

Large-scale coalition formation: application in power distribution systems

by

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Bc., Czech Technical University, 2011

Ing., Czech Technical University, 2013

AN ABSTRACT OF A DISSERTATION

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Abstract

Coalition formation is a key cooperative behavior of a system of multiple autonomous agents. When the capabilities of individual agents are not sufficient for the improvement of well-being of the individual agents or of the entire system, the agents can benefit by joining forces together in coalitions. Coalition formation is a technique for finding coalitions that are best fitted to achieve individual or group goals. This is a computationally expensive task because often all combinations of agents have to be considered in order to find the best assignments of agents to coalitions. Previous research has therefore focused mainly on small-scale or otherwise restricted systems. In this thesis we study coalition formation in large-scale multi-agent systems. We propose an approach for coalition formation based on multi-agent simulation. This approach allows us to find coalitions in systems with thousands of agents. It also lets us modify behaviors of individual agents in order to better match a specific coalition formation application. Finally, our approach can consider both social welfare of the multi-agent system and well-being of individual self-interested agents.

Power distribution systems are used to deliver electric energy from the transmission system to households. Because of the increased availability of distributed generation using renewable resources, push towards higher use of renewable energy, and increasing use of electric vehicles, the power distribution systems are undergoing significant changes towards active consumers who participate in both supply and demand sides of the electricity market and the underlying power grid. In this thesis we address the ongoing change in power distribution systems by studying how the use of renewable energy can be increased with the help of coalition formation. We propose an approach that lets renewable generators, which face uncertainty in generation prediction, to form coalitions with energy stores, which on the other hand are always able to deliver the committed power. These coalitions help decrease the uncertainty of the power generation of renewable generators, consequently allowing the

generators to increase their use of renewable energy while at the same time increasing their profits. Energy stores also benefit from participating in coalitions with renewable generators, because they receive payments from the generators for the availability of their power at specific time slots. We first study this problem assuming no physical constraints of the underlying power grid. Then we analyze how coalition formation of renewable generators and energy stores in a power grid with physical constraints impacts the state of the grid, and we propose agent behavior that leads to increase in use of renewable energy as well as maintains stability of the grid.

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Scott A. DeLoach

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Pavel Janovsky

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Power distribution systems are used to deliver electric energy from the transmission system to households. Because of the increased availability of distributed generation using renewable resources, push towards higher use of renewable energy, and increasing use of electric vehicles, the power distribution systems are undergoing significant changes towards active consumers who participate in both supply and demand sides of the electricity market and the underlying power grid. In this thesis we address the ongoing change in power distribution systems by studying how the use of renewable energy can be increased with the help of coalition formation. We propose an approach that lets renewable generators, which face uncertainty in generation prediction, to form coalitions with energy stores, which on the other hand are always able to deliver the committed power. These coalitions help decrease the uncertainty of the power generation of renewable generators, consequently allowing the

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Chapter 1

Introduction

1.1 Motivation

Power distribution networks were originally designed to deliver electricity from a single utility to large number of customers. Therefore, both the current economical and physical subsystems of power distribution systems (PDS) implement this one-way electricity flow. However, technological advances as well as economical and political pressure push the requirements on PDS to move away from the original model towards more distributed, multi-directional, smarter, and more robust solutions. Some of the main drives for innovations in PDS are:

- *distributed generation*; with technological advances in electricity generation using renewable resources, photo-voltaic and wind generators are becoming available for household-size customers in the distribution network.
- *prosumers*; this term characterizes active consumers in the distribution grid. These consumers are able to generate power and send it back to the grid, consequently becoming an active part of economical and physical subsystems of PDS.
- *large, shiftable load*; increasing production of electric vehicles will pose a significant requirement on the grid by increasing the household load during peak times in the afternoon and evening, and throughout the night. However, all electric vehicles do not

have to be charged at the same time, therefore this shiftable load can be utilized in PDS control.

- *demand-response* is a technique that enables utilities to offer their costumers profit in exchange for temporary reduction in their load. This technique has been used to ensure grid reliability.
- *political demand for higher integration of renewable resources* has been present in recent years in the form of renewable portfolio standards (RPS). For example, the RPS target of the state of California for the year 2020 is that 33% of all generation will come from renewable sources ([Ipakchi and Albuyeh, 2009](#)).

These new requirements will significantly alter the power distribution grid. Because of the increasing amount of distributed generation, the current uni-directional power flow will be replaced by a more complex bidirectional flow, thus creating new complex flow patterns. Customer loads will increase due to increased use of electrical vehicles. Distributed generation will increase on the supply side. The unpredictability of the power produced by renewable generation will have to be balanced by traditional generation or other energy stores. Unless these issues are addressed, they will have undesirable effects on the power distribution network.

Renewable resources are being used for cheap, sustainable, environmentally friendly, and inexhaustible electricity generation. These positive aspects of renewable energy, together with the goal of reducing carbon emissions, drive the electrical grid development towards larger integration of generators that use renewable energy. However, despite the advantages of renewable resources, their use is lower than that of nonrenewable resources such as coal and natural gas. One of main reasons for the low use of renewable resources such as wind and solar power is the unpredictable amount of generation related to dependence on weather ([Holtinen et al., 2009](#)). Currently, electricity is traded in two stages ([Kirschen and Strbac, 2004](#); [Pinson et al., 2007](#)), in a day-ahead energy market that matches supply and demand, and in a real-time market in order to adjust for real-time variations in supply and demand. In the day-ahead market, generators bid amounts of energy that they will be able to generate at

given time slots. Since in the electricity market the supply must always match the demand, a failure to deliver the committed amount of energy can have major negative consequences, and it is therefore penalized (Pinson et al., 2007). Consequently, the renewable generators, due to weather unpredictability, are forced to bid amounts that are lower than the predicted generation, thus decreasing the use of renewable resources (Holttinen et al., 2009).

Technological advances in computational and sensory devices have allowed the emergence of cyber-physical systems (CPS). These physical systems are monitored, coordinated, controlled, and integrated by a system of computing devices (Khaitan and McCalley, 2013; Rajkumar et al., 2010). Some examples of CPS are medical systems, transportation systems, defense systems, and power distribution systems (Rajkumar et al., 2010). A CPS approach is well suited for addressing challenges in power distribution systems (PDS). The physical part of the system includes the physical grid consisting of substations, feeders, laterals, transformers and households. Another physical aspect of such system is the physical state of the grid, which consists of voltage, frequency, and power constraints. The cyber part of the system consists of smart meters and communication links. Also, researchers in the field of multi-agent systems (MAS) have proposed to include MAS in the cyber part of CPS. Agents in MAS can be assigned to various physical parts of PDS, and they can participate in the CPS via grid control and online auctions, forming an intelligent power distribution system (Case, 2015).

In this thesis we investigate how MAS can be deployed to become a part of the PDS. Specifically, we study how agents can form coalitions in order to increase either welfare of the entire system, or their own profit. We focus on two specific parts of PDS: renewable generators and energy stores. We let these two groups form coalitions in a MAS simulation, and then observe significant changes in the system caused by this process.

In order to apply coalition formation in the PDS domain, an approach must be used that can find high-quality coalitions in systems with large numbers of agents. Even though coalition formation has previously been studied, the proposed approaches usually focus on finding solutions with maximum social welfare in small-scale systems. Previous research has focused on small-scale systems mainly because for n agents there are $O(n^n)$ possible solutions (Rah-

wan and Jennings, 2008), and determining the optimal solution has been proven to be an NP-complete problem (Sandholm et al., 1999). Searching for optimal solutions in large-scale systems is therefore infeasible. For coalition formation in the PDS domain we need an approach that will perform optimally or almost-optimally in small-scale systems, while being able to find high-quality solutions in large-scale systems. To this end we investigate the use of MAS simulation as a tool for performing large-scale coalition formation.

MAS simulation is an iterative process in which agents interact with each other and with their environment. It can be used to show a step-by-step evolution of a system without a need for potentially expensive real-world experiments. MAS simulation is typically modeled on a microscopic level by designing behaviors of individual agents in MAS, and an emergence of macroscopic system-level behavior caused by the microscopic-level changes is observed. Similarly, by modeling strategies of individual agents we will observe the resulting coalition formation process of all agents in MAS.

1.2 Thesis Statement

Multi-agent simulation provides a computationally tractable approach for coalition formation of large numbers of agents that finds high quality solutions with respect to social welfare of the multi-agent system and individual welfare of individual agents. Large-scale coalition formation can be used in a power distribution system to increase the use of renewable resources and at the same time increase profits of renewable generators and energy stores.

In this thesis we present an approach that uses multi-agent simulation to perform coalition formation in an iterative manner in order to maximize *social welfare*. Multiagent simulation allows us to observe the process of forming coalitions in an iterative manner. While coalition formation is typically approached as a single-step task, it is beneficial to model coalition formation as a dynamic process where coalitions change over time. In such a process the agents can utilize information about previous and current values of coalitions to support

their decision to leave the current coalition and join a new coalition. We propose a general framework that models coalition formation as a dynamic, iterative process. Our framework can be used to simulate real world applications of coalition formation.

We also present an approach that considers *individual welfare* of individual self-interested agents by improving the stability of coalitions. Stability of the coalitions measures the coalition's ability to de-incentivize any sub-coalition of agents from leaving the coalition. Self-interested agents seek to maximize their own profit rather than the social welfare, which makes social-welfare maximizing solutions unrealistic. Therefore, coalition stability must be addressed as a concept that along with the social welfare influences the coalition formation algorithms and solutions.

Furthermore, we show an approach that uses multi-agent simulation to perform coalition formation in PDS. This approach increases the use of renewable resources by allowing renewable generators to hedge against their own unpredictability by forming coalitions with energy stores such as batteries, capacitors, or any generators that are able to provide exact amounts of power at given times. Inside these coalitions renewable generators purchase availability of energy stores to generate power when needed. Renewable generators use this availability to avoid fees for failure to provide committed generation whenever the current generation is lower than the committed value.

We demonstrate the effectiveness of multi-agent simulation for coalition formation of large numbers of agents by showing that:

1. social welfare of our solutions in small-scale systems is comparable to the social welfare of solutions found by state-of-the-art algorithms that find optimal solutions for small numbers of agents.
2. per-agent social welfare of our solutions does not decrease when increasing the scale of the system. Consequently, the quality of our solutions in large-scale systems is comparable with quality of solutions in small-scale systems.
3. performance of our approach dominates performance of other state-of-the-art approaches for coalition formation in large-scale systems.

4. coalitions found by our approach are stable because agents within the coalitions do not have an incentive to deviate and form separate sub-coalitions.

We demonstrate the usability of our coalition formation approach in the PDS domain by showing that:

1. both renewable generators and energy stores are economically incentivized to participate in coalition formation.
2. application of our approach increases the use of renewable resources in the PDS.
3. our approach respects physical constraints of the PDS.

1.3 Contributions

Contributions of this thesis are:

1. A theoretical framework for large-scale coalition formation, which is presented in Chapter 4.
 - (a) *A framework for evaluating coalition formation strategies that uses multi-agent simulation (Janovsky and DeLoach, 2016a).* The framework can be used to simulate real-world scenarios of coalition formation. We show examples of such scenarios along with their representation in the framework. We also discuss the practical meaning of our framework.
 - (b) *The capability to evaluate large-scale coalition selection strategies on scenarios consisting of thousands of agents (Janovsky and DeLoach, 2016a).* Since the solutions reflect the decision making of single agents in the dynamic coalition formation process, optimality is not guaranteed. Thus, we show how to evaluate proposed strategies by comparing them against optimal solutions for small numbers of agents (up to 20) and then demonstrating that those strategies are stable in large-scale scenarios with up to 10,000 agents. We also show that in majority

of the tested instances our proposed strategies perform similarly or better than a state-of-the-art coalition formation algorithm.

- (c) *An algorithm for large-scale coalition formation of thousands of agents that uses deviations of the agents in order to increase coalition stability* (Janovsky and DeLoach, 2016c,d). Our approach uses multi-agent simulation, in which agents make decisions about joining, leaving, and deviating from coalitions. We show the approach and we discuss a deviation strategy.
 - (d) *An approach for selecting sub-optimal solutions based on their social welfare and coalition stability* (Janovsky and DeLoach, 2016c,d). We discuss the ways to select a solution out of a pool of solutions for which stability is unknown and expensive to compute.
 - (e) *Experimental evaluation of our approach using real-world datasets* (Lichman, 2013; World Trade Organization, n.d.) and comparison with state-of-the-art approaches in small (Cruz-Mencía et al., 2013) and large (Farinelli et al., 2013)-scale systems (Janovsky and DeLoach, 2016a,c,d).
2. An application of the theoretical framework in the PDS domain, which is presented in Chapter 5.
- (a) *An approach to increase use of renewable energy sources in a PDS using coalition formation of renewable generators and energy store owners* (Janovsky and DeLoach, 2016b). We show our model of renewable sources and energy stores, and we describe how to use multi-agent simulation for coalition formation of renewable generators and energy store owners.
 - (b) *Experimental evaluation of the coalition formation process between renewable generators and energy store owners* (Janovsky and DeLoach, 2016b). We show that our approach increases use of renewable resources by increasing profit of renewable generators.
 - (c) *An approach to increase use of renewable energy sources in a PDS with physical*

constraints. We show an approach for coalition formation of renewable generators and energy stores that respects physical constraints of the underlying power grid.

(d) *Experimental evaluation of coalition formation in PDS with physical constraints.*

Using randomly generated scenarios based on a benchmark power grid ([Khatod et al., 2006](#)), we show that coalition formation increases use of renewable energy, and it also improves the physical state of the underlying power grid.

1.4 Overview

This thesis is organized as follows. We introduce the background for coalition formation and PDS in Chapter 2. In that chapter we also formally define concepts relevant to coalition formation and PDS. We overview the related work in coalition formation and PDS in Chapter 3. We present the theoretical framework for large-scale coalition formation in Chapter 4, with Section 4.1 describing social welfare-based optimization, and Section 4.2 presenting coalition stability-based optimization with respect to behavior of self-interested agents. Chapter 4 includes experimental analysis of the presented approach. In Chapter 5 we show the application of the theoretical framework in the PDS domain, with Section 5.1 showing how coalition formation of renewable generators and energy stores can be used to increase use of renewable resources while assuming no physical constraints of the PDS. Section 5.2 then discusses including physical constraints of PDS in the model used in Section 5.1 and shows positive effects that coalition formation has on the physical state of the PDS.

Chapter 2

Background

In this chapter we explain coalition formation and PDS concepts. We give an overview of some typical problems that are being solved in these areas, and we specify how research presented in this thesis fits into this overview. We also define terms that are used throughout this thesis.

2.1 Coalition Formation

A MAS is a system of autonomous agents, which are able to operate either by themselves or as a collective. One of key aspects of agents that work as a group is their ability to coordinate and cooperate with each other. Such agents can take joint, coordinated actions to improve their performance, or to achieve goals that are beyond the capabilities of individual agents (Rahwan et al., 2015). This cooperation can be beneficial both for the social welfare of the whole system as well as well-being of single, possibly selfish, agents. One of the research areas that study multi-agent cooperation is the study of coalition formation. Following are the definitions of a coalition and coalition formation.

Definition 2.1.1 (Coalition). *Coalition is a cooperating group of autonomous agents. Besides a general coalition, there are two specific coalition configurations:*

- *Singleton coalition is a coalition that contains a single agent.*

- *Grand coalition is a coalition that contains all agents.*

Definition 2.1.2 (Coalition formation). *Coalition formation is a process in which multiple autonomous agents create coalitions in order to achieve individual or group goals.*

Some reasons for agents to form groups are to achieve tasks that require their cooperation, to optimize their expenses, to increase their collective power over other agents, or to share resources.

Coalition formation is typically split in three sub-problems, that can be approached independently (Sandholm and Lesser, 1997). These sub-problems are

1. Coalition structure generation,
2. Solving an optimization problem in each coalition,
3. Division of the coalition's profit among its agents.

Coalition structure generation refers to formation of a coalition structure.

Definition 2.1.3 (Coalition structure). *Coalition structure is a set of coalitions such that each agent belongs to a coalition in the coalition structure.*

In a coalition structure, coalitions can be either distinct or overlapping. An example of distinct group formation is when families arrange shared family plans with phone companies in order to save expenses. On the other hand, Facebook users can join multiple groups, thus creating overlapping user coalitions. In this thesis we consider distinct coalitions.

In the coalition structure generation problem, we consider a set of agents $A = \{a_1, a_2, \dots, a_n\}$, where n is the total number of agents. The task is to find a coalition structure $CS = \{C_1, C_2, \dots, C_l\}$, which is a set of l coalitions C_j . Each coalition C_j can be assigned a value $v(C_j)$.

Definition 2.1.4 (Coalition value). *Coalition value $v(C_j)$ is a real number assigned to a coalition C_j as a function of the set of agents that participate in C_j . Coalition value is independent of the state of other coalitions in the coalition structure.*

CS is then assigned a value which corresponds to the aggregate value of all coalitions in CS ,

$$v(CS) = \sum_{C_j \in CS} v(C_j). \quad (2.1)$$

Typically, the task of coalition structure generation is to find the optimal coalition structure.

Definition 2.1.5 (Optimal coalition structure). *Optimal coalition structure CS^* is a coalition structure with the highest value $v(CS)$,*

$$CS^* = \arg \max_{CS \in \Pi^A} v(CS), \quad (2.2)$$

where the argmax function returns CS with the highest value of $v(CS)$ and Π^A denotes the set of all possible coalition structures containing all agents in the set A .

Since there are $O(n^n)$ coalition structures in Π^A (Rahwan and Jennings, 2008), finding the optimal coalition structure is feasible only for small numbers of agents n . Major part of this thesis studies the coalition structure generation problem in large-scale systems with hundreds and thousands of agents.

Solving an optimization problem in each coalition is concerned with the cooperation of agents inside a coalition. Typically, the task is to maximize performance of the coalition (Rahwan et al., 2015). For example, a group of researchers has to decide how to combine their knowledge in order to write a good scientific publication. Research presented in this thesis does not include this coalition formation sub-problem.

Division of the coalition's profit among its agents seeks to find ways to divide coalition's profit among self-interested coalition members. Typical metrics that are applied to solutions of this sub-problem are fairness and stability (Rahwan et al., 2015). Fairness addresses whether agents' rewards correspond to their contributions, and stability verifies that agents in a coalition are not incentivized to selfishly create sub-coalitions in order to increase their profit. The following example illustrates how searching for a social welfare-maximizing coalition structure yields unstable coalitions. In this example we assume that social welfare is equal to sum of coalition values, which are in turn calculated by summing up agents' prof-

its. Three agents x , y , and z , can form coalitions with the following distribution of profit: $\{x = 2, y = 2, z = 3\}$, $\{x = 3, y = 3\}$, $\{x = 1, z = 1\}$, $\{y = 1, z = 1\}$, $\{x = 0\}$, $\{y = 0\}$, and $\{z = 0\}$. The first coalition yields the highest total social welfare of 7. However, agents x and y could jointly deviate from this coalition and form the second coalition in order to maximize their own profit.

Definition 2.1.6 (Coalition stability). *Coalition stability reflects the willingness of self-interested agents to stay in coalitions that were assigned to them.*

Coalition stability is addressed in literature mainly through the concept of a *core*.

Definition 2.1.7 (Coalition core). *Core is a set of allocations to the agents in a coalition, such that these allocations cannot be improved upon by allocations to a subset of these agents. In another words, allocations are in a core if there exists no sub-coalition in which some agents would get higher profit and no agents would get lower profit than in the original coalition.*

While the *core* is a strong concept, its computation requires an evaluation of all $2^{|C_j|}$ possible sub-coalitions of each coalition C_j containing $|C_j|$ agents. In this setting checking whether a solution is in the core is *co-NP-complete* (Greco et al., 2011), and determining whether the core is non-empty is Δ_2^P -complete (Greco et al., 2011). This complexity makes the use of the *core* in large-scale systems with thousands of agents infeasible. This thesis addresses the concept of coalition stability in large-scale systems, however, we will propose to achieve this metric by changing the coalition structure generation process instead of via profit division. We will show that letting the agents know the profit division scheme while forming the coalitions results in more stable coalition structures, and it removes the need for computation of profit division.

2.2 Power Distribution System

Power distribution systems (PDS) are physical systems that carry electricity from a transmission system to individual customers. We show an example of a PDS in Figure 2.1. PDS

connect with the transmission system through a substation, which lowers the voltage. Electricity is then carried by feeders, which further split into laterals. Finally, the laterals are connected to transformers, which further lower the voltage for use in households. PDS form a tree structure in which the substation represents the root node of the tree and households and other consumers are the leaf nodes.

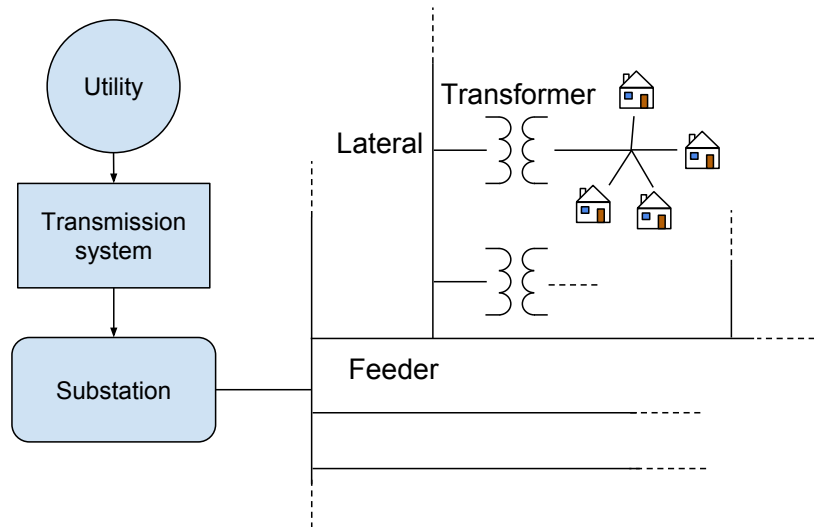


Figure 2.1: *Example of a power distribution system*

PDS are currently undergoing multiple changes that occur mostly on the consumer level. Lowering prices of PV generators along with government subsidies incentivize many homeowners to install small rooftop PV generators. These generators are used to cover household load as well as to sell power to the grid. Furthermore, an increasing number of home appliances, including electric vehicles, are able to shift their load in order to optimize homeowners' expenses. These changes result in the customers becoming more actively involved in both the physical and the economical sub-systems of PDS. *Transactive Energy* is a new term that describes this phenomenon. It was defined in a 2014 report by GridWise Alliance ([GridWise Alliance, 2014](#)) as “the ability for consumers and end devices to buy and sell energy and related services in a dynamic and interactive manner”. The customers will be able to actively participate in the transactive energy market by dynamically buying and selling energy ([GridWise Alliance, 2014](#); [The GridWise Architecture Council, 2015](#)).

Many difficult challenges arise due to these changes in PDS. Because of their increased

capabilities the active consumers will be able to affect the PDS in unpredictable ways. High penetration of active consumers will require new control mechanisms to ensure stability of the grid and prevent congestion. Distributed generation using mainly renewable sources can negatively affect the grid stability due to unpredictability of renewable generation. Currently, the utilities only allow a limited amount of distributed renewable generation because of its unforeseen effects on the grid. However, decreasing costs of small renewable generators and an increasing push towards use of renewable resources will force a significant increase in penetration of distributed generation in PDS. New approaches will be needed for distributed generation control. Furthermore, customers will not be able to make fast and frequent decisions about the actions of their households in energy markets. Therefore, intelligent agents will have to be designed to represent households and autonomously make decisions about load and generation of the household. Security will also be a major concern as the amount of information shared about customers will have to be carefully considered and protected.

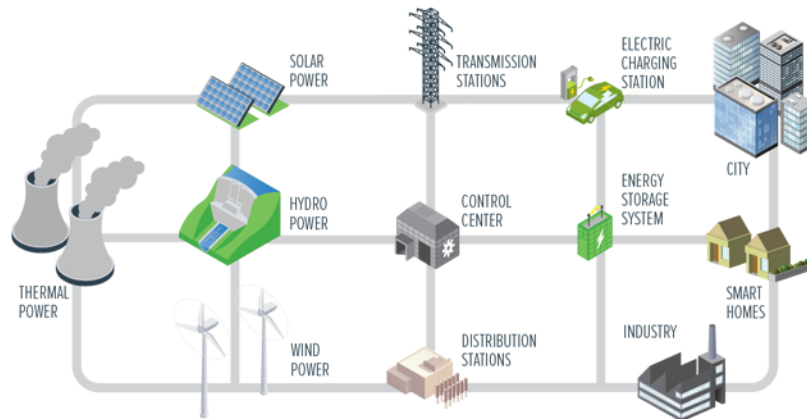


Figure 2.2: *Smart grid: multi-directional power flow (Comsar Energy, 2013).*

Many of these problems are being addressed through the concept of a *smart grid*. The smart grid is *an evolution of electricity networks toward greater reliance on communications, computation, and control* (Camacho et al., 2011). Smart grid includes many autonomously operating stakeholders, including smart homes, energy storages, and power generators, connected to the existing power grid and cooperating via internet. Figure 2.2 shows these active

stakeholders participating in a multi-directional power flow in the power grid. Smart grid is defined more formally by Title XIII of Energy Independence and Security Act of 2007 (EISA), which was approved by the US Congress, as a system that achieves each of the following requirements ([US Congress, 2007](#)):

1. Increased use of digital information and controls technology to improve reliability, security, and efficiency of the electric grid.
2. Dynamic optimization of grid operations and resources, with full cybersecurity.
3. Deployment and integration of distributed resources and generation, including renewable resources.
4. Development and incorporation of demand response, demand-side resources, and energy-efficiency resources.
5. Deployment of ‘smart’ technologies (real-time, automated, interactive technologies that optimize the physical operation of appliances and consumer devices) for metering, communications concerning grid operations and status, and distribution automation.
6. Integration of ‘smart’ appliances and consumer devices.
7. Deployment and integration of advanced electricity storage and peak-shaving technologies, including plug-in electric and hybrid electric vehicles, and thermalstorage air conditioning.
8. Provision to consumers of timely information and control options.
9. Development of standards for communication and interoperability of appliances and equipment connected to the electric grid, including the infrastructure serving the grid.
10. Identification and lowering of unreasonable or unnecessary barriers to adoption of smart grid technologies, practices, and services.

In this thesis we address specifically items 3 and 7 of the EISA smart grid requirements list. We focus on increasing use of distributed generation through renewable resources (item 3) in Chapter 5. We will show that the use of renewable resources can be increased by letting renewable generators form coalitions with energy stores (item 7). The remaining items of the EISA list of requirements for smart grid are out of scope of this work.

2.3 Summary

Coalition formation is a process in which multiple autonomous agents create cooperating groups called coalitions in order to achieve individual or group goals. The goal of coalition formation is usually to increase social welfare or profit of individual agents. Coalition formation includes coalition structure generation, solving coalition's optimization problem, and division of coalition's profit (Sandholm and Lesser, 1997). In this thesis we address coalition structure generation in large-scale MAS. We also study coalition stability, which is related to division of coalition's profit.

PDS carry electricity from a transmission system to individual consumers. Multiple changes on consumer level of PDS are creating a need for new approaches for PDS control. For example, with the rise of small PV generators consumers are able to cover their own load as well as sell power to the grid. This distributed generation using renewable resources creates difficult challenges associated with unpredictability of consumer behavior as well as unpredictable availability of renewable sources. These challenges are being addressed by the concept of a smart grid. Many requirements for a smart grid are listed in Energy Independence and Security Act (US Congress, 2007). In this thesis we focus on integration of distributed generation through renewable resources (item 3 in Title XIII of (US Congress, 2007)). We show that coalition formation with the help of energy stores (item 7 in Title XIII of (US Congress, 2007)) can be used to increase renewable generation.

Chapter 3

Related Work

This chapter provides an overview of related work in the field of coalition formation and use of renewable resources in power distribution systems. We also describe the distinctions between related work and research performed in this thesis in order to show novelty of our work.

3.1 Coalition Formation

This section discusses related work in coalition formation, including multi-agent simulation used for coalition formation, and approaches that consider stability of coalitions. Coalition formation is usually solved by one of the following approaches that will be discussed in the following sections: dynamic programming, graph-based algorithms, heuristic algorithms, or hierarchical clustering. The first two approaches are exact and guaranteed to find optimal solutions. Although the last two approaches do not provide guarantees on solution quality, they are able to solve large problem instances. Our approach of using multi-agent simulation can be classified as a heuristic algorithm because the simulation performs a greedy search in the state-space of CSs.

3.1.1 Dynamic Programming

Dynamic programming (DP) was initially used to solve the coalition formation problem with the first algorithm proposed by Yeh (Yun Yeh, 1986). Yeh’s algorithm directly depends on the following theorem that defines a value of an optimal coalition structure:

Theorem 3.1.1. *Given a coalition C , let $f(C)$ be the value of an optimal coalition structure formed by agents in C . Then*

$$f(C) = \begin{cases} v(C) & \text{if } |C| = 1 \\ \max\{v(C), \max_{\{C_1, C_2\} \in \Pi^C} (f(C_1) + f(C_2))\} & \text{otherwise.} \end{cases} \quad (3.1)$$

In another words, the value of the optimal coalition structure over agents in C can be calculated by considering all possible splittings of coalition C into subcoalitions C_1 and C_2 .

Π^C in Theorem 3.1.1 denotes all coalition structures containing agents in coalition C . Theorem 3.1.1 is implying that any coalition C should be split into some coalitions C_1 and C_2 if the sum of values of optimal coalition structures consisting of agents in coalitions C_1 and C_2 is greater than value of coalition C . Proof of Theorem 3.1.1 is presented in (Rahwan et al., 2015). The DP algorithm works based on Theorem 3.1.1 as follows: it iterates over all possible coalitions in an increasing order of coalition size, and for each coalition C a decision is made whether the highest coalition value can be achieved by the coalition C itself or by splitting the coalition in two sub-coalitions C_1 and C_2 . Since coalitions C_1 and C_2 are smaller than coalition C , their optimal values are calculated before coalition C is considered. The algorithm has to consider all coalitions C_1 and C_2 such that $C_1 \cup C_2 = C$ and $C_1 \cap C_2 = \emptyset$. The algorithm proceeds this way until the grand coalition (see Definition 2.1.1) is reached.

Yeh’s DP algorithm for coalition formation was further improved. Rahwan and Jennings (Rahwan and Jennings, 2008) proposed an algorithm called Improved Dynamic Programming (IDP), an improvement of standard dynamic programming that performs fewer operations (38.7% of operations in problems with 25 agents) and uses less memory (33%-66% of the memory used by the DP algorithm). Even with these improvements, authors of IDP only

perform experiments with up to 25 agents. Recently, Cruz-Mencía et al. (Cruz-Mencía et al., 2013) proposed an optimized implementation of DP and IDP algorithms. This optimized implementation is 10 times faster than the IDP algorithm. Furthermore, (Cruz-Mencía et al., 2013) achieves additional 5-6 times speedup by implementing the IDP algorithm in a distributed way. With these improvements the algorithm still spends several hours finding optimal solutions for systems with 27 agents.

3.1.2 Graph-based Approaches

Graph-based algorithms utilize synergy graphs, which are graphs that encode agents' abilities to cooperate peer to peer. For example, communication, trust, or social constraints between pairs of agents can influence how the coalitions are formed. The DyCE algorithm (Voice et al., 2012) improves upon IDP by recognizing and ignoring infeasible coalitions using the synergy graph. Bistaffa et al. (Bistaffa et al., 2014) proposed a branch and bound algorithm CFSS that searches the state-space by contracting edges of the synergy graph. An edge contraction corresponds to merging coalitions associated to its incident vertices. An example edge contraction is shown in Figure 3.1. This state-of-the-art algorithm is able to find optimal solutions in specific instances of the problem containing up to 50 agents in 100 seconds. Moreover, the algorithm can be used to find sub-optimal solutions in large-scale systems. However, the algorithm is limited to valuation functions that can be expressed as a sum of a monotonic and an anti-monotonic function. Finally, Sless et al. (Sless et al., 2014) have recently proposed a centralized graph-based algorithm in which a central organizer suggests new cooperation between agents by adding edges to the synergy graph. These adjustments of the synergy graph are associated with a given cost. (Sless et al., 2014) then studies this scenario from the aspects of maximizing social welfare as well as finding stable coalitions, and performs experiments on a graph with 20 vertices.



