ANALYSIS OF OPENINGS IN REINFORCED CONCRETE BEAMS

by

SURENDAR MOHAN MALHOTRA

B. E. (Honors) Civil Engineering, 1964

Govt. Engineering College, Jabalpur University

Jabalpur, India

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Major Professor

TABLE OF CONTENTS

GENERAL NOMENCLATURE

A = area.

A_{st} = total area of longitudinal steel.

A_s = area of tension reinforcement.

A'_s = area of compression reinforcement.

A_v = total area of web reinforcement in tension within a distance "S", measured in a direction parallel to the longitudinal reinforcement.

C = compressive force.

D = overall depth of beam.

E_c = modulus of elasticity of concrete.

E_s = modulus of elasticity of steel.

F = direct force.

G = modulus of rigidity.

I = moment of inertia.

K = multiplying factor for shear distribution in top chord of opening.

K_1 = 0.85 for strengths, f'_c, up to 4000 psi.

M = bending moment.

M_u = ultimate bending moment.

M_b = moment capacity at balanced conditions in a column = $P_b\ e_b$.

P = applied load.

P_u = ultimate load.

P_b = axial load capacity of a column at balanced conditions.

P_o = axial load capacity of a column when concentrically loaded.

S = spacing of vertical stirrups.

T = tensile force.

V = shear force.

V_u = total ultimate shear.

V'_u = ultimate shear carried by web reinforcement.

Z = section modulus.

a = depth of equivalent rectangular stress block.

a_b = depth of equivalent rectangular stress block for balanced conditions = $K_1 c_b$.

b = width of beam.

c = distance from extreme compression fiber to neutral axis.

c_b = distance from extreme compression fiber to neutral axis for balanced conditions = $(87000)d/(87000+f_y)$.

d = effective depth.

d' = distance from extreme compression fiber to centroid of compression reinforcement.

d'' = distance from plastic centroid to centroid of tension reinforcement.

e = actual eccentricity.

e_b = eccentricity of load P_b measured from plastic centroid of section.

f'_c = compressive stress of concrete at 28 days.

f''_c = compressive stress of concrete on the day of test.

f_y = yield stress of reinforcement.

ℓ = clear span of the beam.

ℓ' = length of the chords of the opening.

n = ratio of modulus of elasticity of steel (E_s) to that of concrete (E_c).

p = A_s/bd .

p_b = reinforcement ratio producing balanced conditions at ultimate strength.

t = lever arm between C and T.

v = shear stress.

v_c = shear stress carried by concrete.

v_u = nominal ultimate shear stress.

v'_u = ultimate shear stress carried by web reinforcement.

γ_{xy} = unit shearing strain.

ε_a = horizontal unit strain reading in rosette.

ε_b = unit strain reading at 45° with horizontal in rosette.

ε_c = vertical unit strain reading in rosette.

ε_x = longitudinal unit strain.

ε_y = vertical unit strain.

σ_x = longitudinal stress.

τ_{xy} = shearing stress.

SYNOPSIS

This thesis presents an investigation of simply supported, reinforced concrete beams each with one rectangular opening centered at the quarter point of the clear span.

Four beams with different configurations of shear stirrups in the chords of the opening were tested. The design of the longitudinal steel in the chords as well as in the main portion of the beam was on the basis of ultimate load theory and was the same in all four beams.

The experimental results indicate that the shear stress is greater in the top chord than in the bottom chord, although by a small amount. Also the bottom chord deflects more than the top chord in all the cases. The points of contraflexure for each beam were found to be at about the centers of the chords as assumed.

INTRODUCTION

Modern building construction calls for the need to provide passage for utility ducts and pipes under the roof and between the floors. The present practice is to pass these ducts under the beams supporting the slab, thus increasing the space loss between the roof or floor slab and the false ceiling. Therefore it would be desirable to provide for the design of large openings in beams to facilitate passage of ducts and pipes through them instead of under them. This will decrease the dead space in false ceilings and will thus reduce the story heights of multistoried buildings, which will result in a more economical design.

Very little research has been done on the problem of hole reinforcement for reinforced concrete beams. Detailed experimental and theoretical investigations are needed to solve all aspects of this problem.

The basic objects of this thesis are:

(i) To investigate the ratio of distribution of shear stress in the top and bottom chords surrounding the opening of reinforced concrete beams.

(ii) To determine the amount of steel required to resist the shear stresses.

(iii) To observe the deflected shape of such a beam and its mode of failure.

(iv) To determine the ultimate load carrying capacity of the member under different conditions of shear reinforcement in the top and bottom chords of the opening.

In order to attain the objects listed above, tests were carried out on four reinforced concrete beams each with a single hole. All beams were simply supported and loaded at the center. The only variable parameter was the amount of shear reinforcement in the chords around the hole.

REVIEW OF LITERATURE

The research on this problem was first started in Sweden. Most of the literature is published in the Swedish language and only one translation in English was available; a report by Mogens Lorentsen (2) on holes in reinforced concrete girders. He used the method of influence lines and ultimate strength theory for designing the beams with openings. It was found that calculated loads for ultimate failure were less than those obtained by tests though the difference was negligible.

A. Acavalos (1) and H. R. Daniel (1) carried out research on reinforced concrete beams with large openings for their Master's theses at the University of Saskatchewan, Saskatoon, Canada.

A paper was written by Karim W. Nassar (1) in association with the above-mentioned engineers, based on their research.

The basic assumptions for designing a beam with large opening were taken by Nassar, A. Acavalos and H. R. Daniel (1) to be:

1. The top and bottom members of the beam at the opening behave in a manner similar to the chords of a vierendeel panel.
2. The portions of the beams at the opening, when not subjected to transverse loads, have points of contraflexure at their midspans.
3. When portions of the beams at the openings have adequate stirrups, they carry the external shear in proportion to their cross sectional areas.
4. The shear force concentration at the corners is equal to twice the simple shear force.

Test specimens were designed on the basis of these assumptions and Whitney's ultimate load theory. The experimental results showed that the assumptions were substantially correct. However, there were some discrepancies between the observed results and those predicted by the third assumption. There was some difference between the predicted and observed ultimate load. The smallest variation was 6% greater and the largest variation was observed to be 24% less than the theoretical.

Some work was performed on prototype prestressed concrete beams with large openings in them by H. S. Ragan and J. Warwark (3). A T-beam was tested to find the maximum load which could be applied.

In 1963 Edmund P. Segner (4) tested steel beams of I-section and studied rectangular openings parallel to the neutral axis of the beam. The study was limited to the reinforcement design of the web of the steel beam around the holes.

John E. Bower (5) experimented on steel I-beams with holes in the webs and the paper was published in the ASCE Structural Division Journal of Oct. 1966. The work was confined to determining the stress concentration around circular and rectangular holes not exceeding half the web width of steel beams. In the case of round holes the theory of elasticity seemed to apply but for rectangular holes the principle of Vierendeel analysis gave more satisfactory results. However, the extension of Vierendeel analysis to beams with holes having non-rectangular configurations seemed to be questionable because boundary conditions at the holes would not be satisfied.

W. Wright and J. G. Bysne (6) carried out some experimental work on stress concentrations in concrete. Tensile tests were performed on plain thin concrete specimens. The purpose was to ascertain the effects of holes of various shapes on the tensile strength. The results yielded the conclusion that the stress concentration factors was 3 in circular holes, 5.9 in square holes with corner radii of 0.014 d and $[1 + \frac{2a}{b}]$ for elliptical holes, where a and b were the semi-major and semi-minor axes of the ellipse.

THEORY

The design of the solid portion of the beam is based on Whitney's ultimate strength theory which was incorporated in the building code (ACI 318-63) (11).

Equations used from this code:

The balanced longitudinal steel ratio,

$$p_b = \frac{0.85\ K_1\ f'_c}{f_y} \times \frac{87000}{87000 + f_y} \qquad \text{(ACI 16-2)}$$

The actual longitudinal steel ratio,

$$p = \frac{A_s}{bd} \qquad \text{(ACI 1600)}$$

The ultimate bending moment,

$$M_u = \phi[A_s\ f_y(d-a/2)] \qquad \text{(ACI 16-1)}$$

The failure shear stress for unreinforced concrete,

$$v_c = \phi[1.9\sqrt{f'_c} + 2500\ \frac{pV\ d}{M}] \qquad \text{(ACI 17-2)}$$

The spacing of stirrups,

$$s = \frac{\phi\ f_y\ d\ A_v}{V'_u} \qquad \text{(ACI 17-4)}$$

ϕ, the capacity reduction factor is taken equal to one in all calculations.

DESIGN OF TOP AND BOTTOM CHORDS AT THE OPENING

The chord design is based on the following assumptions:

(i) Vierendeel action of the beam chords around the opening. The chords are assumed to be fixed at throat sections.

(ii) Imaginary hinges form at the midspan of the chords at points of contra-flexure. Shear bending and direct stress are considered to act on each chord.

(iii) The shear is shared by both top and bottom chords in some ratio, not necessarily according to the cross sectional areas.

The beams were designed to fail in shear around the opening in order to more clearly observe the effects of the different shear reinforcement ratios used.

The stresses at the roots of the chords around the opening were obtained simply by a summation of two effects at the opening: (i) the gross beam bending moment; and (ii) the shear bending moment. The stresses were calculated as for a cantilever using the above assumptions and simple column theory applied to the chord sections above and below the opening.

The assumed portion of the total shear carried by the top chord, K, was varied in different specimens as

$$\frac{(1-K)}{K} = 1,\ 0.5,\ 0.$$

The direct force in the chord,

$$F = \frac{M_{(midspan)}}{t}\ . \qquad (1)$$

where $M_{(midspan)}$ = bending moment, due to load at the center of beam, at midspan of the chords.

t = lever arm between C and T as shown in Fig. 1.

$$M_{(throat)} = V \times \frac{\ell'}{2}\ . \qquad (2)$$

where $M_{(throat)}$ = bending moment at the throat section due to the shear force in the chords.

ℓ' = length of the chord.

V = shear force at midspan of the chord.

The longitudinal steel in the chords is designed to resist the effects of $M_{(throat)}$ and F. The following equations based on ultimate theory (11) were used for designing the steel:

The "balanced" column load (assuming symmetrical reinforcement),

$$P_b = 0.85\ f'_c\ b\ a_b \qquad \text{(ACI 19-1)}$$

The "balanced" column moment,

$$M_b = P_b \times e_b = \phi[0.85\ f'_c\ b\ a_b(d-d''-\frac{a_b}{2}) + A'_s\ f_y(d-d'-d'') + A_s\ f_y\ d''] \qquad \text{(ACI 19-3)}$$

The "balanced" eccentricity,

$$e_b = \frac{M_b}{P_b} \qquad \text{(ACI 19-3)}$$

The actual eccentricity,

$$e = \frac{M_{(throat)}}{F}$$

When $F < P_b$

or $e > e_b$

the ultimate capacity of the member is controlled by tension.

When $F > P_b$

or $e < e_b$

the capacity is controlled by compression and

$$F = P_u = \frac{P_o}{1 + [(P_o/P_b)-1]e/e_b} \qquad \text{(ACI 19-8)}$$

where

$$P_o = \phi[0.85\ f'_c(A_g-A_{st}) + A_{st}\ f_y] \qquad \text{(ACI 19-7)}$$

DESIGN OF CORNER REINFORCEMENT

Assume a stress concentration at the corner of a rectangular opening to be twice the shear stress in the member. The reinforcement of the corner is designed to resist the diagonal tension and the re-bars are placed at an angle of 45° with the horizontal.

DESCRIPTION OF BEAM TESTS

Before fabricating the test beams an investigation was done on the properties of materials to be used.

For the concrete mix, Type I Portland cement, sand with fineness modulus of 2.8 and aggregate of 3/8" size were used.

Two mixes were tried:

(i) Cement:Sand:Aggregate - 1:2:4. Water cement ratio: 6 gallons per sack of cement.

(ii) Cement:Sand:Aggregate - 1:1.5:3. Water cement ratio: 6 gallons per sack of cement.

These ratios were based on the weight of these materials.

The compressive strength f'_c after 28 days was found to be 2800 psi for Mix I and 4500 psi for Mix II. It was decided to order "Ready Mix" of 1:2:3 with a water cement ratio of 6 gallons per sack of cement. This would give f'_c between 3500 psi and 4000 psi.

Although reinforcing bars were specified by the manufacturer to have a yield strength of 40 ksi, tensile tests were run to determine the actual f_y of the bars.

In order to determine strains on the concrete surface, electric resistance strain rosettes were used. It was found that wire strain gages of 3/4" size glued with Duco cement on a well ground, smooth, prepared surface of concrete will give satisfactory strain measurements up to the point where cracking starts in the concrete. The curing time of Duco cement was found to be within the limits of 24 to 36 hours.

Four test beams were designed on the basis of the assumed theory. They were 6 ft. long, 9 in. deep and 4.5 in. wide with a rectangular opening of the size 15 in. x 3 in. as shown in Fig. 2. The opening was centered on the quarter point of the clear span of the beam. Each beam had only one variable parameter; the amount of shear reinforcement in the chords of the opening. Beam No. 1 had no web shear reinforcement in either of the two chords of the opening. Beam No. 2 had all the web shear in the top chord and none in the bottom chord. In Beam No. 3 equal web shear reinforcement was provided and Beam No. 4 had 66% and 34% of the total web shear reinforcement, in the top and bottom chords respectively.

As determined by the design calculations presented in Appendix IV, 3 No. 6 bars were provided in the tension side of the beam, 2 No. 6 bars at the top to facilitate binding of bars and stirrups, 3 No. 6 bars in the top and bottom of the top and bottom chords of the opening, and 2 No. 4 bars placed at 45° to the horizontal at each corner. All the details of steel reinforcement are illustrated in Figs. 2 and 3.

The forms were made with 3/4 in. thick plywood and 2 in. x 4 in. wooden flats as shown in Fig. 4.

The yield strength of the different bars used as reinforcement obtained by tensile tests were:

#1 bar	$f_y = 78{,}000$ psi
#2 bar	$f_y = 54{,}000$ psi
#4 bar	$f_y = 54{,}000$ psi
#6 bar	$f_y = 44{,}500$ psi

Before the concreting was started, the forms were thoroughly cleaned and ensured to be water tight. A thin varnish coat was given inside the forms and they were left overnight to dry. A thin coating of grease was given to the inside surface of the forms, just prior to the concreting.

The concrete mix came in one batch and was used for all the specimens. Three cylinders were filled according to ASTM standards in order to run a compressive strength test after 28 days. Three test cylinders were filled for each beam in order to determine the actual compressive strength of the beams on the day of testing. The concrete was vibrated by using an immersion type vibrator and extra care was taken in areas around the opening to ensure proper consolidation of the concrete by rodding and surface vibration.

The beams along with the cylinders were moved into the moisture room the following day. Three cylinders were tested after 28 days of curing and f'_c was found to be 3.8 ksi.

After 42 days beams and cylinders were taken out of the moisture room. When the test beams were sufficiently dry, the surface at the midpoints of the top and bottom chords of the opening was ground with the help of a coarse emery disc.

The concrete surface could be ground smooth on the face which was open while concreting. This face will from now onward be designated as "face one". The surface on the form side which looked smooth before grinding, became rough after the grinding operation. This face will be denoted by "face two". The reason for these different surface conditions could be that it was easy to work on "face one" while concreting as compared to "face two" and hence the aggregate was much better bound by cement on "face one" as compared to "face two".

Dial gages were used for measuring the deflections of the top and bottom chords and the center deflection of the beam. Six gages were mounted on the top and bottom chords at a spacing of 3 in. as shown in Fig. 5. This arrangement remained the same for all of the test beams.

A single concentrated load was applied at the center of the beams and the beams were simply supported at the ends. Bearing plates were placed under the beams at the supports. A Tinius Olsen screw type, load testing machine with a capacity of 1000 kips, was used for loading the test beam as shown in Fig. 6.

A Budd Strainsert Model HW-1 portable strain indicator was used to measure the output of the strain gages. The setup of a beam being tested in the machine along with deflection gages, strain gages and strain indicator is shown in Fig. 7.

The strain gages used were BLH Electronics Type AR-2-S6 temperature compensated, 45° wire strain rosettes. They were glued at the midpoints of the top and bottom chords of the opening as illustrated in Fig. 5. Four rosettes were glued on Beam No. 1; two on "face one" of the beam at assumed points of contraflexure of the top and bottom chords and one on "face two" at the point of contraflexure of the bottom member. One rosette was glued on the top of the beam at a distance of 18.5 in. from the end support on the solid side of the beam. Only longitudinal strains were measured by this gage. The main purpose of this gage was to compare the experimental results of stresses to those calculated with conventional theory in order to test the behavior of the strain gage. The rest of the beams had three rosettes on them; two on "face one" at midpoints of top and bottom chords and one on "face two" at the midpoint of the top or bottom chord.

Beam No. 1 was loaded in increments of 500 lbs. until it reached the ultimate load. It was felt that the load increment could be adopted as 1000 lbs. and so the other three beams were loaded with this increment to their ultimate failure load.

Cracks, as they developed, were painted black with a "magic marker" and photographs were taken at different times and stages of different beam tests.

RESULTS OF BEAM TESTS

All test beams were subjected to a concentrated load applied at the mid-span of the beam. The opening in all the beams reduced the cross sectional area by 33 percent. All beams had everything constant except the shear reinforcement in the top and bottom chords of the opening.

Beam No. 1, as shown in Fig. 2 was provided with no shear reinforcement in either of the chords of the opening. Four electrical wire rosette strain gages as described in the last section were glued to the concrete surface. The surface where strain gage No. 1 was applied, was well prepared and smooth. The curing time of the gage pasted with Duco cement was 24 hours at normal room temperature. The experimental stress values obtained from this gage for different load levels show reasonable agreement to those calculated theoretically. Both curves are linear as shown in Fig. 14.

Strain gages No's. 2 and 3 were glued at midspan and middepth of the top and bottom chords of the opening respectively on "face one" of the beam. In the top chord the longitudinal stress calculated experimentally showed agreement with the theoretical up to a load of 6 kips. The curve after 7 kips load was linear up to a load of 10 kips and was almost parallel to the theoretical curve as shown in Fig. 15. The shear stress curve varied linearly with load up to a load value of 9 kips as illustrated in Fig. 16. In the bottom chord the longitudinal stress curve on the basis of strain readings behaved erratically as can be seen in Fig. 17. The stress after falling from a maximum value of 200 psi at 8 kips load to 0 at about 11 kips load, changed sign rapidly to a value of -404 psi at a load of 13 kips. The shear stress curve was linear up to a load of 3 kips followed by an erratic curve as noted in Fig. 18. Rosette No. 4 had a rough surface under it and did not function well; hence the data obtained from this gage was rejected. This was pasted at midpoint of the bottom chord on "face two" of the beam.

The ratio of the shear distribution for the bottom to top chord in general had a tendency to decrease with increasing load, that is, more and more shear was resisted by the top chord as compared to the bottom chord. As seen in Fig. 13 the ratio was 0.88 at a load value of 1 kip and came down to 0.685 at a load value of 12 kips. Although there are some extreme variations at a load of 3 kips and 8 kips, the general trend was that the ratio decreased with increase of load.

The deflected curve of the chords was "S" shaped as predicted; however, the bottom chord deflected more than the top chord as can be seen in Figs. 19 and 20. These figures also indicate that points of contraflexure are located near midspan of the chords.

The corners did not fail and the first corner cracks appeared after the beam had failed in shear around the opening.

The first shear cracks appeared in the top chord at a load of 14 kips followed by more shear cracks in the top and bottom chords as the load was increased. The mode of failure was by shear in the chords of the opening at an ultimate load of 17 kips. A heavier ultimate load than that calculated in the design is attributed to the heavy longitudinal re-bars which are assumed not to share any shear. Shear cracks in the chords after failure are illustrated in Fig. 8.

Beam No. 2 had three rosettes at midpoints of the chords, Nos. 3 and 4 strain gages were mounted at the top and bottom chords respectively on "face one", and No. 2 gage was at the top chord on "face two". Gage Nos. 3 and 4 having a smooth surface under them gave satisfactory readings whereas gage

No. 2 being on a rough surface behaved erratically and the results were rejected.

The curve of longitudinal stress in the top chord versus load was linear up to a load of 8 kips and the values were in agreement with those calculated theoretically as shown in Fig. 23. The shear stress curve was linear up to 8 kips load as shown in Fig. 24. The longitudinal stress curve in the bottom chord behaved almost in the same manner as in beam No. 1. This curve remained linear only up to 2 kips load as shown in Fig. 25. The shear stress curve for this point was linear up to about 6 kips load as can be seen in Fig. 26.

The ratio of the shear distribution between the bottom and top chords, as can be seen in Fig. 22, in general decreased with increase of load. If we neglect the ratios at 1 kip load and 13 kips load, the variation is 0.89 at 2 kips load to 0.378 at 12 kips load as can be seen in Fig. 22.

The chords deflected into an "S" shape again with points of contraflexure around midspans as can be seen in Fig. 27 and 28. The bottom chord also deflected much more than the top chord. This beam had all the shear reinforcement in the top chord and none in the bottom chord hence the top chord was much stiffer than the bottom chord.

A corner crack developed in the upper corner after the shear failure occurred as shown in Fig. 9.

Shear cracks were first noted in the bottom chord at a load of 15 kips followed by cracks in the top chord at 16 kips. The mode of failure was by shear in the chords at an ultimate load of 18.9 kips. The shear cracks in the chords around the opening can be seen in Fig. 9.

Three rosettes were pasted on beam No. 3 at the midpoints of the chords. Rosettes Nos. 2 and 4 in the top and bottom chords at the midpoints respectively on "face one" were cured for 24 hours. No. 3 strain gage was pasted at the midpoint of the top chord on "face two" and once again did not behave well. Thus the data from gage No. 3 was rejected.

Strain measurements in the top member were satisfactory as shown in Figs. 31 and 32. But in the bottom chord linear results could be obtained only up to a load of 4 kips as illustrated in Figs. 33 and 34.

The ratio of shear distribution had in general the same behavior as in the other beams, the ratio in this case varying from 0.777 at a load of 1 kip to 0.726 at a load of 9 kips. Values beyond 9 kips load were quite erratic as shown in Fig. 30 and Table 14.

The deflected shape as shown in Figs. 35 and 36 remained the same as in the other cases showing points of contraflexure around midspan. However, the magnitude of the deflections was smaller than all the other cases. The deflections in the bottom chord were once again greater than those in the top chord.

Shear cracks appeared in the top chord first at a load of 17 kips followed by cracks in the bottom chord. At a load of about 18 kips a shear crack appeared in the main beam and a corner crack in the lower right hand corner of the hole as shown in Fig. 10. The corner crack as well as the shear crack in the main body of the beam did not increase with higher loads. The mode of failure was by shear in the chords around the opening at an ultimate load of 21.8 kips.

Beam No. 4 had strain gages Nos. 2 and 4 glued on the top and bottom chords respectively on "face one" at midpoints of the chords. No. 3 rosette was pasted on "face two" of the top chord at midpoint on a rough surface and also gave erratic results which were not used. Strain gage Nos. 2 and 4 behaved in a manner similar to those at these points in other beams as can be seen in Figs. 39 and 40. The stress curves for the top chord were linear up to 6 kips load and in the case of the bottom chord the curves were linear up to a load of 3 kips as illustrated in Figs. 39, 40, 41 and 42.

The curve showing the ratio of shear distribution between the bottom and top chords in general decreases with increasing loads, as shown in Fig. 38. Refering to Fig. 38 and Table 20 the values are seen to vary from 0.73 at 2 kips load to 0.5 at 9 kips load after touching a lowest value of 0.36 at 7 kips load. The value at 10 kips load is 0.88. All values beyond 10 kips are very erratic.

As shown in Figs. 43 and 44, the deflected shape of the chords indicates that the location of the points of contraflexure are around midspan as assumed. The magnitude of deflections is much more in the bottom chord than in the top chord.

First shear cracks were observed in both the chords at a load of 16 kips followed by corner cracks in the bottom corner on the left hand side of the opening and the top corner on the right hand side of the opening with a shear crack in the main body of the solid side of the beam as illustrated in Fig. 11. The corner cracks and shear crack in the main body of the beam did not increase with load. The mode of failure was shear in the chord around the opening at an ultimate load of 18.8 kips.

In Fig. 12 the deflected shape of the chords and the crack patterns are magnified by loading the beam beyond the ultimate load.

The deflections in the center of the beams, although more than those of a solid beam of the same properties, gave the type of curve one would otherwise expect as illustrated in Figs. 21, 29, 37 and 45.

It is felt that the reason for the failure of the strain gages glued on the rough surface was due to the fact that it was not possible to fill the cavities of the rough surface with Duco cement without having entrapped air. When there is an air bubble under the strain gage it is not perfectly bonded to the concrete surface, hence it cannot give true measurements of the strains.

It is observed that the bottom chord deflected more than the top chord in all the cases. This may be due to the fact that the top chord is in compression whereas the bottom chord is in tension and concrete does not contribute as much resistance in tension as in compression.

In all the beams strain measurements in the bottom chords started becoming erratic at considerably lower loads than those in the top chords. The reason for this could be that the concrete was cracking in tension. The mode of failure of all the beams was due to shear in the chords as designed.

By examining Table 33 it is seen that the ultimate load carrying capacity of the beams varies with the variation in the shear reinforcement in the top and bottom chords of the opening.

CONCLUSIONS

(1) On the basis of the results obtained in this investigation it is observed that in none of the test beams was the ratio of distribution of shear between bottom and top chord more than 0.89 with the lowest ratio being 0.36. The corresponding ratios of shear reinforcement provided in the test beams were 1, 0.5 and 0 and the highest ultimate load was observed for a ratio of 1. Therefore it is felt that a reinforcement ratio of 0.85 to 0.9 should produce a higher ultimate load by utilizing both chords to their full capacity for resisting shear. In other words,

$$\frac{(1-K)}{K} = 0.9 \text{ to } 0.85$$

or

$$K = 0.525 \text{ to } 0.54 \quad .$$

It was also observed that when the value of K was kept equal to 0.5, shear cracks started in the top chord before they were noticed in the bottom chord, whereas when K was one, the bottom chord started having shear cracks. When K was kept equal to 0.66, the bottom chord had shear cracks before the top chord was noticed to have them. Therefore it seems very likely that $0.525 < K < 0.54$ could be the best ratio of shear reinforcement distribution to make a shear failure occur in top and bottom chords at the same time.

(2) Points of contraflexure were observed to be at or very near to the midpoints of the chords of the opening as assumed.

(3) The longitudinal steel for the main beam as well as for the chords can be satisfactorily designed on the basis of the methods given in the ACI Code (11).

(4) The assumption that the corner stress concentration factor is two seems to be valid for rectangular openings in concrete beams.

(5) The ultimate load in all cases of failure was observed to be higher than that predicted.

APPENDIX I

FIGURES

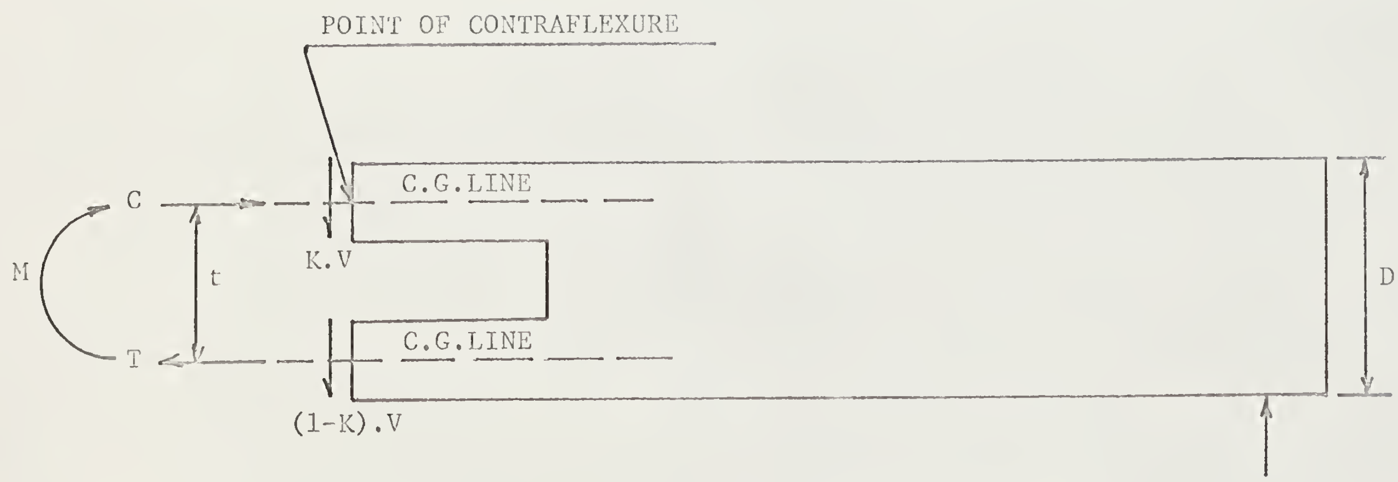


FIG. 1. FREE BODY DIAGRAM BASED ON THE ASSUMED THEORY.

