 0138          IF(NSTORE.GE.NSAV)GO TO 330                                       A0151
ISN 0140          GO TO 120                                                         A0152
ISN 0141      330 CONTINUE                                                          A0153
ISN 0142          WRITE(6,11)IPAGE                                                  A0154
ISN 0143          IPAGE=IPAGE+1                                                     A0155
ISN 0144          YNEFF=Y(N,1)/CRIT*1.D2                                            A0156
ISN 0145          WRITE(6,18)XMACH,YNEFF                                            A0157
ISN 0146          WRITE(6,25)NSTORE                                                 A0158
ISN 0147          DMDR=F(1,R,Y,N)                                                   A0159
```

```
ISN 0148          WRITE(6,22)NSTORE,R(N),Y(N,1),Y(N,2),DMDR
ISN 0149          VALUE=3.D0/4.D0/((2.D0+(GAMMA-1.D0)*Y(N,1)**2)/PSI/Y(N,1)**2/(1.D0
                 $+GAMMA*Y(N,1)**2))**S
ISN 0150          FPSI=FPSI+R(N)**(4.D0*S)*VALUE
ISN 0151          CONST=PSI**S/FPSI
ISN 0152          WRITE(6,23)FPSI
ISN 0153          WRITE(6,17)CONST
ISN 0154          IF(YNEFF.LT.90.)GO TO 345
ISN 0156          DO 333 IP=1,K,1
ISN 0157          PUNCH 3,RPOS(IP),MACH(IP),IP
ISN 0158      333 CONTINUE
ISN 0159          DO 336 IP=1,K,1
ISN 0160          PUNCH 4,RPOS(IP),DENS(IP),IP
ISN 0161      336 CONTINUE
ISN 0162      337 CONTINUE
ISN 0163          WRITE(6,11)IPAGE
ISN 0164          IPAGE=IPAGE+1
ISN 0165          XINIT=0.0D0
ISN 0166          ICOUNT=0
ISN 0167          XMD=2.01410222D0*1.660571D-27
ISN 0168          XME=548.579D-6*1.660571D-27
ISN 0169          XMH=1.00866522D0*1.660571D-27
ISN 0170          A=CONST/(PI*XME)**(1.D0/6.D0)*(XMD/2.059D2)**(4.D0/9.D0)*DSQRT(E)*
                 $(2.35D19)**(2.D0/3.D0)*(4.D0*PI/3.D0)**(5.D0/9.D0)*(1.5D0/1.6D0)**
                 $(7.D0/6.D0)*(3.D0/XMH)**(1.D0/3.D0)/T**(1.D0/6.D0)
ISN 0171          B=9.D0/3.76D3
ISN 0172          C=-4.096D2/3.807D0
ISN 0173          DARG=2.8208D1/1.0575D0
ISN 0174          FVOL=4.D0*PI/3.D0*RP*RP*RP*2.059D2/DN/XMD
ISN 0175      338 CONTINUE
ISN 0176          WRITE(6,26)XINIT
ISN 0177          H=1.D-2
ISN 0178          V1=G(T,H)/(XM/XMH)**(1.D0/3.D0)/FVOL**(5.D0/9.D0)/DN**(2.D0/9.D0)
ISN 0179          WRITE(6,27)V1
ISN 0180          H=1.D-3
ISN 0181          V2=G(T,H)/(XM/XMH)**(1.D0/3.D0)/FVOL**(5.D0/9.D0)/DN**(2.D0/9.D0)
ISN 0182          WRITE(6,27)V2
ISN 0183          XPENET=1.D0-XINIT
ISN 0184          ICOUNT=ICOUNT+1
ISN 0185          PUNCH 28,XPENET,V2,ICOUNT
ISN 0186          XINIT=1.D0-XPENET*2.D-1**1.D-2
ISN 0187          IF(MOD(ICOUNT,6).EQ.0)GO TO 339
ISN 0189          WRITE(6,10)
ISN 0190          GO TO 340
ISN 0191      339 CONTINUE
ISN 0192          WRITE(6,11)IPAGE
ISN 0193          IPAGE=IPAGE+1
ISN 0194      340 CONTINUE
ISN 0195          IF(V2.GT.1.D2)GO TO 338
ISN 0197      345 CONTINUE
ISN 0198          WRITE(6,11)IPAGE
ISN 0199          IPAGE=IPAGE+1
ISN 0200          ITABLE=ITABLE+1
ISN 0201          GO TO 110
ISN 0202      999 CONTINUE
ISN 0203          STOP
ISN 0204          END
```

```
         C*****************************************************************
         C***
         C***
         C***  THE PURPOSE OF F IS TO PROVIDE THE COUPLED DIFFERENTIAL
         C***  EQUATIONS REQUIRED BY RKGSDQ AND HAMMIN.  COMMON BLOCK FDATA IS
         C***  REQUIRED.  S IS THE VALUE OF PSI AND G IS THE VALUE OF GAMMA.
