Holistic approach to stochastic analysis of public transportation networks

by

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Abstract

Evaluating multi-modal public transportation networks is complicated and often the data needed to conduct detailed analysis is not available or difficult to attain. Much research has been done on methods to improve efficiency and redesign networks using mathematical and simulated models, but there is a lack of holistic analysis on the state of current networks. Holistic analysis allows efficient analysis requiring minimal equipment or software and can be used to test different scenarios that affect traffic through the network easily. It can also be used to determine where more critical analysis should be conducted.

This thesis develops a network of queues to evaluate parameters of a transportation network consisting of multiple modes of transportation on a station-by-station basis, focusing on wait times, station utilization, and queue lengths. The first formulation defines the network as a set of nodes and arcs that are used to conduct the analysis. Second, an open Jackson network is used to evaluate the effective arrival rate to then calculate the different performance measures of the network. Finally, simulation using SIMIO is used to determine the interarrival time data of passengers entering each station that is not available in published data sets.

Using a case study and numerical experiments, this thesis analyzes both a network with one mode of transportation and one with multiple modes. It also analyzes the impact of changing aspects of these networks on performance measures of each station within it. We observe what happens when passengers select a different transportation mode and how flow is affected when stations are closed. Overall, we demonstrate that queueing theory provides a method to quickly analyze a network without requiring complicated software.

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Acknowledgments

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Dedication

To my loving husband.

Chapter 1

Introduction

By 2050, it is expected that 70% of the population could be urbanized [2], up from the 56% currently living in urban areas [3]. In the United States, the Census Bureau estimates that currently 80.0% of the U.S. population lives in an urbanized area [4]. While growth is expected to occur mainly in developing countries that may not have an existing public transportation system [2], the importance of the current transportation systems that already exist for the 53 billion people around the world that use them cannot be discounted [1].

While many studies aim to determine how to improve public transportation systems, few aim to evaluate the current systems holistically and instead choose to focus on a single route or a single station to conduct a deep investigation into the efficiency of that part in the system.

Figure 1.1: Passengers waiting at train station in Germany [1].

Holistic analysis of a transportation network can be essential in identifying critical nodes and links within the system as a starting point for where to focus more targeted study [5]. It can also serve as a tool to conduct basic analysis of the effects of disruptions to the network due to unforeseen incidents, whether from natural disasters or external threats to the network [5]. In his book, Taylor [5] describes this type of analysis as vulnerability analysis. The majority of the studies discussed in the section devoted to public transportation focus on rail networks with mathematical models aiming to capture a specific vulnerability metric, defined differently by each study. While this type of analysis can be useful and more robust than what this research aims to show, these studies still require large amounts of data and computation power to solve.

In this research, a Jackson Network is used to develop a queueing model of the transportation network to analyze the utilization of stations within the network as well as identify wait times for passengers at each station. Using stochastic queueing theory allows us to determine the stations most used and easily adjust parameters to study the effect on the network.

1.1 Queueing Models

According to *Factory Physics, 3rd Edition* queueing theory is the science of waiting in a line and a queueing system is a system that combines an arrival process, service process, and queue. Queueing theory is applied to queueing systems to "characterize performance" using different parameters. These parameters include probability of there being a certain number of items at a service station, expected wait time, expected time at a service station, average number of items at a service station, and expected number of items in a queue for a service station [6]. These parameters seek to give information about the steady state of a system. Queueing systems are characterized by their arrival process, service process, and the number of stations available to service an item [6]. Each process is characterized using time distributions, with the most common being deterministic, exponential, and completely general distributions [6].

There are three main topics where queueing theory has been significantly used to analyze systems: information systems, production systems, and service systems [7]. Queueing theory began with Agner K. Erlang in 1909 and his paper on managing congestion in telephone lines [8]. Since then, queueing theory has been instrumental in analyzing information systems to include computer processing and Internet traffic [7]. Around the same time that queueing was gaining popularity in the communications realm, mass production was emerging as was the need to increase and gain efficiency [7]. To better analyze these systems, queueing theory was applied. Much of queueing theory was devised with production and manufacturing systems as the impetus [7]. Lastly, queueing theory started being applied to service systems, like hospitals, call centers, and theme parks to analyze the system and inevitably change or alter the design of them to decrease wait time [7]. While queueing theory applies to systems where only one "service" is required before leaving the system, it is also possible to analyze a network of queues, or when an item requires multiple different services before it exits the system [8]. While there are multiple different ways to analyze a network of queues, the focus of this research is on an open Jackson Network.

Jackson Networks were originally developed to model a queueing system for the overhaul of aircraft engines [8]. However, more recently Jackson Networks have been used for planning in medical centers [9, 10] and in supply chain analysis [11, 12]. A Jackson Network is considered an open network of queues, wherein there is at least one arc allowing customers to enter the network and at least one arc allowing customers to leave the network [13]. An example of this type of network is shown in Figure 1.2. The network is fully defined once the external arrival process and the number of servers at each queue, the routing through the network, the service time at each queue, and the service discipline are defined [13]. In this format, the routing through the network is stochastic and dependent on routing probabilities [13]. It is possible then, that a customer could end up doing loops, or end up back in a queue that they already were in.

Computer simulation is a common tool used to analyze queues. It is defined as a "comprehensive method for studying systems" [14]. Originally developed as a tool to help predict the weather, computer simulation is now used by many different academic disciplines [14]. There are three main categories that simulation is used for: heuristics, predicting data that is

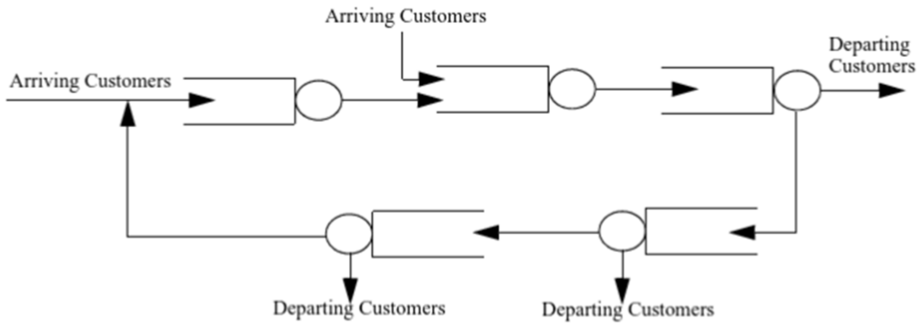


Figure 1.2: Example of an open network of queues. [13]

not available or difficult to collect, and understanding complicated data that currently exists [14]. Simulation is also a valuable tool to validate mathematical models used to explain real stochastic phenomenon and vice versa [15]. Of these purposes, using simulation to generate data and validate the mathematical model developed are utilized in this research.

1.2 Holistic Analysis and Transportation

Using holistic network analysis tools can be an instrumental way of identifying initial issue or problems within an existing public transportation network. It is even more important now in the years following the COVID-19 pandemic as ridership dropped significantly across all modes of transportation [16]. Not only can it help identify issues, but it can also aid in determining how a network responds to different potential situations, like closing a station for repair or personnel manning issues. What if environmental factors change how riders use the network and more people start riding the subway versus riding the bus? Determining how passengers flow through the network and where congestion occurs are imperative to any initial analysis of a public transportation network and there is surprisingly little data to explore without modelling.

Therefore, conducting holistic network analysis through queueing theory is a valuable tool. Queueing theory requires minimal amounts of data and can be applied easily using spreadsheets, allowing for quick and dynamic analysis of multiple different scenarios quickly without significant computational power [17]. Requiring minimal data is a key factor when considering public transportation, as often the data needed to conduct exacting evaluation is not available. There are multiple reasons for the lack of data including limited data collection budgets and inadequate tools for collection, especially in public transportation [18]. Even in cities where data is collected, it can be old, or it can lack the specificity needed to run a full study. For example, Washington D.C. collects ridership data and publishes it monthly. However, the bus data only shows the number of riders on a given route and does not give specifics for where the riders originate or get off the bus [19]. This makes finding any issues or determining ramifications of network disruptions difficult.

1.3 Contributions of Research

This research contributes to the field of study by developing an easily accessible holistic view of a transportation network with respect to the number of customers and their wait time. It integrates simulation and Jackson networks to gain information on multi-modal transportation networks. However, it is also beneficial in comparing the functionality of a single mode of transporation within a transporation network to its function when combined into a multi-modal system. Moreover, it develops an analysis method that is dynamic and can be adjusted easily without the need for specialized software or long simulation runs. This research provides transportation network managers with the ability to quickly check scenarios of different types of station disruptions and make decisions on how to prioritize or move assets around the network. It also allows network managers the ability to identify more significant problems within the network that need detailed analysis.

1.4 Research Tasks

The main objective of this research is to develop a queueing model of a multi-modal public transportation model to allow for holistic analysis of the network using expected wait times, expected queue lengths, and station utilization. Simulation is utilized to create essential arrival data to examine the network at the maximum feasible flow of passengers.

Then the model is applied to a real transportation network as a case study to examine the validity of the model and to conduct holistic analysis of the network. Three experiments are conducted to demonstrate the type of analysis this model provides. Overall, this research investigates the following: RQ1: What is the maximum flow of passengers into the network every hour? RQ2: What are the ramifications on the network if passengers alter their mode of transportation? RQ3: What are the ramifications on the network if stations of varying types are closed for extended periods of time?

Answering these questions will provide understanding of the information that can be collected from this type of analysis and its usefulness as an analytical tool for public transportation networks.

1.5 Thesis Outline

This thesis is organized into five chapters. Chapter 2 provides a literature review of material discussed in this thesis, focusing on applications of queueing theory and simulations to solve transportation problems.

Chapter 3 discusses network and mathematical model formulation, which provides the basic understanding for research questions two and three. It also discusses how to build the simulation model that is used to answer research question 1.

Chapter 4 develops the simulation and mathematical analysis for the case study of a small piece of the Washington D.C. public transportation network. This chapter provides insight into all research questions.

Finally, Chapter 5 summarizes the model, results, and concludes with a discussion of possible future work.

