A PROBABILISTIC AND ECONOMIC ANALYSIS OF A MAJOR COMPONENT SHARED AMONG ELECTRIC UTILITIES

by

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TABLE OF CONTENTS

Page

Page

APPENDIXES

LIST OF FIGURES

Figure | Page

LIST OF TABLES

Chapter 1

INTRODUCTION

Increased competition, coupled with the recessive climate of today's marketplace, require effective control and maintenance of inventory systems. The proper balance between inventory investment and profits can make the difference between the success or failure of a business (1). The fundamental reason for maintaining items in inventory is that it is physically impossible or economically unsound to have an item arrive at the precise instant a demand for that item occurs. Businesses thus typically carry in inventory raw materials, finished goods, and repair or replacement components.

The manufacturing industries realized the need for inventories in the late 1800s because finished products were being produced in lots, production set-up costs were high, and storage of the finished goods in factory warehouses was necessary. Prior to 1940 the inventory systems were treated as deterministic systems since component needs were assumed to be known quantities (2). The simple lot-size formula was derived first by Harris and later by Wilson, who used it to analyze inventory systems (3). During the early 1940s, the "Christmas tree model" or the "newsboy problem" was the first attempt to model stochastically a demand for a product (4). After 1945, the newly developed fields of management science and operations research also investigated the stochastic nature of inventory problems.

Once of the early analytical models developed to describe inventory systems was queueing theory. A queue or waiting line is formed by randomly generated demands at a service facility. Queueing theory can be used to predict the steady-state probability of a system being in a particular state at a specified time. The pioneering work using queueing theory was performed by Erlang during 1901 to 1920 (5), while the stochastic treatment of queueing theory was studied by Kendall during 1951 to 1953 (6,7). Since that time numerous studies have been conducted using queueing theory to model inventory systems. Implicit assumptions of queueing theory are an infinite calling source and the independence of different states of the system (8,9).

There is considerable incentive to evaluate the economic feasibility of spare pool inventory systems for high cost components. A stochastic process is used to determine the demand on the pool by combining two frequently researched areas of inventory systems; namely, the stockpiling of low cost components and the repair or replacement of high cost components. Low cost components are usually stockpiled if their failure could disrupt the system operation. Upon failure of a low cost component, the failed component is replaced with a new component and then discarded. The loss of production is insignificant if the installation time is short. By contrast, high cost components are not usually held in inventory and must be repaired or replaced upon failure. The loss of production can be minimized by shifting manpower and other resources to another production line during the repair/replacement and/or installation time.

A different situation arises for certain component failures in an electrical power plant where the only product is electricity. During component failures that result in shutdown or derating of the plant, manpower and other resources cannot be shifted to an alternate production line. Of particular interest are components with low probabilities of failure and large capital costs (e.g., typically hundreds of dollars). Main power transformers, coolant water pumps, turbine rotors, and nuclear safety equipment failures are examples of components that can cause a significant loss of output capacity. Compounding the problem is the extended length of time a plant could be shut down or derated because of a long repair/replacement time for these components.

A management scheme, known as Pooled Inventory Management, was developed by the General Electric Company (GE) to help prevent extended outages at nuclear power plants (10). In this program, components with small failure probabilities and long repair/replacement times, and whose failure would result in a significant reduction in plant capacity are placed in a spare pool. The costs of the spare pool operation are shared among the members of the pool thereby reducing the added cost of the spare purchase while providing the advantages of a spare component to each pool member.

The economic evaluations of such a spare-component pool is generally quite complex as a result of differing economic procedures used by pool members, differences in the capacity of members' plant components, and the stochastic nature of the failure process. A 1974 study conducted by GE proposed a method to evaluate a spare-component pool for generator step-up transformers at electrical power stations (11). While this study was somewhat simplistic (e.g., infinite pool lifetime, no escalation of costs and all costs are capitalized), it did demonstrate the economic attractiveness of sharing a spare step-up transformer among several utilities. In 1985, Shultis (12) performed a more detailed analysis of the spare-pool problem, in which two component management plans, designated by Plan A and Plan B, were evaluated.

Under Plan A, each plant repairs or replaces its failed component when a failure occurs. Under Plan B, a spare component is purchased and placed in storage prior to failure. If at the time of a component failure at a plant the spare is available, the failed component is removed and replaced by the spare. The spare is used as a temporary substitute until the failed component is repaired or replaced.

Shultis estimated the probability of spare availability using binomial theory to calculate the fraction of each year the spare is available. As a consequence, the spare is used only if it is immediately available at the time of a plant failure and the repair/replacement time is less than one year. Because the components considered in this study have very small failure rates, the probability of multiple failures of a single component was considered negligible. Similarly, the probability of the spare failing during the temporary installation time was neglected. In addition, the economic costs are estimated on a yearly discounted basis with the assumption that all costs are paid at the end of each yearly interval.

The purpose of the present study was to refine some of the probabilistic and economic models used by Shultis and to assess the importance of such refinements. Specific objectives were to:

(a) develop a general Monte Carlo method to describe the spare component availability and fraction downtime (i.e., fraction of the repair/replacement time the plant is shutdown or derated),

(b) develop a model for continuous-time cost analysis in which costs are evaluated at the time they are incurred and not at the end of the year as was done in the earlier study,

(c) compare how different failure-model assumptions affect the subsequent economic analysis of the spare-component pool, and

(d) compare results obtained via the continuous-time cost model with the year-end cost model.

As in the previous study, total costs are estimated for two component plans, designated by Plan A and Plan B. Under Plan A, each plant repairs or replaces the failed component when a failure occurs. The plants are assumed to be operating independently in the sense that a failure in one plant has no effect on another plant. In the case of a component failure, the plant will remain shutdown or derated for a fixed period of time. There is no provision for an operating plant with a failed component to borrow a substitute component from another plant.

Under Plan B, a single spare component is purchased and placed in storage prior to component failure. If at the time of a component failure at a plant the spare is available, the failed component is removed and replaced by the spare and used as a temporary substitute until the failed component is repaired or replaced. The plant with the failed component has the option to use the spare should it become available prior to the arrival of the repaired/replacement component. If the spare is installed during the repair/replacement time the plant avoids a portion of the costs that would have to be paid under Plan A. However, the plant must also pay for a portion of the spare-pool costs in addition to the expenses incurred during removal and installation of the failed component and spare. During the time the spare is not installed, the failed plant incurs costs as if operated under Plan A.

