

TECHNIQUES AND ECONOMICS OF SPENT URANIUM RECYCLE.  
IN LWR FUEL CYCLES

by

Jun Ren Wang

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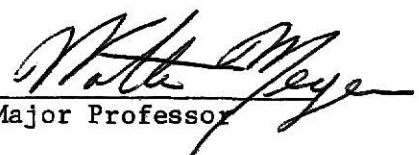
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## NOMENCLATURE

$C$	present worth of the net nuclear fuel cycle cost, \$/kg U
$C_1, C_2, C_3, \text{ etc.}$	defined by Eqs. (2.44)-(2.57) (Sect. 2.3)
$C_1, C_2, C_3, \text{ etc.}$	defined by Eq. (3.16) (Sect. 3.5)
$C_F$	unit cost of the feed, \$/kg U
$C_P$	nuclear fuel cycle cost, Mills/kWh
$C_R$	unit cost of the recycled feed, \$/kg U
$C_S$	unit cost of the separative duty, \$/unit of sep. work
$C_W$	unit cost of the tails, \$/kg U
$DB^2$	leakage rate of thermal neutrons per unit volume per unit thermal neutron flux
$E$	separative work
$E$	fuel burnup, MWD/Tonne
$E$	average of exponential function
$e$	thermal efficiency
$f(\underline{r})$	ratio of flux at $\underline{r}$ to flux at center of reactor
$\bar{f}$	average of ratio of flux to central flux
$F$	flow rate of the feed stream, kg/day
$H$	net mass upflow of isotope $i$ in the cascade
$i$	interest rate or cost of capital
$K_1, K_2, K_3, \text{ etc.}$	defined by Eqs. (2.15)-(2.37) (Sect. 2.3)
$n$	number of years until income is received or expense is paid
$n_1$	number of months of pre-irradiation period
$n_2$	number of months of reactor operation
$N$	concentration, atoms/cm <sup>3</sup>

P	resonance escape probability
P	flow rate of product material, kg/Day
$P_1$	probability of neutron not leaking while slowing down from fission to resonance energy
$P_2$	probability of neutron not leaking while slowing down from resonance to thermal energy
$P_{th}$	probability of neutron not leaking while slowing down from resonance to thermal energy
$P_6$	flow rate of $^{236}\text{U}$ in the product stream, kg/day
$\underline{r}$	position in reactor
R	flow rate of recycle uranium feed, kg/day
R	total present worth revenue, \$/kg U
$R_6$	weight of U-236 in the recycle fuel, kg
T	average residence time of fuel in reactor core, sec
t	time, sec
V	separative potential, defined by Eq. (3.12)
W	flow rate of waste material, kg/day
$W_6$	flow rate of $^{236}\text{U}$ in the waste stream, kg/day
X	weight of $\text{U}_3\text{O}_8$ , lb
$X_F$	weight fraction of U-235 in the feed stream
$X_P$	weight fraction of U-235 in the product stream
$X_R$	weight fraction of U-235 in the recycle stream
$X_W$	weight fraction of U-235 in the waste stream
Y	weight of uranium in the form of $\text{UF}_6$ , kg
z	distance from midplane measured to axis, cm
Z	core length, cm
$Z'$	effective core length, cm

$\alpha$	ratio of capture cross section to fission cross section for fissionable nuclide
$\gamma$	defined by Eq. (2.41)
$\epsilon$	fast fission factor, net fission neutrons produced per neutron produced in thermal fission
$\eta$	fission neutrons produced per thermal neutron absorbed in fissionable material
$\theta$	intermediate flux time, $n/Kb$
$K$	defined by Eq. (2.40)
$\lambda$	decay constant, $\text{sec}^{-1}$
$\nu$	neutrons produced per fission
$\sigma$	microscopic cross section for absorption of thermal neutrons, barns ( $10^{-24} \text{ cm}^2$ )
$\phi$	thermal-neutron flux, $\text{neutrons/cm}^2 \text{ sec}$

## 1.0 INTRODUCTION

Due to the rapid growth of nuclear power capacity in recent years, and the policy of the AEC not to purchase spent uranium recovered from a privately owned fuel, more and more spent nuclear fuel will be either not reprocessed or awaiting sale after reprocessing. Although at the present time, yellow-cake still is available at reasonable prices<sup>(1)</sup>, the utilization of spent uranium may prove useful as a means of reducing nuclear fuel cycle costs. The purposes of this paper, therefore, are to study the methods of spent uranium recycle and to evaluate the relative economic values of the proposed recycle schemes.

The recovered spent fuel can be prepared for recycle in either of two ways<sup>(2,3,4)</sup>, by re-enriching in a gaseous diffusion plant, with incidental, partial removal of the parasitic  $^{236}\text{U}$ , or by blending the reprocessed fuel with highly enriched natural fuel. The problems resulting from the introduction of  $^{236}\text{U}$  into the fuel cycle can be summed up as follows<sup>(3,5)</sup>:

(1) U-236 is a thermal-neutron poison, whereas U-238 is a fertile material; therefore, a fuel containing  $^{236}\text{U}$ , i.e., the recycled, fuel, must be more highly enriched than the  $^{236}\text{U}$  free natural fuel in order to maintain the criticality of the core at the same specific power level. (2) The presence of U-236 increases the amount of separative work expended in a gaseous diffusion plant to produce uranium of a specific U-235 content, since separation of U-235 from U-238 is less costly than separation of U-235 from an equal amount of U-236 due to smaller separation ratio. (3) The buildup of U-237 increases the required uranium product decay time.

As far as the fuel cycle cost alone is concerned, recycling the spent nuclear fuel by blending is more costly than re-enriching, but when other

factors are considered, blending can for practical purposes be favored over re-enriching.

### 1.1 Literature Review

There are several literature citations concerning the recycle of reprocessed spent fuel or the effects of Uranium-236 on nuclear reactors and gaseous diffusion plants. But very few have made a detailed cost analysis of the spent fuel recycle and presented any results that can be of benefit to the power industry. G. A. Garrett et al.<sup>(6)</sup> were the first to explore the impact of Uranium-236 on nuclear power reactors and gaseous diffusion plants. In their report, The Uranium-236 Problem in the Combined Operation of Nuclear Power Reactors and Isotope Separation Plants, they calculated the effect of  $^{236}\text{U}$  on the cost of enriched uranium at different levels of burnup and at several different modes of operation. Their conclusions were (1) the presence of  $^{236}\text{U}$  in material discharged from reactors is potentially capable of causing a significant increase in net reactor fuel costs, but if the nuclear power industry has an adequate rate of growth the  $^{236}\text{U}$  will not cause any significant increase in fuel costs. The U-236 problem is most severe when the reactor burnup is low because the only sink for the U-236 produced in the reactor is the waste stream from the separation cascade. The closer to the waste withdrawal point the U-236 is introduced, the easier it is to remove (the higher the burnup, the lower the concentration of U-235 in the recycle fuel). When the nuclear power industry has an adequate rate of growth (improved technology) much higher fuel burnup can be achieved; therefore, the U-236 problem may be alleviated with time. (2) In complex situations involving reactors and a diffusion plant where the reactors require fuels of different  $^{235}\text{U}$  concentrations, the cost due to

the presence of  $^{236}\text{U}$  are not additive. (3) In such a complex situation, the most economical mode of operation involves the use of more than one diffusion plant, and it may involve blending streams of unequal  $^{235}\text{U}$  concentration.

Pierre Lagrange et al.<sup>(7)</sup> in their paper, Re-Enrichment of Depleted Uranium by Passage Through a Gaseous Diffusion Installation, gave an evaluation of the cost of re-enrichment as computed from the general formula for various economic conditions and also reviewed the case of a typical installation having a certain flexibility of operation and several feed points. As to the buildup of the transmutation products and their effects on the fuel cycle, an article titled Effect of Uranium Recycle on Transuranic Elements Buildup by E. D. Arnald<sup>(5)</sup> has made a very detailed study in this respect. His paper presented a clear picture on how  $^{236}\text{U}$ ,  $^{237}\text{U}$ ,  $^{237}\text{Np}$ , and  $^{238}\text{Pu}$  could build up with the subsequent recycle of nuclear fuel. The transmutation product concentrations are affected by neutron flux, number of cycles, initial fuel enrichment, and the fraction of  $^{236}\text{U}$  removed. The buildup of transmutation products may have many appreciable effects on the design and operation of the fuel cycle. The decay time required will increase as a result of higher concentrations of  $^{237}\text{Np}$ ; chemical separation plants may be required to separate  $^{237}\text{Np}$  as well as uranium, plutonium, and fission products. The buildup of  $^{238}\text{Pu}$  in the plutonium product may create additional biological or handling problems. An important conclusion noted that all problems resulting from isotope buildup in the  $^{236}\text{U}$  buildup chain may be decreased in seriousness by approximately an order of magnitude with removal of about 25% of the  $^{236}\text{U}$  by re-enrichment in a gaseous diffusion plant.

At present, application of isotopes is largely in a development phase. Only  $^{60}\text{Co}$  and fission product cesium have been used as radiation sources for chemical processing and food irradiation. The reprocessing of power reactor fuels can yield a number of potentially useful by-products, such as  $^{85}\text{Kr}$ ,  $^{90}\text{Sr}$ ,  $^{106}\text{Ru}$ ,  $^{137}\text{Cs}$ , neptunium,  $^{236}\text{U}$ , plutonium, americium, and curium.

According to the article, Power Cost Reduction from Isotope Revenue, written by L. W. Lang, et al.<sup>(8)</sup> there will be a large market for isotopes in the future. Growing use of  $^{238}\text{Pu}$ ,  $^{60}\text{Co}$ , and  $^{90}\text{Sr}$  will undoubtedly occur. But before substantial profits can be envisioned from reactor production of isotopes, the isotope industry will have to grow even faster than the remarkable growth rate predicted for the nuclear power industry. In another paper entitled The Effect of  $^{236}\text{U}$  and  $^{237}\text{Np}$  on the Value of Uranium as Feed for Pressurized Water Power Reactors by D. A. Goellner, et al.<sup>(3)</sup> they have evaluated two steady-state uranium-recycle schemes using  $^{237}\text{Np}$  credit and  $^{236}\text{U}$  penalty as two decisive factors in determining whether re-enriching or blending are more economical manners of re-using spent fuel. The general expression for the extra separative work due to the presence of  $^{236}\text{U}$  in the recycle flow was developed by Henri M. Guéron, et al.<sup>(4)</sup> in their paper, Isotopic Composition Along a Diffusion Cascade with a Postirradiation Recycle Flow Effect of the Presence of  $^{236}\text{U}$  on the Separative Work. In this work they derived the equation for extra separative work from the integral proposed by Garrett, et al.<sup>(6)</sup> to calculate the separative work required for multicomponent separation. The authors Guéron and Geller concluded that the extra separative work introduced by the presence of  $^{236}\text{U}$  in the input stream in the diffusion plant constitutes a very small fraction of the total separative work.



As for the  $^{236}\text{U}$  penalty to the power cost, Leonard Geller, et al.<sup>(9)</sup> in another of their jointly published papers,  $^{236}\text{U}$  in the Nuclear Power Universe of the the Next Fifteen Years, gave a prediction of the average unit payments exchanged between the private reactor operators and the enrichment plant owner of the next fifteen years. Their study also reflected the fact that  $^{236}\text{U}$  introduced into the enrichment cascade with one batch of recycle fuel will be distributed among all the product streams for some time to come.

A. MacCragh is one of the few who have performed an economic study of spent uranium recycle. In his paper, The Economics of Blending and Diffusion in Uranium Recycle, an equation has been derived to compare the costs of two schemes, diffusion and blending, for recycling spent fuel to light water reactors. The calculations show that blending is slightly more costly than diffusion. The factors that can chiefly influence this conclusion according to the author are the discharge enrichment and the value of  $^{236}\text{U}$ . A very high burnup would favor diffusion; a very high price for  $^{238}\text{Pu}$  and, therefore, for  $^{236}\text{U}$ , would render blending the desirable route. The author used an over simplified model neglecting many important factors relevant and decisive to the study of spent fuel recycle. Such factors include U-236 penalty, extra separative duty requirement, extra U-235 concentration, and the difference in inventory time between the two schemes. Moreover, A. MacCragh's paper didn't use a burnup code to calculate the final isotopic concentration in the spent fuel. He based his uranium and plutonium credits on assumptions. Further, he failed to consider the overall fuel cycle cost but simply used the cost before reactor operation; this is highly unrealistic. Therefore the conclusions drawn by MacCragh were not well founded, and thus the purpose of this paper is to examine the situations in a more realistic and detailed manner.

## 2.0 REFERENCE-DESIGN REACTORS AND BURNUP EQUATIONS

### 2.1 The Reference-Design Reactors

The reference-design reactors for this study are nominal 1,000 MWe light water reactors. The 1,065 MWe boiling water reactors (BWR), Brown's Ferry Station, Tennessee Valley Authority<sup>(26)</sup> manufactured by General Electric Co. and the 1,000 MWe pressurized water reactor (PWR) of the Delaware Valley Utilities<sup>(26)</sup> manufactured by Westinghouse Electric Corp. have been selected as the two reference-design reactors.

The two most important reasons for choosing 1,000 MWe light water reactors as the reference design reactors are first, light water reactors of either type are very well developed and highly dependable, and second, most of the power plants under construction or on order are between the ratings from 800 MWe to 1,100 MWe<sup>(26)</sup>. Therefore a 1,000 MWe rating light water reactor can well represent these "present generation" reactors.

### 2.2 Neutron Energy Cycle

Consider a unit volume of fuel containing  $N_m$  atoms of fissionable material such as  $^{235}\text{U}$ , or  $^{239}\text{Pu}$  of thermal absorption cross section  $\sigma_m$  and  $N_g$  atoms of fertile material ( $^{238}\text{U}$ ) of thermal absorption cross section  $\sigma_g$ . The rate of absorption of neutrons by fissionable material is  $N_m \sigma_m \phi$ , where  $\phi$  is the thermal neutron flux. The resulting fissions produce fast neutrons at a rate  $\eta_m N_m \sigma_m \phi$ .

The *fast fission factor* is defined as the ratio of the net rate of production of fast neutrons to the rate of production of fast neutrons by thermal fission. The fraction  $\epsilon - 1$  of the fast neutrons comes from fission of fertile material with fast neutrons. The net production rate of fast neutrons from fission is  $\epsilon \eta_m N_m \sigma_m \phi$ <sup>(14)</sup>.

Table 1

PROPERTIES OF THE REFERENCE-DESIGN 1,065 MW(e) BWR

Reactor Power: Thermal	3,293 MW(t)
Electrical	1,065 MW(e)
Enrichment Ratio	2.56%
No. of Fuel Elements	764
No. of Rods per Fuel Element	49
Total Uranium Loading	149,800 kg
H <sub>2</sub> O/UO <sub>2</sub> , Volume Ratio	2.41
Core Dimensions: Radius	237.6 cm
Height	365.8 cm
Effective Core Dimensions: Radius	252.2 cm
Height	388.2 cm
Average Thermal Neutron Flux	$2.0 \times 10^{13} \text{ n/cm}^2\text{-sec}$
Average Burnup	27,500 MWD/Tonne
Fast Fission Factor	1.06736
Non-Leakage Probability:	
Fission-to-Resonance	0.96126
Fission-to-Thermal	0.95164
Resonance Escape Probability	0.72397
Thermal Leakage Factor	$3.23255 \times 10^{-4} \text{ cm}^{-1}$
Average Specific Power	22.0 MW/MTU
Typical Conversion Electrical Efficiency	32.8%
Load Factor	0.80
Reload Frequency	Annual
Fraction Fuel Elements Reloaded	1/4

Table 2

PROPERTIES OF THE REFERENCE-DESIGN 1,000 MW(e) PWR

Reactor Power: Thermal	3,083 MW(t)
Electrical	1,000 MW(e)
Enrichment Ratio	3.3%
No. of Fuel Elements	193
No. of Rods per Fuel Element	204
Total Uranium Loading	88,600 kg
H <sub>2</sub> O/UO <sub>2</sub> , Volume Ratio	1.66
Core Dimensions: Radius	169.8 cm
Height	365.8 cm
Effective Core Dimensions: Radius	183.0 cm
Height	394.2 cm
Average Thermal Neutron Flux	$2.5 \times 10^{13} \text{ n/cm}^2\text{-sec}$
Average Burnup	33,000 MWD/Tonne
Fast Fission Factor	1.06736
Non-Leakage Probability:	
Fission-to-Resonance	0.96230
Fission-to-Thermal	0.95268
Resonance Escape Probability	0.72397
Thermal Leakage Factor	$3.17862 \times 10^{-4} \text{ cm}^{-1}$
Average Specific Power	34.8 MW/MTU
Typical Conversion Electrical Efficiency	32.5%
Load Factor	0.865
Reload Frequency	Annual
Fraction Fuel Elements Reloaded	1/3

The fraction of the fast neutrons which do not escape from the reactor as they degrade from fission to resonance energy is called the *fission-to-resonance nonleakage probability*, and is denoted by  $P_1$ . Hence, the rate at which fast neutrons degrade into the resonance region is  $\epsilon \eta_m N_m \sigma_m \phi P_1$ .

The fraction of the resonance neutrons which are not captured but are degraded to lower energies is called the *resonance escape probability*, which is denoted by  $P$ . Hence,  $\epsilon \eta_m N_m \sigma_m \phi P_1 (1 - P)$  neutrons undergo resonance absorption per unit volume per unit time, and  $\epsilon \eta_m N_m \sigma_m \phi P_1 P$  are degraded to lower energies.

Of the latter, some diffuse to outer surfaces and escape, but the fraction  $P_2$  remains in the reactor and becomes thermal neutrons.  $P_2$  is called the *resonance-to-thermal nonleakage probability*. Finally, the number of neutrons completing an energy cycle is  $\epsilon \eta_m N_m \sigma_m \phi P_1 P P_2$  neutrons per unit volume, per unit time. The product  $P_1 P_2$  is the *fission-to-thermal nonleakage probability*, which is denoted by  $P_{th}$ . Thermal neutrons are consumed by (1) absorption in fissionable material at a rate  $N_m \sigma_m \phi$ ; (2) absorption in nonfissionable material at a rate  $\sum_i N_i \sigma_i \phi$ ; (3) leakage at a rate  $DB^2 \phi$ .

The neutrons balance of a typical thermal reactor can be demonstrated by the neutrons flow sheet of Fig. 1.

### 2.3 Change of Fuel Composition as a Function of Flux Time

For the reference-design light water reactors studied, it is assumed that the fuels are unmixed, and undergo graded irradiation. By graded irradiation it is meant that the fuel channels are divided into groups, each small enough for transverse variations in flux to be negligible. The neutron flux along a fuel rod varies with distance  $z$  from the middle of the

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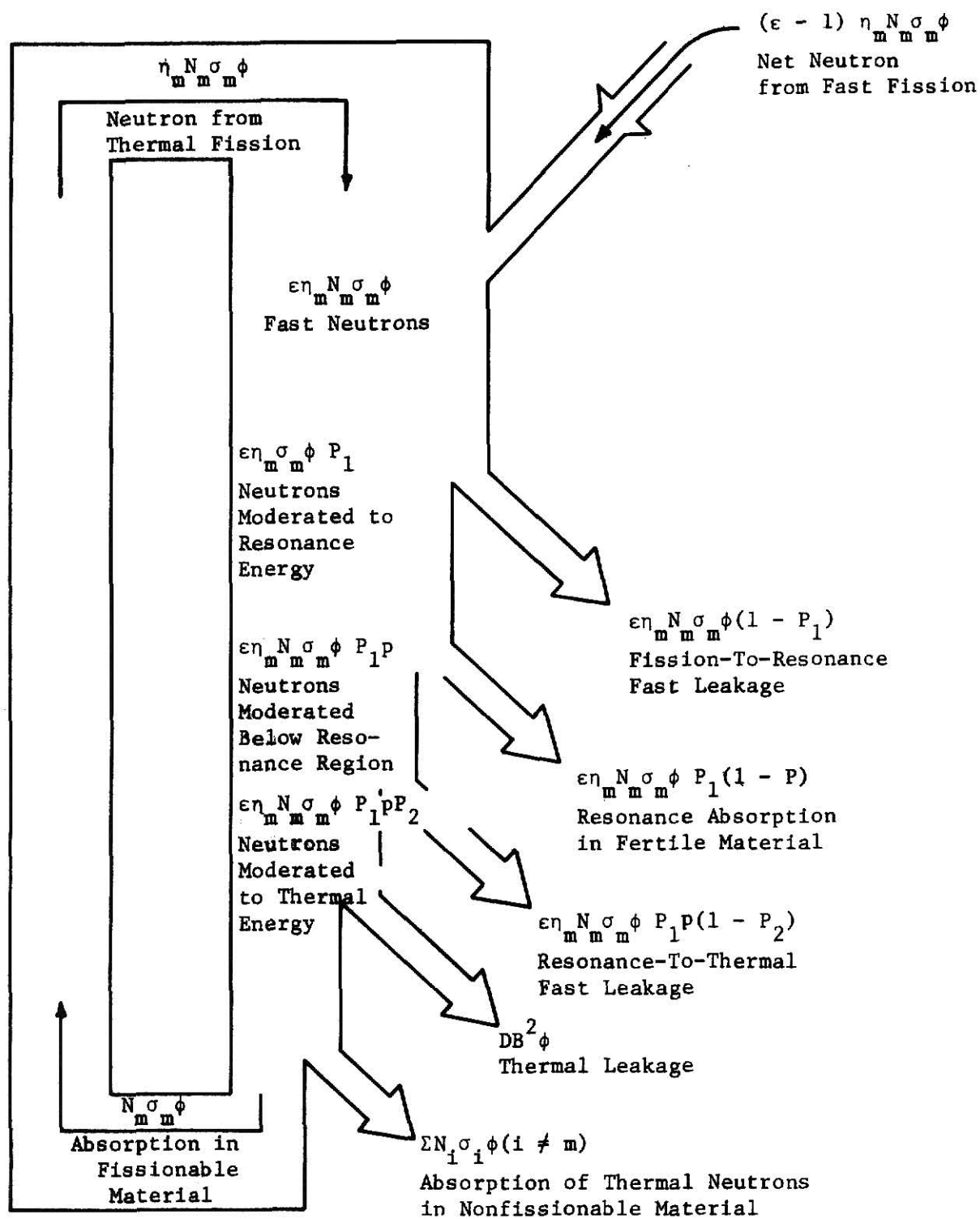


Fig. 1. Flow Sheet for Neutrons in a Thermal Reactor

rod as  $\cos(\pi Z/Z')$ , where  $z$  is the length of the core and  $Z'$  is the effective length<sup>(14)</sup> of the core. The distance  $(Z' - Z)/2$  is called the reflector savings.

For this work first it is assumed that the dependence of flux on position  $\underline{r}$  and time  $t$ ,  $\phi(\underline{r}, t)$ , may be expressed as

$$\phi(\underline{r}, t) = f(\underline{r}) \phi_0(t) . \quad (2.1)$$

$\phi_0(t)$  is the dependence of the neutron flux on time at the center of the core and  $f(\underline{r})$  is the ratio of the flux at  $\underline{r}$  to the flux at the center of the core. From the equations presented in Section II.2, Chapter 3 of Nuclear Chemical Engineering<sup>(13)</sup>, the dependence of local concentration of flux time is of the general form

$$N(\underline{r}, t) = A + C\theta(\underline{r}, t) + \sum_i F_i \exp[-\sigma_i \theta(\underline{r}, t)] \quad (2.2)$$

where  $A$ ,  $C$  and  $F_i$  are constants,  $\theta(\underline{r}, t)$  is the integral  $\int_0^t \phi(t) dt$ , which is assigned to be

$$\theta(\underline{r}, t) = f(\underline{r}) \theta_0(t) . \quad (2.2a)$$

When  $\theta(\underline{r}, t)$  is given by (2.2a), Eq. (2.2) becomes

$$N(\underline{r}, t) = A + Cf(\underline{r}) \theta_0(t) + \sum_i F_i \exp[-\sigma_i f(\underline{r}) \theta_0(t)] . \quad (2.3)$$

The average concentration of a nuclide in the fuel discharged from the reactor is:

$$\bar{N}_0(t) \equiv \frac{\int_{(\text{core})} N(\underline{r}, t) d\underline{r}}{\int_{(\text{core})} d\underline{r}} . \quad (2.4)$$

The volume average concentration of a nuclide in fuel discharged after a graded irradiation to a central flux time of  $\theta$  is



$$N_0^G(\theta_0) = \frac{\int_{-Z/2}^{Z/2} N(\theta_0 \cos \frac{\pi z}{Z'}) dz}{Z} \quad (2.5)$$

Since the local concentration  $N$  is given by Eq. (2.3) the volume average concentration of a nuclide in fuel discharged after graded irradiation to a central flux time  $\theta_0$  is

$$N_0^G(\theta_0) = A + C\theta_0 \bar{f}_0^G + \sum_1 F_1 E_0^G(\sigma \theta_0) \quad (2.6)$$

where

$$\bar{f}_0^G = \frac{\int_{-Z/2}^{Z/2} \cos(\pi z/Z') dz}{Z} \quad (2.7)$$

and

$$E_0^G(\sigma \theta_0) = \frac{\int_{-Z/2}^{Z/2} \exp(-\sigma \theta_0 \cos \frac{\pi z}{Z'}) dz}{Z} \quad (2.8)$$

Note that superscript  $G$  has been used to denote graded irradiation.

Consider a reactor which at time zero is charged with uranium fuel containing  $N_{25}^0$  atoms of  $^{235}\text{U}$  per cubic centimeter,  $N_{26}^0$  atoms of  $^{236}\text{U}$ ,  $N_{28}^0$  atoms of  $^{238}\text{U}$ , and no plutonium or fission products. This fuel is then exposed to a constant thermal neutron flux  $\phi$ . The variation in concentration of each nuclide in this fuel with time is obtained as follows:

#### Uranium 235

The rate of change of the number of atoms of  $^{235}\text{U}$  per unit volume is

$$\frac{dN_{25}}{dt} = -N_{25} \sigma_{25} \phi \quad (2.9)$$

The solution of (2.9), subject to  $N_{25} = N_{25}^0$  at time  $t = 0$ , is

$$N_{25} = N_{25}^0 e^{-\sigma_{25}\theta}$$

By replacing  $e^{-\sigma_{25}\theta}$  with  $E_0^G(\sigma_{25}\theta)$ , following eqn is obtained

$$N_{25} = N_{25}^0 E_0^G(\sigma_{25}\theta) \quad (2.10a)$$

### Uranium 236

Uranium 236 is produced by the non-fission capture of neutrons in  $^{235}\text{U}$ . If fresh fuel (yellow cake) is used the fuel is  $^{236}\text{U}$  free, but if reprocessed fuel (uranium) is used, there will be a certain amount of  $^{236}\text{U}$  present in the fuel before irradiation starts. The net rate of change of  $^{236}\text{U}$  concentration is

$$\frac{dN_{26}}{dt} = \frac{N_{25} \sigma_{25} \alpha_{25} \phi}{1 + \alpha_{25}} - N_{26} \sigma_{26} \phi \quad (2.11)$$

The solution of Eq. (2.11), subject to the condition that  $N_{26} = N_{26}^0$  at  $t = 0$  is

$$N_{26} = \frac{N_{25}^0 \sigma_{25} \alpha_{25}}{(\sigma_{25} - \sigma_{26})(1 + \alpha_{25})} (E_0^G(\sigma_{26}\theta) - E_0^G(\sigma_{25}\theta)) + N_{26}^0 E_0^G(\sigma_{26}\theta) \quad (2.12)$$

### Uranium 237

Uranium 237 is formed by the neutron capture of  $^{236}\text{U}$ , which undergoes  $\beta$  decay with a half-life of 6.75 days, assuming formation from  $^{238}\text{U}$  by an n, 2n reaction is ignored.

The net rate of change of  $^{236}\text{U}$  concentration is

$$\frac{dN_{27}}{dt} = N_{26} \sigma_{26} \phi - \lambda_{27} N_{27} \quad (2.13)$$

the solution of Eq. (2.13), subject to  $N_{27} = 0$  at time  $t = 0$  is

$$N_{27} = \frac{K_1 [E_0^G(\sigma_{26}\theta) - e^{-\lambda_{27}t}]}{\lambda_{27} - \sigma_{26} \phi \bar{f}_0^G} - \frac{K_1 [E_0^G(\sigma_{25}\theta) - e^{-\lambda_{27}t}]}{\lambda_{27} - \sigma_{25} \phi \bar{f}_0^G} + \frac{N_{26}^0 \sigma_{26} \phi [E_0^G(\sigma_{26}\theta) - e^{-\lambda_{27}t}]}{\lambda_{27} - \sigma_{26} \phi \bar{f}_0^G} \quad (2.14)$$

where

$$K_1 = \frac{N_{25}^0 \sigma_{25} \sigma_{26} \alpha_{25} \phi \bar{f}_0^G}{(\sigma_{25} - \sigma_{26})(1 + \alpha_{25})} . \quad (2.15)$$

### Neptunium 237

Neptunium 237, the product of  $\beta^-$  decay of  $^{237}\text{U}$ , exhibits a net rate of change in concentration,

$$\frac{dN_{37}}{dt} = \lambda_{27} N_{27} - N_{37} \sigma_{37} \phi . \quad (2.16)$$

The solution of (2.16) with  $N_{37} = 0$  at  $t = 0$ , is

$$N_{37} = \frac{K_2 [E_0^G(\sigma_{26}\theta) - E_0^G(\sigma_{37}\theta)]}{(\sigma_{37} - \sigma_{26}) \phi \bar{f}_0^G} + \frac{K_3 [E_0^G(\sigma_{25}\theta) - E_0^G(\sigma_{37}\theta)]}{(\sigma_{37} - \sigma_{25}) \phi \bar{f}_0^G} \\ - \frac{(K_2 + K_3) [e^{-\lambda_{27}t} - E_0^G(\sigma_{37}\theta)]}{\bar{f}_0^G \sigma_{37} \phi - \lambda_{27}} \quad (2.17)$$

where

$$K_2 = \frac{\lambda_{27}(K_1 + N_{26}^0 \sigma_{26} \phi)}{\lambda_{27} - \sigma_{26} \phi \bar{f}_0^G} \quad (2.18)$$

$$K_3 = - \frac{K_1 \lambda_{27}}{\lambda_{27} - \sigma_{25} \phi \bar{f}_0^G} \quad (2.19)$$

### Neptunium 238

Neptunium 238 is produced by the non-fission capture of neutrons in  $^{237}\text{Np}$ .

The net rate of change of  $^{238}\text{Np}$  concentration is

$$\frac{dN_{38}}{dt} = N_{37} \sigma_{37} \phi - \lambda_{38} N_{38} - N_{38} \sigma_{38} \phi . \quad (2.20)$$

The solution of Eq. (2.20), subject to  $N_{38} = 0$  at  $t = 0$ , is

$$\begin{aligned}
 N_{38} = & \frac{K_5 [E_0^G(\sigma_{25}\theta) - E_0^G(\sigma_{38}\theta) e^{-\lambda_{38}t}]}{\lambda_{38} + \sigma_{38} \phi_{f0}^G \sigma_{25} \phi_{f0}^G} \\
 & + \frac{K_4 [E_0^G(\sigma_{26}\theta) - E_0^G(\sigma_{38}\theta) e^{-\lambda_{38}t}]}{\lambda_{38} + \sigma_{38} \phi_{f0}^G \sigma_{26} \phi_{f0}^G} \\
 & + \frac{K_6 [e^{-\lambda_{27}t} - E_0^G(\sigma_{38}\theta) e^{-\lambda_{38}t}]}{\lambda_{38} + \sigma_{38} \phi_{f0}^G \lambda_{27}} \\
 & - \frac{(K_4 + K_5 + K_6) [E_0^G(\sigma_{37}\theta) - E_0^G(\sigma_{38}\theta) e^{-\lambda_{38}t}]}{\lambda_{38} + \sigma_{38} \phi_{f0}^G \sigma_{37} \phi_{f0}^G} \quad (2.21)
 \end{aligned}$$

where

$$K_4 = \frac{K_3 \sigma_{37}}{\sigma_{37} - \sigma_{26}} \quad (2.22)$$

$$K_5 = \frac{K_4 \sigma_{37}}{\sigma_{37} - \sigma_{25}} \quad (2.23)$$

$$K_6 = - \frac{(K_3 + K_4) \sigma_{37} \phi}{\sigma_{37} \phi_{f0}^G \lambda_{27}} \quad (2.24)$$

### Plutonium 238

Plutonium 238 is produced by  $\beta^-$  decay of Neptunium 238, and also from an n, 2n reaction on  $^{238}\text{U}$ , but here we are only concerned with the route forward by recycled  $^{236}\text{U}$ . Therefore, only the  $^{238}\text{U}$  produced by  $\beta^-$  decay of  $^{238}\text{Np}$  is considered.