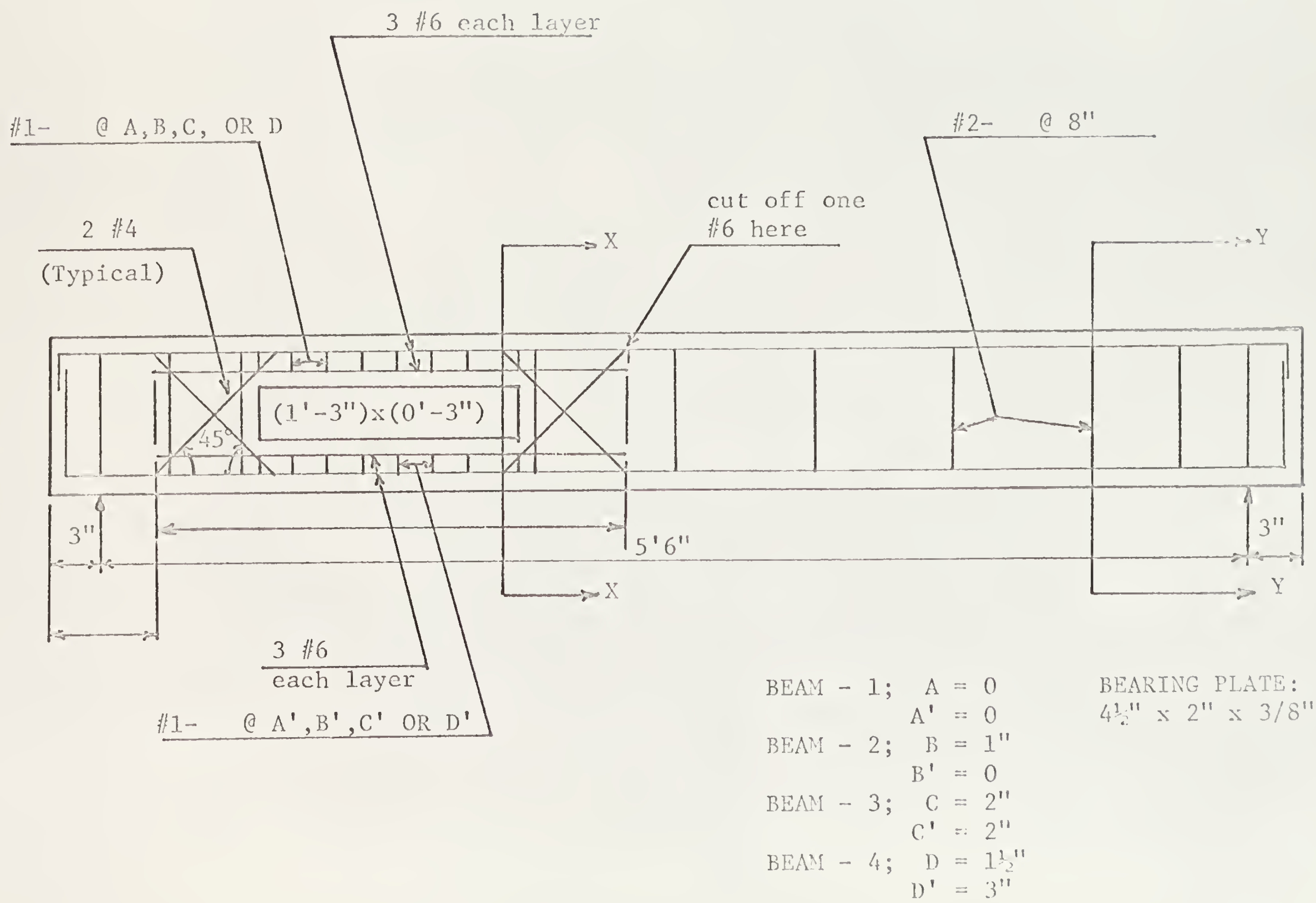
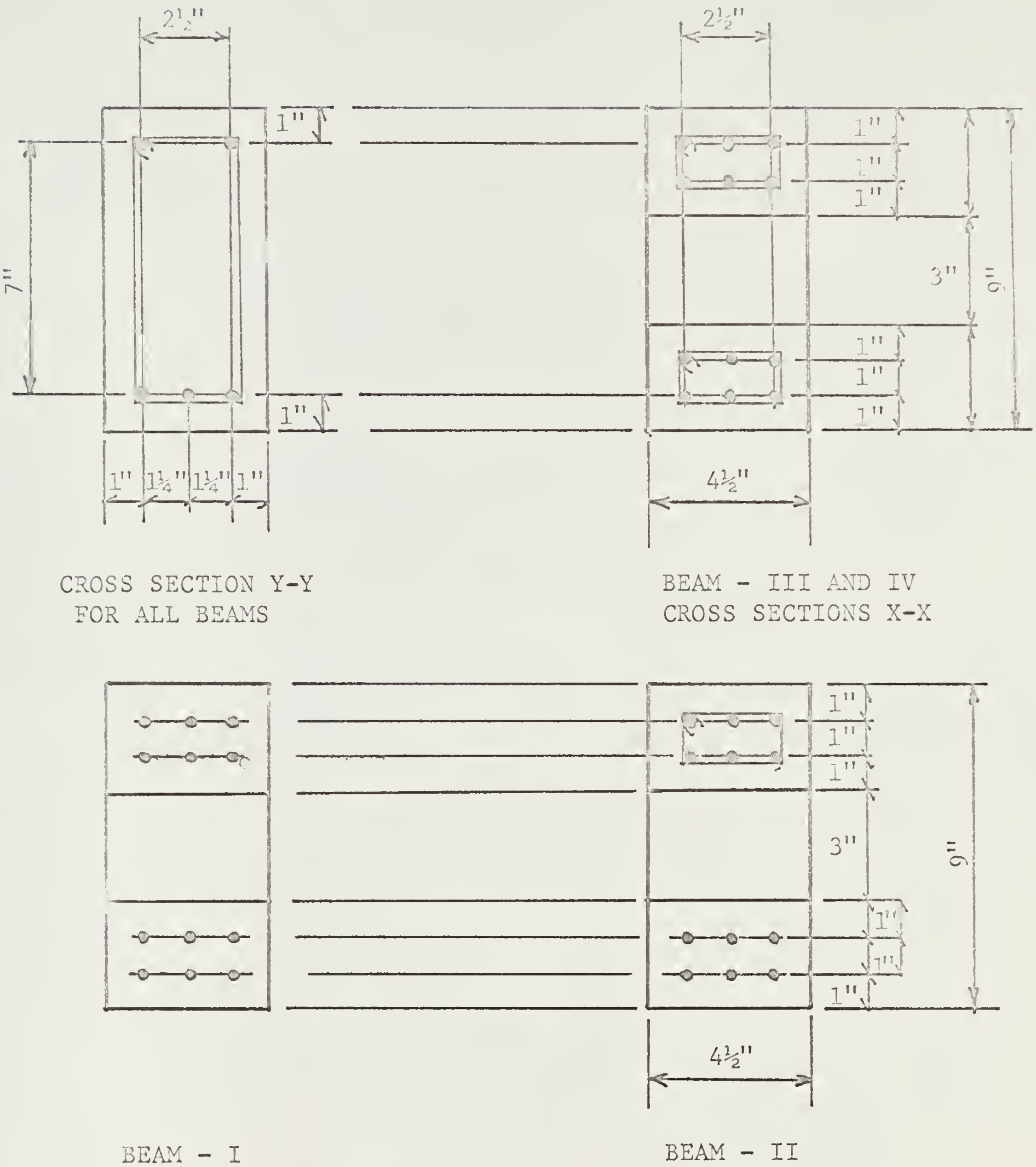


FIG. 2. DIMENSIONS AND REINFORCEMENT DETAILS OF TEST BEAMS.



CROSS SECTIONS X-X

FIG. 3. BEAM CROSS SECTIONS

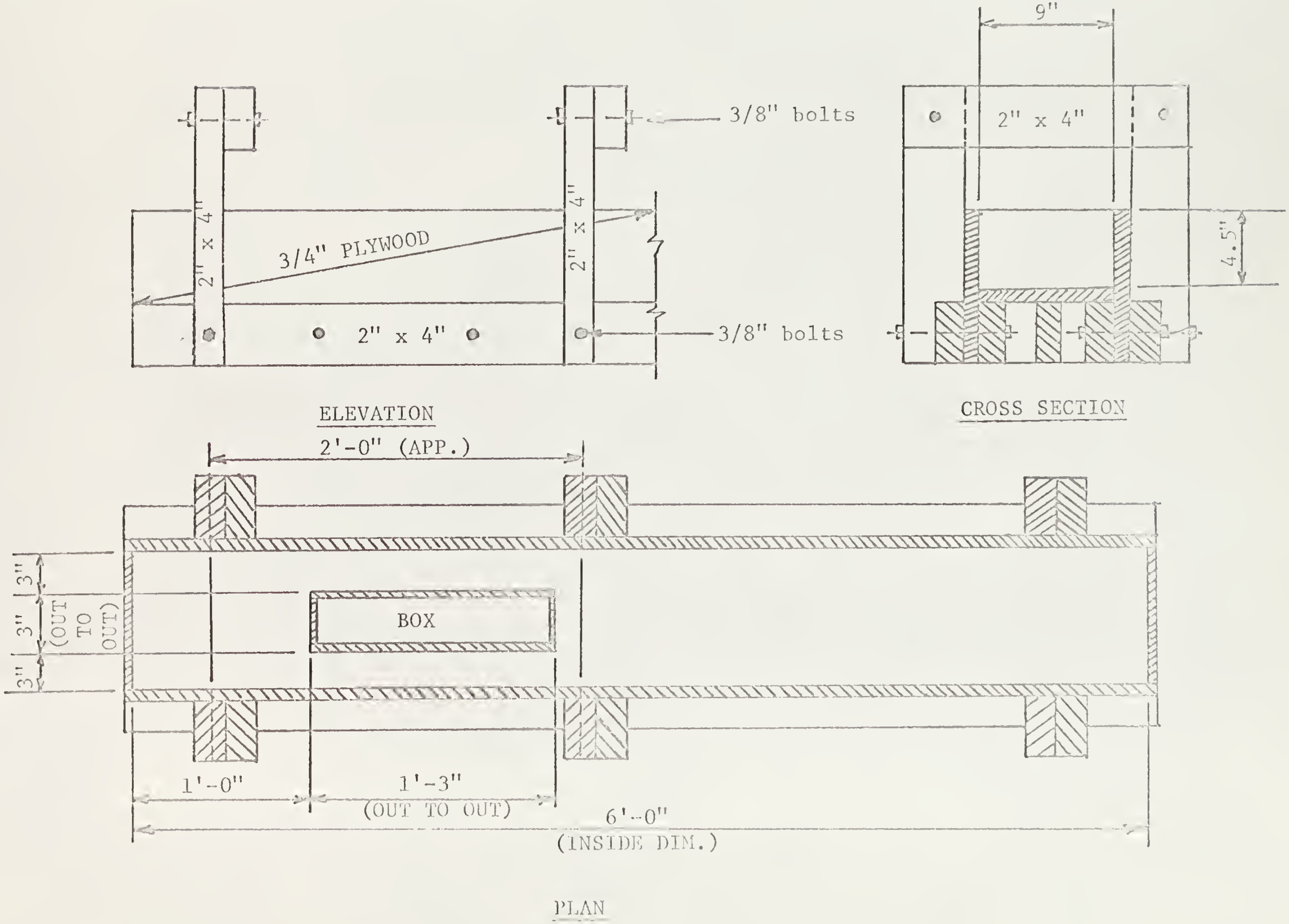

3/8" bolts
3/4" PLYWOOD
2" x 4"
2" x 4"
2" x 4"
3/8" bolts
ELEVATION
9"
2" x 4"
4.5"
CROSS SECTION
2'-0" (APP.)
3"
3"
(OUT
TO
OUT)
3"
BOX
1'-0"
1'-3"
(OUT TO OUT)
6'-0"
(INSIDE DIM.)
PLAN

FIG. 4. FORM

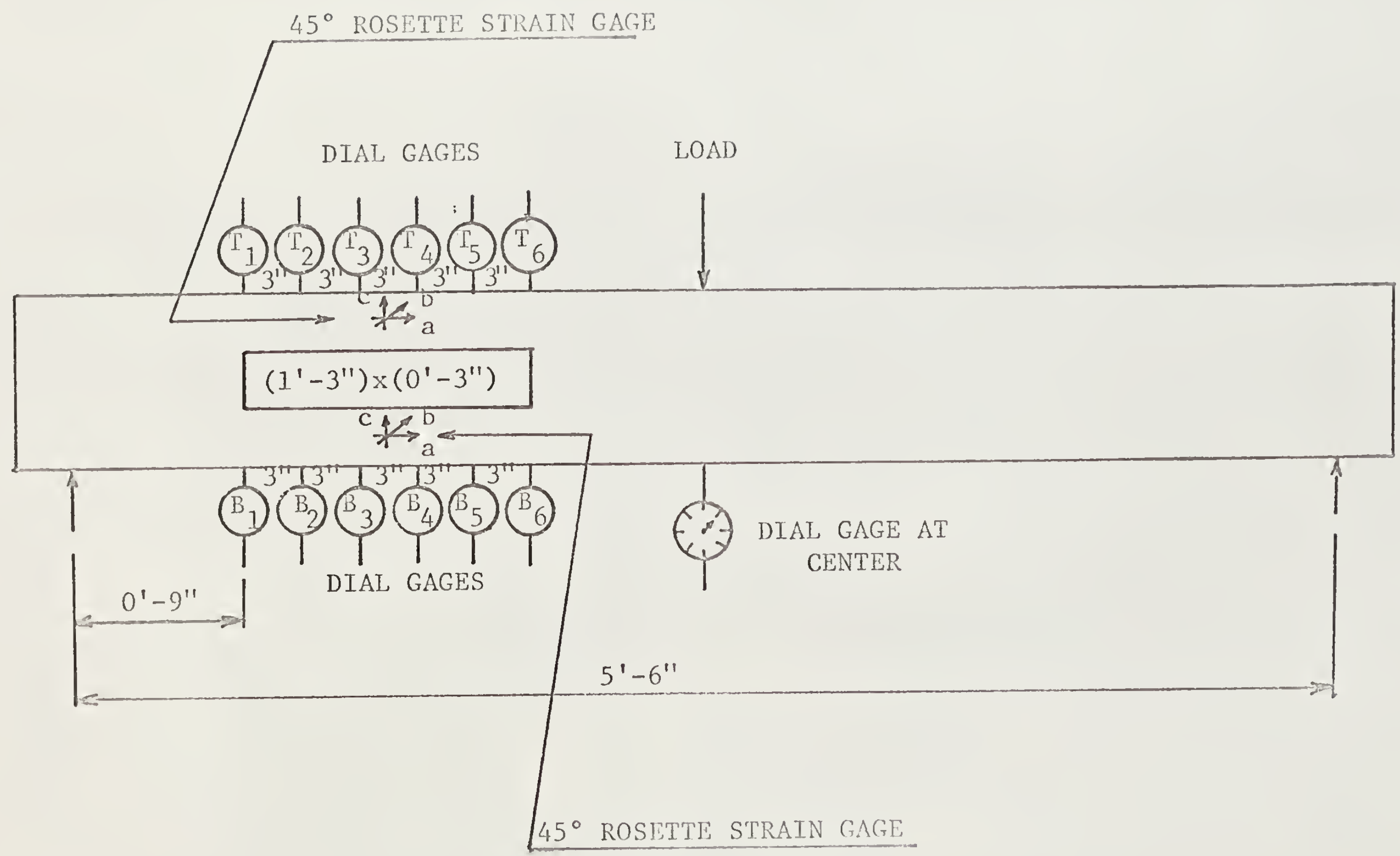


FIG. 5. ARRANGEMENT OF DIAL GAGES AND STRAIN GAGES FOR ALL TEST BEAMS.

FIG. 6. TEST BEAM IN MACHINE

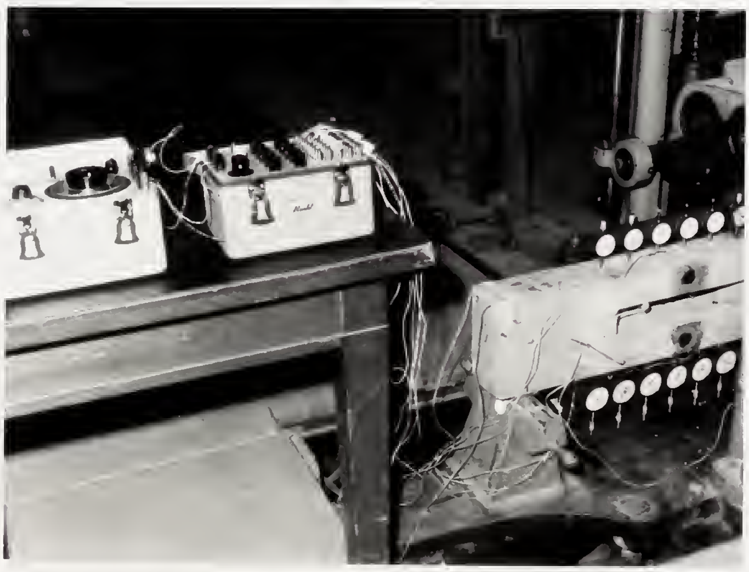


FIG. 7. DEFLECTION AND STRAIN MEASURING INSTRUMENTS WITH A TEST BEAM

FIGURE 8. BEAM #1 AFTER FAILURE

FIGURE 9. BEAM #2 AFTER FAILURE

FIGURE 10. BEAM #3 AFTER FAILURE

FIGURE 11. BEAM #4 AFTER FAILURE

FIGURE 12. BEAM #4 WITH THE DEFLECTED SHAPE OF THE CHORDS ACCENTUATED AFTER FAILURE

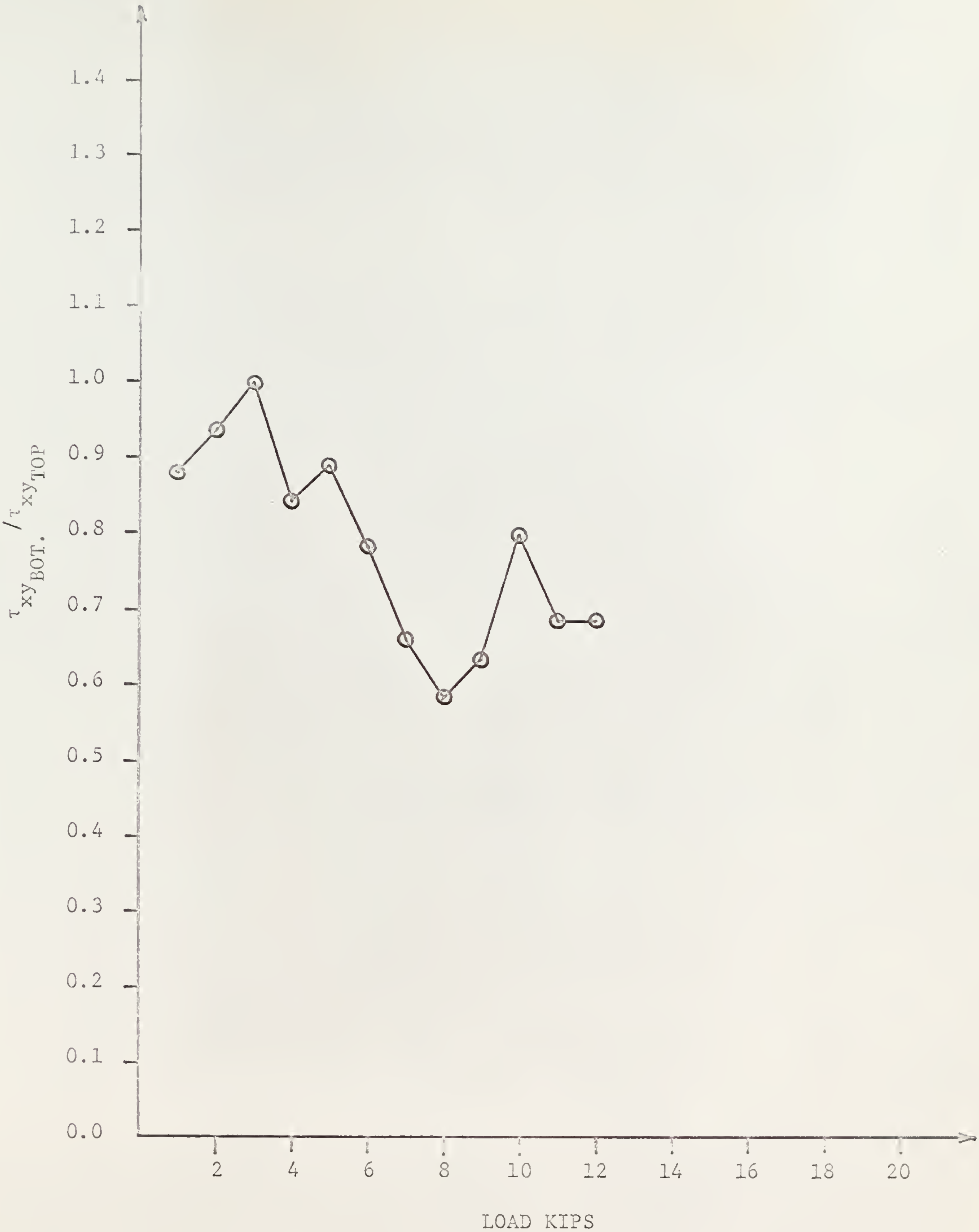


FIG. 13. $\tau_{xy_{BOT.}} : \tau_{xy_{TOP}}$ VS LOAD, BEAM #1.

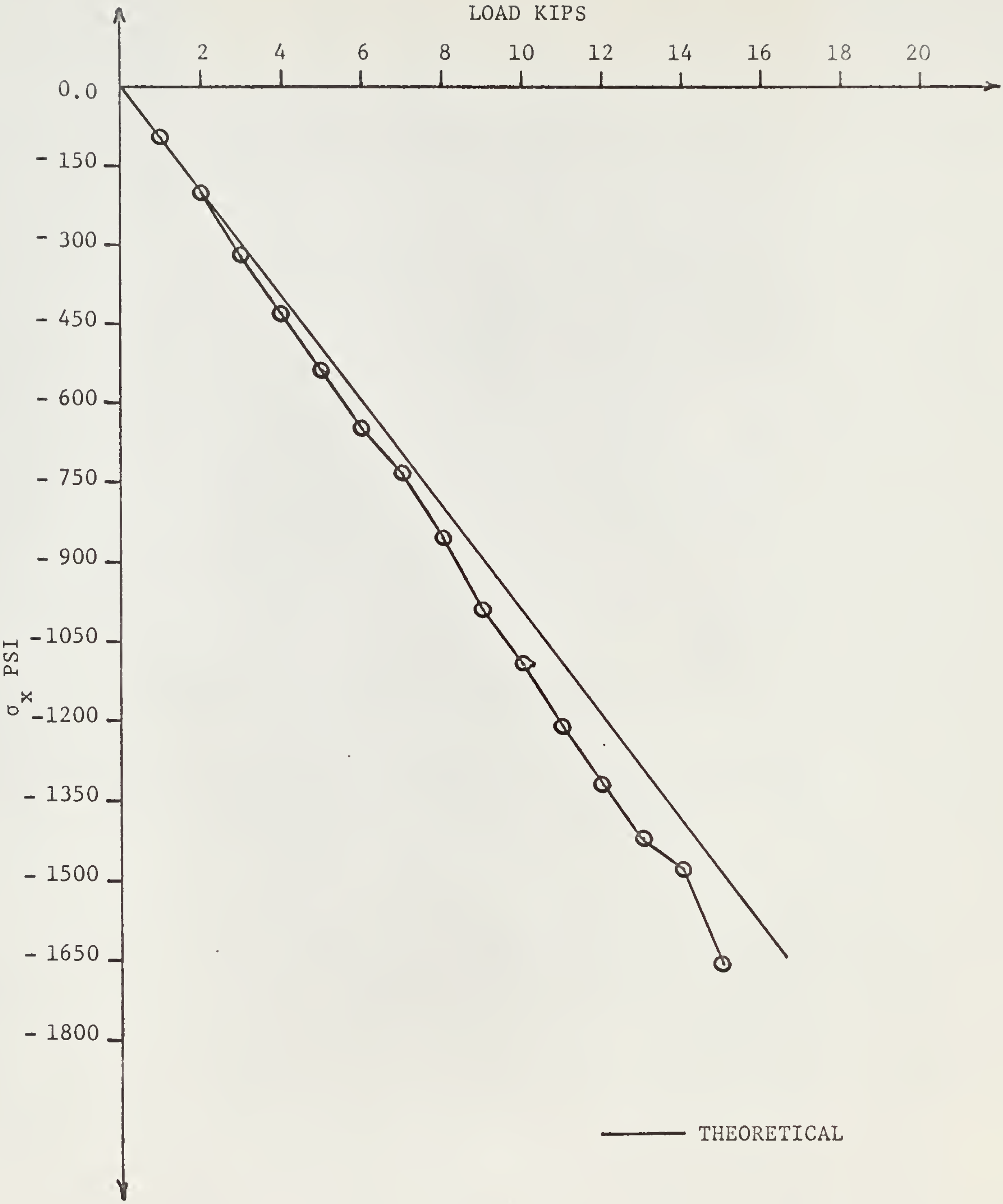


FIG. 14. LONGITUDINAL STRESS σ_x VS LOAD, TOP OF SOLID PORTION OF BEAM, 18½" FROM END SUPPORT.

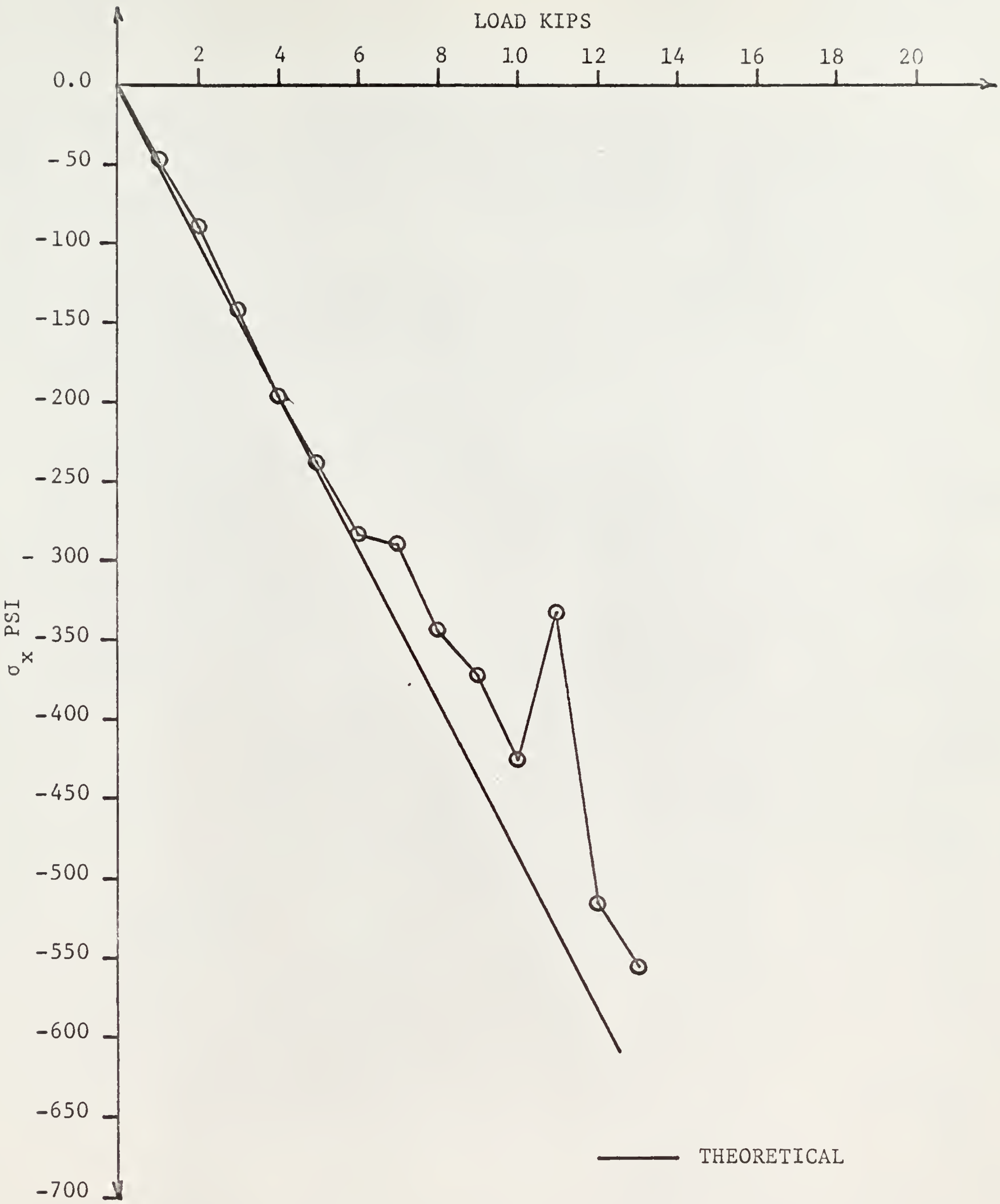


FIG. 15. LONGITUDINAL STRESS σ_x VS LOAD, BEAM #1, MIDPOINT TOP CHORD, STRAIN GAGE #2.

