LOT SIZING IN MULTI-LEVEL MULTI-ECHELON INVENTORY SYSTEM

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I. INTRODUCTION

A. The General Lot Sizing Problem

Management of production inventories represents a major interest of production and operations managers. In many industries, the investment in production inventories comprises a substantial share of the firm's assets. Effective control of these inventories can lead to significant cost savings; ineffective inventory control often results in excessive stock investment or material shortages. This thesis is concerned with the techniques to determine the quantity and timing of production orders to replenish inventories in multi-level manufacturing systems, also known as multi-echelon or multi-stage inventory system. In this thesis it will be referred to as multi-level inventory system.

B. Economic Order Quantity

Originally, inventory systems were based on the principle of having items in stock at all times. There was no systematic cost effective way of ordering and stocking items. The shortcoming inherent in this type of system was the high total cost (overall cost). The total cost as used in this thesis is the sum of the setup cost and the inventory carrying cost. Another shortcoming in the system was unpredicTable inventory levels.

An order size determination technique called Economic Order Quantity (EOQ) was later developed to alleviate some of the aforementioned shortcomings. This technique was also called Economic Lot Size (ELS); whereas the form EOQ was associated with purchased parts, ELS referred to manufactured parts. In both EOQ and ELS, the optimum quantity determined was one that will result in minimum total cost over a finite period.

This EOQ strategy does not perform well in a typical manufacturing demand environment. Ideally in manufacturing, the aim is to have the "raw material" delivered just in time to meet the shop demand for the raw material. The economic lot size model is based on minimizing the total cost for a finite planning period, given total aggregate demand for that period. The model does not consider the actual timing of the total demand over the period. For example, the technique will result in the same ordering strategy for both, a demand pattern that requires 100 units per month for a total of 300 units in 3 months as well as a demand pattern of 100 units, 50 units, and 150 units in 3 months. Thus, the traditional Economic Order Quantity is totally insensitive to the timing and quantity of actual discrete demands over the planning period.

C. Lot Sizing and MRP Systems

At this stage lot sizing techniques were developed which overcame the problem associated with the Economic Order Quantity model. These lot sizing techniques result in lower inventory levels, and are capable of performing well in a manufacturing environment. Further, the lot sizing techniques are sensitive to the timing of the requirement in a finite planning horizon. Both the EOQ model as well as the lot sizing techniques minimize the total cost. The EOQ model is based on the total aggregate demand over a finite planning period; the lot sizing techniques do not consider the total aggregate demand over a finite planning period in the determination of the lot quantity. Even though the total aggregate demand over a planning period remains the same, if the timing and the quantity of this demand over this planning period changes, the lot sizing techniques will result in different ordering strategies, and thereby total overall cost. Thus, under different demand patterns the lot sizing techniques result in different total cost, different number of setups, and different inventory levels.

In manufacturing, a finite time period is required to manufacture an item. Thus, if an item is required at a specific time, the order as well as the material for the item should be issued to the shop or the manufacturing department ahead of the required time. This time interval between the time the raw material is issued and the time the finished good is required is called the lead time of the item. For example, if an item is required on January 1st., and has a lead time of 4 months, the material as well as the orders to make the item would have been issued to the shop on September 1st. This process of back issuing an order is called time phasing, and the orders for an item which are time phased are called "Planned Orders". Attributes such as lot sizing, time phasing, and planned orders lead to a system called the "Material Requirement Planning" (MRP) system. An MRP system is a highly effective tool for inventory management due to the following reasons (7):

1. Investment in inventory can be held to a minimum.
2. Order quantities are related to requirements, changes in forecasted demands, and changes in inventory level.
3. An MRP system also emphasizes meeting the requirements, the timing of the requirements, and the necessary orders to meet such requirements.

The primary objective of an MRP system is to determine when and how much of an item to either purchase or manufacture. This process is based on:

1. The Master Production Schedule.
2. The Bill Of Material.
3. On-Hand Inventory level.

The master production schedule is merely a plan for an end item. This plan is derived based on existing committed orders as well as forecasted orders. From this requirement schedule, requirements for all component items including assemblies, subassemblies and raw materials required for making the end item is derived by the MRP system using the Bill Of Material. The bill of material contains a list and number of all subassemblies required to make the end item. Thus, the bill of material structure for an automobile will contain requirements for 4 tires, one carburetor assembly, etc. The bill of material structure for the carburetor assembly will contain the individual components required to assemble the carburetor. Similarly for the tire assembly, it will contain an order for a rim, nuts and bolts. Granted one now has the bill of material and the demand for the end item, the requirements for each of the subassemblies and component parts can be easily derived.

Thus, if one knows the demand for the end item (cars), one can easily derive from the bill of material the requirements for the carburetors, tires, and other items. These requirements for the various items are then adjusted with their respective on-hand inventories to derive the planned orders for the various items.

Traditionally, the Economic Order Quantity (EOQ) technique was used for lot sizing. This technique yielded good results when applied to items having independent demand. The demand of an item is termed independent, when the item's demand is unrelated to the demand of other items. "There has been a feeling that the results would generally be poor if EOQ is applied to dependent demand items" (4). A demand of an item is termed dependent when the item's demand is directly related to, or is derived from, the demand of another related item. This relationship is defined in the bill of materials.

It has been demostrated by many authors and practitioners that traditional inventory control methods have not performed well in MRP environments. Attempts have been made to examine the effect of various lot sizing methods on the performance of a multi-level inventory system in an MRP environment. This brief discussion of material requirement planning demonstrates its use in planning and control of multi-level production systems.

D. Multi-Level Production Systems

A multi-level production system can be defined as a system in which a final product is built in various stages beginning from the raw material to the final assembly. In each of these stages the material from the previous stage is transformed through either a material removal or assembly process. The final product in a multi-level production system is referred to as the end item. Subassemblies, component parts, or assemblies are referred to as items at the intermediate level. In some studies, the end item level is referred to as the highest level number of a multi-level inventory system. In this thesis, the end item level is referred to as the lowest level number of a multi-level inventory system. Thus, in an hierarchical arrangement of items in a multi-level inventory system , the raw material is given the highest level number, the work in process items are given the various intermedieate level numbers, and the end item is given the lowest level number. Figure 1.1 shows a simplified model of a multi-level inventory system. In this system, the manufacturing related decisions are first made at the end item level. These decisions in turn dictate the decisions at the intermediate levels. The last decision made is at the raw material level.

One characteristic of a multi-level production system is that: the production of a part at any level is dependent on the availability of its component parts. For example, production of one automobile depends on having at least four wheel assemblies on hand, in addition to other constituent components. For each wheel assembly, there must be a tire, a wheel, and lug nuts. The entire production is linked through these precedence relationships. If at any level in the system a component part is unavailable, the manufacturing process must be stopped until a sufficient quantity of that component part is acquired or fabricated.

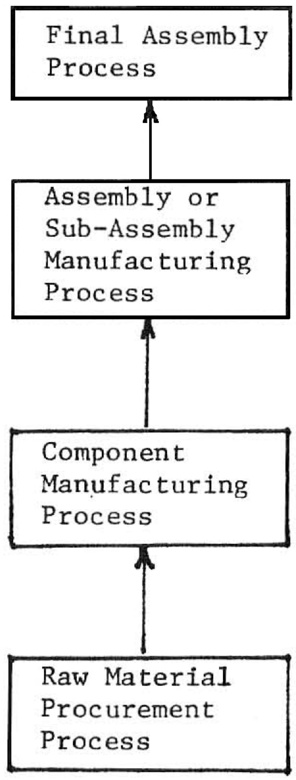

Figure 1.1 Model of multi-level production system

The precedence relationship may be depicted graphically, as shown in Figure 1.2. This is identical to the product structure in a bill of material system. The nodes represent parts, or components, at each level. The arrows denote processing operations, and lead from the constituent part, or subpart to the parent part. A part may be both a parent part and a subpart depending upon its position in the system. The number on the arrows is the usage factor; this number indicates the number of units of each subpart needed for the assembly of one unit of the parent part.

E. Lot sizing Models in Multi-level Systems

The general lot sizing problem is concerned with the determination of a production plan which minimizes the total cost, given the demand forecast for the items to be produced. In a multi-level manufacturing system, the requirements of parts at any level depend on the planned production schedule (lot sizings) of higher level parts, and can be predicted once this production schedule is determined. Till now, researchers have attempted without a great deal of success to ascertain the operating characteristic of various lot sizing models for MRP systems. The purpose of this thesis is to present an evaluation of a limited set of lot sizing alternatives in multi-level inventory systems.

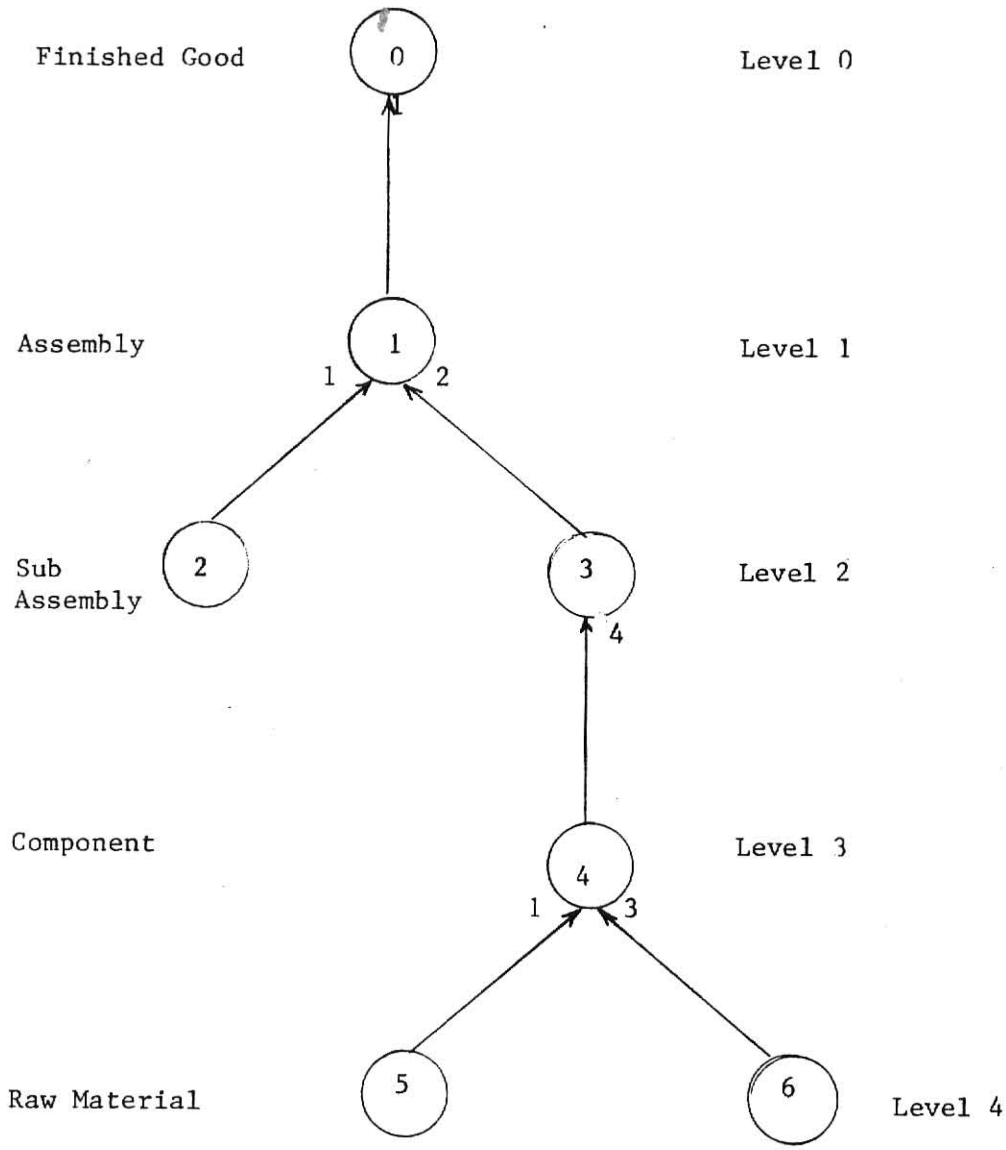

Figure 1.2 Product structure of a multi-level inventory system

A significant amount of work has already been documented in the literature (1) for a single stage system. Given a cost ratio and a demand pattern for a single stage system, there are various lot sizing techniques that will satisfy various measures of performance. According to Collier (4), "Currently MRP users are using single stage lot sizing models such as Economic Order Quantity (EOQ), Lot For Lot (LFL), Periodic Order Quantity (POQ), Least Total Cost (LTC) within an MRP system (1)(2)".

A single-stage lot sizing model may be applied to all the items and levels of a product structure or different lot sizing models to different levels of a product structure. Irrespective of the lot sizing strategy adopted, the common goal in a multi-level inventory system is to achieve the minimum total cost for the whole system, and not the minimum total cost at any particular level. Thus, optimization at individual levels is not as important as the global optimization.

Returning to the example of Figure 1.2, the choice of a lot sizing technique for part 1 affects the gross requirement for parts 2 and 3 , thus affecting the total cost for the whole system. Collier's study (4) also suggests that "a significant degree of interaction occurs between single stage lot sizing model at different levels of a product structure". The degree of interaction refers to the effect, the planned order releases of the end item have on the planned order releases of the higher level items. Therefore, using different lot sizing models at different levels of a product structure can result in different total costs for the whole system and also in different requirements at the various levels. Thus, the selection of a mixed lot sizing strategy is an important managerial decision for an MRP system. It is also important to select an appropriate lot sizing model for each level of a product structure.

F. Problem Description

The problem addressed in this thesis can be briefly stated as follows : Given a muti-level inventory system as is normally present in MRP systems, what is the best mixed lot sizing strategy for the whole system with respect to the total cost?

Simple as this problem may seem, given the several levels and the various lot sizing techniques that can be applied, the combinational explosion of strategies to be tested becomes quickly evident. For example, whereas the 2 level system with 3 lot sizing models will result in 9 different lot sizing strategies, a 3 level system with 3 lot sizing models will result in 27 different lot sizing strategies.

In this thesis rather than evaluate all possible combinations, a restricted set of 9 different mixed lot sizing strategies based on 6 different lot sizing models were evaluated.

A multi-level inventory system consisting of 3 levels and 17 nodes is considered in this thesis. Different demand patterns for the end item are generated, to enable comparisons between alternatives. The various demand patterns include constant, random and seasonal types. It is assumed that all demand for the parts is derived from the master schedule. Thus, only independent demand is considered. Spare parts, replacements, and other types of such requirements not derived from the end item requirements are not considered.

The cost ratio (S/I) of setup cost (S) to annual inventory carrying cost (I) covers the range of cost structures employed in other studies (1)(5)(10). A planning horizon of finite length is considered. Each node of the system represents an item. The lead time for each item is assigned in the beginning. Different lot sizing models are applied at all levels of the product structure. It is assumed that there is sufficient manufacturing capacity to complete all open orders according to schedule.