Figure 3.1: *CFSS: Example of an edge contraction in a triangle graph. The dashed edge is contracted to form the graph on the right, creating coalition $\{A, C\}$ (Bistaffa et al., 2014).*

3.1.3 Approaches for Large-scale Systems

An increasing number of agents causes the search for optimal CS to become infeasible. Suboptimal solutions can then be found using heuristic algorithms. Shehory and Kraus (Shehory and Kraus, 1998) first proposed a greedy algorithm that restricts the allowed size of coalitions to solve task allocation in a multi-agent system.

Other approaches were later used to tackle coalition formation in larger scale setting. A genetic algorithm is used in (Sen and Dutta, 2000), in which a set of coalition structures is considered by iterative evaluation, selection, and recombination of the coalition structures. Simulated annealing-based approach is proposed in (Keinänen, 2009), in which a new coalition structure is iteratively randomly sampled from a neighborhood of the current coalition structure. Then the new coalition structure is set as the current coalition structure if either the former is better than latter, or with some probability related to the current simulated annealing temperature. Finally, a greedy adaptive search was proposed in (Di Mauro et al., 2010). This search iterates in two phases: constructive and local search. In the constructive phase the algorithm considers adding an agent to one of the current coalitions. Then, the local search phase explores different neighborhoods of the current coalition structure. Despite promising results, particularly for the greedy adaptive search, these algorithms focus on instances of the coalition formation problem that contain less than 100 agents.

Farinelli et al. recently proposed C-Link (Farinelli et al., 2013), which is a hierarchical clustering algorithm that addresses large-scale coalition formation. C-Link starts with singleton coalitions, and iteratively merges the most suitable pairs of coalitions based on

criteria inspired by standard clustering approaches. Single-link, complete-link, and average-link clustering approaches can be used with C-Link. These methods consider gain that two coalitions would achieve if they were merged. Furthermore, (Farinelli et al., 2013) also proposed a new clustering criterion called gain-link, which significantly improves the quality of solutions. C-Link finds a suboptimal solution for 2,732 agents in 4 minutes. Although C-Link addresses a similar problem to the problem we address, we focus on the simulation aspect by studying how strategies of single agents affect overall behavior of the system. Unlike C-Link, our framework models systems that change and adapt and it computes the evolution of the coalition structure over time.

3.1.4 Multi-agent Simulation

Multi-agent simulation is a tool for studying complex systems by eliminating the need for infeasible real-world experiments. Such simulation can be used to test hypotheses about emergence of a macroscopic behavior based on behaviors and interactions of individuals (Drogoul and Ferber, 1994). Multi-agent simulation is a suitable approach for coalition formation because of its iterative nature, which is very similar to a dynamic nature of the coalition formation process. Multi-agent simulation can be used to search for a solution without a specific optimizing algorithm. It is often used to show how a real system would evolve over time, thereby decreasing the need for potentially expensive real-world experiments. However, theoretically proving a bound on the solution quality is often difficult in multi-agent simulation. Therefore, a theoretical proof is often substituted by experimental analysis.

Multi-agent simulation studies coalition formation from several viewpoints. In (Merida-Campos and Willmott, 2004), agents randomly choose coalitions in a coalition game in order to perform tasks. After each round, depending on their simple strategies, the agents can decide to leave the coalition or to stay. (Merida-Campos and Willmott, 2004) show that agents can benefit from exploiting knowledge about past successful coalitions. We take this simulation approach further by proposing more complex heuristic strategies and applications. An iterative approach for finding core-stable coalitions was proposed in (Arnold

and Schwalbe, 2002). These agents use best-response strategy to choose new coalitions. While the approach in (Arnold and Schwalbe, 2002) is similar to ours, it can only be used in small scale scenarios due to its high complexity, as was shown in (Bistaffa and Farinelli, 2013), where the algorithm from (Arnold and Schwalbe, 2002) was improved and empirically tested. A physics-motivated algorithm is proposed in (Lerman and Shehory, 2000) to solve the coalition formation problem for large-scale electronic markets. Coalition formation is solved in (Lerman and Shehory, 2000) using a probability-based macroscopic model, which is defined by a series of differential equations. A coalition may form whenever two individuals or coalitions randomly encounter each other. The decisions about leaving coalitions are also made randomly based on some probability. Our framework does not use the macroscopic point of view and can therefore model behavior of single agents. Our agents also utilize more complex strategies. Finally, a recent approach has been proposed in (Pan et al., 2016) to dynamically assemble teams of workers to perform crowdsourcing tasks.

3.1.5 Game Theoretical Approaches

Coalition formation can also be solved using game theory. (Yamamoto and Sycara, 2001) presented an auction-based system for buyer coalition formation in large-scale e-markets. There buyers form coalitions in order to exploit volume discounts. (Yamamoto and Sycara, 2001) also proposes a profit division scheme, which is used to divide coalitions' profit among its members. (Pillai and Rao, 2013) uses game theory to address a more applied problem of grouping servers in order to address requests demanding capabilities exceeding a single server. (Pillai and Rao, 2013) models coalition formation as multiple instances of a zero-sum game. In (Bonnevay et al., 2005), game theoretical perspective of coalition formation is taken in which agents are defined by attraction for gain, stability, and strength of character. Even though the authors of (Bonnevay et al., 2005) provide strong game theoretical background, they only experiment with four agents.

3.1.6 Approaches for Systems with Self-interested Agents

Coalition formation algorithms for systems with self-interested agents need to consider single-agent profit as well as stability of coalitions. Differences between social welfare-maximizing agents and individually rational or self-interested agents and studied in (Jennings and Campos, 1997). There the authors define goals and benefits of the individual members as well as of the society, and they define a socially responsible agent, which is a compromise between agents optimizing social welfare and individual profit.

Theoretical properties of stability in coalition formation have been studied extensively. (Sandholm and Lesser, 1997) provides an overview of social welfare and stability in the coalition formation setting. (Pycia, 2012) studies the existence of core stable coalition structures with respect to profit sharing rules and agents' preferences over coalitions. (Cechlárová and Romero-Medina, 2001) assumes agents' preferences over individual agents, and studies existence of core-stable coalition structures for extensions of these preferences towards preferences over sets of agents. Theoretical properties of profit sharing are studied in (Hoefer and Wagner, 2013). There, bounds are given on the price of stability, which is a ratio between a value of the best Nash equilibrium and the maximum social welfare.

Algorithms have been proposed to find stable coalitions in coalition formation games. (Augustine et al., 2011) proposes algorithms to find Nash equilibria for three profit sharing scheme games: fair value games, in which agent's profit is based on its contribution to the coalition, labor union games, which takes into account the order in which agents joined their coalitions, and Shapley games, which calculate agents' profits based on the Shapely value. Unlike (Augustine et al., 2011), which assumes deviations (situations in which agents leave coalitions) of single agents only, we look for coalitions that are stable with respect to deviations of groups of agents. (Conitzer and Sandholm, 2006) studies how core stable coalitions can be found in various games. (Conitzer and Sandholm, 2006) also proposes algorithms that find a core for superadditive games. Since superadditivity leads to a grand coalition being the somewhat trivial optimal solution, we do not pose this restriction on our scenarios. (Anshelevich and Sekar, 2015) computes profit sharing between agents that grants

stability of the solutions for subadditive games only. There, a polynomial-time approximation algorithm is proposed that achieves coalition stability without sacrificing too much social welfare. Again, we do not require such restrictions on the valuation functions. Coalition stability in a request for proposal domain is studied in (Kraus et al., 2003), in which a negotiation protocol for coalition formation is introduced. There stability is demonstrated by showing that allowing agents to deviate from pure strategy profiles is not beneficial. It is unclear whether the algorithm proposed in (Kraus et al., 2003) can be modified for use in general scenarios.

3.2 Power Distribution Systems

Evolution of PDS and the concept of a smart grid has been widely studied. Overview of the changes that take place in PDS is given in (Ipakchi and Albuyeh, 2009). There the authors stress the need for changes in the capabilities of PDS. In (Khaitan and McCalley, 2013) PDS are described as cyber-physical systems (CPS), highlighting the importance of security and reliability of PDS. A broader perspective is taken in (Rajkumar et al., 2010), which presents motivation for CPS along with many real-world examples including PDS.

Impacts of distributed renewable energy and distributed storage on PDS have also been studied. Overview of distributed renewable generation and energy storage systems is presented in (Toledo et al., 2010). Challenges with integrating distributed storage are studied in (Mohd et al., 2008), which concludes that future focus of grid development should be on large number of small storage devices placed close to distributed generation. Impact of distributed renewable generation on PDS is studied in (Begović et al., 2001), where the authors claim that distributed generation can have positive effects on the grid because it can help keep voltage on feeders in bounds. This claim is supported by extensive simulations. Effects of battery energy storage systems on PDS are studied in (Tang and Zhang, 2015), which shows that battery energy storage can significantly reduce fluctuation of power flowing through the grid. An approach for minimizing energy cost of an energy buyer equipped with a battery energy storage is proposed in (Khalid et al., 2016). The proposed approach uses

dynamic programming, and is evaluated by simulations.

Coalition formation has been proposed in literature to increase the integration of renewable resources. (Zhang et al., 2015a) proposes an approach for increasing bids of renewable generators in day-ahead markets. (Zhang et al., 2015a) use law of large numbers to show that coalitions of renewable generators can benefit from their spacial distribution, since the adverse effects of prediction uncertainty are mitigated. However, large coalitions of renewable generators can gain market power and lead to unintended outcomes. (Zhang et al., 2015a) therefore studies the trade-off between uncertainty and market power, showing that there is an ideal state where coalitions are large enough to reduce uncertainty, but small enough not to gain significant market power. A fundamental difference between our approach and approach in (Zhang et al., 2015a) is the fact that we consider coalitions of renewable generators with energy stores, while (Zhang et al., 2015a) studies homogeneous coalitions of renewable generators.

Use of coalitions of renewable generators for decreasing uncertainty in electricity production is also studied in (Nayyar et al., 2013). There a profit sharing mechanism is proposed that is used to fairly distribute profit after coalitions of renewable generators are formed. This profit sharing mechanism incentivizes formation of coalitions. Under specified conditions the generators are incentivized to bid higher generation amounts in the day-ahead market. Even though the goals of (Nayyar et al., 2013) are similar to our goals, their approach is different since they study homogeneous coalitions of renewable generators.

A more general perspective is taken in (Zhang et al., 2015b), where a market is studied in which producers face production uncertainty. The authors show that producers can benefit by forming coalitions, which takes advantage of their diversity. The problem is specified in terms of Cournot games, and a trade-off between mitigating uncertainty and market power is studied. Analysis in (Zhang et al., 2015b) shows that for n firms the optimal coalition size in terms of mitigating both uncertainty and market power is of the order of $O(\sqrt{n})$.

3.3 Summary

Many algorithms have been proposed that search for a coalition structure with optimal or sub-optimal social welfare. Among them we highlight dynamic programming approaches ([Cruz-Mencía et al., 2013](#); [Rahwan and Jennings, 2008](#); [Yun Yeh, 1986](#)), hierarchical clustering for large numbers of agents ([Farinelli et al., 2013](#)), and approaches that use multi-agent simulation ([Lerman and Shehory, 2000](#); [Merida-Campos and Willmott, 2004](#)). Other approaches were designed for systems with self-interested agents, among these we highlight publications that provide theoretical analysis ([Hoefler and Wagner, 2013](#); [Pycia, 2012](#); [Sandholm and Lesser, 1997](#)) and approaches that find stable coalitions ([Anshelevich and Sekar, 2015](#); [Augustine et al., 2011](#); [Conitzer and Sandholm, 2006](#)). For a broader overview of coalition formation literature we refer the reader to a recent comprehensive survey on coalition structure generation in ([Rahwan et al., 2015](#)).

Overview of current changes in PDS is provided in ([Ipakchi and Albuyeh, 2009](#); [Khaitan and McCalley, 2013](#); [Rajkumar et al., 2010](#)). More specifically, effects of distributed renewable generation and distributed storage in PDS are summarized in ([Begović et al., 2001](#); [Khalid et al., 2016](#); [Mohd et al., 2008](#); [Tang and Zhang, 2015](#); [Toledo et al., 2010](#)). These publications stress the need for new control mechanisms that would allow higher penetration of distributed renewable generation. Research in coalition formation in PDS has mainly focused on forming coalitions between renewable generators in order to mitigate generation uncertainty ([Nayyar et al., 2013](#); [Zhang et al., 2015a,b](#)).

Chapter 4

Coalition Formation

In this chapter we present a theoretical framework for large-scale coalition formation. The goal of coalition formation is typically either maximizing social welfare or improving welfare of individual agents. We propose an approach that seeks to maximize social welfare in Section 4.1. Then, we show an approach that finds coalitions with respect to welfare of single self-interested agents in Section 4.2. We analyze both of these approaches experimentally using real-world datasets, and then compare our results with results of state-of-the-art algorithms.

4.1 Multi-Agent Simulation

In this section we show how multi-agent simulation can be used to perform *large-scale* coalition formation in order to find coalition structures with high social welfare. We formally define this problem in Section 4.1.1. We assume that the number of agents is in range of thousands. For such a problem, state-of-the-art algorithms cannot find the optimal solution in a feasible time, so we propose to find suboptimal solutions using multi-agent simulation. Multi-agent simulation allows us to observe the process of forming coalitions in an iterative manner. While coalition formation is typically approached as a single-step task that finds a coalition structure, it is beneficial to model coalition formation as a dynamic process where

coalitions change over time. In such a process the agents can utilize information about previous and current values of coalitions to support their decision to leave the current coalition and join a new coalition. We propose a general framework that models coalition formation as a dynamic, iterative process. Our framework can be used to simulate real world applications of coalition formation.

4.1.1 Problem Statement

We studied the problem in which large numbers of agents create coalitions. Specifically we considered the number of agents ranging from 2 to 10,000. We refer to the problem as *large-scale coalition formation*, and we define it as follows.

Definition 4.1.1 (Large-scale coalition formation). *Let us consider a set of agents $A = \{a_1, a_2, \dots, a_n\}$, where n is the number of agents, which can be a large number. The task of large-scale coalition formation is to find a coalition structure $CS = \{C_1, C_2, \dots, C_l\}$, which was defined in Definition 2.1.3 as a set of l coalitions C_j , where each agent is contained in a single coalition. This condition is formally defined as*

$$\forall i \in \langle 1, n \rangle \exists! j \in \langle 1, l \rangle : a_i \in C_j. \quad (4.1)$$

In order to measure quality of a coalition structure CS , we further define the value of CS .

Definition 4.1.2 (Value of a coalition structure). *The value of a coalition structure corresponds to the social welfare of MAS. The value of a coalition structure is defined as a sum of values of all coalitions in the coalition structure,*

$$v(CS) = \sum_{C \in CS} v(C), \quad (4.2)$$

where $v(C)$ was defined in Definition 2.1.4 as a value assigned to the coalition C by a valuation function, which is defined in Section 4.1.4.

In order to compare solutions of problem instances, we define the gain of CS .

Definition 4.1.3 (Gain). *The gain of a coalition structure shows how much, on average, each agent in a coalition structure benefits from participating in a coalition. This metric shows the quality of a solution from the perspective of a single agent. Specifically, the gain $g(CS)$ of a coalition structure CS is defined as*

$$g(CS) = \frac{v(CS) - v(CS_0)}{n}. \quad (4.3)$$

CS_0 denotes a coalition structure containing only singleton coalitions (see Definition 2.1.1), which is the initial state before coalition formation takes place.

In order to show how close a solution is to the optimal coalition structure as defined in Definition 2.1.5, we use gain ratio, which was originally defined in (Farinelli et al., 2013).

Definition 4.1.4 (Gain ratio). *Gain ratio $gr(CS) \in \langle 0, 1 \rangle$ expresses the degree of sub-optimality of a coalition structure CS ,*

$$gr(CS) = \frac{g(CS)}{g_{opt}}, \quad (4.4)$$

where g_{opt} denotes the gain of an optimal solution obtained by a dynamic programming algorithm (Cruz-Mencía et al., 2013). A coalition structure CS is optimal if $gr(CS) = 1.0$.

Note that g_{opt} and $gr(CS)$ can only be found in the small-scale scenarios due to limitations of optimal algorithms. We use the gain and gain ratio metrics to compare the quality of our solutions.

4.1.2 Framework

We propose a general framework that can model and solve specific applications of the coalition formation problem. We modeled coalition formation as an iterative process in which the agents leave and join coalitions in an iterative fashion. The algorithm for this process is depicted in Algorithm 1 and works as follows.

First the simulator is initialized. Initialization consists of following steps. Agents are created, and each agent initially forms a singleton coalition (line 1). Interest vectors of size k are then assigned to agents (line 2). Interest vectors are essential to the simulation because elements of an interest vector express specific interests of the agent. In various problem applications these interest vectors may represent the amount of resources owned or requested by the agent, or an electricity load of the agent’s household. Next, a strategy is assigned to each agent (line 3). This strategy is later used to determine whether or not the agent should leave a current coalition and which coalition it should join. A new coalition structure is then created (lines 4 and 5). This structure holds all agents grouped in current coalitions. Finally, the evaluation agent and a valuation function are initialized. The evaluation agent is responsible for evaluating all coalitions and announcing coalition rankings in each iteration based on the specified valuation function. The time complexity of this initialization step (lines 1 to 6) is $O(n \cdot k)$.

Algorithm 1 Multi-agent simulation of coalition formation

Input: number of agents n , number of iterations N , size of interest vectors k .

Output: Coalition structure with highest gain.

```

1: create  $n$  agents
2: assign interest vectors of size  $k$  to agents
3: assign strategies to agents
4: initialize new coalition structure  $CS$ 
5: create a coalition for each agent and add it to  $CS$ 
6: initialize evaluation agent and valuation function
7: for iteration in  $1:N$  do
8:   for all agents in random order do
9:     if agent.strategy.leaveCoalition() then
10:      leave current coalition, update its value
11:      newcoalition  $\leftarrow$  agent.strategy.pickCoalition()
12:      update value of newcoalition
13:     end if
14:   end for
15:   evaluate all coalitions
16:   announce the ranking of coalitions
17:   store current coalition structure
18: end for
19: return best coalition structure

```

After the initialization step, the simulation begins with a first iteration. In every iteration

the agents use their strategies to decide whether to leave their current coalition and, if so, which coalition to join (lines 9 to 11). Then all remaining coalitions are evaluated by the evaluation agent, and the ranking of coalitions is announced (lines 15 and 16). Finally, the current coalition structure is stored (line 17). Each agent in each iteration accesses during its decision making at most n other agents' interest vectors, regardless of the agents' grouping in coalitions. Therefore the worst-case time complexity of Algorithm 1 is $O(N \cdot n^2 \cdot k)$. At the end of the simulation, the best coalition structure is selected out of all stored coalition structures, and returned as the solution (line 19).

4.1.3 Applications

In this section we discuss two general applications of large-scale coalition formation. We also discuss the approach we took to model these applications in our framework. Other applications of coalition formation can be modeled in the framework following our approach.

- **Resource sharing** - Agents operate with several resources, and they can have either a surplus or a shortage of each resource. Agents with a surplus try to form coalitions with agents with a shortage so that the surplus amount can be transferred to (bought by) agents with shortages. We use interest vectors to store surplus or shortage of each resource. The value of a coalition then depends on the amount of resources shared. For example, consider a scenario with three agents and two resources. The amounts of resources that each agent operates with are shown in Table 4.1. Agent 1 is considering joining a coalition with either of the two remaining agents. While agent 1 could share resource 2 with agent 2, agent 1 would benefit more by forming a coalition with agent 3, because agent 1 and agent 3 can share both resource 1 and resource 2.
- **Collective energy purchasing** - Electricity can be bought either at a spot or forward market. A spot market provides electricity according to the current amount requested. However, agents can exploit reduced tariffs at forward markets that sell constant amounts of electricity for long periods of time. In order to exploit the forward market, the aggregate energy profile of the buyers (i.e. the hourly energy requirements

Table 4.1: *Example of a resource sharing scenario. Positive and negative values denote surpluses and shortages respectively.*

| Agent \ Resource | Resource 1 | Resource 2 |
|------------------|------------|------------|
| Agent 1 | 100 | -50 |
| Agent 2 | 50 | 50 |
| Agent 3 | -50 | 50 |

for a day) must be flat. Agents can form coalitions in order to flatten their aggregate daily energy profile. We model energy purchasing by storing the agents' energy profiles in their interest vectors. A value of a coalition then depends on the flatness of its aggregate interest vector. The collective energy purchasing application is relevant to the PDS domain, because it builds on the concept of active consumers who try to optimize their expenses. We will examine another application of coalition formation in the PDS domain in Chapter 5.

4.1.4 Valuation Functions

A valuation function $f : C \rightarrow \mathbb{R}$ assigns a value v to each coalition C . We propose market-based valuation function to represent resource sharing. We also discuss collective energy purchasing (Vinyals et al., 2012) and normally distributed coalition structures (Rahwan et al., 2009) valuation functions.

- **Market-based valuation function** is used in the resource sharing application. The value of a coalition is determined by the amount of resources shared (bought and sold) within the coalition. More specifically,

$$v(C) = \sum_{l=1}^k \min(b_C^+[l], b_C^-[l]) + \kappa(C) \quad (4.5)$$

where $b_C^+[l]$ is the positive balance for resource l , which is the sum of surpluses of resource l over all agents in coalition C , and $b_C^-[l]$ is an absolute value of the negative balance computed with the shortages, respectively. $\kappa(C) = -|C|^\gamma$ was proposed in

(Farinelli et al., 2013) to represent the penalty for the coalition size. The penalty prevents the grand coalition (defined in Definition 2.1.1 as a coalition containing all n agents) from forming, and it represents the difficulty associated with a cooperation of a large number of agents.

- **Collective energy-purchasing valuation function** was proposed in (Vinyals et al., 2012). The value of an expected payment (coalition value) for coalition C is given by

$$v(C) = \sum_{t=1}^T q_S^t(C) \cdot p_S + T \cdot q_F(C) \cdot p_F + \kappa(C) \quad (4.6)$$

where p_S and p_F represent unit prices at the spot and forward markets, respectively, $q_S^t(C)$ represents the amount of energy to be bought at the spot market at time t , and $T \cdot q_F(C)$ represents the total amount of energy to be bought at the forward market for time interval T (in our experiments, $T = 24$ represents a length of a daily energy profile). $\kappa(C) = -|C|^\gamma$ (Farinelli et al., 2013) captures the penalty for the coalition size. Unlike the market-based valuation function, the collective energy-purchasing valuation function creates strong interdependence between the elements of interest vectors.

An algorithm given in (Vinyals et al., 2012) computes optimal energy amounts for a coalition given the coalition’s aggregate energy profile, which we store in the coalition’s aggregate interest vector. This algorithm first sorts the aggregate energy profile in a descending order. The amount of energy to be bought at the forward market $q_F(C)$ is then set to the value at position $p_F/p_S \cdot T$ in the profile, which is the amount of energy covered by at least $\frac{p_F}{p_S}$ of the time interval. Finally, for each time slot the spot quantity $q_S^t(C)$ is calculated as the amount of demanded electricity that exceeds the forward quantity. Using this algorithm, we obtain energy amounts $q_S^t(C)$ and $q_F(C)$ that we use to compute the coalition value $v(C)$.

- **Normally Distributed Coalition Structures (NDCS)** is a challenging valuation function benchmark proposed by (Rahwan et al., 2009). The value of a coalition is drawn from a normal distribution $\mathcal{N}(\mu, \sigma)$ with $\mu = |C|$ and $\sigma = \sqrt{|C|}$. We include

NDCS here for the sake of comparison of our approach with the C-Link algorithm, which is a hierarchical clustering approach for coalition formation proposed in (Farinelli et al., 2013).

4.1.5 Coalition Selection Strategies

Agents use coalition selection strategies to make two decisions: whether to leave a coalition and which (if any) of the existing coalitions to join. We propose *mixed* and *local search* coalition selection strategies. We also discuss *coalition value-based* strategy and *random* strategy. We assume that the agents have complete information about the system including other agents' interest vectors.

- **Coalition value-based strategy** advises the agent to join a coalition that maximally benefits from the addition of the agent. For agent a_i , the new coalition C_{new} is

$$C_{new} \leftarrow \arg \max_{C \in CS} (v(C \cup \{a_i\}) - v(C)). \quad (4.7)$$

An agent leaves a coalition if the coalition is not the agent's current choice. This strategy maximizes marginal contribution of an agent to a coalition. In game theory literature this strategy is often referred to as the best response strategy (Fudenberg and Tirole, 1991).

- **Random strategy**, proposed in (Merida-Campos and Willmott, 2004), makes decisions to leave and join coalitions randomly. Despite its trivial reasoning, this strategy can be used for a fast search of the state-space.
- **Mixed strategy** utilizes decision making of at least two strategies. Whenever a decision on leaving or joining a coalition is needed, this strategy selects a deciding strategy from a list of available strategies, and forwards the decision request to this strategy. The deciding strategy is chosen using a roulette wheel algorithm, which picks a strategy randomly based on given probabilities of the strategies.

- **Local search strategy** performs local optimization with random jumps when a local optimum is reached. This strategy combines the *coalition value-based* and *random* strategies as follows. The *coalition value-based* strategy is used by all agents as long as the resulting coalition structure continues to change. If an iteration yields the same coalition structure as the previous iteration, the *random* strategy is used once by all agents in order to escape the local optimum.

4.1.6 Experimental Analysis

We evaluated our coalition selection strategies and valuation functions experimentally using the gain and gain ratio metrics (Definitions 4.1.3 and 4.1.4). However, the gain ratio metric can only be applied to instances with small numbers of agents (here up to 20) because an optimal solution is used as a baseline. For the baseline, we used an optimized implementation of a dynamic programming algorithm from (Cruz-Mencía et al., 2013). Note that for the dynamic programming algorithm, evaluations of all possible coalitions were generated using the given valuation functions.

We used the following parameter settings for the experiments. In order to achieve reasonable run-times in instances with various numbers of agents n , we used the following numbers of iterations for N . We set the number of iterations to $N = 500$ for small-scale instances with $n \leq 20$, $N = 10$ for instances with $n \in (20, 5000)$ and $N = 3$ for instances with $n > 5000$. These limits were designed to achieve reasonable run-times. We will show that the change in gain of our solutions when adding iterations is very small for N approaching 10. Because our algorithms are any-time, a solution can be returned at any point during the simulation.

For the energy purchasing scenario the interest vectors of length $k = 24$ stored real-world daily energy profiles of households in Portugal (Lichman, 2013) (one value for each hour, T was therefore set to 24). The hourly values were averaged for each agent over all days in January 2014 into a single average January day.

For the resource sharing scenario, we used an international trade dataset provided by the World Trade Organization (World Trade Organization, n.d.). The dataset stores import

and export amounts in US dollars between 167 countries in 17 commodity types, therefore we set $k = 17$. The amount of each resource of each agent was computed as the difference between export and import amounts of the given country in the year 2014. Positive and negative values of the resulting resource amounts denote surplus and shortage respectively.

The parameter γ representing coalition size penalty was set to 1.1 following (Farinelli et al., 2013) for the energy purchasing scenario and to 2 for the resource sharing scenario. The higher value of γ was used in the resource sharing scenario to prevent the grand coalition, which is the trivial solution, from being the optimal solution. As suggested in (Vinyals et al., 2012) and (Farinelli et al., 2013), we fixed the prices for the energy purchasing scenario at $p_S = -80$ and $p_F = -70$. Negative values are used because the coalition value is maximized. The baseline dynamic programming algorithm is implemented in C using integer and long types, which creates a risk of integer overflow for large coalition values. Due to these numeric limitations of the baseline dynamic programming implementation we used randomly generated data for the small scale experiments. Specifically, elements of interest vectors were drawn from a uniform distribution $\mathcal{U}\{0, 10\}$ for the collective energy purchasing scenario and $\mathcal{U}\{-10, 10\}$ for the resource sharing scenario.

We ran our Java implementation of the proposed algorithms on 2.7 GHz Intel Xeon E5 CPU with 2 GB of memory. The 2 GB limit provides sufficient memory for our algorithms used in experiments with up to 10,000 agents. All results were generated by averaging 10 random runs of our algorithms.

4.1.7 Experiment Results in Small-scale Problem Instances

For problem instances containing up to 20 agents, we compared the performance of our coalition selection strategies with optimal solutions. Table 4.2 shows an average gain ratio achieved by the strategies. The table also shows results achieved by a state-of-the-art hierarchical agglomerative clustering algorithm C-Link (Farinelli et al., 2013).

The best average gain ratio was achieved by strategies that combine local search and random approaches, as shown in the first and third highest ranking strategies in Table 4.2.

Table 4.2: Average gain ratio in the small-scale problem instances, with number of iterations $N = 500$. Combination of local and random searches yields results closest to the optimum. Results are averaged over all small-scale experiments with resource sharing, collective energy purchasing, and NDCS scenarios.

| Coalition selection strategy | Average gain ratio |
|--------------------------------------|--------------------|
| Mixed: coalition value-based, random | 0.9399 |
| C-link: Gain Linkage | 0.9289 |
| Local search | 0.9203 |
| Coalition value-based | 0.8700 |
| Random | 0.8097 |

Our two highest ranking strategies used both agents’ best response through the *coalition value-based* strategy and random search through the *random* strategy. While both *coalition value-based* and *random* strategy achieved lower gain ratio, their combination found solutions close to the optimum, because this combination can both utilize the information about the coalition values as well as escape local optima. The highest ranking *mixed* strategy utilized random search more often than the *local search* strategy, it was therefore able to search larger portion of the search space and consequently find better solutions.

The locally optimizing *coalition value-based* strategy achieved a worse gain ratio because it cannot escape local optima and therefore it wastes the remaining iterations after a local optimum is found. The *random* strategy, which performs an uninformed search of the state-space, ranked last.

Figure 4.1 shows gain ratio of our strategies and C-Link in the challenging NDCS scenario for various numbers of agents. Recall from Definition 4.1.4 that if gain ratio $gr(CS) = 1.0$ then the solution is optimal. In this scenario the *local search* strategy ranks first, outperforming both the *mixed* strategy and C-Link, thus showing the effectiveness of the *local search* strategy in challenging small-scale problem instances.

The results in small-scale problem instances showed that the multi-agent simulation approach for coalition formation yields solutions with gain on average up to 94% of the gain of optimal solutions (see Table 4.2) in problem instances where optimal solutions can be obtained using a state-of-the-art optimal algorithm. The results also showed that *local search*

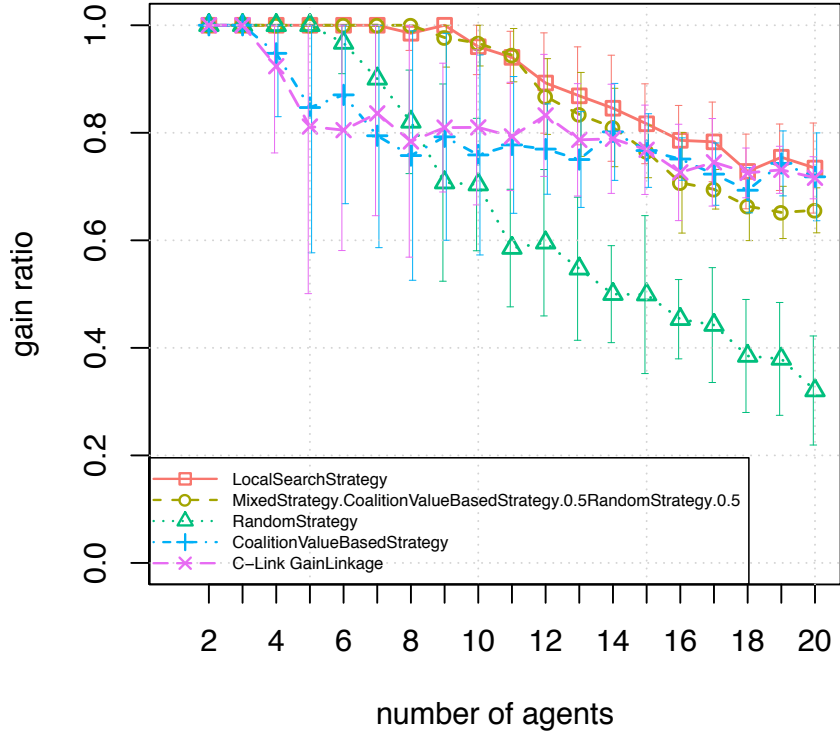


Figure 4.1: Gain ratio in NDCS scenario. The mixed strategy is encoded as “MixedStrategy.StrategyA.probability-of-AStrategyB.probability-of-B.” Error bars show standard deviation of aggregated variables.

and *mixed* strategies find solutions of similar or higher quality than the state-of-the-art algorithm C-Link.

