A STUDY OF MINIMUM WEIGHT PRELIMINARY PLASTIC DESIGN OF UNBRACED FRAMES

by 557

DER-HSIUNG SU

Diploma, Taipei Institute of Technology,
Taiwan, China, 1964

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1971

Approved by

Robert R Snell
Major Professor

TABLE OF CONTENTS

INTRODUCTION

Plastic design is an advantageous replacement for conventional elastic design in many complicated structures. Using concepts of limit analysis in plastic design, several authors have recently explored the problem of designing a structural frame for minimum weight. The preliminary design of the tall steel frame in this report is based on these concepts and considers that the building is not braced against lateral loads. The columns are assumed to remain elastic, whereas plastic hinges are allowed to form in the beams. Such a design is herein called a "Weak-Beam Strong-Column Mechanism" approach to design.

The Simplex Method of Linear Programming is introduced and used to solve a simple frame. A computer program is used to obtain an optimum design for minimum weight of a 10-story building model.

REVIEW OF LITERATURE

In 1904, Mitchell (1)* studied the minimum design of a structure subjected to a fixed loading condition and established the principle of the optimum weight design; Heyman (2) continued this work using concepts of limit analysis and showed that the method of inequalities, which Neal and Symonds (7) had developed for the determination of the load carrying capacity of a frame of given dimensions, could be easily adapted to the problem of designing a frame for minimum weight, if the weight per unit length of a structural member could be taken as proportional to the fully plastic bending moment of this member. Foulkes (3) pointed out that under this assumption the minimum weight design of a structural frame constituted a problem in linear programming. In a later paper (4), the same author established necessary and sufficient conditions for a given design to be a minimum-weight design, assuming again that the unit weight of a member was proportional to its fully plastic moment. Making the same assumption, Livesley (5) indicated a different approach and

*Numbers in parentheses refer to corresponding items in the References studied the programming of the problem for an automatic computer solution. He was able, at most, to treat a four-story frame of one bay, since the memory capacity of the computer did not allow him to solve more complex structures. Heyman and Prager (6) developed a design method that automatically furnishes the minimum weight design which makes more modest demands on the memory capacity of the computer.

In 1960, Heyman (9) proposed the optimum weight design of sway frames on the basis of elastic-plastic behaviour. Methods of optimum elastic-plastic design have been described by Murry and Ostapenko (8), and by Rubinstein and Karagozian (10) in 1966. In the former case, change of geometry effects are approximated by increasing the lateral loads, whereas in the later case, no provision for these effects is made. In both cases the designs obtained are preliminary only, and must be checked for frame and member instability.

OPTIMUM DESIGN OF A STEEL FRAME

(1) Plastic Hinge Concept: The plastic design of beams and frames is based on an idealized relationship between bending moment and curvature. When the plastic moment M_p is reached, rotation takes place with no increase in bending moment [see Fig. 1] , and the section performs as a hinge rotating with constant moment M_p. Such a hinge is called a "plastic hinge"

(2) Collapse Mechanism: When a sufficient number of hinges develop, the structure becomes an unstable system referred to as a "collapse mechanism". The magnitude of the applied load causing the formation of a collapse mechanism is termed the "ultimate load".

(3) Safety Factor in Limit Design: The safety factor in limit design is taken as the ratio between the ultimate load P_u, and service load P, or P_u=(safety factor)P.

(4) Energy Consideration: From energy considerations, the internal energy U, absorbed by the plastic hinges, must be equal to, or greater than the external energy E^*. $U \geq E^*$.

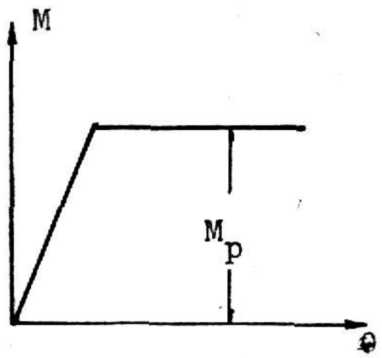


Fig. 1 Moment - Rotation Relationship

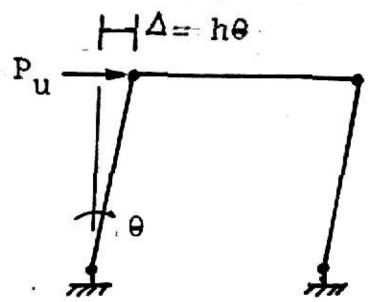


Fig. 2 Frame Under Ultimate Load

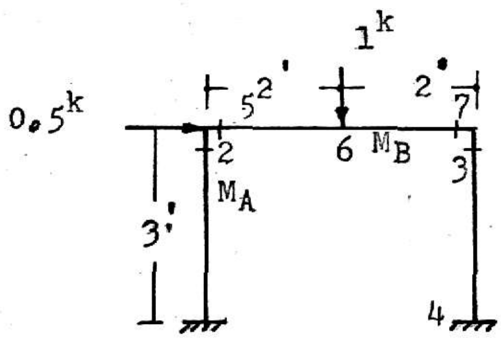


Fig. 3 Frame With Service Load

APPLICATION OF LINEAR PROGRAMMING

(1) Objective: The objective is to select column and beam sections that will make the weight of the frame a minimum with the safety factor $\geq$ 2.

(2) The Objective Function: The weight Q is minimized by minimizing the quantity M_pL. For brevity, we call M_pL the objective function and denote it by Z

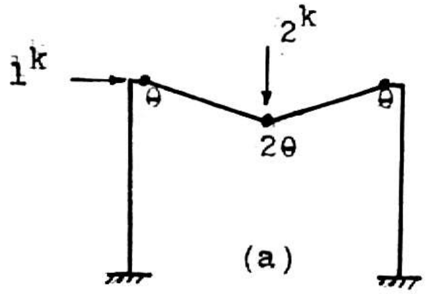
$$Z = \sum M_p L = 6M_A + 4M_B \quad \text{(See Fig. 3)} \tag{1}$$

(3) Inequalities: The first step is to multiply the service load by the safety factor of 2 to yield the ultimate loads. Plastic moments may develop as follows:

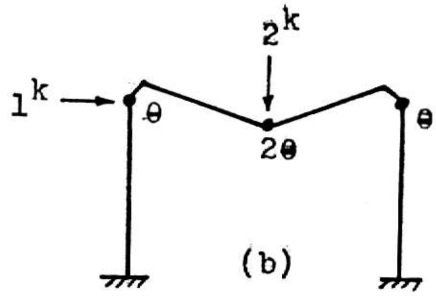
In matrix form

$$[a]\begin{bmatrix} M_A \\ M_B \end{bmatrix} + [r] \geq 0 \tag{2a}$$

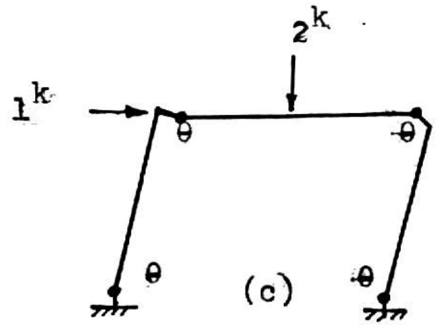
$$\begin{bmatrix} 0 & 4 \\ 2 & 2 \\ 2 & 2 \\ 4 & 0 \\ 2 & 4 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} M_A \\ M_B \end{bmatrix} + \begin{bmatrix} -4 \\ -4 \\ -3 \\ -3 \\ -7 \\ -7 \end{bmatrix} \geq 0 \tag{2b}$$



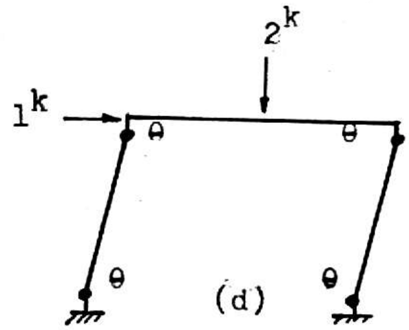
$4\ M_B \geq 4\ (M_A \geq M_B)$



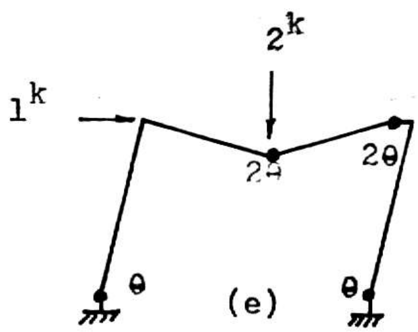
$2\ M_A + 2\ M_B \geq 4\ (M_A \leq M_B)$



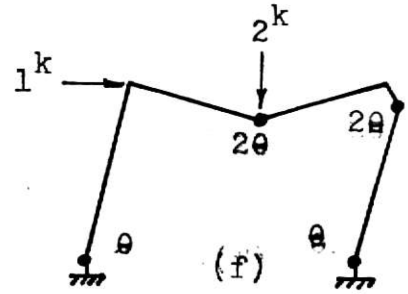
$2\ M_A + 2\ M_B \geq 3 (M_A > M_B)$



$4\ M_A \geq 3\ (M_A \leq M_B)$



$2\ M_A + 4\ M_B \geq 7 (M_A \geq M_B)$



$4\ M_A + 2\ M_B \geq 7 (M_A < M_B)$

Fig. 4 Collapse Mechanisms

(4) <u>The Design Solution:</u>

(a) Graphical Method - The graphical method of solving linear programming is limited to cases which have two or three variables. The procedure for solving a maximization (minimization) problem by this method is as follows:

Step 1: Plot the convex set

Step 2: Draw the objective function

Step 3: Find the extreme point which optimizes the objective function

In the present example, the coordinates of this point M are readily found to be

$M_A = M_B = 7/6$, and $Z = 35/3$

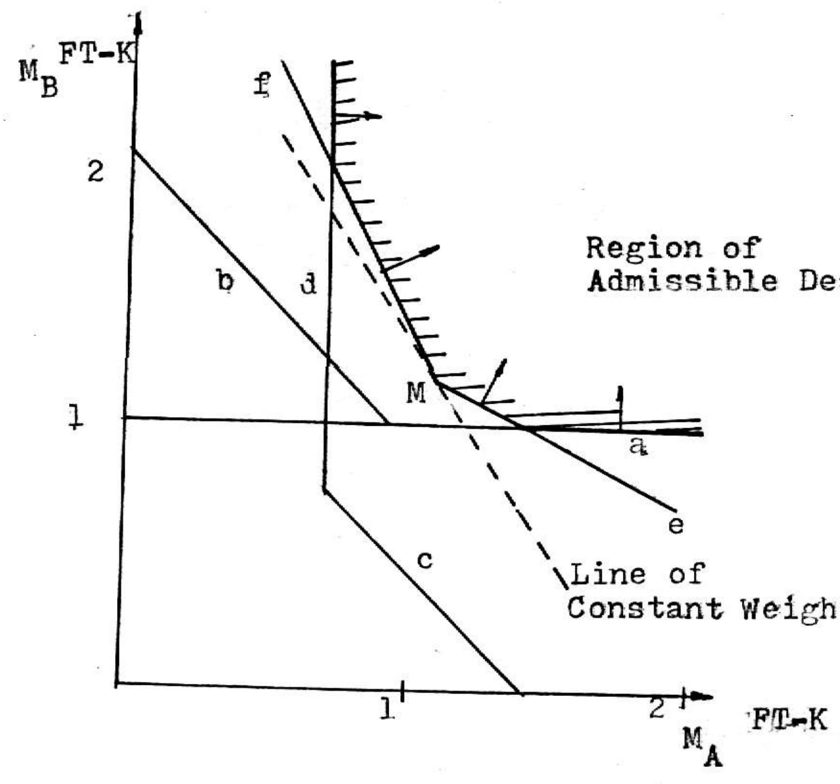


Fig. 5 Region of Admissible Designs

(b) Simplex Method - The simplex method is a very powerful procedure for solving linear programming problems. For problems in which more than three variable plastic moment capacities are involved, the graphical solution shown in Fig. 5 is impossible. In such cases the simplex method is used. Suppose we take the same example as was solved by the graphical method. There are only four equations which affect the solutions, so the systematic numerical procedure is as follows:

Step 1: Convert the inequalities into equalities by subtracting nonnegative slack variables and then adding nonnegative artificial variables.

Step 2: Attach a per-unit cost of zero to each slack variable and of a large number S to each artificial variable in the objective function.

Step 3: Design the initial basic feasible solution.

Step 4: Determine the entering vector (key column). Select the vector with maximum $Z_j - C_j > 0$

Step 5: Determine the departing vector (key row). Select the vector which reaches zero first as the entering variable is increased. By using the θ-rule: (12) For $a_{ik} > 0$

$$\theta_i = \min\left(\frac{b_i}{a_{ik}} \right), \; i = 1, 2, \ldots\ldots m.$$

Step 6: Revise the simplex tableau: Identify the key element which lies at the intersection of the key column and key row.

Return to step 2, 3 until $Z_j - C_j \leq 0$.

This indicates that the given basic feasible solution is optimal.

Note: ◯ indicates the key element determined after the entering vectors and the departing vectors have been selected

□ identifies the basic vector

C_j		6	4	0	0	0	0	S	S	S	S		
C_K	M_K	M_A	M_B	M_C	M_D	M_E	M_F	M_G	M_H	M_I	M_J	M_o	
S	M_G	0	4	-1	0	0	0	$\boxed{1}$	0	0	0	4	$\leftarrow \theta_1 = \frac{4}{4}$
S	M_H	4	0	0	-1	0	0	0	1	0	0	3	$\theta_2 = \frac{4}{0}$
S	M_I	2	4	0	0	-1	0	0	0	1	0	7	$\theta_3 = \frac{7}{2}$
S	M_J	4	2	0	0	0	-1	0	0	0	1	7	$\theta_4 = \frac{7}{4}$
Z_j		10S	10S	-S	-S	-S	-S	S	S	S	S	21S	
$Z_j - C_j$		10S-6	10S-4	-S	-S	-S	-S	0	0	0	0		

Entering Vector: M_B ↑

Departing Vector: M_G ↓

TABLE 1. THE INITIAL SIMPLEX TABLEAU

C_j		6	4	0	0	0	0	S	S	S	S		
C_K	M_K	M_A	M_B	M_C	M_D	M_E	M_F	M_G	M_H	M_I	M_J	M_o	
4	M_B	0	$\boxed{1}$	$-\frac{1}{4}$	0	0	0	$\frac{1}{4}$	0	0	0	1	
S	M_H	4	0	0	-1	0	0	0	$\boxed{1}$	0	0	3	$\theta_1 = \frac{3}{4}$
S	M_I	2	0	1	0	-1	0	-1	0	1	0	3	$\theta_2 = \frac{3}{2}$
S	M_J	4	0	$\frac{1}{2}$	0	0	-1	$-\frac{1}{2}$	0	0	1	5	$\theta_3 = \frac{5}{4}$
$Z_j - C_j$		10S-6	0	$\frac{3}{2}$S-1	-S	-S	-S	$-\frac{5}{2}$S+1	0	0	0	11S+4	
		↑ E.V.							↓ D.V.				

TABLE 2. THE FIRST SIMPLEX TABLEAU

C_j		6	4	0	0	0	0	S	S	S	S		
C_K	M_K	M_A	M_B	M_C	M_D	M_E	M_F	M_G	M_H	M_I	M_J	M_o	
4	M_B	0	1	$-\frac{1}{4}$	0	0	0	$\frac{1}{4}$	0	0	0	1	
6	M_A	$\boxed{1}$	0	0	$-\frac{1}{4}$	0	0	0	$\frac{1}{4}$	0	0	$\frac{3}{4}$	$\theta_1 = \frac{3}{4}$
S	M_I	0	0	1	$\frac{1}{2}$	-1	0	-1	$-\frac{1}{2}$	$\boxed{1}$	0	$\frac{3}{2}$	$\theta_2 = \frac{3}{2}$
S	M_J	0	0	$\frac{1}{2}$	1	0	-1	$-\frac{1}{2}$	-1	0	1	2	$\theta_3 = 4$
$Z_j - C_j$		0	0	$\frac{3}{2}S-1$	$\frac{3}{2}S-\frac{3}{2}$	-S	-S	$-\frac{5}{2}S+1$	$-\frac{5}{2}S+\frac{3}{2}$	0	0	$\frac{7}{2}S+9$	

E.V. D.V.

TABLE 3. THE SECOND SIMPLEX TABLEAU

	C_j	6	4	0	0	0	0	S	S	S	S		
C_K	M_K	M_A	M_B	M_C	M_D	M_E	M_F	M_G	M_H	M_I	M_J	M_o	
4	M_E	0	1	0	$\frac{1}{8}$	$-\frac{1}{4}$	0	0	$-\frac{1}{8}$	$\frac{1}{4}$	0	$\frac{11}{8}$	$\theta_1 = 11$
6	M_A	1	0	0	$-\frac{1}{4}$	0	0	0	$-\frac{1}{4}$	0	0	$\frac{3}{4}$	
0	M_C	0	0	$\boxed{1}$	$\frac{1}{2}$	-1	0	-1	$-\frac{1}{2}$	1	0	$\frac{3}{2}$	$\theta_3 = 3$
S	M_J	0	0	0	$\frac{3}{4}$	$\frac{1}{2}$	-1	0	$-\frac{3}{4}$	$-\frac{1}{2}$	$\boxed{1}$	$\frac{5}{4}$	$\theta_4 = \frac{5}{3}$
$Z_j - C_j$		0	0	0	$\frac{3}{4}S-1$	$\frac{1}{2}S-1$	-S	-S	$-\frac{7}{4}S+1$	$-\frac{3}{2}S+1$	0	$\frac{5}{4}S+10$	
					↑ E.V.						↓ D.V.		

TABLE 4. THE THIRD SIMPLEX TABLEAU

C_j		6	4	0	0	0	0	S	S	S	S	
C_K	M_K	M_A	M_B	M_C	M_D	M_E	M_F	M_G	M_H	M_I	M_J	M_o
4	M_B	0	1	0	0	$-\frac{1}{3}$	$\frac{1}{6}$	0	0	$\frac{1}{3}$	$-\frac{1}{6}$	$\frac{7}{6}$
6	M_A	1	0	0	0	$\frac{1}{6}$	$-\frac{1}{3}$	0	0	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{7}{6}$
0	M_C	0	0	1	0	$-\frac{4}{3}$	$\frac{2}{3}$	-1	0	$\frac{4}{3}$	$-\frac{2}{3}$	$\frac{2}{3}$
0	M_D	0	0	0	$\boxed{1}$	$\frac{2}{3}$	$-\frac{4}{3}$	0	-1	$-\frac{2}{3}$	$\frac{4}{3}$	$\frac{5}{3}$
$Z_j - C_j$		0	0	0	0	$-\frac{1}{3}$	$-\frac{4}{3}$	-S	-S	$-S+\frac{1}{3}$	$-S+\frac{4}{3}$	$\frac{35}{3}$

TABLE 5. THE FINAL SIMPLEX TABLEAU

The Optimal Solutions Are:

$M_A = M_B = \frac{7}{6}$

$Z = \frac{35}{3}$ (Same answers as were found by the graphical method)

FORMULATION OF PRELIMINARY TALL BUILDING DESIGN

1. Assumptions

a. Consider the columns to remain elastic, whereas plastic hinges can form in the beams. This lead to mechanisms which are herein called the "Weak-Beam Strong-Column mechanisms".

b. Assume the structure to have adequate lateral stiffness to limit the sidesway deflection under service loads. A corresponding equation of constraint can be derived which will also be in terms of the plastic moment capacities of the beams only.

c. A conservative stiffness requirement (limiting sidesway deflection) for the beams of the structure is

$$C_b = \frac{P_{total}}{0.6\,\Delta_{allowable}}$$

in which C_b=stiffness factor of beam, P=total vertical load and Δ=lateral displacement.

d. The internal energy capacity of the structure must be greater than the work done by the external loads.

e. There are three possible collapse mechanisms: One vertical mechanism and two sidesway mechanisms.

f. Assume that the wind load governs over the seismic load and that the wind load is constant over the height of the structure.