For purposes of comparison between the strategies, the total setup cost (order cost), total inventory carrying cost, total system cost and the CPU time are recorded for the different lot sizing strategies. However, in the final analysis the total cost becomes the overriding criterion. The next section states the research objective.

G. Research Objective

A mixed lot sizing strategy is one in which more than one lot sizing algorithm is applied to different levels of a product structure. For example, consider a mixed lot sizing strategy POQ/SM/WW. In this strategy the POQ lot sizing model is applied at the end item level, the SM lot sizing model is applied at the subassembly level, and the WW lot sizing model is applied at the raw material level.

The overall objective of the research is to present an evaluation of a set of mixed lot sizing techniques in a multi-level inventory system. The mixed lot sizing techniques to be evaluated are:

1. POQ/POQ/POQ : This lot sizing strategy was chosen because in an earlier study by Choi (3) the author concluded that this strategy was the best performer when evaluated for various demand patterns under different cost ratios in a multi-level system.
2. GT/GT/GT : This lot sizing strategy was chosen

because for a single stage inventory system, the GT model was shown to perform better than the rest of the lot sizing models except for the Wagner-Whitin model.

3. LTC/LTC/LTC : This lot sizing strategy was chosen because in an earlier study by Collier (5), the author proved this strategy to be the best performer among several strategies used by the author in a multi-level inventory system.

4. WW/WW/WW : This lot sizing strategy was chosen because the Wagner-Whitin model was shown to result in the optimum for the single stage system for any demand pattern for any cost ratio.

5. POQ/POQ/LFL : In these 3 lot sizing strategies,the
6. GT/GT/LFL : LFL model is chosen for the lowest
7. LTC/LTC/LFL : component item in the product structure, because it has been postulated by Choi (3) that the ordering policy at the higher levels of a multi-level inventory system resembles the Lot-For-Lot philosophy. The choice of the POQ, LTC and GT lot

sizing models at the lower level is because of the aforementioned reasons for strategies 1, 2 and 3.

8. POQ/SM/WW : These lot sizing strategies were
9. LTC/GT/WW : chosen on a random basis.

The following questions are addressed in this thesis.

(1) Given that the cost ratio is applied uniformly to all the levels, do varying demand patterns have a bearing on the choice of an "optimum" mixed lot sizing strategy?

(2) Given that the different cost ratios are applied at various levels, do varying demand patterns have a bearing on the choice of an "optimum" mixed lot sizing strategy?

(3) The Wagner-Whitin algorithm has been shown to result in the optimum strategy for single stage systems. Will the Wagner-Whitin applied at all levels in a multi-level system result in an overall optimum for the whole system? Thus, the question really being addressed is whether or not the sum of the individual optimums at various levels results in the overall optimum.

(4) At the higher levels in a multi-level inventory system, does the ordering strategy ultimately resembles Lot-For-Lot?

To answer the above questions, the selected mixed lot sizing strategies were evaluated under different demand patterns of the end item. Then the different mixed lot sizing strategies were compared againest the total system cost.

When the total system cost is derived using synthetic data, this is also referred to as total system planned cost. Accordig to Collier (5), this total system cost is equivalent to the total system planned cost, when costs are derived based on synthetic demand data and cost ratio.

H. Report Structure

This research report begins with an introduction, where the MRP based system, problem definition and the research objectives are discussed. In the next chapter, literature relevent to this study is reviewed. The subsequent chapter explains the working procedure of the discrete lot sizing models used in this study. Then the research methodology is presented, followed by the results of the study.

II. REVIEW OF RELEVANT LITERATURE

In chapter I, the multi-level lot sizing problem was introduced along with a description of the problem. A perspective for the thesis is established in this chapter by reviewing the relevant lot sizing literature.

A. Characteristics of Lot Sizing Problems

Several researchers have published articles surveying the various lot sizing techniques. Each offers his own classification scheme for the lot sizing problem. Lot sizing problems can be classified according to the number of levels, the number of end items, the continuity of demand, permissibility of backlogging and the type of production and inventory control system used (7).

1. Number of Levels

This characteristic refers to the number of stages in the manufacturing system. It is assumed that at each level there is a part or a component that may be inventoried. There are two major catagories for this characteristic: single-level and multi-level. Single-level lot sizing problems involve determining the planned orders for a single part; the demands for each part are determined externally. In multi-level lot sizing problems, the planned orders and requirements for the parts are linked through the dependent demand concept. In multi-level production system, the end item demand dictates the requirement of assemblies or subassemblies which in turn dictates the requirement for the raw materials. Therefore, the demand for the raw materials is dependent upon the demand of the subassemblies or assemblies, and the demand for the subassemblies or assemblies depend upon the demand of the end item.

2. Number of end items: Number of parent parts

This characteristic refers to the number of finished goods for which we have master production schedules. Again there are two categories for this characteristic: single end item and multiple end items. A subpart may have a single parent item, or it may have multiple parent items as shown in Figure 2.1. A parent part may occur at any level in the product structure; typically, a part will be a subpart to its parent part, and is a parent part for its own subparts.

3. Continuity of Demand

As previously described, the demand for the particular part may be discrete or continuous. With continuous demand, units are removed from a part's inventory gradually throughout a period. With discrete demand, the part inventory is depleted at the beginning of the period. The difference between these two cases lies in the timing of the

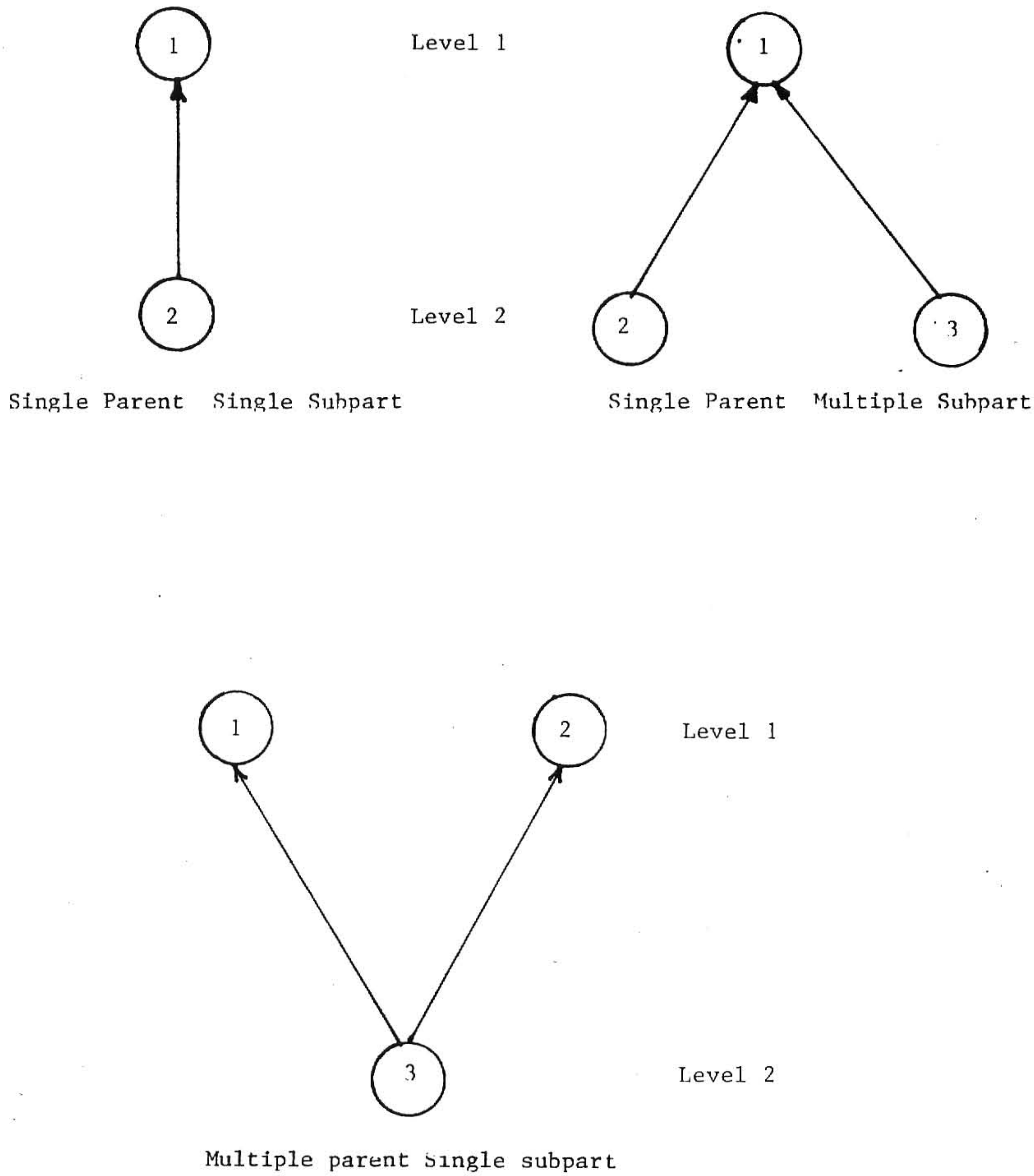

Figure 2.1 Example of single and multiple parts

demand.

4. Backlogging

Backlogging or back orders for parts may or may not be allowed. A backlog is created whenever the inventory of a part is not sufficient to meet the demand for that part. Even though management may not desire backlogs, they still may occur due to unforeseen circumstances.

5. Production Controll System

The use of MRP as a production planning and inventory control tool constitutes another characteristic of lot sizing problems. Before similar tools were available, the predominant production planning and control systems were collectively called the order point systems. In fact, this type of system is still used by a majority of manufacturing companies. Because more and more firms are adopting MRP systems, it is important to consider lot sizing rules for this type of system.

B. Characteristics of the Thesis Problem

The multi-level lot sizing problem studied in this thesis has the following characteristics:

1. There is only one end item.
2. Some items in the product structure have multiple parent items.
3. The demand for the end item is discrete and deterministic.
4. A planning horizon of 12 periods.
5. An unlimited production capacity to manufacture all the required items.

C. Related Research

Many combinations of problem characteristics have been studied. To better relate previous work to the current thesis topic, only the relevant subset will be presented here.

There has been very little work in the area of multi-level lot sizing models that has been documented so far. Multi-stage inventory analysis is frequently approached by using a set of single stage, single product inventory models. It is useful to examine them first.

D. Single-Level, Discrete, Deterministic Demand Models

A Material Requirement Planning (MRP) system requires a lot sizing model to be used for every item under its control. The quantity to be produced at any time period has an impact on inventory control.

In general, inventory ordering policies can be divided into two categories(10). The first category of inventory ordering policy is the fixed or variable order quantity, variable period method. When use has depleted inventory level down to some predetermined point, a fixed or variable quantity is ordered. The traditional Economic Order Quantity methodology belongs to this category. "The second category of inventory ordering policy is the variable quantity, fixed period method"(10). In this procedure, optimal inventory levels are determined. At fixed time periods, the inventory status is reviewed and a variable order quantity is placed to bring the inventory level back to the predetermined optimal level. Almost all researchers investigating lot sizing models have restricted their efforts to the models that order either a fixed or variable order quantity at variable times.

Wagner and Whitin developed a lot sizing model based on the principles of dynamic programming (17). The authors assumed in their model that the manufacturing price and the selling price were constant for all the periods. The authors also assumed that the demands and the costs were all non-negative. By using several theorems, they simplified the required computations in their procedure. Wagner and Whitin showed that in an optimal ordering policy, each order quantity covers an integer number of consecutive periods of a finite planning horizon. The planned orders arrive at the beginning of a period when the inventory level has fallen to zero. To determine a period to place an order, all possible ordering combinations that satisfy the demand requirements of each period in the planning horizon were evaluated. The authors established a minimal cost policy for periods 1 through t based on the following equation:

$$F(t) = \min \left[\begin{array}{l} \min\limits_{i \leq j < t} \left[S_j + \sum\limits_{h=j}^{t-1} \sum\limits_{k=h+1}^{t} i_h d_k + F(j-1) \right. \\ \qquad\qquad S_t + F(t-1) \end{array} \right]$$

d_t = amount demanded

i_t = carrying cost per unit of inventory carried forward to period t+1

X_t = amount ordered

$F(1) = S_I$

$F(0) = 0.$

As per the above equation, if there are T periods in the planning horizon, a maximum T(T+1)/2 cost computation needs to be done. The algorithm starts from the first period and proceeds in the forward direction till the last period. After evaluating all possible combinations of ordering, it chooses the combination that results in the minimum total cost over the entire planning horizon. Wagner and Whitin also showed for the case of a steady state demand, and constant ordering and holding cost, their method resulted in the same total cost as the "square root formula"(EOQ).

The advantage of this method was that it resulted in lower total cost or at worst the same cost as the EOQ algorithm. This was truely independent of the demand pattern and the cost criterion. This method ensured minimum total cost over the planning horizon. There was however a drawback in the algorithm: this approach is optimal only if the requirements beyond the planning horizon are not brought into consideration. In most of the practical situations this is not true, as requirements will continue beyond the planning horizon.

Kaiman (10) in 1969 investegated the performance of the EOQ and the Wagner-Whitin lot sizing model for single stage system. The author used seven sets of demand data in his analysis. These seven sets were based on random, constant, and cyclical demand patterns. To study the effect of different demand patterns on the various lot sizing models, the author determined the coefficient of variation for each demand pattern. The coefficient of variation decribes the degree of variation in the demand pattern. The coefficient of variation was an important factor in Kaiman's study and his subsequent conclusions. As per his study, the choice of an algorithm can be determined by using the coefficient of variation.

The author considered three different cost structure ratios: S=2.5P, S=P, S=P/2. (where S= set up cost and P=price per unit). Kaiman in his article (10) stated that "As more variations are observed in the demand pattern, it becomes evident that the Wagner-Whitin methodology provides a good approximation of the least total cost solution". A graphic presentation of total cost vs coefficient of variation for three different cost structure ratios showed when to switch from Economic Order Quantity to Wagner-Whitin algorithim.

Kaiman concluded by saying that both the methods (EOQ and WW) are applicable under different demand conditions. When variation in the demand pattern was extreme (3.17) (10), the Wagner-Within model performed well. In the case of low variation the EOQ model was preferred because it was easy to use and it had a small advantage in total cost.

