Corrugated web plate girders

by

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Abstract

Built-up plate girders are typically used in construction when there are large loads and long span that rolled shapes do not work. The high web slenderness ratio makes plate girders susceptible to buckling among other limit states. Transverse stiffeners are often required along the span of the girder to prevent local buckling. The use of a corrugated web eliminates the need for transverse stiffeners, which reduces fabrication cost significantly.

The flexural and shear behaviors of Corrugated Web Plate Girders are different from regular plate girders due to its unique web geometry. Various web geometries are available with further research developing design equations for each. The web introduces a transverse moment that the flanges must resist in addition to in-plane loads. Design equations are used in Europe through Eurocode. Researchers have proposed equations for use in North America with similar assumptions as those in Europe, however they are not adopted yet in design code. This report seeks to introduce design methods and practice of corrugated web plate girders in Europe as well as proposed North American design methods.

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Chapter 1: Introduction

Structural steel is a prominent material used for buildings, bridges, and other structures. As a material, steel has high strength-to-weight ratios, ductile behavior, and other properties which influence the behavior of it. Effective in both tension and compression, steel has high strength capacities when compared to other materials. For many projects, steel is an economical choice because of these advantages. Steel structural members often are not limited by shear or flexure strengths but deflection criteria. To meet deflection requirements, larger sections are required.

When specifying structural steel sections, rolled shapes are often preferred. Hot-rolled steel shapes are produced by heating steel to an elevated temperature which allows it to be shaped easily. It is then rolled through a mill into the desired shape. Rolled shapes are desirable as calculations for section properties and strengths are readily available in the *AISC Steel Construction Manual*. Most rolled shapes can sufficiently carry the loads required and satisfy other requirements, however, special cases require built-up shapes known as plate girders.

1.1 Plate Girders

Plate girders are an alternative to rolled sections. Plate girders are built-up shapes consisting of individual steel plates that are constructed to create a larger section than what rolled shapes are, increasing the strength and stiffness. This may be advantageous when high loads and long spans are required of a structure. Plate girders are often used in bridges for this purpose. Plate girders can be constructed in a variety of shapes. With built-up I-shaped members, sections do not have to be doubly symmetric. They can be singly symmetric with AISC 360 *Specification for Structural Steel Buildings* addressing design requirements. Although singly symmetric I-shaped sections are allowed, they are not commonly used in building design.

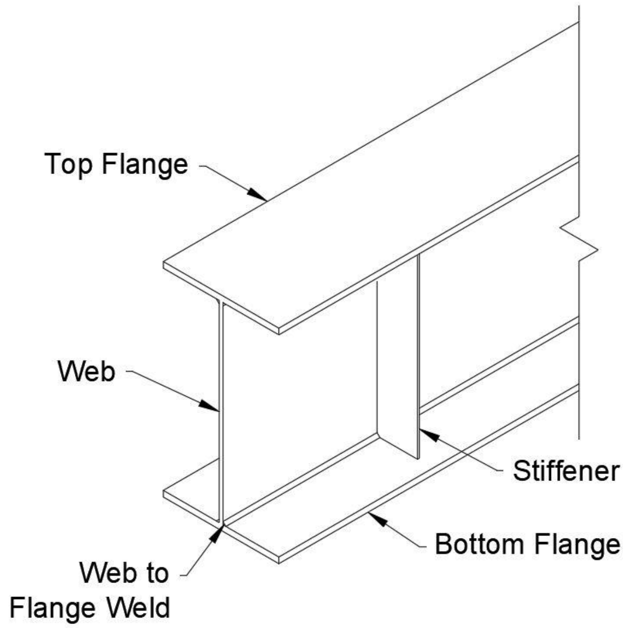

Figure 1.1 Plate Girder

Plate girders first emerged in the late 19th century for uses in railroad bridges. They quickly became economical choices, being able to efficiently use material compared to rolled beams. While plate girders can be either welded, bolted, or riveted during fabrication, welding quickly became the most common choice with improved fabrication methods in the 1950s. Plate girders do not have to use the same material strengths throughout the girder. Sizes of plates may also change along the span length. Changing plate sizes and material strengths along the span length of the girder results in an economical design.

The flexural strength of plate girders depends on the proportions of the flange. Flanges are classified as compact, noncompact, or slender which influences the design procedure. The limit states for flexure are similar to rolled I-shapes but include additional limit states due to the high slenderness. The limit states include compression flange yielding, lateral-torsional buckling, compression flange local buckling, tension flange yielding, and web local buckling.

Increased beam depth results in a higher shear strength due to the cross-sectional area of the web. The shear behavior of plate girders differs compared to rolled sections. With the high depth to web thickness ratio associated with high loads, the slenderness ratio typically is high which introduces issues related to stability. Because plate girders are often designed with transverse stiffeners, post-buckling strength of the web may be realized through a behavior called tension field action. The web of the girder along with the transverse stiffeners act as a truss once the web buckles, providing some additional post-buckling strength. The design of the girder for shear can either consider or not to consider tension field action.

To combat web buckling associated with slender webs, stiffeners are provided to increase the strength and stiffness of the girder. Stiffeners are plates welded to the web and flange of the girder. The plates are not required to be welded to both flanges. There are two types of stiffeners. Stiffeners distributed along the girder span are known as intermediate stiffeners and are used to increase the shear strength. This is done by controlling the buckling strength of the web or increasing the post-buckling strength. Stiffeners form panels with the aspect ratio, a/h, which governs design. Bearing stiffeners are placed at locations of high concentrated loads where the strength of the girder is not adequate. Bearing stiffeners resist the limit states of web local yielding, web local crippling, and web sidesway buckling with compressive forces. Web local yielding and flange local bending must be checked when tension forces are applied. Although it is possible to design plate girders without stiffeners, it is typically not an economical choice. While the benefits of stiffeners on plate girders are significant, they also have disadvantages such as increased steel tonnage and fabrication costs. Stiffeners welded to the web reduce the fatigue life of the girder as well. For plate girders to be economical, they must be designed to optimize both the amount of steel used and the fabrication costs.

1.2 Corrugated Web Plate Girders

Due to the associated amount of steel, plate girders must be carefully designed to be economical. Corrugated web plate girders may offer a solution to reduce the amount of steel while also decreasing fabrication costs compared to flat web plate girders. This innovative idea emerged as a possible solution to increase out-of-plane stiffness, web buckling resistance, and weight reduction without the use of transverse stiffeners.

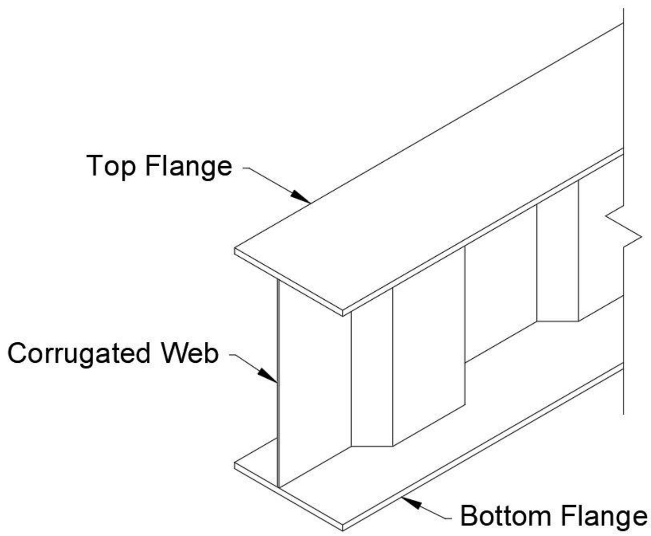

Figure 1.2 Corrugated Web Plate Girder

Corrugated web plate girders offer increased shear resistance, similar to normal plate girders, with a significant reduction in weight. This weight decrease is due to the web thickness being significantly thinner than what rolled shapes or normal plate girders are constructed of. The corrugated web also eliminates the need for transverse stiffeners in many cases by increasing the out-of-plane stiffness and buckling strength of the girder. Corrugated web plate girders are similar in construction to normal plate girders except for a corrugated plate for the web as opposed to a flat plate. Connections between the web and flanges are welded for ease of fabrication and to ensure high strength.

Design of corrugated web plate girders continues to be a highly researched topic. Equations have been proposed based on assumptions and research. Currently, there is no code requirements in AISC 360 *Specification for Structural Steel Buildings* while Eurocode has provisions on the design of corrugated web plate girders.

1.3 Report Structure

This report seeks to introduce an innovative structural system which results in a reduced amount of steel, more importantly reduced fabrication costs, while having the same strength capacities compared to normal plate girders. First, web geometry and behaviors of the web are explained. The assumptions for design methods as well as industry examples are introduced. Next, proposed strength equations by North American researchers and from Eurocode will be explained. An example comparing a corrugated web plate girder designed by the North American proposed method and Eurocode method is presented. Additionally, the design of a flat web plate girder according to AISC 360 is included for comparison purpose. Corrugated web plate girders seek to provide an economical option for long spans with high shear loads.

Chapter 2: Literature Review

Corrugated web plate girders have seen many uses in recent history. It continues to grow in popularity around the world. Most design equations consider the same methods however vary slightly. This chapter will introduce corrugated web plate girders and the design equations used for shear and flexural resistance.

2.1 History of Corrugated Web Plate Girders

Corrugated sheet metal was first patented in 1829. It was used for aircraft in the early 1900s. Corrugated sheets were used as the skin for the fuselage and wings to provide a stiff, lightweight solution. Beams with trapezoidal profiles were first manufactured in Sweden since 1966 and were soon seen in the Soviet Union (Pasternak, 2004). France began development of corrugated web beams in the early 1980s. However, due to high manufacturing costs, it did not gain popularity (Pasternak, 2004).

It soon became clear that for corrugated web plate girders to be economically feasible, manufacturing costs had to be reduced. The first fully automated production of corrugated web beams occurred in 1990 at the company Zeman & Co. in Vienna. Standard sizes of flanges, web, and lengths are available. The thickness of the web ranges from 1.5 mm to 3 mm. Due to the thin web, fillet welds between the web and flanges can be welded on one side only through a special metal active gas (MAG) welding process called transferred ionized molten energy (TIME). The penetration of the weld ensures there is no corrosion-sensitive gap on the non-welded side (Pasternak, 2004). By fully automating the process, corrugated web plate girders became an economical choice for designers to use.

2.2 Web Profile

Many different web profiles have been proposed and used. Some of these geometries included trapezoidal, sinusoidal, triangular, and rectangular. Of these listed, trapezoidal, and sinusoidal geometries are the most popular among the research community and industry. This report will focus on trapezoidal webs and their associated properties. The web profile is one of the most important properties of the girder. The web height, thickness, and corrugation geometry directly affect the shear strength. Different nomenclature is used between the proposed North American method and Eurocode. A representation of typical geometry notation of the corrugated web can be seen in Figure 2-1. The top image is for the proposed North American method while the bottom image is for Eurocode.

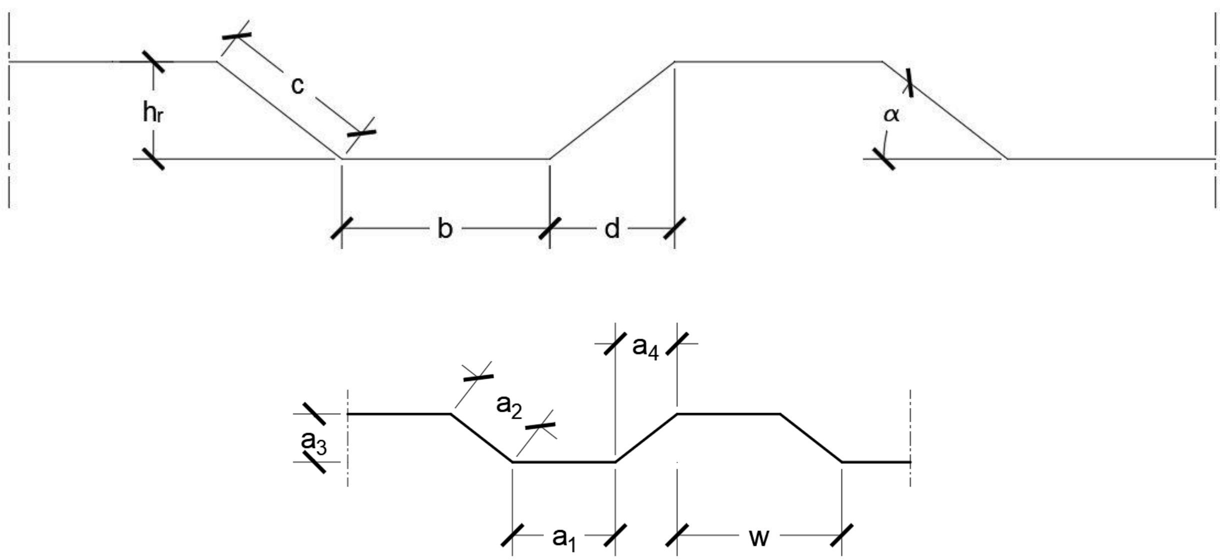

Figure 2.1 Corrugated Web Profile Notation

Each flat piece of the web is known as a panel or "fold". These panels are either classified as inclined or longitudinal panels. The first variables of the web are b (a_1) and c (a_2) which are the lengths of the longitudinal and inclined panels respectively. The next variable is the projected length of the inclined fold in the longitudinal direction, defined as d (a_4). The height of the inclined panel measured in the transverse direction is defined as h_r (a_3). The angle of the web corrugation is known as α. The corrugation angle is typically kept between 30 degrees and 45 degrees (Driver Et al., 2006). The ratio of lengths of longitudinal fold to inclined fold is known as β. The equation can be defined as:

$$\beta = \frac{b}{c}$$

It is recommended for β to be between one and two (Driver Et al., 2006). Satisfying these limits will result in an economical design. With a low value of β, excess web material will be required while a large value will result in low global buckling strength.

