LINEAR FREE VIBRATIONS OF ORTHOTROPIC ANNULAR PLATES OF VARIABLE THICKNESS

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Nomenclature

a,b	outer and inner radii of the plate, respectively
A_1,A_2,A_3,A_4	known coefficients of governing equations
a_{ij}	elastic constants
c	$= a_{11}/a_{22}$
D	flexural rigidity of the plate
D_0	$h_0^3/12a_{22}$
f_1	dimensionless frequency parameter
g	dimensionless transverse displacement
h	plate thickness
h_0	thickness of plate at $r = 0$
M,N	coefficient matrices
M_r,M_θ	radial and circumferential bending moments per unit length
$p(r,t)$	transverse load
r,θ,z	cylindrical coordinates (see fig. 1)
t	time variable
w	transverse displacement of the middle plane
$\bar{Y},\bar{Z},\bar{H}$	$(\bar{4}x1)$ vector functions
β	exponent of thickness expressions
η	function of radial distance
ν	$= -a_{12}/a_{22}$
λ	$= (c-\nu^2)\omega^2$
ρ	mass density per unit volume
ξ,τ	dimensionless space and time variables, respectively
$\varepsilon_r,\varepsilon_\theta$	components of strain
σ_r,σ_θ	components of stress

ζ	admissible variational function
$\bar{\eta}$	adjustable data in the related initial value problem

1. INTRODUCTION

The axisymmetric flexural vibrations of a cylindrically aeolotropic plate has been analysed by several authors [1-6]. The basic differential equations of motion and associated boundary conditions of the nonlinear axisymmetric flexural vibrations of a cylindrically anisotropic circular plate of varying thickness have been developed by Huang [7], using the method of variational calculus. The advantage of this method is that both the differential equations and appropriate boundary conditions can be obtained simultaneously. The present analysis deals with the linear free vibrations of orthotropic annulus of variable thickness.

In this report, the change in frequency parameter is analysed by varying $c = a_{11}/a_{22}$, the ratio of elastic constants. A circular plate with fixed immovable inside and free outside conditions is considered. The governing equation for the free vibration of such a plate is solved by the method of numerical integration by shooting. The thickness of the plate is varied in the radial direction only both linearly and nonlinearly.

Finally, a brief discussion of the results is presented.

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*[ ] Numbers in brackets designate references at the end of report.

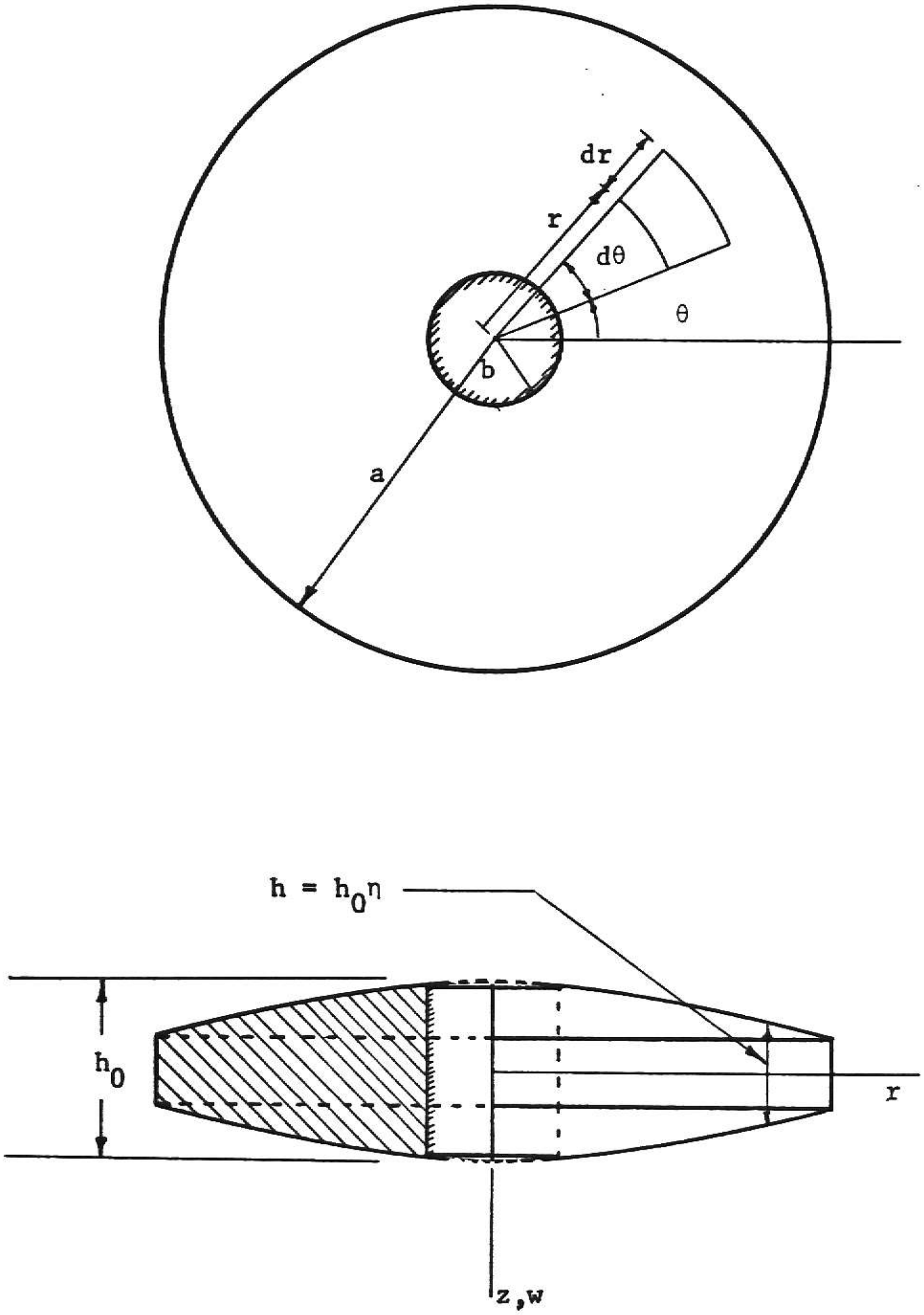
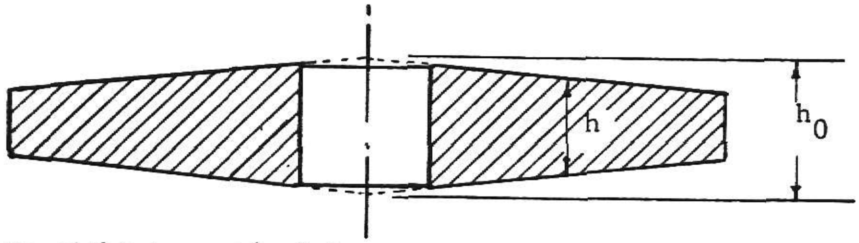
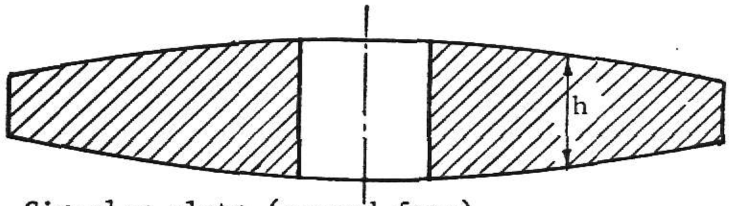