Chapter 2

Literature Review

2.1 Stochastic Analysis and Public Transportation

2.1.1 Network Models

The intial focus of the literature review was on stochastic analysis of public transportation. Most articles focused on different stochastic elements in an optimization model to improve public transportation efficiency or conduct network design. One such study aimed to improve scheduling of public transportation using generalized stochastic Petri nets [20]. Another focused on identifying public transportation routes to deliver freight in accordance with stochastic demand in an urban environment [21].

There were multiple papers using different stochastic applications to analyze rider behavior and choices. One focused on modeling the mode choice of riders in determining how ride-sourcing services could be effectively integrated into the existing public transit network [22]. Zhang et al [22] looked at two different means of transportation from origin to destination, one using a ride-share service to get to the public transportation network, and the other only using available public transportation. The stochastic elements used in this paper were identified to be the wait time and fare for the ride share network. Another study conducted research on mode choice behaviors using game theory [23]. This study aimed to develop a method to determine when a passenger may choose to use public versus private transportation based on an identified indifference zone calculated by estimating the cost of each mode of transportation. This cost was determined by combining the parameters of traffic volume, value of time, travel time, and monetary cost. Then, the probability that a passenger changes to a different mode could be expressed [23].

More recently, studies into incorporating shared autonomous vehicle dispatching into public transportation have become more prevalent. Li et al [24] defined a car-sharing network using autonomous vehicles with nodes representing both customers and vehicles, with arcs connecting the vehicles and customers. Then they defined the waiting time for a customer at a customer node as the probability of an arrival at any given time,

$$t$$

, the interarrival time between two different customers at the node, and a binary variable representing if there is a customer at the node. Then the study continues to determine a methodology to minimize the wait time of customers to solve. Levin [25] expanded on Li et al's [24] work to incorporate public transportation integration into the autonomous ride sharing. However, while aspects of this research could be applied to the questions of this thesis, integration of the public transportation had more effect on the optimization methodology than in initial problem set up [25].

The basics for Li's study were based on a study by Volkov [26], who developed a Markov-based redistribution model for manned taxis. Again, customers would arrive in at nodes throughout different regions in accordance with a Poisson process, where a routed taxi would arrive to pick them up. Again, this was an optimization model to solve a pick up and delivery problem that focused on minimizing the customer wait time [26]. While this arrival principle can be applied to a public transportation network, customers are not waiting for a taxi at a random location, they are waiting on a bus or train at a fixed location that is not changing. Also, routing in the proposed problem is less specific. This study is not concerned with a customer being delivered to an exact location. It is instead concerned at looking at each node customers arrive to and determining network parameters associated with that node.

2.1.2 Queueing Models

Markovian chains and their association with queueing were the most similar stochastic ideas that could be applied to this research. Therefore, using this information the focus of the literature review switched to queueing networks and how they have been applied to the study of public transportation networks. Initial focus was specifically on Jackson Networks, but then broadened to include queueing theory in general.

As with the stochastic analysis literature reviewed, the majority of articles establish a Jackson Network model, and then optimize over a certain set of parameters to solve. For example, Golam Alam et al [27] studied traffic congestion in urban areas by converting the urban road networks into a Jackson Network and optimizing the waiting times to determine appropriate service and arrival rates. In this research, they define each intersection as a node in the network, and the roads with cars as the queues and the service and arrival rates represent the flow of cars through the different intersections [27]. The optimization they do aims to determine how to control the signals allowing traffic to move through the intersections, while minimizing the time spent waiting at each intersection [27]. This is useful for vehicle or car traffic analysis, but it does not help to gain a holistic understanding of the network. It also solely focuses on private vehicles which can be useful in analyzing the road network, but less so in analyzing public transportation. Analysis into public transportation can provide insight into how to further improve those networks and therefore help alleviate private vehicle congestion as public transit becomes easier.

A paper by Choi and Hanaoka [17] that aimed to examine a network holistically was focused on humanitarian efforts and easily identifying wait times of aircraft at an airport during disaster operations. In this study they applied a Jackson Network with servers representing varying aircraft purposes: medical, personnel transport, rescue, refueling, et cetera, and applied different queueing disciplines like first come first serve, priority, and mixed to determine which discipline reduced the waiting time of the aircraft on the ground [17]. Instead of running an optimization on this problem, the goal was to determine the best system numerically using easily accessible spreadsheets that would allow for more dynamic analysis as situations on the ground in emergencies changed [17]. While this paper shows good reasoning for conducting holistic analysis of networks, it uses the Jackson Networks for analyzing the ground network of an airport, not of an urban transit network.

Expanding the search more broadly to queueing and public transportation produced more results, but nothing that aimed to provide holistic analysis of the network. In one paper analysis of individual queues is presented for determination of whether a passenger will continue to wait or will choose to leave the system [28]. Another discusses transient behavior of networks, instead of analyzing the transit network using queueing theory [29]. Other papers focus on using the network analysis and queuing to optimize hub locations, like one paper which discusses the best locations for car sharing system stations using queueing theory [30]. In another paper, they focus on queue analysis at a single bus stop, but use probability of late and on-time buses to determine the number of passengers in the queue [31]. By modeling the queue as a bivariate Markov chain, they could conduct analysis on the distribution of the queue length and the time before the next arrival. The systems defined in this are solved numerically in discrete time [31]. This idea could potentially be applied as an extension to this research, but as the focus was a holistic approach to quickly reviewing and understanding the health of a network and where potential increased observation is required, many of the other ideas examined are beyond the scope of the interest of this study.

2.2 Simulation Models and Public Transportation

Simulation has been broadly used in transportation studies to provide comparison and validation of numerical methods mainly due to the complexity and scale of the networks analyzed. For this research simulation is used not only as a comparison tool to validate mathematical models, but also as a tool to create data that is otherwise not available and too cumbersome to collect. As a review of multiple simulations stated, "it is necessary to rely on tools such as simulation that allow the impact of changes observation without altering the real system" [32].

In one paper forecasting passenger travel behavior, simulation is used to develop a "synthetic population" that allowed for initial estimates of passenger attributes [33]. They then used these attribute distributions in their mathematical model to determine the routes a passenger with a specific selection of attributes would travel. The initial simulation was derived from population data and the resulting simulated population was compared to the original data to validate the results [33]. This is similar to the simulation used in methodology of the thesis, as simulation is used to develop the data needed to solve the mathematical model due to a lack of available data.

Solecka and Zak [34] used simulation in a more traditional sense, using it to design alternative solutions by eliminating and changing components of the system. Then using their results from the simulation, they could compare it to results from the current transportation network. Using the simulation software, they compared travel times, waiting times, number of passengers, and the number of transfers. They also identified the ridership of up to five different modes of transportation [34]. The simulation software used in this thesis has a similar functionality, but as the purpose is to conduct network analysis quickly, running simulations for anything outside of the basic data was beyond the scope of this study.

Another study used simulation data envelopment analysis to evaluate performance of specific bus routes [35]. Using four quantifiable indexes calculated by the authors and a virtual input, a value for operational efficiency was determined, and used to compare the routes during peak hours and off-peak hours [35]. The operational efficiency was calculated using MATLAB and a series of linear programming problems assigning weights to different indexes, maximizing efficiency of each route [35]. While a useful way to analyze and compare individual routes easily, this method failed to examine the network as a whole.

2.3 Gaps in Literature

In reviewing applicable literature, no papers were found integrating simulation with Jackson networks to conduct overall network analysis of public transportation networks. Many methods aimed to optimize specific items in a single mode transportation network, but none analyzed a multi-mode network using Jackson networks. Also, simulation was rarely used to develop data for use in a mathematical model. More importantly, queuing analysis and simulation have not been used to develop an overview of the functioning of a single mode or multi-mode public transportation network. In combining the use of simulation to develop data and Jackson networks to conduct a holistic analysis of multi-mode public transportation networks, this research provides a method to conduct network diagnostics and allow network managers the ability to make decisions quickly.

Chapter 3

Stochastic Model for Public Transportation Network

3.1 Mathematical Model: Multiple Modes of Transportation

A public transportation network can be constructed as a network using a fully connected graph $G = (N, A)$, with a finite set of nodes $N = \{1, 2, 3, \dots, m\}$ and bidirectional arcs $A = \{(i, j), (k, l), \dots, (s, t)\}$ joining nodes in N [36]. In this case, each station represents a node, and the arcs represent the routes connecting one station to another. The arcs are bidirectional as someone could move from station $i, i \epsilon N$ to station $j, j \epsilon N$, but also move from station j back to station i. See Figure 3.1.

A person can be modeled traveling through the network, from node i to node j, using a valid path, P, where P is a sequence of arcs such that the initial node of each arc is the same as the terminal node of the preceding arc and each node is distinct [36]. In the example in Figure 3.1, there are multiple valid paths from node 1 to node 3: $P = \{(1, 3)\}$, $P = \{(1, 2), (2, 3)\}$, and $P = \{(1, 2), (2, 4), (4, 3)\}$ are three examples of valid paths.