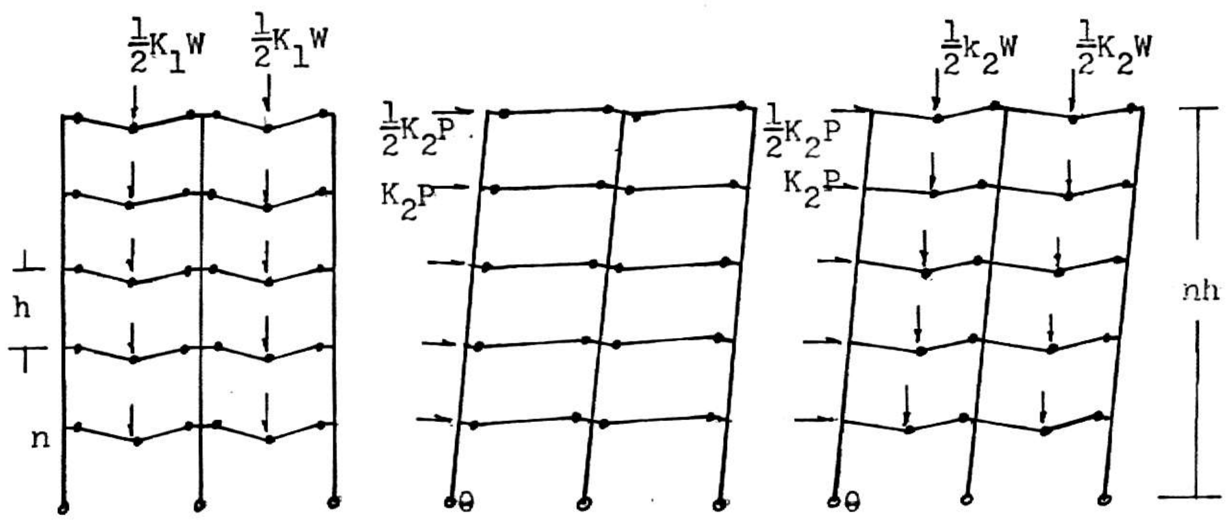


Fig. 6 Weak-Beam, Strong-Column Mechanisms

2. Plastic Analysis

(A) Beam Mechanism for Vertical Load:

The optimum use of the frame in Fig. 6(a) occurs when mechanisms are formed in each beam simultaneously. Constraining the total external work to be less than the internal energy of the structure

$$K_1\frac{W}{2}\left(\frac{L}{2}\theta\right) \le 4M_p\theta$$

$$K_1\frac{WL}{16} \le M_p$$

or

$$M_p \ge \frac{K_1WL}{16} \tag{2}$$

in which K_1 = Safety factor for D.L. + L.L.

W = Vertical concentrated load on beam,

L = Beam span.

(B) Sidesway Mechanism: Weak-Beam, Strong-Column.— The mechanism of Fig. 6(b) occurs when the energy in the frame is predominately the result of the lateral loads.

The external work

$$E^* = K_2\left[\frac{Pnh}{2} + P(n-1)h + P(n-2)h + \ldots .Ph\right]\theta$$

$$= K_2Ph\left[\frac{n}{2} + \frac{(n-1)n}{2}\right]\theta = \frac{1}{2}K_2Phn^2\theta \qquad (3)$$

in which K_2 = Safety factor for lateral loads,

n = number of stories, and h = story height.

The internal energy

$$U = 2m \sum_{r=1}^{n} M_{pr}\theta \qquad (4)$$

in which m = number of bays. Setting $E^* \leq U$ yields

$$\frac{1}{2}K_2Phn^2\theta \leq 2m \sum_{r=1}^{n} M_{pr}\theta$$

or $$\sum_{r=1}^{n} M_{pr} \geq \frac{K_2Phn^2}{4m} \qquad (5)$$

in which r is the reference level from the top of the building. It is assumed that the beam sizes increase linearly as a function of r. At r=1 the moment capacity M_{p1} must be adequate for the vertical load W, or

$$M_{p1} \geq \frac{K_1 WL}{16} \tag{6}$$

The moment capacity M_{pn} at level n is

$$M_{pn} = \sum_{r=1}^{n} M_{pr} - \sum_{r=1}^{n-1} M_{pr} \tag{7}$$

Making this substitution into Eq. (5) gives

$$M_{pn} \geq \frac{K_2 Ph(2n-1)}{4m} \tag{8}$$

The average value of the moment capacity, M_{pa} in the frame is

$$M_{pa} \geq \frac{M_{p1} + M_{pn}}{2} \tag{9}$$

$$\geq \frac{\frac{K_2 Ph(2n-1)}{4m} + \frac{K_1 WL}{16}}{2} \tag{10a}$$

or expressed as an allowable lateral load P

$$P \leq \frac{(2M_{pa} - \frac{K1WL}{16})\ 4m}{K_2 h(2n-1)} \qquad (10b)$$

(C) Combined Mechanism: Weak Beam, Strong-Column. — The vertical and lateral loads combine to form the mechanism shown in Fig. 6(c). The total external work is

$$E^* = K_2\left[\frac{Phn^2}{2} + \frac{mnWL}{4}\right]\theta \qquad (11)$$

and the internal energy is:

$$U = 4M_p mn\theta \qquad (12)$$

For $E^* \leq U$

$$K_2\left[\frac{Phn}{2} + \frac{mWL}{4}\right] \leq 4M_p m \qquad (13)$$

The moment capacity, M_p, in the total frame can be expressed in terms of an average M_{pa}, and also the total internal energy can be expressed as

$$U = 4m\theta \sum_{r=1}^{n} M_{pr}$$

The moment capacity at level n is

$$M_{pn} = \frac{K_2}{4m}\left[\frac{Phn^2}{2} + \frac{mnWL}{4}\right] - \frac{K_2}{4m}\left[\frac{Ph(n-1)^2}{2} + \frac{m(n-1)WL}{4}\right]$$

$$= \frac{K_2}{4m}\left[\frac{Ph(2n-1)}{2} + \frac{mWL}{4}\right] \tag{14}$$

$$M_{p1} = \frac{K_1 WL}{16}$$

Substituting M_{pn}, M_{p1} into Eq. (10a)

$$M_{pa} \geq \frac{1}{2}\left[\frac{K_2}{4m}\left[\frac{Ph(2n-1)}{2} + \frac{mWL}{4}\right] + \frac{K_1 WL}{16}\right]$$

$$= \left[K_2\left[1 + 2\left(\frac{2n-1}{m}\right)\frac{Ph}{WL}\right] + K_1\right]\frac{WL}{32} \tag{15a}$$

Solving for P,

$$P \leq \frac{WLm}{2(2n-1)h}\left[\left(\frac{32M_{pa}}{WL} - K_1\right)\frac{1}{K_2} - 1\right] \tag{15b}$$

The collapse mechanism of Fig. 6(c) is valid only if the moment at the left end of each beam is less than M_p. A typical beam of Fig 6(c) is shown in Fig. 7

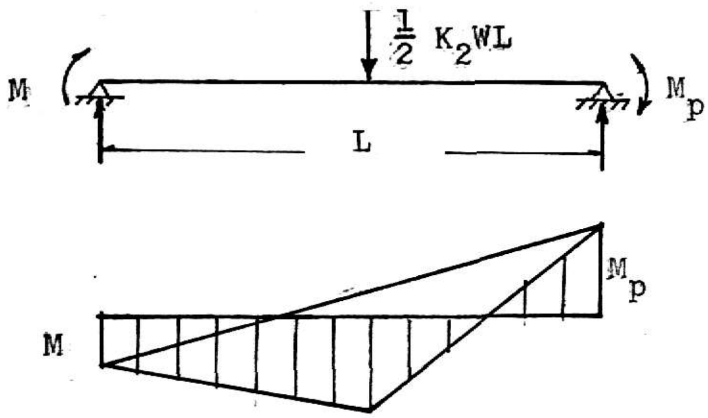


Fig. 7 Typical Beam in Combined Mechanism of Fig. 6(c)

The equilibrium condition for the beam is

$$M_p = K_2 \frac{WL}{8} - \frac{(M_p - M)}{2} \tag{16}$$

As M approaches M_p the above equation shows that the maximum value of M_p approaches $K_2WL/8$. Therefore, Eq. (16) is valid, provided

$$M_{pa} \leq \frac{K_2WL}{8} \tag{17}$$

Substituting M_{pa} into Eq. (15b)

$$P \leq \frac{WLm}{2(2n-1)h}\left[(4K_2 - K_1)\frac{1}{K_2} - 1\right]$$

$$\leq \frac{WLm}{2(2n-1)h}\left(3 - \frac{K_1}{K_2}\right) \qquad (18)$$

(D) Complete Structure: The complete structure consists of a group of frames which support at each level a total lateral load P_T and vertical load W_T. The vertical load path is usually fixed and cannot be significantly altered, the lateral load path can be altered to achieve greater economy.