4.1.8 Experiment Results in Large-scale Problem Instances

Given the promising results of the small-scale experiments, we experimented with higher numbers of agents (up to 10,000) in order to measure the performance of our algorithms in a large-scale setting in which optimal algorithms cannot be applied. Figures 4.2 and 4.3 compare our strategies and C-Link based on the achieved gain. Note that the gain loss between iterations with number of agents $n = 5000$ and $n = 5500$ is caused by the decreased number of iterations N for $n > 5000$. This loss is largest for the *random* strategy, because lowering the iteration limit lowers the number of random samples of the state-space. Strategies that are not based on the *random* strategy are affected as well because the number

of agents’ best responses is also decreased. We decreased the number of iterations N in order to achieve reasonable run-time of our algorithm. Further experiments showed that increasing the number of iterations yields results with both gain and run-time comparable to C-Link.

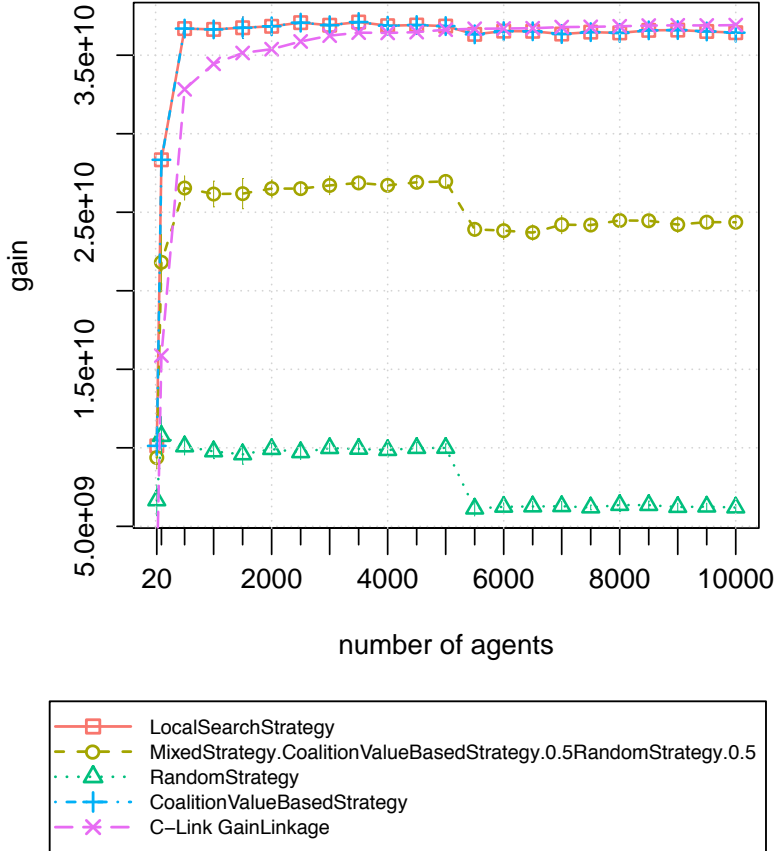


Figure 4.2: Gain achieved by our algorithm and C-Link in resource sharing scenario for given coalition selection strategies and valuation functions, for number of iterations $N = 10$ for $n \leq 5000$ and $N = 3$ for $n > 5000$. Gain reflects how much on average each agent benefits from coalition formation. We cannot compare gain values across scenarios because each valuation function yields a different magnitude of the gain. The mixed strategy is encoded as “MixedStrategy.StrategyA.probability-of-AStrategyB.probability-of-B.” Error bars show standard deviation of aggregated variables.

We highlight the following three observations. First, in collective energy purchasing scenario the gain achieved by *local search* and *coalition value-based* strategies is the same as the gain achieved by C-Link, and in instances of resource sharing scenario with $N = 10$

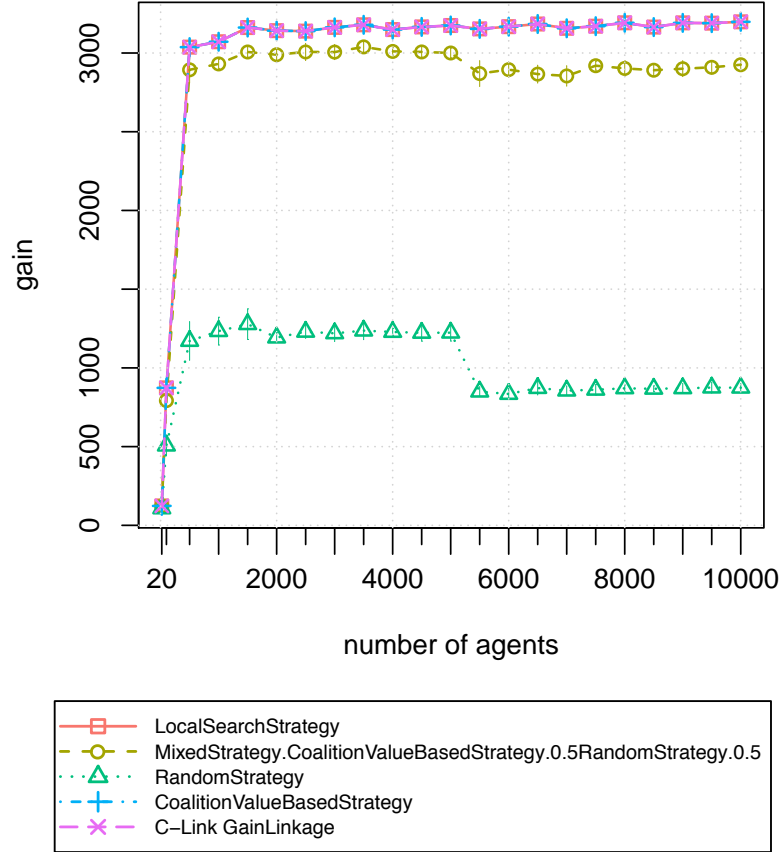


Figure 4.3: Gain achieved by our algorithm and C-Link in collective energy purchasing scenario for given coalition selection strategies and valuation functions, for number of iterations $N = 10$ for $n \leq 5000$ and $N = 3$ for $n > 5000$. Gain reflects how much on average each agent benefits from coalition formation. We cannot compare gain values across scenarios because each valuation function yields a different magnitude of the gain. The mixed strategy is encoded as “MixedStrategy.StrategyA.probability-of-AStrategyB.probability-of-B.” Error bars show standard deviation of aggregated variables.

the gain of these two strategies is greater than the gain of C-Link. Decreasing number of iterations to $N = 3$, which decreases the number of best responses to 3 per agent, causes the gain of our strategies in resource sharing scenario to become slightly lower than the gain of C-Link. However, in these instances the run-time of our strategies is over one order of magnitude lower than the run-time of C-Link, as shown in Figure 4.7.

Second, the gain achieved by *local search* and *coalition value-based* strategies for $n \geq 20$

is greater than the gain at $n = 20$, demonstrating that our algorithms provide stable average gain for the agents with increasing scale of the problem. Therefore agents in large-scale scenarios benefit from coalition formation more than agents in small-scale scenarios in which the solutions are very close to the optimum. The reason for this phenomenon is a more open state-space of the large-scale multi-agent system. Adding agents to the tested problem instances creates better opportunities for the agents and consequently increases the overall gain. This is a promising result given the absence of comparison with optimal solutions.

Third, the *local search* and the *coalition value-based* strategies outperformed other strategies in all scenarios because each agent in each iteration locally optimizes the overall gain. Although in small-scale problem instances (see Table 4.2) this approach is outperformed by quicker strategies that randomly search larger part of the state-space within the given number of iterations, this advantage is no longer as important in large-scale problem instances because the state-space grows exponentially and therefore these quicker strategies can only search a small part of it. Therefore slow, locally optimizing strategies yield solutions with higher gain.

Since the input to Algorithm 1 is the number of iterations N , it is important to study the effect of choosing N . Figure 4.4 shows the gain achieved by our strategies after $N = \langle 1; 10 \rangle$ iterations in the resource sharing scenario. The gain of the highest ranking *local search* and *coalition value based* strategies dominated the gain achieved by C-Link after two iterations, and this gain became stable around fifth iteration. This result shows that the number of iterations used in the resource sharing scenario can be very low. Furthermore, this shows that after only a few iterations solutions of our strategies dominate solutions of a state-of-the-art algorithm. Similar results are shown in Figure 4.5 for the collective energy purchasing scenario. There our strategies achieved gain comparable with C-Link after the first iteration.

Another factor in choosing the number of iterations is the convergence of our algorithm. Convergence analysis is important because it can limit the number of necessary iterations and therefore decrease the run-time of our algorithms. Since the *local search*, *random*, and *mixed* strategies are random-based and therefore do not converge, we only study the convergence of the *coalition value-based* strategy. Figure 4.6 shows the number of iterations

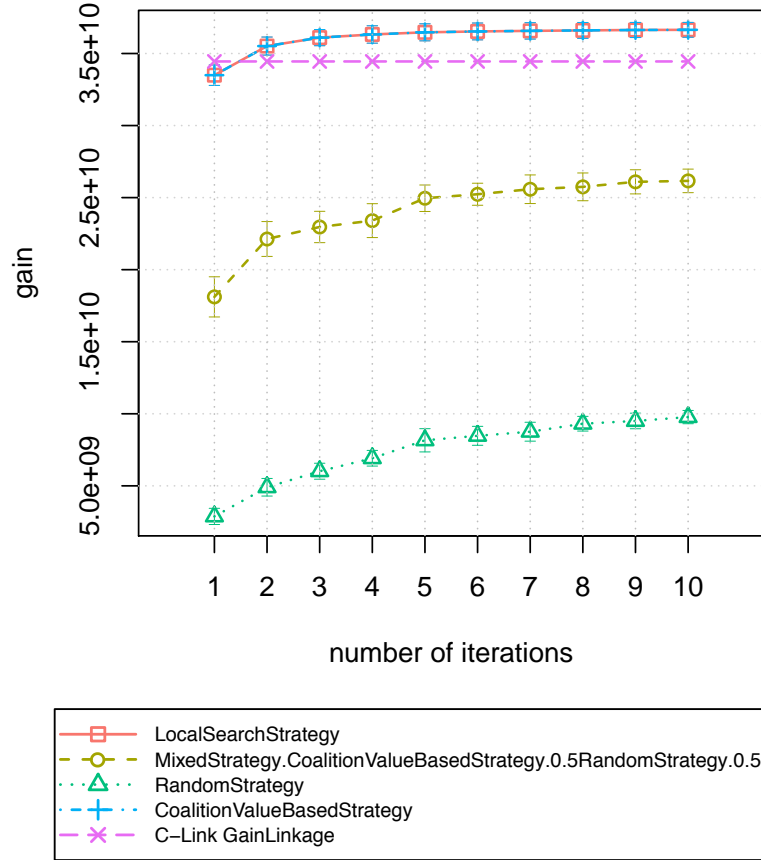


Figure 4.4: Effect of the number of iterations N on gain in the resource sharing scenario with 1000 agents. Local search and coalition value based strategies dominated C-Link after two iterations. The mixed strategy is encoded as “MixedStrategy.StrategyA.probability-of-AStrategyB.probability-of-B.” Error bars show standard deviation of aggregated variables.

until convergence in resource sharing and collective energy purchasing scenarios. The results vary between the two scenarios, indicating that the resource sharing scenario is harder to solve. Interestingly, the number of iterations until convergence does not increase significantly with increasing number of agents in the collective energy purchasing scenario. Overall, the number of iterations needed to reach convergence is relatively small. Figure 4.6 shows the maximum number of iterations that should be considered for the number of iterations N .

Finally, in practice the run-time of the algorithm can be a constraint, it is therefore also a deciding factor in choosing the number of iterations N . Figures 4.7 and 4.8 show run-time

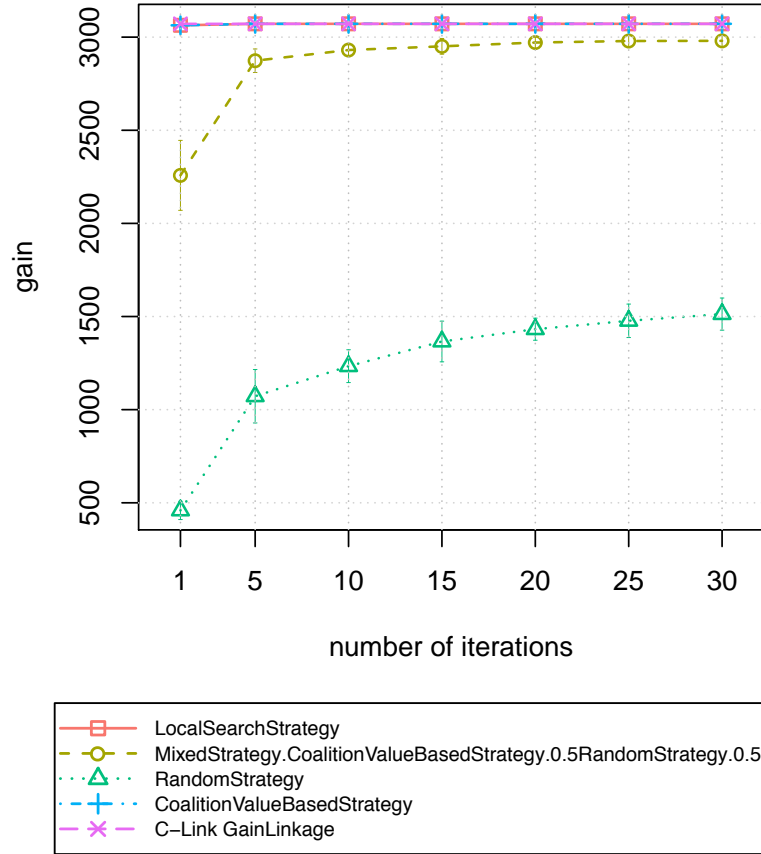


Figure 4.5: Effect of the number of iterations N on gain in the collective energy purchasing scenario with 1000 agents. The mixed strategy is encoded as “MixedStrategy.StrategyA.probability-of-AStrategyB.probability-of-B.” Error bars show standard deviation of aggregated variables.

of our algorithm. Note again that the sudden decrease in run-time between $n = 5000$ and $n = 6000$ is caused by decrease in the number of iterations from $N = 10$ to $N = 3$. C-Link cannot benefit from this decrease because it is not an iterative algorithm. Consequently, it is impossible to affect the run-time of C-Link by changing the input parameters. *Coalition value-based* and *local search* strategies are slower than the other strategies because they more often perform expensive search for a coalition maximizing the marginal contribution. However, in instances with $n > 5000$, in which $N = 3$, these strategies are faster than C-Link. Particularly in the resource sharing scenario with high numbers of agents our algorithm is

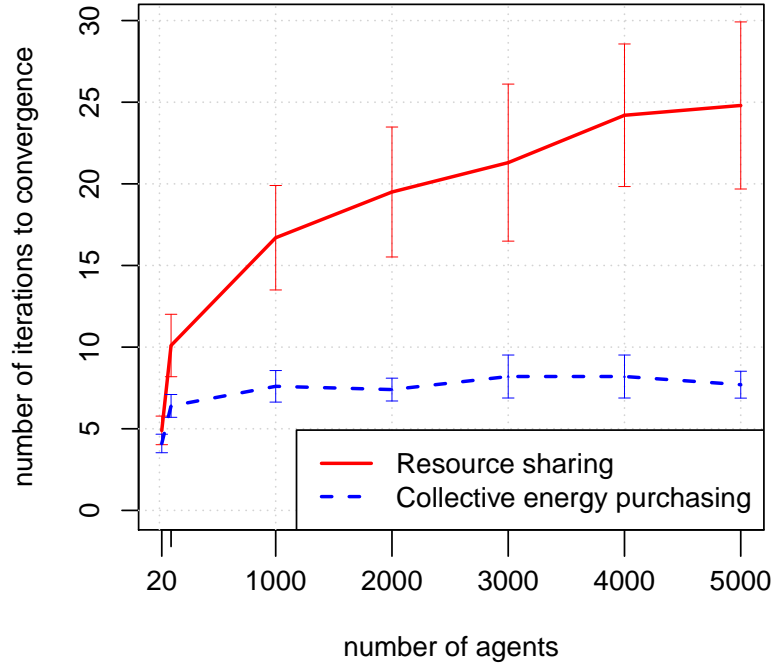


Figure 4.6: Number of iterations it takes the coalition value-based strategy to converge to a local optimum in both scenarios. Error bars show standard deviation of aggregated variables.

faster by one order of magnitude. Together, Figures 4.4, 4.7, and 4.8 show the trade-off between run-time and solution quality. Unlike C-Link, our approach allows for tuning of this trade-off using the number of iterations N .

4.1.9 Discussion

We designed a framework that simulates coalition formation. While the simulator part of the framework is fixed, the valuation functions and agents' strategies are configurable, which gives a user extensive flexibility in modeling various coalition formation scenarios. We showed the usability of the framework by designing and implementing various types of agent behavior along with specific applications of coalition formation. Our design approach can be followed to model other applications by designing and implementing new valuation functions. New agents' behavior can also be modeled with the use of new coalition selection

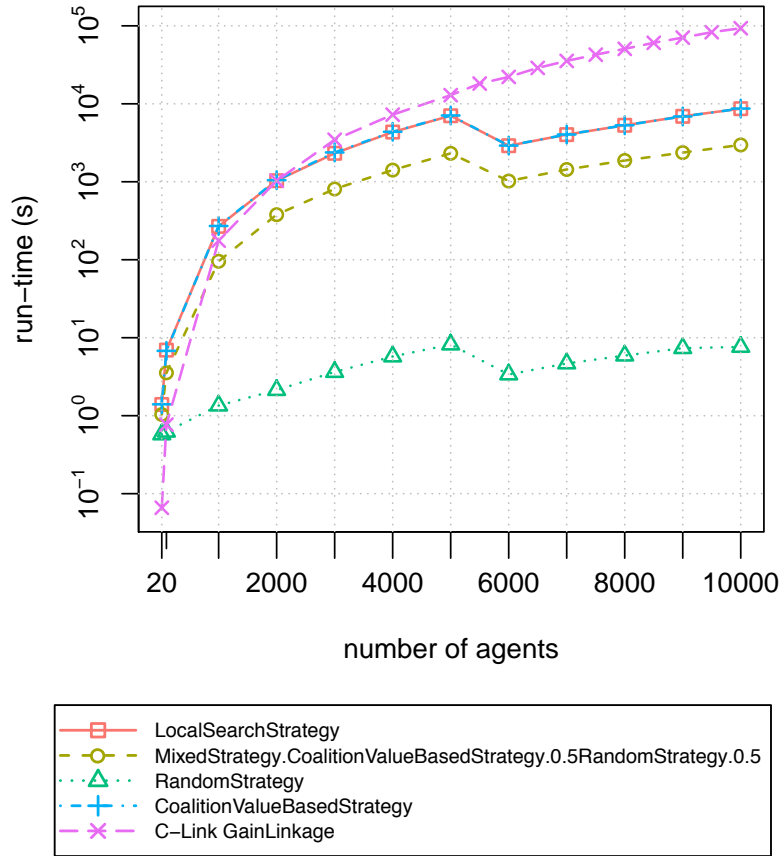


Figure 4.7: Run-time of our algorithm and C-Link in resource sharing scenario. Our algorithm is faster than C-Link in large instances. The mixed strategy is encoded as “MixedStrategy.StrategyA.probability-of-AStrategyB.probability-of-B.” Error bars show standard deviation of aggregated variables.

strategies following our approach.

The framework can be used to simulate scenarios with thousands of agents. In these large-scale scenarios we do not guarantee finding an optimal solution. However, the framework can be used to generate a step-by-step evolution of the multi-agent system based on specific behavior of the agents, which better reflects the real world where global optimum is seldom an achievable goal. We show performance of the framework in simulations of the coalition formation applications. Using a combination of local and random searches, the framework is able to find solutions very close to the optimum in small-scale instances of the problem.

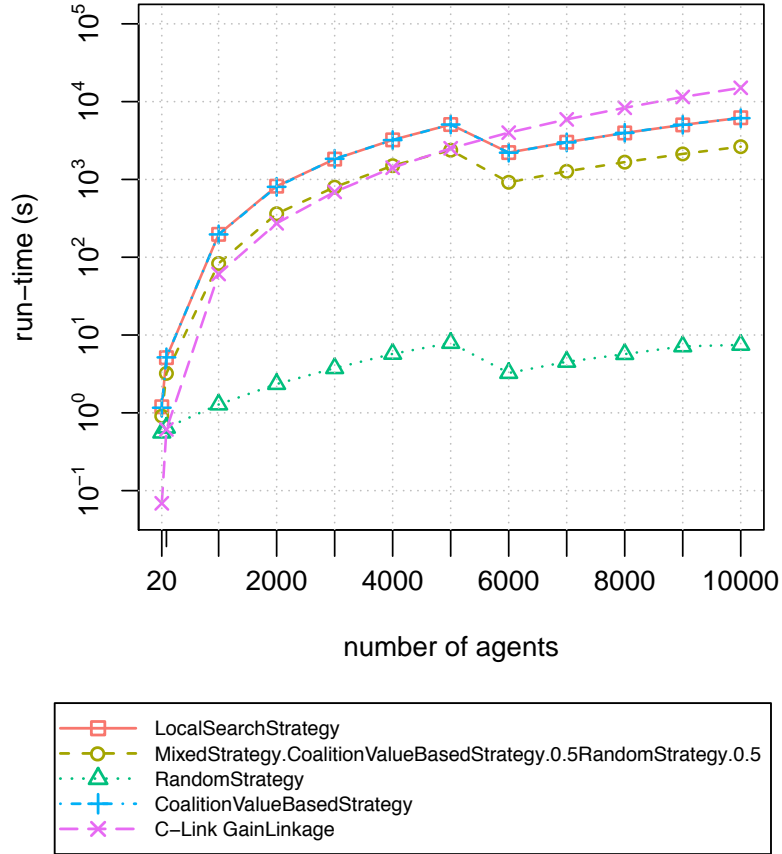


Figure 4.8: Run-time of our algorithm and C-Link in collective energy purchasing scenario. Our algorithm is faster than C-Link in large instances. The mixed strategy is encoded as “MixedStrategy.StrategyA.probability-of-AStrategyB.probability-of-B.” Error bars show standard deviation of aggregated variables.

With the scale of the problem increasing to 10,000 agents, the single agents achieve higher gain than in small-scale instances. For new user-defined coalition formation applications and agents’ strategies it might not be obvious which strategy will yield the best solutions. Our framework can be used to test different strategies of the agents in given applications and determine the performance of the strategies, similarly as we did in Section 4.1.6.

As mentioned in Section 4.1.2, the worst-case time complexity of our algorithm is $O(N \cdot n^2 \cdot k)$, which is lower than the $O(n^3)$ complexity of the C-Link algorithm (Farinelli et al., 2013). An element that could affect the time complexity as well as the behavior of our

algorithm is a cost of communication between agents. The communication cost would have to be included if the approach was distributed among multiple computational units. However, since Algorithm 1 is centralized, we do not include the communication cost in our calculations for the cause of simplicity and to be able to compare our results with results of C-Link, which also does not take communication cost into account.

In order to reuse our framework, several steps have to be taken and several design decisions must be made.

1. Implement Algorithm 1.
2. Decide about strategies that the agents will use. For example, a self-interested agent might utilize a different strategy than a selfless agent.
3. Design a valuation function to represent the specific problem application that is being examined.
4. Decide which coalition structure should be selected as the solution, since one coalition structure is created in every iteration. In our experiments we selected coalition structure with the highest gain, but other options include selecting the last coalition structure or the coalition structure containing a coalition with the highest value.

4.2 Coalition Stability

In this section we propose an approach for finding *stable* coalitions in large-scale MAS. Recall that we consider a coalition stable if no sub-group of its members is incentivized to leave the coalition. Coalition stability is in literature approached mainly using the concept of a core. However, the complexity of determining the core renders it unusable in large-scale systems. Therefore instead of the core, we approach coalition stability using multi-agent simulation. Instead of looking for stable allocations of the coalition value to the agents as is done in the core, we specify an allocation scheme beforehand and let the agents utilize this information to choose more stable coalitions during the iterative process of multi-agent simulation.

4.2.1 Problem Statement

We study the coalition formation problem, in which agents $a_1, a_2, \dots, a_n \in A$ form coalitions C_j such that each agent belongs to exactly one coalition. This problem was defined in Definition 4.1.1. A coalition structure CS is a set of all coalitions C_j that the agents formed (see Definition 2.1.3). The task is to find a coalition structure that maximizes its social welfare (as in Section 4.1) as well as its stability.

In order to measure the social welfare of the formed coalition structure, we reuse the value of a coalition (Definition 2.1.4), the value of a coalition structure (Definition 4.1.2), and the gain of a coalition structure (Definition 4.1.3). In order to use gain in multi-criterial optimization (see Section 4.2.4), we define normalized gain as follows.

Definition 4.2.1 (Normalized gain). *Normalized gain $g_{norm}(CS)$ of a coalition structure CS is computed using gain $g(CS)$ and a set of coalition structures P as follows:*

$$g_{norm}(CS) = \frac{g(CS) - g_{min}}{g_{max} - g_{min}}, \quad (4.8)$$

where

$$g_{min} = \min_{CS \in P} g(CS) \quad (4.9)$$

and

$$g_{max} = \max_{CS \in P} g(CS). \quad (4.10)$$

It holds that

$$\forall CS \in P : g_{norm}(CS) \in \langle 0, 1 \rangle \wedge \min_{CS \in P} g_{norm}(CS) = 0 \wedge \max_{CS \in P} g_{norm}(CS) = 1. \quad (4.11)$$

Coalition stability has to be considered in systems with self-interested agents. These self-interested agents prefer their own profit over the social welfare of the MAS. Self-interested agents maximize their own profit, which we define for agent a_i participating in coalition C_j as

$$p_{C_j}(a_i) = v(C_j \cup \{a_i\}) - v(C_j), \quad (4.12)$$

where the coalition values $v(C_j \cup \{a_i\})$ is computed right after and $v(C_j)$ is computed right before the agent entered the coalition. The profit reflects marginal contributions of agents to the coalitions, (Augustine et al., 2011) describes games that use this profit sharing scheme as Labor Union games. This definition of profit guarantees that the allocation to the agents granted at the point of entry to the coalition will not change later regardless of further changes in the coalition.

In order to measure the stability of coalition structure CS we need to determine stability of all coalitions $C_j \in CS$. Determining the coalition stability is computationally expensive, because it requires evaluation of all $2^{|C|}$ sub-coalitions. We therefore introduce α -stability to approximate the coalition stability.

Definition 4.2.2 (α -stable coalition). *We say that a coalition C is α -stable if no sub-coalition D with $\langle 1, \alpha \rangle$ members can be formed in which some agents would benefit more and no agent would benefit less than in C , formally:*

$$\begin{aligned} C \text{ is } \alpha\text{-stable iff } \nexists D \subset C, |D| \in \langle 1, \alpha \rangle : \\ \exists a_i \in D : p_D(a_i) > p_C(a_i) \\ \wedge \forall a_i \in D : p_D(a_i) \geq p_C(a_i). \end{aligned} \quad (4.13)$$

Definition 4.2.3 (Stability of a coalition structure). *We denote S_α as the set of α -stable coalitions in CS , for which it holds that $\forall \alpha : S_{\alpha+1} \subseteq S_\alpha$. We define stability of a coalition structure in terms of α as*

$$\text{stability}_\alpha(CS) = \frac{|S_\alpha|}{|CS|} \quad (4.14)$$

where $|CS|$ denotes the number of coalitions in CS . It holds that

$$\lim_{\alpha \rightarrow \max_{C_j \in CS}(|C_j|)} \text{stability}_\alpha(CS) = \text{stability}(CS) \quad (4.15)$$

where $\text{stability}(CS)$ is the true stability of CS , which we define as the ratio of stable coalitions in CS .

Since stability_α is non-increasing with respect to α , it can serve as an upper estimate of the coalition structure stability. Figure 4.9 shows how stability_α approaches the true stability with increasing α . The figure also shows the trade-off between the quality of the stability estimate and the run-time required to compute the estimate.

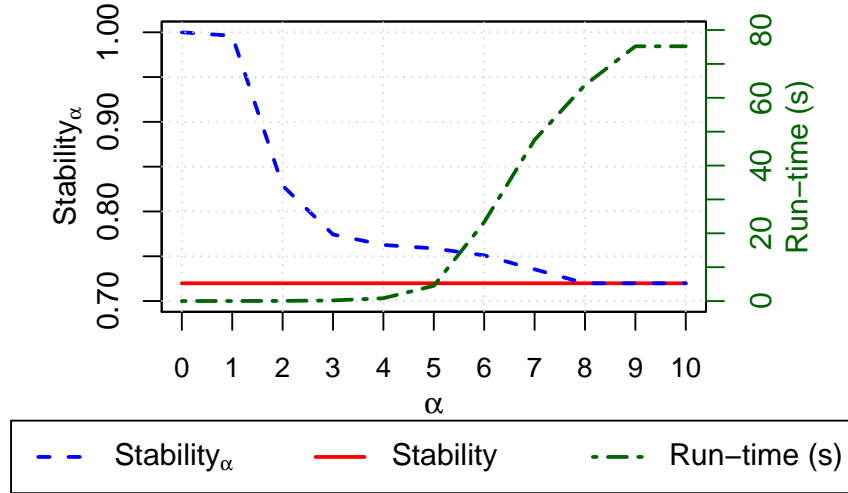


Figure 4.9: Stability_α approaches the true stability with increasing α . Stability_α was measured for coalition structure with 1000 agents after 15 iterations. For $\alpha > 10$ all coalitions were either smaller than α or determined unstable. Run-time denotes the time required for calculation of stability_α .

Finally we use the price of stability

$$PoS(CS_{sw}, CS_{sa}) = \frac{g(CS_{sw})}{g(CS_{sa})} \quad (4.16)$$

to show the ratio between the gain of social welfare maximizing solutions CS_{sw} and the gain of solutions reached by behavior of self-interested agents CS_{sa} .

4.2.2 Algorithm

We find solutions to coalition formation using multi-agent simulation. We extend a multi-agent simulation framework for large-scale coalition formation proposed in Section 4.1, in which the agents maximize the social welfare. In that framework the agents use strategies to decide about leaving their coalitions and joining new coalitions. The coalitions are evaluated by a polynomial-time valuation function $f : C \rightarrow \mathbb{R}$. This process repeats in an iterative fashion, resulting in an agent-driven search of the state-space of coalition structures. While the approach proposed in Section 4.1 shows almost-optimal performance in small-scale scenarios and stable gain in large-scale scenarios, it does not consider stability of the solutions.

In order to increase stability of coalition structures we extend Algorithm 1 from Section 4.1 by first allowing the agents to create more stable sub-coalitions within their coalition by the process of deviation (Section 4.2.3), and second by selecting the best solution out of the pool of solutions generated by the simulation with respect to both social welfare and stability (Section 4.2.4). Algorithm 2 shows our approach with lines 10 to 14 showing the use of deviation, and line 17 representing the solution selection. The concepts of deviation and solution selection are described in the following sections.

4.2.3 Deviation

Deviation allows agents to leave their current coalition along with other agents from the same coalition. We allow the agents to deviate from their coalitions in order to guide the search towards more stable coalition structures. There are two conditions that a sub-coalition must satisfy in order to be able to deviate from a coalition.

Definition 4.2.4 (Deviating sub-coalition). *A sub-coalition of agents $D \subset C$ can deviate from a coalition C if the following two conditions are met:*

$$\forall a_i \in D : p_D(a_i) \geq p_C(a_i), \tag{4.17}$$

Algorithm 2 Multi-agent simulation of coalition formation that maximizes gain and stability

Input: number of agents n , number of iterations N .
Output: coalition structure with highest gain and $stability_\alpha$.

- 1: initiate the simulator
- 2: **for** iteration in $1 : N$ **do**
- 3: **for all** agents in random order **do**
- 4: **if** $agent.strategy.leave$ **then**
- 5: $agent.coalition.recompute$ profit of agents after $agent$
- 6: $agent.coalition \leftarrow agent.strategy.pickCoalition$
- 7: compute $agent$'s new profit
- 8: **end if**
- 9: **end for**
- 10: **for all** agents in random order **do**
- 11: **do**
- 12: boolean deviated $\leftarrow agent.strategy.deviat$ e
- 13: **while** deviated
- 14: **end for**
- 15: store current solution
- 16: **end for**
- 17: choose best solution

and

$$\exists a_i \in D : p_D(a_i) > p_C(a_i). \quad (4.18)$$

These conditions are satisfied by sub-coalitions in which no agent loses profit by deviation and at least one agent gains profit. If these conditions are satisfied, we call coalition D a deviating sub-coalition.