Note that when considering multiple modes of transportation (bus and subway systems),

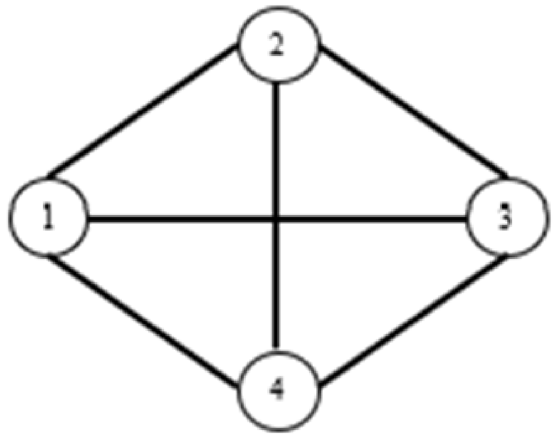
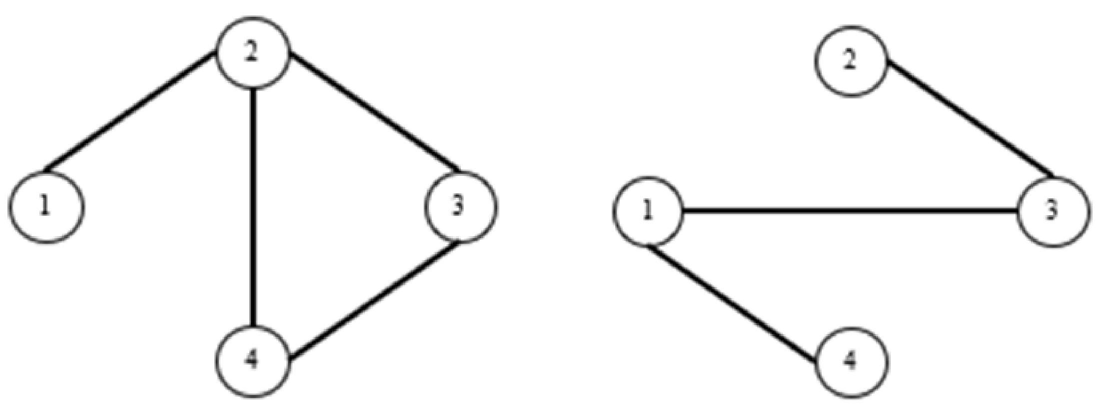


Figure 3.1: Bidirected graph $G = (N, A)$, where $N = \{1, 2, 3, 4\}$ and $A = \{(1, 2), (1, 3), (1, 4), (2, 3)(2, 4), (3, 4)\}$.

separating the full graph into two sub-graphs is necessary, G_b,G_s, where each subgraph $G_l = (N_l, A_l), l = b, s$ may not be fully connected as some paths may only be valid in the bus or subway network. Figure 3.2 presents an example of the two proposed subgraphs. Note that the associated arcs within each sub-graph are different, showing the different routes the transit modes follow, resulting in a complex model.



(a) Subgraph $G_b = (N_b, A_b)$ (b) Subgraph $G_s = (N_s, A_s)$

Figure 3.2: Subgraphs from original graph

Additionally, in a public transportation network, the goal is to move people from a starting point to an end point, but the stochastic nature due to uncertainty in travel times and random choice of transportation modes adds complexity to the model. A stochastic model is needed to fully capture the dynamics of the system. In literature, such models are explained using a Jackson network.

3.2 Jackson Network Model

To calculate parameters of the network, queueing theory must be considered in modeling the problem. As each station has a flow of people external to the network entering the system, the proposed system was modeled with two sub graphs using a Jackson network, following M/G/1 queuing.

The previous subgraphs, G_b and G_s, are defined as networks corresponding to the bus and subway networks respectively. It is assumed that each node $i_b, i_b \epsilon N_b$ and $i_s, i_s \epsilon N_s$, where i_b,i_s represents the bus and subway stations at node $i, i \epsilon N$ in the original graph. The external arrival rate of passengers into the system at node i is annotated to be γ_i, where $\gamma_i = 1/t_i$ and t_i is the mean interarrival time of passengers to station i. Therefore, the arrival rate of passengers into node $i_l, l = b, s$, is represented by $\gamma_{i_l} = \gamma_i r_{i_l}, l = b, s$ and r_{i_l} represents the probability of a passenger selecting transportation mode l on entering node i, $l = b, s$. Then, at any node i_b or i_s the server processes each passenger at a service rate of μ_{i_b} and μ_{i_s} following a general distribution. After being serviced at node i_l passengers proceed to the next station j_l with probability p_{i_l,j_l} [8]. At each station, passengers can leave the network as well, with probability $q_{i_l} = 1 - \sum_{j_l \epsilon N_l} p_{i_l,j_l}$. A depiction of this is shown in Figure 3.3.

Using this general framework, the average arrival rate to each node i_l, λ_{i_l}, is the mean exponential external arrival rate, γ_{i_l}, plus the sum of the external arrival rates to node i_l from nodes within the system as shown in Equation 3.1 [13].

$$\lambda_{i_l} = \gamma_{i_l} + \sum_{j_l \epsilon N_l} \lambda_{j_l} p_{j_l,i_l} \quad , \quad i_l \epsilon N_l \tag{3.1}$$

By letting $\boldsymbol{\lambda_l} = [\lambda_{1_l}, \lambda_{2_l}, \ldots, \lambda_{m_l}]$, $\boldsymbol{\gamma_l} = [\gamma_{1_l}, \gamma_{2_l}, \ldots, \gamma_{m_l}]$, where m_l is the total number of nodes in N_l, and $\boldsymbol{P_l}$ is the $m_l \times m_l$ routing matrix $[p_{i_l,j_l}]$, Equation 3.1 is rewritten to be Equation 3.2 [13].

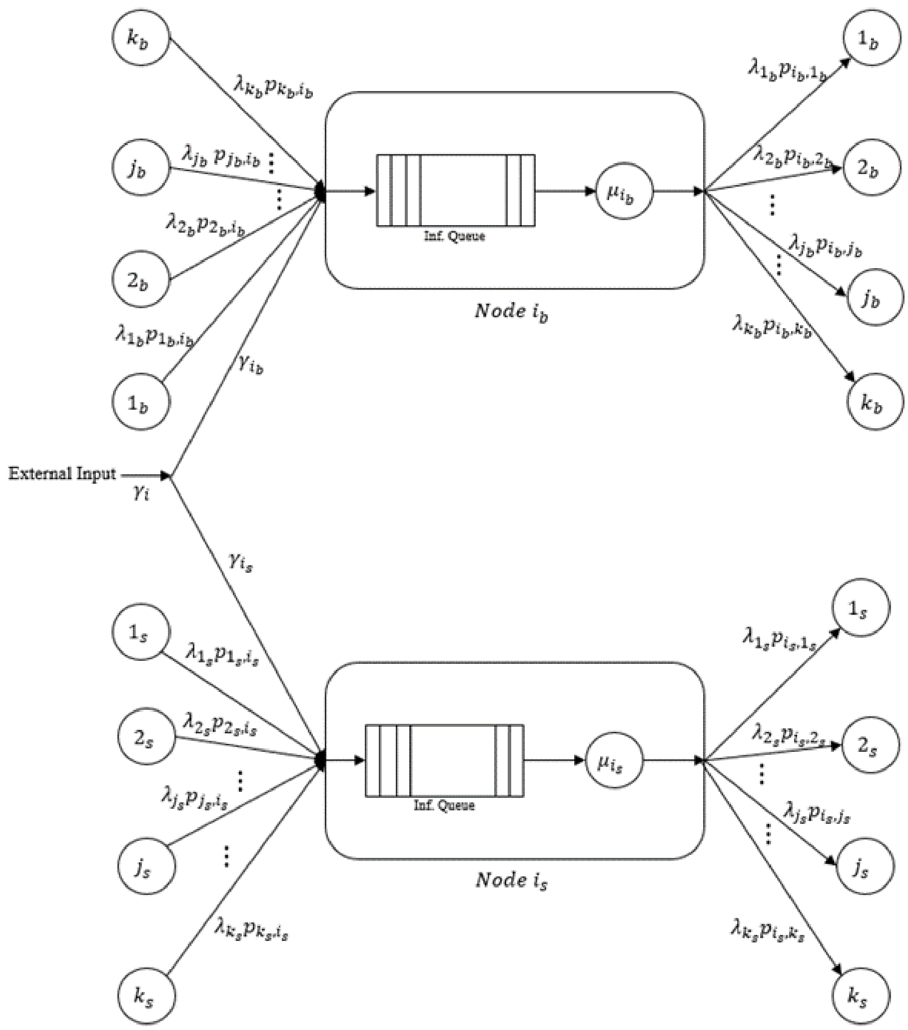


Figure 3.3: Nodal breakdown of proposed Jackson Network.

$$\boldsymbol{\lambda_l} = \boldsymbol{\gamma_l} + \boldsymbol{\lambda_l}\boldsymbol{P_l} \tag{3.2}$$

Solving for $\boldsymbol{\lambda_l}$ results in Equation 3.3 [13]:

$$\boldsymbol{\lambda_l} = \boldsymbol{\gamma_l}[\boldsymbol{I} - \boldsymbol{P_l}]^{-1} \tag{3.3}$$

Now with the average arrival rate performance measures of station utilization, average queue length, average number in the system, and average time in the system can be defined. First, these parameters are defined per station or node i_l. At station i_l the expected service rate distribution and the average arrival rate, λ_{i_l} are known. Using the service rate distribution, the mean service rate, μ_{i_l}, the standard deviation, σ_{i_l}, and the coefficient of variation, c_{v,i_l}, can be determined for each station.

First the utilization of station i_l is defined as Equation 3.4 [37]:

$$\rho_{i_l} = \frac{\lambda_{i_l}}{\mu_{i_l}} \tag{3.4}$$

Next, the average time spent in the queue at station i_l, w_{q,i_l} (Equation 3.5 [37]) and the average time spent at station i_l, w_{i_l} (Equation 3.6 [37]) is calculated.

$$w_{q,i_l} = \frac{1 + {c_{v,i_l}}^2}{2} \cdot \frac{\rho_{i_l}}{1 - \rho_{i_l}} \cdot \frac{1}{\mu_{i_l}} \tag{3.5}$$

$$w_{i_l} = \frac{1}{\mu_{i_l}} + w_{q,i_l} \tag{3.6}$$

Then, the number of waiting passengers, L_{q,i_l} (Equation 3.7), and the number of passengers at station i_l, L_{i_l} (Equation 3.8) are calculated using Little's Law [37].

$$L_{q,i_l} = \lambda_{i_l} w_{q,i_l} \tag{3.7}$$

$$L_{i_l} = \lambda_{i_l} w_{i_l} \tag{3.8}$$

To get the expected time people spend waiting in line for each mode of transportation, $W_{q,l}$, the time spent at each station is summed across all nodes. See Equation 3.9 [6].

$$W_{q_l} = \sum_{i_l \epsilon N_l} w_{q,i_l} \tag{3.9}$$

The expected number of passengers in the queue for each mode of transportation, Q_l, is also calculated by summing the expected number in the queue, L_{q,i_l} (Equation 3.10 [6])

$$Q_l = \sum_{i_l \epsilon N_l} L_{q,i_l} \tag{3.10}$$

Finally, the expected number of passengers in the system waiting for and boarding each mode of transportation, T_l, is calculated by summing L_{i_l} for all nodes (Equation 3.11 [6]).

$$T_l = \sum_{i_l \epsilon N_l} L_{i_l} \tag{3.11}$$

The key to being able to evaluate system performance using the numerical method is to determine the external arrival rate to each node. The external arrival rate is the rate at which passengers not currently in the bus or subway network enter the system. Data collection to determine the external arrival rates is a challenging task and is not found in available data sets. Therefore, a simulation model is developed to determine arrival rates.