Tuite and Anderson (16) pointed out a drawback that favored the Wagner-Whitin algorithm over the EOQ algorithm with respect to total cost. The EOQ model was made to bear the inventory carrying costs on one-half of the order quantity (average inventory) during the period of usage. The Wagner-Whitin algorithm charged the carrying cost only on the inventory held for use in the succeeding periods. However, this algorithm did not charge any carrying cost for the average inventory held for use in the period of usage. Therefore, Tuite and Anderson corrected this error by simply charging a carrying cost on the average inventory held for use in the period of usage. They did not include the carrying cost during the period of usage while determining the ordering policy by the original Wagner-Whitin algorithm. After determining the ordering policy, they added the carrying cost during the period of usage to determine the total cost. The carrying cost during the period of usage was based on the following equation:

$$\sum_{i=1}^{n} \frac{d_i}{2} \cdot C_H$$

Where

n = total number of periods

d_i = demand in period i

C_H = inventory carrying cost/unit

The authors used the same demand data set used by Kaiman for comparison of the EOQ method with the Wagner-Whitin method. Tuite and Anderson concluded by saying that the Wagner-Whitin method was optimal when variability in the demand was large. Thus, inclusion of the carrying cost during the period of usage did not affect the optimum ordering pattern; still, this cost can not be ignored when computing the total cost.

In 1968, DeMatteis (6) formulated a new lot sizing technique called the Part-Period Balancing algorithm wherein ten to thirty times fewer calculations were required as compared to the Wagner-Whitin algorithm (17). Mandoza (12) mathematically proved the validity of the Part-Period Balancing algorithm.

The technique was based on a simple measure called the part-period value. "If the number of parts in inventory are multiplied by the number of periods over which they are held" (6) the result is a unit of measure called part-period value. This is similar to man-hours or man-days which express the work content of a job. The economic part period was obtained by dividing the setup (or ordering) cost by the inventory carrying cost per part per period. This technique performs well when the variation in the demand pattern is very small. They incorporated "look-ahead" and "look-back" tests into the algorithm in order to reduce the carrying cost for demand sets with large variations. Both, the look-ahead and look-back tests can be loosely termed as post optimality tests. The Look-Ahead/Look-Back feature is intended to prevent inventory levels covering peak requirements from being carried for long periods of time, and to avoid orders being keyed to periods with low requirements. Figure 2.2 illustrates the required steps for the Look-Ahead and Look-Back tests.

When the cumulative part-period value exceeds the EPP (Economic Part-Period), a setup is tentatively recommended. If the tentative setup occurred at period S, then the demand at period S+1 would be compared with the demand at period S. If the tentative setup is moved ahead to period S+1, then the part period value in period S would be n*d (n is the number of periods for which inventory carrying costs are charged if d is not produced in the setup at period S).

If tentative setup remained in period S, the part-period value in period S+1 would be 1*d (inventory carrying costs are charged for 1 period). If d was larger than n* , then the tentative setup should be moved ahead to period S+1. This procedure is called the look ahead test, and this can be extended subsequently until the final period. After the look ahead test is executed, the look-back test is performed. The same concept as the look-ahead test is applied in this test. However, as shown in Figure 2.2 only one step of the look-back test is performed by the algorithm.

The Part-Period Balancing algorithm was compared with the Wagner-Whitin and the Least Unit Cost algorithm. The Least Unit Cost algorithm "attempts to compute for various order sizes the cost per unit chargeable to setup and to inventory

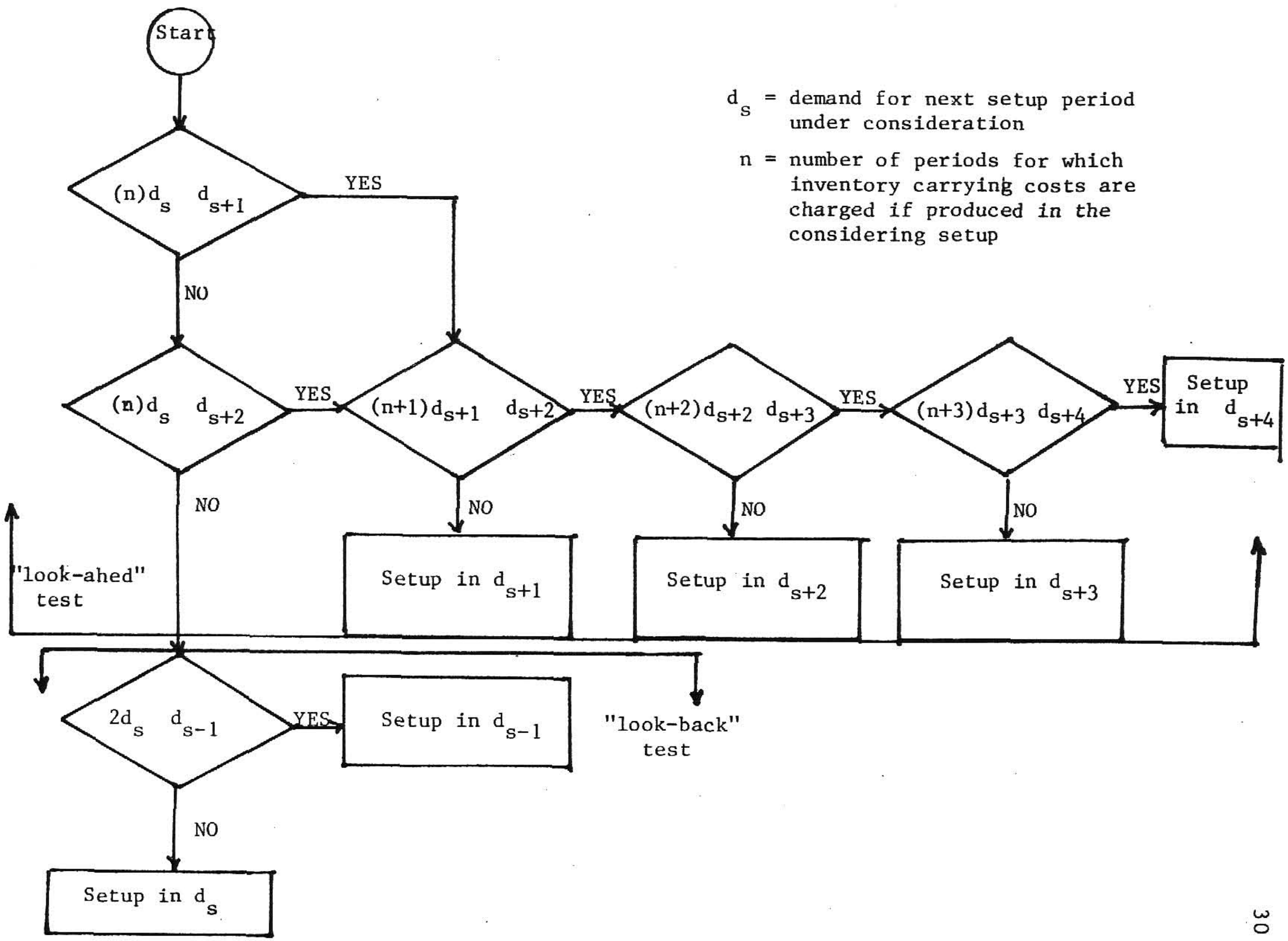

Figure 2.2 Illustration representing the required steps of the Look-Ahead and Look-Back tests

holding cost, and selects a minimum cost value". DeMatteis divided the planning horizon into short horizons (six months) and long horizons (twelve months). According to DeMatteis, for short horizons, the Part-Period Balancing algorithm produced the minimum total cost solution in lesser computational time than the wagner-Whitin algorithm. However for long horizons, whereas the Wagner-Whitin algorithm always resulted in total minimum cost solution, there was no guarantee that the Part-Period Balancing algorithm will result in the minimum total cost solution. The advantages of the part-period balancing technique are as follows:

1. It is particularly well suited for industries where demand is forecasted for a limited number of periods.
2. It is very precise for demands with large variations as well as with small variations.
3. It is inexpensive and quick to implement.
4. It does not consider the demand for those periods for which the setups have already been determined.

In 1969, Silver and Meal (13) introduced a simple formula, which takes into account the variations in the demand rate and achieves a major savings in the total cost, when compared with the Economic Order Quantity model. They modified the static EOQ method under the assumption of varying demand rate for a short time horizon. An advantage of the Silver-Meal algorithm was the reduced computational effort as compared to the Wagner-Whitin algorithm. This advantage resulted from a modification of the EOQ method.

The authors compared the average annual cost resulting from this method with the cost from the EOQ and the Wagner-Whitin methods. They used three different data sets which were taken from other studies by Kaiman (10) and Anderson (16). According to the authors, the Silver-Meal model performed better than both the EOQ model as well as the Wagner-Whitin model with respect to the average annual cost. However, they concluded that "A more exact cost comparison would be achieved by simulating the three methods over a large number of years (with either the annual demand pattern repeating each year or with a pattern that changed from year to year)".

Subsequent to the original development, a serious drawback was pointed out in this model. The model allowed replenishment to be made at any time in the time period, and not necessarily at the beginning of the demand period. This was not valid because the requirements needed during a period should be available on hand at the beginning of the period. Therefore, a few years later they modified their earlier heuristic model (13) to alleviate this drawback.

In their modified version (14), Silver and Meal also tried to find an order quantity Q that minimizes the cost per unit time within a larger total time period that Q lasts. This heuristic model along with the EOQ, PPB, and POQ was compared to the Wagner-Whitin algorithm with respect to the total cost criterion. In their study, Silver and Meal used the system design parameters (demand, cost ratio, etc.) employed by Berry (1) and Kaiman (10) in earlier studies. The authors showed that the total cost obtained by their model was on the average only 0.4 percent higher than the optimum cost achieved by the Wagner-Whitin algorithm. According to the authors, this was a small price to pay for the simplicity of their model in comparison to the Wagner-Whitin model. This model is very attractive for use in coping with deterministic time-varying demand patterns.

Berry (1) in his article presented a framework for comparison between lot sizing procedures such as the Economic Order Quantity, Periodic Order Quantity, Part-Period Balancing and the Wagner-Whitin. A broad range of cost structure ratios and demand patterns were considered, for comparing the performance of these algorithms. Berry suggested the use of the total cost, and the computing time as the performance criteria. However in pilot runs, Berry noticed that the above two criteria were in conflict with each other. Thus an algorithm that performed well with respect to one criteria, failed with respect to the other. Consequently, according to Berry, a compromise has to be made between the criteria based on user's discretion. Berry, however used the total cost alone for his comparison of various lot sizing techniques under different demand patterns and cost structures.

The author based his discussion between the various lot sizing algorithms as a function of two input parameters.

1. The coefficient of variation in the demand rate.
2. The ratio of the Economic Order Quantity (EOQ) to average period demand (D).

A planning horizon of 12 discrete periods was considered by the author. Each period of the planning horizon represented one week. The EOQ/D ratio measures the degree of mismatch between the economic order quantity and the average demand per week. The EOQ/D ratio varied from 0.73 to 1.82. The coefficient of variation of demand data set varied from 0 to 3.31.

A graph was drawn between the total cost and the coefficient of variation. Berry's results differed from the results obtained by Kaiman. This is because Kaiman did not include the carrying cost during the period of usage in computing the total cost for the Wagner-Whitin algorithm. However, Berry showed that the total cost of the EOQ model was the same as that of the Wagner-Whitin model for a constant demand pattern. If the coefficient of variation has a value between 0 - 0.5 (constant demand pattern or almost constant demand pattern), Kaiman showed that the Wagner-Whitin model achieved a lower total cost than the EOQ model. For varying demand patterns, the Wagner-Whitin model resulted in lower total cost than the EOQ model. For a very high coefficient of variation 3.17, the EOQ model and the WW model resulted in the same total cost.

The Part-Period Balancing algorithm resulted in a total cost, which was on the average, only 3.5 percent higher than the total cost as per the Wagner-Whitin algorithm. Finally, Berry concluded by emphasizing three criteria in choosing an inventory procedure for material requirement planning systems: the total cost, computational efficiency, and procedural simplicity.

E. Review of Multi-Level Lot Sizing Literature

Theisen (15) evaluated several lot sizing techniques for a multi-level inventory system in an MRP environment. The lot-sizing techniques considered by the author were the Least Total Cost, Least Unit Cost, Periodic Order Quantity, and the Wagner-Whitin algorithm. The net requirements were determined by the gross requirements minus on-hand inventory minus scheduled receipts. Based on the study, the following recommendations were made by the author:

1. To use a fixed lot size quantity at the end item level. When any unavoidable changes in the requirements have to be made at this level, they should be in the timing only, and not in the quantity. If the changes in the quantity

occurred at the end item level, then the orders already released to the shop or vendor for items at the lower levels have to be expedited or rescheduled.

2. Where setup costs were minimal, intermediate level items such as subassemblies could utlize the Lot-For-Lot approach. The Lot-For-Lot approach would minimize the effect of quantity change at the lower levels.
3. At higher level nodes, a more dynamic lot sizing technique such as the Periodic Order or Least Total Cost technique should be used.

Biggs, Goodman and Hardy (2) investigated the use of heuristic rules for lot-sizing by means of a computer simulation model. Based on multiple performance criteria, the authors evaluated the effect of conventional lot sizing rules on different levels of a multi-level MRP system. The authors used a large scale computer simulation model of a multi-product, multi-level system so that long experiment time horizons could be collapsed into relatively short computer times. Their system simulation model was an adaptation of "Factory-2"(11), which was a multi-level production inventory model developed at Carnegie-Mellon University. The authors in their study made the following assumptions:

1. Deterministic demand patterns.
2. Manpower of uniform ability for all the levels.
3. No preemption of production.

4. One operation at a time at each product structure.

The design of their experiment was a complete factorial design with five lot-sizing rules and 30 replications resulting in a total of 150 runs of the simulation model for each level. Altogether, for three levels, 450 simulation runs were made. They used Monte Carlo type simulation to get 30 replications of demand patterns. The authors evaluated the following lot sizing models:

1. Economic Order Quantity
2. Lot-For-Lot
3. Periodic Order Quantity
4. Part-Period Balancing
5. Wagner-Whitin

All the algorithms except the Lot-For-Lot algorithm tend to combine the requirements for more than one time period. Excepting the EOQ algorithm, all the other algorithm did not split the requirements of a time period into two different lots. The criteria used for analyzing the system were:

1. Total units of stockout for final product.
2. Total number of setups for the entire system.
3. Average inventory carrying cost for the entire system.

Their study showed that the Economic Order Quantity model was better for all the criteria except for the average inventory cost. For this criteria the Lot-For-Lot model performed better. The rationale for this was that the Lot-

For-Lot model has the least investment in the inventory. The surprising result which the authors found was the performance of the Wagner-whitin algorithm. This model did not perform well and was ranked only third for the setup criteria.

Collier (5) demonstrated the effect of lot sizing on work center load. The author addressed two different questions in his study.

1. Which lot sizing model minimizes the total costs which includes setup, inventory carrying cost and the capacity associated cost?
2. What effect do the lot sizing models used by the MRP system have on the planned work center load profile?

To answer these questions, selected lot sizing models were evaluated under four cost ratios. Collier in his study used a product structure of 4 levels and 13 nodes. The author investigated the following 8 lot sizing strategies.