FIG. 16. SHEARING STRESS τ_{xy} VS LOAD, BEAM #1,
TOP CHORD, STRAIN GAGE #2.

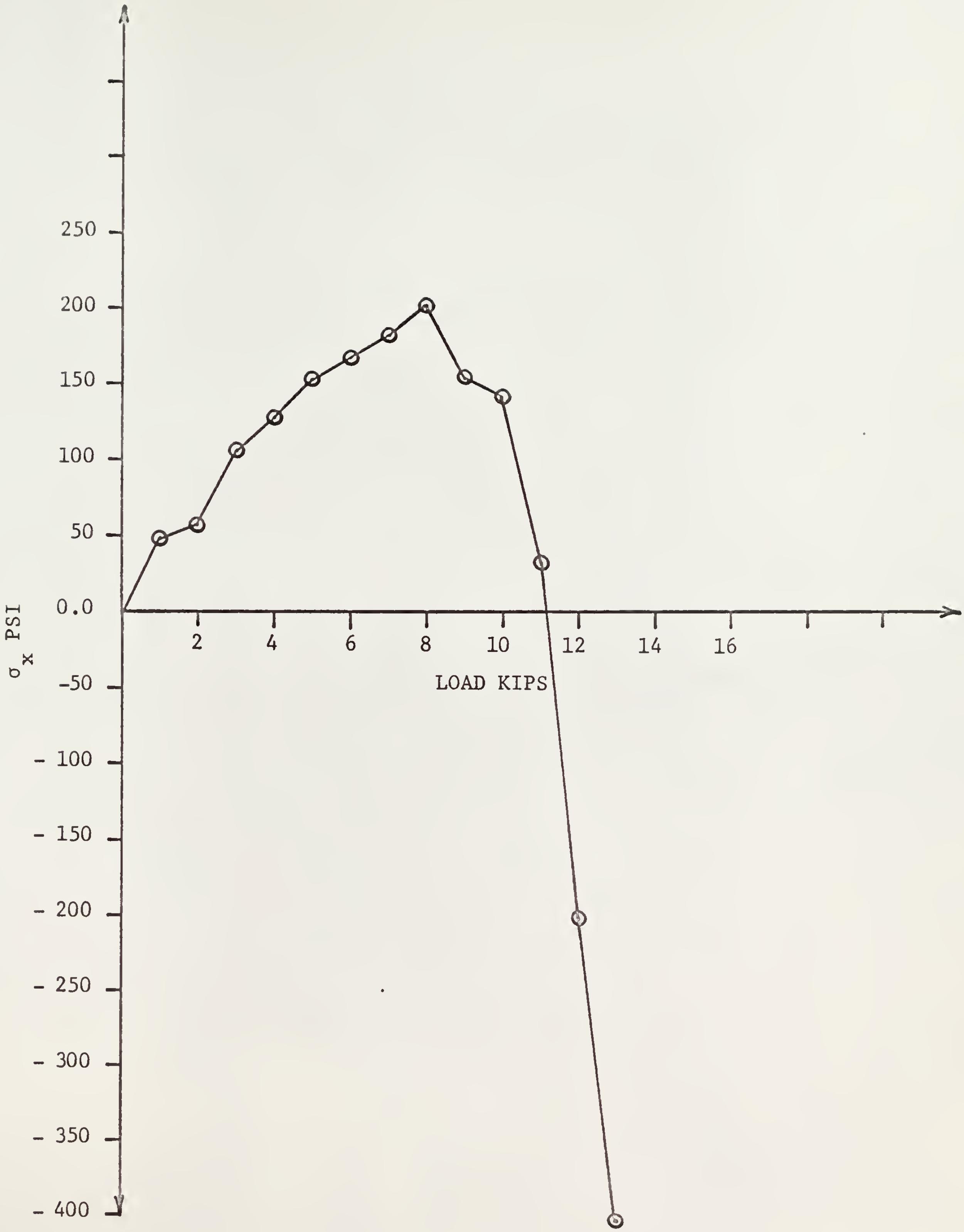


FIG. 17. LONGITUDINAL STRESS σ_x VS LOAD, BEAM #1, STRAIN GAGE #3, MIDPOINT BOTTOM CHORD.

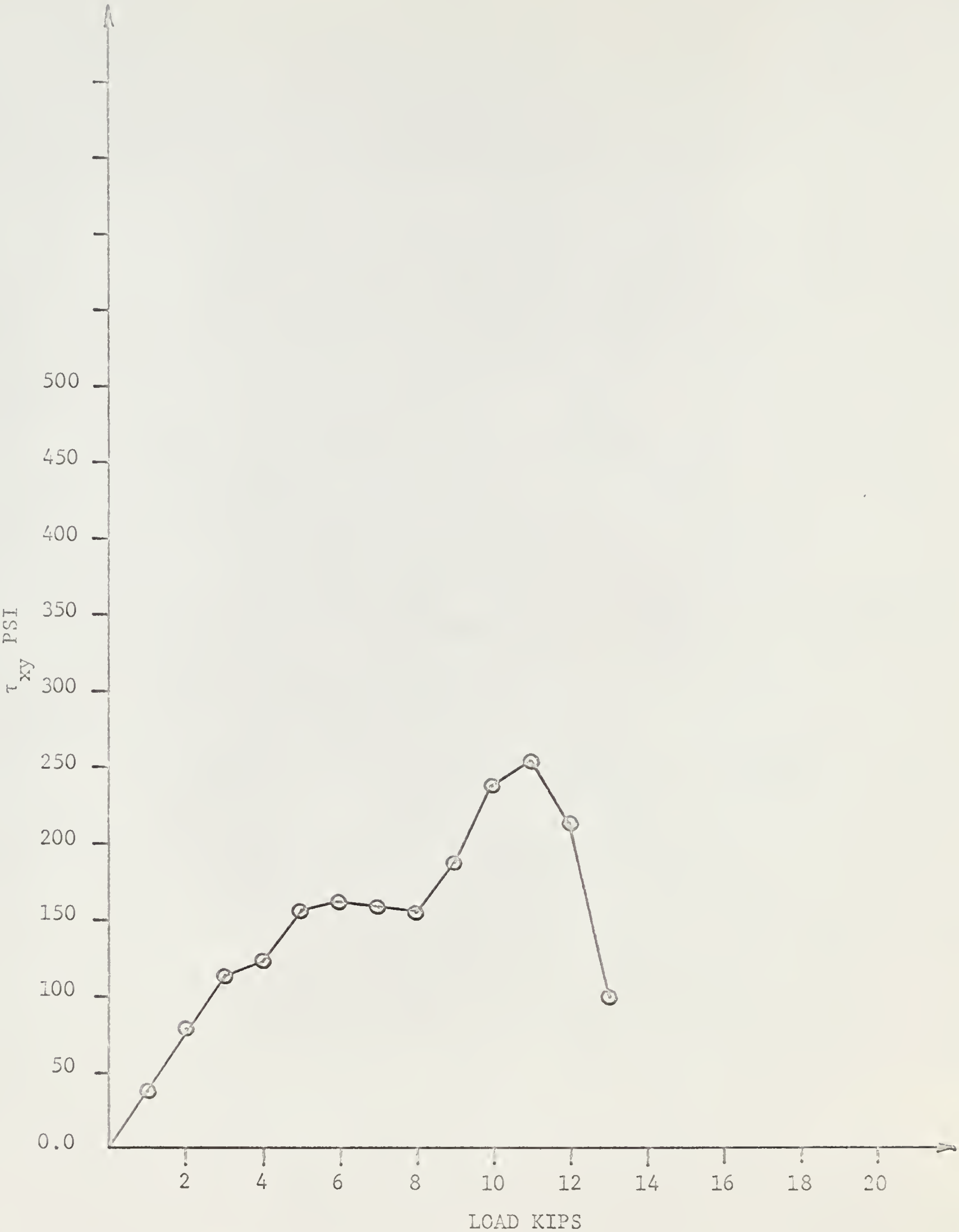


FIG. 18. SHEARING STRESS τ_{xy} VS LOAD, BEAM #1, STRAIN GAGE #3, MIDPOINT BOTTOM CHORD.

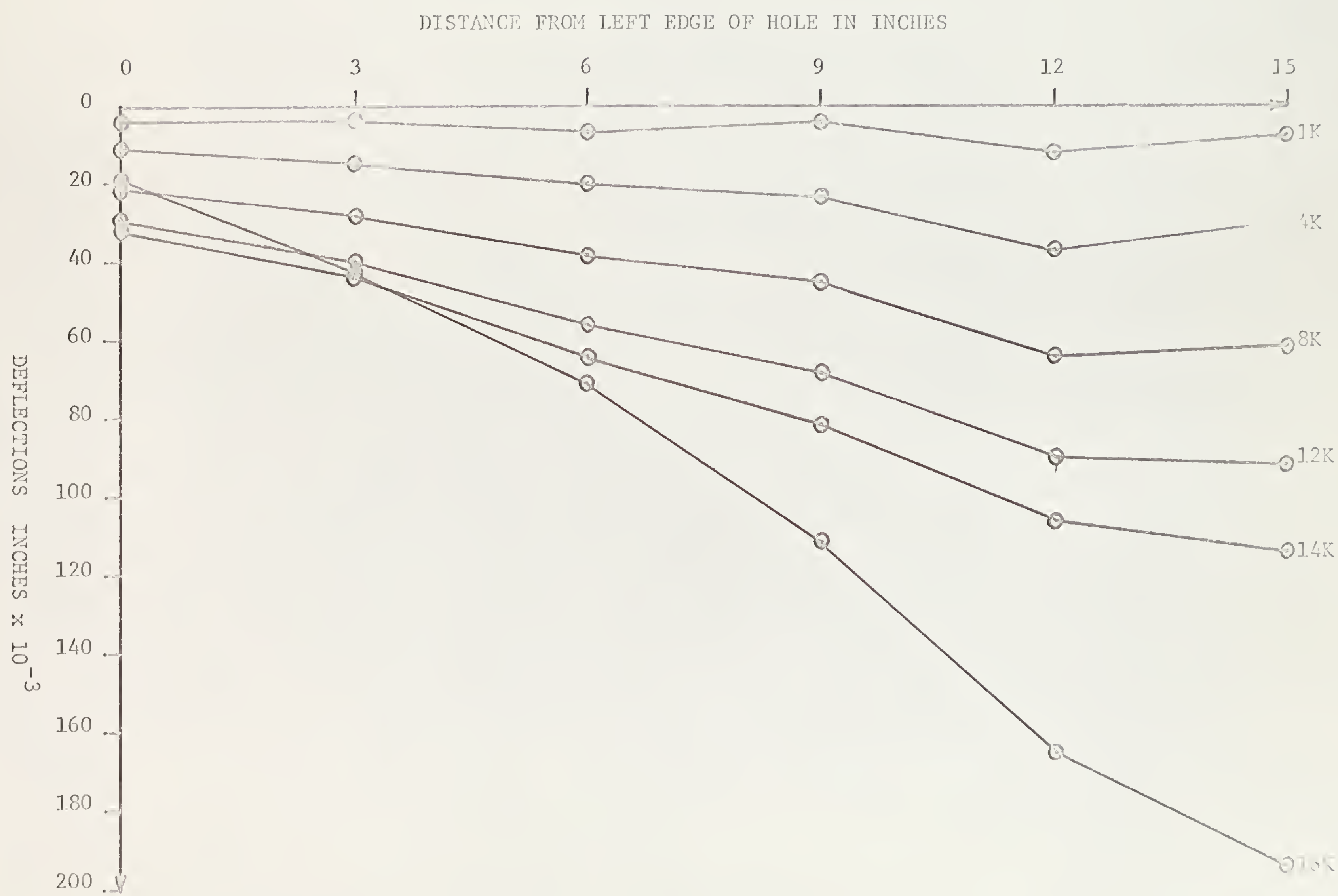


FIG. 19. DEFLECTIONS, TOP CHORD, BEAM #1.

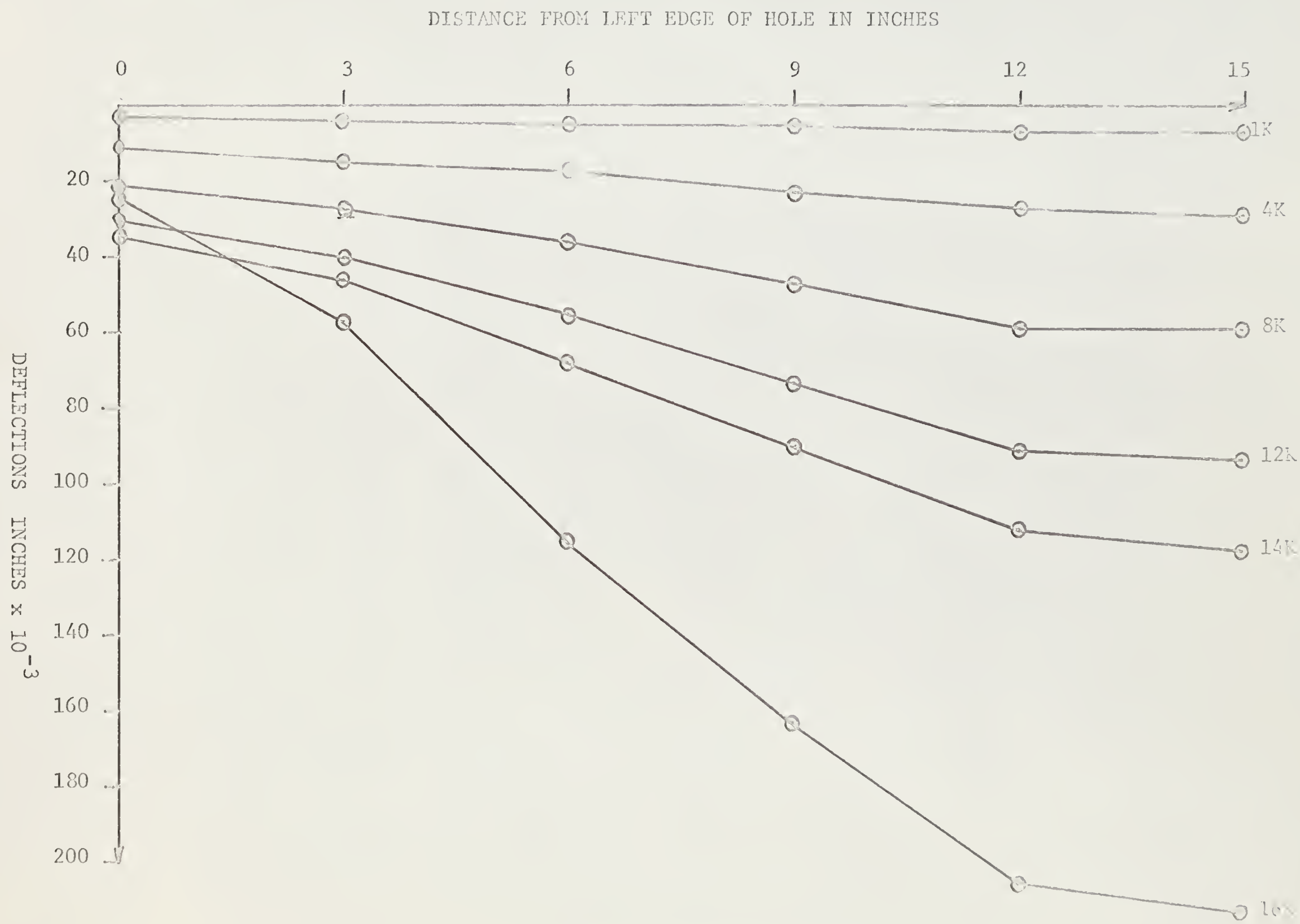


FIG. 20. DEFLECTIONS, BOTTOM CHORD, BEAM #1.

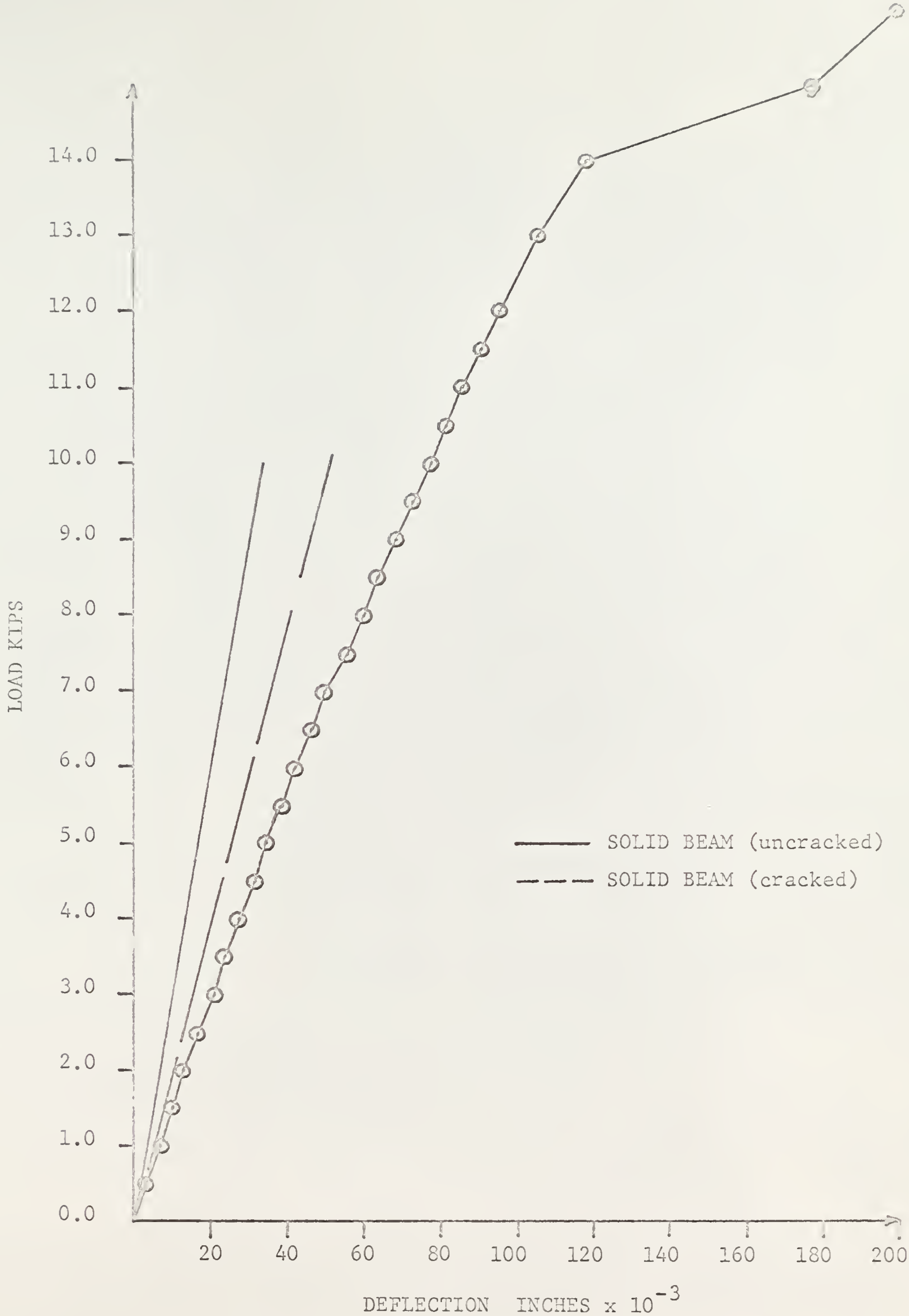


FIG. 21. DEFLECTIONS AT CENTER OF THE BEAM VERSUS LOAD BEAM #1.

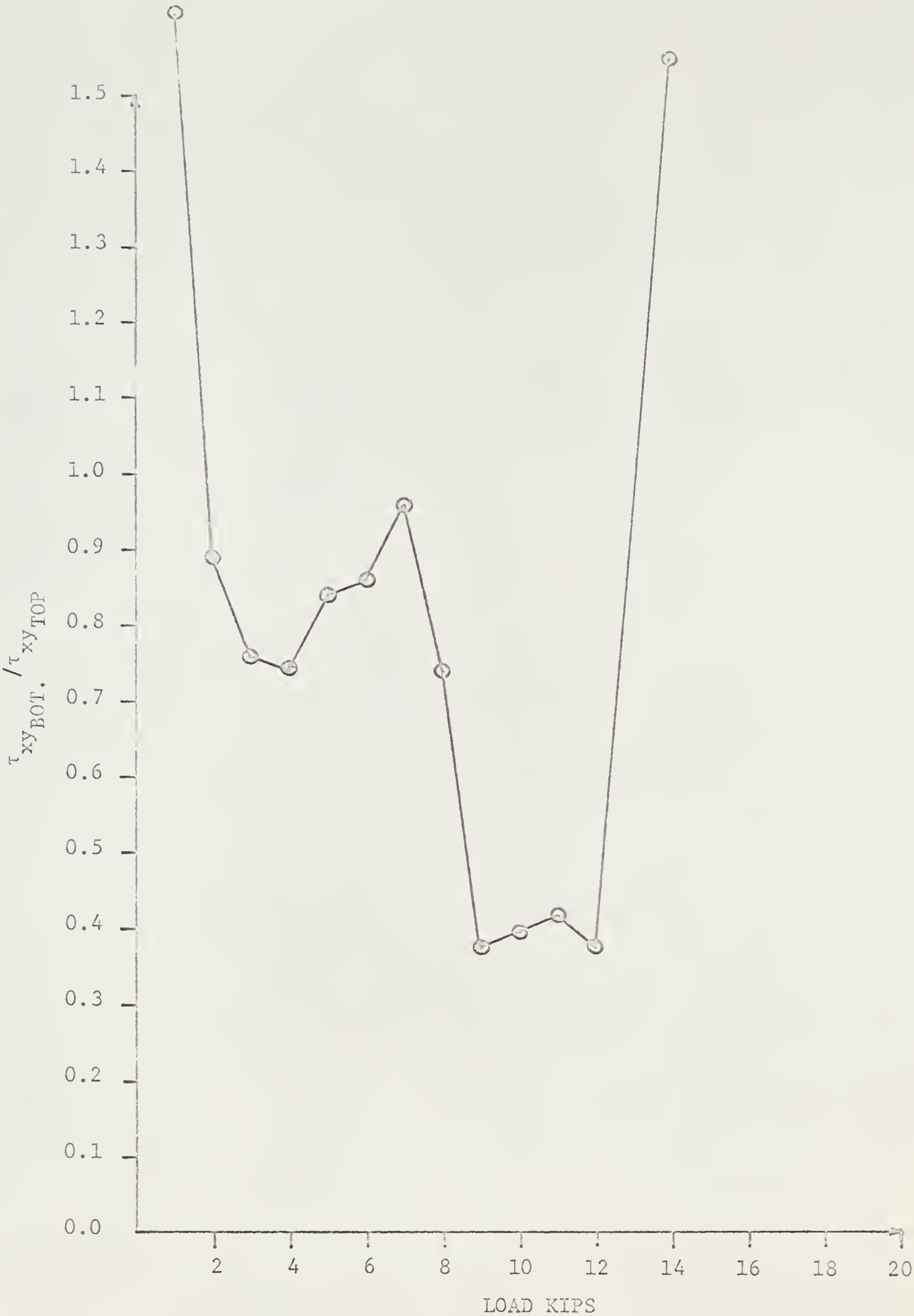


FIG. 22. $\tau_{xy_{BOT.}} : \tau_{xy_{TOP}}$ VS LOAD, BEAM #2.

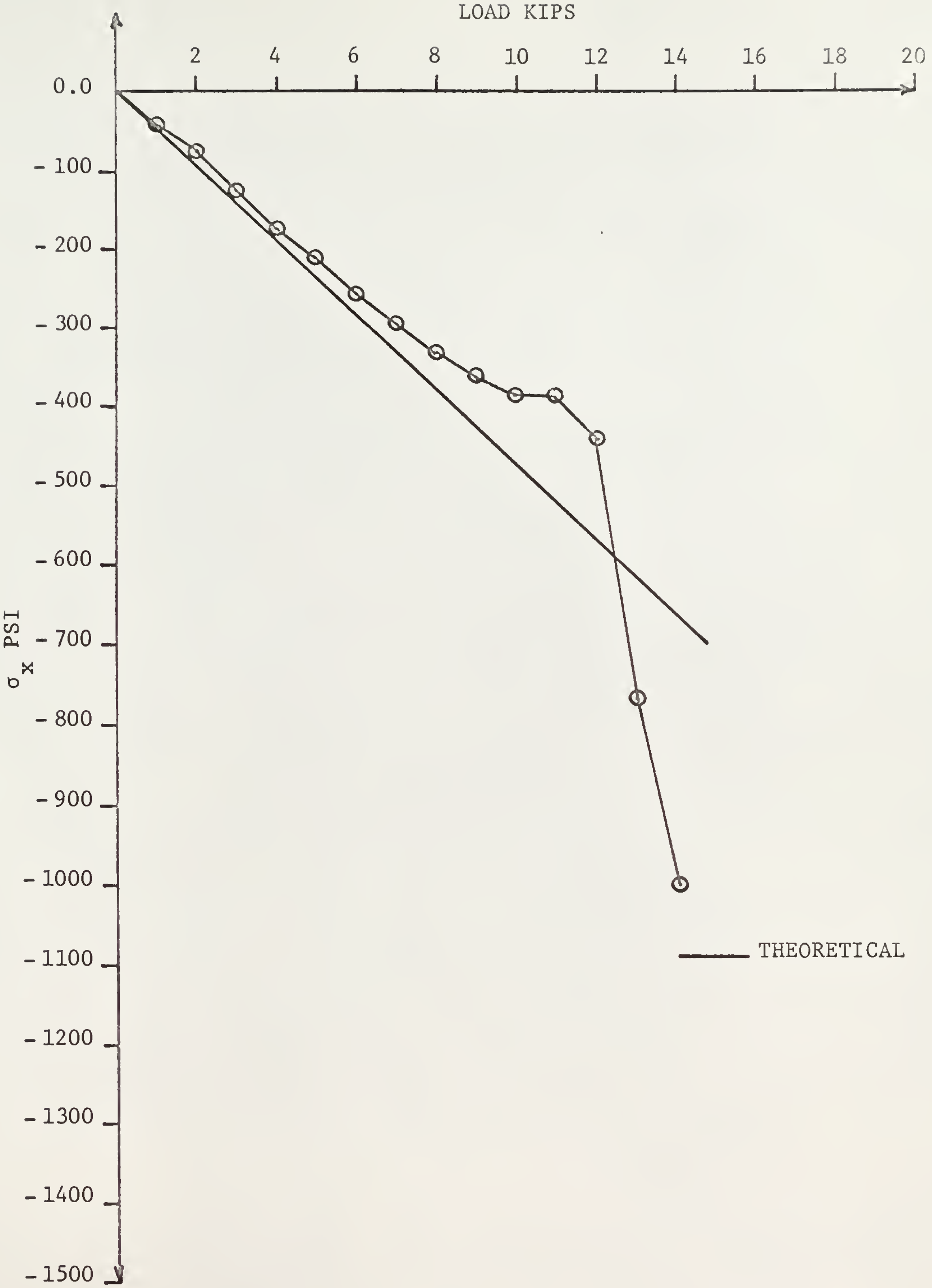


FIG. 23. LONGITUDINAL STRESS σ_x VS LOAD, BEAM #2, STRAIN GAGE #3, MIDPOINT TOP CHORD.