3.3 SIMIO Modelling

The purpose of constructing a simulation model for this problem is to determine the maximum feasible flow of passengers into the network due to a lack of data allowing direct calculation and data analysis. As the primary purpose for research is holistic network analysis, optimization software included in the simulation program to optimize flow was used. The utilization of each station is maximized by iterating over the mean interarrival time of passengers to each station. The rest of this section describes how the simulation model was built.

Each station has an external flow of passengers into the system of people not already in and flowing through the network. As discussed in Section 3.2, to conduct analysis on all stations for all modes of transportation, it is necessary to determine the effective arrival rate λ_{i_l} to each station or node i_l. To do this, the overall external arrival rate γ_i to each node i must be determined first. Therefore, the network is modeled and the mean interarrival time, t_i, is optimized.

First, the stations or nodes of the network are modeled as servers, separating out the sub-graph nodes into different servers. For example, if node 1 exists in both N_b and N_s, then two nodes 1_b and 1_s are modeled as two servers, represented as BusStation1 and SubwayStation1. This allows for an easier method to manage the different processing times of the modes. Servers were connected by unidirectional time paths, specifying the average time between each station in accordance with the associated sub-graphs. Unidirectional paths were used to ensure the correct travel between servers was maintained, and to ensure proper flow of units in the system. Maintaining the example from Figure 3.2, the SIMIO model for the two different sub-graphs is shown in Figure 3.4. Note that this only includes the servers and paths or arcs between them. For this research, it is assumed that once someone selects a mode of transportation, they remain in it and do not switch out.

Each server was connected to a sink, allowing for the representation of a passenger leaving the system from any node or station in the network. Time was not taken into account for theses connections as once a passenger chose to leave the network, the time it took for them to leave was inconsequential. To determine routing within this model, output nodes on the servers were set to be routed “By Link Weight”. The link weights associated between the nodes were the transition matrix $\boldsymbol{P_l}$ defined in Section 3.2, and the link weights to the sinks from server i_l are q_{i_l}. Figure 3.5 shows the model with sinks connected and path weights defined.

Then, the external sources were connected into the network. Each source was connected to its associated servers. For example, in a two-mode network as proposed, “Source1” would be connected to “BusStation1” and “SubwayStation1” as it provides the external flow for both modes of transportation. If there were three modes of transportation at any given

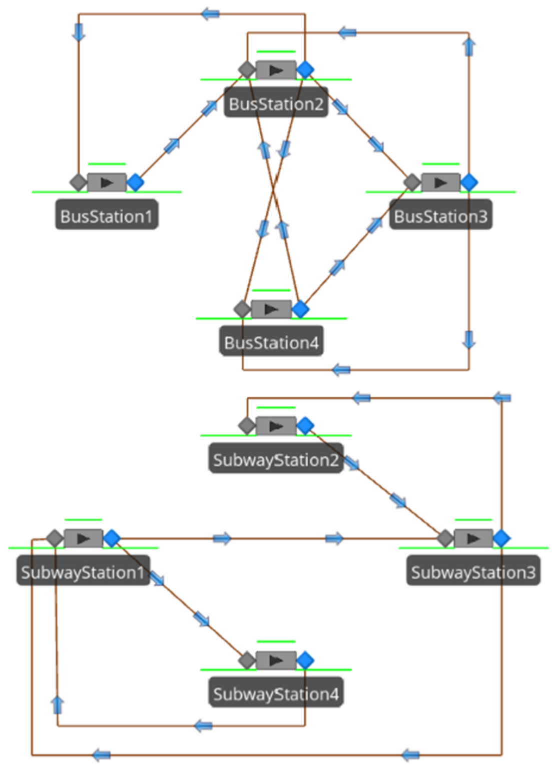


Figure 3.4: SIMIO model of servers and time paths showing the sub-graphs as specified in Fig. 3.2

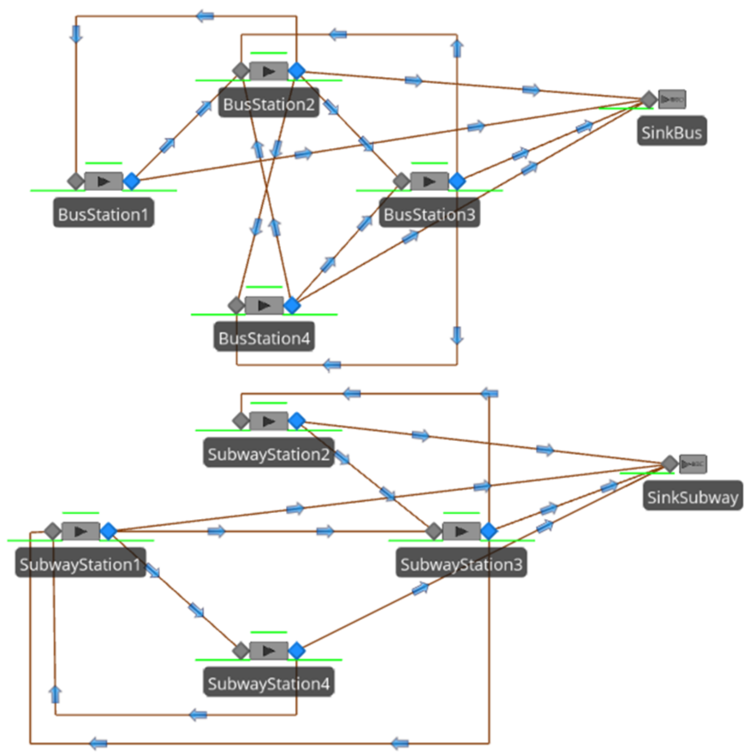


Figure 3.5: Partial SIMIO model of servers and sinks example.

station, the source would be connected to three different servers. Figure 3.6 shows the full model built in SIMIO for the example network. The sources all had exponentially distributed interarrival times, t_i, that were modeled as a state property, allowing the internal SIMIO software to optimize t_i for each source.

Once the base model was built, optimization was conducted using the OptQuest for SIMIO add-in. The control, or independent variables were the different interarrival times, t_i, defined earlier as state properties in SIMIO. Response or dependent variables were set corresponding to each node i_l to be the simulated utilization of each station. A multi-objected weighted optimization model, with the subway stations weighted twice as much as the bus stations was used to prioritize maximizing utilization of the subway network over the bus network. An upper bound, ub_l, was established for each response variable corresponding to the maximum utilization of each station, or the percentage utilization that would ensure infinite queues would not occur. The simulation run time was set to be a large value, M, allowing the simulation to reach steady state and to relate to the steady state calculations

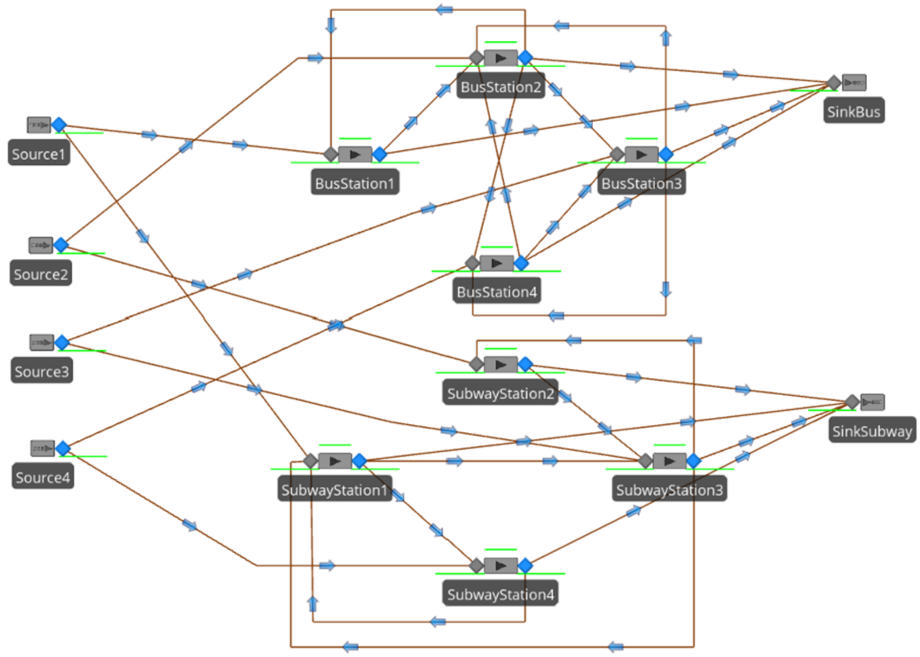


Figure 3.6: Full SIMIO model example.

of queueing theory. A warm-up time of $M/20$ was established to ensure the system reached steady state before data was collected. The resulting optimal interarrival times were used to calculate the external arrival rates.