A vertical plane is formed by the centerline of the web. The top and bottom flanges are symmetrically attached to this centerline, known as the middle plane. The middle plane serves as a reference point for the girder. When analyzing the cross section of the girder, an idealized cross section consisting of thin-walled elements is used. An origin point is chosen along the middle plane based on the centroid of the flanges. The origin point will always lie on the middle plane but may change position vertically if flange dimensions vary from each other. Figure 2-2 demonstrates the idealized cross section with the x-axis in the horizontal direction and y-axis in the vertical direction.

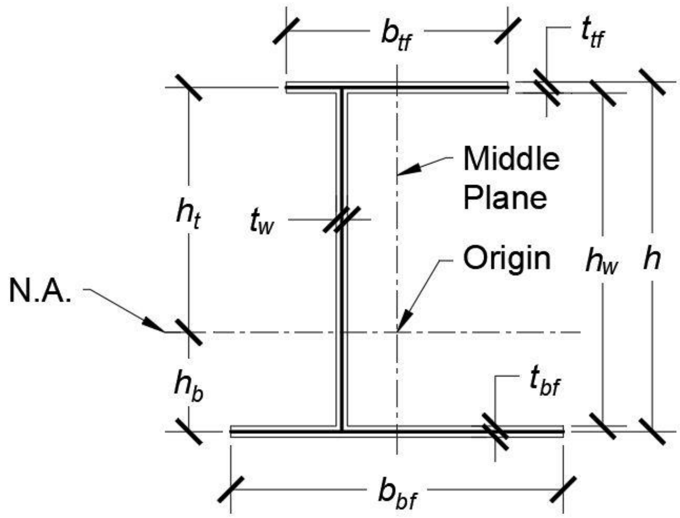

Figure 2.2 Idealized Cross Section

With the idealized cross section, the following equations for the area of the top and bottom flanges are given by:

$$A_{tf} = b_{tf} t_{tf}$$

$$A_{bf} = b_{bf} t_{bf}$$

Once flange areas have been determined, the distance to the neutral axis from either the top or bottom of the idealized section is given by:

$$h_t = \frac{A_{bf} h}{A_{bf} + A_{tf}}$$

$$h_b = \frac{A_{tf} h}{A_{bf} + A_{tf}}$$

Finally, the moment of the inertia about the x-axis is given by:

$$I_x = \frac{A_{bf} A_{tf} h^2}{A_{bf} + A_{tf}}$$

Loads applied to the middle plane are known as in-plane loads while loads applied transversely to this plane are known as out-of-plane loads. In-plane loads cause a bending moment, about the x-axis, as well as a shear force, in the y-direction.

2.3 Shear Strength

With the production of corrugated web plate girders starting in the 1960's, many design equations have been proposed to estimate the shear strength of the web. Experimental tests and finite element analysis models were used to predict the shear strength of corrugated web plate girders. The current European design standard *Eurocode 3 – Design of steel structures – Part 1-5: Plated structural elements* includes design rules for plate girders with corrugated webs. While currently no standards exist in North America, Driver Et al. (2006) proposed equations to estimate the shear strength.

2.3.1 North America Proposed Method - Driver Et al., (2006)

Shear strength tests conducted on small scale specimens with corrugated webs are readily available across the world. Three limit states for shear must be analyzed, global shear buckling, local shear buckling, and web yielding. Driver Et al. (2006) suggested that many previously proposed equations overestimate the shear strength of the web due to the small-scale tests of these girders. As a result, finite element analysis and full-scale girder tests were performed to obtain more accurate predictions for the shear strength.

Shear strength is a function of the web thickness, web height, corrugation geometry, and material properties. The main assumptions associated with corrugated web plate girders are that the bending moment is carried by the flanges with no contribution from the web while shear is carried by the web with no contribution from the flanges. These assumptions come from thin-walled beam theory that cross-sectional forces of the web and flanges are carried only in their own plane (Driver Et al., 2006). Experimentally, longitudinal stresses due to bending were shown to not be significant in the web. From these results, the assumption that moment-shear interaction can be neglected was confirmed (Driver Et al., 2006). With these assumptions, the web is subject to pure shear stress which allows the shear strength to be predicted using plate stability theory. Stiffeners are provided on slender flat web plate girders, increasing the stiffness and strength. With corrugated web plate girders, the web possesses increased stiffness and strength eliminating the need for stiffeners.

The web is subject to the limit states of web yielding and shear buckling. Shear buckling can be classified either as global buckling or local buckling. Global buckling occurs across a large portion of the girder while local buckling occurs within a single panel. When local buckling occurs, an interactive failure may result, propagating into adjacent folds. Local buckling is considered a function of the slenderness ratio of an individual panel while global buckling is a function of the entire web. The theory of shear strength formulas assumes each fold is simply supported by adjacent panels as well as by the flanges (Driver Et al., 2006). When adjacent folds have different widths, local shear buckling of the wider fold is restrained by the narrower width fold. The narrower fold will have the greater local shear buckling strength. β determines the governing panel for local shear buckling. When $\beta = 1$, both panels are equally critical for local shear buckling. When $\beta > 1$, the longitudinal folds are critical and restrained by the inclined folds. When $\beta < 1$, the inclined folds are critical and restrained by the longitudinal folds. Slender webs will typically fail in shear buckling with slender flanges leading to ineffective cross-section utilization.

The first limit state to check for shear strength is local elastic shear buckling. The local elastic shear buckling stress is determined by plate stability theory. Each panel is assumed to be supported by adjacent panels on vertical sides and by the flanges along the horizontal sides. The equation used by Driver Et al., (2006) is as follows:

$$\left(\tau_{cr,L}\right)_{el} = k_L \frac{\pi^2 E}{12(1-v^2)(w/t_w)^2}$$ (Driver Et al., (2006) Eq. 1)

Where E is the modulus of elasticity, v is Poisson's ratio, w is the maximum panel width, t_w is the web thickness, and k_L is the local shear buckling coefficient. The local shear buckling coefficient depends on the aspect ratio of the panel and the boundary conditions being assumed of the panel. If the panel is considered simply supported, the value is 5.34. If the panel is assumed to be fixed, the value is 8.98. Research has indicated that the true value is between a simply supported condition and fixed condition (Driver Et al., 2006). To be conservative, the simply supported value is typically chosen.

To determine the global elastic buckling stress, the web of the girder is treated as an orthotropic flat web. This gives the equation:

$$\left(\tau_{cr,G}\right)_{el} = k_G \frac{E t_w^{1/2} b^{3/2}}{12 h_w^2} F(\alpha, \beta)$$ (Driver Et al., (2006) Eq. 2)

Where b is the longitudinal panel width, h_w is the web height, and k_G is the global shear buckling coefficient. Once again, the coefficient depends on the considered boundary conditions for the web with simply supported being 31.6 and 59 being fixed. *F(α, β)* is a coefficient that characterizes the geometry of the web and is given by:

$$F(\alpha, \beta) = \sqrt{\frac{(1+\beta)\sin^3\alpha}{\beta + cos\alpha}} \cdot \left\{\frac{3\beta+1}{\beta^2(\beta+1)}\right\}^{3/4}$$ (Driver Et al., (2006) Eq. 3)

Where α is the angle of the inclined panel and β is the ratio of longitudinal fold width to inclined fold width. A large β value will result in low global buckling strength while a low value will result in uneconomical design. The values for β are commonly between 1 and 2 while the values for α are typically between 30 and 45 degrees with recommendations to be no less than 30 degrees (Driver Et al. 2006). If values of β are less than 1, excess web material will be used. If values of β are greater than 2, the web will have a low post-buckling strength.

When elastic shear buckling stress exceeds 80% of the shear yield stress, the following inelastic equation may be used for the critical of local or global buckling:

$$(\tau_{cr})_{inel} = \sqrt{0.8\tau_y(\tau_{cr})_{el}} \leq \tau_y \qquad \text{(Driver Et al., (2006) Eq. 4)}$$

With the yield stress given by:

$$\tau_y = \frac{F_y}{\sqrt{3}} \qquad \text{(Driver Et al., (2006) Eq. 5)}$$

Low post-buckling strength remains a problem with corrugated web plate girders. Due to low post-buckling strength, it is recommended to prevent global buckling. By rearranging equations for yield stress, inelastic shear stress, and the elastic global buckling stress, yielding of the web will control the shear strength if the web height to thickness ratio satisfies the following equation:

$$\frac{h_w}{t_w} \leq 1.91\psi\sqrt{\frac{E}{F_y}\left(\frac{b}{t_w}\right)^{1.5} F(\alpha,\beta)} \qquad \text{(Driver Et al., (2006) Eq. 11)}$$

Where the factor ψ provides a factor of safety and with a recommend value of 0.9.

Design equations are developed based on the local buckling slenderness ratio. The local buckling slenderness ratio is given by:

(Driver Et al., (2006) Eq. 15)

$$\lambda_L = \frac{w}{t_w}\sqrt{\frac{F_y}{E}}$$

Where w is the maximum of either the longitudinal panel width or inclined panel width. Once the local buckling slenderness ratio has been determined, the nominal shear strength may be calculated based on the limit state of the web.

For yielding: $\lambda_L \leq 2.586$

(Driver Et al., (2006) Eq. 12)

$$V_n = 0.707\left(\frac{F_y}{\sqrt{3}}\right)h_w t_w$$

For inelastic buckling: $2.586 < \lambda_L \leq 3.233$

(Driver Et al., (2006) Eq. 13)

$$V_n = \sqrt{\frac{1}{1 + 0.15\lambda_L^2}}\left(\frac{F_y}{\sqrt{3}}\right)h_w t_w$$

For elastic buckling: $3.233 < \lambda_L$

(Driver Et al., (2006) Eq. 14)

$$V_n = \sqrt{\frac{1}{1 + 0.014\lambda_L^4}}\left(\frac{F_y}{\sqrt{3}}\right)h_w t_w$$

To maximize the shear local buckling strength and ensure the previous equations are applicable, the following equation must also be satisfied:

$$\frac{b}{t_w} \leq 2.586\sqrt{\frac{E}{F_y}}$$ (Driver Et al., (2006) Eq. 16)

Where b is the width of the longitudinal panel.

By satisfying both the web height to thickness as well as the longitudinal width to web thickness limits, the girder's shear strength will be maximized through yielding. Satisfying both limits results in an economical girder design for shear strength.

2.3.2 Eurocode

There are code provisions in current European standards for the design of plate girders with corrugated webs. *Eurocode 3 – Design of steel structures – Part 1-5: Plated structural elements* includes a section in Annex D which details design rules. Both trapezoidal and sinusoidal web geometries are included in this code. Section D.1 gives general information about variables that differ slightly than those discussed above.

Like other research, local and global shear buckling limit states are checked. The critical stress for local buckling is found as a long plate while global buckling is derived from orthotropic plate theory. The commentary states that some authors define the global buckling factor (D_x) without the factor (1-v^2), where v is Poisson's ratio. However, including the factor is more accurate. Panel edges once again are assumed to either be simply supported or fixed. Simply supported conditions are often chosen as the flanges are not rigid enough to restrain rotation. EN 1993-1-5 includes two checks for local and global buckling to show the post-critical strength which is not present in global buckling.

Section D.2.2 of EN 1993-1-5 details shear strength requirements. Shear strength is given as:

$$V_{bw,Rd} = \chi_c \frac{f_{yw}}{\gamma_{M1}\sqrt{3}} h_w t_w \qquad \text{(EN 1993-1-5 Eq. D.4)}$$

Where χ_c is the lesser of the reduction factors for local buckling or global buckling and γ_{M1} is a safety factor. The local buckling factor is calculated from (EN 1993-1-5 Eq. D.5) as:

$$\chi_{c,l} = \frac{1.15}{0.9 + \bar{\lambda}_{c,l}} \leq 1.0 \qquad \text{(EN 1993-1-5 Eq. D.5)}$$

Where the local slenderness ratio ($\bar{\lambda}_{c,l}$) is calculated as:

$$\bar{\lambda}_{c,l} = \sqrt{\frac{f_{yw}}{\tau_{cr,l}\sqrt{3}}} \qquad \text{(EN 1993-1-5 Eq. D.6)}$$

Where f_{yw} is the yield stress of the web and $\tau_{cr,l}$ is the critical local shear stress given by:

$$\tau_{cr,l} = 4.83E\left[\frac{t_w}{a_{max}}\right]^2 \qquad \text{(EN 1993-1-5 Eq. D.7)}$$

Where a_{max} is the larger of a_1 and a_2.

