

ULTIMATE STRENGTH PREDICTIONS FOR BEAMS
WITH WEB OPENINGS

by

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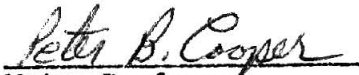
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INTRODUCTION

The initial objective of this investigation was to develop an ultimate strength analysis of composite beams with web openings by extending the method used by Redwood [1]*, Congdon [2], Frost [3] and Wang [4]. This ultimate strength analysis would give an interaction diagram establishing theoretical values of moment and shear acting concurrently to cause failure at the opening. Major difficulties were encountered in this initial effort and a continuous interaction diagram could not be obtained. A re-examination of the method and assumptions was called for. A problem was identified in that the assumptions were found to be incorrect for eccentric web openings. Wang encountered similar difficulties in his analysis of noncomposite beams with reinforced eccentric web openings but did not identify the cause [4].

At this stage in the investigation an alternative method of analysis by McCormick became available [5]. McCormick discussed many aspects of the analysis of beams with web openings, including the variables eccentricity and composite beams, but presented an application only for concentric web openings. A comparison of Redwood's method [1] with McCormick's clarified the problem in the assumptions Wang had used in the eccentric case, and led to the justification of McCormick's method, which was finally adopted for the ultimate strength analysis of beams with eccentric web openings.

In this report the two methods of analysis are reviewed, compared, and the problem encountered with eccentric openings is identified. McCormick's method is applied to eccentric, unreinforced openings. Comments on the calculations for the interaction diagram follow, and a computer solution is

*References are listed by number in brackets.

presented. Finally the results of this analysis are compared with the results of laboratory tests on beams with eccentric rectangular web openings.

LITERATURE SURVEY

Since 1968, several analyses of the ultimate strength of beams with web openings have been made. Generation of moment-shear interaction diagrams showing the relationship between moment and shear values causing failure was an integral part of these analyses. These values of moment and shear were divided by corresponding values for the beam without an opening showing the decrease in strength due to the opening.

The most general case of a reinforced eccentric web opening is shown in Fig. 1. The beam is defined by d , t_w , b and t . Section 1 is the high moment edge of the opening and Section 2 is the low moment edge. The opening size is defined by h and a and its location by e and L . If $e=0$, the opening is concentric. The position of the reinforcement is defined by u . The reinforcing area is defined by c and q such that if $c=t_w$, the opening is unreinforced.

The following assumptions were adopted by all of the investigators:

1. Equilibrium must be satisfied.
2. Plastic hinges are assumed at the edges of the opening in the sections above and below the opening. This constitutes a localized failure mechanism shown in Fig. 2.
3. Perfectly plastic action is assumed at each hinge.
4. Von Mises' yield condition defines failure in combined bending and shear. $(F_y^2 = f^2 + 3f_v^2)$
5. Secondary bending is caused in the sections above and below the opening by the presence of shear.
6. The possibility of failure due to instability and the beneficial effects of strain hardening were not considered.

Bower developed a means of predicting the strength of beams with concentric rectangular openings [6]. The secondary moment in the tee sections was expressed in the form of axial forces of equal magnitude and opposite direction applied to the top of the flange and next to the opening in the web. Assuming the points of contraflexure in the secondary moments were at the center of the opening gave the magnitude of the secondary moments and the stress reversal points at the plastic hinges. Shear was assumed to act concurrently with the normal force due to secondary moment in the web. This is illustrated in Fig. 3.

Redwood used a different approach to analyze the problem of concentric rectangular web openings [1]. Stress distributions were assumed at each hinge and static equilibrium conditions were imposed. Two stress reversal cases which Redwood called low shear and high shear were encountered. The stress reversal points at Section 2 were always in the flange, but for low shear, both stress reversal points at Section 1 were in the web and for high shear, both stress reversal points at Section 1 were in the flange. Although Redwood did not assume the location of the points of contraflexure, it will be shown from the resulting equations that they are at the center of the opening. Redwood and other investigators assumed that the shear force is evenly distributed throughout the webs of the tee sections. Redwood's approach is summarized in Appendix A.

Congdon [2], Frost [3] and Wang [4] expanded Redwood's work to include two other variables, reinforcement and eccentricity of the web opening. Congdon considered beams with reinforced concentric rectangular openings [2]. Congdon assumed that the reinforcement does not resist shear. Frost attempted a solution for eccentric web openings [3]. Two stress reversal cases were developed as in Redwood's work. Frost clearly omitted a stress

reversal case inherent in the eccentricity of his problem. Wang developed an analysis of beams with reinforced eccentric openings and considered all stress reversal combinations [4]. Congdon's approach to reinforcing was adopted. Wang encountered a problem at low values of shear and described it on page 18. "When the problem starts with very small (or zero) values of V , one of the roots will be positive and less than 1 and the other root will be negative but close to zero. In this case, the other k -values cannot be evaluated within their limits from the former. Therefore, the technique adopted is to set the latter equal to zero in order to obtain a solution."

Richard, as did Wang and Frost, approached the problem of eccentric rectangular web openings [7]. Following Bower's method, Richard assumed the points of contraflexure were at the center of the opening to determine stress reversal locations. A shear value was assumed on the top tee and shear on the bottom tee was found by imposing equilibrium. Richard stated on page 20 that "assuming a significant value of V_t for the initial increment sometimes could, and in fact did, result in interaction curves with no plotted points for low shear values up to 60% of the range of the shear ordinate."

McCormick presented a comprehensive study of web openings on I-beams [5]. Approaches to the analysis of non-composite and composite beams with single and multiple web openings were outlined. Openings of varying shapes and eccentricities were included. Two innovations of note were McCormick's treatment of shear stress and eccentricity. Instead of reducing the value of the normal stress in the web due to shear, he reduced the web thickness and retained the normal yield stress. To treat eccentricity a moment acting on the larger tee section was introduced. Solutions for beams with concentric

openings were illustrated. The point of contraflexure in the moment due to shear was assumed to be at the center of the web opening. McCormick's method is summarized in Appendix B.

Two basic approaches are encountered in these investigations. The methods of analysis differ in the representation of forces at the individual plastic hinges. The next two sections review these approaches separately.

REDWOOD'S METHOD

Redwood described a method of estimating the ultimate strength of a beam with a concentric, unreinforced rectangular web opening [1]. Figures and equations involved in his solution are contained in Appendix A. Congdon [2], Frost [3] and Wang [4] utilized Redwood's approach to estimate the strength of reinforced concentric holes, unreinforced eccentric holes and reinforced eccentric holes, respectively. Their work will be reviewed in this section.

Plastic stress distributions were assumed at each plastic hinge. The stress reversal locations were defined by the variables k_1 at Section 1 and k_2 at Section 2. Due to symmetry k_1 and k_2 have the same values in the top and bottom tee sections.

The location of stress reversal points varies with shear. In all cases the points of stress reversal at Section 2 are in the flange. The points of stress reversal at Section 1 are at the edge of the opening when shear is zero and move up the web into the flange as shear is increased.

Redwood developed equations for two stress reversal cases which he called low shear and high shear. Using Wang's notation, low shear is Case WW since the stress reversal points at Section 1 in the top and bottom tees are both in the web, and high shear is Case FF since the stress reversal points at Section 1 are in the flanges.

A shear force was assumed at the opening and the corresponding normal forces were evaluated for the assumed stress reversal case. Due to symmetry half of the total shear force was assigned to the top tee and half to the bottom tee. The shear was distributed uniformly throughout the clear depth of the tee web. The corresponding normal stress in the web satisfied Von Mises' yield criterion. The normal stress in the flange was always equal to

the yield stress. The resultant normal forces and their locations were expressed in terms of the cross-sectional parameters as well as the variables k_1 at Section 1 and k_2 at Section 2.

Equilibrium conditions were imposed to obtain relationships between bending moments and shear forces. Isolating the tee sections shows that the normal and shear forces at Sections 1 and 2 are equal in magnitude and opposite in direction. This permits k_1 to be expressed in terms of k_2 . Similarly, to satisfy equilibrium at each section, the normal force in the top tee is equal and opposite to the normal force in the bottom tee. Therefore the resultant normal force is constant at each hinge. At each section, the opposing normal forces constitute the moment at that section.

Since the total shear force is constant across the opening length $2a$, overall moment equilibrium requires that $M_1 = M_2 + 2Va$. Substitution into this equation results in a quadratic equation which was solved for k_2 . Once k_2 was known, k_1 could be found.

The value of k_1 indicated whether the correct stress reversal case had been assumed. If not, the equations for normal force, etc. corresponding to the other stress reversal case would be applied. Case WW equations were used at low values of shear. As shear increased, k_1 increased. When k_1 exceeded one, the high shear Case FF was adopted.

After the stress reversal locations were known, the moment at the centerline of the opening could be found. This combination of shear and moment gave a point on an interaction diagram between moment and shear. By varying shear, additional points on the interaction diagram could be generated.

Congdon analyzed concentric, reinforced web openings [2]. The reinforcing was assumed to resist no shear so the normal stress on the reinforcing was always equal to the yield stress. Following Redwood's procedure, four stress reversal cases were encountered; SS, RR, WW and FF indicating both stress reversal points at Section 1 were in the web stub adjacent to the opening, the reinforcement, the clear web and the flange, respectively. As shear increased the stress reversal cases were encountered in the order listed above.

Frost extended Redwood's approach to include beams with eccentric rectangular holes [3]. Quadratic equations in terms of k_{2T} were derived for stress reversal Cases FF and WW. Introducing these derivations on page 9, Frost wrote, "Equations for other possible combinations of neutral axes locations are not given but could be similarly developed." From this statement it could be assumed that stress reversal Case FW was considered. Case FW would provide for a stress reversal point in the flange in the top tee at Section 1 and in the web in the bottom tee at Section 1. As in all cases, both stress reversal points were in the flange at Section 2.

Due to eccentricity, the shear could not be assumed to be equally distributed between the top and bottom tee sections. A shear ratio was defined as the shear force in the top tee divided by the shear force in the bottom tee. Only one shear ratio would satisfy equilibrium. Frost noted on page 7 that any other shear ratio would not cause failure and suggested a method of successive approximations to determine the correct shear ratio.

Wang's solution included both eccentricity and reinforcement of rectangular web openings [4]. The reinforcement was assumed to resist no shear. The correct shear ratio was found by successive approximation.

The complexity of Wang's problem is evident by the stress reversal cases considered. With eccentricity toward the top flange, Cases SS, SR, SW, RR, RW, WW, FW and FF were encountered. At zero shear there was no stress reversal point in the top section and in the bottom tee section the point of stress reversal was at the equal area axis of the cut reinforced section. Since Case WF was not encountered it could be seen that the stress reversal point moved into the top flange before it moved into the bottom flange. This was true for all cases except concentric cases, where the stress reversal points were at the same positions in the top and bottom tees. There is no evidence to indicate that Frost used Case FW, although Wang found this case was necessary to generate complete interaction diagrams.

Since the stress reversal locations varied between the top and bottom tee sections, Wang chose to use k_{2T} as the unknown in his quadratic equations. The larger root of the quadratic never resulted in compatible values for other stress reversal locations, so the smaller root was chosen. At low values of shear the smaller root was negative and close to zero. To obtain a solution, Wang let k_2 equal zero.

Imposing conditions of static equilibrium on an assumed stress reversal case is the basis of Redwood's method of solution. A shear force is assumed and the corresponding moment is found by identifying the location of stress reversal points. In the eccentric case a trial and error method of solution must be used to determine the ratio of the shear forces assigned to each tee section. Wang encountered small negative values for the stress reversal location at low values of shear, but was able to generate a continuous interaction diagram by setting these values equal to zero.