Chapter 4

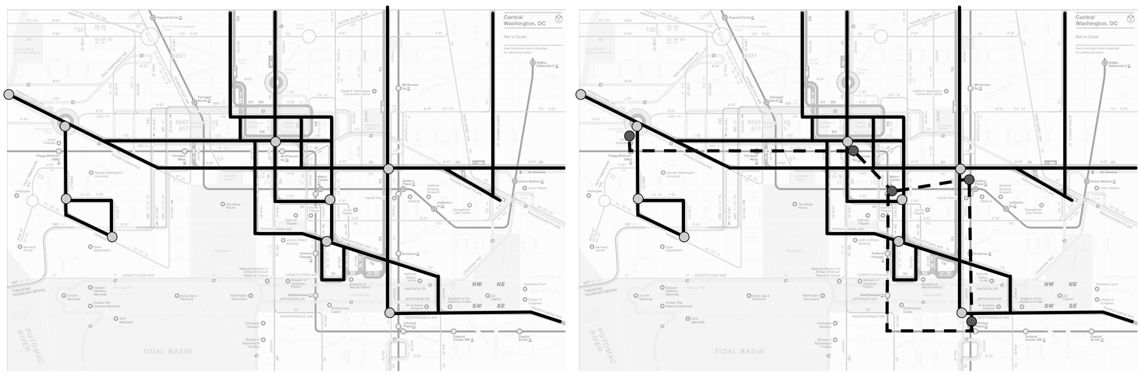
Numerical Studies

4.1 Case Study for Bus and Subway Network

To demonstrate the effectiveness of this model, this research considered a simplified network of the bus and subway systems of Washington D.C. and uses realistic ridership data to calculate the performance measures using the proposed model.

The Washington D.C. public transportation network is vast and sprawling, with subway and bus routes stretching miles outside the city to accommodate commuter traffic, as well as containing many smaller local routes running across the city. To narrow the scope of the research and reduce complexity, the focus was restricted to the commuter bus routes, shown as thick black lines in Figure 4.1a [38]. Commuter routes were selected as it is expected that those routes would have the most passengers on any given day. The analysis only considers stations where multiple bus routes converge and a passenger could change between bus routes. The stations selected are shown as light gray circles in Figure 4.1a [38].

Then the subway network was incorporated. Within the area considered for the bus network, there were multiple subway stops. Like the bus network, to limit complexity, subway stations that connected to the identified bus network and stations where a passenger could change routes were the only ones considered. The resulting subway network is laid over the bus network in Figure 4.1b [38], with dashed lines representing the subway lines and darker gray circles corresponding to the subway stations. Note that the map is not to scale and the subway stations are generally underneath the bus stations.



(a) Bus network used for the case study. (b) Subway network overlaid on bus network.

Figure 4.1: Bus and subway networks used as a part of the case study. The bus network is shown in thick black lines, with selected stations shown as light gray circles. The subway network is shown as dashed black lines, with selected stations shown as dark gray circles.

The resulting network is comprised of nine bus stations and five subway stations that moved passengers throughout the busiest areas of Washington D.C. Figure 4.2 shows the simplified network not on a map. The bus routes are shown as solid lines and the subway routes are shown as dashed lines. For simplicity, the subway and bus stations that overlap are combined in this drawing.

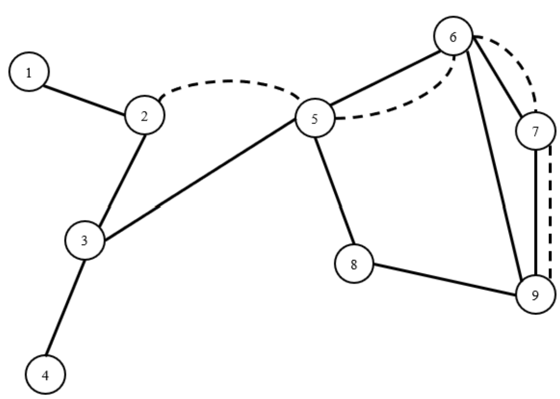


Figure 4.2: Full case study network including both bus, shown as solid black lines, and subway routes, black dashed lines.

Using this basic network, the following undirected graph $G = (N, A)$, where

$$N = \{1, 2, \ldots, 9, L\}, A = \left\{ \begin{array}{l} (1,2), (1,L), (2,3), (2,5), (2,L), (3,4), (3,5), \\ (3,L), (4,L), (5,6), (5,8), (5,L), (6,7), (6,9), \\ (6,L), (7,9), (7,L), (8,9), (8,L), (9,L) \end{array} \right\}$$

is shown in Figure 4.3. In this graph, arcs to node L represent passengers choosing to leave the network. Subgraphs, G_b and G_s are shown in Figures 4.4a and 4.4b, respectively. The nodes represent the stations and the undirected arcs are the routes between those stations.

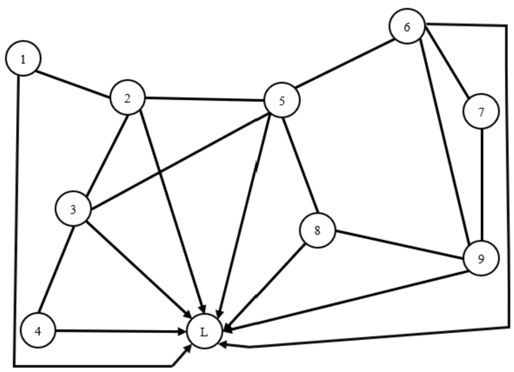
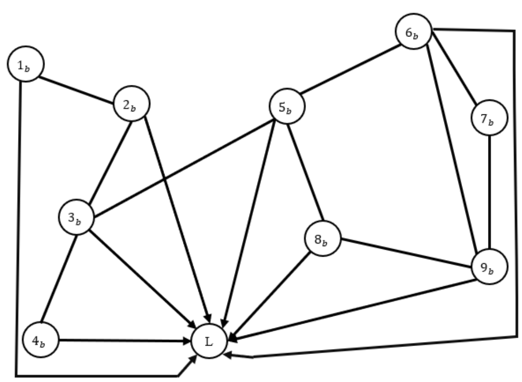
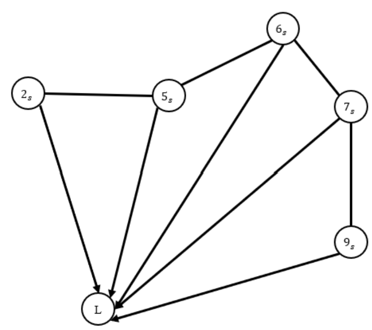


Figure 4.3: Full graph of proposed network, $G = (N, A)$.



(a) Subgraph, $G_b = (N_b, A_b)$.



(b) Subgraph, $G_s = (N_s, A_s)$.

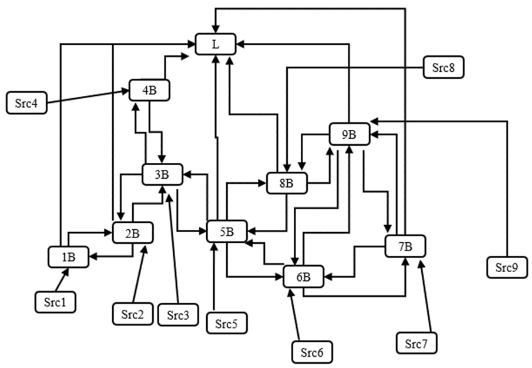
Figure 4.4: Subgraphs of proposed networks: $G_b = (N_b, A_b)$ and $G_s = (N_s, A_s)$.

4.2 Experiment 1: Bus Only Network

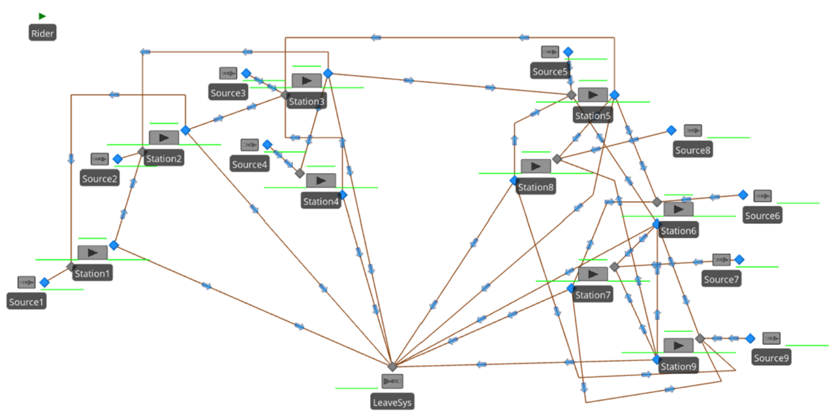
The simulation model for the bus only network was set up as described in Section 3.3, with the travel time as 15 minutes, the average time between the different stations as per the Washington D.C. transit authority website [19]. See Figure 4.5a for simplified SIMIO bus model construction. Each server was modeled to have a service time corresponding to the time it takes for a person to load a bus. Stephen Arhin et al [39] presented time data for the length of time it takes to board a bus, that is modified in this research to also accommodate for the capacity of a bus and the time between bus arrivals. In this way, changes in bus capacity or in arrival times of buses can be accounted for within the service time of each station. The determined service time follows a triangular distribution with the minimum time of 0.33 minutes, the maximum time of 0.54 minutes, and the most likely time of 0.43 minutes. The proposed SIMIO model is shown in Figure 4.5b.

Ridership data and educated judgements were used to assign probabilities to the decisions a passenger would make at each station. The ridership data used was a filtered subset from the Washington Metropolitan Area Transit Authority (WMATA) ridership data [19], filtered by bus route from January 2015 to October 2022, which provided the total number of passengers per day on a specific route. Bus routes used in this analysis were: 31, 38B, 32, 42, 52, S2, X2, 80, and 70.

To determine the probabilities of a passenger moving throughout the network, the passengers flowing along a specific route was considered. For example, say a passenger is currently at Station 2 in the proposed network. They can either leave the network or move to a different station. The linkages between Station 2 and Stations 1 and 3 may be composed of different bus routes. Therefore, the total number of passengers traveling along routes into and out of Station 2 are summed together, then the specific link is analyzed. So, if there were a total of 1000 passengers traveling into and out of routes that Station 2 services, and only 100 passengers travel the routes between Station 1 and Station 2, the probability of a passenger moving from Station 2 to Station 1 is 10%. Applying this method to the entire network the probability matrix $\boldsymbol{P_b}$ and $\boldsymbol{q_b}$ were defined.



(a) Simplified simulation model.



(b) Full simulation model.