Consider a structure with external and internal frames as shown in Fig. 8. The total lateral load P_T can be related to the load applied to the individual frames.

$$(2P_e + NP_i) = P_T \qquad (19)$$

in which N = the number of internal frames, and

P_e = external laternal load

P_i = internal lateral load respectively

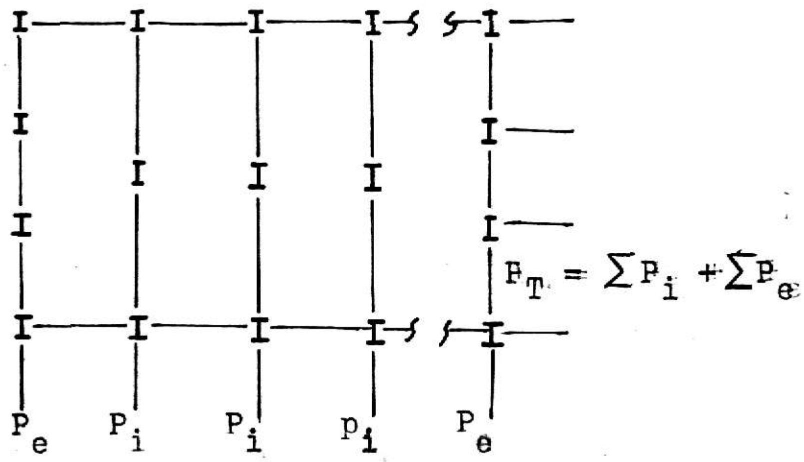


Fig. 8 - Structure with Two Exterior and Three interior Frames

Relating the total internal energy in the structure to the work done by the external forces leads to the following inequality:

$$2\left[\frac{2(M_{pae} - \frac{K_1 W_e L}{16})4m}{K_2 h(2n-1)}\right] + N\left[\frac{2(M_{pai} - \frac{K_1 W_i L}{16})4m}{K_2 h(2n-1)}\right]$$

$$\geq P_T \tag{20}$$

From Eq. (17), the M_{pae} and M_{pai} to be valid must satisfy the following inequalities

$$M_{pai} \leq \frac{K_2 W L_i}{8} \tag{20a}$$

$$M_{pae} \leq \frac{K_2 W L_e}{8} \tag{20b}$$

but from Eq. (2) we obtain the following two additional constraint equations

$$M_{pai} \geq \frac{K_1 W L_i}{16} \tag{21a}$$

$$M_{pae} \geq \frac{K_2 W L_e}{16} \tag{21b}$$

(E) Lateral Displacement Constraint: Consider first the sidesway deflection due to the flexibility of the beams. The actual frame is replaced by that shown in Fig. 9, in which the columns are assumed pinned at their feet but are otherwise rigid.

If each column deflects through an angle θ because of the action of the wind loads, the bending moment M_r induced at the ends of each beam at level r is:

$$M_r = \frac{6EI_r\theta}{L} \tag{22}$$

The strain energy stored in the beam at level r is therefore:

$$U_r = m \frac{M_r{}^2 L}{6EI_r} = m \frac{6EI_r}{L} \theta^2 \tag{23}$$

and the total strain energy stored is

$$U = m\,\frac{6E\theta^2}{L} \sum I_r \tag{24}$$

in which θ = the rotation of each beam end and E = the modulus of elasticity.

The work done by the lateral load is

$$E^* = \frac{1}{2}\left[(\tfrac{1}{2}P)nh\theta + P(n-1)h\theta\right] = \tfrac{1}{4}Phn^2\theta \tag{25}$$

In the elastic range, $E^* = U$

$$\frac{m6E\theta^2}{L} \sum I_r = \tfrac{1}{4}Phn^2\theta \tag{26}$$

solving for θ

$$\theta = \frac{PhL}{24}\left(\frac{n^2}{m}\right)\frac{1}{E\sum I_r} \tag{27}$$

The stiffness factor $C = P/\Delta = P/nh\theta$

$$C = \frac{24mE\sum I_r}{h^2Ln^3} \tag{28}$$

A plot of plastic moment capacity M_p versus moment of inertia I for common W sections in Fig. 10 indicates an approximately linear relationship for the most economical Wide Flange sections.. From Fig.10 two relationship are derived.

$I = 2.5\ M_p$, for $M_p < 250$

and

$I = 6\ M_p - 1050$, for $M_p > 250$

In which I is in^4 and M_p is ft-kips

The stiffness C now can be expressed in terms of M_p

$$C = \frac{24\ mE \sum (2.5M_p)}{h^2 Ln^3} \text{ for } M_p < 250 \tag{29a}$$

$$C = \frac{24mE \sum (6M_p - 1050)}{h^2 Ln^3} \text{ for } M_p > 250 \tag{29b}$$

The displacement limitation constraint in terms of M_p is

$$2\ C_e + N\ C_i \geq \frac{P_{total}}{0.6\Delta_{allowable}} \tag{29c}$$

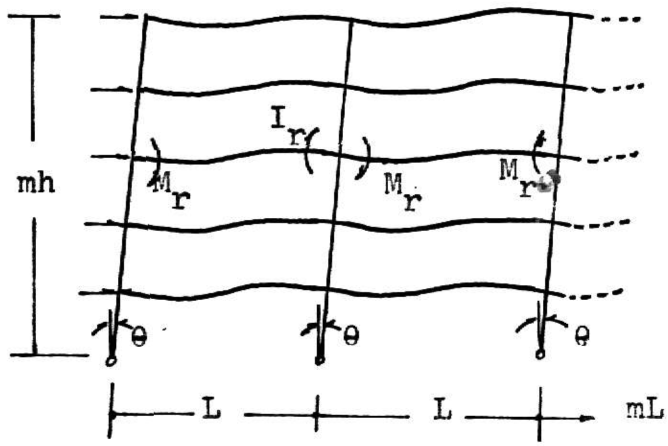


Fig. 9 Deflection Mode - Flexible Beams and Infinitely Stiff Column

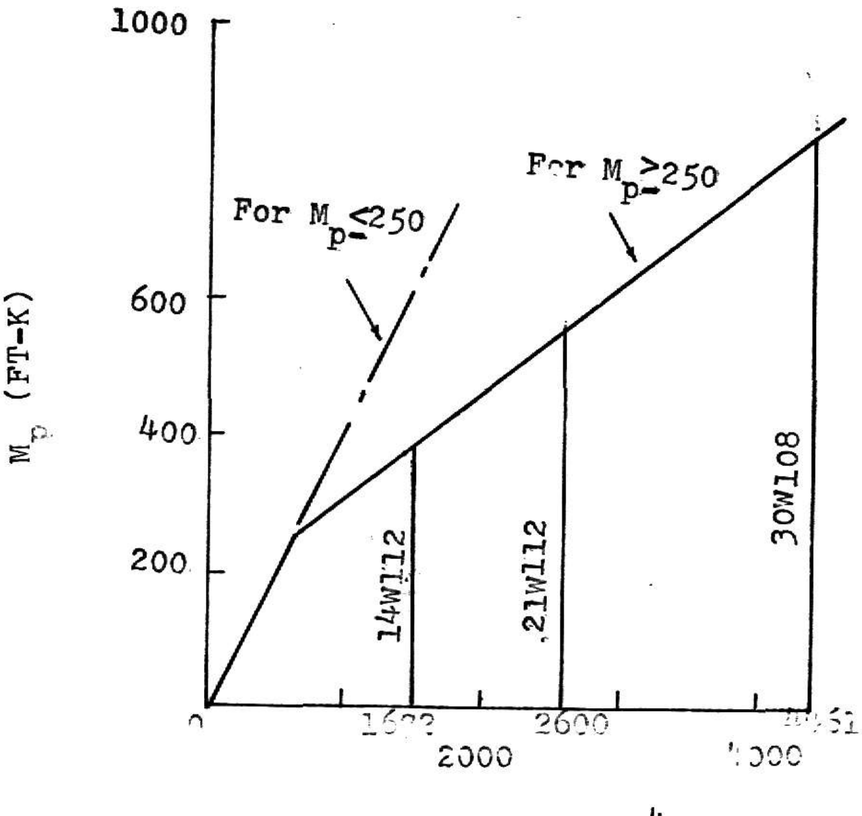


Fig. 10 Moment Capacity Versus Moment of Inertia (Elastic Stiffness)

(F) <u>Objective Function</u>: The optimization is in terms of the beams only. The objective function is related to the plastic moment capacities of the beams only, or

$$Z = m_e L_e M_{pae} + m_i L_i M_{pai}$$

in which

m = number of bays,

L = beam span,

M_{pa} = average moment capacity for beams

(G) <u>Column Loads</u>: The column moments M_{cr} and axial loads V_c caused by the lateral load are calculated as follows: From Fig. 11 and Eq. (16)