The reduction factor for global buckling is given by:

$$\chi_{c,g} = \frac{1.5}{0.5 + \bar{\lambda}_{c,g}^2} \leq 1.0 \qquad \text{(EN 1993-1-5 Eq. D.8)}$$

Where the global slenderness ratio ($\bar{\lambda}_{g,l}$) is calculated as:

$$\bar{\lambda}_{c,g} = \sqrt{\frac{f_{yw}}{\tau_{cr,g}\sqrt{3}}} \qquad \text{(EN 1993-1-5 Eq. D.9)}$$

Where $\tau_{cr,g}$ is the critical global shear stress given by:

$$\tau_{cr,g} = \frac{32.4}{t_w h_w^2} \sqrt[4]{D_x D_z^3} \qquad \text{(EN 1993-1-5 Eq. D.10)}$$

Where the factor D_x is given by:

$$D_x = \frac{Et_w^3}{12(1-v^2)}\frac{w}{s} = \frac{Et_w^3}{12(1-v^2)}\frac{a_1 + a_4}{a_1 + a_2} \qquad \text{(EN 1993-1-5 Commentary Eq. 13.10)}$$

Where s is the longitudinal projected length of one-half wavelength, w is the gross length of one-half wavelength, and v is Poisson's ratio.

The factor D_z is given by:

$$D_z = \frac{EI_z}{w} = \frac{Et_w a_3^2}{12}\frac{3a_1 + a_2}{a_1 + a_4} \qquad \text{(EN 1993-1-5 Commentary Eq. 13.11)}$$

Where I_z is the second moment of area of one corrugation w.

Next, flange induced buckling must be considered. Eurocode does not allow for flange induced buckling and provides a limit that must be satisfied. The effective area of the compression flange and the buckling factor must be determined. The effective area is to be determined according to section 4.4 in the same way normal plate girders are. However, the buckling factor differs and is taken as the larger of the following:

$$k_\sigma = 0.43 + \left(\frac{b}{a}\right)^2 \qquad \text{(EN 1993-1-5 Eq. D.2)}$$

$$k_\sigma = 0.6 \qquad \text{(EN 1993-1-5 Eq. D.3)}$$

Where b is the maximum outstand width from edge of weld and $a = a_1 + 2a_2$.

Section 4.4 gives requirements for calculating the effective area of the compression flange. The following equation is used to calculate the effective cross-sectional area:

$$A_{e,ff} = \rho A_e \qquad \text{(EN 1993-1-5 Eq. 4.1)}$$

Where ρ is the reduction factor for plate buckling and A_e is the gross cross-sectional area.

The reduction factor is found from section 4.4(3) from outstand compression elements with the given limits where:

$$\rho = 1.0 \; for \; \bar{\lambda}_p \leq 0.748 \qquad \text{(EN 1993-1-5 Eq. 4.3)}$$

$$\rho = \frac{\bar{\lambda}_p - 0.188}{\bar{\lambda}_p^2} \leq 1.0 \; for \; \bar{\lambda}_p > 0.748$$

The slenderness ratio is found from section 4.4(3) by:

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\bar{b}/t}{28.4\varepsilon\sqrt{k_\sigma}} \qquad \text{(EN 1993-1-5 Eq. 4.3)}$$

Where k_σ is the buckling factor, t is the thickness of the flange, $\bar{b}$ is equal to c: the distance from the edge of the flange to the web for outstand flanges, and ε is given by:

$$\varepsilon = \sqrt{\frac{235}{f_y\left[\frac{N}{mm^2}\right]}} \qquad \text{(EN 1993-1-5 Eq. 4.3)}$$

With the effective area of compression flange determined, the following limit can be solved:

$$\frac{h_w}{t_w} \leq k\frac{E}{f_{yf}}\sqrt{\frac{A_w}{A_{fc}}} \qquad \text{(EN 1993-1-5 Eq. 8.1)}$$

Where k is a factor based on whether plastic rotation, plastic moment resistance, or elastic moment resistance is utilized. A_w is the cross-sectional area of the web and A_{fc} is the effective cross-sectional area of the compression flange. With this limit satisfied, flange induced buckling is avoided.

2.4 Flexural Strength

2.4.1 North America Proposed Method - Abbas Et al. (2006)

As introduced earlier, shear is carried entirely by the web and the web has no contribution to the flexural strength of the girders. This is due to the high flexibility of the web compared to conventional plate girders. Based on this assumption, the flanges carry the entire bending moment. This flexibility caused by web corrugation is known as the accordion effect. The accordion effect is the expansion and contraction of the corrugated web due to low axial stiffness. An expression that compares the flexibility of the web to a normal flat web is given by:

$$FR = 2\left(\frac{h_t}{t_w}\right)\left(\frac{3\beta + 2}{\beta + cos\alpha}\right) \qquad \text{(Abbas Et al., (2006) Eq. 1)}$$

The high values of FR indicate that the web does not carry significant axial stresses compared to a flat plate web which leads to the following assumptions essential for predicting the behavior of the girder (Abbas Et al., 2006):

1. The flanges resist bending moment with no contribution from the web.
2. The web resists shear with no contribution from the flanges.
3. Cross sections do not deform.
4. For the web and flanges, plane section remains plane after bending.
5. Saint Venant's torsion contribution is negligible.
6. The girder is constructed of materials that are elastic, homogenous, and isotropic.
7. Strains, deformations, and deflections are small with equilibrium in the undeformed state.

Due to low axial rigidity of the web, little interaction occurs between the web and flanges. Web contribution to the flexural capacity is assumed to be zero, although some additional resistance may be provided by the web. Profiles with unequal fold widths have a higher web participation and moment capacities than web profiles with equal fold widths. The accordion effect is significantly higher in slender webs than compact webs.

Since the web of the girder is not always in the same plane as the load, it causes eccentricity. Due to this eccentricity, the flange must resist stresses due to in-plane bending and flange transverse bending. This eccentricity will cause the girder to twist out-of-plane at the same time deflects in-plane due to in-plane loads. To analyze the behavior of the flanges, conventional beam theory can be used for in-plane bending while flange transverse bending is analyzed by applying equivalent out-of-plane loads due to the web eccentricity. As seen in Figure 2-3, resultant forces on the flanges occur due to shear flow from the web. The greatest moment occurs when the web is farthest away from the centerline while the greatest shear occurs at the intersection of the web and the centerline.

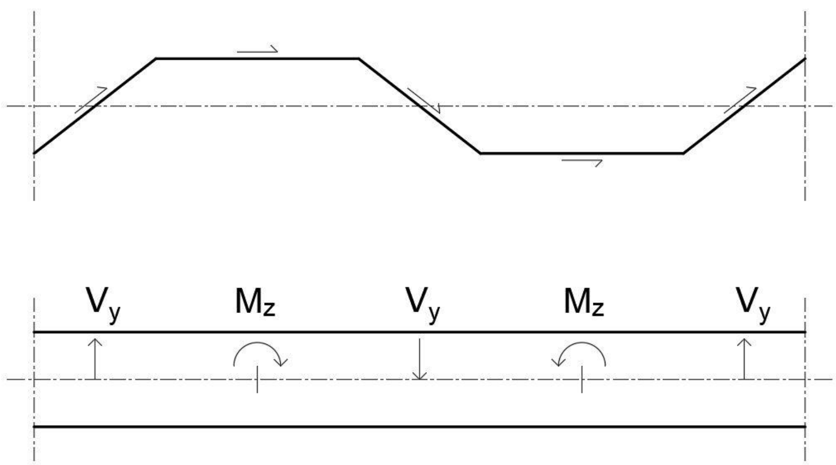

Figure 2.3 Resultant Forces Due to Shear Flow

A primary bending moment and a primary shear are produced by in-plane loads. When calculating the bending stresses due to in-plane bending, the conventional assumption follows: two equal and opposite flange normal forces resist the bending moment. The distributed flange normal stresses are given by:

$$\sigma_b = \frac{M_x Y}{I_x} \qquad \text{(Abbas Et al., (2006) Eq. 22)}$$

Where M_x is the primary bending moment, Y is the distance from the neutral axis, and I_x is the moment of inertia of the two flanges about the X-axis.

When calculating the moment of inertia, only the flanges are considered without any contribution of the web. The shear flow in the flanges and web due to changes of the uniformly distributed flange normal stresses is given by the relationship:

$$q = \frac{V_y Q_x}{I_x}$$

The next assumption is that the web carries the entire shear force with no bending moment. This gives the shear stress constant over the height of the web and is given by:

$$\tau_w = \frac{V_y}{h t_w}$$

with the corresponding shear flow given by:

$$q_w = \frac{V_y}{h}$$

Due to eccentricity caused by the web, *e*, a force of the resultant shear stresses in the flanges is related to the primary shear by:

$$V_b = V_y \frac{e}{h}. \qquad \text{(Abbas Et al., (2006) Eq. 2)}$$

In-plane loads cause a torsional moment in addition to bending moment about the major axis. Due to this, the beam must be analyzed for in-plane bending as well as flange transverse bending. The flange transverse bending problem may be solved using either flange transverse displacement or as a torsion problem. The results by Abbas Et al., (2006) do not provide flexural strength equations like the shear strength equations. Instead, the theoretical normal stresses in the bottom flange are determined due to both in-plane bending as well as flange transverse bending. Using superposition, the total stresses within the flanges can be calculated.

Stresses due to flange transverse bending must also be considered. The stresses due to flange transverse bending is calculated from:

$$\sigma_t = \frac{M_t x}{I_t} \qquad \text{(Abbas Et al., (2006) Eq. 22)}$$

Where M_t is the flange transverse bending moment, x is the location along the flange in the transverse direction, and I_t is the moment of inertia of the flange about its major axis, (coincides with the global y-axis).

To calculate the transverse bending moment, two methods are introduced: the fictitious load method and the C-factor method. Abbas Et al., (2007) introduces the simplified C-factor method to calculate the transverse bending moment which will be discussed. The C-factor method relates the transverse bending moment of a sinusoidally corrugated web girder to other web geometries.

The C-factor method involves the relationship of transverse shear and transverse moment acting on the girder. The flange transverse shear is found by the relation:

$$V_t = \frac{dM_t}{dz} = -\frac{2V_y e}{h} + A_1 \qquad \text{(Abbas Et al., (2007) Eq. 1)}$$

Where V_y is the primary shear, h is the effective depth of the section considering thin-walled sections, e is the web eccentricity, and A_1 is a constant of integration based on the boundary conditions of the flanges.

The flange transverse moment can be found by the moment-curvature analysis relationship:

$$M_t = -EI_t \frac{d^2u_t}{dz^2} \qquad \text{(Abbas Et al., (2007) Eq. 2)}$$

Where E is the modulus of elasticity, I_t is the moment of inertia of the flange about the y-axis, and u_t is the flange transverse displacement.

For calculating the transverse bending moment using the C-factor method, Abbas Et al., (2007) solves the above equations for two cases: a simply supported span with a uniformly distributed load and a simply supported span with a concentrated end moment. Both equations apply to a sinusoidally corrugated web girder as the base beam. The transverse bending moment of a sinusoidal corrugated web plate girder subject to a uniformly distributed load is found from:

$$M_t = \frac{p_y L^2 e_0}{2\pi n h}\left\{\left[1 - 2\frac{z}{L}\right]\cos\left(2\pi n\frac{z}{L}\right) + \frac{2}{2\pi n}\sin\left(2\pi n\frac{z}{L}\right) + \left[\cos(2\pi n) - \frac{2}{2\pi n}\sin(2\pi n) + 1\right]\frac{z}{L} - 1\right\} \qquad \text{(Abbas Et al., (2007) Eq. 3)}$$

Where e_0 is the amplitude of corrugations or $h_r/2$, z is the longitudinal position, n is the number of corrugations, and L is the length of the span.

Abbas Et al., (2007) indicated that the transverse bending moment is related to the accumulated area under the corrugation profile. As a result, a ratio known as the C-factor is introduced which relates the area of a sinusoidal corrugated web to other web geometries. The C-factor for various web geometries can be found in the following table:

Table 2-1 C-Factors

Profile	C-factor
Sinusoidal	1
Trapezoidal	$\pi\left[\frac{b + d/2}{L_0}\right]$
Triangular	$\pi/4$
Rectangular	$\pi/2$

Once the transverse bending moment of a sinusoidal corrugated web plate girder has been calculated, it can be directly related to other geometries using the corresponding C-factors by the relation:

$$\frac{M_t^I}{M_t^J} = \frac{C^I}{C^J} \qquad \text{(Abbas Et al., (2007) Eq. 31)}$$

Where I and J represent the two profiles being related.

