Procedural fluency in mathematics: Exploring elementary teachers' knowledge, understanding, and application in classroom practices

by

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B.S., Emporia State University, 2005
M.S., Emporia State University, 2009

AN ABSTRACT OF A DISSERTATION

submitted in partial fulfillment of the requirements for the degree

DOCTOR OF EDUCATION

Curriculum and Instruction
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Abstract

Throughout the history of public education in the United States of America, the teaching of mathematics and its focus have been topics of reform and calls for change. Students need a balanced and intentional approach to learning mathematics, which supports each student in understanding and applying mathematical concepts. The purpose of this study was to determine the level of knowledge and understanding of teachers related to procedural fluency and how components of procedural fluency are applied in classroom practices. In addition, it is important to understand that computational fluency and basic fact fluency are embedded in the larger proficiency strand of procedural fluency. The study employed a multiple case study format as well as multiple data sources to answer the research questions. The overarching research was: How does what a teacher knows and understands about procedural fluency translate into classroom practices during core instruction? The study also included the following subquestions: (1) What does a teacher know and understand about computational fluency in mathematics? (2) In what ways does a teacher plan for computational fluency development? (3) In what ways does a teacher explicitly connect conceptual understanding to computational fluency? (4) What does a teacher do to ensure the development of procedural fluency in the classroom? (5) What does fluency practice look like in the classroom? (6) What components of fluency do the practices present in the classroom address?

The research design was qualitative in nature and included six cases with each participant representing one case. The researcher collected data related to teachers' knowledge and understanding of fluency using a survey, an interview, and a follow-up interview. The researcher then completed three classroom observations for core math instructional block and three debriefs sessions with each participant to determine the level of application of classroom practices that

supported the development of fluency. The data was organized in a codebook composed of a priori codes and was analyzed for themes within each case and across all the cases for themes. The study looked to place each participant in one of four categories based on their knowledge and application level: unconsciously unaligned, consciously unaligned, unconsciously aligned, and consciously aligned.

The researcher found that five cases were unconsciously unaligned in terms of practices that support fluency, and one case was consciously aligned. Most teachers did not have a strong knowledge base or understanding of procedural fluency. As a result, purposeful instruction needed to support the development of procedural fluency through quality practice and assessment aligned with fluency components and actions did not appear to be occurring based on the data collected. The results suggest that additional professional development, especially in quality fluency practice, is needed to strengthen the knowledge and understanding of the teacher before application can happen in the classroom.

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Dr. Sherri Martinie

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"If I have seen further, it is by standing on the shoulders of giants".

-Isaac Newton

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Finally, I would like to thank the people who participated in the study. Thank you for sharing your classrooms, students, and stories with me. Your dedication, passion, and desire for continuous growth is inspiring. You each taught me more than you will ever know!

Dedication

This dissertation is dedicated to the possibility that one day, in the near future, all students will see themselves as learners and doers of math.

Chapter 1 - Introduction

Overview

Depending on who you ask, the idea of mathematics can create anxiety, fear, love, hate, confidence but rarely neutral feelings. It is the role of public education to work to make high quality math instruction accessible for all students through the development of a positive math identity and a balanced approach to the strands of mathematical proficiency. Mathematical proficiency occurs when curriculum, instruction, and assessment intertwine conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (National Research Council (NRC), 2001).

One of the key responsibilities of a school leader is to reduce the variability of teaching quality within their school (Hattie, 2015). This can be achieved through focusing on What Matters Most Framework as outlined by McRel International (2010) which includes the integration of guarantee challenging and intentional instruction. In order to achieve this, there must be high expectations and challenging instruction for each student, intentionally matching of strategies to learning outcomes, and planning for learning.

A component of math instruction that needs to be guaranteed for each student is the development of procedural fluency through explicit strategy instruction, the use of visual representations or tools, quality fluency practice, and assessment (Bay-Williams & SanGiovanni, 2021). For this to occur, educators must have a comprehensive understanding of fluency and the components needed in order to implement practices that are coherent, rigorous, and accessible for all students.

Background of the Problem

Since math became a consistent component of the elementary curriculum at the turn of the twentieth century, it has been the subject of reform efforts. Major efforts to reform mathematics education in the United States have occurred about every twenty years caused by unrest with the current status quo. With each new reform effort came an aggressive change in the purpose and components of what was viewed to be quality math instruction. There has been movement back and forth between two major schools of thought: math that focused on basic fact instruction and math that focused on the application/construction of mathematical knowledge by the students. However, research (Huinker & Bill, 2017; National Council of Teachers of Mathematics (NCTM), 2014; NCR, 2001) shows that to be proficient in math, you must have understanding and application. There is a clear need to provide balance between these two ideals in order to sustain a more lasting, comprehensive approach to mathematics education in the United States of America.

Statement of the Problem

In the Kansas College and Career Ready for Mathematics, the term *fluently* is found thirteen times in the standards for kindergarten through eighth grade (with the term fluency used at first grade) with at least one standard dedicated to this component of mathematical proficiency at each level. After each use of the term, the terms efficiently, accurately and flexibility are included in parentheses. The standards for elementary are included below in table one to illustrate the clear progress of mathematical procedures.

When clicking on the link attached after each use of the word fluently, the user is taken to an article entitled, Fluency *is More than Mere Speed*, produced by the department of education for the state of Kansas (2013) to explain their stance on fluency as it relates to mathematics. The definition of fluency from the state is clear, "performing a skill flexibility, accurately, and efficiently." Even though our most current set of standards were updated in 2017, more than four years later many educators still equate fluency with solely speed (Boaler & Zoido, 2016). When all components of fluency are not emphasized, students cannot achieve mathematical proficiency (Huinker & Bill, 2017).

Table 1.1

Kansas College and Career Ready Standards Related to Fluency at the Elementary Level

Grade	Standard Stem	Description of Standard
Kindergarten	K.OA.5	Fluently (efficiently, accurately, and flexibly) add and subtract within 5
First Grade	1.0A.6	Add and subtract within 20, demonstrating fluency (efficiently, accurately, and flexibly) for addition and subtraction within 10. Use mental strategies such as counting on; making *ten (e.g.)*; decomposing a number leading to a ten *(e.g.);* using the relationship between addition and subtraction *(e.g. knowing that, one knows)*; and creating equivalent but easier or known sums *(e.g. adding by creating the known equivalent)*.
Second Grade	2.OA.2	Fluently (efficiently, accurately, and flexibly) add and subtract within 20 using mental strategies (counting on, making a ten, decomposing a number, creating an equivalent but easier and known sum, and using the relationship between addition and subtraction) Work with equal groups of objects to gain foundations for multiplication
	2.NBT.5	Fluently (efficiently, accurately, and flexibly) add and subtract

		within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction *(e.g. composing/decomposing by like base-10 units, using friendly or benchmark numbers, using related equations, compensation, number line, etc.).*
Third Grade	3.OA.7	Fluently (efficiently, accurately, and flexibly) multiply and divide with single digit multiplications and related divisions using strategies *(e.g. relationship between multiplication and division, doubles, double and double again, half and then double, etc.)* or properties of operations
	3.NBT.2	Fluently (efficiently, accurately, & flexibly) add and subtract within 1000 using strategies *(e.g. composing/decomposing by like base-10 units, using friendly or benchmark numbers, using related equations, compensation, number line, etc.)* and algorithms (including, but not limited to: traditional, partial-sums, etc.) based on place value, properties of operations, and/or the relationship between addition and subtraction.
Fourth Grade	4.NBT.4	Fluently (efficiently, accurately, and flexibly) add and subtract multi-digit whole numbers using an efficient algorithm (including, but not limited to: traditional, partial-sums, etc.), based on place value understanding and the properties of operations.
Fifth Grade	5.NBT.5	Fluently (efficiently, accurately, and flexibly) multiply multi-digit whole numbers using an efficient algorithm *(ex., traditional, partial products, etc.)* based on place value understanding and the properties of operations

Although there is an emphasis on the importance of procedural fluency and mathematical proficiency in the state standards, it is not represented in the legislative policies related to professional development. Beginning in 2020, each school in the state of Kansas is required by the Legislative Task Force on Dyslexia and the Kansas State Board of Education to provide annual training related to structured literacy including six hours of initial training (Kansas State Department of Education, 2021). The purpose of the training is to ensure that staff members have the skills and knowledge necessary to identify students who are struggling in reading and provide evidence-based interventions as a result of the student's academic need. However, there are not mandated professional development requirements in the area of mathematics to ensure that teachers have the skills and knowledge needed to provide a balanced, structured math curriculum as well as identify and provide appropriate supports for students struggling in math. The content that we spend time on can demonstrate what we value as time is a limited resource in education.

Purpose of the Study

The purpose of the study is to use multiple case studies to determine what teachers know and understand about procedural fluency and how this translates into classroom practices. The study will occur in an elementary school with a focus on six teachers who were selected using convenience sampling. To determine both knowledge and application of practices that develop procedural fluency, the study was created utilizing a combination of two theoretical frameworks, The Fostering a Community of Learner Model (Shulman & Shulman, 2004) and Knowledge and Teaching (Shulman, 1986) to focus how teachers gain content and pedagogical knowledge as well as serving as a guide to develop teachers.

Significance of the Study

This study serves to inform future research on aspects of connection between content knowledge and how it is linked to what occurs in the classroom. This study is significant because it works to discover what is occurring in classrooms to support the development of fluency as it relates specifically to the teacher’s current knowledge and understanding level. Instructional leaders need to understand how knowledge translates into classroom practices in order to enhance the level of content and pedagogical knowledge to belief and identity. According to Dilts, the learning levels of belief and identity create the reasons behind what you do and become how you see yourself. Only when we move past compliance and conformity can we facilitate math instruction that answers the calls of reformers and educators alike.

Primary Research Question

The study was designed to be qualitative in nature to answer the following research questions.

Overarching Research Question

How does what a teacher knows and understands about procedural fluency translate into classroom practices during core instruction?

Subquestions

- What does a teacher know and understand about computational fluency in mathematics?
- In what ways does a teacher plan for computational fluency development?
- In what ways does a teacher explicitly connect conceptual understanding to computational fluency?
- What does a teacher do to ensure the development of procedural fluency in the classroom?
- What does fluency practice look like in the classroom?
- What components of fluency do the practices present in the classroom address?

Educators will fall into one of four levels related to their knowledge of procedural fluency and how it impacts their classroom practices. These four levels come from the Stages of Competence Model, which was first introduced by Noel Burch in the 1970's and has since seen many adaptations based on the discipline of study. These four levels include unconsciously unaligned, consciously unaligned, unconsciously aligned, and consciously aligned (see table 1.2).

Table 1.2

Stages of Competence Model (revised)

Unconsciously Unaligned Not able to articulate his or her knowledge and understanding related fluency components and actions that support the development of procedural fluency **-and-** No or limited classroom practices that equate to the development and support of procedural fluency	**Unconsciously Aligned** Not able to articulate his or her knowledge and understanding related fluency components and actions that support the development of procedural fluency **-but-** Implements classroom practices that equate to the development and support of procedural fluency
Consciously Unaligned Able to articulate his or her knowledge and understanding related fluency components and actions that support the development of procedural fluency	**Consciously Aligned** Able to articulate his or her knowledge and understanding related fluency components and actions that support the development of procedural fluency

-but-	**-and-**
No or limited classroom practices that equate to the development and support of procedural fluency	Implements classroom practices that equate to the development and support of procedural fluency

Summary of Research Design

The research design was empirical in nature and works to answer how questions through the collection of qualitative data. The data was organized and analyzed in a multiple case study format to determine similarities and differences occurring through the progression of the study. The researcher completed a survey to gather information about what teachers know and understanding related to procedural fluency. Based on the information gleaned from the survey, semi-structured interview was conducted to ensure a comprehensive understanding of what the teacher knew and understood about fluency was obtained.

The researcher then conducted three observations over a two-week period for each participant. The observations on the instruction occurring the core instructional math block in order to study how the knowledge was translated into classroom practices. After each observation, an oral debrief was completed by the participant and researcher to gather additional information related to how the participant believed fluency was developed before, during, and after the observed lessons. As data was collected a rubric was used to determine the level of expertise of each educator in the terms of knowledge and understanding of fluency as well as the classroom practices used to develop fluency.

Assumptions and Scope of Study

The researcher made several assumptions at the onset of this study. It was assumed that the participants would be open and honest to the best of their ability during the interview and debriefing conversation. In addition, it was assumed that teachers would provide information on the survey that was based on their professional experience and accurately reflected their knowledge and understanding of procedural fluency

The scope of the study will be six participants in an elementary school in Kansas. The study is based on standards that were developed originally by Common Core Standards Initiative group which forty-one states still use some form of across the United States. In addition, research shows that the majority of students' academic needs are not being met in the area of mathematics (National Center for Education Statistics, 2019).

Definition of Terms

(As each term relates to procedural fluency)

Accuracy: using a mathematical procedure to get the correct answer (Bay-Williams & Stokes, 2017)

Appropriateness: selecting the appropriate strategy to get the correct answer in the most efficient way (Bay-Williams & Stokes, 2017)

Balanced: practice that is intentionally planned so there is balance related to focus on all the components of fluency (efficiency, flexibility (appropriate strategy selection), and accuracy)

Basic Fact Fluency: fluency with the four operations (addition, subtraction, multiplication, and division) involving single-digit numbers- a component of procedural fluency (Bay-Williams & SanGiovanni, 2021)

Centers: a physical space in the room where students participate in activities that promote the development of fluency components and actions

Computational Fluency: fluency in the scope of the four operations (addition, subtraction, multiplication, and division)- a component of procedural fluency (Bay-Williams & SanGiovanni, 2021)

Conceptual Understanding: an integrated and functional grasp of mathematical ideas, which allow students to use mathematical procedures purposefully because they know when to use them and why they work

Connected: practice that is intentionally planned in order to help students see relationships and connections

Direct- Strategy Instruction: the implementation of instruction that includes demonstration, modeling, and practice related to strategies that can be used to solve procedures

Distributive Practice: review that takes place after the original learning has occurred

Efficiency: the ability to judge a variety of strategies to determine which one best fit the problem (Bay-Williams & Stokes, 2017)

Explicit- Strategy Instruction: the implementation of instruction that is clear and precise in order to help students use strategies to complete fluency actions

Focused: practice that is intentionally planned so students have opportunities to practice accuracy by learning to use a procedure and get the right answer

Feedback: providing timely, specific information related to student progress of procedural fluency

Flexibility: the ability to transfer procedures to different problems and contexts (NCTM, 2014)

Fluency Actions: strategy selection, reasonable time, trades out/ adapts strategy, application of strategy, complete steps, and correct answer

Fluency Components: accuracy, efficiency, and flexibility

Games: the use of activities that promote the development of procedural fluency by providing opportunities for students to select, make choices about, and use the most efficient strategy

Intentionally Planned: the implementation of instruction that is deliberate and purposeful in the development, use, and analysis of strategies used to solve procedures

Procedural Fluency: fluency in the scope of procedures in mathematics, which encompasses both basic fact and computational fluency (Bay-Williams & SanGiovanni, 2021)

Processed: practice that is intentionally planned so students have opportunities to process their learning through reflection

Routines: the specific use of instructional time dedicated to think, problem-solve, and discuss mathematical procedures

Varied: practice that is intentionally planned so student have a variety of experiences related to cognitive demand, focus on component of fluency, and engagement

Visual Models: pictures and images that help students to interpret numbers

Worked Examples: Problems that are already solved (or partially solved) used to structure discussion focused on when and why a strategy works and makes sense to use

Chapter Summary

Overall, the concept of reform and change is a prevalent topic when it comes to mathematical education. There is a need to balance our approach as students must understand

and apply mathematical concepts to be successful. It takes the intentional implementation of classroom practices that develop fluency to create learning experiences that are rigorous and promote healthy math identities. The next chapter will provide a review of the literature related to mathematics education and how this connects to procedural fluency.

Chapter 2 - Review of Literature

Fluency Practices in the Classroom

The way a math teacher approaches instruction in the classroom is impacted by their content knowledge, their own experiences as a math student, and their beliefs and values (Cartwright, 2018). This study will focus on how fluency practices that occur in the classroom are specifically related to the way teachers define fluency and its purpose within the development of mathematical concepts in the classroom. If fluency is viewed with a singular lens of automaticity, understanding cannot occur. The purpose of the literature review is to outline the need for not only a balance of conceptual understanding and procedural fluency, but also the need to recognize the iterative nature of conceptual and procedural knowledge (Rittle-Johnson et al, 2001). Thinking of mathematical proficiency as a swinging rope, it stands to reason that the most efficient way to reach the top is the use of a hand over hand climbing technique (Rittle-Johnson, 2019). Likewise, conceptual understanding and procedural fluency working in tandem or hand-over-hand fashion helps students to achieve mathematical proficiency. However, when there is a focus on only one (conceptual understanding or procedural fluency), it is like climbing the rope with one hand. Some students might reach the top, but it will be much more difficult, and some may not reach the level of proficiency needed at all. In addition, if there is not intentional, continual connection of conceptual understanding and procedural fluency, classroom instruction becomes less strategy-focused and lacks the components that develop mathematical proficiency.

The literature review begins by describing the history of math education to show how fluency has been addressed as well as integrated over the last century. The review also provides a foundation for the definition of "rigor," which fluency is a part of, and proceeds to detail how it fits into the bigger picture of math proficiency. There is a need for educators to have a true understanding of fluency and the components necessary to ensure students have the skills and knowledge necessary to interact with mathematical concepts proficiently. The idea of fluency is much more complex than how it is often translated into instructional practices.

In order for procedural fluency to occur in the classroom, there needs to be a foundation of conditions that foster fluency development. A description of these conditions and how educators can create a learning environment that supports students' mathematical understanding is also included in this review. These conditions must be in place to serve as foundation for quality fluency practice to be built upon. The review transitions into and offers insight on mathematics teaching practices, essential foundations, and instructional approaches that support and enhance the development of procedural fluency. Even under the best conditions, challenges implementing fluency practices exist. As a result, the review concludes by identifying some of the challenges facing educators who are attempting to implement those fluency practices.

History of Mathematics Education in the United States

From the twentieth century to now, the teaching of mathematics in the United States has undergone six distinct phases (Lambdin & Walcott, 2010). Each reform movement in mathematics education influenced the core instruction that students receive in the area of mathematics. As a result, this has greatly determined students' mathematical understanding, attitudes towards the subject, and their ability to use math in real life contexts.

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mathematics. As a result, this has determined students' mathematical understanding, attitudes towards the subject, and their ability to use math in real life contexts.

Phases in the History of Mathematics Education

Phase One (1900s, 1910s, and 1920s): Drill and Practice. This phase of mathematical education focused on drill and practice as the primary focus on math instruction instead of viewing it as one component of the math curriculum. Mathematics was "taught by concentrating on drills with skills that had been segmented into small, distinct, easily mastered units" (Lambdin & Walcott, 2010, p. 7). These drills were provided in isolation so little connection was made between mathematical concepts by the students. In this phase, the focus was on memorization, which was being able to quickly recall the answer to a fact or complete a procedure without connections (Lambdin & Walcott, 2010). Speed was the main focus, not conceptual understanding, and application, so as a result, students could not apply their knowledge in various situations or to solve problems beyond simple arithmetic.

Phase Two: Meaningful Arithmetic (1930s and 1940s). In order to address the issue of students not being able to apply their learning at deeper levels, math education transitioned to the Meaningful Arithmetic Phase where the focus shifted to developing math in a meaningful way. Although the term meaningful was interpreted differently by educators, in general, this phase was known for teaching and using math in context (Lambdin & Walcott, 2010), which opened the door for student generated strategies. A belief that was prominent during this stage of instruction was that students could through incidental interaction with math concepts instead of the skills needing to be taught in a systematic and cumulative fashion (Lambdin & Walcott, 2010).

Although the focus in each of these phases was distinctly different, the common bond was the lack of systematic teaching, which led to gaps and an inability for students to apply these concepts at a deeper level. Another pivotal point in the reform of mathematical instruction was caused by World War II and the launch of Sputnik. In both cases, it appeared that Americans did not have the mathematical knowledge necessary to protect our nation and be competitive in the world of technology (Lambdin & Walcott, 2010; Klein, 2003).

Phase Three: New Math (1950s and 1960s). During the New Math Phase, mathematicians intervened at this point and helped restructure what was being taught and how mathematics was taught through preservice teacher preparation and textbooks (Lambdin & Walcott, 2010; Klein, 2003). The two major ideals within this phase were the idea of a spiraling curriculum infused with discovery learning. Discovery learning provided opportunities for students to create their own theories on how to solve the problem. There was also a focus on building understanding from concrete models to representational forms and then abstract models was the end goal (Lambdin & Walcott, 2010). These ideas related to theory of teaching and learning demonstrated the need for conceptual understanding in order to develop procedural fluency and utilization of different strategies.

Phase Four: Back to Basics (1970s and early 1980s). With a decrease in achievement scores, the next movement was Back to Basics. This phase provided a noticeable divide in mathematical education. Some educators and educational institutions quickly abandoned the new math materials and went to teaching using demonstration, drill, and continuous practice. Others still held onto the new math approaches but included some traditional instruction on more basic concepts (Lambdin & Walcott, 2010). Overall, this movement promoted a swift return to drill and practice in classrooms across the United States (Ellis & Berry, 2005). The focus on drill and practice once again looked at math in terms of speed and traditional algorithms instead of strategy development and understanding.

Phase Five: Problem Solving (1980s). With a realization that teaching the basics did not prepare students for success in the workforce, a new era emerged known as the Problem-Solving Phase (Dossey, et al., 2016; Lambdin & Walcott, 2010). This encouraged the use of cooperative learning and opportunities for students to share about their thinking. The focus of this phase centered around all students' ability to use mathematics and theories related to constructivist design were integrated into the teaching of mathematics. This approach focused on students developing their own strategies and having opportunities for discourse within the classroom. In relation to the development of procedural fluency, these focuses promoted flexibility and efficiency related to finding the best strategy to solve a problem.

Phase Six: Standards and Accountability (1990s to present). Our current stage relates to standards and accountability. The standards presented in this phase provided focus, coherence, and rigor. Rigor is defined as the balance of procedural fluency, conceptual understanding, and application. In addition, research related to the eight mathematical practices that should be developed in students was introduced by the Council of Chief State School Officers and the National Governors Association Center for Best Practices in the early 2010's. These mathematical practices are outlined in figure 1 below from the National Council of Teachers of Mathematics (2015). As stated in Lambdin and Walcott (2010), "A visitor (to a contemporary American classroom) is likely to see evidence of most of the major phases through which mathematical education passed during the twentieth century, as well as influences of theories of undergirding these curricular phases" (p. 20). As a result, the transfer of these shifts is not consistently present in the classroom especially in terms of the balance needed for rigor.

Table 2.1

Eight Mathematical Practices (NCTM, 2015)

Standards for Mathematical Practices	
1.	Make sense of problems and persevere in solving them
2.	Reason abstractly and quantitatively.
3.	Construct viable arguments and critique the reasoning of others.
4.	Model with mathematics.
5.	Use appropriate tools strategically.
6.	Attend to precision.
7.	Look for and make use of structure.
8.	Look for and express regularity in repeated reasoning.

Connection to Procedural Fluency

Throughout the history of American education, the path of math instruction is riddled with educational reform and frequent debates related to content and pedagogy (Fey & Graeber, 2003). Each shift has developed from discontentment with the current reality. This has resulted in the aggressive pendulum swings outlined above between a focus on basic facts and a focus on understanding. In the middle of this continuum of educational philosophies stands a concept that can bridge these ideals together to ensure that mathematics is taught in a systematic, explicit manner: procedural fluency. The concept is prevalent in core shifts proposed during the Standards and Accountability Era. Procedural fluency stems from the need for students to have both fluency and understanding regarding mathematical concepts in order to be mathematically proficient.

What is Fluency?

Fluency is an educational term that is commonly used when describing educational outcomes related to both reading and mathematical instruction. The term fluency often has many different definitions, interpretations, and perceived purposes (Simensen, 2010). As a result, this creates confusion on how fluency instruction should be facilitated in the classroom as well as what constitutes fluency. Often educators are more familiar with the term fluency as it relates to reading. This discussion on fluency will begin with a model representing fluency in reading and how to connect this knowledge to a model representing fluency in mathematics (Singer & Strasser, 2017; Balhinez & Shaul, 2019).

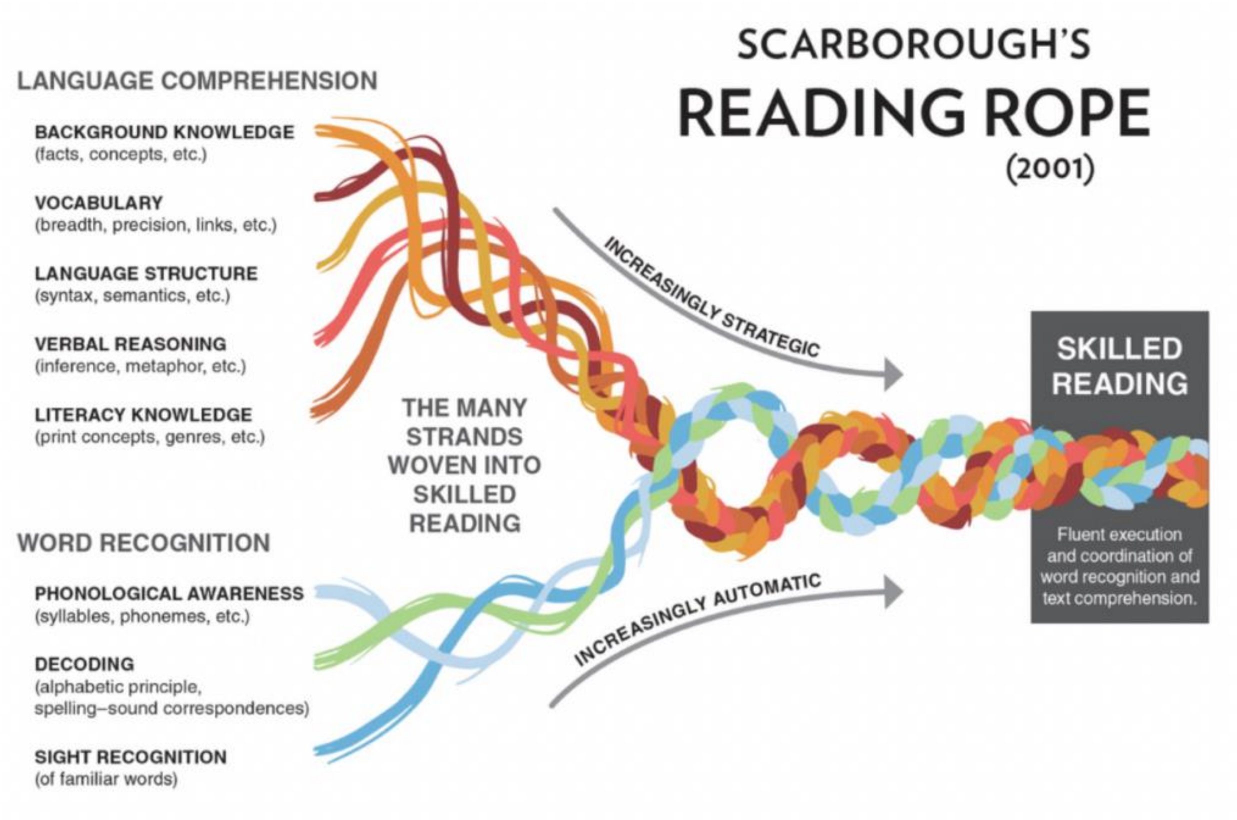
Fluency in Reading - The Science of Reading and the Reading Rope

There has been a great deal of research completed on how the brain develops reading skills and this has been coined the Science of Reading (Lyon & Chhabra, 2004; Moats, 2020; Ordetx, 2021). From this knowledge, a continuum of skills has been developed in order to develop fluent readers. This includes a focus on the five components of reading as outlined by the National Reading Panel. These components include phonemic awareness, phonics, fluent text reading, vocabulary, and comprehension. Not only must these components be included to develop fluent readers, but reading instruction must also be explicit, systematic, engaging, and intensive (data-driven and focused on essential skills) (Stewart, 2020).

Scarborough (2001) introduced the “Reading Rope,” (see figure 1) which demonstrates the language, comprehension, and word recognition skills that must be interwoven in instruction to develop skilled reading. Skilled reading is defined as the “fluent execution of word recognition and text comprehension” (Scarborough, 2001, p. 98). In addition, when any one strand (skill) is not acquired, it weakens the strength of the rope in its entirety, thus impacting a student's ability to be a skilled reader (Scarborough, 2001).

Figure 2.1

Strands of Reading Proficiency (Scarborough, 2001)



The term fluency is most synonymous with the teaching of reading. Fluency in reading often is related to the speed that students read a text. However, it was clear that just because a student can read a passage quickly does not mean that he/she can understand or comprehend it (Hasbrouck & Glaser, 2018). In a study conducted by Bigozzi et al. (2017), the authors conclude that reading rapidity did not help students process information, but instead accuracy has a much larger impact on comprehension. The National Reading Panel determined in a report released in

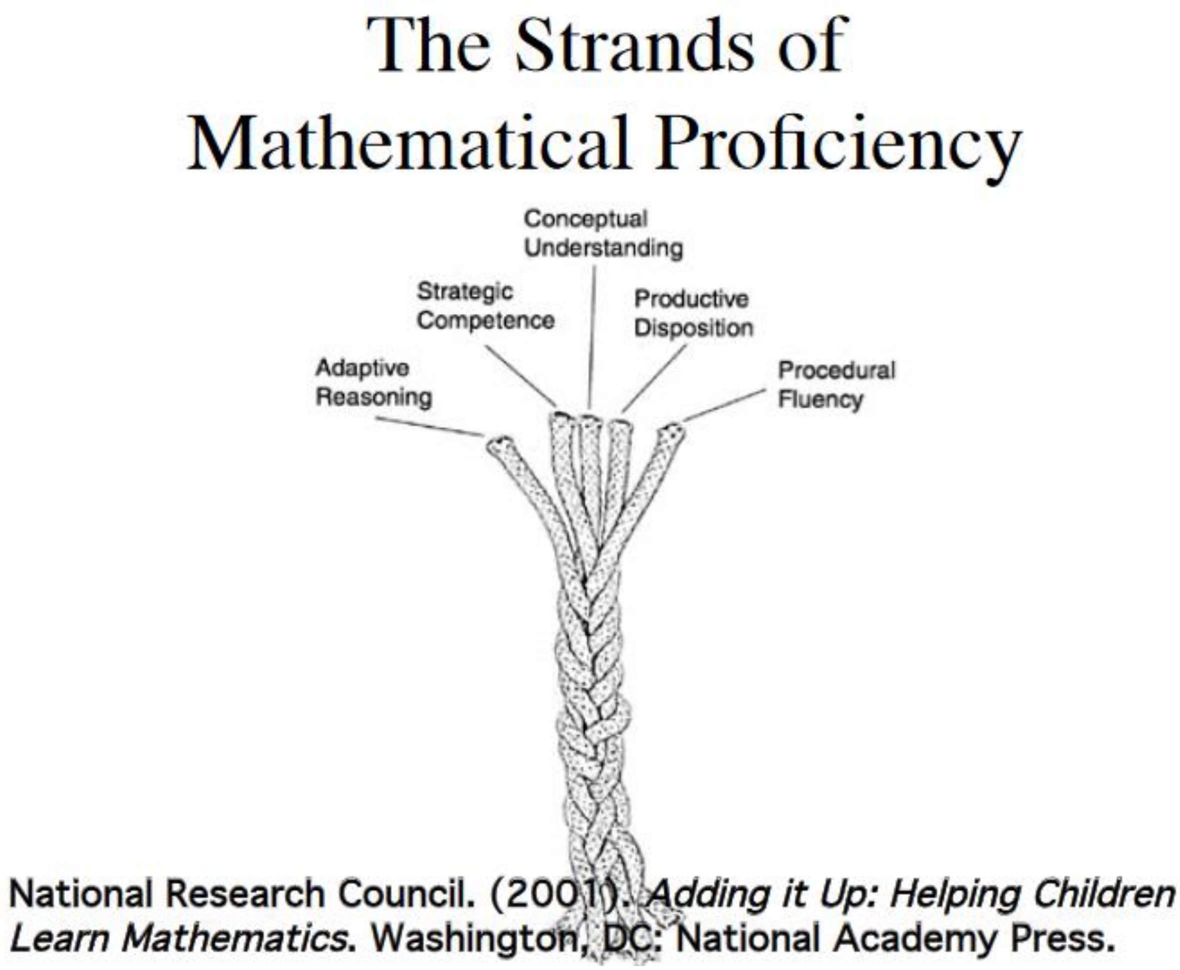
2000, that fluency instruction must go beyond word recognition and needs to include the comprehension processes as well. The Professional Development Institute (2009) goes on to describe fluency as the "end result of the decoding and comprehension processes" (p. 2-9). As a result, the definition of fluency was expanded to include not only pace, but also accuracy, smoothness, phrasing, and expression.