McCORMICK'S METHOD

A method of plastic analysis of beams with web openings was developed by McCormick [5]. The key to McCormick's analysis is the representation of the plastic hinge capacity in terms of normal and moment forces. Appendix B contains the figures and equations relating to McCormick's approach.

McCormick expressed the capacity of the tee sections as a function of the forces acting at each plastic hinge. A shear force, a normal force and a secondary moment due to shear were applied at each hinge.

The shear force was distributed uniformly throughout the clear depth of the tee section web in combination with the normal forces. Von Mises' yield condition was satisfied by reducing the thickness of the web so the normal stress would equal the yield stress throughout the tee section.

Using the reduced web thickness, the location of the equal area axis is determined. This equal area axis is also referred to as the plastic neutral axis. The summation of forces above equals the summation of forces below this axis.

For nearly all rolled shapes, the area of the flanges is greater than the area of the web, resulting in an equal area axis in the flange. For this reason it is the only plastic neutral axis location considered in Appendix B, although McCormick treated the case where the equal area axis is in the web in a similar manner.

A moment due to shear acts in conjunction with a normal force to form a plastic hinge in the reduced tee section. The normal force is applied at the equal area axis. Equal areas above and below that assigned to the normal force have the same stress and therefore involve the same force. These forces act in opposite directions to comprise the secondary moment.

due to shear. This moment can be expressed in terms of the normal force and the reduced tee section properties. For eccentric openings, McCormick suggested the introduction of an additional moment in the larger tee section.

Two derivations result depending on whether the normal force is confined to the flange or extends into the web. For concentric web openings, the first of these corresponds to Case FF and the latter to Case WW in Redwood's analysis.

Equilibrium conditions give the relationship between the bending moment and shear force which cause a localized failure. Isolating the tee sections shows that the normal and shear forces at Sections 1 and 2 are equal in magnitude and opposite in direction. Since shear is constant, the cross section properties are constant across the tee sections. This combined with a constant normal force results in secondary moments having the same magnitude at Sections 1 and 2. Thus the secondary moment is zero at the center of the opening. Moment equilibrium of the tee sections gives the value and direction of the secondary moment. Once the secondary moment is known, the normal force may be found from the cross sectional properties of the tee sections. Equilibrium at Sections 1 and 2 requires the normal force to be constant at each hinge and the sum of shear forces in the top and bottom tees to equal the total shear force. The moment at the center of the opening is the normal force times the distance between the equal area axes in the top and bottom tees.

McCormick provided one example of an "exact" plastic analysis in which a concentric web opening was considered. His solution was approximate in that the location of the equal area axes was not varied with shear. A shear force was assumed, giving the value of the secondary moment. The normal

force was determined from the solution of a quadratic equation. McCormick calculated the moment at Section 1 for various values of shear.

This section on McCormick's ultimate strength analysis of beams with web openings completes the review of the literature. In the following sections the two basic methods of analysis will be compared and the problems in applying Redwood's method to the eccentric case will be defined.

COMPARISON OF REDWOOD'S METHOD WITH MCCORMICK'S METHOD

The methods of analysis developed by Redwood [1] and McCormick [5] will be compared in this section to form a basis for further analytical research. To simplify this comparison, the same notation was used in the development of both analyses where applicable. Methods for the analysis of concentric web openings will be discussed first.

The only significant difference between the two methods is McCormick's assumption that the point of contraflexure due to secondary moment is at the center of the opening. As pointed out in the discussion of McCormick's method, this is necessary due to the plastic properties of the tee sections. If the normal forces are the same at Sections 1 and 2, the secondary moments must have the same magnitude so the point of contraflexure in this moment is at the center of the opening. Although Redwood did not make this assumption, imposing equilibrium led to the same results. Appendix C shows that the point of contraflexure in Redwood's solution is at the center of the web opening, thus verifying this simplifying assumption.

The treatment of secondary moment also differs. McCormick portrayed moment due to shear in the tee sections by resisting moments at the hinges. Redwood's secondary moment is generated by the differing locations of the resultant normal forces at Sections 1 and 2.

The differences between the two approaches do not affect the interaction diagrams generated. McCormick's method gives a clearer and easier formulation due to the representation of forces acting at each hinge and the assumption that the point of contraflexure is at the center of the web opening.

There is no satisfactory solution for eccentric web openings in the literature. As noted in the literature survey, Richard [7] and Wang [4] both encountered difficulties at low shear values. Richard found that assuming a small value of shear acting on the top tee and solving for shear in the bottom tee resulted in an incomplete interaction diagram. No points could be plotted for low values of total shear. Wang bypassed this problem by setting a small negative root of a quadratic equation in k_{2T} equal to zero to obtain a solution.

The attempted solutions for the strength of beams with eccentric web openings used by Wang and Richard contained a single stress reversal location at each hinge. It will be shown that a problem in the low shear range of the interaction diagram arises due to the presence of more than one stress reversal at Section 2 in the bottom tee. Frost used the same formulation and would have had similar problems if he generated an exact interaction diagram; however, he did not report them [3].

The presence of more than one stress reversal point in the larger tee at Section 2 for low values of shear will be demonstrated by looking first at the plastic analysis of a beam with an eccentric web opening subjected to pure bending (Fig. 4a). A stress reversal in the larger tee section is necessary to have a constant normal force at each hinge. The equal area axis gives the initial stress reversal location in the web of the bottom tee. None of the solutions based on Redwood's approach considered a stress reversal point in the web at Section 2. Wang recognized this as the location of the initial stress reversal point at Section 1 but not at Section 2. As in the concentric case, a stress reversal due to shear moves toward the flange at Section 1 and toward the web from the outer edge of the flange at Section 2. For

eccentric openings, three stress reversal locations are indicated at Section 2 in the bottom tee (Fig. 4b).

McCormick's method is therefore preferable to Redwood's. Adapting Redwood's method to the indicated stress reversal combinations would involve developing stress reversal cases WW, FW and FF (indicating the location of stress reversal points at Section 1) for both stress reversal cases at Section 2. Using McCormick's method the location of the stress reversal points is not needed. McCormick's use of a moment due to eccentricity as well as a moment due to shear in the bottom tee is a valid representation of the forces acting at low values of shear (Fig. 4b). McCormick suggested the use of this additional moment but did not explain its application.

APPLICATION OF McCORMICK'S METHOD TO BEAMS WITH ECCENTRIC WEB OPENINGS

Using McCormick's formulation, the method of solution is similar for concentric and eccentric web openings. The moment capacity of the tee sections reduced for normal force must be determined for the following two cases: when the normal force is confined to the flange and when it extends into the web. In the eccentric case the hinges in the smaller tee are subjected to normal force and moment due to shear while the hinges in the larger tee are subjected to normal force and moment due to shear as well as moment due to eccentricity. A shear ratio is assumed and secondary moment is found for each hinge. The normal force is calculated from the smaller tee and applied to the larger tee so the moment due to eccentricity may be computed. The moment due to eccentricity is the difference between the moment due to shear and the moment capacity of the larger tee section reduced for the normal force. The moment at the centerline of the opening is equal to the magnitude of the normal forces times the distance between them plus the moment due to eccentricity.

Each shear ratio will give a moment for the corresponding shear force. The shear ratio giving the maximum moment is the correct one. In a concentric case the maximum moment is found when shear is distributed evenly to the top and bottom tees, with a moment due to eccentricity of zero. In an eccentric case the largest normal force satisfying equilibrium gives the maximum total moment. Assigning all the shear to the larger tee gives the largest normal force. Due to the plastic properties of the tee section, as shear increases, the moment due to shear increases and the moment due to eccentricity decreases while the normal force is constant.

At a certain value of shear, the moment due to shear will result in a constant normal force without a moment due to eccentricity, and shear must be applied to the smaller tee section. With the moment due to eccentricity eliminated, a single stress reversal exists at each hinge as in Redwood's approach [1]. The range of shear for which the moment due to eccentricity applies is the same range in which Wang and Richard encountered difficulties.

A computer program was formulated to generate points on an interaction diagram using this method of solution. The flow diagram and program in Fortran are included in Appendix D.

EXPERIMENTAL RESULTS

The results of five ultimate load tests on beams with unreinforced eccentric web openings are found in the literature. Test results are reported by Frost [3] on four beams and by McNew [8] on one beam. The pertinent test parameters of the beams are tabulated below.

<u>Ref.</u>	<u>Beam</u>	<u>No.</u>	<u>a</u>	<u>h</u>	<u>e</u>	<u>M/V</u>
Frost	W 16 x 40	1	6.4	3.2	1	40
Frost	W 16 x 40	2	6.4	3.2	2	40
Frost	W 16 x 40	5	6.4	3.2	1	0
Frost	W 16 x 40	6	6.4	3.2	2	0
McNew	W 16 x 45	1	4.5	3.0	2	30

Moment shear interaction curves were generated for these beams using the computer program in Appendix D. To compare the data with theories, both Frost and McNew corrected the experimental ultimate loads to eliminate the effects of strain hardening by using load-deflection curves. The corrected test results are plotted on the interaction curves in Fig. 5. It can be seen that the interaction curves furnish reliable but conservative predictions of the experimental ultimate loads.

SUMMARY

The main contributions of this report are the identification of a source of error in the previous ultimate strength analyses of beams with eccentric web openings, and the extension and application of McCormick's method to beams with eccentric web openings. A moment due to eccentricity acting on the larger tee was shown to be necessary for low values of shear. When a moment due to eccentricity acts concurrently with a moment due to shear, more than one stress reversal is necessary at one of the hinges.

Two minor contributions relate to Redwood's method. The first pertains to the application of Redwood's method to the eccentric case. Since the exact location of stress reversal points must be found, the presence of more than one stress reversal point at a hinge makes this unnecessarily complex if the moments involved are not identified. The second is the observation that the point of contraflexure for the moment due to shear is at the center of the opening, although this fact was not assumed nor pointed out in Redwood's work.

McCormick's method of analysis was found to be better suited for extension to the eccentric case. The values of the forces causing failure at an opening are determined instead of the locations of stress reversal points. A clearer picture of the forces causing failure results.

RECOMMENDATIONS FOR FURTHER STUDY

The logical extension of the method of analysis presented here would be the analysis of beams with reinforced eccentric web openings. The plastic properties of the cross sections at the plastic hinges would have to be developed. Centering the normal load at the equal area axis would provide the possible stress reversal cases.

The eccentric analysis could also be applied to composite beams with web openings. It must be noted that tension in the concrete at Section 2 is caused by secondary moment. Since concrete has a negligible tensile strength, the effective section above the opening at Section 2 will be smaller than that at Section 1. This is a major source of difficulty in the analysis of the composite case.

Further experimental tests would be helpful in determining the reliability and accuracy of the ultimate strength analysis of beams with eccentric web openings.