The net rate of change of  $^{238}\text{Pu}$  concentration is

$$\frac{dN_{48}}{dt} = \lambda_{38} N_{38} - N_{48} \sigma_{48} \phi \quad (2.25)$$

The solution of (2.25), subject to  $N_{48} = 0$  at  $t = 0$  is

$$\begin{aligned}
 N_{48} = & \frac{K_7 [E_0^G(\sigma_{26}\theta) - E_0^G(\sigma_{48}\theta)]}{(\sigma_{48} - \sigma_{26}) \phi \bar{f}_0^G} + \frac{K_8 [E_0^G(\sigma_{25}\theta) - E_0^G(\sigma_{48}\theta)]}{(\sigma_{48} - \sigma_{25}) \phi \bar{f}_0^G} \\
 & + \frac{K_9 [e^{-\lambda_{27}t} - E_0^G(\sigma_{48}\theta)]}{\bar{f}_0^G \sigma_{48} \phi - \lambda_{27}} + \frac{K_{10} [E_0^G(\sigma_{37}\theta) - E_0^G(\sigma_{48}\theta)]}{(\sigma_{48} - \sigma_{37}) \phi \bar{f}_0^G} \\
 & - \frac{(K_7 + K_8 + K_9 + K_{10}) [E_0^G(\sigma_{38}\theta) e^{-\lambda_{38}t} - E_0^G(\sigma_{48}\theta)]}{\sigma_{48} \phi \bar{f}_0^G - \lambda_{38} - \sigma_{38} \phi \bar{f}_0^G} \quad (2.26)
 \end{aligned}$$

where

$$K_7 = \frac{K_4 \lambda_{38}}{\lambda_{38} + \sigma_{38} \phi \bar{f}_0^G - \sigma_{26} \phi \bar{f}_0^G} \quad (2.27)$$

$$K_8 = \frac{K_5 \lambda_{38}}{\lambda_{38} + \sigma_{38} \phi \bar{f}_0^G - \sigma_{25} \phi \bar{f}_0^G} \quad (2.28)$$

$$K_9 = \frac{K_6 \lambda_{38}}{\lambda_{38} + \sigma_{38} \phi \bar{f}_0^G - \lambda_{27}} \quad (2.29)$$

$$K_{10} = \frac{-(K_4 + K_5 + K_6) \lambda_{38}}{\lambda_{38} + \sigma_{38} \phi \bar{f}_0^G - \sigma_{37} \phi \bar{f}_0^G} \quad (2.30)$$

### Plutonium 239

First consider  $^{239}\text{Pu}$  produced by non-fission neutron capture of  $^{238}\text{Pu}$  ( $\beta^-$  decay product of  $^{238}\text{Np}$ ) the rate of accumulation of  $^{239}\text{Pu}$  is given by

$$\frac{dN'_{49}}{dt} = \frac{N_{48} \sigma_{48} a_{48} \phi}{1 + a_{48}} - N'_{49} \sigma_{49} \phi \quad (2.31)$$

The solution of (2.31), subject to  $N'_{49} = 0$  at  $t = 0$  is

$$\begin{aligned}
 N'_{49} = & \frac{K_{11}[E_0^G(\sigma_{26}\theta) - E_0^G(\sigma_{49}\theta)]}{(\sigma_{49} - \sigma_{48}) \phi \bar{f}_0^G} + \frac{K_{12}[E_0^G(\sigma_{25}\theta) - E_0^G(\sigma_{49}\theta)]}{(\sigma_{49} - \sigma_{25}) \phi \bar{f}_0^G} \\
 & + \frac{K_{13}[e^{-\lambda_{27}t} - E_0^G(\sigma_{49}\theta)]}{\sigma_{49} \phi \bar{f}_0^G - \lambda_{27}} + \frac{K_{14}[E_0^G(\sigma_{37}\theta) - E_0^G(\sigma_{49}\theta)]}{(\sigma_{49} - \sigma_{37}) \phi \bar{f}_0^G} \\
 & + \frac{K_{15}[E_0^G(\sigma_{38}\theta) e^{-\lambda_{38}t} - E_0^G(\sigma_{49}\theta)]}{\sigma_{49} \phi \bar{f}_0^G \lambda_{38} - \sigma_{38} \phi \bar{f}_0^G} \\
 & - \frac{(K_{11} + K_{12} + K_{13} + K_{14} + K_{15})[E_0^G(\sigma_{48}\theta) - E_0^G(\sigma_{49}\theta)]}{(\sigma_{49} - \sigma_{48}) \phi \bar{f}_0^G} \quad (2.32)
 \end{aligned}$$

where

$$K_{11} = \frac{K_7 \sigma_{48} \alpha_{48}}{(1 + \alpha_{48})(\sigma_{48} - \sigma_{26})} \quad (2.33)$$

$$K_{12} = \frac{K_8 \sigma_{48} \alpha_{48}}{(1 + \alpha_{48})(\sigma_{48} - \sigma_{25})} \quad (2.34)$$

$$K_{13} = \frac{K_9 \sigma_{48} \alpha_{48} \phi \bar{f}_0^G}{(1 + \alpha_{48})(\sigma_{48} \phi \bar{f}_0^G - \lambda_{27})} \quad (2.35)$$

$$K_{14} = \frac{K_{10} \sigma_{48} \alpha_{48}}{(1 + \alpha_{48})(\sigma_{48} - \sigma_{37})} \quad (2.36)$$

$$K_{15} = \frac{(K_7 + K_8 + K_9 + K_{10}) \sigma_{48} \alpha_{48} \phi \bar{f}_0^G}{(1 + \alpha_{48})(\sigma_{48} \phi \bar{f}_0^G - \lambda_{38} - \sigma_{38} \phi \bar{f}_0^G)} \quad (2.37)$$

Now consider the  $^{239}\text{Pu}$  produced from absorption of thermal neutron in  $^{238}\text{U}$ , as well as from absorption of resonance neutrons of  $^{239}\text{Pu}$ ,  $^{235}\text{U}$ ,  $^{241}\text{Pu}$  fission in  $^{238}\text{U}$ .

The rate of accumulation of  $^{239}\text{Pu}$  is given by

$$\begin{aligned} \frac{dN''_{49}}{dt} = & N_{28}^0 \sigma_{28} \phi + \eta_{25} \epsilon P_1 (1 - P) N_{25} \sigma_{25} \phi \\ & + \eta_{49} \epsilon P_1 (1 - P) N''_{49} \sigma_{49} \phi + \eta_{41} \epsilon P_1 (1 - P) N_{41} \sigma_{41} \phi \\ & - N''_{49} \sigma_{49} \phi . \end{aligned} \quad (2.38)$$

Eq. (2.38) can be written as

$$\frac{dN''_{49}}{d\theta} = N_{28}^0 \sigma_{28} + K_{25} N_{25} \sigma_{25} - \gamma N_{49} \sigma_{49} + K_{41} N_{41} \sigma_{41} \quad (2.39)$$

where

$$K_m = \eta_m \epsilon P_1 (1 - P) \quad (2.40)$$

$$\gamma = 1 - K_{49} . \quad (2.41)$$

Due to the fact that  $\gamma N_{49} \sigma_{49} \gg K_{41} N_{41} \sigma_{41}$ , the formation of Pu-239 by absorption of resonance neutrons from  $^{241}\text{Pu}$  can be neglected. Equation 2.39 reduced to

$$\frac{dN_{49}}{d\theta} = N_{28}^0 \sigma_{28} + K_{25} N_{25} \sigma_{25} - \gamma N_{49} \sigma_{49} . \quad (2.42)$$

with  $N_{25}$  given by (2.10), the solution of this equation, subject to  $N''_{49} = 0$  at  $t = 0$ , is

$$N''_{49} = C_1 + C_2 E_0^G(\sigma_{25}\theta) - (C_1 + C_2) E_0^G(\sigma_{49}\gamma\theta) \quad (2.43)$$

where

$$C_1 = \frac{N_{28}^0 \sigma_{28}}{\sigma_{49} \gamma} \quad (2.44)$$

$$C_2 = \frac{K_{25} N_{25}^0 \sigma_{25}}{\sigma_{49} \gamma - \sigma_{25}} . \quad (2.45)$$

The overall final  $^{239}\text{Pu}$  concentration is

$$N_{49} = N'_{49} + N''_{49} \quad . \quad (2.46)$$

#### Plutonium 240

The rate of change of concentration of  $^{240}\text{Pu}$  is

$$\frac{dN_{40}}{d\theta} = \frac{\alpha_{49} N_{49} \sigma_{49}}{1 + \alpha_{49}} - N_{40} \sigma_{40} \quad . \quad (2.47)$$

with  $N_{49}$  given by (2.46), the solution of Eq. (2.47) subject to  $N_{40} = 0$  at  $t = 0$  is

$$N_{40} = C_3 + C_4 E_0^G(\sigma_{25}\theta) + C_5 E_0^G(\sigma_{49}\gamma\theta) - (C_3 + C_4 + C_5) E_0^G(\sigma_{40}\theta) \quad (2.48)$$

where

$$C_3 = \frac{N_{28}^0 \sigma_{28} \alpha_{49}}{\sigma_{40} \gamma (1 + \alpha_{49})} \quad (2.49)$$

$$C_4 = \frac{C_2 \sigma_{49} \alpha_{49}}{(\sigma_{40} - \sigma_{25}) (1 + \alpha_{49})} \quad (2.50)$$

$$C_5 = \frac{C_3 \sigma_{40}}{\sigma_{40} \gamma - \sigma_{40}} + \frac{C_4 (\sigma_{40} - \sigma_{25})}{\sigma_{49} \gamma - \sigma_{40}} \quad . \quad (2.51)$$

#### Plutonium 241

The rate of change of concentration of  $^{241}\text{Pu}$  is

$$\frac{dN_{41}}{d\theta} = N_{40} \sigma_{40} - N_{41} \sigma_{41} \quad . \quad (2.52)$$

with  $N_{40}$  given by (2.48), the solution of Eq. (2.52), subject to  $N_{41} = 0$  at  $t = 0$  is,

$$\begin{aligned} N_{41} = & C_6 + C_7 E_0^G(\sigma_{25}\theta) + C_8 E_0^G(\sigma_{49}\gamma\theta) + C_9 E_0^G(\sigma_{40}\theta) \\ & - (C_6 + C_7 + C_8 + C_9) E_0^G(\sigma_{41}\theta) \end{aligned} \quad (2.53)$$



where

$$C_6 = \frac{C_3 \sigma_{40}}{\sigma_{41}} \quad (2.54)$$

$$C_7 = \frac{C_4 \sigma_{40}}{\sigma_{41} - \sigma_{25}} \quad (2.55)$$

$$C_8 = \frac{C_5 \sigma_{40}}{\sigma_{41} - \sigma_{49} \gamma} \quad (2.56)$$

$$C_9 = \frac{(C_3 + C_4 + C_5) \sigma_{40}}{\sigma_{40} - \sigma_{41}} \quad (2.57)$$

### Plutonium 242

The rate of accumulation of  $^{242}\text{Pu}$  is

$$\frac{dN_{42}}{dt} = \frac{\alpha_{41} N_{41} \sigma_{41} \phi}{1 + \alpha_{41}} - N_{42} \sigma_{42} \phi \quad (2.58)$$

with the change in concentration of  $N_{41}$  with time given by (2.53), the solution of Eq. (2.58) subject to  $N_{42} = 0$  at  $t = 0$  is:

$$\begin{aligned} N_{42} = & \frac{\alpha_{41} \sigma_{41}}{1 + \alpha_{41}} \frac{C_6 [1 - E_0^G(\sigma_{42} \theta)]}{\sigma_{42}} + \frac{C_7 [E_0^G(\sigma_{25} \theta) - E_0^G(\sigma_{42} \theta)]}{\sigma_{42} - \sigma_{25}} \\ & + \frac{C_8 [E_0^G(\sigma_{49} \gamma \theta) - E_0^G(\sigma_{42} \theta)]}{\sigma_{42} - \sigma_{49} \gamma} + \frac{C_9 [E_0^G(\sigma_{40} \theta) - E_0^G(\sigma_{42} \theta)]}{\sigma_{42} - \sigma_{40}} \\ & - \frac{(C_6 + C_7 + C_8 + C_9) [E_0^G(\sigma_{41} \theta) - E_0^G(\sigma_{42} \theta)]}{\sigma_{42} - \sigma_{41}} \quad (2.59) \end{aligned}$$

### Fission Products

The rate of formation of fission products from the fission of  $^{235}\text{U}$  is

$$\frac{dN_F(25)}{dt} = \frac{N_{25} \sigma_{25} \phi}{1 + \alpha_{25}} \quad (2.60)$$

With the variation  $N_{25}$  with time given by (2.10), the solution of Eq. (2.60), subject to  $N_F(25) = 0$  at  $t = 0$ , is

$$N_F(25) = \frac{N_{25}^0 [1 - E_0^G(\sigma_{25} \theta)]}{1 + \alpha_{25}} \quad (2.61)$$

The rate of formation of fission products from  $^{239}\text{Pu}$  fission is

$$\frac{dN_F(49)}{dt} = \frac{N_{49} \sigma_{49} \phi}{1 + \alpha_{25}} \quad (2.62)$$

With the variation of  $N_{49}$  with time given by (2.43), the solution of this equation subject to  $N_F(49) = 0$  at  $t = 0$  is:

$$N_F(49) = \frac{\sigma_{49}}{1 + \alpha_{49}} C_1 f_0^G \theta + C_2 \frac{[1 - E_0^G(\sigma_{25} \theta)]}{\sigma_{25}} - (C_1 + C_2) \frac{[1 - E_0^G(\sigma_{49} \gamma \theta)]}{\sigma_{49} \gamma} \quad (2.63)$$

The rate of formation of fission products from  $^{241}\text{Pu}$  fission is

$$\frac{dN_F(41)}{dt} = \frac{N_{41} \sigma_{41} \phi}{1 + \alpha_{41}} \quad (2.64)$$

By comparing Eq. (2.64) with Eq. (2.58), we see

$$\frac{dN_F(41)}{dt} = \frac{dN_{42}}{\alpha_{41} dt} ; \quad (2.65)$$

therefore

$$N_F(41) = \frac{N_{42}}{\alpha_{41}} \quad (2.66)$$

By integration of Eq. (2.60), the following solution is obtained:

$$N_F(25) = \frac{\sigma_{25} \theta}{1 + \alpha_{25}} N_{25} \quad (2.67)$$

In the same manner, the following equations are obtained:

$$N_F(49) = \frac{\sigma_{49} \theta}{1 + \alpha_{49}} N_{49} \quad (2.68)$$

$$N_F(41) = \frac{\sigma_{41} \theta}{1 + \alpha_{41}} N_{41} \quad . \quad (2.69)$$

The inventory of fission product pairs from U-238 is obtained by considering the rate of production of neutrons in fast fission of U-238. The net rate of production of neutron from  $^{238}\text{U}$  fission is  $\nu_{28} - 1$  times the net rate of fission of U-238, which at steady state is  $(\nu_{28} - 1) N_F(28)$ . The net rate of production of neutrons by fast fission of U-238 is also  $\epsilon - 1$  times the rate of production of neutrons in fission of U-235, Pu-239, and Pu-241.

Therefore

$$(\nu_{28} - 1) N_F(28) = (\epsilon - 1) [\eta_{25} N_{25} \sigma_{25} \phi + \eta_{49} N_{49} \sigma_{49} \phi + \eta_{41} N_{41} \sigma_{41} \phi] \quad . \quad (2.70)$$

From (2.70) with (2.67), (2.68), (2.69), and since

$$\eta_m = \frac{\nu_m}{1 + \alpha_m} \quad ,$$

it follows that

$$N_F(28) = \frac{\epsilon - 1}{\nu_{28} - 1} [\nu_{25} N_F(25) + \nu_{49} N_F(49) + \nu_{41} N_F(41)] \quad . \quad (2.71)$$

### Uranium 238

The equation for the decrease in  $^{238}\text{U}$  concentration is obtained by considering the processes by which  $^{238}\text{U}$  is used up:

$$\begin{aligned} N_{28}^0 - N_{28} &= N_{28}^0 \sigma_{28} \theta + \nu_{25} \epsilon P_1 (1 - P) N_F(25) \\ &\quad + \nu_{49} \epsilon P_1 (1 - P) N_F(49) + N_F(28) \end{aligned} \quad (2.72)$$

$$N_{28}^0 \sigma_{28} \theta = \text{Absorption of thermal neutrons.}$$

$$\nu_{25} \epsilon P_1 (1 - P) N_F(25) = \text{Absorption of resonance neutrons from fission of } ^{235}\text{U}$$

$$v_{49} \epsilon P_1 (1 - P) N_F(49) = \text{Absorption of resonance neutrons from fission of } {}^{239}\text{Pu}$$

$$N_F(28) = \text{Fast fission of } {}^{238}\text{U}$$

Equations developed in this section are incorporated into the computer burnup codes for final fuel composition and concentration calculations.

### 3.0 NUCLEAR FUEL CYCLES AND NUCLEAR FUEL CYCLE COSTS

#### 3.1 Natural Uranium Fuel Cycle

A nuclear power reactor consumes nuclear fuel to produce power. After the fuel is consumed, some residual values which can be reclaimed for credit remain in the spent fuel. The processing of nuclear fuel from its natural or reprocessed state through its use in the nuclear reactor and the subsequent reclamation of the residual values comprise the nuclear fuel cycle.

The present day natural uranium fuel cycle without uranium or plutonium recycle, see Fig. 2, starts with the mining of uranium ore. The ore is then processed to "yellow cake," which is crude oxide or salt concentrate assaying 70-90 percent  $U_3O_8$ <sup>(16)</sup>. Natural uranium as it is mined and milled contains only 0.711 percent U-235 by weight<sup>(19)</sup>. To meet the  $^{235}U$  concentration necessary for typical light water reactor operation, enrichment is required. In preparing the uranium for enrichment, the yellow cake must be purified further and converted into  $UF_6$  (uranium hexafluoride). This step is called conversion or fluorination. After  $U_3O_8$  is converted into  $UF_6$ , it is shipped to one of the three gaseous diffusion plants operated by AEC, where  $UF_6$  is enriched to the desired concentration.

Nuclear fuels can be divided into two categories as far as ownership is concerned. The private reactor operators have the option to either buy or lease the fuel from AEC or private operators. If the nuclear fuel is leased from AEC, the nuclear fuel cycle may be considered to start with the leasing of enriched  $UF_6$  from the AEC.

The next step in the nuclear fuel cycle after U-235 enrichment is fabrication, which includes conversion of enriched  $UF_6$  into  $UO_2$ . The  $UO_2$  powder is pressed into pellets and sintered. The pellets are ground to size

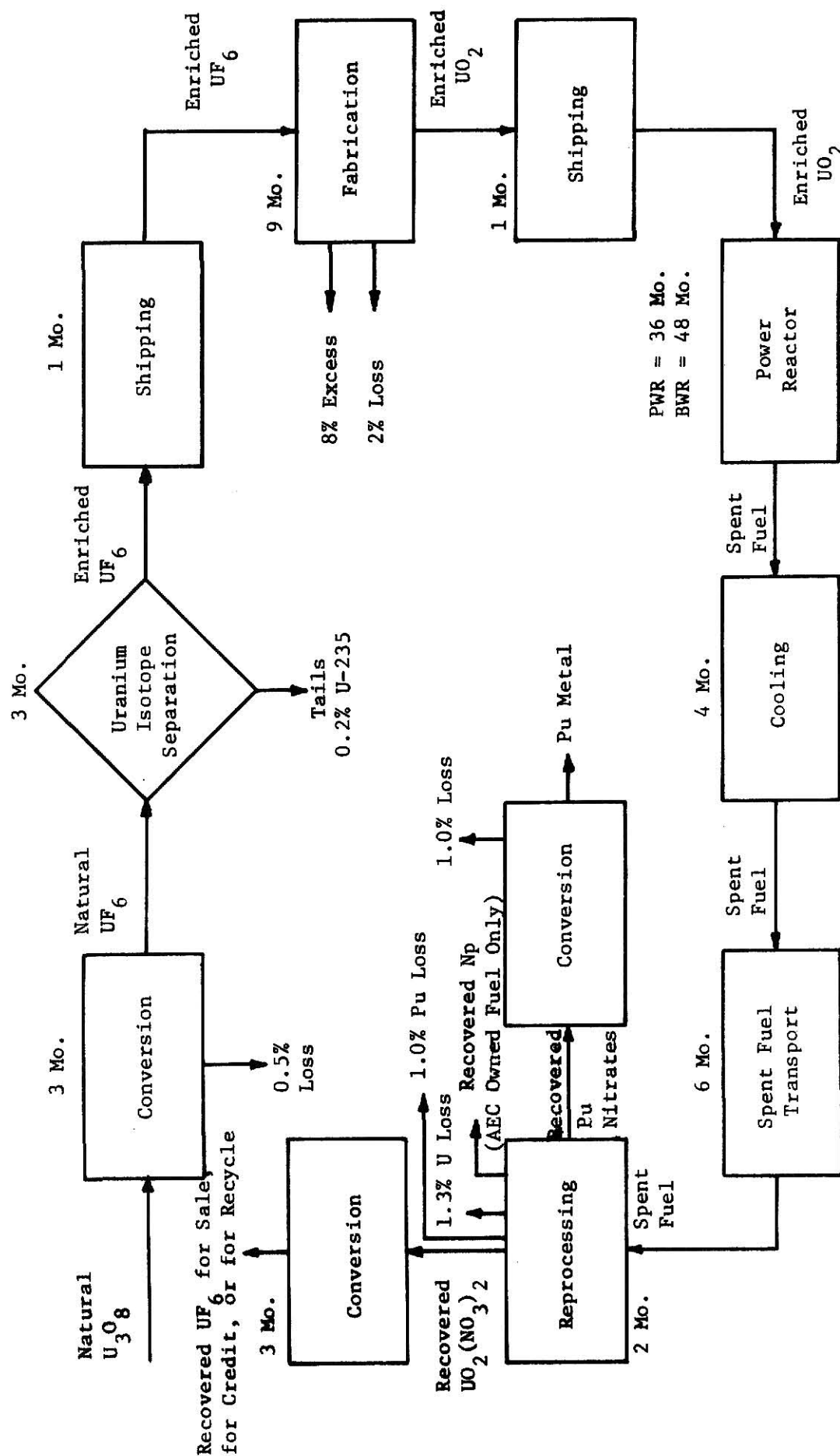


Fig. 2. Natural uranium fuel cycle flow sheet.

and encapsulated in zircaloy or stainless steel rods. These rods are mechanically fastened into bundles called fuel assemblies.

Fuel assemblies are shipped to the nuclear power plant and are loaded into the reactor where the fuel undergoes irradiation to produce power. When the fuel element reaches a certain estimated degree of burnup, based on a measured average burnup, it is removed from the core. Due to the buildup of fission products during the irradiation, the spent fuel discharged from the reactor is extremely radioactive. It must be "cooled" in shielded storage to permit radiation level and decay heat load to diminish substantially before the spent fuel can be transported to a spent fuel reprocessing facility. Shipment of spent fuel from the utility cooling pond to the reprocessing plant must be carried out in massive, shielded casks. (Transport of spent nuclear fuel is thus a costly as well as time consuming operation.)

In reprocessing, the uranium isotopes, plutonium isotopes, neptunium, waste, and fission products are separated chemically by solvent extraction, fluorination, and ion exchange processes. If the fuel is owned by AEC, then Np-237 will be separated and recovered from the spent nuclear fuel to be used as a target material for Pu-238 production in AEC facilities<sup>(30)</sup>. On the contrary, if the fuel is privately owned, the neptunium is not recovered but is discarded with the waste fission products<sup>(30)</sup>.

The radioactive wastes are concentrated. Under an AEC procedure adopted last year, the wastes could be stored as liquids for up to five years, and then are required to be calcined to a dry oxide solid form. They are then canned in high integrity, stainless steel containers. After a suitable observation and cooling period, (maximum duration is five years) these containers must be shipped to a federal waste repository for perpetual storage<sup>(17)</sup>.

The uranium and plutonium isotopes recovered from the reprocessing process are converted respectively into uranium hexafluoride and plutonium nitrate or metal before they are available for sale, for credit claim, or for recycle.

### 3.2 Spent Uranium Recycle by the Re-Enriching Procedure

When the spent uranium is to be recycled by the re-enriching procedure (see Fig. 3), the nuclear fuel cycle starts with the feeding of recovered  $\text{UF}_6$  along with the natural  $\text{UF}_6$  into the enrichment cascade to achieve the desired  $^{235}\text{U}$  concentration.

Special consideration must be given to the fuel enrichment ratio when spent uranium is to be recycled. Since U-236 can act as a thermal neutron poison, when reprocessed spent uranium is used, the final  $^{235}\text{U}$  concentration must be enriched to a higher level than indicated in the reference-design (see Tables 1 and 2) to obtain a fuel with the desired nuclear lifetime<sup>(9)</sup>. The excess  $^{235}\text{U}$  concentration required can be calculated from a neutron balance analysis. In the re-enriching scheme, in spite of the fact that a portion of the recycled  $^{236}\text{U}$  leaves the cycle in the diffusion plant tails, a large quantity of the recycled  $^{236}\text{U}$  still can be found in the product stream of the enrichment cascade. Therefore, a higher  $^{235}\text{U}$  concentration is still needed. For the rest of the nuclear fuel cycle, every step of the re-enriching scheme is identical to the nominal natural uranium fuel cycle as stated in Section 3.1 and demonstrated by the flow sheet, Fig. 3.

### 3.3 Spent Uranium Recycle by the Blending Procedure

When the spent uranium is to be recycled by blending, see Fig. 3, the nuclear fuel cycle starts with the blending of recycled  $\text{UF}_6$  with



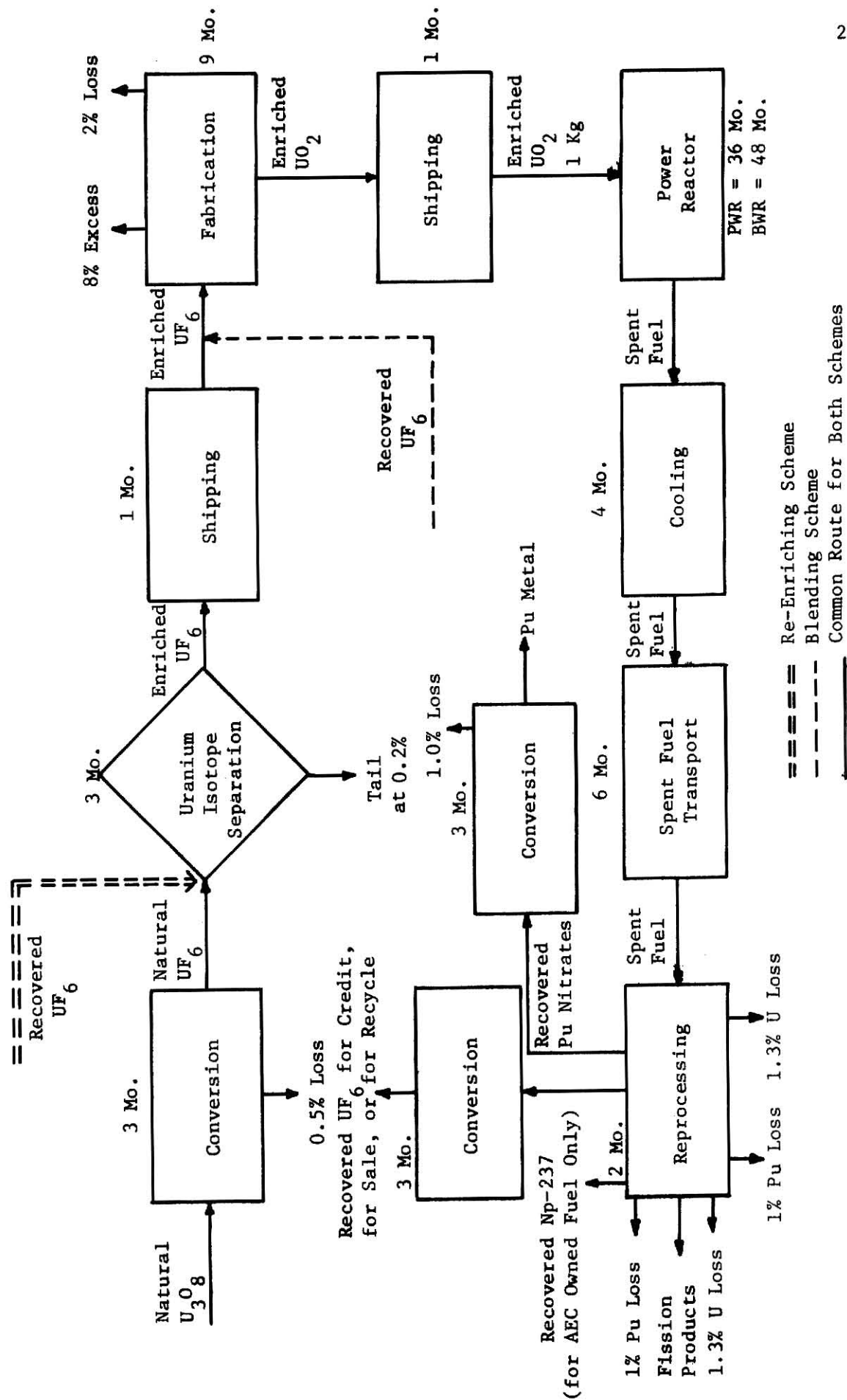


Fig. 3. Spent uranium recycle flow sheet.

makeup fuel of natural enriched uranium having a high  $^{235}\text{U}$  concentration. The makeup uranium may be obtained either by purchasing it directly from AEC or by assuming private ownership and shipping the natural  $\text{UF}_6$  feed to a gaseous diffusion plant and using the "toll enrichment" service of AEC to have it enriched to the desired  $^{235}\text{U}$  concentration. In the blending scheme, all recycled  $^{236}\text{U}$  reenters the reactor. Therefore the blending scheme requires a more highly enriched fuel than the re-enriching scheme.

After the recycled  $\text{UF}_6$  is blended with the makeup natural  $\text{UF}_6$  having a sufficiently high  $^{235}\text{U}$  concentration, the next step is the fabrication of fuel elements. From this point on every step in the blending scheme is similar to the re-enriching scheme (see Fig. 3).

### 3.4 Nuclear Fuel Ownership

Nuclear fuels, in general, can be classified into two categories as either leased fuel or privately owned fuel. When the fuel is privately owned, after the cooling period following burning is over the spent nuclear fuel must be removed from the power plant (so that the cooling pond will not be filled)<sup>(29)</sup>. Now a choice is necessary. The reactor operator can either move the spent fuel from the reactor to a perpetual storage facility by paying the waste disposal service charge, or he can ship the spent fuel to a reprocessing plant requiring the payment of a reprocessing charge generally higher than the perpetual storage charge. In this latter case, however, the reactor operator can expect a sales return or credit from the recovered uranium, plutonium, and neptunium. Only if the value of the recovered uranium, plutonium, and neptunium exceeds the net reprocessing charge (total charge minus disposal charge), is the spent fuel recovery a reasonable business proposition<sup>(29)</sup>. Since the privately owned fuel is not leased from

AEC or other private lessors, no one is obliged to purchase the recovered uranium. Under this circumstance, the reactor operator must find a buyer for his recovered uranium before having the spent fuel reprocessed<sup>(3)</sup>.

Therefore, the uranium credit is an uncertainty in the case of privately owned fuel. In this paper, in spite of the indefinite character of uranium credits, the uranium credit was assumed to be calculated in the same manner as for leased fuel but the U-236 penalty was also taken into consideration.

If the fuel is leased, when the spent fuel is discharged from the reactor, it is fully depreciated, in other words, it is valueless. The plutonium credit and the uranium credit have already been entered on the books. (Even though the cash income would not be received until the spent fuel is reprocessed<sup>(28)</sup>.) The reason for this is that AEC or private lessor has the obligation to purchase the recovered uranium.

### 3.5 The Fuel Cycle Material Requirements

The quantity of uranium needed at each step of the fuel cycle is based on one kg U as enriched  $\text{UO}_2$  loaded into the reactor of an equilibrium core. Starting with reactor operation, the uranium requirements at each preceding step will be derived by working backward through the fuel cycle (see Figures 2 and 3). For example in the fabrication step, about 10 percent extra uranium is required<sup>(16)</sup> for two reasons. Excess uranium is needed to cover the scrap requirements that arise in the manufacturing process due to chipped pellets, grinding waste, etc. This scrap is largely recovered and subsequently reused. A small amount of material is lost during the process, which is unavoidable. Generally speaking, the recoverable scrap amounts to about 8 percent of the extra uranium while the losses are near 2 percent. Therefore, for each kg uranium loaded into the reactor, 1.1 kg must be supplied to the fabrication plant.

The step preceding fabrication is enrichment. The material requirements here are calculated using a model of the  $\text{UF}_6$  diffusion process<sup>(13,18)</sup>. Each stage within the enrichment cascade receives a feed stream which it separates into two output streams: (1) an "enriched" stream which contains a higher concentration of  $^{235}\text{U}$  than the feed stream, and (2) a "depleted" stream which contains a lower concentration of  $^{235}\text{U}$  than does the feed stream. The mathematical analysis of a gaseous diffusion plant consists of writing mass balances for all of the uranium and for the fissile components of the uranium. Assumptions made are that flows of different concentrations of  $^{235}\text{U}$  are not mixed and the presence of  $^{234}\text{U}$  can be neglected. Eight variables are included in the analysis: the flow rate of natural  $\text{UF}_6$  feed, the flow rate of recycled  $\text{UF}_6$  feed, the flow rate of enriched product, the flow rate of waste, and concentrations of each of these streams.