The Five Strands of Mathematical Proficiency

As we think about how fluency with reading parallels with fluency in mathematics, the similarities can be easily viewed by comparing the "Reading Rope" with the Five Strands of Mathematical Proficiency (National Research Council (NRC), 2001). The five strands include: procedural fluency, conceptual understanding, strategic competence, adaptive reasoning, and productive disposition (see figure 2). The idea that fluency only applies to solving math problems quickly is outdated and can be a common misconception when teaching fluency in math as well as reading. In addition, procedural fluency can be viewed as interwoven with conceptual understanding.

Figure 2.2

Strands of Mathematical Proficiency (National Research Council, 2001)



According to the National Research Council (2001) procedural fluency is defined as "skill in carrying out procedures flexibly, accurately, efficiently and appropriately" (p. 5). This demonstrates that the concept of fluency must expand beyond speed and recall in order to intentionally develop mathematical proficiency.

Procedural Fluency

There is also some confusion about the use of the word "procedure." Various terms are used interchangeably related to procedure when they do not mean the same thing (Bay-Williams & Stokes-Levine, 2017). According to Bay-Williams & Stokes-Levine (2017), fluency encompasses all of the components described below (see table 2) so it is important to be clear about how the terms are related to one another. However, it is even more critical to understand how each term has a different purpose in the acquisition of procedural fluency.

Table 2.2

Clarification of Terms Used in Conjunction with the Word Procedure (Bay-Williams & Stokes-Levine, 2017)

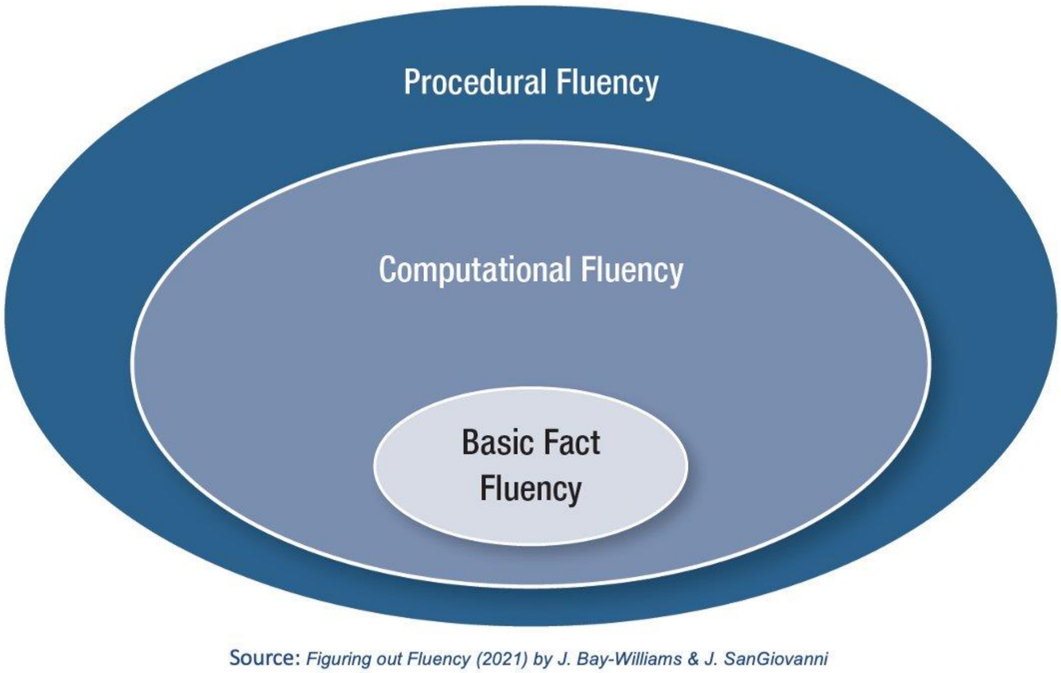
Procedural Term	Definition
Procedural knowledge	knowing step-by-step how to complete a procedure
Procedural skills	being able to carry out those steps
Procedural understanding	knowing how and why the procedure works
Procedural flexibility	knowing when to use a particular procedure and being able to adapt the procedure when needed
Procedural efficiency	being able to quickly and appropriately use a procedure

Each of these components (knowledge, skills, understanding, flexibility, and efficiency) contributes to the characteristics of procedurally fluent mathematicians. Procedural knowledge and skills ensure that the answer is accurate. Procedural understanding is needed to determine efficiency, appropriateness of the strategy, and flexibility related to the strategy used.

In addition, fluency can be used to mean different things when combined with other words like basic fact, computational, and procedure (see figure 2.3). Procedural fluency includes a variety of content domains that must be considered when planning, developing, and assessing procedural fluency in the classroom. These content domains include basic facts, equivalents and comparisons, algorithms and procedures, equations and formulas, conversions, and operations (Bay-Williams and SanGiovanni, 2021).

Figure 2.3

Fluency Terms (Bay-Williams & SanGiovanni, 2021)

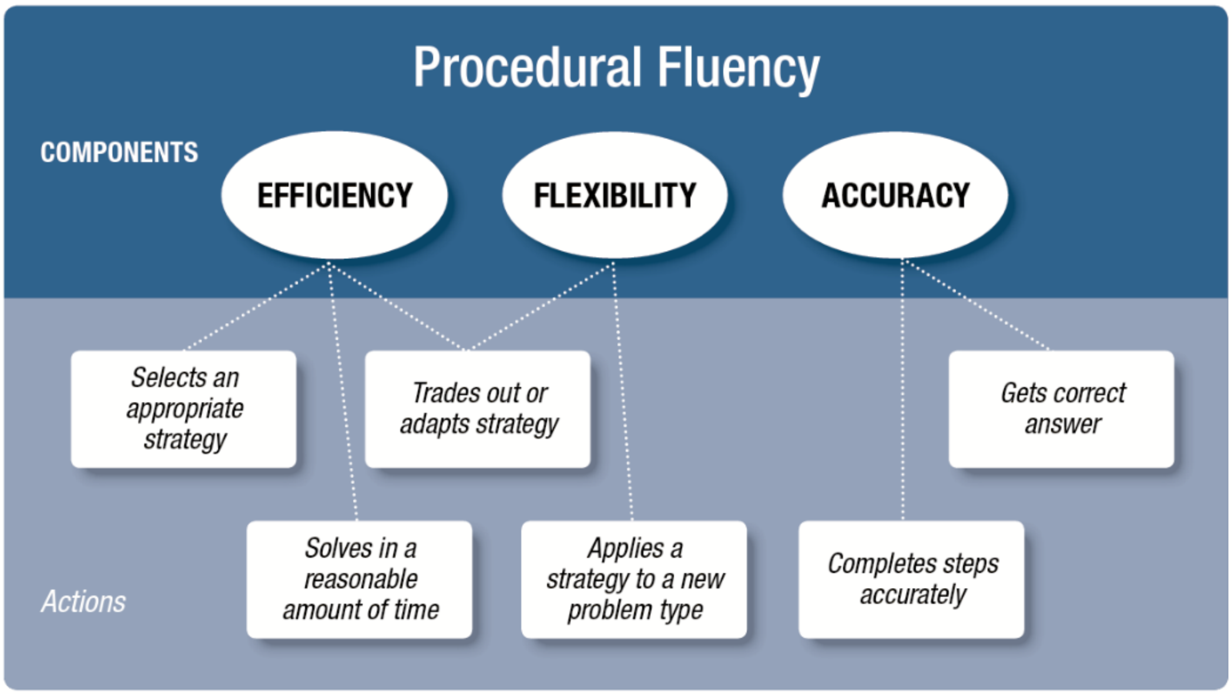


As with reading fluency, oftentimes people equate the term fluency with speed and accuracy (Boaler & Zoido, 2016). However, in reality, this is really being fluent with basic facts, which is a component of fluency but not at the same level. "Fluency is more comprehensive than being able to solve a problem with speed and accuracy" (Bay-Williams & Stokes-Levine, 2017, p. 62). Bay-Williams and Stokes-Levine (2017) describe procedural fluency as students "having a deep knowledge... and applying that knowledge in carrying out procedures flexibly, accurately, efficiently, and appropriately (see figure 4). The National Council of Teachers of Mathematics (2014) goes on to say that flexibility is a student's ability to "transfer procedures to different problems and contexts" (p. 1). Efficiency is defined as "being able to judge the repertoire to determine which one best fits the problem given" (Bay-Williams & Stokes-Levine, 2017, p. 65).

As a result, procedural fluency goes beyond the memorization of facts and procedures (NCTM, 2014). Automaticity is important (Baker & Cuevas, 2018), but fluency is not simply automaticity. Automaticity is a part of efficiency and efficiency is part of fluency (Bay-Williams, 2018). Fluency must be assessed and taught with a more comprehensive view.

Figure 2.4

Components of Procedural Fluency (Bay-Williams & SanGiovanni, 2021)



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As we look to develop procedural fluency in the classroom, it must go beyond traditional avenues such as flash cards, worksheets, and low-level questions. Students need opportunities to develop their thinking related to math through "experience in integrating concepts and procedures and building on familiar procedures as they create their own informal strategies and procedures" (NCTM (*Procedural Fluency in Mathematics*), 2014, p. 1). A teacher must explicitly plan in order to develop fluency by focusing on the why and when, comparing procedures and problems, and making explicit connections (Bay-Williams & Stokes- Levine, 2017). In addition, there is often a rush to move to fluency before students have developed conceptual understanding related to the concept especially as it relates to fact fluency. In conclusion, there are many components that develop the holistic definition of procedural fluency, and this impacts how it is developed, facilitated, and assessed during classroom instruction.

Classroom Conditions that Support Procedural Fluency

The development and cultivation of procedural fluency cannot occur in a vacuum or through the use of isolated strategies and practices. In the following section, the attributes needed to foster the conditions necessary to support procedural fluency are outlined as well as how each component contributes towards the development of procedural fluency for elementary students. The subsections are organized to progress from the most general to the most specific conditions as they relate to the support of procedural fluency.

Creating a Learning Environment that Supports Students and the Learning of Mathematics

The creation of the norms and culture of a learning environment models what it means to do and learn mathematics as well as what is valued in relation to learning in a classroom (Selling, 2016). Student achievement is impacted by the student's perception related to the fairness and support of the teacher (Adnan et al., 2014). This is further supported in a study by Turner et al (2002) where the researchers suggest that students who were supported (helped students build understanding, provided motivation, and provided opportunities to demonstrate new competencies) by their teacher used avoidance strategies less frequently when interacting with mathematical concepts. Teachers can support students with the integration of classroom conditions that promote a growth mindset for both the students and teacher, develop positive mathematical identity and agency, and ensure equity and accessibility for all students. These conditions are described in greater detail below.

Growth Mindset. Cherry (2021) defines mindset as “a set of beliefs that shape how you make sense of the work and yourself in it. It influences how you think, feel, and behave in any given situation” (p. 1). According to Dweck (2006), there are two types of mindsets: fixed (traits cannot be changed) and growth (traits can develop over time), which plays a critical role in student (and all people’s) success and failure.

Teacher Mindset. It is essential for teachers to model and have a growth mindset especially in terms of the learning of math (Boaler, 2013). In addition, staff must believe that all students can successfully learn math with the right support in place. This is further reinforced in *Principles to Action* (2014) as it is stated that “in excellent math programs, teachers hold themselves and each other accountable for the mathematical success of every student” (p. 59). Collective efficacy begins with the teachers’ self-efficacy in which a teacher believes that he or she can create an instructionally sound learning environment and this belief is strengthened by leaders who can unite a staff around a strong purpose (Bandura, 1993). Collective efficacy is needed to establish and maintain a school community where teachers believe that together, they have the capabilities needed to positively impact student learning and help all students to learn (Goddard et al., 2004). Brock and Hundley (2016) add “Teachers must offer up learning experiences that are engaging, valuable, accessible, and meaningful in order for growth mindset to make a true difference in learning outcomes” (p. 20).

There are notions that there are math people and not math people. Oftentimes, people who are good at math are thought to be born that way and are in turn naturally good at math (Boaler, 2016). This can be described as the entity theory of intelligence which states traits are fixed. In a study conducted by Rattan et al. (2011), the researchers found when teachers believed that traits were fixed, they were much more likely to utilize different teaching strategies when students were struggling with a mathematical concept. This resulted in the use of strategies and feedback that was intended to comfort the student instead of reinforcing the idea that math can be achieved by all through arduous work and perseverance. Abandoning continuous high expectations when students struggle can result in reduced levels of student engagement and achievement (Rattan et al., 2011).

A fixed mindset on the part of the teacher can also lead to students believing that to be good at math is to answer questions the fastest and with little effort. Instead, math is a skill that can be developed through arduous work and perseverance (Boaler, 2016). The development of grit occurs “when students see how deliberate practice (and a healthy amount of failure) is almost always required to rise to the top of any field, the idea of growth mindset can better be cemented as a viable for success” (Brock and Hundley, 2016, p. 134). It is critical for the teacher to provide feedback in a way that supports the process of learning instead of intellectual level (Mueller & Dweck, 1998). In a comprehensive report that include six studies related to the type of teacher feedback (based on performance or effort), Mueller & Dweck (1998) concluded that students who receive feedback based on effort are more likely to select problems that provided increased learning opportunities whereas students who were given feedback on their ability continued to select problems they knew they could get right.

Student Mindset. Students need support in developing strong growth mindsets. This is supported by the teacher providing opportunities for students to make mistakes within safety parameters so they can learn and grow. When a growth mindset is purposefully taught and encouraged, students are more likely to be engaged in learning, which positively impacts student achievement (Brock and Hundley, 2016). In order for this to occur, students must have access to tasks and questions in math that have "plenty of space for learning and give students the opportunity to explore, create, and grow" (Boaler, 2016, p. 180).

Mindset and Procedural Fluency. Procedural fluency ensures that students have the conceptual understanding necessary to look at a problem and determine the best way to solve it in terms of accuracy and efficiency. This requires students to be open to seeing the variety of ways a problem can be solved which requires a growth mindset. In contrast, a fixed mindset sees one way of solving the problem usually related to standard algorithms. A growth mindset allows students to have the conversations necessary to realize the many avenues that can be taken to arrive at a solution while still focusing on the attributes of fluency within mathematics.

Mathematics Identity and Agency. All students have a math identity (Aguirre et al., 2013). The creation of this identity begins in elementary schools where students first establish beliefs "about the nature of learning mathematics and about their own abilities to learn mathematics" (Huinker & Bill, 2017, p. 238). It is the role of the teacher to help students develop positive identities related to math. This occurs when students understand what they are doing and as a result, identify as someone who can do math (Bay-Williams & SanGiovanni, 2021). Strong mathematical agency is "the capacity and willingness to engage mathematically" and stems from conceptual understanding especially related to ideas, relationships, and operations (Huinker & Bill, 2017, p. 68). Identity and agency become critical components of creating an environment where students will engage with math at a level necessary to develop procedural fluency by using their knowledge to solve problems that they do not necessarily know how to do (NCTM, 2014). A study related to strategy choice revealed that American students had a greater tendency to refuse to solve math problems that they did not have previous exposure to in comparison with students in Taiwan who completed the same problems (Vasilyeva et al., 2015).

Access and Equality. One of the guiding principles for effective school mathematics as outlined in *Principles to Action* is access and equity for all students. This requires “all students to have access to high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential” (NCTM, 2014, p. 59). In order for a math program to be a vision of access and equality it “requires being responsive to students’ backgrounds, experiences and knowledge when designing, implementing and assessing the effectiveness” of the overall program (NCTM, 2014, p. 60). In terms of procedural fluency, “each student is capable of developing repertoire of strategies and learning skills at applying those strategies flexibility, efficiently, and accurately” (Bay-Williams & SanGiovanni, 2021, p. 17).

In summary, the learning environment contributes significantly to the students’ beliefs and values related to their ability to engage in and continue to work through more complex mathematical tasks. As the learning environment is structured to support students in their development of mindset, identity and agency, the teacher must implore specific teaching practices that contribute to the development of procedural fluency. A great body of research supports evidence that mathematics teaching enhances the development of procedural fluency when the teacher facilitates meaningful math discourse, implements tasks that promote reasoning and problem solving, pose purposeful questions and support students in productive struggle. The subsection below summarizes each practice and its relationship to the development of procedural fluency.

Evidence of Mathematics Teaching Practices that Enhance the Development of Procedural Fluency

"An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically" (NCTM, 2014, p. 5). The integration of specific mathematics teaching practices used in tandem with one another creates the learning environment needed for students to develop the skills necessary to use procedures fluently. This is done through the intentional integration of mathematical tasks that provide opportunities to discuss a variety of strategies, promote reasoning, encourage students to explain and reflect on their thinking, and are appropriately challenging in order for students to engage in productive struggle. These ideals are described in greater detail below.

Facilitate Meaningful Math Discourse. Effective teaching "facilitates discourse among students to build a shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (NCTM, 2014, p. 29). By having opportunities to discuss, respond to and question one another, conceptual understanding is developed and reinforced. The discourse community is a necessary condition in mathematics instruction to ensure students have experiences in determining how different approaches to solve a task are similar and how they are different (NCTM, 2014). John Hattie has conducted studies to determine the effect size of classroom practices on student learning. His research found that an effect size of .4 is the point that indicates an appropriate rate of learning will occur or the equivalent of one year of academic growth (Hattie, 2011). In *Visible Learning for Mathematics*, classroom discussion or discourse has an effect size of .82 or approximately two years of growth when students have opportunities to collaborate with each other to solve rigorous tasks (Hattie, et a1, 2017). Procedural fluency is achieved when students can efficiently get the correct answer by selecting and using the most appropriate strategy. To develop students' flexibility and efficiency, math discourse must be present, so a variety of strategies are presented, discussed, and evaluated.

Implement Tasks that Promote Reasoning and Problem Solving. Tasks with high levels of cognitive demand provide students with the "opportunity to engage actively in reasoning, sense making and problem solving so that they develop a deep understanding of mathematics" (NCTM, 2014, pg. 20). In addition, Wearne and Hiebert (1994) found classrooms that emphasize a conceptual approach, using tasks with meaningful problem situations, and engaging students in discourse, showed higher levels of performance on an end of year test and more growth as measured by a pre- and post-test. As stated earlier, rigor is the balance of conceptual understanding, procedural fluency, and application. For this balance, and ultimately a rigorous curriculum within our classrooms to occur, students must have access to math tasks that can be solved in different ways, are worthy of extended discussion related to strategy use, and engage students in strategy choices related to efficiency and accuracy.

Pose Purposeful Questions. When used in a purposeful way, a well-timed question can motivate, challenge, and inspire students. The use of questions in mathematics to develop procedural fluency is a critical component. In this way, "effective math teaching relies on questions that encourage students to explain and reflect on their thinking" (NCTM, 2014, p. 35). When developing and asking questions, it is important to intentionally ask distinct types of questions in different patterns. When implemented at high levels, questions ensure that students are "reflecting on and justifying their reasoning, not just simply providing answers" (NCTM, 2014, p. 41). The design of purposeful questions can be especially beneficial when they are focused on helping students to build their conceptual understanding and strategy selection.

Relating back to figure four, the use of questions is beneficial in order to help students adapt and determine the appropriateness of the strategies selected.

Support Productive Struggle. Procedural fluency occurs when students are able to apply strategies and knowledge in new situations (NCR, 2001; NCTM, 2014; Huinker & Bill, 2017). Productive struggle refers to a student's "effort to make sense of mathematics, to figure something out that is not immediately apparent" (Hiebert & Grouws, 2007, p. 287). This can only occur if there are opportunities for students to struggle with content related to mathematical ideas and relationships. Teachers must provide time with tasks and facilitate productive struggle by asking questions instead of stepping in to do the work for the students (NCTM, 2014). Unproductive struggle, by contrast, refers to a situation in which a student "make[s] no progress towards sense-making, explaining, or proceeding with a problem or task at hand" (Warshauer 2011, p. 21). Hiebert and Grouws (2007) identified student struggle as one of the critical features in teaching that promotes conceptual understanding. Specifically, by supporting productive struggle in the classroom, teachers help students develop perseverance and stamina in solving problems.

In summary, the teaching practices discussed must be interwoven throughout the mathematics curriculum to provide students with frequent, ongoing opportunities to make sense of math and how it relates to the flexible selection of appropriate strategies to solve problems accurately and efficiently. In this subsection, the focus was on the teaching practices that relate to the development of procedural fluency. In the following subsection, the foundational supports that must be present when planning, facilitating, and assessing mathematical instruction in conjunction with the teaching practices outlined above in order to develop strong, sustainable procedural fluency will be discussed.

Foundations needed for Procedural Fluency

It has been stated that a house must have a strong foundation in order to ensure the structure that it is built on is sound. Likewise, in mathematical instruction, there are components that are imperative in creating a solid foundation for students before a structure like procedural fluency can be added. The elements below must be combined through carefully prepared instruction to create the mathematical foundation needed to engage in mathematics proficiently and develop procedural fluency.

Conceptual Understanding. As noted in the rope model in figure 2 as well as by National Research Council (2001), conceptual understanding relates specifically to the "comprehension of mathematical concepts, operations, and relations" (p. 5). Students with deep conceptual understanding "are able to use mathematical procedures meaningfully because they understand why they work and when it is appropriate to use them" (Huinker & Bill, 2017, p. 68). Procedural fluency is built on a foundation of conceptual understanding. "Conceptual understanding of any topic is developed through the use of manipulatives and other concrete tools, visuals, drawings, and connections to meaningful situations" (Bay-Williams & SanGiovanni, 2021, p. 49). In addition, when procedural knowledge and skills are explicitly and systematically built from conceptual understanding, math makes sense to students and is viewed as a connected discipline as opposed to isolated skills and steps.

State Standards and Learning Progressions. Educators must be well-informed of the state standards and how these standards specifically build towards the learning progressions needed to develop mathematical proficiency. With this knowledge, teachers can plan, implement, and assess learning tasks that promote reasoning and problem solving. These experiences help students move towards more advanced usage of both formal and student created strategies. In addition, learning progressions work to ensure that students have the conceptual understanding needed to move to procedural fluency. The use of different strategies is a cornerstone of procedural fluency development, which occurs when there are intentional opportunities for students to reflect on the use of strategies in terms of accuracy, efficiency, and appropriateness in order to support the growth of flexibility related to strategy use.

Establishment of Mathematics Goals and Purpose. Learning goals for mathematics explicitly indicate "what mathematics students are to learn and understand as a result of instruction" (NCTM, 2014, p. 12). This is a key principle of action related to effective teaching and learning identified as a way to ensure success for all students in the text, *Principles to Actions.* The goal must be aligned to the standards and learning progression and communicated with students. By determining the goal or purpose of the lesson, teachers have a better understanding of what must be achieved in order to reach this goal. This is reflected directly in the selecting of tasks, instructional strategies, sequencing, and assessment of the goal. In addition, goals provide focus and "roadmap for instruction" that should be used to "inform what we look and listen for as students engage with a task" (Huinker & Bill, 2017, p. 24). Through the establishment of a goal and purpose of the lesson, teachers can strategically focus on the five strands of mathematical proficiency to ensure an approach that is balanced and systematic. Purposeful, targeted instruction ensures that students have the skills, knowledge, and dispositions necessary to apply their understanding of concepts within the context of the five strands of mathematical proficiency.

Vocabulary Instruction. Vocabulary development is important because it helps students "develop a mindset for thinking mathematically" (Hattie et al., 2017, p. 120). A math class needs to be structured so students "talk, discuss, reason, and argue then the clear and exact language will emerge naturally" (Zager, 2017, p. 85). The use of academic language aligns with mathematical task six, which requires students to address precision. Vocabulary knowledge has an impact on student's comprehension of an academic discipline (an effect size of .67 according to Hattie et al., 2017, p. 235). This is further explained by Marzano and Pickering (2005) as they explain that vocabulary has a profound effect because "people's knowledge of any topic is encapsulated in the terms they know that are relevant to a topic" (p. 2). In addition, the vocabulary instruction must be systematic in order to be effective (Marzano & Pickering, 2005; Zager, 2017). The common math language that is developed helps students to have access to precise words when describing strategy use, their justification in using the strategy, and their conceptual understanding. In addition, as the formal language is developed and expanded, students can use these terms to describe their use of strategies and how the use of this strategy is procedurally fluent.

Strategic Competence. Strategic competence occurs based on students' "ability to formulate mathematical problems, represent, and solve them" (NRC, 2001, p. 116). Students need to understand and explain their approaches and strategies, which include traditional computational algorithms as well as student-generated strategies (NCTM, 2014). Strategic competence helps students make choices related to the efficiency, accuracy, and the overall appropriateness of the strategy. Through this process, students may use the traditional algorithm to solve problems should it be the most efficient way to do so. However, strategic competence prompts an understanding that the use of computational algorithms are not always the most efficient and other strategies developed by the students' conceptual understanding can be more appropriate.

Estimation. The development of computational estimation helps students focus on the reasonableness of the strategy and answer. Reasonableness is a critical component of procedural fluency because it helps students to determine the efficiency of the particular strategy and its impact on accuracy. The use of strategies related to place value concepts provides students with a tool to establish reasonableness. Students should be encouraged to use flexibility as they access strategies and decide which strategy is most efficient. Computational estimation can be developed by rounding to estimate (the use of rounding before solving a problem or instead of computing the problem), front-end estimation, compatible numbers, and range (Bay-Williams & SanGiovanni, 2021). Estimation needs to be practiced "frequently and consistently" because "estimation is thinking" (Bay-Williams & SanGiovanni, 2021, p. 72).

In summary, the teacher is responsible for the implementation of learning experiences that systematically contribute to the formation of a strong foundation related to mathematics. Only when this foundation is in place, can students begin to look for ways to solve problems more accurately and efficiently. Once a learning environment has been established that supports students in building foundational concepts needed for procedural fluency through the use of effective teaching practices, teachers can begin to incorporate targeted instructional approaches that support the development of procedural fluency. Instructional approaches that have been shown to develop procedural fluency are discussed in the following subsection.

Instructional Approaches that Support the Development of Procedural Fluency

In the previous subsections, there has been a focus on more generalized conditions that contribute to the overall development of procedural fluency. In the subsection below, the focus shifts to specific, targeted instructional approaches that can be integrated by mathematics teachers to enhance students' procedural fluency. These approaches provide the explicit and systematic instruction and practice needed to develop procedural fluency.

Strategy instruction. Students need direct, explicit instruction related to strategies, especially in relation to their efficiency. The teacher must intentionally plan instruction to ensure the **development** of how a strategy works and when to use it. Equally importantly, instruction must also **practice** how a strategy works and when to use it (Bay-Williams & SanGiovanni, 2021). Strategy instruction should relate to each component represented in figure four to ensure procedural fluency is developed methodically.

Visual Representations. One way that teachers can support the use of appropriate strategies is through the purposeful integration of visual representations. Visual representations can help clarify students' thinking and follow the mathematical thinking of their classmates (Huinker & Bill, 2007). Representations can highlight specific aspects of a mathematical concept allowing students to make better sense of the structure of the mathematics. Representations provide students access to abstract mathematical ideas and ultimately the opportunity to learn to carry out procedures flexibly, accurately, efficiently, and appropriately.

Quality Fluency Practice. Primarily, quality fluency practices must be balanced, engage students, and attend to all six fluency actions. These actions include, "(1) selects an appropriate strategy, (2) solves in a reasonable time, (3) trades out or adapts strategy, (4) applies a strategy to a new problem type, (5) completes steps accurately, and (6) gets correct answer" (Bay-Williams & SanGiovanni, 2021, p. 130).

Quality fluency practice must be focused, varied, processed, and connected. Focused fluency practice provides students with multiple opportunities to gain experience in how to use a strategy in isolation and get the correct answer. Varied fluency practice, or practice that is changed by cognitive demand, focused on the component of fluency and type of engagement, "help students understand the strategy, use the strategy, and know when to choose the strategy (Bay-Williams & SanGiovanni, 2021, p. 134). Quality fluency practice requires opportunities for students to process and reflect which offers closure for the experience (Bay-Williams & SanGiovanni, 2021). Fluency practice is connected when the teacher strategically plans so that students can see the relationships and connections.

Before fluency is practiced, "students need a secure understanding of the concepts and processes inherent in the strategy" (Bay-Williams &SanGiovanni, 2021, p. 138). There is no set amount of time that guarantees fluency for all students. As a result, as fluency is planned, attention should be paid to the purpose, the students' familiarity with the strategy, and time needed for the child to develop automaticity. Regular opportunities for quality practice after a strategy has been learned can involve routines, worked examples, games, and centers.

Routines. Routines are an instructional practice that purposefully sets time aside for thinking, problem solving, and discussion. When determining specific routines to enhance procedural fluency, it is important for the teacher to focus on when the routine will be used, how often the routine will be used, and how the routine will be facilitated (Bay-Williams &SanGiovanni, 2021). In a study conducted by Rajotte Marcotte and Bureau-Levasseur (2016), the researchers concluded that the use of daily routines in math increased students' extrinsic motivation when it came to solving problems.

An example of a routine that could be used within a classroom learning environment is number talks. Numbers talks are a daily activity that can be used with students related to the use of a variety of strategies to solve problems mentally. Using this as a routine promotes structure related engagement, participation. and celebration of the use of multiple strategies. "As students engage in number talks, they explain the mathematics behind their thinking, using and connecting strategies flexibly while striving for efficiency" (Berger, 2017, p. 6).