NOMENCLATURE

a	Half length of opening
b	Flange width
c	Total width of reinforcing bars (including web thickness)
Case	Stress reversal case given by two letters, the first indicating the stress reversal location in the top tee at Section 1 and the second indicating the stress reversal location in the bottom tee at Section 1
d	Depth of beam
d_c	Distance between normal forces in top and bottom tees in McCormick's method
e	Distance from the middepth of the beam to the middepth of the opening
EAA	Equal Area Axis of a tee section
f	Normal stress reduced for shear
f_B	Normal stress in bottom tee reduced for shear
f_T	Normal stress in top tee reduced for shear
f_v	Shear stress
f_{vB}	Shear stress in bottom tee
f_{vT}	Shear stress in top tee
F_y	Yield stress
h	Half opening depth
k	Indicator of stress reversal location. The subscripts 1 and 2 refer to Sections 1 and 2; T and B refer to top and bottom tees, respectively
L	Longitudinal distance from opening centerline to nearest support
M	Moment at centerline of opening
M_1	Moment at Section 1
M_2	Moment at Section 2
M_e	Moment due to eccentricity
M_v	Moment due to shear

- n Normal force divided by normal yield force in tee. The subscripts T and B refer to the top and bottom tees, respectively
- n^* A specific value of n when the normal force, centered at the equal area axis, extends to the web-flange junction
- P One of the forces comprising the secondary moment at a hinge. The subscripts 1 and 2 refer to Sections 1 and 2
- q The thickness of the reinforcing bars
- Q Resultant normal force
- Q^* A specific value of Q which extends to the web-flange junction when centered at the equal area axis
- Q_y Normal yield force for tee section
- s Depth of web in the tee section. The subscripts T and B refer to the top and bottom tee sections, respectively
- t Flange thickness
- t_w Web thickness
- u Distance between opening and reinforcing bar
- V Total shear force. The subscripts T and B refer to the top and bottom tee sections, respectively
- w Web thickness reduced for shear. The subscripts T and B refer to the top and bottom tee sections, respectively
- $\bar{y}$ Distance from edge of opening to resultant normal force. The subscripts T and B refer to the top and bottom tee sections, respectively

**THIS BOOK
CONTAINS
NUMEROUS PAGES
WITH DIAGRAMS
THAT ARE CROOKED
COMPARED TO THE
REST OF THE
INFORMATION ON
THE PAGE.**

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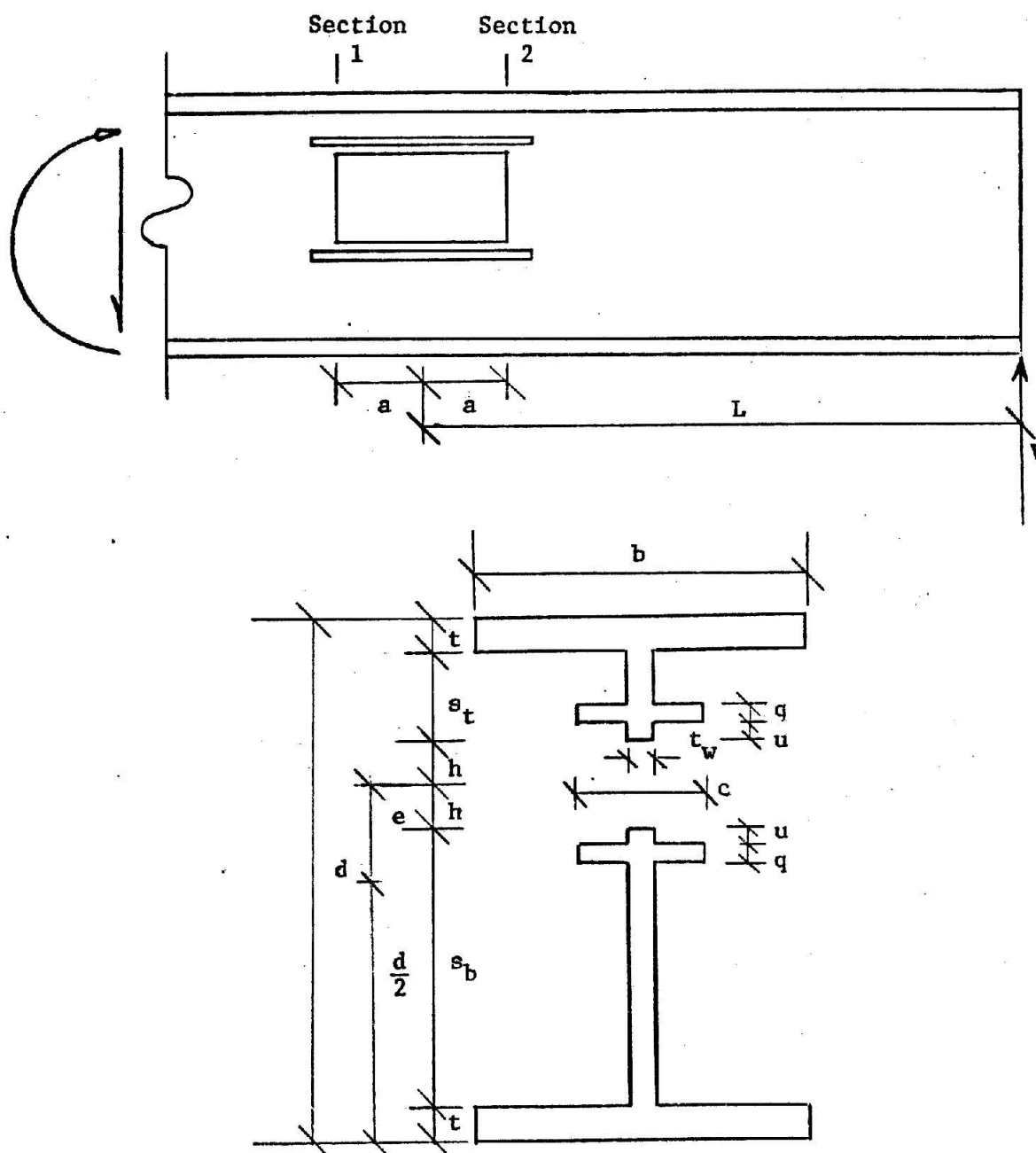


Fig. 1. Beam with Web Opening

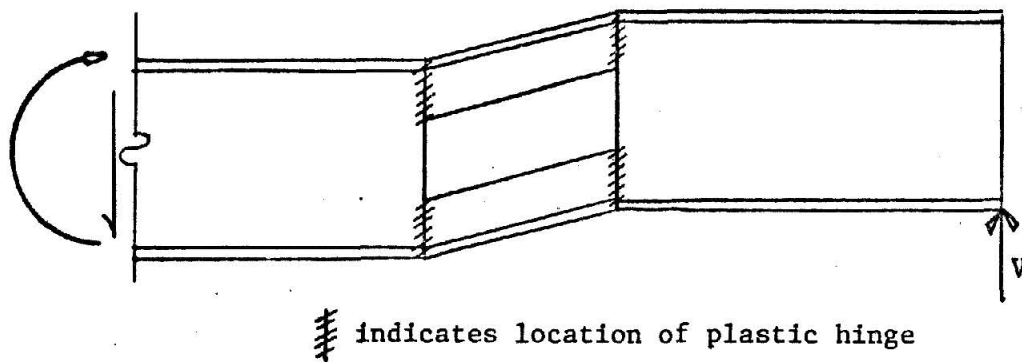


Fig. 2. Localized Failure Mechanism

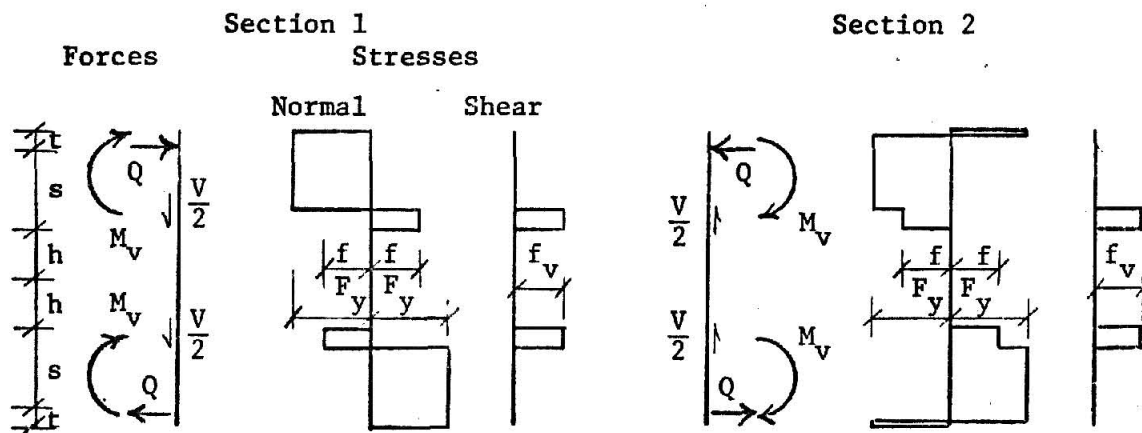
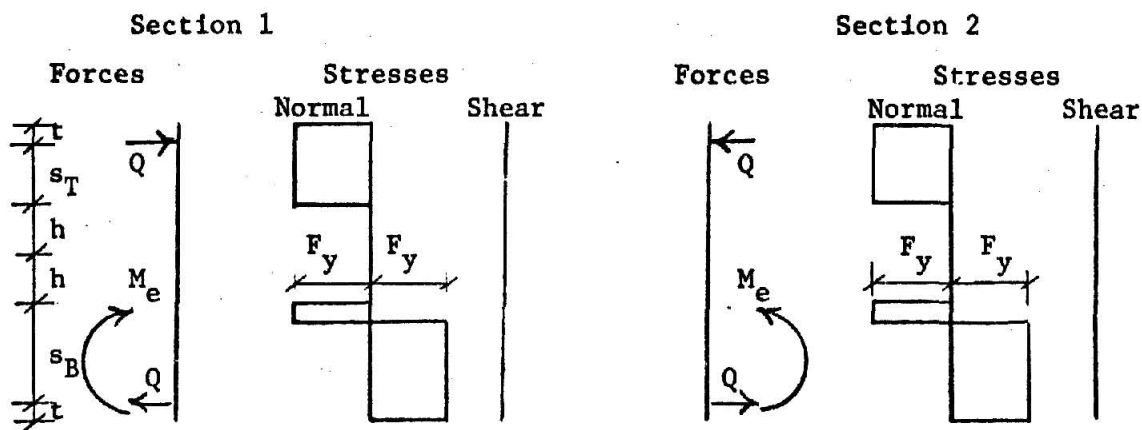
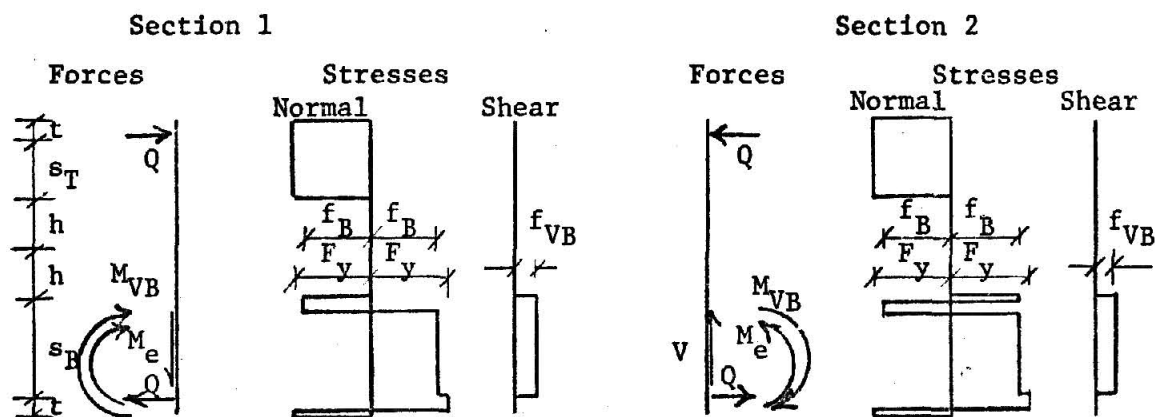
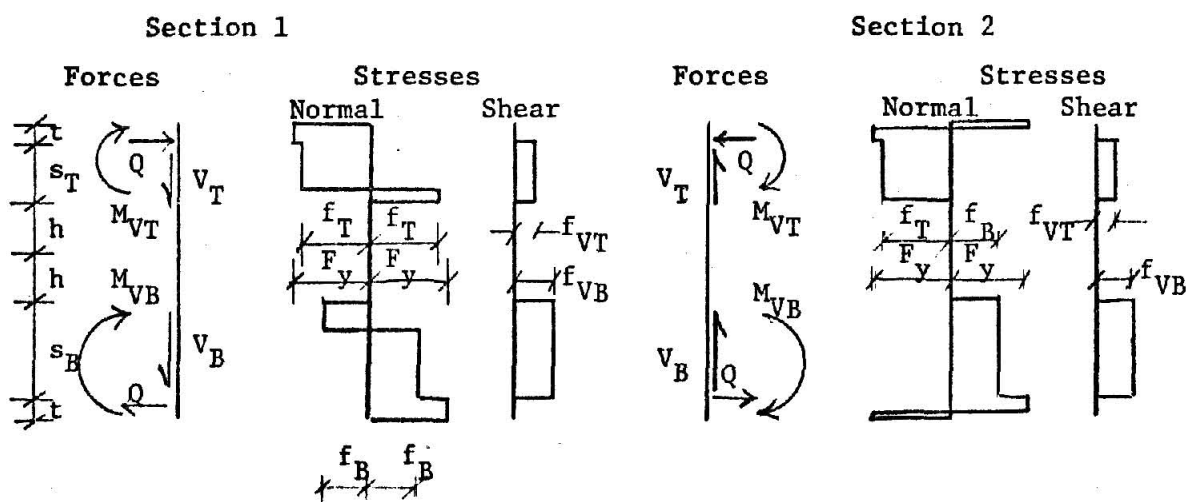