This study considers only the case when the repair/replacement time is a fixed period of time. In addition, the repair/replacement time is assumed to be the same for each plant with a similar component regardless of the component's capacity or operating history.

For the Plan B analysis, the fraction of the repair/replacement time the plant is shut down or derated due to the failure of the component under consideration was designated as the "fraction downtime". Because of the addition of a spare to the system and a finite number of plants in the pool, standard queueing theory models cannot be used to estimate the fraction downtime. An exact analytical solution for the fraction downtime that takes into account all possible combinations of pool lifetimes, removal and installation times, component repair/replacement times, etc. appears to be, if even possible, very difficult. However, because of the uncertainties in the component failure rates and the simplified engineering economic analysis used, the complexity involved in a detailed theoretical approach may not be warranted.

An approximate solution for the fraction downtime can be obtained using renewal theory techniques (13). In a system described by standard renewal theory, a plant can be in only one of two states; that is, a plant operating with an original or repaired/replacement component is said to be in the "on" state. A plant that is down or operating with the spare installed is in the "off" state. The spare is available at the time of a plant failure only if all plants in the system are in the "on" state excluding the "just failed" plant. Standard renewal theory is, however, limited to estimating the fraction downtime for the case when the spare is used only by the "just failed" plant if immediately available at the time of failure and the spare failure rate is zero.

Another approximate method to estimate the expected fraction downtime can be obtained by simulating the component-failure problem using a Monte Carlo analysis (14,15). An approach similar to that used by Widawsky (14) and Emshoff and Sisson (15) was used where the variable time-increment method was used. The random failure times were simulated by generating a sequence of random numbers from the appropriate probability distributions characteristic of the desired failure model. The simulated failure times for each pool history were then analyzed to determine when the spare would be available for use as well as the fraction of the repair/replacement time for which the spare could be installed at a failed plant. A large number of pool histories were simulated and the results of each history were then averaged to estimate the probability of spare availability and the fraction downtime given an initial set of conditions.

A computer code, SIMULATION, was written based on the Monte Carlo simulation technique for three different failure models. Model 1 had the following properties: (i) the spare has zero failure probability, (ii) the components are installed and removed instantaneously, and (iii) the spare is used when it becomes available. Model 2 was identical to Model 1 with the exception that the spare is used only if it is available at the time of a component failure. Model 3 was also similar to Model 1 with the exception that the spare has a failure rate equal to the failure rate of the components installed in the plants.

To verify the computer simulation the results of Model 1 were compared to the results predicted by renewal theory. Renewal theory can determine the probability that a system is in either an "on" or "off" condition. For the case of a spare with zero failure probability and a fixed repair/replacement time (Model 1) the probability of spare availability at a given time to that estimated by the computer simulation should agree with the probability that all plants are operating at time t as estimated by renewal theory.

As a second phase of evaluating the effectiveness of a spare-component pool, the failure models were then combined with economic models to determine the present worth costs of the two component-management plans. A simple economic analysis approach was used that allowed for yearly varying escalation and discount rates (16). Unlike similar analyses, the economic model formulated in this research project estimated present worth costs based on when the costs were incurred instead of the typical discrete year-end method.

The economic model used in this study divided the costs into four major groups. The failure-dependent variable costs included the cost of purchasing replacement energy during a failure. The variable costs during plant operation included the operation and maintenance costs associated with an operating component. The failure-dependent fixed costs included the repair/replacement component costs, the failed component salvage value, and the cost of temporarily installing the spare. The annual fixed costs included reserve capacity costs, spare-pool maintenance costs, spare salvage value, and the used component salvage value.

A computer code, KSUSPARE, was written to calculate the estimated Plan A and Plan B present worth costs. The analytical methods used in KSUSPARE were then compared to the analytical methods used in PC-SPARE. The fraction downtime estimated by SIMULATION for all three simulation models and renewal theory were used in KSUSPARE to estimate Plan A and Plan B present worth costs. Finally, an example problem compared the numerical differences between PC-SPARE (12) and KSUSPARE for three variations of initial conditions.

The present study is divided into three major sections. Chapters II and III present the development of the failure and economic models, respectively, while in Chapter IV the continuous-time and year-end cost models are compared. Chapter V summarizes the study and suggests areas for further study.

Chapter 2

FAILURE MODELS

To assess the economic feasibility of creating a spare-component pool, it is first necessary to estimate the expected or average downtime for a given plant over the spare-pool lifetime. The downtime estimate includes the interdependence of the possible failure times for all plants in the spare pool and availability of the spare in the repair/replacement time interval. In this study the repair/replacement time for the failed component is a fixed time. Simplifying assumptions are that similar plant components have equal failure rates and equal repair/replacement times.

An exact analytical solution for the average downtime that accounts for all possible combinations of pool lifetimes, component removal and installation times, component repair/replacement times, etc. appears to be very difficult. For a negligible spare component failure rate and spare usage assumptions, the average downtime can be estimated by renewal theory. The average downtime for the more general case conditions must be estimated using Monte Carlo techniques.

2.1 Plant Operation as a Renewal Process

Consider a single plant with a component failure rate λ when the original component operates. The plant is either "on" (i.e., operating) or "off" (i.e., shutdown). A system that can be described as being in one of two states, "on" or "off", is a renewal process.