1. The EOQ method for node 1 (end item) and LFL method for other nodes (EOQ/LFL).
2. The EOQ method for all the nodes (EOQ/EOQ).
3. The POQ method for node 1 and LTC method for other nodes (POQ/LTC).
4. The POQ method for all the nodes (POQ/POQ).
5. The LFL method for all the nodes (LFL/LFL).

6. The LTC method for all the nodes (ITC/LTC).
7. The WW method for node 1 and LFL method for other nodes (WW/LFL).
8. The WW method for all the nodes (WW/WW).

Collier analyzed the relationship between the lot sizes, lead times, and the shop load. The author related the manufacturing lead time with the shop load. The rationale for doing so was because the production of each component part requires a manufacturing lead time and the lot quantity determines the work load. Therefore, "how these lot size quantities and resulting loads are allocated over the manufacturing lead time directly affects the nature of each work center load profile". Collier calculated the manufacturing lead time by summing the lead times for all the operations. The author used the following notations in his formula:

Q_i = the lot size quantity for node i

r_{ij} = the run time per unit for node i and operation j

S_{ij} = the setup time for node i and operation j

m_{ij} = the transit time a associated with node i and operation j

W_{ij} = the waiting time associated with node i and operation j

n_{ik} = the set of required operations for node i at work center k

I = the total number of nodes i

K = the total number of work centers for which a load profile was developed

ℓ_{ij} = the planned manufacturing lead time for node i and operation j

$$\ell_{ij} = (Q_{ij} \cdot r_{ij} + S_{ij} + m_{ij} + W_{ij})$$

$$j = 1,2,\ldots, n_{ik}, \quad i \in I, \quad k \in K$$

M_{ij} = the load associated with Q_i for operation j

$$M_{ij} = (Q_i \cdot r_{ij} + S_{ij})$$

$$j = 1, 2, \ldots, n_{ik}, \quad i \in I, \quad k \in K$$

The manufactured items in Collier's study had to undergo only one operation at one work center. Only one work center load profile was generated because all manufactured items were processed through a single work center.

The author assumed the demand for the end item to be constant at 510 units per week. To evaluate the multi-level lot sizing strategies, the author considered four different cost ratios (5/1, 30/1, 55/1 and 80/1). The cost ratios were used uniformly at all the levels of a multi-level inventory system.

The performance criteria included the total planned setup cost, total planned inventory carrying cost, total planned overtime hours, average weekly load in machine hours, and coefficient of aggregate load variation. The first two performance measures represent the traditional cost associated with lot sizing. The latter three criteria relate to the nature of the planned load schedule for the work center. The coefficient of aggregate load variation was defined as the standard deviation of weekly planned load divided by the average weekly planned load for a work center. It reflects on a relative basis, the amount of variability in the load profile and the degree of management replanning required to smoothen out the load.

The performance data were collected over a planning horizon of 52 weeks. The performance of the various lot sizing strategies were evaluated on the following measures:

1. A relative performance index based on total setup and inventory cost.
2. A relative performance index based on total setup, inventory carrying, and capacity change costs.
3. Total planned setup and inventory carrying cost.
4. Total planned overtime hours based on a regular time capacity of 400 machine hours per week for the single work center.
5. Coefficient of aggregate load variation.
6. Average weekly load in machine hours.

Table 2.1 presents the best strategy for the different performance criteria.

In 1981, Choi (3) presented an evaluation of lot sizing alternatives in a multi-level inventory system. Choi in his

Table 2.1 Ranking of algorithm by performance criteria

Cost Ratio → / Criteria ↓	5/1	30/1	55/1	80/1
1	LTC/LTC	WW/LFL	LTC/LTC	LTC/LTC
2	LTC/LTC	WW/LFL	LTC/LTC	LTC/LTC
3	LFL/LFL WW/LFL	EOQ/EOQ	WW/EOQ	EOQ/LFL
4	LFL/LFL	LFL/LFL	LFL/LFL	LFL/LFL
5	WW/EOQ POQ/POQ	LFL/LFL	LFL/LFL	LFL/LFL
6	LTC/LTC	WW/LFL	WW/EOQ	WW/EOQ

study used a product structure of 3 levels and 21 nodes. The author investigated the following lot sizing algorithms:

1. Economic Order Quantity (EOQ)
2. Economic Order Quantity/Economic Production Hybrid Method (EEH)
3. Lot-For-Lot (LFL)
4. Periodic Order Quantity (POQ)
5. Least Unit Cost (LUC)
6. Least Total Cost (LTC)
7. Part-Period Balancing (PPB)
8. Silver-Meal algorithm (SM)
9. Wagner-Whitin algorithm (WW)

The author in his study made the following assumptions:

1. A deterministic demand pattern.
2. A planning horizon of 12 periods, each being 1 month long.
3. No shortages in the system.
4. Integer lead times and production ratios.

The author used 9 sets of demand data in his analysis. These 9 sets were based on random, constant,varying and seasonal demand patterns. A constant pattern was obtained using a constant rate of 50 units/period. The random demand pattern was obtained from an uniform distribution U(0,200). The varying demand pattern was obtained using normal distribution and the seasonal (cylical) pattern was obtained using the SINE function. After generating each demand pattern using various functions or distributions, each demand pattern was normalized to get the same total aggergate demand over the planning horizon.

The author used different cost ratios for each node. The values of these cost ratios were taken from Collier's study (5). The lead time and the production ratio used by the author were generated using an uniform distribution U(1,5). The author used the same lot sizing technique for all the levels of the product structure.

The total setup cost, total inventory cost and the total system cost served as the performance criteria for comparisons between different lot sizing strategies. However, in his final analysis the total system cost became the overriding criterion. Table 2.2 presents Choi's ranking of each strategy with respect to total cost for various demand patterns.

Table 2.2 Ranking of algorithm for each demand pattern

Type → / Rank ↓	Constant	Random	Varying	Varying	Varying	Seasonal	Seasonal	Combination	Actual
1	POQ	POQ	POQ	EOQ/EPQ	POQ	LUC	POQ	POQ	PPB
2	LUC LTC PPB	LUC	LUC LTC PPB	POQ	LUC LTC PPB	POQ	EOQ/EPQ	EOQ/EPQ	LTC
3	SM	EOQ/EPQ	SM	LUC LTC PPB	SM	EOQ/EPQ	LUC	LUC	SM
4	EOQ	LTC PPB	EOQ/EPQ	SM	WW	EOQ	PPB	PPB	POQ
5	EOQ/EPQ	SM	EOQ	EOQ	EOQ	PPB	LTC	LTC	LUC
6	LFL	WW	WW	WW	EOQ/EPQ	LTC	SM	SM	WW
7	WW	EOQ	LFL	LFL	LFL	WW	EOQ	EOQ	EOQ
8		LFL				SM	WW	WW	EOQ/EPQ
9						LFL	LFL	LFL	LFL

III. REVIEW OF DISCRETE LOT SIZING ALGORITHMS

A. Multi-Level Inventory System

As discussed in chapter one, the objective of this study is to evaluate mixed lot sizing strategies for a multi-level inventory system. In a multi-level inventory system the demand for the end item is always known. From this known demand for the end item, demand for the component items and raw materials are generated by an explosion of the Bill of Material product structure. Before showing in detail the demand generating process for items at the higher levels of a multi-level inventory system, some important terms used in a multi-level inventory system need to be defined.

The term "gross requirements" of a component node is the quantity that is required to support the requirements of a parent node. Unless and until the parent node is not the end item, this gross requirement quantity is not directly related to the end item demand. The gross requirement of the end item is the market requirement.

The term " scheduled receipts" represents the quantity of units that are received in a period based on orders placed either in the current period or the previous periods.

The term "lead time" is defined as the time interval between the time the orders are issued and the time the orders are received.

The term "lead time" is defined as the time interval between the time, the orders are issued and the time the orders are received.

The term "production ratio" reflects the quantity relationship between the node and it's predecessor nodes. The ratio can be expressed as follows:

$$\text{Ratio } (j) = \frac{Q_j}{Q_{p(j)}} \qquad \text{Ratio } (j) > 0$$

Where

j = the j^{th} node

$p(j)$ = the predecessor of the j^{th} node

$Q_{p(j)}$ = the quantity to be ordered for the $p(j)$ node

Q_j = the quantity required for the j^{th} node

The cost ratio is the ratio between the setup (order) cost and the annual carrying cost. It is expressed as follows:

$$\text{Cost Ratio} = \frac{\text{Setup Cost per Setup}}{\text{Annual Carrying Cost}}$$

B. Demand For the Higher Level Items of a Multi-Level System

Consider a multi-level system consisting of two levels and three nodes as shown in figure 3.1.

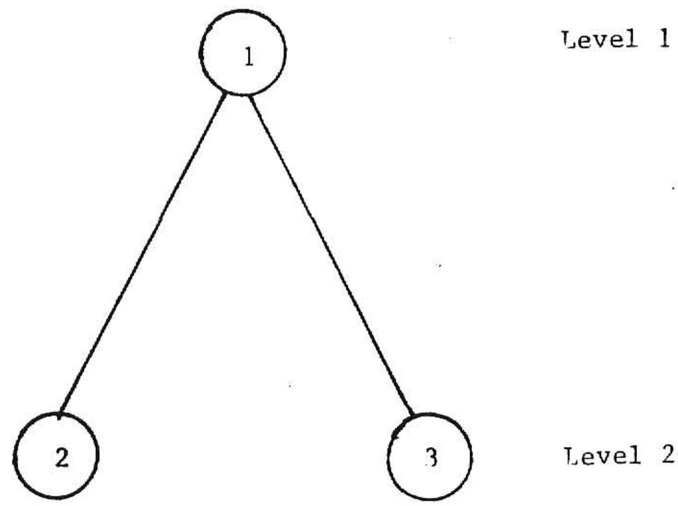

Figure 3.1 A multi-level inventory system

In figure 3.1 each of the nodes 1, 2 and 3 represent item 1, 2 and 3 respectively. Whereas item 1 represents the end item, items 2 and 3 represent the purchased parts required to manufactured item 1. The demand for the end item is deterministic and known. The process of determining the demand for the items at the higher levels of the multi-level inventory system is explained in this section. For the multi-level inventory system shown in figure 3.1, consider the following parameters:

1. Planning horizon of 12 periods, each period being one month long.
2. A setup cost per setup equals to $100

3. Carrying cost/unit/period = $1

4. Annual carrying cost/unit = $12

5. Production ratio between items 1 and 2 = 1:2

6. Production ratio between items 1 and 3 = 1:1

7. End item demand is deterministic.

8. Instantaneous replenishment i.e., no lead time.

9. EOQ model is used to determine the planned order releases for the end item.

Based on the aforementioned cost parameters the Economic Order Quantity for the requirements of periods 1-12 as shown in Table 3.1 is 100 units.

$$EOQ = 2*S*F/2 \qquad \text{equation 3.1}$$

Where EOQ = economic lot size

S = Setup cost/Setup

F = annual demand rate

I = Annual inventory carrying cost

Table 3.1 shows the planned order releases for the end item using the EOQ model. The EOQ model resulted in planned order releases for the end item in periods 1, 3, 6, 7, 8, and 11. Due to zero lead time, the orders are received in the same period in which they are released. To manufacture one unit of item 1, two units of item 2 and one unit of item 3 are required. Table 3.1 shows a planned order of 100 units for item 1 in period 1. To manufacture 100 units of item 1, 200 units of item 2 and 100 units of item 3 are required in period 1. Thus, to meet the planned order releases for the end item, 200 units of item 2 and 100 units of item 3 are required in periods 1, 3, 6, 7, 8, and 11.

Therefore, the requirement for items in the higher levels of a multi-level inventory system is determined from the planned order releases for the end item. Table 3.2 and 3.3 shows the requirement (demand) pattern for items 2 and 3.

Once, the demand for items in the higher levels is known, the next step is to determine the planned order releases for these items. For example, consider item 2. Assume that the Lot-For-Lot model is used to determine the planned order releases for this item.

The Lot-For-Lot model is the most straightforward of all the lot sizing models. In this model the lot size quantity is same as the requirements of a period. In the LFL model each lot is consumed in the same period in which it is received. Table 3.4 and 3.5 show the planned order releases as per the LFL model for the requirements shown in Table 3.2 and 3.3. As can be seen from Tables 3.2 - 3.5 the requirements are directly related to the planned order releases of the end item.

Table 3.1 Planned Order Releases using EOQ

Periods	1	2	3	4	5	6
Requirements	50	25	75	50	00	100
Scheduled receipts	100		100			100
Beginning inventory	100	50	125	50		100
On-hand inventory	50	25	50	0	0	0
Average inventory	75	37.5	82.5	25	0	50
Order releases	100		100			100

Periods	7	8	9	10	11	12
Requirements	50	75	25	40	60	50
Scheduled receipts	100	100			100	
Beginning inventory	100	150	75	50	10	50
On-hand inventory	50	75	50	10	50	
Average inventory	75	125	62.5	30	80	25
Order releases	100	100			100	

Table 3.2 Demand Pattern For Item 2

Periods	1	2	3	4	5	6
Requirements	200		200			200

Periods	7	8	9	10	11	12
Requirements	200	200			200	

Table 3.3 Demand Pattern For Item 3

Periods	1	2	3	4	5	6
Requirements	100		100			100

periods	7	8	9	10	11	12
Requirements	100	100			100	

Table 3.4 Planned Order Releases using LFL

Periods	1	2	3	4	5	6
Requirements	200		200			200
Scheduled receipts	200		200			200
Beginning inventory	200		200			200
On-Hand inventory	0		0			0
Average inventory	100		100			100
Order releases	200		200			200

Periods	7	8	9	10	11	12
Requirements	200	200			200	
Scheduled receipts	200	200			200	
Beginning inventory	200	200			200	
On-Hand inventory	0	0			0	
Average inventory	100	100			100	
Order releases	200	200			200	

Table 3.5 Planned Order Releases using LFL

Periods	1	2	3	4	5	6
Requirements	100		100			100
Scheduled receipts	100		100			100
Beginning inventory	100		100			100
On-Hand inventory	0		0			0
Average inventory	50		50			50
Order releases	100		100			100

Periods	7	8	9	10	11	12
Requirements	100	100			100	
Scheduled receipts	100	100			100	
Beginning inventory	100	100			100	
On-Hand inventory	0	0			0	
Average inventory	50	50			50	
Order releases	100	100			100	

The above example explains the process of generating demand for the items at the higher level of a multi-level inventory system. In the above example no lead time was considered. Practically this is not true because for a manufactured item or a purchased item, the shop or the vendors need time to manufacture and deliver the requested item. Therefore, lead times should be incorporated in determining the planned order releases for any item in a multi-level inventory system.