Worked examples. Worked examples are problems that demonstrate a step-by-step solution to a problem or task and can be posed in three ways (see table three). This focused instructional method provides students with purposeful opportunities to discuss "why a strategy works and when a strategy makes sense" (Bay-Williams & SanGiovanni, 2021, p. 144). In addition, worked examples can build procedural fluency because there is a clear focus on efficiency, accuracy, and flexibility (Bay-Williams & SanGiovanni, 2021; Hattie, Fisher, & Frey, 2017; Sweller, 2016; Atkinson, Derry, Renkl, & Wortham, 2000). According to Hattie, Fisher, and Frey (2017), using worked examples in the classroom once students have a strong conceptual understanding can have an effect size of 0.57 in relation to influence on student learning. One teaching strategy outlined by Boaler (2016) focuses on teachers identifying their favorite mistakes (also sometimes referred to as favorite no) utilizing student work. The teacher will select work with conceptual mistakes and use this to begin a discussion about the error in a way that celebrates the mistake as an opportunity for the brain to grow.

Table 2.3
Types of Worked Examples (Bay-Williams & SanGiovanni, 2021)

Type of Example	Fluency Focus	What Occurs
Correctly Worked Examples	Efficiency Flexibility	*Selects Appropriate Strategy *Applies a New Strategy to a New Problem Types
Partially Worked Examples	Efficiency Accuracy	*Selects Appropriate Strategy *Completes steps accurately; gets correct answer

Incorrectly worked Examples	Accuracy	*Completes steps accurately; gets correct answer

Games. The use of games related to practicing fluency can promote many of the focuses of fluency as well as increase student engagement. However, there are specific considerations that must be made when utilizing games in this manner in order to ensure accuracy and high levels of rigor. In order for games to promote the development of procedural fluency, they must include opportunities for students to select and make choices about the best strategy to use related to efficiency. When implementing fluency games, there must be a clear focus, accountability built-in, and opportunities to check for accuracy (Bay-Williams & SanGiovanni, 2021).

Centers. The use of centers is appropriate for any grade level as they set up a physical space in the classroom that promotes engagement, challenge, and motivation. These centers can be individual or collaborative in nature depending on the purpose of the centers. Center activities could include "sorting tasks, choice problems, or games that can be played independently (Bay-Williams & SanGiovanni, 2021, p. 150).

Assessment. "Assessing fluency means attending to all three components of fluency and ensuring that this attention is visible to students (and parents or caregivers) to communicate the real meaning of (and goal of) procedural fluency" (Bay-Williams & SanGiovanni, 2021, p. 154). Traditional fluency tests only measure accuracy, which contributes to the belief that fluency is simply the memorization of facts and the speed in which they can be recalled. The integration of checklists and rubrics is one way to ensure that all components of fluency are being assessed and more importantly that students are receiving timely, specific feedback based on their progress related to procedural fluency.

In summary, quality fluency practice, visual representation and tools, and strategy instruction provide students with the multiple opportunities to manipulate content that develops their ability to view the use of strategies through different lenses in order to determine the best strategy as it relates to accuracy and efficiency. Although there are clear conditions that support the development of procedural fluency, the focus on appropriate fluency practices is not present in all elementary classrooms. The following section will examine the challenges related to implementing fluency practices.

Challenges Implementing Fluency Practices

There is a strong body of research and evidence-based practices related to quality math instruction including the utilization of a balanced, connected approach that focuses on conceptual understanding and procedural fluency which has been in circulation for the last twenty years. The National Council of Teachers of Mathematics has been advocating for a vision of learning where students are active learners, students have opportunities to construct their knowledge of mathematics through exploration, discussion, and reflection for over twenty-five years with specific policy changes (NCTM, 1989, 2000, 2006, 2017). However, the integration of these principles is not wide-spread or observable in most elementary classrooms in the United States of America. In fact, the National Research Council reviewed educational research studies that involved surveying teachers and the utilization of observational studies (both in person and recorded) to determine what practices were occurring in the classroom. The results from the observations that have been compiled for more than a century determined "that the core of teaching- the way in which the teacher and students interact about the subject being taught- has changed very little over time" (NRC, 2001, p. 48). This relates to all subject areas and is common in all parts of the country. The most ordinary form of teaching in the United States is known as recitation as a cycle where the teacher presents information, asks questions, the students respond usually with one word and one at a time, and the teacher provides feedback in the form of right or wrong. This continues until all the material for the lesson has been covered and then students typically work independently on an assignment that requires them to practice skills from the earlier lesson (NRC, 2001). This learning cycle is summarized as extensive teacher-directed explanation and questioning, followed by student seatwork on paper and pencil assignments (Welch, 1978). The lack of change in the elementary classroom mathematics instruction has been consistently demonstrated in studies over time (Hiebert et al., 2003; NACOME, 1975; Weiss, 1977).

There are several factors that contribute to this perpetual cycle of mathematics education. One cause identified in several texts is that mathematical instruction is often based on personal beliefs and values related to math based on our own accumulated experiences in math classes instead of research. As a result, teachers often teach math how they were taught, which typically resembles recitation (Hattie et al., 2017; Zager, 2017). This default can be attributed to the fact that “many teach math with their own fear of the subject” (Boaler, 2006, p. 8) and McAnallen (2010) “found that 33% of elementary school teachers (224 of 678 respondents)” reported to have math anxiety (p. 36). The study also found that factors that contributed to their anxiety according to the respondents was lack of conceptual understanding, limited ways to solve problems, learning the rules of math (but not the why), and negative teacher interactions (McAllen, 2010). Jackson and Legginwell (1999) found that only 7% of preservice teachers (11 of the 157 of those surveyed) “used positive language to describe their experiences as math students” based on a study conducted using written responses to a prompt (p. 583).

Another contributing factor is the teacher's fixed mindset related to the capabilities of students to use math at higher levels. This often results in teachers providing step-by-step instruction focused on the procedure instead of developing conceptual understanding. Students are spoon fed procedures when they are "shown or told how to solve problems and not encouraged to employ their own reasoning or strategy" (Bay-Williams & SanGiovanni, 2021, p. 17). Unproductive beliefs related to students' ability to interact with higher level math results in lower expectations and narrower curricula (NCTM, 2014).

During the accountability and standards movement, there has been focus on high stakes testing that typically focuses on more basic mathematical concepts. Due to the importance placed on standardized testing in the United States, this accountability measure puts pressure on teachers to rush to speed and accuracy over conceptual understanding. In addition, there must be a balanced approach to the math curriculum that involves all five stands, but typically more is devoted to more isolated instruction due to the structure of current testing at the state and national levels.

The comprehensive report of how children learn math in *Adding It Up*, finds that "very few teachers currently have the specialized knowledge needed to teach mathematics" as it has been envisioned (NRC, 2001, p. 428). In a review of over 1,300 studies, the Regional Educational Laboratory at Edvance Research (2007), found that professional development affects student achievement only when professional development enhances teacher knowledge and skill, and this new knowledge is applied to classroom teaching based on the motivation, beliefs, and ability to apply the professional development of the teacher. The time, money and resources dedicated to professional development cannot always ensure that these experiences are provided at the depth needed to impact the factors that impact the translation of new knowledge into classroom practices. In fact, “teachers frequently feel as though professional development is something that is done to them instead of something done for them” (NCTM, 2014, p. 101). When teachers are not actively engaged and involved in professional development that is based on their needs, classroom practice is not impacted.

In summary, due to the factors listed above, there is not a conscious and widespread knowledge of how procedural fluency is and should be developed in the classroom. We know that the effectiveness of the teacher has the greatest impact on student learning (Marzano, 2010). In order to address the implications and challenges that prevent fundamental change on what is taught in mathematics and the learning experiences that determine the complexity in which students are able to engage with mathematical concepts, there must be a more comprehensive acknowledgement of what teachers know and understand in terms of fluency and how this knowledge and understanding is translated into classroom practices.

Chapter Summary

Through the course of mathematics education in the United States, there have been two major schools of thought with contradictory beliefs related to the teaching and learning of mathematics that have dictated reform efforts. These views can be summarized as a focus on the basic computation of mathematics through fact fluency and the understanding of mathematical concepts through a constructivist model approach. However, each extreme view has proven to cause disenchantment with the current status quo and a new era of reform. As a result, there has been extraordinarily minor change related to how mathematics is actually taught in elementary classrooms across the nation due to several challenges including teachers’ values, beliefs and mindset related to math, a focus on high stakes testing, and a lack of the content and pedagogical knowledge needed to facilitate high quality math instruction.

Through the development of procedural fluency, students can have a deep understanding of mathematical concepts and the skills necessary to solve a variety of problems. Instead of conflicting views of mathematical education, procedural fluency provides a balance of procedure and understanding to support students when engaging in mathematical tasks that develop the skills, knowledge, understanding, and dispositions needed to be proficient mathematically. Procedural fluency is developed through several conditions including the learning environment, teaching practices, the intentional focus on the foundational components needed for procedural fluency, and the use of evidence-based teaching approaches. When students are procedural fluent, they can solve problems using a variety of strategies flexibly, appropriately, accurately, and efficiently.

In order to ensure procedural fluency is incorporated into math instruction effectively, it is important to determine how teachers' current understanding and skills related to procedural fluency translate into classroom practices. The next chapter will provide an overview of the methods used to answer the research questions as well as rationale for the design of the study.

Chapter 3 - Research Methodology

Chapter Introduction

The purpose of this study was to gain contextual, concrete, and in-depth knowledge about fluency practices occurring during core mathematical academic time in elementary classrooms. This chapter explains the methodological framework and research design that were used to examine what teachers know and understand about fluency and how this translates into classroom practices. The chapter begins by stating the research question and subquestions, followed by the design of the study, the participants and setting, and concludes with the steps of the research procedure including a description of the instruments, data collection and data analysis.

Research Question

This study sought to answer the following research question and subquestions:

- How does what a teacher knows and understands about procedural fluency translate into classroom practices during core instruction?
    - What does a teacher know and understand about computational fluency in mathematics?
    - In what ways does a teacher plan for computational fluency development?
    - In what ways does a teacher explicitly connect conceptual understanding to computational fluency?
    - What does a teacher do to ensure the development of procedural fluency in the classroom?
    - What does fluency practice look like in the classroom?
    - What components of fluency do the practices present in the classroom address?

Design of Study

A qualitative study, empirical in nature, was selected as the research method for this study. This study was designed to determine what teachers know and understand about procedural fluency in mathematics, first through the use of a survey and an interview. Then, to study how this knowledge translates into practice, data was collected through classroom observations during mathematics lessons that occurred over the period of three class periods, a debriefing conservation after each observation, and a reflection conversation after all three observations had occurred. In order to understand the application in the classroom, the actions of the teachers, not just what they articulated in a survey or interview, had to be considered. The data was analyzed using qualitative research methodology. It was critical that procedures were implemented intentionally and systematically in order to ensure rigor and quality throughout the study. The use of qualitative research allowed the researcher to understand the justification behind the responses provided, and the design added additional depth to the context of the study.

Case Study

The data was collected using a multiple case study design with converging evidence from multiple sources. The convergence of evidence offered the triangulation of data and strengthened the construct validity of the case studies (Yin, 2014). For this study, a case was defined as an individual teacher and each teacher was included in the multiple case study as modeled by *Adaptive Teaching in Literacy Instruction: Case Studies of Two Teachers* (Parsons, 2012) and reinforced in *Case Study Research and Application: Design and Methods* (Yin, 2014). In order for the case studies not to become too broad, it was important to bind the case (Baxter & Jack, 2008). The case was binded by place (a single school), time (established observation blocks as well as number of observations, and geographical location (Kansas). In addition to the format of multiple case studies, the case studies were instrumental in nature because the case or teacher "is of secondary interest, it plays a supportive role, and facilitates our understanding of something else" (Stake, 2000, p. 437). In this instance, the case studies were meant to describe the circumstance of how knowledge and understanding was translated into classroom practices rather than the study of each individual teacher.

A multiple case study approach allowed for the study of this occurrence in different contexts (Stake, 2006). Since there were multiple cases included, it provided an opportunity to examine individual data by examining within case themes and cross data analysis of all the cases to determine similarities and differences (Creswell, 2007). This design complimented the need for classroom observation in order to determine what was truly occurring in the classroom, especially in conjunction to how knowledge and understanding translated into classroom practices. In addition, case studies are used to answer how and why questions as they relate to the research questions (Yin, 2014). The incorporation of a case study in the methodology allowed for direct observation and ability to interact with participants to provide a richer insight to transfer of knowledge and understanding in the classroom (Merriam & Tisdell, 2016). Overall, designing the study using a multiple case format allowed for in-depth analysis related to the research question as well as the subquestions.

Participants and Setting

Participants. The participants in this study were six teachers from same elementary school. One teacher taught kindergarten, two teachers taught second grade, one teacher taught fifth grade, one teacher taught sixth grade, and one teacher provided tiered instructional support for all grades levels in the building. The participants were selected using convenience sampling based on the number of respondents (Merriam & Tisdell, 2016). Each participant volunteered to participate after the principal of the school discussed the study with the staff in its entirety during a staff meeting and sent out a message with specific details related to the study from the researcher.

Setting. The setting of the study was a Title I elementary school in a small town located near the center of Kansas. The town population was around 1,040 community members. The school was a pre-kindergarten through sixth grade school with approximately 230 students and thirty staff members (certified and classified). There were two sections at each grade level with a student to teacher ratio of approximately 16 to 1 depending upon the enrollment for each grade level. The school demographics showed that a majority of the students are white (87%), followed by Hispanic (9%) and remaining students are represented in all other ethnic groups (4%). A total of 55% of students are from low-income families.

There was a different instructional block of time (kindergarten through fourth grade classes have approximately 50-60 minutes while fifth and sixth grade classes are scheduled for 90 minutes) each day for students dedicated to engaging all students in learning activities related to the mathematical standards by grade level. In addition, there was a 30-minute block of time each day to provide tiered mathematical instruction based on student readiness. Student readiness was determined by a universal screening that is given to all students three times each year and ongoing progress monitoring throughout the school year. The resource used in the school was iReady Math which was used with varying levels of fidelity by grade level. Teachers used supplemental materials found from various sources (mostly online) to address mathematical concepts when the district provided resource was not utilized.

Procedures

Description of and Rationale for Procedures

Step One: Participants completed a survey. Each participant completed an electronic survey with a combination of close and open-ended questions. The survey approach was imperative for the quick turnaround related to the collection of data. Data based on the participants' responses on the survey was needed to create and conduct a semi-structured interview. In addition, the completion of a survey by each participant was critical to the study of the research question because the responses were analyzed to determine the teacher's current knowledge and understanding of procedural fluency. The open-ended nature included in the survey allowed for a more in-depth analysis of the knowledge and understanding teachers had related to procedural fluency.

Step Two: Researcher conducted semi-structured interview with each participant. The researcher conducted an interview with each individual participant in order to gather background related to their personal experiences as a math student and as a math teacher. This information was critical in determining how procedural fluency was likely to be implemented and supported in the classroom based on these experiences. The interview ensured that the researcher had a comprehensive view of each individual participant's level of knowledge and understanding. A rubric was completed to determine the overall knowledge and understanding related to fluency for each participant.

Step Three: Researcher observed math lesson and used field notes to complete rubric. The next step procedurally was for the researcher to observe the whole group instructional block of each participant for a total of three consecutive lessons over the course of three school days. The researcher used a template for taking field notes related to observable teacher and student actions during each lesson observation. The lesson was recorded in its entirety. The recording of each lesson was viewed to ensure the field notes were accurate and fairly represented the level of alignment of the instruction to the components needed for fluency development.

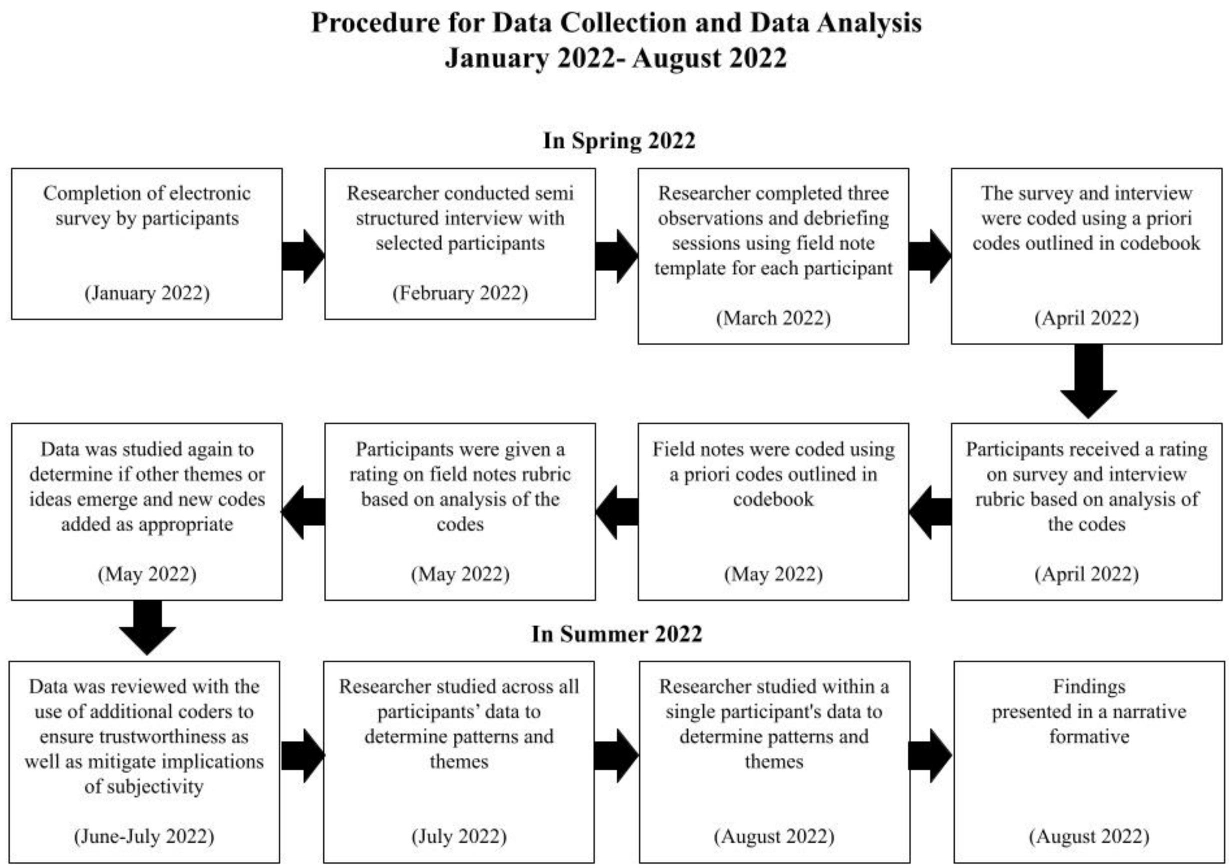
The classroom observations were essential to determine how what teachers know and understand was represented in the classroom through instructional practices, student interactions, and the use of instructional time. Each math lesson was observed in its entirety; however, only components related to development of procedural fluency were analyzed. The information compiled in the field notes and verified through additional viewings of the lesson was used to complete the rubric. The rubric provided a score that was used to determine the alignment of the participant’s instructional practices as they relate to practices that support the development of computational and procedural fluency.

Step Four: Debriefing. The researcher met with each participant after the observation with the purpose to debrief the lesson was taught. The debrief conversation was designed to include a semi-structured interview. The interview was structured with specific questions to gather additional information in relation to how the teacher believed fluency was addressed in his/her classroom during lesson, what was done before the lesson to prepare for the development of procedural fluency, and what will be done in the future. In addition, the researcher provided feedback to continue to develop the teacher’s knowledge and understanding related to procedural fluency. The feedback was provided using the principles and structure of coaching conversations based on the work of Jim Knight. These principles include equality (belief that coaches and teachers are equal partners), choice (options are provided related to what and how they learn), voice (professional learning empowers teachers), dialogue (professional learning creates opportunities for authentic conversations), reflection, praxis (application in real context), and reciprocity (expectation to get the same as what is put in).

The questions were created using the SUN protocol (Sexton, 2019) where the coaching conversation focused on summarizing (what occurred during the lesson), unpacking (what led to specific actions and responses during the lesson), and new learning and direction setting (next steps were agreed upon for subsequent lessons). The summarizing and unpacking components were completed during the debrief of the first two lessons. The summarizing, unpacking, and new learning and direction setting components were completed after the third lesson as a comprehensive debrief of the final lesson. The final debriefs also provided an opportunity to discuss all three observations in tandem. Responses were recorded on the field note template and used as a piece of evidence as the rubric for each participant was completed.

Figure 3.1

Procedure for Data Collection and Analysis



Instruments and Data Collection

Data was collected at each phase of the study in order to determine trends related to teachers' knowledge and understanding of procedural fluency and how it translated into classroom practices.

Surveys. The primary purpose for the survey was to determine what teachers know and understand about fluency. The survey also provided information related to how the participants believed they were currently addressing fluency instruction in the classroom.

Design. The survey was designed with a combination of open-ended (four) and closed question types (seven) with a total of eleven questions. The first five questions were designed to gather demographic information related to experience, current grade level, and confidence level related to math instruction. Questions six through nine were open-ended and designed to gather information related to what teachers know and understand about procedural fluency as well as what fluency instruction currently looks like in their classrooms. Questions ten and eleven were meant to ensure that teachers have shared all the information necessary to accurately determine the level of knowledge and understanding each teacher has in relation to fluency development.

Protocol. Each participant was emailed the link to survey as well as description of the purpose of the survey. Participants completed the survey electronically and used a naming protocol, so their identity remained anonymous to everyone outside of the researcher. The researcher used information from the survey to structure and drive the questioning of the semi-structured interview. For this reason, it was necessary to have a way to link the survey back to the respondent.

Interviews. The interview questions were an extension of the survey questions. Virtual semi-structured individual interviews were conducted in order to elicit more comprehensive and in-depth information on how teachers think about the application of their knowledge and understanding of concepts as they plan and facilitate lessons. The open-ended nature of the interview questions facilitated the ability to build on participant's responses to complex issues in order to explore his or her knowledge and understanding of procedural fluency at a deeper level. The use of interviews was also beneficial to determine what the teacher believes constitutes the development of procedural fluency in the classroom. Yin (2018) explains that "Interviews can be especially helpful by suggesting explanations (i.e. the “hows” and “whys”) of key events, as well as the insights reflecting participants’ relativist perspective” (p. 118).

Protocol. Each case study participant engaged in a remote interview using the platform Zoom in order to gain additional information related to their experiences and knowledge in the area of procedural fluency. Each participant was asked five experience and behavior specific questions. This was done to provide the researcher with a historical perspective as well as a better understanding of the participants’ actions, behaviors, and activities related to fluency practices (Merriam & Tisdell, 2016). Additional questions were asked based on the participants' responses on the survey. The interviewer used an interview guide, which was tailored to match the responses from the survey of each participant. The interview guide, which was utilized by the researcher, was a modified version of the sample interview protocol provided on page 191 of *Research Design: Quantitative, Qualitative, and Mixed Method Approaches* (Creswell & Creswell, 2018). The generic interview guide that was differentiated for each participant can be found in appendix C.

Data Collection. The interview was recorded using the record feature in Zoom. This ensured that the interview could be accurately transcribed after the interview. During the facilitation of the interview, the researcher also took short notes based on the participant’s responses that were used to go back to certain points of the interview that were especially interesting or helpful in determining the level of knowledge and understanding of each participant. The interview was transcribed using the online application, Grain. The researcher then reread the transcript while watching the interview and made any revisions/corrections necessary to the document to ensure the accuracy and integrity of the transcript.

Observations. The role of the observer in this context was that of an observer as a participant. With this role in mind, the gathering of information and the activities of the observer were known to the group (both teacher and students). According to Merriam and Tisdell, "Regardless of the stance, an investigator cannot help but affect and be affected by the setting, and this interaction may lead to some distortion of the situation as it exists under non research conditions" (p. 161). In order to minimize this distortion, it was important for the teacher and students to know when the observations were and provided an opportunity to prepare accordingly.

Protocol. Each observation was focused on the physical setting, participants, actions and interactions, conversations, subtle factors (such as nonverbal communication), and the behavior of the teacher and students (Merriam & Tisdell, 2016). The information related to what was observed was recorded using a field notes template. The use of field notes as a collection tool meant that the gathered data from each observation was descriptive in nature. The field notes were divided into two sections. The purpose of the first column (which was split into two parts) was to record descriptive notes related to the actions of the teachers and students. The descriptive notes included the specific information noted above for the teacher and students. The second column provided a place for the researcher to record reflective notes. The reflective portion included the observer's comments in regard to "researcher's feelings, reactions, hunches, initial interpretation, speculations, and working hypotheses" (Merriam & Tisdell, 2016, p. 151). The field note template was a modified version of observation protocol template from Creswell (2007). The template can be found in appendix D. A concentrated effort was made to ensure full notes in a narrative format were constructed as soon after the observation as possible.

Data Analysis and Scoring of Surveys, Interviews, and Observations

Data Analysis. At the conclusion of each round of data collection, surveys, interviews, and observations were coded using a priori codes. These codes were developed based on what was available in the literature and research related to procedural fluency. The use of deductive coding was selected because it allowed for the efficient identification of relevant data. In order to avoid bias and miss valuable insight as a result of the predetermined codes, the data was analyzed several times to determine if additional themes or ideas emerged. The purpose of these additional reviews was to add new codes, if necessary, to comprehensively represent the data collected.

As each data source was collected, the researcher analyzed the data in two stages. In the first stage, the purpose of the coding process was to study the data available to get a general overview and understanding of the data collected. The codes from the codebook were used to code evidence within the data of knowledge, understanding, or application of instructional practices that supported the development of procedural fluency. In vivo coding and process coding were also important during this stage of analysis since examples of the participant's own words and actions were used when the data was coded and analyzed (Manning, 2017). In the next stage, the researcher conducted line by line coding in order to examine the data at a more in-depth and comprehensive level. Examples from the data were included in the codebook, which created sources of organized information that were more efficiently studied for themes within and across the cases.

Based on the data included in the codebook, the researcher worked to identify, clearly articulate, and develop themes. The development of themes created meaning from the data and started to create the narrative of what the data was demonstrating.

Scoring. The rubrics were constructed based on the information in the literature review in regard to what effective fluency development entails. A rubric was used to determine the knowledge and understanding level of each teacher related to fluency. A second rubric was used during each observation in order to determine how the teachers' knowledge and understanding was translated into their classroom practices. The use of both rubrics created an overall rating for each case study participant related to their knowledge and understanding of procedural fluency and an overall rating related to what procedural fluency development looked like in practice (based on classroom observations). Each element of the rubric was linked to the research questions and codebook to ensure proper alignment in data collection and analysis procedures.

Case Study Data Analysis

The data collected was analyzed in two ways. First, the researcher studied across all cases to determine patterns and themes. Then, the researcher studied within in each individual case to determine if growth or change in relationship the integration of effective classroom practices needed to develop procedural fluency were present.

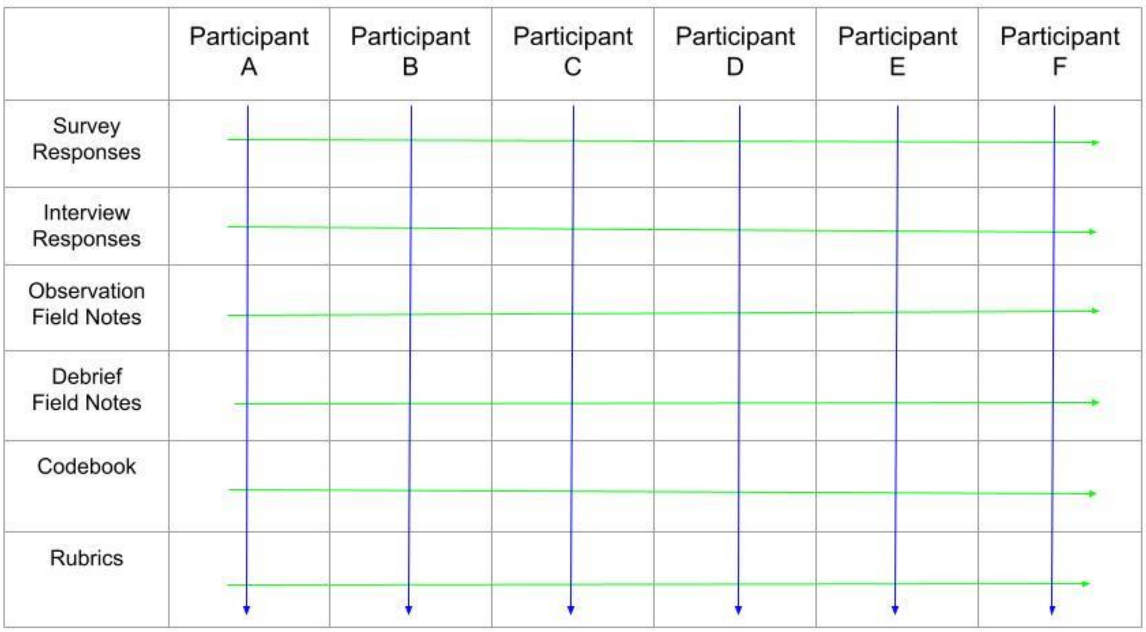
The findings were checked using data triangulation based on the purposeful integration of a variety of methods to collect data (Henning & Roberts, 2016). In this study, surveys, interviews, and field notes were used to determine the ratings on the rubric as well as to develop the overarching themes for each case study individually and all case studies as a whole. In addition, the use of different data sources allowed the researcher to use several data sources to build justification for the overall themes (Creswell & Creswell, 2017).

Presentation of Findings

The findings of the study were presented in two ways in chapter four. A summary of the data for each case and the corresponding findings were written in a narrative format. These findings also described connections within each case study as represented by the blue arrows in Figure 3.2. The findings were also presented in terms of themes that emerged based on all the cases as represented by the green arrows in Figure 3.2.

Figure 3.2

Presentation of Findings



Trustworthiness

The trustworthiness of data content analysis depended on the inclusion of rich and well-saturated data. Credibility, dependable, transferability, and confirmability are key aspects of qualitative research (Creswell, 2009). This study achieved trustworthiness by implementing multiple validity strategies as recommended by Creswell (2009): (1) triangulation of data sources, (2) use of rich, thick descriptions to convey findings, (3) inclusion of negative findings, and (4) prolonged time in the field by the researcher.

The triangulation of data sources was achieved by the convergence of different data sources of information, which added validity of the study. Rich, thick descriptions were used to describe findings, illustrate the different perspectives of the participants, and provide realistic insight into how procedural fluency knowledge and understanding translated into classroom practice. In qualitative research, the researcher is the key instrument for collecting and analyzing data (Creswell, 2009). The researcher responded to the environment and adapted circumstances as appropriate during the data collection phase of the study. In addition, the researcher processed data promptly and can explored anomalous responses in a timely fashion because of this. The role and background of the researcher ensured the credibility of the study.