Fig. 3. Bower's Representation of Plastic Hinge Forces

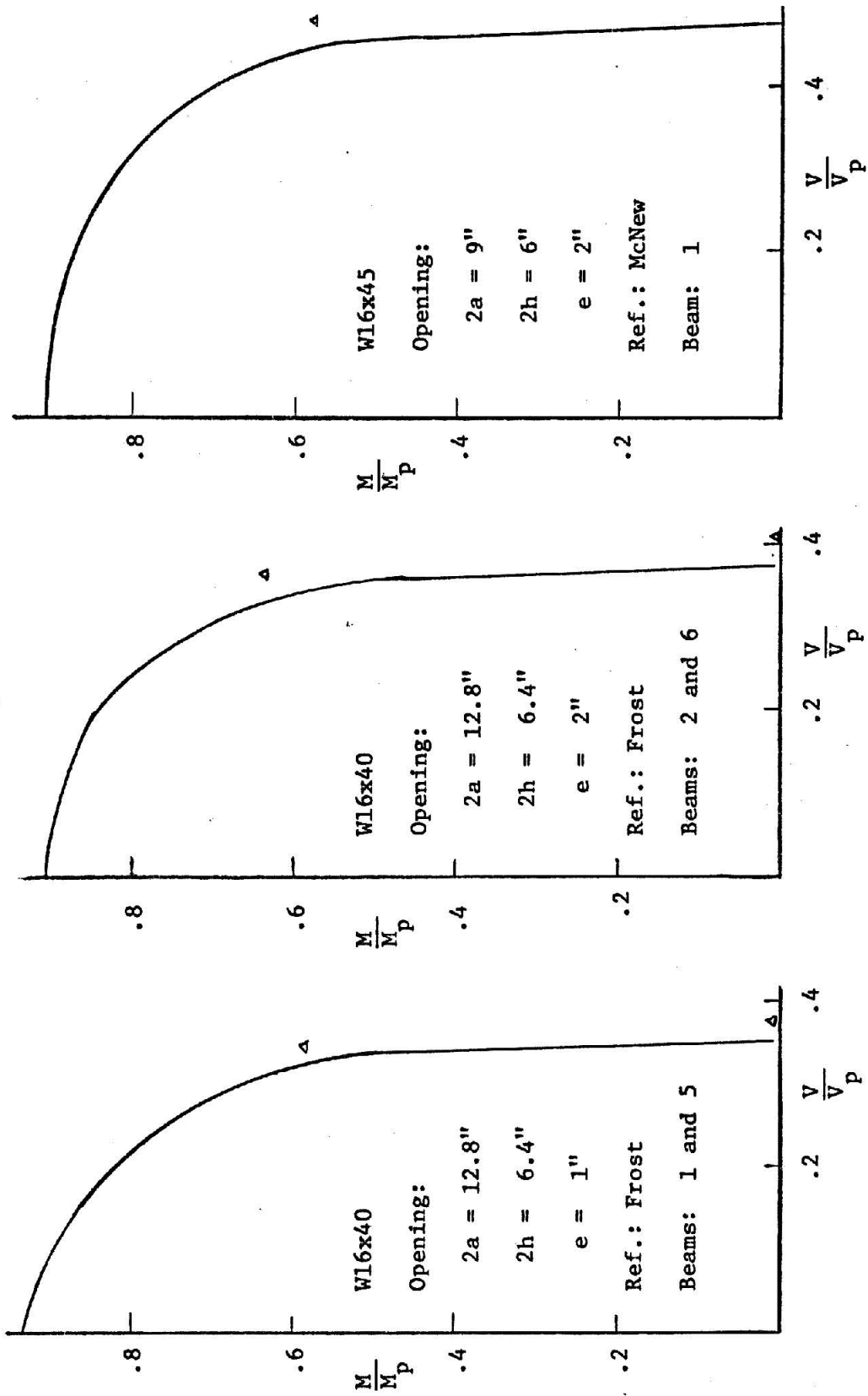
(a) Failure due to pure bending ($V=0$)

(b) Failure with a small shear force



(c) Failure with an intermediate shear force

Fig. 4. Beams with Eccentric Web Openings at Failure



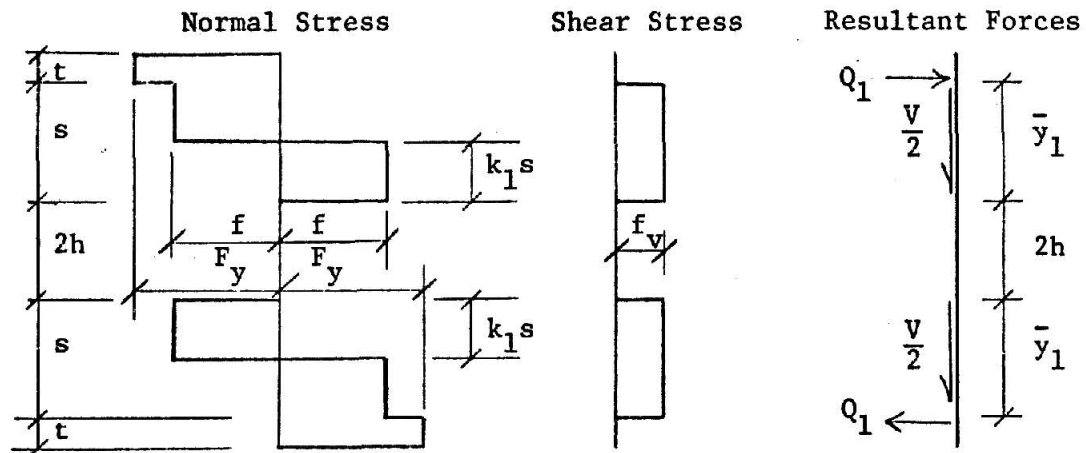
Δ indicates test results corrected for strain hardening

Fig. 5. Test Results Plotted on Interaction Diagrams

APPENDIX A

REDWOOD'S METHOD

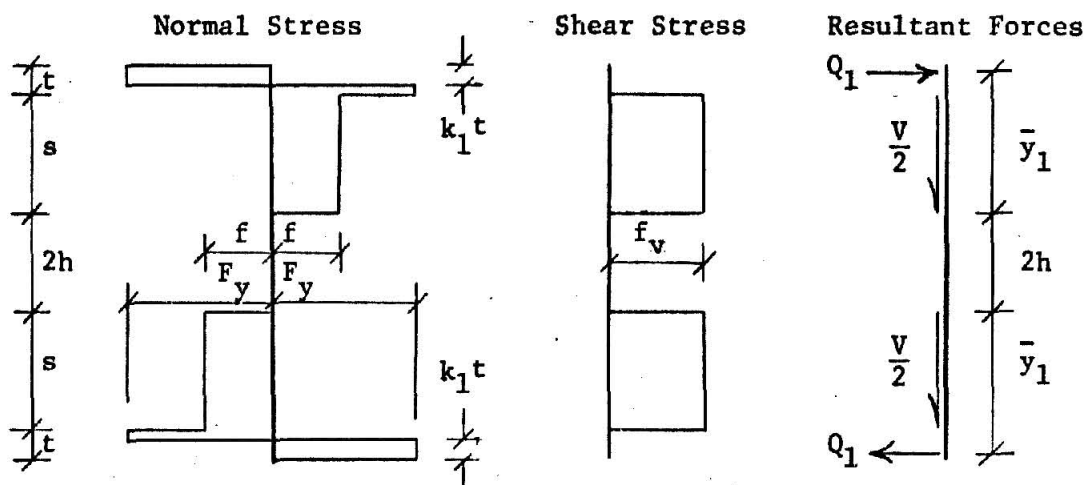
Redwood's method of ultimate strength analysis of beams with web openings involves imposing static equilibrium on assumed stress reversal cases. For Redwood's concentric case there are two cases of stress reversal locations at Section 1 and one at Section 2. The stress reversal locations are illustrated below with their corresponding resultant forces and equations. In the concentric case, half of the total shear force is assigned to the top tee and half to the bottom tee. In all cases the shear stress is distributed evenly throughout the clear depth of the web. The corresponding normal stress must satisfy Von Mises' yield criterion, i.e. $F_y^2 = f^2 + 3f_v^2$, where F_y = yield stress, f = normal stress and f_v = shear stress. Cross sectional properties are shown in Fig. 1.