To illustrate the effects of non-zero lead times, assume a lead time of 3 periods for the end item of the multi-level inventory system shown in Figure 3.1. Also assume as before that the EOQ model is used for determining the lot sizes. Table 3.6 shows the planned order releases for the end item using the EOQ model.

The differences between the planned order releases with (Table 3.6) and without lead time (Table 3.1) are:

1. With a lead time of 3 periods, the order is received in period 4. But the order for this lot is released in period 1. Therefore, the system should have enough initial inventory for item 1 to meet the requirement for periods 1, 2, and 3. If there is no lead time, the first order quantity placed in period 1 is received in the same period (as shown in Table 3.1). Thus no initial inventory is required in this case.

Table 3.6 Planned Order Releases using EOQ

Periods	1	2	3	4	5	6
Requirements	50	25	75	50	0	100
Scheduled receipts				100		100
Beginning inventory	150	100	75	100	50	150
On-Hand inventory	100	75	0	50	50	50
Average inventory	125	87.5	37.5	75	50	75
Order releases	100		100		100	

Periods	7	8	9	10	11	12
Requirements	50	75	25	40	60	50
Scheduled receipts		100		100		100
Beginning inventory	50	100	25	100	60	100
On-Hand inventory	0	75	0	60	0	50
Average inventory	25	87.5	12.5	80	30	75
Order releases	100		100			

2. In the case of non-zero lead time, the number of planned order releases in the planning horizon are less than the case, where zero lead time is considered. This lesser number of planned order releases equals the number of orders placed during the first 'X' periods (X equals the non-zero lead time).

In practice, lead times are incorporated for all items at all levels in a multi-level inventory system. Table 3.7 and 3.8 show the planned order releases for item 2 and 3 with lead times of 2 and 1 period, respectively. For the end item, the planned order releases are different in Tables 3.7 and 3.8 in comparison to Tables 3.2 and 3.3, respectively.

The following section describes the algorithimic procedure for the various discrete lot sizing models used in this study.

C. Discrete Lot Sizing Models

This section details the procedure for using the following lot sizing models in single level inventory systems.

1. Periodic Order Quantity (POQ)
2. Wagner-Whitin (WW)
3. Gaither (GT)
4. Least Total Cost (LTC)
5. Silver Meal (SM)

Table 3.7 Planned Order Releases using LFL

Periods	1	2	3	4	5	6
Requirements	200		200		200	
Scheduled receipts			200		200	
Beginning inventory	200		200		200	
On-Hand inventory	0		0		0	
Average inventory	100		100		100	
Order releases	200		200		200	

Periods	7	8	9	10	11	12
Requirements	200		200			
Scheduled receipts	200		200			
Beginning inventory	200		200			
On-Hand inventory	0		0			
Average inventory	100		100			
Order releases	200					

Table 3.8 Planned Order Release using LFL

Periods	1	2	3	4	5	6
Requirements	100		100		100	
Scheduled receipts			100		100	
Beginning inventory	100		100		100	
On-Hand inventory	0		0		0	
Average inventory	50		50		50	
Order releases		100		100		100

Periods	7	8	9	10	11	12
Requirements	100		100			
Scheduled receipts	100		100			
Beginning inventory	100		100			
On-Hand inventory	0		0			
Average inventory	50		50			
Order releases		100				

The above models are called discrete lot sizing models because, each model generates a lot size quantity that equals the net requirement for an integer number of consecutive planning periods. The discrete lot sizing techniques do not create "remnants",i.e.,quantities that would be carried in inventory for some length of time without being sufficient to cover a infull the requirement of a future periods.

Parameters of a sample problem

The following are the parameters of a sample problem that will be used to explain various discrete lot sizing models listed above.

1. Setup cost (s) = $100
2. Annual carrying cost (I) = $12/unit.
3. Carrying cost per unit per period = $1
4. A total aggregate demand of 600 units for a planning horizon of 12 periods.
5. Zero lead time.
6. Requirements for the end item as shown in Table 3.9.

1. Periodic Order Quantity

This model is, basically, a modified version of the classical EOQ model. The modifications are made to suit the requirements of a discrete period demand. Using known future demands as represented by the requirements schedule of a given item, the EOQ is first computed using Equation 3.1. The EOQ quantity obtained is then used to determine the number of orders to be placed in the planning horizon. The number of periods in the planning horizon are then divided by the number of orders to be placed in the planning horizon, to determine the ordering interval. At each ordering period an order quantity that will cover the requirements untill the next ordering period is placed.

1.1 POQ for the example problem

As per Equation 3.1 the EOQ is

$$2*600*100/12 = 100$$

and the number of orders to be placed in the planning horizon

$$= 600/100 = 6.$$

Thus, the ordering interval = 12/6 = 2 periods.

Using these values, the planned order releases for the item is shown in Table 3.10

2. Least Total Cost

The Least Total Cost approach is based on the rationale that the sum of the setup and the inventory carrying cost for all lots will be minimized if these costs are set equal.

Table 3.9 Requirement Pattern

Periods	1	2	3	4	5	6
Requirements	50	25	75	50	0	100

Periods	7	8	9	10	11	12
Requirements	50	75	25	40	60	50

Table 3.10 Planned Order Releases using POQ

Periods	1	2	3	4	5	6
Requirements	50	25	75	50	0	100
Order releases	75		125			150

Periods	7	8	9	10	11	12
Requirements	50	75	25	40	60	50
Order releases		100		100		50

This objective is achieved by producing in quantities such that the setup cost equals the carrying cost for the quantity associated with the setup. This quantity is called the economic part period (EPP).

The (EPP) is defined as the quantity carried in inventory that will result in the carrying cost and the setup cost being equal. The EPP is determined by dividing the the inventory carrying cost per unit per period (c/unit/period) into setup (order) cost (S) represented by Equation 3.3.

$$EPP = s/c \qquad \text{Equation 3.3}$$

Where:

S = Setup cost per setup

c = inventory carrying cost/unit/period

EPP = 100/1 =100

A measure called part-period is used in determining the setup quantity. The part-period is defined as the product between the quantity and the number of periods it is carried in inventory. For example, one unit carried in inventory for three periods will have a part-period value of 3, and so will a quantity of 3 carried in inventory for one period. The LTC model chooses a quantity whose part-period comes closest to EPP. An example of LTC computation appears in Table 3.11, and table 3.12 shows the planned order releases for the example problem using the LTC model.

3. Wagner-Whitin

This model uses an optimizing procedure based on the principles of dynamic programming. Basically, it evaluates all possible ordering combinations that satisfy the demand requirements of each period in the planning horizon. Table 3.14 contains the specific calculations and the planned order releases for the item.

The following briefly explains the logical progression of the algorithm when applied to the example problem. In period one, there is only one possible way of ordering i.e., order in period one itself. Therefore, the optimal plan for ordering during period one is to order in period one alone thereby incurring an ordering cost of $100.

Two possibilities must be evaluated for period two: either order in period two, and use the best policy for period one (at a cost of $100+$100 = $200), or order in period one for both periods, and carry inventory into period two (at a cost of $100+$25 = $125). The better policy is the latter one.

The algorithm proceeds along this manner and evaluates all ordering combinations for each period of the planning horizon. Akin to a dynamic programing model, the optimal planned order releases are determined by working backwards

Table 3.11 Computation of LTC

Periods	Net requirements	Carried in inventory	Lot size	Part-Periods (cummulative)
1	50	0	50	0
2	25	1	75	25
3	75	2	150	175

Table 3.12 Planned Order Releases using LTC

Periods	1	2	3	4	5	6
Requirements	50	25	75	50	0	100
Order releases	75		125			150

Periods	7	8	9	10	11	12
Requirements	50	75	25	40	60	50
Order releases		100		100		50

from the final period to the first period. This backward determination is best explained with an example shown in Table 3.13.

The rationale for the planned order releases shown in Table 3.13 can be explained as follows: The minimum cost achieved for the entire system is $780 as shown in Table 3.13 under period 12. This cost is achieved by adopting the optimal ordering policy for periods 1 through 10 and ordering a lot in period 11 that covers the requirement for periods 11 and 12. The minimum cost for periods 1 through 10 is $630 as shown in Table 3.13 under period 10. This is achieved by adopting the optimal ordering policy for periods 1 through 7, and ordering a lot quantity in period 8 that covers the requirements for periods 8, 9, and 10. In a similar manner, the ordering periods and the quantities in the periods can be determined. For this example, ordering will take place in periods 1, 3, 6, 8 and 11 and the quantities will be 75, 125, 150, 140 and 110 respectively.

4. Gaither

The following explains in a stepwise fashion the workings of the Gaither model. An example problem is also worked to better explain the workings of the algorithm.

Table 3.13 Planned Order Releases using WW

periods	1	2	3	4	5	6
Requirements	50	25	75	50	0	100
	100	200	225	325	275	375
		125	275	275	275	525
			275		275	
Minimum cost	100	125	225	275	275	375
order releases	75		125			150

Periods	7	8	9	10	11	12
Requirements	50	75	25	40	60	50
	475	525	625	650	730	810
	425	550	550	665	710	780
		575		630	785	810
						810
Minimum cost	425	525	550	630	710	780
Order releases		140			110	

Step 0

Establish the first non-zero demand period as a setup period T.

From Table 3.14 T = 1.

Step 1

Initially set the lot size (Q) equal to the requirement R(T) of period T. Also set i=1. Q = 50.

Step 2

Consider the requirement of period T+i (R(T+i)). Compute the cost of carrying R(T+i) for i periods. If this cost is less than the setup cost go to step 3, else go to step 4.

i = 1, R(T+i)=25.

Carrying cost of R(T+i) for i periods = 25*1 = $25.

Setup cost = $100.

Since $25 < $100, Go to step 3.

Step 3

Set Q = Q+R(T+i) = 50+25 = 75

i = i+1 = 1+1 = 2

Go To Step 2

Step 2

i = 2, R(T+i) = 75

Carrying cost of R(T+i) for i periods = $150

Setup cost = $100

Since $150 > $100, Go To Step 4

Table 3.14 Planned Order Release using GT

Periods	1	2	3	4	5	6
Requirements	50	25	75	50	0	100
Order releases	75		125			150

Periods	7	8	9	10	11	12
Requirements	50	75	25	40	60	50
Order releases		140			110	

Step 4

Establish t as a tentative setup period where t = T+i. Compute the cost of carrying R(t-1) for t-T-1 periods (CCt). If this carrying cost is less than the cost of carrying R(t) for 1 period go to step 6, else go to step 5.

t = T+i = 1+2 = 3

R(t-1) = R(3-1) = R(2) = 25.

t-T-1 = 3-1-1 = 1

CCt = R(t-1)*(t-T-1) = 25*1 = 25

Cost of Carrying R(t) for 1 period = 75*1 75

Since $25 < $75, Go to step 5.

Step 5

Set the lot size of setup in period T = Q.

Set T = t, Q = 0, and i = 1.

Lot size for setup in period T = 1, is Q = 75.

T = 3 , Go to step 2.

Step 6

Set q = R(t) and set k = 1

Step 7

Consider the requirement for period R(t+k). Compute the cost of carrying R(t+k) for k periods. If this cost is less than the setup cost go to step 8, else go to step 9.

Step 8

Set q = q+R(t+k). Set k = k+1. Go to step 7.

Step 9

Compute the cost of carrying q for 1 period. If this cost is lower than the CCt calculated in step 4, then go to step 10, else go to step 11.

Step 10

Establish the lot size for the setup period T = Q-R(t-1)

Set T = t-1, Q = q and i = k

Go to step 2

Step 11

Set the lot size for the setup at period T = Q.

set T = t, Q = 0 and i = 1. Go to step 2.

This process is continued until the demand in each period of a planning horizon has been allocated to a particular order. Table 3.14 shows the periods in which the planned orders are released using the Gaither lot sizing model.

5. Silver-Meal

The Silver-Meal model chooses a lot size that minimizes the average total cost per period between two consecutive setups. The demand for the consecutive periods are accumulated into a tentative lot until the average total cost (sum of setup and carrying cost for this tentative lot) per period increases. In the computation of this average cost, the periods considered are only those whose requirements are included in the tentative lot size. When the average cost increases at a certain period a new setup is established at this period and the computation restarted.

The following illustrates the working of the SM model for the example problem.

By making a setup in period 1 the average total cost for period 1 is T(1)=100. Adding the next period demand in this setup the lot quantity becomes 75. This lot carries the requirements for two periods.

T(2)= (100+25*1)/2 = 62.5

T(2) < T(1). Therefore add the requirement of period 2 to setup in period 1. Now consider adding the requirement of period 3 to setup in period 1. The lot quantity becomes 200. The average cost now becomes (100+25*1+75*2)/3 = 91.67.

T(3) = 91.67

T(3) > T(2). Therefore, demand of period 3 will not be included in the lot that is to be setup in period 1. A new setup is made in period 3. The lot size for the first setup = 75. This lot will cover the requirements of periods 1 and 2. The same procedure is followed untill the requirements of each period of the planning horizon has been satisfied. Table 3.15 shows the periods in which the planned orders are releasesd using the Silver-Meal algorithm.

Table 3.15 Planned Order Releases using SM

Periods	1	2	3	4	5	6
Requirements	50	25	75	50	0	100
Order Releases	75		125		150	

Periods	7	8	9	10	11	12
Requirements	50	75	25	40	60	50
Order Rerleases		100		100		50

D. Determination of The Total Cost for a Multi-Level System

The total cost of a multi-level inventory system is the sum of the total costs for each level of a multi-level inventory system. The total cost for any level is the sum of the following two costs for that level.

1. Total setup cost for all nodes at that level.
2. Total inventory carrying cost for all nodes at that level (Inventory carrying cost is based on the average inventory level per period over the planning horizon).

<u>Node 1</u>

For the multi-level system shown in Figure 3.1, the EOQ lot size model was used for node 1. Therefore, the total cost for node 1 is determined by using the following relation:

TC = F*S/Q + Q/2 *I

Where:

TC = Total Cost

S = Setup Cost per setup

F = Annual Demand

Q = Economic Order Quantity

I = Annual Inventory Cost

TC = 600*100/100 +100*12/2 =1200

<u>Node 2</u>

Six orders were released as shown in Table 3.4, resulting in the total setup cost of $600

Total carrying cost = Average inventory level for each period * inventory carrying cost/unit/period.

100+100+100+100+100+100+100 = 600*$1 = $600

Total cost for node 2 = $600 + $600 = $1200

<u>Node 3</u>

Six orders were released as shown in Table 3.5 , resulting in the total setup cost of $600

Total carrying cost = Average inventory level for each period * inventory carrying cost/unit/period.

50+50+50+50+50+50 = 300*$1 =$300

Total cost for level 2 = Total cost for node 2 and node 3

Total cost for level 2 = $1200 + $900 = $2100

Total cost for the multi-level inventory system = $1200 + $2100 = $3300

The following chapter details the research methodology.