A renewal process is a series of points on the interval $(0,\infty)$ where the times between failures are independent and identically distributed random variables (17). Let T_{on} be a random variable defined as the time before failure of the plant component. The cumulative distribution function (CDF) for the random variable T_{on} is given by

$$F_{on}(t) = \int_{-\infty}^{t} f_{on}(x)dx, \tag{2.1}$$

where $f_{on}(x)$ is the associated probability density function. Let T_{off} be the random variable defined as the repair/replacement time. The plant, therefore, operates for a random time T_{on} and is then down for a random time T_{off}. Let T be the time the plant begins operating after a repair (i.e., where $T = T_{on} + T_{off}$, and let f(x) be its density function. The function g(t) is defined as the probability the system is operating at time t given by

$$g(t) = \int_{0}^{\infty} \text{Prob}\{\text{plant operating at } t \mid T = x\}\ f(x)\ dx. \tag{2.2}$$

For the case $x \leq t$ the probability the plant is operating at time t is equal to the probability the plant was operating at time t-x (see Fig. 2.1). For the case $x > t$ the probability that the plant is operating at time t is equal to the probability that t is in the interval $[0,T_{on}]$. Thus,

$$\text{Prob}\{\text{plant operating at } t \mid T{=}x\} = \begin{cases} g(t-x) & , \quad x \leq t \\ \text{Prob}\{t < T_{on} \mid T{=}x\} & , \quad x > t. \end{cases} \tag{2.3}$$

Substitution of this result into Eq. (2.2) yields

$$g(t) = \int_{0}^{t} g(t-x)\ f(x)\ dx + \int_{t}^{\infty} \text{Prob}\{t < T_{on} \mid T < x\}\ f(x)\ dx. \tag{2.4}$$

The second integral in the above equation is the mathematical expression for Prob $\{t < T_{on}$ and $t < T\}$. Because $T_{on} \leq T$,

$$\int_t^{\infty} \text{Prob}\{t<T_{on} \mid T=x\}\, f(x)dx = \text{Prob}\{t < T_{on} \text{ and } t < T\} = \text{Prob}\{t < T_{on}\}.$$

Thus, the results from the above expression can be combined with Eq. (2.1) to give,

$$\int_t^{\infty} \text{Prob}\{t < T_{on} \mid T=x\}\, f(x)dx = 1 - \int_{-\infty}^{t} f_{on}(x)dx. \tag{2.5}$$

The above expression can now be combined with Eq. (2.4) to give

$$g(t) = 1 - \int_{-\infty}^{t} f_{on}(x)dx + \int_0^t g(t-x)\ f(x)\ dx\ . \tag{2.6}$$

Equation (2.6) is a renewal equation for which expressions for $f_{on}(t)$ and $g(t-x)$ must be found.

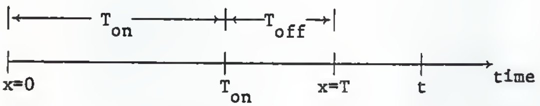

FIG. 2.1. Graphical representation of single plant operation.

Because the component failure rate is constant, the time intervals between each component failure are independently and identically distributed according to the negative exponential distribution (17). The value of x is defined only for positive values of time, thus

$$1 - \int_{-\infty}^{t} f_{on}(x)dx = 1 - \int_0^t f_{on}(x)dx = 1 - \int_0^t \lambda\ e^{-\lambda x}\ dx = e^{-\lambda t}. \tag{2.7}$$

The $\text{Prob}\{T \le t\} = 0$ for $t \le T_{off}$ because the plant must have completed a full operating cycle (i.e., $T_{on}+T_{off}$) before operating again. The random variable T_{on} can equal zero but the minimum value of T_{off} is the fixed repair/replacement time. Thus, the CDF for T is

$$F(T) = \mathrm{Prob}\{T \leq t\} = \mathrm{Prob}\{T_{on} \leq t-T_{off}\}$$

$$= \begin{cases} 0 & , \quad t \leq T_{off} \\ 1 - \exp[-\lambda(t-T_{off})] & , \quad t > T_{off} \, . \end{cases} \tag{2.8}$$

The density function f(T) found by differentiation of F(T) is

$$f(T) = \begin{cases} 0 & , \quad t \leq \quad T_{off} \\ \lambda \exp[-\lambda(t-T_{off})] & , \quad t > \quad T_{off}. \end{cases} \tag{2.9}$$

The above result for the density function can now be substituted into Eq. (2.7) to give

$$g(t) = \begin{cases} e^{-\lambda t} & , \; t \leq T_{off} \\ e^{-\lambda t} + \lambda \exp(\lambda \, T_{off}) \int_{T_{off}}^{t} e^{-\lambda x} \, g(t-x)dx & , \; T_{off} < t \leq \infty. \end{cases} \tag{2.10}$$

2.2 Relationship of Renewal Process to Spare-Component Problem

Consider the case of one plant in the spare pool. The failure rate of a one plant system is λ, when the original component operates. If the spare component is perfect, i.e., has zero failure rate, the system failure rate is zero when the plant is shutdown or operating with the spare installed. For these specific conditions the system is either "on" (i.e., operating without the spare component) or "off" (i.e., plant is shutdown or operating with its original component). A system that can be described as either "on" or "off" is a renewal process. Thus, T_{off} equals the plant component repair/ replacement time during which the plant is shutdown (or operates with the perfect spare as a temporary substitute). Hence, the probability that the plant is operating in its original state is given by Eq. (2.10) with $T_{off} = T_{repl}$. A substitution of variables can be made where $x = t - u$ and $dx = - du$. Equation (2.10) then becomes

$$g(t) = \begin{cases} e^{-\lambda t} & , \; 0 < t \le T_{repl} \\ e^{-\lambda t} + \lambda \exp(\lambda T_{repl}) \int_0^{t-T_{repl}} \exp[-\lambda(t-u)] g(u) du, & T_{repl} < t \le \infty . \end{cases} \tag{2.11}$$

2.3 Solution Techniques for the Renewal Equation

The evaluation of Eq. (2.10) can be accomplished by using three methods: analytical, numerical, and Laplace transforms.

2.3.1 Analytical Solution to the Renewal Equation for the First Three Intervals

The renewal equation of Eq. (2.11) can be cast into a somewhat simpler form by introducing the variables $\beta = \lambda T_{repl}$, $\tau = \lambda t$, and $\tau' = \lambda u$. Equation (2.11) then becomes

$$g(\tau) = \begin{cases} e^{-\tau} & , \quad 0 < \tau \le \beta \\ e^{-\tau} + e^{\beta} \int_0^{\tau-\beta} e^{-(\tau-\tau')} g(\tau') \, d\tau' & , \quad \beta < \tau \le \infty . \end{cases} \tag{2.12}$$

Let $g_i(\tau)$ be the solution to Eq. (2.12), where $g_i(\tau) = g(\tau)$ for $(i-1)\beta < \tau \le i\,\beta$. The solution to Eq. (2.12) for the first three τ intervals is listed below.