The necessary equations are

$$F + R = P + W \quad (\text{Mass Balance of Total Uranium}) \quad (3.1)$$

and

$$X_F F + X_R R = X_P P + X_W W \quad (\text{Mass Balance of Fissile Uranium-}^{235}\text{U}) \quad (3.2)$$

where

$F$  = Flow rate of natural uranium feed, kg/day

$R$  = Flow rate of recycle uranium feed, kg/day

$P$  = Flow rate of product material, kg/day

$W$  = Flow rate of waste materials, kg/day

$X_F$  = Weight fraction of  $^{235}\text{U}$  in feed

$X_R$  = Weight fraction of  $^{235}\text{U}$  in recycle feed

$X_P$  = Weight fraction of  $^{235}\text{U}$  in product

$X_W$  = Weight fraction of  $^{235}\text{U}$  in waste.

Equations (3.1) and (3.2) can be combined and expressed in terms of flow rate of waste materials and flow rate of natural uranium feed in the following two equations:

$$W = \frac{P(X_P - X_F) - R(X_R - X_F)}{X_F - X_W} \quad (3.3)$$

$$F = \frac{P(X_P - X_W) - R(X_R - X_W)}{X_F - X_W} \quad (3.4)$$

If only natural  $UF_6$  is used to feed the cascades, in other words, no recovered spent fuel is recycled back to cascades, then Equations (3.3) and (3.4) are reduced to the general forms

$$W = P \frac{X_P - X_F}{X_F - X_W} \quad (3.5)$$

$$F = P \frac{X_P - X_W}{X_F - X_W} \quad (3.6)$$

when the spent fuel is recycled. The flow rate of  $^{236}U$  in the waste stream,  $W_6$ , is given by Guéron et al.<sup>(4)</sup> as

$$W_6 = R_6 \frac{\left(\frac{1}{X_R}\right)^{1/3} - \left(\frac{1}{X_P}\right)^{1/3}}{\left(\frac{1}{X_W}\right)^{1/3} - \left(\frac{1}{X_P}\right)^{1/3}} \quad (3.7)$$

where  $R_6$  is the weight of  $^{236}U$  in the recycle fuel. The schedule of base charges and standard table of enriching services published by AEC<sup>(19)</sup> are computed on the basis of tails (waste) assay of 0.2 weight percent  $^{235}U$ . This is also the value of weight fraction of  $^{235}U$  in waste assigned in this paper.

A loss of about 0.5 percent<sup>(16)</sup> is incurred in the  $U_3O_8$  to  $UF_6$  conversion or  $UO_2(NO_3)_2$  to  $UF_6$  conversion step. When yellow cake is used, the

usual case, its mass is commonly given in pounds. The following equation may be used to convert the mass of  $\text{UF}_6$  in kg of  $\text{U}_3\text{O}_8$  (yellow cake) needed in pounds.

$$x = y \times \frac{2.205 \text{ lb}}{\text{kg}} \times \frac{1 \text{ kg U}}{0.848 \text{ kg U}} \quad (3.8)$$

where

$x$  = weight of  $\text{U}_3\text{O}_8$ , lb.

$y$  = weight of uranium in the form of  $\text{UF}_6$ , kg

### 3.6 The Nuclear Fuel Cycle Costs

Fuel cycle costs are incurred for the purchase of fuel materials, conversion, enrichment services, shipping, and spent fuel reprocessing; credits are claimed for residual values in the spent fuel.

The cost per unit of material required at each step of the fuel cycle is tabulated in Table 3.1. A price of \$8/lb for  $\text{U}_3\text{O}_8$ <sup>(15,20,21)</sup> is assigned. The cost of  $\text{U}_3\text{O}_8$  to  $\text{UF}_6$  conversion is taken to be \$2.20/kg U<sup>(21)</sup>.

Uranium enrichment cost is obtained by summing up the cost of separative duty and the cost of feed materials then subtracting the credit allowed for the tail stream<sup>(13,15,18)</sup>. That is

$$\text{Cost (\$)} = C_S E + C_F F + C_R R - C_W W \quad (3.9)$$

where

$F$  = flow rate of the feed stream

$R$  = flow rate of the recycle stream

$W$  = flow rate of the tails stream

$E$  = separative work

$C_S$  = unit cost of the separative duty

$C_F$  = unit cost of the feed

$C_W$  = unit cost of the tails

$C_R$  = unit cost of the recycle feed

The separative work is given by Garrett et al.<sup>(6)</sup> as

$$E = \int_{X_W}^{X_P} \frac{H_5 - HX}{[X(1 - X) - \frac{2}{3} XZ]^2} dX \quad (3.10)$$

with

$i = 5, 6, 8$  representing  $^{235}\text{U}$ ,  $^{236}\text{U}$ , and  $^{238}\text{U}$

$H_i$  = net mass upflow of isotope  $i$  in the cascade

$X$  = weight fraction of  $^{235}\text{U}$

$Z$  = weight fraction of  $^{236}\text{U}$

$X_P$  = weight fraction of  $^{235}\text{U}$  in the product

$X_W$  = weight fraction of  $^{235}\text{U}$  in the waste.

If there is no  $^{236}\text{U}$  present, Eq. (3.10) reduces to the familiar

$$E = WV_W + PV_P - FV_F - RV_R \quad (3.11)$$

with

$$V_i = (2X_i - 1) \text{Log} \frac{X_i}{1 - X_i} \quad (3.12)$$

$X_i$  = composition of  $i$ th flow stream fraction.

The function  $V_i$  defined by (3.12) is called the separative potential. It is a function only of composition and is dimensionless. The separative work  $E$  describes the relative difficulty of accomplishing the particular isotopic separations task, and is described in units of kg. The extra separative work contributed by the presence of  $^{236}\text{U}$  is given by Guéron et al.<sup>(4)</sup> as

$$\Delta E = 4 W_6 \text{Log} \left( \frac{X_W}{X_R} \right) + 4 P_6 \text{Log} \left( \frac{X_P}{X_R} \right) \quad (3.13)$$

Table 3

## COST AND DURATION OF STEPS IN THE NUCLEAR FUEL CYCLE

(Equilibrium Core)

Fuel Cycle Step	Product	Duration	Material Cost
1. Mine and Mill	$U_3O_8$	---	\$8.00/lb of $U_3O_8$ (15, 20, 21)
2. $U_3O_8$ to $UF_6$ Conversion	$UF_6$	3 Months	\$2.20/kg of U (21)
3. Enrichment (Natural $UF_6$ is used)	Enriched $UF_6$	3 Months	\$32.00/Unit of (19) Separative Work
4. Enrichment (Recycle $UF_6$ is used)	Enriched $UF_6$	6 Months	\$32.00/Unit of (19) Separative Work
5. Enrichment to Fabrication Shipment		1 Month	---
6. Fabrication	Fabricated $UO_2$	9 Months	\$70.00/kg of U (22)
7. Fabrication to Reactor Shipment		1 Month	\$3.00/kg of U (15)
8. Reactor Operation	Irradiated Fuel	36 Months (PWR) 48 Months (BWR)	---
9. Cooling		4 Months	---
10. Spent Fuel Transport to Assemble a Batch at Reprocessing Site		6 Months	\$5.00/kg of U
11. Reprocessing	Uranium, Plutonium Fission Products	2 Months	\$32.00/kg of U (15)
12. $UO_2(NO_3)_2$ to $UF_6$ Conversion	$UF_6$	3 Months	\$5.60/kg of U (15)
13. $Pu(NO_3)_3$ to Pu Conversion	Pu	3 Months	\$1.50/g of Pu (27)



where

$W_6$  = flow rate of  $^{236}\text{U}$  in the tails

$P_6$  = flow rate of  $^{236}\text{U}$  in the product.

The charge per kilogram unit of separative work is \$32 as published by USAEC<sup>(19)</sup>, the cost of natural  $\text{UF}_6$  having an assay of 0.711 weight percent  $^{235}\text{U}$  is set by AEC to be \$23.46/kg  $\text{U}$ <sup>(19)</sup>, no value is assigned to the tails assay of 0.2 weight percent  $^{235}\text{U}$ . For the base charges on  $\text{UF}_6$  and the amount of feed material, separative work required per kg of  $\text{UF}_6$  at different enrichment ratios, see Table 2 which is published by USAEC.

The typical charge for fabricating an equilibrium core is set at \$70/kg  $\text{U}$ <sup>(22)</sup>. The fabrication costs may include some conversion steps, such as  $\text{UF}_6$  to  $\text{UO}_2$ ,  $\text{UO}_2(\text{NO}_3)_2$  to  $\text{UF}_6$ , and  $\text{UO}_2(\text{NO}_3)_2$  to  $\text{UO}_2$ . The fabrication cost also includes the cost of cladding and assembly hardware. During the operation period of power reactor (fuel burning), no additional capital outlays are required. In this phase of the cycle, the fuel is productive, electricity is generated, and revenues are collected.

After the fuel is discharged from the reactor, the irradiated fuel has to be "cooled" for a period of time. Since a depreciation charge is not taken into the account, no cost is incurred during this step of the fuel cycle. Shipping costs are calculated from unit cost and quantity data in the same manner as fabrication costs. Cost of shipment from the enrichment plant to the fabricator is included in the fabrication cost; therefore, zero value is assigned. A value of \$3/kg  $\text{U}$ <sup>(15)</sup> is used for the cost of shipment from the fabrication plant to the reactor. After the cooling period is over, the spent fuel will be shipped to the reprocessing plant. The cost of spent fuel transportation can vary significantly depending upon where the reprocessing plant is located. A uniform cost of \$5/kg  $\text{U}$  is assumed in this study.

Table 4. Schedule of Base Charges and Standard Table of Enriching Services Published by AEC (March 1971)

Assay (Weight Percent U-235)	Schedule of Base Charges (\$/Kg U as UF <sub>6</sub> )	Standard Table of Enriching Service	
		Feed Component (Normal) (Kgs U Feed/ Kg U Produced)	Separative Work Component (Kgs W U/ Kg U Product)
0.20	3.00		
0.25	3.00	0.098	-0.100
0.30	3.00	0.196	-0.158
0.35	3.00	0.294	-0.189
0.38	3.00	0.352	-0.197
0.40	3.49	0.391	-0.198
0.42	4.46	0.431	-0.197
0.44	5.46	0.470	-0.194
0.46	6.52	0.509	-0.189
0.48	7.63	0.548	-0.182
0.50	8.81	0.587	-0.173
0.52	10.01	0.626	-0.163
0.54	11.27	0.0665	-0.151
0.56	12.61	0.705	-0.137
0.58	13.92	0.744	-0.123
0.60	15.30	0.783	-0.107
0.65	18.89	0.881	-0.062
0.70	22.60	0.978	-0.012
0.711	23.46	1.000	0.000
0.75	26.65	1.076	0.044
0.80	30.87	1.174	0.104
0.85	35.22	1.272	0.168
0.90	39.69	1.370	0.236
0.95	44.26	1.468	0.307
1.00	48.90	1.566	0.380
1.10	58.43	1.761	0.535
1.20	68.25	1.957	0.698
1.30	78.29	2.153	0.868
1.40	88.52	2.348	1.045
1.50	98.95	2.544	1.227
1.60	109.50	2.740	1.413
1.70	120.15	2.935	1.603
1.80	130.96	3.131	1.797
1.90	141.86	3.327	1.994
2.00	152.86	3.523	2.194
2.20	175.09	3.914	2.602
2.40	197.57	4.305	3.018
2.60	220.30	4.697	3.441
2.80	243.24	5.088	3.871
3.00	266.33	5.479	4.306
3.20	289.61	5.871	4.746
3.40	313.02	6.262	5.191
3.60	336.52	6.654	5.638
3.80	360.16	7.045	6.090
4.00	383.86	7.436	6.544
4.50	443.50	8.415	7.690
5.00	503.59	9.393	8.851
5.50	564.03	10.372	10.022
6.00	624.77	11.350	11.203
7.00	746.97	13.307	13.587
8.00	869.93	15.264	15.995
9.00	993.51	17.221	18.422
10.00	1,117.54	19.178	20.863
12.00	1,336.76	23.092	25.782
14.00	1,617.14	27.006	30.737
16.00	1,868.39	30.920	35.719
18.00	2,120.37	34.834	40.724
20.00	2,372.93	38.748	45.747
25.00	3,006.37	48.532	58.369
30.00	3,642.16	58.317	71.064
35.00	4,279.78	68.102	83.816
40.00	4,918.92	77.887	96.616
50.00	6,201.33	97.456	122.344
60.00	7,488.93	117.025	148.235
70.00	8,782.18	136.595	174.302
80.00	10,082.97	156.164	200.605
85.00	10,737.71	165.949	213.892
90.00	11,397.63	175.734	227.341
92.00	11,664.01	179.648	232.796
93.00	11,798.05	181.605	235.550
94.00	11,932.86	183.562	238.328
96.00	12,233.13	187.476	244.842
98.00	13,129.41	191.389	269.982

The cost of reprocessing can be calculated based on two different options. The first option is a unit cost approach and is identical in method to the fabrication cost. The quantity of fuel reprocessed is based on the quantity of fuel charged into the reactor. The cost of \$33/kg of uranium loaded into the reactor is assigned to the first option<sup>(16)</sup>. The second option is a cost model of a reprocessing plant patterned after the Nuclear Fuel Services (NFS)<sup>(24)</sup> plant. This model describes a plant having a basic throughput for initial fuel enrichment up to 3%, and a throughput decreasing with higher initial enrichment. The costs are calculated as follows:

$$\text{Reprocessing Cost} = (\text{Cost/Day}) \frac{\text{Quantity Reprocessed}}{(\text{Reprocessing Throughput})(1 + \text{Cleanup Time})} \quad (3.14)$$

where

Cleanup Time = 8 days or 1/3 of the reprocessing time,  
whichever is greater.

Cost/Day = \$24,000/Day

Reprocessing Throughput = 1,000 kg/day

Quantity Reprocessed = quantity of fuel charged into the reactor.

This is equivalent to a unit cost of \$32/kg U<sup>(15)</sup> for reprocessing. There is no charge for fission product waste disposal; this charge has already been included in the reprocessing cost.

The reprocessing plant delivers the recovered uranium as uranyl nitrate,  $\text{UO}_2(\text{NO}_3)_2$ , and plutonium as one of its nitrates. If the fuel is leased from AEC, before the uranium can be returned to AEC for credit, it must be converted to  $\text{UF}_6$ . A price of \$5.60/kg U<sup>(15)</sup> is given for  $\text{UO}_2(\text{NO}_3)_2$  to  $\text{UF}_6$  conversion. To claim the credit for plutonium the plutonium must be converted to a metallic form from one of its nitrates. A cost of

\$1.50/g Pu<sup>(27)</sup> is charged by AEC for the plutonium conversion. If the nuclear fuel is owned by a private reactor operator, the uranium credit cannot be computed from the schedule of price versus enrichment given in Table 2, as leased from AEC for the following reasons: (1) U-236 is a neutron poison whereas U-238 is a fertile material, so they affect reactivity life time differently. (2) The presence of U-236 increases the amount of separative work expended in a gaseous diffusion plant to produce uranium of a specified U-235 content since separation of the U-235 from U-238 is less costly than separation of U-235 from an equal amount of U-236. Therefore, in this paper, the uranium credit for privately owned fuel is computed by treating <sup>236</sup>U in the recovered uranium as <sup>238</sup>U. Then the value of recovered uranium can be found from Table 2 by subtracting the cost of extra separative work induced by the presence of the <sup>236</sup>U in the spent fuel. The extra separative work required to completely eliminate <sup>236</sup>U is difficult to calculate. As a matter of fact, with the separation ratio between <sup>235</sup>U and <sup>236</sup>U being so small, it is impossible to completely remove <sup>236</sup>U from the rest of uranium isotopes. Therefore, to simplify the problem, <sup>236</sup>U is treated as <sup>235</sup>U, and the extra separative duty performed is interpreted as the extra cost charged by the gaseous diffusion plant to achieve higher enrichment of the product necessary to offset the presence of the <sup>236</sup>U.

The AEC commitment to purchase plutonium expired at the end of 1970. From now until breeder reactors are commercially available, the plutonium market will be limited to (1) research needs for breeder reactor development and (2) recycle plutonium to replace some of the <sup>235</sup>U in nuclear fuel for light water reactors. The determination of a market value for plutonium after AEC purchase ceased is difficult because many of the factors important

to the evaluation are not accurately known. Such factors include the cost of plutonium fabrication, the future price of  $U_3O_8$  which plutonium displaces  $^{235}U$ , and the nuclear behavior of plutonium-uranium fuel in light water reactors. A plutonium value of \$6/gm of fissile material is hence arbitrarily assigned here, so the plutonium credit can be calculated. Another product recovered from the reprocessing plant is Neptunium-237, which is a precursor of  $^{238}Pu$ . At the present time,  $^{238}Pu$  has been selected for use in many isotope applications, but mainly as a power source where its long half life, high power density and lack of significant photon emission are important.

Depreciation expenses usually are not considered in the nuclear fuel cycle. But since the two recycle schemes have different processing times for material durations in the enrichment cascade, it is necessary to deduct for the re-enriching scheme the depreciation expense of the extra three months required in this case for processing of the spent fuel. The rate of depreciating the spent fuel after it is discharged from the core is taken as 16%, a charge used by Southern California Edison Company<sup>(28)</sup>.

### 3.7 Duration of the Nuclear Fuel Cycle

The conversion  $U_3O_8$  to  $UF_6$  conversion about three months. The subsequent enrichment step required another three months when starting with natural  $UF_6$  feed material, but if the feed material includes the recycle stream of recovered  $UF_6$  in addition to the natural  $UF_6$  feed stream, the duration becomes six months due to the presence of  $^{236}U$  in the recycle stream. Fabrication, including conversion of enriched  $UF_6$  to  $UO_2$ , pelletizing, rod loading, and assembly takes about nine months.

The shipment of enriched  $UF_6$  from the gaseous diffusion plant to the fabrication plant, as well as the shipment of nuclear fuel assemblies from fabrication plant to reactor site are both assumed to have a duration of one month. For the next step, reactor operation, the duration is dependent upon the type of light water reactor. For an equilibrium core, a PWR has an average core life of 36 months, while a BWR has an average core life of 48 months. The cooling time for discharged spent fuel is set as four months (120 days). This is a generally used time period for spent fuel cooling.

The majority of the nuclear power plants in operation or under construction do not have direct access to rail for shipment of the irradiated fuel. Therefore, in many cases the spent fuel is shipped by truck. Due to the payload limitation on even overweight truck shipments only a few fuel assemblies can be shipped in one cask. It is estimated for a 1,000 MW(e) light water reactor, six months will be required to complete spent fuel transportation to the reprocessing site.

A time period of two months is required to separate uranium, plutonium, and neptunium from the waste and fission products. After uranium and plutonium have been recovered from the spent fuel, they are in the form of nitrate salts which must be converted to  $UF_6$  and Pu metal respectively to claim the credits. Three months have been assigned for these uranium and plutonium conversions.

### 3.8 The Present Worth Method

Section 3.6 presented and discussed the cost of the individual steps in the nuclear fuel cycle. These costs can be looked upon as dollars paid by the utility to its suppliers for materials and services rendered. However, most of these payments are made long before the utility collects revenues

from its customers for the electric power produced from the fuel. Therefore the utility has capital invested in nuclear fuel for many months before the receipt of revenue. Thus in addition to dollars paid to suppliers, fuel cycle costs should include a capital charge or interest on such advanced payments.

A typical annual capital charge for private electric utilities would be 10 percent<sup>(15,16)</sup> of the outstanding working capital. This includes a 6.5 percent return to the bondholders and stockholders, 3.0 percent for federal income taxes and another 0.5 percent for miscellaneous taxes and insurance. The net present worth method<sup>(25)</sup> has been adopted here to calculate the total nuclear fuel cycle costs when considering the effects of capital charge. This method assumes the anticipated cash outlays are discounted at the cost of capital to obtain the present worth of the cash outlays. Similarly, the anticipated cash incomes are discounted at the cost of capital to obtain the present worth of the incomes. The net present worth method is expressed by the following equation:

$$\text{Present Worth} = \frac{1}{(1 + i)^n} (\text{Cash Flow}) \quad (3.15)$$

where

$i$  = interest rate or cost of capital

$n$  = number of years until income is received or expense is paid.

$(1 + i)^n$  = present worth factor

Cash Flow = receipt or payment in the  $n^{\text{th}}$  year.

### 3.9 The Net Nuclear Fuel Cycle Cost

The nuclear fuel cycle cost in mills/kilowatt hour is calculated by dividing the present worth of revenue for a given time period with the amount of total energy sold per kg uranium loaded into the reactor during the same time period. In a nuclear power plant, income is derived from sales of energy, uranium credit, plutonium credit, possible neptunium credit, and the sale of excess uranium to fuel fabricators. The cash outlays are for fuel element costs, shipping costs, reprocessing costs, and conversion costs.

The present worth of revenue is equal to the present worth of the net fuel cycle cost, in \$/kg U charged to the reactor. For a once-through feed of uranium oxide fuel elements, the present worth of the net nuclear fuel cycle cost,  $C$ , is:

$$\begin{aligned}
 C = & C_1 \quad \text{costs of UF}_6 \text{ feed (include natural or recycled)} \\
 & +C_2 \quad \text{pre-irradiation shipping costs} \\
 & +C_3 \quad \text{enrichment costs} \\
 & +C_4 \quad \text{fabrication cost} \\
 & +C_5 \quad \text{spent fuel transportation cost} \\
 & +C_6 \quad \text{reprocessing cost} \\
 & +C_7 \quad \text{UO}_2(\text{NO}_3)_2 \text{ to UF}_6 \text{ conversion cost} \\
 & +C_8 \quad \text{Pu nitrates to Pu metal conversion cost} \\
 & -C_9 \quad \text{credit for uranium in spent fuel} \\
 & -C_{10} \quad \text{credit for plutonium in spent fuel} \\
 & -C_{11} \quad \text{credit for uranium recovered from the fabrication process} \\
 & -C_{12} \quad \text{potential credit for neptunium in spent fuel.} \quad (3.16)
 \end{aligned}$$

In the above equation, all the costs or credits are present worth values.

Revenue is collected monthly during the reactor operation time; therefore,



for a PWR, revenues are collected 36 times throughout the nuclear fuel cycle.

By the same procedure revenues are collected 48 times for a BWR.

The total present worth revenue is given by

$$R = \frac{C}{\frac{\left(\frac{1}{1+i}\right)^{n_1} \left[1 - \left(\frac{1}{1+i}\right)^{n_2}\right]}{1 - \left(\frac{1}{1+i}\right)}} \quad (3.17)$$

where

$n_1$  = number of months of pre-irradiation period

$n_2$  = number of months of reactor operation

$i$  = interest rate or capital charge per month

and

$$\frac{\left(\frac{1}{1+i}\right)^{n_1} \left[1 - \left(\frac{1}{1+i}\right)^{n_2}\right]}{1 - \left(\frac{1}{1+i}\right)}$$

is the present worth factor of total revenue.

The contribution of the nuclear fuel cycle to the cost of electric power,  $C_p$ , in mills/kWh, is then given by

$$C_p(\text{Mills/kWh}) = \frac{n_2 \times R(\$/\text{kg U}) \times 1,000(\text{kg/Tonne}) \times 1,000(\text{Mills}/\$)}{E(\text{MWD/Tonne}) \times 24,000(\text{kWh/MWD}) \times e(\text{kWh Elec./kWh Heat})}$$

where

$E$  = burnup of fuel

$e$  = thermal efficiency of the power plant

#### 4.0 RESULTS AND DISCUSSION

Figures 4 through 7 present the calculated results of the contribution of the nuclear fuel cycle cost to the total cost of electric power from either natural uranium fuels or recycled spent uranium fuels. The figures were plotted from the information listed in Tables 4 through 7 of Appendix B.

For the reference-design 1,000 MWe PWR at average burnup, the re-enriching scheme was 4.44 percent cheaper than the blending scheme for privately owned fuel, and was 3.56 percent cheaper for leased fuel. With respect to the reference-design 1,065 MWe BWR at average burnup, the re-enriching scheme was 5.72 percent cheaper than the blending scheme for privately owned fuel, and was 4.83 percent cheaper for leased fuel.

The effects of varying the nuclear fuel cycle variables, one variable at a time are shown in Figures 8 through 13. There are of course other factors in the nuclear fuel cycle that may vary other than those used here, but those factors including shipping cost, fabrication cost, and reprocessing cost would be constant for both the spent uranium recycle schemes studied. Since variations in these factors would not alter the final result, they were omitted from the present discussion.

Changes in the price of yellow cake, the discharged fuel enrichment, or the cost of separative work as can be seen from Figures 8-13 have no influence on the choice between re-enriching and blending.

Under present technology Np-237 is the preferred target material for the production of Pu-238. An argument exists whether or not a credit should be assigned to neptunium recovered from spent fuel reprocessing. At the present time, all Np-237 target material used by the AEC to produce Pu-238 is recovered from AEC owned fuels and AEC has no plans to purchase privately

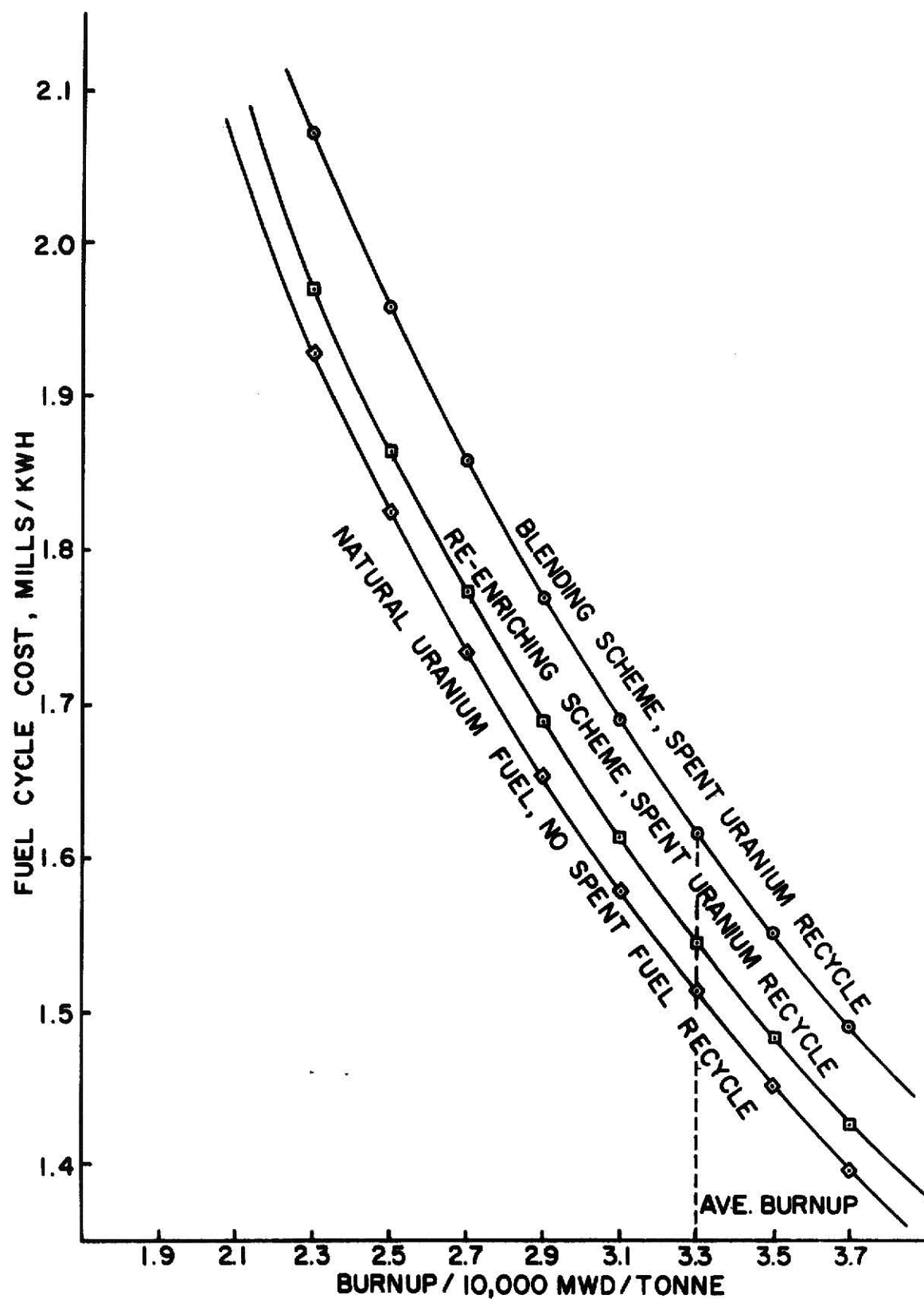


Fig. 4. 1,000 MWe PWR fuel cycle costs for privately owned fuel.

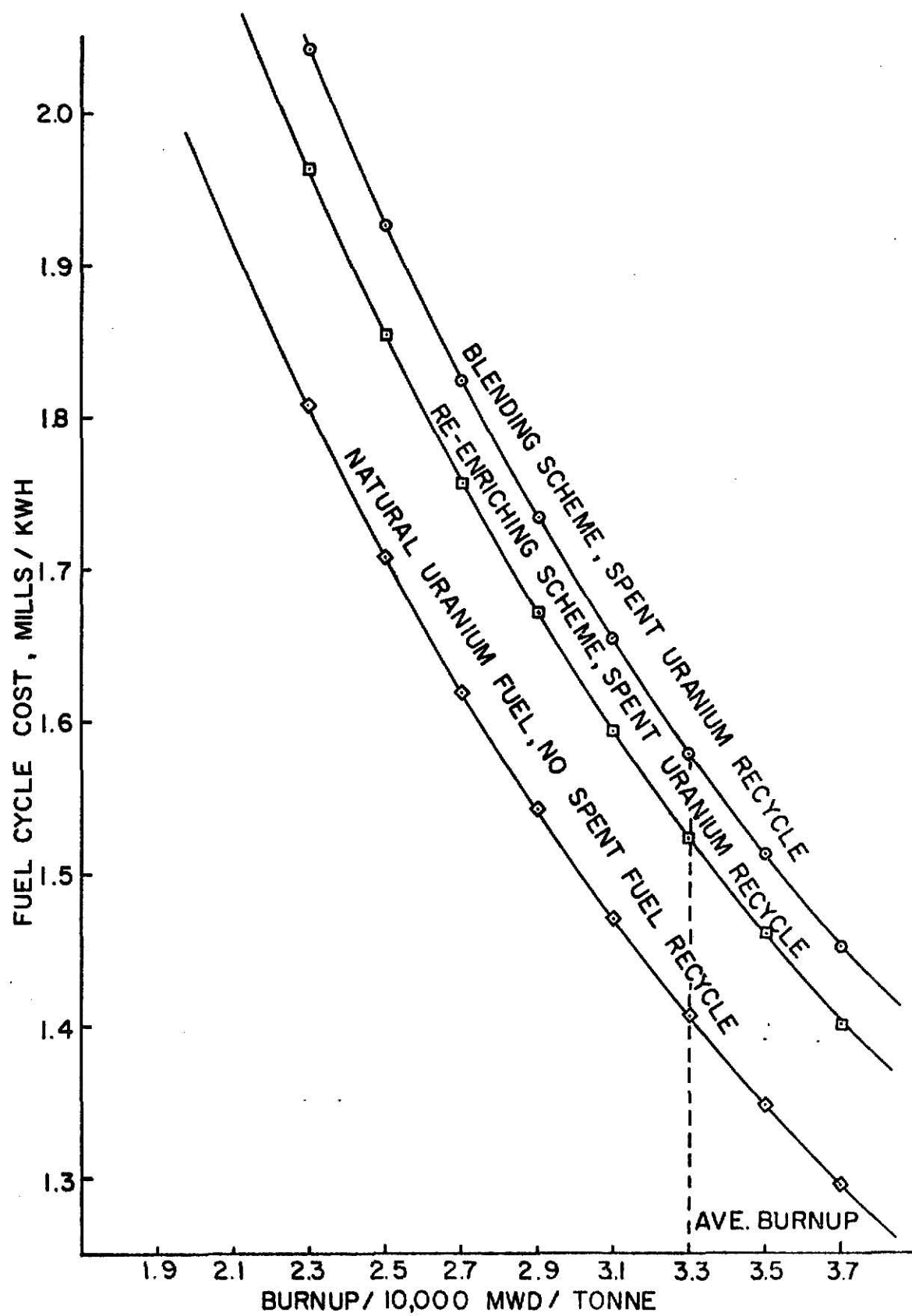


Fig. 5. 1,000 MWe PWR fuel cycle costs for leased fuel.