         C***  R IS THE DISTANCE FROM PELLET CENTER, RELATIVE TO PELLET RADIUS.
         C***
         C***  STATEMENT  NO.       PURPOSE OF STATEMENT
         C***  -------------        ----------------------------------------
         C***
         C***       10              DIFFERENTIAL EQUATION FOR MACH NUMBER, M
         C***       20              DIFFERENTIAL EQUATION FOR RELATIVE DENSITY, P
         C***       30              TEST FOR CONTINUITY AT M = 1.D+00
         C***       40              EQUATION FOR G(T,H)
         C***
         C***
         C*****************************************************************
ISN 0002       FUNCTION F(L,X,Y,J)
ISN 0003       IMPLICIT REAL*8(A-H,O-Z)
ISN 0004       REAL*8 M
ISN 0005       COMMON/FDATA/S,G,T,A,B,C,D
ISN 0006       COMMON/HAMCAT/CRIT,IFRKG,LG
ISN 0007       DIMENSION X(1),Y( 1005,1)
ISN 0008       CAPX(T,RA)=(1.D0-(RA)**2)**2*T
ISN 0009       IF(LG.EQ.4)GO TO 40
ISN 0011       A=1.D+00
ISN 0012       B=2.D+00
ISN 0013       M=Y(J,1)
ISN 0014       P=Y(J,2)
ISN 0015       R=X(J)
ISN 0016       IF(L-1)1,10,20
ISN 0017     1 WRITE(6,6)J,L
ISN 0018     6 FORMAT(1H0,2X,'...ERROR...AT STEP = ',I3,', DIFF EQ CALLED = ',I3)
ISN 0019       F=0.D+00
ISN 0020       RETURN
ISN 0021    10 F=(S*(P*M*R*R)**3*(A+G*M*M)-M/R*(B+(G-A)*M*M))/(A-M*M)
ISN 0022       RETURN
ISN 0023    20 CONTINUE
ISN 0024       IF(L-3)25,30,1
ISN 0025    25 F=B*P*M*M/(A-M*M)*(A/R-S*(P*R*R)**3)
ISN 0026       RETURN
ISN 0027    30 CONTINUE
ISN 0028       F=(S*(P*M*R*R)**3*(A+G*M*M)-M/R*(B+(G-A)*M*M))
ISN 0029       RETURN
ISN 0030    40 CONTINUE
ISN 0031       RA=X(J)
ISN 0032       IF(RA.GT.9.99999999D-01)GO TO 45
ISN 0034       DNUM=CAPX(T,RA)+B*CAPX(T,RA)**2+C/CAPX(T,RA)+D
ISN 0035       DENO=(1.D0/CAPX(T,RA)+B-4.D0*C/CAPX(T,RA)**3)**(1.D0/3.D0)
ISN 0036       F=DNUM/DENO*A
ISN 0037       RETURN
ISN 0038    45 CONTINUE
ISN 0039       F=0.D0
ISN 0040       RETURN
ISN 0041       END
```

```
          C*************************************************************************GTH01
          C***                                                                   ***GTH02
          C***                                                                   ***GTH03
          C***    THE PURPOSE OF THIS FUNCTION IS TO AID IN THE CALCULATION OF THE  ***GTH04
          C***    REQUIRED PELLET VELOCITY BY CORRECTING FCR TEMPERATURE AND        ***GTH05
          C***    DENSITY DISTRIBUTION EFFECTS.                                      ***GTH06
          C***                                                                   ***GTH07
          C***                                                                   ***GTH08
          C*************************************************************************GTH09
ISN 0002        FUNCTION G(T,H)                                                     GTH10
ISN 0003        IMPLICIT REAL*8(A-D,F-H,O-Z)                                        GTH11
ISN 0004        REAL*8 K1(1),K2(1),K3(1),K4(1)                                      GTH12
ISN 0005        COMMON/HAMDAT/CRIT,IFRKG,L                                          GTH13
ISN 0006        COMMCN X(1005),Y(1005,2),E(2),XINIT                                 GTH14
ISN 0007        M=1005                                                              GTH15
ISN 0008        NC=1                                                                GTH16