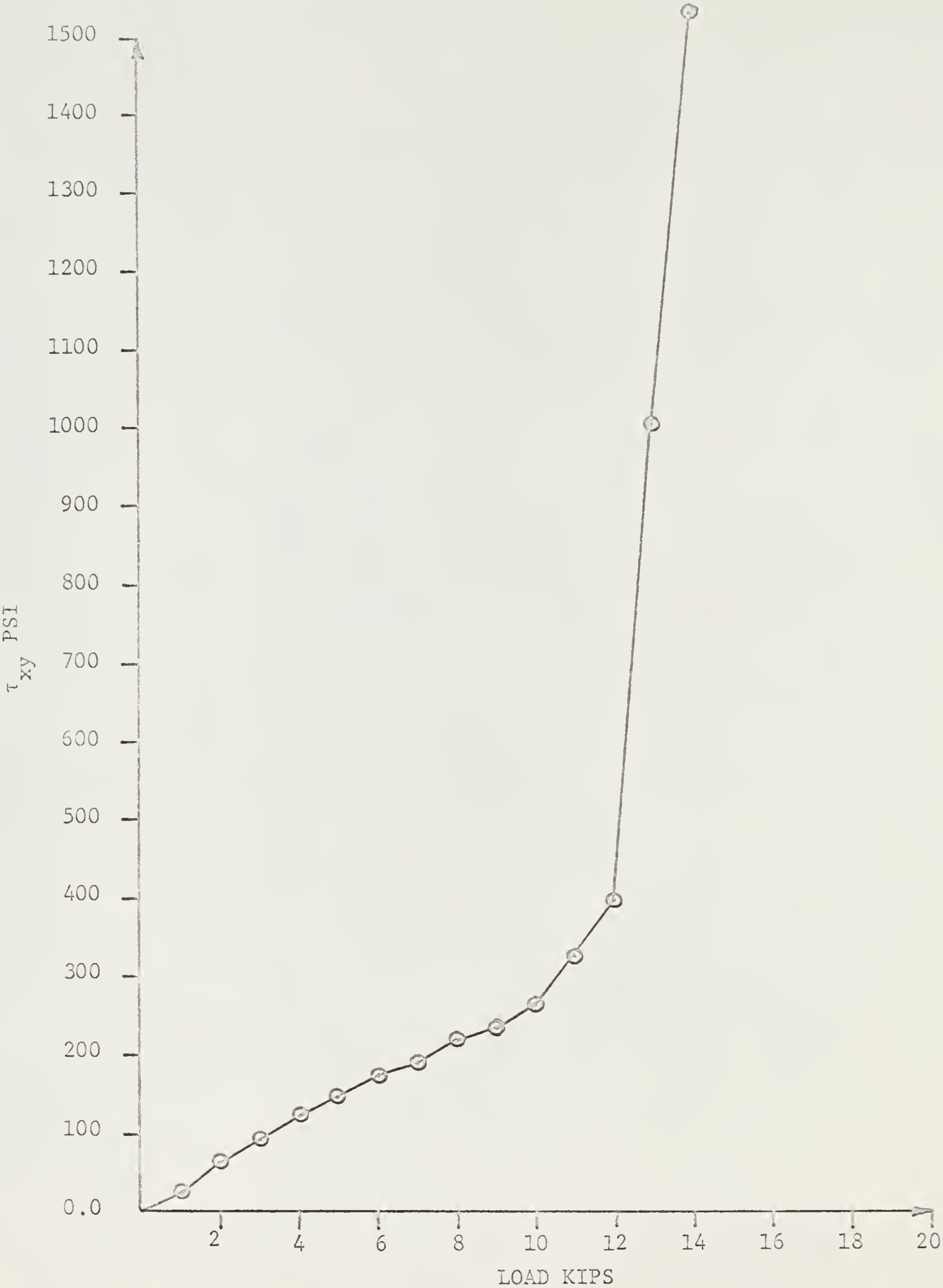


FIG. 24. SHEARING STRESS τ_{xy} VS LOAD, BEAM #2, STRAIN GAGE #3, MIDPOINT TOP CHORD.

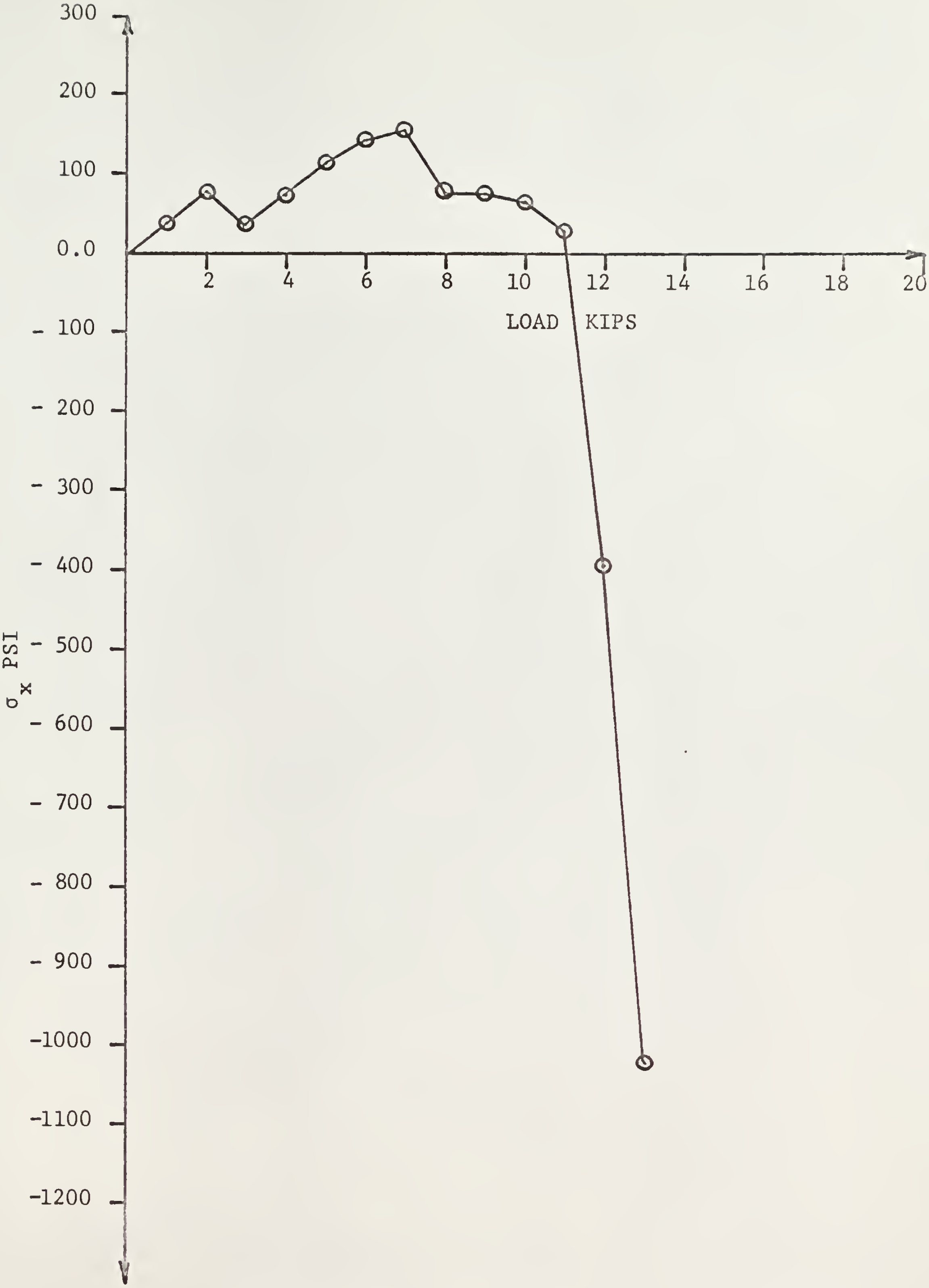


FIG. 25. LONGITUDINAL STRESS σ_x VS LOAD, BEAM #2 STRAIN GAGE #4, MIDPOINT BOTTOM CHORD.

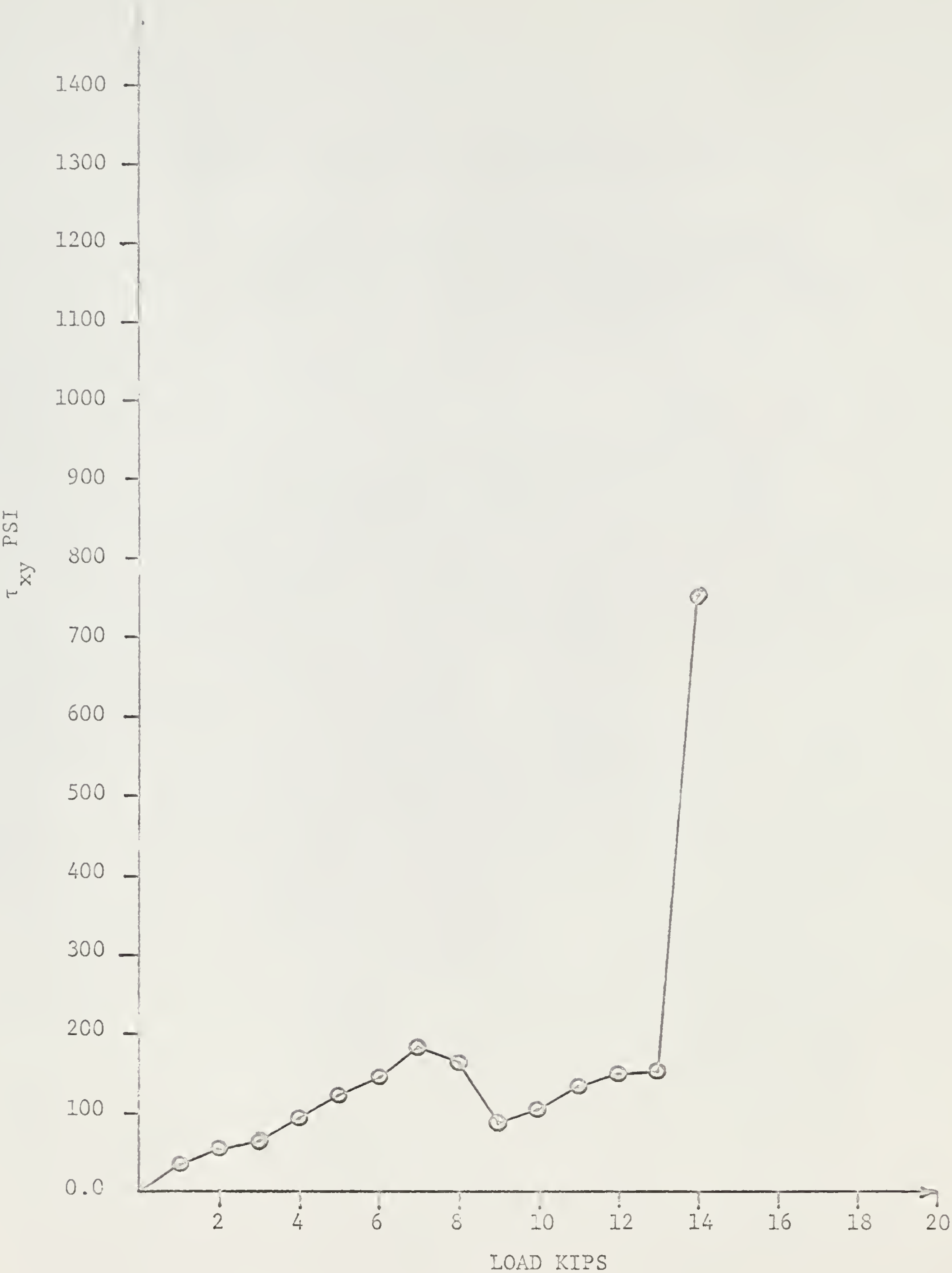


FIG. 26. SHEARING STRESS τ_{xy} VS LOAD, BEAM #2, STRAIN GAGE #4, MIDPOINT BOTTOM CHORD.

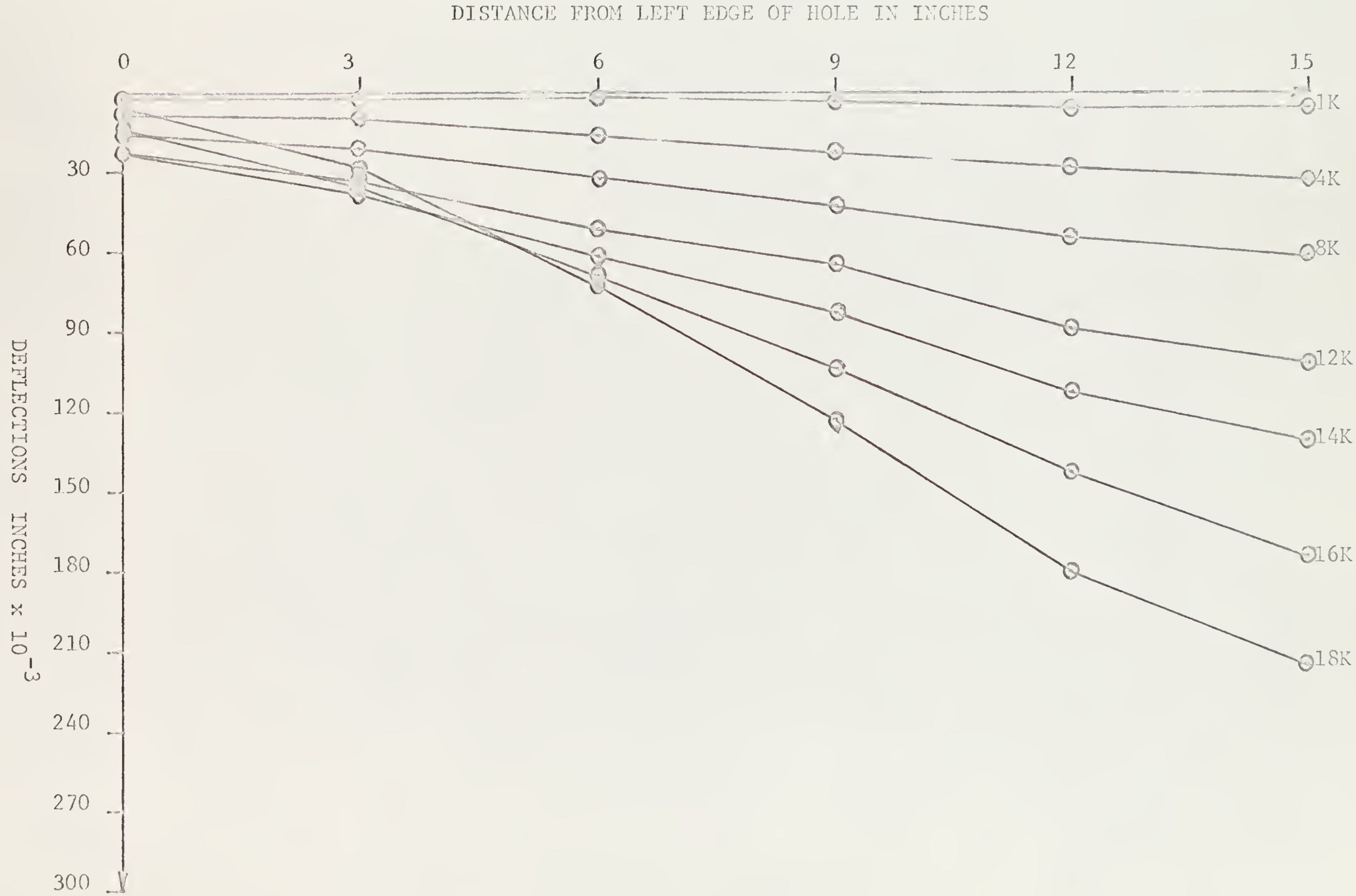


FIG. 27. DEFLECTIONS, TOP CHORD, BEAM #2

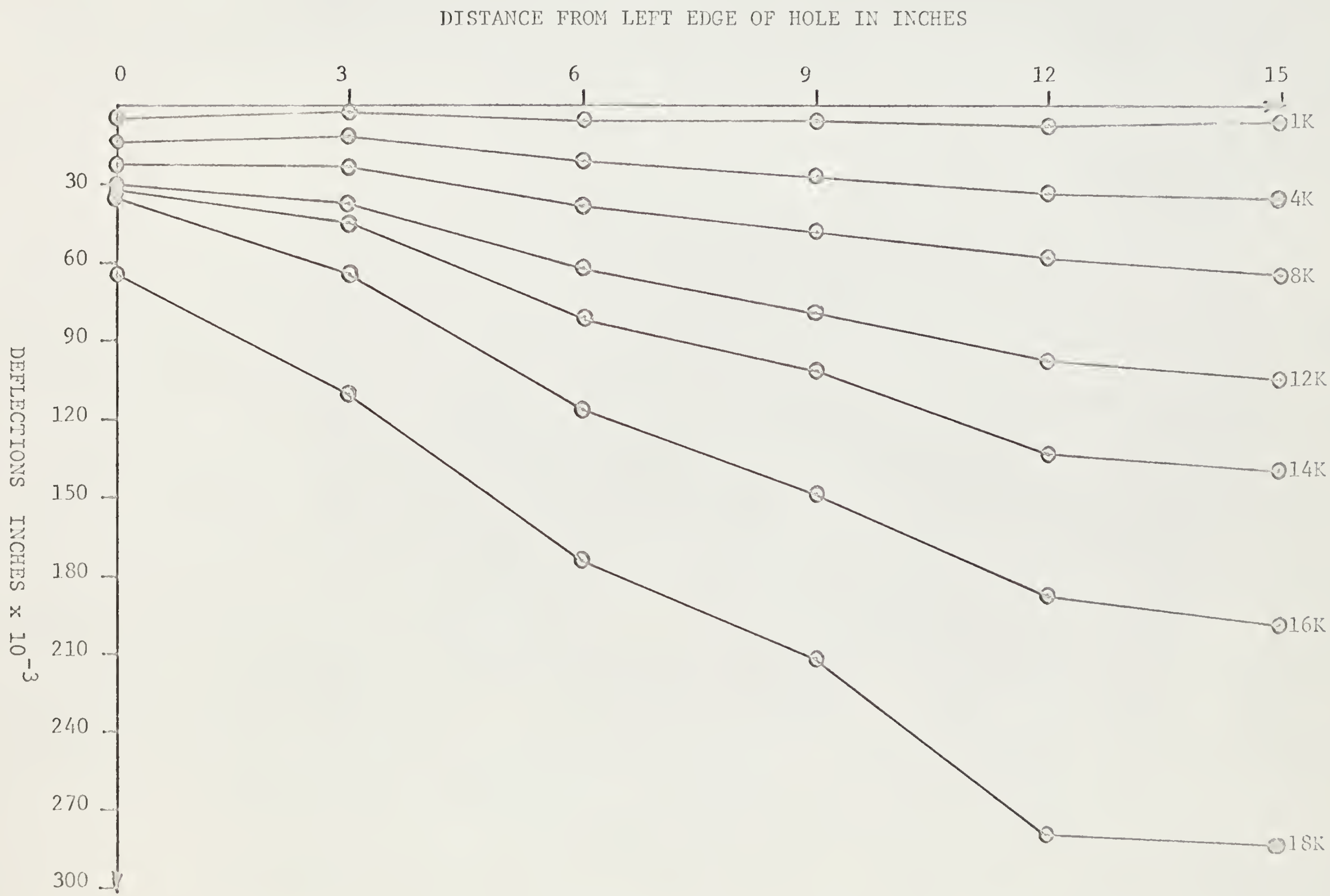


FIG. 28. DEFLECTIONS, BOTTOM CHORD, BEAM #2

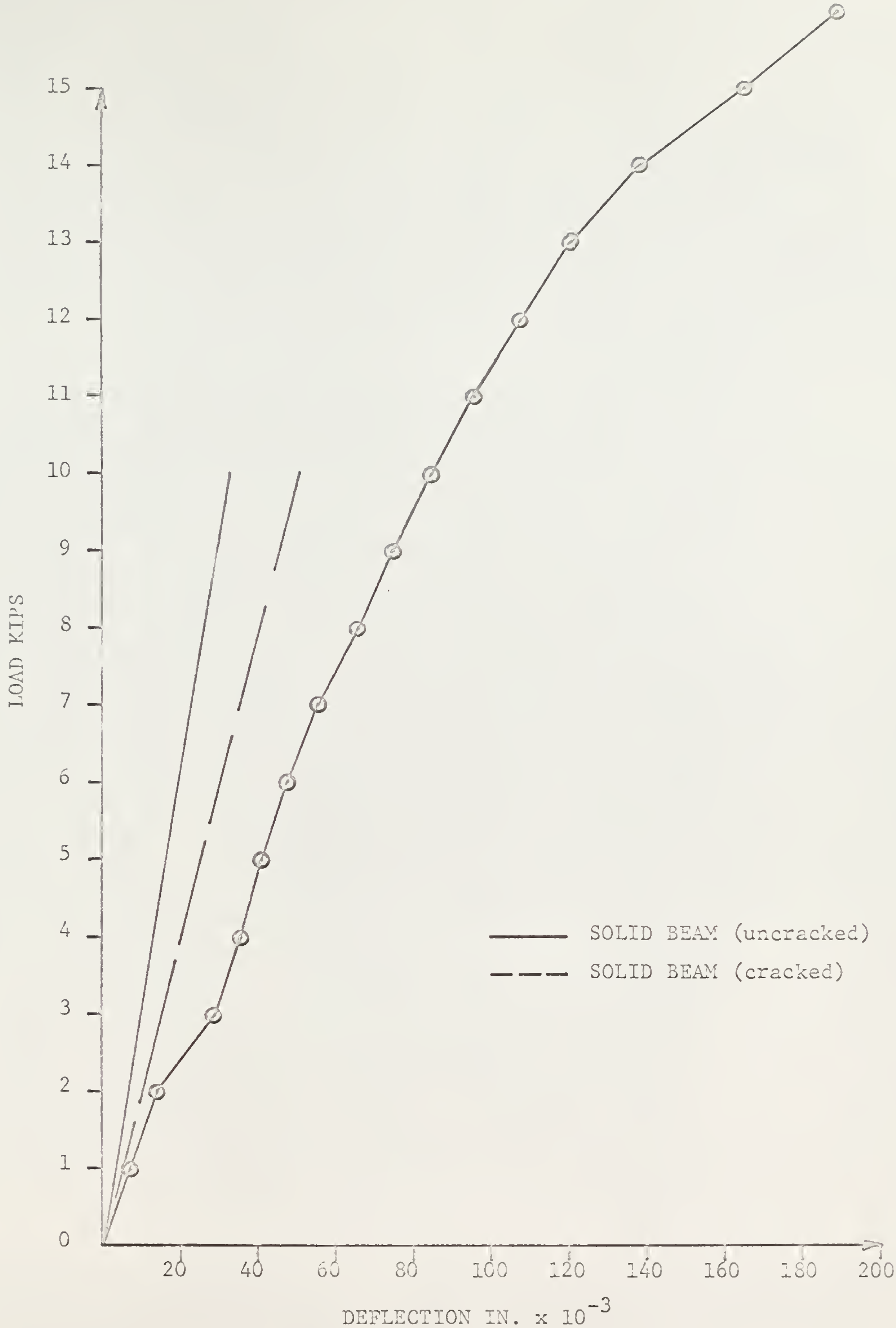


FIG. 29. DEFLECTIONS AT CENTER VS LOAD, BEAM #2

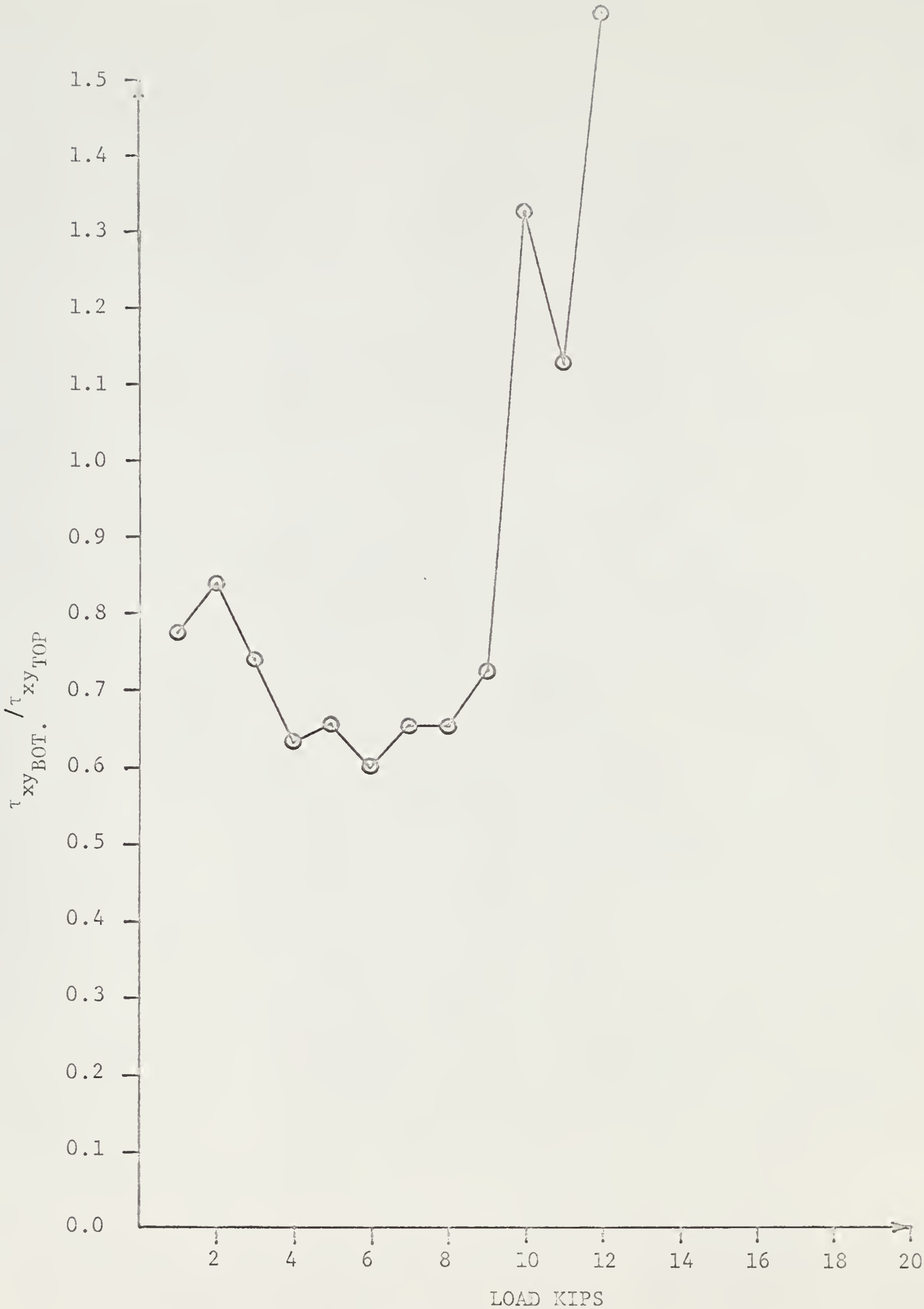


FIG. 30. $\tau_{xy_{BOT.}} : \tau_{xy_{TOP}}$, BEAM #3

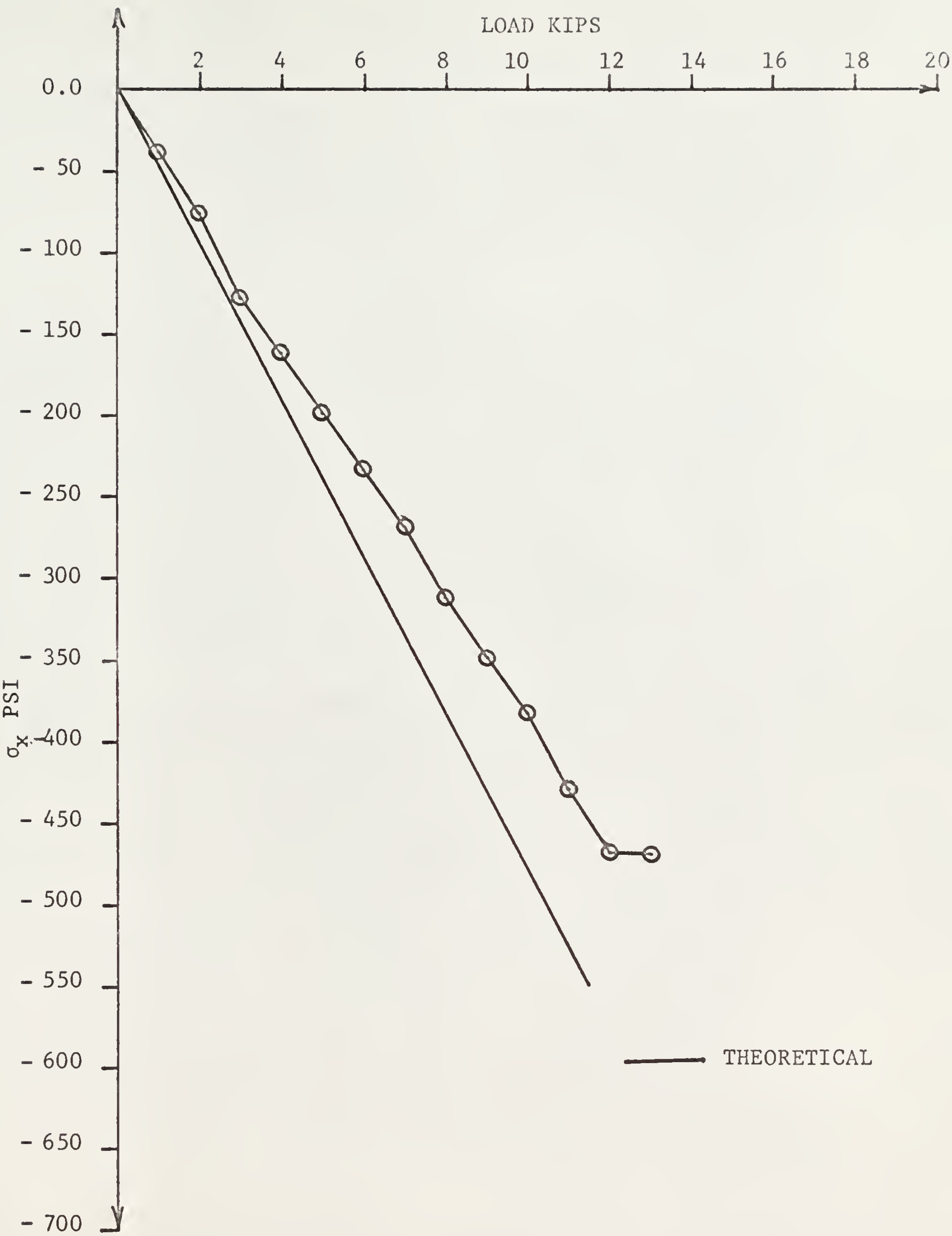


FIG. 31. LONGITUDINAL STRESS σ_x VS LOAD, BEAM #3
STRAIN GAGE #2, MIDPOINT TOP CHORD.