Figure 4.5: Simplified and full simulation models of bus only network.

$$
\boldsymbol{P_b} = \begin{bmatrix} 0 & 0.50 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.50 & 0 & 0.23 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.20 & 0.48 & 0.27 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.59 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.08 & 0 & 0 & 0.36 & 0 & 0.20 & 0 \\ 0 & 0 & 0 & 0 & 0.40 & 0 & 0.42 & 0 & 0.16 \\ 0 & 0 & 0 & 0 & 0 & 0.42 & 0 & 0 & 0.29 \\ 0 & 0 & 0 & 0 & 0.50 & 0 & 0 & 0 & 0.29 \\ 0 & 0 & 0 & 0 & 0 & 0.16 & 0.29 & 0.13 & 0 \end{bmatrix}, \boldsymbol{q_b} = \begin{bmatrix} 0.50 \\ 0.27 \\ 0.05 \\ 0.41 \\ 0.36 \\ 0.02 \\ 0.29 \\ 0.21 \\ 0.42 \end{bmatrix}
$$

An initial run was conducted for this initial study and determined that the most utilized stations were 3, 5, and 7. Therefore, as optimization of the interarrival times was conducted, we set the weights for the SIMIO optimization to prioritize the overloaded bus stations to be twice that of the other stations. The upper bound for all response variables were set to 80%. The optimal interarrival times found through SIMIO experimentation are found in Table 4.1.

Table 4.1: Optimized interrival times for bus only network in minutes and associated utilization with 95% confidence.

Station	1_b	2_b	3_b	4_b	5_b
t_{i_b}	1.0	2.8	2.4	2.5	3.7
ρ_{i_b} (%)	77.6 ± 0.6	68.4 ± 0.6	70.6 ± 0.3	51.1 ± 0.4	79.9 ± 1.0

Station	6_b	7_b	8_b	9_b
t_{i_b}	4.6	2.6	4.5	1.5
ρ_{i_b} (%)	79.7 ± 0.8	71.2 ± 0.3	35.1 ± 0.5	72.6 ± 0.4

Then using Equation 3.3, the effective arrival rate, λ_{i_b} , could be calculated. The effective arrival rates for the initial case study for one mode of transportation is in Table 4.2. Overall, based on this effective arrival rate, 236 passengers per hour enter the system on average.

Then, the performance measures were calculated. On average, in the steady state of the system, the utilization of a station is 67.3%, a passenger spends 8.85 minutes waiting for

Table 4.2: Effective arrival rate, λ_{i_b}, in people per minute.

Station	1_b	2_b	3_b	4_b	5_b	6_b	7_b	8_b	9_b
λ_{i_b}	1.00	0.36	0.42	0.40	0.27	0.22	0.38	0.22	0.67

and boarding the bus, there are a total of 8 passengers waiting for a bus, and a total of 14 passengers waiting for and actively boarding the bus at any given time. Table 4.3 shows the totaled results.

Table 4.3: Expected wait time, expected number in queue, and expected number of passengers waiting and boarding in the bus network.

W_{q_b}	8.85 min
Q_b	8
T_b	14

Note, the utilization calculated using queueing theory and the simulated values have an average difference of 0.17%, therefore validating the mathematical model. Also, the utilizations of Stations 5 and 6 are at the upper bound. This means that Stations 5 and 6 are not able to tolerate an increase in flow, and that more buses may be required to drop the service times of those stations. As expected, the other parameters associated with both Station 5 and 6 are also higher than those of the other stations. In fact, the time at Stations 5 and 6 accounts for 30% of the total time spent in a queue or boarding. Table 4.4 shows the full results for each station.

Table 4.4: Performance measures for bus only network by station.

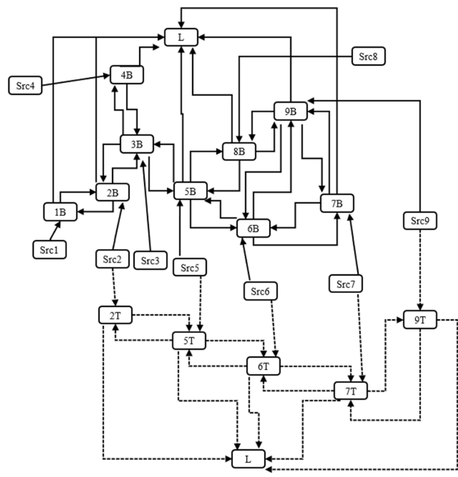
Station	1_b	2_b	3_b	4_b	5_b	6_b	7_b	8_b	9_b
ρ_{i_b}	0.77	0.68	0.70	0.51	0.80	0.80	0.71	0.35	0.72
w_{q,i_b} (min)	0.75	0.47	0.52	0.23	0.88	0.86	0.54	0.12	0.58
w_{i_b} (min)	1.19	0.90	0.95	0.66	1.32	1.30	0.97	0.55	1.01
L_{q,i_b}	1.34	0.74	0.84	0.27	1.63	1.59	0.89	0.10	0.96
L_{i_b}	2.12	1.42	1.54	0.78	2.43	2.38	1.60	0.45	1.69

4.3 Experiment 2: Bus and Train Network

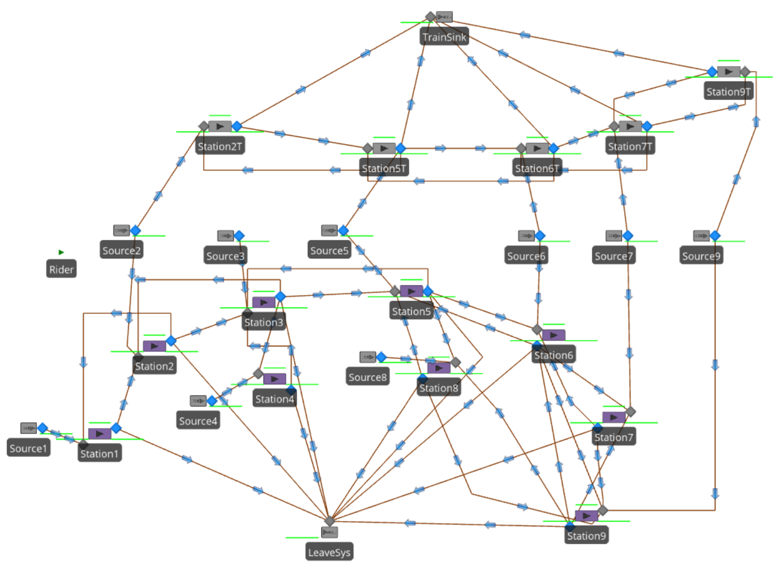
Once a single mode of transportation was validated, the two-mode model for the bus and subway networks was built. All aspects of the bus only network remained the same in this model. As stated previously, it was assumed that passengers did not switch their mode of transportation once they entered it. The simplified SIMIO model is shown in Figure 4.6a. The dashed lines in the figure represent the flow through the subway network and solid lines represent flow through the bus network. In this model, Sources 2, 5, 6, 7, and 9 supply passengers to the bus network and the subway network because at these stations both a bus stop and subway station exist. Therefore, a passenger can choose which mode of transportation to take. The full SIMIO model is shown in Figure 4.6b.

The processing time at the subway stations was calculated using ridership data from the WMATA ridership data [19], filtered by station from January 2015 to October 2022. The stations used were: Foggy Bottom, McPherson Square, L'Enfant Plaza, Metro Center, and Gallery Place. The ridership data gave the number of passengers entering a station during rush hour and a daily total number of passenger entered. Using the number of passengers entering and the capacity of the train, the time to board was determined by averaging the time in between trains by the number of passengers that would need to board it. By calculating this value for the two rush hours and daily total, the minimum, maximum, and most likely time to board the train was determined.

Just like the bus network, ridership data was used to determine the likelihood a passenger moved from one station to another resulting in probability matrix, $\boldsymbol{P_s}$. In this model, $\boldsymbol{P_b}$, remained the same as given in Section 4.2. To determine the probability that a passenger would take either the bus or subway, the ratio of total riders across the stations examined was used. Therefore, from shared sources in the simulation, 71% of passengers were directed into the subway network, and 29% were directed into the bus network.



(a) Simplified simulation model.



(b) Full simulation model.

Figure 4.6: Simplified and full simulation models of bus and train networks.

$$\boldsymbol{P_s} = \begin{bmatrix} 0 & 0.50 & 0 & 0 & 0 \\ 0.33 & 0 & 0.33 & 0 & 0 \\ 0 & 0.33 & 0 & 0.33 & 0 \\ 0 & 0 & 0.33 & 0 & 0.33 \\ 0 & 0 & 0 & 0.5 & 0 \end{bmatrix}, \boldsymbol{q_s} = \begin{bmatrix} 0.50 \\ 0.34 \\ 0.34 \\ 0.34 \\ 0.50 \end{bmatrix}$$

Then the same procedure was followed as described in Section 3.3 to determine feasible interarrival rates. The results from the SIMIO optimization are found in Table 4.5.

Table 4.5: Optimized interrival times for bus and subway network in minutes and associated utilization with 95% confidence.

Station	1	2	3	4	5
t_{i_b}	1.5	0.4	2.5	3.6	1.7
ρ_{i_b} (%)	68.9 ± 0.4	79.7 ± 0.6	68.5 ± 0.5	44.8 ± 0.5	79.8 ± 0.6
ρ_{i_s} (%)	-	25.6 ± 0.1	-	-	23.7 ± 0.2

Station	6	7	8	9
t_{i_b}	1.3	1.2	1.7	0.7
ρ_{i_b} (%)	72.9 ± 0.5	58.9 ± 0.4	49.4 ± 0.4	60.9 ± 0.5
ρ_{i_s} (%)	20.3 ± 0.2	21.1 ± 0.2	-	17.1 ± 0.2

Solving for the effective arrival rates using Equation 3.3, the effective arrival rates, λ_{i_b} and λ_{i_s}, were calculated. The effective arrival rates for the bus and subway network is in Table 4.6. This results in an average of 438 passengers entering the system every hour, almost doubling that of the bus only network.

Table 4.6: Effective arrival rate, λ_{i_b} and λ_{i_s}, in people per minute.