<u>First interval $(0 < \tau \le 1)$</u>

$$g_1(\tau) = e^{-\tau} . \tag{2.13}$$

<u>Second interval ($1 < \tau \leq 2$)</u>

$$g_2(\tau) = g_1(\tau) + e^{\beta} \int_0^{\tau-\beta} e^{-(\tau-\tau')} g_1(\tau')d\tau',$$

which, upon evalution gives

$$g_2(\tau) = e^{-\tau} [1+e^{\beta}(\tau-\beta)] . \tag{2.14}$$

<u>Third interval ($2 < \tau \leq 3$)</u>

$$g_3(\tau) = g_1(\tau) + e^{\beta} \left\{ \int_0^{\beta} e^{-(\tau-\tau')} g_1(\tau')d\tau' + \int_{\beta}^{\tau-\beta} e^{-(\tau-\tau')} g_2(\tau')d(\tau') \right\},$$

which evaluates to

$$g_3(\tau) = e^{-\tau} \left\{ 1 + e^{\beta} \left[\tau-\beta + e^{\beta} [(\tau/2-\beta)(\tau-2)] \right] \right\} . \tag{2.15}$$

The above integral equation demonstrates the dependence of $g(\tau)$ upon $g_{i-1}(\tau)$. At each evaluation time τ, the value for $g(\tau)$ depends upon the previous values of $g(\tau)$ over the interval $(0,(i-1)t^{i})$. Thus, the expressions for $g_i(\tau)$ get more complex as i increases. An alternative for evaluating $g_i(\tau)$ analytically, especially for large τ, is to solve Eq. (2.12) numerically.

2.3.2 Numerical Solution to the Renewal Equation

The method of numerical integration chosen to evalute the renewal equation was the trapezoid rule. Consider an integrable function $f(x)$ that is to be evaluated over the interval $a \leq x \leq b$. The integral I is defined as

$$I = \int_a^b f(x)\, dx. \tag{2.16}$$

The interval [a,b] is then divided into N equal subintervals of width Δx, where

$$\Delta x = \frac{b-a}{N} . \tag{2.17}$$

The area under the curve in each subinterval bounded by x_{j-1} and x_j can be approximated by

$$\int_{x_{j-1}}^{x_j} f(x)dx \cong \frac{f_{j-1} + f_j}{2} (\Delta x) \tag{2.18}$$

where f_{j-1} and f_j denote the function f(x) evaluated at x_{j-1} and x_j respectively. Thus, Eq. (2.16) can be approximated by (18)

$$\int_a^b f(x)dx \cong \frac{\Delta x}{2} (f_1 + f_{N+1} + 2 \sum_{j=2}^{N} f_j) , \tag{2.19}$$

where $f_1 = f(a)$ and $f_{N+1} = f(b)$.

The trapezoid rule can be applied to the renewal equation. Starting at time zero each repair/replacement interval β is split into N equal subintervals (see Fig. 2.2).

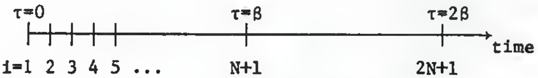

FIG. 2.2. The time line broken into interval lengths of β with N subintervals.

From Figure 2.2 it can be seen $\tau_i = i\Delta\tau$ for $i = 0, 1 \ldots \infty$. For the case of $i \leq N$

$$g(\tau_i) \equiv g_i = e^{-\tau_i} = e^{-i\Delta\tau_i} \quad , \quad \tau_i \leq \beta \tag{2.20}$$

while for i > N

$$g(\tau_i) = g_i = e^{-\tau_i} \left\{ 1 + e^{\beta} \int_0^{\tau_i - \beta} e^{\tau'} g(\tau') d\tau' \right\}, \ \beta < \tau_i \leq \infty. \tag{2.21}$$

Applying the trapezoid approximation to Eq. (2.21) gives

$$g_i \cong e^{-\tau_i} \left\{ 1 + e^{\beta} \frac{\Delta\tau}{2} \left(g_1 + e^{\tau_i - \beta} g_{i-N+1} + 2 \sum_{j=2}^{i-N} e^{\lambda\tau_i} g_j \right) \right\}, \ i > N. \tag{2.22}$$

Because the analytical solution to the renewal equation is known for $\tau \leq \beta$, the first intervals of i up to N are calculated using Eq. (2.20). The remaining intervals are calculated using the results from i up to N and Eq. (2.22).

A computer code was written to evaluate Eq. (2.22). The numerical integration was performed for a maximum time such that the result for $g(\tau \geq 4)$ was within 0.01% of the asymptotic value (see Section 2.3.3). The result from the numerical integration was then compared to the analytical results for $g(\tau)$ derived in Eq. (2.13), (2.14), and (2.15) for the first three intervals. The results indicate excellent agreement between the analytical and the numerical approximation for a very small number of subintervals. It is this numerical procedure that was used in the subsequent simulation of plant failures outlined in Section 2.5.

The trapezoid solution technique is exact when compared to the analytical solution over the first three intervals. The $g_i(\tau)$ functions being integrated over these intervals are equal to zero, a constant, and a linear term, respectively, for which a straight line representation is exact. Thus, a fine quadrature mesh is necessary only for intervals at time $\tau \geq 3$.

The results of an example problem for a single plant are represented graphically in Figure 2.3. A study of this figure indicates a negative exponential behavior in the first repair/replacement time interval, followed by oscillations in the g(t) value. After several repair/replacement time intervals the value for g(t) stabilizes to an asymptotic value. Thus, for long lifetimes, the approximation can be made that the probability a plant is operating at time t equals the asymptotic solution to the renewal equation.