IV. METHOD OF ANALYSIS

A. Summary of Similar Works

In Chapter II, previous work by authors such as Biggs, Goodman and Hardy (3), Collier (2) and Choi (16) was reviewed. Table 4.1 shows some important differences between this study and those of the aforementioned authors.

B. Product Structure of the Problem

Figure 4.1, shows the actual product structure of a multi-level inventory system of 3 levels and 17 nodes used in this study. As shown in Figure 4.1, the product structure contains only one end item. A few items at level 3 of the product structure have multiple parent items. The requirement for such items is derived from the planned order releases of both the parent items.

C. Assumptions for the Multi-level system

For comparison between the mixed lot sizing strategies used in this study, the following assumptions have been made:

1. A planning horizon of 12 periods is assumed. Each period of the planning horizon represents one month.
2. All the demands for the end item is derived from the master schedule. Spare parts, replacements and

Table 4.1 Differences between similar works.

Authors → Characteristics ↓	Biggs Goodman Hardy	Collier	Choi	Birla
Product Structure	Single end item and one parent node for each node	Single end item and one parent node for each node	single end item and one parent node for each node	single end item and multiple parent nodes
Number of levels	3 levels	4 levels	3 levels	3 levels
Number of nodes	3 nodes	13 nodes	21 nodes	17 nodes
Planning Horizon	30 periods	52 periods	12 periods	12 periods
Demand Pattern	Varying (deterministic)	Constant (deterministic)	Constant and varying (deterministic)	Constant and varying (deterministic)

	Biggs Goodman Hardy	Collier	Choi	Birla
Shortages	Allowed	Not allowed	Not allowed	Not allowed
Vehicle for analysis	Large scale computer simulation model	A deterministic simulation model	Fortran IV based algorithmic model	Fortran IV based algorithmic model
Pefrormance Criteria	1-Total number of stockouts of final product. 2-Total number of setups of total system 3-Average inventory carrying cost of total system	1-Total planned setup cost. 2-Total inventory carrying cost. 3-Total average weekly load in machine hours. 4-Total coefficient of aggregate load variation.	1-Total inventory carrying cost. 2-Total setup cost. 3-Total system cost.	1-Total System Cost 2-Total CPU Time

	Biggs Goodman Hardy	Collier	Choi	Birla
Cost Ratio	Uniform for all the levels	Uniform for all the levels	Different for different nodes	Constant for all the levels. Different for different levels
Lot sizing strategies	EOQ/EOQ/EOQ	EOQ/EOQ/EOQ	EOQ/EOQ/EOQ	POQ/POQ/POQ
	LFL/LFL/LFL	LFL/LFL/LFL	EOQ/EPQ/EPQ	GT/GT/GT
	POQ/LFL/LFL	POQ/POQ/POQ	POQ/POQ/POQ	LTC/LTC/LTC
	PPB/PPB/PPB	LTC/LTC/LTC	LFL/LFL/LFL	POQ/POQ/LFL
	WW/WW/WW	EOQ/LFL/LFL	LTC/LTC/LTC	GT/GT/LFL
		WW/LFL/LFL	LUC/LUC/LUC	LTC/LTC/LFL
		WW/EOQ/EOQ	PPB/PPB/PPB	POQ/SM/WW
		POQ/LTC/LTC	SM/SM/SM	LTC/GT/WW
			WW/WW/WW	WW/WW/WW

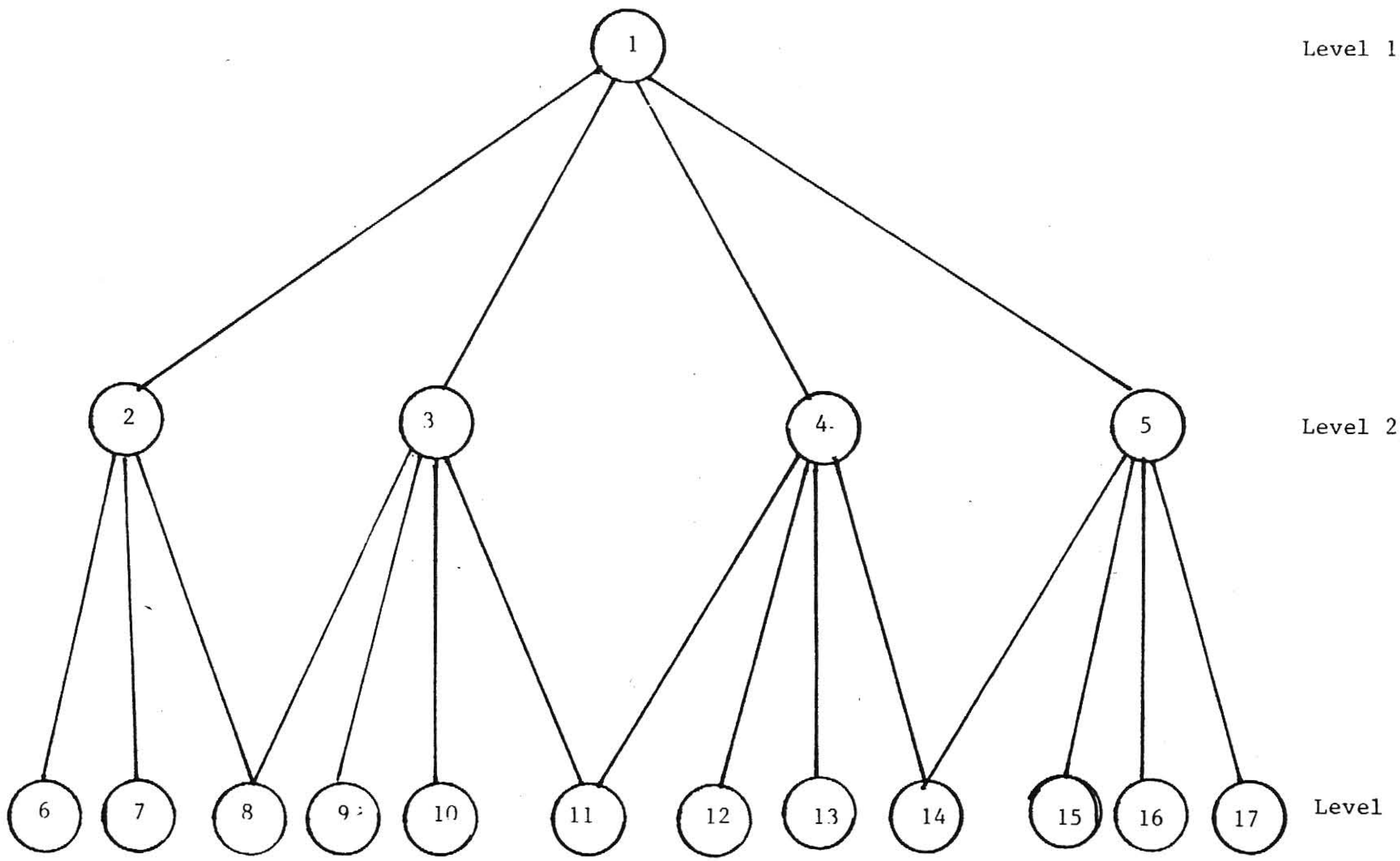

Figure 4.1 A product structure of 3 levels and 17 nodes for the multi-level inventory system

other types of such requirements not derived from the end item requirements are not considered.

3. The demand for the end item is deterministic and known.
4. The requirements for each period of the planning horizon must be met at the beginning of the period. The planned orders are therefore received at the beginning of the period.
5. A total demand quantity of 600 units for a planning horizon of 12 periods.
6. All the requirements for the items in any period must be met. No shortages are allowed.
7. The ordering decisions are assumed to occur at the beginning of the period.
8. The lead times and the production ratios are assumed to be integer numbers, and are generated by using an uniform distribution in the range 1-5.
9. The inventory carrying cost for each lot sizing model is based on the average inventory level in each period of the planning horizon.

D. Generation of Demand Data

The actual process of generating a demand data set consists of two steps. First, a generalized probability distribution or predetermined function for each set is established and an original demand pattern is obtained. The second step is to normalize the original demand pattern based on a total aggregate demand of 600 units over the planning horizon. This ensures consistency in demand units for all the demand patterns.

For purpose of this study, seven different demand data sets were synthetically generated using the Monte-Carlo principle. The seven sets were based on either generalized probability distributions (uniform and normal) or on predetermined functions (SINE and CONSTANT).

The first set was generated based on a constant demand pattern. The magnitude was set at 50 units per period for a total of 600 units in the planning horizon.

The second set was generated based on an uniform distribution in the range of 1 to 100.

Sets 3, 4 and 5 were generated based on a normal distribution with a mean of 100 and standard deviation of 0 = 5, 0 = 10 and 0 = 15 respectively.

Sets 6 and 7 were generated based on a SINE function as explained in equation 4.1.

$$Y = \beta_0 + \beta_1 * \frac{\text{Sin } (2\pi t)}{L} \qquad \text{eq. } 4.1$$

Where

Y = a requirement point at a period t

β_0 β_1 = coefficients for determining Y

L = Number of cycles for the planning horizon

t = a period ($t = 1,2,\ldots,12$)

The values for B , B and L were selected to be 100, 50 and 4 for set 6 and 100, 99 and 3 for set 7.

The original values as well as the normalized values for each demand pattern are presented in table 4.2. The fractional values are rounded off to integer values. Table 4.3 shows the standard deviation and the coefficient of variation for each demand pattern.

The coefficient of variation which describes the variation in the demand pattern is calculated according to equation 4.2. The larger the value of coefficient of variation,

$$V = \sigma/\overline{r} \qquad \text{eq. 4.1} \quad \text{eq. 4.2}$$

where

$$\overline{r} = \frac{\sum_{t=1}^{T} r_t}{T} .$$

$$\sigma = \sqrt{\frac{\sum_{t=1}^{T} r_t^2}{T} - \left(\frac{\sum_{t=1}^{T} r_t}{T} \right)^2}$$

the greater the variablity in the demand pattern. For a constant demand pattern, the coefficient of variation is 0.

E. Determination of the Cost Ratio

The setup cost and the annual inventory carrying cost are used in defining the cost ratio. These terms are related to each other according to equation 4.3.

Table 4.2 Seven demand requirement patterns

	Pattern	Set	1	2	3	4	5	6	7	8	9	10	11	12	Total	Mean
1	Constant	NN	50	50	50	50	50	50	50	50	50	50	50	50	600	50
		N	50	50	50	50	50	50	50	50	50	50	50	50	600	50
2	Random	NN	97	26	77	57	84	4	99	60	90	38	1	59	692	58
		N	84	23	66	49	73	4	86	52	78	33	1	51	600	50
3	Varying	NN	83	99	96	95	103	97	110	98	105	99	105	101	1190	100
		N	42	50	48	48	52	49	55	49	53	50	53	51	600	50
4	Varying	NN	92	90	98	104	97	92	105	87	96	99	113	113	1188	100
		N	47	45	50	52	49	47	53	44	49	50	57	57	600	50
5	Varying	NN	84	102	92	119	99	106	124	127	108	100	83	113	1255	100
		N	40	48	44	57	47	51	59	60	52	48	40	54	600	50
6	Seasonal	NN	151	100	49	100	151	100	49	100	151	100	49	100	1200	100
		N	75	50	25	50	75	50	25	50	75	50	25	50	600	50
7	Seasonal	NN	21	179	99	21	179	99	21	179	99	21	179	99	1195	100
		N	11	90	49	11	90	49	11	90	49	11	90	49	600	50

NN = Non-Normalized Set;
N = Normalized Set.

Table 4.3 Coefficient of variation and standard devation of each data set

	Pattern	Specification	Parameter 1	Parameter 2	Coefficient of variation
1	Constant	--	--	$\mu = 50$, $\sigma = 0$	0.00
2	Random	Uniform Distribution		$\mu = 50$, $\sigma = 28.173$	0.5634
3	Varying	Normal Distribution	$\mu = 100$, $\sigma_1 = 4.999$	$\mu = 50$, $\sigma =$ B..215	0.0643
4	Varying	Normal Distribution	$\mu = 100$, $\sigma_2 = 9.999$	$\mu = 50$, $\sigma = 4.074$	0.0814
5	Varying	Normal Distribution	$\mu = 100$, $\sigma_3 = 14.999$	$\mu = 50$, $\sigma = 6.567$	0.1313
6	Seasonal	Sine Function	$\beta_0 = 100$, $\beta_1 = 50.99$ $L = 4$	$\mu = 50$, $\sigma = 18.027$	0.3605
7	Seasonal	Sine Function	$\beta_1 = 100$, $\beta_2 = 99.99$ $L = 3$	$\mu = 50$, $\sigma = 32.246$	0.6451

μ = mean, σ = standard deviation, β_0 , β_1 = the coefficients for SINE function,
L = the cycle interval , parameter 1 = the value of parameters to generate each demand set,
parameter 2 = the value of parameters after normalization of each demand set.

$$\text{Cost Ratio} = \frac{\text{Setup cost/setup}}{\text{Annual Inventory Carrying Cost/unit}} \qquad \text{Eq. 4.3}$$

The values of the cost ratios applied in this thesis are taken from other studies by Collier (2), and Kaiman (5). Collier in his study used four cost ratios: 5/1, 30/1, 55/1 and 80/1. Kaiman in his study used three cost ratios: 2/1, 5/1 and 12.5/1. This study evaluates the mixed lot sizing strategies for two cases.

Case I. The cost ratios are applied uniformly to all the levels of the multi-level inventory system.

Case II. Different cost ratios are applied at different levels of the multi-level inventory system.

For case I, the mixed lot sizing strategies are evaluated for four different cost ratios: 5/1, 30/1, 55/1 and 80/1. For case II, three different cost ratios are selected. These are as follows:

For level 1: The cost ratio applied is 5/1.

For level 2: The cost ratio applied is 12.5/1.

For level 3: The cost ratio applied is 30/1.

As the level number decreases, the value of cost ratios also decreases. The rationale for this being, as one moves towards the end item level (lower level number) the cost of the item increases. Consequently the cost of carrying inventory increases. Thus the ratio of the setup to the carrying cost is smaller in comparison to the same ratio at the component level (higher level numbers).

F. Lead Time and the Production Ratio

The lead time and the production ratio for this study are determined using an uniform distribution U(1,5). All derived values for the lead time and the production ratio are restricted to integers. Once these values are determined, they are fixed for the entire simulation. Table 4.4 shows the lead time for each node of the multi-level inventory system. A part explosion diagram showing the production ratio for each node is presented inFigure 4.2.