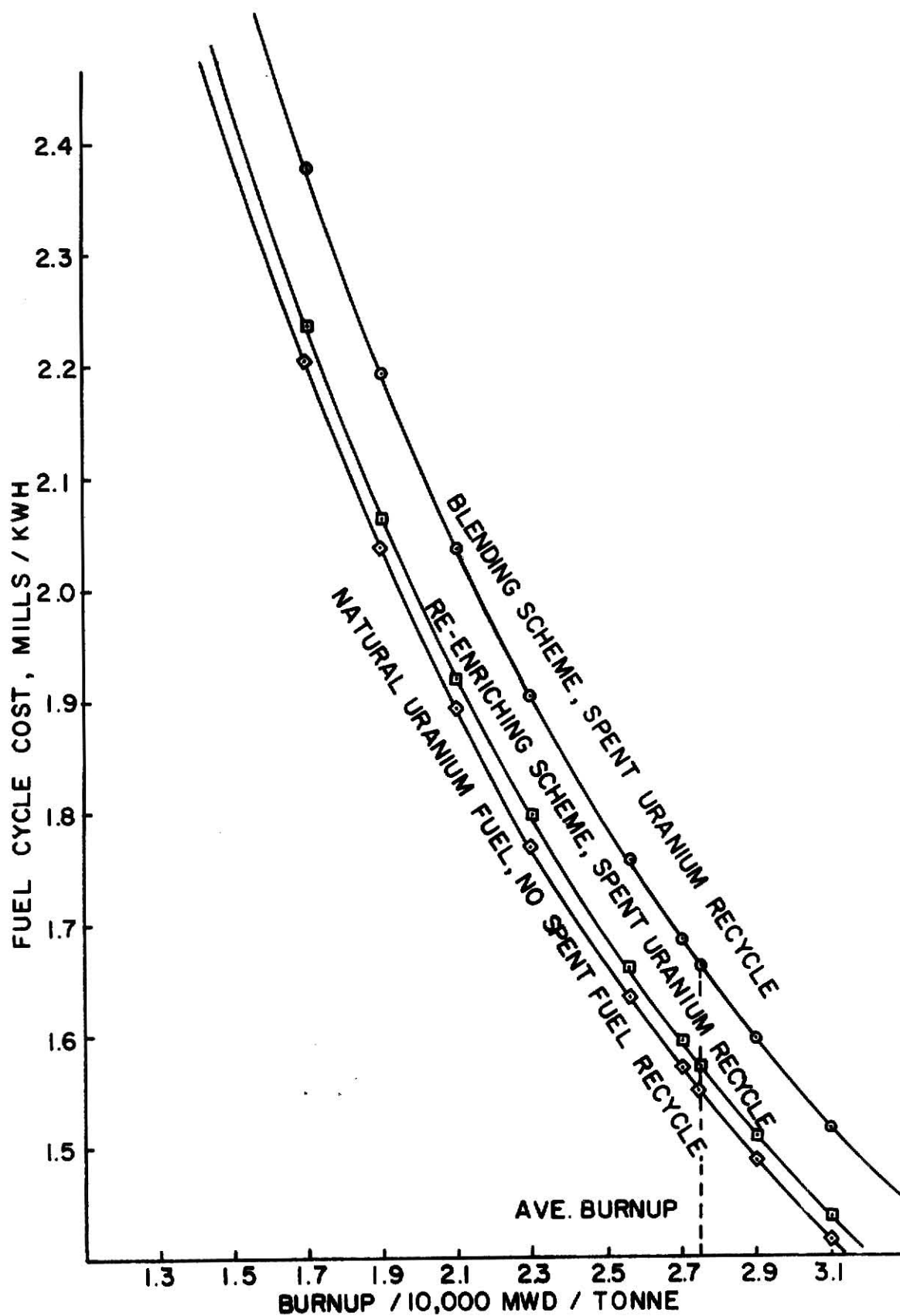


Fig. 6. 1,065 MWe BWR fuel cycle costs for privately owned fuel.

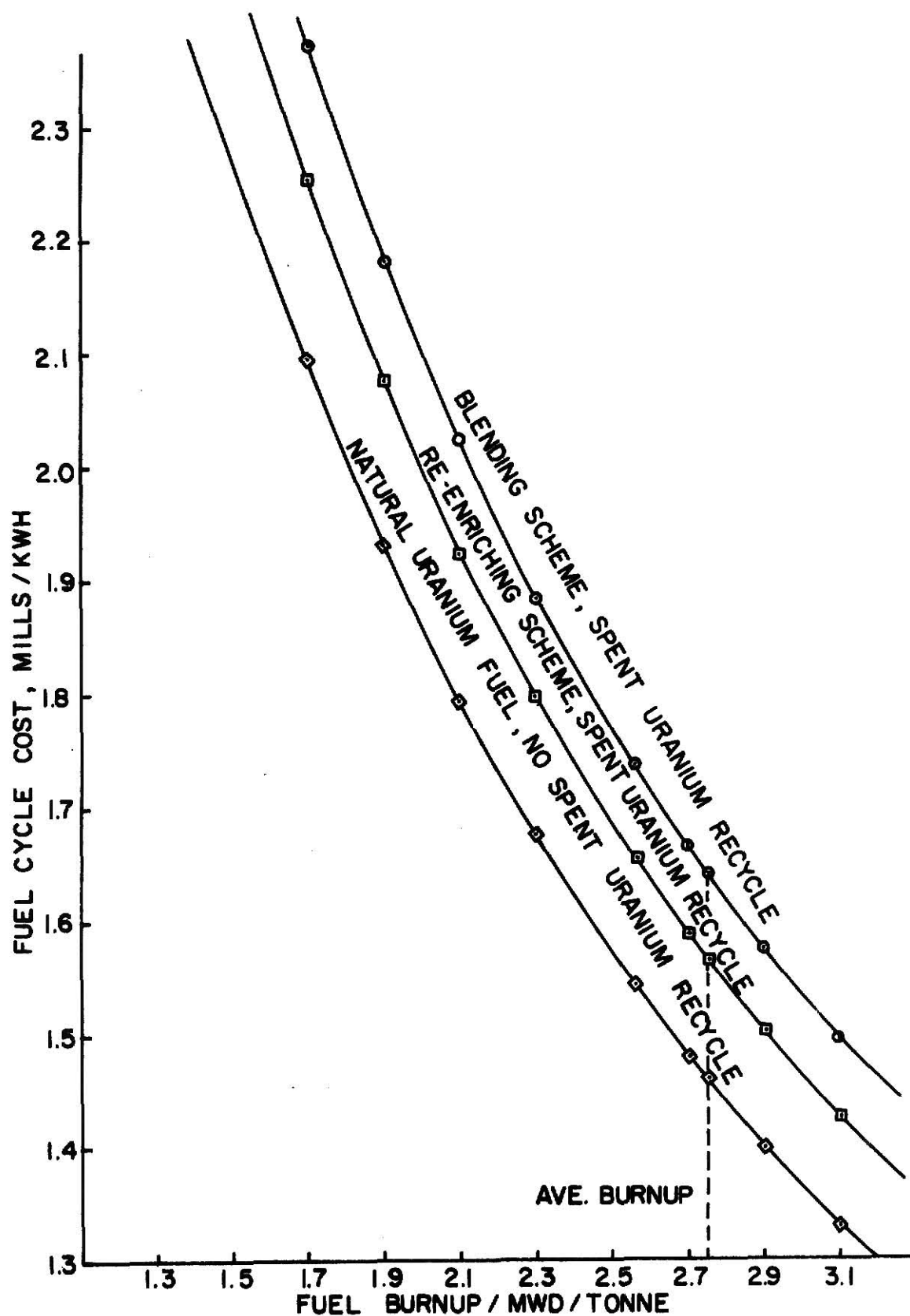


Fig. 7. 1,065 MWe BWR fuel cycle costs for leased fuel.

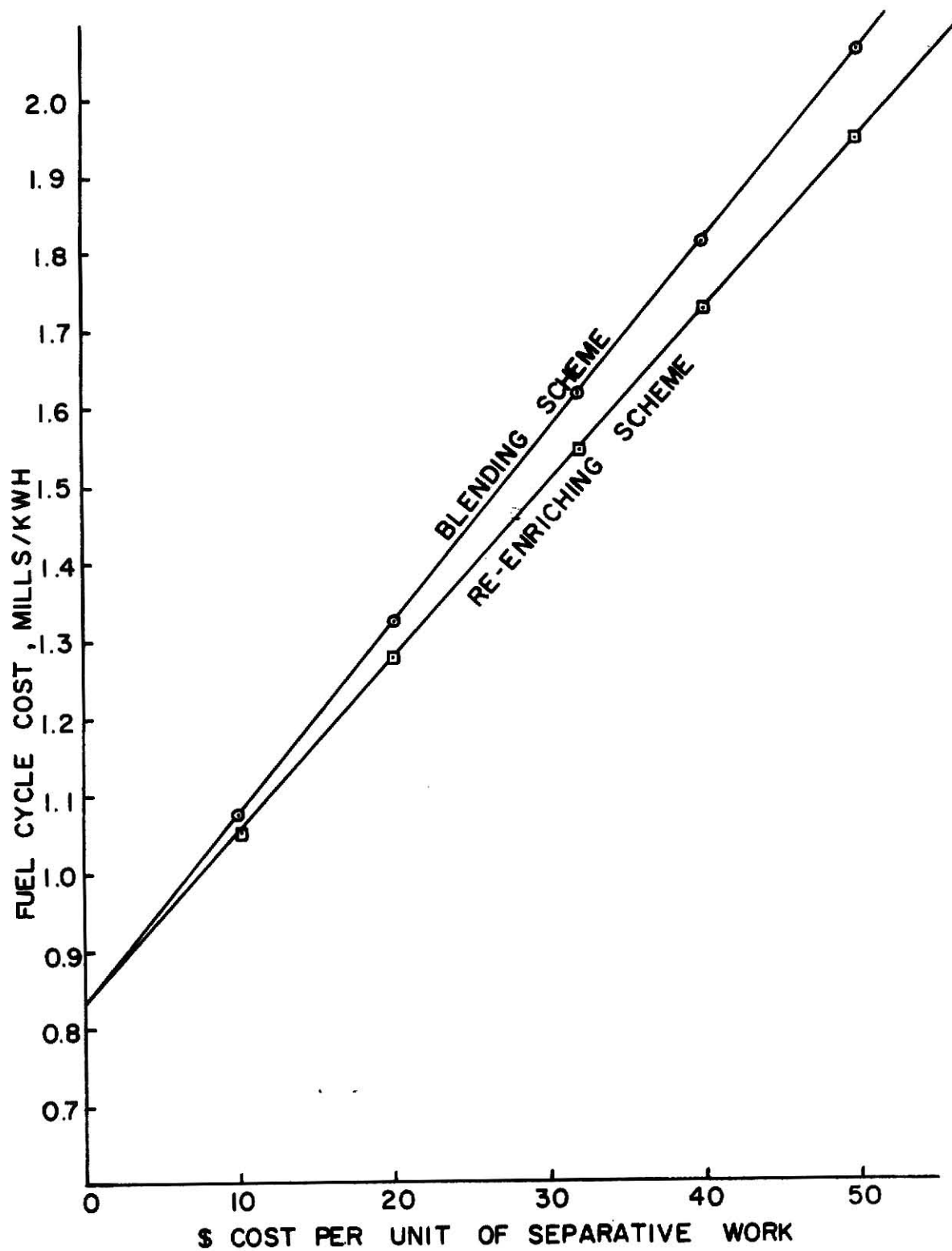


Fig. 8. Effect of separative work on fuel cycle cost for a 1,000 MWe PWR privately owned fuel.

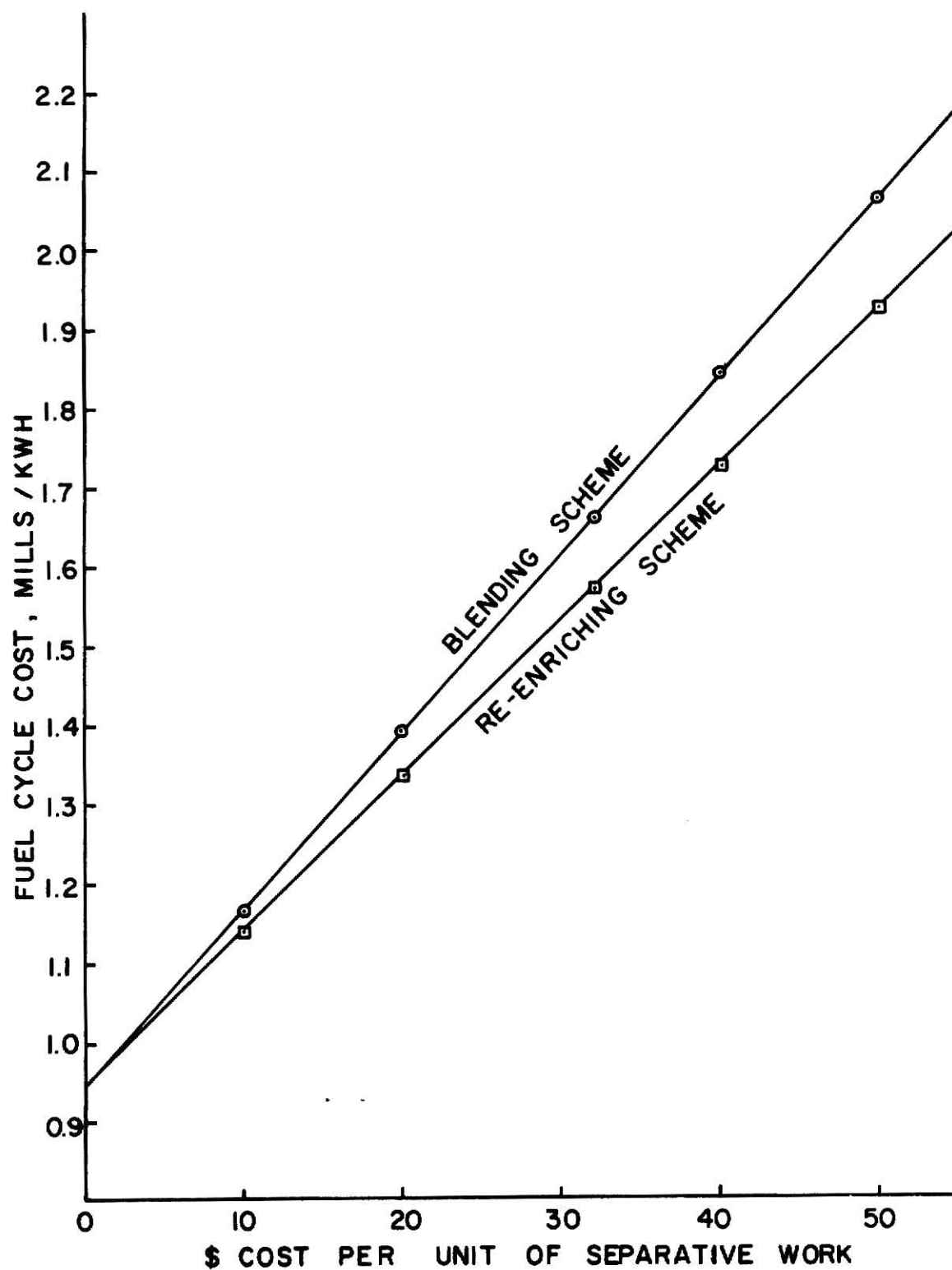


Fig. 9. Effect of separative work on fuel cycle cost for a 1,065 MWe BWR privately owned fuel.



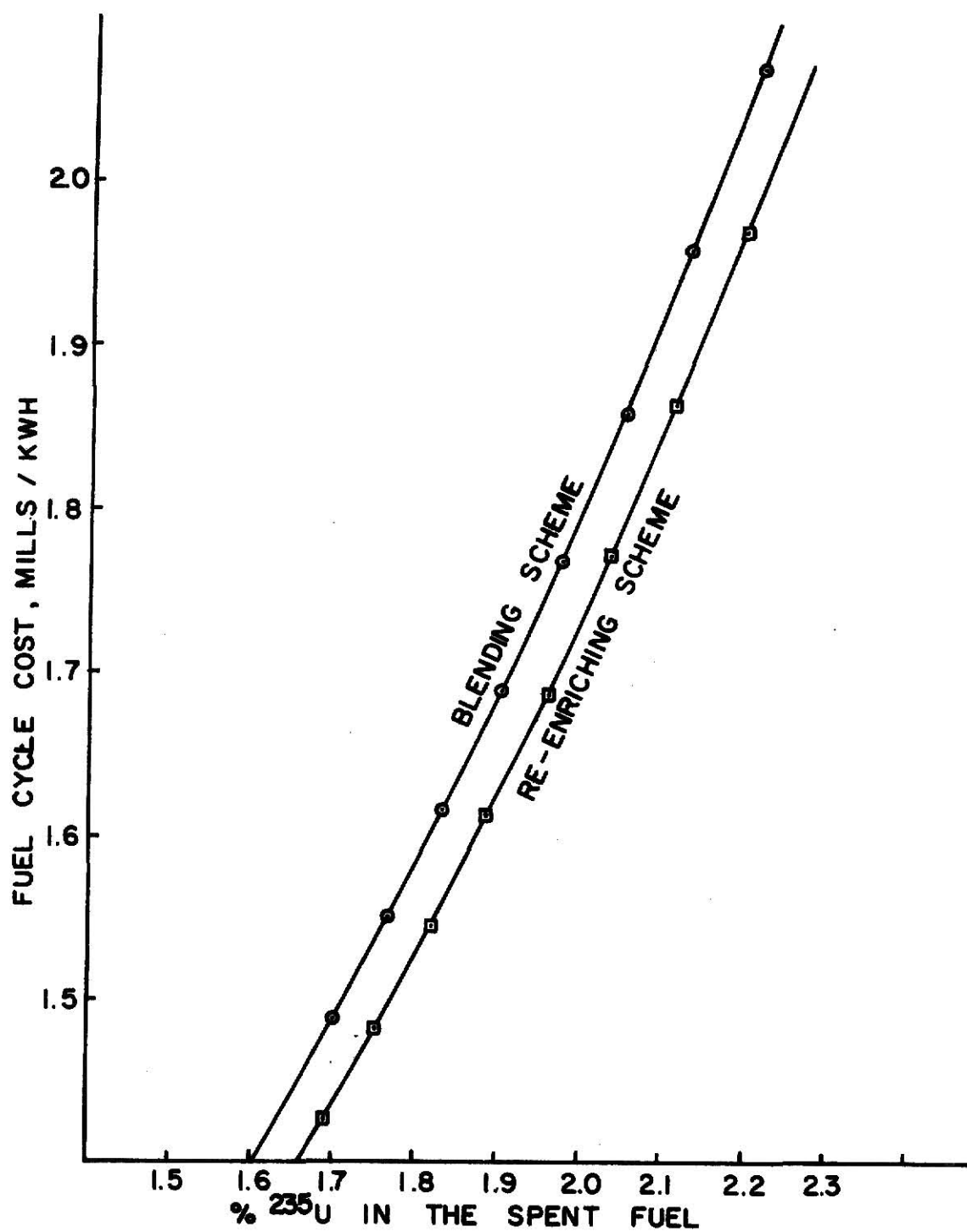


Fig. 10. Effect of product fuel enrichment on fuel cycle cost for a 1,000 MWe BWR.

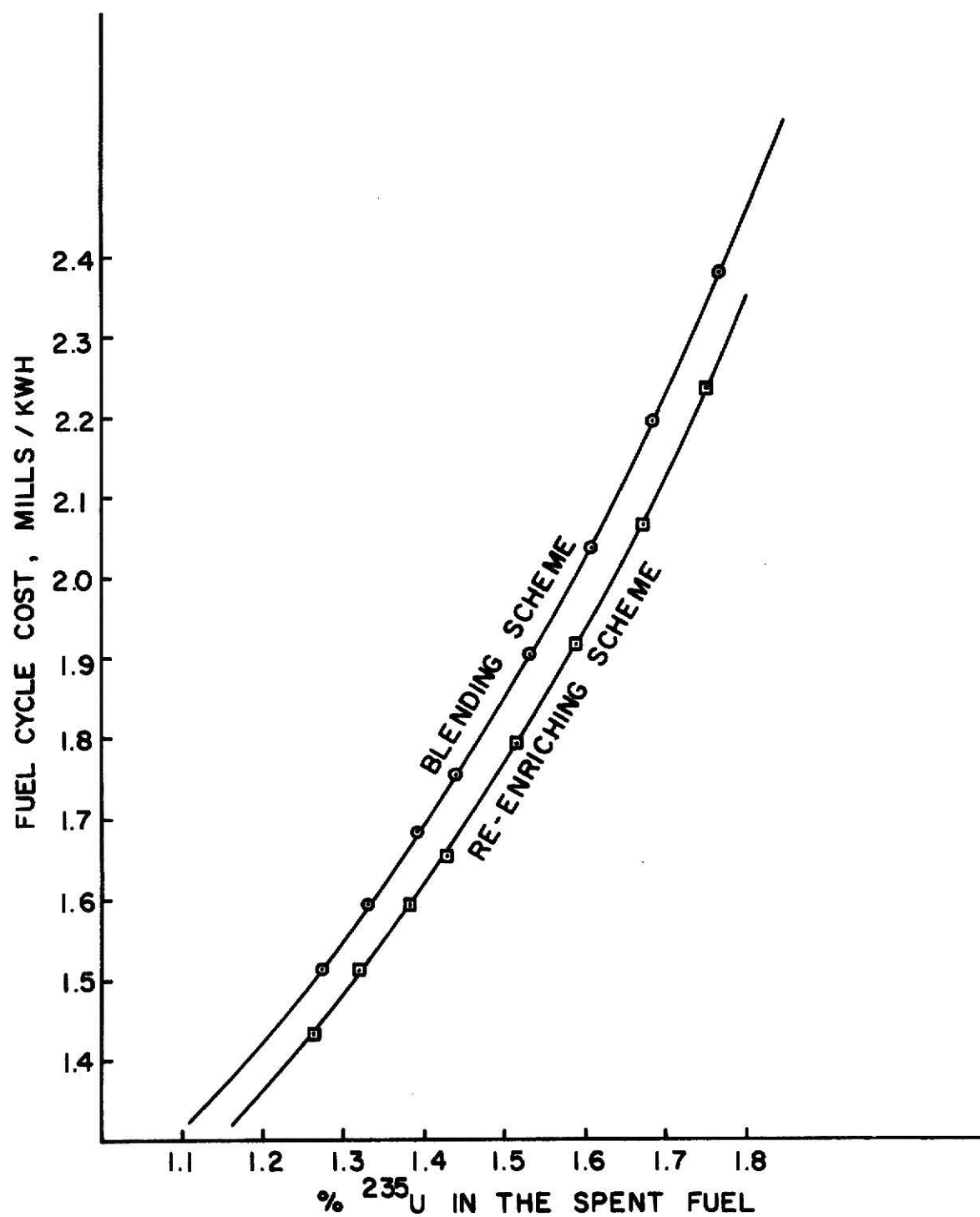


Fig. 11. Effect of discharged fuel enrichment on fuel cycle costs for a 1,065 MWe BWR.

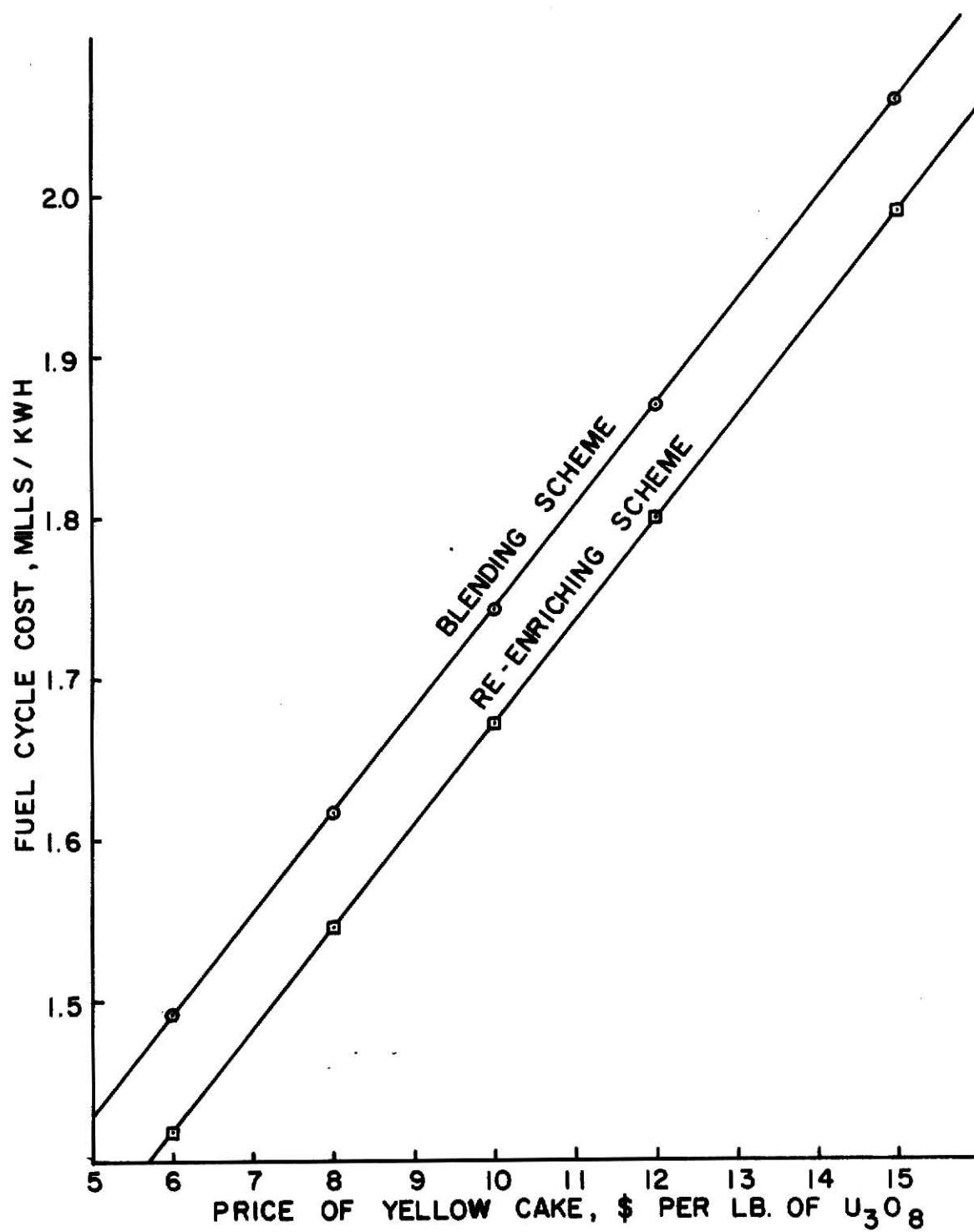


Fig. 12. Effect of yellow cake cost on fuel cycle cost for a 1,000 MWe PWR, privately owned fuel.

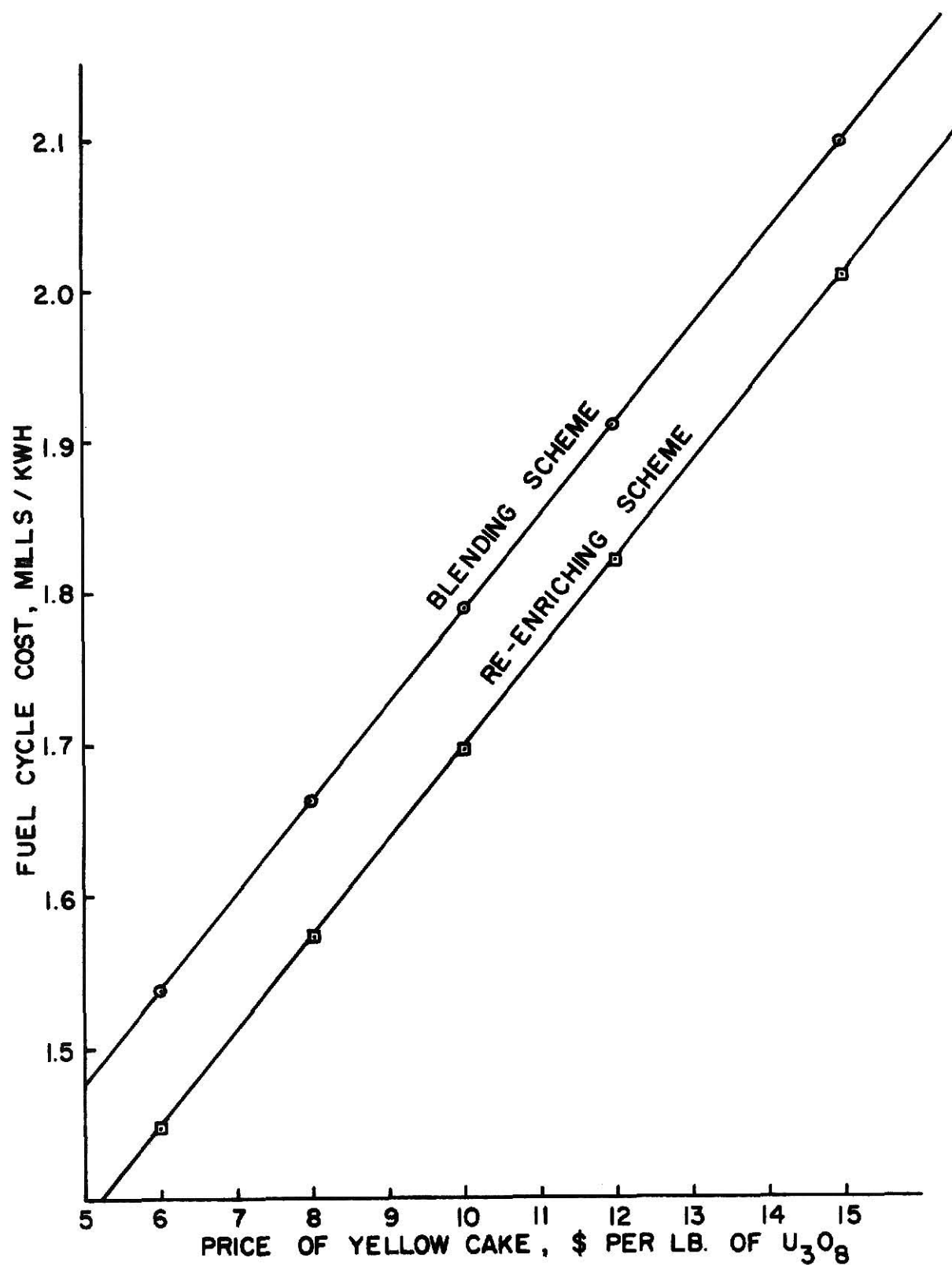


Fig. 13. Effect of yellow cake cost on fuel cycle cost for a 1,065 MWe BWR, privately owned fuel.

owned  $\text{Np-237}^{(30)}$ . Therefore in this study neptunium did not produce any revenue or credits for the present fuel cycles.

For the reference-design 1,000 MWe PWR at average burnup, privately owned natural uranium fuel was 2.20 percent cheaper than fuel produced by the re-enriching scheme, and 11.75 percent cheaper than blending scheme fuels. For the same reactor natural uranium fuel was 8.28 percent cheaper than re-enriching scheme fuel and 12.28 percent cheaper than blending scheme fuel when considering leased fuel.

For the reference-design 1,065 MWe BWR at average burnup, natural uranium fuel was 1.47 percent cheaper than re-enriching scheme fuel, and 7.27 percent cheaper than blending scheme fuel for the case of privately owned fuel. For the same reactor, natural uranium fuel was 7.23 percent cheaper than re-enriching scheme fuel, and 12.41 percent cheaper than blending scheme fuel for the case of leased fuel. The conclusion can be drawn of course that the natural uranium fuel cycle is cheaper than either of the spent uranium recycle schemes. Therefore, the nominal natural uranium fuel cycles are still economically favored over spent uranium recycle with one exception occurring under the two conditions described below that will favor the re-enriching scheme. The first condition occurs when the price of yellow cake rises above \$14.00 per pound; the second condition occurs when instead of feeding both natural and recycle to the uranium cascade, only recycle  $\text{UF}_6$  is used as the feed. The present re-enriched policy employed by AEC calls for reenrichment of the spent uranium as part of the "toll enriching service" offered by AEC (the quantity of the recycled irradiated  $\text{UF}_6$  is a small fraction of the feed stream<sup>32</sup>.) The second condition requires the assumption that there will be a sufficient amount of recovered spent uranium fuel accumulated so that it alone will serve as a cascade feed material.

Further, AEC must agree to accept this situation which will lead to gross contamination of the cascade. It appears unlikely that two conditions described above will be met simultaneously within the next twenty years. Of the two uranium recycle schemes studied, uranium blending in every case is more expensive than re-enriching. Blending has been successfully accomplished in AEC plants over a period of years<sup>(31)</sup>; however, it requires special equipment and skill in performing isotopic analyses. (This is another disadvantage of the blending scheme.) The only foreseeable situation that may make blending an economically attractive alternative will occur if a highly enriched uranium, now primarily stockpiled for weapon purposes, can be obtained at a sale price relatively lower than the regular prices published by AEC.