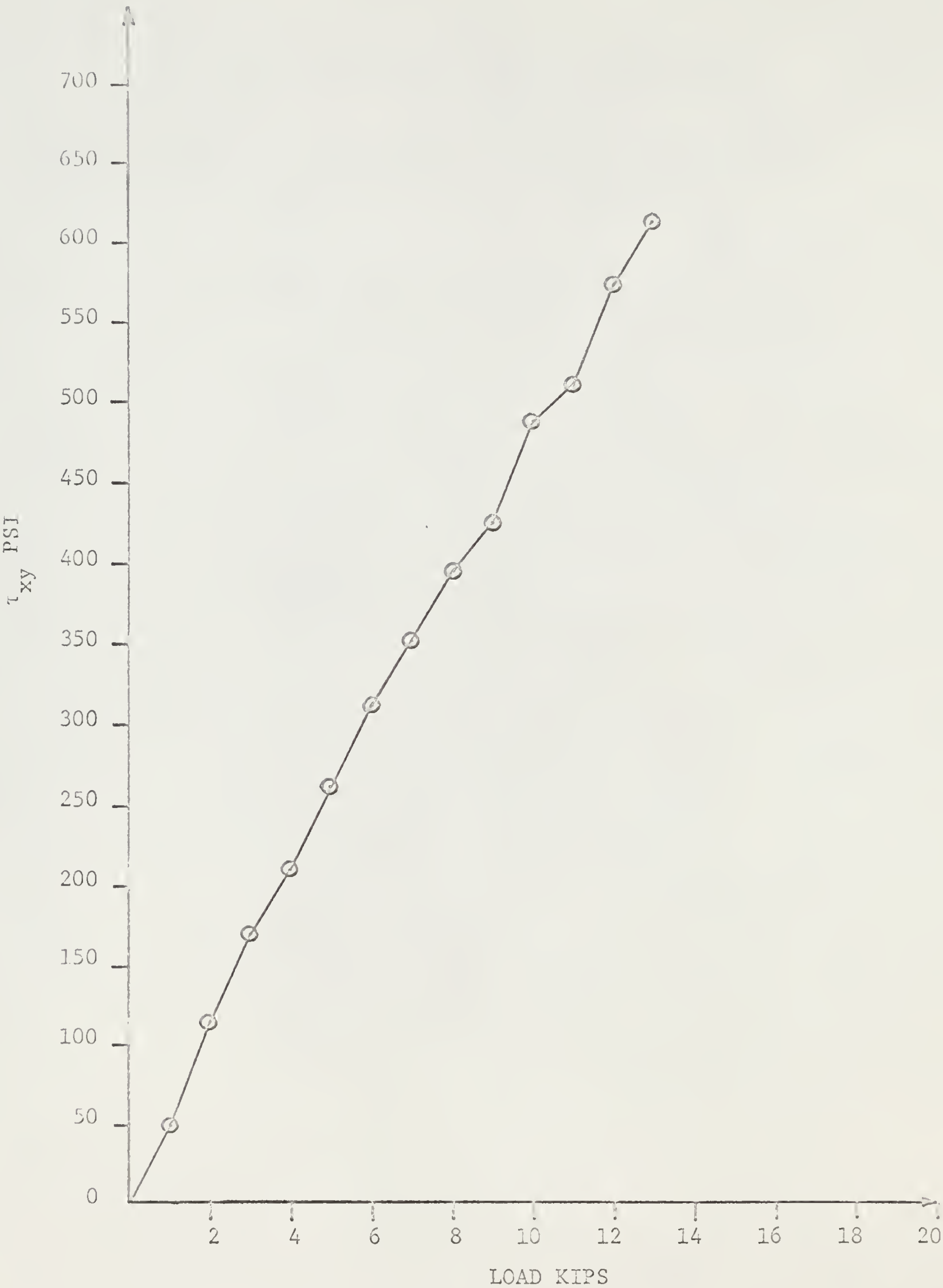


FIG. 32. SHEARING STRESS τ_{xy} VS LOAD, BEAM #3
STRAIN GAGE #2, MIDPOINT TOP CHORD.

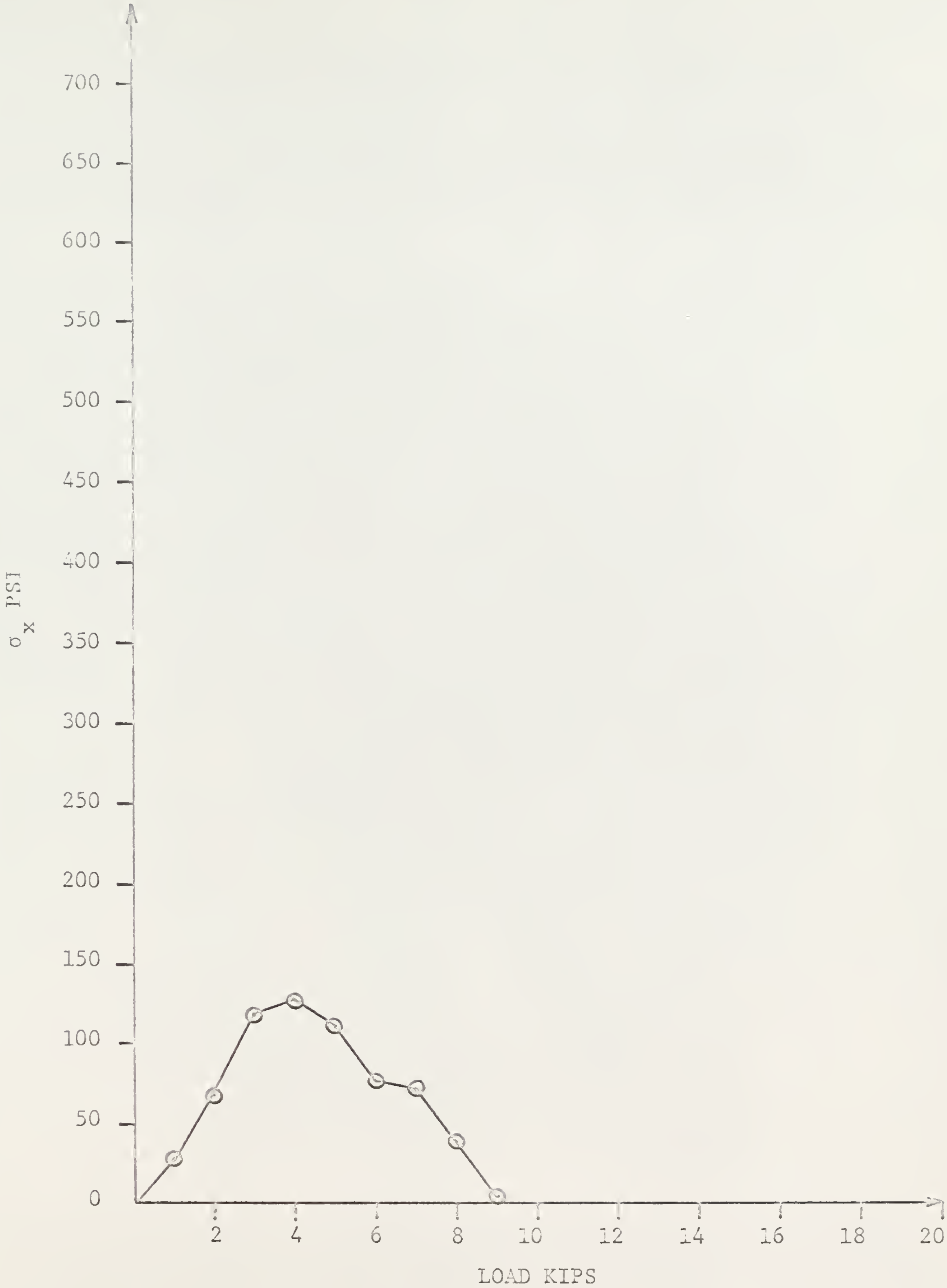


FIG. 33. LONGITUDINAL STRESS σ_x VS LOAD, BEAM #3 STRAIN GAGE #4, MIDPOINT BOTTOM CHORD.

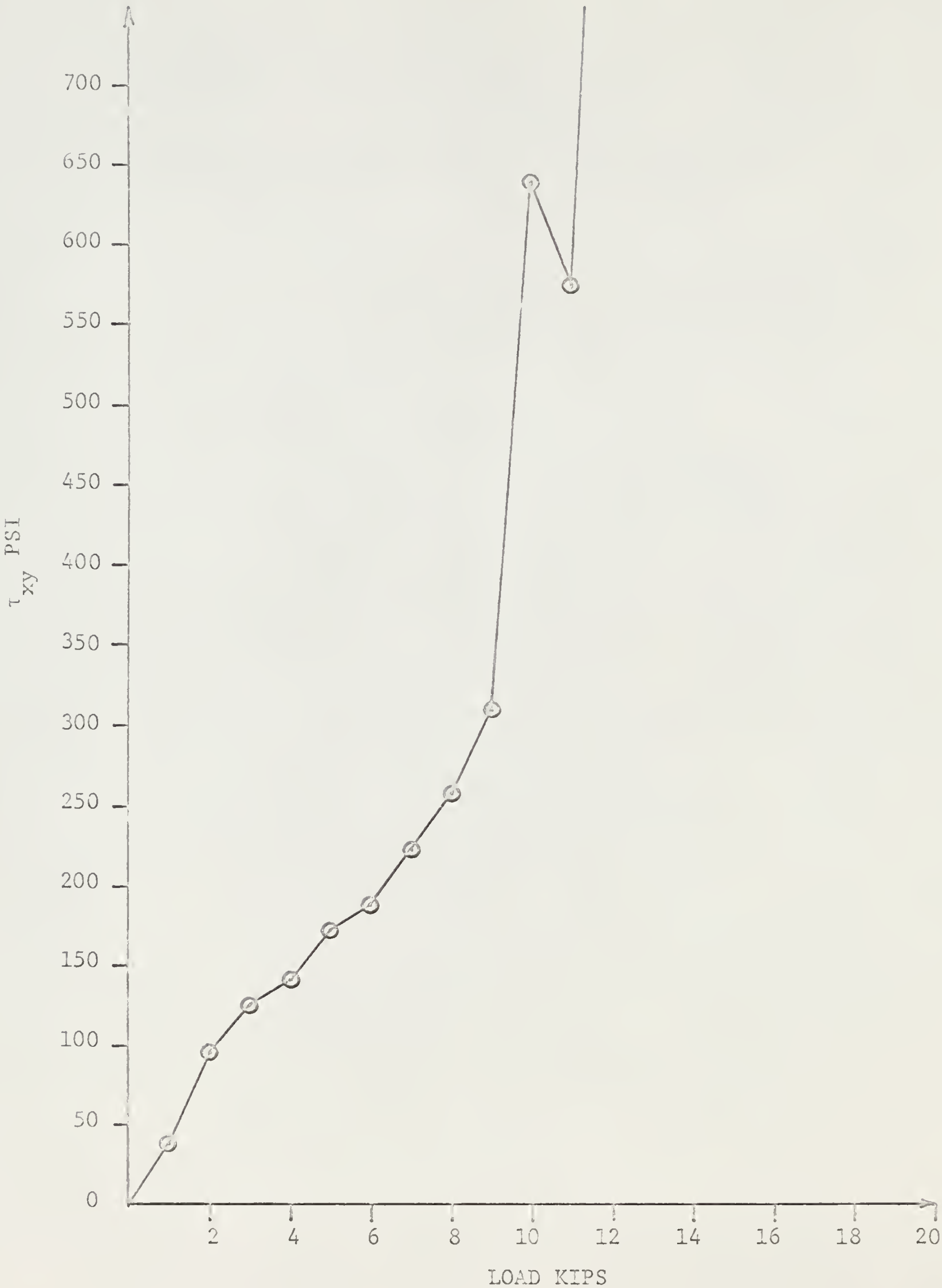


FIG. 34. SHEARING STRESS τ_{xy} VS LOAD, BEAM #3
STRAIN GAGE #4, MIDPOINT BOTTOM CHORD.

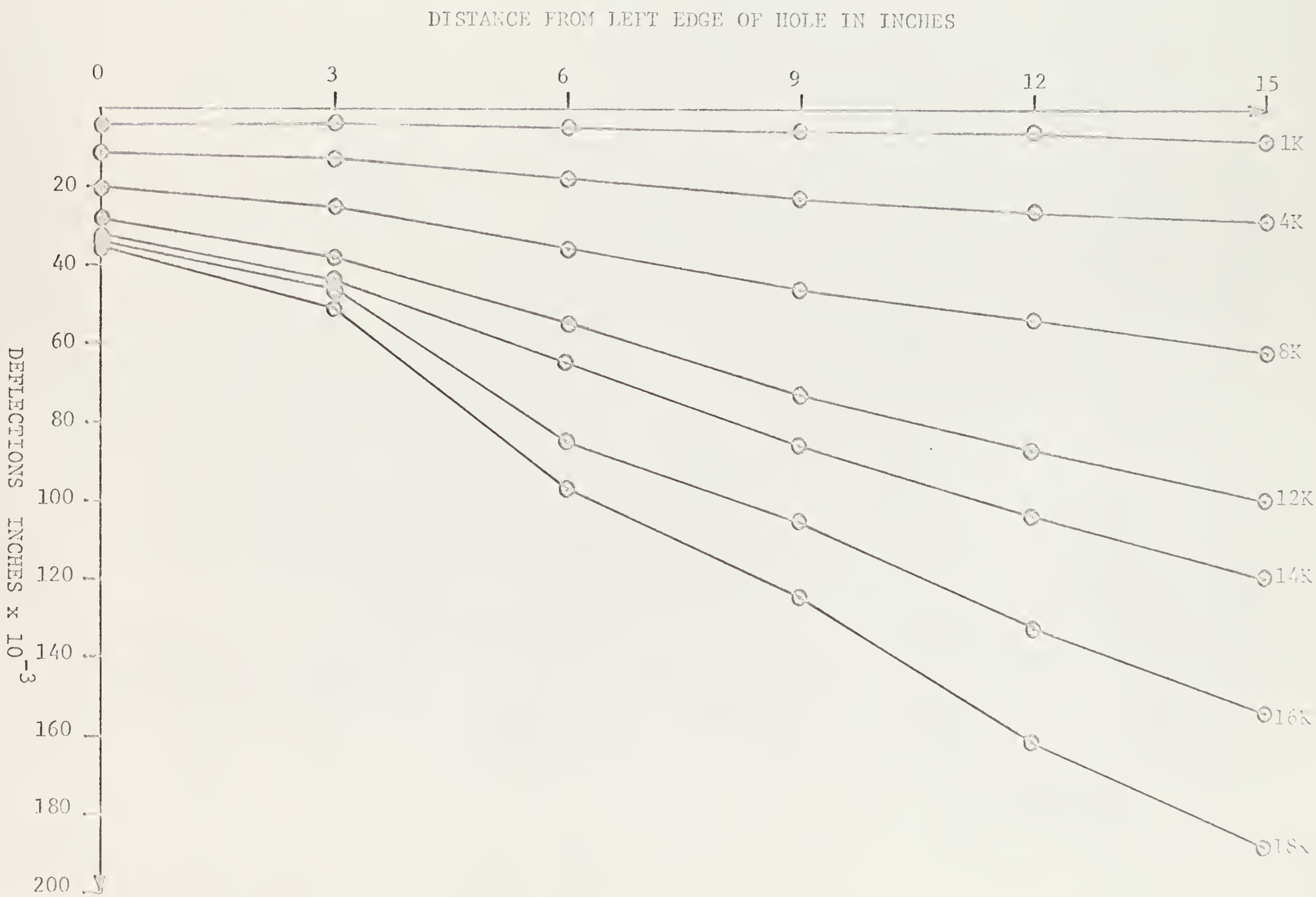


FIG. 35. DEFLECTIONS, TOP CHORD, BEAM #3

FIG. 36. DEFLECTIONS, BOTTOM CHORD, BEAM #3

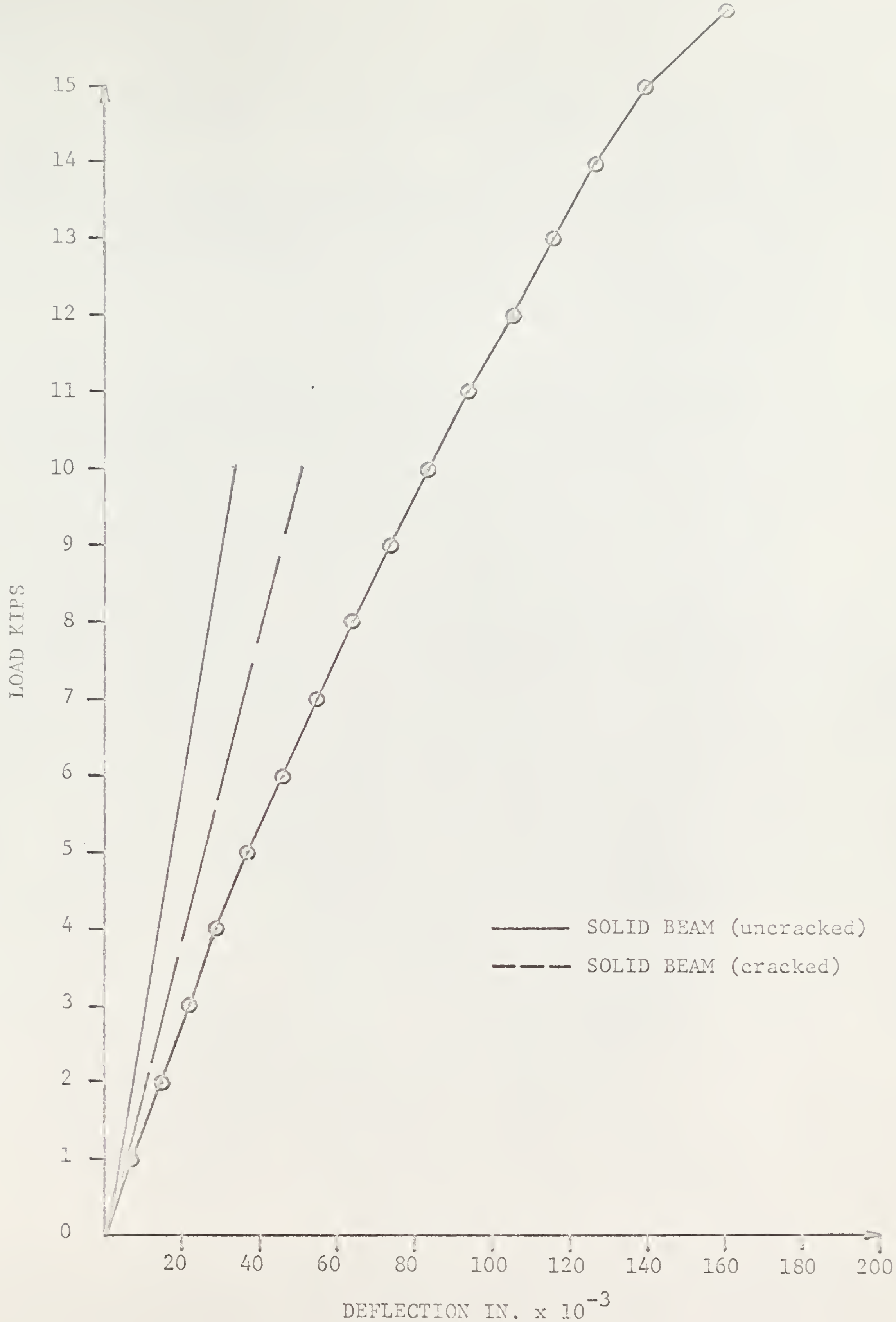


FIG. 37. DEFLECTIONS AT CENTER VS LOAD, BEAM #3

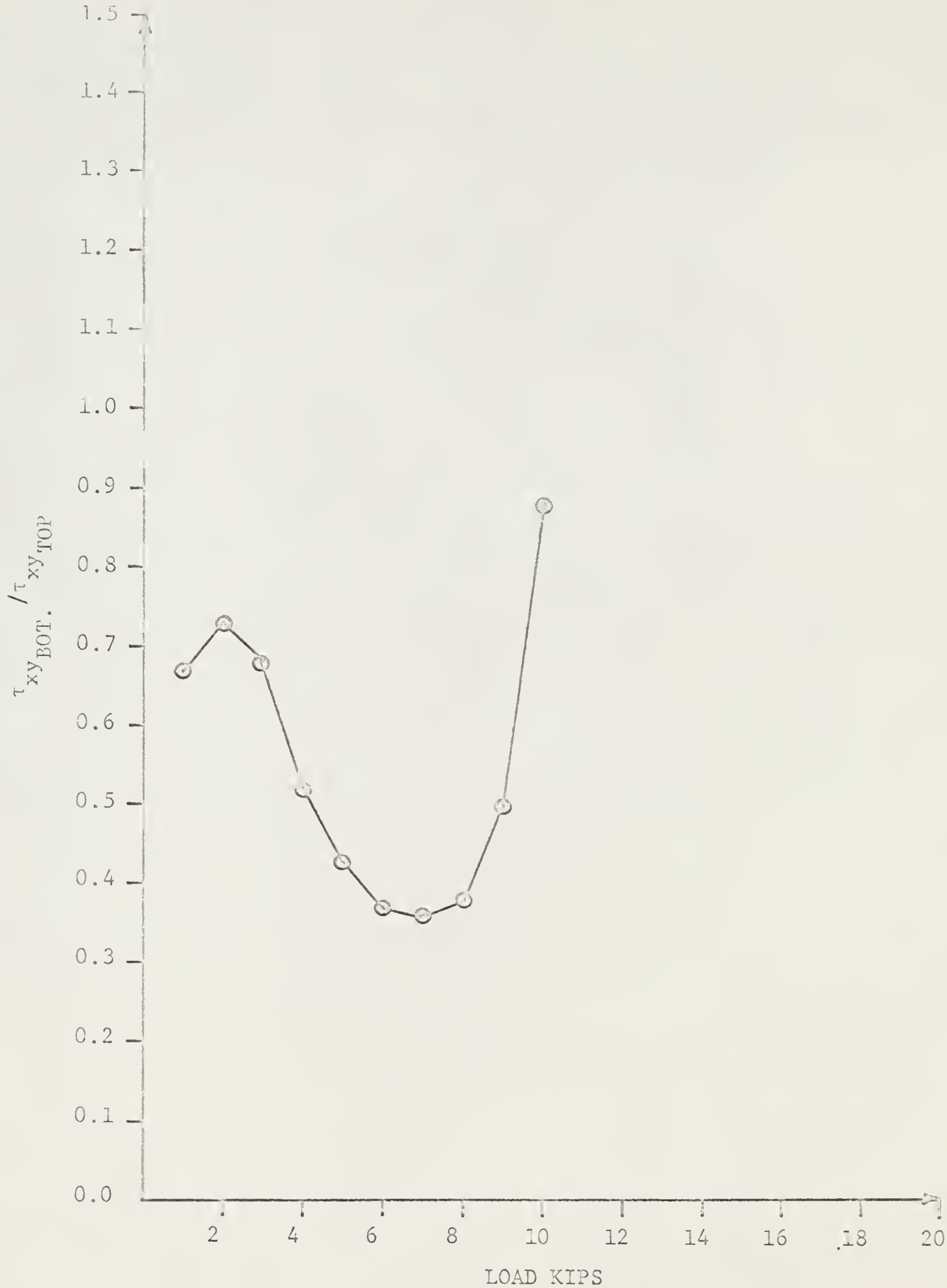


FIG. 38. $\tau_{xy_{BOT.}} : \tau_{xy_{TOP}}$ VS LOAD, BEAM #4

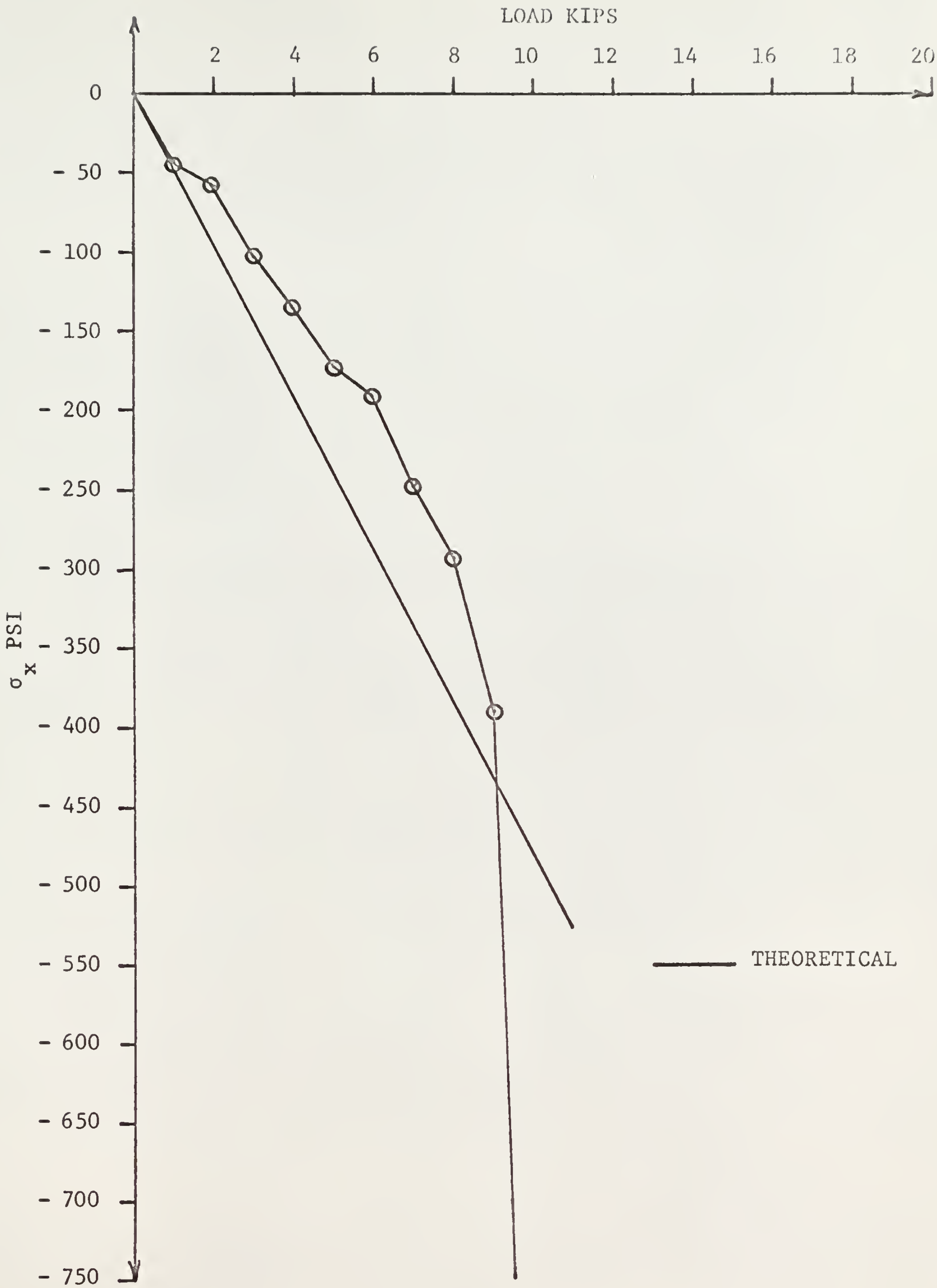


FIG. 39. LONGITUDINAL STRESS σ_x VS LOAD, BEAM #4
STRAIN GAGE #2, MIDPOINT TOP CHORD.