ISN 0009        L=4                                                                 GTH17
ISN 0010        X(1)=XINIT                                                          GTH18
ISN 0011        Y(1,1)=0.D0                                                         GTH19
ISN 0012        NSAV=(1.D0-X(1))/H+1.1D0                                            GTH20
ISN 0013     50 CONTINUE                                                            GTH21
ISN 0014        N=NSAV                                                              GTH22
ISN 0015        IF(NSAV.GT.1000)N=1000                                              GTH23
ISN 0017        DO 100 J=2,N,1                                                      GTH24
ISN 0018        CALL RKGSDQ(J,X,Y,NC,H,E,K1,K2,K3,K4,M)                             GTH25
ISN 0019    100 CONTINUE                                                            GTH26
ISN 0020        IF(NSAV.LE.1000)GO TO 150                                           GTH27
ISN 0022        NSAV=NSAV-999                                                       GTH28
ISN 0023        X(1)=X(N)                                                           GTH29
ISN 0024        Y(1,1)=Y(N,1)                                                       GTH30
ISN 0025        GO TO 50                                                            GTH31
ISN 0026    150 CONTINUE                                                            GTH32
ISN 0027        HNEW=1.D0-X(N)                                                      GTH33
ISN 0028        N=N+1                                                               GTH34
ISN 0029        CALL RKGSDQ(N,X,Y,NC,HNEW,E,K1,K2,K3,K4,M)                          GTH35
ISN 0030        G=Y(N,1)                                                            GTH36
ISN 0031        RETURN                                                              GTH37
ISN 0032        END                                                                 GTH38
```

```
      C*******************************************************************
      C***
      C***
      C***   THE PURPOSE OF HAMMIN IS TO SOLVE A SET OF SIMULTANEOUS FIRST
      C***   ORDER FUNCTIONALS OF Y WITH RESPECT TO X USING HAMMING'S FIFTH
      C***   ORDER PREDICTOR-CORRECTOR.  FOURTH ORDER RUNGA-KUTTA-GILL
      C***   PROVIDES STARTING VALUES WHICH ARE ITERATED UPON BY THREE OTHER
      C***   INTEGRATION FORMULAES UNTIL A CONSISTENT STARTER IS OBTAINED.
      C***
      C***
      C***   XO = INITIAL VALUE OF X.
      C***
      C***   YO = ARRAY OF INITIAL Y-VALUES.
      C***
      C***   NC = NUMBER OF COUPLED EQUATIONS.
      C***
      C***   H = INTEGRATION INCREMENT
      C***
      C***   N = NUMBER OF INTEGRATIONS TO BE PERFORMED.
      C***
      C***   A1,A2 = PREDICTOR-CORRECTOR PARAMETERS TO BE CHOSEN BY USER.
      C***
      C***   ERR = CONVERGENGE PARAMETER FOR STARTER.
      C***
      C***   X = ARRAY OF X-VALUES
      C***
      C***   Y = 2-DIMENSIONAL ARRAY OF Y-VALUES.
      C***
      C***   IER = ERROR PARAMETER.  IF IER=0, INTEGRAL INSIDE OF ASSYMPTOTES.
      C***                           IF IER=1, STARTER FAILED TO CONVERGE.
      C***                           IF IER=2, INTEGRAL DIVERGES.
      C***
      C***   YC,W1,W2,W3,W4,E = WORK ARRAYS OF LENGTH NC.