Agents' profit in Equations 4.17 and 4.18 is calculated in Algorithm 2, lines 5 and 7. If an agent finds a deviating sub-coalition, this sub-coalition will deviate from its current coalition C and form a new coalition, thus increasing the stability of the original coalition. Considering all $2^{|C|-1}$ possible sub-coalitions that an agent can be part of is infeasible, therefore agents use a heuristic to guide their search. Some possible heuristics are adding agents to the sub-coalition in order of increasing and decreasing profit, and in random order. Our experiments showed that most stable coalitions were found using the increasing profit heuristic. We therefore let the agents to form the sub-coalitions by adding other agents in order of increasing profit. An agent keeps adding other agents to the new sub-coalition as

long as the above-mentioned conditions are met. Algorithm 3 shows the deviation process of a single agent. In Algorithm 3, lines 2 to 5 show how an agent considers other members of its coalition for deviation by calculating their profit in a sub-coalition D (line 3). This sub-coalition is extended in each iteration (line 4). Finally, if the resulting sub-coalition D contains other agents, this sub-coalition will deviate from the original coalition C_j (line 7), since all members of D are either indifferent or benefiting by this deviation.

Algorithm 3 Deviation of an agent a_i in coalition C_j : boolean $\text{deviate}(a_i, C_j)$

Input: agents a_i and coalition C_j , such that $a_i \in C_j$.

Output: true iff agent a_i deviated.

```

1:  $D \leftarrow$  singleton coalition with  $a_i$ 
2: for all agents  $a_k$  in  $C_j \setminus a_i$  ordered by heuristic do
3:   if  $p_{D \cup a_k}(a_k) < p_{C_j}(a_k)$  then break end if
4:    $D \leftarrow D \cup a_k$ 
5: end for
6: if  $|D| = 1$  then return false end if
7:  $C_j \leftarrow C_j \setminus D$ 
8: return true

```

Deviation is performed in our model after the agents decide on leaving and joining coalitions. Each iteration of the simulation therefore consists of two steps: social welfare maximization by leaving and joining coalitions (Algorithm 2, lines 3 to 9), and stability maximization by deviation (Algorithm 2, lines 10 to 14). The agents deviate recursively, which means they try to deviate from the new coalition created by their deviation.

4.2.4 Solution Selection

An inherent advantage of using multi-agent simulation for coalition formation is the fact that it creates a pool of solutions by storing all coalition structures encountered during the search. At the end of the simulation, Algorithm 1 selects from this pool a solution that maximizes the gain. We propose to select a solution based on both gain and stability metrics. However, computing stability of a coalition structure is computationally expensive, therefore we use stability_α to estimate the true stability of the solutions.

The computation of stability_α is shown in Algorithms 4 and 5. Algorithm 4 computes

Algorithm 4 Computation of $stability_\alpha$

Input: α_{max} , list of coalition structures $list_{CS}$.

Output: $stability_\alpha$ computed for all coalition structures in $list_{CS}$ for $\alpha = \alpha_{max}$.

```
1: for  $\alpha \in \langle 1, \alpha_{max} \rangle$  do
2:   for all  $CS$  in  $list_{CS}$  do
3:     if  $CS.computationComplete$  then
4:       continue
5:     end if
6:      $\alpha$ -stable( $\alpha, CS, CS.closeList$ )
7:   end for
8:   if  $\forall CS : CS.computationComplete$  then
9:     return
10:  end if
11: end for
```

$stability_\alpha$ of coalition structures generated by multi-agent simulation in an iterative fashion for increasing $\alpha \in \langle 1, \alpha_{max} \rangle$ by calling Algorithm 5 (Algorithm 4, line 6). Algorithm 5 computes $stability_\alpha$ for given α and coalition structure CS by checking whether all permutations of all combinations of agents in each coalition $C_j \in CS$ are dominated by C_j using Equation 4.13 (Algorithm 5, lines 8-15). All permutations must be considered because the order in which agents join the coalitions determines their profit (see Equation 4.12). We only have to determine whether a coalition is α -stable if it is $(\alpha - 1)$ -stable. Algorithm 5 therefore avoids evaluation of coalitions that have already been marked as not stable for lower α (Algorithm 5, lines 4-6). For the same reason Algorithm 4 skips stability computation for any coalition structure for which $stability_\alpha$ of all its coalitions is already determined (Algorithm 4, lines 3-5). Finally, Algorithm 5 counts the number of α -stable coalitions (Algorithm 5, line 18), and it computes $stability_\alpha$ for the given coalition structure (Algorithm 5, line 20).

After $stability_\alpha$ of all coalition structures is computed, a multi-criteria optimization is used to select a best coalition structure based on its gain and $stability_\alpha$. Common approaches of multi-criteria optimization are finding Pareto optimal solutions and designing a fitness function. Pareto optimal solutions do not prefer any of the considered criteria, therefore in our experiments we used a simple fitness function that allows us to give preference to any of

Algorithm 5 Computation of α -stable($\alpha, CS, CS.closeList$)

Input: α , coalition structure CS , list of closed coalitions in CS $CS.closeList$.
Output: $stability_\alpha$ of CS for given α .

- 1: $CS.computationComplete \leftarrow true$
- 2: $stableCount \leftarrow 0$
- 3: **for all** $C_i \in CS$ **do**
- 4: **if** $CS.closeList$ contains C_i **then**
- 5: continue
- 6: **end if**
- 7: **if** $|C_i| \geq \alpha$ **then**
- 8: **for all** $comb \leftarrow$ combinations of agents in C_i of size α **do**
- 9: **for all** $perm \leftarrow$ permutations of $comb$ **do**
- 10: **if** $perm$ dominates C_i **then**
- 11: add C_i to $CS.closeList$
- 12: continue (go to line 3)
- 13: **end if**
- 14: **end for**
- 15: **end for**
- 16: $CS.computationComplete \leftarrow false$
- 17: **end if**
- 18: $stableCount ++$
- 19: **end for**
- 20: $stability_\alpha(CS) \leftarrow stableCount/|CS|$

the criteria:

$$f(CS, \alpha) = w_g \cdot g_{norm}(CS) + w_s \cdot stability_\alpha(CS), \quad (4.19)$$

where $g_{norm}(CS) \in \langle 0, 1 \rangle$ is a normalized gain of CS as defined in Definition 4.2.1, $\alpha \in \langle 1, n \rangle$ is an input parameter that represents the trade-off between quality of solution stability estimate and computation time, and w_g and w_s are weights assigned to the two criteria. Note that given the values of $g_{norm}(CS)$ and $stability_\alpha(CS)$ for each CS , Pareto optimal solutions can also easily be found. Finally, the best coalition structure is returned, such that

$$CS_{best} = \operatorname{argmax}_{CS} f(CS, \alpha), \quad (4.20)$$

where the argmax function returns CS with the highest value of $f(CS, \alpha)$.

The effect of solution selection on the resulting solution depends on the specific coalition formation application, however, Theorem 4.2.1 gives guarantees on the quality of the

resulting solution.

Theorem 4.2.1 (Effect of solution selection). *When used with the multi-agent simulation approach for coalition formation (Algorithm 1), solution selection will never decrease $stability_\alpha$ of the resulting solution.*

Proof. We will use the proof by contradiction. Let Algorithm 1 return solution CS_1 with gain $g(CS_1)$ and stability $stability_{\alpha,1}$, and let Algorithm 2 used without deviation return solution CS_2 with gain $g(CS_2)$ and stability $stability_{\alpha,2}$. Let us assume that $stability_\alpha$ decreased because of solution selection, i.e.

$$stability_{\alpha,1} > stability_{\alpha,2}. \quad (4.21)$$

Since solution selection picks the resulting solution using Equation 4.20, we can imply that regardless of values of w_g and w_s , solution CS_2 could have been chosen only because

$$g(CS_1) < g(CS_2). \quad (4.22)$$

However, Equation 4.22 is in contradiction with the description of Algorithm 1 which states that the coalition structure with the highest gain is returned (see Algorithm 1). This contradiction completes the proof of Theorem 4.2.1. \square

$Stability_\alpha$ is used in our approach both by the solution selection algorithm and as a metric that represents the quality of the solution. We therefore use two values of α , and we specifically denote α_{ss} the value of α used by the solution selection algorithm. Algorithm 4 is also used to evaluate the $stability_\alpha$ of the selected best solution. In that case the list of coalition structures $list_{CS}$ contains only the single best solution, and a value of α which is greater than α_{ss} is used to provide a better stability estimate for the single coalition structure.

Figure 4.10 shows the effect all possible combinations of deviation and solution selection.

- With no deviation and no solution selection, a coalition structure is selected randomly

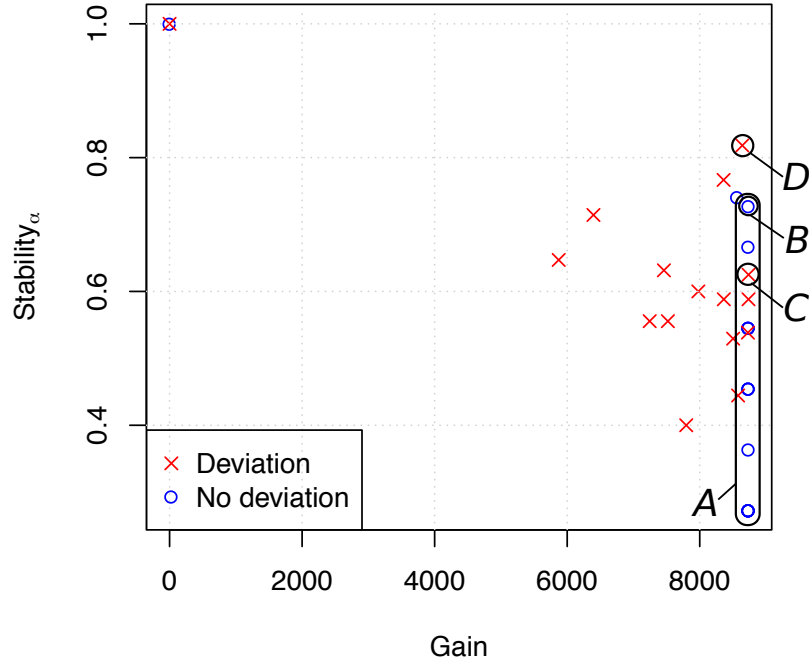


Figure 4.10: *Gain and stability $_{\alpha}$ of coalition structures generated by two simulations (with and without deviation) with 100 agents, 15 iterations, and $\alpha = 4$.*

from the set A , since only the gain is maximized and agents are not allowed to deviate. This setting therefore always returns a solution with highest gain regardless of the coalition stability.

- Solution selection without deviation returns solution B , because that solution achieved the maximum value of $f(CS, \alpha)$ (see Equations 4.19 and 4.20) among solutions in which agents are not allowed to deviate.
- Deviation without solution selection returns solution C , because coalition stability is not considered when selecting the solution, but agents are allowed to deviate.
- Deviation used along with solution selection returns solution D , because both criteria are considered, and the agents are allowed to deviate.

A combination of both of these approaches yields solutions with higher stability while only

sacrificing a small fraction of the gain.

4.2.5 Experimental Analysis

We tested our algorithm in two coalition formation scenarios: collective energy purchasing and resource sharing. These scenarios were described in Section 4.1.3. Because the use of $\kappa = -|C|^\gamma$ (see Section 4.1.4) as a coalition size penalty causes agents to form small coalitions, we define κ using an offset μ as

$$\kappa = \min(-|C| + \mu, 0)^\gamma, \quad (4.23)$$

which effectively allowed us to increase the average coalition size and thus make the problem harder to compute due to its exponential complexity. In our experiments we set $\mu = 10$ to encourage agents to form coalitions of at least that size. Following Section 4.1.6 we set $\gamma = 1.1$ in the collective energy purchasing scenario and $\gamma = 2.0$ in the resource sharing scenario. The effect of Equation 4.23 is depicted in Figure 4.11.

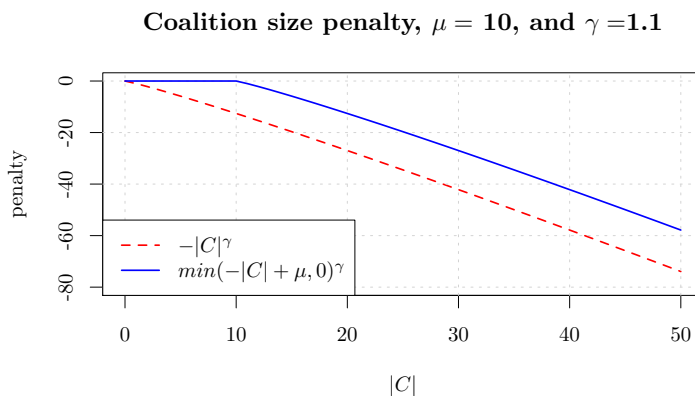


Figure 4.11: Coalition size penalty κ that gives preference to larger coalitions

Several agent strategies were studied in Section 4.1. In our experiments we use the *local search* strategy, in which the agents perform a best response move to new coalitions (i.e. the agents select coalitions which grant them maximal marginal profit). If the search reaches a local optimum for all agents, a random jump is applied by all agents in order to escape this

optimum.

We used two values of α for evaluation of $stability_\alpha$. For the solution selection algorithm, we set $\alpha_{ss} = 3$ to allow the algorithm to quickly compute $stability_\alpha$ of multiple solutions, and for the final stability verification we set $\alpha = 4$ to obtain a better final stability estimate. In order to give equal preference to both gain and stability we set the weights $w_g = w_s = 1$.

In order to achieve reasonable run-times of our algorithm, we used the following number of iterations N in our experiments. For instances with number of agents $n < 100$ we set $N = 100$ and for instances with $n > 100$ we set $N = 10$. We ran our Java implementation of the proposed algorithms on 2.7 GHz Intel Xeon E5 CPU with 2 GB of memory. We averaged our results over 10 random runs. Random runs are necessary because agents make decisions in random order.

4.2.6 Experiment Results

We compared results of our algorithms with the baseline multi-agent simulation algorithm for coalition formation from Section 4.1 using the $stability_\alpha$ (Definition 4.2.3) and *price of stability* (Equation 4.16) metrics. Average values of $stability_\alpha$ and *price of stability* are shown in Table 4.3. The first row of Table 4.3 shows results of the baseline algorithm. The following rows show how the average $stability_\alpha$ increases when we plug in the proposed stability-increasing methods. As expected, the average *price of stability* is increasing with the increase in $stability_\alpha$, but the increase in *price of stability* is very low compared to the significant improvement in $stability_\alpha$. Table 4.3 therefore shows that our algorithms find solutions with much higher stability while only sacrificing a fraction of the social welfare.

Table 4.3: Trade-off between average stability and average price of stability achieved by our algorithms with $\alpha = 4$ and $n \in \langle 20, 5000 \rangle$.

| Algorithm | | Results | |
|-----------|--------------------|----------------------------|---------------|
| Deviation | Solution selection | Average $stability_\alpha$ | Average PoS |
| No | No | 0.3914 | - |
| YES | No | 0.6299 | 1.0308 |
| No | YES | 0.6665 | 1.0210 |
| YES | YES | 0.8185 | 1.0629 |

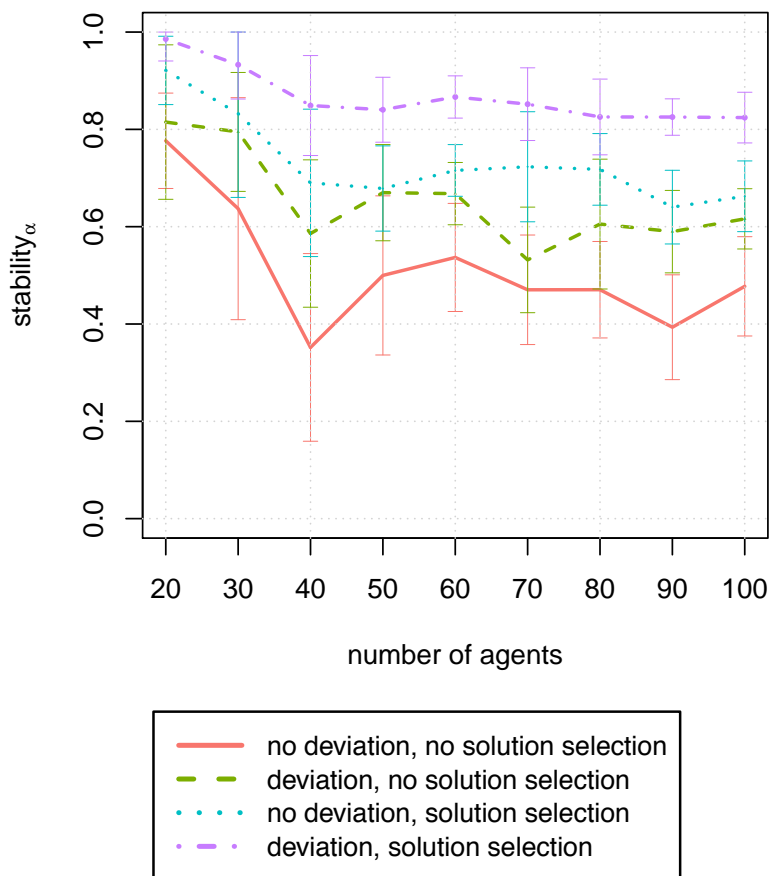


Figure 4.12: Stability achieved by our algorithms and the baseline algorithm in the collective energy purchasing scenario: combination of deviation and solution selection algorithms yields highest stability.

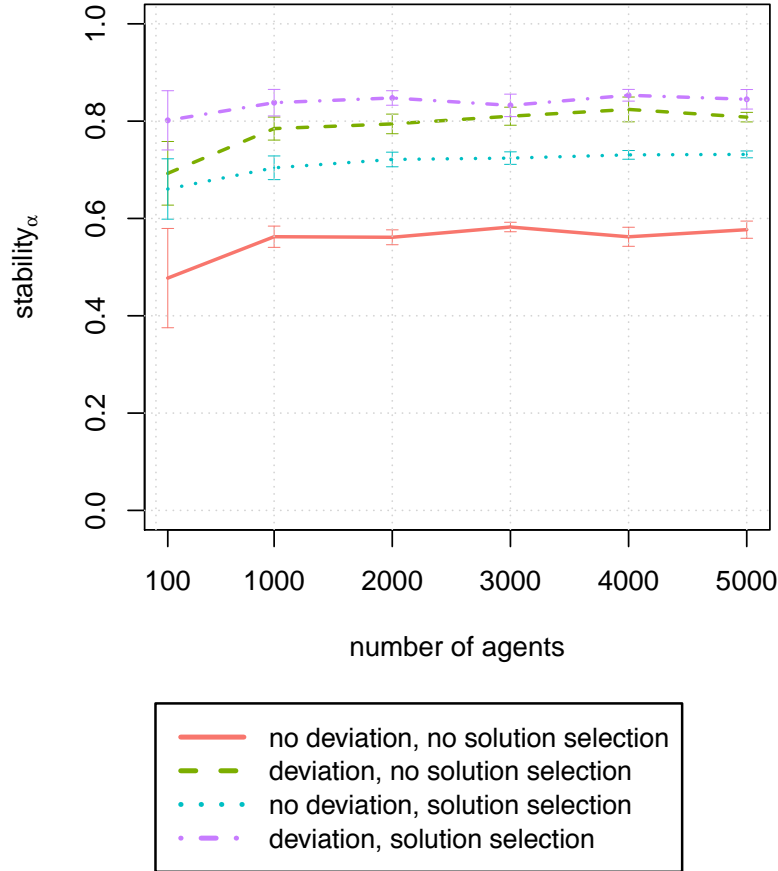


Figure 4.13: *Stability achieved by our algorithms and the baseline algorithm in the collective energy purchasing scenario: combination of deviation and solution selection algorithms yields highest stability.*

Stability of our solutions is depicted in Figures 4.12 and 4.13 in collective energy purchasing scenario and in Figures 4.14 and 4.15 in resource sharing scenario. As stated in Theorem 4.2.1, the use of solution selection algorithm never decreases the stability of the solutions, therefore the solutions generated by the solution selection algorithm always dominate the baseline algorithm with respect to stability. Unfortunately, this dominance is not guaranteed by the deviation algorithm. However, in most instances the deviation algorithm achieves higher $stability_\alpha$ than the baseline algorithm. Finally, the highest increase in $stability_\alpha$ is achieved in majority of instances when both the deviation and the solution selection algorithms are used together. As shown in Table 4.3, the average $stability_\alpha$ in-

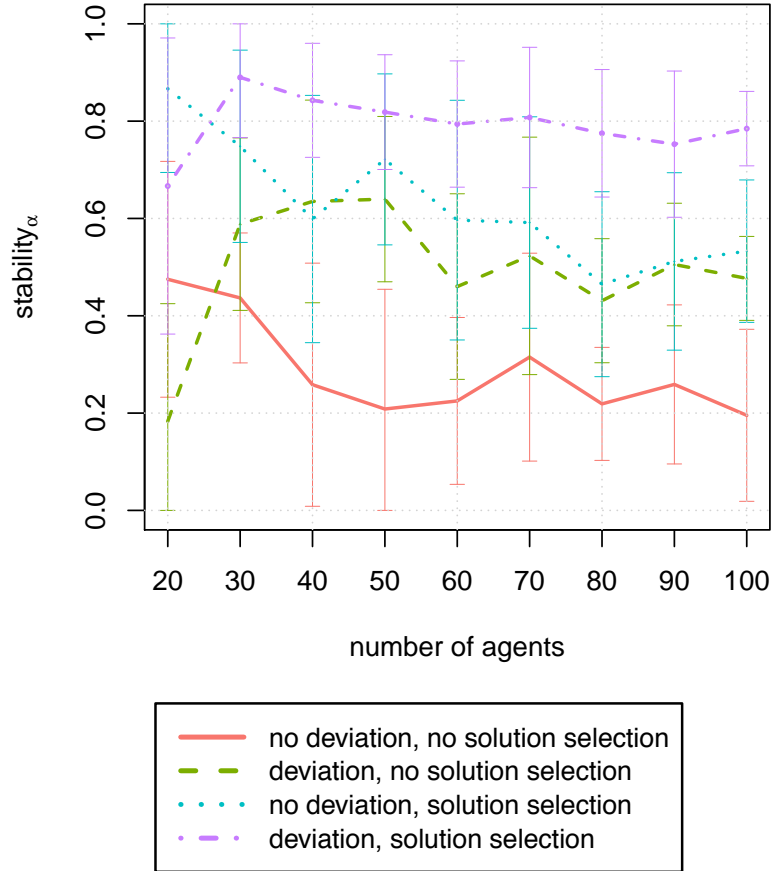


Figure 4.14: *Stability achieved by our algorithms and the baseline algorithm in the resource sharing scenario: combination of deviation and solution selection algorithms yields highest stability.*

creases from 39% achieved by the baseline algorithm to 82% achieved by the combination of deviation and solution selection algorithms.

The solution selection algorithm evaluates $stability_\alpha$ of all coalition structures for given α_{ss} . Figure 4.16 shows $stability_\alpha$ and gain for varying values of α_{ss} , where $\alpha = 5$. As expected, the $stability_\alpha$ of the selected solution is increasing with increasing α_{ss} , since higher α_{ss} provides a better stability estimate. However, due to the inherent trade-off between coalition stability and social welfare, the gain decreases with increasing α_{ss} . Figure 4.16 only shows algorithms that include solution selection and are therefore affected by changing α_{ss} .

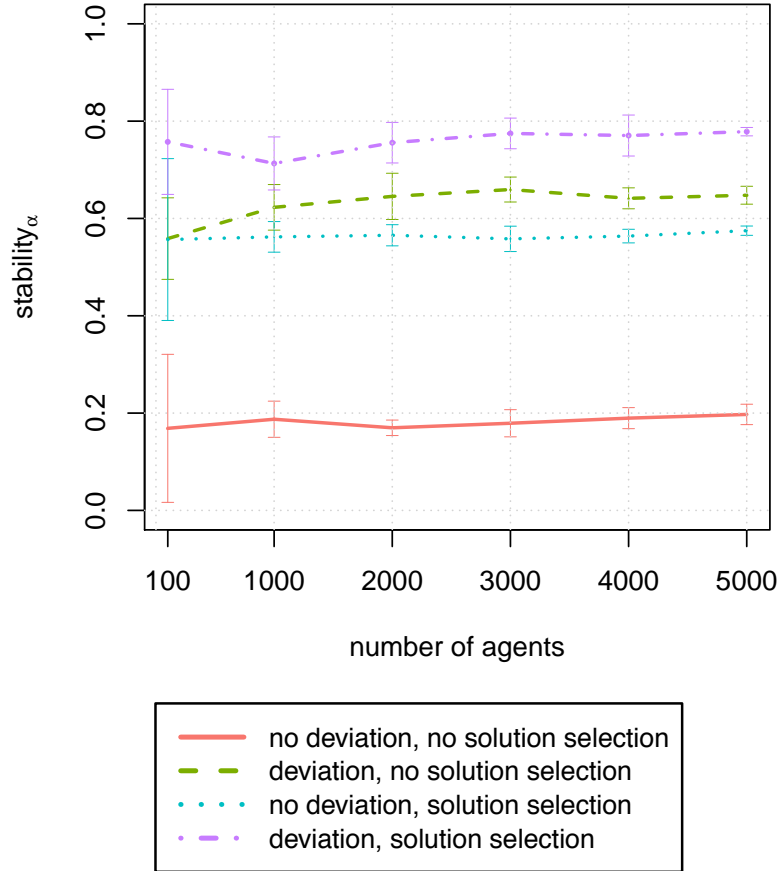


Figure 4.15: *Stability achieved by our algorithms and the baseline algorithm in the resource sharing scenario: combination of deviation and solution selection algorithms yields highest stability.*

The number of iterations N affects the quality of the resulting coalition structure. We show $stability_\alpha$ and gain of our algorithm for varying numbers of iterations N in Figure 4.17. With the increasing number of iterations the agents have more opportunity to cooperate by creating coalitions, which leads to an increase in gain. However, higher social welfare might result in lower stability of the coalitions. This effect is most obvious in the results of the baseline algorithm, in which due to the increase in gain the $stability_\alpha$ drops significantly. However, when we plug in the stability-increasing approaches proposed in Sections 4.2.3 and 4.2.4, the decrease in $stability_\alpha$ is much slower.

In practice the run-time of an algorithm is an important factor. Figure 4.18 shows the

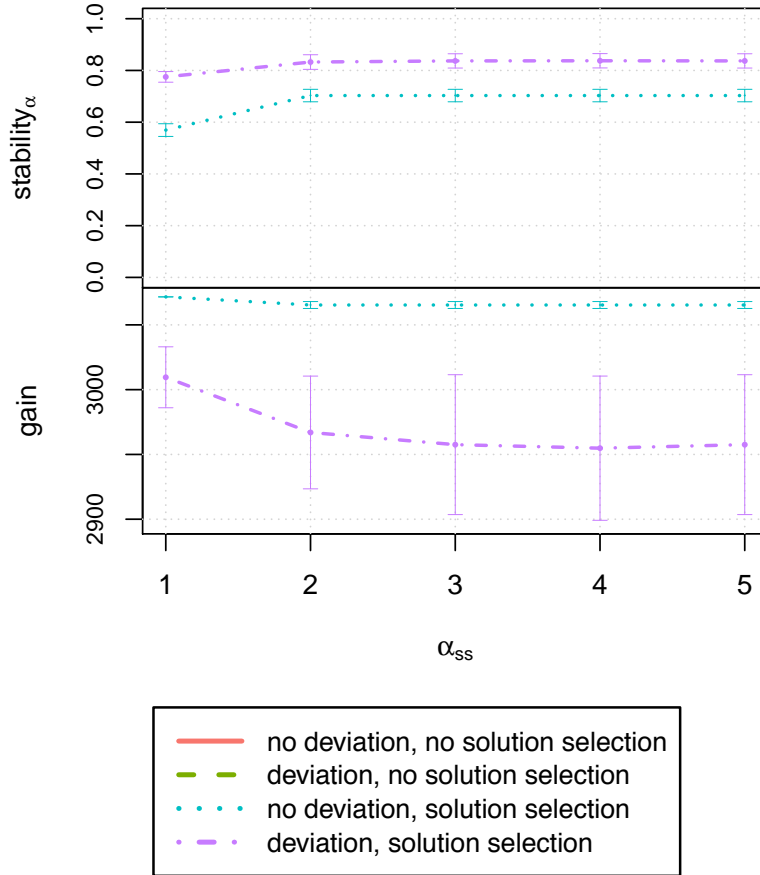


Figure 4.16: *Effect of α_{ss} on stability $_{\alpha}$ and gain in collective energy purchasing scenario with 1000 agents and $\alpha = 5$: higher α_{ss} yields better stability estimate and therefore increases stability of the selected solution.*

run-time of our algorithm for increasing numbers of agents. Interestingly, the run-time does not change significantly when we plug in the proposed stability-increasing algorithms. Deviation of the agents has a higher impact on run-time than solution selection, because it is executed by all agents in each iteration. Run-time of our algorithm can be decreased by decreasing the number of iterations N , however such an approach might yield solutions of lower quality, as shown in Figure 4.17.

We also experimented with the state-of-the-art algorithm for coalition formation C-Link (Farinelli et al., 2013) in order to determine its ability to create stable coalitions. C-Link, like our approach, can also be used with arbitrary valuation functions, and, as we showed in

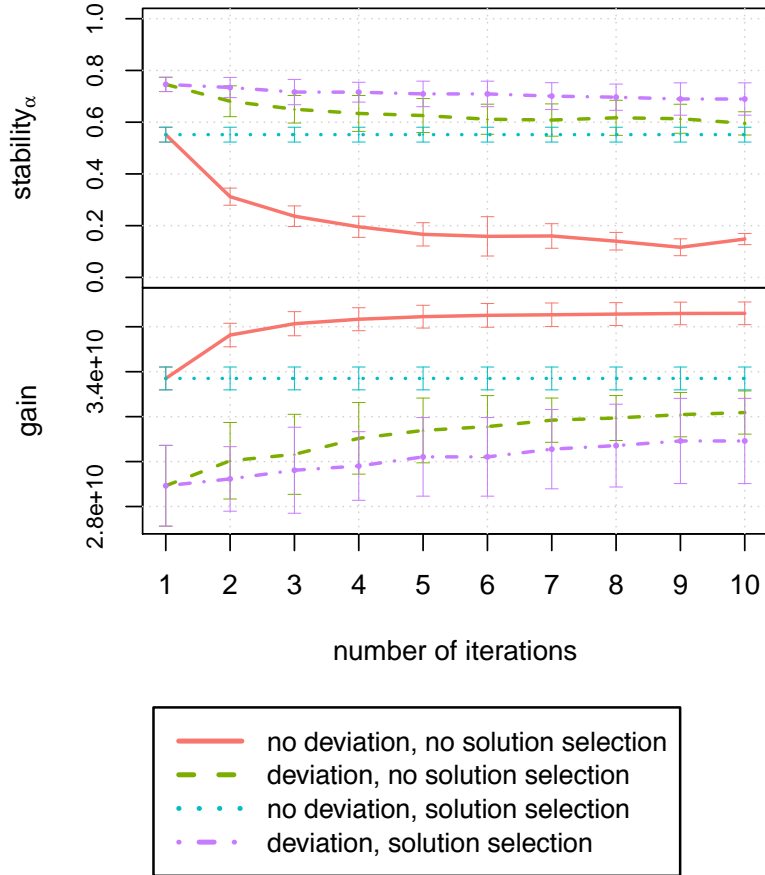


Figure 4.17: *Effect of number of iterations N on stability_α and gain: higher N yields higher gain because the agents have more opportunity to form coalitions, which naturally leads to a decrease in stability. Decrease in stability_α achieved by our algorithms is significantly lower than the decrease achieved by the baseline algorithm.*

Section 4.1, social welfare of its solutions is comparable with results of the baseline algorithm. Even though C-Link was not designed for use with self-interested agents, the algorithm might still inherently create stable coalitions. However, our experiments showed that the only stable coalitions in solutions generated by C-Link in the collective energy purchasing and resource sharing scenarios are singleton coalitions, which by definition in Equation 4.13 are always stable. This result shows that multi-agent simulation, especially along with the stability-increasing methods proposed in Sections 4.2.3 and 4.2.4, is better suitable for coalition formation of self-interested agents than other state-of-the-art coalition formation

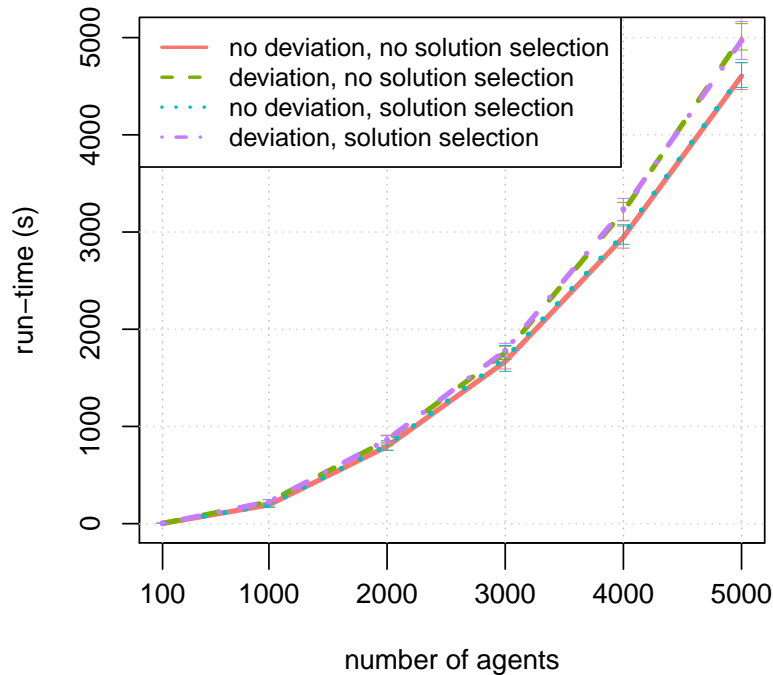


Figure 4.18: Run-time of our algorithms and the baseline algorithm in collective energy purchasing scenario with number of iterations $N = 10$.

algorithms.