$$M_r^* = 3M_{pr} - \frac{K_2 WL}{4} \tag{30a}$$

$$M_r = 3M_{pr} - \frac{K_2 WL}{4} \tag{30b}$$

and

$$M^*_{cr-1} + M^*_{cr} = M^*_r \quad , \qquad M^*_{cr-1} \leq M^*_{cr}$$

Therefore,

$$M^*_{cr-1} \leq \frac{M_r^*}{2} \tag{31}$$

Similarly, $M_{cr-1} + M_{cr} = M_r + M_{pr}$

and

$$M_{cr-1} \leq M_r + M_{pr}/2$$

The axial load in each column caused by lateral load is

$$V_c^* = \frac{M_r + M_{pr}}{L} \tag{32a}$$

and

$$V_c = \frac{M_{pr} + M_r}{L} + \frac{M_{pr} + M_r}{L} \tag{32b}$$

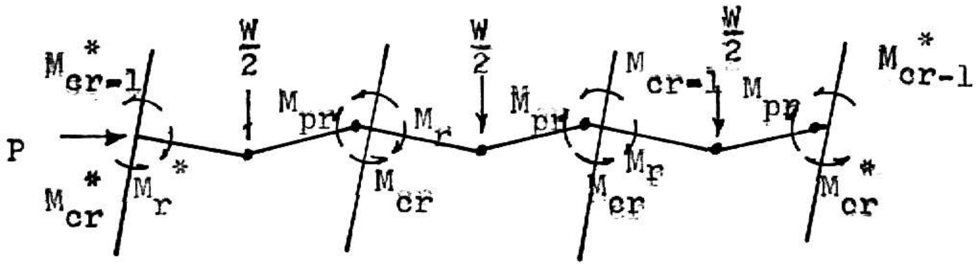


Fig. 11 Calculation of Column Moments

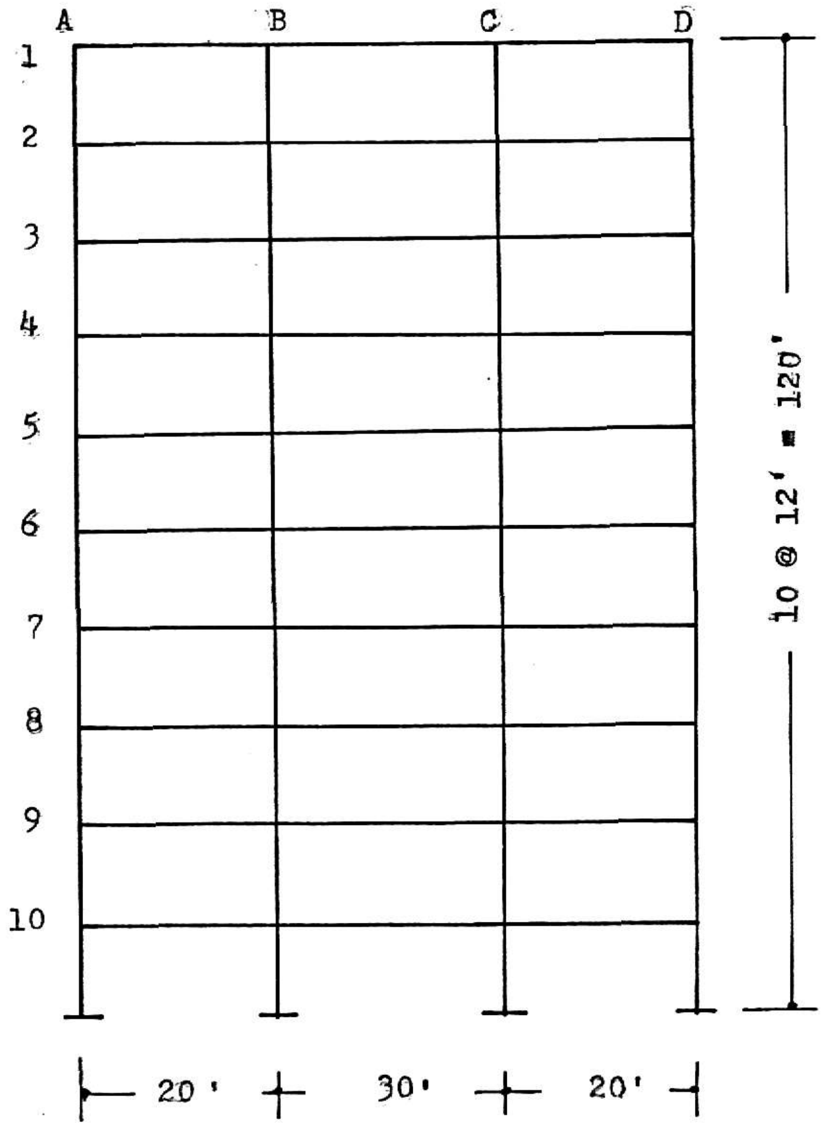


Fig. 12 Elevation of Building

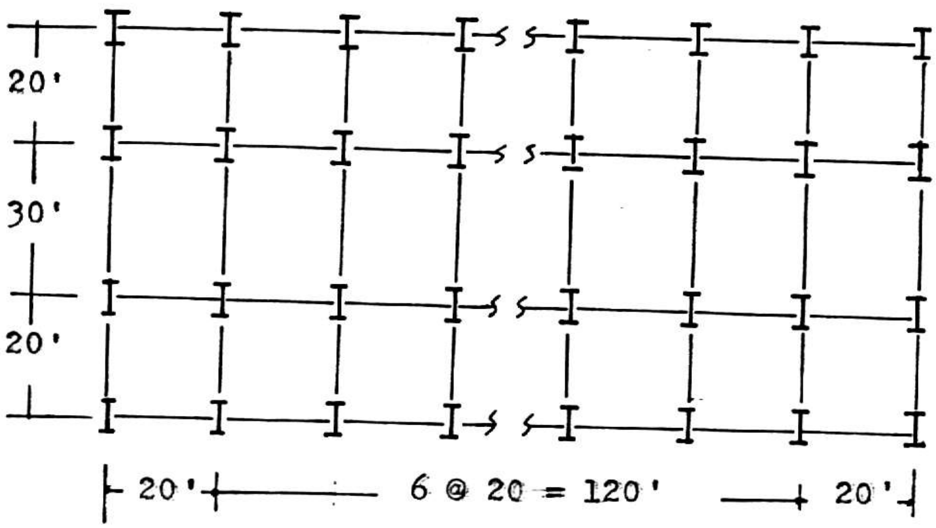


Fig. 13 Typical Plan

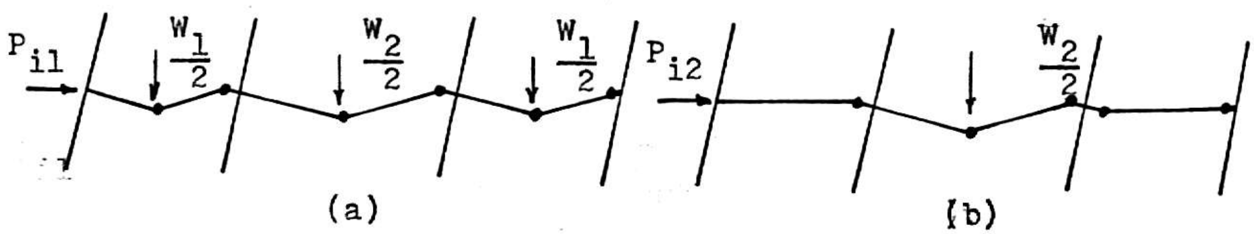


Fig. 14 Collapse Mechanism for Interior Frame

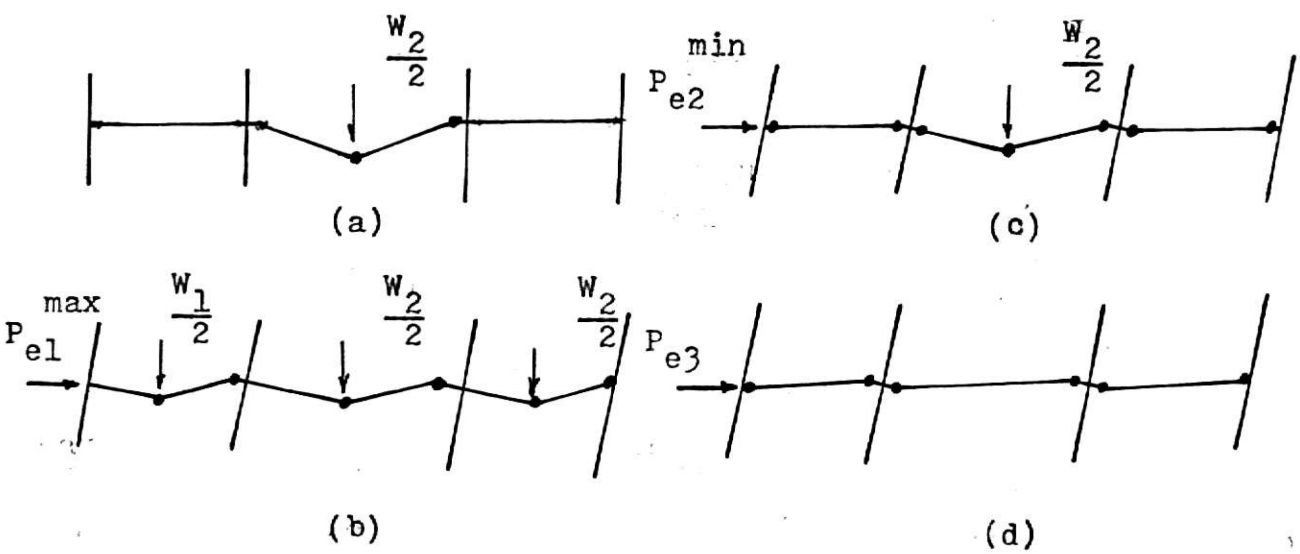


Fig. 15 Collapse Mechanism for Exterior Frame

NUMERICAL EXAMPLE

A 10-story rigid frame building is used in this example of preliminary design. A plan view is shown in Fig. 13

$$n = 10 \quad h = 12\text{ft} \quad L_1 = 20\text{ft} \quad L_2 = 30\text{ft}$$

Lateral Load: For wind load $P_{tot} = 66^{\text{kips}}$ per level

Vertical Load: For live load + dead load

AISC SPECIFICATIONS, 1969 (11)

(a) Interior Frame: $W_1 = 150 \times 20 \times 20 = 60^{\text{kips}}$

$W_2 = 150 \times 30 \times 20 = 90^{\text{kips}}$

(b) Exterior Frame: $W_1' = 30^{\text{kips}}$

$W_2' = 45^{\text{kips}}$

Lateral Deflection Limit: $\Delta_{\text{allowable}} = 0.18\ hn$

$= 0.18 \times 12 \times 10 \times 12$

$= 2.6$ in

Safety Factor: $K_1 = 1.7$ (For D.L. + L.L.)