Under current policy, AEC's gaseous diffusion plants accept any feed material as long as it is in the form of uranium hexafluoride<sup>(31)</sup>. The current feed consists of normal uranium owned by toll enrichment customers and uranium recovered from irradiated reactor fuel<sup>(31)</sup>. Since the recovered spent uranium contains a certain amount of  $^{236}\text{U}$  (varies with the degree of fuel burnup), it contaminates the enrichment cascade forever once it is introduced into the cascade. As a matter of fact, the cascades of AEC's gaseous diffusion plants have already been contaminated. But because the quantity of recovered spent  $\text{UF}_6$  which has been fed into the cascade is almost negligible compared to the main feed stream of normal  $\text{UF}_6$ , the products show only a trace of  $^{236}\text{U}$ <sup>(32)</sup>.

At the present time, no penalty charge is asked by AEC for cascade contamination. AEC has not yet decided whether a penalty will be charged or not in the future<sup>(32)</sup>. With the fast growth of nuclear power it is expected that much more recovered spent fuel will be recycled. Thus it can also be

expected that a larger detectable  $^{236}\text{U}$  concentration will be found in each successive batch of product passing through the diffusion plants. If a penalty is charged by then, and if the penalty happens to be very costly, then to recycle the recovered spent uranium by blending might be favored. As indicated by the Figures 4 through 7 the re-enriching scheme is cheaper than the blending scheme and the natural uranium fuel cycle is cheaper than either re-enriching scheme for both leased and privately owned fuels. For the private reactor operators that own their fuel, if they can't find a buyer for their recovered spent uranium or if the type of warranty covering their fuel purchase is not a comprehensive fuel supply agreement, in other words, if the recovered spent uranium cannot be sold or returned to the vender for credit, then it virtually has a zero value. If this is the case, the only solution to this problem is to recycle the recovered spent uranium by the re-enriching process.

## 5.0 SUGGESTIONS FOR FURTHER STUDY

At the present time, neptunium in spent nuclear fuel does not receive a credit, but in the future, as Pu-238 requirements for space and marine undersea power applications, heart pacemakers, heart assist and artificial hearts as well as other applications increase, a considerable quantity (as proposed and estimated by several agencies)<sup>(8)</sup>, will be required and will result in significant prices for both Np-237 and Pu-238. It is suggested here that a future study should be directed to investigate the effects of neptunium on the nuclear fuel cycle costs when a demand and market price for recovered neptunium from privately owned spent nuclear fuel is established.

It is also suggested that a study be undertaken to determine the economics of mixing recycled plutonium with some of the recycled uranium and upgrading the rest of the recycled uranium by re-enriching or blending to take advantage of the situation that AEC had stopped purchasing plutonium and it is currently available at a low price.



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## 8.0 APPENDICES

## APPENDIX A

Two computer programs were used in this study. The code DRAGON was designed to calculate the isotopic concentration of the spent nuclear fuel discharged from the core after a specific burnup. The code FULCYC was developed to compute the nuclear fuel cycle cost.

### DRAGON--The Nuclear Fuel Burnup Code

This computer program was used to compute the final concentration of various isotopes, i.e. U-235, U-236, U-238, Pu-239, Pu-240, Pu-241, Pu-242, and Np-237. Existing in the irradiated fuel discharged from a reactor. In this paper, the code "DRAGON" was used to determine the composition of the spent natural uranium fuel. The spent fuel was then reprocessed and recycled back to the reactor with the natural uranium make up fuel. After the fuel was loaded in the reactor the new core burnup calculation for either natural or recycled fuel would be performed by the built-in subroutine of another code, FULCYC. The graded irradiation was chosen to be the standard irradiation method. Comment cards presented in the program aid in understanding the logic. The variables were defined in the FORTRAN listing and in the input data below.

### Input Data

Card 1: Format (8E 10.4)

S(25), S(26), S(28), S(49), S(40), S(41), S(42), S(38) - Thermal neutron absorption cross section of U-235, U-236, U-238, Pu-239, Pu-240, Pu-241, Pu-242 and Np-238 in  $\text{cm}^2$ .

Card 2: Format (2E 10.4, 5F 10.0)

S(37), S(48) - Thermal neutron absorption cross section of Np-237  
and Pu-238 in  $\text{cm}^2$ .

$P_1$  - Fission-to-resonance nonleakage probability  
P - Resonance escape probability  
Pth - Resonance-to-thermal nonleakage probability  
EPS - Fast fission factor  
D49 - Ratio of capture cross section to fission cross section  
of Pu-239

Card 3: Format (6F 10.0, E 10.4)

D25, D48, D41 - Ratio of capture cross section to fission cross section  
of U-235, Pu-238, and Pu-241.

E25, E41, E49 - Fission neutrons produced per thermal neutron absorbed  
in U-235, Pu-241, and Pu-239.

PHI - Thermal neutron flux in  $\text{n/cm}^2\text{-sec}$

Card 4: Format (4F 10.0, 2E 10.3)

V25, V28, V49, V41 - Neutrons produced per fission by U-235, U-238, Pu-239,  
and Pu-241 nuclides.

DC27, DC38 - Decay constants of U-237 and Np-238.

Card 5: Format (3F 10.0, II 10, F 10.0)

AU25, AU26, AU28 - The weight of U-235, U-236, and U-238 loaded into  
the reactor.

N - Number of sets of data to be computed

THW - Reactor power, thermal, in MW(t)

```

C *****
C THIS PROGRAM IS TO CALCULATE THE COMPOSITION IN UNMIXED FUEL
C DISCHARGED AFTER GRADED IRRADIATION TO A SPECIFIC FLUX TIME.
C
C THE LIST OF MAIN VARIABLES:
C   ABSORPTION CROSS SECTION, SQ. CM.
C   S(25) = OF U-235
C   S(26) = OF U-236
C   S(28) = OF U-238
C   S(48) = OF PU-238
C   S(49) = OF PU-239
C   S(40) = OF PU-240
C   S(41) = OF PU-241
C   S(42) = OF PU-242
C   S(37) = OF NP-237
C   S(38) = OF NP-238
C   P1    = FISSION-TO-RESONANCE NONLEAKAGE PROBABILITY.
C   P      = RESONANCE ESCAPE PROBABILITY.
C   PTH    = FISSION-TO-THERMAL NONLEAKAGE PROBABILITY.
C   EPS    = FAST FISSION FACTOR
C   RATIO  OF CAPTURE CROSS SECTION TO FISSION CROSS SECTION.
C   D25    = FOR U-235
C   D48    = FOR PU-238
C   D49    = FOR PU-239
C   FISSION NEUTRONS PRODUCED PER THERMAL NEUTRON ABSORBED IN
C   FISSIONABLE MATERIAL.
C   E25    = FOR U-235
C   E41    = FOR PU-241
C   E49    = FOR PU-239
C   NEUTRONS PRODUCED PER FISSION.
C   V25    = BY U-235
C   V28    = BY U-238
C   V49    = BY PU-239
C   V41    = BY PU-241
C   DC27   = DECAY CONSTANT OF U-237.
C   DC38   = DECAY CONSTANT OF NP-238.
C   WEIGHT OF THE URANIUM ISOTOPES IN THE CORE,KG.
C   AU25   = OF U-235
C   AU26   = OF U-236
C   AU28   = OF U-238
C   PHI    = NEUTRON FLUX,N/SQ. CM.
C   THW    = REACTOR POWER, THERMAL,MWT.
C   BU(I)  = BURNUP OF THE FUEL,MWD/TONNE.
C   Z      = CORE HEIGHT,CM.
C   ZP     = EFFECTIVE CORE HEIGHT,CM.
C   N      = NO. OF SETS OF DATA TO BE COMPUTED.
C *****
C
C DIMENSION S(50),B(50,50),C25T(50),C26T(50),C28T(50),C49T(50),
C 1C40T(50),C41T(50),C42T(50),BA(50,50),BU(50),RATIO(50),C37T(50),

```



```

2TIME(50),FLU(50),DF(300),DG(300),SUM(300),SUM1(300),FLT(10)
C
  READ (5,10) S(25),S(26),S(28),S(49),S(40),S(41),S(42),S(38)
10  FORMAT (PE10.4)
  READ (5,11) S(37),S(48),P1,P,PTH,EPS,D49
11  FORMAT (2E10.4,5F10.0)
  READ (5,12) D25,C48,D41,E25,E41,E49,PHI
12  FORMAT (6F10.0,110.4)
  READ (5,13) V25,V28,V49,V41,DC27,DC38
13  FORMAT (4F10.0,2E10.3)
  READ (5,14) AU25,AU26,AU28,N,THW
14  FORMAT (7F10.0,110,F10.0)
  READ (5,15) (BU(I), I=1,N)
15  FORMAT (10F8.0)
  READ (5,16) Z,ZP
16  FORMAT (2F10.0)
  DO 20 I=1,N
C
C  FLUX TIME CALCULATION.
C
  TIME(I)=PU(I)*(AU25+AU26+AU28)*1.E-03*8.64E+04/THW
  FLU(I)=PHI*1.E-24*TIME(I)
C
C  B(D,I) AND RA(49,I) CALCULATIONS BY APPLYING SIMPSON RULE.
C
  S(27)=DC27*TIME(I)
  A=3.14159/ZP
  DO 40 J=1,25
  D=J+24
    IF (D-27) 50,110,60
60    IF (D-29) 50,40,70
70    IF (D-37) 40,50,80
80    IF (D-39) 130,40,90
90    IF (D-42) 50,50,100
100   IF (D-48) 40,50,50
50    WW=S(D)*FLU(I)*1.E 24
      GO TO 120
130   WW=S(D)*FLU(I)*1.E 24+DC38*TIME(I)
      GO TO 120
110   WW=S(D)
120   DX=7/2.
      F49=E49*EPS*P1*(1.-P)
      AL=1.-F49
      WY=AL*S(49)*FLU(I)*1.E 24
      N1=0
      N1D=Z/2
      N1=N1+N1D
      XM1=FLOAT(N1)
      DX=DX/XN1
      N2=N1+1

```

```

      DO 150 M=1,N2
      ZM=FLNAT(M)
      ZN=(ZM-1)*DB
      DG(M)=EXP(-WY*COS(A*ZN))
      IF (WW.GT.100) GO TO 140
      DF(M)=EXP(-WW*COS(A*ZN))
      GO TO 150
140   DF(M)=0.
150   CONTINUE
      DO 160 K=1,N2
      SUM(K)=DF(1)+DF(N2)
      SUM1(K)=DG(1)+DG(N2)
160   CONTINUE
      DO 170 L=2,N1,2
      SUM(L)=4.*DF(L)
      SUM1(L)=4.*DG(L)
170   CONTINUE
      N3=N1-1
      DO 180 L=3,N3,2
      SUM(L)=2.*DF(L)
      SUM1(L)=2.*DG(L)
180   CONTINUE
      XSUM=0.
      XSUM1=0.
      DO 190 IK=1,N1
      XSUM=XSUM+SUM(IK)
      XSUM1=XSUM1+SUM1(IK)
190   CONTINUE
      XSUM=2.*XSUM*DB/(3.*Z)
      XSUM1=2.*XSUM1*DB/(3.*Z)
      R(D,I)=XSUM
      RA(49,I)=XSUM1
40   CONTINUE
C
C   ISOTROPIC CONCENTRATIONS CALCULATION.
C
      A25=AU25*238./(AU28*235.)
      A26=AU26*238./(AU28*236.)
      A28=1.
C
C   U-235 CONCENTRATION CALCULATION.
C
      C25=A25*P(25,I)
      C25T(I)=C25*AU28*235./238.
C
C   U-236 CONCENTRATION CALCULATION.
C
      C26A=A25*S(25)*D25*(B(26,I)-B(25,I))/(S(25)-S(26))*(1.+D25)
      C26B=A26*B(26,I)
      C26=C26A+C26B

```

C26T(I)=C26\*AU28\*236./238.

C  
C  
C

PU-239 CONCENTRATION CALCULATION.

C1=A2P\*S(28)/(S(49)\*AL)  
 F25=E25\*FPS\*P1\*(1.-P)  
 C2=F25\*A25\*S(25)/(S(49)\*AL-S(25))  
 C49A=C1+C2\*B(25,I)-(C1+C2)\*BA(49,I)  
 PQT=A25\*S(25)\*S(26)\*D25\*PHI/((S(25)-S(26))\*(1.+D25))  
 WA03=DC27\*(PQT+A26\*S(26)\*PHI)/(DC27-S(26)\*PHI)  
 WA04=-PQT\*DC27/(DC27-S(25)\*PHI)  
 WA05=WA03\*S(37)/(S(37)-S(26))  
 WA06=WA04\*S(37)/(S(37)-S(25))  
 WA07=-(WA03+WA04)\*S(37)\*PHI/(S(37)\*PHI-DC27)  
 WA09=WA05\*DC38/(DC38+PHI\*(S(38)-S(26)))  
 WA10=WA06\*DC38/(DC38+PHI\*(S(38)-S(25)))  
 WA11=WA07\*DC38/(DC38+S(38)\*PHI-DC27)  
 WA12=-(WA05+WA06+WA07)\*DC38/(DC38+PHI\*(S(38)-S(37)))  
 WA13=WA09\*S(48)\*C48/((1.+D48)\*(S(48)-S(26)))  
 WA14=WA10\*S(48)\*C48/((1.+D48)\*(S(48)-S(25)))  
 WA15=WA11\*S(48)\*C48\*PHI/((1.+D48)\*(S(48)\*PHI-DC27))  
 WA16=WA12\*S(48)\*C48/((1.+D48)\*(S(48)-S(37)))  
 WARE=(WA09+WA10+WA11+WA12)\*S(48)\*D48\*PHI  
 WINE=(1.+D48)\*(S(48)\*PHI-DC38-S(38)\*PHI)  
 WA17=WARE/WINE  
 WA18=WA17\*(B(26,I)-B(49,I))/(PHI\*(S(49)-S(26)))  
 WA19=WA17\*(B(25,I)-B(49,I))/(PHI\*(S(49)-S(25)))  
 WA20=WA17\*(B(27,I)-B(49,I))/(S(49)\*PHI-DC27)  
 WA21=WA17\*(B(37,I)-B(49,I))/(PHI\*(S(49)-S(37)))  
 WA22=WA17\*(B(38,I)-B(49,I))/(S(49)\*PHI-DC38-S(38)\*PHI)  
 WA23A=-(WA13+WA14+WA15+WA16+WA17)\*(B(48,I)-B(49,I))  
 WA23B=PHI\*(S(49)-S(48))  
 WA23=WA23A/WA23B  
 C49B=WA18+WA19+WA20+WA21+WA22+WA23  
 C49=C49A+C49B  
 C49T(I)=C49\*AU28\*239./238.

C  
C  
C

NP-237 CONCENTRATION CALCULATION.

WA25=WA02\*(B(26,I)-B(37,I))/(S(37)-S(26))\*PHI  
 WA26=WA02\*(B(25,I)-B(37,I))/(S(37)-S(25))\*PHI  
 WA27=-(WA03+WA04)\*(B(27,I)-B(37,I))/(S(37)\*PHI-DC27)  
 C37=WA25+WA26+WA27  
 C37T(I)=C37\*AU28\*237./238.

C  
C  
C

PU-240 CONCENTRATION CALCULATION.

C3=A2R\*S(28)\*D49/(S(40)\*AL\*(1.+D49))  
 C4=C2\*S(49)\*D49/((S(40)-S(25))\*(1.+D49))  
 C5=C3\*S(40)/(S(49)\*AL-S(40))+C4\*(S(40)-S(25))/(S(49)\*AL-S(40))

```
C
C
C
C40=C3+C4*B(25,I)+C5*BA(49,I)-(C3+C4+C5)*B(40,I)
C40T(I)=C40*AU28*240./238.

C
C
C
PU-241 CONCENTRATION CALCULATION.

C6=C3*S(40)/S(41)
C7=C4*S(40)/(S(41)-S(25))
C8=C5*S(40)/(S(41)-S(49)*AL)
C9=(C3+C4+C5)*S(40)/(S(40)-S(41))
C41=C6+C7*B(25,I)+C8*BA(49,I)-(C6+C7+C8+C9)*B(41,I)
C41T(I)=C41*AU28*241./238.

C
C
C
PU-242 CONCENTRATION CALCULATION.

BEN1=C6*.366E-01*FLU(I)*1.E 24+C7*(1.-B(25,I))/S(25)
BEN2=C8*(1.-BA(49,I))/(S(49)*AL)
BEN=BEN1+BEN2
PIG=C9*(1.-B(40,I))/S(40)-(C6+C7+C8+C9)*(1.-B(41,I))/S(41)
C42=D41*C(41)*(BEN+PIG)/(1.+D41)
C42T(I)=C42*AU28*242./238.

C
C
C
U-238 CONCENTRATION CALCULATION.

FP25=A25*(1.-B(25,I))/(1.+D25)
GEN1=C1*FLU(I)*1.E 24+C2*(1.-B(25,I))/S(25)
GEN2=-(C1+C2)*(1.-BA(49,I))/(S(49)*AL)
GEN=GEN1+GEN2
FP49=S(40)*GEN/(1.+D49)
FP41=C42/D41
FP28=(EP*-1.)*(V25*FP25+V49*FP49+V41*FP41)/(V28-1.)
C28=A28-FP28-C49-FP49-C40-C41-C42-FP41
C28T(I)=AU28*C28

C
C
C
FINAL ENRICHMENT RATIO CALCULATION.

RATIO(I)=C25T(I)/(C28T(I)+C26T(I)+C25T(I))
FLT(I)=FLU(I)*1.E 03

CONTINUE
FORMAT ('H1,40X,' THE CONCENTRATION OF ISOTOPES IN THE BURNUP FUEL
1')
WRITE (6,33)
FORMAT ('H0,3X,' FLUX TIME          U235          U238          U236
1      PU'239          PU240          PU241          PU242          NP2
237' /)
WRITE (6,18)
FORMAT ('E14.7)
WRITE (6,17) (FLT(I),C25T(I),C28T(I),C26T(I),C49T(I),C40T(I),
1C41T(I),C42T(I),C37T(I), I=1,N)
32 FORMAT ('H0,40X,' THE FINAL ENRICHMENT RATIO'/)
WRITE (6,32)
```

```
31  FORMAT (10F12.7)
    WRITE (6,31) (RATIO(I), I=1,N)
    STOP
    END
```

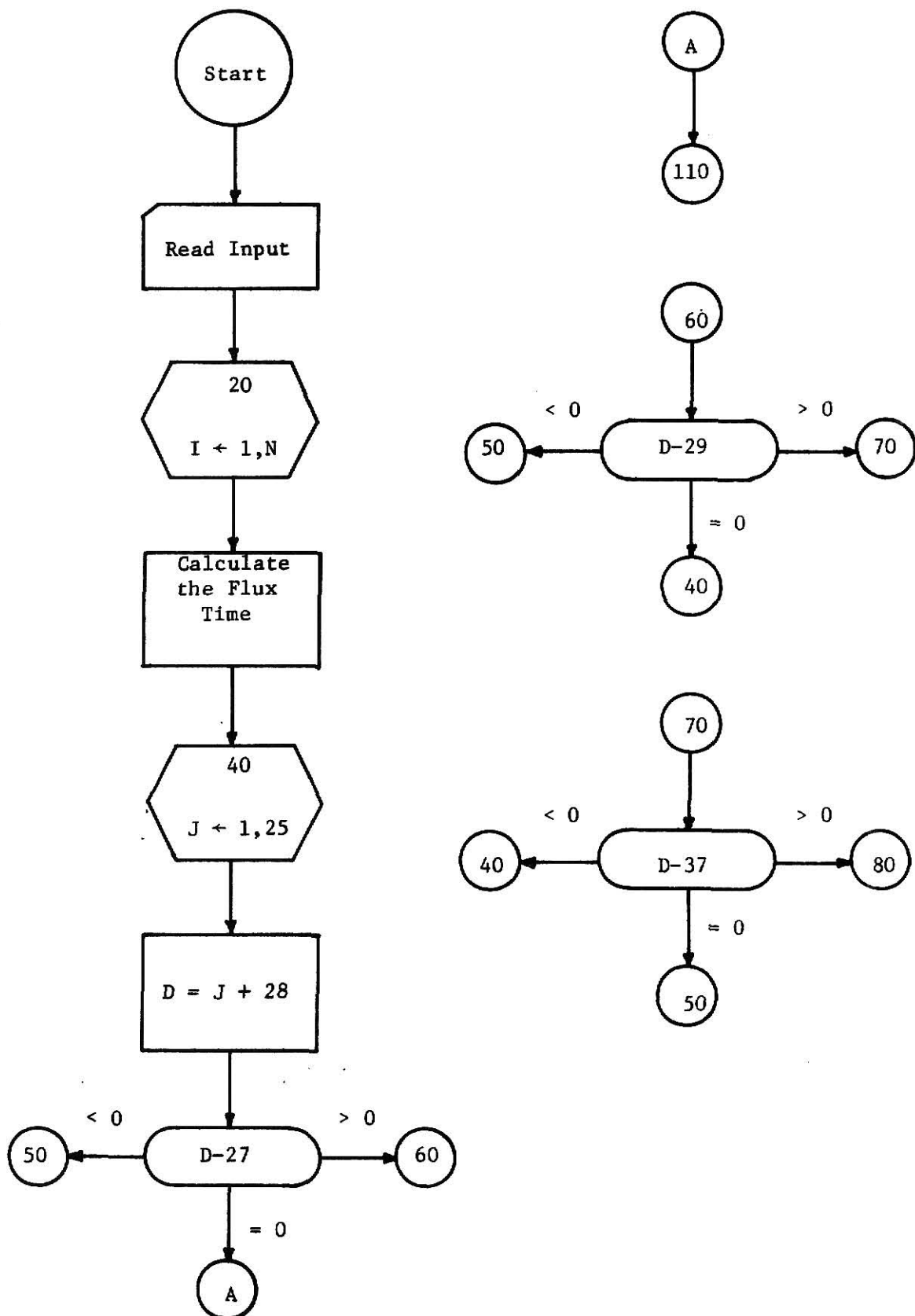


Fig. A-1. "DRAGON" computer program flow sheet.

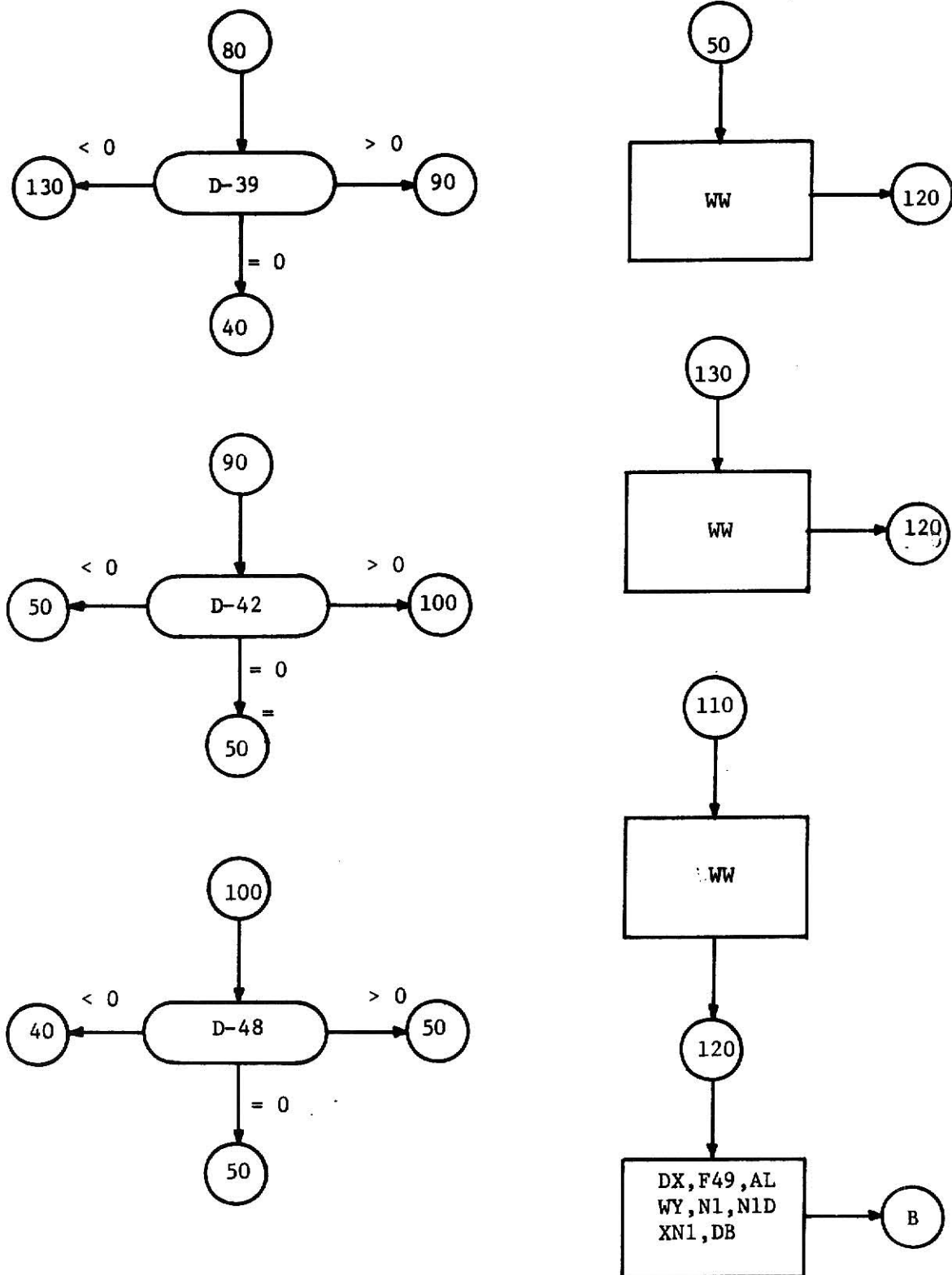


Fig. A-2 (Continued)

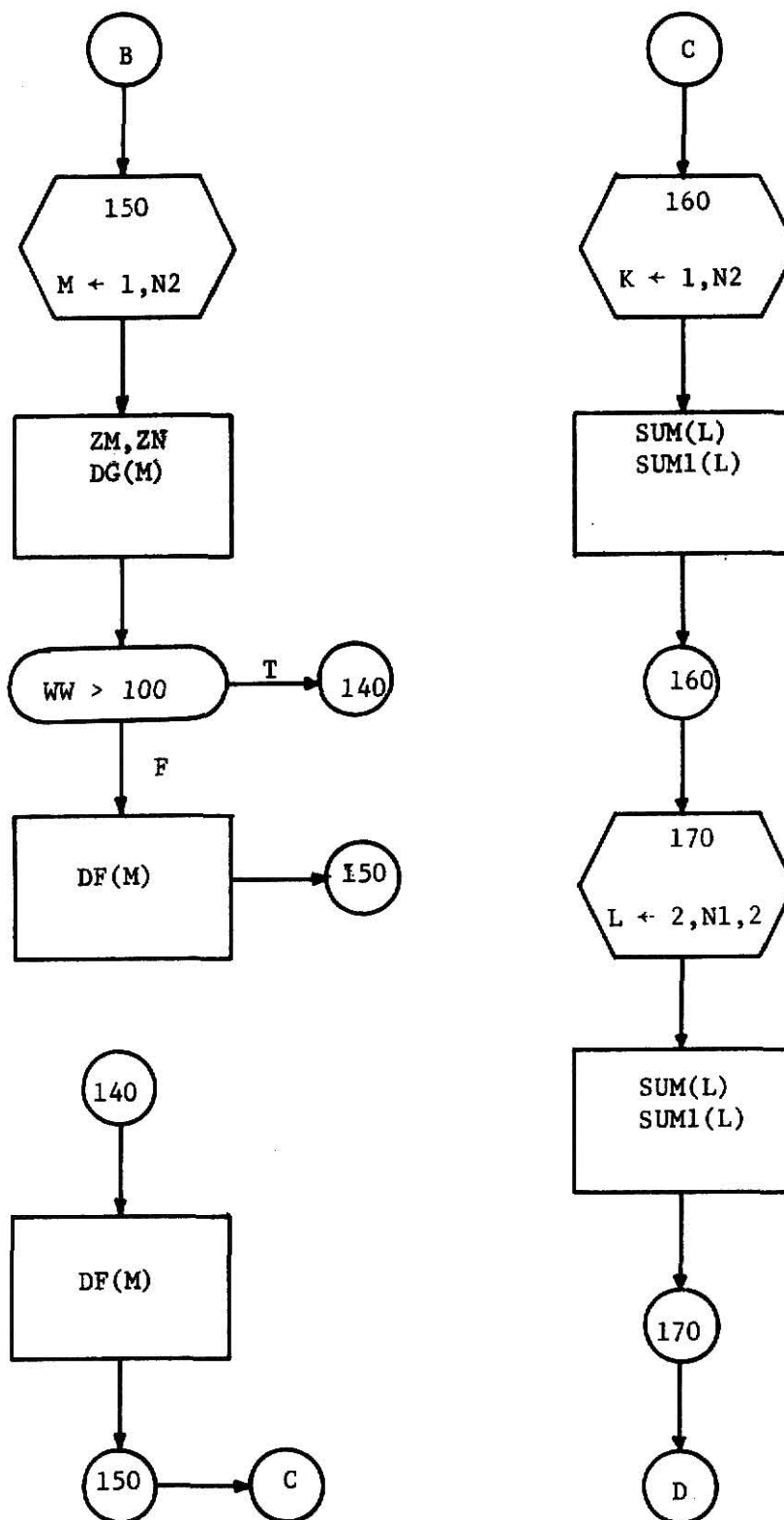


Fig. A-3 (Continued)



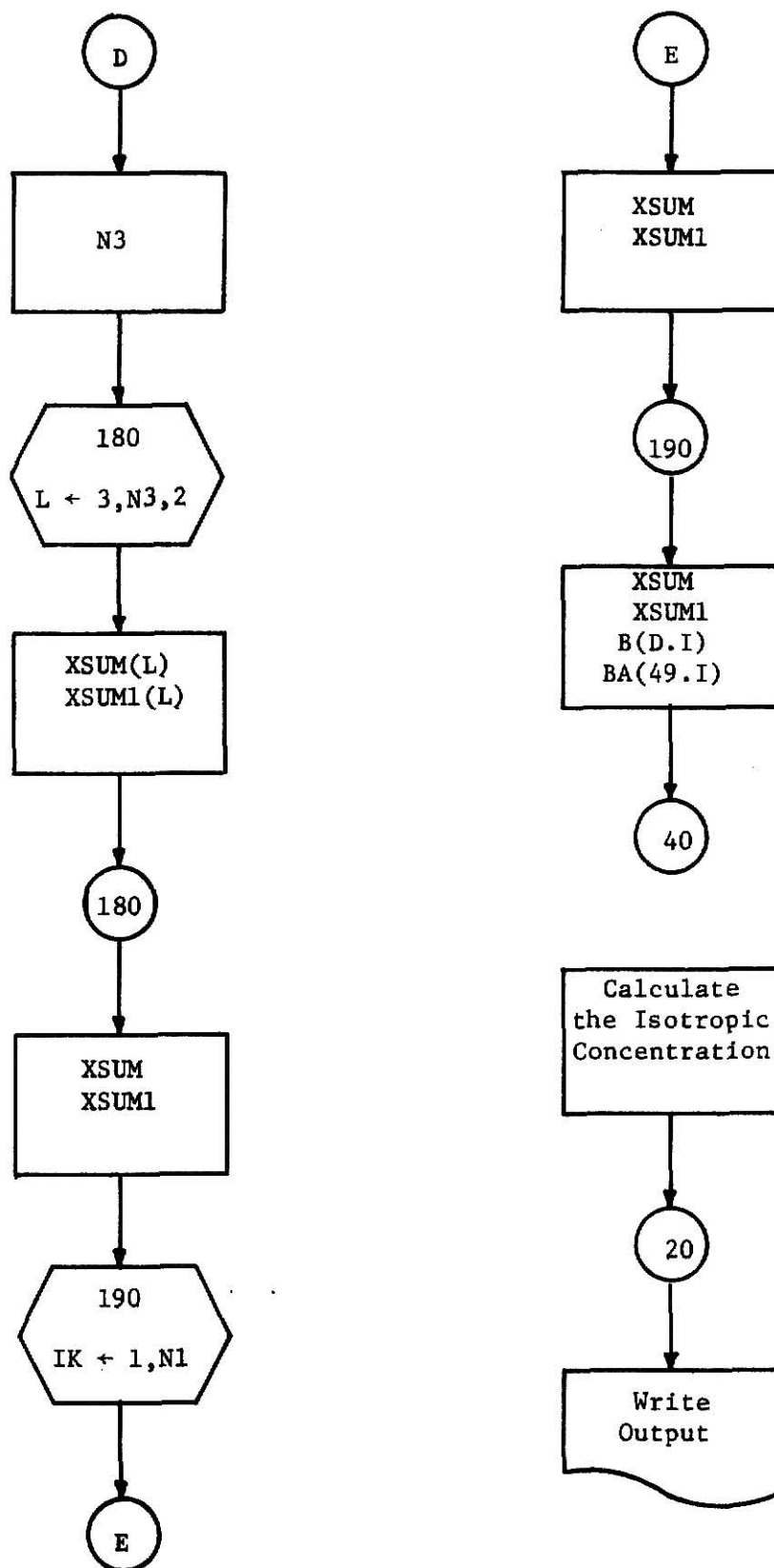


Fig. A-4 (Continued)

Card 6: Format (10F 8.0)

BU(I) - Nuclear fuel burnup in MWD/Tonne

Card 7: Format (2F 10.0)

Z - Core height in cm

ZP - Effective core height in cm

FULCYC - A computer code for computing nuclear fuel cycle costs

This computer program was developed to calculate the unit cost of nuclear fuel, not the total nuclear power cost. Because expenses including plant capital cost, operating cost, and maintenance cost were not taken into consideration, since the fuel cycle cost is the main concern of this research, the discounted cash flow method was used for the calculation of fuel cost. The code, FULCYC, consisted of a main program and a subroutine--BURNUP. The main program read in the reactor core physics and operating data, unit cost of the materials and services required for the nuclear fuel cycle. Then the contribution of the fuel cycle to the cost of electric power was calculated. The subroutine was designed to generate the fuel consumption information and the composition of discharged fuel at some specific burnup. The spent fuel compositions calculated by the subroutine, BURNUP, were fed back to the main program for the credits calculation. Since the nuclear fuel might be privately owned or leased, the fuel cycle might start with natural uranium or recovered spent uranium, the type of reactor might be BWR or PWR. "FULCYC" was tailored to handle all possible nuclear fuel cycles involving these variables.