4.2.7 Discussion

In this section we discuss some design choices that have to be made when designing a multi-agent system for coalition formation of self-interested agents. We discuss various profit sharing schemes, definitions of stability, and behaviors of self-interested agents. Then we analyze time complexity and convergence of our algorithms. Finally, we discuss practical usefulness of our approach.

Several profit sharing schemes have been proposed in the literature. *Equal sharing* (Pycia, 2012) divides the coalition value equally among all its agents, *fair value sharing* (Augustine et al., 2011) defines agents' payoff as marginal contribution to the coalition, *labor union sharing* (Augustine et al., 2011), which we use in our experiments, defines agents' payoff as

agents' marginal contribution to the coalition at the time of entry, and *Shapley sharing* (Augustine et al., 2011) assigns payoffs based on agents' Shapley values. Among these sharing schemes, labor union is the only one in which consequent additions to the coalition do not affect agent's payoff assigned at the time of coalition entry. In all other sharing schemes agent's profit changes after it joins the coalition, since profit in equal sharing depends on the size of the coalition, fair value sharing uses agents' marginal contribution in the current coalition, and Shapely value considers all orderings of agents in a coalition including agents that joined the coalition after the agent. Therefore labor union sharing does not require additional computation after agent's profit is calculated when the agent joins a coalition. Labor union sharing is the only sharing scheme that models marginal contribution and at the same time is reasonably computationally efficient, which is why we used it in our experiments.

Several concepts have been used to describe coalition stability. Nash equilibrium describes a state in which no agent has an incentive to unilaterally deviate. A stronger concept is a core, which is a set of profit assignments to agents, such that no subset of agents in a coalition has an incentive to jointly deviate from the coalition. Our definition of stability in Equation 4.13 follows the concept of the core. One might also consider a stricter version of the definition, in which all deviating agents must benefit by the deviation, i.e. $\forall a_i \in D : p_D(a_i) > p_C(a_i)$. However, this strict definition of stability yields high stability values for arbitrary coalitions, and therefore renders the problem less interesting.

Our algorithm searches the state-space of coalition structures using three actions of the agents: leave a coalition, join a coalition, and deviate from a coalition. We designed these actions in order to search for coalition structures with high values of both social welfare and coalition stability. However, the space of possible agents' actions is not limited to these actions. For example, agents from multiple coalitions could jointly deviate, agents could decide whether to allow other agents to enter their coalition, agents could force other agents in their coalition to leave, etc. Adding new actions to the agents' action space will lead to new behavior of the multi-agent system. New actions do not extend the state-space of coalition structures, however, actions such as unilateral deviation from multiple coalitions can be computationally expensive, and their addition might therefore significantly increase

the run-time of the algorithm. When designing agents' actions we must take into account the specific problem that is being solved, the effect of the actions on agents' behavior, and the computational complexity of the designed actions.

A major advantage that our approach has over other coalition formation approaches (see Chapter 3 for overview of related work) is its ability to model wide range of agent behaviors. Coalition formation approaches usually seek to optimize social welfare or find stable coalitions. These approaches often assume individual rationality, which means that each agent makes rational decisions in order to maximize its profit or profit of the entire MAS. However, researchers pointed out that the concept of individual rationality is unrealistic when used to model human behavior (Gigerenzer and Selten, 2002). It is therefore important to allow for models of more realistic human behavior. Our approach can easily meet this requirement, because it directly models decision making of individual agents through agents' strategies, and agents' actions through the multi-agent simulation.

Searching the exponential state-space of coalition structures can lead to exponential worst-case time complexity of the search algorithms. However, we show a polynomial time complexity of both deviation and solution selection algorithms with respect to number of agents n and a constant α_{ss} bound. Deviation of a single agent requires sorting the agents in the coalition by marginal profit. The agents can deviate recursively, therefore the complexity of deviation of a single agent is $O(\sum_{i=1}^n (i \cdot \log(i)))$, for which an upper bound is $O(n^2 \log(n))$. Solution selection searches through all permutations of all combinations of size $\langle 1, \alpha_{ss} \rangle$ of agents in a coalition. Evaluating a single coalition therefore requires $O(\sum_{i=1}^{\alpha_{ss}} (i! \cdot \binom{n}{i}))$ steps. For $\alpha_{ss} \ll n$ it holds that

$$\sum_{i=1}^{\alpha_{ss}} \left(i! \cdot \binom{n}{i} \right) \leq \alpha_{ss} \cdot \alpha_{ss}! \cdot \binom{n}{\alpha_{ss}} \leq \alpha_{ss} \cdot n^{\alpha_{ss}}, \quad (4.24)$$

therefore the worst time complexity of finding $stability_{\alpha}$ for a single coalition with $\alpha = \alpha_{ss}$ is $O(n^{\alpha_{ss}})$. The worst-case time complexity of the solution selection algorithm is therefore $O(N \cdot n^{\alpha_{ss}+1})$, given the input of N coalition structures, each containing at most n coalitions. Worst-case time complexity of the baseline coalition formation algorithm is $O(N \cdot n^2)$, as

was shown in Section 4.1.

Table 4.4: *Worst-case time complexity of our algorithms as a function of number of iterations N , number of agents n , and a small constant α_{ss} . In our experiments we set $\alpha_{ss} = 3$.*

| Algorithm | | Worst-case time complexity |
|-----------|--------------------|--|
| Deviation | Solution selection | |
| NO | NO | $O(N \cdot n^2)$ |
| YES | NO | $O(N \cdot n^3 \cdot \log(n))$ |
| NO | YES | $O(N \cdot n^{\max(\alpha_{ss}+1,2)})$ |
| YES | YES | $O(N \cdot n^{\max(\alpha_{ss}+1,3)} \cdot \log(n))$ |

Building on the complexity analysis above, Table 4.4 shows worst-case time complexity of our algorithms. Since we treat α_{ss} as a small constant, we get a polynomial complexity for all proposed algorithms. The analysis in Table 4.4 is very conservative, since we assume that each coalition structure contains n coalitions, each coalition is composed of n agents, and each sub-coalition deviating from coalition C is of size $|C| - 1$. The analysis does not include complexity of the valuation functions, as these are given as an input to the simulation. However, both collective energy purchasing and resource sharing valuation functions require constant time with respect to the number of agents n , and therefore do not affect the complexity analysis. Our algorithm is centralized, therefore we do not assume any additional cost of communication between agents, as would be the case with algorithms that distribute the computation among the agents.

An important aspect of an algorithm is its convergence behavior. The baseline algorithm converges if the random jump is not used (see Figure 4.6). Similarly, if the agents were only allowed to deviate, the simulation would converge from any initial state because each deviation splits coalitions and decreases the average coalition size. Therefore separate maximization of social welfare, as well as separate maximization of coalition stability, is guaranteed to converge. However, combining these steps in order to maximize both metrics does not guarantee convergence, because maximization of social welfare and maximization of coalition stability drive the search towards two different local optima.

Our algorithm can be used in real-world scenarios where coalition stability has to be considered. Unlike the social welfare-maximizing solutions, solutions with high coalition stability respect preferences of individual self-interested agents by considering their profit in coalitions. Solutions with high stability are therefore often more realistic than social welfare-maximizing solutions. Decision making of individual agents might lower the social welfare of the solution, but as we showed in Table 4.3, the *price of stability* of our solutions only slightly increases with major increase in stability. Furthermore, the weights w_g and w_s in Equation 4.19 can be adjusted to give preference to either the social welfare or stability.

4.3 Summary

In Section 4.1 we proposed a general framework for finding suboptimal solutions for a large-scale coalition formation problem containing thousands of agents using a multi-agent simulation. We modeled coalition formation as an iterative process in which agents leave and join coalitions based on the information from the current and previous iterations. We presented example applications of coalition formation: resource sharing and collective energy purchasing, along with valuation functions that model them by assigning values to the coalitions. We discussed coalition selection strategies that the agents can use in their decision making to leave and join coalitions. Finally, we analyzed our algorithms experimentally by comparing performances of the strategies in various problem settings using synthetic and real-world data.

We showed that our algorithms perform almost optimally in small-scale problem instances in which our best strategies performed similarly or better than the state-of-the-art algorithm for coalition formation C-Link. We also showed that the performance of our algorithms is stable in large-scale instances in which comparison with an optimal solution is infeasible. In these large-scale instances the quality of solutions found by our algorithm is greater or equal to the quality of solutions found by C-Link in majority of instances, and in remaining instances our algorithm yields run-time lower than run-time of C-Link by one order of magnitude, while still keeping solution quality similar to the quality of solutions found by

C-Link.

We found that the best performance is achieved by strategies that combine local search with random jumps. Our strategies found solutions with values, on the average, up to 94% of the optimum in small-scale problem instances and maintained a steady gain per agent in large-scale problem instances.

Algorithms that find stable coalition structures are often proposed for settings that restrict the properties of the valuation functions. Practical aspects of the high complexity of finding stable coalitions for large-scale multi-agent systems are often not considered. In Section 4.2 we proposed an approach for increasing coalition stability in large-scale coalition formation with self-interested agents and arbitrary valuation functions. We modeled agent behavior using multi-agent simulation, in which we let agents to choose profitable coalitions and deviate from unstable coalitions. At the end of the simulation, we selected a solution out of a pool of generated coalition structures based on its social welfare and stability. We experimentally showed that our approach is able to increase the stability of the solutions in two real-world scenarios. We also showed that the necessary price for this increase in stability that our algorithm incurs to the social welfare is very low.

Chapter 5

Power Distribution Systems

In this chapter we propose an approach which uses large-scale coalition formation algorithms from Chapter 4 to increase use of renewable sources in PDS by letting renewable generators and energy stores to form coalitions in order to mitigate uncertainty in prediction of renewable generation amounts. Section 5.1 shows our approach in a setting with no physical constraints of the underlying power grid. Section 5.2 then studies the effects of this approach in a PDS with physical constraints represented by an underlying power grid. Finally, Section 5.2 also shows how coalition formation of renewable generators and energy stores can improve the physical state of PDS.

5.1 Increasing Use of Renewable Resources

Solar and wind power provide a source of renewable energy, which can be used for electricity generation at costs comparable with traditional generators that utilize coal, natural gas, or nuclear energy. However, renewable sources are not used to their full potential for electricity generation. One of main reasons for the low use of these renewable resources is the unpredictable amount of generation related to dependence on weather ([Holttinen et al., 2009](#)).

In this section we focus on the weather-related unpredictability of the generation by

renewable resources. Electricity is traded ahead of time in a day-ahead energy market. In this market generators bid amounts of energy that they will be able to generate at given time slots. Since in the electricity market the supply must always match the demand, a failure to deliver the committed amount of energy can have major negative consequences, and it is therefore penalized. Consequently, the renewable generators, due to weather unpredictability, are forced to bid amounts that are lower than the predicted generation, thus decreasing the use of renewable resources (Holttinen et al., 2009).

We propose an approach that can be used to increase the use of renewable resources by forming coalitions between renewable generators and a high number of energy stores. We assume energy stores to be of any kind. For example, energy stores are often implemented as batteries, capacitors, solar batteries, or mechanical flywheels, and they usually store energy in a range of kilowatt hours to megawatt hours. Furthermore, traditional generators can also be considered as energy stores since their generation is predictable. We are not concerned with the type of energy stores, however we require them to be able to provide energy at a time that they commit to. This capability can be viewed as a commodity for which there is demand among the renewable generators. A renewable generator can buy coverage of some portion of the generation that it commits to. If the renewable generator is able to produce the entire amount of committed energy, it only pays the store owners for the coverage it ordered. On the other hand, if the renewable generator is not able to generate the committed energy, it can use part of the ordered coverage in order to avoid fees for failure to provide energy. A store owner is paid for the uncertainty coverage as well as for the amount of energy provided to the grid.

5.1.1 Model

We model the renewable generators and energy store owners as agents in a multi-agent system. We describe all variables used in our model in Table 5.1. Renewable generators are modeled using a triplet $(g_c(t), g_r(t), u_r)$ for time t corresponding to the time slots. This triplet represents the generation committed and actually generated by the renewable generator, and

a coverage uncertainty that the generator is willing to pay for. The energy store owners are modeled as a triplet (s_b, s_e, m) which represents the beginning and end of a time interval within which the store can provide power, and the maximum total amount of provided power.

Table 5.1: *Description of model variables*

| | |
|-------------|--|
| $p_r(t)$ | profit of a renewable generator r that participates in coalition formation |
| $p_{r0}(t)$ | profit of a renewable generator r that does not participate in coalition formation |
| $g_c(t)$ | generation committed by a renewable generator |
| $g_e(t)$ | estimated generation of a renewable generator |
| $g_r(t)$ | real generation achievable by a renewable generator, not observable |
| $g_{sc}(t)$ | generation committed by an energy store |
| $g_{sr}(t)$ | real generation provided by an energy store |
| $u_r[\%]$ | percentage of g_c to be requested as coverage for uncertainty |
| $u(t)[\%]$ | percentage of $g_c(t)$ granted by energy stores as coverage for uncertainty |
| m | maximum total amount of power to be distributed by an energy store |
| s_b, s_e | energy store availability begin, end |
| p_g | price for generation |
| p_c | price for uncertainty coverage |
| p_f | price for failure to provide committed generation |
| $c_c(t)$ | cost of uncertainty coverage |
| $c_f(t)$ | cost of failure to provide committed generation |
| $c_0[\%]$ | commitment of a renewable generator that does not participate in coalition formation |
| t | time |

5.1.2 Renewable Generators

In order to increase the use of renewable resources, an incentive has to be given to renewable generators to bid higher energy amounts in the day-ahead market. In this section we derive this incentive in a form of a profit function. This profit function should reflect the fact that the renewable generator is paid for its generation, which is assumed to be the minimum of committed and real generation. However, the generator has to pay for uncertainty coverage, which is provided to the generator by energy stores. The generator also has to pay for failure to provide committed generation in case it fails to deliver all the committed energy.

Formally, the profit function is expressed as

$$p_r = \sum_t p_r(t) = \sum_t (p_g \cdot \min(g_c(t), g_r(t)) - c_c(t) - c_f(t)), \quad (5.1)$$

This profit function assumes that $\forall t : g_c(t) = g_e(t)$, i.e. that the renewable generator will always commit to generate the estimated amount of energy.

The cost of uncertainty coverage $c_c(t)$ is a product of price for uncertainty, percentage of committed generation, and committed generation, as follows:

$$c_c(t) = p_c \cdot u(t) \cdot g_c(t). \quad (5.2)$$

Note that the uncertainty cost is independent of the real generation $g_r(t)$, since the uncertainty coverage is paid for before the value of $g_r(t)$ is known. The cost of uncertainty coverage $c_c(t)$ is calculated using the percentage of the coverage actually provided $u(t)$, for which it holds that $\forall t : u(t) \leq u_r$, since the store owners can commit less than the value requested by a renewable generator.

The cost of failure to provide committed generation $c_f(t)$ is determined using a difference between amounts of generation committed and actually provided. The amount of generation that is provided by a renewable generator can be expressed as $g_r(t) + u(t) \cdot g_c(t)$. The generator has to pay the cost of failure to provide committed generation only if the amount of committed generation is greater than the amount of generation provided by the generator. Therefore, the cost of failure to provide committed generation is expressed as

$$c_f(t) = p_f \cdot \max(g_c(t) - (g_r(t) + u(t) \cdot g_c(t)), 0). \quad (5.3)$$

This cost is eliminated at time t if and only if

$$g_c(t) - (g_r(t) + u(t) \cdot g_c(t)) \leq 0, \quad (5.4)$$

which is equivalent to

$$u(t) \geq 1 - \frac{g_r(t)}{g_c(t)}. \quad (5.5)$$

For example, if the real generation $g_r(t)$ reached 80% of the committed amount $g_c(t)$, the cost of failure to provide generation would be zero if u_r was set to at least 20% of the estimated generation and the generator was able to purchase uncertainty coverage $u(t) = u_r$ at time slot t .

Combining Equations 5.1, 5.2, and 5.3, we can express the profit of a renewable generator in the coalition formation setting as

$$p_r = \sum_t p_r(t) = \sum_t \left(p_g \cdot \min(g_c(t), g_r(t)) - p_c \cdot u(t) \cdot g_c(t) - p_f \cdot \max(g_c(t) - (g_r(t) + u(t) \cdot g_c(t)), 0) \right). \quad (5.6)$$

Without coalition formation the renewable generator is forced to bid a lower amount in order to prevent paying the cost for failure to provide committed energy, consequently generating less power and receiving lower profit for generation. On the other hand, the uncertainty coverage cost does not apply to this renewable generator, thus removing a negative portion of the profit function in Equation 5.1 related to the cost of uncertainty coverage. This renewable generator will be able to generate either $g_r(t)$ or $c_0 \cdot g_e(t)$, whichever is smaller. The generator will have to pay cost for failure to provide committed generation, which is proportional to the difference between committed and real generation. The profit of a renewable generator that does not participate in coalition formation with energy store owners is therefore

$$p_{r0} = \sum_t p_{r0}(t) = \sum_t \left(p_g \cdot \min(c_0 \cdot g_e(t), g_r(t)) - p_f \cdot \max(c_0 \cdot g_e(t) - g_r(t), 0) \right). \quad (5.7)$$

As long as $p_r > p_{r0}$, the renewable generator is given an incentive to participate in the coalition formation with energy store owners and bid higher amounts of generation in the day-ahead market. In Section 5.1.5 we experimentally compare the profit of renewable generators that participate in coalition formation with energy store owners, and the profit

of renewable generators that do not participate in coalition formation, and we show that the former achieve higher profits and higher generation.

5.1.3 Energy Stores

In order to simplify the problem we assume that energy stores are always able to sell their stored energy, either to renewable generators inside the coalitions, or to some other party. We also assume that stores are always able to provide the committed amounts of energy. This assumption eliminates factors such as different types of stores, or whether stores buy the power from other subjects. Consequently, our approach works with any kind of energy stores, as long as they are able to provide committed amounts of power at the specified time slots.

The profit function of an energy store owner consists of the payment for provided generation $p_g \cdot g_{sr}(t)$ and payment for committed generation $p_c \cdot g_{sc}(t)$,

$$p = \sum_{t=s_b}^{s_e} (p_g \cdot g_{sr}(t) + p_c \cdot g_{sc}(t)). \quad (5.8)$$

This profit function must satisfy the following constraints:

$$0 \leq \sum_{t=s_b}^{s_e} g_{sc}(t) \leq m \wedge \forall t : g_{sr}(t) \leq g_{sc}(t), \quad (5.9)$$

which limits the total amounts of energy committed and provided by the store. An energy store is incentivized to participate in coalition formation with renewable generators if $\sum_{t=s_b}^{s_e} p_c \cdot g_{sc}(t)$ is positive, which happens when a renewable generator purchases the uncertainty coverage from this store.

When an energy store joins a coalition with a renewable generator, it distributes its power to available time slots using Algorithm 6, which works as follows. First, available amounts of energy are computed for each time slot based on requirements of the renewable generator and amounts of energy already committed by other energy stores (lines 1-8). Then the power is

distributed to available time slots (lines 10-13). These steps are repeated as long as there is some power distributed to any time slot. Finally, the newly committed amount of energy is used to increase profit of the energy store (line 17) and uncertainty coverage of the renewable generator (line 18).

Algorithm 6 Distribution of generation commitment from an energy store to a coalition with a renewable generator

Input: energy store: s_b, s_e, m ;
renewable generator: u_r, g_e , amount of power already committed by stores:
 $\forall t : \sum \hat{g}_{sc}(t)$
Output: energy store: profit; renewable generator: updated uncertainty coverage

- 1: $\forall t : \text{available}(t) \leftarrow u_r \cdot g_e(t) - \sum \hat{g}_{sc}(t); g_{sc}(t) = 0$
- 2: $\text{toDistribute} \leftarrow m$
- 3: **while** changed **do**
- 4: changed $\leftarrow false$
- 5: available spots \leftarrow find spots where $\text{available}(t) > 0$
- 6: split $\leftarrow \text{toDistribute} / |\text{available spots}|$
- 7: **for** time slot = s_b to s_e **do**
- 8: add $\leftarrow \min(\text{available}(\text{time slot}), \text{split})$
- 9: **if** add > 0 **then**
- 10: available(time slot) $-=$ add
- 11: toDistribute $-=$ add
- 12: $g_{sc}(\text{time slot}) +=$ add
- 13: changed $\leftarrow true$
- 14: **end if**
- 15: **end for**
- 16: **end while**
- 17: $\forall t : \text{increase energy store profit by } p_c \cdot g_{sc}(t)$
- 18: $\forall t : \text{increase uncertainty coverage by } g_{sc}(t)$

Since the distribution of power in Algorithm 6 is performed frequently in our approach, it is important to analyze its time complexity. Algorithm 6 is performed in an iterative fashion. In each iteration except the last one at least one time slot is completely filled. In the last iteration the entire available amount is used. Therefore the number of iterations is limited to $s_e - s_b$ (lines 3-16). In each iteration $s_e - s_b$ time slots are traversed, and for each time slot operations of constant time complexity are performed. Therefore, Algorithm 6 performs $O((s_e - s_b)^2)$ operations. Given that in a day-ahead market with hourly time slots it holds that $s_e - s_b \leq 24$, Algorithm 6 has a constant time complexity.

5.1.4 Coalition Formation using Multi-agent Simulation

To simulate coalition formation of renewable generators and store owners we use the multi-agent simulation approach proposed in Section 4.1. There a simulator is proposed in which agents leave and join coalitions in an iterative manner. This coalition formation process is used to search the state space of coalition structures, which are sets of coalitions, in order to find coalition structures with high social welfare. During the simulation agents use strategies to decide whether to leave their coalitions and which coalition to join. The strategy that achieves the best results combines agent's best response, in which an agent searches for a coalition to which it can bring most benefit, with random jumps whenever a local optimum is reached. In this work we base the agents' strategy on that strategy. The coalitions are assigned values by a specified valuation function. Quality of a solution is represented in Section 4.1 by a social welfare, which is a sum of values of all coalitions in a coalition structure. The simulation generates a pool of coalition structures, from which a coalition structure with the highest social welfare is selected as the solution.

There are several differences between the approach in Section 4.1 and coalition formation of renewable generators with energy store owners. First, unlike in Section 4.1, coalition formation between renewable generators and energy store owners is not concerned with social welfare. Both renewable generators and energy store owners are self-interested agents, which seek only to maximize their own profit. This difference can be implemented by changing agents' strategies to find best fitting coalitions based on agents' profit instead of coalition value, which corresponds to using a valuation function which penalizes difference between amounts of power requested by the renewable generator, and the amounts of power provided by the energy stores. We call this valuation function *renewable energy valuation function*, and we define it as follows:

Definition 5.1.1 (Renewable energy valuation function). *Renewable energy valuation function is a valuation function that assigns value v to a coalition C as follows:*

$$v(C) = - \sum_{t_k=1}^T \max((u_r - u(t_k)) \cdot g_e(t_k), 0). \quad (5.10)$$

Second, each coalition in this setting must always contain exactly one renewable generator. In terms of coalition formation we call these agents coalition leaders. Coalition leaders do not leave or join coalitions. On the other hand coalition leaders affect the profit of agents joining the coalitions, which consequently affects the behavior of the other agents. In our scenario, coalition leaders define the requested uncertainty coverage for each time slot. Regular agents then join coalitions in which they can sell their power according to the requested uncertainty coverage.

Finally, since the iterative coalition formation process yields a pool of coalition structures, a single coalition structure must be selected as the solution. Unlike in Section 4.1, where the solution is selected based on social welfare, in our setting we select a solution with the highest profit of renewable generators gained from participation in coalition formation. Formally, we return the coalition structure for which it holds that

$$CS^* = \operatorname{argmax}_{CS \in \mathcal{S}} \left(\sum_{r \in R} \sum_t p_r(t) \right), \quad (5.11)$$

where \mathcal{S} is a set of all coalition structures generated by the coalition formation simulator, R is a set of all renewable generators, and $p_r(t)$ is a profit of renewable generator r in coalition $C \in \mathcal{CS}$.

The simulator for coalition formation of renewable generators and energy store owners is shown in Algorithm 7, and it works as follows. First, renewable generators are created and assigned estimated generation values for each time slot. Then energy stores are created and assigned the beginning and end of availability s_b and s_e , and maximum total amount of power m (line 1). Then we create coalitions, each containing one renewable generator (line 2). After this initialization step the simulation begins. In the simulation energy stores are deciding whether they should leave their coalitions and which coalitions they should join. When an energy store joins a coalition, its profit is increased, and the coalition is updated (lines 5-8), according to Algorithm 6. If an energy store leaves a coalition, profit of all stores that joined the coalition after this store is recalculated, since the distribution of their power might have changed after the removal (line 6). Finally, we select the best coalition structure

according to Equation 5.11 (line 13).

Algorithm 7 Simulation of coalition formation between renewable generators and energy stores

Input: number of energy stores, number of renewable generators, number of iterations N , number of time slots.
Output: coalition structure according to Equation 5.11.

- 1: initiate energy stores and renewable generators
- 2: create a coalition for each renewable generator
- 3: **for** iteration in $1 : N$ **do**
- 4: **for all** energy stores in random order **do**
- 5: **if** *energy_store.strategy.leave* **then**
- 6: *energy_store.coalition.recompute* profit of energy stores after *energy_store*
- 7: *energy_store.coalition* \leftarrow *energy_store.strategy.pick_coalition*
- 8: *energy_store.profit* \leftarrow Algorithm 6
- 9: **end if**
- 10: **end for**
- 11: save current coalition structure
- 12: **end for**
- 13: choose best coalition structure according to Equation 5.11

The time complexity of the original algorithm for n agents and N iterations is $O(N \cdot n^2 \cdot k)$, where k is a size of a so called interest vector associated with each agent (see Section 4.1.2). In our setting an interest vector of a renewable generator contains values $u_r \cdot g_c(t)$, where $t \in \langle 0, 24 \rangle$ in the day-ahead market with hourly slots, and interest vectors of energy stores contain values of s_b , s_e , and m . Size of interest vectors k is therefore in our setting a small constant. Consequently, the time complexity of Algorithm 7 is $O(N \cdot n^2)$.

5.1.5 Experimental Analysis

We experimentally evaluate the coalition formation algorithm in the renewable domain in order to show that our approach creates profit incentives for renewable generators to participate in coalition formation. We will also show that our coalition formation approach increases use of renewable resources.

The experiments were performed using the following setup. We tested our approach in scenarios with 50 renewable generators, which means that the coalition structure contained 50 coalitions. The number of coalitions affects the quality of solutions indirectly through its

ratio compared to the number of energy stores. This ratio shows how many energy stores on average are members of a single coalition with a renewable generator. We therefore vary the number of energy stores in order to observe the effect of this ratio on the resulting coalition structure. Generation amounts for 24 time slots corresponding to 24 hourly slots in one day were generated for each renewable generator at random from a uniform distribution $\mathcal{U}(0, 100)$ representing percentage, and the generators were given estimates of these amounts drawn from a normal distribution with standard deviation $\sigma = 20$ because according to (Zhang et al., 2015a) a day-ahead prediction error of wind and solar generators is around 20% to 25%. Parameters of the stores s_b , s_e , and m were generated randomly from uniform distributions $\mathcal{U}(0, 23)$ (corresponding to 24 hourly time slots) and $\mathcal{U}(0, 100)$ (corresponding to percentage) respectively. Note that this setting assumes that renewable generators as well as energy stores are of the same type with respect to generation amounts. Unless stated otherwise, we set price for generation $p_g = 50$, price for uncertainty coverage $p_c = 10$, and price for failure to provide committed generation $p_f = 100$. These prices were set so that price for uncertainty coverage is lower than price for generation, which in turn is lower than price for failure to provide committed generation. Figure 5.3 further shows results of a case where price for failure to provide committed generation is lower than price for generation. As a baseline we use a scenario in which renewable generators do not participate in coalition formation, in which case renewable generators only bid commitment percentage $c_0 = 80\%$ of predicted generation $g_e(t)$ to hedge against uncertainty. This value of c_0 is based on the 20% to 25% estimated day-ahead prediction error (Zhang et al., 2015a). We let the simulation run for 10 iterations and then selected a coalition structure with highest sum of profit of renewable generators. All results were averaged over 10 random runs.

Figures 5.1, 5.2, 5.3, and 5.4 compare profit of renewable generators that participate in coalition formation p_r with their profit in case they did not participate p_{r0} . Recall that profit of renewable generators participating in coalition formation consists of generation profit, cost of uncertainty coverage $c_c(t)$ and cost of failure to provide committed generation $c_f(t)$. Figure 5.1 shows summarized profit of 50 renewable generators in scenarios with 100 to 1000 energy stores. Generation profit is constant, since it reflects the generation

of the renewable generator and is therefore independent of the number of energy stores. On the other hand, cost of uncertainty coverage and cost of failure to provide committed generation depend significantly on the number of energy stores. With increasing number of energy stores the cost of failure to provide committed generation decreases and the cost of uncertainty coverage increases because the generators can cover more uncertainty. Since the decrease in the cost of failure to provide committed generation is larger than the increase in the cost of uncertainty coverage, the resulting profit is increasing. In scenarios with over 300 energy stores the profit of 50 renewable generators is greater if they participate in coalition formation.

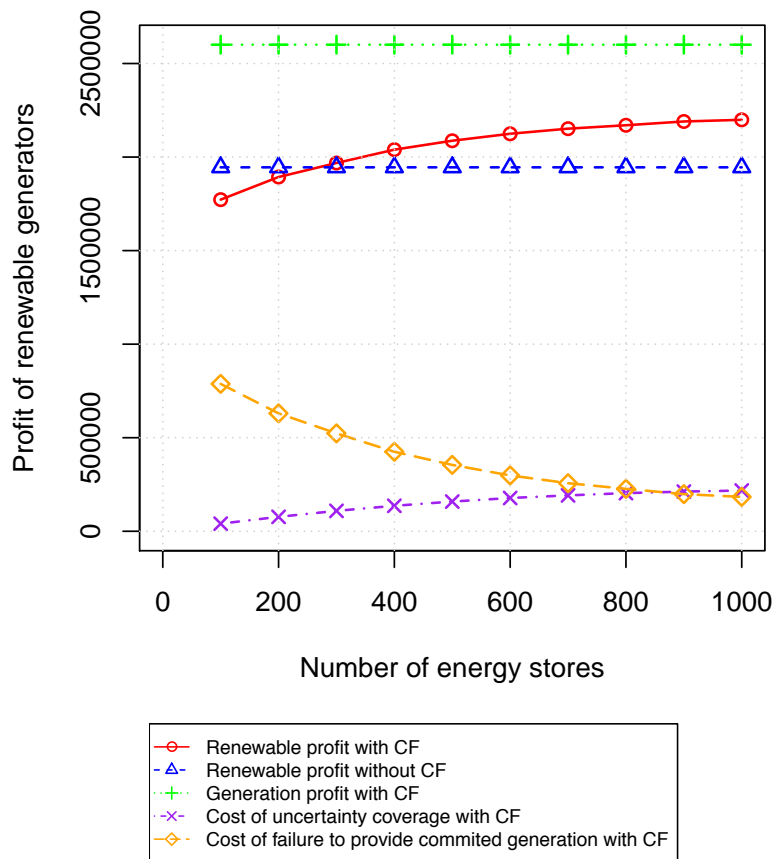


Figure 5.1: Summarized profit of 50 renewable generators with 100 to 1000 energy stores.

