APPLICATION OF STATE INCREMENT DYNAMIC PROGRAMMING TO INDUSTRIAL MANAGEMENT SYSTEMS

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SYMBOLS

f	scalar function
$\underline{f}$	vector function
h	profit function
s	dummy variable for time
t	time, independent variable
t_o, t_f	initial and final time respectively
x	scalar state
$\underline{x}$	state vector
z	scalar control
$\underline{z}$	control vector
B	block designation
$G^{(k)}$	profit resulting from control $z^{(k)}$ over incremental time
$H^{(k)}$	interpolated optimal profit at state $x^{(k)}$
$I(x,t)$	optimal profit at state x and time t
$J(x,z,t)$	profit function
K	total number of control increments
M	total number of time increments
S	total number of time increments Δt within a block
Z	set of admissible controls
X	set of possible states
α^-, α^+	bounds on control variables

β^-, β^+	bounds on state variables
Δx	fixed increment in state variable
Δt	fixed increment in time
ΔT	interval in independent variable time covered by a block
Δz	fixed increment in control variable

1. INTRODUCTION

Dynamic programming is a useful tool in optimizing multistage optimization problems. The method of dynamic programming is based on the mathematical notion of recursion. The main characteristics of optimization problems to which it can be applied is that several decisions have to be made to optimize the overall performance of a system. Furthermore, the system consists of several stages and the decision made at the later stage do not affect the performance of the earlier stages.

For a multistage process, the optimal overall policy cannot be arrived at by considering the effect of each decision at each stage separately. In other words, it is not true that the overall optimum is the same optimum obtained by optimizing each individual stage. It might, for example be that a bit of sacrifice in the first stage may place us in a much stronger position in the second stage and thus a higher total profit is obtained. This will be achieved if the method of dynamic programming is used.

The subject owes much of its development to Bellman and his associates at the Rand Corporation. The concept on which dynamic programming [1-5] is based is the principle of optimality which states:

> "An optimal policy has the property that, whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with respect to the state resulting from the first decision".