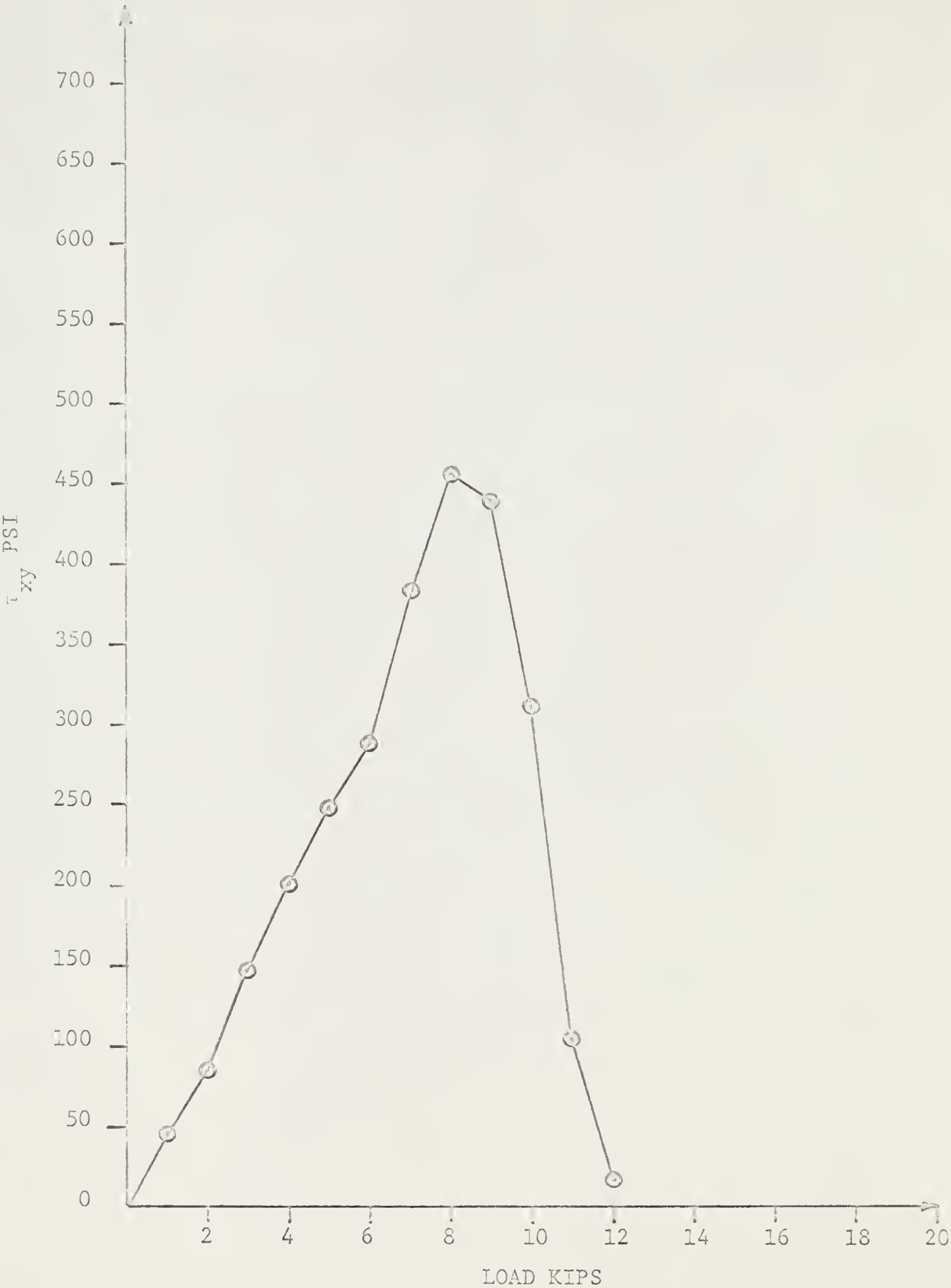


FIG. 40. SHEARING STRESS τ_{xy} VS LOAD, BEAM #4
STRAIN GAGE #2, MIDPOINT TOP CHORD.

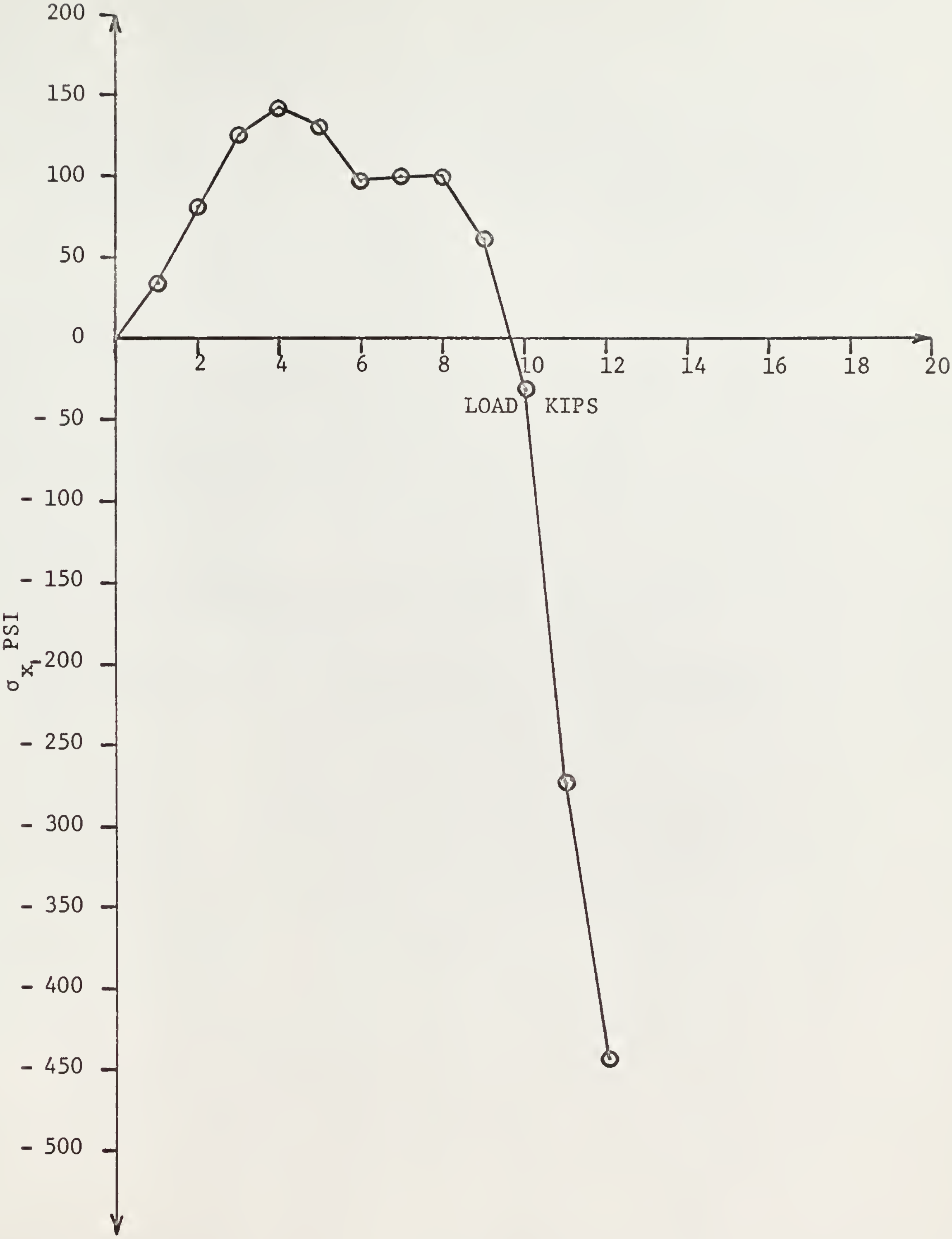


FIG. 41. LONGITUDINAL STRESS σ_x VS LOAD, BEAM #4
STRAIN GAGE #4, MIDPOINT BOTTOM CHORD.

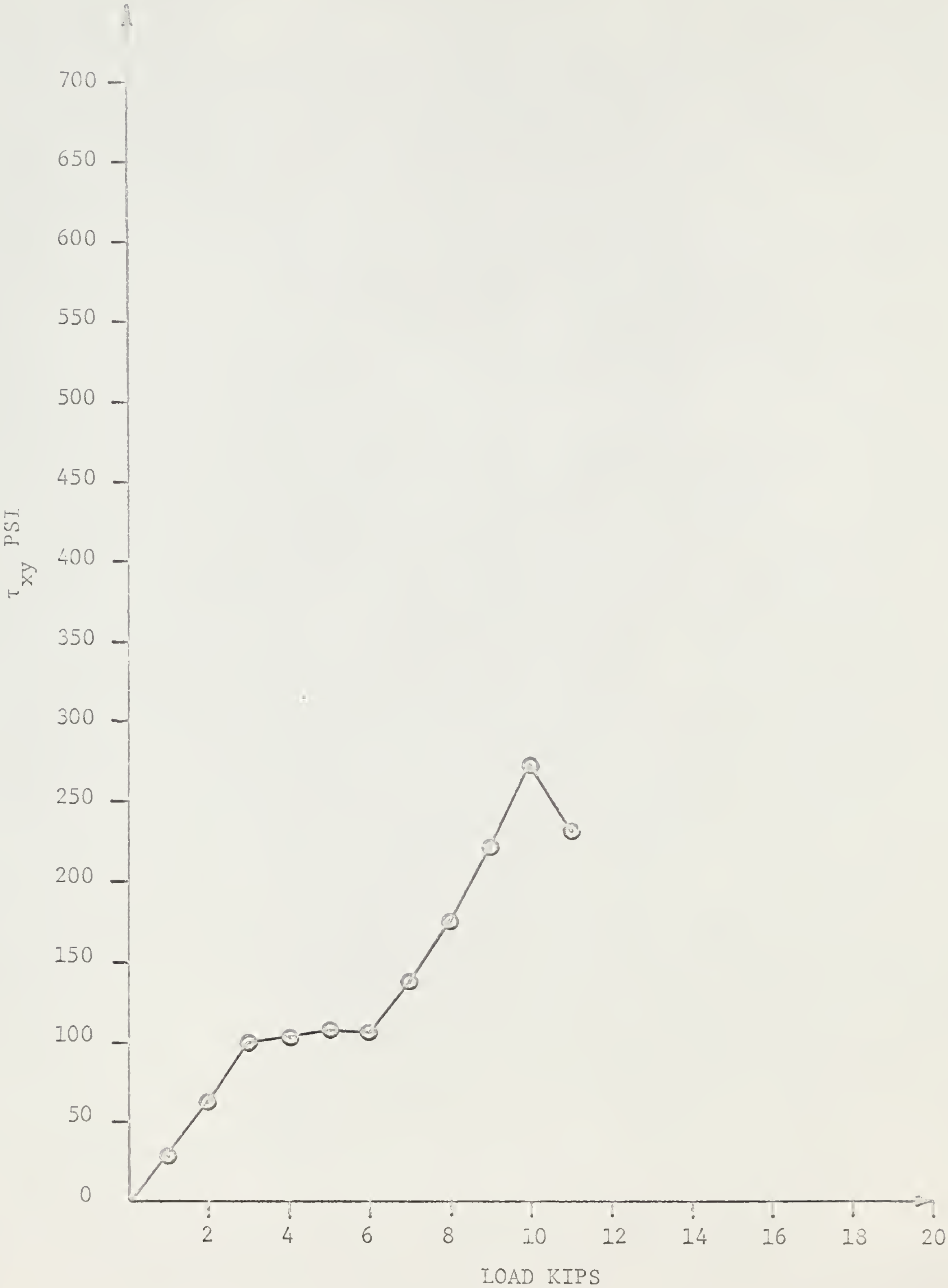


FIG. 42. SHEARING STRESS τ_{xy} VS LOAD, BEAM #4
STRAIN GAGE #4, MIDPOINT BOTTOM CHORD.

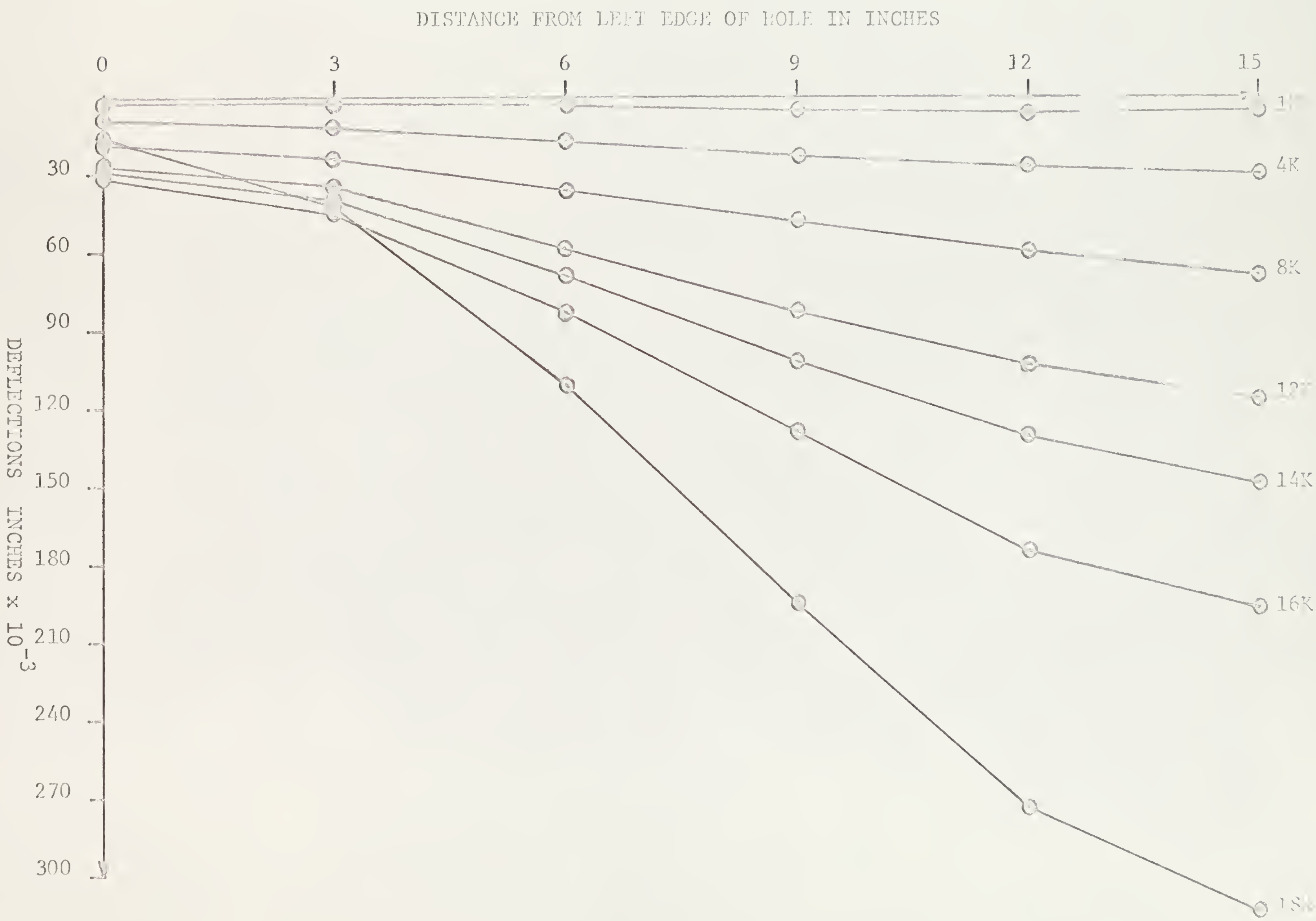


FIG. 43. DEFLECTIONS, TOP CHORD, BEAM #4

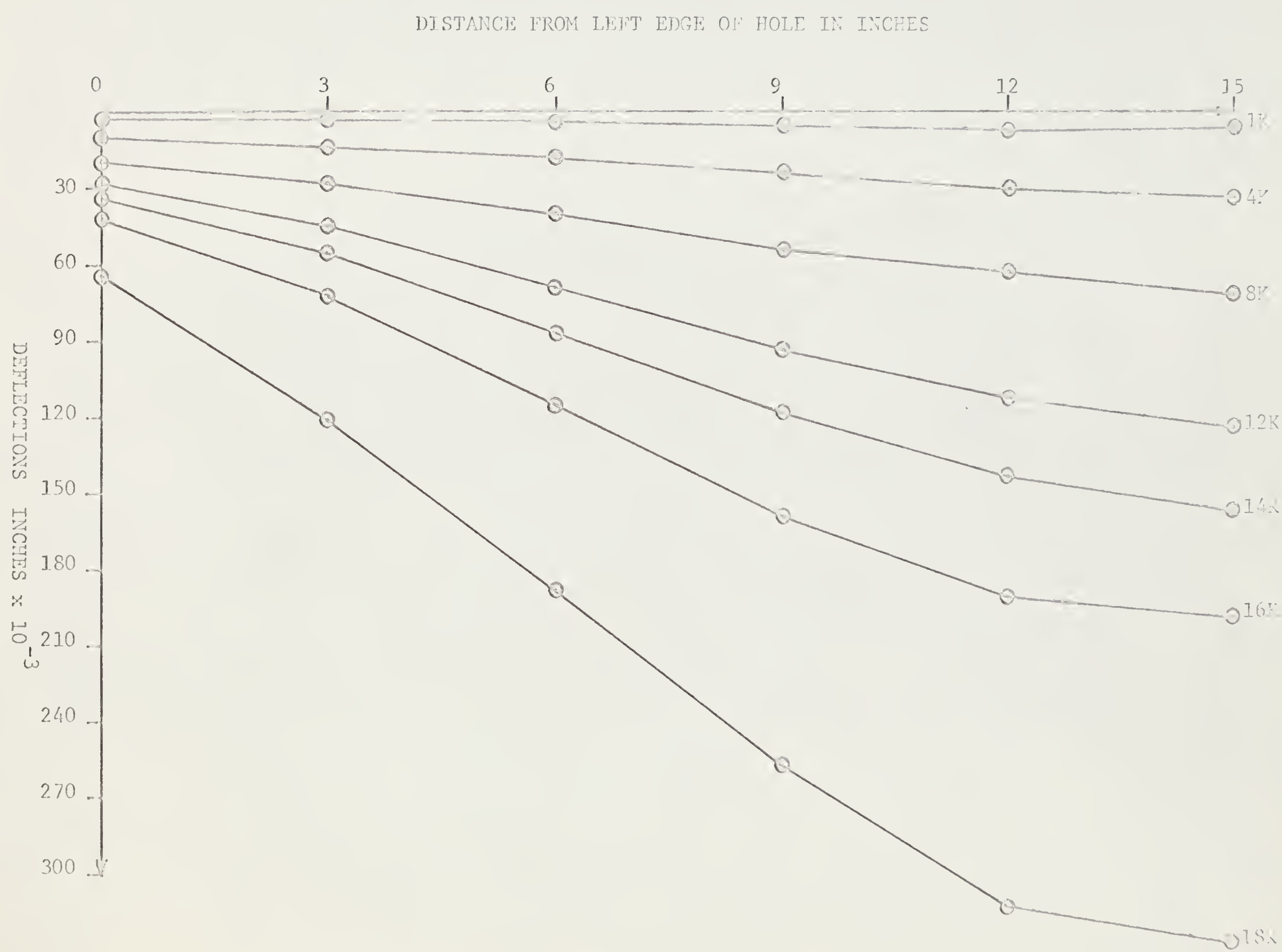


FIG. 44. DEFLECTIONS, BOTTOM CHORD, BEAM #4

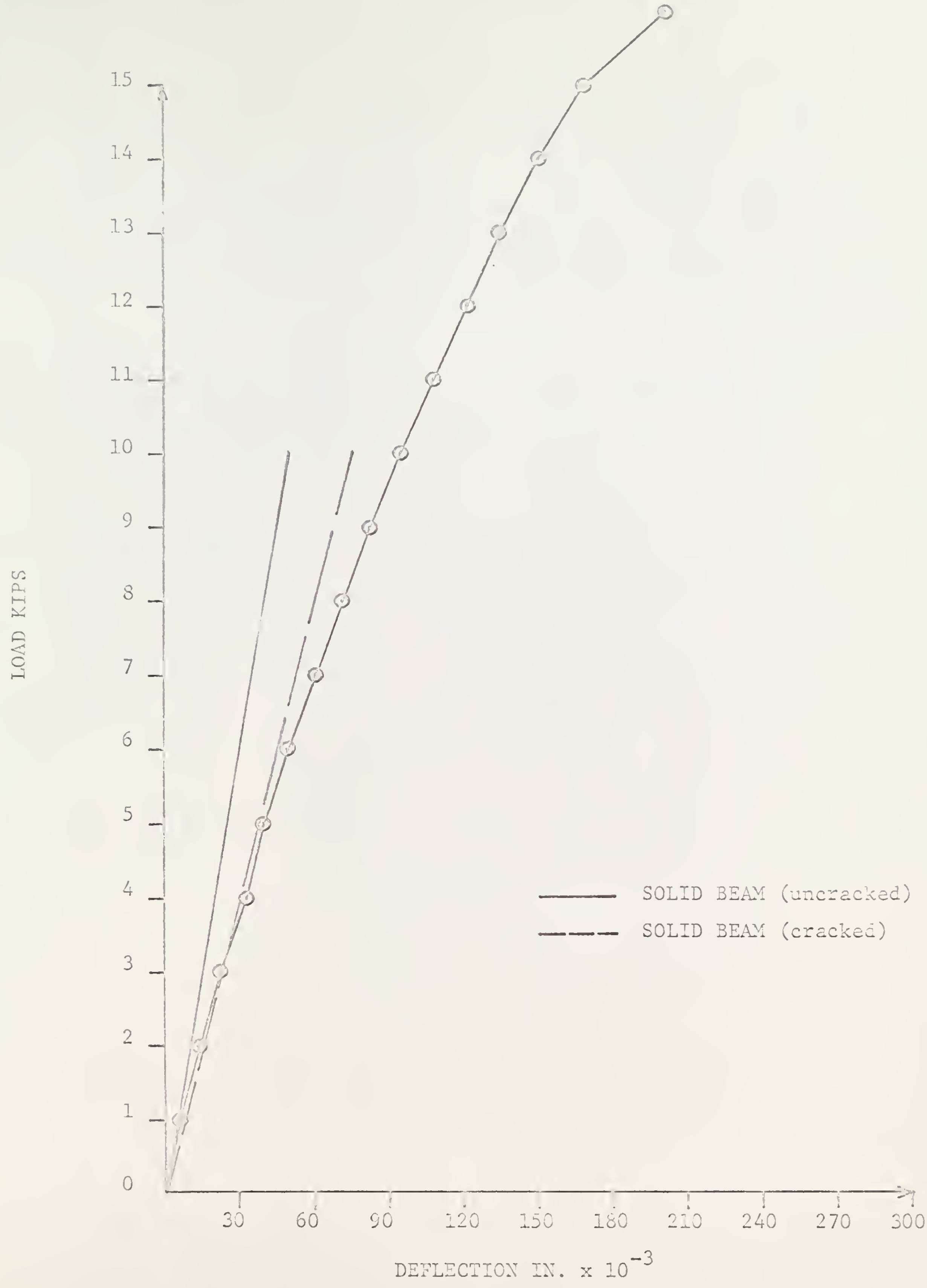


FIG. 45. DEFLECTIONS AT CENTER VS LOAD, BEAM #4

APPENDIX II

TABLES

TABLE 1

$\gamma_{xy_{BOT.}} : \gamma_{xy_{TOP}}$, BEAM #1, MIDPOINTS OF CHORDS.

NO SHEAR REINFORCEMENT PROVIDED IN CHORDS.

LOAD KIPS	$\gamma_{xy\,(TOP)}$ IN./IN. x 10^{-6}	$\gamma_{xy\,(BOT.)}$ IN./IN. x 10^{-6}	RATIO $\frac{\gamma_{xy\,(BOT.)}}{\gamma_{xy\,(TOP)}}$
1	25	22	0.880
2	48	45	0.936
3	65	65	1.000
4	83	70	0.844
5	100	89	0.890
6	119	93	0.781
7	136	90	0.661
8	150	88	0.586
9	168	107	0.636
10	169	135	0.800
11	210	144	0.685
12	178	122	0.685

TABLE 2

σ_x, BEAM #1, STRAIN GAGE #1 GLUED ON TOP SURFACE OF BEAM AT 18½" FROM SUPPORT, SOLID SIDE OF BEAM

LOAD KIPS	ε_x IN./IN. x 10^{-6}	$\sigma_x = 1000\ f'_c\ \varepsilon_x$ PSI
0.5	- 12	- 48.6
1.0	- 24	- 97.2
1.5	- 30	- 121.5
2.0	- 50	- 202.5
2.5	- 62	- 251.0
3.0	- 78	- 316.0
3.5	- 91	- 368.0
4.0	- 107	- 433.0
4.5	- 121	- 490.0
5.0	- 133	- 539.0
5.5	- 146	- 591.0
6.0	- 160	- 648.0
6.5	- 171	- 692.0
7.0	- 181	- 733.0
7.5	- 200	- 810.0
8.0	- 211	- 855.0
8.5	- 226	- 915.0
9.0	- 245	- 991.0
9.5	- 256	-1039.0
10.0	- 270	-1091.0
10.5	- 295	-1195.0

TABLE 2 (continued)

LOAD KIPS	ε_x IN./IN. x 10^{-6}	$\sigma_x = 1000\ f'_c\ \varepsilon_x$ PSI
11.0	- 298	-1209.0
11.5	- 310	-1258.0
12.0	- 326	-1320.0
13.0	- 350	-1420.0
14.0	- 365	-1480.0
14.5	- 400	-1620.0
15.0	- 410	-1660.0

TABLE 3

STRAINS, BEAM #1, STRAIN GAGE #2 GLUED ON TOP CHORD AT MIDPOINT, FACE ONE

LOAD KIPS	$\varepsilon_a = \varepsilon_x$ IN./IN. x 10^{-6}	ε_b IN./IN. x 10^{-6}	$\varepsilon_c = \varepsilon_y$ IN./IN. x 10^{-6}	$\gamma_{xy} = 2\varepsilon_b - \varepsilon_a - \varepsilon_c$ IN./IN. x 10^{-6}
0.5	- 6	- 4	+ 4	- 6
1.0	- 12	- 15	+ 7	- 25
1.5	- 15	- 20	+ 9	- 34
2.0	- 22	- 31	+ 8	- 48
2.5	- 29	- 39	+ 10	- 59
3.0	- 35	- 44	+ 12	- 65
3.5	- 42	- 51	+ 12	- 72
4.0	- 49	- 60	+ 12	- 83
4.5	- 52	- 65	+ 16	- 94
5.0	- 59	- 70	+ 19	-100
5.5	- 62	- 78	+ 20	-114
6.0	- 70	- 84	+ 21	-119
6.5	- 70	- 81	+ 26	-118
7.0	- 71	- 88	+ 31	-136
7.5	- 80	- 98	+ 31	-147
8.0	- 85	-102	+ 31	-150
8.5	- 96	-108	+ 34	-154
9.0	- 92	-110	+ 40	-168
9.5	-100	-112	+ 42	-166

TABLE 3 (continued)

LOAD KIPS	$\varepsilon_a = \varepsilon_x$ IN./IN. x 10^{-6}	ε_b IN./IN. x 10^{-6}	$\varepsilon_c = \varepsilon_y$ IN./IN. x 10^{-6}	$\gamma_{xy} = 2\varepsilon_b - \varepsilon_a - \varepsilon_c$ IN./IN. x 10^{-6}
10.0	-105	-116	+ 42	-169
10.5	-110	-123	+ 44	-180
11.0	- 82	-123	+ 46	-210
11.5	-120	-125	+ 47	-177
12.0	-127	-129	+ 47	-178
13.0	-137	-133	+ 39	-168
14.0	-305	-225	+ 31	-176

TABLE 4

σ_x AND τ_{xy}, BEAM #1, STRAIN GAGE #2

LOAD KIPS	$\sigma_x = E_c \varepsilon_x$ PSI	$\tau_{xy} = G \gamma_{xy}$ PSI
1	- 48.5	44.0
2	- 89.0	84.5
3	- 141.9	114.4
4	- 198.2	146.0
5	- 239.0	176.0
6	- 284.0	209.0
7	- 288.0	239.0
8	- 344.0	264.0
9	- 372.0	296.0
10	- 425.0	296.5
11	- 332.0	370.0
12	- 515.0	313.0
13	- 555.0	296.0

TABLE 5

STRAINS, BEAM #1, STRAIN GAGE #3 GLUED ON BOTTOM CHORD AT MIDPOINT, FACE ONE

LOAD KIPS	$\varepsilon_a = \varepsilon_x$ IN./IN. x 10^{-6}	ε_b IN./IN. x 10^{-6}	$\varepsilon_c = \varepsilon_y$ IN./IN. x 10^{-6}	$\gamma_{xy} = 2\varepsilon_b - \varepsilon_a - \varepsilon_c$ IN./IN. x 10^{-6}
0.5	+ 4	+ 9	0	12
1.0	+ 12	+ 17	0	22
1.5	+ 15	+ 24	- 1	34
2.0	+ 14	+ 28	- 3	45
2.5	+ 20	+ 36	- 4	56
3.0	+ 26	+ 41	- 9	65
3.5	+ 28	+ 40	- 10	62
4.0	+ 31	+ 42	- 19	70
4.5	+ 32	+ 42	- 21	71
5.0	+ 38	+ 50	- 27	89
5.5	+ 40	+ 47	- 31	85
6.0	+ 41	+ 49	- 36	93
6.5	+ 43	+ 50	+ 36	93
7.0	+ 45	+ 49	+ 37	90
7.5	+ 42	+ 50	- 40	98
8.0	+ 50	+ 48	- 42	88
8.5	+ 39	+ 42	- 48	93
9.0	+ 38	+ 50	- 45	107
9.5	+ 39	+ 55	- 48	119

TABLE 5 (continued)

LOAD KIPS	$\varepsilon_a = \varepsilon_x$ IN./IN. x 10^{-6}	ε_b IN./IN. x 10^{-6}	$\varepsilon_c = \varepsilon_y$ IN./IN. x 10^{-6}	$\gamma_{xy} = 2\varepsilon_b - \varepsilon_a - \varepsilon_c$ IN./IN. x 10^{-6}
10.0	+ 35	+ 61	- 48	135
10.5	+ 21	+ 59	- 40	137
11.0	+ 8	+ 60	- 32	144
11.5	- 28	+ 50	- 15	143
12.0	- 50	+ 34	- 4	122
13.0	-100	- 6	+ 32	56
14.0	+378	+1885	+1729	1663

TABLE 6

σ_x AND τ_{xy}, BEAM #1, STRAIN GAGE #3

LOAD KIPS	$\sigma_x = E_c\ \varepsilon_x$ PSI	$\tau_{xy} = G\ \gamma_{xy}$ PSI
1	+ 48.6	38.7
2	+ 56.7	79.1
3	+ 105.2	114.4
4	+ 125.8	123.0
5	+ 154.0	156.5
6	+ 166.0	163.5
7	+ 182.0	158.1
8	+ 202.5	155.0
9	+ 154.0	188.0
10	+ 142.0	238.0
11	+ 32.4	254.0
12	- 202.5	214.0
13	- 404.0	98.5

*TABLE 7

STRAINS, BEAM #1, STRAIN GAGE #4 GLUED ON
BOTTOM CHORD AT MIDPOINT, FACE TWO

LOAD KIPS	$\varepsilon_a = \varepsilon_x$ IN./IN. x 10^{-6}	ε_b IN./IN. x 10^{-6}	$\varepsilon_c = \varepsilon_y$ IN./IN. x 10^{-6}	$\gamma_{xy} = 2\varepsilon_b - \varepsilon_a - \varepsilon_c$ IN./IN. x 10^{-6}
0.5	+ 11	+ 7	- 4	7
1.0	+ 16	+ 20	- 7	31
1.5	+ 24	+ 28	- 9	41
2.0	+ 30	+ 36	- 10	52
2.5	+ 40	+ 48	- 11	67
3.0	+ 46	+ 56	- 16	82
3.5	+ 50	+ 70	- 18	108
4.0	+ 35	+ 99	- 20	183
4.5	+ 30	+112	- 22	216
5.0	+ 30	+147	- 22	286
5.5	+ 16	+171	- 23	349
6.0	+ 3	+198	- 24	417
6.5	- 11	+240	- 20	511
7.0	- 26	+275	- 20	596
7.5	- 40	+324	- 20	708
8.0	- 52	+364	- 22	802
8.5	- 94	+484	- 13	1075
9.0	-132	+601	- 5	1339
9.5	-150	+658	- 3	1469

*TABLE 7 (continued)

LOAD KIPS	$\varepsilon_a = \varepsilon_x$ IN./IN. x 10^{-6}	ε_b IN./IN. x 10^{-6}	$\varepsilon_c = \varepsilon_y$ IN./IN. x 10^{-6}	$\gamma_{xy} = 2\varepsilon_b - \varepsilon_a - \varepsilon_c$ IN./IN. x 10^{-6}
10.0	-178	+702	+ 2	1580
10.5	-210	+760	+ 22	1708
11.0	-242	+798	+ 49	1789
11.5	-259	+870	+135	1864
12.0	-271	+911	+184	1909
13.0	-330	+1060	+340	2110
14.0	-518	+1166	+509	2341

*This strain gage being on a rough surface did not behave well, hence the data was rejected.