$K_2 = 1.3$ (For D.L. + L.L. + W.L.)

(A) <u>Interior Beam Design</u>

$$M_{pl} \geq \frac{K_1 W_1 L_1}{16} \geq \frac{1.7 \times 60 \times 20}{16} \geq 127.5^{\text{ k-ft}} \qquad (33)$$

$$M_{p2} \geq \frac{K_1 W_2 L_2}{16} \geq \frac{1.7 \times 90 \times 30}{16} \geq 287 \text{ k-ft} \tag{34}$$

Case A: shown in Fig. 14(a)

For an n story mechanism

$$K_2\left[\frac{P_i h n^2}{2} + \left(\frac{W_1 L_1}{2}\right)n + \left(\frac{W_2 L_2}{4}\right)n\right]$$

$$\leq \sum_{r=1}^{n} 8M_{p1r} + \sum_{r=1}^{n} 4M_{p2r} \tag{35}$$

For an n-1 story mechanism

$$K_2\left[\frac{P_i h(n-1)^2}{2} + \frac{(W_1 L_1)(n-1)}{2} + \frac{(W_2 L_2)(n-1)}{4}\right]$$

$$\leq \sum_{r=1}^{n-1} 8M_{p1r} + \sum_{r=1}^{n-1} 4M_{p2r} \tag{36}$$

Therefore, at story n

$$K_2\left[\frac{(2n-1)ph}{2} + \frac{W_1 L_1}{2} + \frac{W_2 L_2}{4}\right]$$

$$\leq 8M_{p1n} + 4\, M_{p2n} \tag{37}$$

And

$$M_{pa1} = \frac{M_{p1} + M_{p1n}}{2}, \quad M_{pa2} = \frac{M_{p2} + M_{p2n}}{2}$$

or

$$M_{p1n} = 2M_{pa1} - M_{p1}, \quad M_{p2n} = 2M_{pa2} - M_{p2}$$

Substituting the values of M_{p1n} and M_{p2n} into Eq.(37)

$$K_2 \left[\frac{P_i h(2n-1)}{2} + \frac{W_1 L_1}{2} + \frac{W_2 L_2}{4} \right]$$

$$\leq 16\, M_{pa1} - 8M_{p1} + 8M_{pa2} - 4M_{p2}$$

From Eq. (17), the maximum values of M_{pa1} and M_{pa2} are

$$M_{pa1} \leq \frac{K_2 W_1 L_1}{8} \leq \frac{1.3 \times 60 \times 20}{8}$$

$$\leq 195^{\text{ k-ft}} \tag{38}$$

$$M_{pa2} \leq \frac{K_2 W_2 L_2}{8} \leq \frac{1.3 \times 90 \times 30}{8}$$

$$\leq 439^{\text{ k-ft}} \tag{39}$$

hence Eq. (36) becomes

$$114\ P_i + 1275 \leq 3440, \quad P_i \leq 18.9 \text{ kips}$$

Substituting the value of P_i back into Eq. (36) gives

$$2M_{pa1} + M_{pa2} \leq 829 \tag{40}$$

Case B: Shown in Fig. 14(b)

For an n story mechanism

$$K_2 \left[\frac{P_{i2} h n^2}{2} + \frac{n W_2 L_2}{4} \right] \leq \sum_{r=1}^{n} 4M_{p1r} + \sum_{r=1}^{n} 4M_{p2r}$$

For the nth story mechanism

$$K_2 \left[\frac{(2n-1) P_{i2} h}{2} + \frac{W_2 L_2}{4} \right] \leq 4M_{p1n} + 4M_{p2n} \tag{41}$$

$$M_{p1n} = 2M_{pa1} - M_{p1} = 262.5 \text{ k-ft}$$

$$M_{p2n} = 2M_{pa2} - M_{p2} = 591 \text{ k-ft}$$

Therefore, Eq. (41) becomes

$$1.3\left[\frac{19 \times 12\ P_{i2}}{2} + 675\right] \leq 3414$$

$P_{i2} \leq 17.2$ kips

(B) <u>Exterior Beam Design</u>

From Fig. 15(a)

$$M_{p3} \geq \frac{K_1 W_2' L_2}{16} \geq \frac{1.7 \times 45 \times 30}{16}$$

$$\geq 143.5 \text{ k-ft} \qquad (42)$$

The other mechanisms are as follows:

Case A: Shown in Fig. 15(b)

For first story mechanism is

$$K_2\left[\frac{P_{e1} hn}{2} + \frac{W_1' L_1}{2} + \frac{W_2' L_2}{4}\right] \leq 12\ M_{p3} \qquad (43)$$

$$1.3\left[\frac{P_{e1} \times 12 \times 10}{2} + \frac{30 \times 20}{2} + \frac{45 \times 30}{4}\right]$$

$\leq 12 \times 143.5$ $P_{e1} = 11.5$ kips

Case B: Shown in Fig. 15(c), for first story mechanism

$$K_2\left[\frac{P_{e2}hn}{2} + \frac{W_2' L_2'}{4}\right] \leq 8 M_{p2}$$

$$1.3\left[60 P_{e2} + 337.5\right] \leq 1150$$

$$P_{e2} = 9.1 \text{ kips}$$

Case C: Shown in Fig. 15(d), for first story mechanism

$$K_2\left[\frac{P_{e3}hn}{2}\right] \leq 6 M_{p3}$$

$$1.3 \left[60 P_{e3}\right] = 862$$

$$P_{e3} = 11.0 \text{ kips}$$

Comparing these three case gives

$$P_{e\ min} = P_{e2} = 9.1 \text{ kips}$$

Referring to Fig. 15(b) to find the maximum limitation on P_e and M_{pa3}.

For an n story mechanism

$$K_2\left[\frac{P_e h n^2}{2} + 2\left(\frac{W_1' L_1}{4}\right)n + \left(\frac{W_2' L_2}{4}\right)n\right]$$

$$\leq 12 \sum_{r=1}^{n} M_{p3r}$$

At story n

$$K_2\left[\frac{P_e h(2n-1)}{2} + \frac{W_1' L_1}{2} + \frac{W_2' L_2}{4}\right]$$

$$\leq 12\ M_{p3n} \qquad (44)$$

$$M_{p3n} \leq \frac{K_2 W_2' L_2}{8} = \frac{1.3 \times 45 \times 30}{8}$$

$$\leq 219^{\text{ k-ft}} \qquad (45)$$

$$M_{p3n} \leq 2M_{pa3} - M_{p3} = 438 - 143.5 \leq 294.5$$

Substituting the value of M_{p3n} into Eq. (44)

$$1.3\left[114\ P_e + 300 + 337.5\right] \leq 12 \times 294.5$$

$$P_e = 18.3^{\text{ kips}}$$

(C) <u>Lateral Displacement Constraint</u>

Referring to Fig. 14 and using the minimum value of M_{pl} and M_{p2}, $P_{i\ min}$ is obtained

$$K_2\left[\frac{P_{i\ min}\ hn}{2} + 2\left(\frac{W_1L_1}{4}\right) + \frac{W_2L_2}{4}\right]$$

$$\leq 8(127.5) + 4(287)$$

$$1.3\left[\ 60\ P_{i\ min} + 1275\ \right] = 2168$$

$$P_{i\ min} = 6.6 \text{ kips}$$

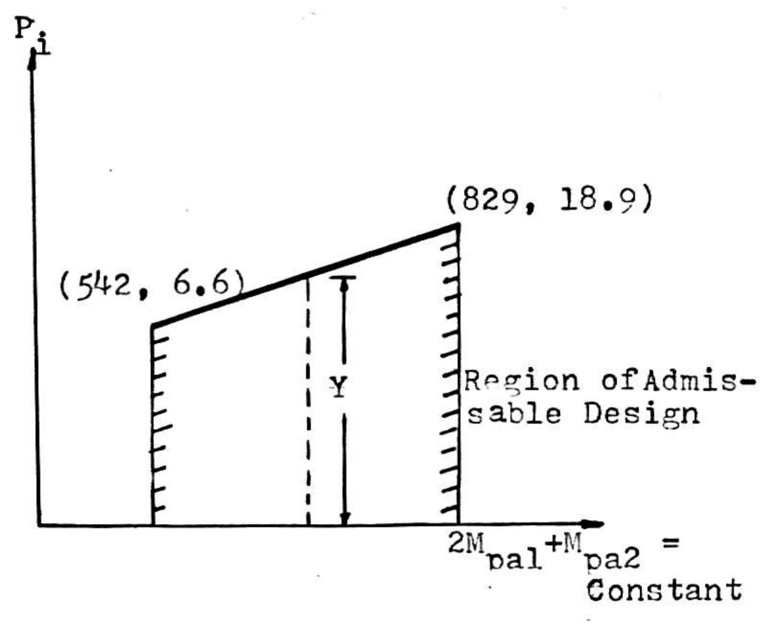


Fig. 16 Solution of P_i

To find the general constraint on P_i

From Fig. 16

$$P_i = Y = 6.6 + \frac{18.9 - 6.6}{829 - 542} (2M_{pa1} + M_{pa2} - 542)$$

$$= 6.6 + 0.0429 (2M_{pa1} + M_{pa2} - 542)$$

$$= 0.0858 M_{pa1} + 0.0429 M_{pa2} - 16.7$$

In a similar fashion

$$P_e \doteq 9.1 + \frac{18.9 - 9.1}{294.5 - 143.5} (M_{pa3} - 143.5)$$

$$= 9.1 + 0.061 (M_{p3a} - 143.5)$$

$$= 0.061 M_{pa3} + 0.34$$

And $2 P_e + 6 P_i \geq 66$

$$5.16 M_{pa1} + 2.50 M_{pa2} + 1.22 M_{pa3} \geq 1655 \qquad (46)$$

The elastic stiffness criteria from Eq. (29c) is

$$2 C_e + 6 C_i \geq \frac{66}{0.6 \times 2.6} = 42.3 \text{ K/in} \qquad (47)$$

Substituting Eq. (29a) and Eq. (29b) into Eq. (47) yields

$$\frac{48E}{h^2n^3}\sum\left[\frac{2}{L_1}(2.5\ M_{pa3}) + \frac{1}{L_2}(2.5\ M_{pa3})\right] +$$