Another technique that was utilized by the researcher to ensure trustworthiness of the study was the use of multiple raters. Multiple raters (two in total) brought different perspectives to the interpretation of data in order to help mitigate the implications of subjectivity in qualitative methods. A training was held with both raters to share the overview of the study and help familiarize them with data collection and analysis tools. Each rater used the codebook to code the survey, interview, and fields for a case. The researcher and both participants met to discuss the codes that were identified as well as the data used to support the label of each code. If there were any discrepancies, the reasoning and data was discussed in detail. In each instance where there were conflicting views of the coding, consensus was reached through discussion. The process was repeated for a second case in order to ensure alignment of the research and second raters' understanding and interpretation of each code. In some cases, it was necessary to use the descriptions found in the codebook to determine if that data aligned with the assigned code. For the final meeting, each rater was given two additional cases to code. The researcher met with each rater to compare the findings. From the multiple rater experience, it became apparent that a follow-up interview was needed to get additional details related to classroom practices marked in question eleven of the survey. A follow-up interview was conducted with five of the six participants to gain additional insights into the responses they provided on the survey.

Chapter Summary

The design of research stemmed from the use of procedures and tools that best fit with research questions. Each type of data supported the triangulation of data sources to ensure the portrait of each teacher in the case study has a narrative that is an accurate, comprehensive view of his or her knowledge of procedural fluency and how this knowledge is present in classroom practices. In the next chapter, the findings of the study will be shared.

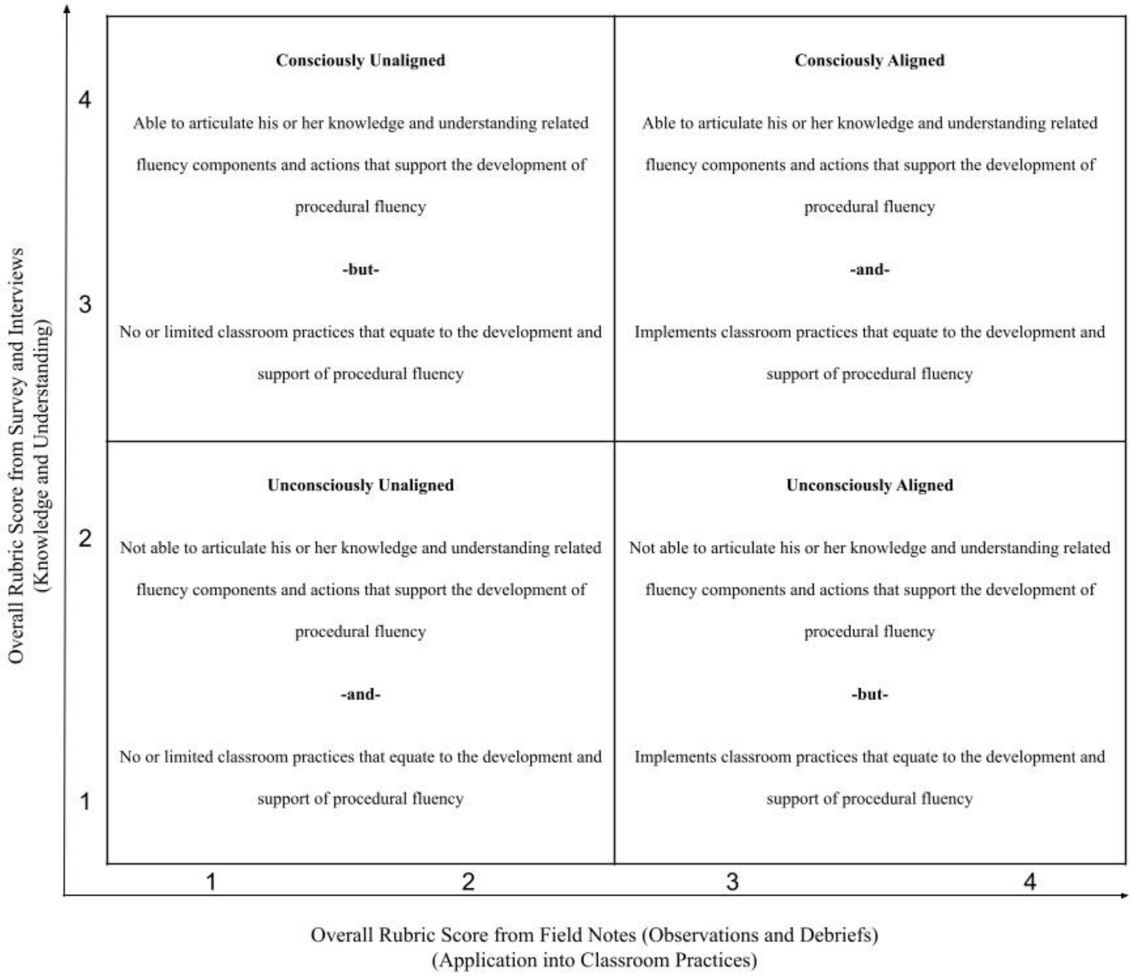
Chapter 4 - Presentation of Findings

Chapter Introduction

The purpose of this study was to develop a contextual, concrete, and in-depth knowledge about fluency practices occurring during core mathematical academic learning time in elementary classrooms. The findings articulated in the following sections are based on the analysis of the data collected through a survey, interviews, observations, and debriefing conversations. In order to explain the findings for the overarching research question, *how does what a teacher knows and understands about procedural fluency translate into classroom practices during core instruction?*, each participant was placed into one of four categories. These categories included unconsciously unaligned, consciously unaligned, unconsciously aligned, and consciously aligned. The categories are described in greater detail in Figure 4.1. The numbers on each axis represent the scores on the two rubrics, which can be found in appendix H and appendix I. This rating will demonstrate their knowledge and understanding of procedural fluency and the level of impact this knowledge and understanding has on classroom instructional practices that support and develop fluency components and integrate fluency actions.

Figure 4.1

Stages of Competence Model (revised) with Correlation to Rubric Scores



The chapter is organized by case study. The findings related to the over-arching research question as well as each subquestion is discuss for each case. Each narrative case description also highlights themes that emerge within the case. The chapter concludes by presenting the findings and themes from across all six cases. As each case is described, there are parts of particular cases that have sections that read very similarly to one another. The fact that the school is using a district provided curriculum resource at all grade levels and the short time frame in which the observations, resulted in similar lessons being observed from classroom to classroom.

Case A

Introduction

Participant A has served as the title math teacher for thirty of the thirty-six total years of his/her career. In this role, the teacher serves students in kindergarten through sixth grade using small group instruction to pre-teach skills and address potential holes in their learning. In terms of the confidence level rating related to the teaching of math at the assigned grade level, Participant A provided a self- identified ranking of four because of the support and training that the teacher has received over the course of his/her career from a colleague. There were several discrepancies in the collection of the data for this case. The survey was completed after the interview and due to scheduling conflicts only two observations and debriefing sessions occurred. In addition, the teacher retired at the end of the school year and was unavailable for a follow-up interview. The participant was determined to be unconsciously unaligned to fluency practices based on the data collected, analyzed, and shared below. In addition, the rubric score for each component is recorded in the tables that follow for Participant A.

Knowledge and Understanding of Computational Fluency in Mathematics

Participant A's knowledge and understanding of fluency focused on the importance of accuracy when solving problems. The participant indicated that "independent work and accuracy are good indicators" to determine a students' level of skill related to a particular procedure. The participants' response during initial interview confirmed the focus of accuracy within his or her classroom practices. The participant explains that he or she sometimes uses songs to help students learn basic facts. However, there is one student who always skips four times seven, which causes the rest of the song to be incorrect. In this instance, the participant shared that it is important that students know how to solve a problem correctly and not rely on other mechanisms. There was no indication of knowledge related to the need for students to solve procedures in a reasonable time, know multiple strategies that students can apply or adapt to solve a procedure, or select the appropriate strategy in relation to efficiency and flexibility. This, combined with the indicators of purposeful planning (described below), resulted in a score of 0.50 on the rubric.

Explicit Connection of Conceptual Understanding to Computational Fluency

Participant A was able to articulate what conceptual understanding was through an example of his/her own schooling experience. In this example, the teacher explained completing steps and processes without understanding the why behind it made it so he or she did not have a deeper understanding of the procedure. Although participant A was able to articulate what conceptual understanding is as well as how procedural fluency is built from conceptual understanding, the participant's responses in the survey and interview did not show evidence of using this information for pacing. This resulted in an overall score of 2.00 for this subquestion based on the rubric.

How Computational Fluency Development is Planned For

Participant A plans for fluency development through the use of varied practice. In the survey, the participant indicated that "using several different techniques is usually more effective" and went on to list examples including "computer practice, flash cards, and games." In terms of planning for quality fluency practice the survey and interview responses did not indicate that the learning activities utilized were focused, processed, connected, or balanced. In addition, the learning activities did not address any of the fluency actions. This resulted in a rubric score of 0.33 for this subquestion.

What the Teacher Does to Ensure the Development of Procedural Fluency

Explicit Strategy Instruction. In terms of strategy-based instruction, participant A indicated during the debrief that he/she focuses on strategy instruction during small group instruction. The observations and subsequent debriefs did not indicate that strategy instruction in the classroom was explicit, intentionally planned, or provided opportunities for students to generate their own strategies for solving problems.

Visual Representations and Tools. Participant A included visualization and connection in the lessons facilitated for the students. Pictorial representations were used to help students to visual fractions. Visualization was also used during both observations to help students compare fractions as well as find equivalent fractions. A specific example of this occurred during the second observation, the teacher drew shapes broken into equal parts. There was also a purposeful connection to the visual to illustrate the relationship between the numerator and denominator as evident in the second observation. The teacher and students created pictures to represent different fractions to review the meaning of the terms numerator and denominator focusing on what each part of the fraction represents.

Quality of Fluency Practices. Participant A provided students opportunities to practice using learning activities that were focused. The use of focused practice helped students practice accuracy by learning to use a practice and get the correct answers. This was evident in observation one when the teacher prompted students to divide their pictures into equal parts in order to accurately compare fractions. In the second observation, the use of the review game helped students to practice with a focus on accuracy. There were no other examples that would indicate learning activities that were varied, processed, connected, or balanced. The learning activities also did not require students to attend to all six fluency actions.

What Fluency Practices Looks Like in the Classroom

Examples of Quality Fluency Practices. Throughout the observations and debriefs, participant A demonstrated quality fluency practice by integrating a game to review components of fractions that students had been learning in previous lessons including adding fractions with like denominators, comparing fractions, and recognizing equivalent fractions. This shows a focus on purposeful practice because students had to determine the correct answer to proceed in the game. There were no examples of worked examples, routines, or centers represented during the observed time or shared during the debrief sessions.

Assessment and Feedback. Participant A demonstrated practices that relate to the use of formative assessments as well as providing feedback to students. During the first observation, the teacher used an individual exit ticket where students had to solve a multiplication problem to check fact fluency. In addition, the second observation included the teacher monitoring the game play and addressing misconceptions as they arose. Participant A provided feedback to the students related to their understanding of fractions as it related to one whole in the first observation. However, the assessment of procedural fluency was not evident in the observations or debriefs. Rather the assessment and feedback focused solely on the development and support of accuracy.

Components of Fluency Addressed in Classroom Practices

There was no direct, intentional implementation of learning activities for students to focus on the selection of appropriate strategy that helps them to solve the problem in a reasonable time. The learning activities observed did not focus on when it is appropriate to select a particular strategy in order to solve the problem correctly. In addition, the learning activities observed did not have students select strategies based on efficiency and accuracy.

How does what a teacher knows and understands about procedural fluency translate into classroom practices during core instruction?

Participant A demonstrated the following scores for each rubric item related to his or her knowledge and understanding of procedural fluency. The ratings demonstrated in the table are compiled based on the data collect during the initial survey and interview. Based on the survey and interview responses, participant A does not appear to have knowledge and skills related to fluency in alignment with the literature. This shows the participant is unconsciously unaware of the knowledge and understanding needed when it comes to this strand needed for mathematical proficiency.

Table 4.1

Survey and Interviews Data Analysis for Case A

Research Subquestions	Indicator/Measure	Case	
		A	
		S	**w/ I**
What does a teacher know and understand about fluency in mathematics?	Knowledge and Understanding of Procedural Fluency	1	**1**
	Purposeful Planning for Quality Fluency Practice - Focused, varied, processed, connected	1	**1**
	Purposeful Planning for Quality Fluency Practice- efficiency, accuracy, and flexibility	0	**0**
	Purposeful Planning for Quality Fluency Practice- Fluency Actions	0	0
	Overall Rating for Subquestion	**0.50**	**0.50**
In what ways do teachers explicitly connect conceptual understanding to procedural fluency?	Understanding of the link between conceptual understanding and procedural fluency	2	**2**

	Overall Rating for Subquestion	**2.00**	**2.00**
In what ways do teachers plan for fluency development?	Purposeful Planning for Quality Fluency Practice - Focused, varied, processed, connected	1	**1**
	Purposeful Planning for Quality Fluency Practice- efficiency, accuracy, and flexibility	0	**0**
	Purposeful Planning for Quality Fluency Practice- Fluency Actions	0	0
	Overall Rating for Subquestion	**0.33**	**0.33**
How does what a teacher knows and understands about fluency translate into classroom practices during core instruction?	**Overall Rating**	**0.80**	**0.80**

Participant A demonstrated the following overall rubric scores. The table organized below show the level in which fluency practices are translated into the classroom. The rubric scores were a compilation of the pertinent data collected using field notes during each observation and debriefing conversation. This illustrates the participant practices are not aligned with quality fluency practices as outlined in the literature review. Overall, the participant's practices are unconsciously unaligned with the practices needed for quality, intentional fluency instruction to occur in the classroom setting.

Table 4.2

Field Notes (Observations/Debriefs) Data Analysis for Case A

Research Subquestions	Indicator/Measure	Case	
		A	
		O	w/D

What does a teacher do to ensure the development of procedural fluency in the classroom?	Explicit Strategy Instruction	0	**1**
	Visual Representations and Tools	2	**2**
	Overall Quality of Fluency Practices- Focused, varied, processed, connected and six fluency actions	1	**1**
	Overall Quality of Fluency Practice- efficiency, accuracy, and flexibility	0	**0**
	Overall Rating for Subquestion (Mean)	**0.75**	**1.00**
What does practice for fluency look like in the classroom?	Examples of Quality Fluency Practice: Routines	0	**0**
	Examples of Quality Fluency Practice: Worked Examples	0	**0**
	Examples of Quality Fluency Practice: Centers	0	**0**
	Example of Quality Fluency Practice: Games	1	**1**
	Assessment Type	0	**0**
	Feedback based on Assessment	0	0
	Overall Quality of Fluency Practices	1	**1**
	Overall Rating for Subquestion (Mean)	**0.29**	**0.29**
What components of fluency do the practices present in the classroom address?	Overall Quality of Fluency Practice- efficiency, accuracy, and flexibility	0	**0**
	Overall Rating for Subquestion (Mean)	**0.00**	**0.00**
How does what a teacher knows and understands about fluency **translate into classroom practices during core instruction**?	**Overall Rating**	**0.44**	**0.56**

Themes within Case A

There were several themes that emerged when studying and analyzing the data from Case A. Throughout the survey and interview, this participant equated the idea of fluency with memorization as well as the component of accuracy. This participant defined procedural fluency as focus on accuracy and answering problems quickly because they have been memorized. This definition does not align with the idea that mathematical fluency is actually multi-faceted and must include a focus on all components (efficiency, accuracy, and flexibility) in order to positively impact mathematical proficiency. In addition, there was a clear correlation between the participants' more limited knowledge and understanding of procedural fluency demonstrated in the data and what was occurring in the classroom. There were many components and learning activities that support the development of fluency that were not present in the participant's understanding or application in classroom practices. Some of examples of instructional practices that were not utilized include a focus on the fluency actions, the use of varied tasks to promote fluency like centers, routines, and worked examples, and a focus on all components of procedural fluency.

Case B

Introduction

Participant B is a kindergarten teacher and completing his/her first year of teaching. In this role, the teacher serves fourteen students in one of two kindergarten classrooms at the school. The participant uses a combination of whole group and small group instruction during the core math block. In terms of the confidence level rating related to the teaching of math at the assigned grade level, Participant B provided a self-assigned ranking of four because of the support and training that the teacher has received from the company related to implementation of the math resource, iReady Math at the beginning of the year. The participant was determined to be unconsciously unaligned to fluency practices based on the data collected, analyzed, and shared below. In addition, the rubric score for each component is recorded in the tables that follow for Participant B.

Knowledge and Understanding of Computational Fluency in Mathematics

Participant B's knowledge and understanding of fluency touched on the importance of accuracy when solving problems. The participant indicated that fluency occurs "when they can generalize the math concept into real world scenarios and do so successfully." There was no indication of knowledge related to the need for students to solve procedures in a reasonable time, know multiple strategies that students can apply or adapt to solve a procedure, or select the appropriate strategy in relation to efficiency and flexibility. This, combined with the indicators of purposeful planning (described below), resulted in a score of 1.25 on the rubric.

Explicit Connection of Conceptual Understanding to Computational Fluency

Participant B was able to define conceptual understanding on his or her survey response as "conceptual understanding breaks mathematical concepts down and demonstrates where the numbers are coming from." The survey and interviews responses did not indicate that the participant understood the link between procedural fluency and conceptual understanding. In addition, the responses did not demonstrate how this connection impacts pacing and instruction. This resulted in an overall score of 1.00 for this subquestion based on the rubric.

How Computational Fluency Development is Planned For

The data collected through the survey and interviews showed that the participant does not have specific knowledge or understanding related to the planning for the development of computational fluency. The responses did not focus on attending to all components of fluency, the fluency actions, or the characteristics of quality fluency practice. The participant did include a focus on providing practice to help students with memorization, however. On the survey, the participant described quality fluency practice in the mathematic classroom as, "I envision memorizing addition, subtraction, multiplication, and division facts being considered as a version of procedural fluency in the classroom". Instead of describing procedural or computational fluency, the participant has honed in his or her focus on the idea of automaticity, which is part of overall procedural fluent picture, but it is not the whole picture.

What the Teacher Does to Ensure the Development of Procedural Fluency

Explicit Strategy Instruction. In terms of strategy-based instruction, participant B demonstrated in the observations that strategy instruction is intentionally planned for, and instruction related to the strategy was direct and explicit. An example of direct strategy instruction occurred in observation three when the participant used counters and a tens frame to model adding two numbers. This was explicit because it helped students to ensure that their answer was accurate. The integration of the strategy instruction was deliberate as the teacher had the materials ready to model how they can be used to solve the problems. The observations and subsequent debriefs did not indicate that strategy instruction in the classroom provided opportunities for students to generate their own strategies for solving problems.

Visual Representations and Tools. Participant B included visualization as an instructional practice in he or she facilitated for the students. The idea of using tools and strategies to help students to visualize a problem was present in all three classroom observations. An example of visual representation used during observation two was when the students used two-colored counters to represent numbers in an addition equation. The counters were used to specifically to help students to understand and verbalize what each number and symbol meant in the equation that was created in relationship to a word problem. The participant commented during the debrief of the same observation that the counters as well as a direct link to the parts of an equation helped students to successfully write and solve equations.

Quality of Fluency Practices. Participant B provided students with opportunities to practice using learning activities that were focused. The learning activities were defined as focused because they helped student practice accuracy by learning to use a practice and get the correct answer. The use of focused learning tasks was evident in all three observations. An example of focused fluency practice from observation one was during the third rotation. Students worked to answer addition and subtraction problems. After finding the solution to the card, students sorted the card into piles based on the answers. As students worked to solve each problem using the strategy of their choice, the teacher monitored the progress of the students to ensure they were accurately solving each problem. If the answer was incorrect, the teacher used prompts and cues to redirect the students. Most errors resulted for the students confusing the procedure associated with the addition and subtraction signs. This likely indicates that students did not have conceptual understanding necessary to solve the problem accurately and efficiency. In addition, the fluency practice was connected because they helped students to see relationships and make connections. A specific example of connected practiced occurred during lesson two when students used counters to model addition problems and write equations. The use of the counters supported students in seeing the connection of how addition are represented and solved. There were no other examples that would indicate learning activities that were varied, processed, or balanced. In addition, the learning activities also did not specifically require students to attend to all six fluency actions

What Fluency Practices Looks Like in the Classroom

Examples of Quality Fluency Practice. Participant B provided students with opportunities to practice fluency through the use of centers. The teacher structured the core instructional block to be taught using small group instruction and learning centers. In two of three lessons observed, students rotated three times with one station being teacher-led. In the other observed lesson, students did have center rotations after a whole group lesson. In one rotation of the first observation, students practiced skip counting by fives to hundred in diverse ways including the use of manipulatives, a video, a song, floor spots, and a hundreds chart). In the other rotation, students used an iPad to complete assigned tasks that required them to subitize and add with dominoes. During the second and third observations, the technology station remained the same, and the second rotation had students add or subtract an equation and sort the card based on the answer.

Based on the centers observed, the centers provided engagement through active student participation, varied learning tasks, and individual accountability. At the technology station, students were monitored by a paraeducator to ensure they were answering questions accurately as well as utilizing their time wisely. In addition, the students turned in their work at the end of the station so the teacher could provide feedback, assign new tasks, and ensure students were in academic learning time throughout the station. The second rotation was also facilitated by a paraeducator. There were a variety of activities that all students could participate in, and adult monitored to ensure each student was interacting with the content appropriately. Throughout the center, the paraeducator listened to individual students count or answer the equations to ensure that they were practicing the intended skills correctly.

In addition, the centers were challenging for students since students' placements in the group were based on student readiness. The tasks at the teacher-led table as well as the other centers were differentiated based on content. This provided the appropriate level of challenge because the tasks were aligned with the students' zone of proximal development. The lessons observed and the information shared during the debriefs did not indicate that routines, worked examples, or games were used to practice fluency during the core block of instruction.

Assessments and Feedback. During the observed lessons, participant B often used formative assessments such as every pupil response (where the students used a thumbs up, thumb to the side, or a thumbs down to indicate their understanding of the learning objective) and teacher observation to determine the students' level of understanding. Based on the information gathered, the teacher would often provide prompts and cues or individual support to help address student misunderstanding. However, the assessment of fluency was not evident in the observations or debriefs. In addition, any feedback provided was not specific to the development of procedural fluency.

Components of Fluency Addressed in Classroom Practices

Participant B implemented learning activities that required students to focus on the appropriate strategy that helps them solve the problem correctly in all three observed lessons. An example from lesson two was students practiced writing and solving addition and subtraction equations. The teacher asked students for several ways the problem could be solved. After students shared several examples of strategies, they had learned from the math resource, the teacher asked students to determine what strategy they could use to make sure they got the right answer. This demonstrates a focus on accuracy as it relates to how students solve problems. The lessons did not include learning activities that required students to select and use appropriate strategies based on solving the problem in a reasonable time. Students also did not engage in learning tasks that required selecting and using strategies flexibly based on accuracy and efficiency.

How does what a teacher knows and understands about fluency translate into classroom practices during core instruction?

Participant B demonstrated the following scores for each rubric item related to his or her knowledge and understanding of procedural fluency. The ratings demonstrated in the table are compiled based on the data collect during the initial survey and interview. Based on the survey and interview responses, participant B does not appear to have knowledge and skills related to fluency in alignment with the literature. This shows the participant is unconsciously unaware of the knowledge and understanding needed when it comes to this strand needed for mathematical proficiency.

Table 4.3

Survey and Interviews Data Analysis for Case B

Research Subquestions	Indicator/Measure	Case	
		B	
		S	w/ I
What does a teacher know and understand about fluency in mathematics?	Knowledge and Understanding of Procedural Fluency	1	1
	Purposeful Planning for Quality Fluency Practice - Focused, varied, processed, connected	1	1
	Purposeful Planning for Quality Fluency Practice- efficiency, accuracy, and flexibility	0	0

	Purposeful Planning for Quality Fluency Practice- Fluency Actions	0	0
	Overall Rating for Subquestion	**0.50**	**0.50**
In what ways do teachers explicitly connect conceptual understanding to procedural fluency?	Understanding of the link between conceptual understanding and procedural fluency	1	1
	Overall Rating for Subquestion	**1.00**	**1.00**
In what ways do teachers plan for fluency development?	Purposeful Planning for Quality Fluency Practice - Focused, varied, processed, connected	0	2
	Purposeful Planning for Quality Fluency Practice- efficiency, accuracy, and flexibility	0	0
	Purposeful Planning for Quality Fluency Practice- Fluency Actions	0	2
	Overall Rating for Subquestion	**0.00**	**0.44**
How does what a teacher knows and understands about fluency translate into classroom practices during core instruction?	**Overall Rating**	**0.60**	**1.50**

Participant B demonstrated the following overall rubric scores. The table organized below show the level in which fluency practices are translated into the classroom. The rubric scores were a compilation of the pertinent data collected using field notes during each observation and debriefing conversation. This illustrates the participant practices are not aligned with quality fluency practices as outlined in the literature review. Overall, the participant's practices are unconsciously unaligned with the practices needed for quality, intentional fluency instruction to occur in the classroom setting.

Table 4.4

Field Notes (Observations/Debriefs) Data Analysis for Case B

Research Subquestions	Indicator/Measure	Case	
		B	
		O	w/D
What does a teacher do to ensure the development of procedural fluency in the classroom?	Explicit Strategy Instruction	3	**3**
	Visual Representations and Tools	1	**1**
	Overall Quality of Fluency Practices- Focused, varied, processed, connected and six fluency actions	3	**3**
	Overall Quality of Fluency Practice- efficiency, accuracy, and flexibility	2	**2**
	Overall Rating for Subquestion (Mean)	**2.25**	**2.25**
What does practice for fluency look like in the classroom?	Examples of Quality Fluency Practice: Routines	0	**0**
	Examples of Quality Fluency Practice: Worked Examples	0	**0**
	Examples of Quality Fluency Practice: Centers	3	**3**
	Example of Quality Fluency Practice: Games	0	**0**
	Assessment Type	0	**0**
	Feedback based on Assessment	0	**0**
	Overall Quality of Fluency Practices	3	**3**
	Overall Rating for Subquestion (Mean)	**0.86**	**0.86**
What components of fluency do the practices present in the classroom address?	Overall Quality of Fluency Practice- efficiency, accuracy, and flexibility	2	**2**
	Overall Rating for Subquestion (Mean)	**2.00**	**2.00**
How does what a teacher knows and understands about fluency **translate into classroom practices**	**Overall Rating**	**1.20**	**1.20**

during core instruction?			

Themes Within Case B

The analysis of Case B revealed several themes within the case. The participant knowledge of procedural fluency centered around the idea of accuracy and memorization. This translated into classroom practices as the fluency practice was focused on completing the steps of a problem correctly as well as getting the correct answer. Although the participant did not demonstrate a high level of knowledge and understanding related to procedural fluency, there were many practices that supported the development of procedural fluency present in the classroom. The participant follows the curriculum with fidelity, so this application could be based on the practice provided in the book. It will be necessary to increase this participant to the conscious category so he or she knows what is being implemented in the classroom and why. This knowledge is essential to ensure that procedural fluency is developed and supported intentionally. When a teacher understands the why behind what is included in a resource, he or she is able to implement that learning activity with the targeted purpose of developing fluency with a focus beyond accuracy.

Case C

Introduction

Participant C has served as a second-grade teacher for nine of the fourteen total years of his/her career. In this role, the teacher serves eleven students in one of two second grade sections in the school. In terms of the confidence level rating related to the teaching of math at the assigned grade level, Participant C provided a self-determined ranking of three based on the fact that sometimes he or she feels like the students are getting it and then all of the sudden they do not. As a result, the participant indicated that this shift related to the students' understanding could be caused by his or her planning and instruction. The participant was determined to be unconsciously unaligned to fluency practices based on the data collected, analyzed, and shared below. In addition, the rubric score for each component is recorded in the tables that follow for Participant C.

Knowledge and Understanding of Computational Fluency in Mathematics

Participant C's knowledge and understanding of computational fluency centered around the concept of accuracy. The participant indicated that fluency was the "ability to solve mathematical problems accurately" in his or her survey response to describing procedural fluency. There was no indication of knowledge or understanding related to the need for students to solve procedures in a reasonable time, know multiple strategies that can be applied or adapted to solve a procedure, or select the appropriate strategy in relation to efficiency and flexibility. This, combined with the indicators of purposeful planning (described below), resulted in a score of 1.25 on the rubric.

Explicit Connection of Conceptual Understanding to Computational Fluency

Participant C was able to define conceptual understanding on his or her survey response as well as demonstrate his or her knowledge of the link between conceptual understanding and procedural fluency. In the survey, Participant C shared, "procedural fluency is having the skill or knowing the steps to solve a problem and conceptual understanding is knowing the why behind it." The survey and interview responses did not demonstrate an understanding of how this connection impacts pacing and instruction. This resulted in an overall score of 2.00 for this subquestion based on the rubric.

How Computational Fluency Development is Planned For

Participant C plans learning activities that develop the components and actions of computational fluency through focused, processed, and connected practice. In his or her follow-up interview, Participant C described focused instruction as “teaching the strategy, giving them time to practice, and choose what works best for them.” Students are provided opportunities to process and reflect with the use of Number Talks within the core block of instruction based on the survey response. In the initial interview, Participant C highlighted connected practice by stating a focus on “how numbers are related to each other and how they build on (how they work together and the patterns they have).” The survey and interview responses did not indicate the integration of practice that was varied or balanced. The survey showed one of the six fluency actions being attended to as the participant identified the need to complete steps in order to be fluent. The other five fluency actions were not explicitly addressed in the data collected. This resulted in a rubric score of 1.33 for this subquestion.

What the Teacher Does to Ensure the Development of Procedural Fluency

Explicit Strategy Instruction. In terms of strategy-based instruction, participant C demonstrated in the observations that strategy instruction is intentionally planned for, and instruction related to the strategy was direct and explicit. An example of direct strategy instruction occurred in observation three when the participant had students use base ten pieces to model the value of each number when writing the expanded form on the number. This was explicit because it helped students to ensure that the equation they created to represent the number was accurate. The integration of the strategy instruction was deliberate as the teacher had the materials ready for students to use the represent each number. The observations and subsequent debriefs did not indicate that strategy instruction in the classroom provided opportunities for students to generate their own strategies for solving problems.

Visual Representations and Tools. The idea of using tools and strategies to help students to visualize a problem was present in all three classroom observations. An example of visual representation used during observation two was the use of base-ten pieces to represent the values of each number based on their place. The use of the base-ten pieces helped students to visualize the number related the value of each digit, which is a critical component of number sense as well as building a foundation for strong conceptual understanding.

Quality of Fluency Practices. Participant C provided students with opportunities to practice using learning activities that were focused. The practices were coded as focused because they helped students to practice accuracy by learning to use a practice and get the correct answers. This was evident in all three observations. An example of focused fluency practice from observation two was when the teacher provided students with multiple ways to represent the same number. This was focused because it helped students to practice determining the value of each digit of the number and representing in the same number in different ways. There were no other examples that would indicate learning activities that were varied, processed, connected, or balanced. In addition, the learning activities also did not specifically require students to attend to all six fluency actions.