Section 1. Case WW (both stress reversal points are in the web at Section 1)

$$Q_1 = btF_y + st_w(1-2k_1)f$$

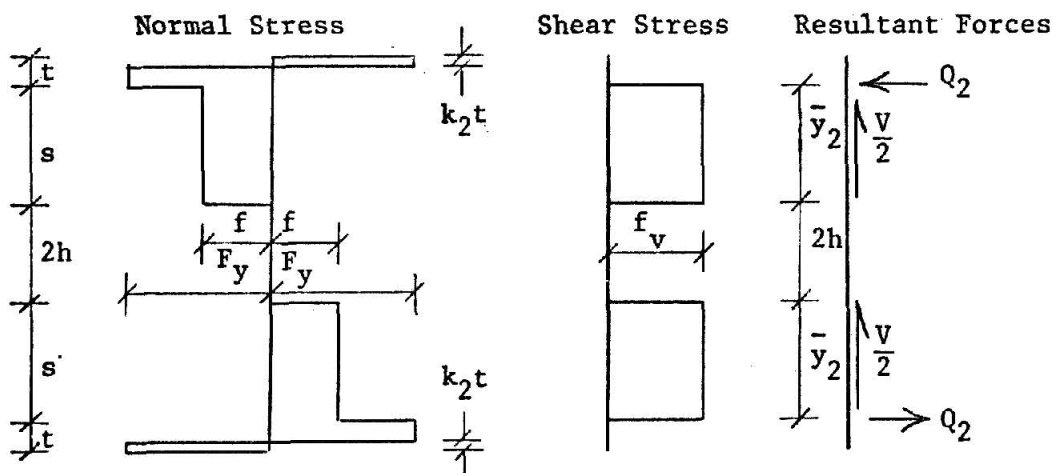
$$Q_1 \bar{y}_1 = btF_y(s + .5t) + .5s^2 t_w f(1-2k_1^2)$$



Section 1. Case FF (both stress reversal points are in the flanges at Section 1)

$$Q_1 = btF_y(2k_1 - 1) - st_w f$$

$$Q_1 \bar{y}_1 = -.5s^2 t_w f + btF_y [2k_1(s+t) - (s+.5t) - tk_1^2]$$

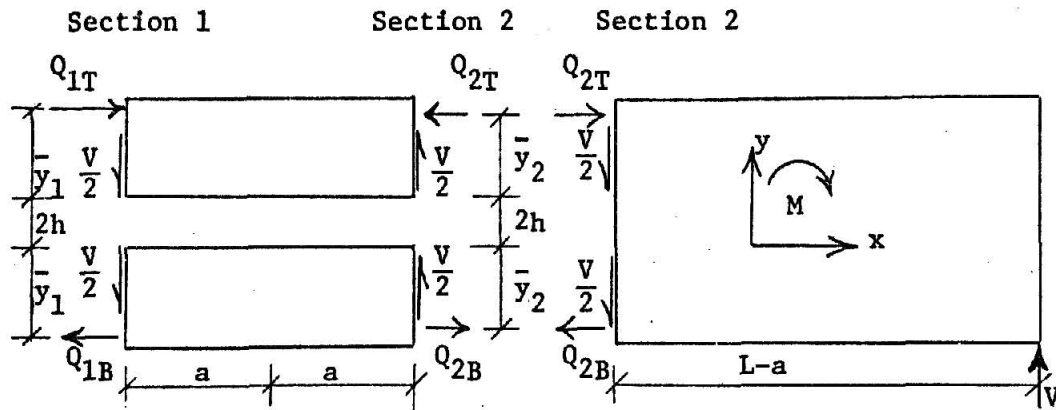


Section 2. Case WW or Case FF

$$Q_2 = st_w f + btF_y(1 - 2k_2)$$

$$Q_2 \bar{y}_2 = .5s^2 t_w f + btF_y [(s+.5t) - 2(s+t)k_2 + tk_2^2]$$

Imposing equilibrium on the free body diagrams below gives the following relationships.



tee sections:

$$\Sigma F_x; 0 = Q_{1T} - Q_{2T}; \quad Q_{1T} = Q_{2T} = Q_T$$

$$0 = -Q_{1B} + Q_{2B}; \quad Q_{1B} = Q_{2B} = Q_B$$

$$\Sigma F_y; 0 = \frac{V}{2} - \frac{V}{2}$$

Shear force is constant across the tee section.

$$\Sigma M_A; 0 = Q_T \bar{y}_1 - Q_T \bar{y}_2 - Va;$$

$$0 = Q_B \bar{y}_1 - Q_B \bar{y}_2 - Va.$$

Section 1:

$$\Sigma F_x; 0 = Q_T - Q_B; \quad Q_T = Q_B = Q$$

The normal force is constant at each hinge.

$$\Sigma F_y; 0 = V - \frac{V}{2} - \frac{V}{2}$$

Shear in top and bottom tees must sum to the total shear.

$$\Sigma M_A; 0 = Q\bar{y}_1 + Q(2h+\bar{y}_1) - V(L+a)$$

$$= 2Q(h+\bar{y}_1) - V(L+a) = M_1 - V(L+a)$$

$$\text{where } M_1 = 2Q(h+\bar{y}_1)$$

Section 2:

$$\Sigma F_x; 0 = Q - Q$$

$$\Sigma F_y; 0 = V - \frac{V}{2} - \frac{V}{2}$$

$$\Sigma M_B; 0 = Q\bar{y}_2 + Q(2h+\bar{y}_2) - V(L-a)$$

$$= 2Q(h+\bar{y}_2) - V(L-a) = M_2 - V(L-a)$$

$$\text{where } M_2 = 2Q(h+\bar{y}_2).$$

Subtracting the moment equilibrium equation at Section 1 from that at Section 2, gives the following relationship: $M_1 = M_2 + 2Va$. Substitution into this equation gives the following quadratic equation in k_2 :

Case WW:

$$k_2^2 t \left[1 - \frac{bF_y}{t_w f} \right] - (d-2h)k_2 + \frac{Va}{btF_y} = 0$$

Case FF:

$$k_2^2 - (1+B)k_2 + \left[\frac{Va}{2btF_y} - \frac{Bs}{2t} + \frac{B^2}{2} \right] = 0$$

$$\text{where } B = \frac{st_w f}{btF_y}.$$

Solving the quadratic gives the value of k_2 and the final variable, k_1 , may be found. Adding the moment equilibrium equations at Sections 1 and 2 gives the moment at the center line of the opening, M : $M = VL = .5(M_1 + M_2)$. A point on the interaction diagram is plotted as

$$\left(\frac{V}{V_p}, \frac{M}{M_p} \right)$$

where

$$M_p = F_y [bt(d-t) + .25t_w(d-2t)^2]$$

and

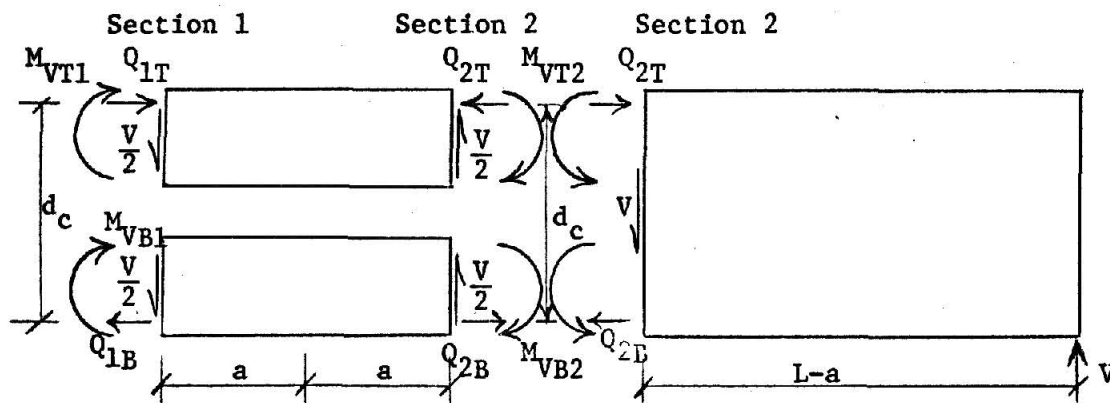
$$V_p = \frac{1}{\sqrt{3}} [t_w(d-2t) F_y]$$

An interaction diagram is generated by varying the assumed shear force and calculating the corresponding moments.

APPENDIX B

McCORMICK'S METHOD

McCormick identified the internal forces acting on the tee sections above and below an opening as shear forces, normal forces and secondary moments due to shear. These are illustrated in the following free body diagrams.



Where

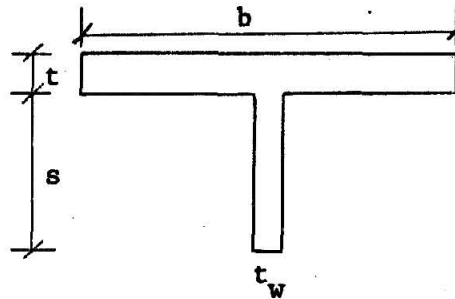
M_v = moment due to shear, and

d_c = distance between resultant normal forces.

The values of normal force, shear force and moment acting concurrently to form a plastic hinge may be found by considering the plastic properties of the tee sections. Equilibrium conditions are used to relate the forces acting at the four plastic hinges.

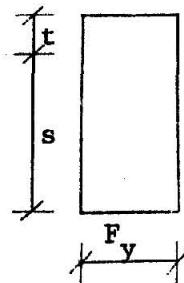
McCormick developed the following plastic properties of the tee sections for his analysis of beams with web openings.

Plastic Properties of Tee Sections



Cross Sectional Parameters

Normal Stress

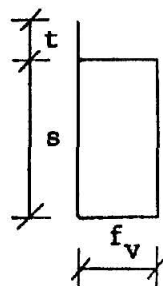


Pure Normal Load

Q_y = normal yield force with a pure normal load.

$$= F_y (bt + st_w).$$

Shear Stress



Pure Shear Load

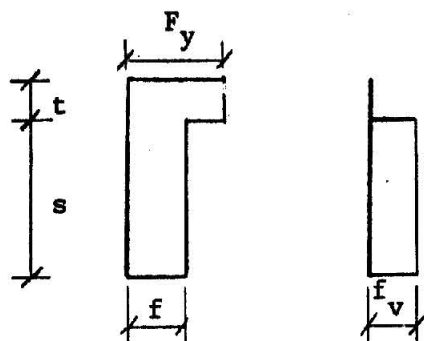
V_y = shear yield force with a pure shear load.

Von Mises yield criterion: $F_y^2 = f^2 + 3f_v^2$ so $f_v = \frac{1}{\sqrt{3}} F_y$.

$$V_y = \frac{1}{\sqrt{3}} F_y st_w.$$

Normal Stress

Shear Stress



Combined Shear and Normal Loads

Q = axial load reduced for shear

$$= F_y bt + st_w f.$$

where $f = \sqrt{F_y^2 - 3f_v^2}$ by Von Mises yield condition so that

$$Q = btF_y + st_w F_y \sqrt{1 - \frac{3f_v^2}{F_y^2}}$$

Parameters

Normal Stress

Shear Stress

$$y = t_w \sqrt{1 - \frac{3f_v^2}{F_y^2}}$$

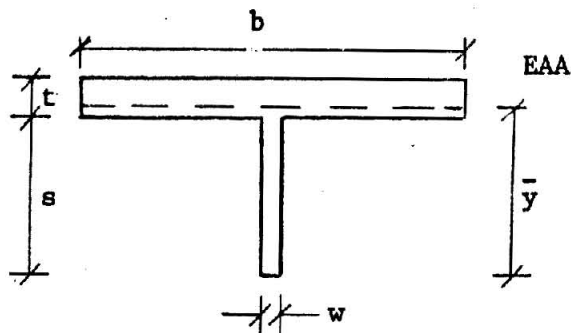
Adopted Cross Sectional Properties

$$Q_y = F_y (bt + sw)$$

Once the web area is reduced, the shear force need not be further considered. The axial stress is equal to the yield stress, F_y , in the flange and in the web.

Combined Bending and Normal Forces

The normal force is centered at the equal area axis (EAA) with the remainder of the tee section used to resist moment.



Location of EAA (for most W shapes, the equal area axis is in the flange)

$$\bar{y} = s + .5t(1 - \frac{sw}{bt})$$

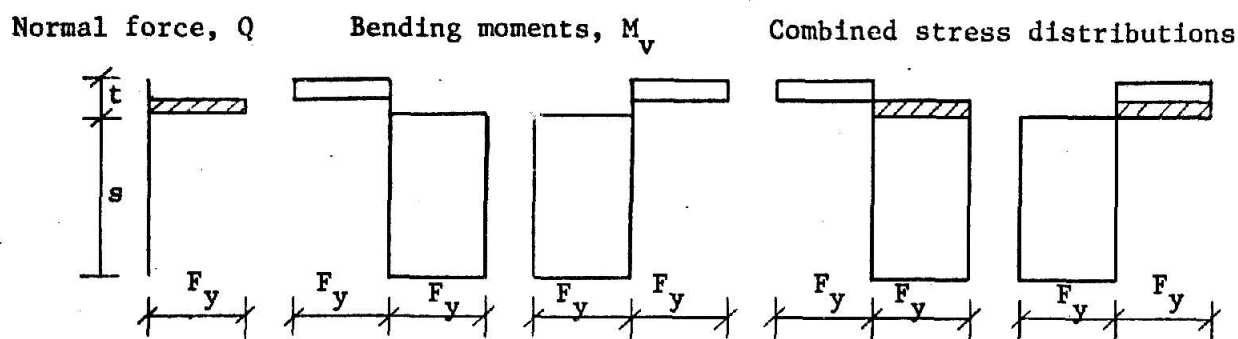
Since this case is most commonly found in practice and corresponds to Redwood's analysis, it is the only case developed here. McCormick's report includes the cases where the equal area axis is at the web-flange junction as well as in the web.