2.3.3 Solving the Renewal Equation Using Laplace Transforms

The renewal equation for our one component system can be rewritten as

$$g(t) = \begin{cases} e^{-\lambda t} & 0 < T \leq T_{repl} \\ e^{-\lambda t} + \lambda \exp(\lambda T_{repl}) \displaystyle\int_{T_{repl}}^{t} e^{-\lambda x} g(t-x)dx \;, & T_{repl} < t \leq \infty. \end{cases} \tag{2.23}$$

Recall the Laplace transformation is defined as (19)

$$\mathcal{L}\{f(t)\} = \int_0^\infty dt\ e^{-st} f(t) \equiv \bar{f}(s). \tag{2.24}$$

The Laplace transform of Eq. (2.23) thus is

$$\bar{g}(s) = \int_0^\infty dt\ e^{-st} e^{-\lambda t} + \lambda \exp(\lambda T_{repl}) \int_{T_{repl}}^\infty dt\ e^{-st} \int_{T_{repl}}^{t} dx\ e^{-\lambda x} g(t-x). \tag{2.25}$$

The first term on the right hand side in the above equation is

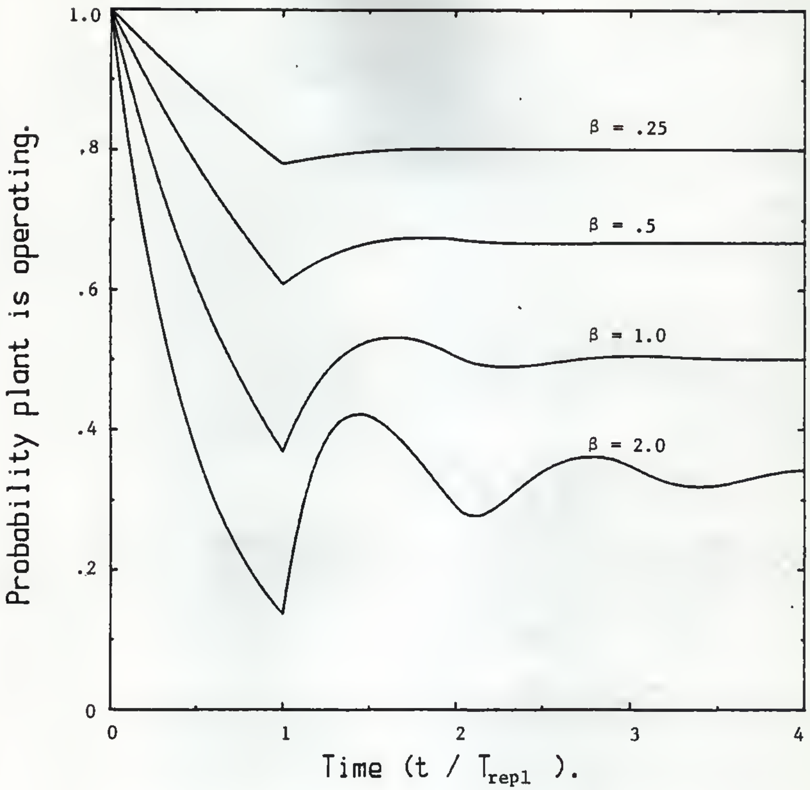

FIG. 2.3. The numerical solution to the renewal equation for a single plant and different β values.

$$\int_0^\infty dt\, e^{-st}\, e^{-\lambda t} = L\{e^{-\lambda t}\} = \frac{1}{\lambda+s}\ . \tag{2.26}$$

To evaluate the second term in Eq. (2.25), a variable is introduced such that $w = x - T_{repl}$ or $x = w + T_{repl}$. Thus,

$$\int_{T_{repl}}^{t} dx\, e^{-\lambda x} g(t-x)dx = \int_0^{t-T_{repl}} dw\, e^{-\lambda w} \exp(-\lambda\, T_{repl})\ g(t-w-T_{repl}). \tag{2.27}$$

A second change in variables is now introduced, namely $v = t - T_{repl}$ so that the second term on the right hand side of Eq. (2.25) becomes

$$\lambda \int_0^\infty dw\, e^{-s(v+T_{repl})} \int_0^v dw\, e^{-\lambda w}\ g(v-w).$$

Recall from Laplace transforms (19)

$$L\left\{\int_0^t f(u)\ g(t-u)du\right\} = L\{f(u)\}L\{g(u)\} = \bar{f}(u)\ \bar{g}(u), \tag{2.28}$$

where f(u) and g(t-u) are some arbitrary functions. The second term on the right hand side of Eq. (2.15) can now be rewritten as

$$\lambda \exp(-s\ T_{repl}) \int_0^\infty dv\, e^{-sv} \int_0^v dw\, e^{-\lambda w}\ g(v-w) = \lambda \exp(-s\ T_{repl})$$

$$L\left\{\int_{w=0}^{v} e^{-\lambda w}\ g(v-w)dw\right\} = \lambda \exp(-s\ T_{repl})\ L\left\{e^{-\lambda w}\right\} \bar{g}(s)$$

$$= \lambda \exp(-s\ T_{repl})\ \frac{1}{s+\lambda}\ \bar{g}(s).$$

Equation (2.25) can now be written as

$$\bar{g}(s) = \frac{1}{s+\lambda} + \lambda \exp(-s\, T_{repl}) \frac{\bar{g}(s)}{s+\lambda} , \qquad (2.29)$$

which upon solving for $\bar{g}(s)$ gives

$$\bar{g}(s) = \frac{1}{s + \lambda - \lambda \exp(-s\, T_{repl})} . \qquad (2.30)$$

The solution of the renewal equation g(t) can, in principle, be obtained by taking the inverse Laplace transform of $\bar{g}(s)$, i.e.,

$$g(t) = L^{-1} \left\{ \frac{1}{s + \lambda - \lambda \exp(-s\, T_{repl})} \right\} , \quad T \geq T_{repl} \qquad (2.31)$$

This inversion appears to be very difficult, or if possible, to perform analytically. However, it is possible to obtain from it the asymptotic solution. The asymptotic solution for g(t) as $t \to \infty$ can now be found by application of the final value theorem of Laplace transforms (19)

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s\, \bar{f}(s) , \qquad (2.32)$$

where f(t) is some arbitrary function and $\bar{f}(s) = L\{f(t)\}$.