What Fluency Practices Looks Like in the Classroom

Examples of Quality Fluency Practices. Participant C provided opportunities for students to develop their skills related to procedural fluency through the use of centers and discussion routines. The first observed lesson was structured using centers and small group instruction. Students rotated through four rotations with one station being teacher-led. During one rotation, students worked independently to match the digital time to the analog clock. Students had access to an individual clocks to model the time, if needed. In the second rotation, students practiced telling times that were a quarter past and a quarter to the hour. The final independent station required student to write the digital time if it was represented on an analog and draw the time on an analog clock when a digital time was represented. During the teacher led station, students demonstrated different times on a clock and determined whether scenarios would occur in the am, pm, or both. Each station required active student engagement and contained a component of student accountability since work from each station was turned in for the teacher to check. In addition, the centers included a variety of learning tasks.

Participant C also dedicated time to the discussion where there was conversation related to strategy use. This occurred during number talks in observation two and three. In both examples, students were encouraged to share different ways that they solved each problem. In terms of other types of quality fluency, there did not appear to be opportunities for students to practice using games or worked examples.

Assessment and Feedback. During the observed lessons, participant C often used formative assessments and teacher observation to check the students understanding of the objective. Most commonly during the lesson, this was accomplished by students completing the problem on a whiteboard and then holding up the whiteboard when prompted to do so. This allowed the teacher to quickly determine the level of understanding of each student and the class as a whole. Based on the information gathered, the teacher would often ask additional questions to help provide clarity to the misconception or individual support to help address a student's specific misunderstanding. However, the assessment of fluency was not evident in the observations or debriefs. In addition, any feedback provided was not specific to the development of procedural fluency.

Components of Fluency Addressed in Classroom Practices

Participant C implemented learning activities that required students to focus on selecting the appropriate strategy that helps them solve the problem correctly in all three observed lessons. An example from lesson three occurred when the students had to represent numbers in three different ways including standard form, using base-ten pieces, and expanded form. The teacher carefully monitored students' progress as they worked to represent the number in each way on their marker boards and using base-ten pieces. This activity assessed the level of each students' knowledge of place value, which is important when using strategies to solve problems in order to ensure the problem is completed accurately and in a reasonable amount of time.

The teacher also focused on flexibility during lessons two and three. Both lessons included a number talk. In lesson two, student were shown the number 42 and asked to determine different ways that the number could be represented. Students provided answers such as 20+21+1, 30+10+2, and 22+ 20. The ability to decompose numbers flexibility is an important skill as students can apply this knowledge when selecting strategies to use when solving problems. The lessons did not include learning activities that required students to select and use appropriate strategies based on solving the problem in a reasonable time. The lessons could have been tweaked to develop fluency a greater level by asking students to elaborate on their selection, use, and defense of the strategy they shared during routine of Number Talks especially related to the choice and use of strategies based on getting the correct answer in a reasonable amount of time.

How does what a teacher knows and understands about fluency translate into classroom practices during core instruction?

Participant C demonstrated the following scores for each rubric item related to his or her knowledge and understanding of procedural fluency. The ratings demonstrated in the table are compiled based on the data collect during the initial survey and interview. Based on the survey and interview responses, participant C does not appear to have knowledge and skills related to fluency in alignment with the literature. This shows the participant is unconsciously unaware of the knowledge and understanding needed when it comes to this strand needed for mathematical proficiency.

Table 4.5

Survey and Interviews Data Analysis for Case C

Research Subquestions	**Indicator/Measure**	**Case**	
		C	
		S	**w/ I**
What does a teacher know and understand about fluency in mathematics?	Knowledge and Understanding of Procedural Fluency	1	**1**
	Purposeful Planning for Quality Fluency Practice - Focused, varied, processed, connected	1	**3**
	Purposeful Planning for Quality Fluency Practice- efficiency, accuracy, and flexibility	0	**0**
	Purposeful Planning for Quality Fluency Practice- Fluency Actions	1	**1**
	Overall Rating for Subquestion	**0.75**	**1.25**

In what ways do teachers explicitly connect conceptual understanding to procedural fluency?	Understanding of the link between conceptual understanding and procedural fluency	2	**2**
	Overall Rating for Subquestion	**2.00**	**2.00**
In what ways do teachers plan for fluency development?	Purposeful Planning for Quality Fluency Practice - Focused, varied, processed, connected	1	**3**
	Purposeful Planning for Quality Fluency Practice- efficiency, accuracy, and flexibility	0	**0**
	Purposeful Planning for Quality Fluency Practice- Fluency Actions	1	**1**
	Overall Rating for Subquestion	**0.66**	**1.33**
How does what a teacher knows and understands about fluency translate into classroom practices during core instruction?	**Overall Rating**	**1.00**	**1.40**

Participant C demonstrated the following overall rubric scores. The table organized below show the level in which fluency practices are translated into the classroom. The rubric scores were a compilation of the pertinent data collected using field notes during each observation and debriefing conversation. This illustrates the participant practices are not aligned with quality fluency practices as outlined in the literature review. Overall, the participant's practices are unconsciously unaligned with the practices needed for quality, intentional fluency instruction to occur in the classroom setting.

Table 4.6

Field Notes (Observations/Debriefs) Data Analysis for Case C

Research Subquestions	Indicator/Measure	Case
		C

		O	w/D
What does a teacher do to ensure the development of procedural fluency in the classroom?	Explicit Strategy Instruction	3	**3**
	Visual Representations and Tools	1	**1**
	Overall Quality of Fluency Practices- Focused, varied, processed, connected and six fluency actions	1	**1**
	Overall Quality of Fluency Practice- efficiency, accuracy, and flexibility	3	**3**
	Overall Rating for Subquestion (Mean)	**2.00**	**2.00**
What does practice for fluency look like in the classroom?	Examples of Quality Fluency Practice: Routines	2	**2**
	Examples of Quality Fluency Practice: Worked Examples	0	**0**
	Examples of Quality Fluency Practice: Centers	1	**1**
	Example of Quality Fluency Practice: Games	0	**0**
	Assessment Type	0	**0**
	Feedback based on Assessment	0	**0**
	Overall Quality of Fluency Practices	1	**1**
	Overall Rating for Subquestion (Mean)	**0.57**	**0.57**
What components of fluency do the practices present in the classroom address?	Overall Quality of Fluency Practice- efficiency, accuracy, and flexibility	3	**3**
	Overall Rating for Subquestion (Mean)	**3.00**	**3.00**
How does what a teacher knows and understands about fluency **translate into classroom practices during core instruction**?	**Overall Rating**	**1.10**	**1.10**

Themes Within Case C

The analysis of Case C revealed several themes within the case. The participant knowledge of procedural fluency centered around the idea of accuracy. This translated into classroom practices as the fluency practice was focused on completing the steps correctly as well as getting the correct answer. The participant did include learning activities such as numbers talks that contributed to development of other components of procedural fluency such as flexibility. From the survey and interview responses, however; the participant may not be intentionally or consciously attending to the development of fluency. It will be necessary to increase this participant to the conscious category so he or she knows what is being implemented in the classroom and why. This knowledge is essential to ensure that procedural fluency is developed and supported intentionally. When a teacher understands the why behind what is included in a resource, he or she is able to implement that learning activity with the targeted purpose of developing fluency with a focus beyond accuracy.

Case D

Introduction

Participant D has served as a second-grade teacher for four of the seventeen total years of his/her career. In this role, the teacher serves eleven students in one of the two second grade sections at this grade. In terms of the confidence level rating related to the teaching of math at the assigned grade level, Participant D provided a self-assigned ranking of three because the participant felt as if there was always room for improvement. This improvement occurs through continuous learning by the teacher and the integration of more effective instructional strategies during lessons. The participant was determined to be unconsciously unaligned to fluency practices based on the data collected, analyzed, and shared below. In addition, the rubric score for each component is recorded in the tables that follow for Participant D.

Knowledge and Understanding of Computational Fluency in Mathematics

Participant D's knowledge and understanding of computational fluency included flexibility and accuracy based on his or her responses on the survey and during the interviews. In terms of flexibility, the participant indicated in the survey that students are fluent with a particular procedure when "they understand that different problems are solved in different ways." In the initial interview, the participant highlighted the importance of accuracy by stating "there are so many different strategies that they can use as long as they can explain why they got the right answer." There was no indication of knowledge or understanding related to the need for students to select the appropriate strategy in relation to efficiency and flexibility or that a problem needs to be solved in a reasonable amount of time. This, combined with the indicators of purposeful planning (described below), resulted in a score of 0.75 on the rubric

Explicit Connection of Conceptual Understanding to Computational Fluency

Participant D was able to define conceptual understanding on his or her survey response as "conceptual understanding is showing and explaining how you get the answer or knowing that the answer you have is the correct answer." The survey and interviews responses did not indicate that the participant understood the link between procedural fluency and conceptual understanding. In addition, the responses did not demonstrate how this connection impacts pacing and instruction. This resulted in an overall score of 1.00 for this subquestion based on the rubric.

How Computational Fluency Development is Planned For

Participant D plans learning activities that develop the components and actions of computational fluency through a focus on the fluency action of completing steps in order to solve problems accurately. In the survey this was demonstrated when asked to describe what quality fluency practice looks like in a math classroom. The participant responded, "the continuous practicing of solving different types of math problems to practice the procedure and steps to solve that type of problem". The responses in the survey and interview did not show the participant consciously planned for practice that was focused, varied, connected, processed, or balanced. In addition, quality practice was focused on the fluency actions that developed accuracy rather than a more balanced approach that reflects all six fluency actions. This resulted in a rubric score of 0.33 for this subquestion.

What the Teacher Does to Ensure the Development of Procedural Fluency

Explicit Strategy Instruction. In terms of strategy-based instruction, participant D demonstrated in the observations that strategy instruction is intentionally planned for, and instruction related to the strategy was direct and explicit. An example of direct strategy instruction occurred in observation two when the participant used base ten pieces to model the value of each number when writing the expanded form on the number. This was explicit because it helped students to ensure that the equation they created to represent the number was accurate. The integration of the strategy instruction was deliberate as the teacher had the materials ready to model the use of these materials to represent the values of different places in a three-digit number. The observations and subsequent debriefs did not indicate that strategy instruction in the classroom provided opportunities for students to generate their own strategies for solving problems.

Visual Representations and Tools. Participant D included visualization in the lesson instruction that was provided for students. The idea of using tools and strategies to help students to visualize a problem as well as make connections based on the visuals provided was present in all three lessons. An example of visual representation was when the teacher used base-ten pieces to represent numbers in expanded form during the second lesson. This helped the students accurately visualize the number based on the value of each digit. In addition, the participant helped the students to make connections using visuals. In the third observation, the teacher connected the base-ten pieces to that of denominations of bills. This connection helped students to focus on the reasonableness of their expanded form based on the value of the bill as well as the base-ten pieces.

Quality of Fluency Practices. Participant D provided students with opportunities to practice using learning activities that were focused. The inclusion of focused practice supported the students in practicing a procedure to get the correct answer. This was evident in all three observations. An example of focused fluency practice from observation two when student solved problems meant to reinforce the concept of expanded for. The teacher encourage students to use of the two tools presented in the book to solve the problem. The students were encouraged to pick the strategy that would help them get the correct answer. The observations and debriefing session did not indicate that students have opportunities to practice when it is varied, connected, processed, or balanced. In addition, the practice that was observed did not require students to attend to all six fluency actions.

What Fluency Practices Looks Like in the Classroom

Examples of Quality Fluency Practices. During the three classroom observations, there were limited opportunities for students to develop their procedural fluency through purposeful practice. The learning activities that did promote fluency were not facilitated using games, worked examples, routines, or centers. Rather the development of fluency was addressed through teacher-centered questioning and guided practice.

Assessment and Feedback. During the observed lessons, participant D often used quick checks for understanding as well as teacher observation to determine the students' level of understanding. Based on the information gathered, the teacher would often provide individual support to students to help address his or her misunderstanding. In addition, during the first observation, the teacher gave the students a quiz and during the debrief shared that the information from the assessment would be used to reteach any skills needed as a result. The assessment focused on accuracy as it was based solely on whether or not the students arrived at the correct answers. There was no opportunity for students to select, use, and justify their use of strategies. As a result, the comprehensive assessment of fluency was not evident in the observations or debriefs. In addition, any feedback provided was not specific to the development of procedural fluency.

Components of Fluency Addressed in Classroom Practices

Participant D implemented learning activities that required students to focus on the appropriate strategy selection in the second observation. The criteria for the selection of the appropriate strategy was based on what would support them in solving the problem correctly. An additional example from this lesson was students practiced solving addition problems that were connected based on place value. The teacher asked questions as they worked through each step of the problems to ensure that the problems were being solved correctly. Some examples of the type of questions asked include, "You have 200, how many hundreds is that? If you have 70, how many tens is that? If you have 5, how many ones do you have?". This demonstrates a focus on accuracy as it relates to how students solve problems and interpret the value of each number based its place value. The lessons did not include learning activities that required students to select and use appropriate strategies based on solving the problem in a reasonable time. Students also did not engage in learning tasks that required selecting and using strategies flexibly based on accuracy and efficiency.

How does what a teacher knows and understands about fluency translate into classroom practices during core instruction?

Participant D demonstrated the following scores for each rubric item related to his or her knowledge and understanding of procedural fluency. The ratings demonstrated in the table are compiled based on the data collect during the initial survey and interview. Based on the survey and interview responses, participant D does not appear to have knowledge and skills related to fluency in alignment with the literature. This shows the participant is unconsciously unaware of the knowledge and understanding needed when it comes to this strand needed for mathematical proficiency.

Table 4.7

Survey and Interviews Data Analysis for Case D

Research Subquestions	**Indicator/Measure**	**Case**	
		D	
		S	**w/ I**
What does a teacher know and understand about fluency in mathematics?	Knowledge and Understanding of Procedural Fluency	1	**2**
	Purposeful Planning for Quality Fluency Practice - Focused, varied, processed,	0	**0**

	connected		
	Purposeful Planning for Quality Fluency Practice- efficiency, accuracy, and flexibility	0	**0**
	Purposeful Planning for Quality Fluency Practice- Fluency Actions	1	**1**
	Overall Rating for Subquestion	**0.50**	**0.75**
In what ways do teachers explicitly connect conceptual understanding to procedural fluency?	Understanding of the link between conceptual understanding and procedural fluency	1	**1**
	Overall Rating for Subquestion	**1.00**	**1.00**
In what ways do teachers plan for fluency development?	Purposeful Planning for Quality Fluency Practice - Focused, varied, processed, connected	0	**0**
	Purposeful Planning for Quality Fluency Practice- efficiency, accuracy, and flexibility	0	**0**
	Purposeful Planning for Quality Fluency Practice- Fluency Actions	1	**1**
	Overall Rating for Subquestion	**0.33**	**0.33**
How does what a teacher knows and understands about fluency translate into classroom practices during core instruction?	**Overall Rating**	**0.60**	**0.80**

Participant D demonstrated the following overall rubric scores. The table organized below show the level in which fluency practices are translated into the classroom. The rubric scores were a compilation of the pertinent data collected using field notes during each observation and debriefing conversation. This illustrates the participant practices are not aligned with quality fluency practices as outlined in the literature review. Overall, the participant's practices are unconsciously unaligned with the practices needed for quality, intentional fluency instruction to occur in the classroom setting.

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Table 4.8

Field Notes (Observations/Debriefs) Data Analysis for Case D

Research Subquestions	Indicator/Measure	Case D O	Case D w/D
What does a teacher do to ensure the development of procedural fluency in the classroom?	Explicit Strategy Instruction	3	**3**
	Visual Representations and Tools	3	**3**
	Overall Quality of Fluency Practices- Focused, varied, processed, connected and six fluency actions	2	**2**
	Overall Quality of Fluency Practice- efficiency, accuracy, and flexibility	1	1
	Overall Rating for Subquestion (Mean)	**2.50**	**2.50**
What does practice for fluency look like in the classroom?	Examples of Quality Fluency Practice: Routines	0	**0**
	Examples of Quality Fluency Practice: Worked Examples	0	**0**
	Examples of Quality Fluency Practice: Centers	0	**0**
	Example of Quality Fluency Practice: Games	0	**0**
	Assessment Type	0	**0**
	Feedback based on Assessment	0	**0**
	Overall Quality of Fluency Practices	2	**2**
	Overall Rating for Subquestion (Mean)	**0.29**	**0.29**

What components of fluency do the practices present in the classroom address?	Overall Quality of Fluency Practice- efficiency, accuracy, and flexibility	1	**1**
	Overall Rating for Subquestion (Mean)	**1.00**	**1.00**
How does what a teacher knows and understands about fluency **translate into classroom practices during core instruction**?	**Overall Rating**	**1.00**	**1.00**

Themes Within Case D

The analysis of Case D revealed several themes within the case. The participant knowledge of procedural fluency centered around the idea of accuracy. This translated into classroom practices as the fluency practice was focused on completing the steps correctly as well as getting the correct answer. The participant did not provide opportunities to develop procedural fluency in the use of games, routines, centers, or worked examples. By increasing what the teacher knows and understands about procedural fluency more intentional learning tasks could be integrated in the core math instructional block to support the development of all components of fluency as well as attend to all six fluency actions.

Case E

Introduction

Participant E is serving his or her first year as sixth grade teacher, although the participant has twenty-two total years of experience in his/her career. In this role, the grade level is departmentalized, so the participant teaches both sections of sixth grade students' math. In each class there are approximately seventeen students. In terms of the self-determined confidence level rating related to the teaching of math at the assigned grade level, Participant E provided a ranking of five because of his or her experience level, the support and math training that has been provided over the years, and the participant's belief that he or she has knowledge and skills to help all student learn math. The participant was determined to be unconsciously unaligned to fluency practices based on the data collected, analyzed, and shared below. In addition, the rubric score for each component is recorded in the tables that follow for Participant E.

Knowledge and Understanding of Computational Fluency in Mathematics

Participant E's knowledge and understanding of computational fluency included efficiency, appropriate strategy selection, and flexibility based on his or her responses on the survey and during the interviews. In terms of efficiency, the participant shared in his or her initial interview that it is critical that students are comfortable with the idea that math does not have to be done in a specific way and it is important to think about numbers in different ways. This helps students to produce ideas on how to solve problems in efficient ways. In terms of flexibility, the participant indicated on several questions in the survey the importance of students using "a wide array of procedures and strategies to answer problems." Additional examples of responses aligned with the idea of flexibility include "they are able to use the procedure in different situations without prompting" and "when you see them using strategies from a month or two in the past, it shows you they really have a strong understanding of how and when to apply that procedure." The survey responses of this participant also supported the appropriate selection of strategies. Some statements from the survey that were aligned with appropriateness were when students "choose strategies that allow them to solve problems in different and original ways" and when students can "verbalize why they selected the particular strategy." There was no indication of knowledge and understanding related to the need for students to correctly solve a problem in relational to computational fluency. This, combined with the indicators of purposeful planning (described below), resulted in a score of 2.00 on the rubric.

Explicit Connection of Conceptual Understanding to Computational Fluency

Participant E was able to explain conceptual understanding on his or her survey response in the following way, "as students become more fluent and have a larger amount of strategies at their disposal, they become better problem solvers and their conceptual understanding deepens." In addition, the link between conceptual understanding and procedural fluency as well as how it impacts pacing was described by Participant E when he or she shared "without conceptual understanding, students will struggle to obtain procedural fluency." The survey and interviews responses did not indicate that the participant understood how the pacing related to procedural fluency and conceptual understanding impacted instruction. This resulted in an overall score of 3.00 for this subquestion based on the rubric.

How Computational Fluency Development is Planned For

When participant E plans for the development of computational fluency in the classroom, he or she ensures that students have access to learning activities that are focused and connected. In the follow-up interview, the participant shared "providing a deep understanding of how and why problems are solved this way and giving them multiple strategies that they can draw from instead of telling them to just do it this way" when asked to describe what focused practice looked like in the classroom. In addition, on the survey, the participant demonstrated knowledge and understanding of connected practiced when he or she responded, "the questions were chosen and constructed to solve a wide range of different types of practice". The survey and interview responses did the not indicated the planning of learning activities centered around varied, processed, or balanced practice.

In addition to purposefully planning of computational fluency in terms of focused and connected classroom practices, the participant also had responses that aligned with two of the six fluency actions. These actions included trades out or adapts strategy, complete steps as well as application of a strategy. In the survey response, the participant supported the fluency action related trades out or adapts when he or she responded, “give them problems and encourage them to come up with unique strategies to solve the problems”. In the initial interview, the participant described learning activities that develop students’ ability to apply a strategy to a new problem type in this way, “it is important to manipulate numbers and problems in different ways that makes sense to you and allows students to figure out different ways to solve problems”. The responses did not indicate a direct link to the other four fluency actions (strategy selection, reasonable time, complete steps, and correct answer).

What the Teacher Does to Ensure the Development of Procedural Fluency

Explicit Strategy Instruction. In regard to strategy instruction, there was evidence of direct, explicit, and intentionally planned strategy instruction during each observation. An example of explicit strategy instruction occurred during the second observation. The book presented two strategies that students could have used to solve problem. These methods included modeling using algebra tiles and analyzing it. Modeling it required students to visualize the problems using tiles and the analyze it method involved using the expression in conjunction with the distributive property. Participant E explained both strategies and why they worked. This was intentionally planned as it was a component of the district provided resource, which is used with fidelity by Participant E. The introduction and explanation of different strategies is direct because there was demonstration and modeling as the participant illustrated why each strategy worked through clear and precise instruction. The observations did not include instructional practices that allowed students to create and discussed their own strategies for solving problem as the strategies were presented in the textbook and explained by the teacher.

Visual Representations and Tools. In each of the lesson session, there were specific examples where the problems were presented using a visual representation. The presentation of the visual representation was the catalyst to a deeper discussion and understanding during discussions, which provided students with structure and access to abstract concepts. For example, this occurred in the third observation when a pictorial representation was used to combine like terms. It provided an opportunity for students to discuss how to combine like terms and why. Each visual representation was provided within the context of the textbook.

Quality of Fluency Practices. When there were opportunities for student practice, learning activities were focused and processed. In terms of focused practice, there were ample opportunities for students to learn to use a practice to get the correct answer. Specifically, an example in the second observation occurred when there was a focus on using the distributive property to find equivalent fractions. Students had to complete several problems where they had to apply this practice using the prompts provided in the textbook. In this same lesson, students were then asked to share what strategy they liked better and record their preference in the workbook. This indicates that the practice was processed because students had the opportunity to reflect after the practice. The characteristic of processed practice could have been enhanced in its quality and better aligned with the components of procedural fluency, if students were asked to determine what strategy they liked best based on efficiency. Students could describe what strategy they like best because it helped them to get the answer in a reasonable and helped ensured the answer was accurate.

What Fluency Practices Looks Like in the Classroom

Examples of Quality Fluency Practices. In all three observed lessons, there was a clear routine in place related to student discussion and problem solving. For example, in the first observed lesson, students began by working a problem from the resource label as “try it” independently. After a designated amount of work time, students shared their thinking with a partner by referencing the work they had completed in their workbook. Individual students then came up to front of the room as displayed their work to share their answer and how they got the answer. In this particular example, there were a number of different ways of solving the problem and many different ways were shared by the students. In the subsequent two lessons, the same pattern was followed when “try it’ problems appeared in the book, which was typically at the beginning of each new session. This structure allowed students to access problems that could be solved in different ways and promoted the exchange of ideas. During the lessons the use of worked examples, games, and centers was not observed or referred to during the debriefing conversations.

Assessment and Feedback. The participant used the students’ independent work as well as their answers during class discussions as an indicator of the level of student understanding related the objective of the lesson. During the debrief after the second lesson, the participant shared that based on the observations of student knowledge and understanding, student would benefit from additional time on the topic. In the next lesson, the teacher spent the first five minutes of the lesson reteaching concepts from the previous lesson using direct instruction and modeling. Specifically, the teacher focused on what each number and variable actually represented in the expression. Based on the formative assessment, the teacher felt students did not have a strong enough understanding of the distributive property, commutative property, associative property, and the vocabulary. The intention of review was to reiterate important components of each property and ensure accurate understanding of the tier three (content specific) vocabulary words. The teacher was able to accurately reflect on student achievement based on the teacher observation and intervene appropriately. This ability will provide a good foundation for using assessments to support and develop fluency components and actions. The teacher can monitor student progress related to the strategies they chose, use, and the justification they provide regarding their strategy selection and provide specific target based on efficiency, accuracy, and flexibility.

Components of Fluency Addressed in Classroom Practices

The components of procedural fluency that were addressed in classroom practices during the observations were accuracy and flexibility. In lessons one and two, students were encouraged to solve problem by selecting the strategy that worked best for them to complete the problem. The strategies were not selected based on efficiency, but rather on what the students liked best. For example, in the first lesson students had access to tiles and grid paper (as recommended by the book) to aid in the solving the problem. For this problem, one student decided to use the grid paper and five students used the tiles. The focus on accuracy came when students shared their thinking with the class. If the problem was solved incorrectly and the student sharing knew why, the student would share why their answer was wrong. If a student got the problem wrong and did not know, the teacher asked clarifying questions to the class to help determine the error. The fluency components addressed could have been enhanced if students had opportunities to select strategies based on efficiency and had time to process why the strategy worked best for that type of problem.

How does what a teacher knows and understands about fluency translate into classroom practices during core instruction?

Participant E demonstrated the following scores for each rubric item related to his or her knowledge and understanding of procedural fluency. The ratings demonstrated in the table are compiled based on the data collect during the initial survey and interview. Based on the survey and interview responses, participant E does not appear to have knowledge and skills related to fluency in alignment with the literature. This shows the participant is unconsciously unaware of the knowledge and understanding needed when it comes to this strand needed for mathematical proficiency.

Table 4.9

Survey and Interviews Data Analysis for Case E

Research Subquestions	**Indicator/Measure**	**Case**	
		E	
		S	**w/ I**
What does a teacher know and understand about fluency in mathematics?	Knowledge and Understanding of Procedural Fluency	2	**3**
	Purposeful Planning for Quality Fluency Practice - Focused, varied, processed, connected	1	**2**
	Purposeful Planning for Quality Fluency Practice- efficiency, accuracy, and flexibility	1	**1**
	Purposeful Planning for Quality Fluency Practice- Fluency Actions	1	**2**
	Overall Rating for Subquestion	**1.25**	**2.00**
In what ways do teachers explicitly connect conceptual understanding to procedural fluency?	Understanding of the link between conceptual understanding and procedural fluency	3	**3**
	Overall Rating for Subquestion	**3.00**	**3.00**
In what ways do teachers plan for fluency development?	Purposeful Planning for Quality Fluency Practice - Focused, varied, processed, connected	1	**2**

	Purposeful Planning for Quality Fluency Practice- efficiency, accuracy, and flexibility	1	**1**
	Purposeful Planning for Quality Fluency Practice- Fluency Actions	1	**2**
	Overall Rating for Subquestion	**1.00**	**1.66**
How does what a teacher knows and understands about fluency translate into classroom practices during core instruction?	**Overall Rating**	**1.60**	**2.20**

Participant E demonstrated the following overall rubric scores. The table organized below show the level in which fluency practices are translated into the classroom. The rubric scores were a compilation of the pertinent data collected using field notes during each observation and debriefing conversation. This illustrates the participant practices are not aligned with quality fluency practices as outlined in the literature review. Overall, the participant's practices are unconsciously unaligned with the practices needed for quality, intentional fluency instruction to occur in the classroom setting. However, the participant's score were on the higher end in both categories noting that his or her knowledge and understanding as well as application is more developed than some of the other participants in this category.

Table 4.10

Field Notes (Observations/Debriefs) Data Analysis for Case E

Research Subquestions	**Indicator/Measure**	**Case**	
		E	
		O	**w/D**
What does a teacher do to ensure the development	Explicit Strategy Instruction	3	**3**

of procedural fluency in the classroom?	Visual Representations and Tools	1	**1**
	Overall Quality of Fluency Practices- Focused, varied, processed, connected and six fluency actions	2	**2**
	Overall Quality of Fluency Practice- efficiency, accuracy, and flexibility	3	**3**
	Overall Rating for Subquestion (Mean)	**2.25**	**2.25**
What does practice for fluency look like in the classroom?	Examples of Quality Fluency Practice: Routines	2	**2**
	Examples of Quality Fluency Practice: Worked Examples	0	**0**
	Examples of Quality Fluency Practice: Centers	0	**0**
	Example of Quality Fluency Practice: Games	0	**0**
	Assessment Type	0	**0**
	Feedback based on Assessment	0	**0**
	Overall Quality of Fluency Practices	2	**2**
	Overall Rating for Subquestion (Mean)	**0.57**	**0.57**
What components of fluency do the practices present in the classroom address?	Overall Quality of Fluency Practice- efficiency, accuracy, and flexibility	3	**3**
	Overall Rating for Subquestion (Mean)	**3.00**	**3.00**
How does what a teacher knows and understands about fluency **translate into classroom practices during core instruction**?	**Overall Rating**	**1.10**	**1.10**

Themes Within Case E

When analyzing the data for Case E, several themes surfaced. One such theme is the idea of student discussion. In all three observed lessons as well as the responses in the survey and interviews, it is clear that the participant values providing academic learning time for student discussion. Students had the opportunity to solve problems in different ways as well as share their thinking with others. This classroom practice provides a strong foundation to extend the focus on strategy selection to efficiency. Another theme that emerged was that the participant had a higher score for his or her knowledge and this did not necessarily translate into classroom practices. This could because it is the participant's first year at the grade and he or she used the district provided resource with fidelity.

Case F

Introduction

Participant F has served all twenty-eight years of his or her career teaching fifth grade. In this role, the grade level is departmentalized, so the participant teaches both sections of fifth grade students' math. There are approximately sixteen students in each class. In terms of the confidence level rating related to the teaching of math at the assigned grade level, Participant F provided a self-determined ranking of four because the participant has worked to specialize in the area of math but feels there is always ways to grow as an educator. The participant was determined to be consciously unaligned to fluency practices based on the data collected, analyzed, and shared below. In addition, the rubric score for each component is recorded in the tables that follow for Participant F.

Knowledge and Understanding of Computational Fluency in Mathematics

Participant F's knowledge and understanding of computational fluency included accuracy, appropriate strategy selection, and flexibility based on his or her responses on the survey and during the interviews. On the survey, when the participant to explain procedural fluency in his or her own words, the response stated, "I think of procedural fluency as having at least one dependable algorithm that can be appropriately, consistently, and accurately applied to solve a problem". This demonstrates that the participant's knowledge and understanding of computational fluency included a focus on accuracy. In terms of flexibility, the participant referred to this concept in his or her responses related to the definition of procedural fluency as well as what quality fluency looks like the classroom. The participant stated that procedural fluency and the practice of fluency is "the ability to modify a procedure to fit a particular situation" and being able to "find that different methods might work better for them in different situations". This idea of flexibility was also present in the initial interview when the participant stated, "we have to be open to letting kids explore what works for them". The concept of selecting the appropriate strategy was also found in the survey responses, when the participant stated a teacher knows a student is fluent with a procedure when they can "seamlessly apply appropriate strategies for the given problem". There was no direct correlation to the concept of students selecting and using strategies based on efficiency in the survey or interview responses. This, combined with the indicators of purposeful planning (described below), resulted in a score of 3.00 on the rubric

Explicit Connection of Conceptual Understanding to Computational Fluency

Participant F was able to explain conceptual understanding on his or her survey response in the following way, "conceptual understanding directly links to the student being able to choose an appropriate procedure or procedures that fit a situation, the ability to discern what is." In addition, the link between conceptual understanding and procedural fluency as well as how it impacts pacing was described by Participant F when he or she shared, "Procedural fluency should always be built on conceptual understanding, we know which should come first and effectively, has to come first in quality math instruction." The survey and interviews responses did not indicate that the participant understood how the pacing related to procedural fluency and conceptual understanding impacted instruction. This resulted in an overall score of 3.00 for this subquestion based on the rubric.