Fig. 1. Variable thickness plate with fixed inside boundary.



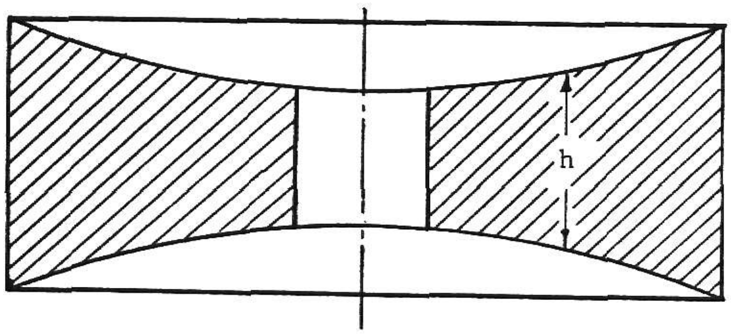
Case I, Straight tapered plate.

$h = h_0\eta$; $\eta = (1-k\xi)$, $k = 0.5$



Case II, Circular plate (curved form)

$h = h_0\eta$; $\eta = e^{-\beta\xi^2/6}$, $\beta = 4$



Case III, Circular plate, (curved form)

$h = h_0\eta$; $\eta = e^{-\beta\xi^2/6}$, $\beta = -4$

Fig 2. Different cases of plate shapes.

2. DERIVATION OF GOVERNING EQUATION

In what follows, the circular plate is analysed by small deflection theory of plate and the governing equation of motion is derived along with appropriate boundary conditions. The following simplifying assumptions are made:

(i) Normals to the middle plane before bending remain straight and normal to the middle plane after bending.

(ii) Normal stresses, σ_z, are small compared with other stress components and may be neglected in stress-stain relations.

(iii) The stretching of the median surface of the plate is considered negligible, since the deflections are small as compared with the local thickness of the plate.

In accordance with these assumptions, the components of strain for axisymmetric case are found to be

$$\begin{aligned} \varepsilon_r &= -zw_{rr} \\ \varepsilon_\theta &= -zr^{-1}w_r \\ \gamma_{r\theta} &= 0 \end{aligned} \qquad (1)$$

In the above equations, w(r,t) is the transverse component of the mid-plane displacement and w_r and w_{rr} are first and second derivatives of w with respect to r, respectively.

The stress strain relationships in polar coordinates for orthotropic axisymmetric plates are [10]