TABLE 8

$\gamma_{xy_{BOT.}} : \gamma_{xy_{TOP}}$, BEAM #2, MIDPOINTS OF CHORDS.

ALL SHEAR REINFORCEMENT IN TOP, NONE IN BOTTOM CHORD.

LOAD KIPS	γ_{xy} (TOP) IN./IN. x 10^{-6}	γ_{xy} (BOT.) IN./IN. x 10^{-6}	RATIO $\frac{\gamma_{xy}(BOT.)}{\gamma_{xy}(TOP)}$
1	13	21	1.61
2	36	32	0.89
3	50	38	0.76
4	67	50	0.745
5	81	68	0.84
6	94	81	0.86
7	104	100	0.96
8	120	89	0.74
9	130	49	0.377
10	144	57	0.396
11	178	74	0.415
12	217	82	0.378
13	55	85	1.55

TABLE 9

STRAINS, BEAM #2, STRAIN GAGE #3 GLUED ON TOP CHORD AT MIDPOINT, FACE ONE

LOAD KIPS	$\varepsilon_a = \varepsilon_x$ IN./IN. x 10^{-6}	ε_b IN./IN. x 10^{-6}	$\varepsilon_c = \varepsilon_y$ IN./IN. x 10^{-6}	$\gamma_{xy} = 2\varepsilon_b - \varepsilon_a - \varepsilon_c$ IN./IN. x 10^{-6}
1	- 10	- 11	+ 1	13
2	- 18	- 25	+ 4	36
3	- 30	- 37	+ 6	50
4	- 41	- 50	+ 8	67
5	- 50	- 61	+ 9	81
6	- 61	- 72	+ 11	94
7	- 70	- 82	+ 10	104
8	- 79	- 95	+ 9	120
9	- 86	-106	+ 4	130
10	- 92	-121	- 6	144
11	- 93	-140	- 9	178
12	-105	-163	- 4	217
13	-183	-143	+ 52	55
14	-238	-118	+840	838
15	-385	-162	+1643	1582

TABLE 10

σ_x AND τ_{xy}, BEAM #2, STRAIN GAGE #3

LOAD KIPS	$\sigma_x = E_c\ \varepsilon_x$ PSI	$\tau_{xy} = G\ \gamma_{xy}$ PSI
1	- 42.0	23.8
2	- 75.5	65.9
3	- 126.0	91.5
4	- 172.0	122.8
5	- 210.0	148.0
6	- 256.0	172.0
7	- 294.0	190.0
8	- 332.0	220.0
9	- 361.0	238.0
10	- 386.0	264.0
11	- 390.0	326.0
12	- 440.0	397.0
13	- 769.0	100.8
14	-1000.0	1530.0

TABLE 11

STRAINS, BEAM #2, STRAIN GAGE #4 GLUED ON BOTTOM CHORD AT MIDPOINT, FACE ONE

LOAD KIPS	$\varepsilon_a = \varepsilon_x$ IN./IN. x 10^{-6}	ε_b IN./IN. x 10^{-6}	$\varepsilon_c = \varepsilon_y$ IN./IN. x 10^{-6}	$\gamma_{xy} = 2\varepsilon_b - \varepsilon_a - \varepsilon_c$ IN./IN. x 10^{-6}
1	+ 9	- 8	- 4	21
2	+ 18	- 10	- 6	32
3	+ 8	- 19	- 8	38
4	+ 17	- 22	- 11	50
5	+ 27	- 27	- 13	68
6	+ 34	- 30	- 13	81
7	+ 37	- 41	- 19	100
8	+ 18	- 46	- 21	89
9	+ 18	- 81	- 71	49
10	+ 15	- 57	- 72	57
11	+ 7	- 75	- 83	74
12	- 94	-123	- 70	82
13	-166	-145	- 39	85
14	-243	- 59	+539	414
15	- 75	- 9	0	57

TABLE 12

σ_x AND τ_{xy}, BEAM #2, STRAIN GAGE #4

LOAD KIPS	$\sigma_x = E_c\ \varepsilon_x$ PSI	$\tau_{xy} = G\ \gamma_{xy}$ PSI
1	+ 37.8	38.4
2	+ 75.5	58.5
3	+ 33.6	69.5
4	+ 71.5	91.5
5	+ 113.5	124.4
6	+ 143.0	148.0
7	+ 155.8	183.0
8	+ 75.5	163.0
9	+ 75.5	89.5
10	+ 63.0	104.2
11	+ 29.4	135.3
12	- 395.0	150.0
13	-1020.0	155.5
14	--	756.0

*TABLE 13

STRAINS, BEAM #2, STRAIN GAGE #2 GLUED ON
TOP CHORD AT MIDPOINT, FACE TWO

LOAD KIPS	$\varepsilon_a = \varepsilon_x$ IN./IN. x 10^{-6}	ε_b IN./IN. x 10^{-6}	$\varepsilon_c = \varepsilon_y$ IN./IN. x 10^{-6}	$\gamma_{xy} = 2\varepsilon_b - \varepsilon_a - \varepsilon_c$ IN./IN. x 10^{-6}
1	- 7	+ 7	+ 2	19
2	- 13	+ 17	+ 4	43
3	+ 7	+150	+ 89	204
4	0	+164	+ 94	234
5	- 7	+179	+102	263
6	- 10	+210	+118	312
7	- 15	+278	+166	405
8	+ 10	+450	+271	620
9	+ 35	+620	+331	874
10	+156	+959	+593	1209
11	+351	+1389	+850	1577
12	+564	+1882	+1196	2004
13	+888	+2542	+1630	2566
14	+1219	+3440	+2270	3391
15	+1677	+4954	+3251	4980

*This strain gage being on a rough surface did not behave well, hence the data was rejected.

TABLE 14

$\gamma_{xy_{BOT.}} : \gamma_{xy_{TOP}}$, BEAM #3, MIDPOINTS OF CHORDS.

HALF SHEAR REINFORCEMENT IN TOP CHORD AND HALF IN BOTTOM CHORD.

LOAD KIPS	γ_{xy} (TOP) IN./IN. x 10^{-6}	γ_{xy} (BOT.) IN./IN. x 10^{-6}	RATIO $\frac{\gamma_{xy}\text{(BOT.)}}{\gamma_{xy}\text{(TOP)}}$
1	27	21	0.777
2	62	52	0.840
3	92	68	0.740
4	120	76	0.634
5	143	94	0.658
6	170	102	0.600
7	191	121	0.655
8	214	140	0.655
9	231	168	0.726
10	259	346	1.330
11	277	312	1.130
12	312	496	1.590
13	333	726	2.180

TABLE 15

STRAINS, BEAM #3, STRAIN GAGE #2 GLUED ON
TOP CHORD AT MIDPOINT, FACE ONE

LOAD KIPS	$\varepsilon_a = \varepsilon_x$ IN./IN. x 10^{-6}	ε_b IN./IN. x 10^{-6}	$\varepsilon_c = \varepsilon_y$ IN./IN. x 10^{-6}	$\gamma_{xy} = 2\varepsilon_b - \varepsilon_a - \varepsilon_c$ IN./IN. x 10^{-6}
1	- 9	+ 11	+ 4	27
2	- 18	+ 24	+ 4	62
3	- 30	+ 34	+ 6	92
4	- 38	+ 46	+ 10	120
5	- 47	+ 53	+ 10	143
6	- 55	+ 63	+ 11	170
7	- 63	+ 70	+ 12	191
8	- 73	+ 77	+ 13	214
9	- 82	+ 83	+ 17	231
10	- 90	+ 94	+ 19	259
11	-101	+ 97	+ 18	277
12	-110	+112	+ 22	312
13	-110	+127	+ 31	333
14	-123	+147	+ 48	369

TABLE 16

σ_x AND τ_{xy}, BEAM #3, STRAIN GAGE #2

LOAD KIPS	$\sigma_x = E_c \ \varepsilon_x$ PSI	$\tau_{xy} = G \ \gamma_{xy}$ PSI
1	− 38.2	50
2	− 76.4	115
3	− 127.5	170
4	− 161.5	212
5	− 200.0	264
6	− 234.0	314
7	− 268.0	354
8	− 310.0	396
9	− 348.0	427
10	− 382.0	480
11	− 430.0	512
12	− 467.0	577
13	− 467.0	615

TA[illegible] 17

STRAINS, BEAM #3, [illegible]GE #4 GLUED ON
BOTTOM CHORD AT MID[illegible]FACE ONE

LOAD KIPS	$\varepsilon_a = \varepsilon_x$ IN./IN. x 10^{-6}	ε_b IN./IN. x 10^{-6}	$\varepsilon_c = \varepsilon_y$ IN./IN. x 10^{-6}	$\gamma_{xy} = 2\varepsilon_b - \varepsilon_a - \varepsilon_c$ IN./IN. x 10^{-6}
1	+ 7	+ 12	- 4	21
2	+ 16	+ 32	- 4	52
3	+ 28	+ 44	- 8	68
4	+ 30	+ 45	- 16	76
5	+ 26	+ 49	- 22	94
6	+ 18	+ 47	- 26	102
7	+ 17	+ 54	- 30	121
8	+ 9	+ 57	- 35	140
9	+ 1	+ 67	- 35	168
10	+440	+411	+ 36	346
11	+599	+480	+ 49	312
12	+689	+639	+ 93	496
13	+804	+830	+130	726
14	+960	+1176	+246	1146

TABLE 18

σ_x AND τ_{xy}, BEAM #3, STRAIN GAGE #4

LOAD KIPS	$\sigma_x = E_c \; \varepsilon_x$ PSI	$\tau_{xy} = G \; \gamma_{xy}$ PSI
1	+ 29.8	38.8
2	+ 68.0	96.0
3	+ 119.0	126.0
4	+ 127.8	140.5
5	+ 110.5	174.0
6	+ 76.5	189.0
7	+ 72.2	224.0
8	+ 38.2	258.0
9	+ 4.25	310.0
10	+1870.0	640.0
11	+2540.0	576.0
12	+2920.0	916.0
13	+3420.0	1345.0

*TABLE 19

STRAINS, BEAM #3, STRAIN GAGE #3 GLUED ON TOP CHORD AT MIDPOINT, FACE TWO

LOAD KIPS	$\varepsilon_a = \varepsilon_x$ IN./IN. x 10^{-6}	ε_b IN./IN. x 10^{-6}	$\varepsilon_c = \varepsilon_y$ IN./IN. x 10^{-6}	$\gamma_{xy} = 2\varepsilon_b - \varepsilon_a - \varepsilon_c$ IN./IN. x 10^{-6}
1	- 8	- 5	+ 1	3
2	- 27	- 11	+ 3	2
3	- 38	- 15	+ 6	2
4	- 52	- 23	+ 9	3
5	- 66	- 31	+ 10	6
6	- 81	- 40	+ 11	10
7	- 96	- 50	+ 14	18
8	-109	- 60	+ 15	26
9	-124	- 71	+ 18	36
10	-145	- 85	+ 20	45
11	-163	-101	+ 20	59
12	-183	-114	+ 25	70
13	-205	-135	+ 30	95
14	-231	-160	+ 46	135

*This strain gage being on a rough surface did not behave well, hence the data was rejected.

TABLE 20

$\gamma_{xy_{BOT.}} : \gamma_{xy_{TOP}}$, BEAM #4, MIDPOINTS OF CHORDS.

0.66 SHEAR REINFORCEMENT IN TOP AND 0.34 IN BOTTOM CHORD.

LOAD KIPS	γ_{xy} (TOP) IN./IN. x 10^{-6}	γ_{xy} (BOT.) IN./IN. x 10^{-6}	RATIO $\frac{\gamma_{xy}(BOT.)}{\gamma_{xy}(TOP)}$
1	24	16	0.67
2	45	33	0.73
3	78	53	0.68
4	107	55	0.52
5	132	57	0.43
6	152	56	0.37
7	203	73	0.36
8	242	93	0.38
9	233	117	0.50
10	165	145	0.88
11	55	123	2.24

TABLE 21

STRAINS, BEAM #4, STRAIN GAGE #2 GLUED ON
TOP CHORD AT MIDPOINT, FACE ONE

LOAD KIPS	$\varepsilon_a = \varepsilon_x$ IN./IN. x 10^{-6}	ε_b IN./IN. x 10^{-6}	$\varepsilon_c = \varepsilon_y$ IN./IN. x 10^{-6}	$\gamma_{xy} = 2\varepsilon_b - \varepsilon_a - \varepsilon_c$ IN./IN. x 10^{-6}
1	- 10	+ 6	- 2	24
2	- 13	+ 17	+ 2	45
3	- 24	+ 28	+ 2	78
4	- 31	+ 38	0	107
5	- 40	+ 44	- 4	132
6	- 44	+ 51	- 6	152
7	- 57	+ 70	- 6	203
8	- 67	+ 87	- 1	242
9	- 89	+ 94	+ 44	233
10	-203	+ 5	+ 48	165
11	-300	- 66	+113	55
12	-339	-127	+ 77	8

TABLE 22

σ_x AND τ_{xy}, BEAM #4, STRAIN GAGE #2

LOAD KIPS	$\sigma_x = E_c\ \varepsilon_x$ PSI	$\tau_{xy} = G\ \gamma_{xy}$ PSI
1	- 43.5	45.4
2	- 56.5	85.0
3	- 104.2	147.5
4	- 135.0	202.0
5	- 174.0	249.5
6	- 191.5	287.0
7	- 248.0	384.0
8	- 292.0	457.0
9	- 388.0	440.0
10	- 882.0	312.0
11	-1305.0	104.0
12	-1475.0	15.1

TABLE 23

STRAINS, BEAM #4, STRAIN GAGE #4 GLUED ON
BOTTOM CHORD AT MIDPOINT, FACE ONE

LOAD KIPS	$\varepsilon_a = \varepsilon_x$ IN./IN. x 10^{-6}	ε_b IN./IN. x 10^{-6}	$\varepsilon_c = \varepsilon_y$ IN./IN. x 10^{-6}	$\gamma_{xy} = 2\varepsilon_b - \varepsilon_a - \varepsilon_c$ IN./IN. x 10^{-6}
1	+ 8	+ 11	- 2	16
2	+ 19	+ 23	- 6	33
3	+ 29	+ 33	- 16	53
4	+ 33	+ 33	- 22	55
5	+ 30	+ 33	- 21	57
6	+ 22	+ 26	- 26	56
7	+ 23	+ 32	- 32	73
8	+ 23	+ 38	- 40	93
9	+ 14	+ 43	- 45	117
10	- 6	+ 56	- 27	145
11	- 63	+ 48	+ 36	123
12	-102	- 5	+ 60	32

TABLE 24

σ_x AND τ_{xy}, BEAM #4, STRAIN GAGE #4

LOAD KIPS	$\sigma_x = E_c\ \varepsilon_x$ PSI	$\tau_{xy} = G\ \gamma_{xy}$ PSI
1	+ 34.8	30.02
2	+ 82.5	62.5
3	+ 126.0	100.0
4	+ 143.5	104.0
5	+ 131.0	108.0
6	+ 95.5	106.0
7	+ 100.0	138.0
8	+ 100.0	176.0
9	+ 61.0	221.0
10	- 26.1	274.0
11	- 274.0	232.0

*TABLE 25

STRAINS, BEAM #4, STRAIN GAGE #3 GLUED ON TOP CHORD AT MIDPOINT, FACE TWO

LOAD KIPS	$\varepsilon_a = \varepsilon_x$ IN./IN. x 10^{-6}	ε_b IN./IN. x 10^{-6}	$\varepsilon_c = \varepsilon_y$ IN./IN. x 10^{-6}	$\gamma_{xy} = 2\varepsilon_b - \varepsilon_a - \varepsilon_c$ IN./IN. x 10^{-6}
1	- 8	+ 10	+ 3	25
2	- 15	+ 26	+ 4	63
3	- 13	+ 73	+ 30	129
4	+ 3	+221	+107	332
5	- 2	+350	+192	510
6	+ 7	+544	+345	736
7	+ 29	+740	+517	934
8	+ 75	+1010	+722	1223
9	+109	+1294	+975	1504

*This strain gage being on a rough surface did not behave well, hence the data was rejected.

TABLE 26

DEFLECTIONS IN INCHES AT CHORDS AND CENTER OF THE BEAM #1

DIAL GAGE / LOAD KIPS	T_1	T_2	T_3	T_4	T_5	T_6	B_1	B_2	B_3	B_4	B_5	B_6	CENTER
0.0	-	-	-	-	-	-	-	-	-	-	-	-	-
0.5	0.004	0.004	0.007	0.004	0.012	0.007	0.003	0.003	0.004	0.004	0.005	0.004	0.003
1.0	0.004	0.004	0.007	0.006	0.015	0.008	0.003	0.004	0.005	0.005	0.007	0.007	0.007
1.5	0.005	0.006	0.009	0.009	0.019	0.011	0.004	0.006	0.007	0.008	0.010	0.009	0.010
2.0	0.006	0.007	0.010	0.011	0.022	0.014	0.006	0.007	0.008	0.010	0.013	0.013	0.013
2.5	0.007	0.009	0.013	0.014	0.026	0.018	0.007	0.009	0.011	0.013	0.017	0.017	0.017
3.0	0.009	0.011	0.015	0.017	0.030	0.022	0.008	0.011	0.013	0.016	0.020	0.021	0.021
3.5	0.010	0.013	0.017	0.019	0.032	0.025	0.009	0.012	0.015	0.019	0.023	0.024	0.024
4.0	0.011	0.015	0.020	0.023	0.037	0.030	0.011	0.015	0.017	0.023	0.027	0.029	0.028
4.5	0.012	0.016	0.022	0.025	0.040	0.034	0.012	0.016	0.019	0.025	0.031	0.031	0.032
5.0	0.013	0.018	0.024	0.028	0.043	0.036	0.013	0.017	0.021	0.028	0.034	0.034	0.035
5.5	0.014	0.019	0.026	0.030	0.046	0.040	0.014	0.019	0.023	0.031	0.038	0.038	0.039
6.0	0.015	0.021	0.028	0.033	0.049	0.043	0.015	0.020	0.025	0.034	0.041	0.041	0.042
6.5	0.017	0.022	0.031	0.036	0.053	0.048	0.017	0.022	0.028	0.037	0.045	0.045	0.047

TABLE 26 (continued)

LOAD KIPS \ DIAL GAGE	T_1	T_2	T_3	T_4	T_5	T_6	B_1	B_2	B_3	B_4	B_5	B_6	CENTER
7.0	0.018	0.024	0.033	0.039	0.056	0.051	0.018	0.024	0.030	0.040	0.049	0.049	0.050
7.5	0.019	0.026	0.036	0.042	0.061	0.057	0.020	0.026	0.033	0.044	0.054	0.054	0.056
8.0	0.021	0.028	0.038	0.045	0.064	0.061	0.021	0.027	0.036	0.047	0.059	0.059	0.060
8.5	0.022	0.029	0.040	0.048	0.067	0.064	0.022	0.029	0.038	0.050	0.062	0.063	0.064
9.0	0.023	0.031	0.043	0.051	0.071	0.069	0.024	0.031	0.041	0.054	0.067	0.069	0.069
9.5	0.024	0.033	0.045	0.054	0.074	0.073	0.025	0.032	0.043	0.057	0.070	0.072	0.073
10.0	0.025	0.034	0.047	0.057	0.077	0.077	0.026	0.034	0.045	0.060	0.074	0.077	0.078
10.5	0.026	0.035	0.049	0.059	0.080	0.080	0.027	0.035	0.048	0.063	0.079	0.080	0.082
11.0	0.027	0.037	0.051	0.062	0.083	0.083	0.028	0.036	0.050	0.066	0.082	0.086	0.086
11.5	0.028	0.038	0.053	0.065	0.087	0.087	0.029	0.038	0.053	0.070	0.087	0.089	0.091
12.0	0.029	0.040	0.056	0.068	0.090	0.092	0.030	0.040	0.055	0.073	0.091	0.094	0.096
13.0	0.031	0.042	0.060	0.074	0.097	0.102	0.032	0.043	0.061	0.080	0.100	0.104	0.106
14.0	0.032	0.044	0.064	0.081	0.106	0.114	0.034	0.046	0.068	0.090	0.112	0.118	0.119
15.0	0.020	0.044	0.068	0.106	0.153	0.177	0.025	0.052	0.102	0.145	0.184	0.188	0.178
16.0	0.019	0.043	0.071	0.111	0.165	0.194	0.024	0.057	0.115	0.163	0.206	0.214	0.200

TABLE 27

DEFLECTIONS IN INCHES AT CHORDS AND CENTER OF THE BEAM #2

LOAD KIPS \ DIAL GAGE	T_1	T_2	T_3	T_4	T_5	T_6	B_1	B_2	B_3	B_4	B_5	B_6	CENTER
0.0	-	-	-	-	-	-	-	-	-	-	-	-	-
1.0	0.002	0.001	0.001	0.003	0.005	0.005	0.004	0.002	0.005	0.005	0.007	0.006	0.007
2.0	0.004	0.003	0.005	0.007	0.011	0.012	0.007	0.004	0.010	0.011	0.013	0.014	0.014
3.0	0.007	0.008	0.013	0.017	0.023	0.026	0.012	0.010	0.018	0.022	0.027	0.030	0.029
4.0	0.008	0.010	0.016	0.022	0.028	0.032	0.014	0.012	0.021	0.027	0.033	0.035	0.036
5.0	0.010	0.013	0.019	0.026	0.033	0.038	0.015	0.014	0.025	0.030	0.038	0.040	0.041
6.0	0.011	0.015	0.022	0.030	0.039	0.044	0.017	0.017	0.028	0.035	0.043	0.047	0.048
7.0	0.013	0.018	0.027	0.035	0.046	0.052	0.019	0.019	0.033	0.041	0.050	0.055	0.056
8.0	0.015	0.021	0.032	0.042	0.054	0.061	0.022	0.023	0.038	0.048	0.058	0.065	0.066
9.0	0.017	0.024	0.036	0.048	0.061	0.070	0.024	0.026	0.044	0.054	0.066	0.070	0.075
10.0	0.019	0.027	0.041	0.055	0.070	0.079	0.026	0.030	0.050	0.061	0.075	0.081	0.085
11.0	0.020	0.030	0.047	0.061	0.079	0.090	0.028	0.034	0.056	0.070	0.085	0.093	0.096
12.0	0.022	0.033	0.051	0.064	0.088	0.101	0.030	0.037	0.062	0.079	0.097	0.105	0.108
13.0	0.023	0.035	0.055	0.073	0.098	0.114	0.031	0.041	0.069	0.090	0.111	0.119	0.121

TABLE 27 (continued)

DIAL GAGE / LOAD KIPS	T_1	T_2	T_3	T_4	T_5	T_6	B_1	B_2	B_3	B_4	B_5	B_6	CENTER
14.0	0.022	0.037	0.061	0.082	0.112	0.131	0.032	0.045	0.081	0.101	0.133	0.139	0.139
15.0	0.019	0.036	0.065	0.092	0.129	0.154	0.031	0.052	0.096	0.126	0.161	0.170	0.166
16.0	0.017	0.035	0.069	0.103	0.143	0.174	0.035	0.064	0.116	0.148	0.188	0.199	0.190
17.0	0.013	0.034	0.071	0.113	0.161	0.198	0.046	0.085	0.143	0.179	0.240	0.237	0.224
18.0	0.006	0.028	0.072	0.123	0.180	0.215	0.064	0.110	0.174	0.212	0.280	0.283	0.262

TABLE 28

DEFLECTIONS IN INCHES AT CHORDS AND CENTER OF THE BEAM #3

LOAD KIPS \ DIAL GAGE	T_1	T_2	T_3	T_4	T_5	T_6	B_1	B_2	B_3	B_4	B_5	B_6	CENTER
0.0	-	-	-	-	-	-	-	-	-	-	-	-	-
1.0	0.004	0.004	0.005	0.006	0.006	0.008	0.004	0.004	0.005	0.005	0.007	0.008	0.007
2.0	0.006	0.007	0.009	0.012	0.014	0.015	0.006	0.007	0.009	0.011	0.013	0.016	0.015
3.0	0.009	0.010	0.014	0.017	0.020	0.022	0.009	0.010	0.013	0.017	0.019	0.023	0.022
4.0	0.011	0.013	0.018	0.023	0.026	0.028	0.011	0.013	0.018	0.023	0.027	0.031	0.029
5.0	0.013	0.015	0.022	0.028	0.033	0.036	0.013	0.016	0.022	0.029	0.035	0.038	0.037
6.0	0.015	0.019	0.027	0.035	0.040	0.045	0.016	0.020	0.027	0.036	0.044	0.047	0.046
7.0	0.018	0.022	0.031	0.040	0.047	0.054	0.018	0.023	0.032	0.043	0.051	0.055	0.055
8.0	0.020	0.025	0.036	0.046	0.054	0.062	0.021	0.026	0.037	0.050	0.060	0.063	0.064
9.0	0.022	0.028	0.040	0.053	0.062	0.071	0.024	0.030	0.042	0.057	0.069	0.072	0.074
10.0	0.024	0.031	0.045	0.059	0.070	0.081	0.026	0.033	0.048	0.064	0.078	0.082	0.084
11.0	0.026	0.034	0.049	0.065	0.078	0.090	0.028	0.037	0.054	0.070	0.086	0.091	0.094
12.0	0.028	0.038	0.055	0.073	0.087	0.100	0.031	0.042	0.063	0.079	0.096	0.103	0.106
13.0	0.030	0.041	0.060	0.079	0.096	0.109	0.033	0.047	0.069	0.088	0.106	0.114	0.116

TABLE 28 (continued)

LOAD KIPS \ DIAL GAGE	T_1	T_2	T_3	T_4	T_5	T_6	B_1	B_2	B_3	B_4	B_5	B_6	CENTER
14.0	0.032	0.044	0.065	0.086	0.104	0.120	0.035	0.051	0.076	0.096	0.116	0.125	0.127
15.0	0.033	0.045	0.069	0.093	0.116	0.134	0.037	0.056	0.084	0.108	0.130	0.140	0.140
16.0	0.033	0.046	0.085	0.105	0.132	0.154	0.039	0.063	0.096	0.126	0.153	0.161	0.161
17.0	0.033	0.048	0.090	0.114	0.145	0.170	0.041	0.068	0.105	0.140	0.170	0.178	0.178
18.0	0.035	0.051	0.097	0.124	0.161	0.188	0.044	0.076	0.115	0.154	0.185	0.186	0.197
19.0	0.037	0.045	0.107	0.140	0.186	0.219	0.049	0.087	0.133	0.180	0.222	0.222	0.230