The moment capacity of the tee section reduced for normal force is the moment due to shear, M_v , in the concentric case. To obtain a solution, M_v is found in terms of the adopted cross-sectional parameters and the normal force. Two derivations result depending on whether the normal force is confined to the flange, Case FF, or extends into the web, Case WW. The quantity n^* is defined as the normal force Q^* , which is the limit between these two cases, divided by the axial yield force Q_y .

n^* : Q^* is centered at the EAA and extends to the web-flange junction so that

$$\frac{Q^*}{Q_y} = \frac{1 - \frac{sw}{bt}}{1 + \frac{sw}{bt}} = n^*$$

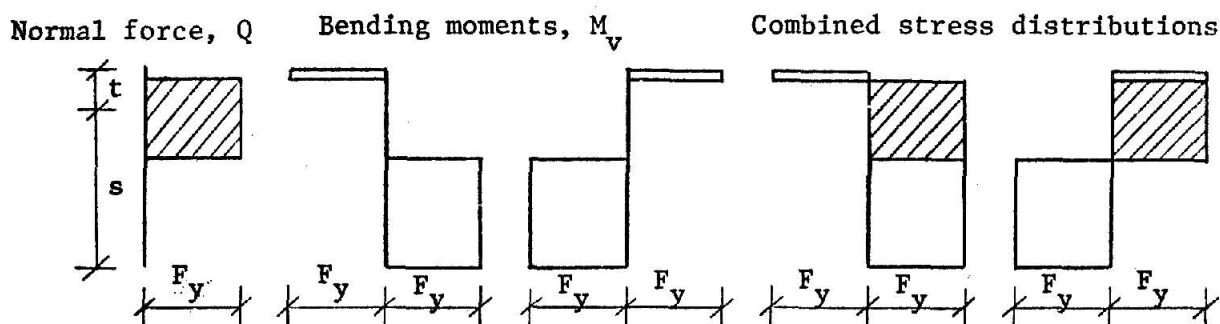
$n = \frac{Q}{Q_y}$; if $n < n^*$ Case FF applies.
 if $n > n^*$ Case WW applies.

Case FF ($n < n^*$)

$$M_v = F_y \left\{ \frac{sw}{2} (s+t) \left[1 + \frac{t}{s+t} \left(1 - \frac{sw}{bt} \right) + \frac{1}{2} \frac{bt}{sw} \frac{t}{s+t} \left(1 - \frac{sw}{bt} \right)^2 \right] - \frac{bt^2}{4} \left(1 + \frac{sw}{bt} \right)^2 n^2 \right\}$$

d_c = distance between the normal force resultants in the top and bottom tees,

$$= 2(s+h) + t \left(1 - \frac{sw}{bt} \right).$$

Case WW ($n > n^*$)

$$M_v = F_y \left\{ \frac{sw}{2} (s+t) \left[1 + \frac{t}{s+t} \left(1 - \frac{sw}{bt} \right) \right] + \frac{bt^2}{8} \left[\left(1 - \frac{sw}{bt} \right) - n \left(1 + \frac{sw}{bt} \right) \right] \left[\left(1 - \frac{sw}{bt} \right) \left(3 - \frac{b}{w} \right) + n \left(1 + \frac{sw}{bt} \right) \left(1 + \frac{b}{w} \right) \right] \right\}$$

$$\begin{aligned}
d_c = 2h + \frac{F_y}{Q} & \left(\frac{bt^2}{4} - \frac{swt}{2} - \frac{b^2 t^2}{4w} + \frac{bts}{2} + \frac{s^2 w^2}{4b} - \frac{s^2 w}{4} \right) \\
& + \frac{F_y}{Q_y} \left(2hts + \frac{bt^2}{2} + \frac{b^2 t^2}{2w} + \frac{3s^2 w}{2} - \frac{s^2 w^2}{2b} \right) \\
& + \frac{QF_y}{2} \left(\frac{bt^2}{4} + \frac{swt}{2} - \frac{b^2 t^2}{4w} - \frac{sbt}{2} + \frac{s^2 w^2}{4b} - \frac{s^2 w}{4} \right)
\end{aligned}$$

Equilibrium

These equations relate to the free body diagrams presented at the beginning of this Appendix.

On tee sections.

$$\Sigma F_x = 0 = Q_{1T} - Q_{2T} ; \quad Q_{1T} = Q_{2T} = Q_T$$

$$\Sigma F_x = 0 = -Q_{1B} + Q_{2B} ; \quad Q_{1B} = Q_{2B} = Q_B$$

$$\Sigma F_y = 0 = -V_{T1} + V_{T2} ; \quad V_{T1} = V_{T2} = V_T$$

$$\Sigma F_y = 0 = -V_{B1} + V_{B2} ; \quad V_{B1} = V_{B2} = V_B$$

$$\Sigma M_{Q_T} = 0 = M_{VT1} + M_{VT2} - 2aV_T$$

$$\Sigma M_{Q_B} = 0 = M_{VB1} + M_{VB2} - 2aV_B$$

From the plastic properties of the tee sections; if $Q_{T1} = Q_{T2}$
 $M_{VT1} = M_{VT2} = M_{VT}$ and if $Q_{B1} = Q_{B2}$, $M_{VB1} = M_{VB2} = M_{VB}$. From the
moment equilibrium equations, $M_{VT} = aV_T$ and $M_{VB} = aV_B$.

On Section 1.

$$\Sigma F_x = 0 = Q_{1T} - Q_{1B} ; \quad Q_{1T} = Q_{1B} = Q$$

Therefore the resultant normal force is the same at each hinge.

$$\Sigma F_y = 0 = -V_T - V_B + V ;$$

Therefore, shear in top and bottom tees must sum to total shear. For concentric cases it is assumed that $V_T = V_B = \frac{V}{2}$.

$$\Sigma M_Q = 0 = Qd_c + M_{VT} + M_{VB} - V(L+a) .$$

On Section 2.

$$\Sigma F_x = 0 = -Q_{2T} + Q_{2B} , \quad Q_T = Q_B = Q$$

$$\Sigma F_y = 0 = -V_{T2} - V_{B2} + V ; \quad V_T + V_B = V$$

$$\Sigma M_Q = 0 = Qd_c - M_{VT} - M_{VB} - V(L-a)$$

The moment at the center of the opening, M , gives the ordinate to one point on an interaction diagram. Adding the moment equilibrium equations at Sections 1 and 2 gives $M = VL = Qd_c$.

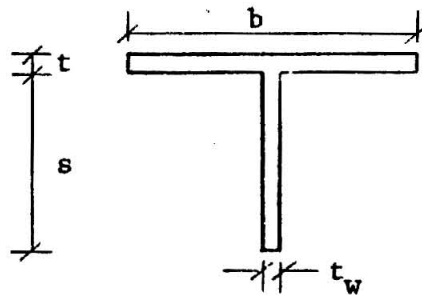
Procedure for concentric openings

1. Assume shear force V . $V_T = V_B = \frac{V}{2}$.
2. Calculate w and M_V ($M_{VT} = M_{VB} = M_V$).
3. Calculate Q from section properties.
4. $M = Qd_c$.

APPENDIX C

SECONDARY MOMENTS IN REDWOOD'S METHOD

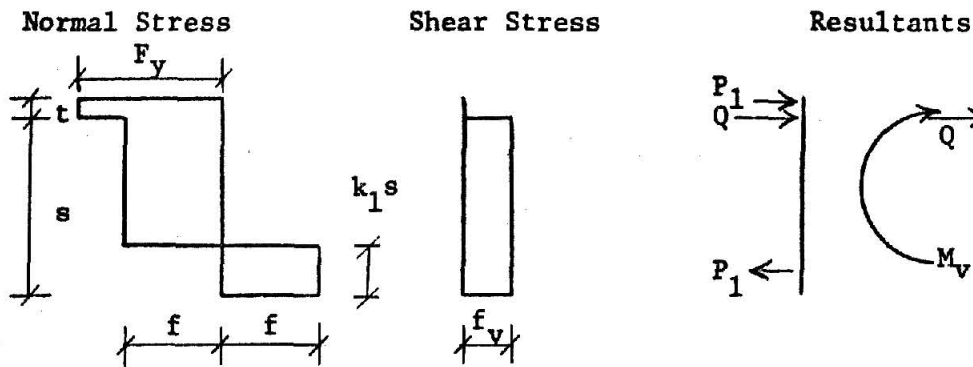
The secondary moments at Sections 1 and 2 must be shown to be equal to demonstrate that the point of contraflexure is at the center of the opening in Redwood's solution. This will be done by using Redwood's stress reversal locations and separating the normal forces into a resultant normal force and a secondary moment as McCormick did. Since the flange areas of most W-shapes are larger than the web areas, the resultant normal force will be applied in the flange. The forces comprising the secondary moment will be determined from the stress reversal location. If these forces are the same at Sections 1 and 2, the secondary moments are the same. Only the top tee will be shown because of symmetry.



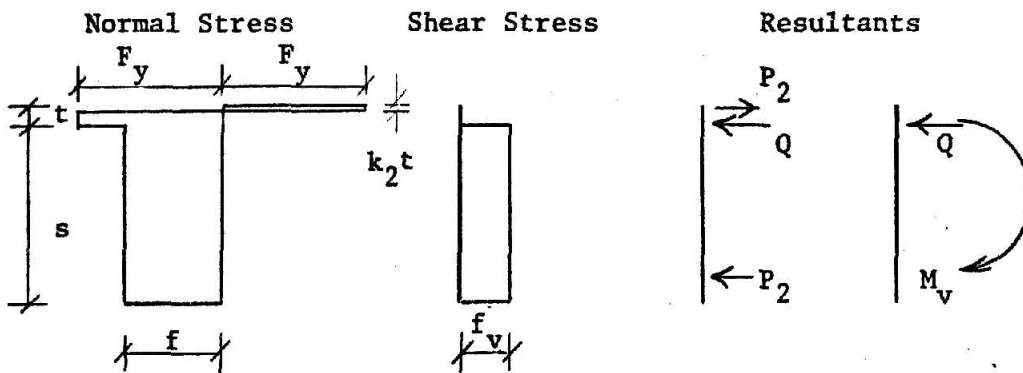
Cross-Sectional Dimensions of Tee Sections

The values of the resultant forces may be expressed in terms of the cross-sectional properties and the stress reversal location. Since the P forces comprising the moment due to shear have the same magnitude, the normal force is the resultant normal force.