The renewable generators have to decide what percentage of their generation will be cov-

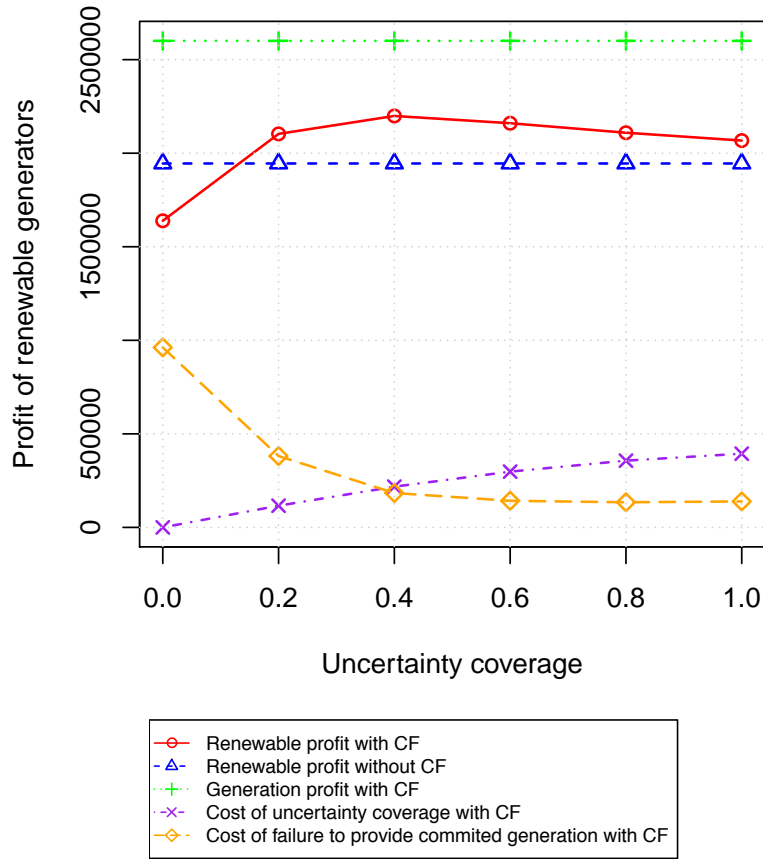


Figure 5.2: Summarized profit of 50 renewable generators with 1000 energy stores and varying uncertainty coverage u_r .

ered by uncertainty coverage. Figure 5.2 shows summarized profit of 50 renewable generators with 1000 energy stores for various uncertainty coverage percentages u_r . For $u_r \geq 20\%$ the profit of renewable generators is greater when participating in coalition formation. Given our experimental setting, the highest profit was achieved when approximately 40% of the renewable generation was covered by uncertainty coverage.

Price for generation p_g , price for uncertainty coverage p_c , and price for failure to provide committed generation p_f affect the resulting profit of renewable generators. Figure 5.3 compares profit of renewable generators with and without coalition formation when the price for failure to provide committed generation is set lower than price for generation, specifically $p_f = 30$. This change brings higher profit to renewable generators not participating in

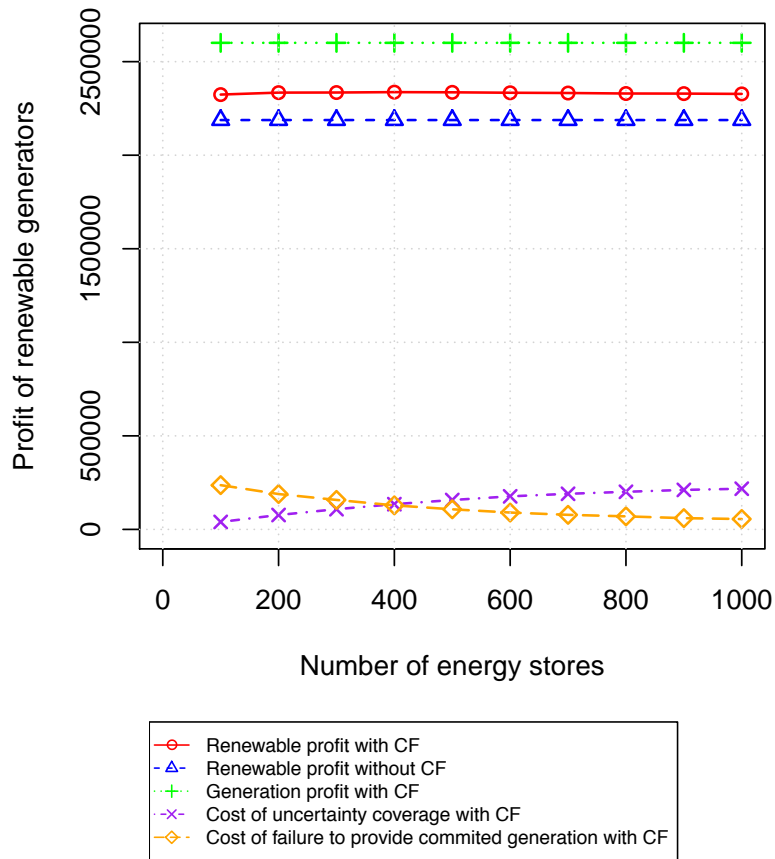


Figure 5.3: Summarized profit of 50 renewable generators with 100 to 1000 energy stores and low cost for failure to provide committed energy, $p_f = 30$.

coalition formation. Even though the increase in profit of generators participating in coalition formation is lower than in Figure 5.1, it will still incentivize the renewable generators to participate in coalition formation.

The commitment percentage of renewable generators not participating in coalition formation c_0 affects the profit of these renewable generators. In Figure 5.4 we compare profit of renewable generators with and without coalition formation with varying commitment percentage c_0 . While higher commitment yields higher generation profit, the cost of failure to provide committed generation also increases, which results in a decrease in the overall profit. Therefore the profit achieved by renewable generators not participating in coalition formation is lower than profit of renewable generators that utilize coalition formation, regardless

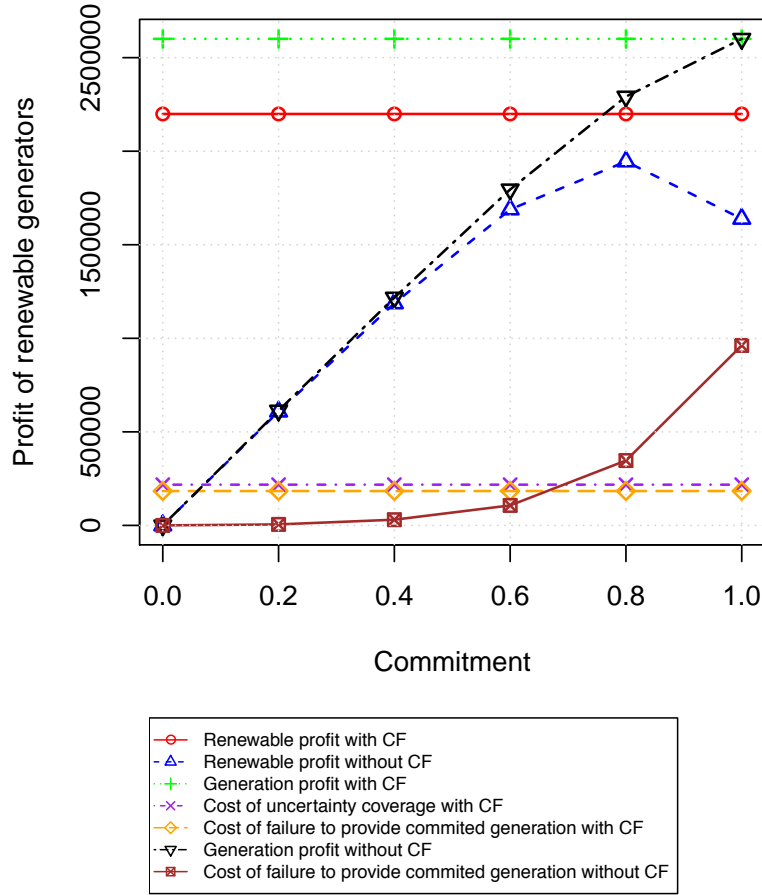


Figure 5.4: Summarized profit of 50 renewable generators with 1000 energy stores and varying commitment of generators that do not participate in coalition formation c_0 .

of the commitment percentage.

Over all, the amount of renewable generation is increased because the generators are able to bid higher values. In Figures 5.1, 5.2, and 5.3 the generators not participating in coalition formation only bid 80% of the predicted generation $g_e(t)$ (we vary this percentage in Figure 5.4), while generators utilizing coalition formation bid 100% of $g_e(t)$. We simulate bidding 80% of predicted generation because according to (Zhang et al., 2015a) a day-ahead prediction error of a wind farm can be up to 25%, and somewhat less for solar generation. Table 5.2 shows amounts of renewable generation with and without coalition formation as well as energy store use and total generation. Our approach yields a 13.5% increase in total

renewable generation from 45,827 units to 52,012 units. Energy stores are used to produce 7,780 units, which amounts to 13% of the total generation. Total generation of renewable generators and energy stores is increased by 30.4% due to coalition formation.

Table 5.2: *Increase in renewable generation caused by coalition formation (CF) of 50 renewable generators and 1000 energy stores*

| Generation type | Generation amount | Increase in generation |
|---------------------------------|-------------------|------------------------|
| Renewable generation without CF | 45,837 | – |
| Renewable generation with CF | 52,012 | 13.5% |
| Energy store generation with CF | 7,780 | – |
| Total generation with CF | 59,792 | 30.4% |

5.1.6 Discussion

Our approach can be used to simulate a possible real-world energy market scenario. In a real world scenario energy stores would also choose coalitions with renewable generators in an iterative fashion. However, the solution of the real-world scenario, in which one iteration of coalition formation would take place every day, would be the last coalition structure created (because unlike in the multi-agent simulation, we are not able to backtrack in time in the real world). In our experiments we select the coalition structure with best profit for renewable generators according to Equation 5.11. This coalition structure represents the best solution from the point of view of renewable generators. In order to verify that our algorithm simulates behavior of potential real-world energy stores, we performed experiments in which we selected the coalition structure created in the last iteration. Surprisingly, the results are almost equivalent to results in Figure 5.1. Our algorithm therefore yields comparable results in both renewable generator-oriented and energy store-oriented setting.

Another application of our approach is to show that coalition formation between renewable generators and energy stores increases use of renewable resources, and it yields higher profits for both participating parties. It can further be used to determine the specific conditions under which coalition formation increases profits, such as number of energy stores,

amount of energy available per store, quality of renewable generation estimates etc.

The coalition formation algorithm can be extended to include more real-world characteristics, such as influence of physical distance between energy stores and renewable generators, energy grid limits, real-time uncertainty coverage purchasing during the day based on hour-ahead forecast, or wider action space of energy stores, which could include actions like purchasing energy or charging the store.

We made a design choice to use multi-agent simulation for coalition formation. However, coalition structures can be found by various techniques including clustering, dynamic programming, graph-based approaches, auction-based approaches, and multi-agent simulation. We used multi-agent simulation because it can model behavior of self-interested agents. Simulation also provides a per-agent interface for setting specific real-world parameters of the renewable generators and energy stores. Finally, multi-agent simulation shows a dynamic coalition formation process, which is beneficial when studying how parameters of single agents influence the behavior of the entire multi-agent system.

In this work we made several assumptions about the energy market domain. First, we assumed that energy stores are always able to provide committed energy, and that they are always able to sell their energy elsewhere in case it is not needed by a renewable generator. Removing the first assumption would extend the problem by introducing penalties for energy stores. Strategies would then have to be developed in order for the energy stores to decide which actions to take to avoid the penalties for failure to provide committed generation. The second assumption is justifiable since the energy stores can use the real-time market and sell the energy based on real-time prices. However, the real-time market is undesirable for the energy stores because its prices are very volatile. Second, we assumed that all renewable generators are of a same type, since their generation amounts are drawn from the same distribution. In a real-world scenario various photo-voltaic systems and wind turbines will have a wide generation range. This difference however does not change the main experimental results, since higher generation only changes the number of energy stores required to cover uncertainty. Third, we assumed a profit function of a renewable generator consisting of generation profit, cost of uncertainty coverage, and cost of failure to provide

committed generation (see Equation 5.1). However, the profit function of a real renewable generator can consist of multiple other elements. The profit function in Equation 5.1 can be extended to model profit of a specific renewable generator. Finally, we assumed constant prices for generation, uncertainty coverage and failure to provide committed generation. In a real-world market these prices would dynamically change. However, since we do not allow renewable generators to form coalitions with each other, they cannot gain significant market power, which would allow them to affect the market prices. Therefore we can model prices as variables that are independent of behavior of renewable generators.

5.2 Increasing Use of Renewable Energy in PDS with Physical Constraints

Section 5.1 analyzed effect of coalition formation between renewable generators and energy stores, and showed that forming coalitions can increase the use of renewable energy as well as the profit of participating renewable generators and energy stores. However, the approach in Section 5.1 does not take into account the underlying power grid and its physical constraints. It is possible that actions of energy stores can result in undesirable voltage levels at various nodes of the power grid. In this section we therefore examine how actions of energy stores and renewable generators affect the physical state of the power grid.

More specifically, the first part of the analysis provided in this section describes use of the coalition formation approach proposed in Section 5.1 in a standard IEEE 69-bus test system from (Khatod et al., 2006), and shows how actions of agents affect voltages on the grid nodes. To do this, we will use a *load-flow* algorithm, which is widely used in power systems for voltage calculations. Given a grid with v nodes and a power vector $P \in \mathbb{R}^{v \times 1}$, the *load-flow* algorithm calculates a vector of voltages $V \in \mathbb{R}^{v \times 1}$. The vector P corresponds to the amount of generation and load at each grid node, and it can be constructed for every time slot t considered in the approach in Section 5.1 using real power from renewable generation $g_r(t)$ and real generation provided by energy stores $g_{sr}(t)$ (see Table 5.1 for description of

model variables). We then design metrics to measure how desirable the effects of coalition formation between renewable generators and energy stores are for the power grid.

The second part of the analysis of the effects that coalition formation poses on the power grid is to design approaches that improve the physical state of the power grid. These approaches include new strategies for the agents, which increase the desirable effect of coalition formation on the power grid. Since current strategies do not include any information about the underlying power grid, these strategies will be used as a baseline approach. We assume that by allowing the agents to use information such as the structure of the grid or estimated effect of their actions on the grid, we will achieve a more stable power grid, while still increasing use of renewable energy as well as increasing profits of individual agents.

Consequently, the approach for increasing the use of renewable energy in PDS with physical constraints ties together approaches for coalition formation proposed in Sections 4.1 and 4.2 with the approach for coalition formation of renewable generators and energy stores proposed in Section 5.1. This section shows that coalition formation can solve a problem with real-world constraints in the PDS domain.

5.2.1 Model of the Power Distribution Grid

Section 5.1.1 described the model of renewable generators and energy stores, and their interaction in the coalition formation process. This section builds on Section 5.1.1 by specifying the model of the power grid as a physical environment in which renewable generators and energy stores operate. We first describe the IEEE 69-bus system along with the types of nodes that we consider in our analysis. We then show how the voltage can be calculated for each node and discuss how this voltage can be used to measure the physical state of the power grid.

The IEEE 69-bus system is a power distribution system that was previously used in (Khatod et al., 2006). The system is shown in Figure 5.5. The grid is connected to the substation through node 1, and the power is distributed to eight branches. The impedance at each line is specified.

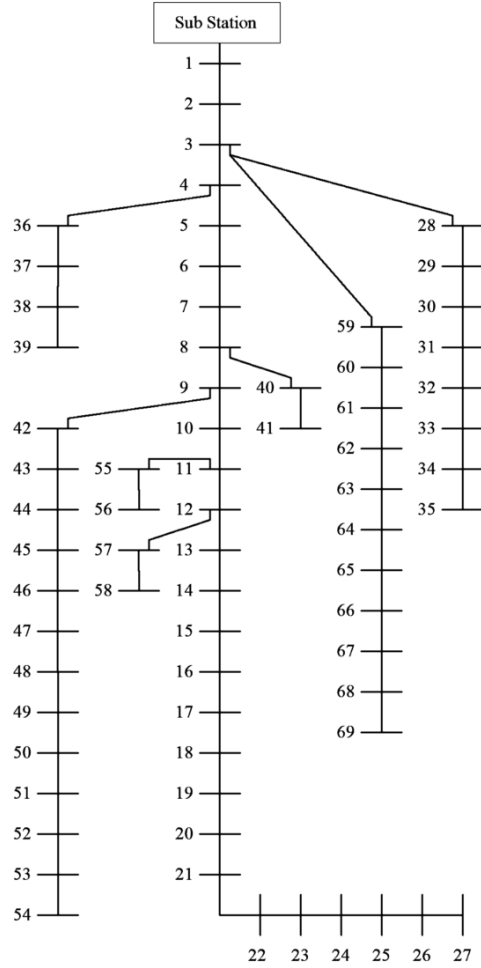


Figure 5.5: *IEEE 69-bus system (Khatod et al., 2006)*

We will consider three types of nodes:

1. **Loads** represent regular households or any grid elements that draw power from the grid. We assume that load nodes never send power to the grid. Load nodes can therefore lower the voltage of nodes in their proximity, but they can never increase voltage of the neighboring nodes.
2. **Renewable generators** use renewable energy to generate power. Renewable generators face generation uncertainty caused by weather prediction inaccuracy. A model of the renewable generator was described in Section 5.1.2. We assume that renewable generator nodes never draw power from the grid. Renewable generator nodes can therefore increase the voltage of the neighboring nodes, but are not able to decrease

this voltage.

3. **Energy stores** can generate predictable amounts of power. Energy stores are incentivized to sell the capability to predict generation to the renewable generators, as was described in Section 5.1. A model of the energy store was shown in Section 5.1.3. Similarly to the renewable generator nodes, we assume that energy store nodes never draw power from the grid. Therefore energy store nodes can also only increase voltage of the nodes in their proximity.

An output of coalition formation in the PDS domain is a power vector $P \in \mathbb{R}^{v \times 1}$, which holds the power generated or used by each node in a single time slot. If we consider hourly slots, coalition formation yields 24 power vectors. These power vectors represent the physical impact of coalition formation on the power grid. However, the physical state of the power grid is better described by changes in voltage. We therefore use a *load-flow* algorithm to calculate voltage vectors based on power vectors. The *load-flow* algorithm is widely used in power systems for voltage calculation. The algorithm finds voltages by solving a system of power-flow equations.

The physical state of the power grid can be well represented by the voltage levels on the grid nodes. Another significant physical characteristic of the grid is line capacity. However, PDS are designed in a very robust way, so line capacity violations occur very rarely. We therefore focus our analysis mainly on voltage characteristics. The power grid operates with base voltage of 12.66 kV and base power of 10 MW. Voltage on each grid node differs from the base voltage, however, there are limits to the difference of the node voltage and the base voltage. Voltage on each node should lie between 95% and 105% of the base voltage. If the voltage exceeds these limits, a violation occurs. A violation is negative if the voltage is less than 95% of the base voltage, and positive if the voltage exceeds 105% of the base voltage. These violations represent an undesirable state of the power grid. We can therefore use the number of violations as a metric to determine the effect of coalition formation on the power grid. An ideal outcome of coalition formation would be a grid with no voltage violations, however, such state might not be achievable. We will show the effect of coalition formation

on the number of voltage violations in Section 5.2.3.

The voltage vector can also be used to guide agents' decision making throughout the coalition formation process. The coalition formation process takes place ahead of time, so that renewable generators can use the results in the day-ahead market. The real-time voltage vector is therefore unknown during coalition formation. However, we assume that the agents can use a voltage vector that would occur if coalition formation did not occur. We call this voltage the *initial voltage* and define it as follows:

Definition 5.2.1 (Initial voltage). *The initial voltage is a set of voltage vectors $V \in \mathbb{R}^{v \times 1}$. Each voltage vector corresponds to a single time slot, and is obtained using the load-flow algorithm. The input of the load-flow algorithm is a power grid with the same configuration of nodes as is used in the coalition formation process, with two exceptions:*

1. *Renewable generators do not participate in coalition formation, therefore they commit to c_0 percent of the estimate generation $g_e(t)$.*
2. *Energy stores do not participate in coalition formation, therefore they do not generate any power.*

The *initial voltage* can be used as a prediction of the voltages at each node in each time slot during coalition formation. Section 5.2.4 will describe the way in which renewable generators and energy stores use this voltage prediction in order to improve the physical state of the power grid.

Note that a real PDS includes network elements that are designed to prevent voltage violations. We are not considering these elements in our analysis due to their high complexity. Results of this analysis are still significant, because lowering number of violations in a distribution grid without network elements that prevent violations implies a decreased need for these devices in a real grid, which lowers their use and necessary cost associated with their wear and tear.

5.2.2 Experimental Analysis

The following sections analyze effects of coalition formation on the power grid described in Section 5.2.1. This analysis is supported by experiment results, which have been obtained using the following experimental setting. Parameters not mentioned in this section were assigned the same values as in Section 5.1.5. In our experiments we use the IEEE 69-bus grid, which contains 69 grid nodes. We performed experiments with varying numbers of renewable generators, energy stores, and loads. Unless stated otherwise, in the rest of this chapter we fixed number of energy stores to 20 for varying numbers of renewable generators, and we fixed number of renewable generators to 5 for varying numbers of energy stores. In each simulation we assigned agents representing renewable generators and energy stores to the grid nodes randomly. The unassigned nodes represent loads. Figure 5.6 shows our implementation of the power grid, or more specifically, a snapshot of a single time slot in a random configuration of 5 renewable generators, 20 energy stores, and 44 load nodes.

The generation amounts of renewable generators were created randomly from a uniform distribution $\mathcal{U}(0, 20 \text{ kW})$. The estimated amounts were then generated by adding a noise value drawn from a normal distribution $\mathcal{N}(\mu = 0, \sigma = 0.2 \cdot 20 \text{ kW})$ to the real generation amounts. The constant 0.2 corresponds to 20% day-ahead prediction error for wind and solar generators published in (Zhang et al., 2015a). For energy stores, the total amount of power m was drawn from a uniform distribution $\mathcal{U}(0, \frac{1}{4} \cdot T \cdot 20 \text{ kW})$, where $T = 24$ represents the number of hourly time slots. This way, if the energy store can distribute its power among all 24 time slots, there will be at most 5 kW available for each time slot. In a more average case of distributing power among 12 time slots in the day, there will be at most 10 kW available for each time slot. The load values of the load nodes were drawn for each time slot from a uniform distribution $\mathcal{U}(-20 \text{ kW}, 0)$. The load values are negative to reflect the opposite flow of power compared to the generation nodes. The upper limit of 20 kW is a reasonable assumption for a PDS consisting of households and small generators. Each line in the power grid has an associated impedance, which is a measure of the line’s resistance. We show specific impedance values for each line in Appendix A, Table A.1. We let the coalition

formation process run for 1000 iterations, and we averaged all results over 10 random runs.

In a real power system, we have to take into account active power P and reactive power Q . However, coalition formation of renewable generators and energy stores specified in Section 5.1 considers the active power only. The relation between active and reactive power is defined by a *power factor* as follows:

$$\text{power factor} = \frac{P}{\sqrt{P^2 + Q^2}}. \quad (5.12)$$

In our experiments we fixed the *power factor* for load nodes to 0.85, which allowed us to calculate the reactive power Q on these nodes. Reactive power on the energy store nodes depends on the type of energy store, which our approach does not restrict. Therefore we set the *power factor* to 1, which corresponds to using batteries as energy stores. The reactive power on energy store nodes is then $Q = 0$. Finally, similar reasoning let us to set $Q = 0$ for the renewable generator nodes, which corresponds to using solar power PV generators, and wind generators in their basic state (state in which the wind generators are not adjusting the *power factor* to control voltage). Many generators adjust their *power factor* in response to the state of the power grid. We do not include this behavior in our model because of its high complexity.

It is sometimes helpful to display the results of our experiments using a percentage compared to an initial value. In the following sections we show the *percentage of number of violations* achieved by a given technique, compared to a number of violations achieved without coalition formation. Specifically, we define the *percentage of number of violations* as

$$\text{percentage of number of violations} = \frac{nv_t}{nv_0}, \quad (5.13)$$

where nv_t denotes the number of violations achieved by a given technique, and nv_0 denotes the number of violations achieved in the same scenario, but without coalition formation. Values 0% and 100% of the *percentage of number of violations* therefore denote an improvement achieving state without violations and no improvement respectively.

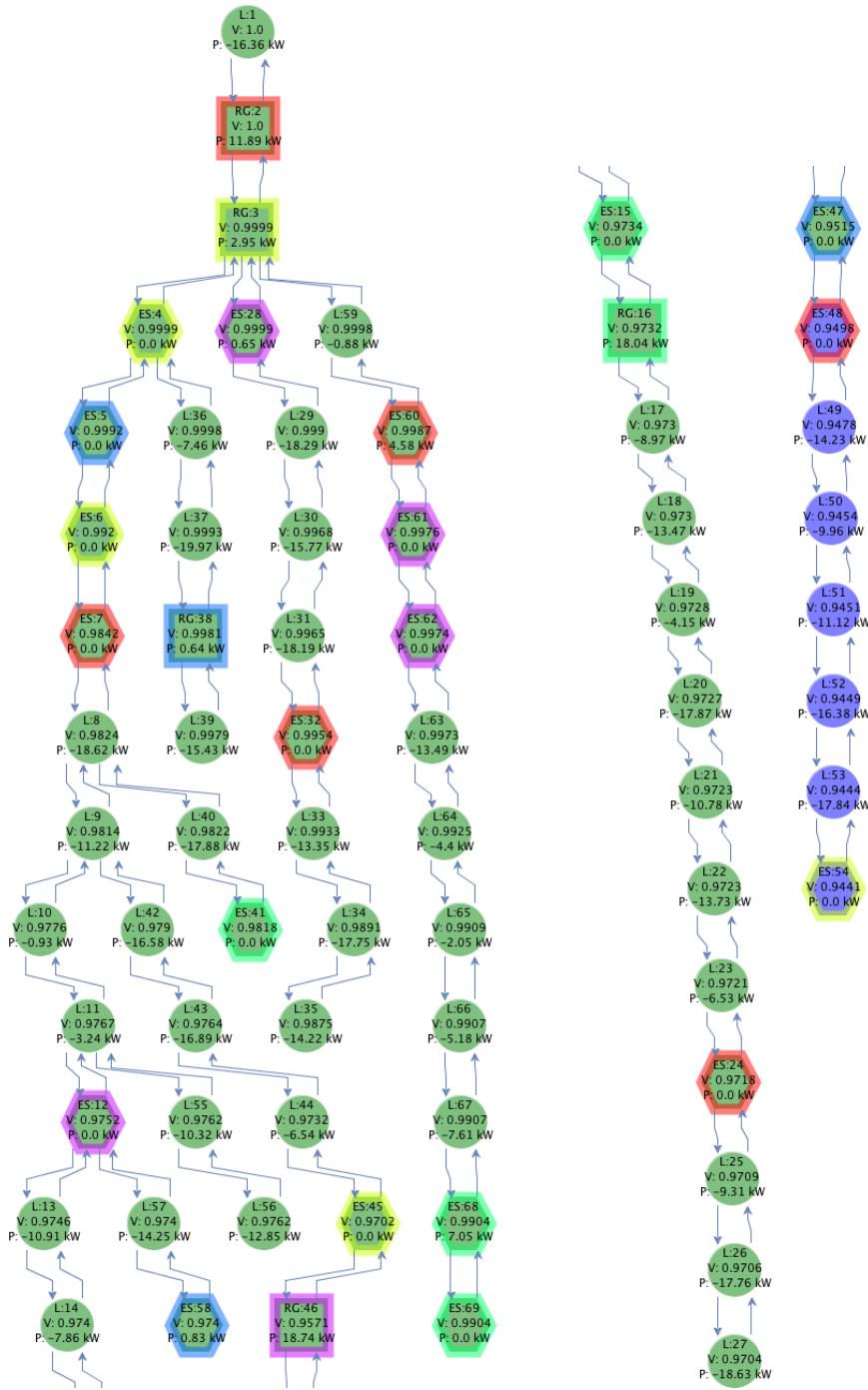


Figure 5.6: IEEE 69-bus system implementation, snapshot of a single time slot. Circular nodes denote loads (L), rectangular nodes denote renewable generators (RG), and hexagonal nodes denote energy stores (ES). Text description includes node type (L/RG/ES), node number (1-69), current fraction of base voltage (V), and current real power (P). Nodes with green color are not experiencing a violation, nodes with blue color are experiencing a negative violation. The outline color denotes membership in a coalition. Nodes 14 and 15, and nodes 46 and 47 are connected.

5.2.3 Effect of Coalition Formation on the Physical State of the Power Distribution Grid

Coalition formation of renewable generators and energy stores has a significant effect on the voltage state of the power grid. The main result of Section 5.1 was an increase in use of renewable energy due to coalition formation. The extra renewable generation was supported by added energy store generation. Consequently, the coalition formation process described in Section 5.1 adds more generation to the system. This effect is beneficial if the grid consists of higher number of loads than generation nodes. However, if the majority of nodes are generation nodes, added generation can hurt the system.

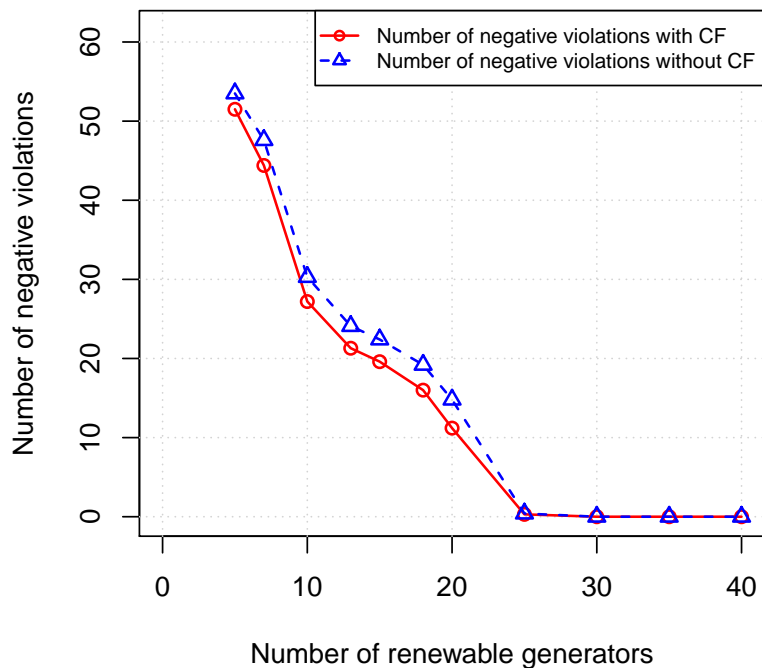


Figure 5.7: Comparison of the number of negative violations for various numbers of renewable generators between scenarios with and without coalition formation (CF)

We show results of experiments focusing on the effect of coalition formation between renewable generators and energy stores for varying numbers of renewable generators and 20

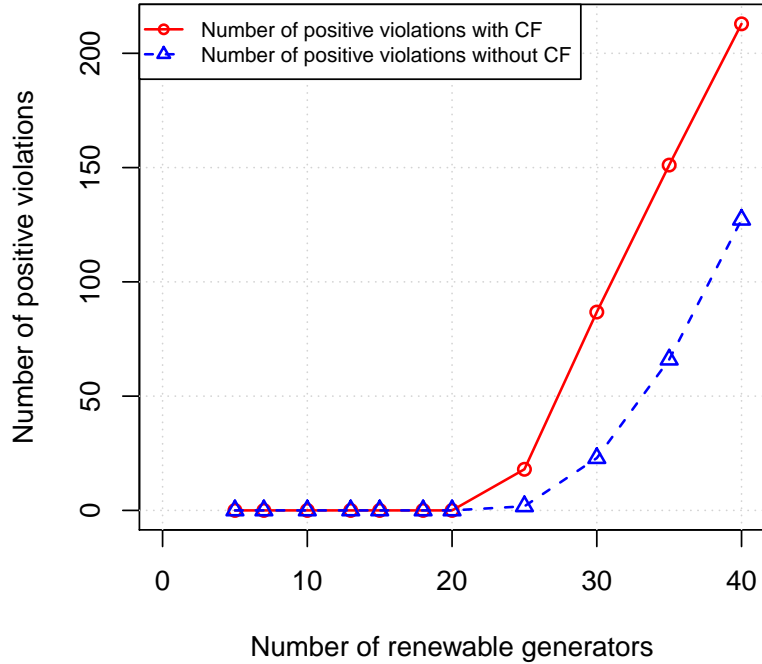


Figure 5.8: Comparison of the number of positive violations for various numbers of renewable generators between scenarios with and without coalition formation (CF)

energy stores in Figures 5.7, 5.8, and 5.9. Figure 5.7 shows the number of negative violations as a function of the number of renewable generators. With an increasing number of renewable generators, the number of negative violations decreases, because the total amount of generation in the grid is increased. Coalition formation incentivizes renewable generators to generate more power, which further decreases the number of negative violations. Therefore, the overall effect of coalition formation on the number of negative violations is positive. However, as shown in Figure 5.8, the added generation can increase the number of positive violations. Figure 5.8 shows that increasing the number of renewable generators increases the number of positive violations. Coalition formation further amplifies this effect, resulting in a significantly higher increase in the number of positive violations. Figure 5.9 shows the total number of violations, which is a sum of negative and positive violations. The figure shows that as long as the number of renewable generators is less than 25, a system with 20

energy stores will benefit from coalition formation. This conclusion is reasonable because with 25 renewable generators and 20 energy stores, the number of generating nodes is greater than the number of loads in the grid, which is generally an undesirable state because the supply of power exceeds its demand. Given the result in Figure 5.9, we focus the rest of our analysis on power grids with lower numbers of renewable generators. For these power grids coalition formation can decrease the number of voltage violations, thus improving the physical state of the power grid.