Station	1	2	3	4	5	6	7	8	9
λ_{i_b}	1.58	1.83	1.58	1.04	1.84	1.68	1.35	1.14	1.41
λ_{i_s}	-	2.55	-	-	2.36	2.02	2.12	-	1.71

The effective arrival rates allow us to solve the system parameters we are interested in. On average, in the steady state of the system, the utilization of a bus station is 64.7%, while the average utilization of a subway station was 21.5%. Passengers should expect to spend 8.18 minutes waiting for and boarding the bus, but only 0.58 minutes waiting for and boarding the subway. Also, there are a total of 7 passengers waiting for the bus as opposed to only one passenger waiting on a subway at any given time. Overall performance measures of both networks are shown in Table 4.7.

Table 4.7: Expected wait time, expected number in queue, and expected number of passengers waiting and boarding in the bus and subway network.

W_{q_b}	8.18 min	W_{q_s}	0.58 min
Q_b	7	Q_s	1
T_b	13	T_s	2

Adding in the subway network, the average utilization of bus stations does not significantly change. In this case, this is because the interarrival times were decreased, resulting in more passengers overall entering the network. Note that the average difference between utilization calculated using the mathematical model and the simulation results was less than half a percent. Table 4.8 shows the results for each bus station.

Table 4.8: Performance measures for bus stations in the two-mode network model.

Station	1_b	2_b	3_b	4_b	5_b	6_b	7_b	8_b	9_b
ρ_{i_b}	0.69	0.79	0.68	0.45	0.80	0.73	0.59	0.49	0.61
w_{q,i_b} (min)	0.48	0.84	0.47	0.18	0.86	0.58	0.31	0.21	0.34
w_{i_b} (min)	0.91	1.28	0.91	0.61	1.29	1.02	0.74	0.65	0.77
L_{q,i_b}	0.76	1.55	0.75	0.18	1.57	0.98	0.42	0.24	0.48
L_{i_b}	1.44	2.34	1.43	0.63	2.37	1.71	1.01	0.74	1.09

From the utilization calculated from each subway station, the subway can facilitate the flow of more passengers, making it less vulnerable to disruptions in the network. As with the bus network, the calculated utilization is less than half a percent different than the simulated values, validating the mathematical model. The full results for each train station are in Table 4.9.

In validating this model against the simulation there is only an average difference of 0.20% between the simulated utilizations found in SIMIO and the calculated utilizations using the mathematical model. Therefore, the mathematical model is valid and can be used

Table 4.9: Performance measures for subway stations in the two-mode network model.

Station	2_s	5_s	6_s	7_s	9_s
ρ_{i_s}	0.26	0.24	0.20	0.21	0.17
w_{q,i_s} (min)	0.02	0.02	0.01	0.01	0.01
w_{i_s} (min)	0.12	0.12	0.11	0.11	0.11
L_{q,i_s}	0.05	0.04	0.03	0.03	0.02
L_{i_s}	0.30	0.28	0.23	0.24	0.19

to analyze the network. Also of note, is that by adding in the subway network twice as many people can enter the network within an hour, however it does little to decrease the wait time or number of people in the queues for buses.

4.4 Experiment 3: Increasing Percentage in Train Network

After running two experiments to determine the system parameters, what changes would occur if the ridership changed, and more riders elected to ride the subway? Therefore, the model was so that 85% of passengers entering the network would choose to ride the subway, $r_{i_s} = 0.85$ and $r_{i_b} = 0.15$. All other parameters remained the same.

Keeping the same interarrival times determined in Section 4.3 to give accurate comparison, then if more people elect to take the subway, unsurprisingly, the subway station utilization increases, and the bus station utilization decreases. On average, the utilization of bus stations decreased by 22.8% and the utilization of subways increased by 19.7%. See Table 4.10 for overall system parameters.

Table 4.10: Expected wait time, expected number in queue, and expected number of passengers waiting and boarding in the bus and subway network if the ridership percentage of the subway is increased.

W_{q_b}	5.95 min	W_{q_s}	0.60 min
Q_b	2	Q_s	1
T_b	7	T_s	2

With less utilization in the bus network, the total time waiting and boarding decreased by 27.3% and the total number of passengers waiting in queues dropped 64.2%. However, the decrease in bus stations not connected to subway stations saw less of a change. On average, utilization of stations not connected to subway stations decreased by only 13%, while the utilization of stations connected to subway stations dropped 30%. The remaining performance measures showed similar trends as shown in Table 4.11. Therefore, as ridership on subways increases, more attention should be paid to the standalone bus stations.

Table 4.11: Performance measures for bus stations if subway ridership increases.

Station	1_b	2_b	3_b	4_b	5_b	6_b	7_b	8_b	9_b
ρ_{i_b}	0.57	0.57	0.59	0.40	0.61	0.49	0.38	0.43	0.41
w_{q,i_b} (min)	0.29	0.29	0.32	0.15	0.34	0.21	0.13	0.16	0.15
w_{i_b} (min)	0.73	0.72	0.75	0.58	0.78	0.65	0.57	0.60	0.58
L_{q,i_b}	0.39	0.37	0.43	0.14	0.48	0.24	0.12	0.16	0.14
L_{i_b}	0.96	0.94	1.02	0.54	1.09	0.74	0.50	0.59	0.55

While significant differences occurred in the bus network by dropping the percent of the passengers entering the network, the changes in the subway network were more subtle. The total time spend waiting and boarding only increased by an average 3.6% and, though the total number of passengers waiting in the queue doubled, the number of passengers rounded to the nearest whole person was still only one. The full results for the subway network are shown in Table 4.12. In this way, the subway network remains less vulnerable to network disruption and an increase in ridership by 14% does not significantly affect it.

Table 4.12: Performance measures for subway stations if subway ridership is increased.

Station	2_s	5_s	6_s	7_s	9_s
ρ_{i_s}	0.31	0.28	0.24	0.25	0.21
w_{q,i_s} (min)	0.02	0.02	0.02	0.02	0.01
w_{i_s} (min)	0.12	0.12	0.12	0.12	0.11
L_{q,i_s}	0.07	0.06	0.04	0.05	0.03
L_{i_s}	0.38	0.34	0.29	0.30	0.23

4.5 Experiment 4: Impact of Disruptions in Proposed Network

The third experiment examined the effect of service disruptions within the network using the base interarrival times and bus versus subway routing decisions used in Section 4.2. The three disruptions examined were in a train station, a bus station, and in both the train and bus stations. Station 7 was selected for disruption as it was one with the highest utilization, but not the highest, which would theoretically create more changes in the networks.

4.5.1 Disruptions in Train Stations

In this first scenario, subway Station 7 was shut down. Passengers were allowed to enter the bus station, but not the subway station. Also, the subway was allowed to go through the station, but no passengers were able to leave the network from Station 7. To analyze this scenario, parameter changes were required. Probability matrix, $\boldsymbol{P_s}$, was changed as shown below to remove the option to leave the network from the subway station. Utilization and mean service time were forced to 0. Also, all passengers arriving to Station 7 entered the bus network. The probability matrix, $\boldsymbol{P_b}$, remained the same as in previous experiments.

$$\boldsymbol{P_s} = \begin{bmatrix} 0 & 0.50 & 0 & 0 & 0 \\ 0.33 & 0 & 0.33 & 0 & 0 \\ 0 & 0.33 & 0 & 0.33 & 0 \\ 0 & 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0 & 0.5 & 0 \end{bmatrix}, \boldsymbol{q_s} = \begin{bmatrix} 0.50 \\ 0.34 \\ 0.34 \\ 0 \\ 0.50 \end{bmatrix}$$

If it is assumed all passengers who would normally take the subway from Station 7 take the bus instead, then bus stations 5, 6, and 7 reach maximum utilization levels. All three stations have utilization over 90%, with station 7 utilization reaching 99.5%. Due to the high utilization, at any given time, a passenger waits at Station 7 for 48 minutes. Stations 5 and 6 also see significant increases in wait times, up from less than one minute for both to over 2 minutes and 7 minutes, respectively. See Table 4.13 for full results. Therefore, if a train station needs to be shut down, more buses should travel along routes into and out of Stations 5, 6, and 7 to accommodate the increase in traffic. Or, if possible, focus on shutting down the train station during non-peak hours to reduce the traffic that enters the bus network.

Table 4.13: Performance measures for bus stations if subway station is disrupted.

Station	1_b	2_b	3_b	4_b	5_b	6_b	7_b	8_b	9_b
ρ_{i_b}	0.69	0.80	0.70	0.46	0.92	0.97	1.00	0.54	0.78
w_{q,i_b} (min)	0.48	0.86	0.51	0.18	2.58	7.46	48.22	0.26	0.78
w_{i_b} (min)	0.92	1.30	0.94	0.62	3.01	7.90	48.65	0.69	1.21
L_{q,i_b}	0.77	1.59	0.82	0.19	5.49	16.73	110.77	0.32	1.40
L_{i_b}	1.45	2.39	1.52	0.65	6.41	17.70	111.77	0.86	2.18

Eliminating external flow to the subway network at Station 7, removing ability to stop at the station and option to leave the network at subway station 7, resulted in a small increase in utilization among all other subway stations. Nothing as dramatic as in the bus network, but an increase all the same. Wait times at all stations increased by about 0.01 minutes, too. Table 4.14 shows the full results for the subway stations. Overall, shutting down the subway station had a much greater impact on the bus network than on the subway network and no significant changes need to be made to accommodate this disruption in the subway network.

Table 4.14: Performance measures for subway stations if subway station is disrupted.