Comment cards presented in the program aid in understanding the logic. The variables were defined in the FORTRAN listing and in the input data below.

## Input Data

Card 1: Format (8E 10.4)

S(25), S(26), S(28), S(49), S(40), S(41), S(42), S(38) - Thermal neutron absorption cross section of U-235, U-236, U-238, Pu-239, Pu-240, Pu-241, Pu-242 and Np-238 in  $\text{cm}^2$

Card 2: Format (7E 10.4)

S(37), S(48) - Thermal neutron absorption cross section of Np-237 and Pu-238

$P_1$  - Fission-to-resonance nonleakage probability

P - Resonance escape probability

PTH - Resonance-to-thermal nonleakage probability

EPS - Fast Fission Factor

D49 - Ratio of capture cross section to fission cross section of Pu-239

Card 3: Format (6E 10.4)

D25, D48, D41 - Ratio of capture cross section to fission cross section of U-235, Pu-239, and Pu-241

E25, E41, E49 - Fission neutrons produced per thermal neutron absorbed in U-235, Pu-241, and Pu-239

Card 4: Format (6E 10.4)

V25, V28, V49, V41 - Neutrons produced per fission by U-235, U-238, Pu-239, Pu-241 nuclides

DC27, DC38 - Decay constants of U-237 and Np-238 in  $\text{sec}^{-1}$

Card 5: Format (2E 10.4)

Z - Core height in cm

ZP - Effective core height in cm

Card 6: Format (3E 12.6, I8)

AV25, AV26, AV28 - Weight of U-235, U-236, U-238 in the fuel discharged  
from the reactor in kg

N - No. of sets of data to be computed

Card 7: Format (3F 10.0, E 10.2)

AI25, AI28 - Reference design load of U-235 and U-238 in Kg

THW - Reactor power, thermal in MW(t)

PHI - Neutron flux in  $n/cm^2$ -sec

Card 8: Format (8E 10.4)

PE - Plant efficiency

SEPT - Reprocessing plant operation cost in \$/day

THRT - Reprocessing plant throughput in kg/day

UHCON -  $UO_2(NO_3)_2$  to  $UF_6$  conversion charge in \$/day

CP1 - Fuel shipping charge from gaseous diffusion plant to fabrication  
plant in \$/kg U

CP2 - Fuel element shipping charge from fabrication plant to reactor  
site in \$/kg U

CP3 - Spent fuel shipping charge from reactor site to reprocessing plant  
in \$/kg U

CFX - 0.711 weight percent uranium cost in \$/kg

Card 9: Format (8E 10.4)

FABL - Fabrication loss ratio

FEX - Fabrication excess ratio

PRPU - Price of fissile plutonium, \$/gm

RLP - Reprocessing loss ratio of plutonium

FABT - Fabrication charge, \$/kg

XW1 - Optimum tail stream composition

XF2 - Natural uranium feed stream composition

CG - Unit cost of separative duty, \$/kg U separative work

Card 10: Format (10F 8.0)

Duration of Steps in the Fuel Cycle in Month

TM1 - Duration of  $UF_6$  conversion

TM2 - Duration of enrichment service

TM3 - Duration of transportation from gaseous diffusion plant to fuel  
fabrication plant

TM4 - Duration of fuel fabrication

TM5 - Duration of transportation from fabrication plant to reactor site

TM6 - Duration of fuel in a PWR

TM7 - Duration of spent fuel cooling

TM8 - Duration of spent fuel transportation from reactor site to fuel  
reprocessing plant

TM9 - Duration of spent fuel reprocessing

TM10 - Duration of re-enriching the recovered spent fuel

Card 11: Format (F 8.0)

TM11 - Duration of fuel in a BWR in month

Card 12: Format (10F 8.0)

BU(I) - Fuel burnups in MWD/Tonne

Card 13: Format (6E 10.4)

REPLU - Reprocessing loss ratio of uranium

CONL - Conversion loss ratio

RATE - Capital interest rate  
XP - Reference design enrichment ratio  
DEPR - Ratio of depreciation on the spent fuel  
CNP - Pu nitrates to Pu metal conversion loss ratio

Card 14: Format (F 4.0, 2I 4)

AM - 0, for spent uranium recycle cases  
- 1, for natural uranium no recycle cases  
LM - 1, for PWR  
- 2, for BWR  
NP - 0, AEC leased fuel  
- 1, privately owned fuel

Card 15: Format (3E 10.4)

UNP - Unit cost of Np-237 in \$/gm  
PL - Reprocessing loss ratio of Np-237  
CONLP - Pu conversion loss ratio

# **ILLEGIBLE DOCUMENT**

**THE FOLLOWING  
DOCUMENT(S) IS OF  
POOR LEGIBILITY IN  
THE ORIGINAL**

**THIS IS THE BEST  
COPY AVAILABLE**

```

C *****
C
C THE LIST OF MAIN VARIABLES:
C   AI25 = WEIGHT OF REFERENCE DESIGN REACTOR U-235 CONTENT, KG.
C   AI28 = WEIGHT OF REFERENCE DESIGN REACTOR U-238 CONTENT, KG.
C   AM   = 0, FOR RECYCLE CASES.
C         = 1, FOR FRESH FUEL CASES.
C   WEIGHT OF THE URANIUM ISOTOPES IN THE CORE.
C   AU25 = OF U-235
C   AU26 = OF U-236
C   AU28 = OF U-238
C   AV25 = WEIGHT OF U-235 IN THE SPENT FUEL WHEN FRESH FUEL IS
C         USED, KG.
C   AV26 = WEIGHT OF U-236 IN THE SPENT FUEL WHEN FRESH FUEL IS
C         USED, KG.
C   AV28 = WEIGHT OF U-238 IN THE SPENT FUEL WHEN FRESH FUEL IS
C         USED, KG.
C   BU(I) = BURNUP OF THE FUEL.
C   CFX   = 0.711 WEIGHT PERCENT URANIUM COST, $/KG.
C   CG    = UNIT COST OF SEPARATIVE DUTY, $/KG U SEP. WORK.
C   CNP   = CONVERSION COST OF PU METAL FROM ONE OF ITS NITRATES,
C         $/G OF PU.
C   CONL  = CONVERSION LOSS RATIO.
C   CONLP = CONVERSION LOSS OF PU.
C   SHIPPING OR TRANSPORTATION CHARGES, $/KG.
C   CP1   = FROM ENRICHMENT PLANT TO FABRICATION PLANT.
C   CP2   = FROM FABRICATION PLANT TO REACTOR.
C   CP3   = FROM REACTOR TO REPROCESSING PLANT.
C   RATIO OF CAPTURE CROSS SECTION TO FISSION CROSS SECTION.
C   D25   = FOR U-235
C   D48   = FOR PU-238
C   D49   = FOR PU-239
C   DC27  = DECAY CONSTANT OF U-237.
C   DC38  = DECAY CONSTANT OF NP-238.
C   DEPR  = RATIO OF DEPRECIATION ON THE REPROCESSED SPENT FUEL.
C   FISSION NEUTRONS PRODUCED PER THERMAL NEUTRON ADSORBED IN
C   FISSIONABLE MATERIAL.
C   F25   = FOR U-235
C   E41   = FOR PU-241
C   E49   = FOR PU-239
C   EPS   = FAST FISSION FACTOR
C   FABL  = FABRICATION LOSS RATIO.
C   FABT  = FABRICATION CHARGE, $/KG.
C   FEX   = FABRICATION EXCESS RATIO.
C   LM    = 1 FOR PWR.
C         = 2 FOR BWR.
C   N     = NO. OF SETS OF DATA TO BE COMPUTED.
C   NP    = 0 AEC-LEASED FUEL.
C         = 1 PRIVATE OWNED FUEL.
C   P     = RESONANCE ESCAPE PROBABILITY.

```



C P1 = FISSION-TO-RESONANCE NONLEAKAGE PROBABILITY.  
 C P2 = WEIGHT OF FRESH ENRICHED URANIUM TO BE MIXED WITH  
 C REPROCESSED FUEL, KG.  
 C PE = PLANT EFFICIENCY.  
 C PHF = NEUTRON FLUX, N/SQ. CM.  
 C PL = REPROCESSING LOSS OF NP-237.  
 C PRPU = PRICE OF PLUTONIUM, \$/GM.  
 C PTH = FISSION-TO-THERMAL NONLEAKAGE PROBABILITY.  
 C RATE = CAPITAL INTEREST.  
 C REPLU = REPROCESSING LOSS RATIO OF URANIUM.  
 C RLP = REPROCESSING LOSS OF PLUTONIUM.  
 C SEPT = REPROCESSING PLANT OPERATING COST, \$/DAY.  
 C ABSORPTION CROSS SECTION, SQ. CM.  
 C S(25) = OF U-235  
 C S(26) = OF U-236  
 C S(28) = OF U-238  
 C S(48) = OF PU-238  
 C S(49) = OF PU-239  
 C S(40) = OF PU-240  
 C S(41) = OF PU-241  
 C S(42) = OF PU-242  
 C S(37) = OF NP-237  
 C S(38) = OF NP-238  
 C THRT = REPROCESSING PLANT THROUGHPUT, KG/DAY.  
 C THW = REACTOR POWER, THERMAL.  
 C DURATION OF STEPS IN THE FUEL CYCLE, MONTH.  
 C TM1 = DURATION OF CONVERSION.  
 C TM2 = DURATION OF ENRICHMENT.  
 C TM3 = DURATION OF TRANSPORTATION FROM ENRICHMENT PLANT TO  
 C FABRICATION PLANT.  
 C TM4 = DURATION OF FABRICATION.  
 C TM5 = DURATION OF TRANSPORTATION FROM FABRICATION TO REACTOR.  
 C TM6 = DURATION OF IRRADIATION OF A PWR.  
 C TM7 = DURATION OF COOLING.  
 C TM8 = DURATION OF TRANSPORTATION FROM REACTOR TO FUEL  
 C REPROCESSING PLANT.  
 C TM9 = DURATION OF REPROCESSING.  
 C TM10 = DURATION OF ENRICHMENT FOR REENRICHING THE REPROCESSED FUEL.  
 C TM11 = DURATION OF IRRADIATION OF A BWR.  
 C UHCON = UF6 CONVERSION CHARGE FROM UO2(NO3)2, \$/KG.  
 C UNP = UNIT COST OF NP-237, \$/G.  
 C NEUTRONS PRODUCED PER FISSION.  
 C V25 = BY U-235  
 C V28 = BY U-238  
 C V49 = BY PU-239  
 C V41 = BY PU-241  
 C XF2 = 0.711 WEIGHT PERCENT FEED STREAM COMPOSITION.  
 C XP = ENRICHMENT RATIO OF REFERENCE DESIGN REACTOR.  
 C XW1 = OPTIMUM TAIL STREAM COMPOSITION.  
 C Z = CORE HEIGHT.

```

C      7P      = EFFECTIVE CORE HEIGHT.
C
C      *****
COMMON AU25(2),AU26(2),AU28(2),C25T(10,2),C28T(10,2),C26T(10,2),
1C49T(10,2),C40T(10,2),C41T(10,2),C42T(10,2),FLT(10),BU(10),N,
2P1,P,PTH,EPS,D49,D25,D48,E25,E41,E49,PHI,V25,V28,V49,V41,DC27,
3DC38,THW,Z,ZP,S(50),AI,D41,P6,AM,AV25,AV26,AV28,AI25,AI28,FR,
4C37T(10,2),CONL
  DIMENSION PUC(10,2),UCRT(10,2),XP3(2),SPR3(2),DEX3(2),
1PPUT(10,2),PURT(10,2),CS(10,2),PR(10,2),RR(10,2),FCC(10,2),TEX(2),
2XP4(2,2),DEX4(2,2),TEY(2,2),PC(10,2),P4(2,2),PPC(10,2),
3CON(10,2),PUCON(10,2),PCON(10,2),PPU(10,2)
  READ (5,10) S(25),S(26),S(28),S(49),S(40),S(41),S(42),S(38)
10  FORMAT (F10.4)
  WRITE (6,10) S(25),S(26),S(28),S(49),S(40),S(41),S(42),S(38)
  READ (5,11) S(37),S(48),P1,P,PTH,EPS,D49
11  FORMAT (7E10.4)
  WRITE (6,11) S(37),S(48),P1,P,PTH,EPS,D49
  READ (5,12) D25,D48,D41,E25,E41,E49
12  FORMAT (F10.4)
  WRITE (6,12) D25,D48,D41,E25,E41,E49
  READ (5,13) V25,V28,V49,V41,DC27,DC38
13  FORMAT (F10.4)
  WRITE (6,13) V25,V28,V49,V41,DC27,DC38
  READ (5,16) Z,ZP
16  FORMAT (2E10.4)
  WRITE (6,16) Z,ZP
  READ (5,61) AV25,AV26,AV28,N
61  FORMAT (2E12.6,I8)
  WRITE (6,61) AV25,AV26,AV28,N
  READ (5,81) AI25,AI28,THW,PHI
81  FORMAT (2F10.0,E10.2)
  WRITE (6,81) AI25,AI28,THW,PHI
  READ (5,64) PE,SEPT,THRT,UHCON,CP1,CP2,CP3,CFX
64  FORMAT (F10.4)
  WRITE (6,64) PE,SEPT,THRT,UHCON,CP1,CP2,CP3,CFX
  READ (5,64) FABL,FEX,PRPU,RLP,FABT,XW1,XF2,CG
  WRITE (6,64) FABL,FEX,PRPU,RLP,FABT,XW1,XF2,CG
  READ (5,66) TM1,TM2,TM3,TM4,TM5,TM6,TM7,TM8,TM9,TM10
66  FORMAT (10F8.0)
  WRITE (6,66) TM1,TM2,TM3,TM4,TM5,TM6,TM7,TM8,TM9,TM10
  READ (5,65) TM11
65  FORMAT (F8.0)
  WRITE (6,65) TM11
  READ (5,68) (BU(I), I=1,N)
68  FORMAT (10F8.0)
  WRITE (6,68) (BU(I), I=1,N)
  READ (5,12) REPLU,CONL,RATE,XP,DEPR,CNP
  WRITE (6,12) REPLU,CONL,RATE,XP,DEPR,CNP
  READ (5,62) AM,LM,NP

```

```

62  FORMAT (F4.0,2I4)
    WRITE (6,62) AM,LM,NP
    READ (5,F3) UNP,PL,CONLP
83  FORMAT (2E10.4)
    WRITE (6,83) UNP,PL,CONLP
    DO 300 I=1,N
    DO 300 J=1,2

C
C    THE CALCULATION OF POWER SOLD PER KG URANIUM INPUT.
C
    POWER=PE*BU(I)*24.

C
C    CALCULATION OF SHIPPING COST AND SPENT FUEL TRANSPORTATION,
C    $/KG U .
C
    FR=FABL+FEX
    PQ=1.+FB
    CP=CP1*(1.+FB)

C
C    CALCULATION OF FABRICATION CHARGE, $/KG U.
C
    FAR=FABT*1.

C
C    CALCULATION OF REENRICHMENT COST OF REPROCESSED FUEL, $/KG U.
C    FIRST, CALCULATE THE EXTRA SEPARATIVE WORK CONTRIBUTED BY THE
C    PRESENCE OF U-236.
C
    AV=AV25+AV26+AV28
    IF (LM.EC.1) GO TO 220
    FR=1./4.
    GO TO 230
220  FR=1./3.
230  AI=(AI25+AI26)*FR
    R=AV*(1.-CONL)/AI
    R6=R*AV26/AV
    XF1=AV25/AV
    WL=XF1**(-1./3.)*XP**(-1./3.)
    WP=XW1**(-1./3.)*XP**(-1./3.)
    W6=R6*WL/WP
    P6=R6-W6
    SP2=4.*W6*ALOG(XW1/XF1)+4.*P6*ALOG(XP/XF1)
    CALL SUBPUP

C
C    CALCULATION OF NP-237 CREDIT.
C
    PC(I,J)=UNP*C37T(I,J)*1000.*(1.-PL)/AI
    IF (AM) 200,310,330

C
C    THEN, ASSUME THERE ARE NO U-236 IN THE FEED STREAM AND CALCULATE
C    THE SEPARATIVE WORK REQUIRED.

```

```

C
310  XP1=AU25(1)/AI
      F1=(PQ*(XP1-XW1)-R*(XF1-XW1))/(XF2-XW1)
      W1=F1+R-PQ
      VXP1=(2.*XP1-1.)*ALOG(XP1/(1.-XP1))
      VXF1=(2.*XF1-1.)*ALOG(XF1/(1.-XF1))
      VXW1=(2.*XW1-1.)*ALOG(XW1/(1.-XW1))
      VXF2=(2.*XF2-1.)*ALOG(XF2/(1.-XF2))
      SP1=W1*VXW1+PQ*VXP1-F1*VXF2-R*VXF1
      SPR1=SP1+SP2
      DEX1=CG*SPR1
      FCT1=CFX*F1
      TE1=DEX1+FCT1

C
C    CALCULATION OF ENRICHMENT COST BY BLENDING REPROCESSED FUEL
C    WITH FRESH FUEL.
C
      P2=PQ-AV*(1.-CONL)/AI
      XP2=PQ*AU25(2)/(AI*P2)-R*XF1/P2
      F2=P2*(XP2-XW1)/(XF2-XW1)
      W2=F2-P2
      VXP2=(2.*XP2-1.)*ALOG(XP2/(1.-XP2))
      VXW2=VXW1
      SPR2=W2*VXW2+P2*VXP2-F2*VXF2
      DEX2=CG*SPR2
      FCT2=CFX*F2
      TE2=DEX2+FCT2

C
C    CALCULATION OF URANIUM CREDIT, $/KG U.
C
330  UOT=C25T(I,J)+C26T(I,J)+C28T(I,J)
      UIN=AI
      XP3(1)=C25T(I,J)/UOT
      XP3(2)=AV25/AV
      P3=1.
      DO 320 K=1,2
        F3=(XP3(K)-XW1)/(XF2-XW1)
        W3=F3-1.
        VXP3=(2.*XP3(K)-1.)*ALOG(XP3(K)/(1.-XP3(K)))
        VXF3=(2.*XF2-1.)*ALOG(XF2/(1.-XF2))
        VXW3=(2.*XW1-1.)*ALOG(XW1/(1.-XW1))
        SPR3(K)=W3*VXW3+P3*VXP3-F3*VXF3
        DEX3(K)=CG*SPR3(K)
        FCT3=CFX*F3
        TE3(K)=DEX3(K)+FCT3
320  CONTINUE

C
C    CALCULATION OF UC2(NO3)2 TO UF6 CONVERSION COST.
C
      CON(I,J)=UOT*UHCCN*(1.-REPLU)/UIN

```

```

C
C   CALCULATION OF PU METAL CONVERSION COST FROM ONE OF ITS NITRATES.
C
SST=C49T(I,J)+C40T(I,J)+C41T(I,J)+C42T(I,J)
PUCON(I,J)=SST*CNP*(1.-RLP)/UIN
C
C   CALCULATION OF REPROCESSING CHARGES,$/KG U AS 1.KG U INPUT BASED
C   ON NFS PLANT MODEL.
C
RCT=SEPT*4./{THRT*3.}
C
C   CALCULATION OF PLUTONIUM CREDITS,$/KG U.
C
CPU=C41T(I,J)+C49T(I,J)
PUT=CPU*(1.-RLP)*(1.-CONLP)/UIN
PUC(I,J)=PRPU*PUT*1000.
C
C   CALCULATION OF URANIUM CREDIT IN THE SPENT FUEL.
C
UCRT(I,J)=UOT*(1.-REPLU)*(1.-CONL)*TEX(1)/UIN
C
C   COST OF REPROCESSED FUEL NEEDED FOR FABRICATION.
C
UF=TEX(2)*AV*(1.-CONL)/AI
IF (NP.EC.0) GO TO 336
C
C   CALCULATION OF U-236 PENALTY IF THE FUEL IS NOT LEASED FROM AEC,
C   IN OTHER WORDS, THE PRIVATE OWNERSHIP IS THE CASE.
C
DO 335 M=1,2
DO 335 K=1,2
XP4(1,1)=(AV25+AV26)/AV
XP4(1,2)=XF1
XP4(2,1)=(C25T(I,J)+C26T(I,J))/UOT
XP4(2,2)=C25T(I,J)/UOT
P4(1,K)=P
IF (M.EQ.1) GO TO 334
P4(2,K)=UOT*(1.-REPLU)/UIN
334 F4=(XP4(M,K)-XW1)/(XF2-XW1)
W4=F4-P4(M,K)
VXP4=(2.*XP4(M,K)-1.)*ALOG(XP4(M,K)/(1.-XP4(M,K)))
VXF4=VXF2
VXW4=VXW3
SPR4=W4*VXW4+P4(M,K)*VXP4-F4*VXF4
DEX4(M,K)=CG*SPR4
FCT4=CFX*F4
TEY(M,K)=DEX4(M,K)+FCT4
335 CONTINUE
TS4=TEY(1,1)-TEY(1,2)
TS6=TEY(2,1)-TEY(2,2)

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      GO TO 337
336  T14=0.
      T15=0.
      IF (AM.C7.0) GO TO 340
C
C  DEPRECIATION EXPENSE INCURRED TO REENRICHENET FUEL TO THE XLWA
C  TIME REQUIRED IN GASLUS DIFFUSION PLANT.
C
337  DEP=DEPR*UF/4.
      IF (AM.CC.0) GO TO 360
C
C  THE CALCULATION OF FUEL COST IF CORE IS LOADED WITH ENRICH FUEL.
C
340  F5=PQ*(XF-XW1)/(XF2-XW1)
      V5=F5-PQ
      VXF5=(2.*XF2-1.)*ALOG(XF2/(1.-XF2))
      VXP5=(2.*XP-1.)*ALOG(XP/(1.-XP))
      SP5=V5*VXW3+PQ*VXP5-F5*VXF5
      DFX5=CG*SP5
      FCT5=CFX*F5
      T5=DFX5+FCT5
C
C  THE CALCULATION OF THE PRESENT WORTH OF COSTS.
C
360  PT=1.+FATE/12.
      T1=-T11
      IF (LM.CC.2) TM6=TM11
      IF (AM.CC.1) GO TO 361
      IF (J.CC.1) TM2=TM10
361  T2=-(TM1+TM2)
      T3=-(TM1+TM2+TM3)
      T4=-(TM1+TM2+TM3+TM4)
      T5=-(TM1+TM2+TM3+TM4+TM5)
      T6=-(TM1+TM2+TM3+TM4+TM5+TM6)
      T7=-(TM1+TM2+TM3+TM4+TM5+TM6+TM7)
      T8=-(TM1+TM2+TM3+TM4+TM5+TM6+TM7+TM8)
      T9=-(TM1+TM2+TM3+TM4+TM5+TM6+TM7+TM8+TM9)
      IF (AM.CC.1) GO TO 365
      PUF=UF*PT**0.
      PCP=PCP*PT**12
      PT1=T1*PT**T2
      PT2=T2*PT**T2
      PR(I,1)=(T11+UF-T14)*FEX*PT**T3
      PR(I,2)=(T22+UF-T14)*FEX*PT**T3
      IF (AM.CC.0) GO TO 366
365  PT15=T15*PT**T2
366  PCP=CP*PT**T3
      PCON(I,J)=CON(I,J)*PT**T9
      PPO(I,J)=PPO(I,J)*PT**T9
      PT14=T14*PT**0.

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```

PTF6=TE6*PT**T9
PFAB=FA1*PT**T3
PCP2=CP2*PT**T5
PCP3=CP3*PT**T8
PACT=ACT*PT**T9
PPC(I,J)=PC(I,J)*PT**T9
PPUT(I,J)=PUC(I,J)*PT**T9
PUCT(I,J)=UCT(I,J)*PT**T9

C
C   CALCULATION OF FUEL CYCLE COST, MILLS/KW HR.
C
IF (AM) 200,280,370
350 CS1=PFAB+PCP+PFAB+PCP2+PCP3+PACT-PPUT(I,1)+PCON(I,1)
   CS2=-PUCT(I,1)+PUF-PRI(I,1)+PEP-PT14-PPC(I,1)+PT16+PPU(I,1)
   CS(I,1)=CS1+CS2
   IF (J.EQ.1) GO TO 350
   CS3=PT14+PCP+PFAB+PCP2+PCP3+PACT-PPUT(I,2)+PCON(I,2)
   CS4=-PUCT(I,2)+PUF-PRI(I,2)-PT14-PPC(I,2)+PT16+PPU(I,2)
   CS(I,2)=CS3+CS4
   GO TO 350
370 PFA=TE5*EX*PT**T5
   CS(I,J)=PT15+PCP+PFAB+PCP2+PCP3+PACT-PPUT(I,J)-PUCT(I,J)
   1-PPC(I,J)+PT16+PCON(I,J)+PPU(I,J)
380 L(I,J)=CS(I,J)*(1.-1./(1.+PT))**TH6/(PT**T5-PT**T6)
   FCC(I,J)=KR(I,J)*1000./POWER
390 CONTINUE
   IF (AM.EQ.1.) GO TO 495
490 FORMAT ('H1,44X,' TABLE OF PRESENT WORTH COSTS')
492 FORMAT ('H0,' TURNUP CONVERSION DURATION CREDIT PLUPTON CRED
1IT APPROPRIATED FUEL REFRESHMENT FRESH FUEL TOTAL COST')
   WRITE (6,490)
   WRITE (6,492)
   DO 498 I=1,N
   WRITE (6,496) BU(I),PCON(I,1),PUCT(I,1),PPUT(I,1),PUF,PT11,CS(I,1)
496 FORMAT ('8.0,F12.5,F16.5,F17.5,F17.5,F17.5,F23.5')
498 CONTINUE
   DO 494 I=1,N
   WRITE (6,497) BU(I),PCON(I,2),PUCT(I,2),PPUT(I,2),PUF,PT12,CS(I,2)
497 FORMAT ('8.0,F12.5,F16.5,2F17.5,17X,F14.5,F14.5')
494 CONTINUE
31 FORMAT ('H1,40X,' THE CONCENTRATION OF ISOTOPES IN THE BURNUP FUEL
1')
495 WRITE (6,33)
18 FORMAT ('H0,5X,' FLUX TIME          U235          U238          U236
1          PU239          PU240          PU241          PU242          PU243
137' /)
   WRITE (6,18)
17 FORMAT ('8.14.7')
   DO 500 I=1,N
   WRITE (6,17) FL(I),COST(I,1),UCT(I,1),C26T(I,1),C49T(I,1),

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1C40T(I,1),C41T(I,1),C42T(I,1),C37T(I,1)
500 CONTINUE
    IF (AM.EC.1.) GO TO 410
    DO 501 I=1,N
    WRITE (6,17) FLT(I),C25T(I,2),C28T(I,2),C26T(I,2),C49T(I,2),
1C40T(I,2),C41T(I,2),C42T(I,2),C37T(I,2)
501 CONTINUE
410 WRITE (6,71)
71 FORMAT (1H1,40X,' THE FUEL CYCLE COST'/)
    IF (AM.EC.1.) GO TO 420
    WRITE (6,72)
72 FORMAT (F4X,' FUEL CYCLE COST, BY WAY OF')
    IF (AM.EC.0.) GO TO 440
420 WRITE (6,91)
91 FORMAT (28X,' BURNUP          FLUX TIME          FUEL CYCLE COST')
    IF (AM.EC.1.) GO TO 460
440 WRITE (6,73)
73 FORMAT (28X,' BURNUP          FLUX TIME          REENRICHMENT    BLENDING')
460 WRITE (6,74)
74 FORMAT (28X,' (MWD/TONNE)    (N/KB)          (MILLS/KW HR)')
    IF (AM.EC.1.) GO TO 480
    DO 502 I=1,N
470 WRITE (6,75) BU(I),FLT(I),FCC(I,1),FCC(I,2)
75 FORMAT (29X,F6.0,E16.7,E16.7,E15.7)
502 CONTINUE
    IF (AM.EC.0.) GO TO 505
480 DO 503 I=1,N
    WRITE (6,92) BU(I),FLT(I),FCC(I,1)
92 FORMAT (29X,F6.0,E16.7,E22.7)
503 CONTINUE
505 STOP
    END
    SUBROUTINE BURNUP
C
C   THIS PROGRAM IS TO CALCULATE THE COMPOSITION IN UNMIXED FUEL
C   DISCHARGED AFTER GRADED IRRADIATION TO A SPECIFIC FLUX TIME.
C
    COMMON AU25(2),AU26(2),AU28(2),C25T(10,2),C28T(10,2),C26T(10,2),
1C49T(10,2),C40T(10,2),C41T(10,2),C42T(10,2),FLT(10),BU(10),N,
2P1,P,PTH,EPS,D49,D25,D48,E25,E41,E49,PHI,V25,V28,V49,V41,DC27,
3DC38,THW,Z,ZP,S(50),AI,D41,P6,AN,AV25,AV26,AV28,AI25,AI28,FR,
4C37T(10,2),CONL
    DIMENSION B(50,10),BA(50,10),TIME(10),FLU(10),DF(300),DG(300),
1SUM(300),SUM1(300),EU(2)
    DO 20 I=1,N
C
C   FLUX TIME CALCULATION.
C
    TIME(I)=PU(I)*AI*1.E-03*8.64E 04/(THW*FR)
    FLU(I)=P/I*1.E-24*TIME(I)

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S(27)=DC27*TIME(I)
A=3.14159/ZP
DO 40 J=1,25
C
C      B(D,I) AND BA(49,I) CALCULATIONS BY APPLYING SIMPSON RULE.
C
      D=J+24
      IF (D-27) 50,110,60
60      IF (D-29) 50,40,70
70      IF (D-37) 40,50,80
80      IF (D-39) 130,40,90
90      IF (D-42) 50,50,100
100     IF (D-48) 40,50,50
50      WW=S(D)*FLU(I)*1.E 24
      GO TO 120
130     WW=S(D)*FLU(I)*1.E 24+DC38*TIME(I)
      GO TO 120
110     WW=S(D)
120     DX=7/2.
      F49=349*EPS*P1*(1.-P)
      AL=1.-F49
      WY=AL*S(49)*FLU(I)*1.E 24
      N1=0
      N1D=7/2
      N1=N1+N1D
      XN1=FLOAT(N1)
      DX=DX/XN1
      N2=N1+1
      DO 150 M=1,N2
      ZM=FLOAT(M)
      ZN=(ZM-1)*DB
      IF (D-49) 122,121,150
121     DG(M)=EXP(-WY*COS(A*ZN))
122     IF (WW.GT.100) GO TO 140
      DF(M)=EXP(-WW*COS(A*ZN))
      GO TO 150
140     DF(M)=0.
150     CONTINUE
      DO 160 K=1,N2
      IF (D-49) 161,162,160
162     SUM1(K)=FG(1)+DG(N2)
161     SUM(K)=DF(1)+DF(N2)
160     CONTINUE
      DO 170 L=2,N1,2
      IF (D-49) 171,172,170
172     SUM1(L)=4.*DG(L)
171     SUM(L)=4.*DF(L)
170     CONTINUE
      N3=N1-1
      DO 180 L=3,N3,2

```

```

      IF (D-49) 181,182,180
182  SUM1(L)=2.*DG(L)
181  SUM(L)=2.*DF(L)
180  CONTINUE
      XSUM=0.
      XSUM1=0.
      DO 190 IK=1,N1
      IF (D-49) 191,192,190
192  XSUM1=XSUM1+SUM1(IK)
191  XSUM=XSUM+SUM(IK)
190  CONTINUE
      XSUM=2.*XSUM*DB/(3.*Z)
      XSUM1=2.*XSUM1*DB/(3.*Z)
      F(D,I)=XSUM
      BA(49,I)=XSUM1
40  CONTINUE
C
C  RECYCLE FUEL INITIAL CONCENTRATIONS CALCULATION.
C
      DO 20 J=1,2
      IF (AM) 20,21,22
21  AU26(1)=P6*AI
      AU26(2)=AV26*(1.-CONL)
      EU1=AU26(J)*(S(28)+S(26))*235.
      EU2=(S(27)*(E25*PTH*P-1.))+S(28))*236.
      FU(J)=EU1/EU2
      AU25(J)=FU(J)+AI25*FR
      AU28(J)=AI-AU26(J)-AU25(J)
      GO TO 210
22  AU26(J)=0.
      AU25(J)=AI25*FR
      AU28(J)=AI28*FR
C
C  ISOTROPIC CONCENTRATIONS CALCULATION.
C
210  A25=AU25(J)*238./(AU28(J)*235.)
      A26=AU26(J)*238./(AU28(J)*236.)
      A28=1.
C
C  U-235 CONCENTRATION CALCULATION.
C
      C25=A25*P(25,I)
      C25T(I,J)=C25*AU28(J)*235./238.
C
C  U-236 CONCENTRATION CALCULATION.
C
      C26A=A25*S(25)*D25*(B(26,I)-B(25,I))/(S(25)-S(26))*(1.+D25)
      C26B=A26*B(26,I)
      C26=C26A+C26B
      C26T(I,J)=C26*AU28(J)*236./238.