How Computational Fluency Development is Planned For

When participant F plans for the development of computational fluency in the classroom, he or she ensures that students engage in learning activities that are varied and processed. In the survey, the participant stated that quality fluency practice is “allowing and providing ample opportunities for students to try various strategies and find which one works”. This was reinforced in the interview when the participant shared that is important to make sure student have multiple opportunities to practice a procedure, which occurs through student discussion, use of visual representation like fractions sticks, and games. In the survey, Participant F explained that a teacher knows when a student is fluent with a procedure when they have “the ability to analyze, critique, and explain their own work as well as the work of others”. This shows the need for students to have opportunities to process and reflect related to the procedures that are being used. The responses on the survey and interviews did not indicate that focused, connected, and balanced opportunities for practice were intentionally planned for.

What the Teacher Does to Ensure the Development of Procedural Fluency

Explicit Strategy Instruction. In all three lessons that were observed, the teacher implemented learning activities that provided students access to strategy instruction that was direct, explicit, and intentionally planned. An example of direct, explicit instruction occurred during the second lesson when the teacher modeled and demonstrated how to use cross-cancellation as a way to simplify before multiplying fractions. Participant F modeled the strategy using a think aloud and included information about why the process worked. It was clear that it was intentionally planned for because the teacher shared his or her intention to introduce this concept in the previous debrief after lesson one. The participant viewed the ability to utilize cross-cancellation as the next step needed by the students related to the multiplication of fractions. In the observed lessons, students did not have the opportunity to create and discuss their own strategies for solving procedures. In each instance where the students worked to solve a problem, they were given specific choices related to what strategies they could utilize.

Visual Representations and Tools. The use of visual representation was demonstrated through the integration of pictorial models. In the first observed lesson, students were working to multiply fractions, the problem was first presented using a rectangular model by the teacher. The debrief indicated that this was one of the ways that students had learned to multiply fractions in previous lessons. As students worked to solve problems independently towards the end of the lesson, most chose to use the rectangular model when multiplying fraction. The use of this visual helped students to visualize what is actually happen when you are solving problems such as 1/3 of ¼.

Quality of Fluency Practices. When students were engaged in learning tasks that required them to practice fluency, these tasks were focused, processed, and connected. The participant use of connected practice was modeled during the debrief after lesson one when he or she responded that one thing that went well during the lesson is when students understood the relationship between multiplication and division and were able to provide good rationale aligned with efficiency. This illustrated that the learning activities helped students to see relationships and make connections. The practiced was processed because students had to the opportunity to refine their understanding of and reflect on the efficiency during the first lesson. A specific example related to the concept of processed practice was when students reviewed problems from a previous lesson to determine if multiplication or division would be the most efficient way to solve the problem. In terms of focused practice, there were learning activities in all three lessons that helped students practice accuracy by learning to use a practice and get the correct answer. Specifically in lesson three, the teacher demonstrated how to use the equivalent fraction when converting fractions to decimals. Students had several opportunities to reason through and practice this skill in subsequent examples. In the observed lessons, the practice was not varied or balanced.

What Fluency Practices Looks Like in the Classroom

Examples of Quality Fluency Practices. When analyzing the data for classroom practices and learning activities that develop fluency, Participant F utilized games and routines. In the first and third observations, students engaged in a game on their electronic device from the NCTM Illuminations website that required them to eliminate all the cards in as few of moves as possible. Cards could be eliminated if there were equal to or less than the card drawn. This game was focused, purposeful, and included student accountability. The game focused on helping students to increase their accuracy related to equivalent fractions. The teacher indicated in the debrief after the third lesson that the purpose of the game was to help students accurately identify equivalent fractions in a reasonable amount of time, which was a concept they have been targeting for most of the year. The game also included student accountability because as they finished the round, they showed their score to the teacher who encouraged each student to try again for a lower score. The use of games in the classroom could be enhanced by including specific opportunities for students to select and makes choices about the best strategy to use.

Participant F also utilized routines that provided students with access to problems that could be solved in different ways. An example of this type of routine occurred in the first and second observed lesson when the teacher facilitated a problem of the day activity. In one instance, the students had the opportunity to solve a problem independently and the next lesson, they solve the problem with a partner. In both lessons, the students created a plan for how to solve the problem and shared their plan with a partner before working through the steps needed to find the solution for the problem. The teacher then led a class discussion related to how the problem could be solved. The use of this routine provided students with regular opportunities to engage in learning activities that required deeper thinking and discussion.

Assessment and Feedback. When thinking about the instructional practices of assessment and feedback, Participant F utilized formative assessments as well as timely, specific feedback related to procedural fluency progress especially in the area of efficiency. In terms of formative assessment, the teacher implemented the use of teacher observation and student work to determine the students' level of mastery related to the lesson objective. Based on the assessment, the teacher would reteach concepts using visual models, additional examples, or more direct instruction to mediate student misunderstanding. During the first lesson when students were providing justification for their strategy use, the teacher provide feedback related to the level of efficiency the strategy adhered to.

Components of Fluency Addressed in Classroom Practices

Within the classroom, the observed lesson addressed efficiency and accuracy In terms of accuracy, an example of this component occurred during the first observation. Students were engaged in a problem of the day, which began with the teacher providing the problem with the numbers excluded and covered in the picture. Students worked independently to devise a plan related to how they might solve the problem and then shared their plan with partner. Students worked independently to implement their plan once the numbers were revealed. The teacher noticed that many students added instead of multiplying causing them to get the incorrect answer. The teacher addressed the misconception using a visual example involving the students in the class and then used that activity to link to the rectangular model included in the problem. In this case, accuracy was attended to through teacher observation and focusing on the reasonableness of the answer.

The concept of efficiency was purposefully targeted in the first lesson as well. This occurred when students were working to solve problems related to multiplying fractions. The teacher shared that a multiplication problem can be rewritten as a division problem and provided examples of when this might aid in solving the problem more efficiently. As a class, they then reviewed the problems from the previous day's assignment to determine if division or multiplication would be the most efficient way to solve the problem. When sharing the strategy they selected, students provided rationale supporting their choice. Then, during their independent work, students analyzed each problem to determine the most efficient process to use. This was a clear, intentional focus on helping students to select and use strategies for solving problems based on efficiency. The focus did not attend to flexibility because the students were not encouraged to select strategies based on what would help them to get the correct answer, instead the focus was on how to solve the problem in a reasonable amount of time.

How does what a teacher knows and understands about fluency translate into classroom practices during core instruction?

Participant F demonstrated the following scores for each rubric item related to his or her knowledge and understanding of procedural fluency. The ratings demonstrated in the table are compiled based on the data collect during the initial survey and interview. Based on the survey and interview, participant F does appear to have knowledge and skills related to fluency in alignment with the literature. This shows the participant is consciously aware of the knowledge and understanding needed when it comes to this strand needed for mathematical proficiency.

Table 4.11

Survey and Interviews Data Analysis for Case F

Research Subquestions	Indicator/Measure	Case F S	Case F w/ I
What does a teacher know and understand about fluency in mathematics?	Knowledge and Understanding of Procedural Fluency	3	4
	Purposeful Planning for Quality Fluency Practice - Focused, varied, processed, connected	2	3
	Purposeful Planning for Quality Fluency Practice- efficiency, accuracy, and flexibility	2	3
	Purposeful Planning for Quality Fluency Practice- Fluency Actions	2	3
	Overall Rating for Subquestion	**2.25**	**3.25**
In what ways do teachers explicitly connect conceptual understanding to procedural fluency?	Understanding of the link between conceptual understanding and procedural fluency	3	3
	Overall Rating for Subquestion	**3.00**	**3.00**
In what ways do teachers plan for fluency development?	Purposeful Planning for Quality Fluency Practice - Focused, varied, processed, connected	2	3
	Purposeful Planning for Quality Fluency Practice- efficiency, accuracy, and flexibility	2	3
	Purposeful Planning for Quality Fluency Practice- Fluency Actions	2	3
	Overall Rating for Subquestion	**2.00**	**3.00**

How does what a teacher knows and understands about fluency translate into classroom practices during core instruction?	Overall Rating	2.40	3.20

Participant F demonstrated the following overall rubric scores. The table organized below show the level in which fluency practices are translated into the classroom. The rubric scores were a compilation of the pertinent data collected using field notes during each observation and debriefing conversation. This illustrates the participant practices are not aligned with quality fluency practices as outlined in the literature review. Overall, the participant's practices are consciously unaligned with the practices needed for quality, intentional fluency instruction to occur in the classroom setting.

Table 4.12

Field Notes (Observations/Debriefs) Data Analysis for Case F

Research Subquestions	Indicator/Measure	Case	
		F	
		O	w/D
What does a teacher do to ensure the development of procedural fluency in the classroom?	Explicit Strategy Instruction	3	**3**
	Visual Representations and Tools	1	**1**
	Overall Quality of Fluency Practices- Focused, varied, processed, connected and six fluency actions	3	**3**
	Overall Quality of Fluency Practice- efficiency, accuracy, and flexibility	3	**3**
	Overall Rating for Subquestion (Mean)	**2.50**	**2.50**
What does practice for fluency look like in the	Examples of Quality Fluency Practice: Routines	1	1

classroom?	Examples of Quality Fluency Practice: Worked Examples	0	0
	Examples of Quality Fluency Practice: Centers	0	0
	Example of Quality Fluency Practice: Games	2	3
	Assessment Type	0	0
	Feedback based on Assessment	1	1
	Overall Quality of Fluency Practices	3	3
	Overall Rating for Subquestion (Mean)	**1.00**	**1.14**
What components of fluency do the practices present in the classroom address?	Overall Quality of Fluency Practice- efficiency, accuracy, and flexibility	3	3
	Overall Rating for Subquestion (Mean)	**3.00**	**3.00**
How does what a teacher knows and understands about fluency **translate into classroom practices during core instruction**?	**Overall Rating**	**1.40**	**1.50**

Themes Within Case F

The collection and analysis of the data collected for Case F revealed many themes. The teacher had strong knowledge and understanding related to procedural fluency, but this was not fully apparent in the observed lessons. The inclusion of a variety of ways to practice fluency related to centers and worked examples could compliment the knowledge and understanding of this participant. In addition, the games that were utilized did not allow for students to choose and use strategies which is another way the practice occurring the classroom could be more targeted in development of procedural fluency in the classroom. The participant was the only teacher to intentionally provide instruction related to and opportunities for practice related to efficiency.

Findings Across All Cases

Below are the overall findings and trends of all six cases related to the study. This information highlights patterns related to the knowledge and understanding of participants and how this knowledge was translated into classroom practices.

Knowledge and Understanding of Computational Fluency in Mathematics

The knowledge and understanding related to computational fluency were based on the participant's responses on the survey and during interviews in relations to the components of fluency which include accuracy, flexibility, and efficiency. The analysis of the data also included a focus on appropriate strategy selection. Five participants attributed fluency to accuracy, or the students' getting the correct answer. Three of participants who only identified accuracy in their definition and understanding also focused on memorization as being a key component of fluency, which is not supported by the literature. One participant recognized efficiency or being able to solve a procedure or problem in a reasonable time as a component of fluency. Three participants identified flexibility as a part of fluency while two participants shared about selecting and using the appropriate strategy. Overall, one participant had a complete view of fluency, which included all the components of fluency, as indicated by the data collected.

Explicit Connection of Conceptual Understanding to Computational Fluency

Conceptual understanding is a necessary foundational support of procedural fluency. This focuses on the meaningful use of mathematical procedures because the students know why strategies work and when it is appropriate to use them. All six participants were able to define conceptual understanding and four participants were able to describe the link between conceptual understanding and procedural fluency. This link highlights the knowledge that procedural fluency is built from conceptual understanding. Two participants shared responses that aligned to the concept that procedural fluency cannot occur without conceptual understanding. However, no participants demonstrated knowledge that conceptual understanding cannot be rushed and can impact the pacing of the lesson. Overall, the participants in each case understand what conceptual understanding is. In order to support the development of fluency in a more impactful way, the participants could be more intentional about using this knowledge of conceptual understanding to support fluency through the planning and pacing of their instruction.

How Computational Fluency Development is Planned For

To determine how computational fluency development is planned for, data was analyzed for quality fluency practices and fluency actions. Two participants shared responses that demonstrated focused practice where students have opportunities to learn to use a practice in order to get the correct answer. Two participants described learning activities that allow students to practice fluency in a variety of ways. One participant identified the idea of processed practice where student have the opportunity to reflect and process after fluency practice. Three of participants explained the importance of practice that helps students see relationships and make connections. None of the participants provided responses that indicated that fluency practice needs to be balanced based on the components of fluency. An additional component of fluency practice that two participants identified related to the idea of memorization as the end goal of fluency practice.

What the Teacher Does to Ensure the Development of Procedural Fluency

To determine how a teacher ensures the development of procedural fluency in the classroom, data was collected and analyzed in the areas of explicit strategy instruction, visual representations and tools, and the overall quality of fluency practice. All six cases included examples of direct strategy instruction. Furthermore, five of the six cases integrated strategy instruction that was also clear and precise in order to help students use strategies and was intentionally planned. Intentionally planned strategy instruction was evident through deliberate and purposeful development and use of strategies.

Five cases implemented tools and visuals that helped students to visualize a problem. In two cases, participants implemented processed that connected visual representation to strategy. In one case, the participant implemented processes that use visual representations and tools to strengthen the effectiveness of student understanding during a classroom discussion to provide structure and access to abstract concepts.

The quality of fluency practiced was assessed by the implementation of learning activities that are planned to develop fluency. Aligned and quality fluency practice can be focused, varied, processed, connected, and balanced. In addition, quality fluency practice should attend to all six fluency actions (strategy selection, reasonable time, trades out/adapts strategy, application of strategy, complete steps, and getting the correct answers). All six cases had examples of focused practice where learning activities help students to practice accuracy by learning to use a practice to get the correct answer. Two participants included processed practice. This type of practice provided students with the opportunity to process and reflect after they practiced a procedure. There were two participants that implemented learning activities that provided opportunities for connected practice. This type of practice helped students to see relationships and make connections. During the observations for each case, no participants implemented learning activities that were varied or balanced. With more intentional integration of practice that is varied and balanced, student will have more opportunities to develop the components and actions of fluency.

What Does Practice for Fluency Look Like in the Classroom?

When considering what quality fluency practice looks like in the classroom, educators can leverage the use of routines, worked examples, centers, games, and assessments that focus on all the components of fluency. In terms of the use of routines, three participants have implemented routines that relate to solve problems in different ways and/or promote the exchange of ideas through extended conversation related to strategies. Two participants used centers in order to practice procedural fluency. Two participants have incorporated games in their classrooms. In terms of worked examples, there was no evidence that these of problems have been implemented. There was no evidence in the classroom observations or debriefs that participants were assessing or providing feedback related to fluency.

What Components of Fluency Do the Practices Present in the Classroom Address?

The components of fluency practice include efficiency, accuracy, and flexibility. Five of the participants had learning activities that promoted accuracy by helping students focus on strategies that helped them to get the correct answer. In terms of flexibility, two participants included learning activities that supported students in selecting the appropriate strategy. One of the participants included learning activities that encouraged students to select and use strategies based on how they could solve the problem in any efficient manner.

Overall the data that was collected, organized, analyzed from the survey, interviews, observations, and debriefs, it is determined that five participants would be categorized as unconsciously unaligned in relation to fluency practices and one participant was categorized as consciously aligned in relation to fluency practices. These findings indicate that five participants do not have knowledge and understanding needed to effectively implement classroom practices that support the development of procedural fluency. The specifics related to each component of the rubric across the cases is represented in table 4.13 and table 4.14.

Table 4.13

Combined Survey and Interview Data Analysis for All Cases

Research Subquestions	Indicator/Measure	Case Study					
		A	**B**	**C**	**D**	**E**	**F**
What does a teacher know and understand about fluency in mathematics?	Knowledge and Understanding of Procedural Fluency	1	1	1	2	3	4
	Purposeful Planning for Quality Fluency Practice - Focused, varied, processed, connected	1	0	3	0	2	3
	Purposeful Planning for Quality Fluency Practice- efficiency, accuracy, and flexibility	0	0	0	0	1	3
	Purposeful Planning for Quality Fluency Practice- Fluency Actions	0	0	1	1	2	3
	Overall Rating for Subquestion (Mean)	**0.50**	**0.25**	**1.25**	**0.75**	**2.00**	**3.25**
In what ways do teachers explicitly connect conceptual understanding to procedural fluency?	Understanding of the link between conceptual understanding and procedural fluency	2	1	2	1	3	3
	Overall Rating for Subquestion (Mean)	**2.00**	**1.00**	**2.00**	**1.00**	**3.00**	**3.00**
In what ways do teachers plan for fluency development?	Purposeful Planning for Quality Fluency Practice - Focused, varied, processed, connected	1	1	3	0	2	3
	Purposeful Planning for Quality Fluency Practice- efficiency, accuracy, and flexibility	0	0	0	0	1	3
	Purposeful Planning for Quality Fluency Practice- Fluency Actions	0	0	1	1	2	3
	Overall Rating for Subquestion (Mean)	**0.33**	**0.33**	**1.33**	**0.33**	**1.66**	**3.00**

How does what a teacher knows and understands about fluency translate into classroom practices during core instruction?	**Overall Rating**	**0.80**	**0.50**	**1.40**	**0.80**	**2.20**	**3.20**

Table 4.14

Combined Field Notes (Observations/Debriefs) Data Analysis for All Cases

Research Subquestions	**Indicator/Measure**	**Case Study**					
		A	**B**	**C**	**D**	**E**	**F**
What does a teacher do to ensure the development of procedural fluency in the classroom?	Explicit Strategy Instruction	1	3	3	3	3	3
	Visual Representations and Tools	2	1	1	3	1	1
	Overall Quality of Fluency Practices- Focused, varied, processed, connected and six fluency actions	1	3	1	2	2	3
	Overall Quality of Fluency Practice- efficiency, accuracy, and flexibility	0	2	3	2	3	3
	Overall Rating for Subquestion (Mean)	**1.00**	**2.25**	**2.00**	**2.50**	**2.25**	**2.50**
What does practice for fluency look like in the classroom?	Examples of Quality Fluency Practice: Routines	0	0	2	0	2	0
	Examples of Quality Fluency Practice: Worked Examples	0	0	0	0	0	0
	Examples of Quality Fluency Practice: Centers	0	3	1	0	0	0
	Example of Quality Fluency Practice: Games	1	0	0	0	0	3
	Assessment Type	0	0	0	0	0	0
	Feedback based on Assessment	1	0	0	0	0	1

	Overall Quality of Fluency Practices	1	3	1	2	2	3
	Overall Rating for Subquestion (Mean)	**0.71**	**0.86**	**0.57**	**0.29**	**0.57**	**1.00**
What components of fluency do the practices present in the classroom address?	Overall Quality of Fluency Practice- efficiency, accuracy, and flexibility	0	2	3	2	3	3
	Overall Rating for Subquestion (Mean)	**0.00**	**2.00**	**3.00**	**2.00**	**3.00**	**3.00**
How does what a teacher knows and understands about fluency **translate into classroom practices during core instruction**?	**Overall Rating**	**0.60**	**1.20**	**1.10**	**1.00**	**1.10**	**1.50**
Rating on Consciously Aligned Continuum	**Combination of Ratings from Both Tables**	**U/U**	**U/U**	**U/U**	**U/U**	**U/U**	**C/U**

Themes Across the Cases

There were many themes that materialized through the analysis of the findings across all cases. These themes include:

- When fluency practice is implemented in the classroom, the activities and learning tasks seemed to be more on end that elicit more teacher-centered learning practices. For example, all participants implemented direct strategy instruction in their classroom but did not provide opportunities for student generated strategies.
- The participants with the highest knowledge and understanding and application scores taught upper grade levels and were departmentalized (meaning that they were responsible for teaching the math for the entire grade level).
- The use of centers was not present in the intermediate grade levels.
- The participants' knowledge and understanding and application of fluency practices in the classroom equated most often to activities that promoted accuracy.

- There were several terms where the researcher and participants defined differently which impacted the application of classroom practices. This specifically impacted the practices of worked examples and routines as they related to quality fluency practice.

Chapter Summary

The findings show that there is a need to focus on the development of procedural fluency for not only students but teachers as well. The overall findings show that all but one participant do not have the knowledge and understanding necessary to implement classroom practices aligned with the development of procedural fluency. The final chapter will discuss the findings as the impact the teaching and learning of mathematics, implications based on the findings, and recommendations for future research.

Chapter 5 - Discussion of Findings, Implications, and Recommendations

Chapter Introduction

One component of an effective school, which has been determined to be the most influential are the teachers within the school (Marzano, 2010). With this statistic in mind, the content and pedagogical knowledge of teacher greatly impacts what occurs in the classroom and quality of the learning experiences for the students. This extends the concept of procedural fluency as it provides balance to math instruction through a focus on understanding and application (NCR, 2001). As a result, it is important to recognize what a teacher understands and knows about a topic as well as how this knowledge translates into classroom practices that could positively impact student achievement. The chapter will begin with a summary of the purpose of the study and methodology. After a review of the study, the findings of the over-arching research question and each subquestion will be summarized and implications of these findings will be discussed. It will conclude with recommendations for future studies.

Discussion of Findings

The purpose of the study was to determine what teachers know and understand in regard to procedural fluency. Then, based on the teacher's knowledge and understanding, the study researched the instructional practices integrated into the classroom to support the development of procedural fluency. Six educators in a pre-kindergarten through sixth grade elementary building in Kansas volunteered to participate in the study. A survey and interviews were used to collect data related to each participant's understanding and knowledge of the components and fluency actions related to procedural fluency. The application of knowledge and understanding into classroom practices was studied through classroom observations and debriefs with the teachers compiled in field notes. The data was analyzed using a priori codes and organized using a codebook. Using the data compiled in the codebook, rubrics were utilized to categorize the level of each participant's knowledge, understanding, and application of practices that align with the research and support the development of procedural fluency. In the paragraphs below, there is a comprehensive, synthesized summary of the findings related to the overarching research question.

What do Teachers Know and Understanding in Regard to Procedural Fluency?

The first purpose of the study was to determine what teachers know and understand related to procedural fluency. This was an important component of the study because a teacher could be implementing classroom practices that align with the development and support of the fluency components and actions but not be consciously aware of why these practices are critical. This could occur if the teacher is following the textbook with fidelity or implementing practices because they are the next big thing. If the teacher is consciously aware of how the practice aligns with the development of fluency, the instruction is planned and implemented with intentionality in regard to the desire outcome of the fluency practice. In addition, a comprehensive view of a teacher's knowledge and understanding of a particular topic helps to know how, where, and when to intervene in terms of professional development and instructional coaching that would need to be provided.

When looking at the knowledge and understanding of how teachers would define, practice, and assess fluency, only one participant had a wholistic view of procedural fluency (see Table 5.1 below). Procedural fluency encompasses both basic fact fluency and computation fluency and is fluency in the scope of procedures in mathematics (Bay-Williams & SanGiovanni, 2021). In addition, in for a student to be procedural fluent with a concept, they must have a deep knowledge and be able to “apply knowledge in carrying out procedures flexibly, accurately, efficiently, and appropriately” (Bay-Williams & Stoke-Levine, 2017, p.62). The data shows that teachers still have a narrow view of fluency, and three participants related it just the component of accuracy and included memorization as a component of fluency development. These findings supports literature that states that many educators still equate fluency with solely speed (Boaler & Zoido, 2016). The cause of this gap of knowledge may be explained by teachers not having access to materials and training that build a more updated and comprehensive idea of fluency. As a result, they continue to focus on fluency as it aligns with their understanding of it. The findings demonstrated that specific professional development related to what procedural truly is needed in order for teachers to become more consciously aware of how procedural fluency is built and supported in the classroom.

Table 5.1

Findings- Knowledge and Understanding Related to What Procedural Fluency is

	Description	**Case A**	**Case B**	**Case C**	**Case D**	**Case E**	**Case F**
Efficiency	The ability of students to solve a procedure in a reasonable time					X	
Flexibility	The ability of students to know multiple strategies as well as apply or adapt these strategies to solve a procedure				X	X	X
Appropriateness	The ability of students to select the appropriate strategy in relation to efficiency and flexibility					X	X
Accuracy	The ability of students to correctly solve a procedure	X	X	X	X		X
Memorization	The inclusion of memorization in relation	X	X	X	X		

	to how procedural fluency is defined						

Teachers must have knowledge and understanding related to conceptual understanding because procedural fluency cannot occur without conceptual understanding (NCTM, 2014). All six participants had knowledge and understanding related to what conceptual understanding is and four participants understood the link between procedural fluency and conceptual understanding. This knowledge and understanding is linked to the research because procedural fluency is linked to conceptual understanding and conceptual understand cannot be rushed (NRC, 2001). However, only two participants were able to articulate how this link impacts pacing, and no participants were able to connect this pacing to instructional decision-making (see Table 5.2).

This knowledge and understanding related to how conceptual understanding impacts pacing, planning, and instruction is especially important in the development of computational and procedural fluency. This is because "when we prematurely ask students to learn procedures without a solid understanding, students begin to see mathematics as isolated bits of knowledge to memorized rather than viewing it a s connected and coherent discipline" (Huinker & Bill, 2017, p. 57). The knowledge and understanding of this ideal needs to be an intentional focus of professional development efforts.

Table 5.2

Findings-Link Between Conceptual Understanding and Computational Fluency

	Description	**Case A**	**Case B**	**Case C**	**Case D**	**Case E**	**Case F**
Explanation	Conceptual understanding allows students to use mathematical procedures purposefully because they know when to use them and why they work	X	X	X	X	X	X

Link	Procedural fluency is built from conceptual understanding	X		X		X	X
Pacing	Procedural fluency cannot occur without conceptual understanding and conceptual understanding cannot be rushed					X	X

The development of computational and procedural fluency at the level needed for students to be mathematically proficient does not occur incidentally in the classroom. This level of development takes careful, targeted planning on the part of the teacher. Balanced and engaging fluency practices includes the planning of learning tasks that are focused, varied, processed, connected, and attend to all fluency components (Bay-Williams & SanGiovanni, 2021). There are a variety of different levels of understanding of what quality practice by the participants includes and no participants recognize the importance of incorporating all the fluency components. In addition, two participants describe practices related to fluency with the sole goal of memorization (see table 5.3). There is a need to increase the participants' knowledge and understanding of what constitutes quality fluency practices. This increased level of knowledge and understanding will be benefit students and teachers by helping teachers understand that "practice cannot be measured in the number of problems or exercises on a page but rather the number of times one is exposed to a situation, the frequency of that exposure, and the interaction or processing that embodies the experience" (Bay-Williams and SanGiovanni, 2021, p. 131). Thus, high quality and purposeful practice is just as important, if not more important, than amount of time or problems dedicated to practice (Schwartz, 2017).

Table 5.3

Findings- Purposeful Planning for Quality Fluency Practices

	Description	Case A	Case B	Case C	Case D	Case E	Case F
Focused	The description of learning activities that are planned to help students practice accuracy by learning to use a practice and get the correct answer			X		X	
Varied	The description of learning activities that are planned to help students practice fluency in a variety of ways related to cognitive demand, focus on components of fluency, and type of engagement	X					X
Processed	The description of learning activities that are planned to provide students will the opportunity to process and reflect after fluency practice			X			
Connected	The description of learning activities that are planned purposefully to help students see relationships and make connections			X		X	X
Balanced	The description of learning activities that are planned to integrate all components of fluency (efficiency, flexibility, accuracy, and appropriateness)						
Memorization	The description of learning activities that are focused on memorization as the end goal		X		X		

Another component that must be considered when planning is that of fluency actions that assist students in selecting the best strategy to solve a problem efficiently, accurately, and flexibly. Three participants' responses did not connect with any of the fluency actions while two participants inclusion of fluency actions was limited to completing the steps of a problem, which supports the development of accuracy. Only one participant was able to reference more than one fluency action as he or she conveyed his or her knowledge and understanding related to purposeful planning for fluency practice (see Table 5.4). Fluency actions can be developed through “the understanding of different strategies, practice with each of those strategies, and practice selecting between strategies” (Bay-William, 2021, p. 4). The planning of learning activities that attend to all six fluency actions in turn build and support the development of procedural fluency. There needs to be a concerted effort to increase the exposure of teachers to the six fluency actions in order to improve the quality of fluency practice.

Table 5.4

Findings- Purposeful Planning for Quality Fluency Practice- Fluency Actions

	Description	Case A	Case B	Case C	Case D	Case E	Case F
Strategy Selection	The description of learning activities that require students to appropriate strategies based on efficiency						
Reasonable Time	The description of learning activities that require students to solve problems in a reasonable time						
Trades out/ Adapts Strategy	The description of learning activities that require students to trade or adapt strategies					X	
Application of Strategy	The description of learning activities that develop students' ability to apply a strategy to a new problem type					X	
Complete Steps	The description of learning activities that develop students' ability to use an algorithm or strategy correctly			X	X		
Correct Answer	The description of learning activities that develop students' ability to get the correct answer even when there is more than one correct answer, or the goal is a reasonable						

	answer						

The findings of the study clearly demonstrate that teachers can benefit from opportunities to increase their knowledge and understanding related to procedural fluency. One significant finding of this study that aligns with the literature is that even though there is a great deal of literature to support the need for procedural fluency in the classroom in order to promote mathematical proficiency, it is still not occurring in many classrooms past fact fluency and memorization (Bay-Williams & SanGiovanni, 2021). From the survey and interview responses, the participants indicated that they had not had exposure to concept of procedural fluency because it has not been a focus of their professional development or their own educational experiences. Essentially, they did not know what they did know. This can be remedied by dedicating professional learning time to increase what teachers know and understand about how mathematical proficiency can be developed in the classroom at any grade level.

How Does What Teachers Know and Understanding related to Procedural Fluency Translate into Classroom Practices?

The second purpose of the study was to determine how what teachers knew and understood related to procedural fluency translated into classroom practices. This was an important component of the study because a teacher could have strong knowledge and understanding related to procedural knowledge but not translating into classroom practices. This could occur if the teacher is following the textbook with fidelity, but it does not consistently implement opportunities for purposeful practice. This could also occur in the teacher has the knowledge and understanding but does not see the purpose or value in implementing opportunities for students to practice. If the teacher is implementing classroom practices and learning activities that align with the development of fluency, the students have multiple opportunities to interact with components and actions of fluency regularly. An increased exposure to purposeful and high-quality practice will result in students with strong math identifies and agencies because they can see themselves as a learner and doer of math (NCTM, 2014).