Thus,

$$\lim_{t\to\infty} g(t) = \lim_{s\to 0} s\, \bar{g}(s) = \lim_{s\to 0} \frac{s}{s + \lambda - \lambda \exp(-sT_{repl})} . \qquad (2.33)$$

Application of L'Hospital's rule to the above limit yields

$$\lim_{s\to 0} s\, \bar{g}(s) = \frac{1}{1 + \lambda\, T_{repl}} = \frac{1/\lambda}{1/\lambda + T_{repl}} . \qquad (2.34)$$

Note that $1/\lambda$ is the expected time the system is operational and T_{repl} is the expected time the system is down. This result is reasonable because the system should stabilize after a certain number of repair/replacement time intervals. Thus, the probability the plant is operating is equal to the mean time to failure divided by the total cycle time. This asymptotic solution for g(t) agrees with the asymptotic probability derived by Parzen (17) for a general distribution $F_{on}(t)$ given by

$$\lim_{t\to\infty} g(t) = \frac{1}{\lambda}\int_0^\infty [1 - F_{on}(t)]dt = \frac{E[T_{on}]}{E[T_{on} + T_{off}]} \; . \qquad (2.35)$$

2.3.4 Consideration of Multiple Plant Spare-Component Problem

The renewal process analogy can be extended to include the possibility of more than one plant belonging in the spare-component pool. The N members of the spare-component pool comprise a system of N independent plants where the probability the entire system is operating with original (or repaired/replacement) plant components at time t, denoted by $g_s(t)$, is simply

$$g_s(t) = [g(t)]^N, \qquad (2.36)$$

where g(t) is the probabililty of a single plant operating with an original (or repaired/replacment) plant component at time t. The quantity $g_s(t)$ is thus the probability the spare component is unused and available should it be needed. For the special case of a perfect spare (i.e., zero failure rate), g(t) is given by the renewal equation solution presented in the previous section. Thus, the asymptotic probability that the entire system is operational at time t is given by

$$g_s(t) = \left(\frac{1}{1+\lambda T_{repl}}\right)^N \quad , \quad \lim_{t\to\infty} \; . \qquad (2.37)$$

2.3.5 Probability of Spare Availability

The probability that the spare is available at any time t estimated by renewal theory is the probability that all plants are operating at time t given by Eq. (2.36). The results of an example problem are graphically represented in Figure 2.4. A study of the figure indicates that the relative minimum value and successive oscillations in the $g_s(t)$ values are sensitive to the product of the component failure rate and the repair/replacement time and insensitive to the number of plants in the spare-component pool.

2.3.6 Consideration of Multiple Spares Problem

The renewal process analogy can also be extended to include the possibility a system of N plants will be operating when two spare components are available. The probability that one spare will be available at the time of a plant failure is the sum of the probability that all plants in the system are operating plus the probability that at most one plant is down. The probability of spare availability for the case of two spares is thus

$$g_s(t) = [g(t)]^N + N[g(t)]^{N-1}[1-g(t)]. \qquad (2.38)$$

Substituting the analytical solution for g(t) for $t < T_{repl}$ and using the asymptotic solution to g(t) for large t, the probability the entire system is initially operating or operating after the pool has been in place for a long time is

$$g_s(t) = \begin{cases} e^{-N\lambda t} + N\, e^{-(N-1)\lambda t}\,(1 - e^{-\lambda t}) \quad , & t < T_{repl} \\ \left(\dfrac{1}{1+\lambda\ T_{repl}}\right)^N + N \left(\dfrac{1}{1+\lambda\ T_{repl}}\right)^{N-1} \left(1 - \dfrac{1}{1+\lambda\ T_{repl}}\right) , & t >> T_{repl} \end{cases} \qquad (2.39)$$

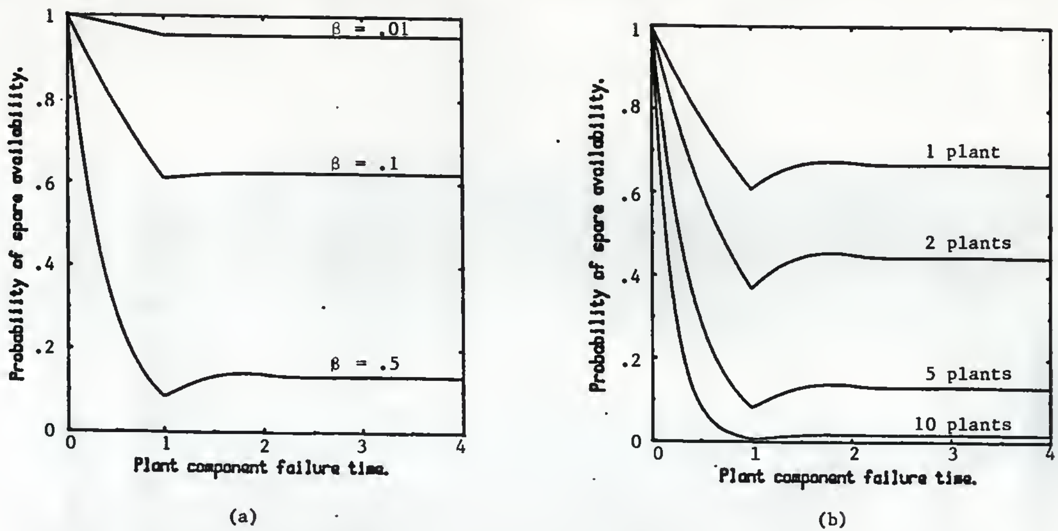

FIG. 2.4. Probability of spare availability estimated by renewal theory for (a) different values of β with 5 plants in the spare pool and (b) different numbers of plants in the spare pool with $\beta = .5$.

In general, for the case of k spares the probability that at least one spare will be available is

$$g_s(t) = \sum_{j=0}^{k-1} \binom{N}{j} [g(t)]^{N-j} [1-g(t)]^j \tag{2.40}$$

so that the initial and asymptotic system operational probabilities become

$$g_s(t) = \begin{cases} \sum_{j=0}^{k-1} \binom{N}{j} e^{-(N-j)\lambda t}(1-e^{-\lambda t})^j \quad , \quad 0 < t < T_{repl} \\ \sum_{j=0}^{k-1} \binom{N}{j} \left(\frac{1}{1+\lambda\ T_{repl}}\right)^{N-j} \left(1 - \frac{1}{1+\lambda\ T_{repl}}\right)^j \quad , \ t >> T_{repl} \ . \end{cases} \tag{2.41}$$

For the remainder of this present study only the case of a single spare-component pool is considered.