$$\frac{144E}{h^2n^3}\sum\left[\frac{2}{L_1}(2.5\ M_{pa1}) + \frac{1}{L_2}(6M_{pa2} - 1050)\right]$$

$$\geq 42.3$$

or

$$36M_{pa1} + 28.8M_{pa2} + 16M_{pa3} \geq 40250 \tag{48}$$

(D) The Objective Function and The Simplex Method Solution

Referring to Fig. 13, the objective function is

$$Z = 240\ M_{pa1} + 180\ M_{pa2} + 140\ M_{pa3} \tag{49}$$

In summary the collapse constraints are:

$$
\left[\begin{array}{llllr}
M_{pa1} & & & \geq 127.5 & \text{(Eq.33)} \\
 & M_{pa2} & & \geq 287 & \text{(Eq.34)} \\
M_{pa1} & & & \leq 195 & \text{(Eq.39)} \\
 & M_{pa2} & & \leq 439 & \text{(Eq.40)} \\
2M_{pa1} & + M_{pa2} & & \leq 829 & \text{(Eq.41)} \\
 & & M_{pa3} & \geq 143.5 & \text{(Eq.42)} \\
 & & M_{pa3} & \leq 219 & \text{(Eq.45)} \\
5.16M_{pa1} & + 2.58M_{pa2} & + 1.22M_{pa3} & \geq 1655 & \text{(Eq.46)} \\
36M_{pa1} & + 28.8M_{pa2} & + 16M_{pa3} & \geq 40250 & \text{(Eq.48)}
\end{array}\right.
$$

After using the Simplex Method of Linear Programming, we obtain, from the computer output, the following results:

$M_{pa1} = 195^{\text{ k-ft}}$, $M_{pa2} = 439^{\text{ k-ft}}$, $M_{pa3} = 219^{\text{ k-ft}}$

$Z = 156{,}480.0$

The interior frame beams will be sized for

$M_{p1} = 127.5^{\text{ k-ft}}$ for $r = 1$

$M_{p1} = 195 \times 2 - 127.5 = 262.5^{\text{ k-ft}}$ for $r = 10$

$M_{p2} = 287^{\text{ k-ft}}$ for $r = 1$

$M_{p2} = 439 \times 2 - 287 = 591^{\text{ k-ft}}$ for $r = 10$

The exterior frame beam will be sized for

$M_{p3} = 143.5^{\text{ k-ft}}$ for $r = 1$

$M_{p3} = 219 \times 2 - 143.5 = 294.5$ for $r = 10$

(E) Column Design: Design is in the elastic range

(a) For Interior Frame

$$M_{cr}^{*} = \frac{M_{p1r}}{2}$$

$$M_{cr} = \frac{M_{p1r} + M_{p2r}}{2}$$

(b) For Exterior Frame

At level r

$$(m - 1)\ 2M_{cr} + 2x2M_{cr}^{*} = \frac{K_2 P_e h}{2}(2r - 1)$$

$$4M_{cr} + 4M_{cr}^{*} = \frac{K_2 P_e h}{2}(2r - 1) \tag{49}$$

The value of P_e is determined based on

$$\frac{P_e}{P} = \frac{C_e}{C}$$

$$P_e = \frac{P}{C} C_e = \frac{66}{42.3} \times \frac{48E}{h^2 n^3} \times$$

$$\left[\frac{2}{L_1}(2.5\ M_{pa3}) + \frac{1}{L_2}(2.5\ M_{pa3})\right]$$

$$= \frac{66}{42.3} \times \frac{48 \times 30 \times 10^3}{144 \times 144 \times 1000} \times$$

$$\left[2.5\ M_{pa3}\left(\frac{2}{20} + \frac{1}{30}\right)\right]$$

$$= \frac{66 \times 1440}{42.3 \times 144 \times 144} (0.9\ M_{pa3})$$

$$= \frac{66 \times 9}{42.3 \times 144} M_{pa3}$$

$$= 0.0973\ (219) = 21.3\ ^{kip}$$

Assuming that $2M_{cr}^{*} = M_{cr}$, therefore, from Eq. 4

$$M_{cr}^{*} = \frac{K_2 P_e h}{24}(2r - 1)$$

$$= \frac{1.3 \times 21.3 \times 12}{24} (2r - 1)$$

$$= 13.8(2r - 1) \tag{50}$$

$$M_{cr} = \frac{K_2 P_e h}{12}(2r - 1) = 27.6\ (2r - 1) \tag{51}$$

The axial load in the column due to the lateral load is approximated by:

$$\text{Interior Frame:}\quad V_{cr}^{*} = \frac{M_r^{*} + M_{p1r}}{20} \leq \frac{2M_{p1r}}{20}$$

$$= \frac{M_{p1r}}{10}$$

$$V_{cr} = \mp \frac{2M_{p1r}}{20} \pm \frac{2M_{p2r}}{30}$$

Exterior Frame: $V_{cr}^{*} = \frac{4M_{cr}^{*}}{20}$

$$V_{cr} = \pm \frac{M_{cr}}{20} \mp \frac{M_{cr}}{30}$$

(F) Beam Section Selection: Use ASTM A36 Steel

	Interior						Exterior Frame		
Story	M_{p1} (k-ft)	Required Z (in^3)	Trial Section	M_{p2} (k-ft)	Required Z (in^3)	Trial Section	M_{p3} (k-ft)	Required Z (in^3)	Trial Section
1	127.5	42.6	W14X30	287	95.7	W18X50	143.5	48	W14X34
2	127.5	42.6	W14X30	287	95.7	W18X50	143.5	48	W14X34
3	127.5	42.6	W14X30	345	115	W21X62	143.5	48	W14X34
4	127.5	42.6	W14X30	418	139	W21X62	152.3	50.8	W14X34
5	127.5	42.6	W14X30	492	164	W24X76	176	58.8	W16X40
6	127.5	42.6	W14X30	566	188	W24X76	200	66.7	W16X40
7	148	49.3	W16X36	591	197	W24X76	224	74.7	W16X45
8	186	62	W16X36	591	197	W24X76	247.5	82	W16X45
9	225	75	W18X45	591	197	W24X76	271.5	90	W18X50
10	264	88	W18X45	591	197	W24X76	294.5	98	W18X50

(G) Column Section Selection: (1) Use All 14W Sections, A36 Steel. (2) Including Wind Load, the Permitted Reduction in Axial Load is 25 Percent. (3) Check AISC Specification. (11) (Assume K = 1.0, C_m = 0.85)

(i) Interior Frame Column A and D

Tier	P (kips)	M k-in	Trial Section	$P' = B_x x M$ (kips)	$P_1 = P + P'$ (kips)	$P_y = AF_y$ k-in	$M_p = \sigma_y Z$ k-in
$T_1 \sim T_2$	19.1	766	W14X43	153	172.1	456	2510
$T_3 \sim T_4$	38.4	766	W14X43	153	191.4	456	2510
$T_5 \sim T_6$	57.3	766	W14X43	153	210.3	456	2510
$T_7 \sim T_8$	76.7	1116	W14X61	218	294.7	646	3680
$T_9 \sim T_{10}$	95.5	1584	W14X68	307	402.5	720	4130

Tier	$\frac{P_1}{P_y}+\frac{M}{1.18M_p}$	$\frac{b_f}{2t_f}$	$\frac{Kl}{r_x}$	$\frac{Kl}{r_y}$	F_a (ksi)	Per= $1.7AF_a$ (kips)	F_e' (ksi)	$P_e= 1.92AF_e$ (kip)	$\frac{P_1}{P_{cr}}+\frac{C_mM}{M_p(1-\frac{P_1}{P_e})}$
T_1 ∫ T_2	0.637<1 o.k.	0.75< 0.85 o.k.	24.8	76.2	15.75	340	242	5820	0.74<1 o.k.
T_3 ∫ T_4	0.679<1 o.k.	0.756< 0.85 o.k.	24.8	76.2	15.75	340	242	5820	0.8<1 o.k.
T_5 ∫ T_6	0.719<1 o.k.	0.756< 0.85 o.k.	24.8	76.2	15.75	340	242	5820	0.86<1 o.k.
T_7 ∫ T_8	0.715<1 o.k.	0.78< 0.85 o.k.	24.1	58.8	17.55	536	255	8550	0.81<1 o.k.
T_9 ∫ T_{10}	0.752<1 o.k.	0.7< 0.85 o.k.	24	58.7	17.6	598	260	9980	0.98<1 o.k.