Table 4.2 Lead times for each node

node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Lead time	1	5	5	1	1	1	4	3	5	4	2	1	3	4	5	1	2

H. Performance Criteria

Seven lot sizing strategies are evaluated with respect to the total system cost for the entire planning horizon. In this thesis, the total system cost is considered as the major performance criteria because the total system cost considers all costs incurred in manufacturing a unit. Thus, in any organization it meets the objective of minimizing the global cost as opposed to minimizing subcosts. Further, the total system cost is considered as a standard measure for

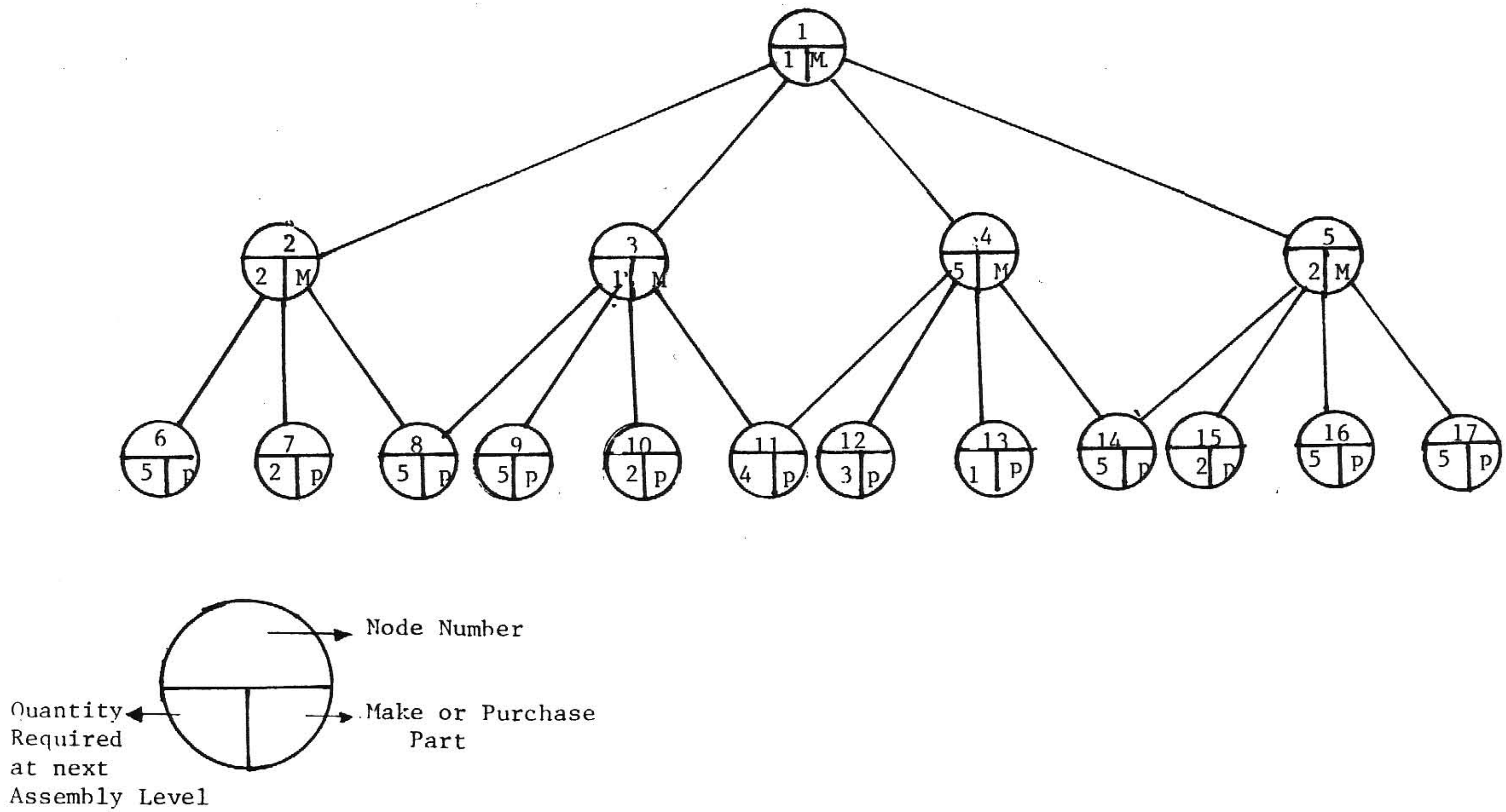

Figure 4.2 Part explosion diagram for product ratios

comparing lot sizing alternatives by different authors. The total system cost as used in this thesis is expressed as follows:

The Total System Cost = Total Cost / Node $1 < i < 17$

Total cost / node = Total number of setup * setup cost/setup + Total inventory carrying cost

The setup cost is incurred in those periods in which the the planned orders are released, and not in those periods in which the planned orders are received. The total inventory carrying cost is the sum of the inventory carrying cost for each period of the planning horizon. The inventory carrying cost for each period is based on the average inventory level for that period. The average inventory level is expressed as follows:

$$\text{Average inventory} = \frac{\text{(Beginning + Ending) inventory}}{2}$$

Tuite and Anderson (9), discussed the reason for using average inventory level in computing the inventory carrying cost. Their discussion has been paraphrased in chater II.

When two lot sizing strategies result in the same total cost, or if there is not a significant difference between the total cost obtained by two different strategies, then the CPU time provides a secondary criterion for selecting a mixed lot sizing strategy. The reason the CPU time is chosen as a secondary criterion is because of the bearing it has on the total computer time needed to perform an MRP requirement generation. In the real world, MRP systems generally consist of several parts and components and so the choice of a lot sizing strategy could significantly affect the total CPU time needed to process the items.

In addition to the CPU time, one can also select a lot sizing straegy which is easy to implement and easy to understand by the shop personnel.

The inventory carrying cost can become an important secondary criterion if management is concerned about the investment in inventory or is constrained by storage space (generally higher inventory costs imply more number of units in storage).

The setup cost / setup can become an important secondary criterion if management wants to reduce nonproductive times on machines (nonproductive includes all time excepting time spent on actual metal removal).

In this thesis the inventory carrying cost and the setup cost / setup are not considered as secondary performance criteria because the choice of these two criteria is subjected to various management styles and operating environments.

In this thesis the total system cost is considered as the major performance criteria and the total CPU time as the secondary criterion.

V. ANALYSIS OF RESULTS

A. Analysis of Result Based On Total System Cost

This section discusses the results for the two different cases of cost ratio application discussed in section E of chapter II.

Case I. Uniform application of cost ratios

The total system cost of the nine multi-level lot sizing strategies for various cost ratios and demand patterns is shown in Tables 5.1-5.7. As shown in Table 5.1, all lot sizing strategies resulted in the same total cost for a cost ratio of 5/1. This can be explained as follows.

For a cost ratio of 5/1, the annual inventory carrying cost is very high compared to the setup cost. Therefore, all lot sizing strategies attempt to avoid a large inventory. Thus the setups are more frequent and in many cases resemble the Lot-For-Lot ordering policy.

For a cost ratio of 80/1, table 5.1 shows that the total cost obtained by most of the lot sizing strategies is the same. This can be explained as follows. For a cost ratio of 80/1, the annual inventory carrying cost is very less compared to the setup cost. Therefore at such a high cost ratio, larger lot sizes are generated and fewer setups are made in all the strategies. Consequently, all the lot sizing strategies result in almost the same number of setups

Table 5.1 Total system cost for constant demand pattern

Cost Ratio → Strategies ↓	5/1	30/1	55/1	80/1
POQ/POQ/POQ	3467.09	3996.25	3650.82	2822.09
GT/GT/GT	3467.09	2324.98	583.3	563.33
LTC/LTC/LTC	3467.09	3996.25	3344.16	2822.09
POQ/POQ/LFL	3467.09	3996.25	3650.82	2822.09
GT/GT/LFL	3467.09	2324.98	583.3	563.33
LTC/LTC/LFL	3467.09	3996.25	3344.16	2822.09
POQ/SM/WW	3467.09	3996.25	3811.66	2822.09
LTC/GT/WW	3467.09	3996.25	3245.41	2822.09
WW/WW/WW	3838.74	3996.25	2472.08	2822.09

Table 52 Total system cost for random demand pattern

Cost Ratio - > Strategies ↓	5/1	30/1	55/1 55/1	80/1
POQ/POQ/POQ	3106.44	3717.7	2263.95	2780.06
GT/GT/GT	3979.56	2599.32	500.63	525.83
LTC/LTC/LTC	3979.56	3664.03	2899.32	2613.57
POQ/POQ/LFL	3106.44	3717.7	2263.95	2780.06
GT/GT/LFL	3979.56	2599.32	500.63	525.83
LTC/LTC/LFL	3979.56	3664.03	2899.32	2613.57
POQ/SM/WW	3145.47	3717.7	2263.95	2780.06
LTC/GT/WW	3979.56	3664.32	2899.32	2613.57
WW/WW/WW	3979.56	2599.32	2899.32	3199.29

Table 5.3 Total system cost for varying demand pattern-1

Cost Ratio → Strategies ↓	5/1	30/1	55/1	80/1
POQ/POQ/POQ	3547.26	4087.09	3724.75	2860.76
GT/GT/GT	3919.34	2160.76	517.75	572.75
LTC/LTC/LTC	3547.26	4087.09	3420.79	2860.76
POQ/POQ/LFL	3547.26	4087.09	3724.75	2860.76
GT/GT/LFL	3919.34	2160.76	517.75	572.75
LTC/LTC/LFL	3547.26	4087.09	3420.79	2860.76
POQ/SM/WW	3547.26	4087.09	3885.54	2860.76
LTC/GT/WW	3547.26	4087.09	3420.79	2860.76
WW/WW/WW	3919.34	3951.25	3005.79	3305.79

Table 5.4 Total system cost for varying demand pattern-2

Cost Ratio → Strategies ↓	5/1	30/1	55/1	80/1
POQ/POQ/POQ	3523.67	4032.44	3710.5	2865.28
GT/GT/GT	3893.83	2424.56	547.41	573.41
LTC/LTC/LTC	3523.67	4032.44	3424.5	2865.28
POQ/POQ/LFL	3523.67	4032.44	3710.5	2865.28
GT/GT/LFL	3893.83	2424.56	547.41	573.41
LTC/LTC/LFL	3523.67	4032.44	3424.5	2865.28
POQ/SM/WW	3523.67	4032.44	3872.75	2865.28
LTC/GT/WW	3523.67	4032.44	3424.5	2865.28
WW/WW/WW	3893.83	4032.44	2999.49	3299.5

Table 5.5 Total system cost for varying demand pattern-3

Cost Ratio → Strategies ↓	5/1	30/1	55/1	80/1
POQ/POQ/POQ	3582.33	4155.36	3733.16	2850.41
GT/GT/GT	3755.32	2150.41	547.17	572.17
LTC/LTC/LTC	3582.33	4155.36	3437.16	2850.41
POQ/POQ/LFL	3582.33	4155.36	3733.16	2850.41
GT/GT/LFL	3755.32	2150.41	547.17	572.17
LTC/LTC/LFL	3582.33	4155.36	3437.16	2850.41
POQ/SM/WW	3582.33	4155.36	3893.66	2850.41
LTC/GT/WW	3582.33	4155.36	3437.16	2850.41
WW/WW/WW	3582.33	4097.12	3017.12	3317.12

Table 5.6 Total system cost for seasonal demand pattern-1

Cost Ratio → / Strategies ↓	5/1	30/1	55/1	80/1
POQ/POQ/POQ	3275.42	3811.88	3481.02	3054.14
GT/GT/GT	3457.92	2101.25	517.51	542.51
LTC/LTC/LTC	3905.43	3811.88	3174.37	2801.25
POQ/POQ/LFL	3275.42	3811.88	3481.02	3054.14
GT/GT/LFL	3457.92	2101.25	517.51	542.51
LTC/LTC/LFL	3905.43	3811.88	3174.37	2801.25
POQ/SM/WW	3275.42	3811.88	3641.86	3054.14
LTC/GT/WW	3905.43	3811.88	3174.37	2801.25
WW/WW/WW	3730.41	3839.16	2451.25	2801.25

Table 5.7 Total system cost for seasonal demand pattern-2

Cost Ratio → Strategies ↓	5/1	30/1	55/1	80/1
POQ/POQ/POQ	3325.58	4021.71	3677.5	3037.89
GT/GT/GT	3254.64	2337.89	2315.64	588.99
LTC/LTC/LTC	3827.67	4021.71	2687.89	3037.89
POQ/POQ/LFL	332 .58	4021.71	3677.5	3037.89
GT/GT/LFL	3254.64	2337.89	2315.64	588.99
LTC/LTC/LFL	3827.67	4021.71	2687.89	3037.89
POQ/SM/WW	3325.58	4021.71	3833.41	3037.89
LTC/GT/WW	3827.67	4021.71	2687.89	3037.89
WW/WW/WW	4270.62	4220.62	2687.89	3037.89

thereby resulting in the same total cost.

Thus for a constant demand pattern, lot sizing strategies are insensitive for these extreme values of cost ratios. The lot sizing strategies are sensitive to the cost ratios having values in between these extreme values. As shown in Table 5.1, for a cost ratio of 55/1, different lot sizing strategies resulted in different total cost.

A closer inspection of total cost for all the demand patterns (Tables 5.1 -5.7) under a cost ratio of 55/1 reveals that any mixed lot sizing strategy using the POQ model at the end item level resulted in similar total costs. In like manner, any mixed lot sizing strategy using the LTC model at the end item level resulted in similar total costs. There are two reasons for the above. First, at this cost ratio, the LTC model and the POQ model result in a limited number of setups for the end item. Secondly, when these setups are passed on to the higher levels of a multi-level inventory system any lot sizing technique applied at these higher levels resemble the Lot-For-Lot philosophy. This phenomena is observed for most of the demand patterns when the cost ratio is set at 55/1.

Irrespective of the demand pattern, as can be seen in Tables 5.1 - 5.7, the GT/GT/GT and GT/GT/LFL lot sizing strategies achieved the lowest total cost for the cost ratios 30/1, 55/1 and 80/1. In particular, for the cost ratios 55/1 and 80/1 these strategies achieved a total cost which is 4 times lower than the total cost achieved by other lot sizing strategies. The reason for this is as follows. According to the Gaither algorithm, the cost of carrying any future period requirement in the current setup is compared with the cost of an addtional setup. If the cost of carrying the future period requirement is less than the additional setup cost, then the future period requirement is added to the current setup and carried in inventory. For high setup cost ratios (30/1, 55/1, 80/1), the Gaither algotithm results in very few setups. The requirements of the end item for the entire planning horizon is covered by one or two setups only. This fact can be seen in Table 5.8.

The planned order releases for the end item becomes the requirement schedule for the component items. These requirments of the component items have two characteristics. First, due to lumpiness of the demand there are very few distinct number of requirements, thereby reducing the total setup cost at these levels. Secondly, this small number of requirements forces the Gaither algorithm to resemble the Lot-For-Lot algorithm, thereby reducing the inventory carrying cost. Thus the total cost for the higher level items is very small which causes the GT/GT/GT strategy to achieve a very small total cost for the entire system.