2.3.7 Estimation of Fractional Downtime Using Renewal Theory

The fraction downtime, denoted as f_{DT}, is defined as the effective fraction of the repair/replacement time that a plant is shut down or derated due to a failure. The fraction downtime can be expressed as

$$f_{DT} = \frac{\text{effective plant shutdown or derated time due to a failure}}{T_{repl}} \ .$$

For Plan A, the plant is shut down the entire repair/replacement time (i.e., fraction downtime = 1). For Plan B, the plant may be shutdown or derated only a portion of the repair/replacement time depending on the availability of the spare.

In this section the fraction downtime is derived for the following assumptions:

1) The removal time is the same for the failed component and the spare. The installation time is the same for the repaired/replacement component and the spare.
2) The spare's failure rate is zero (i.e., a perfect spare).
3) The spare is removed and returned to storage (and thus made available to other plants) when the repaired/replacement component is installed and returned to operation.

Under the assumptions listed above, the fractional downtime can be expressed as

$$f_{DT} = \left\{\begin{array}{l}\text{fraction of the interval} \\ T_{repl} \text{ required to remove} \\ \text{the failed component, in-} \\ \text{stall and remove the spare,} \\ \text{and install the repair/} \\ \text{replacement component}\end{array}\right\} + \text{Prob}\left\{\begin{array}{l}\text{spare is not} \\ \text{available} \\ \text{at the time} \\ \text{the failed} \\ \text{component is} \\ \text{removed}\end{array}\right\}\left\{\begin{array}{l}\text{remaining fraction} \\ \text{of the } T_{repl} \\ \text{interval not used} \\ \text{for removal or} \\ \text{installation}\end{array}\right\},$$

where

$$\text{Prob}\left\{\begin{array}{l}\text{spare is not} \\ \text{available} \\ \text{at the time} \\ \text{the failed} \\ \text{component is} \\ \text{removed}\end{array}\right\} = 1 - \text{Prob}\left\{\begin{array}{l}\text{spare is not} \\ \text{installed in} \\ \text{either of the} \\ \text{other plants} \\ \text{in the spare} \\ \text{pool}\end{array}\;\middle|\;\begin{array}{l}\text{spare has not} \\ \text{failed during} \\ \text{the time it is} \\ \text{installed in a} \\ \text{plant}\end{array}\right\}.$$

The above expression for spare availability can be approximated by renewal theory as

$$\text{Prob}\left\{\begin{array}{l}\text{spare is not} \\ \text{available at the} \\ \text{time the failed} \\ \text{component is} \\ \text{removed}\end{array}\right\} \cong 1 - [g(t)]^{(N-1)}(1), \qquad (2.42)$$

where t is the time the failed component is removed, N is the number of plants in the spare pool, and the probability the spare has not failed equals to 1.

Let T_{temp} be the maximum temporary time the spare can be installed and T_{ex} be the sum of the failed component removal and spare installation times. Thus, the repair/replacement time interval is made up of a removal and installation time at the beginning of the interval, a temporary spare use time in the middle, and a removal and installation time at the end of the interval (i.e., $T_{repl} = T_{temp} + 2T_{ex}$). The fraction downtime can now be estimated by

$$f_{DT}(N,t) \cong \frac{T_{repl} - T_{temp}}{T_{repl}} + \left\{1 - [g(t+T_{ex})]^{(N-1)}\right\} \frac{T_{temp}}{T_{repl}} . \tag{2.43}$$

For the case of N = 1, $f_{DT}(1,t) = (T_{repl} - T_{temp})/T_{repl}$ while for the case of a very large number of plants ($N \to \infty$), $f_{DT}(\infty,t) \to 1$, i.e., the spare is always in use and never available. For the case of constant g(t) (i.e. asymptotic pool operation) the fractional downtime is

$$f_{DT}(N) \cong \frac{T_{repl} - T_{temp}}{T_{repl}} + \left\{1 - \left(\frac{1}{1+\lambda T_{repl}}\right)^{N-1}\right\} \frac{T_{temp}}{T_{repl}} . \tag{2.44}$$

2.3.8 Sample Problem For a Five Component System

The fractional downtime as a function of time was calculated for a system of five plants with $\lambda = .01\ yr^{-1}$, $T_{repl} = 2$ yr., and $T_{temp} = 1.833$ yr ($T_{ex} = 2$ mo). The fractional downtime was calculated using the numerical solution for g(t) derived in Section 2.3.2 and the asymptotic solution for g(t) derived in Section 2.3.3. The results for both numerical and asymptotic g(t) are listed in Table 2.1.

Table 2.1 Fractional downtime estimated by renewal theory with component failure rate = .01(yr^{-1}), repair/replacement time = 1.833 yr, and 5 plants in the spare pool.

time[1]	fractional downtime asymptotic g(t)	actual g(t)
0.0	0.1533	0.0866
0.1	0.1533	0.0938
0.2	0.1533	0.1011
0.4	0.1533	0.1153
0.6	0.1533	0.1294
0.8	0.1533	0.1432
1.0	0.1533	0.1540
1.2	0.1533	0.1537
1.4	0.1533	0.1535
1.6	0.1533	0.1534
1.8	0.1533	0.1533
2.0	0.1533	0.1533
2.2	0.1533	0.1533
2.4	0.1533	0.1533
2.6	0.1533	0.1533
2.8	0.1533	0.1533
3.0	0.1533	0.1533

[1]The time is dimensionless (i.e., time = t/T_{repl})

2.4 PC-SPARE Probability of Spare Availability

The results for the fractional downtime using renewal theory to describe the probability of spare availability can be compared to the method using binomial theory outlined in PC-SPARE (12). Recall that the probability of j failures in time t, denoted by $P_f(j,t)$, is given by

$$P_f(j,t) = \frac{(\lambda\ t)^j\ e^{-\lambda t}}{t!}\ . \tag{2.45}$$

For the case of constant repair/replacement time and N plants in the spare pool the probability that the spare is available at time t is constant and dependent on the number of failures in the repair/replacement interval being analyzed. The probability of spare availability, denoted by P_{avail}, can be expressed as

$$P_{avail} = \sum_{k=1}^{N} \text{Prob}\left\{\begin{array}{l|l} \text{(k-1) failures in} & \text{only one} \\ T_{repl} \text{ in the other} & \text{failure can} \\ \text{(N-1) plants} & \text{occur per plant} \end{array}\right\} \text{Prob}\left\{\begin{array}{l} \text{of any failed} \\ \text{plant claiming} \\ \text{the spare} \end{array}\right\}.$$

