

MODELING AND SIMULATION OF DEEP BED FILTRATION:
A Stochastic Compartmental Model

by

310

SONG-TIEN CHOU

B. S., National Taiwan University, 1974

M. S., Kansas State University, 1982

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Statistics

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1984

Approved by:

R. Nassar

Major Professor

LD
2668
R4
1984
C55
C. 2

A11202 659165
LIST OF CONTENTS

ii

LIST OF TABLE 1	iii
LIST OF FIGURES	iv
I. INTRODUCTION	1
II. MATHEMATICAL MODEL	3
1. Compartmental Model	3
2. Internal Particle Distribution	9
3. Distribution of Particle in the Effluent	14
III. NUMERICAL SIMULATION AND PARAMETER ESTIMATION	15
1. Numerical Simulation	16
2. Parameter Estimation	17
IV. DISCUSSION AND CONCLUSIONS	19
NOMENCLATURE	29
REFERENCES	30
APPENDIX A: TRANSITION PROBABILITY FUNCTIONS FOR SINGLE COMPARTMENT MODEL	33
APPENDIX B: THE RANDOM SEARCH METHOD -- USING THE SHRINKING STRATEGY	37
APPENDIX C: PROGRAM USED TO SEARCH FOR THE OPTIMAL SET OF PARAMETERS FOR THE MODEL WHICH FITS ELIASSEN'S EXPERIMENTAL DATA	40
ACKNOWLEDGMENTS	49
ABSTRACT	52

**THIS BOOK
CONTAINS
NUMEROUS PAGES
WITH THE ORIGINAL
PRINTING BEING
SKEWED
DIFFERENTLY FROM
THE TOP OF THE
PAGE TO THE
BOTTOM.**

**THIS IS AS RECEIVED
FROM THE
CUSTOMER.**

**THIS BOOK
CONTAINS
NUMEROUS PAGES
WITH DIAGRAMS
THAT ARE CROOKED
COMPARED TO THE
REST OF THE
INFORMATION ON
THE PAGE.**

**THIS IS AS
RECEIVED FROM
CUSTOMER.**

Table 1: Parameters of the Model Used for Describing Eliassen's
Experimental Data 22

LIST OF FIGURES

Figures

1. Schematic diagram of a deep-bed filter represented by a n-compartment model 23
2. Transition diagram for a deep-bed filtration process 24
3. Simulation of the concentration dynamics of filtrates sampled from a deep-bed filter which is divided into two compartments with the following parameters to demonstrate the backmixing effect:
 - I : $m_{12}=3.0/h$, $m_{13}=2.0/h$, $m_{21}=0.0/h$, $m_{24}=1.0/h$,
 $m_{31}=0.5/h$, $m_{42}=0.25/h$, $\mu_2=1.5/h$.
 - II: $m_{12}=3.0/h$, $m_{13}=2.0/h$, $m_{21}=0.15/h$, $m_{24}=1.0/h$,
 $m_{31}=0.5/h$, $m_{42}=0.25/h$, $\mu_2=1.5/h$ 25
4. Simulation of the concentration dynamics of filtrates sampled from a deep-bed filter which is divided into two compartments with the following parameters to demonstrate the exit effect:
 - I : $m_{12}=3.0/h$, $m_{13}=2.0/h$, $m_{21}=0.0/h$, $m_{24}=1.0/h$,
 $m_{31}=0.5/h$, $m_{42}=0.25/h$, $\mu_2=1.5/h$.
 - II: $m_{12}=1.0/h$, $m_{13}=2.0/h$, $m_{21}=0.0/h$, $m_{24}=1.0/h$,
 $m_{31}=0.5/h$, $m_{42}=0.25/h$, $\mu_2=1.0/h$ 26
5. Simulation of the concentration dynamics of filtrates sampled from a deep-bed filter which is divided into two compartments with the following parameters to show the characteristics of the filter bed with absorbing solid media:
 - I : $m_{12}=2.5/h$, $m_{13}=1.5/h$, $m_{24}=1.0/h$, $\mu_2=2.0/h$,
 $m_{21}=m_{31}=m_{42}=0.0/h$.
 - II: $m_{12}=2.5/h$, $m_{13}=2.5/h$, $m_{24}=1.0/h$, $\mu_2=2.0/h$,
 $m_{21}=m_{31}=m_{42}=0.0/h$ 27
6. Fitting the present model to Eliassen's data(1935):
 sand diameter(0.051 cm), bed depth(60 cm), flow rate (0.136 cm/sec), hydrous ferric floc suspension(dia., 0.00124 cm;con., 0.0050 vol%). 2 compartments. Run 6:
 $m_{12}=29.677/h$, $m_{13}=436.600/h$, $m_{31}=0.1385/h$, $\mu_2=16.713/h$,
 $m_{24}=18.432/h$, $m_{42}=0.0107/h$ 28