TABLE 29

DEFLECTIONS IN INCHES AT CHORDS AND CENTER OF THE BEAM #4

LOAD KIPS \ DIAL GAGE	T_1	T_2	T_3	T_4	T_5	T_6	B_1	B_2	B_3	B_4	B_5	B_6	CENTER
0.0	-	-	-	-	-	-	-	-	-	-	-	-	-
1.0	0.002	0.002	0.003	0.005	0.006	0.005	0.002	0.003	0.004	0.005	0.005	0.007	0.006
2.0	0.002	0.004	0.006	0.009	0.010	0.011	0.003	0.005	0.007	0.009	0.011	0.014	0.014
3.0	0.006	0.008	0.011	0.016	0.018	0.020	0.006	0.009	0.012	0.016	0.020	0.024	0.023
4.0	0.009	0.012	0.017	0.023	0.027	0.030	0.010	0.014	0.018	0.024	0.030	0.034	0.033
5.0	0.011	0.014	0.021	0.028	0.033	0.038	0.012	0.016	0.022	0.030	0.036	0.041	0.040
6.0	0.014	0.018	0.026	0.035	0.042	0.048	0.014	0.020	0.028	0.038	0.044	0.051	0.050
7.0	0.016	0.021	0.031	0.042	0.052	0.059	0.016	0.024	0.035	0.046	0.054	0.061	0.061
8.0	0.018	0.024	0.036	0.048	0.060	0.069	0.019	0.028	0.040	0.054	0.063	0.072	0.072
9.0	0.020	0.027	0.041	0.056	0.072	0.080	0.021	0.033	0.047	0.063	0.073	0.084	0.083
10.0	0.023	0.030	0.046	0.063	0.082	0.091	0.023	0.035	0.053	0.072	0.085	0.095	0.095
11.0	0.025	0.033	0.052	0.072	0.094	0.104	0.027	0.041	0.062	0.083	0.099	0.110	0.109
12.0	0.026	0.035	0.058	0.082	0.104	0.117	0.028	0.045	0.069	0.093	0.112	0.124	0.122
13.0	0.027	0.037	0.063	0.091	0.116	0.131	0.031	0.049	0.077	0.103	0.125	0.138	0.135

TABLE 29 (continued)

LOAD KIPS \ DIAL GAGE	T_1	T_2	T_3	T_4	T_5	T_6	B_1	B_2	B_3	B_4	B_5	B_6	CENTER
14.0	0.027	0.039	0.069	0.102	0.131	0.149	0.034	0.055	0.087	0.118	0.143	0.157	0.151
15.0	0.027	0.040	0.074	0.111	0.145	0.167	0.036	0.060	0.097	0.133	0.159	0.175	0.169
16.0	0.029	0.044	0.085	0.129	0.175	0.197	0.042	0.072	0.115	0.159	0.191	0.199	0.203
17.0	0.030	0.047	0.093	0.144	0.197	0.222	0.046	0.081	0.131	0.180	0.218	0.227	0.229
18.0	0.017	0.040	0.111	0.195	0.274	0.315	0.064	0.121	0.188	0.257	0.313	0.328	0.315

TABLE 30

YIELD STRENGTH OF DIFFERENT REINFORCING BARS

BAR SIZE	AREA OF CROSS SECTION SQ. IN.	YIELD LOAD LBS.	YIELD STRESS f_y PSI
#1			78,000
#2	0.05	2,710	54,000
#4	0.2	10,950	54,000
#6	0.44	19,650	44,500

TABLE 31

COMPRESSIVE STRESS OF CONCRETE AT 28 DAYS

CYLINDER NO.	f'_c PSI	AVERAGE f'_c PSI
1	3,890	
2	4,014	
3	3,590	
		3,800

TABLE 32

COMPRESSIVE STRESS OF CONCRETE ON THE DAY OF BEAM TEST

	f''_c PSI			
CYLINDER NO.	BEAM 1	BEAM 2	BEAM 3	BEAM 4
1	4,000	4,100	4,300	4,386
2	4,100	4,300	4,200	4,280
3	4,050	4,200	4,250	4,400
AVERAGE f''_c PSI	4,050	4,200	4,250	4,350

TABLE 33

ULTIMATE LOAD PREDICTED AND ULTIMATE LOAD MEASURED

BEAM NO.	P_u (THEORETICAL) KIPS	P_u (MEASURED) KIPS	$\frac{P_u \text{(MEASURED)}}{P_u \text{(THEORETICAL)}}$
1	7.4	17.0	2.30
2	15.1	18.9	1.25
3	15.1	21.8	1.44
4	15.1	18.8	1.24

APPENDIX III

DATA REDUCTION

BEAM #1

$$f''_c = 4.05 \text{ ksi}$$

where f''_c = compressive stress of concrete based on cylinder test on the day of the beam test.

$$\sigma_x = E_c \ \varepsilon_x = 1000 \ f''_c \ \varepsilon_x$$

where σ_x = longitudinal stress

E_c = modulus of elasticity of concrete

ε_x = longitudinal unit strain

$E_c = 4.05 \times 10^6$

σ_x [experimental] $= 4.05 \times 10^6 \ \varepsilon_x$

σ_x [theoretical] $= \frac{M}{Z}$

where M = moment at the point

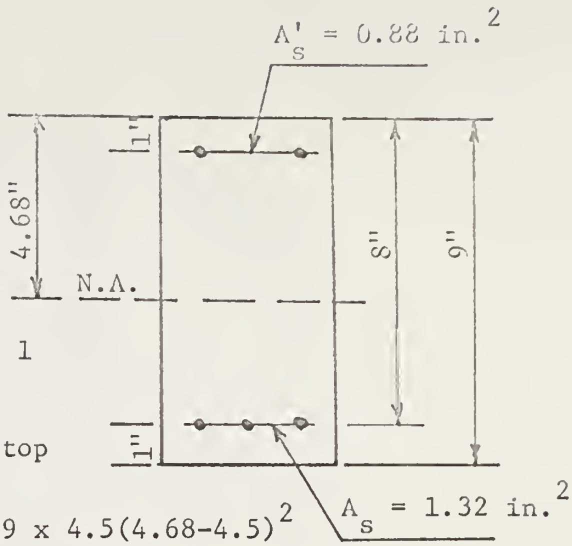
Z = section modulus

$M = \frac{P}{2} \times 18.5$ [for strain gage #1 glued at 18.5" from support on top surface of solid beam]

$E_s = 29 \times 10^6$ psi

$E_c = 4.05 \times 10^6$ psi

$n = \frac{29}{4.05} = 7.15$



$$A = 9 \times 4.5 + (n-1)(A'_s + A_s)$$

$$= 40.5 + 6.16 \times 2.2$$

$$A\,\bar{y}\ [\text{uncracked}] = 9 \times 4.5 \times 4.5 + (n-1)A_s \times 8 + (n-1)A'_s \times 1$$

$$\bar{y} = 4.68 \text{ in. from top}$$

$$I = \frac{1}{12}(9)^3\, 4.5 + 9 \times 4.5(4.68-4.5)^2 + (n-1)A_s(8-4.68)^2 + (n-1)A'_s(4.68-1)^2$$

$$I = 437.62 \text{ in.}^4$$

$$\sigma_x = P/2\left[\frac{18.5 \times 4.68}{437.62}\right]$$

$$= -0.099\ P \text{ psi}$$

where P is the load applied in lbs.

For strain gage #2 at midpoint of top chord the direct force in the chord is given by Eq. (1) as:

$$F = \frac{P}{2} \times 16.5 \times \frac{1}{6}$$

$$= 1.375\ P$$

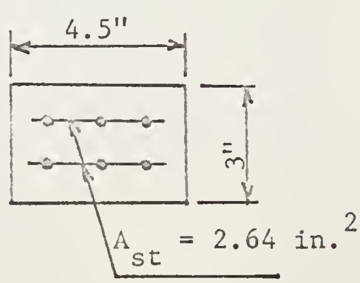
$$\sigma_x = \frac{F}{A}$$ [since midpoint of chords is point of contraflexure and thus has only direct force F and no moment]

$$A = 4.5 \times 3 + (n-1)(2.64)$$

$$= 29.71 \text{ in.}^2$$

$$\sigma_x = \frac{1.375}{29.71} \times P$$

$$= -0.0463\ P$$



$$G = \frac{E_c}{2(1+\mu)}$$

where G = modulus of ridigity

μ = Poisson's ratio = 0.15 (assumed)

$$G = \frac{4.05 \times 10^6}{2 \times 1.15} = 1.76 \times 10^6 \text{ psi}$$

$$\tau_{xy} = \gamma_{xy}\, G = 1.76 \times 10^6\, \gamma_{xy}$$

where τ_{xy} = shear stress

γ_{xy} = unit shear strain

BEAM #2

f''_c = 4200 psi

$E_c = 4.2 \times 10^6$ psi

$$n = \frac{29 \times 10^6}{4.2 \times 10^6} = 6.9$$

$A = 29.1 \text{ in.}^2$

$$\sigma_x = \frac{1.375}{29.1} P = -0.0472 P$$

$$G = \frac{4.2 \times 10^6}{2.3} = 1.83 \times 10^6$$

BEAM #3

f''_c = 4250 psi

$E_c = 4.25 \times 10^6$ psi

n = 6.82

$A = 28.9 \text{ in.}^2$

$$\sigma_x = \frac{1.375}{28.9} P = -0.0475 P$$

$G = 1.85 \times 10^6$ psi

BEAM #4

$$f''_c = 4350 \text{ psi}$$

$$E_c = 4.35 \times 10^6 \text{ psi}$$

$$n = 6.66$$

$$A = 28.5 \text{ in.}^2$$

$$\sigma_x = \frac{1.375}{28.5} P = -0.0482 P$$

$$G = 1.89 \times 10^6 \text{ psi}$$

DEFLECTION AT CENTER OF THE SOLID BEAM WITH SAME DIMENSIONS AND STEEL

Equation for finding neutral axis of cracked section:

$b = 4.5$ in.

$d = 8$ in.

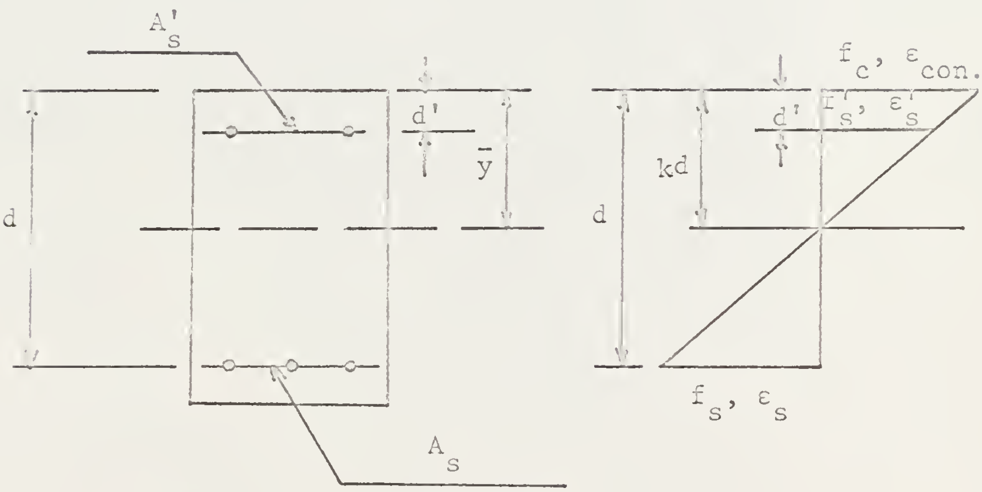
$d' = 1$ in.

$\bar{y}$ = distance from extreme compression fibre to neutral axis = kd

$f_s = E_s \, \varepsilon_s$

$f'_s = E_s \, \varepsilon'_s$

$f_c = E_c \, \varepsilon_{con.}$



$$\frac{kd}{\varepsilon_{con.}} = \frac{d - kd}{\varepsilon_s}$$

$$\frac{\varepsilon'_s}{kd - d'} = \frac{\varepsilon_{con.}}{kd}$$

$$f_s \, A_s = f'_s \, A'_s + \frac{1}{2} f_c \, b \, kd$$

$$A_s[E_s\ \varepsilon_s] = A'_s[E_s\ \varepsilon'_s] + \frac{1}{2}\ [E_c\ \varepsilon_{con.}]b\ kd$$

$$A_s\ E_s\ [\ \frac{d-kd}{kd}\ ]\ \varepsilon_{con.} = A'_s\ E_s\ [\ \frac{kd-d'}{kd}\ ]\ \varepsilon_{con.} + \frac{1}{2}\ b\ kd\ E_c\ \varepsilon_{con.}$$

$$n\ A_s(d-kd) = n\ A'_s(kd-d') + \frac{1}{2}\ b(kd)^2$$

$$n\ A_s(1-k) = n\ A'_s(k-\frac{d'}{d}) + \frac{bd}{2}\ k^2$$

$$(\frac{bd}{2})k^2 + (n\ A'_s)k - n\ A'_s\ \frac{d'}{d} - n\ A_s + n\ A_s\ k = 0$$

$$(\frac{bd}{2})k^2 + n(A'_s+A_s)k - n(A'_s\ \frac{d'}{d} + A_s) = 0$$

For Beam No. 1:

$E_c = 4.05 \times 10^6$ psi

$n = 7.15$

$k = 0.475$ (cracked)

$\bar{y} = 3.8$ in. (cracked)

$$I_{(\text{transformed cracked})} = \frac{1}{3} \times 4.5 \times (3.8)^3 + (n-1)A'_s(3.8-1)^2 + n\ A_s(8-3.8)^2$$

$$= 291.1\ \text{in.}^4$$

Deflection (Beam No. 1, cracked section) $= \dfrac{P\ell^3}{48\ E_c I} = \dfrac{(66)^3\ P}{48 \text{x} 4.05 \text{x} 10^6 \text{x} 291.1}$

$$= 5.08 \times 10^{-6}\ P$$

where P is in lbs. and deflection is in inches.

$$I_{(\text{transformed uncracked})} = 437.62\ \text{in.}^4$$

Deflection (Beam No.1, uncracked section) $= \dfrac{(66)^3 P}{48 \times 4.05 \times 10^6 \times 437.62}$

$= 3.38 \times 10^{-6}\ P$

For Beam No. 2:

$E_c = 4.2 \times 10^6$ psi

$n = 6.9$

$k = 0.429$ (cracked)

$\bar{y} = 3.43$ in. (cracked)

$I_{(\text{transformed cracked})} = 281.9\ \text{in.}^4$

$I_{(\text{transformed uncracked})} = 430.24\ \text{in.}^4$

Deflection (cracked) $= 5.06 \times 10^{-6}\ P$

Deflection (uncracked) $= 3.3 \times 10^{-6}\ P$

For Beam No. 3:

$E_c = 4.25 \times 10^6$ psi

$n = 6.82$

$k = 0.417$ (cracked)

$\bar{y} = 3.34$ in. (cracked)

$I_{(\text{transformed cracked})} = 280\ \text{in.}^4$

$I_{(\text{transformed uncracked})} = 427.37\ \text{in.}^4$

Deflection (cracked) $= 5.04 \times 10^{-6}\ P$

Deflection (uncracked) $= 3.3 \times 10^{-6}\ P$

For Beam No. 4:

$E_c = 4.35 \times 10^6$ psi

$n = 6.66$

$k = 0.428$ (cracked)

$\bar{y} = 3.42$ in. (cracked)

$I_{(\text{transformed cracked})} = 274.2 \text{ in.}^4$

$I_{(\text{transformed uncracked})} = 421.37 \text{ in.}^4$

Deflection $= 5.02 \times 10^{-6}$ P
(cracked)

Deflection $= 3.27 \times 10^{-6}$ P
(uncracked)

APPENDIX IV

DETAILED DESIGN CALCULATIONS FOR TEST BEAMS

(a) BALANCED DESIGN FOR MAIN LONGITUDINAL REINFORCEMENT:

f_y = 44.5 ksi for #6 bars

f_y = 54 ksi for #4 and #2 bars

f_y = 78 ksi for #1 bars

f'_c = 3.8 ksi

b = 4.5"; d = 8"

$$p = A_s/bd = p_b = \frac{0.85\ K_1\ f'_c}{f_y} \times \frac{87000}{87000 + f_y} \qquad \text{(ACI 16-2)}$$

$$p_b = 0.478\ \frac{3.8}{44.5} = 0.04$$

$A_s/bd = 0.04$

$A_s = 0.04 \times 4.5 \times 8 = 1.44\ \text{in.}^2$

(3 #6 provided; $A_s = 1.32\ \text{in.}^2$)

p = 0.0366

$M_{ult.} = A_s\ f_y(d-a/2)$

where $a = A_s\ f_y/0.85\ f'_c\ b$

$$M_{ult.} = 1.32 \times 44.5\ [8 - \frac{1.32 \times 44.5}{2 \times 0.85 \times 3.8 \times 4.5}]$$

= 29.25 K-Ft.

$$M_{ult.} = \frac{P_u}{2} \times \frac{SPAN}{2}$$

SPAN = 5'-6"

$$P_u = \frac{29.25 \times 2}{2.75} = 21.3\ \text{kips}$$

(b) ULTIMATE LOAD BASED ON SHEAR FAILURE OF BEAM WITHOUT SHEAR REINFORCEMENT:

$$v_c = \phi(1.9\sqrt{f'_c} + 2500 \frac{p\ V\ d}{M}) \qquad \text{(ACI 17-2)}$$

where $\phi = 1$

$p = 0.0366 = A_s/bd$

V = shear force at the section

d = effective depth

M = moment at the section

v_c = shear stress carried by concrete

At center, $v_c = 1.9\sqrt{3800} + 2500 \times 0.0366 \times \frac{21300}{2} \times \frac{8}{351500}$

$= 117 + 22.2 = 139.2 \approx 139$ psi

$V_c = 139 \times 4.5 \times 8 = 5010$ lbs.

$= 5.01$ kips

$P_{u\,\text{(unreinforced web)}} = 10.02$ kips

(c) DESIGN OF SHEAR REINFORCEMENT IN THE MAIN SECTION OF THE BEAM:

$$V_u = \frac{P_u}{2}$$

$$v_u = \frac{V_u}{bd}$$

where V_u = shear force based on ultimate load calculated in (a) for longitudinal steel provided.

v_u = shear stress in section based on the above shear force.

Allowable v_u = 139 psi (based on allowable value at center)

$$V_u = \frac{21.3}{2} = 10.65 \text{ kips}$$

$$v_u = \frac{10.65}{4.5 \times 8} = 296 \text{ psi}$$

STRESS TAKEN BY STIRRUPS = v'_u

$v'_u = 296 - 139 = 157$ psi

$$S = \text{Spacing} = \frac{f_y \times d \times A_v}{V'_u}$$

where A_v = total cross sectional area of vertical stirrups at spacing "S" using #2 = 2 x 0.05

$$S = \frac{54000 \times 8 \times 2 \times 0.05}{157 \times 4.5 \times 8} = 7.7 \text{ in.}$$

But spacing provided in test beams calculated before concreting in the preliminary design is #2 @ 8" centers.

$$V'_u \text{ for 8" spacing} = \frac{54000 \times 8 \times 2 \times 0.05}{8}$$

$$= 5400 \text{ lbs.}$$

V'_u required to keep P_u same as before

(for spacing 7.7") = 157 x 4.5 x 8 = 5650 lbs.

Reduction in V'_u due to a larger spacing = 5650 - 5400 = 250 lbs.

P_u (based on shear failure for 8" spacing of #2)

= 21.3 - 2 x 0.25 = 20.8 kips

(d) <u>ULTIMATE LOAD BASED ON STIRRUP CONFIGURATION IN CHORDS AROUND OPENING:</u>

(i) Beam #1: No shear stirrups in top as well as bottom chords.

$$v_c = \phi(1.9\sqrt{f'_c} + 2500 \frac{p\ V\ d}{M}) \qquad \text{(ACI 17-2)}$$

$$p = \frac{\text{total area of cross section of steel}}{b \times d}$$

$b = 4.5"$

$d = 2"$

$$p = \frac{2.64}{4.5 \times 2} = 0.293$$

M = moment at the section (center of chord)

$= V_u$ (based on (a)) x distance of center of chord from support of beam.

$= \frac{21.3}{2} \times 16.5 = 176$ K-in.

$V = 10.65$ kips

$\phi = 1$

$$v_c = 1.9\sqrt{3800} + 2500 \times \frac{0.293 \times 10650 \times 2}{176000}$$

$$= 117 + 88.7 = 205.7 \text{ psi}$$

V_u (considering v_c calculated above as nominal ultimate shear stress for unreinforced chords)

$= 2[205.7 \times 4.5 \times 2]$

$= 3.7$ kips

P_u (based on unreinforced chords)

$= 7.4$ kips

(ii) Stirrups only in the top chord; Beam #2.

#1 - $\boxed{s}$ @ 1" centers.

Allowable shear in concrete as calculated in (i) = 3.7 kips

$$\text{Total shear} = 3.7 + \frac{A_v f_y d}{S}$$

$$= 3.7 + \frac{0.0123 \times 2 \times 78 \times 2}{1}$$

$$= 3.7 + 3.84 = 7.54 \text{ kips}$$

P_u (based on this case) = 15.08 kips.

(iii) Stirrups distributed evenly in top as well as bottom chord; Beam #3.

#1 - $\boxed{s}$ @ 2" centers.

(iv) Beam #3

#1 - @ 1.5" centers in the top chord.

#1 - @ 3" centers in the bottom chord.

(e) DESIGN OF LONGITUDINAL REINFORCEMENT IN THE CHORD:

Moment of midspan of the top chord

$$= M_{midspan} = \frac{P_u}{2} \times \text{distance from end support}$$

Reaction at end support $= V_u = \frac{P_u}{2}$

[P_u as calculated in (a)]

$$M_{(midspan)} = \frac{21.3}{2} \times \frac{16.5}{12} = 14.65 \text{ K-Ft.}$$

Axial force in chords $= F = \frac{M_{(midspan)}}{t}$

where t = lever arm between C and T as shown in Fig. 1.

$$F = \frac{14.65 \times 12}{6} = 29.3 \text{ K}$$

Moment at throat section $= M_{throat} = V_u \times \frac{\ell'}{2}$

where V_u = shear force as calculated in (a)

ℓ' = length of the chord

$$M_{(throat)} = \frac{21.3}{2} \times 7.5 = 80 \text{ K-in.}$$

Design top chord for

$M = 80$ K-in.

$F = -29.3$ kips

$P_b = 0.85 f'_c\, b\, a_b$ (ACI 19-1)

where P_b = axial load capacity at balanced conditions.

a_b = depth of equivalent rectangular stress block for balanced conditions = $K_1 \; C_b$

K_1 = 0.85 for strengths, f'_c, up to 4000 psi

C_b = distance from extreme compression fiber to neutral axis for balanced conditions = $(87000)d/(87000+f_y)$

$$a_b = 0.85 \times 2 \times \frac{87}{131.5} = 1.128''$$

$$P_b = 0.85 \times 3.8 \times 4.5 \times 1.128 = 16.4 \text{ K.}$$

$$M_b = P_b \times e_b = \phi[0.85 \; f'_c \; b \; a_b(d-d''-\frac{a_b}{2}) + A'_s \; f_y(d-d'-d'') + A_s \; f_y \; d''] \quad \text{(ACI 19-3)}$$

M_b = moment capacity at balanced conditions

e_b = eccentricity of load P_b measured from plastic centroid of section.

ϕ = 1

d' = distance from extreme compression fiber to centroid of compression reinforcement

d'' = distance from plastic centroid to centroid of tension reinforcement.

A'_s = area of "compression" reinforcement

A_s = area of "tension" reinforcement

$$M_b = 16.4(2-0.5-0.564) + 1.32 \times 44.5(2-1)$$
$$= 15.36 + 58.7$$
$$= 74.1 \text{ K-inches.}$$

$$e_b = \frac{M_b}{P_b} = \frac{74.1}{16.4} = 4.52''$$

$$\text{Actual } e = \frac{M_{(throat)}}{F} = \frac{80}{29.3} = 2.73''$$

$$P_o = \phi[0.85 \; f'_c(A_g-A_{st}) + A_{st} \; f_y] \quad \text{(ACI 19-7)}$$

where P_o = axial load capacity of actual member when concentrically loaded

A_g = gross area of section

A_{st} = total area of longitudinal reinforcement.

$$P_o = 0.85 \times 3.8(3x4.5-2.64) + 2.64 \times 44.5$$

$$= 35.1 + 117.4 = 152.5 \text{ K.}$$

$$P_u = \frac{P_o}{1 + [(P_o/P_b)-1]e/e_b} \qquad \text{(ACI 19-8)}$$

where P_u = axial load capacity under combined axial load and bending.

= F

$$F = \frac{152.5}{1 + [\frac{152.5}{16.4} - 1]\ \frac{2.73}{4.52}} = 25.3 \text{ K.}$$

$$F = -25.3 \text{ kips}$$

Reaction at the supports = $V_u = \frac{P_u \text{ (from (a))}}{2}$

$$M_{(midspan)} = F \times 6 = 0.5\ P_u \times 16.5$$

$$P_u = \frac{25.3 \times 6}{8.25} = 18.5 \text{ kips.}$$

P_u (based on longitudinal steel in chord) = 18.5 kips.

P_u on the basis of shear stirrups provided in the top chord = 15.08 K.

Following current practice, the bottom chord is designed with the same longitudinal reinforcement as the top chord.

Summary of P_u based on different conditions:

(a) P_u based on longitudinal steel in main beam = 21.3 K

(b) P_u based on unreinforced web of solid beam = 10.02 K

(c) P_u based on shear stirrups provided in main beam = 20.8 K

(d) P_u based on stirrup configuration in chords:

(i) no stirrups at all Beam #1 = 7.4 K

(ii), (iii) or (iv) different distribution of stirrups in top and bottom chords = 15.1 kips.

(e) P_u based on longitudinal steel in chords $\geq$ 18.5 K as this value is arrived at by assuming all the shear force is acting in the top chord, which is unlikely.

DESIGN OF CORNER REINFORCEMENT:

Reinforcing bars placed at 45° with the horizontal must resist:

$$2(\sqrt{2} \times V_u)$$

Multiplying factor for shear concentration at corners = 2 (assumed) (1,6)

$$\text{Tension force} = 2\sqrt{2} \times \frac{21.3}{2} \text{ (from (a))}$$

$$= 30 \text{ K}$$

$$A_s = \frac{30}{f_{y\,(\text{for } \#4)}} = \frac{30}{54} = 0.55 \text{ in.}^2$$

2 #4 bars are provided in each corner of the opening; that is, 4 bars on each side of the opening as shown in Fig. 5.

$$A_s = 0.63 \text{ in.}^2$$

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BIBLIOGRAPHY

1. Nasser, Karim W.; Acavalos, A.; Daniel, H. R. "Behaviour and design of large openings in reinforced concrete beams." A.C.I. Journal, Jan. 1967, p. 25.

2. Lorentsen, Mogens. "Holes in reinforced concrete." BYGGMASTAREN, Vol. 41, No. 7, July 1962, p. 141-152.

3. Ragan, H. S.; Warwark, J. "Tee members with large openings." Prestressed Concrete Institute, Journal, August 1967, Vol. 12, No. 14, p. 52.

4. Segner, E. P. "Reinforcement requirements for girder web openings." ASCE (structural division) June 1964, p. 147.

5. Bower, J. E. "Experimental stresses in wide-flenge beams with holes." ASCE (structural division) October 1966, p. 167.

6. Wright, W.; Bysne, J. G. "Stress concentration in concrete." Nature, Vol. 203, Sept. 26, 1964, p. 1374-1375.

7. Kani, G. N. J. "Basic facts concerning shear failure." A.C.I., Journal, June 1966, p. 675.

8. Saad, S.; Hendry, A. W. "Stresses in deep beams with a central concentrated load." Experimental Stress Analysis, Proceedings, Vol. XVIII, No. 1, p. 192.

9. Arora, J. S. "Experimental stress analysis of an I-beam with rectangular web cut out." Master's Thesis, Kansas State University, Department of Civil Engineering, Manhattan, Kansas, 1967.

10. Guralnick, Sidney A. "Shear strength of reinforced concrete beams." ASCE (structural division), January 1959, p. 1.

11. Building code requirements for reinforced concrete (ACI 318-63) June 1963.

ANALYSIS OF OPENINGS IN REINFORCED CONCRETE BEAMS

by

SURENDAR MOHAN MALHOTRA

B. E. (Honors) Civil Engineering, 1964

Govt. Engineering College, Jabalpur University

Jabalpur, India

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ABSTRACT

This thesis presents the results of an experimental study of simply supported, reinforced concrete beams having rectangular openings with different amounts of shear reinforcement around the openings. The beams were loaded with a concentrated load at the center of the clear span.

Longitudinal and shearing strains were measured with electrical resistance strain gages, which were 45° wire rosettes, bonded to the concrete surface at midpoints of the top and bottom chords of the opening. Deflections were measured by using dial gages placed 3" apart at the top and bottom chords and at the center of the beam.

The experimental results were compared with those calculated on the basis of Vierendeel panel behavior of the chords at the opening.

The results indicate that the theory assumed for the design of such beams is valid in general. It was found that the shear stress is greater in the top chord as compared to the bottom chord although the difference between them is small. The shear stress ratio of bottom to top ranges between 0.85 to 0.9.

It was also found that the bottom chord deflects more than the top chord even when they are equally reinforced. The deflected shape of the chords is an "S" curve with the point of contraflexure at about the midspan of the chords as assumed in the Vierendeel Truss theory.

The assumed stress concentration factor of two at the corners appears to be reasonable.