```

C  
C  
C

## PU-239 CONCENTRATION CALCULATION.

$C1 = A28 * S(28) / (S(49) * AL)$   
 $F25 = F25 * FPS * P1 * (1. - P)$   
 $C2 = F25 * A25 * S(25) / (S(49) * AL - S(25))$   
 $C49A = C1 + C2 * B(25, I) - (C1 + C2) * BA(49, I)$   
 $PNT = A25 * S(25) * S(26) * D25 * PHI / ((S(25) - S(26)) * (1. + D25))$   
 $WA03 = DC27 * (PNT + A26 * S(26) * PHI) / (DC27 - S(26) * PHI)$   
 $WA04 = -PNT * DC27 / (DC27 - S(25) * PHI)$   
 $WA05 = WA03 * S(37) / (S(37) - S(26))$   
 $WA06 = WA04 * S(37) / (S(37) - S(25))$   
 $WA07 = -(WA03 + WA04) * S(37) * PHI / (S(37) * PHI - DC27)$   
 $WA09 = WA07 * DC38 / (DC38 + PHI * (S(38) - S(26)))$   
 $WA10 = WA07 * DC38 / (DC38 + PHI * (S(38) - S(25)))$   
 $WA11 = WA07 * DC38 / (DC38 + S(38) * PHI - DC27)$   
 $WA12 = -(WA05 + WA06 + WA07) * DC38 / (DC38 + PHI * (S(38) - S(37)))$   
 $WA13 = WA09 * S(48) * C48 / ((1. + D48) * (S(48) - S(26)))$   
 $WA14 = WA10 * S(48) * C48 / ((1. + D48) * (S(48) - S(25)))$   
 $WA15 = WA11 * S(48) * C48 * PHI / ((1. + D48) * (S(48) * PHI - DC27))$   
 $WA16 = WA12 * S(48) * C48 / ((1. + D48) * (S(48) - S(37)))$   
 $WAF = (WA09 + WA10 + WA11 + WA12) * S(48) * D48 * PHI$   
 $WINE = (1. + D48) * (S(48) * PHI - DC38 - S(38) * PHI)$   
 $WA17 = WAF / WINE$   
 $WA18 = WA17 * (B(26, I) - B(49, I)) / (PHI * (S(49) - S(26)))$   
 $WA19 = WA17 * (B(25, I) - B(49, I)) / (PHI * (S(49) - S(25)))$   
 $WA20 = WA17 * (B(27, I) - B(49, I)) / (S(49) * PHI - DC27)$   
 $WA21 = WA17 * (B(37, I) - B(49, I)) / (PHI * (S(49) - S(37)))$   
 $WA22 = WA17 * (B(38, I) - B(49, I)) / (S(49) * PHI - DC38 - S(38) * PHI)$   
 $WA23A = -(WA13 + WA14 + WA15 + WA16 + WA17) * (B(48, I) - B(49, I))$   
 $WA23B = PHI * (S(49) - S(48))$   
 $WA23 = WA23A / WA23B$   
 $C49B = WA18 + WA19 + WA20 + WA21 + WA22 + WA23$   
 $C49 = C49A + C49B$   
 $C49T(I, J) = C49 * AU28(J) * 239. / 238.$

C  
C  
C

## NP-237 CONCENTRATION CALCULATION.

$WA25 = WA07 * (B(26, I) - B(37, I)) / ((S(37) - S(26)) * PHI)$   
 $WA26 = WA07 * (B(25, I) - B(37, I)) / ((S(37) - S(25)) * PHI)$   
 $WA27 = -(WA03 + WA04) * (B(27, I) - B(37, I)) / (S(37) * PHI - DC27)$   
 $C37 = WA25 + WA26 + WA27$   
 $C37T(I, J) = C37 * AU28(J) * 237. / 238.$

C  
C  
C

## PU-240 CONCENTRATION CALCULATION.

$C3 = A28 * S(28) * D49 / (S(40) * AL * (1. + D49))$   
 $C4 = C2 * S(49) * D49 / ((S(40) - S(25)) * (1. + D49))$   
 $C5 = C3 * S(40) / (S(49) * AL - S(40)) + C4 * (S(40) - S(25)) / (S(49) * AL - S(40))$   
 $C40 = C3 + C4 * B(25, I) + C5 * BA(49, I) - (C3 + C4 + C5) * B(40, I)$

```
C
C
C      C40T(I,J)=C40*AU28(J)*240./238.
      PU-241 CONCENTRATION CALCULATION.
      C6=C3*S(40)/S(41)
      C7=C4*S(40)/(S(41)-S(25))
      CR=C5*S(40)/(S(41)-S(49)*AL)
      C9=(C3+C6+C5)*S(40)/(S(40)-S(41))
      C41=C6+C7*B(25,I)+C8*BA(49,I)-(C6+C7+C8+C9)*B(41,I)
      C41T(I,J)=C41*AU28(J)*241./238.
C
C
C      PU-242 CONCENTRATION CALCULATION.
      BEN1=C6*.366E-01*FLU(I)*1.E 24+C7*(1.-B(25,I))/S(25)
      BEN2=C8*(1.-BA(49,I))/(S(49)*AL)
      BEN=BEN1+BEN2
      PIG=C9*(1.-R(40,I))/S(40)-(C6+C7+C8+C9)*(1.-B(41,I))/S(41)
      C42=D41*S(41)*(BEN+PIG)/(1.+D41)
      C42T(I,J)=C42*AU28(J)*242./238.
C
C
C      PU-238 CONCENTRATION CALCULATION.
      FP25=A25*(1.-B(25,I))/(1.+D25)
      GEN1=C1*FLU(I)*1.E 24+C2*(1.-B(25,I))/S(25)
      GEN2=-(C1+C2)*(1.-BA(49,I))/(S(49)*AL)
      GEN=GEN1+GEN2
      FP49=S(40)*GEN/(1.+D49)
      FP41=C42/D41
      FP28=(EPS-1.)*(V25*FP25+V49*FP49+V41*FP41)/(V28-1.)
      C28=A28-FP28-C49-FP49-C40-C41-C42-FP41
      C28T(I,J)=AU28(J)*C28
      FLT(I)=F(U(I))*1.E 03
2C. CONTINUE
      RETURN
      END
```

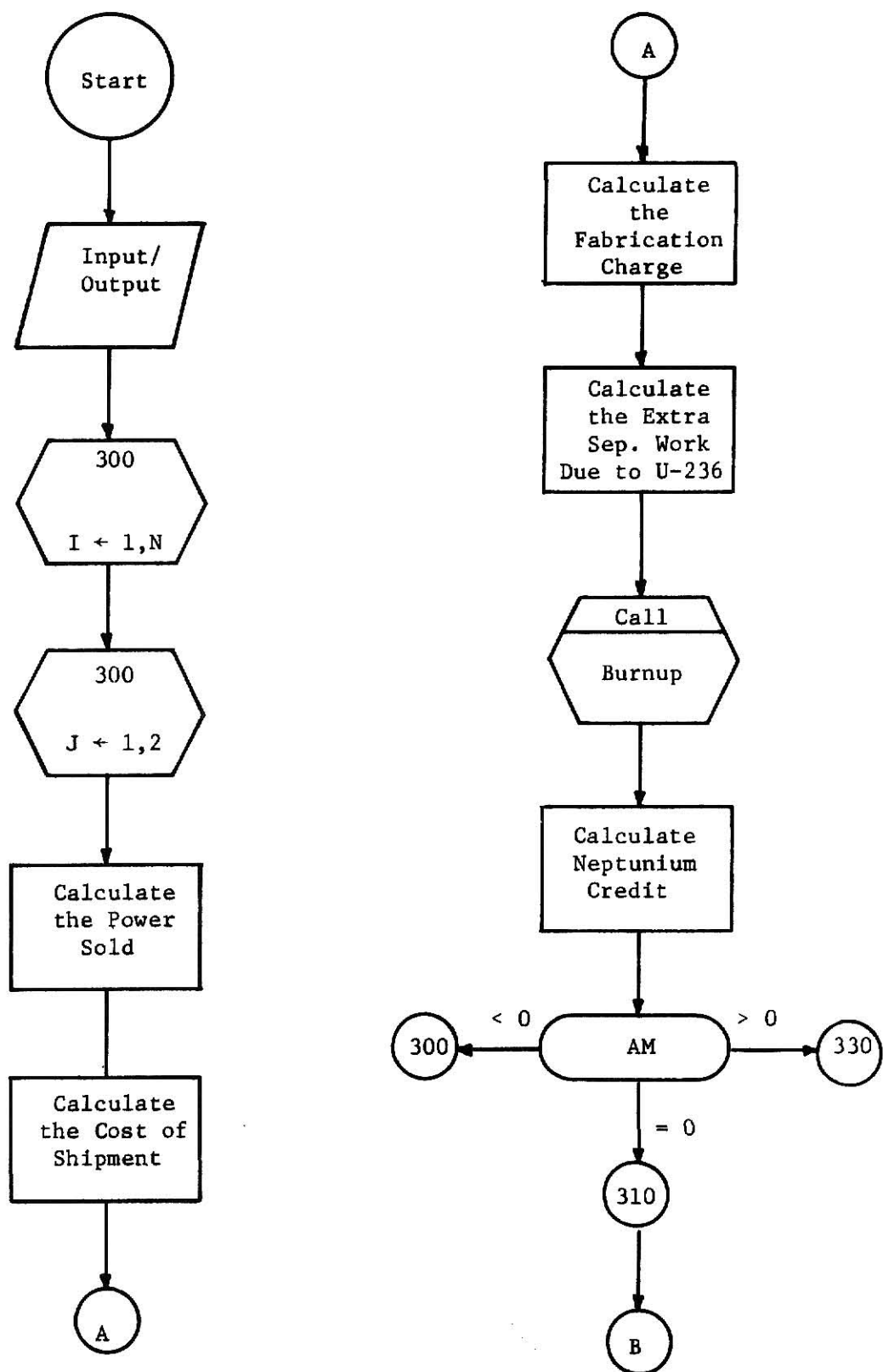


Fig. A-5. "FULCYC" computer program flow sheet.

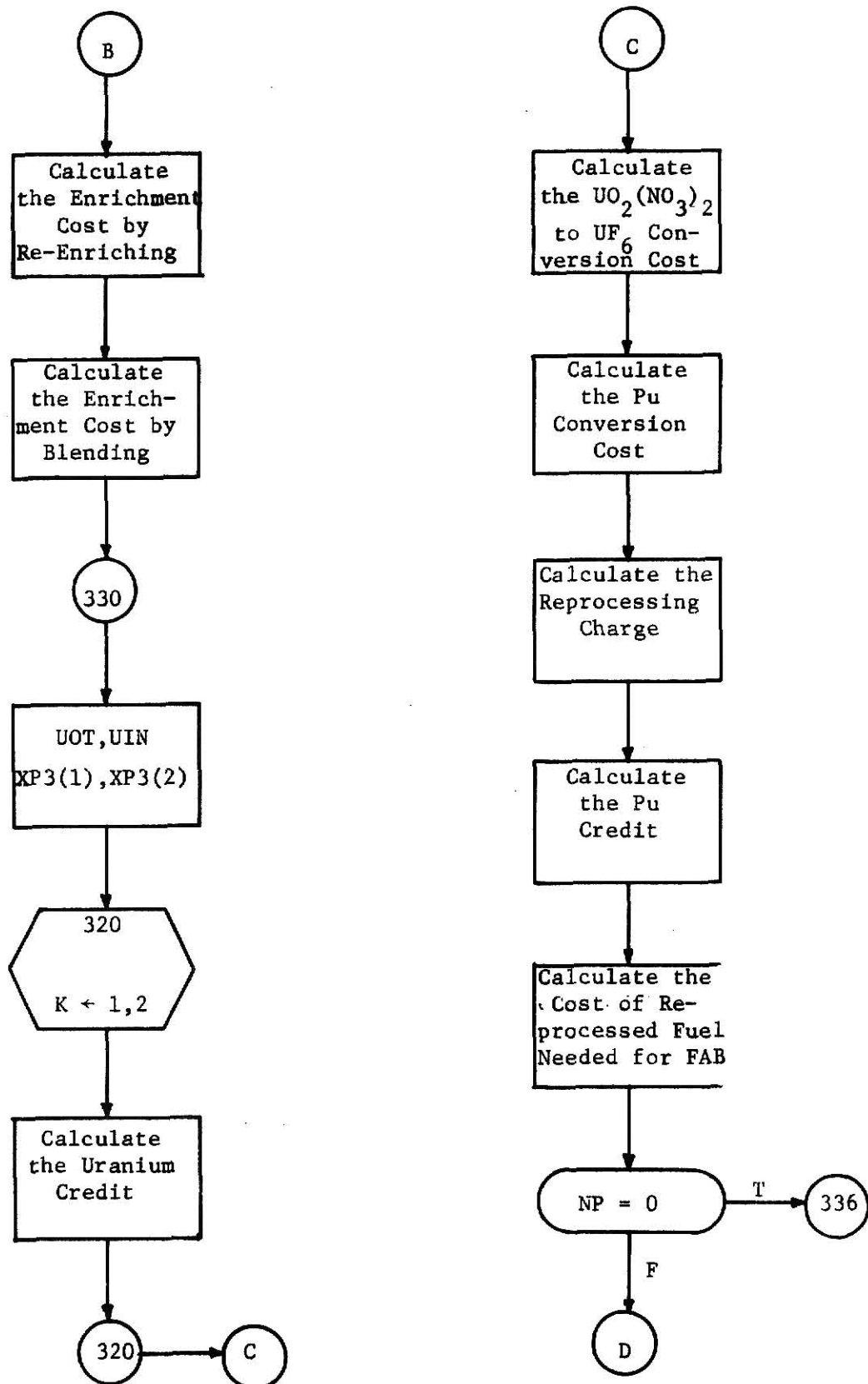


Fig. A-6 (Continued)

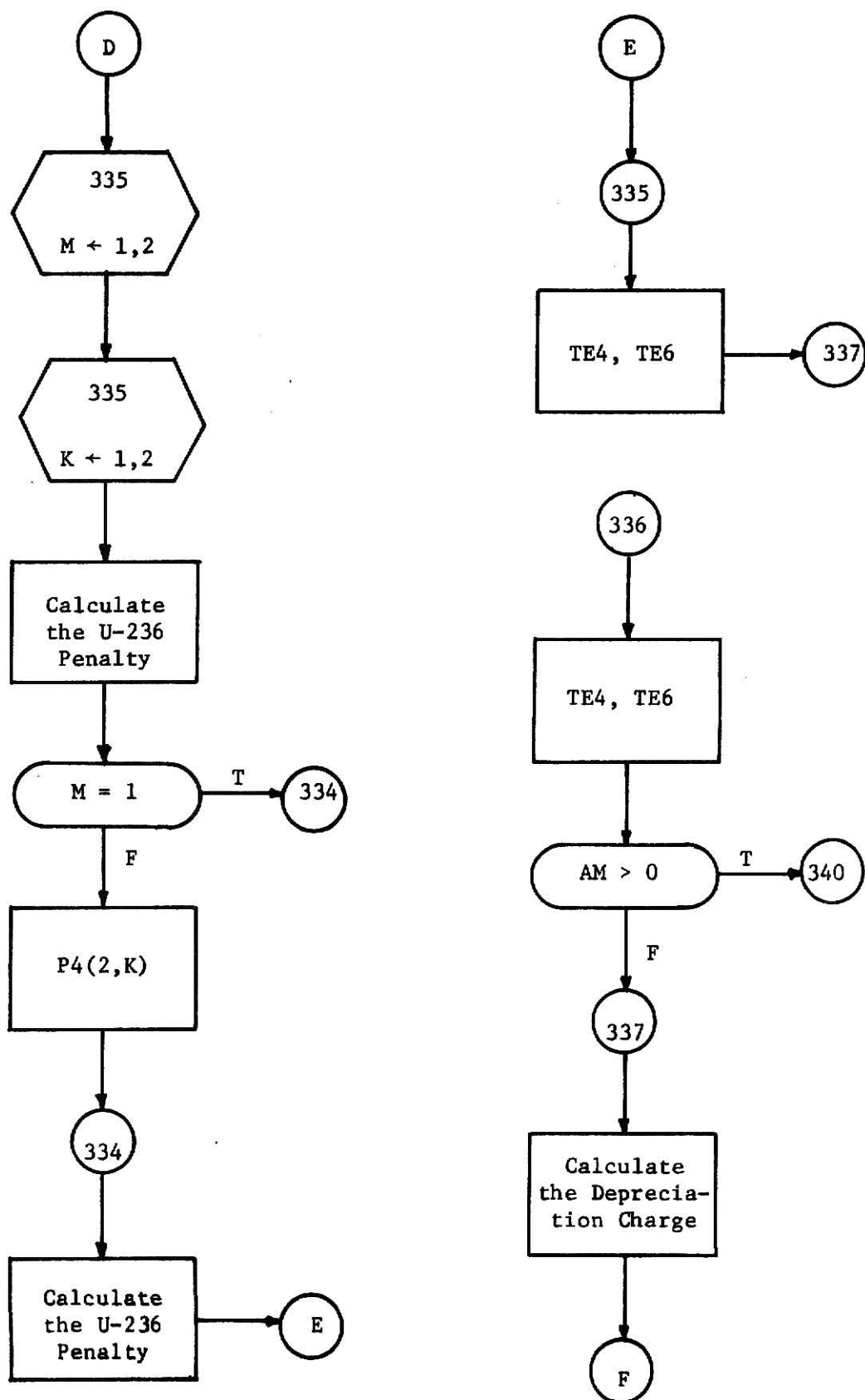


Fig. A-7 (Continued)

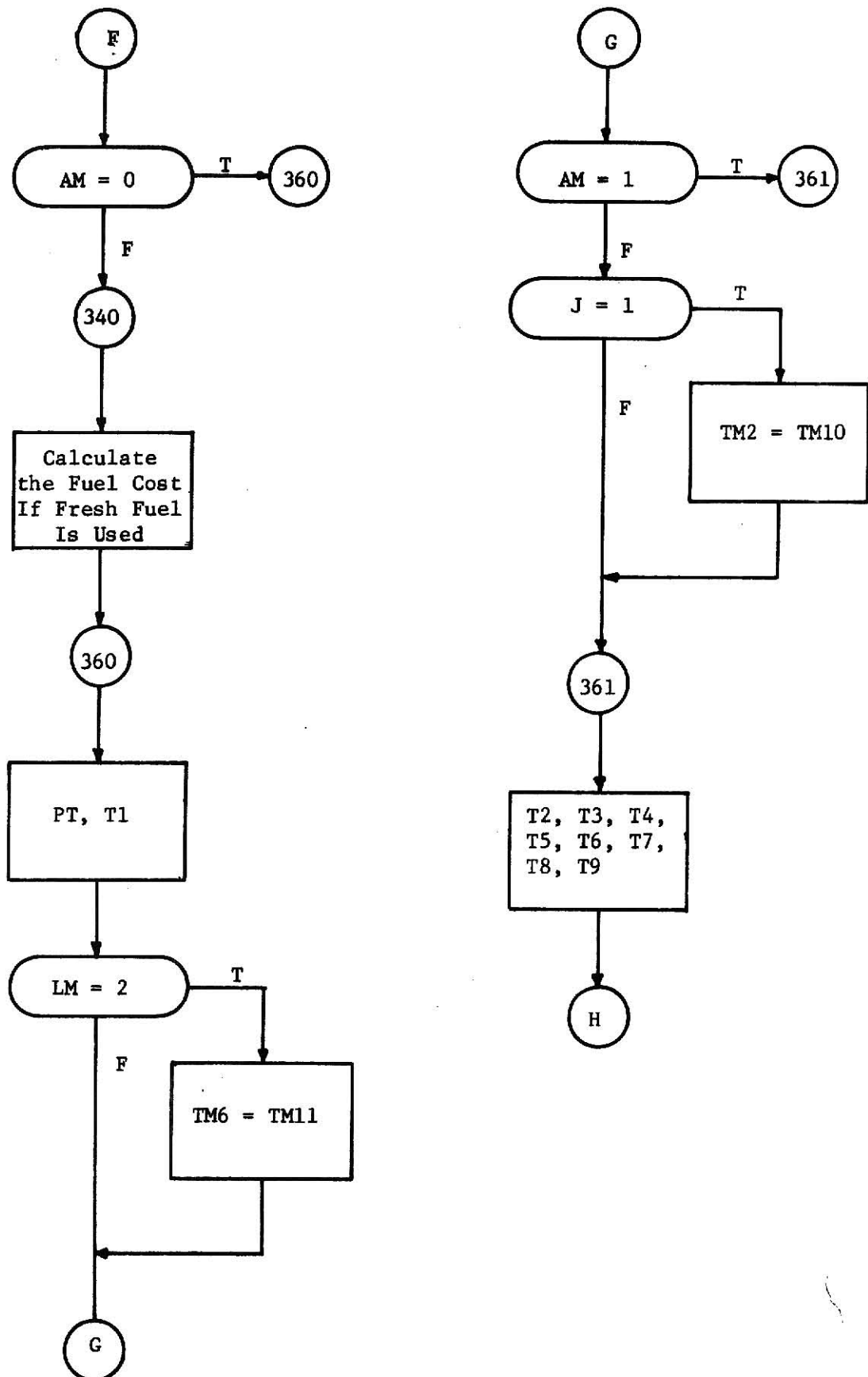


Fig. A-8 (Continued)



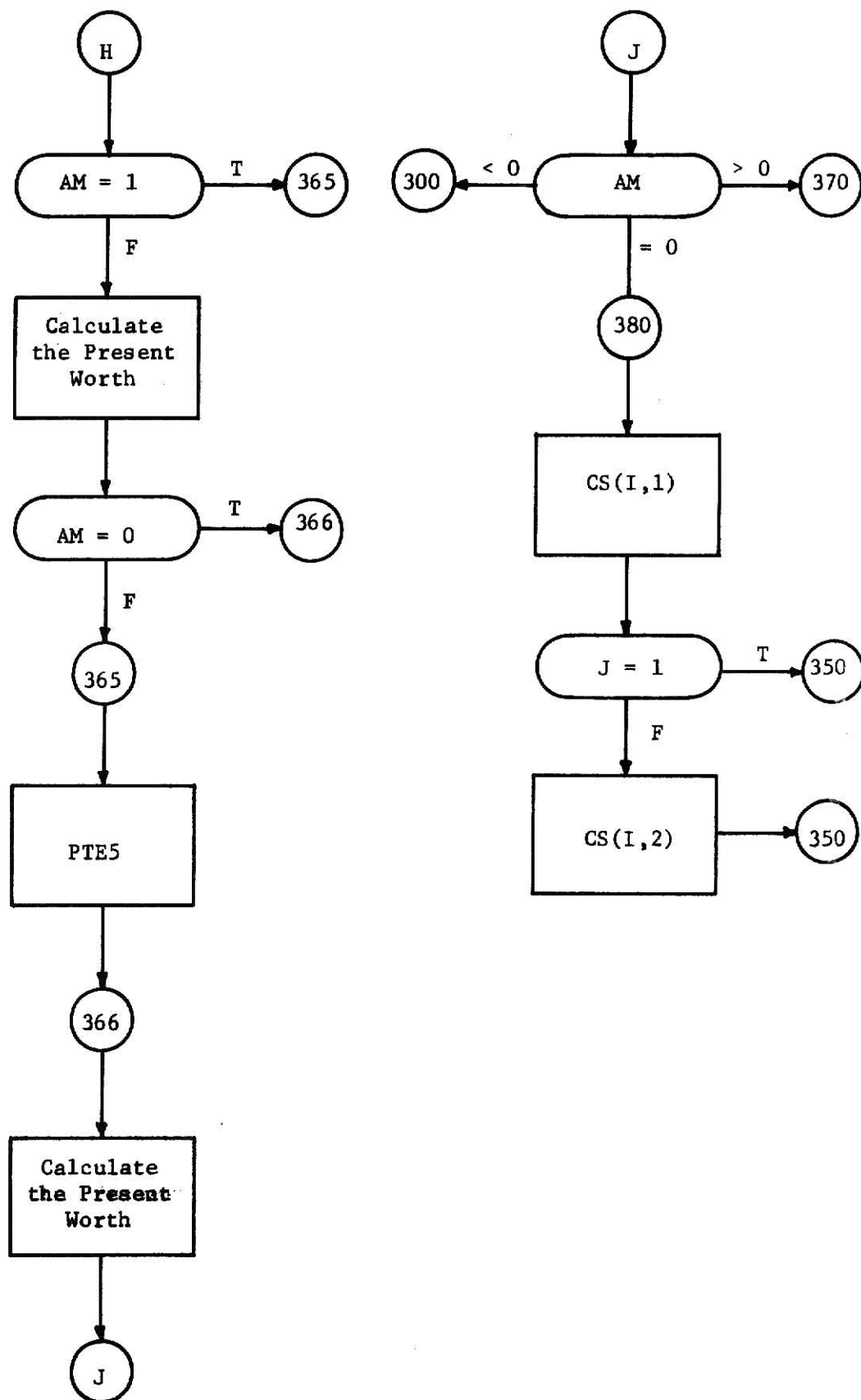


Fig. A-9 (Continued)

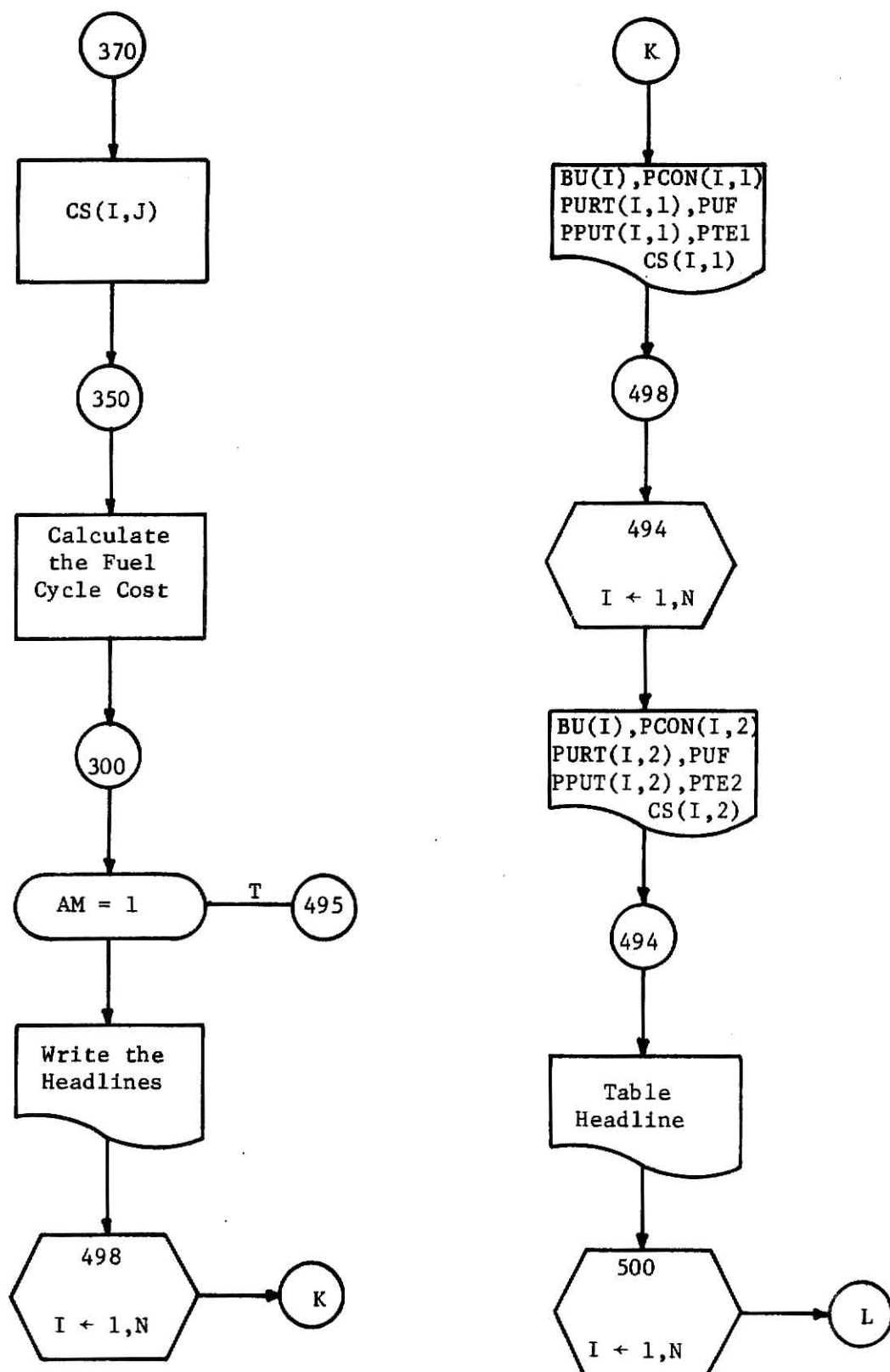


Fig. A-10 (Continued)

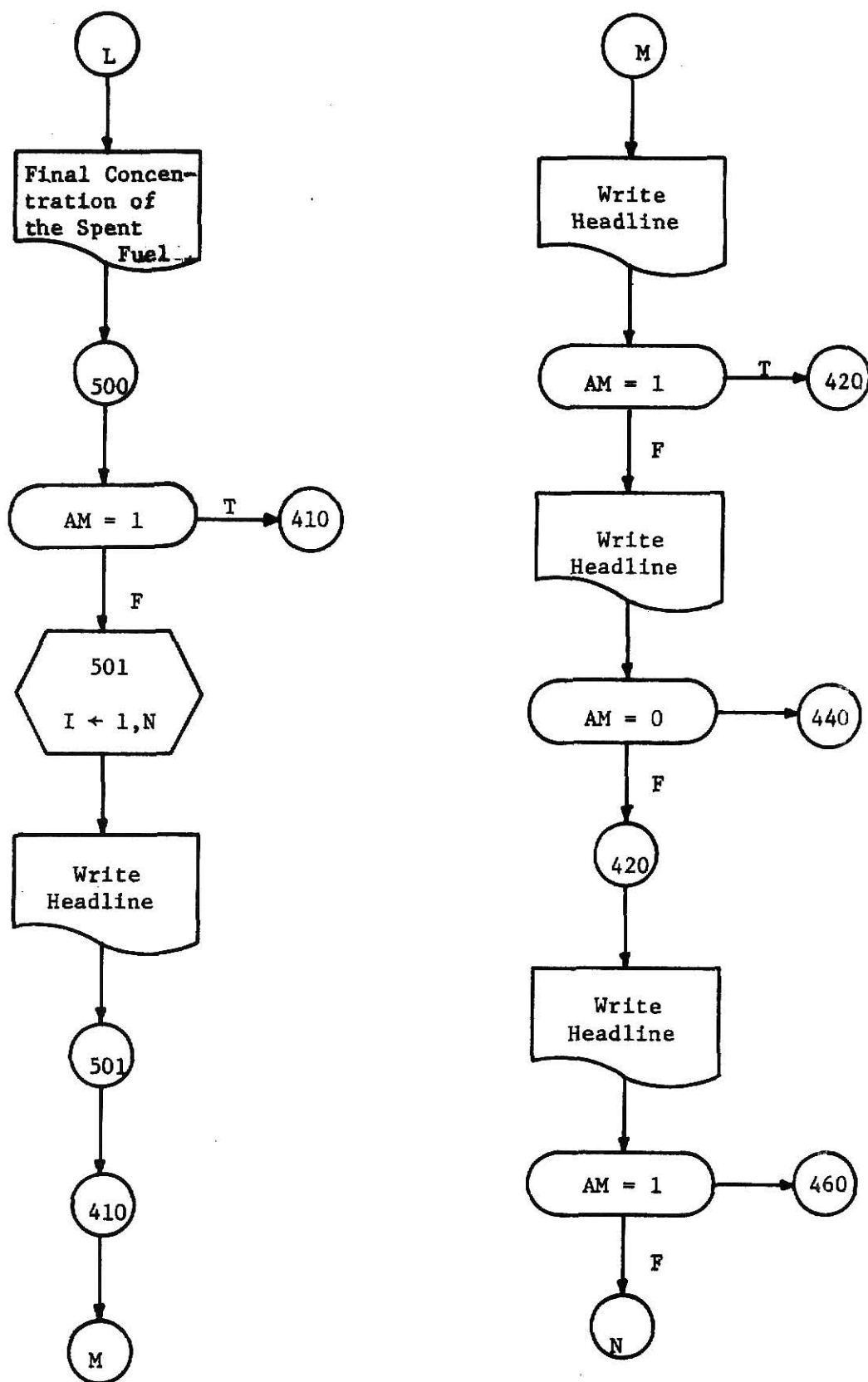


Fig. A-11 (Continued)