      C***
      C***   YP,ER  = WORK ARRAY OF MINIMUM SIZE (4,NC).
      C***
      C***   ND = DIMENSION SIZE OF Y IN MAIN PROGRAM.
      C***
      C***   TO GET DOUBLE PRECISION VERSION, REMOVE C'S FROM PC240 AND PC245.
      C***
      C***
      C*******************************************************************
ISN 0002    SUBROUTINE HAMMIN(XO,YO,NC,N,H,A1,A2,ERR,X,Y,IER,YP,YC,W1,W2,W3,W4
           $,E,ER,ND)
ISN 0003    REAL*8 XO,YO,H,A1,A2,X,Y,YP,YC,W1,W2,W3,W4,C1,C2,C3,C4,C5,C6,C7,EP
           $,EC,AO,BO,B1,B2,B3,BCN1,BCO,BC1,BC2,CRIT,F
ISN 0004    COMMON/HAMCAT/CRIT,IFRKG,LG3333
ISN 0005    DIMENSION YO(1),X(1),Y(ND,1),YP( 4,1),YC(1),E(1),W1(1),W2(1),W3(1)
           $,W4(1),ER( 4,1)
ISN 0006    IER=0
ISN 0007    C1=9.D+00
ISN 0008    C2=19.D+00
ISN 0009    C3=5.D+00
ISN 0010    C4=24.D+00
ISN 0011    C5=4.D+00
ISN 0012    C6=3.D+00
```

```
ISN 0013            C7=8.D+00
ISN 0014            EP=(2.51D+02-C2*A1-C7*A2)/6.D+00
ISN 0015            EC=(-C2+11.D+00*A1-C7*A2)/6.D+00
ISN 0016            A0=1.D+00-A1-A2
ISN 0017            B0=(55.D+00+C1*A1+C7*A2)/C4
ISN 0018            B1=(-59.D+00+C2*A1+32.D+00*A2)/C4
ISN 0019            B2=(37.D+00-C3*A1+C7*A2)/C4
ISN 0020            B3=(A1-C1)/C4
ISN 0021            BCN1=(C1-A1)/C4
ISN 0022            BC0=(C2+13.D+00*A1+C7*A2)/C4
ISN 0023            BC1=(-C3+13.D+00*A1+32.D+00*A2)/C4
ISN 0024            BC2=(1.D+00-A1+C7*A2)/C4
ISN 0025            IF(IFRKG.GE.0)GO TO 155
ISN 0027            X(1)=X0
ISN 0028            DO 100 L=1,NC,1
ISN 0029            Y(1,L) =Y0(L)
ISN 0030        100 CONTINUE
ISN 0031            DO 115 J=2,4,1
ISN 0032            CALL RKGSDQ(J,X,Y,NC,H,E,W1,W2,W3,W4,ND)
ISN 0033        115 CONTINUE
ISN 0034            JCHECK=0
ISN 0035        120 CONTINUE
ISN 0036            JCHECK=JCHECK+1
ISN 0037            DO 130 L=1,NC,1
ISN 0038            YP(2,L)=Y(1,L) +H*(C1*F(L,X,Y,1)+C2*F(L,X,Y,2)-C3*F(L,X,Y,3)+F(L,
                   $X,Y,4))/C4
ISN 0039            YP(3,L)=Y(1,L) +H*(F(L,X,Y,1)+C5*F(L,X,Y,2)+F(L,X,Y,3))/C6
ISN 0040            YP(4,L)=Y(1,L) +H*(F(L,X,Y,1)+C6*(F(L,X,Y,2)+F(L,X,Y,3))+F(L,X,Y
                   $,4))/C7*C6
ISN 0041            DO 125 J=2,4,1
ISN 0042            ER(J,L)=(Y(J,L)-YP(J,L))/Y(J,L)
ISN 0043            ER(J,L)=ABS(ER(J,L))
ISN 0044        125 CONTINUE
ISN 0045        130 CONTINUE
ISN 0046            ITEST=0
ISN 0047            DO 140 L=1,NC,1
ISN 0048            DO 135 J=2,4,1
ISN 0049            IF(ER(J,L).GT.ERR)ITEST=1
ISN 0051            Y(J,L)=YP(J,L)
ISN 0052        135 CONTINUE
ISN 0053        140 CONTINUE
ISN 0054            IF(JCHECK.GE.500)GO TO 145
ISN 0056            IF(ITEST.EQ.1)GO TO 120
ISN 0058        145 DO 150 L=1,NC,1
ISN 0059            YP(1,L)=0.D+00
ISN 0060            YC(L)=0.D+00
ISN 0061        150 CONTINUE
ISN 0062        155 CONTINUE
ISN 0063            DO 300 J=4,N,1
ISN 0064            X(J+1)=X(J)+H
ISN 0065            DO 175 L=1,NC,1
ISN 0066            YP(2,L)=YP(1,L)
ISN 0067        175 CONTINUE
ISN 0068            DO 200 L=1,NC,1
ISN 0069            YP(1,L)=A0*Y(J,L)+A1*Y(J-1,L)+A2*Y(J-2,L)+H*(B0*F(L,X,Y,J)+B1*F(L,
                   $X,Y,J-1)+B2*F(L,X,Y,J-2)+B3*F(L,X,Y,J-3))
ISN 0070        200 CONTINUE
ISN 0071            DO 225 L=1,NC,1
```

```
ISN 0072           Y(J+1,L)=YP(1,L)-(EP/(EP-EC))*(YP(2,L)-YC(L))                        PC575