The fictitious load method applies loads transversely to the flange to determine the transverse shear, moment, and displacement of the flanges (Abbas Et al., 2006). The fictitious load is given as a load per unit length by:

$$p_t = \frac{2}{h}\left[V_y \frac{de}{dz} + e\frac{dV_y}{dz}\right] \qquad \text{(Abbas Et al., (2007) Eq. 5)}$$

Where h is the effective depth of the section considering thin-walled elements, V_y is the primary shear, and e is the web eccentricity. Figure 2.4 demonstrates this fictitious loading for a triangular web profile.

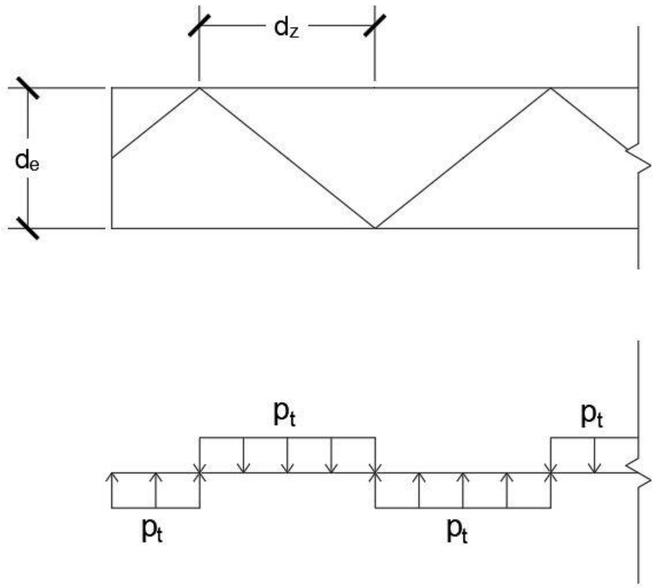

Figure 2.4 Fictitious Loading on Triangular Profile

Two special conditions must also be considered when calculating the transverse load. The first involves when the web eccentricity is discontinuous. This only occurs in webs with rectangular profiles. The fictitious load for this point is found from:

$$P_t = \frac{2}{h} V_y \Delta e$$ (Abbas Et al., (2007) Eq. 6)

Where Δe is the change of web eccentricity.

The second condition is when the primary shear is discontinuous. This occurs at locations with point loads. The fictious load is then found from:

$$P_t = \frac{2}{h} e \Delta V_y$$ (Abbas Et al., (2007) Eq. 7)

Where ΔV_y is the change in primary shear.

Once these fictitious loads are determined, they are applied transversely to the flange. From these new loads, the transverse shear, moment, and displacement may be found using conventional structural analysis methods. Finally, the superposition of stresses due to in-plane bending and flange transverse bending may be calculated by:

$$\sigma = \frac{M_x Y}{I_x} + \frac{M_t x}{I_t} \qquad \text{(Abbas Et al., (2006) Eq. 22)}$$

2.4.2 Eurocode

Eurocode also has standards for flexural strength derived with some of the same assumptions. Section D.2.1 is used to determine flexural strength (EN 1993-1-5: Eurocode 3, Design of steel structures, Part 1-5: Plated structural elements). Nominal moment strength is derived as the smallest axial resistances of either of the flanges times the distance between the centroids of the flanges. Lateral-torsional buckling may influence the strength if the compression flange is not braced adequately. However, studies have found that the corrugated web provides some additional resistance against lateral-torsional buckling compared to flat web girders. Flexural strength must also consider a reduction of yield stress due to transverse moment resulting from the shear flow in the flanges.

If there is a large shear force in the cross section, the axial resistance may be influenced by lateral bending. The maximum transverse moment is found where the inclined part of the web intersects the center on the flange. The maximum transverse moment can be found from:

$$M_{z,max} = \frac{V a_3}{4h_w}(2a_1 + a_4) \qquad \text{(EN 1993-1-5 Commentary Eq. 13.1)}$$

Flange induced buckling must also be considered and prevented. Rules from EN 1993-1-5 section 4.4(1) and 4.4(2) are used when calculating flange induced buckling with new buckling coefficients introduced for the corrugated web. Two flange buckling modes are possible. One considers plate buckling while the other considers torsional buckling of the flange around the centerline. The new buckling coefficient considers a hinged support along the web. Instead of calculating the buckling coefficient, a conservative value of 0.60 is also provided.

Similar, to other methods, Eurocode considers only the flanges when determining the flexural strength of the girder. The flexural strength is the smallest value calculated by the following equations:

For yielding of the tension flange:

$$M_{y,Rd} = \frac{b_2 t_2 f_{yf,r}}{\gamma_{M0}}\left(h_w + \frac{t_1 + t_2}{2}\right) \quad \text{(EN 1993-1-5 Eq. D.1)}$$

For yielding of the compression flange:

$$M_{y,Rd} = \frac{b_1 t_1 f_{yf,r}}{\gamma_{M0}}\left(h_w + \frac{t_1 + t_2}{2}\right) \quad \text{(EN 1993-1-5 Eq. D.1)}$$

For the compression flange considering lateral-torsional buckling:

$$M_{y,Rd} = \frac{b_1 t_1 \chi f_{yf}}{\gamma_{M1}}\left(h_w + \frac{t_1 + t_2}{2}\right) \quad \text{(EN 1993-1-5 Eq. D.1)}$$

Where b is the flange width, t is the flange thickness, h_w is the height of the web, and $f_{yf,r}$ is the reduced yield stress of the flanges due to transverse moment which can be found by:

$$f_{yf,r} = f_{yf} f_T \quad \text{(EN 1993-1-5 Eq. D.1)}$$

Where the reduction factor is given by:

$$f_T = 1 - 0.4\sqrt{\frac{\sigma_x(M_z)}{\frac{f_{yf}}{\gamma_{M0}}}} = 1 - 0.4\sqrt{\frac{6 M_z \gamma_{M0}}{f_{yf} b_f^2 t_f}} \quad \text{(EN 1993-1-5 Commentary Eq. 13.2)}$$

Where $\sigma_x(M_z)$ is the stress in the flange due to transverse moment, χ is a reduction factor for out of plane buckling from section 6.3 of EN 1993-1-1, and γ_{M0} is a safety factor. The transverse moment, M_z, is due to the shear flow of the flanges, as shown above.

2.5 Accordion Effect

The accordion effect is the stretching and contraction of the corrugated web due to the low axial stiffness. Due to the low rigidity, previously derived equations for flexural strength do not consider the interaction between the web and flanges. Recent research has found that some flexural strength may be attributed to the web (Innam Et al., 2022). Since the web contribution is assumed to be zero in design equations, additional flexural strength from the web may be considered. However, this increase in moment resistance is often minimal and typically neglected. The main influences on the accordion effect are the corrugation angles, flange proportions, and web proportions. If the web contribution is to be considered, a web participation factor is defined to quantify the accordion effect. Web participation is the inverse of the accordion effect with coarse corrugations undergoing a higher accordion effect than dense corrugations (Innam Et al., 2022).

Web participation decreases with an increase in corrugation angle. The web participation in moment resistance remains the same when the flanges width increases. However, web participation increases significantly when the flange thickness increases. The accordion effect increases with an increase in web height, decreasing the web participation. If the accordion effect is low, the web contribution can be considered when calculating flexural strength. Innam Et al., (2022) proposed conditions that must be met to consider web participation.

2.6 Fatigue Life

The initiation and formation of cracks in metals under cyclic stresses is known as fatigue. Fatigue is the result of repetitive loading, often seen in plate girders. Welds are prone to crack initiation. Stiffeners on normal plate girders are welded to the web and flanges. The details in these locations result in a reduced fatigue life due to stress concentrations. At these connections, the fatigue strength is determined by factors including the number of cycles, applied stress range, and connection type (Ibrahim Et al., 2006).

Due to the elimination of stiffeners, the use of a corrugated web provides an increased fatigue life compared to normal plate girders requiring stiffeners. Fatigue life of corrugated web plate girders is 50-80% longer than plate girders with full-depth stiffeners and 30-50% longer than plate girders with stiffeners cut short of the tension flange (Ibrahim Et al., 2006). Due to repetitive loading, stresses have a high variation through the life of the girder, making bridges a good application for corrugated web plate girders. Longer fatigue life results in lower inspection costs, reduction in material, and labor costs (Ibrahim Et al., 2006).

Corrugated web plate girders subject to fatigue result in cracks occurring in the tension flange. With fillet welds being used to connect the flanges to the webs, cracks begin at the toe of the web-to-flange weld at the connection between the parallel and inclined web panels. The crack then spreads in the direction perpendicular to the longitudinal stress in the tension flange. These cracks may either spread across the whole width of the flange or only through part of it, additionally spreading into the web.

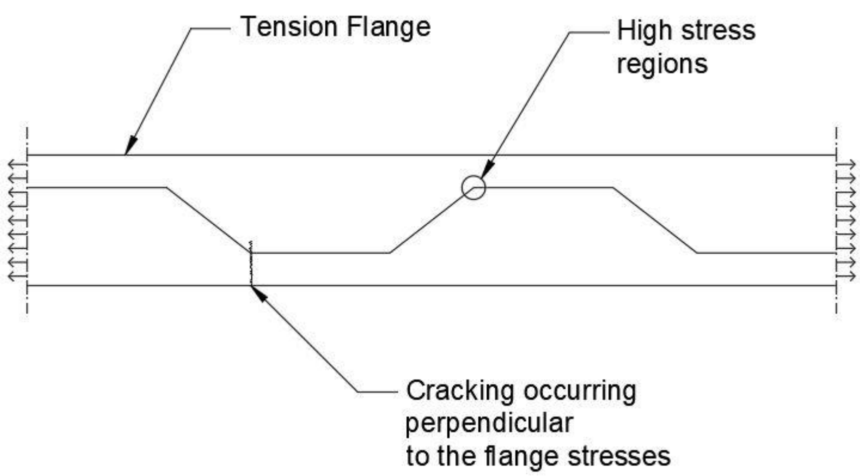

Figure 2.5 Fatigue in the Tension Flange

Longitudinal stresses measured in the parallel folds are 2.5-3.0 times what is measured in the inclined folds of the web. Due to the "accordion effect" and flexibility of the web, these stresses decrease significantly the farther the distance from the flange. Dividing the bending moment by the average cross-section height results in a good approximation of the force in the flanges (Ibrahim Et al. 2006).

2.7 Examples in Industry

Examples of corrugated web plate girders are seen in Europe and Asia. Most uses occur in composite bridge girders. The following are a few examples in industry of corrugated webs around the world.

Located in France, the Cognac Bridge is the first corrugated steel web bridge in the world. Completed in 1986, the bridge is a three-span continuous box girder bridge. The longest span of the bridge is 42.91 meters while the total length is 107.82 meters. The thickness of the web is 8 mm with a web height of 1.771 meters (Sayed-Ahmed 2001).

Figure 2.6 Cognac Bridge (The Constructor)

Another bridge that uses corrugated webs is the Dole Bridge. Also located in France, the bridge has seven spans with a total length of 496 meters and longest span of 80 meters. Another composite box girder bridge, the webs are tapered ranging with a thickness of 8 mm to 12 mm near supports. The depth of the tapered bridge varies from 2.5 meters at midspan to 5.5 meters at the supports (Sayed-Ahmed 2001).

Figure 2.7 Dole Bridge (Sayed-Ahmed, 2003)

The longest span corrugated web bridge is located in Japan. The Aigawa Bridge consists of two separate bridges, one with the longest span of 179 meters. The depth of the bridge is 11.5 meters. The corrugated web design was chosen to lower seismic forces and have a lightweight structure. Japan has quickly become a popular area to use corrugated webs with over 200 uses in bridges since the early 1990s (Structurae 2018).

Chapter 3: Corrugated Web Plate Girder Design Examples

The following chapter will present calculations for shear and moment strength of a corrugated web plate girder designed using proposed North American equations as well as Eurocode equations. Results will also be compared to a flat web plate girder designed using AISC 360 *Specification for Structural Steel Buildings*. Detailed calculations for each girder can be found in Appendices A, B, and C.

A 100 foot simply supported beam subject to uniform load of 5 k/ft was used to design the girders. The girder is assumed to be braced against lateral-torsion buckling. Table 3-1 presents the results of the calculations. Flange dimensions, web dimensions, and weights for each girder are presented. The same web corrugation geometry was used for both designs of corrugated web plate girders. A yield stress of $F_y = 36\ ksi$ was used across all three girders.