Section 1:



Section 2:



Low Shear Case WW

$$P_1 = st_w f k_1$$

$$P_2 = bt F_y k_2$$

P_1 must equal P_2 for the secondary moments to be equal. To get k_1 in terms of k_2 :

$$Q_1 = bt F_y + st_w f (1 - 2k_1)$$

$$Q_2 = st_w f + bt F_y (1 - 2k_2)$$

since

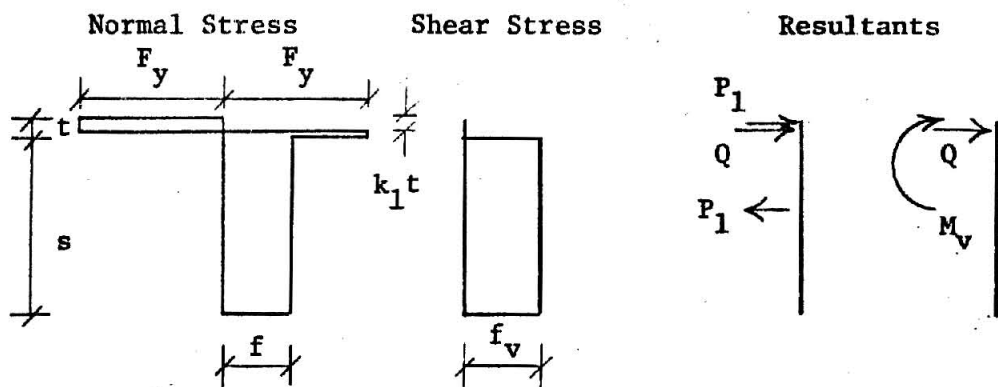
$$Q_1 = Q_2, \quad k_1 = \frac{bt F_y}{st_w f} k_2$$

therefore

$$P_1 = k_1 st_w f = \frac{btF_y}{st_w f} k_2 st_w f = btF_y k_2 = P_2$$

$P_1 = P_2$ so the secondary moments are the same at Sections 1 and 2 and the point of contraflexure is at the center line of the opening in the low shear case.

Section 1:



Section 2 is the same as in Case WW

High Shear Case FF

As in the low shear case, the forces comprising the moment due to shear are expressed in terms of the cross-sectional properties and stress reversal locations, then shown to be equal.

$$P_1 = st_w f + btF_y(1-k_1)$$

$$P_2 = btF_y k_2$$

To get k_1 in terms of k_2 :

$$Q_1 = btF_y(2k_1-1) - st_w f$$

$$Q_2 = st_w f + btF_y(1-2k_2)$$

since

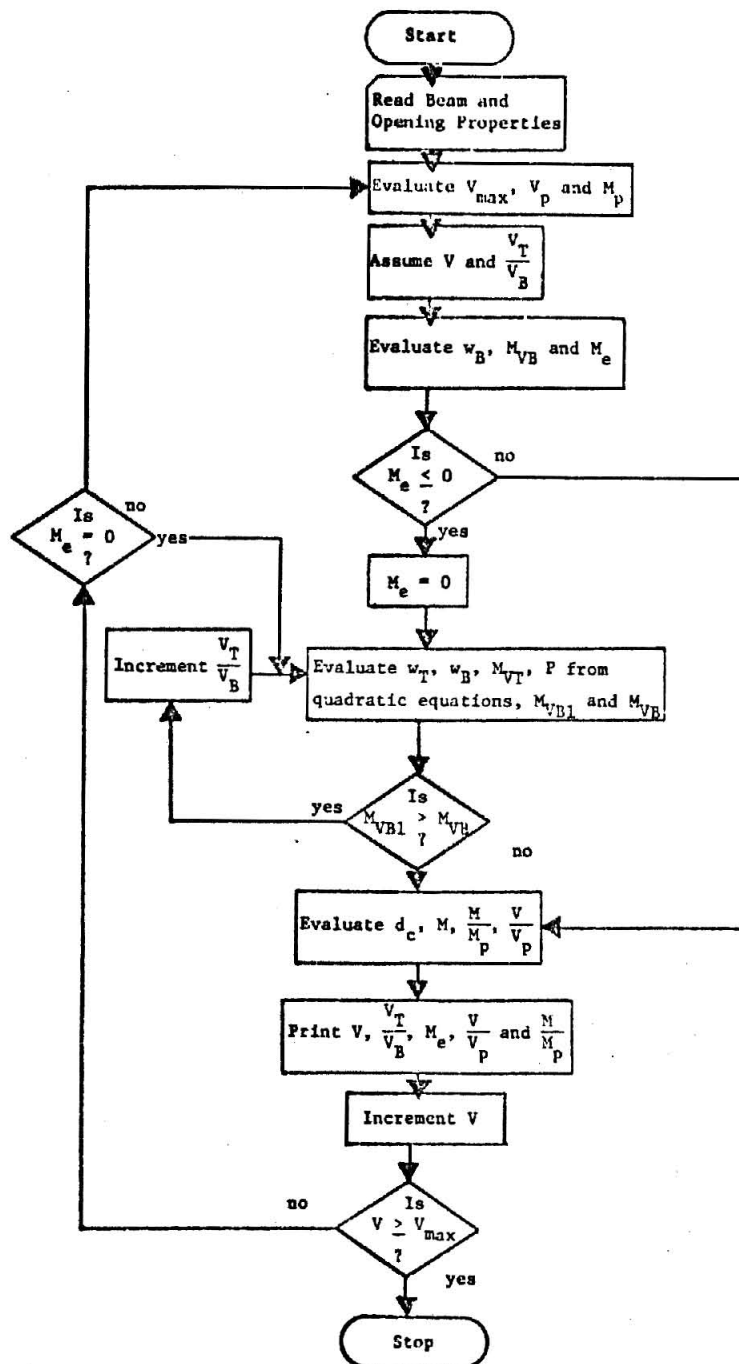
$$Q_1 = Q_2, \quad k_1 = 1 + \frac{st_w^f}{btF_y} - k_2$$

therefore

$$P_1 = st_w^f + btF_y \left[1 - \left(1 + \frac{st_w^f}{btF_y} - k_2 \right) \right] = btF_y k_2 = P_2$$

The point of contraflexure is at the center line of the opening in Redwood's solution.

APPENDIX D
COMPUTER SOLUTION



Computer Flow Diagram

ILLEGIBLE DOCUMENT

**THE FOLLOWING
DOCUMENT(S) IS OF
POOR LEGIBILITY IN
THE ORIGINAL**

**THIS IS THE BEST
COPY AVAILABLE**

```

1      3JCB      TIME=11.0)
2      REAL MVT,AVR,ME,M,NG,NT,MEP,MVRCNE, MVBTWO, NSTAR, NSTART
3      READ (5,1) NP
4      1 FORMAT (I5)
5      DO 2000 I=1,NB
6      READ (5,2) FY, R, D,T,M,M,A,E
7      2 FORMAT (F10.3)
8      VTB=1.
9      ST=0.72*-E-N-T
10     SA=0.72*-E-N-T
11     RT=RT
12     AWE=SA*TK
13     IF (RT .GE. AMR) GO TO 4
14     WRITE (6,3)
15     3 FORMAT (I1X,2H)SINCE THE WFF AREA IS GREATER THAN THE FLANGE AREA T
16     THIS SOLUTION IS NOT APPLICABLE)
17     GO TO 2000
18     4 WRITE (6,5) R,D,T,ST,SB,TW,5,M,A,FY
19     5 FORMAT (I10,5X,I4,11X,I10,11X,I10,2H)ST,10X,2H)SB,11X,2H)TW,11X,
20     C11X,11X,I10,11X,I10,11X,I10,2H)FY,11X,11X,I10,3,2X))
21     VBTW=(1-2)*T*EY/SQRT(3.)
22     VMAX=TA*(C-2)*T-2.*T*EY/SQRT(3.)
23     PV=EY*(RTW*(C-T)+TK*(C-2)*T)**2/4.)
24     WRITE (6,6) VP,VMAX,PV
25     6 FORMAT (I10,5X,2H)VP,8X,4H)VMAX,10X,2H)P,11X,11X,I10,3,2X))
26     WRITE (6,7)
27     7 FORMAT (I10,5X,3H)V,7X,5H)NT/VR,10X,2H)VC,11X,11X,I10,2H)VE,8X,4H)VVP,
28     CEX,4H)XAD)
29     VBT=ST*TK+EY/SQRT(3.)
30     VVE=SB*TK+EY/SQRT(3.)
31     NT=1.
32     NR=1.
33     V=0.
34     VINC=1.
35     PVST=FY*(BT+ST*TK)
36     PVGST
37     VT=(BT*(1ST+1/2.)*ST+2*TW/2.)/(BT+ST*TK)
38     IF (E .EQ. 0.1) GO TO 14
39     VTB=0.
40     C FOR ECCENTRIC OPENINGS, SHEAR ON THE TOP TEE IS INITIALLY ZERO
41     C AND THE AXIAL FORCE IS THE AXIAL YIELD FORCE ON THE TOP TEE
42     8 VAEV
43     WATW=SQRT(1.-(VR/VVB)**2)
44     SAE=SP*WR
45     PVSA=FY*(BT+ST*TK)
46     FR=SWO/PT
47     FM=1.-FR
48     FDB=1.-FR
49     WVA=VPA
50     NSTAG=FWB/FPB
51     A=0/PVSP
52     C THE VALUE OF N COMPARED TO NSTAR FOR THE TEE SECTION DETERMINES WHICH
53     C EQUATION WILL BE USED TO CALCULATE THE MOMENT DUE TO ECCENTRICITY, ME.
54     IF (A .LT. NSTAR) GO TO 9
55     ME=FY*(SA*(C+1/2.)*NR+2*EY/2.+(NR+2*EY/2.)*BT/8.+(FVB**2*(3.-B/WR)*NB*
56     CFPB**2*(2.-3/4-2.1-NB**2*EY/2.)*BT/8.+(FVB**2*(1.48/WR)))- MVB
57     C SINCE WE CANNOT BE NEGATIVE, IT IS SET EQUAL TO ZERO FOR SUBSEQUENT
58     C CALCULATIONS AND SHEAR IS APPLIED TO THE TOP AND BOTTOM TEE SECTIONS
59     IF (ME .LE. 0.1) GO TO 14
60     YB=FY/P*(BT-SW/2.)*SB+1/2.-(SW/2.)*BT/2.+(NB*EY/2.)*BT/2.+(NB*EY/2.)*BT/2.