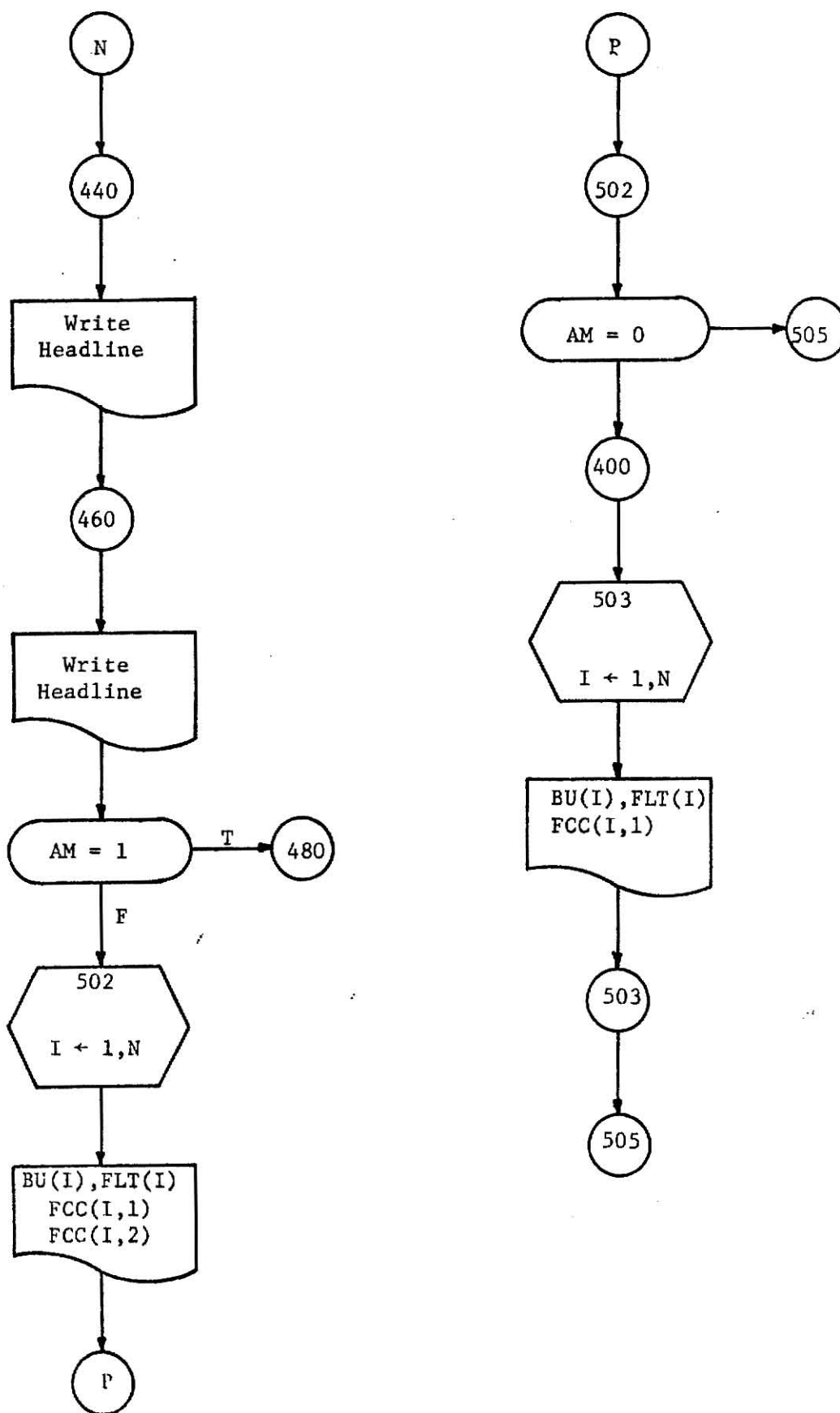


Fig. A-12 (Continued)

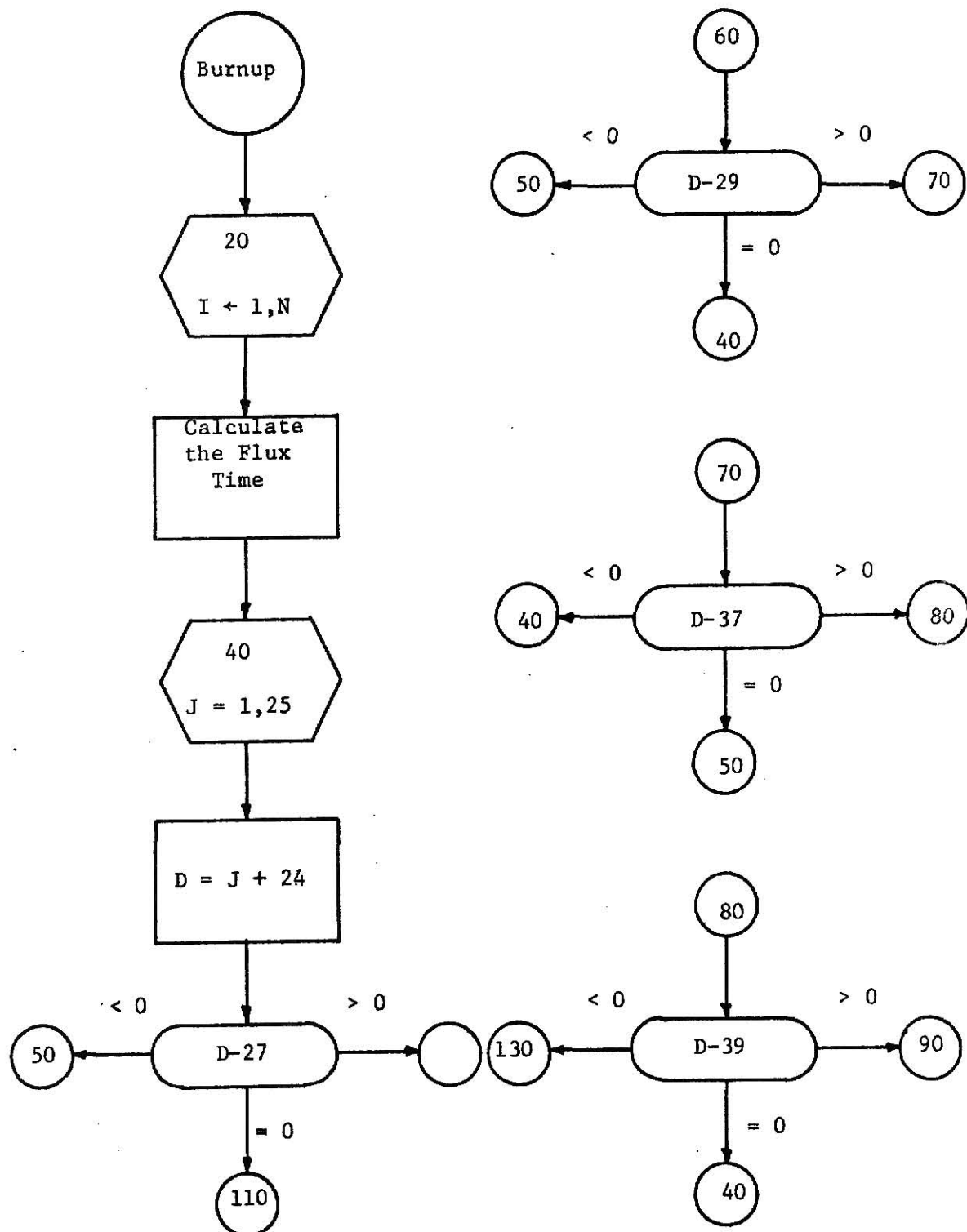


Fig. A-13 (Continued)

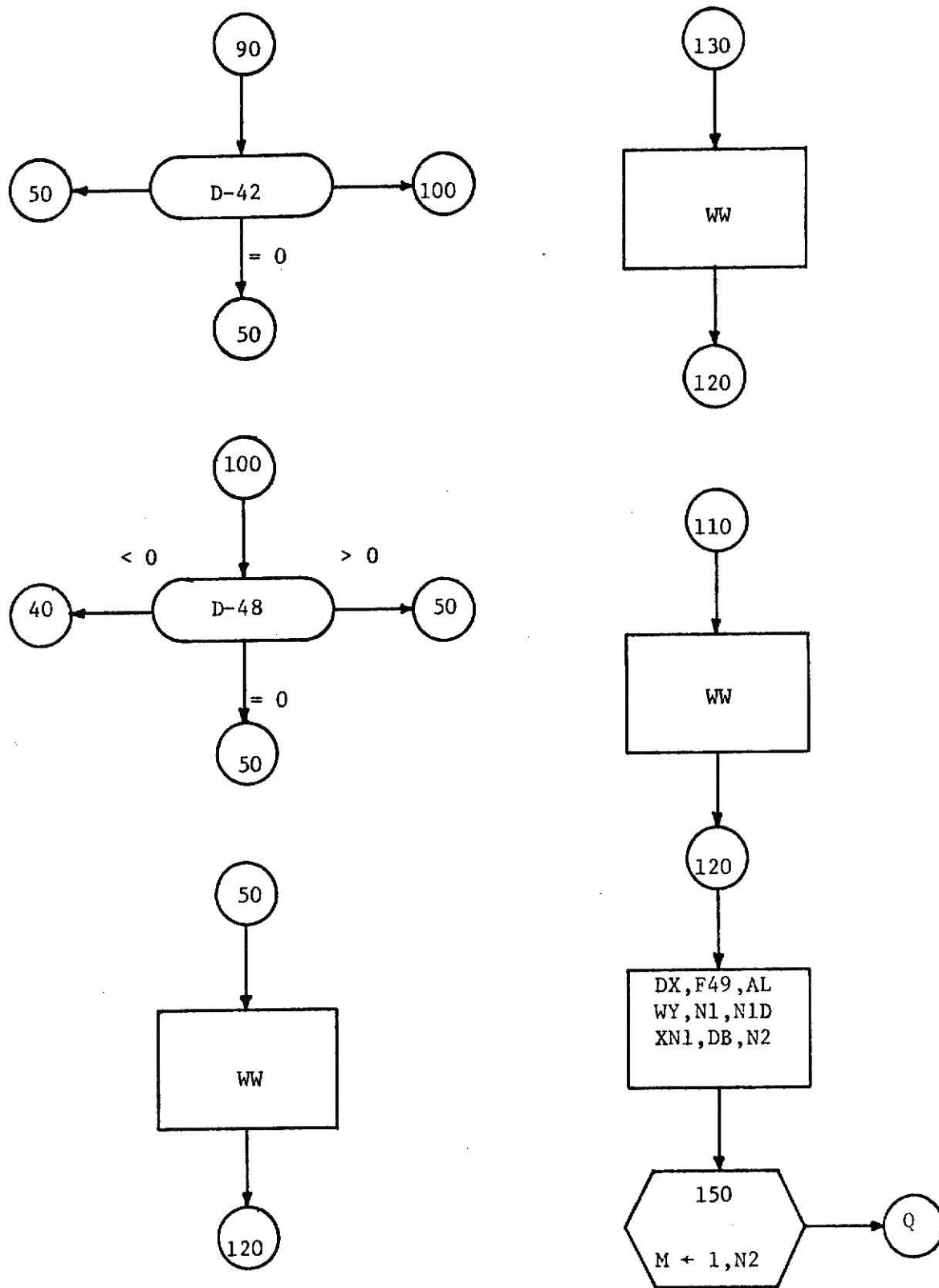


Fig. A-14 (Continued)

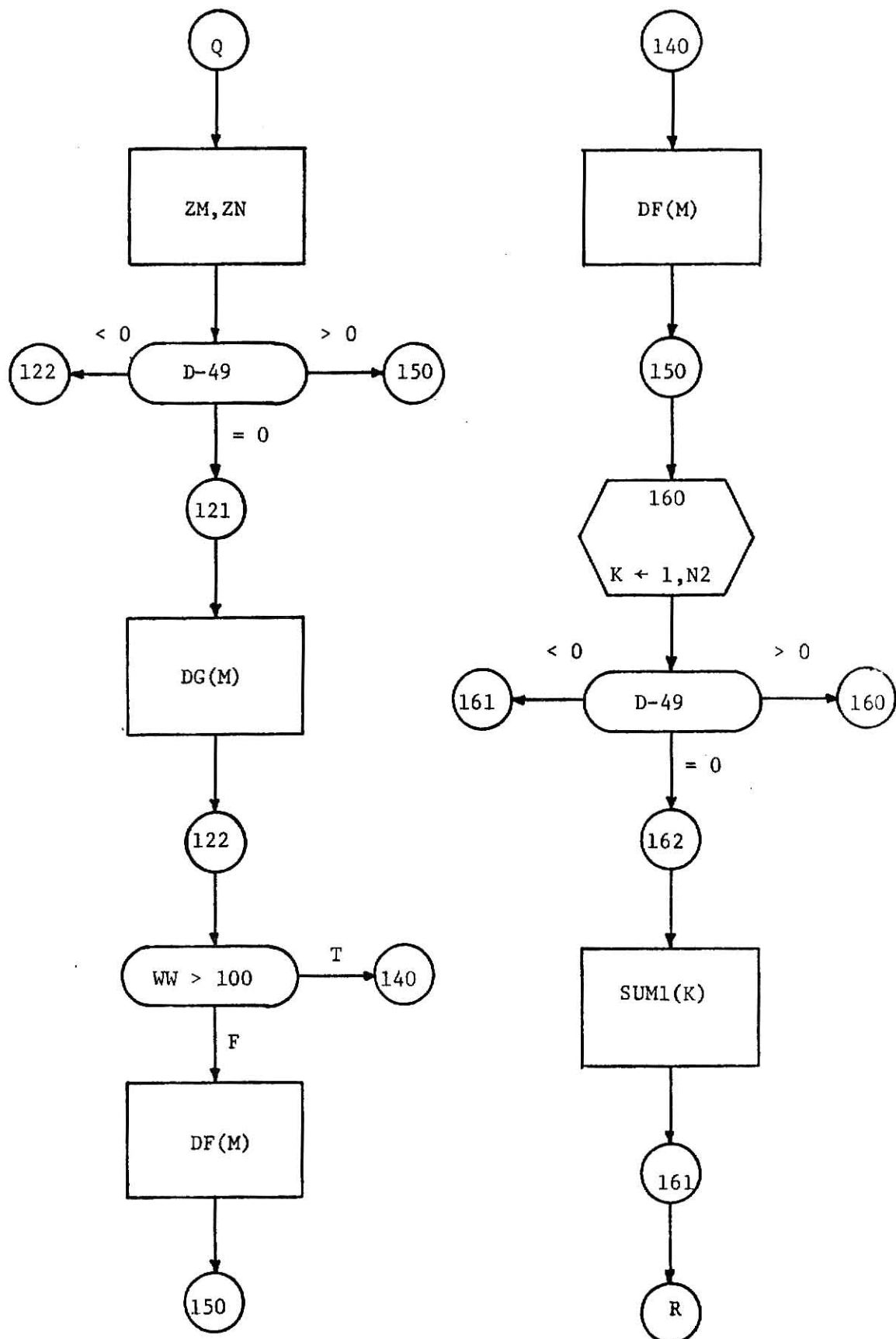


Fig. A-15 (Continued)

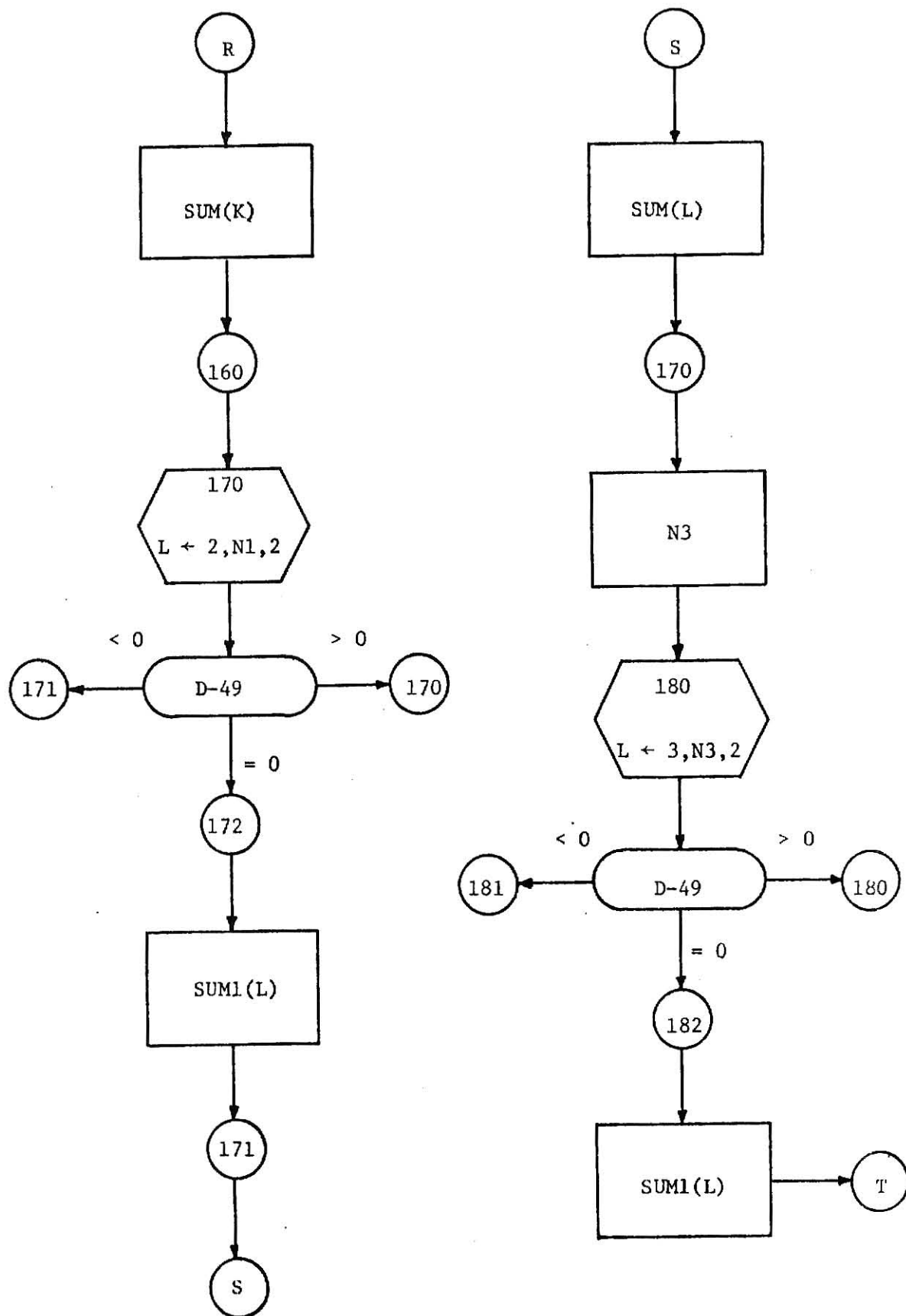


Fig. A-16. (Continued)



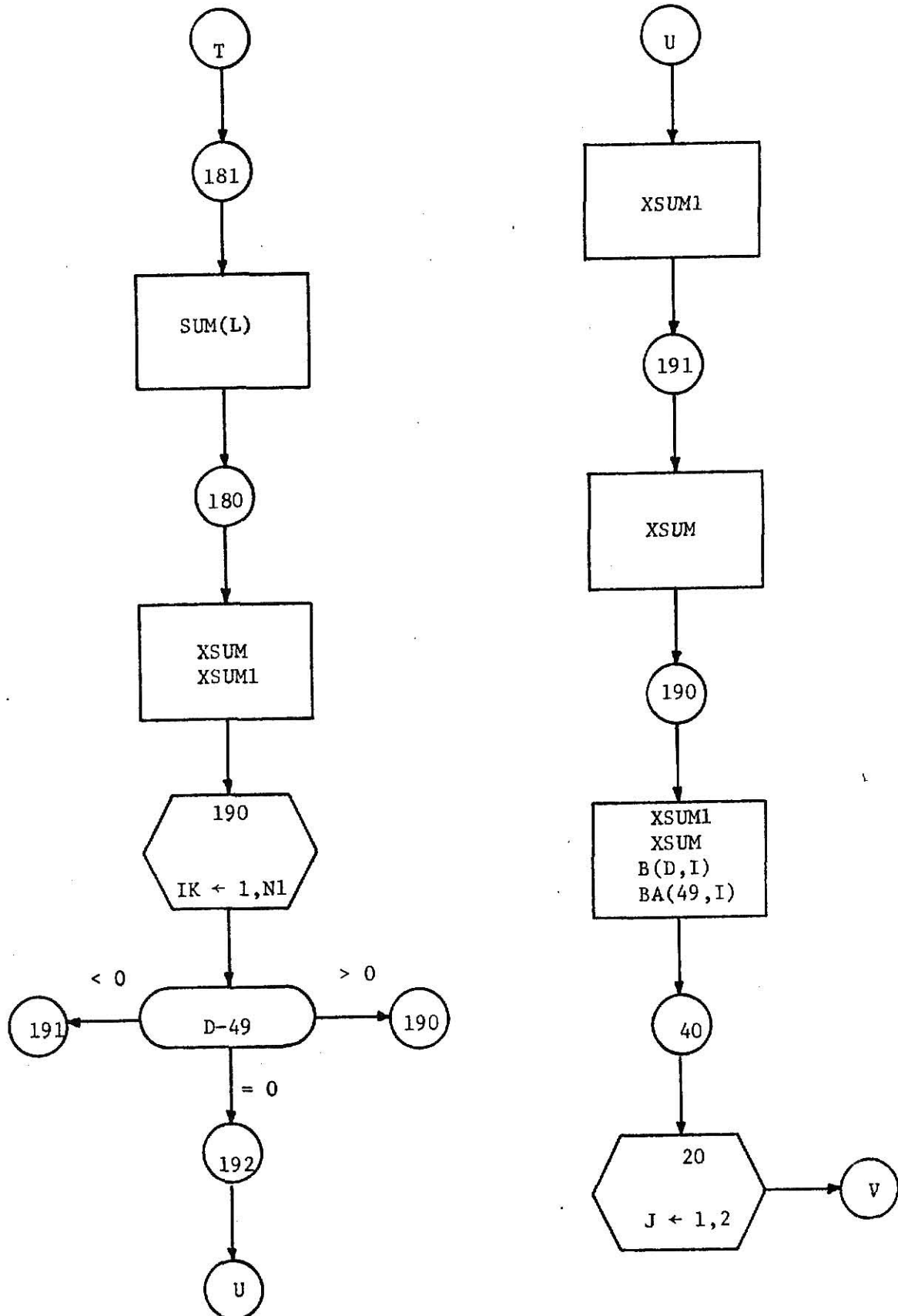


Fig. A-17 (Continued)

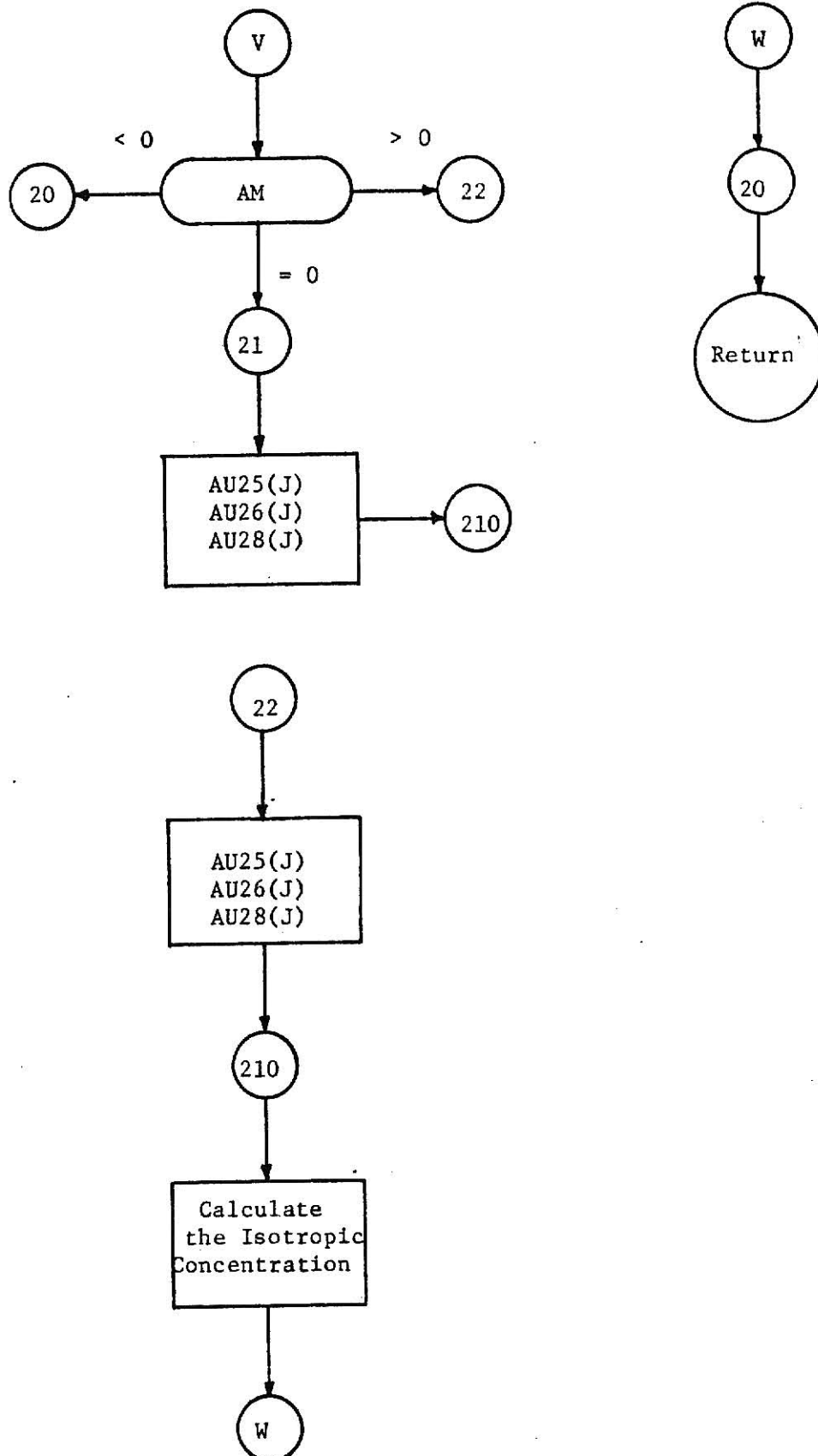


Fig. A-18 (Continued)

## APPENDIX B

Table 5. Nuclear Fuel Cycle Cost of Reference-Design  
1,000 MWe PWR, Privately Owned Fuel

Burnup	Fuel Cycle Costs		
	Natural Uranium	Spent Uranium Re-Enriching Mills/kWh	Recycle Blending
MWD/Tonne			
23,000	1.928291	1.970108	2.072048
25,000	1.825041	1.864845	1.958881
27,000	1.733638	1.771605	1.858882
29,000	1.651987	1.688269	1.769699
31,000	1.578482	1.613201	1.689528
33,000	1.511866	1.545145	1.616973
35,000	1.451144	1.483080	1.550905
37,000	1.395504	1.426182	1.490436

Table 6. Nuclear Fuel Cycle Costs of Reference-Design  
1,000 MWe PWR, Leased Fuel

Burnup	Fuel Cycle Costs		
	Natural Uranium	Spent Uranium Re-Enriching	Recycle Blending
23,000	1.806293	1.963435	2.042458
25,000	1.706713	1.853362	1.926443
27,000	1.618849	1.756439	1.824430
29,000	1.540614	1.670293	1.733879
31,000	1.470407	1.593108	1.652843
33,000	1.406982	1.523482	1.579820
35,000	1.349342	1.460286	1.513604
37,000	1.296683	1.402619	1.453237

Table 7. Nuclear Fuel Cycle Costs of Reference-Design  
1,065 MWe BWR, Privately Owned Fuel

Burnup  MWD/Tonne	Fuel Cycle Costs		
	Natural Uranium	Spent Uranium Re-Enriching Mills/kWh	Recycle Blending
17,000	2.205367	2.235423	2.379304
19,000	2.035604	2.064041	2.193108
21,000	1.892697	1.919633	2.036671
23,000	1.770356	1.795904	1.902972
25,600	1.634970	1.658863	1.755267
27,000	1.570958	1.594028	1.685523
27,500	1.549392	1.572175	1.662038
29,000	1.488338	1.510299	1.595592
31,000	1.414496	1.435423	1.515299

Table 8. Nuclear Fuel Cycle Costs of Reference-Design  
1,065 MWe BWR, Leased Fuel

Burnup  MWD/Tonne	Fuel Cycle Costs		
	Natural Uranium	Spent Uranium Re-Enriching  Mills/kWh	Recycle Blending
17,000	2.096155	2.254553	2.374237
19,000	1.930752	2.074989	2.182594
21,000	1.792010	1.924593	2.022388
23,000	1.673649	1.796461	1.886118
25,600	1.543181	1.655406	1.736351
27,000	1.481708	1.589010	1.665944
27,500	1.461026	1.566685	1.642283
29,000	1.402579	1.503613	1.575476
31,000	1.332086	1.427591	1.495025

Table 9. Effect of Separative Work Costs on Fuel Cycle Cost

Cost of Separative Work, \$/kg U of Separative Work	Fuel Cycle Cost, Mills/KWH			
	1,000 MWe PWR	Re-Enriching 1,065 MWe BWR	1,000 MWe PWR	Blending 1,065 MWe BWR
10	1.057059	1.141432	1.076497	1.167095
20	1.278918	1.337224	1.322168	1.392070
32	1.545145	1.562175	1.616973	1.662038
40	1.722631	1.728807	1.813509	1.842012
50	1.944490	1.924599	2.059182	2.066987



Table 10. Effect of  $U_3O_8$  Costs on Fuel Cycle Cost

Price of Yellow Cake, \$/lb of $U_3O_8$	Fuel Cycle Cost, Mills/KWH			
	Re-Enriching		Blending	
	1,000 MWe PWR	1,065 MWe BWR	1,000 MWe PWR	1,065 MWe BWR
6	1.418527	1.447740	1.491042	1.538111
8	1.545145	1.572175	1.616973	1.662038
10	1.671766	1.696609	1.742903	1.785964
12	1.798141	1.820807	1.868591	1.909650
15	1.987951	2.007338	2.057366	2.095422

Table 11. Effect of Discharged Fuel Enrichment on Fuel Cycle Cost

Discharged Fuel Enrichment	Fuel Cycle Cost 1,000 MWe PWR, Re-Enriching 1,000 MWe BWR, Mills/kWh	Discharged Fuel Enrichment	Fuel Cycle Cost 1,065 MWe BWR, Re-Enriching Mills/kWh
1.26	1.435	1.69	1.426
1.27		1.70	1.490
1.32	1.510	1.75	1.483
1.33		1.77	1.551
1.38	1.594	1.82	1.545
1.39		1.83	1.617
1.43	1.659	1.89	1.613
1.44		1.90	1.690
1.52	1.796	1.96	1.688
1.53		1.97	1.770
1.59	1.917	2.03	1.772
1.60		2.05	1.859
1.67	2.064	2.11	1.865
1.68		2.13	1.959
1.75	2.235	2.20	1.970
1.77		2.21	2.072

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IN LWR FUEL CYCLES

by

Jun Ren Wang

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AN ABSTRACT OF A MASTER'S THESIS

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## ABSTRACT

The uranium recovered from spent nuclear fuel can be recycled back to the reactor again after being upgraded to the necessary enrichment.

The spent uranium can be recycled by either of two schemes. In scheme one, recycled uranium is re-enriched by blending it with makeup feed natural uranium having a high  $^{235}\text{U}$  concentration; in scheme two, the recycled uranium is re-enriched through the cascade of a gaseous diffusion plant with natural feed uranium. The calculations show that the blending scheme is more costly than the re-enriching scheme. By re-enriching the spent uranium containing  $^{236}\text{U}$  through an enrichment cascade the cascade would subsequently be contaminated with  $^{236}\text{U}$ ; however, in the calculations presented here no penalty is assessed for cascade contamination. Re-enriching scheme costs less to recycle and is favored over the blending scheme.

The calculations also reveal that the natural uranium feed nuclear fuel cycles are less costly than either of the recycled spent uranium feed nuclear fuel cycles. In other words, it is still cheaper to use natural uranium fuel instead of recycling the spent uranium.