Based on the data collected, direct and explicit strategy instruction is occurring in classroom. In addition, in most of the cases, strategy instruction is intentionally planned for as well. A commonality amongst all six cases was students did not have the opportunity to generate their own strategies (see Table 5.5). An important teacher action related to building procedural fluency from conceptual understanding is "providing students with opportunities to use their own reasoning strategies and methods for solving problems" (NCTM, 2014, p. 47). This is an imperative step in the development of fluency because fluency is built from exploration to informal strategies to solving problems (Huinker & Bill, 2017). This can be accomplished in classroom practices by focusing on selecting tasks that can be solve in many ways and allowing students to work through the solving problem without being shown how to complete the problem step by step. Overall, "learning different strategies opens the door to procedural fluency and learning to choose among those methods allows students to pass through the door" (Bay-Williams & SanGiovanni, 2021, pg. 16).

Table 5.5

Findings- Strategy Instruction

	Description	**Case A**	**Case B**	**Case C**	**Case D**	**Case E**	**Case F**
Direct	The description and implementation of instruction that includes demonstration, modeling, and practice related to strategies that could be used to solve procedures	X	X	X	X	X	X

Explicit	The description and implementation of instruction that clear and precise in order to help students use strategies to complete fluency actions		X	X	X	X	X
Intentionally Planned	The description and implementation of instruction that is deliberate and purposeful in the development, use and analysis of strategies used to solve procedures		X	X	X	X	X
Student Generated	The description and implementation of instructional practices that allow students to create and discuss their own strategies for solving procedures						

The targeted and purposeful use of visual representations and tools are necessary in the support of appropriate selection and use by students (Huinker & Bill, 2017). However, visual representations and tools are often promoted as a strategy in classroom, which is not the case (Bay-Williams & SanGiovanni, 2021). Visual representation and tools help students to visualize and make connections to a problem but not solve it. A majority (five out of six) cases implemented and described the use of visual representations that helped students to visualize a problem. In addition, two participants facilitated the use of visual representations to connect to specific strategy use while only one participant to strengthen student understanding during a discussion (see Table 5.6). A consistent use of visual representations and tools can support deeper understanding of the concept, help students to make connections to strategy use, compare and contrast strategies, and aid students in the move to more advance strategy use (Huinker & Bill, 2017). The teacher can use a variety of visual representations and tools including hundreds chart, open number lines, and pictures to support the use of strategies in the classroom.

Table 5.6

Findings- Visual Representation and Tools

	Description	Case A	Case B	Case C	Case D	Case E	Case F
Visualization	The description and implementation of tools and visuals that help students to visualize a problem but is not a strategy or action	X	X	X	X		X
Connection	The description and implementation processes that connect representation (tools) to strategies	X			X		
Discussion	The description and implementation processes that use visual representation and tools to strengthen the effectiveness of student understanding during discussion to provide structure and access to abstract concepts					X	

Practice can only support the development of procedural fluency if it is of high quality. Quality fluency practices include the following distinctions: focused, varied, processed, processed, connected, and balanced (Bay-Williams & SanGiovanni, 2021). In addition, effective practice attends to all six of the fluency actions. Besides focused practice, many qualities of quality practice including the fluency actions were sporadically implemented or not at all. All participants did implement focused practice that required students to practice a procedure in order to get the correct answer (see Table 5.7). This aligns with the knowledge and understanding of participants as their definition of procedural fluency focused mainly on accuracy. This correlation implies that there could be a link between what teachers know and understand about procedural fluency and how it translates into how fluency is practiced for and for what purpose. The implications of this finding are discussed in more detail in the section entitled, *Implication of Findings*.

Table 5.7

Findings- Quality of Fluency Practices

	Description	Case A	Case B	Case C	Case D	Case E	Case F
Focused	The implementation of learning activities that are planned to help students practice accuracy by learning to use a practice and get the correct answer	X	X	X	X	X	X
Varied	The implementation of learning activities that are planned to help students practice fluency in a variety of ways related to cognitive demand, focus on components of fluency, and type of engagement						
Processed	The implementation of learning activities that are planned to provide students will the opportunity to process and reflect after fluency practice					X	X
Connected	The implementation of learning activities that are planned purposefully to help students see relationships and make connections		X				X
Balanced	The implementation of learning activities that are planned with all to integrate all components of fluency (efficiency, flexibility, accuracy, and appropriateness)						
Fluency Actions	The implementation of learning activities that require students to attend to all six fluency actions						

Procedural fluency focuses on the components of accuracy, flexibility, and efficiency when selecting and using strategies to solve problems (NRC, 2001). As with their knowledge and understanding of procedural, the type of fluency component present in most classrooms (five out of six) was accuracy (see Table 5.8). Only half of the participants focused either of the other two components of fluency and no participant has classroom practices that incorporated all three. After the knowledge and understanding of procedural fluency is enhanced through professional development, there needs to be integration of classroom-based coaching to ensure that the newly acquired knowledge and understanding is translated into classroom practices effectively and intentionally. There are additional details related to the implications of this finding under the section heading, *Implications of Findings*.

Table 5.8

Findings- Fluency Components in Classroom Practices

	Description	**Case A**	**Case B**	**Case C**	**Case D**	**Case E**	**Case F**
Efficiency	The implementation of learning activities that requires students to focus on the selection of the appropriate strategy that helps them to solve the problem in a reasonable time.						X
Accuracy	The implementation of learning activities that requires students to focus on the selection of the appropriate strategy that helps them to solve the problem correctly.		X	X	X	X	X
Flexibility	The implementation of learning activities that requires students to focus on strategy instruction and requires students to select appropriate strategy based on efficiency and accuracy.			X		X	

Quality fluency practice can be achieved in the classroom with the integration of routines, worked examples, centers, and games (Bay-Williams & SanGiovanni, 2021). In terms of classroom application, three participants included components of routines (see Table 5.9), no participants utilized worked examples (see table 5.10), two participants integrated centers into their core instruction (see 5.11), and two participants leveraged the use of games (see Table 5.12). This is an area where professional development and instructional coaching should be focused because these are specific structures that can be implemented to purposefully practice all components and actions of fluency. Under *Implications of Finding*, there is greater detail about the implication of this finding and how it impacts how fluency is being addressed and practiced by students.

Table 5.9

Findings- Example of Quality Fluency Practice: Routines

	Description	**Case A**	**Case B**	**Case C**	**Case D**	**Case E**	**Case F**
Problem Solving	The description and implementation of instructional routines that provide students access to problems that can be solved in different ways					X	X
Discussion	The description and implementation of instructional routines that promote the exchange of ideas and are worthy of extended conversation related to strategy use			X		X	
When	The description and implementation of instructional routines that occur after a skill or concept is understood						
How Often	The description and implementation of instructional routines that occur often in the						

	classroom setting (example 3-5 times a week)						
Facilitation	The description and implementation of instructional routines that are organized to include groups of three constant partners.						
Purpose	The description and implementation of instructional routines that are used to provide structure in math classrooms with the establishment of specific student expectations for engagement and participation						
Focus on Strategy Selection	The description and implementation of instructional routines that engage student in strategy choices especially in relation to efficiency and accuracy						

Table 5.10

Findings- Example of Quality Fluency Practice: Worked Examples

	Description	Case A	Case B	Case C	Case D	Case E	Case F
Correctly	The description and use of examples that have been worked correctly to focus on efficiency and flexibility						
Partially	The description and use of examples that have been partially completed to focus on efficiency and accuracy						
Incorrectly	The description and use of examples that have been completed incorrectly to focus on accuracy						
Problem Selection	Problems are intentionally selected to encourage the use of appropriate strategy selection, the correct						

	completion of steps, getting the correct answer, and applying a strategy to a new problem type						

Table 5.11

Findings- Example of Quality Fluency Practice: Centers

	Description	**Case A**	**Case B**	**Case C**	**Case D**	**Case E**	**Case F**
Engagement	The description and implementation of learning task that require active student participation and integrate accountability		X	X			
Challenge	The description and implementation of learning tasks that are differentiated in order to provide the appropriate level of challenge based on the student's zone of proximal development		X				
Motivation	The description and implementation of learning tasks that are designed to motivate the student through choice, student interests, learning styles, enjoyment, high expectations, and opportunities for success						
Learning Tasks	The description and implementation of learning tasks that develop procedural fluency which could include by are not limited to sorting tasks, choice problems and games that can be played independently or collaboratively		X	X			

Table 5.12

Findings- Example of Quality Fluency Practice: Games

	Description	Case A	Case B	Case C	Case D	Case E	Case F
Focus	The planning and implementation of games that purposefully targets a strategy or component of fluency student have been learning	X					X
Purpose	The planning and implementation of games that include practice that promotes high levels of rigor and accuracy						X
Accountability	The planning and implementation of games that include accountability for the student(s) related to the fluency actions so each student can be held responsible for their own thinking						
Strategy Selection	The planning and implementation of games that include opportunities for students to select and make choices about the best strategy to use related to efficiency						
Check for Accuracy	The planning and implementation of games that include opportunities for students to check for accuracy (they are completing the steps and getting the correct answer)						X

The purpose of assessments is fundamentally about the communication (Liljedahl, 2021). Assessments communicate what is important, what has been taught, and determines the level to which learning has occurred. All six cases use formative assessments to determine student understanding and adjust their teaching as a result. In two of the cases, the teachers provide students with feedback to support their learning (see Table 5.13). The assessments and feedback did not relate to the fluency components beyond that of accuracy. In the quiz that was given in participant B's classroom, for example, the assessment asked students to solve procedures focused on accuracy or getting the correct answer. A shift that can occur is to use other forms of formative and summative assessments to critique the level of student understanding. One example that can be utilized is journal prompts that provide students with the opportunity to write about their thinking related to their strategy selection and use (SanGiovanni, Bay-Williams, & Serrano, 2022). By focusing on all components of fluency in the way students are assessed communicates to students and their parents that procedural fluency encompasses more speed and accuracy (Bay-Williams & SanGiovanni, 2021).

In terms of feedback, the purpose is to help “close the gap between students’ current level of understanding or performance and the expected level of performance” (Hattie, Fisher, & Frey, 2017). A small, but powerful tweak that can be made by educators is to focus feedback on all components related to procedural fluency. This could be done with the use of a fluency rubric, which integrates all the components of fluency, to clearly communicate the expectations and help student understand what it takes to get to the next level (Bay-Williams & SanGiovanni, 2021). Overall it is important to gather data related to procedural from many sources and provide targeted, specific feedback to students regularly, so they know what they have done well and where they need to go next (Hattie, Fisher, & Frey, 2017).

Table 5.13

Findings- Assessment and Feedback

	Description	Case A	Case B	Case C	Case D	Case E	Case F
Focus	Assessment of fluency focuses on efficiency, flexibility and accuracy related to appropriate strategy selection						
Purpose	Assessment data is used to plan and evaluate the effectiveness of activities used for fluency practice						

Formative	Assessments are ongoing and used to monitor student progress	X	X	X	X	X	X
Summative	Assessment tools that are used to determine the level of student learning that has occurred						
Feedback	Assessments are used to provide students with timely, specific information related to procedural fluency progress	X					X
Parents	What we assesses communicates what is important-Ensuring all fluency components are attended to and visible communicated the real goal of procedural fluency						

The findings of the study clearly demonstrate that once a teacher's knowledge and understanding related to procedural has been increased through professional development, there is still additional work that needs to be done by educational leaders to ensure that this knowledge and understanding is translated effectively in classroom practices. There were instances from the study where the teacher had the knowledge and understand related to a classroom practice that supported the development of procedural fluency but was not reflected in their classroom practices. One significant finding of this study is there is not a clear understanding of what quality practice looks like related to procedural fluency. From the observation and debrief conversation, there needs to be a focus on developing a common language and expectation for many terms related to procedural fluency. It is imperative to get everyone on the same page, so teachers understand what classroom practices need to be implemented in their classrooms and why. This can be addressed by dedicated time to instructional coaching to provide teachers with the support need to transfer learning for professional development sessions into classroom practices that are used consistently and intentionally.

Implications of Findings

The study found that teachers do not have a clear understanding of procedural fluency or an understanding of the current best practices for how it should be developed in the classroom. As a result, the instructional practices being utilized do not align with fluency components and actions needed to development procedural fluency effectively. In education, there is a push to improve reading instruction with a focus on the science of reading and mandated professional development. Many principles of structured literacy are just as prevalent when teaching math including the need for math instruction to be direct, explicit, systematic, and cumulative. The need to shift from idea of the science of reading to looking at the research as the science of quality instruction may provide the focus on mathematical professional development that should be occurring in classrooms across the state of Kansas.

The findings suggest that there is a need to continue to develop the knowledge and understanding of procedural fluency for teachers in order for classroom practices to change. This can be accomplished through quality professional development and classroom based instructional coaching. In the subsequent paragraphs in this section, the instructional shifts that must be focused on during professional learning opportunities are discussed. These assertations are based on what is currently occurring the classroom and the shifts that need to be integrated to develop and support the development of fluency components and actions more purposefully.

Strategy Instruction

The participants consistently employed the incorporation of strategy instruction within their classrooms. There was a focus on the demonstration, modeling, and practicing related to strategies through clear, precise instruction represented in all six cases. In addition, there was intentionality in planning opportunities for students to engage in purposeful, deliberate instruction related to the development of strategies used to solve procedures. In order to continue to align and strengthen strategy instruction with the development of fluency, students need opportunities to create and discuss their own strategies for solving procedures (NCTM, 2014). The way that this could look in practice is by having students solve problems without being given a specific strategy to solve it. Through conversation with peers, students can work to solve problems by choosing and using the best strategy for the particular problem (Huinker & Bill, 2017).

Fluency Components

The participants focused on fluency components especially related to accuracy. In order to continue to support the development of procedural fluency in the classroom, there needs to be purposeful planning to provide opportunities for students to select and use strategies based on solving the problem in a reasonable amount of time. The focus on efficiency when choosing and using strategies will also strengthen students' ability to trade out and adapt strategies. These fluency components are connected to a large body of literature (Bay-Williams & SanGiovanni 2021; Huinker & Bill, 2017; NCTM, 2014; NRC, 2001). In classroom practices, students will have the opportunity to practice a strategy that is typically teacher directed and focused on accuracy. Even when strategy selection is integrated into the lesson, there is not a discussion about the appropriate of the strategy based on efficiency. One way that this can shift in classroom instruction is providing time for academic conversation where students discuss why a particular strategy works best for a certain type of problem whenever possible.

Quality Fluency Practice

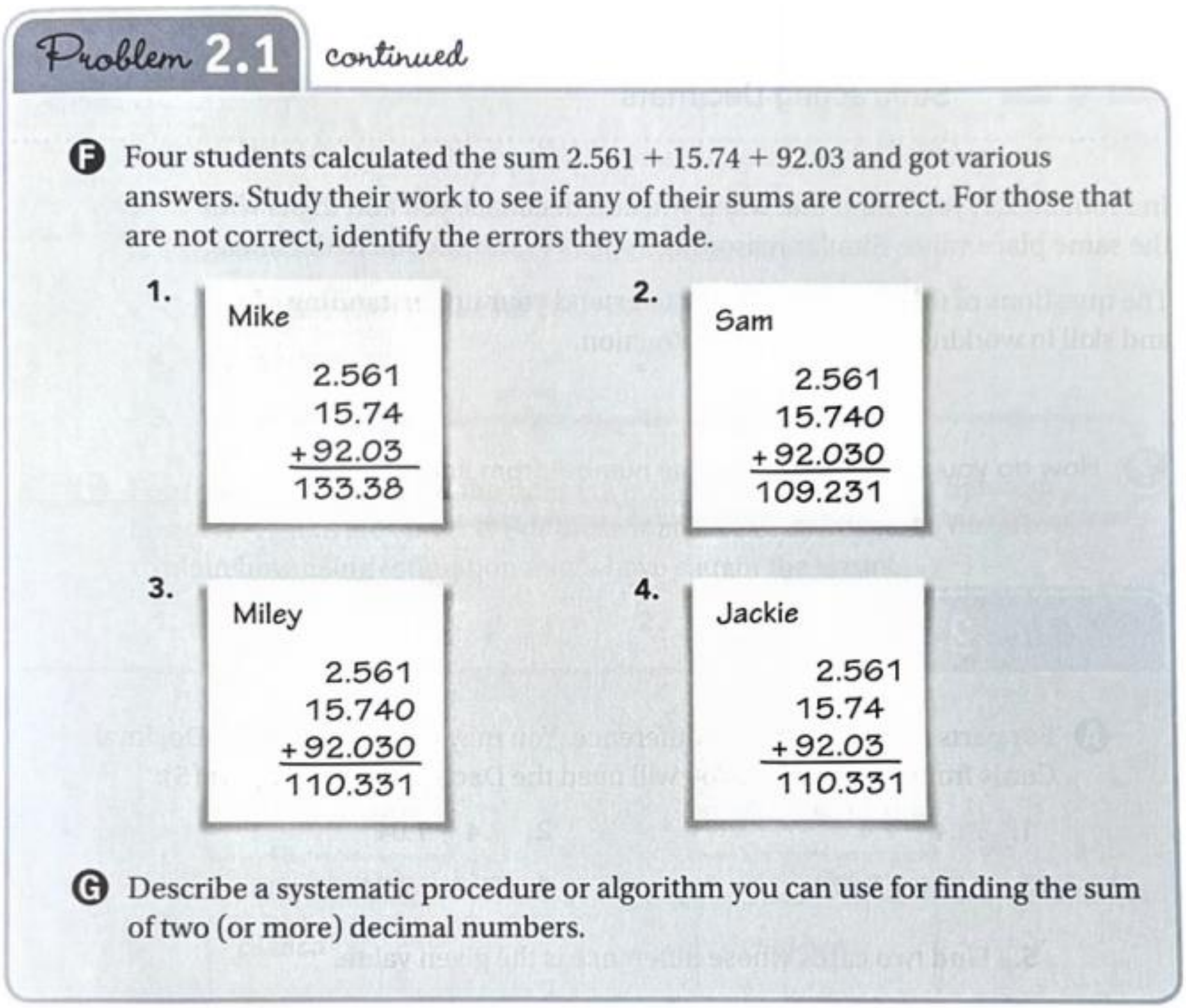
Routines. When analyzing the results for the data collected a theme emerged related to understanding and use of routines in classroom practices. The participants equate routines with parts of the lesson design that are followed each day. For example, participant F shared, that he or she follows a routine of whole group, table group, and individually when working through problems in class. However, routines can hold a much more critical role in classroom practices. The foundation of routines is providing students with access to problems that can be solved in different ways and are worthy of extended conversation related to strategy use (Berger, 2017; Bay-Williams & SanGiovanni, 2021). By intentionally setting aside time during the core instructional block for routines, students have the opportunity to make connections as well as process their learning. A shift towards this type of practice might be to dedicate instructional time each day to Number Talks (Parrish, 2014). This use of number talks has specific expectations for student engagement and participation. The teacher will need to determine when and how often the routine will occur. This structure can support a balanced approach to procedural fluency in the classroom.

Worked Examples. Themes from throughout the survey responses, interview responses, observations, and debriefing conversations demonstrated that there is not a clear understanding of what quality practices looks like in terms of fluency development especially related to structures that can be implemented. One specific example of this materializes when analyzing the responses on the survey and interview. On the survey, four participants marked that they used worked examples in class. However, their understanding of worked examples related to teacher demonstrating examples and students practicing problems through direct instruction. The sentiment of the cases was best summed up in an interview when one of the participants stated, “I thought that was just like their work, like your math lesson whatever we’re doing on their whiteboards. I don’t even know what that is. I just thought it looked good, so I clicked it” when asked about examples of how worked examples are utilized in the classroom. The participants were unsure of what the researcher was looking for when answering the question and asked for reassurance that what they were sharing was correct. There was a clear discrepancy between the definition of the worked examples supported by the literature and the participants’ knowledge and understanding related to the component of quality fluency practice.

This discrepancy highlights the need for shifts to occur in the way fluency is developed and supported in the classroom related to worked examples related to teacher knowledge and application into classroom practices. An example related to current practice in the classrooms observed during this study might focus on the adding of decimals in this way. The teacher writes the problem on the board and students work in pairs or independently to solve it on a piece of paper/marker board. Then, one student or in some cases the teacher will explain how to solve it. The idea that students were solving examples met the definition of worked examples in the cases studied. The research though defines a worked examples as problems that are already solved (or partially solved) used to structure discussion focused on when and why a strategy works and makes sense to use (Hattie, Fisher, & Frey, 2017; Bay-Williams &SanGiovanni, 2021). So instead of the example outlined above, a task like the one found in figure 5.1 could be used. In the instance, students analyze different problems (some worked correctly, or some worked incorrectly) in order to determine the correct answer. Students would work in small groups to determine the correct answer, create rationale for why it is correct as well as rationale for how they know which sums are incorrect. By structuring the task in the way, the learning activity helps students to develop the fluency actions related to complete steps and getting the correct answer. In addition, quality fluency practice is connected and focused. The task is designed to help students practice accuracy and see relationships. Overall, it is critical that teachers have the skills and knowledge needed to analyze problems, determine how the problems promote (or do not promote) fluency development, and see value in the dedicating academic learning time to these types of tasks.

Figure 5.1

An example of a problem with worked examples


Problem 2.1 continued

F Four students calculated the sum 2.561 + 15.74 + 92.03 and got various answers. Study their work to see if any of their sums are correct. For those that are not correct, identify the errors they made.

1. Mike

2.561
15.74
+92.03
133.38

2. Sam

2.561
15.740
+92.030
109.231

3. Miley

2.561
15.740
+92.030
110.331

4. Jackie

2.561
15.74
+92.03
110.331

G Describe a systematic procedure or algorithm you can use for finding the sum of two (or more) decimal numbers.

This was also true in relation to the participants’ responses aligned with routines. Participants identified commonalities in their schedule and the structure of the lesson. In addition, assessment is not assessing the fluency components beyond using a procedure to get an accurate answer.

The use of quality fluency practices and targeted assessment would support the development of procedural fluency in the classroom at a much deeper and sustained level.

Games. The use of games in an effective way to build procedural fluency when implemented with this purpose in mind (Bay-Williams & SanGiovanni, 2021). The games that were applied in the classroom lacked an opportunity for students to select and use the appropriate strategy in terms of efficiency. The purpose of the games was typically focused on accuracy. For example, in one case, the teacher utilized a game to practice concepts related to fractions including adding fractions with like denominators, comparing fractions, and identifying equivalent fractions. During the game, the focus was on getting the correct answer in order to win the game. The game might have been enhanced to further support the development of procedural fluency by having students discuss the strategy they used to solve the problem and why. There are specific procedures that need to be utilized in order for students to compare, add, and find equivalent fractions. By providing opportunities for students to discuss their strategy selection the practice becomes focused and connected because students can make connections between strategies and problem types. Then, after the completion of the game it is important to provide students with an opportunity to reflect on the appropriateness and efficiency of the strategies they selected to solve each problem type. By dedicating time for students to reflect, the practice becomes processed in nature. With addition of student discussion, the game now is purposeful, focuses on strategy selection, and is designed to support student accountability since they must justify their answer.

With a specific focus on how current classroom practices can be tweaked or shift to better align with the best practices for the development of procedurally fluency, the quality of the practice will improve. In turn, students will feel more confident about their ability to solve problems even if they deem them to be difficult (NCTM, 2014).

Recommendations for Future Research

This study provides an in-depth analysis of six participants' knowledge and understanding of procedural fluency and how this translates into classroom practices. A theme that emerged from findings was the lack of center use in the intermediate elementary grades. Future research could study the use of centers by grade level to determine if it is something that teachers perceive as not appropriate or valuable as students grow older. Another theme that emerged from the study was stronger knowledge and understanding based on the analysis of the data using the rubric of participants that were departmentalized and taught upper elementary grades. Future research is needed to determine the correlation between departmentalize and upper grade level content as it relates to the knowledge and understanding of the teacher.

The knowledge, understanding, and level of application related to classroom practices of a teacher are all important components aligned with quality instruction. This study found that educators may not have the knowledge and understanding of fluency components and fluency actions to effectively develop procedural fluency and math proficiency in the classroom. The study provides a way of categorizing the current reality of teachers' content and pedagogical into a matrix of four categories (unconsciously unaligned, consciously unaligned, unconsciously aligned, and consciously aligned. These categories can be determined through the use the codebook and rubrics. This information can be used to inform decision-making about future professional development and provide a focus for instructional coaching. The use of instructional coaching is necessary to ensure knowledge and understanding are translated into classroom practices.

Conclusion

There have aggressive pendulum swings between a focus on basic facts and a focus on understanding when teaching mathematics. In the middle of this continuum of educational philosophies stands a concept that can bridge these ideals together to ensure that mathematics is taught in a systematic, explicit manner: procedural fluency. This study was significant because prior research had determined what best practices related to procedural fluency are. The study concluded that there is a relationship between what teachers know and understanding about procedural fluency and how procedural fluency is developed through classroom practices. There is a need to increase the level of knowledge and understanding of procedural fluency through professional development opportunities and ensure that the knowledge and understanding translates into classroom practices through instructional coaching.

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Appendix A - Informed Consent Form

Procedural Fluency in Mathematics: Exploring Elementary Teachers' Knowledge, Understanding, and Application in Classroom Practices

Principal Investigator:
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The case study will examine your knowledge and understanding of mathematical procedural fluency. The study will also observe how this knowledge and understanding of procedural fluency is translated into classroom practices as you plan, provide instruction, assess, and reflect. We hope by studying your experience and collaborating with you, we can better understand how content knowledge and understanding correlates with the instructional practices occurring in the classroom.

This study is designed so we can collaborate with you in a variety of ways in order to gain insight related to classroom practices that support procedural fluency. You will:

1. Complete an electronic survey to share your current level of knowledge and understanding related to procedural fluency and how you believe you currently promote fluency instruction in your classroom.
2. Participate in an online interview to follow up with your survey responses and gather more in-depth details.
3. Be observed teaching three math lessons. During the observation, we will be identifying practices that promote the development of procedural fluency.
4. Debrief with me after the observations to capture your thoughts and feelings related to the instruction and fluency practices.

5. Participate in a final debriefing session in order to reflect on all the lessons overall and formulate a plan for next steps.

We will compensate you with a copy of the Figuring Out Fluency companion book of your choice (Addition and Subtraction with Whole Numbers, Multiplication and Division with Whole Numbers, Addition and Subtraction with Fractions and Decimals or Multiplication and Division with Fractions and Decimals) and that best aligns with the mathematics standards at your grade level. If you withdraw from the project, you will still receive a copy of the book you selected.

RESEARCH PARTICIPANT INFORMED CONSENT FORM

No foreseeable risks or discomforts are anticipated throughout the duration of this project. Findings will be used to enhance our cognizance of how teachers' knowledge and understanding related to procedural fluency is applied in the form of classroom practices. The mathematical educational community will benefit from the study of how knowledge and understanding is translated into classroom practices. In addition, the data collected, analyzed, and reported from this study could be utilized as a guide to determine appropriate professional development experiences related to quality fluency practices moving forward. The results of the study will be shared with each participant.

The research involves a survey (in electronic form), interviews (using a remote meeting platform), and observations (in person). The identity of the participant, including names and other identifiers, will be kept confidential. Pseudonyms will be used when data is collected and when results are published in any written form including, but not limited to journals and dissertation work or presented at a conference. Data will be made available for reporting purposes only and will not be used or distributed for future research studies without additional informed consent. Responses and recording (both audio and video) will be maintained on secure computers.

Institutional Review Board Chair Contact Information:

- Rick Scheidt, Chair, Committee on Research Involving Human Subjects, 203 Fairchild Hall, Kansas State University, Manhattan, Kansas 66506, 785-532-3224.
- Cheryl Doerr, Associate Vice President for Research, Compliance, 203 Fairchild Hall, Kansas State University, Manhattan, Kansas 66506, 785-532-3224.

I understand that the project is research, and my participation is completely voluntary. I also understand that if I decide to participate in this study, I may withdraw my consent at any time and stop participating at any time without explanation, penalty, loss of benefits, or academic standing to which I may otherwise be entitled.

I verify that my signature below indicates that I have read and understand this consent form. In addition, my signature verifies I willingly agree to participate in this study under the terms described. My signature also acknowledges that I have received a signed and dated copy of this consent form.

Participant Name (print please): ____

Participant Signature: ________________________________ Date: ____________________

Witness Signature: __________________________________ Date: ____________________

Appendix B - Procedural Fluency Survey Questions

Start of Block: Demographic Information

Q1 Identification Code:

Q2 Including this year, how many years have you taught?

Q3 What grade level(s) do you currently teach?

- Kindergarten (1)
- First Grade (2)
- Second Grade (3)
- Third Grade (4)
- Fourth Grade (5)
- Fifth Grade (6)

Q4 Including this year, how many years have you taught this grade level(s)?

Q5 How confident do you feel in regard to teaching math at your assigned grade level(s) with one being not very confident and five being very confident?

End of Block: Demographic Information

Start of Block: Block 1

Q6 Explain mathematical procedural fluency in your own words.

Q7 What is the relationship between procedural fluency and conceptual understanding?

Q8 Describe what quality fluency practice looks like in the math classroom.

Q9 What are some indicators that a student is fluent with a particular math procedure?

End of Block: Block 1

Start of Block: Block 2

Q10 What instructional practices do you currently use to teach, practice, and assess fluency in your math classroom? Please check all that apply.

- Games (1)
- Routines (2)
- Timed Tests (3)
- Centers (4)
- Flash Cards (5)
- Memorization Strategies (6)
- Worked Examples (7)
- Rubrics or Checklists (8)
- Visual Representations or Tools (9)
- Number Tricks (10)

Q11 During your whole group math time, over the course of a unit, how often do students...

	Never or Almost Never	Some Lessons	Most Lessons	Every Lesson

	(1)	(2)	(3)	(4)
have access to visual models to support their understanding of general methods (1)	○	○	○	○
have opportunities for distributed practices of procedures (2)	○	○	○	○
discuss and explain why the procedures they are using work to solve particular problems (3)	○	○	○	○
focus on efficiency of strategies utilized (5)	○	○	○	○
focus on accuracy of strategies utilized (6)	○	○	○	○
focus on the appropriateness of strategies utilized (7)	○	○	○	○

End of Block: Block 2

Appendix C - Generic Initial Interview Guide

Interview Information

Name of Interviewee:
Date:
Time:
Location:
Name of Audio File:

Interview Introduction

- Interviewer introduces herself
- Explains purpose of the study
- Obtain copy of signed consent form
- Provide general overview of related to how the interview will be structured

Specific Questions

(All participants will be asked this list of questions)

1. Describe your experiences as a math student.
2. Describe your experiences related to learning to teach math.
3. What do you believe are the most important things students should learn in math at your grade level?
4. How do you determine what needs to be taught throughout a unit of study?
5. How do you determine how you will deliver and facilitate learning during a unit of study?

Additional Open-Ended Questions

(Questions asked in this section will vary based on the survey response from each case study participant).

1. You marked (number selected on survey) in relations to your confidence teaching math of your grade level. What factors lead to your rating? (Based on question four)

2. Follow up on instructional practices looking for specific examples of implementation based on practices selected by the respondent. (Based on question ten)

   A. If *games* is selected…. Describe an example and examples of how games were used to teach, practice, and/or assess fluency in your math classroom. How did the use of games impact student learning related to fluency? When do you use this practice in the scope and sequence of the curriculum/unit?

B. If *routines* is selected.... Describe an example or examples of routines that you have used to teach, practice and/or assess fluency in your math classroom. How did the use of routines impact student learning related to fluency? When do you use this practice in the scope and sequence of the curriculum/unit?