(ii) Interior Frame Column B and C

Tier	P (kips)	M k-in	Trial Section	$P' = B_x x M$ (kips)	$P_1 = P + P'$ (kips)	$P_y = AF_y$ k-in	$M_p = \sigma_y Z$ k-in
$T_1 \sim T_2$	9.43	2490	W14X84	471	480.4	889	5220
$T_3 \sim T_4$	28.4	3260	W14X111	603	631.4	1175	7050
$T_5 \sim T_6$	62	4210	W14X142	780	842	1500	9200
$T_7 \sim T_8$	96	4660	W14X167	857	953	1365	10900
$T_9 \sim T_{10}$	118	5130	W14X193	940	1058	2040	12800

Tier	$\frac{P_1}{P_y} + \frac{M}{1.18M_p}$	$\frac{Kl}{r_x}$	$\frac{Kl}{r_y}$	F_a (ksi)	Per= $1.7AF_a$ (kips)	F_e' (ksi)	$P_e=$ $1.92AF_e$ (kip)	$\frac{P_1}{P_{cr}} + \frac{C_m M}{M_p(1-\frac{P_1}{P_e})}$	
$T_1 \int T_2$	0.943<1 o.k.	23.5	47.7	18.5	777	270	12800	0.995<1.0	o.k.
$T_3 \int T_4$	0.93<1 o.k.	23.1	38.6	19.3	1070	280	17550	0.99<1.0	o.k.
$T_5 \int T_6$	0.95<1	22.8	36.3	19.48	1380	288	23100	0.996<1.0	o.k.
$T_7 \int T_8$	0.903<1 o.k.	22.4	36	19.5	1640	292.5	27200	0.95<1.0	o.k.
$T_9 \int T_{10}$	0.858<1 o.k.	22.1	35.6	19.54	1880	360	33000	0.91<1.0	o.k.

CONCLUSIONS

(1) Optimum elastic-plastic design corresponds to neither optimum rigid-plastic nor optimum elastic design. It can be regarded as a mixture of both types of problem.

(2) William Prager (7) indicated that in actual fact the unit weight of practical sections is more nearly proportional to $M^{0.6}$. In this report, the weight was assumed to be proportional to $M^{1.0}$. However, the plastic section modulus was assumed to have a linear variation with story level. These two assumptions tend to minimize to overall error (13).

(3) The design approach presented offers a practical and rational preliminary design of a multi-story frame structure. The design indicates the most efficient stiffness distribution for the frame of the structure.

(4) No attempt has been made in the design process to consider the possiblity of lateral torsional buckling of the columns. Therefore they should be checked to see if they meet the requirements of the AISC Specification.

(5) The design of buildings braced against wind forces has not been dealt with in this report. If wind forces can be neglected, and only vertical loads considered, a plastic design procedure for the Weak-Beam Strong-Column assumption is essentially straight forward, although the actual calculations may be tedious.

RECOMMENDATIONS FOR FURTHER STUDY

The general theory and design developed in this report have been limited to structures which have a great deal of regularity. However, a more general approach to design could be applied to irregular frames, which have different bay spans and different story heights.

ACKNOWLEDGMENTS

The author wishes to express his sincere gratitude and deep appreciation to his major professor, Dr. Robert R. Snell, for his valuable assistance, guidance, and encouragement during the preparation of this report.

Appreciation is also due to Dr. Jack B. Blackburn, Head of the Department of Civil Engineering, and to members of the advisory committee.

REFERENCES

1. Mitchell, A. G. M., "The Limit of Economy of Material in Frame Structures", Phil. Mag., Series 6, Vol. 8, No. 47, London, England, November, 1904

2. Heyman, J., "Plastic Design of Beams and Plane Frames for Minimum Material Consumption", Quarterly of Applied Mathematics, Vol. 8, P. 373-381, 1951

3. Foulkes, J., "Minimum Weight Design and Theory of Plastic Collapse", Quarterly of Applied Mathematics, Vol. 10, P 374-358, 1953

4. Folukes, J., "Minimum Weight Design of Structural Frames", Proceedings of the Royal Society, London, A223, P. 482-494, 1954

5. Liveseley, R. K., "The Automatic Design of Structural Frames", Quarterly Journal of Mechanics and Applied Mathematics, Vol. 9, P 257-278, 1956

6. Heyman, J. and Prager, W., "Automatic Minimum Weight Design of Steel Frames", Journal of the Franklin Institute, Vol. 226, P. 339-364, 1958

7. Neal, B. G. and Symonds P. S., "The Calculation of the Collapse Loads for Framed Structures", Journal of the Institution of Civil Engineers, Vol. 35, PP. 21-40, 1950

8. Murry, T. M. and Ostapenko, A., "Optimum Design of Unbraced Multi-Story Frame by Plastic Theory", Leigh University, Fritz Engineering Laboratory Report, No. 273. 42, 1966

9. Bigelow, R. H. "Design of Steel Frames for Minimum Weight", Journal of the Structural Division, ASCE, Vol. 93, No. ST6, December, 1967

10. Rubinstein, M. F. and Karagozian, J., "Building Design Using Linear Programming", Journal of the Structural Division, ASCE, Vol, 92, No. ST 6, December, 1966, PP. 223-245

11. AISC Specification for the Design, Fabrication & Erection of Structural Steel for Buildings, 1969

12. Ashour, S., Linear Programming, Department of Industrial Engineering Classnotes, Kansas State University, 1969

NOTATION

$[a]$	matrix of coefficients
C	stiffness factor P/Δ
C_K	key row of simplex method
E^*	work done by external forces
E	modulus of elasticity
F	objective function
h	story height
I	moment of inertia
K_1	safety factor for dead plus live load
K_2	safety factor for lateral loads
L	beam span
M	moment
M_p	plastic moment
M_{pa}	average moment capacity for beams in a frame
m	number of bays
n	number of stories
P, P'	load
P_e	lateral force at any level for exterior frame
P_i	lateral force at any level for interior frame
P_u	ultimate load
P_T	total lateral force at any level

S	column vector of simplex method
U	interal energy
V	axial column load caused by lateral load
W_T	vertical load at any level
W_u	ultimate vertical load
$\triangle$	lateral displacement at a floor level times the number of stories n
θ	rotation

APPENDIX I

CONTROL PROGRAM COMPILER - MPS/360 V2-M8

```
C001                    PROGRAM
C002                    INITIALZ
C064                    TITLE('10-STORY BUILDING DESIGN')
C065                    MOVE(XDATA,'EXAMPLE')
C066                    MOVE(XPBNAME,'PBFILE')
0067                    CONVERT('CHECK','SUMMARY')
C068                    SETUP('MIN')
0069                    BCDOUT('ONE')
C070                    MOVE(XOBJ,'WEIGHT')
C071                    MOVE(XRHS,'LIMITS')
C072                    PRIMAL
C073                    SOLUTION
C074                    EXIT
C075          SV1        DC(0.0)
CC76          SV2        DC(0.0)
0077                    PEND
```

10-STORY BUILDING DESIGN

```
NAME          EXAMPLE
ROWS
 N   WEIGHT
 G   ROW1
 G   ROW2
 L   ROW3
 L   ROW4
 L   ROW5
 G   ROW6
 L   ROW7
 G   ROW8
 G   ROW9
COLUMNS
     COL1         WEIGHT          240.00000
     COL1         ROW1              1.00000
     COL1         ROW3              1.00000
     COL1         ROW5              2.00000
     COL1         ROW8              5.16000
     COL1         ROW9             36.00000
     COL2         WEIGHT          180.00000
     COL2         ROW2              1.00000
     COL2         ROW4              1.00000
     COL2         ROW5              1.00000
     COL2         ROW8              2.58000
     COL2         ROW9             28.80000
     COL3         WEIGHT          140.00000
     COL3         ROW6              1.00000
     COL3         ROW7              1.00000
     COL3         ROW8              1.22000
     COL3         ROW9             16.00000
RHS
     LIMITS       ROW1            127.50000
     LIMITS       ROW2            287.00000
     LIMITS       ROW3            195.00000
     LIMITS       ROW4            439.00000
     LIMITS       ROW5            829.00000
     LIMITS       ROW6            143.50000
     LIMITS       ROW7            219.00000
     LIMITS       ROW8           1655.00000
     LIMITS       ROW9          40250.00000
ENDATA
```

10-STORY BUILDING DESIGN

SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY
1	WEIGHT	BS	156480.00000	156480.00000-
2	ROW1	BS	195.00000	67.50000-
3	ROW2	BS	439.00000	152.00000-
4	ROW3	UL	195.00000	.
5	ROW4	UL	439.00000	.
6	ROW5	BS	829.00000	.
7	ROW6	BS	219.00000	75.50000-
8	ROW7	UL	219.00000	.
9	ROW8	BS	2406.00000	751.00000-
10	ROW9	**	23167.20000	17082.80000

..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
NONE	NONE	1.00000
127.50000	NONE	.
287.00000	NONE	.
NONE	195.00000	240.00000-
NONE	439.00000	180.00000-
NONE	829.00000	.
143.50000	NONE	.
NONE	219.00000	140.00000-
1655.00000	NONE	.
40250.00000	NONE	.

10-STORY BUILDING DESIGN

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..
11	COL1	BS	195.00000	240.00000
12	COL2	BS	439.00000	180.00000
13	COL3	BS	219.00000	140.00000

..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
.	NONE	.
.	NONE	.
.	NONE	.

10-STORY BUILDING DESIGN

SOLUTION (INFEASIBLE)

TIME = 0.48 MINS. ITERATION NUMBER = 3

...NAME...	...ACTIVITY...	DEFINED AS
FUNCTIONAL RESTRAINTS	156480.00000	WEIGHT LIMITS

A STUDY OF MINIMUM WEIGHT PRELIMINARY PLASTIC DESIGN OF UNBRACED FRAMES

by

DER-HSIUNG SU

Diploma, Taipei Institute of Technology,

Taiwan, China, 1964

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1971

ABSTRACT

This report presents a preliminary approach to the optimum design of tall steel buildings. The problem considered is that of tall steel frames which are not braced against wind loads. The basic tool used in the derivation of a possible design method is simple plastic theory. The results obtained must be checked to see that the assumptions of plastic theory are satisfied.

The basic concepts of the building design are as follows:

(1) Consider that plastic hinges form only in the beams.

(2) Assume a linear variation of beam sizes with story height.

After using a computer program for linear programming to obtain an optimum design for the minimum weight of a 10-story building, it is concluded that the method developed is relatively simple and practical to apply in steel frame design.