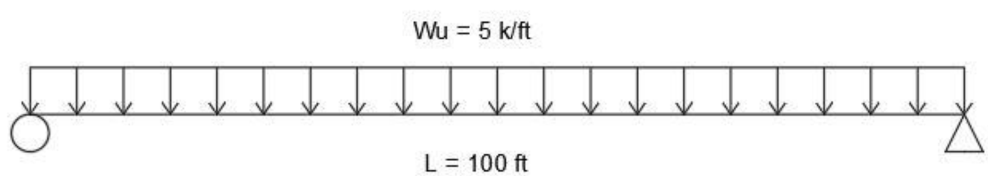

Figure 3.1 Beam Loading

The first step is to determine the loads the girder must resist.

$$M_u = \frac{wl^2}{8} = \frac{(5\ k/ft)(100\ ft)^2}{8} = 6250\ k - ft$$

$$V_u = \frac{wl}{2} = \frac{(5\ k/ft)(100\ ft)}{2} = 250\ k$$

Accounting for shear and moment due to self-weight of the corrugated web plate girders results in the loads both girders must resist (girder weights are from the final design, see Table 3-1):

Table 3-1 Combined Eurocode and North American Designs

	Flat web plate girder with stiffeners	Corrugated web plate girder (Eurocode)	Corrugated web plate girder (North American)
Flange Width	28 in.	26 in. (0.6604 m)	26 in.
Flange Thickness	1.125 in.	1.125 in. (0.0286 m)	1.125 in.
Web Height	82 in.	82 in. (2.0828 m)	82 in.
Stiffeners	(32) 82 in. x 6 in. x 5/16 in.	---	---
Web Thickness	0.25 in.	0.25 in. (0.00635 m)	0.25 in.
Weight	29952.1 lbs.	27558.3 lbs.	27558.3 lbs.

For Eurocode:

$$M_u = 1.2\frac{\left(\frac{27796.5\ lbs}{1000 * 100}\right)(100\ ft)^2}{8} = 416.9\ k - ft + 6250k - ft$$

$$= 6667\ k - ft\ or\ 9039\ kN - m$$

$$V_u = 1.2\frac{27796.5\ lbs}{2000} = 16.7\ k + 250\ k = 266.7\ k\ or\ 1186\ kN$$

For the North American Method

$$M_u = 1.2\frac{\left(\frac{27796.5\ lbs}{1000 * 100}\right)(100\ ft)^2}{8} = 416.9\ k - ft + 6250k - ft$$

$$= 6667\ k - ft$$

$$V_u = 1.2\frac{27796.5\ lbs}{2000} = 16.7\ k + 250\ k = 266.7\ k$$

The flange dimensions for both the Eurocode and North American girders will be 26 in. (0.6604 m) wide and 1.125 in. (0.0286 m) thick. The web dimensions are 82 in. (2.0828 m) tall and 0.25 in. (0.00635 m) thick. The corrugation geometry for the web and cross section are as follows:

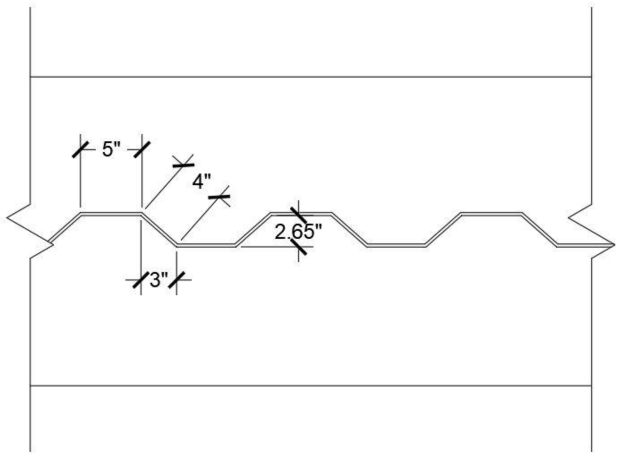

Figure 3.2 Example Girder Web Geometry

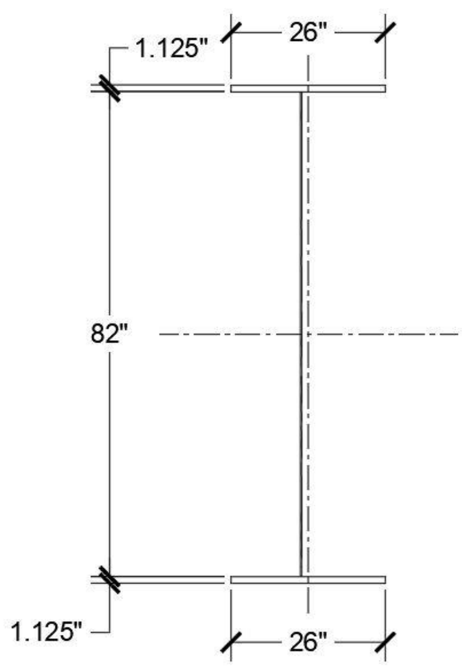

Figure 3.3 Example Girder Cross-Section

3.1 Eurocode Design

Flexure will be designed first. The transverse moment due to the web geometry must first be calculated.

$$M_{z,max} = \frac{V a_3}{4h_w}(2a_1 + a_4) \qquad \text{(EN 1993-1-5 Commentary Eq. 13.1)}$$

$$M_{z,max} = \frac{(1186\ kN)(0.0672\ m)}{4(2.0828\ m)}(2(0.127\ m) + 0.0762m) = 3.2\ kN - m$$

Next, the yield stress of the flanges must be reduced due to the transverse moment.

$$f_T = 1 - 0.4\sqrt{\frac{\sigma_x(M_z)}{\frac{f_{yf}}{\gamma_{M0}}}} = 1 - 0.4\sqrt{\frac{6M_z\gamma_{M0}}{f_{yf}b_f^2 t_f}} \qquad \text{(EN 1993-1-5 Commentary Eq. 13.2)}$$

$$f_T = 1 - 0.4\sqrt{\frac{6(3.2\ kN - m)(1)}{(2.5 * 10^5\ kPa)(0.6604\ m)^2(0.0286\ m)}} = 0.969$$

$$f_{yf,r} = f_{yf} f_T \qquad \text{(EN 1993-1-5 Eq. D.1)}$$

$$f_{yf,r} = (2.5 * 10^5\ kPa)(0.969) = 242201\ kPa$$

With the reduced yield stress, the flexural strength of the girder may now be determined. Since both of the flanges are the same dimensions and the girder is restrained against lateral-torsional buckling. Only one limit state must be checked.

$$M_{y,Rd} = \frac{b_2 t_2 f_{yf,r}}{\gamma_{M0}}\left(h_w + \frac{t_1 + t_2}{2}\right) \qquad \text{(EN 1993-1-5 Eq. D.1)}$$

$$M_{y,Rd} = \frac{(0.6604\ m)(0.0286\ m)(242201\ kPa)}{(1)}\left(2.0828m + \frac{(0.0286\ m + 0.0286\ m)}{2}\right)$$

$$M_{y,rd} = 9650\ kN - m$$

$$M_u = 9039\ kN - m \leq 9650\ kN - m$$

The calculated moment is greater than the ultimate moment resulting in an adequate design for flexure.

The first step for shear is to determine the governing panel width.

$$a_{max} = 0.13\ m$$

Using this value, the critical local shear stress is determined by:

$$\tau_{cr,l} = 4.83E\left[\frac{t_w}{a_{max}}\right]^2$$ (EN 1993-1-5 Eq. D.7)

$$\tau_{cr,l} = 4.83(2.1 * 10^8 kPa)\left[\frac{0.00635\ m}{0.13\ m}\right]^2 = 2.54 * 10^6\ kPa$$

The local slenderness ratio is then determined by:

$$\bar{\lambda}_{c,l} = \sqrt{\frac{f_{yw}}{\tau_{cr,l}\sqrt{3}}}$$ (EN 1993-1-5 Eq. D.6)

$$\bar{\lambda}_{c,l} = \sqrt{\frac{2.5 * 10^5\ kPa}{(2.54 * 10^6)\sqrt{3}}} = 0.24$$

Using this local slenderness ratio, the safety factor for local buckling if found from:

$$\chi_{c,l} = \frac{1.15}{0.9 + \bar{\lambda}_{c,l}} \leq 1.0$$ (EN 1993-1-5 Eq. D.5)

$$\chi_{c,l} = \frac{1.15}{0.9 + 0.26} = 1.01 \leq 1.0$$

With the safety factor for local buckling now determined, the safety factor for global buckling must be calculated. First, the factors D_x and D_z must be calculated from:

$$D_x = \frac{Et_w^3}{12(1-v^2)}\frac{w}{s} = \frac{Et_w^3}{12(1-v^2)}\frac{a_1 + a_4}{a_1 + a_2}$$ (EN 1993-1-5 Commentary Eq. 13.10)

$$D_x = \frac{(2.1 * 10^8\ kPa)(0.00635\ m)^3}{12(1 - 0.28^2)}\frac{0.127\ m + 0.0762\ m}{0.127\ m + 0.1\ m} = 4.32\ kN - m^3$$

$$D_z = \frac{EI_z}{w} = \frac{Et_w a_3^2}{12} \frac{3a_1 + a_2}{a_1 + a_4}$$ (EN 1993-1-5 Commentary Eq. 13.11)

$$D_z = \frac{(2.1 * 10^8\ kPa)(0.00635\ m)(0.048\ m)^2}{12} \frac{3(0.127\ m + 0.1\ m)}{(0.127\ m + 0.0762\ m)} = 1191.9\ kN - m^3$$

Using these values, the critical global shear stress may be calculated from:

$$\tau_{cr,g} = \frac{32.4}{t_w h_w^2} \sqrt[4]{D_x D_z^3}$$ (EN 1993-1-5 Eq. D.10)

$$\tau_{cr,g} = \frac{32.4}{(0.00635\ m)(2.0828\ m)^2} \sqrt[4]{(4.32\ kN - m^3)(1191.9\ kN - m^3)^3} = 3.44 * 10^5\ kPa$$

This value is used to determine the global slenderness ratio:

$$\bar{\lambda}_{c,g} = \sqrt{\frac{f_{yw}}{\tau_{cr,g}\sqrt{3}}}$$ (EN 1993-1-5 Eq. D.9)

$$\bar{\lambda}_{c,g} = \sqrt{\frac{2.5 * 10^5\ kPa}{(3.44 * 10^5\ kPa)\sqrt{3}}} = 0.648$$

With the global slenderness ratio determined, the safety factor for global buckling finally may be determined from:

$$\chi_{c,g} = \frac{1.5}{0.5 + \bar{\lambda}_{c,g}^2} \leq 1.0$$ (EN 1993-1-5 Eq. D.8)

$$\chi_{c,g} = \frac{1.5}{0.5 + (0.648)^2} = 1.6 \leq 1.0$$

The local buckling safety factor governs and is used to determine the shear strength:

$$V_{bw,Rd} = \chi_c \frac{f_{yw}}{\gamma_{M1}\sqrt{3}} h_w t_w$$ (EN 1993-1-5 Eq. D.4)

$$V_{bw,Rd} = (1.0) \frac{(2.5 * 10^5\ kPa)}{(1)\sqrt{3}} (2.0828\ m)(0.00635\ m) = 1909\ kN$$

$$V_u = 1186\ kN \leq 1909\ kN$$

The calculated value for shear resistance is greater than the ultimate shear resulting in an adequate design.

The final check for Eurocode involves checking for flange induced buckling. The buckling factor must first be determined by the greater of:

$$k_\sigma = 0.43 + \left(\frac{b}{a}\right)^2 \qquad \text{(EN 1993-1-5 Eq. D.2)}$$

$$k_\sigma = 0.43 + \left(\frac{\frac{0.6604\ m + 0.0672\ m}{2}}{0.127\ m + 2(0.0762\ m)}\right)^2 = 2.13$$

$$k_\sigma = 0.6 \qquad \text{(EN 1993-1-5 Eq. D.3)}$$

Next, the plate slenderness is determined:

$$\varepsilon = \sqrt{\frac{235}{f_y\left[\frac{N}{mm^2}\right]}} \qquad \text{(EN 1993-1-5 Eq. 4.3)}$$

$$\varepsilon = \sqrt{\frac{235}{250\ MPa}} = 0.97$$

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\bar{b}/t}{28.4\varepsilon\sqrt{k_\sigma}} \qquad \text{(EN 1993-1-5 Eq. 4.3)}$$

$$\bar{\lambda}_p = \frac{0.36\ m/0.0286\ m}{28.4(0.97)\sqrt{2.13}} = 1.4$$

With the plate slenderness calculated, a reduction factor may be determined and applied to the gross area of the compression flange to find the effective area.

$$\rho = \frac{\bar{\lambda}_p - 0.188}{\bar{\lambda}_p^2} \leq 1.0\ for\ \bar{\lambda}_p > 0.748 \qquad \text{(EN 1993-1-5 Eq. 4.3)}$$

$$\rho = \frac{1.4 - 0.188}{1.4^2} = 0.61 \leq 1.0$$

$$A_{e,ff} = \rho A_e \qquad \text{(EN 1993-1-5 Eq. 4.1)}$$

$$A_{e,ff} = 0.6(0.6604\,m)(0.0286\,m) = 0.0115\,m^2$$

Finally, the limit for flange induced buckling is checked with the following equation:

$$\frac{h_w}{t_w} \leq k\frac{E}{f_{yf}}\sqrt{\frac{A_w}{A_{fc}}} \qquad \text{(EN 1993-1-5 Eq. 8.1)}$$

$$\frac{2.0828\,m}{0.00635\,m} = 328 \leq (0.55)\frac{(2.1 * 10^8\,kPa)}{(2.5 * 10^5\,kPa)}\sqrt{\frac{(2.0828\,m)(0.00635\,m)}{(0.0115\,m^2)}} = 496$$

The final design for the corrugated web plate girder design using Eurocode can be found in Table 3-1.