```

```

50      CT=FBP+T/4.*( NB*FPB-FMB)-BT/(4.*NB)*( NB*FPB-FMB))
51      GO TO 10
52      9  VE=FY*(SWR/2.*(S9+T)+SW8*T*FMB/2.+BT*T*(FMB**2-( NB*FPR)**2)/4.-
      CVP
53      IF (VE .LE. 0.) GO TO 14
54      YR=T*FMP/2.+SB
55      10  CC=2.*YR*VT
56      M=DN(C+ME
57      K=M*M/PM
58      VV=V/VF
59      WRITE (6,11) V,VTB,CC,M,ME,VVP,PPM
60      11  FOSVAT (IX,7(F10,4,2X))
61      V=V+VINC
62      IF (V .GE. VMAX) GO TO 12
63      IF (VE .EQ. 0.) GO TO 15
64      GO TO 9
65      12  WRITE (6,13)
66      13  FOSVAT (IX,13HV +AS REACHED VMAX)
67      GO TO 2000
68      14  ME=0.
69      15  VR=V/(1.+VTR)
70      VT=V-VB
71      IF (VR .LE. VYR) GO TO 16
72      VTB=VTR+.001
73      GO TO 15
74      16  IF (VT .LE. VYT) GO TO 17
75      VTB=VTR+.001
76      GO TO 15
77      17  AT=TA*SQRT(1.-(VT/VYT)**2)
78      W3=TA*SQRT(1.-(VB/VYB)**2)
79      SNT=STANT
80      SNO=SB*NB
81      PYST=FY*(RT+SWT)
82      PYSB=FY*(BT+SWB)
83      FT=SWT/RT
84      FBT=1.-FT
85      FB=SWB/RT
86      FME=1.-FB
87      FDR=1.-FB
88      VVT=2*VT
89      NSTART=FVT/FPY
90      NSTARR=FW3/FPB
91      IF ( NT .LT. NSTART) GO TO 20
92      APT=RT*(FPT**2*(1.+B/WT)/(8.*PYST**2)
93      PBT=RT*(FPT**2*(1.+B/WT)/(8.*PYST)
94      CPT=- VVT/FY+SWT*(ST+T)/2.+SNT*1*FMT/2.+RT*1*FMT**2*(3.-B/WT)/8.
95      CSQ=BPBT**2-4.*APT*CPT
96      IF (CSQ .GE. 0.) GO TO 19
97      WRITE (6,18)
98      18  FORMAT(1H0,42MSQUARE ROOT IN QUADRATIC FOR P IS NEGATIVE)
99      GO TO 20
100     19  P=(-APT-SQRT(CSQ))/12.*APT)
101     NT=P/PYST
102     YT=FY/P*(18T-SNT)*(ST+T)/2.-SNT/(2.*B)+( NT*FPT-FMT)*(2.*ST+
      CTEFMT+T/4.*( NT*FPT-FMT)-BT/(4.*WT))*( NT*FPT-FMT))
103     IF ( NT .GT. NSTART) GO TO 23
104     20  CSQ=(- VVT/FY+SWT*(ST+T)/2.+SNT*1*FMT/2.+BT*1*FMT**2/4.)/(RT*FPT
      C**2/(4.*PYST**2))
105     IF (CSQ .GE. 0.) GO TO 22

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```

106 WRITE (6,21)
107 21 FORMAT(1H0.38HVALUE IN SQUARE RECY FOR P IS NEGATIVE)
108 GO TO 2070
109 22 P=SQRT (CSQ)
110 23 YI=T*WT/2.5T
111 WHEN SHEAR INCREMENT BECOMES .1, STRESS REVERSAL IN THE TOP TEE IS IN THE FLANG
112 24 YI=T*WT/2.5T
113 25 VINC=.1
114 26 MVRN=FVR*#A
115 27 NR=P/PYSR
116 28 IF (NR.LT. .5TAPR) GO TO 25
117 29 MVR=FVR*(SWR/2.5T+SWR**FMR/2.5T+BT/9.5*(FMR**2*(3.5P/WR)+NR*
118 30 CDP**FMR*(2.5P/WR-2.5T-NR**2FPR**2*(1.5B/WR)))
119 31 IF (E-FC. 0.1) GO TO 24
120 32 IF (MVR.NE. .GT. MVR) GO TO 27
121 33 24 YI=FY/P*(BT-SWR)*(S3+T/2.5T+SWR/12.5B)+BT/2.5T*(N4*FPR-FMR)*(2.5S+
122 34 CDP**FMR/4.5T NR*FPR-FMR)-BT/(4.5WR)*(NR*FPR-FMR))
123 35 GO TO 10
124 36 MVR=FY*(SWR/2.5T+SWR**FMR/2.5T+BT*FMR*(FMR**2-(NR*FPR)**2)/4.5)
125 37 IF (E-FC. 0.1) GO TO 26
126 38 IF (MVR.NE. .GT. MVR) GO TO 27
127 39 26 YI=FY/P*(2.5T+
128 40 29 F3MAT(1X,46HSTRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE)
129 41 GO TO 10
130 42 27 VTP=VTR+GOC1
131 43 C THE SMALLER THE SHEAR RATIO INCREMENT, THE GREATER THE ACCURACY AND COST.
132 44 GO TO 15
133 45 2000 CCNTINUE
134 46 STOP
135 47 END

```

SENTOP									
A	D	T	ST	SB	TH	F	H	A	FV
7.000	16.000	0.503	3.297	5.297	0.307	1.000	3.200	6.400	36.000
VP	VMAX	WP							
95.675	54.937	2535.515							
V	VT/WR	DC	M	ME	V/VP	M/MP			
0.0000	0.0000	14.3588	2401.6080	58.3334	0.0000	0.9289			
1.0000	0.0000	14.3589	2395.1610	51.8699	0.0105	0.9264			
2.0000	0.0000	14.3592	2388.6200	45.2798	0.0209	0.9238			
3.0000	0.0000	14.3597	2381.5870	38.5621	0.0314	0.9213			
4.0000	0.0000	14.3604	2375.2510	31.7156	0.0418	0.9187			
5.0000	0.0000	14.3613	2368.4210	24.7391	0.0523	0.9160			
6.0000	0.0000	14.3624	2361.4920	17.6302	0.0627	0.9134			
7.0000	0.0000	14.3637	2354.4610	10.3872	0.0732	0.9106			
8.0000	0.0000	14.3652	2347.3270	3.0069	0.0836	0.9079			
9.0000	0.0330	14.3426	2333.1030	0.0000	0.0541	0.9024			
10.0000	0.0830	14.4113	2313.6750	0.0000	0.1045	0.8949			
11.0000	0.1274	14.4407	2293.6070	0.0000	0.1150	0.8871			
12.0000	0.1670	14.4709	2272.8710	0.0000	0.1254	0.8791			
13.0000	0.2026	14.5021	2251.3350	0.0000	0.1359	0.8708			
14.0000	0.2347	14.5341	2229.0590	0.0000	0.1463	0.8621			
15.0000	0.2637	14.5671	2205.9690	0.0000	0.1568	0.8532			
16.0000	0.2901	14.6011	2181.5370	0.0000	0.1672	0.8439			

17.0000	0.3142	14.6162	2156.8940	0.0000	0.1777	0.8342
18.0000	0.3361	14.6724	2130.8160	0.0000	0.1881	0.8241
19.0000	0.3563	14.7099	2103.4870	0.0000	0.1986	0.8136
20.0000	0.3747	14.7486	2074.8960	0.0000	0.2090	0.8025
21.0000	0.3915	14.7887	2044.7730	0.0000	0.2195	0.7909
22.0000	0.4072	14.8304	2012.8910	0.0000	0.2294	0.7785
23.0000	0.4214	14.8735	1979.0110	0.0000	0.2404	0.7654
24.0000	0.4345	14.9184	1942.8010	0.0000	0.2508	0.7514
25.0000	0.4464	14.9651	1903.8280	0.0000	0.2613	0.7363
26.0000	0.4572	15.0138	1861.2490	0.0000	0.2714	0.7199
27.0000	0.4671	15.0646	1814.1710	0.0000	0.2822	0.7017
28.0000	0.4757	15.1177	1761.1250	0.0000	0.2927	0.6812
29.0000	0.4831	15.1734	1699.2230	0.0000	0.3031	0.6572
30.0000	0.4887	15.2312	1623.6960	0.0000	0.3136	0.6280
31.0000	0.4917	15.2904	1515.7910	0.0000	0.3240	0.5878
SQUARE ROOT IN CUBIC FOR P IS NEGATIVE						
32.0000	0.4917	15.3413	1420.2190	0.0000	0.3345	0.5106
33.0000	0.4920	15.3439	1292.4940	0.0000	0.3355	0.4999
34.0000	0.4932	15.3452	1253.8960	0.0000	0.3366	0.4850
STRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE						
35.0000	0.4948	15.3454	1209.1400	0.0000	0.3376	0.4677
STRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE						
36.0000	0.4953	15.3457	1163.7100	0.0000	0.3386	0.4501
STRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE						
37.0000	0.4979	15.3459	1115.0150	0.0000	0.3397	0.4313
STRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE						
38.0000	0.4994	15.3461	1065.2690	0.0000	0.3407	0.4120
STRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE						
39.0000	0.5010	15.3464	1011.4690	0.0000	0.3419	0.3912
STRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE						
40.0000	0.5025	15.3466	955.5651	0.0000	0.3428	0.3697
STRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE						
41.0000	0.5041	15.3469	895.2749	0.0000	0.3439	0.3462
STRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE						
42.0000	0.5056	15.3471	831.9530	0.0000	0.3449	0.3216
STRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE						
43.0000	0.5071	15.3474	762.3523	0.0000	0.3460	0.2949
STRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE						
44.0000	0.5086	15.3476	685.9329	0.0000	0.3470	0.2653
STRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE						
45.0000	0.5101	15.3479	595.5354	0.0000	0.3481	0.2319
STRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE						
46.0000	0.5115	15.3481	498.0430	0.0000	0.3491	0.1926
STRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE						
47.0000	0.5131	15.3484	369.1411	0.0000	0.3501	0.1428
STRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE						
48.0000	0.5146	15.3486	155.2679	0.0000	0.3512	0.0601
VALUE IN SQUARE ROOT FOR P IS NEGATIVE						
7.000	16.000	0.503	2.297	6.297	0.307	3.200
VP	VMAX	MP		ME	V/VP	M/VP
95.675	54.937	2585.515				
V	VT/VB	DC				

FY

A

H

E

TM

SR

ST

T

D

R

36.000

6.400

2.000

2.000

0.307

6.297

2.297

0.503

16.000

7.000

M/VP

V/VP

ME

M

DC

VT/VB

V

34.8998	0.2434	15.3536	846.0076	0.0000	0.3648	0.3349
STRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE						
34.9299	0.2453	15.3539	797.8017	0.0000	0.3658	0.3086
STRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE						
35.0999	0.2471	15.3541	725.7746	0.0000	0.3669	0.2807
STRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE						
35.1698	0.2489	15.3544	645.2715	0.0000	0.3679	0.2456
STRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE						
35.2398	0.2507	15.3546	552.9159	0.0000	0.3690	0.2139
STRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE						
35.3098	0.2526	15.3549	436.4155	0.0000	0.3700	0.1688
STRESS REVERSAL IN BOTTOM TEE IS IN THE FLANGE						
35.4398	0.2544	15.3551	280.5644	0.0000	0.3710	0.1087

VALUE IN SQUARE FEET FOR P IS NEGATIVE

CORE USAGE OBJECT CODE= 7860 BYTES, ARPA AREA= 0 BYTES, TOTAL AREA AVAILABLE= 174112 BYTES

DIAGNOSTICS NUMBER OF EPRCS= 0, NUMBER OF WARNINGS= 0, NUMBER OF EXTENSIONS= 0

COMPILE TIME= 0.56 SEC, EXECUTION TIME= 18.21 SEC, WATFIV - JUL 1973 VIL4 12.41.35 TUESDAY 8 APR 75

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ULTIMATE STRENGTH PREDICTIONS FOR BEAMS
WITH WEB OPENINGS

by

ROBERTA GWYN SCRITCHFIELD

B.S., Kansas State University, 1973

AN ABSTRACT OF A MASTER'S THESIS

Submitted in partial fulfillment of the
requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1975

ABSTRACT

A source of error in previous ultimate strength analyses of beams with eccentric web openings is identified, and a method of analysis is developed which eliminates this source of error. The ultimate strength of the beam is defined by a failure mechanism consisting of plastic hinges in the tee sections above and below each edge of the opening. Values of ultimate moment and shear acting concurrently at the center of the opening are determined.

Previous methods of analysis were based on assumed stress distributions causing failure at each of the four plastic hinges. These analyses neglected the possibility of more than one stress reversal point at a hinge. It is shown that multiple stress reversal points must be considered in the eccentric case for low values of shear.

A representation of the hinge capacity in terms of normal forces and moments is adopted. A moment is assumed to act at each hinge due to shear in the tee sections. The assumption that the point of contraflexure of the moment due to shear is at the center of the opening is verified. An additional moment is assumed to act on the larger tee section at low values of shear to account for its greater capacity and the additional stress reversal points. This method of analysis is described and then applied to numerical examples using a computer solution. Test results show the analysis is reliable, yet somewhat conservative.