ISN 0073      225 CONTINUE                                                              PC580
ISN 0074           DO 250 L=1,NC,1                                                      PC585
ISN 0075           YC(L)=A0*Y(J,L)+A1*Y(J-1,L)+A2*Y(J-2,L)+H*(BCN1*F(L,X,Y,J+1)+BC0*F   PC590
                  $(L,X,Y,J)+BC1*F(L,X,Y,J-1)+BC2*F(L,X,Y,J-2))                         PC595
ISN 0076      250 CONTINUE                                                              PC600
ISN 0077           DO 275 L=1,NC,1                                                      PC605
ISN 0078           Y(J+1,L)=YC(L)-(EC/(EP-EC))*(YP(1,L)-YC(L))                          PC610
ISN 0079      275 CONTINUE                                                              PC615
ISN 0080           IF(J-N)285,300,400                                                   PC620
ISN 0081      285 CONTINUE                                                              PC625
ISN 0082      300 CONTINUE                                                              PC635
ISN 0083           IER=ITEST                                                            PC640
ISN 0084      400 N=N+1                                                                 PC645
ISN 0085           RETURN                                                               PC650
ISN 0086           END                                                                  PC670
```

```
         C*************************************************************************
         C***
         C***
         C***   THE PURPOSE OF RKGSDQ IS TO SOLVE, USING RUNGA-KUTTA-GILL, A SET  ***
         C***   OF SIMULTANEOUS DIFFERENTIAL EQUATIONS, WHICH ARE FIRST ORDER IN  ***
         C***   A COMMON (DUMMY) VARIABLE.  THE QUADRATURE IS FOURTH ORDER.       ***
         C***                                                                     ***
         C***                                                                     ***
         C***   J IS THE ITERATE OF Y BEING SOLVED.                               ***
         C***                                                                     ***
         C***   X(J) IS THE DUMMY VARIABLE.                                       ***
         C***                                                                     ***
         C***   Y(L,J) IS THE SOLUTION TO THE L'TH DIFFERENTIAL EQUATION.         ***
         C***                                                                     ***
         C***   N IS THE NUMBER OF SIMULTANEOUS DIFFERENTIAL EQUATIONS.           ***
         C***                                                                     ***
         C***   H IS THE INCREMENT TO BE ADDED TO X(J-1) TO GET X(J).             ***
         C***                                                                     ***
         C***   E IS A PARAMETER USED IN THE RULE OF COLLATZ, OF LENGTH,N.        ***
         C***                                                                     ***
         C***   K1 IS A REAL WORK ARRAY OF LENGTH, N.                             ***
         C***                                                                     ***
         C***   K2 IS A REAL WORK ARRAY OF LENGTH, N.                             ***
         C***                                                                     ***
         C***   K3 IS A REAL WORK ARRAY OF LENGTH, N.                             ***