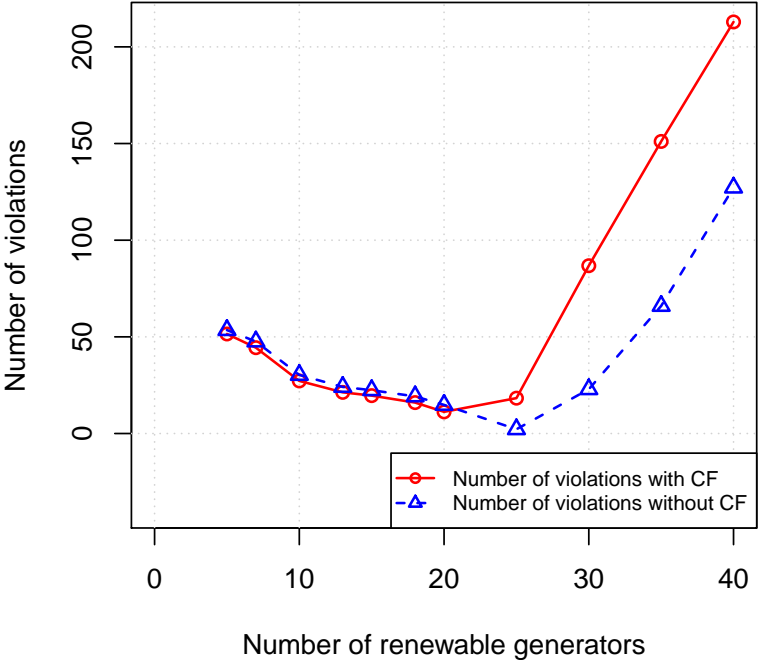


Figure 5.9: Comparison of the total number of violations for various numbers of renewable generators between scenarios with and without coalition formation (CF)

Varying the number of energy stores also affects the number of violations. Figure 5.10 shows the effect of coalition formation on the number of violations as a function of the number of energy stores in a grid with 5 renewable generators. The number of violations in Figure 5.10 is strictly decreasing because in our experimental setting the energy stores do not create any positive violations. Figure 5.10 shows that the impact of varying the number

of energy stores in scenarios with 5 renewable generators is positive, but very small.

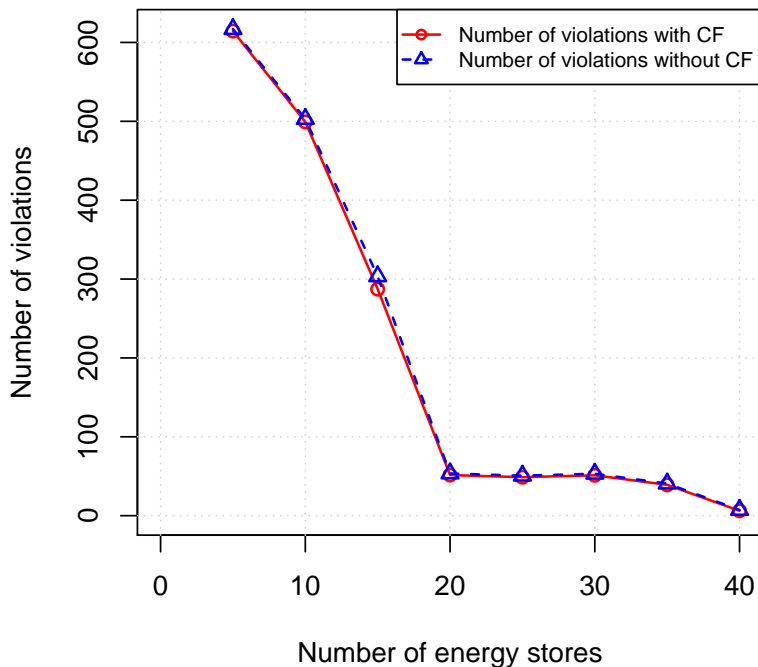


Figure 5.10: Comparison of the total number of violations for various numbers of energy stores between scenarios with and without coalition formation (CF)

Finally, Figure 5.11 shows the profit of 5 renewable generators while varying the number of energy stores. The figure compares the total profit of renewable generators that do not participate in coalition formation (see Equation 5.7) with the total profit of renewable generators that do participate in coalition formation (see Equation 5.1). The figure shows that at least 10 energy stores are needed to incentivize the 5 renewable generators to participate in coalition formation, since in scenarios with at least 10 energy stores the profit of renewable generators that participate in coalition formation exceeds the profit they would receive if they did not participate in coalition formation. This result is significant because it validates results from Section 5.1 in a setting with a power grid with physical constraints.

Over all, coalition formation positively affects the number of negative violations, but can increase the number of positive violations. Our experimental results show a positive effect

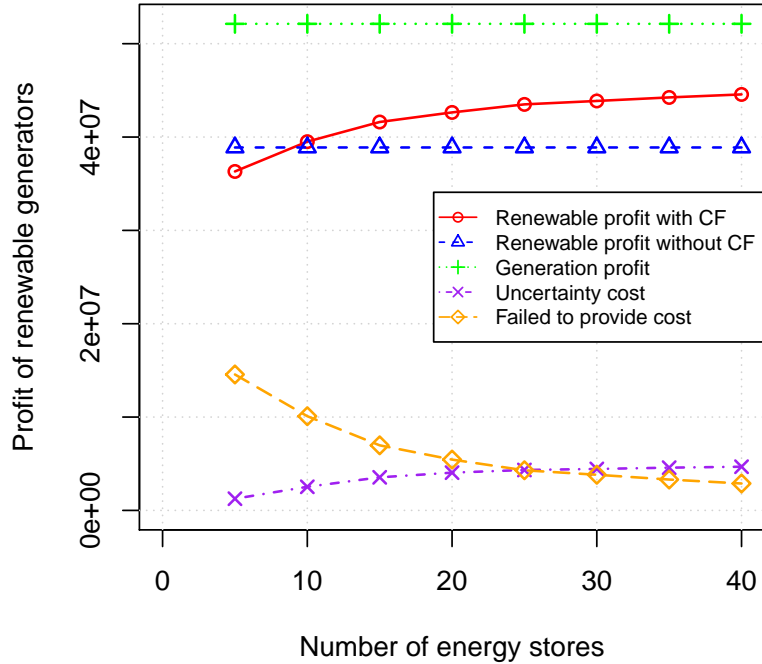


Figure 5.11: Profit of renewable generators in a setting with a power grid with physical constraints.

of coalition formation in power grids with the number of generating nodes not exceeding the number of loads. Given these positive results, the following section investigates how the number of violations can be further decreased.

5.2.4 Improving the Physical State of the Power Distribution Grid

Section 5.2.3 showed the effect of coalition formation on the power grid. This effect is positive in the most important part of the problem space, where the number of generating nodes is lower than the number of loads. In this section we investigate which actions of the agents representing renewable generators and energy stores further decrease the number of violations. We first examine actions of the energy stores. Then we show how actions of renewable generators can decrease the number of violations. Finally, we investigate how the coalition formation process itself can be modified in order to positively affect the physical

state of the grid.

An important aspect of improving the physical state of the PDS is that most decisions of the agents are made ahead of time, based on estimates of voltage values. The ahead-of-time calculation is necessary because commitments have to be made in the day-ahead market. The need for ahead-of-time calculation makes improving the physical state of PDS a challenging task.

5.2.4.1 Actions of Energy Stores

As stated in Section 5.1.3, an energy store is defined by total amount of power m and time slots s_b and s_e between which the power can be distributed. An energy store can form a coalition with a renewable generator by committing specific amounts of power in each time slot between s_b and s_e . The total amount of power distributed by an energy store can be expressed as

$$m_c = \sum_{t=s_b}^{s_e} g_{sc}(t). \quad (5.14)$$

This amount cannot exceed the amount m , however, it is possible for the energy store to commit amount of power that is lower than m . The remaining power can then be used to decrease the number of negative violations. Energy stores can be incentivized to distribute the remaining power by payments from the utility, which is responsible for maintaining stability of the grid. As long as these payments are lower than payments offered by renewable generators, energy stores will be incentivized to participate in coalition formation. The process of distributing the remaining power is described in Algorithm 8, which works in the following way. The input to Algorithm 8 is the energy store node, the uncommitted power $m - m_c$, and the *initial voltage*, which is a voltage state of the power grid without coalition formation (see Definition 5.2.1). First, we find out which time slots between s_b and s_e could experience a negative violation (line 1). This is done by examining the *initial voltage* and collecting all time slots in which the voltage of the energy store drops below 95% of the base voltage. This approach is inaccurate, because we are not using the current state of the grid. However, such information is not available ahead of time. Second, the uncommitted power is

distributed evenly between time slots that could experience a negative violation (lines 3-5). This way, an extra amount of power

$$m_d = \frac{m - m_c}{|S_{nv}|} \quad (5.15)$$

is generated in each of the $|S_{nv}|$ time slots that could experience a negative violation.

Algorithm 8 Distribution of remaining power by an energy store

Input: *grid-node, remaining-power, initial-voltage.*

Output: *remaining-power* is distributed to the grid

- 1: $S_{nv} \leftarrow$ find time slots in which *grid-node* could experience negative violations using *initial-voltage*
 - 2: $\Delta \leftarrow \frac{\text{remaining-power}}{|S_{nv}|}$
 - 3: **for all** time slots t_k in S_{nv} **do**
 - 4: assign power Δ to time slot t_k
 - 5: **end for**
-

Furthermore, we can reach a theoretical bound on the number of negative violations that can be removed by actions of energy stores, if we assume that the energy stores are able to perfectly predict the amount of power that will be required of them in the future by the renewable generators. This assumption is not realistic, but it helps us to show how close our approach can get to the theoretical minimum in number of negative violations with respect to actions of energy stores. Given this assumption, the energy stores are able to calculate the exact amount of power that will be required by the renewable generator as

$$m_r = \sum_{t=s_b}^{s_e} g_{sr}(t). \quad (5.16)$$

Algorithm 8 can then be used with the input of $m - m_r$ remaining power. In this case, an extra amount of power

$$m_{db} = \frac{m - m_r}{|S_{nv}|} \quad (5.17)$$

will be distributed in each of the time slots that could experience negative violation. Since m_{db} is a theoretical bound on the amount of extra power that can be distributed by an energy

store, it holds that

$$|S_{nv}| \cdot m_d \leq |S_{nv}| \cdot m_{db} \leq m. \quad (5.18)$$

Figure 5.12 compares improvement in the number of violations in a scenario with coalition formation with a scenario in which we allow energy stores to distribute the remaining power $|S_{nv}| \cdot m_d$, and a scenario in which we allow energy stores to perfectly predict future requirements of the renewable generators, so they could distribute power $|S_{nv}| \cdot m_{db}$. Figure 5.12 shows a significant improvement for the scenario in which energy stores are allowed to distribute the uncommitted power. Further improvement is achieved in the case of the theoretical bound. However, the theoretical bound provides a relatively small improvement when compared to the improvement achieved by distributing uncommitted power. Practically, this means that using the power from energy stores that was not committed to renewable generators can significantly increase the ability of coalition formation to improve the physical state of the grid. Recall that the energy stores are assigned to random grid nodes. When a real power grid is designed, positions of the energy stores can be determined using an analysis similar to the one we used in Figure 5.12 in order to achieve the best effect on the physical state of the power grid.

5.2.4.2 Actions of Renewable Generators

Given the promising results of the analysis of energy stores' actions, we show that further improvement can also be achieved using actions of renewable generators. A renewable generator that participates in coalition formation commits to generate an estimated amount of power, formally

$$\forall t : g_c(t) = g_e(t). \quad (5.19)$$

However, the real amount of power that can be generated in a given time slot, $g_r(t)$, can differ from the estimated amount. If the committed amount $g_c(t)$ is lower than the actual amount available $g_r(t)$, the renewable generator will only generate the committed amount. Using the information about the *initial voltage* on the generator node, the generator can predict if there will be a negative violation in a given time slot. If the *initial voltage* of the

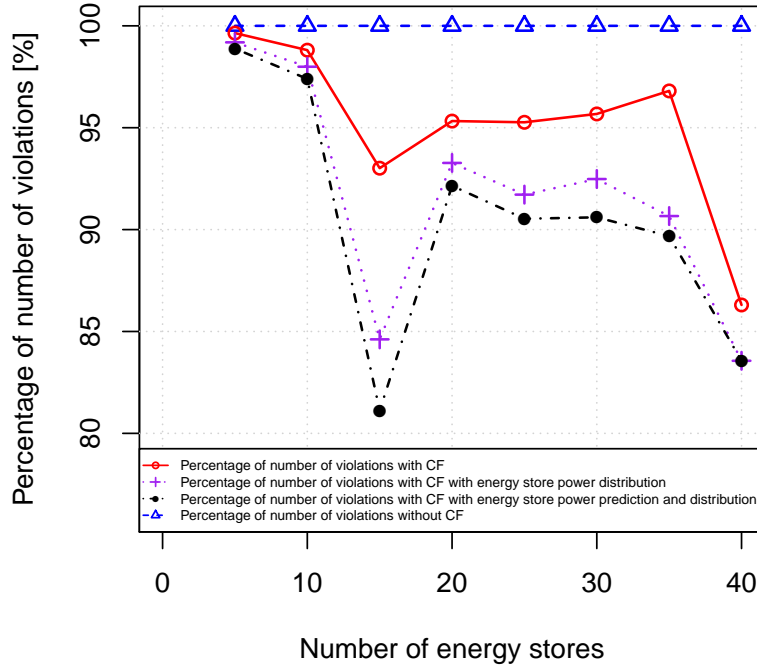


Figure 5.12: *Improvement in the number of violations caused by the use of extra energy store power. Figure shows percentage of number of violations compared to number of violations without coalition formation (see Equation 5.13 for definition of percentage of number of violations and Figure 5.10 for numbers of violations in the scenario without coalition formation). Comparison is between a scenario with no extra actions of energy stores, a scenario with energy stores distributing uncommitted power (energy store power distribution), and a scenario with energy stores distributing uncommitted and unused power (energy store power prediction and distribution).*

renewable generator node drops below 95%, the generator can release all available power, which might remove the negative violation. This increased generation can be incentivized by the utility, which is responsible for maintaining stability of the power grid. Figure 5.13 shows the effect of allowing 15 renewable generators to use the extra power in time slots that might experience negative violations. This effect is positive, but fairly low. The reason for the small percentage improvement is because the difference between the estimated generation and the real generation is drawn from a normal distribution $\mathcal{N}(\mu = 0, \sigma = 0.2 \cdot 20 \text{ kW})$ (see Section 5.2.2). Recall that the constant 0.2 corresponds to 20% day-ahead prediction error

for wind and solar generators published in (Zhang et al., 2015a). The renewable generator can therefore only generate at most 20% extra power, which is often not sufficient to remove the negative violation.

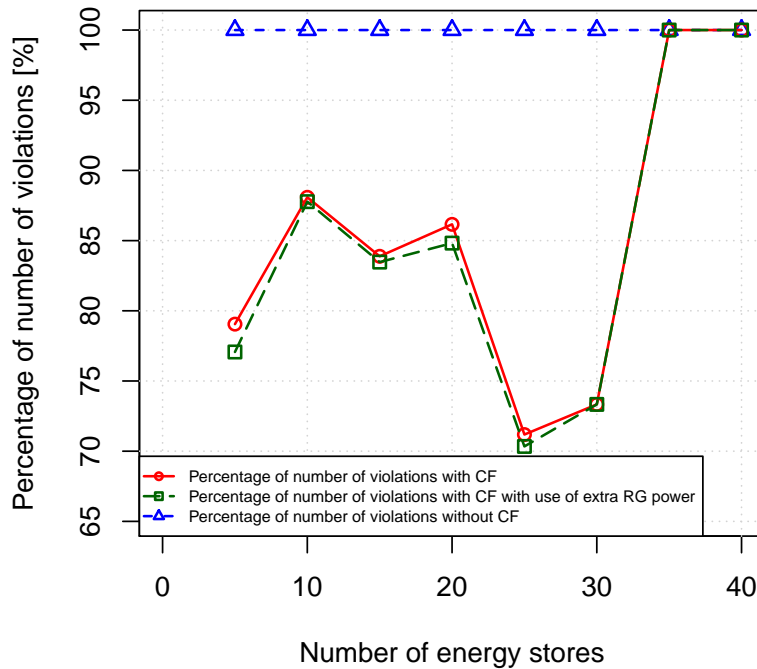


Figure 5.13: Improvement in the number of violations caused by the use of extra renewable generator power. Figure shows percentage of number of violations for various numbers of energy stores and 15 renewable generators compared to number of violations without coalition formation (see Equation 5.13 for definition of percentage of number of violations and Figure 5.10 for numbers of violations in the scenario without coalition formation). Comparison is between a scenario with no extra actions of renewable generators and a scenario with renewable generators using available extra power. 15 renewable generators were used to increase the effect on the number of violations.

5.2.4.3 Changes in the Coalition Formation Process

During the coalition formation process energy stores form coalitions with renewable generators. Membership in a coalition determines the amounts of power $g_{sr}(t)$ that an energy store will generate between time slots s_b and s_e . The generation of energy stores in turn affects

the voltage on the grid nodes. It is therefore possible that changes in the coalition formation process will change the number of violations in the power grid.

The number of violations could be decreased if the energy stores generated more power in time slots in which the nodes in the close proximity of the energy store experience negative violations. On the other hand, extra energy store generation can increase the number of violations if the grid is experiencing positive violations. Let us define a neighborhood of a grid node as follows:

Definition 5.2.2 (Neighborhood of a grid node h of size k). *Let us define ancestors of a grid node h as a set of grid nodes that can be accessed by traveling the grid upwards (towards root of the grid tree shown in Figure 5.5). Let us also define descendants of the grid node h as a set of grid nodes that can be accessed by traveling the grid downwards (towards leafs of the grid tree shown in Figure 5.5). The neighborhood of the grid node h of size k contains all ancestors of the grid node that are accessible by traveling upward at most k edges in the grid tree starting from h , and all descendants of the grid node h that are accessible by traveling downward at most k edges in the grid tree. Note that while all ancestors of h belong to the same branch of the grid tree, descendants of h can belong to multiple branches.*

It is beneficial to let the energy stores examine the *initial voltage* and determine the state of their neighborhoods in a following way. First, each energy store finds its neighborhood nodes using Definition 5.2.2. Each node in the neighborhood is then considered for increase in up-votes or down-votes. Up-votes are given to the energy store by all nodes in the neighborhood that experience negative violations, while down-votes are given by all nodes in the neighborhood that experience positive violations. This process is described formally in Algorithm 9. First, a neighborhood of the node is found (line 1). Then we count number of up-votes and down-votes for each neighboring node and each time slot (lines 3-12).

As described in Sections 4.1 and 5.1.4, coalitions are assigned values by valuation functions. An agent then joins a coalition for which the increase in value caused by addition of this agent is the highest. Coalition formation in the PDS domain uses *renewable energy valuation function* (see Definition 5.1.1), which penalizes a coalition for time slots in which

Algorithm 9 Calculation of neighborhood violations

Input: *grid-node*, *initial-voltage*.

Output: *up-votes*, *down-votes*

```
1: neighborhood(grid-node)  $\leftarrow$  find-neighborhood
2: up-votes  $\leftarrow$  0, down-votes  $\leftarrow$  0
3: for all grid nodes  $n_i$  in neighborhood(grid-node) do
4:   for all time slots  $t_j$  do
5:     if initial-voltage on grid node  $n_i$  at time  $t_j$  < 95% of base voltage then
6:       up-votes  $\leftarrow$  up-votes + 1
7:     end if
8:     if initial-voltage on grid node  $n_i$  at time  $t_j$  > 105% of base voltage then
9:       down-votes  $\leftarrow$  down-votes + 1
10:    end if
11:  end for
12: end for
```

the energy stores do not cover all power that was requested for coverage by the renewable generator. The number of violations could be decreased if the energy stores form coalitions in a way that allows them to generate power in time slots with high number of up-votes and low number of down-votes. In order to incentivize this behavior of the energy stores, we design a valuation function that takes into account knowledge about the physical state of the power grid. We call this valuation function the *prioritized valuation function*, and we define it as follows:

Definition 5.2.3 (Prioritized valuation function). *The prioritized valuation function is a valuation function that assigns a value v to a coalition C as follows:*

$$v(C) = w_p \cdot \sum_{t_k=1}^T g_{sc}(t_k) \cdot (\text{up-votes} - \text{down-votes}) + v_r(C), \quad (5.20)$$

where w_p denotes weight of the correlation between generation of the energy store $\sum_{t_k=1}^T g_{sc}(t_k)$ summed over T time slots and up-votes and down-votes, which are aggregated over all energy stores in coalition C . $(\text{up-votes} - \text{down-votes})$ represents prioritizing grid nodes with high up-votes and low down-votes, and $v_r(C)$ denotes output of the renewable energy valuation function.

The *prioritized valuation function* is designed to optimize both the number of violations

and the use of renewable energy. The parameter w_p can be used to set higher preference to either of the two criteria. The coalition formation generates a pool of coalition structures, from which we choose one as a solution based on some criteria. In Section 5.1 we selected the coalition structure with the highest profit of renewable generators (see Equation 5.11). In order to benefit fully from the effect of the *prioritized valuation function*, here we select the coalition structure with the highest value (see Definition 4.1.2).

The *prioritized valuation function* computes a priority of energy stores based on an estimate of how much their generation benefits the grid. However, this estimate, which is based on the *initial voltage*, is different in each time slot. Therefore this valuation function might be beneficial only in power grids in which the power does not change significantly between time slots. To investigate this theory we performed experiments with varying numbers of time slots. Lower numbers of time slots result in lower total number of changes between time slots. Moreover, lowering the number of time slots makes the original problem of increasing use of renewable energy, which was described in Section 5.1, easier to solve. Since *prioritized valuation function* tries to optimize the original problem as well as decrease the number of violations, simplifying one of these criteria should improve the performance of this valuation function. Figure 5.14 shows the results of this experiment. Indeed, the *prioritized valuation function* performs better in scenarios with lower number of time slots. Unfortunately, the figure shows that for 24 hourly time slots the *prioritized valuation function* does not always improve the number of violations.

Given the results in Figure 5.14, we further focus only on scenarios with 1 time slot. These scenarios represent situations where power does not change over time. Practically, this means that coalition formation would be performed every time we expect a change in power. This is a realistic assumption, as in our scenario with a day-ahead market and 24 hourly time slots coalition formation could happen not once, as proposed in this chapter, but 24 times, once for each single hourly time slot. In such a setting, results shown in Figure 5.14 for a single time slot can be achieved.

We show the percentage improvement caused by the *prioritized valuation function* in a scenario with one time slot in Figure 5.15. In this simplified version of the problem we can

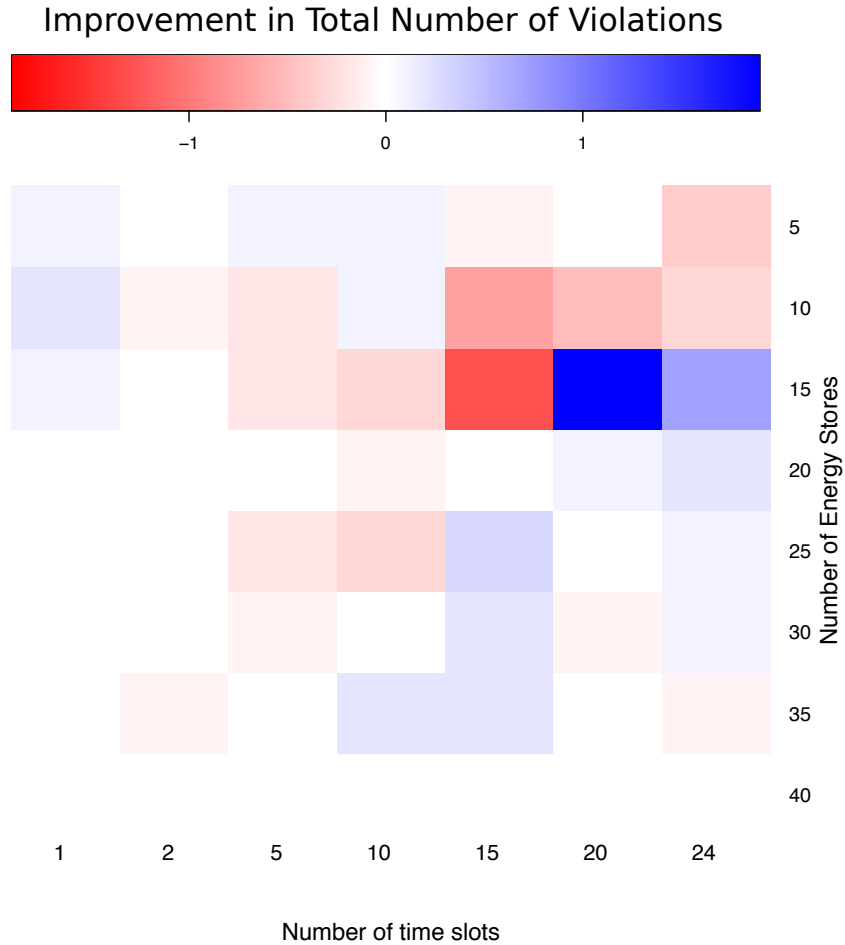


Figure 5.14: *Improvement in the total number of violations (blue is good) caused by prioritizing energy stores for varying number of time slots, number of energy stores, and $w_p = 0.0075$.*

achieve approximately 1% improvement by giving higher priority in the coalition formation process to energy stores whose neighboring nodes are expected to experience most negative violations. The effect of changing the coalition formation process on the physical state can be positive, but is very small. Actions of agents studied in Sections 5.2.4.1 and 5.2.4.2 impact the physical state of the power grid more significantly without the need for simplification of the problem.

We conclude the analysis of the impact of coalition formation on the power grid by showing that even after we change the coalition formation process by using the *prioritized valuation function*, the renewable generators still obtain higher profit when they participate in coalition formation. This result is important because this increased profit will incentivize

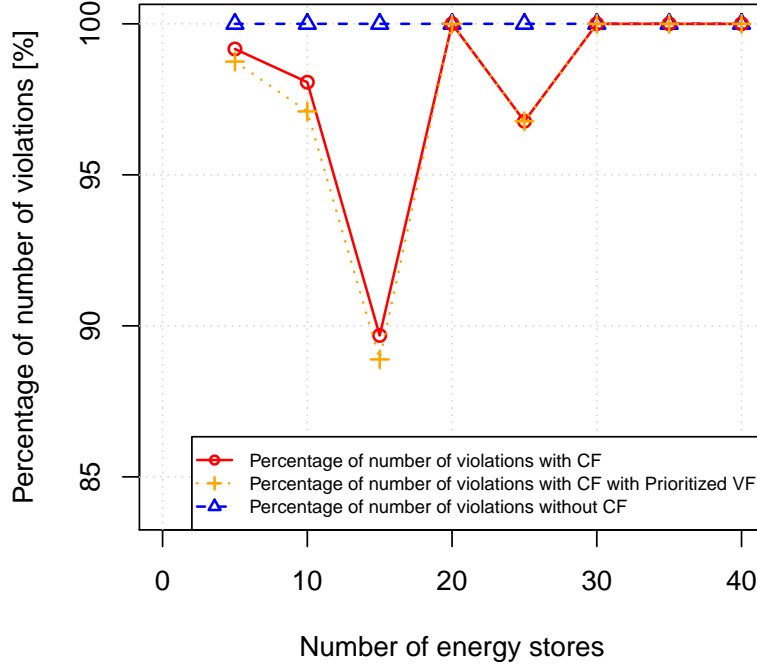


Figure 5.15: Improvement in the total number of violations caused by prioritizing energy stores for $w_p = 0.0075$. See Equation 5.13 for definition of percentage of number of violations.

all agents to participate in coalition formation, which will increase the amount of renewable energy used, as shown in Section 5.1, as well as improve the physical state of the underlying power grid, as shown in Section 5.2.4. Figure 5.16 shows profit of renewable generators with and without participation in coalition formation. Observe that for the given experimental setting the profit of renewable generators is always greater when they participate in coalition formation, even for the lowest number of 5 energy stores. Figure 5.17 shows this profit when we use the *prioritized valuation function*. The profit in this case is almost the same as in the case of using the *renewable energy valuation function* in Figure 5.16. Therefore changing the coalition formation process by designing the new valuation function improved the number of voltage violations, and it did not significantly change the profit of renewable generators.

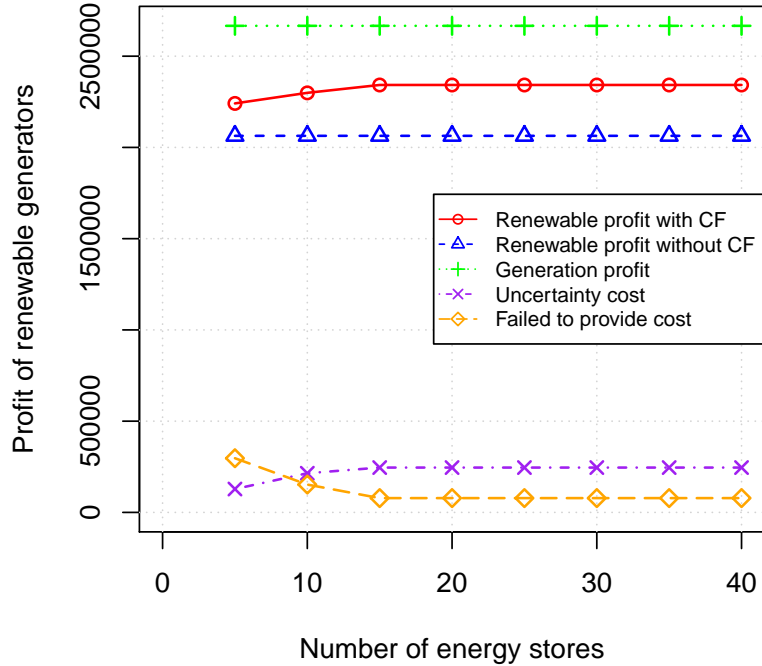


Figure 5.16: Profit of renewable generators participating in coalition formation with renewable energy valuation function.