3.2 North American Proposed Method Design

Shear will be designed for first using the proposed North American methods. The girder has the same dimensions as the girder design using Eurocode. First, the factor that defines the web geometry must first be calculated using values for α and β.

$$\alpha = \cos^{-1}\left(\frac{3\,in}{4\,in}\right) = 41.4°$$

$$\beta = \frac{b}{c} = \frac{5\,in}{4\,in} = 1.25$$

$$F(\alpha,\beta) = \sqrt{\frac{(1+\beta)\sin^3\alpha}{\beta + cos\alpha}} \cdot \left\{\frac{3\beta + 1}{\beta^2(\beta+1)}\right\}^{3/4} \qquad \text{(Driver Et al., (2006) Eq. 3)}$$

$$F(\alpha,\beta) = \sqrt{\frac{(1+1.25)\sin^3(41.4)}{1.25 + \cos\,(41.4)}} \cdot \left\{\frac{3(1.25) + 1}{1.25^2(1.25+1)}\right\}^{3/4} = 0.51$$

With this coefficient determined, the limit for web yielding is checked:

$$\frac{h_w}{t_w} \leq 1.91\psi\sqrt{\frac{E}{F_y}\left(\frac{b}{t_w}\right)^{1.5} F(\alpha,\beta)}$$ (Driver Et al., (2006) Eq. 11)

$$\frac{82\text{ in}}{0.25\ in} \leq 1.91(0.9)\sqrt{\frac{29000\ ksi}{36\text{ ksi}}\left(\frac{5\ in}{0.25\ in}\right)^{1.5}(0.51)}$$

$$328 \leq 328.1$$

Also satisfying the following limit will result in economical web material usage:

$$\frac{b}{t_w} \leq 2.586\sqrt{\frac{E}{F_y}}$$ (Driver Et al., (2006) Eq. 16)

$$\frac{5\ in}{0.25\text{ in}} \leq 2.586\sqrt{\frac{29000\ ksi}{36\text{ ksi}}}$$

$$20 \leq 73.4$$

Next, the local slenderness ratio can be calculated from the following equation:

$$\lambda_L = \frac{w}{t_w}\sqrt{\frac{F_y}{E}}$$ (Driver Et al., (2006) Eq. 15)

$$\lambda_L = \frac{5\ in}{0.25\text{ in}}\sqrt{\frac{36\text{ ksi}}{29000\ ksi}} = 0.7$$

This slenderness ratio falls withing the limit for web yielding which can be calculated from:

$$V_n = 0.707\left(\frac{F_y}{\sqrt{3}}\right)h_w t_w$$ (Driver Et al., (2006) Eq. 12)

$$V_n = 0.707\left(\frac{36\text{ ksi}}{\sqrt{3}}\right)(82\text{ in})(0.25\text{ in}) = 301.2\ k$$

Applying a resistance factor of $\phi = 0.9$ results in the design strength of the girder.

$$\phi V_n = 0.9(301.2\ k) = 271.1\ k$$

$$271.1\ k\ \geq 267\ k$$

Finally, the design for flexure will be shown using the C-factor method for calculating transverse moment. The first step involves calculating the transverse moment for a sinusoidally corrugated girder with the same wavelength. The maximum transverse moment will be calculated at the end of the girder. A corrugation wavelength is equal to 16 in. The total number of wavelengths (n) is equal to 75. The longitudinal position (z) that results in the maximum transverse moment is one half of a wavelength. The longitudinal position along the wavelength for this example is equal to 8 in.

$$M_t = \frac{p_y L^2 e_0}{2\pi n h}\left\{\left[1 - 2\frac{z}{L}\right]\cos\left(2\pi n\frac{z}{L}\right) + \frac{2}{2\pi n}\sin\left(2\pi n\frac{z}{L}\right) + \left[\cos(2\pi n) - \frac{2}{2\pi n}\sin(2\pi n) + 1\right]\frac{z}{L} - 1\right\}$$ (Abbas Et al., (2007) Eq. 3)

$$M_t = \frac{(5\ k/ft)(100\ ft)^2(\frac{2.65}{2}in)}{2\pi(75)(84.25\ in)}\left\{\left[1 - 2\frac{(8\ in/12)}{(100\ ft)}\right]\cos\left(2\pi(75)\frac{(8\ in/12)}{(100\ ft)}\right) + \frac{2}{2\pi(75)}\sin\frac{(8\ in/12)}{(100\ ft)} + \left[\cos(2\pi(75)) - \frac{2}{2\pi(75)}\sin(2\pi(75)) + 1\right]\frac{(8\ in/12)}{(100\ ft)} - 1\right\} = 3.29\ k - ft$$

Using the C-factors for trapezoidal and sinusoidal profiles, the transverse moment for the girder can be calculated using the following relationship:

For sinusoidal profiles:

$$C = 1$$

For trapezoidal profiles:

$$C = \pi \left[\frac{b + d/2}{L_0}\right] = \pi \left[\frac{5\ in + 3\ in/2}{16\ in}\right] = 1.3$$

$$\frac{M_t^I}{M_t^J} = \frac{C^I}{C^J}$$ (Abbas Et al., (2007) Eq. 31)

$$M_t = \frac{1.3}{1} * 3.29\ k - ft = 4.2\ k - ft$$

With the transverse moment now determined, the superposition of bending stress and flange transverse bending stress may be determined:

$$I_t = \frac{(26\ in)(1.125\ in)^3}{12} + (26\ in)(1.125\ in)\left(\frac{(82\ in + 1.125\ in)}{2}\right)^2 = 50531\ in^4$$

$$x = \frac{2.65\ in}{2} = 1.32\ in$$

$$\sigma_t = \frac{M_t x}{I_t} = \frac{(4.2\ k - ft)(12)(1.32\ in)}{50530.7\ in^4} = 0.0013\ ksi$$

$$I_x = \frac{A_{bf} A_{tf} h^2}{A_{bf} + A_{tf}} = \frac{2(26\ in)(1.125\ in)(82\ in + 2 * 1.125\ in)}{(26\ in)(1.125\ in) + (26\ in)(1.125\ in)} = 103809\ in^4$$

$$Y = \frac{(82\ in + 2 * 1.125\ in)}{2} = 42.1\ in$$

$$\sigma_b = \frac{M_x Y}{I_x} = \frac{(6667\ k - ft)(12)(42.1\ in)}{103809\ in^4} = 32.5\ ksi$$

$$\sigma_{total} = 32.5\ ksi + 0.0013\ ksi = 32.5013\ ksi$$

Applying a resistance factor of $\phi = 0.9$ results in the stress of the girder:

$$36\ ksi \geq 36\ ksi$$

Table 3-1 summarizes the results of the two corrugated web plate girder design as well as their total weights. The design for the flat web plate girder with stiffeners can be found in Appendix A.

The corrugated web plate girder designed using the proposed North American equations and Eurocode resulted in the same weight. The weight of the corrugated web plate girders is less than the weight of the flat web plate girder with stiffeners. This comparison used the same web thickness and heights. The thickness of the corrugated web plate girder designed using Eurocode could be reduced to a 3/16 in. thick web, resulting in even less steel. Table 3-2 shows the design strengths and the ratio of ultimate load to design strengths.

Table 3-2 Girder Strengths Comparison

	Shear		Flexural	
	ϕV_n	$\frac{V_u}{\phi V_n}$	$\phi M_n \ or \ F_y$	$\frac{M_u}{\phi M_n} or \frac{\sum f}{F_y}$
Flat web plate girder with stiffeners	278.3 k	0.96	6686.6 k-ft	0.99
Corrugated web plate girder (Eurocode)	429 k (1909 kN)	0.62	7117.5 k-ft (9650 kN-m)	0.94
Corrugated web plate girder (North American)	271.1 k	0.98	36 ksi	1.0

As seen in Table 3-2, the shear strength of the corrugated web plate girder using Eurocode utilizes much less compared to the proposed North American Method. As a result, less web material may be used. The use of a corrugated web may result in a lower self-weight compared to a flat web plate girder for certain applications. The self-weight of the girder will also depend on the code being used. Automated production of the girder may also result in reduced manufacturing costs, avoiding welding individual stiffeners along the web.

Chapter 4: Conclusion

Steel is a common choice of material for various construction projects. Plate girders can be good choices for projects when rolled shapes are not feasible. Plate girders typically have a high resistance to shear and moment as the plate sizes are specifically designed to resist the required loads, often changing sizes for longer spans. To resist high shear loads, stiffeners are provided. However, as they add weight and increase fabrications costs significantly, as well as worsen fatigue problems in the girder. Corrugated web plate girders offer an innovative approach to plate girders without stiffeners, reducing the total amount of steel and particularly fabrication cost.

With the new geometry of the girder associated with the web, different behaviors occur. The web is subject to the limit states including global shear buckling, local shear buckling, and shear yielding. Compared to normal plate girders, the web has a lower post buckling strength. As a result, factors are applied within the shear strength equations based on both experimental and finite element analysis results. For the design to be optimized, yielding of the web is the preferred limit state. The equations derived from various researchers to estimate the shear strength of the corrugated web differ slightly but result in similar values. Flexural strength is determined based on the flanges only with no contribution from the web. Due to the low axial rigidity of the web known as the accordion effect, the web does not contribute to the flexural strength.

Currently, all applications of corrugated web plate girders are seen in Europe and Asia. *Eurocode 3 – Design of steel structures – Part 1-5: Plated structural elements* is the main code used to design corrugated web plate girders. Some standard size girders are available in Europe in both trapezoidal and sinusoidal web geometries. There are no code provisions in the United

States, and further research must be performed to include them into North American codes. While some research to implement them into AISC and AASHTO is available, it is lacking as compared to the information available for EN 1993-1-5. Most research on corrugated web plate girders involve the shear strength of the girder but not the flexural strength.

Steel plate girders are not the only use for corrugated webs. Other applications of corrugated web are also undergoing research. One of these is the use of prestressed composite box girders with corrugated webs. These girders use reinforced concrete for flanges and corrugated steel for webs results in a low structural weight and a quicker construction time. Another application of corrugated webs is at the reduced beam section in moment frames. Reduced beam sections are used to weaken a beam near the beam-column connection in a moment frame. This reduction of strength is to create a hinge which experiences a nonlinear behavior. Typically, both flanges are cut to reduce the flexural strength and cause this failure. Creating a corrugated section known as an accordion-web reduced beam section is a new method proposed. Since a corrugated web does not contribute to the flexural capacity of a beam, the reduced beam section will result in a lower flexural capacity.

While corrugated web plate girders are not commonly used, they have promising potential. The main obstacle as to why these are not seen in North America is that there are no available codes. Standard web sizes and geometries would also benefit usage allowing to easily choose sizes. The other main disadvantage is manufacturing costs. With automatic processes like what is available in Austria, manufacturing costs may be reduced allowing for corrugated web plate girders to become an economical option. Corrugated web plate girders have many advantages compared to normal plate girders.

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Appendix A – AISC Plate Girder Design Example

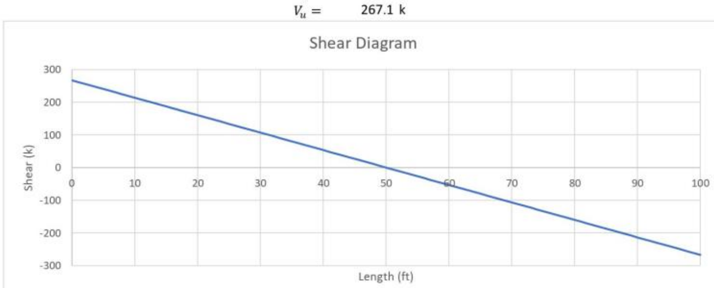
The following appendix demonstrates a normal plate girder design example based on AISC 360.

Plate Girder Design Example	Master's Report	Derek Leamer

The simply supported girder is 100 feet long with a uniformly distributed load. A design example is shown to calculate the moment and shear strengths of the girder.

Conservatively designing the girder using AISC 360 section F5. Assuming Restrained against lateral-torsional buckling.