C. If *timed tests* is selected.... Describe how you have used timed tests to teach, practice and/or assess fluency in your math classroom. How did the use of timed tests impact student learning related to fluency? When do you use this practice in the scope and sequence of the curriculum/unit?

D. If *centers* is selected…. Describe how you have used centers to teach, practice and/or assess fluency in your math classroom. How did the use of timed tests impact student learning related to fluency? When do you use this practice in the scope and sequence of the curriculum/unit?

E. If *flash cards* is selected…. Describe how you have used flash cards to teach, practice and/or assess fluency in your math classroom. How did the use of timed tests impact student learning related to fluency? When do you use this practice in the scope and sequence of the curriculum/unit?

F. If *memorization strategies* is selected…. Describe how you have used memorization strategies to teach, practice and/or assess fluency in your math classroom. How did the use of memorization strategies impact student learning related to fluency? When do you use this practice in the scope and sequence of the curriculum/unit?

G. If *worked examples* is selected…. Describe how you have used worked examples to teach, practice and/or assess fluency in your math classroom. What types of worked examples do you typically use and how do use decide what to use as an example? How did the use of worked examples impact student learning related to fluency? When do you use this practice in the scope and sequence of the curriculum/unit?

H. If *rubrics or checklists* is selected…. Describe how you have used rubrics or checklists to teach, practice and/or assess fluency in your math classroom. How did the use of rubrics or checklists impact student learning related to fluency? When do you use this practice in the scope and sequence of the curriculum/unit?

I. If *visual representations or tools* is selected…. Describe how you have used visual representations or tools to teach, practice and/or assess fluency in your math classroom. How did the use of visual representations or tools impact student learning related to fluency? When do you use this practice in the scope and sequence of the curriculum/unit?

J. If *number tricks is selected*…. Describe how you have used number tricks to teach, practice and/or assess fluency in your math classroom. How did the use of number tricks impact student learning related to fluency? When do you use this practice in the scope and sequence of the curriculum/unit?

Probes

(The following probes will be used ask for more information or additional explanation as appropriate throughout the interview)

- "Please tell me about"
- "Could you provide more details regarding"
- "Could you explain your response in greater detail"
- "What do you mean when you say_________"

Interview Conclusion

- Thank the interviewee for their time and assistance.
- Ask the interviewee if he or she has any question regarding the interview process/procedure
- Ensure confidentiality of responses

Appendix D - Field Notes Template for Observations and Debriefing Conversations

Observation Field Notes		
Length of Activity: Date: Number of Students: Number of Adults:		
Descriptive Notes		**Reflective Notes**
Teacher Actions	Student Actions	

Debriefing Conversation Field Notes

Date: Time: Attendees:		
Summarizing the Lesson- **Opportunity for Reflection**	**Descriptive Notes**	**Reflective Notes**
What do you think went well?		
Unpacking of Lesson- **Focus on determining reasons behind actions and responses related to the development of fluency**	**Descriptive Notes**	**Reflective Notes**
What did you do in previous lessons to prepare students for this?		
How did you plan for the teaching, practicing, or assessment of fluency?		
What are you planning to do in the next lesson?		

Final Debriefing Conservation Field Notes **(After all lessons have been observed)**		
New Learning and Direction- **Focus on what was learning from the lesson and goal setting**	**Descriptive Notes**	**Reflective Notes**
Thinking about all three lessons, how did you plan for fluency development?		
What aspect(s) of fluency did these lessons focus on (efficiency, flexibility, accuracy?)		
What are your next steps in supporting fluency development in the classroom?		
What support do you need moving forward?		

Appendix E - Generic Follow-up Interview Guide

Interview Information

Pseudonym of Interviewee:
Date:
Time:
Location:
Name of Audio/Video File:

Interview Introduction

- Interviewer introduces herself
- Explains purpose of the study
- Obtain copy of signed consent form
- Provide general overview of related to how the interview will be structured

Open Ended Questions

(Questions asked in this section will be based upon an answer of *some lessons*, *most lessons*, and *every lesson*).

1. Follow up on instructional practices to identify specific examples of implementation based on practices selected by the respondent. (Based on question eleven)
    A. If the participant indicates some, most, or every related to *visual models*…. On the survey, you indicated that students have access to visual models to support their understanding of general methods during (response selected on survey) lesson(s). Please describe what that looks like in your classroom.

       **AA1. Research describes visual models as pictures and images that help students to interpret numbers. With this definition in mind, how does this look in your classroom? How often do students have access to visual models over the course of a unit?

    B. If the participant indicates some, most, or every related to *distributed practice*…. On the survey, you indicated that students have opportunities for distributed practices of procedures during (response selected on survey) lesson(s). Please describe what that looks like in your classroom.

       ** BB1. Distributive practice occurs when review takes place after the original learning has occurred. With this definition in mind, how does this look in your classroom? How often do students have access to distributed practice over the course of a unit?

** BB2. Research also states that quality practice should be focused, varied, processed, connected, and balanced. We will discuss each of these components in greater detail related to your classroom practices.

** BB2a. (Focused). When students have opportunities to practice, the practice should be intentionally planned so students have opportunities to practice accuracy by learning to use procedure and get the right answer. With this definition in mind, how does this look in your classroom? How often do students have access to focused practice over the course of a unit?

** BB2b. (Varied). When students have opportunities to practice, the practice should be intentionally planned so students have a variety of experiences related to the cognitive demand, focus on the components of fluency, and engagement. With this definition in mind, how does this look in your classroom? How often do students have access to varied practice over the course of a unit?

** BB2c. (Processed). When students have opportunities to practice, the practice should be intentionally planned so students have opportunities to process their learning through reflection. With this definition in mind, how does this look in your classroom? How often do students have access to processed practice over the course of a unit?

** BB2d. (Connected). When students have opportunities to practice, the practice should be intentionally planned in order to help students see relationships and connections. With this definition in mind, how does this look in your classroom? How often do students have access to connected practice over the course of a unit?

** BB2e. (Balanced). When students have opportunities to practice, the practice should be intentionally planned so there is a balance related to the components of fluency: efficiency, flexibility, accuracy, and appropriate strategy selection. With this definition in mind, how does this look in your classroom? How often do students have access to balanced practice over the course of a unit?

C. If the participant indicates some, most, or every related to *discuss and explain why the procedures work….* On the survey, you indicated that students discuss and explain why the procedures they are using work to solve particular problems during (response selected on survey) lesson(s). Please describe what that looks like in your classroom.

D. If the participant indicates some, most, or every related to *opportunity to use own reasoning strategies and methods….* On the survey, you indicated that students have the opportunity to use their own reasoning strategies and methods for solving problems during (response selected on survey) lesson(s). Please describe what that looks like in your classroom.

E. If the participant indicates some, most, or every related to *focus on efficiency….* On the survey, you indicated that students focus on the efficiency of strategies utilized during (response selected on survey) lesson(s). Please describe what that looks like in your classroom.

** EE1. Research indicates that efficiency is the ability of students to solve a procedure in a reasonable time. With this definition in mind, how does this look in your classroom? How often do students focus on the efficiency of strategies over the course of a unit?

F. If the participant indicates some, most, or every related to *focus on accuracy….* On the survey, you indicated that students focus on the accuracy of strategies utilized during (response selected on survey) lesson(s). Please describe what that looks like in your classroom.

** FF1. Research indicates that accuracy is the ability of students to correctly solve a problem. With this definition in mind, how does this look in your classroom? How often do students focus on the accuracy of strategies over the course of a unit?

G. If the participant indicates some, most, or every related to *focus on appropriateness….* On the survey, you indicated that students focus on the appropriateness of strategies utilized during (response selected on survey) lesson(s). Please describe what that looks like in your classroom.

**G1. Research indicates that appropriateness is the ability of the students to select the most appropriate strategy in relation to efficiency and accuracy. With this definition in mind, how does this look in your classroom? How often do students focus on appropriate strategy selection over the course of a unit?

Specific Questions

(Questions asked in this section will vary based on the survey response from each case study participant).

1. Is there anything else related to procedural fluency that we have not discussed that you would like to share?

Probes

(The following probes will be used ask for more information or additional explanation as appropriate throughout the interview)

- “Please tell me about”
- “Could you provide more details regarding”
- “Could you explain your response in greater detail”
- “What do you mean when you say_________”

Interview Conclusion

- Thank the interviewee for their time and assistance.
- Ask the interviewee if he or she has any question regarding the interview process/procedure
- Ensure confidentiality of responses
- Inquiry about the classroom resource, supply, or book, the participant would like for the upcoming school year.

Appendix F - List of A Priori Codes

Code	Meaning
A1	Knowledge of Procedural Fluency- Efficiency
A2	Knowledge of Procedural Fluency- Flexibility
A3	Knowledge of Procedural Fluency- Appropriateness
A4	Knowledge of Procedural Fluency- Accuracy
B1	Knowledge of Conceptual Understanding- Explanation of Conceptual Understanding
B2	Knowledge of Conceptual Understanding- Link Between Conceptual Understanding and Procedural Fluency
B3	Knowledge of Conceptual Understanding- Pacing
C1	Knowledge of Elements of Quality Fluency Practice: Purposeful Planning- Focused
C2	Knowledge of Elements of Quality Fluency Practice: Purposeful Planning- Varied
C3	Knowledge of Elements of Quality Fluency Practice: Purposeful Planning- Processed
C4	Knowledge of Elements of Quality Fluency Practice: Purposeful Planning- Connected
C5	Knowledge of Elements of Quality Fluency Practice: Purposeful Planning- Balanced
D1	Knowledge of Elements of Quality Fluency Practice: Fluency Actions- Strategy Selection
D2	Knowledge of Elements of Quality Fluency Practice: Fluency Actions- Reasonable Time
D3	Knowledge of Elements of Quality Fluency Practice: Fluency Actions- Trades Out/ Adapts Strategy
D4	Knowledge of Elements of Quality Fluency Practice: Fluency Actions- Application of Strategy
D5	Knowledge of Elements of Quality Fluency Practice: Fluency Actions- Completes Steps
D6	Knowledge of Elements of Quality Fluency Practice: Fluency Actions- Correct Answer
E1	Application of Strategy Instruction- Direct
E2	Application of Strategy Instruction- Explicit
E3	Application of Strategy Instruction- Intentionally Planned
E4	Application of Strategy Instruction- Student Generated
F1	Application of Visual Representation and Tools- Visualization
F2	Application of Visual Representations and Tools- Connection
F3	Application of Visual Representations and Tools- Discussion
G1	Application of Elements of Quality Fluency Practice- Focused
G2	Application of Elements of Quality Fluency Practice- Varied

G3	Application of Elements of Quality Fluency Practice- Processed
G4	Application of Elements of Quality Fluency Practice- Connected
G5	Application of Elements of Quality Fluency Practice- Balanced
G6	Application of Elements of Quality Fluency Practice- Fluency Actions
H1	Application of Routines- Problem Solving
H2	Application of Routines- Discussion
H3	Application of Routines- When
H4	Application of Routines- How Often
H5	Application of Routines- Facilitation
H6	Application of Routines- Purpose
H7	Application of Routines- Focus on Strategy Selection
I1	Application of Worked Examples- Correctly
I2	Application of Worked Examples- Partially
I3	Application of Worked Examples- Incorrectly
I4	Application of Worked Examples- Problem Selection
J1	Application of Centers- Engagement
J2	Application of Centers- Challenge
J3	Application of Centers- Motivation
J4	Application of Centers- Learning Tasks
K1	Application of Games- Focus
K2	Application of Games- Purpose
K3	Applications of Games- Accountability
K4	Application of Games- Strategy Selection
K5	Application of Games- Check for Accuracy
L1	Assessments- Focus
L2	Assessments- Purpose
L3	Assessments- Formative
L4	Assessments- Summative
L5	Assessments- Feedback
L6	Assessments- Parents

Appendix G - Codebook

Case Study Participant:

What does the teacher know and understand about procedural fluency?
Survey and Interview

	Overarching Theme		Code	Description	Example(s) from Survey	Example(s) from Interviews
A	**Knowledge of Procedural Fluency** The ability of students to carry out procedures flexibility, accurately, efficiently, and appropriately	1	Efficiency	The ability of students to solve a procedure in a reasonable time		
		2	Flexibility	The ability of students to know multiple strategies as well as apply or adapt these strategies to solve a procedure		
		3	Appropriateness	The ability of students to select the appropriate strategy in relation to efficiency and flexibility		
		4	Accuracy	The ability of students to correctly solve a procedure		
M		D	Memorization	The inclusion of memorization in relation to how procedural fluency is defined		
B	**Knowledge of Conceptual Understanding** The meaningful use of mathematical procedures by students because they know why strategies work and when it is appropriate to use them	1	Explanation	Conceptual understanding allows students to use mathematical procedures purposefully because they know when to use them and why they work		
		2	Link	Procedural fluency is built from conceptual understanding		
		3	Pacing	Procedural fluency cannot occur without conceptual understanding and conceptual understanding cannot be rushed		

C	**Knowledge of Elements of Quality Fluency Practice- Purposeful Planning** Planning for learning activities that attends to all the fluency components with engaging and meaningful classroom practices	1	Focused	The description of learning activities that are planned to help students practice accuracy by learning to use a procedure and get the correct answer		
		2	Varied	The description of learning activities that are planned to help students practice fluency in a variety of ways related to cognitive demand, focus on components of fluency, and type of engagement		
		3	Processed	The description of learning activities that are planned to provide students with opportunities to process and reflect after fluency practice		
		4	Connected	The description of learning activities that are planned purposefully to help students see relationships and make connections		
		5	Balanced	The description of learning activities that are planned to integrate all components of fluency (efficiency, flexibility through appropriate strategy selection, and accuracy)		
D	**Knowledge of Elements of Quality Practice- Fluency Actions** Learning activities that attend to the fluency actions	1	Strategy Selection	The description of learning activities that require students to select appropriate strategies based on efficiency		
		2	Reasonable Time	The description of learning activities that require students to solve problems in a reasonable time		

		3	Trades Out/ Adapt Strategy	The description of learning activities that require students to trade or adapt strategies		
		4	Application of Strategy	The description of learning activities that develop students' ability to apply a strategy to a new problem type		
		5	Complete Steps	The description of learning activities that develop students' ability to use an algorithm or strategy correctly		
		6	Correct Answer	The description of learning activities that develop students' ability to get the correct answer even when there is more than one correct answer, or the goal is a reasonable answer		

Case Study Participant:

How does this knowledge and understanding of fluency translate into classroom practices?
Field Notes- Observations and Debriefs

	Overarching Theme		Code	Description	Example(s) from Field Notes- Observation	Example(s) from Field Notes- Debrief
E	**Application of Strategy Instruction** Direct, explicit instruction related to mathematical strategies by the teacher, so students know how and when to use a strategy	1	Direct	The implementation of instruction that includes demonstration, modeling, and practice related to strategies that could be used to solve procedures		
		2	Explicit	The implementation of instruction that is clear and precise in order to help students use strategies to complete fluency actions		
		3	Intentionally Planned	The implementation of instruction that is deliberate and purposeful in the development, use, and analysis of strategies used to solve procedures		
		4	Student Generated	The implementation of instructional practices that allow students to create and discuss their own strategies for solving procedures		
F	**Application of Visual Representations and Tools** The use of visuals to highlight specific aspects of math that is focused on structure and providing access to abstract concepts	1	Visualization	The implementation of tools and visuals that help students to visualize a problem (but is not a strategy or action)		
		2	Connection	The implementation processes that connect representation (tools) to strategies		

		3	Discussion	The implementation of processes that use visual representation and tools to strengthen the effectiveness of student understanding during discussion		
G	**Application of Elements of Quality Fluency Practices** The implementation of classroom practices and learning activities that develop fluency	1	Focused	The implementation of learning activities that are planned to help students practice accuracy by learning to use a procedure and get the correct answer		
		2	Varied	The implementation of learning activities that are planned to help students practice fluency in a variety of ways related to cognitive demand, focus on components of fluency, and type of engagement		
		3	Processed	The implementation of learning activities that are planned to provide students with opportunities to process and reflect after fluency practice		
		4	Connected	The implementation of learning activities that are planned to help students see relationships and make connections		
		5	Balanced	The implementation of learning activities that are planned with to integrate all components of fluency (efficiency, flexibility through appropriate strategy selection, and accuracy)		
		6	Fluency Actions	The implementation of learning activities that require students to attend to all six fluency actions		

		7	Efficiency	The implementation of learning activities that require students to focus on the selection of the appropriate strategy, which helps them to solve the problem in a reasonable time		
		8	Accuracy	The implementation of learning activities that require students to focus on the selection of the appropriate strategy, which helps them to solve the problem correctly		
		9	Flexibility	The implementation of learning activities that require students to select an appropriate strategy based on efficiency and accuracy		
H	**Application of Routines** (Example of Quality Fluency Practice) The specific use of instructional time to think, problem solve, and discuss mathematical concepts	1	Problem Solving	The implementation of instructional routines that provide students access to problems that can be solved in different ways		
		2	Discussion	The implementation of instructional routines that promote the exchange of ideas and are worthy of extended conversation related to strategy use		
		3	When	The implementation of instructional routines that occur after a skill or concept is understood		
		4	How Often	The implementation of instructional routines that occur often in the classroom setting (example 3-5 times a week)		

		5	Facilitation	The implementation of instructional routines that are organized to include groups of three constant partners.		
		6	Purpose	The implementation of instructional routines that are used to provide structure in math classrooms with the establishment of specific student expectations for engagement and participation		
		7	Focus on Strategy Selection	The implementation of instructional routines that engage student in strategy choices especially in relation to efficiency and accuracy		
I	**Application of Worked Examples** (Example of Quality Fluency Practice) Problems that are already solved and used to structure discussion focused on when and why a strategy works as well as when it makes sense to use it	1	Correctly	The use of examples that have been worked correctly to focus on efficiency and flexibility		
		2	Partially	The use of examples that have been partially completed to focus on efficiency and accuracy		
		3	Incorrectly	The use of examples that have been completed incorrectly to focus on accuracy		
		4	Problem Selection	Problems are intentionally selected to encourage the use of appropriate strategy selection, the correct completion of steps, getting the correct answer, and applying a strategy to a new problem type		

J	**Application of Centers** (Example of Quality Fluency Practice) A physical space in the room where students participate in activities that promote the development of fluency components and actions	1	Engagement	The implementation of learning tasks that require active student participation and integrate individual accountability		
		2	Challenge	The implementation of learning tasks that are differentiated to provide the appropriate level of challenge based on the student's zone of proximal development		
		3	Motivation	The implementation of learning tasks that are designed to motivate the student through choice, student interests, learning styles, enjoyment, high expectations, and multiple opportunities for success		
		4	Learning Tasks	The implementation of learning tasks that develop procedural fluency which could include by are not limited to sorting tasks, choice problems, and games that can be completed independently or collaboratively		
K	**Application of Games** (Example of Quality Fluency Practice) The use of games to promote the development of procedural fluency by providing opportunities for students to select and make choices about the best strategy	1	Focus	The implementation of games that purposefully targets a strategy or component of fluency student have been learning		
		2	Purpose	The implementation of games that include practice that promotes elevated levels of rigor and accuracy		

	to use	3	Accountability	The implementation of games that include accountability for the students related to the fluency actions so each student can be held responsible for their own thinking		
		4	Strategy Selection	The implementation of games that include opportunities for students to select, make choices about, and use the best strategy based on efficiency		
		5	Check for Accuracy	The implementation of games that include opportunities for students to check for accuracy (By completing the steps and getting the correct answer)		
L	**Assessment** The use of assessments that focuses on all components of procedural fluency	1	Focus	Assessment of fluency focuses on efficiency, flexibility and accuracy related to appropriate strategy selection		
		2	Purpose	Assessment data is used to plan and evaluate the effectiveness of activities used for fluency practice		
		3	Formative	Assessments are ongoing and used to monitor student progress		
		4	Summative	Assessment tools that are used to determine the level of student learning that has occurred		

		5	Feedback	Assessments are used to provide students with timely, specific information related to procedural fluency progress		
		6	Parents	What we assess communicates what is important; ensuring all fluency components are attended to and visible communicate the real goal of procedural fluency		

Appendix H - Rubric- Survey and Interviews

Procedural Fluency Rubric: Survey and Interview Responses

Category	0	1	2	3	4
Knowledge and Understanding of Procedural Fluency **-------------------** Alignment to Research Question: Analysis of what teachers know and understanding about computational fluency Rating Based on Codebook Codes: A1, A2, A3, A4	Knowledge and understanding of procedural fluency related to the components and strategy selection is not evident.	The definition and description of fluency focuses on **one** component of fluency (Efficiency, accuracy, and flexibility) The definition and description of fluency may or may not include appropriate strategy selection.	The definition and description of fluency focuses on **two** components of fluency (Efficiency, accuracy, and flexibility) The definition and description of fluency may or may not include appropriate strategy selection.	The definition and description of fluency focuses on **three** components of fluency (Efficiency, accuracy, and flexibility) The definition and description of fluency does not include appropriate strategy selection. OR The definition and description of fluency focuses on **two** components of fluency (Efficiency, accuracy, and flexibility) The definition and description of fluency includes appropriate strategy selection.	The definition and description of fluency focuses on **all** components of fluency (Efficiency, accuracy, and flexibility) The definition and description of fluency includes appropriate strategy selection.
Understanding of the link between conceptual understanding and procedural fluency **--------------------** Alignment to Research Question:	Descriptions do not indicate knowledge of the concepts or a link between conceptual understanding and procedural	Descriptions indicate an understanding of procedural fluency and/or conceptual understand but does not indicate	Descriptions indicate a link between conceptual understanding and procedural fluency but not how this impacts	Descriptions indicate a link between conceptual understanding and procedural fluency and how this impacts	Descriptions indicate a link between conceptual understanding and procedural fluency as well as how this link

Analysis of how teachers explicitly connect conceptual understanding to computational fluency Rating Based on Codebook Codes: B1, B2, B3	fluency	how they are linked	planning and instruction	planning **or** instruction	impacts both planning **and** instruction
Knowledge and Understanding of What Constitutes Quality Fluency Practice- Purposeful Planning (Focused, varied, processed, connected) -------------------- Alignment to Research Questions: Analysis of what teachers know and understanding about computational fluency Analysis of how teachers plan for computational fluency development Rating Based on Codebook Codes: C1, C2, C3, C4	Description of fluency practices based on the components for purposeful planning are not evident.	Descriptions of fluency practices includes **one** of the four components (Focused, varied, processed, and connected)	Descriptions of fluency practices includes **two** of the four components (Focused, varied, processed, and connected)	Descriptions of fluency practices includes **three** of the four components (Focused, varied, processed, and connected)	Descriptions of fluency practices includes **all four** components (Focused, varied, processed, and connected)
Knowledge and Understanding of What Constitutes Quality Fluency Practice- Purposeful Planning (Efficiency, accuracy, and flexibility) --------------------- Alignment to Research Questions: Analysis of what teachers know and	Description of fluency practices do not show evidence of the components of fluency or strategy selection.	Description of fluency practices included **one** of the components of fluency (Efficiency, accuracy, and flexibility) Description of fluency practices may or may not include appropriate	Description of fluency practices included **two** of the components of fluency (Efficiency, accuracy, and flexibility) Description of fluency practices may or may not include appropriate	Description of fluency practices included **all** components of fluency (Efficiency, accuracy, and flexibility) Description of fluency practices does not include appropriate strategy	Description of fluency practices included **all** components of fluency (Efficiency, accuracy, and flexibility) Description of fluency practices includes appropriate strategy

understanding about computational fluency Analysis of how teachers plan for computational fluency development Rating Based on Codebook Code: C5		strategy selection.	strategy selection	selection. OR Description of fluency practices included **two** of the components of fluency (Efficiency, accuracy, and flexibility Description of fluency practices includes appropriate strategy selection	selection
Knowledge Understanding of What Constitutes Quality Fluency Practice- Purposeful Planning (Fluency Actions) **---------------------** Alignment to Research Questions: Analysis of what teachers know and understanding about computational fluency Analysis of how teachers plan for computational fluency development Rating Based on Codebook Codes: D1, D2, D3, D4, D5, D6	Descriptions of fluency practice do not show evidence of engaging students in the six fluency actions.	Descriptions of fluency practice engage students in **one** of the six fluency actions	Descriptions of fluency practice engage students in **two or three** of the six fluency actions	Descriptions of fluency practice engage students in **four or five** of the six fluency actions	Descriptions of fluency practice engage students in **all** six fluency actions

Appendix I - Rubric: Observations and Debriefs

Procedural Fluency Rubric: Field Notes from Observations and Debriefing Conversations

Category	0	1	2	3	4
Explicit Strategy Instruction ------------------------- Alignment to Research Question: Analysis of what teachers do to ensure the development of procedural fluency in the classroom Rating based on Codebook Codes: E1, E2, E3, E4	The focus on strategy instruction is not evident	Instruction may develop the use of strategies, but it is not planned for.	Instruction is planned to develop use of strategies, but it does not include direct and explicit instructional practices.	Instruction is intentionally planned to develop use of strategies through direct and explicit instructional practices. Students do not have the opportunity to generate and discuss their own strategies.	Strategy instruction is direct and explicit. Instruction is intentionally planned to develop use of strategies in relation to each fluency focus. Students have the opportunity to generate and discuss their own strategies.
Visual Representations and Tools ------------------------- Alignment to Research Question: Analysis of what teachers do to ensure the development of procedural fluency in the classroom Rating based on Codebook Codes: F1, F2, F3	The use of visual representations and/or tools is not evident.	Visual representations and tools are used by the teacher to help students visualize a problem ***or*** there is a clear connection of the representation or tool to the strategy.	Visual representations and tools are used by the teacher to help students visualize a problem ***and*** there is a clear connection of the representation or tool to the strategy.	Visual representations and tools are used by the teacher to help students visualize a problem ***and*** there is a clear connection of the representation or tool to the strategy. The use of visual representations and tools is used during discussions to strengthen students' understanding.	Visual representations and tools are used by the ***teacher and students*** to help visualize a problem ***and*** ensure there is a clear connection of the representation or tool to the strategy. The use of visual representations and tools is used during discussions to strengthen students' understanding, clarify students' thinking and follow the

						thinking of their classmates related to strategy use.
Examples of Quality Fluency Practices Alignment to Research Question: Analysis of what fluency practice looks like in the classroom	**Routines** Rating based on Codebook Codes: H1, H2, H3, H4, H5, H6, H7	The use of routines in classroom practices is not evident.	Routines are identified and may or may not- be used regularly in the classroom to devote time to thinking, problem solving, and discussions related to use of strategies.	Routines are identified and used regularly. They do not focus on strategy selection and problems are not purposefully selected that can be solved in different ways.	Clear routines are in place and used regularly to solve problems in different ways. Routines promote thinking, problem solving, and discussion related to use/selection of strategies but are done in a whole group setting	Clear routines are in place and used regularly to solve problems in different ways. Routines promote thinking, problem solving, and discussion related to use/selection of strategies. Students have the opportunity to work in small groups with constant partners.
	Worked Examples Rating based on Codebook Codes: I1, I2, I3, I4	The use of worked examples in classroom practices is not evident	While worked examples are utilized, the teacher explains why a strategy works and when a strategy makes sense	Students have the opportunity to discuss why a strategy works and when a strategy makes sense, but problems are ***not*** purposefully selected to attend to the fluency actions	Students have the opportunity to discuss why a strategy works and when a strategy makes sense. Problems are purposefully selected to attend to the fluency actions	A variety of worked example types are used and selected to attend to the fluency actions. Students have opportunities to discuss why a strategy works and when a strategy makes sense.
	Centers Rating based on Codebook Codes: J1, J2, J3, J4	The use of mathematical centers in classroom practices is not evident	Centers are present but include tasks that do not motivate, engage, and challenge students.	Centers are systematically designed to motivate, engage, and challenge students. There is no method for individual accountability.	Centers are systematically designed to motivate, engage, and challenge students with the proper integration of accountability. There is a limited variety of learning tasks and opportunities for	Centers are systematically designed to motivate, engage, and challenge students with the proper integration of accountability. There are a variety of learning tasks, and centers provide

					student choice are not integrated throughout the design of the centers.	opportunity for student choice through task, activity, or problem selection.
	Games Rating based on Codebook Codes: K1, K2, K3, K4, K5	The use of mathematical games in classroom practices is not evident	There may not be a clear focus to the use of the game and students do not focus on appropriate strategy selection.	Students may have to use strategies to solve problems, but there is no clear purpose evident.	Games are clearly focused and provide opportunities for students to select and make the best choice related to strategy use. Games do not include accountability and/or checks for accuracy	Games are clearly focused and provide opportunities for students to select and make the best choice related to strategy use. Games have accountability and opportunities to check for accuracy.
Overall Quality of Fluency Practices ------------------------- Alignment to Research Question: Analysis of what teachers do to ensure the development of procedural fluency in the classroom Rating based on Codebook Codes: G1, G2, G3, G4, G5, G6, G7, G8, G9		Fluency practices are not evident in classroom practices	Fluency practices has **one** of the four components (Focused, varied, processed, and connected) Fluency practice is not designed to engage student in all six fluency actions	Fluency practices has **two** of the four components (Focused, varied, processed, and connected) Fluency practice may or may not be designed to engage students in all six fluency actions	Fluency practices has **three** of the four components (Focused, varied, processed, and connected) Fluency practice may or may not be designed to engage students in all six fluency actions	Fluency practices has **all four** components (Focused, varied, processed, and connected) Fluency practice is designed to engage students in all six fluency actions
Analysis of what fluency components are present in classroom practice that support fluency development Rating based on Codebook Codes: G7, G8, G9		Fluency practices are not evident in classroom practices	Fluency practices are limited to one fluency component (Efficiency, accuracy, or flexibility) and do **not** focus on strategy selection	Fluency practices are limited to **one** fluency component (Efficiency, accuracy, or flexibility) and do focus on strategy selection.	Fluency practices include **two** fluency components (Efficiency, accuracy, or flexibility) focused on strategy selection.	Fluency practices include **all three** fluency components (efficiency, accuracy, or flexibility) focused on strategy selection.
		The assessment of	Teachers only assess fluency	Teachers use a single method	Teachers use a variety of	Teachers use a variety of

Assessment Type Alignment to Research Question: Analysis of what fluency practice looks like in the classroom Rating based on Codebook Codes: L1, L2, L3, L4	fluency is not evident.	in traditional ways that focus only on accuracy.	to assess the components of fluency.	methods (including formative and summative) to assess all components of fluency.	methods (including formative and summative) to assess all components of fluency. -AND- Data from assessments is used to plan for and evaluate the effective of classroom and instructional practices related to the development of fluency
Feedback Based on Assessment Alignment to Research Question: Analysis of what fluency practice looks like in the classroom Rating based on Codebook Codes: L5, L6	Feedback based on fluency assessments is not evident	Feedback is provided to students, however; it is not timely or specific based on fluency assessments.	Feedback is provided to students in a timely or specific format based on fluency assessments.	Feedback is provided to the student based on fluency assessments with timely, specific information related to procedural fluency.	Feedback is provided to the student based on fluency assessments with timely, specific information related to procedural fluency. -AND- Focus of all components of procedural fluency is communicated and visible to parents through the assessment and feedback utilized