5.2.5 Discussion

Two important conclusions can be made from the analysis in Section 5.2. First, coalition formation can solve a real-world problem with physical characteristics and constraints. Even though our results are based on simulations, considering the voltage characteristics of the grid allows for possible future application in a real PDS. This conclusion is significant, because it shows a possible real-world application of coalition formation approaches that we proposed in Section 4. Second, the use of renewable energy in a PDS can be increased without harming the physical state of the PDS. This conclusion validates our approach proposed in Section 5.1, where coalition formation was used without considering the underlying power grid.

The approach proposed in Section 5.2 can be used to measure the impact of coalition formation on the PDS. More specifically, the analysis proposed in Section 5.2 can be followed to study effects of forming coalitions on profit of the renewable generators, overall use of

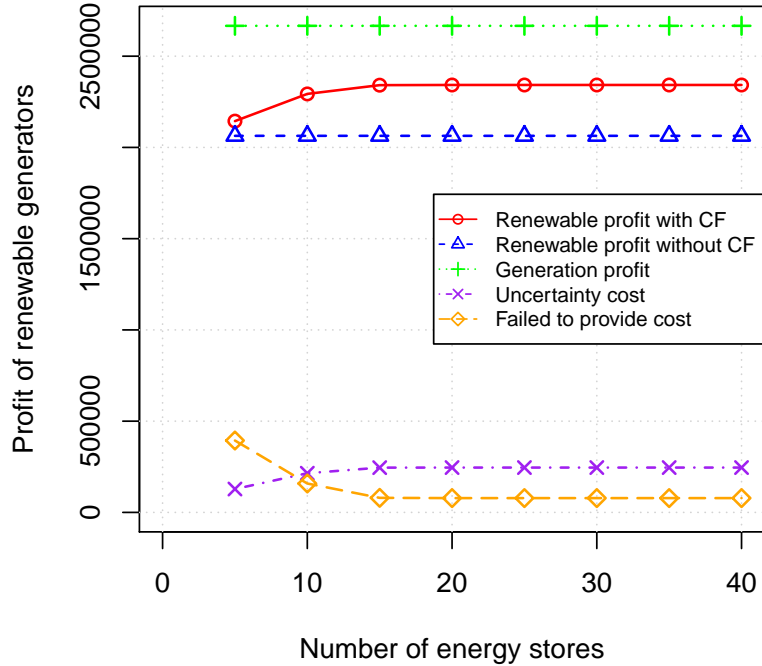


Figure 5.17: Profit of renewable generators participating in coalition formation with prioritized valuation function.

renewable energy, and physical state of the power grid. Furthermore, our approach can be used to organize the PDS in a way that will benefit the power grid the most, while still increasing the use of renewable energy. Our experiments showed that number of renewable generators, number of energy stores, and cooperation of renewable generators and energy stores have significant impact on the physical state of the power grid. Our approach can therefore be used to find the best configuration of nodes that will improve the physical state of the power grid.

Section 5.2 makes several assumptions. First, we assume that the energy stores are charged at the beginning of the day. This assumption is justifiable because we do not restrict the type of energy stores. For example, a battery store would have to be charged, while a diesel generator would not. Including energy store charging in our model would significantly restrict the domain of energy stores that we consider. Second, we assume that

the substation connected to node 1 does not act in any way to maintain stability of the grid. A real substation would actively change the voltage on node 1 in order to stabilize the grid. However, including this behavior in our model would make it harder to observe the impact of coalition formation itself on the power grid. Third, we allow agents to use the information about the *initial voltage*, which is voltage that would occur without coalition formation (see Definition 5.2.1). This voltage serves as an estimate of the voltage state on each node in each future time slot. Any other estimate, for example historical data, could be used instead of the *initial voltage* to help agents predict the physical state of the power grid. Finally, we used a fixed *power factor* when calculating reactive power Q (see Equation 5.12). While this is a reasonable assumption for the load nodes, real generators often dynamically change their *power factor* in order to maintain the grid voltage. As stated in Section 5.2.2, we do not include this behavior in our model because of its high complexity. Including such behavior would also require us to choose specific generator types, which would significantly restrict the domain of both renewable generators and energy stores that our approach considers.

5.3 Summary

In this chapter we proposed an approach to increase use of renewable sources by allowing renewable generators to hedge against uncertainty by forming coalitions with energy stores. In these coalitions energy stores offer to cover generation that renewable generators committed to, but are unable to deliver due to prediction uncertainty. We model renewable generators and energy stores as self-interested agents, and we use multi-agent simulation to create coalition structures by allowing energy stores to leave and join coalitions based on their preference.

We experimentally show that our approach increases use of renewable resources. With the support of coalition formation with energy stores, renewable generators can afford to bid higher amounts of generation in the day-ahead market. In our experiments we show that renewable generators can bid 100% of the predicted generation and still profit, even when facing high fees for failure to provide committed generation. In our experimental setting,

our approach increased the use of renewable resources by 13.5%. We also show that forming coalitions with energy stores increases profit of renewable generators, which incentivizes them to increase renewable generation.

This chapter also studies the impact of coalition formation between renewable generators and energy stores on an IEEE 69-bus power grid with physical constraints (Khatod et al., 2006). We experimentally show that this impact is positive in power grids that contain more load nodes than generation nodes. This is a positive result because it covers the majority of current PDSs. Furthermore, we analyze how actions of renewable generators and energy stores, and changes in the coalition formation process, can improve the physical state of the grid. We show how specific actions of the agents as well as changes in the coalition formation process decrease the number of voltage violations in the power grid.

Consequently, this chapter shows that coalition formation approaches proposed in Chapter 4 can be used to solve a real-world problem with physical and economical characteristics.

Chapter 6

Conclusion

This chapter concludes the thesis by summarizing the current state of the coalition formation research and ongoing PDS changes in Section 6.1. Contributions of the thesis are then summarized in Section 6.2. Furthermore, Section 6.3 discusses limitations of our work, and finally Section 6.4 summarizes the future work in the field of large-scale coalition formation and its application in PDSs.

6.1 Current State

Coalition formation is a process in which multiple agents in a MAS form groups called coalitions in order to achieve individual or group goals. It has been previously studied, but the focus was usually on finding solutions that maximize the social welfare in small-scale MASs with at most a few dozen agents. The previous research has focused mainly on small-scale systems because there are $O(n^n)$ solutions of the coalition formation problem (Rahwan and Jennings, 2008), and finding the optimal solution has been proven to be an NP-complete problem (Sandholm et al., 1999). Approaches that find the optimal coalition structure are mainly dynamic programming-based and graph-based algorithms. Other approaches, such as heuristic algorithms or clustering algorithms, can be used in larger-scale systems to find sub-optimal solutions. While searching for an optimal solution is useful, some applications

require finding "good-enough" solutions on a much larger scale. For example, in the PDS domain the number of households and generators can be in the range of thousands. Many of the current algorithms are not designed for such a scale.

Furthermore, coalition formation is usually addressed in previous research as a single-step task, i.e. the input for the problem is the MAS and once coalitions are formed, they are returned as the output. However, in applications that model human behavior it is beneficial to view coalition formation as a dynamic continuous process in which coalitions repeatedly form, change, and dissolve. A limited amount of research has been done on use of multi-agent simulation to address the dynamic coalition formation problem. Multi-agent simulation can be modified to model many coalition formation applications, such as applications in various domains, applications with self-interested and selfless agents, or applications with multiple types of agents. However, these types of applications have not been extensively studied.

Finally, a few proposed coalition formation approaches study systems with self-interested agents, which seek to increase their own benefit instead of the overall social welfare. These self-interested agents might desire to leave a coalition proposed by a coalition formation algorithm in order to increase their benefit. Therefore, stability of a coalition, which is the coalition's ability to deincentivize its members from leaving the coalition, must be taken into account. Approaches have been proposed for finding stable coalitions, however, many of them either restrict the coalition formation problem to specific sub-problems, or do not address the high complexity of finding stable coalitions in large-scale systems.

PDSs are currently undergoing many changes that significantly alter the power distribution grid. Many households become active consumers by installing small renewable generators and participating in the energy market. This distributed generation changes the electricity flow from the original uni-directional flow to a much more complex multi-directional flow. A political demand for a higher integration of renewable resources further amplifies this effect.

The use of renewable energy is challenging because of its weather-related unpredictability. Electricity is currently traded in a day-ahead market. Generators commit to specific generation amounts for hourly time slots in the next day. Since supply and demand of elec-

tricity must always match, generators are incentivized to generate the committed amounts of power by receiving fees for failure to provide the committed generation. These fees introduce significant challenges for renewable generators because their generation is only predictable with a 20%-25% error (Zhang et al., 2015a). Renewable generators avoid these fees by committing lower amounts of power. Consequently, renewable energy is not utilized to its full potential. The unpredictability of renewable generation has been addressed in previous research mainly by proposing coalitions of renewable generators, and benefiting from a lower prediction uncertainty caused by their spacial distribution.

6.2 Summary of Contributions

Given the current state of research in coalition formation and PDS described in Section 6.1, we summarize contributions that this thesis makes to the state-of-the-art research in both fields.

To address the lack of approaches for coalition formation that find solutions in large-scale systems in which agents dynamically join and leave coalitions, we proposed a multi-agent simulation framework for large-scale coalition formation in Section 4.1 (Janovsky and DeLoach, 2016a). This framework can be used to model various large-scale applications of coalition formation by designing valuation functions that assign values to coalitions, and agents' strategies, that decide on behalf of the agents about joining and leaving coalitions. Section 4.1 shows how agents' strategies can be evaluated in scenarios with various valuation functions by a comparison with optimal solutions, which can be found in small-scale scenarios using existing approaches, and by observing the quality of the solutions in large-scale scenarios in which optimal solutions cannot be found. We compared our strategies with a state-of-the-art algorithm and showed the superior performance of our coalition formation framework.

Furthermore, to enable our framework to model scenarios with self-interested agents, we proposed a multi-agent simulation approach that finds stable coalitions in large-scale systems containing thousands of self-interested agents in Section 4.2 (Janovsky and DeLoach,

2016c,d). We showed that coalition stability can be increased by 1) extending the space of agents' actions by a deviation action, and 2) selecting a coalition structure with a high stability estimate. Agents can use the deviation action to deviate from their coalitions with other members of the coalition as long as all members of this new sub-coalition benefit at least as much as in the original coalition. Including this action in the multi-agent simulation helps break unstable coalitions and form more stable coalitions. Calculating coalition stability is a computationally expensive task, because all permutations of all combinations of coalition members must be considered for creation of a sub-coalition. Therefore, we proposed an approach for calculation of an estimate of coalition stability, which can be used to quickly decide which coalitions are likely stable. We combined the agent deviation and coalition structure selection techniques and showed that this combination significantly increases the stability of the resulting coalitions. We also showed that the decrease in the social welfare, which is necessary because we are considering benefits of individual agents, is very small compared to the improvement in the coalition stability.

This thesis addresses the challenges of using renewable energy caused by the weather-related unpredictability in Section 5.1 (Janovsky and DeLoach, 2016b) by proposing a coalition formation process between renewable generators, which face generation unpredictability, and energy stores, which can generate predictable amounts of power. In a coalition, energy stores can sell their ability to generate predictable amounts of power to the renewable generators. Renewable generators use this ability as a coverage against uncertainty. Energy stores profit from participating in coalitions because they are getting paid by the renewable generators for the ability to generate predictable amounts of power as well as for the actual generation. Renewable generators benefit from participating in coalition formation because it allows them to avoid fees for failure to provide committed generation. Consequently, the renewable generators can commit to higher generation amounts thus increasing the use of renewable energy. We showed that participating in coalition formation increases the profits of all agents. We also showed that coalition formation results in an increased use of renewable energy.

Finally, to show that coalition formation can be applied in a real-world problem, in

Section 5.2 we demonstrated how it can be used in a real PDS by analyzing effects of the approach proposed in Section 5.1 on an IEEE 69-bus test system (Khatod et al., 2006). In this test system we randomly assigned nodes to renewable generators, energy stores, and loads. We then let renewable generators form coalitions with energy stores and observed the effect of the resulting generation on the physical state of the power grid. We showed that coalition formation increases generation of renewable generators and energy stores, which results in increased voltage on grid nodes. Consequently, coalition formation improves physical state of power grids that contain more load nodes than generation nodes, because in these grids increased generation decreases the number of negative voltage violations. Furthermore, we showed that the actions of agents and changes in the coalition formation process result in further improvement of the physical state of the power grid.

Over all, this thesis proposes coalition formation approaches for large-scale MASs that consider both the social welfare of the MAS and the individual profit of agents. It also validates the applicability of these approaches by proposing their application in the PDS domain, and shows that they can increase the use of renewable energy and improve the physical state of the PDS.

6.3 Limitations

There are several limitations to the approaches that are proposed in this thesis. First, the coalition formation framework proposed in Chapter 4 uses multi-agent simulation in a centralized way. The agents make decisions about leaving and joining coalitions one by one, with the knowledge of the impact of previous agents' actions. For many applications including those described in this thesis a centralized approach is sufficient. However, some applications, such as coalition formation of a team of robots, require distributed decision making. Furthermore, a distributed algorithm could benefit from using multiple processing units, thus decreasing the computation time. Since distributed applications were not focus of this work, we leave a distributed multi-agent simulation-based coalition formation algorithm for future work.

Second, the approaches proposed in Chapter 4 allow each agent to be a member of exactly one coalition at a time. While this is generally assumed in most coalition formation research, there are many applications that require the ability to assign agents to multiple groups. For example, a social media account can participate in multiple friend groups. Participation in multiple coalitions adds an extra layer of complexity to the coalition formation problem because the configuration space of possible coalition structures is much larger. Since none of the applications for which the approaches in Chapter 4 were proposed require participation in multiple coalitions, we did not investigate this area of coalition formation.

Third, approaches proposed in Chapter 5 made several assumptions that limit their applicability in a real PDS. We assumed that energy stores are charged in the beginning of the day. Further study of the dynamics of specific real world energy stores, including charging constraints and strategies, would be required before deploying our approach in a real PDS. We also assumed no actions of the utility and renewable generators towards stabilizing the power grid. Further study of these actions of the utility and renewable generators in a real PDS would be required before our approach can be deployed. Finally, prices in a real energy market react to the ratio of supply and demand. In our model we assumed fixed prices in order to show the effect of coalition formation without a variable outside input. An energy market would have to be designed to show use of coalition formation in a more realistic setting.

6.4 Future Work

Approaches proposed in this thesis can be extended in multiple ways. First, limitations summarized in Section 6.3 can be addressed. Specifically, a distributed coalition formation algorithm can be designed. In this algorithm the agents would make decisions iteratively based on the coalition structure found in a previous iteration. This way the order of agents does not affect the resulting coalition structure, because agents' decision making is based on the knowledge of the previous coalition structure, while their actions affect a new coalition structure. Consequently, we can achieve determinism in the distributed environment. Since

the agents' knowledge of the system will differ between the centralized and the distributed approaches, the outcome of coalition formation will likely also be different. Agents can also be allowed to join multiple coalitions. To do this, the solution quality and agents' decision making will have to be defined differently. In the PDS domain, specific real-world energy stores and renewable generators can be studied to design more realistic models of the agents.

Second, a behavior of different types of agents can be studied. For example, agents that do not behave rationally can significantly affect the result of the coalition formation process. Furthermore, restricting the agents' knowledge about the environment to a knowledge about agents' local neighborhoods could lead to more localized coalitions. Both of these examples would extend the ability of our approaches to model human behavior as well as other real-world applications.

Third, all decision making in our approach is performed by the agents. However, it might be beneficial to include some decision making at the coalition level. A coalition could decide collectively whether to accept a new member or which members should leave the coalition. Different models of collective decision making can be considered, thus increasing the domain of our approaches.

Finally, grid elements could be added to the PDS to improve the applicability of our approach proposed in Chapter 5. These elements can include devices that maintain the stability of the grid, or realistic transformers. Including these and other devices would reduce the gap between simulation and real-world experiments.

6.5 Summary

This chapter summarizes the current state of research in coalition formation and PDS and points out several areas that have not received sufficient focus from the research community. It then shows how this thesis addresses these areas by proposing new approaches for coalition formation and their application in PDS. Finally, we summarize limitations of our approaches, and propose some areas of possible future work.

Bibliography

- E. Anshelevich and S. Sekar. Computing stable coalitions: Approximation algorithms for reward sharing. *Web and Internet Economics: 11th International Conference, WINE 2015*, 2015.
- T. Arnold and U. Schwalbe. Dynamic coalition formation and the core. *Journal of Economic Behavior & Organization*, 2002.
- J. Augustine, N. Chen, E. Elkind, A. Fanelli, N. Gravin, and D. Shiryayev. Dynamics of profit-sharing games. *IJCAI'11: 21st International Joint Conference on Artificial Intelligence*, 2011.
- M. Begović, A. Pregelj, A. Rohatgi, and D. Novosel. Impact of renewable distributed generation on power systems. *34th Hawaii International Conference on System Sciences*, 2001.
- F. Bistaffa and A. Farinelli. A fast approach to form core-stable coalitions based on a dynamic model. *2013 IEEE/WIC/ACM International Conference on Intelligent Agent Technology, IAT 2013*, 2013.
- F. Bistaffa, A. Farinelli, J. Cerquides, J. Rodríguez-Aguilar, and S. D. Ramchurn. Any-time coalition structure generation on synergy graphs. *13th International Conference on Autonomous Agents and Multiagent Systems*, 2014.
- S. Bonnefoy, N. Kabachi, and M. Lamure. Agent-based simulation of coalition formation in cooperative games. *IEEE/WIC/ACM International Conference on Intelligent Agent Technology*, 2005.
- E. F. Camacho, T. Samad, M. Garcia-Sanz, and I. Hiskens. Control for renewable energy and smart grids. *The Impact of Control Technology*, 2011.

- D. M. Case. *Engineering Complex Systems with Multigroup Agents*. PhD thesis, Kansas State Univeristy, 2015.
- K. Ceclárová and A. Romero-Medina. Stability in coalition formation games. *International Journal of Game Theory*, 2001.
- V. Conitzer and T. Sandholm. Complexity of constructing solutions in the core based on synergies among coalitions. *Artificial Intelligence*, 2006.
- F. Cruz-Mencía, J. Cerquides, and A. Espinosa. Optimizing performance for coalition structure generation problems' idp algorithm. *2013 International Conference on Parallel and Distributed Processing Techniques and Applications*, 2013.
- N. Di Mauro, T. M. A. Basile, S. Ferilli, and F. Esposito. Coalition structure generation with grasp. *14th International Conference on Artificial Intelligence: Methodology, Systems, and Applications, AIMSA '10, Springer, Berlin/Heidelberg*, pages 111–120, 2010.
- A. Drogoul and J. Ferber. Multi-agent simulation as a tool for modeling societies: Application to social differentiation in ant colonies. *Artificial Social Systems*, 1994.
- A. Farinelli, M. Bicego, S. Ramchurn, and M. Zucchelli. C-link: A hierarchical clustering approach to large-scale near-optimal coalition formation. *Twenty-Third International Joint Conference on Artificial Intelligence*, 2013.
- D. Fudenberg and J. Tirole. *Game Theory*. MIT Press, 1991. ISBN 9780262061414.
- G. Gigerenzer and R. Selten. *Bounded Rationality: The Adaptive Toolbox*. MIT Press, 2002. ISBN 0-262-57164-1.
- G. Greco, E. Malizia, L. Palopoli, and F. Scarcello. On the complexity of the core over coalition structures. *Twenty-Second International Joint Conference on Artificial Intelligence*, 2011.
- M. Hoefler and L. Wagner. Designing profit shares in matching and coalition formation games. *Web and Internet Economics: 9th International Conference, WINE 2013*, 2013.

- H. Holttinen, P. Meibom, A. Orths, B. Lange, O. Mark, J. Olav, A. Estanqueiro, E. Gomez, L. Sder, G. Strbac, J. C. Smith, and F. van Hulle. Impacts of large amounts of wind power on design and operation of power systems , results of iea collaboration. *8th International Workshop on Large-Scale Integration of Wind Power into Power Systems as well as on Transmission Networks of Offshore Wind Farms*, 2009.
- A. Ipakchi and F. Albuyeh. Grid of the future. *IEEE Power and Energy Magazine*, 2009.
- P. Janovsky and S. A. DeLoach. Multi-agent simulation framework for large-scale coalition formation. *2016 IEEE/WIC/ACM International Conference on Web Intelligence*, 2016a.
- P. Janovsky and S. A. DeLoach. Increasing use of renewable energy by coalition formation of renewable generators and energy stores. *European Conference on Multi-agent Systems EUMAS 2016*, 2016b.
- P. Janovsky and S. A. DeLoach. Forming stable coalitions in large systems with self-interested agents. *14th European Conference on Multi-agent Systems EUMAS 2016*, 2016c.
- P. Janovsky and S. A. DeLoach. Increasing coalition stability in large-scale coalition formation with self-interested agents. *22nd European Conference on Artificial Intelligence ECAI 2016*, 2016d.
- N. R. Jennings and J. Campos. Towards a social level characterisation of socially responsible agents. *IEE Proceedings - Software Engineering*, 1997.
- K. Jhala, B. Natarajan, and A. Pahwa. Probabilistic voltage sensitivity analysis (PVSA) - a novel approach to quantify impact of active consumers. *Submitted to IEEE Transactions on Power Systems*, 2016.
- H. Keinänen. Simulated annealing for multi-agent coalition formation. *Third KES International Symposium on Agent and Multi- Agent Systems: Technologies and Applications, KES-AMSTA '09, Springer, Berlin/Heidelberg*, pages 30–39, 2009.

- S. K. Khaitan and J. D. McCalley. Cyber physical system approach for design of power grids: A survey. *2013 IEEE Power & Energy Society General Meeting*, 2013.
- M. Khalid, A. V. Savkin, and V. G. Agelidis. A method for minimizing energy cost in a microgrid with hybrid renewable power generation using controlled battery energy storage. *35th Chinese Control Conference*, 2016.
- D. K. Khatod, V. Pant, and J. Sharma. A novel approach for sensitivity calculations in the radial distribution system. *IEEE Transactions on Power Delivery*, 2006.
- D. S. Kirschen and G. Strbac. *Fundamentals of Power System Economics*. New York, NY, USA:Wiley, 2004.
- S. Kraus, O. Shehory, and G. Taase. Coalition formation with uncertain heterogeneous information. *Proceedings of the second international joint conference on Autonomous agents and multiagent systems*, 2003.
- K. Lerman and O. Shehory. Coalition formation for large-scale electronic markets. *Fourth International Conference on MultiAgent Systems*, 2000.
- M. Lichman. UCI machine learning repository, 2013. URL <https://archive.ics.uci.edu/ml/datasets/ElectricityLoadDiagrams20112014>. Accessed: 2016-09-28.
- Comsar Energy. Smart grids, 2013. URL <http://comsar.com/business-areas/transmission-distribution/smart-grids>. Accessed: 2016-11-04.
- GridWise Alliance. The future of the grid: Evolving to meet americas needs. *Final Report*, 2014.
- The GridWise Architecture Council. Gridwise transactive energy framework version 1.0. *PNNL-22946*, 2015.
- US Congress. Energy independence and security act of 2007 (eisa-2007), 2007.

- World Trade Organization, n.d. URL <http://stat.wto.org/StatisticalProgram/WSDBStatProgramSeries.aspx>. Accessed: 2016-03-03.
- C. Merida-Campos and S. Willmott. Modelling coalition formation over time for iterative coalition games. *Third International Joint Conference on Autonomous Agents and Multi-agent Systems AAMAS 2004*, 2004.
- A. Mohd, E. Ortjohann, A. Schmelter, N. Hamsic, D. Morton, S. Westphalia, A. Sciences, D. Soest, and L. Ring. Challenges in integrating distributed energy storage systems into future smart grid. *2008 IEEE International Symposium on Industrial Electronics*, 2008.
- A. Nayyar, K. Poolla, and P. Varaiya. A statistically robust payment sharing mechanism for an aggregate of renewable energy producers. *2013 European Control Conference (ECC)*, 2013.
- Z. Pan, H. Yu, C. Miao, and C. Leung. Efficient collaborative crowdsourcing. *Thirtieth AAAI Conference on Artificial Intelligence*, 2016.
- P. S. Pillai and S. Rao. A resource allocation mechanism using coalition formation and the uncertainty principle of game theory. *2013 IEEE International Systems Conference (SysCon)*, 2013.
- P. Pinson, C. Chevallier, and G. Kariniotakis. Trading wind generation from short-term probabilistic forecasts of wind power. *IEEE Transactions on Power Systems*, 2007.
- M. Pycia. Stability and preference alignment in matching and coalition formation. *Econometrica*, 2012.
- T. Rahwan and N. R. Jennings. An improved dynamic programming algorithm for coalition structure generation. *7th International Conference on Autonomous Agents and Multiagent Systems*, 2008.
- T. Rahwan, S. D. Ramchurn, and N. R. Jennings. An anytime algorithm for optimal coalition structure generation. *Journal of Artificial Intelligence Research*, 2009.

- T. Rahwan, T. P. Michalak, M. Wooldridge, and N. R. Jennings. Coalition structure generation: A survey. *Artificial Intelligence*, 2015.
- R. Rajkumar, I. Lee, L. Sha, and J. Stankovic. Cyber-physical systems: The next computing revolution ragunathan. *47th ACM/IEEE Design Automation Conference DAC '10*, 2010.
- T. Sandholm, K. Larson, M. Andersson, O. Shehory, and F. Tohmé. Coalition structure generation with worst case guarantees. *Artificial Intelligence*, 1999.
- T. W. Sandholm and V. R. Lesser. Coalitions among computationally bounded agents. *Artificial Intelligence*, 1997.
- S. Sen and P. Dutta. Searching for optimal coalition structures. *Fourth International Conference on Multiagent Systems*, pages 287–292, 2000.
- O. Shehory and S. Kraus. Methods for task allocation via agent coalition formation. *Elsevier Science*, 1998.
- L. Sless, N. Hazon, S. Kraus, and M. Wooldridge. Forming coalitions and facilitating relationships for completing tasks in social networks. *13th International Conference on Autonomous Agents and Multiagent Systems*, 2014.
- W. Tang and J. Y. Zhang. Optimal battery energy storage system control in microgrid with renewable energy generation. *2015 IEEE International Conference on Smart Grid Communications (SmartGridComm): Data Management, Grid Analytics, and Dynamic Pricing*, 2015.
- O. M. Toledo, D. Oliveira Filho, and A. S. A. C. Diniz. Distributed photovoltaic generation and energy storage systems: A review. *Renewable and Sustainable Energy Reviews*, 2010.
- M. Vinyals, F. Bistaffa, A. Farinelli, and A. Rogers. Coalitional energy purchasing in the smart grid. *2012 IEEE International Energy Conference and Exhibition, ENERGYCON 2012*, 2012.

- T. Voice, S. D. Ramchurn, and N. R. Jennings. On coalition formation with sparse synergies. *11th International Conference on Autonomous Agents and Multiagent Systems*, 2012.
- J. Yamamoto and K. Sycara. A stable and efficient buyer coalition formation scheme for e-marketplaces. *AGENTS '01 Proceedings of the fifth international conference on Autonomous agents*, 2001.
- D. Yun Yeh. A dynamic programming approach to the complete set partitioning problem. *BIT Numerical Mathematics*, 1986.
- B. Zhang, R. Johari, and R. Rajagopal. Competition and coalition formation of renewable power producers. *IEEE Transactions on Power Systems*, 2015a.
- B. Zhang, R. Johari, and R. Rajagopal. Cournot games with uncertainty: Coalitions, competition, and efficiency. *ArXiv e-prints*, 2015b.

Appendix A

Impedance values for IEEE 69-bus test system

Chapter 5.2 introduced coalition formation of renewable resources and energy stores in an IEEE 69-bus test system (Khatod et al., 2006). This system is used for evaluation of coalition formation in a real power grid. Setting of the power grid was described in Section 5.2.2. Each line in the power grid has a specific impedance. We show the impedance that we used in our experiments in Table A.1 below. These values are also widely used in the power systems field.

Table A.1: Active (Z_{real}) and reactive (Z_{imag}) impedance values for IEEE 69-bus test system for power lines between nodes **N1** and **N2**. Values are shown per unit, a base impedance unit Z_b in a power grid with the base voltage $U_b = 12.66$ kV and the base power $P_b = 10$ MW is $Z_b = \frac{U_b^2}{P_b} = 16.03 \Omega$.

| N1 | N2 | Z_{real} | Z_{imag} | N1 | N2 | Z_{real} | Z_{imag} |
|-----------|-----------|-------------|-------------|-----------|-----------|--------------|-------------|
| 1 | 2 | 0.000311963 | 0.00074871 | 4 | 36 | 0.002121346 | 0.005240972 |
| 2 | 3 | 0.000311963 | 0.00074871 | 36 | 37 | 0.053096042 | 0.129963638 |
| 3 | 4 | 0.000935888 | 0.002246131 | 37 | 38 | 0.180813549 | 0.442425422 |
| 4 | 5 | 0.015660525 | 0.01834340 | 38 | 39 | 0.051286659 | 0.125471376 |
| 5 | 6 | 0.228356656 | 0.116299674 | 8 | 40 | 0.057900267 | 0.029511666 |
| 6 | 7 | 0.237777928 | 0.121103899 | 40 | 41 | 0.207080803 | 0.069505277 |
| 7 | 8 | 0.057525912 | 0.029324489 | 9 | 42 | 0.108563 | 0.055279781 |
| 8 | 9 | 0.030759517 | 0.015660525 | 42 | 43 | 0.126656834 | 0.064513875 |
| 9 | 10 | 0.510994811 | 0.168896576 | 43 | 44 | 0.177319567 | 0.090281989 |
| 10 | 11 | 0.116798814 | 0.038620975 | 44 | 45 | 0.175510184 | 0.089408494 |
| 11 | 12 | 0.44386045 | 0.146684835 | 45 | 46 | 0.992041209 | 0.332988927 |
| 12 | 13 | 0.642643047 | 0.212134598 | 46 | 47 | 0.488970249 | 0.164092351 |
| 13 | 14 | 0.651378001 | 0.215254225 | 47 | 48 | 0.189798073 | 0.062766884 |
| 14 | 15 | 0.660112955 | 0.218124281 | 48 | 49 | 0.240897554 | 0.073124044 |
| 15 | 16 | 0.122663712 | 0.040555144 | 49 | 50 | 0.316642084 | 0.161284687 |
| 16 | 17 | 0.233597628 | 0.077241951 | 50 | 51 | 0.060770323 | 0.030946694 |
| 17 | 18 | 0.002932449 | 0.00099828 | 51 | 52 | 0.090469167 | 0.046045686 |
| 18 | 19 | 0.204397925 | 0.067571109 | 52 | 53 | 0.443298918 | 0.225798562 |
| 19 | 20 | 0.131398666 | 0.0434252 | 53 | 54 | 0.649506226 | 0.330805188 |
| 20 | 21 | 0.213132879 | 0.070441165 | 11 | 55 | 0.125533768 | 0.038121835 |
| 21 | 22 | 0.008734954 | 0.002870056 | 55 | 56 | 0.002932449 | 0.000873495 |
| 22 | 23 | 0.099266513 | 0.03281847 | 12 | 57 | 0.461330358 | 0.152487341 |
| 23 | 24 | 0.216065327 | 0.071439446 | 57 | 58 | 0.002932449, | 0.00099828 |
| 24 | 25 | 0.467195256 | 0.154421509 | 3 | 59 | 0.002745271 | 0.006738393 |
| 25 | 26 | 0.192730522 | 0.063702772 | 59 | 60 | 0.039931218 | 0.097644308 |
| 26 | 27 | 0.10806386 | 0.035688527 | 60 | 61 | 0.065699333 | 0.076742811 |
| 3 | 28 | 0.002745271 | 0.006738393 | 61 | 62 | 0.018967329 | 0.022149348 |
| 28 | 29 | 0.039931218 | 0.097644308 | 62 | 63 | 0.001123066 | 0.001310243 |
| 29 | 30 | 0.24819748 | 0.082046175 | 63 | 64 | 0.454404788 | 0.530898028 |
| 30 | 31 | 0.043799555 | 0.014475067 | 64 | 65 | 0.193416839 | 0.226048132 |
| 31 | 32 | 0.218997776 | 0.072375333 | 65 | 66 | 0.025580937 | 0.029823629 |
| 32 | 33 | 0.523473317 | 0.175697361 | 66 | 67 | 0.005740113 | 0.007237533 |
| 33 | 34 | 1.065664393 | 0.352268218 | 67 | 68 | 0.067945464 | 0.085664942 |
| 34 | 35 | 0.919665876 | 0.304038793 | 68 | 69 | 0.000561533 | 0.00074871 |