Table 5.8 MRP table for the GT model

Level = 1, Lead Time = 1, Production Ratio = 1, Frequency of Gross Requirement = 12.

Period	1	2	3	4	5	6	7	8	9	10	11	12
Gross Requirement	75	50	25	50	75	50	25	50	75	50	25	50
Schedule Receipts	0	525	0	0	0	0	0	0	0	0	0	0
On-Hand	0	475	450	400	325	275	250	200	125	75	50	0
Planned Order Release	525	0	0	0	0	0	0	0	0	0	0	0

Level = 2, Lead Time = 5, Production Ratio = 2, Frequency of Gross Requirement = 1-1 = 0
Initial Inventory Level = 1050, Lot Size Technique Used = GT

Period	1	2	3	4	5	6	7	8	9	10	11	12
Gross Requirement	1050	0	0	0	0	0	0	0	0	0	0	0
Schedule Receipts	[illegible]	0	0	0	0	0	0	0	0	0	0	0
On-Hand	0	0	0	0	0	0	0	0	0	0	0	0
Planned Order Release	0	0	0	0	0	0	0	0	0	0	0	0

Level = 3, Lead Time = 1, Production Ratio = 3, Frequency of Gross Requirement = 0

Initial Inventory Level = 0, Lot Size Technique Used = GT

Period	1	2	3	4	5	6	7	8	9	10	11	12
Gross Requirement	0	0	0	0	0	0	0	0	0	0	0	0
Schedule Receipts	0	0	0	0	0	0	0	0	0	0	0	0
On-Hand	0	0	0	0	0	0	0	0	0	0	0	0
Planned Order Release	0	0	0	0	0	0	0	0	0	0	0	0

CASE II: Different cost ratios at different levels

The total cost obtained for different lot sizing strategies for various demand patterns using different cost ratio for different levels of a multi-level inventory system is shown in Table 5.9. Table 5.10 shows the relative performance index for each mixed lot sizing strategy under various demand patterns. The index for a particular strategy under a particular demand pattern was determined by dividing the total cost of that strategy by the minimum total cost obtained by any mixed lot sizing strategy for that demand pattern. Table 5.11 shows the relative ranking of each strategy using data from Tables 5.9 and 5.10.

As seen from Table 5.11, the WW/WW/WW mixed lot sizing strategy was amongst the worst performer for a multi-level inventory system. This indicates that in a multi-level inventory system the sum of the individual optimum does not add to the global optimum.

Also from Table 5.11, it can be seen that the GT/GT/GT and GT/GT/LFL lot sizing strategies always resulted in the same total cost for various demand patterns. The same is true for the strategies LTC/LTC/LTC and LTC/LTC/LFL. In both these pairs, it is apparent that the lot sizing model used at highest component level had little bearing on the total cost. This can be explained by the few number of setups that both the LTC and the GT lot sizing model generate at the end item level, allowing the higher level

Table 5.9 Comparison of total system cost for each lot sizing strategy

Demand / Strategy	Constant 1	Random 2	Varying 3	Varying 4	Varying 5	Seasonal 6	Seasonal 7
POQ/POQ/POQ	4305.41	4027.79	4383.58	4357.34	4428.66	4130.42	4266.41
GT/GT/GT	4481.24	4886.54	4839.74	4813.83	4641.83	4302.9	3945.62
LTC/LTC/LTC	4481.24	4886.54	4562.75	4547.17	4601.83	4825.41	4443.58
POQ/POQ/LFL	4268.02	4035.12	4344.92	4316.00	4400.00	4110.42	4227.08
GT/GT/LFL	4481.24	4899.54	4839.74	4813.84	4641.83	4302.92	3945.62
LTC/LTC/LFL	4481.24	4899.54	4562.75	4547.17	4601.83	4825.41	4443.58
POQ/SM/WW	4487.04	4886.54	4567.25	4543.67	4601.83	4295.42	4403.58
LTC/GT/WW	4481.24	4109.95	4562.75	4547.17	4601.83	4820.41	4340.58
WW/WW/WW	4758.74	4886.54	4839.74	5020.62	4601.83	5028.32	5020.62

Table 5.10 Relative performance index of each lot sizing strategy

Demand sStrategy	Constant 1	Random 2	Varying 3	Varying 4	Varying 5	Seasonal 6	Seasonal 7
POQ/POQ/POQ	1.008	1.000	1.009	1.009	1.006	1.005	1.081
GT/GT/GT	1.049	1.213	1.114	1.115	1.055	1.047	1.000
LTC/LTC/LTC	1.049	1.213	1.050	1.054	1.046	1.174	1.126
POQ/POQ/LFL	1.000	1.002	1.000	1.000	1.000	1.000	1.071
GT/GT/LFL	1.049	1.214	1.114	1.11	1. 55	1.047	1.000
LTc/LTC/LFL	1.049	1.214	1.051	1.052	1.046	1.045	1.116
POQ/SM/WW	1.051	1.213	1.051	1.032	1.046	1.045	1.116
LTC/GT/WW	1.049	1.020	1.050	1.054	1.046	1.173	1.100
WW/WW/WW	1. 10	1.213	1.114	1.161	1.046	1.221	1.271

Table 5.11 Performance rank of each lot size strategy

Type > No > Rank ∨	Constant 1	Random 2	Varying 3	Varying 4	Varying 5	Seasonal 6	Seasonal 7
1	POQ/POQ/LFL (4268.02)	POQ/POQ/POQ (4027.79)	POQ/POQ/LFL (4344.92)	POQ/POQ/LFL (4316.00)	POQ/POQ/LFL (4400.00)	POQ/POQ/LFL (4110.42)	GT/GT/GT GT/GT/LFL (3945.62)
2	POQ/POQ/POQ (4305.41)	POQ/POQ/LFL (4035.12)	POQ/POQ/POQ (4383.58)	POQ/POQ/POQ (4357.34)	POQ/POQ/POQ (4428.66)	POQ/POQ/POQ (4130.42)	POQ/POQ/LFL (4227.08)
3	GT/GT/GT GT/GT/LFL LTC/GT/WW LTC/LTC/LTC LTC/LTC/LFL (4481.24)	L LTC/GT/WW (4109.95)	LTC/GT/WW LTC/LTC/LTC LTC/LTC/LFL (4562.75)	POQ/SM/WW (4543.67)	WW/WW/WW POQ/SM/WW LTC/GT/WW LTC/LTC/LTC LTC/LTC/LFL (4601.83)	POQ/SM/WW (4295.42)	POQ/POQ/POQ (4266.41)
4	POQ/SM/WW (4487.04)	GT/GT/GT WW/WW/WW POQ/SM/WW LTC/LTC/LTC (4886.54)	POQ/SM/WW (4567.25)	LTC/LTC/LTC LTC/LTC/LFL LTC/GT/WW (4547.17)	GT/GT/GT GT/GT/LFL (4641.83)	GT/GT/GT GT/GT/LFL (4302.92)	LTC/GT/WW (4340.58)
5	WW/WW/WW (4758.74)	GT/GT/LFL LTC/LTC/LFL (4899.54)	GT/GT/GT GT/GT/LFL WW/WW/WW (4839.74)	GT/GT/GT GT/GT/LFL (4813.83)		LTC/GT/WW (4820.41)	POQ/SM/WW (4403.58)
6				WW/WW/WW (5020.62)		LTC/LTC/LTC LTC/LTC/LFL (4825.41)	LTC/LTC/LTC LTC/LTC/LFL (4483.58)
7						WW/WW/WW (5028.32)	WW/WW/WW (5020.62)

items to follow the LFL strategy. Except for demand patterns with a high coefficient of variation, even the LTC/GT/WW lot sizing strategy produced results identical to LTC/LTC/LTC and LTC/LTC/LFL. In this case then, the lot sizing model chosen at the intermediate level did not significantly impact the total cost. The results indicate that under certain conditions if one uses the LTC or GT lot sizing models at the end item level, one can be indifferent in the choice of a lot sizing model at the highest component level of a multi-level inventory system.

The above phenomenon is not necessarily true for any lot sizing model adopted at the end item level. For example, the mixed lot sizing strategy POQ/POQ/LFL which achieved the best result for 5 of the demand patterns as shown in Table 5.11 does not result in the same cost achieved by the POQ/POQ/POQ strategy. This indicates that for various demand patterns, if the POQ lot sizing model is used at the end item level, one cannot be indifferent in the choice of a lot sizing model at the highest level of a multi-level inventory system.

The lot sizing strategies for which the lot sizing models were randomly selected, performed moderately well for all cases of the demand pattern. As seen from Table 5.11, these strategies always achieved total cost between the lowest and the highest total cost achieved by other strategies. They were neither the best nor the worst performers.

B. Computational Efficiency

The computional efficiency of the various strategies should also be considered. Since all lot sizing strategies are computerized, the aforesaid criterion largely relates to the CPU time used. Different lot sizing strategies require different amount of CPU time to determine the planned order releases. If two lot sizing strategies produce near identical total cost, the user might opt for the strategy that requires less CPU time for computation. Table 5.15 shows the CPU time required by each lot sizing strategy for various demand patterns. In the event the CPU time is not of very high concern, the user can base his choice on the relative ease in understanding the different strategies.

In the next chapter the conclusions derived from this study on lot sizing strategies for a multi-level inventory system are presented.

Table 5.12 CPU time for each strategy

Lot sizing strategies	Total CPU time in (sec.)
POQ/POQ/POQ	2.07
GT/GT/GT	2.28
LTC/LTC/LTC	2.22
POQ/POQ/LFL	1.96
GT/GT/LFL	2.11
LTC/LTC/LFL	2.11
POQ/SM/WW	3.10
LTC/GT/WW	3.21
WW/WW/WW	5.47

VI CONCLUSIONS

Collier (2), in his study applied the cost ratios uniformly to all the levels of a multi-level inventory system and concluded that the LTC/LTC/LTC lot sizing strategy was the best performer. Choi (3), in his study used different cost ratios for different nodes at various levels of a multi-level inventory system. The author concluded that the POQ/POQ/POQ lot sizing strategy was the best performer. Both Choi and Collier have implicitly stated that a quasi mixed lot sizing strategy (one in which the same lot sizing model is used at all levels) is a better performer than a pure mixed lot sizing strategy (one in which different lot sizing models are used at various levels). Results obtained from this study cast doubts on the validity of their statement.

In this study, both the cases of the cost ratio i.e., using a cost ratio uniformly at all the levels and using different cost ratios at different levels of a multi-level inventory system were studied. For the case where the cost ratios were applied uniformly to all the levels of the multi-level system, this study showed that the GT/GT/GT and GT/GT/LFL lot sizing strategy resulted in the lowest total cost for 3 out of 4 cost ratios under all the demand patterns. In this case then, the quasi mixed lot sizing strategy (GT/GT/GT) did not perform better, but performed only as well as a pure mixed lot sizing strategy (GT/GT/LFL).

For the second case where different cost ratios were applied at various levels of the multi-level system, this study showed that POQ/POQ/LFL performed consistently better than the POQ/POQ/POQ. Thus in this case also, a quasi mixed lot sizing strategy (POQ/POQ/POQ) did not perform better than a pure mixed lot sizing strategy (POQ/POQ/LFL).

Based on the results of this study, the four questions addressed in this thesis in Chapter I, can now be answered.

Question 1: Given that the cost ratio is applied uniformly to all the levels, do varying demand patterns have a bearing on the choice of an "optimum" mixed lot sizing strategy ?

Conclusion: Varying demand patterns generally do not affect the choice of an "optimum" mixed lot sizing strategy. As shown in this study, the GT/GT/GT or the GT/GT/LFL strategy yielded the minimum total cost for all the different demand patterns for most of the cost ratios.

Question 2: Given that different cost ratios are applied at various levels, do varying demand patterns have a bearing on the choice of an "optimum" mixed lot sizing strategy ?

Conclusion: Varying demand patterns have no bearing on the choice of an optimum mixed lot sizing strategy, if different cost ratios are applied at various levels of a multi-level inventory system. As seen from Table 5.11, POQ/POQ/POQ strategy consistently performed better than other mixed lot sizing strategies. But the other mixed lot sizing strategies did perform differently under different demand patterns.

Question 3: The Wagner-Whitin algorithm has been shown to result in the optimum strategy for single stage systems. Will the Wagner-Whitin applied at all the levels in a multi-level system result in an overall optimum for the whole system. Thus, the question really being addressed is whether or not the sum of the individual optimums at various levels results in the overall optimum ?

Conclusion: The sum of the individual optimums at various levels does not add up to a global optimum. As shown in Tables 5.1 - 5.11 under all demand patterns the WW/WW/WW strategy never resulted in the lowest total cost.

Question 4: At higher levels in a multi-level system, does the ordering strategy ultimately resemble Lot-For-Lot ?

Conclusion: At the higher levels in a multi-level system the ordering strategy does not necessarily resemble the Lot-For-Lot strategy. If the cost ratios are applied uniformly to all the levels, and if the LTC or GT lot sizing model is applied at the end item level, then the ordering strategy ultimately resembles Lot-For-Lot philosophy.

The study was conducted using limited sythetic data and a short planning horizon . Thus the conclusions stated herein have to be adopted considering the environment in which this study was conducted.

Future work should concentrate on larger systems (a larger number of levels and nodes). Further, the time span for the planning horizon should be increased to achieve better cost comperison between the lot sizing strategies. All possible mixed lot sizing strategies should be evaluated to determine the conditions that bias one strategy in favor of the other strategies.

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LOT SIZING IN MULTI-LEVEL MULTI-ECHELON INVENTORY SYSTEM

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ABSTRACT

Considerable research has already been conducted on lot sizing models in single stage inventory systems. Given certain parameters (cost ratio, demand pattern) there are single stage lot sizing models available that can satisfy one's performance measure. But very little work has been conducted on the effects of mixed lot sizing strategies in multi-level inventory systems.

The selection of lot sizing strategies for a multi-level inventory system is an important decision for production and materials management. This thesis evaluates a restricted set of mixed ot sizing strategies for a multi-level inventory system consisting of 3 levels and 17 nodes. The total system cost was the major performance criteria for comparing the lot sizing strategies. For comparison between the strategies, seven different demand patterns were generated. Lot sizing strategies were compared for each of the seven demand patterns under two cases of cost ratios. First, the cost ratios were applied uniformly to all the levels of the multi-level inventory system. Second, different cost ratios were applied to different levels of the multi-level inventory system.

The results of this study highlight the operating characteristic of specific lot sizing models considering the

total cost. This study showed that when the cost ratios are applied uniformly, a quasi mixed lot sizing strategy did not perform better, but performed only as well as a pure mixed lot sizing strategy. Similarly, when different cost ratios were applied at various levels in a multi-level inventory system, a quasi lot sizing strategy did not perform better than a pure mixed lot sizing atrategy.