Step Description	Calculations	Reference
		AISC 360 UNO
	$w_u =$ 5 k/ft	
Required Moment		
	L = 100 ft	
	$w_u =$ 5 k/ft	
	$M_u = \frac{wl^2}{8} =$ 6250 k-ft	Steel Manual Table 3-23
	$V_u = \frac{wl}{2} =$ 250 k	
	Density of A36 Steel = 0.284 lb/in^3	
	Area of Flanges = 63 in^2	
	Area of Web = 20.5 in^2	
	Self Weight of Girder = 23.7 lb/in	
	284.6 lb/ft	
	$V_{SW} =$ 14.23 k	
	$1.2V_{SW} =$ 17.07 k	
	$M_{SW} =$ 355.7 k-ft	
	$1.2M_{SW} =$ 426.9 k-ft	
	$M_u =$ 6676.9 k-ft	
	$V_u =$ 267.1 k	
Shear Diagram		
Moment Diagram		



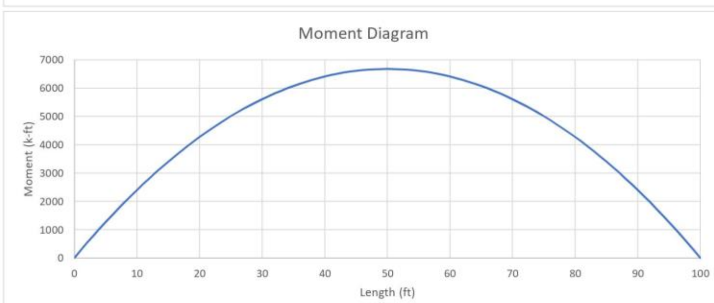


Plate Girder Design Example	Master's Report	Derek Leamer

Step Description	Calculations	Reference
Flexural Strength	Flange: Width (b_f) 28 in; Height (t_f) 1.125 in Web: Width (t_w) 0.25 in; Height (h_c) 82 in E 29000 ksi F_y 36 ksi	
Web Slenderness Limits	$\lambda = \frac{h}{t_w} =$ 328	Table B4.1b Case 15
	Compact/Noncompact $\lambda_p = 3.76\sqrt{\frac{E}{F_y}} =$ 106.7	
	Noncompact/Slender $\lambda_r = 5.70\sqrt{\frac{E}{F_y}} =$ 161.8	
	Web is Slender	
1. Compression Flange Yielding	$h_c =$ 82 in	
	$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} =$ 0.65	(F4-12)
Moment of Inertia	$I_x\ web = \frac{1}{12}bh^3 =$ 11486.83 in^4	
	$I_x\ flange = 2\left(\frac{1}{12}bh^3 + Ad^2\right) =$ 108835.5 in^4	
	$I_x =$ 120322.3 in^4	
	$S_x = \frac{I_x}{(\frac{h_c}{2} + t_f)} =$ 2856.3 in^3	
	$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \leq 1.0$	(F5-6)
	$R_{pg} =$ 0.922	
	$M_n = R_{pg}F_yS_{xc} =$ 7904.6 k-ft	
2. Lateral-Torsional Buckling	$r_t = \frac{b_{fc}}{\sqrt{12\left(1 + \frac{1}{6}a_w\right)}} =$ 7.68 in	(F4-11)
	$L_p = 1.1r_t\sqrt{\frac{E}{F_y}} =$ 239.7 in or 20.0 ft	(F4-7)
	$L_r = \pi r_t\sqrt{\frac{E}{0.7F_y}} =$ 818.2 in or 68.2 ft	(F5-5)
	$L_b \leq L_p$	
	Assuming the girder is continuously braced, lateral-torsional buckling does not apply.	

Plate Girder Design Example	Master's Report	Derek Leamer

Step Description	Calculations	Reference
3. Compression Flange Local Buckling	$b = \frac{b_f}{2}$ $t = t_f$	Table B4.1b Case 11
	$\lambda = \frac{b}{t} =$ 12.44	
	Compact/Noncompact Flanges $\lambda_p = 0.38\sqrt{\frac{E}{F_y}} =$ 10.8	Table B4.1b
	$F_L = 0.7F_y =$ 25.2 ksi	Table B4.1b
	Noncompact/Slender Flanges $\lambda_r = 0.95\sqrt{\frac{k_c E}{F_L}} =$ 19.1	Table B4.1b
	Flange is Noncompact	
Noncompact Flanges	$F_{cr} = \left[F_y - (0.3F_y)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)\right] =$ 33.8 ksi	(F5-8)
Slender Flanges	$F_{cr} = \frac{0.9Ek_c}{\left(\frac{b_f}{2t_f}\right)^2} =$ 59.0 ksi	(F5-9)
	$M_n = R_{pg}F_{cr}S_{xc}/12 =$ 7429.4 k-ft	(F5-7)
4. Tension Flange Yielding	Doubly Symmetric Tension Flange and Compression Flange are the same	
	$S_{xt} \geq S_{xc}$ Limit State Does Not Apply	
	Governing Nominal Moment: M_n 7429.4 k-ft	
	ϕ_b 0.9	
	ϕM_n 6686.5 k-ft	
	$M_u =$ 6676.9 k-ft $\leq$ $\phi M_n =$ 6686.5 k-ft	

Step Description	Calculations	Reference

Shear Strength of Web Without Tension Field Action

h = 82 in
t_w = 0.25 in
F_y = 36 ksi
E = 29000 ksi
a = 2 ft
a/h = 0.292683

For Webs with Transverse Stiffeners

$$k_v = 5 + \frac{5}{\left(\frac{a}{h}\right)^2} = \quad 63.37$$ (G2-5)

$$When \frac{a}{h} > 3.0, use\ k_v = 5.34$$

$$\frac{h}{t_w} \leq 1.10\sqrt{k_v E/F_y}$$

328.0 < 248.5 Use (G2-4)

$$C_{v1} = \quad 1$$ (G2-3)

$$C_{v1} = \frac{1.10\sqrt{k_v E/F_y}}{h/t_w} = \quad 0.76$$ (G2-4)

$$A_w = h * t_w = \quad 20.5\ in^2$$

$$V_n = 0.6 F_y A_w C_{v1} = \quad 335.5\ k$$ (G2-1)

ϕ_v 0.9 G1.a

$\phi_v V_n$ 302.0 k

V_u = 267.1 k ≤ ϕV_n = 302.0 k

First stiffener must be 2' from the end of the girder

Shear Strength of Interior Web Panels with a/h ≤ 3 Considering Tension Field Action

h = 82 in
t_w = 0.25 in
F_y = 36 ksi
E = 29000 ksi
a = 6.00 ft
t_f = 1.125 in
b_f = 28 in

$$k_v = 5 + \frac{5}{\left(\frac{a}{h}\right)^2} = \quad 11.49$$

$$When \frac{a}{h} > 3.0, use\ k_v = 5.34$$

$$When\ h/t_w \leq 1.10\sqrt{k_v E/F_y}$$

328.0 ≤ 105.8 Does Not Apply

$$V_n = 0.6 F_y A_w = \quad 442.8\ k$$

$$When\ h/t_w > 1.10\sqrt{k_v E/F_y}$$

$$When\ 2A_w/(A_{fc} + A_{ft}) \leq 2.5, h/b_{fc} \leq 6.0, and\ h/b_{ft} \leq 6.0$$

$2A_w/(A_{fc} + A_{ft}) =$ 0.7 ≤ 2.5 OK

$h/b_{fc} =$ 2.9 ≤ 6 OK

$h/b_{ft} =$ 2.9 ≤ 6 OK

Use (G2-7)

Web Shear Buckling Coefficient

$1.10\sqrt{k_v E/F_y}$ $1.37\sqrt{k_v E/F_y}$

105.8 < 328.0 ≤ 131.8

Use (G2-11)

Step Description		Calculations	Reference
		$C_{v2} =$ 1	(G2-9)
		$C_{v2} = \frac{1.10\sqrt{k_v E/F_y}}{h/t_w} =$ 0.32	(G2-10)
		$C_{v2} = \frac{1.51 k_v E}{(h/t_w)^2 F_y} =$ 0.13 Use	(G2-11)
		$C_{v2} =$ 0.13	
		$V_n = 0.6F_y A_w \left[C_{v2} + \frac{1-C_{v2}}{1.15\sqrt{1+(a/h)^2}}\right] =$ 309.3 k	(G2-7)
		ϕ_v 0.9	
		$\phi_v V_n$ 278.3 k	
		$V_u =$ 267.1 k ≤ $\phi V_n =$ 278.3 k	
		Place stiffeners at 5 feet O.C.	
Intermediate Stiffeners Check (G2.3)	Note a:	$h/t_w \leq 2.46\sqrt{E/F_y}$	
		328.0 ≤ 69.8 Stiffeners Required	G2.3a
	Note d:	$b_{st} =$ 6 in	G2.3d
		$t_{st} =$ 0.3125 in	
		E = 29000 ksi	
		A36 Steel $F_{yst} =$ 36 ksi	
		$\left(\frac{b}{t}\right)_{st} \leq 0.56\sqrt{E/F_{yst}}$	(G2-12)
		19.2 ≤ 15.89	
		OK	
	Note e:	A36 Steel $F_{yst} =$ 36 ksi	G2.3e
		A36 Steel $F_{yw} =$ 36 ksi	
		E = 29000 ksi	
		h = 78 in	
		$\rho_{st} = Greater\ of:\ F_{yw}/F_{yst}$ 1.00 Governs	
		1	
		$I_{st1} = \frac{h^4 \rho_{st}^{1.3}}{40}\left(\frac{F_{yw}}{E}\right)^{1.5} =$ 40.47 in^4	(G2-14)
		a = 90 in	
		b_p Smaller of 82 in Governs	
		or 90 in	
		$t_w =$ 0.3125 in	
		$I_{st2} = \left[\frac{2.5}{\left(\frac{a}{h}\right)^2} - 2\right] b_p t_w^3 \geq 0.5 b_p t_w^3$	(G2-15)
		-0.306 in^4 1.25 in^4	
		$I_{st2} =$ 1.25 in^4	
		$V_{c1} =$ 309.3 k From Above	
		$V_{c2} =$ 57.5 k From Above using C_{v2}	
		$V_r =$ 267.1 k Required Shear Strength	

Plate Girder Design Example	Master's Report	Derek Leamer

Step Description	Calculations	Reference
	$\rho_w = \dfrac{V_r - V_{c2}}{V_c - V_{c2}} =$ 0.832	G2.3€
	Note C $I_{st2} + (I_{st1} - I_{st2})\rho_w =$ 33.9 in^4	(G2-13)
	t_{st} = 0.3125 in	
	W = 12.25 in	
	$I_{st} = \dfrac{t_w W^3}{12} =$ 47.87 in^4 OK	
Final Design	Length = 100 ft	
	Flange Thickness = 1.125 in; Web Thickness = 0.25 in	
	Flange Width = 28 in; Web Height = 82 in	
	Amount of steel for web and flanges: Flanges = 75600 in^3	
	Web = 24600 in^3	
	Stiffner Width = 6 in	
	Stiffener Thickness = 0.3125 in	
	Stiffener Height = 82 in	
	Number of Stiffeners = 36	
	Volume of One Stiffener = 146.25 in^3	
	Total Volume of Stiffeners = 5265 in^3	
	Total = 105465 in^3	
	Density of A36 Steel = 0.284 lb/in^3	
	Weight = 29952.1 lbs	

Flange Width =	28 in
Flange Thickness =	1.125 in
Web Height =	82 in
Web Thickness =	0.25 in
Weight =	29952.1 lbs

Appendix B – EN 1991-1-5 Design Example

The following appendix demonstrates a corrugated web plate girder design example based on EN 1991-1-5.

Eurocode Corrugated Web Plate Girder Design Example	Master's Report	Derek Leamer

Design of a corrugated web plate girder according to EN 1993-1-5. Using A36 Steel (Yield Strength = 250 MPa). Using the same conditions as the normal plate girder example.

Assuming retrained against lateral-torsional buckling.