Station	2_s	5_s	6_s	7_s	9_s
ρ_{i_s}	0.26	0.24	0.22	0.00	0.18
w_{q,i_s} (min)	0.02	0.02	0.02	0.00	0.01
w_{i_s} (min)	0.12	0.12	0.12	0.00	0.11
L_{q,i_s}	0.05	0.04	0.03	0.00	0.02
L_{i_s}	0.31	0.28	0.25	0.00	0.21

4.5.2 Disruptions in Bus Stations

For the second case, bus Station 7 was shut down. Like before, passengers were allowed to enter the train station, but not the bus station. Also, the bus route would still go through Station 7, but the bus would not stop, and no one would get on or off the bus. The updated transition matrix P_b is below. The transition matrix for the subway network was as in the experiment discussed in Section 4.2. As in section 4.5.1, it was assumed all who would normally get on the bus at Station 7 would get on the subway instead. Utilization and mean service time of bus Station 7 were forced to 0.

$$\boldsymbol{P_b} = \begin{bmatrix} 0 & 0.50 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.50 & 0 & 0.23 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.20 & 0.48 & 0.27 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.59 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.08 & 0 & 0 & 0.36 & 0 & 0.20 & 0 \\ 0 & 0 & 0 & 0 & 0.40 & 0 & 0.42 & 0 & 0.16 \\ 0 & 0 & 0 & 0 & 0 & 0.565 & 0 & 0 & 0.435 \\ 0 & 0 & 0 & 0 & 0.50 & 0 & 0 & 0 & 0.29 \\ 0 & 0 & 0 & 0 & 0 & 0.16 & 0.29 & 0.13 & 0 \end{bmatrix}, \boldsymbol{q_b} = \begin{bmatrix} 0.50 \\ 0.27 \\ 0.05 \\ 0.41 \\ 0.36 \\ 0.02 \\ 0 \\ 0.21 \\ 0.42 \end{bmatrix}$$

Shutting down bus Station 7 slightly increased the utilization of all bus stations, though the main increases were in bus stations 6 and 9, with an average increase of 14.0%. Queue wait times, however, increased significantly. The wait time for station 6 increased by 75.5% and the wait time for station 9 increased by 49.4%. Though stations 5 and 6 have utilizations above 80%, the system is not fully out of control, though queues will start trending toward infinity. However, the wait times are still under a minute and a half. Full results for the bus stations are given in Table 4.15. No significant changes need to be made to the bus network when a bus station is closed. There is some flexibility in the network, and it can handle this type of disruption.

Table 4.15: Performance measures for bus stations if a bus station is disrupted.

Station	1_b	2_b	3_b	4_b	5_b	6_b	7_b	8_b	9_b
ρ_{i_b}	0.69	0.80	0.69	0.45	0.85	0.82	0.00	0.52	0.70
w_{q,i_b} (min)	0.48	0.85	0.49	0.18	1.22	1.02	0.00	0.67	0.94
w_{i_b} (min)	0.91	1.29	0.92	0.61	1.65	1.46	0.00	0.67	0.94
L_{q,i_b}	0.76	1.56	0.78	0.19	2.38	1.95	0.00	0.28	0.82
L_{i_b}	1.45	2.36	1.47	0.64	3.23	2.77	0.00	0.79	1.52

Though the amount of people entering the subway station increased, utilization of the subway stations did not increase significantly. The largest increase in station utilization was at subway Station 7, which increased by 16.1%. These minor changes to the flow through the subway network otherwise did not affect the remaining parameters for the subway network in any significant way. The wait times only increased by 0.01 minutes, but the average total time spent waiting and boarding throughout the network remained the same. The full results for the subway stations are below in Table 4.16.

Table 4.16: Performance measures for subway stations if a bus station is disrupted.

Station	2_s	5_s	6_s	7_s	9_s
ρ_{i_s}	0.26	0.24	0.22	0.25	0.18
w_{q,i_s} (min)	0.02	0.02	0.02	0.02	0.01
w_{i_s} (min)	0.12	0.12	0.12	0.12	0.11
L_{q,i_s}	0.05	0.04	0.03	0.04	0.02
L_{i_s}	0.31	0.28	0.25	0.29	0.21

Therefore, there is not significant strain placed on the network when a bus station is disrupted and no significant changes need to be considered or explored.

4.5.3 Disruptions in both Bus and Train Stations

For the last case, both the bus and train stations at Station 7 were shut down. For this, the interarrival time, mean service time, and utilization for Station 7 was set to 0. Interarrival times for stations surrounding Station 7 were not decreased as it was assumed passengers elected to find alternate means of transportation instead of entering other stations in the network. Probability matrices were also changed as shown below.

$$\boldsymbol{P_b} = \begin{bmatrix} 0 & 0.50 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.50 & 0 & 0.23 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.20 & 0.48 & 0.27 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.59 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.08 & 0 & 0 & 0.36 & 0 & 0.20 & 0 \\ 0 & 0 & 0 & 0 & 0.40 & 0 & 0.42 & 0 & 0.16 \\ 0 & 0 & 0 & 0 & 0 & 0.565 & 0 & 0 & 0.435 \\ 0 & 0 & 0 & 0 & 0.50 & 0 & 0 & 0 & 0.29 \\ 0 & 0 & 0 & 0 & 0 & 0.16 & 0.29 & 0.13 & 0 \end{bmatrix}, \boldsymbol{q_b} = \begin{bmatrix} 0.50 \\ 0.27 \\ 0.05 \\ 0.41 \\ 0.36 \\ 0.02 \\ 0 \\ 0.21 \\ 0.42 \end{bmatrix}$$

$$\boldsymbol{P_s} = \begin{bmatrix} 0 & 0.50 & 0 & 0 & 0 \\ 0.33 & 0 & 0.33 & 0 & 0 \\ 0 & 0.33 & 0 & 0.33 & 0 \\ 0 & 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0 & 0.5 & 0 \end{bmatrix}, \boldsymbol{q_s} = \begin{bmatrix} 0.50 \\ 0.34 \\ 0.34 \\ 0 \\ 0.50 \end{bmatrix}$$

The results in the bus network were surprising. When the station was removed and there was no external flow into the bus network from Station 7, stations 6 and 9 had their utilization increase by 36.7% and 37.7% respectively. The utilization of station 5 increased by 17.6%. With these increases, the utilization of both station 5 and 6 surpassed 90%, with station 5 reaching a utilization of 93.7% and station 6 reaching 99.4% utilization. While the utilization of all stations did not reach the same levels as when the subway station was closed, this is still a significant increase. The expected queue time at station six became to 37.74 minutes, as compared to 0.58 minutes in the base network scenario. This also demonstrates the inability of the system to bring in excess capacity when a station is closed, at least on the bus network side. Full results for the bus network are in Table 4.17.

Table 4.17: Performance measures for bus stations if a bus and train station are disrupted.

Station	1_b	2_b	3_b	4_b	5_b	6_b	7_b	8_b	9_b
ρ_{i_b}	0.69	0.80	0.70	0.46	0.94	0.99	0.00	0.55	0.84
w_{q,i_b} (min)	0.48	0.87	0.51	0.18	3.24	37.30	0.00	0.27	1.14
w_{i_b} (min)	0.92	1.30	0.95	0.62	3.67	37.74	0.00	0.70	1.57
L_{q,i_b}	0.77	1.60	0.83	0.19	7.00	85.58	0.00	0.34	2.20
L_{i_b}	1.46	2.40	1.53	0.65	7.93	86.58	0.00	0.89	3.04

By closing station 7, the largest increase in utilization of the subway stations from all experiments occurred. All subway station utilization increased, with stations 6 and 9 increasing by 36.7% and 37.7%, respectively. However, even with larger increases, no subway utilization was greater than 30% so there were not significant changes to the wait times or number of passengers in the queues. Results for the subway network are found in Table 4.18.

Table 4.18: Performance measures for subway stations if a bus and train station are disrupted.

Station	2_s	5_s	6_s	7_s	9_s
ρ_{i_s}	0.27	0.27	0.28	0.00	0.24
w_{q,i_s} (min)	0.02	0.02	0.02	0.00	0.02
w_{i_s} (min)	0.12	0.12	0.12	0.00	0.12
L_{q,i_s}	0.05	0.05	0.06	0.00	0.04
L_{i_s}	0.32	0.32	0.34	0.00	0.28

To accommodate closing an entire station, more buses need to be available so that the time in between bus arrivals is shortened. Also, getting the subway station open and operational should be prioritized, as then the system behaves like in Section 4.5.2, where the system remains in control.

Chapter 5

Conclusions and Future Work

In this thesis, a network of queues model was developed to evaluate parameters of a two-mode public transportation network. The parameters of expected wait time, station utilization, and queue length were used to conduct a holistic analysis to identify potential concerns within the network quickly without needing cumbersome software and very little data. This is necessary when dealing with complicated public transportation networks to aid in quick decision making. Public transportation networks also often have data that is lacking, whether because the data is old or does not have the specificity needed for statistical anaylsis.

This research differentiates itself by developing an easily accessible model with few data requirements that provides a holistic view of a public transportation network. This type of analysis is essential in identifying critical stations and arcs within the network and as an intial step in more detailed analysis. It is a dynamic method of analysis which allows for multiple different scenarios to be compared quickly. The analysis from the model allows network managers to compare multiple scenarios easily and identify scenarios that cause the least amount of disruption to the system. It also provides insight to where, if there is a significant problem, more detailed analysis should be conducted.

First, the public transportation network was modeled as a series of nodes and arcs, representing stations and routes between the stations, respectively. Due to complexity, the overall network was separated into subgraphs for each mode of transporation within the network to accommodate differences within the network due to the different transportation modes. Then, the subgraphs were modeled as a Jackson network, to allow for the flow of passengers entering the system at every station.

Once modeled as an open network of queues, the system was modeled in simulation software to determine the optimal interarrival time of passengers into the network, by maximizing utilization of the stations. The modeling in simulation software was necessary as interarrival time data to each station is not available. If interarrival data is available for statistical analysis the simulation model is not required.

With maximum feasible external arrival rates, the performance of the network could be evaluated, focusing on station utilization, expected wait times, and queue lengths. This performance evaluation gave information on which areas of the network were most congested and where assets should be moved to within the system.

In the case study and experiments a real two-mode transit network was analyzed. In this case study, bus stations were the most utilized and through experimentation the bus network was most susceptible to disruptions. The train network was not significantly vulnerable to network disruptions, most likely due to the increased capacity of the trains in comparison to the buses.

Future work for this research may incluce expanding the realism of the study by including batching into the queueing network. This could be a more realistic application of mode capacity within the network. The model could also be adjusted to allow for changing of transportation modes during travel. Finally, optimization of the service times of each station could allow for better decision making of where to flex or assign assets in the network.

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