         C***                                                                     ***
         C***   K4 IS A REAL WORK ARRAY OF LENGTH, N.                             ***
         C***                                                                     ***
         C***                                                                     ***
         C*************************************************************************
ISN 0002        SUBROUTINE RKGSDQ(J,X,Y,N,H,E,K1,K2,K3,K4,M)
ISN 0003        REAL*8  X,Y,H,K1,K2,K3,K4,AMD,ADD,AMS,AM,AD,AX,F
         C      REAL K1,K2,K3,K4
ISN 0004        DIMENSION X(1),Y(M,1),E(1),K1(1),K2(1),K3(1),K4(1)
ISN 0005        AMD=2.D+00
ISN 0006        ADD=1.D+00+DSQRT(5.D-01)
ISN 0007        AMS=5.D-01-DSQRT(5.D-01)
ISN 0008        AM=2.D+00-DSQRT(2.D+00)
ISN 0009        AD=2.D+00+DSQRT(2.D+00)
ISN 0010        AX=6.D+00
ISN 0011        X(J)=X(J-1)
ISN 0012        DO 100 L=1,N,1
ISN 0013        Y(J,L)=Y(J-1,L)
ISN 0014    100 CONTINUE
ISN 0015        DO 110 L=1,N,1
ISN 0016        K1(L)=F(L,X,Y,J)*H
ISN 0017    110 CONTINUE
ISN 0018        DO 120 L=1,N,1
ISN 0019        Y(J,L)=Y(J,L)+(K1(L)/AMD)
ISN 0020    120 CONTINUE
ISN 0021        X(J)=X(J)+(H/AMD)
ISN 0022        DO 130 L=1,N,1
ISN 0023        K2(L)=F(L,X,Y,J)*H
ISN 0024    130 CONTINUE
ISN 0025        DO 140 L=1,N,1
ISN 0026        Y(J,L)=Y(J,L)+(K2(L)-K1(L))*(AMD-ADD)
```

```
ISN 0027      140 CONTINUE
ISN 0028          DO 150 L=1,N,1
ISN 0029          K3(L)=F(L,X,Y,J)*H
ISN 0030      150 CONTINUE
ISN 0031          DO 160 L=1,N,1
ISN 0032          Y(J,L)=Y(J,L)+AMS*K1(L)-K2(L)+ADD*K3(L)
ISN 0033      160 CONTINUE
ISN 0034          X(J)=X(J-1)+H
ISN 0035          DO 170 L=1,N,1
ISN 0036          K4(L)=F(L,X,Y,J)*H
ISN 0037      170 CONTINUE
ISN 0038          DO 180 L=1,N,1
ISN 0039          Y(J,L)=Y(J-1,L)+(K1(L)+AM*K2(L)+AD*K3(L)+K4(L))/AX
            C     E(L)=(K2(L)-K3(L))/(K2(L)-K1(L))
            C     E(L)=ABS(E(L))
ISN 0040      180 CONTINUE
ISN 0041          RETURN
ISN 0042          END
```

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CONCEPTUAL DESIGN OF A COMMERCIAL-TOKAMAK-HYBRID-REACTOR FUELING SYSTEM

by

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ABSTRACT

A conceptual design of a fuel injection system for CTHR (Commercial Tokamak Hybrid Reactor) is discussed. Initially, relative merits of the cold-fueling concept are compared with those of the hot-fueling concept; that is, fueling where the electron temperature is below 1 eV is compared with fueling where the electron temperature exceeds 100 eV. It is concluded that cold fueling seems to be somewhat more free of drawbacks than hot fueling. Possible implementation of the cold-fueling concept is exploited via frozen-pellet injection. Several methods of achieving frozen-pellet injection are discussed and the light-gas-gun approach is chosen from these possibilities. A modified version of the ORNL Neutral Gas Shielding Model is used to simulate the pellet injection process. From this simulation, the penetration-depth dependent velocity requirement is determined. Finally, with the velocity requirement known, a gas-pressure requirement for the proposed conceptual design is established. The cryogenic fuel-injection and fuel-handling systems are discussed. A possible way to implement the conceptual device is examined along with the attendant effects on the total system.