Step Description	Calculations	Reference
	Top Flange Width (b_1) = 0.6604 m; Top Flange Thickness (t_1) = 0.028575 m; Bottom Flange Width (b_2) = 0.6604 m; Bottom Flange Thickness (t_2) = 0.028575 m; Yield Strength of Flange (f_{yf}) = 250000 kPa; Yield Strength of Web (f_{yw}) = 250000 kPa; Modulus of Elasticity (E) = 2.10E+08 kPa; Poisson's Ratio (v) = 0.28 Web Height (h_w) = 2.0828 m; Web Thickness (t_w) = 0.00635 m; a_1 = 0.127 m; a_2 = 0.10 m; a_3 = 0.0672 m; a_4 = 0.0762 m; V = 1186 kN; M = 9039 kN-m	EN 1993-1-5 U.N.O.
Partial Factors For Buildings	$\gamma_{M0} =$ 1 $\gamma_{M1} =$ 1	EN 1993-1-1 (pg 45)
Transverse Moment	$M_{z,max} = \frac{Va_3}{4h_w}(2a_1 + a_4) =$ 3.2 kN-m	Commentary (13.1)
Reduced Yield Stress Due to Transverse Moment	$f_T = 1 - 0.4\sqrt{\frac{\sigma_x(M_z)}{\frac{f_{yf}}{\gamma_{M0}}}} = 1 - 0.4\sqrt{\frac{6M_{z,max}\gamma_{M0}}{f_{yf}b_f^2 t_f}} =$ 0.969	Commentary (13.2)
	$f_{yf,r} = f_{yf} f_T =$ 242200.4 kPa Area of one flange = 0.018871 m^2	
Tension Flange	$\frac{b_2 t_2 f_{yf,r}}{\gamma_{M0}}\left(h_w + \frac{t_1 + t_2}{2}\right) =$ 9650.1 kN-m	(D.1)
Compression Flange	$\frac{b_1 t_1 f_{yf,r}}{\gamma_{M0}}\left(h_w + \frac{t_1 + t_2}{2}\right) =$ 9650.1 kN-m	(D.1)
Reduction Factor	$\chi =$ 1 Assuming the girder is restrained against lateral-torsional buckling.	EN 1993-1-1 6.3.2.3
Lateral-Torsional Buckling of Compression Flange	$\frac{b_1 t_1 \chi f_{yf}}{\gamma_{M1}}\left(h_w + \frac{t_1 + t_2}{2}\right) =$ 9960.9 kN-m	(D.1)
	Governing Flexural Strength = 9650 kN-m > 9039 kN-m OK	
Shear Strength	$a_{max} =$ 0.13 m *Where a_{max} is the greater of a_1 and a_2*	
	$\tau_{cr,l} = 4.83E\left[\frac{t_w}{a_{max}}\right]^2 =$ 2.54E+06 kPa	(D.7)
	$\bar{\lambda}_{c,l} = \sqrt{\frac{f_{yw}}{\tau_{cr,l}\sqrt{3}}} =$ 0.24	(D.6)
Local Buckling	$\chi_{c,l} = \frac{1.15}{0.9 + \bar{\lambda}_{c,l}} \leq 1.0$ 1.01 ≤ 1	(D.5)
	$D_x = \frac{Et_w^3}{12(1-v^2)}\frac{w}{s} = \frac{Et_w^3}{12(1-v^2)}\frac{a_1 + a_4}{a_1 + a_2} =$ 4.322 kN-m^3	Commentary (13.10)
	$D_z = \frac{EI_z}{w} = \frac{Et_w a_3^2}{12}\frac{3a_1 + a_2}{a_1 + a_4} =$ 1191.9 kN-m^3	

Step Description	Calculations	Reference
	$\tau_{cr,g} = \frac{32.4}{t_w h_w^2}\sqrt[4]{D_x D_z^3} =$ 344012.4 kPa	(D.10)
	$\bar{\lambda}_{c,g} = \sqrt{\frac{f_{yw}}{\tau_{cr,g}\sqrt{3}}} =$ 0.648	(D.9)
Global Buckling	$\chi_{c,g} = \frac{1.5}{0.5+\bar{\lambda}^2{}_{c,g}} \le 1.0$ 1.6 ≤ 1	(D.8)
	$\chi_c =$ 1.00	
	Where χ_c is the lesser of local buckling and global buckling	
Shear Strength	$V_{bw,Rd} = \chi_c \frac{f_{yw}}{\gamma_{M1}\sqrt{3}} h_w t_w =$ 1909.0 kN > 1186 OK	(D.4)
Buckling Factor	$a = a_1 + 2a_4 =$ 0.279401 m	
	Distance from flange edge to web (c) = 0.363801 m	(D.2)
	$k_\sigma = 0.43 + \left(\frac{c}{a}\right)^2 =$ 2.13	
	$k_\sigma =$ 0.6	(D.3)
	$k_\sigma =$ 2.13 Governs	
	$\varepsilon = \sqrt{\frac{235}{f_y}} =$ 0.97	
	$\bar{b} =$ 0.36 m	
Slenderness	$\bar{\lambda}_p = \frac{\bar{b}/t}{28.4\varepsilon\sqrt{k_\sigma}} =$ 1.4	
	$\rho = \frac{\bar{\lambda}_p - 0.188}{\bar{\lambda}_p^2} \le 1.0\ for\ \bar{\lambda}_p > 0.748$	(4.3)
	$\rho = 1.0\ for\ \bar{\lambda}_p \le 0.748$	
Reduction Factor	$Use\ \rho =$ 0.61	
	Gross Cross Sectional Area $(A_e) =$ 0.01887 m^2	
	$A_{eff} = \rho A_e =$ 0.0115 m^2	(4.1)
Compression Flange Buckling	$\frac{h_w}{t_w} \le k\frac{E}{f_{yf}}\sqrt{\frac{A_w}{A_{eff}}}$	(8.2)
	k = 0.55 Using Standard 0.55 Values for k	
	328 ≤ 496	
	Compression Flange Buckling Will Not Occur	

Corrugated Web Plate Girder Design Example	Master's Report	Derek Leamer

Step Description	Calculations	Reference
Total Steel		

Length of Flanges = 30.48 m
Length of Web = 34.29 m

Flange Width = 0.6604 m   Flange Thickness = 0.028575 m

Web Height = 2.0828 m   Web Thickness = 0.00635 m

Volume of Flanges = 1.150 m^3

Volume of Web = 0.454 m^3

Total Volume = 1.604 m^3

Total Volume (in^2) = 97875 in^3

Density of A36 Steel = 0.284 lb/in^3

Weight of Girder in Pounds = 27796 lb

Flange Width = 26 in   Web Height = 82 in
Flange Thickness = 1.125 in   Web Thickness = 0.25 in

Flange Width =	26 in
Flange Thickness =	1.125 in
Web Height =	82 in
Web Thickness =	0.25 in
Weight =	27796.5 lb

Appendix C – Proposed North American Design Example

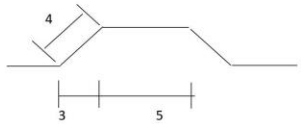
The following appendix demonstrates a corrugated web plate girder design example based on the proposed methods of Driver Et al. (2006) and Abbas Et al. (2006).

Corrugated Web Plate Girder Design Example	Master's Report	Derek Leamer

Design of a corrugated web plate girder using equations derived by Driver Et al. (2006) and Abbas Et al. (2006). Using the same conditions as the normal plate girder example.

Assuming restrained against lateral-torsional buckling.

Step Description	Calculations	Reference



Web Height = 82 in

Inclined Panel Angle (α) = 41.4 degrees

Web Thickness (t_w) = 0.25 in

d = 3 in

Longitudinal Fold Width (b) = 5.00 in

Maximum Panel Width (w) = 5 in

Inclined Fold Width (c) = 4.00 in

F_y = 36 ksi

Longitudnal Fold to Inclined Fold Ratio (β) = 1.25

Modulus of Elasticity (E) = 29000 ksi

Poisson's Ratio (ν) = 0.28

Local Elastic Buckling Stress

Local Shear Buckling Coefficient (k_L) = 5.34

$$(\tau_{cr,L})_{el} = k_L \frac{\pi^2 E}{12(1-\nu^2)(w/t_w)^2} = \quad 345.5 \text{ ksi}$$

Driver Et al. (2006) Equation 1

Global Elastic Buckling Stress

$$F(\alpha,\beta) = \sqrt{\frac{(1+\beta)\sin^3\alpha}{\beta + cos\alpha}} * \left\{\frac{3\beta+1}{\beta^2(\beta+1)}\right\}^{\frac{3}{4}} = \quad 0.51$$

Driver Et al. (2006) Equation 3

Global Shear Buckling Coefficient (k_G) = 31.6

$$(\tau_{cr,G})_{el} = k_G \frac{E t_w^{\frac{1}{2}} b^{\frac{3}{2}}}{12 h_w^2} F(\alpha,\beta) = \quad 32.1 \text{ ksi}$$

Driver Et al. (2006) Equation 2

Yield Stress

$$\tau_y = \frac{F_y}{\sqrt{3}} = \quad 20.8 \text{ ksi}$$

$$(\tau_{cr})_{inel} = \sqrt{0.8\tau_y(\tau_{cr})_{el}} \leq \tau_y$$

Driver Et al. (2006) Equation 4

23.1 ≤ 20.8

Web Yielding Limit

If the following equation is satisfied, web yielding will control for shear

ψ = 0.9

$$\frac{h_w}{t_w} \leq 1.91\psi \sqrt{\frac{E}{F_y}\left(\frac{b}{t_w}\right)^{1.5} F(\alpha,\beta)}$$

Driver Et al. (2006) Equation 11

328 ≤ 328.1 Web Yielding Controls

Slenderness Limit

If the following slenderness limit is satisified, econmical design

$$\frac{b}{t_w} \leq 2.586\sqrt{\frac{E}{F_y}}$$

Driver Et al. (2006) Equation 16

20 < 73.4 Economical

Corrugated Web Plate Girder Design Example	Master's Report	Derek Leamer

Step Description	Calculations	Reference
	Nominal Shear Strength Based on slenderness ratio	
	$\lambda_L = \frac{w}{t_w}\sqrt{\frac{F_y}{E}} =$ 0.7	Driver Et al. (2006) Equation 15
	$For\ Yielding\text{: } \lambda_L \leq 2.586$	Driver Et al. (2006) Equation 12
	$V_n = 0.707\left(\frac{F_y}{\sqrt{3}}\right)h_w t_w =$ 301.2 k	
	$For\ Inelastic\ Buckling\text{: } 2.586 < \lambda_L \leq 3.233$	Driver Et al. (2006) Equation 13
	$V_n = \sqrt{\frac{1}{1+0.15\lambda_L{}^2}}\left(\frac{F_y}{\sqrt{3}}\right)h_w t_w =$ 411.1 k	
	$For\ Elastic\ Buckling\text{: } 3.233 < \lambda_L$	Driver Et al. (2006) Equation 14
	$V_n = \sqrt{\frac{1}{1+0.14\lambda_L{}^2}}\left(\frac{F_y}{\sqrt{3}}\right)h_w t_w =$ 412.0 k	
	Governing Shear Strength = 271.1 k > 267 k Applying ϕ = 0.9 for LFRD	
Flexural Strength	Maximum stresses occur at half of a corrugation wavelength	
	Flange Width (b_f) = 26 in; Flange Thickness (t_f) = 1.125 in	
	Girder Length (L) = 100 ft; Uniform Load (p_y)= 5 k/ft	
	Distance along girder (z) = 8 in; Corrugation Wavelength (L_0) = 16 in	
	Number of Wavelengths (n) = 75.00; Girder Height (h) = 84.25 in	
	h_r = 2.65 in; $e_0 = \frac{h_r}{2} =$ 1.32 in	
Transverse Moment	$M_t = \frac{p_y L^2 e_0}{2\pi n h}\left\{\left[1-2\frac{z}{L}\right]\cos\left(2\pi n\frac{z}{L}\right)+\frac{2}{2\pi n}\sin\left(2\pi n\frac{z}{L}\right)+\left[\cos(2\pi n)-\frac{2}{2\pi n}\sin(2\pi n)+1\right]\frac{z}{L}-1\right\}$ M_t = -3.29 k-ft	
	Maximum Stresses Due to Transverse Moment Occur Near the Edge of the Flange	
	M_t = 3.29 k-ft; C-Factor for Sinusoidal Profile = 1	
	C-Factor for Trapezoidal Profile = $\pi\left[\frac{b+d/2}{L_0}\right] =$ 1.3; $M_t^{Trap} = M_t^{Sin} * \frac{C^{Trap}}{C^{Sin}} =$ 4.20 k-ft	
	Distance from Centerline (x) = 1.32 in; Moment of Inertia of One Flange (I_t)= 50530.7 in^4	
	$\sigma_t = \frac{M_t x}{I_t} =$ 0.001 ksi	
Bending Stresses	Maximum Stresses Due to Bending Occur at Midspan	
	M_x = 6667 k-ft; Distance from N.A. to Edge (Y) = 42.125 in	
	I_x = 103809.2 in^4; $\sigma_b = \frac{M_x Y}{I_x} =$ 32.5 ksi	
Superpostion of Stresses	$\sigma_{total} =$ 32.5 ksi	
	36 ksi ≤ 36 ksi Applying ϕ = 0.9 for LFRD	

Corrugated Web Plate Girder Design Example	Master's Report	Derek Leamer

Step Description	Calculations	Reference
Final Design	Length of flanges = 100 ft Length of web = 112.50 ft Flange Thickness = 1.125 in Web Thickness = 0.25 in Flange Width = 26 in Web Height = 82 in Amount of steel for web and flanges: Flanges = 70200 in^3 Web = 27675 in^3 Total = 97875 in^3 Density of A36 Steel = 0.284 lb/in^3 Weight of Girder = 27796.5 lb	

Flange Width =	26 in
Flange Thickness =	1.125 in
Web Height =	82 in
Web Thickness =	0.25 in
Weight =	27796.5 lb