Each plant is limited to one failure because the plant cannot return to service (i.e., original component installed) by the end of T_{repl} should a failure occur. The probability of a failure in time t is

$$P_f(1,t) = P_f = \frac{(\lambda\ t)^1\ e^{-\lambda t}}{1!} = \lambda\ t\ e^{-\lambda t}\ . \tag{2.46}$$

Thus, according to binomial theory

$$\text{Prob}\left\{\begin{array}{l|l} \text{(k-1) failures in} & \text{only one} \\ T_{repl} \text{ in the other} & \text{failure can} \\ \text{(N-1) plants} & \text{occur per plant} \end{array}\right\} = \binom{N-1}{k-1} P_f^{k-1}(1-P_f)^{N-k}$$

$$= \frac{(N-1)!}{(k-1)!(N-k)!} \left(\lambda\ T_{repl}\ e^{-\lambda T_{repl}}\right)^{k-1} \left(1 - \lambda\ T_{repl}\ e^{-\lambda T_{repl}}\right)^{N-k}\ . \tag{2.47}$$

The assumption is made that each plant that fails has equal claim to the spare, thus

$$\text{Prob}\left\{\begin{array}{l} \text{of any failed} \\ \text{plant claiming} \\ \text{the spare} \end{array}\right\} = \frac{1}{k}\ .$$

The probability of spare availability can now be written as

$$P_{avail} = \sum_{k=1}^{N} \frac{(N-1)!}{k!(N-k)!} \left(\lambda\ T_{repl}\ e^{-\lambda T_{repl}}\right)^{k-1} \left(1 - \lambda\ T_{repl}\ e^{-\lambda T_{repl}}\right)^{N-k}\ . \tag{2.48}$$

The method outlined above allows for a failure in the interval T_{repl} and a plant still has a probability of obtaining the spare. The value of P_{avail} is the average probability that a plant would have use of the spare if the plant were to fail sometime in the interval. The method says nothing about the order of the failures and hence the decision as to whom gets the spare is similar to waiting until the end of the interval and all the failed plants have "drawn numbers from a hat" to decide which plant obtains rights to the spare.

In contrast, renewal theory awards the spare to a plant on the "first come, first served" basis. Any failure in the previous T_{repl} of the last failed plant and the spare is not available. For this reason the probability of availability estimated by PC-SPARE should be larger than the probability of availability estimated by renewal theory. For the case of a low probability of a failure occurring in the T_{repl} interval (e.g., small failure rates, small failure interval, and small number of plants) the results for probability of spare availability estimated by PC-SPARE and renewal theory should be approximately the same.

2.4.1 Sample Problem Comparing Probability of Spare Availability Estimates.

The following example cases were calculated using both renewal theory and PC-SPARE to predict the probability of spare availability (see Table 2.2). As expected, the two methods for probability of spare availability are approximately equal for the cases of a low probability of a failure occurring in the T_{repl} interval. The PC-SPARE method estimates the average probability of spare availability for all plants in the pool. In contrast, the renewal theory method awards the spare to a plant on the "first come, first served" basis thereby ignoring instances when the spare is not available for other members of the spare pool that fail in the T_{repl} interval.

Table 2.2 Probability of spare availability estimates using renewal theory and PC-SPARE methods.

Failure rate (yr^{-1})	Repair/replacement time (yr)	Number of plants in spare pool	Probability of spare availability: Renewal theory	Probability of spare availability: PC-SPARE
0.01	0.1	5	.998004	.996010
		20	.990567	.981188
		100	.952128	.905784
	0.5	5	.990099	.980248
		20	.954118	.909588
		100	.789354	.610323
	0.9	5	.982320	.964796
		20	.919634	.843468
		100	.663473	.411887
0.1	0.5	20	.654579	.395735

2.5 Evaluating Probability of Spare Availability and Fractional Downtime Using Computer Simulation Techniques

The fractional downtime can be determined analytically only for the simple case of a perfect spare and the condition that the spare is only used by the "recently failed" plant if immediately available. The theoretical description that takes into account all possible combinations of pool life-time, component repair time, etc. is presently not available. Consequently, a computer code using Monte Carlo techniques was developed to predict the fractional downtime by simulating random Poisson failures.

Monte Carlo techniques are applied to dynamic problems for which a closed form mathematical solution is difficult or impossible to construct (15). An approach similar to that developed by Widawsky (14) and Emshoff and Sisson (15) was considered where fixed and variable time-increment methods were considered. In the fixed time-increment method (also known as "time slicing") time is incremented by a small constant interval. At the end of each small time step the system is checked to see if any failures have occurred during that interval. The advantage of this method is that a record is not needed of the sequence of events because at each time increment the possibility of each event is checked (14). The disadvantages are 1) the time increments must be short in comparison to the mean time between failures and 2) because failures and repairs are randomly distributed throughout time there are many intervals where no failures will occur.

In the variable time-increment method (also known as "event sequencing") time is variably incremented to the next predicted failure. The advantage of this method is that the time increments may be long as well as short and that a failure occurs only at the beginning and end of the discrete intervals. The disadvantage is that a record of the failures that occur must be kept for the more general conditions of unequal failure rates. Because of the fewer computations required the variable time-increment method was selected for use in this computer simulation code.

The simulation is achieved by generating a sequence of random numbers from the appropriate probability distributions. A Poisson process can be simulated by recalling that the interarrival times, denoted by Δt, have a cumulative distribution function (CDF) of (15)

$$F(\Delta t) = 1-e^{-\lambda\Delta t} \tag{2.49}$$

where λ is the individual component failure rate. For a system of n components with the same failure rate λ operating at the beginning of the operation interval Δt

$$F(\Delta t) = 1 - e^{-n\lambda\Delta t} \ . \tag{2.50}$$

Thus, the time to the next failure is

$$\Delta t = \frac{-\ln[1-F(\Delta t)]}{n\lambda} \ . \tag{2.51}$$

Because $F(\Delta t)$ lies in the interval [0,1], the value for Δt_i can be simulated by randomly selecting a random number u_i from a uniform distribution over the interval [0,1] (see Fig. 2.5).

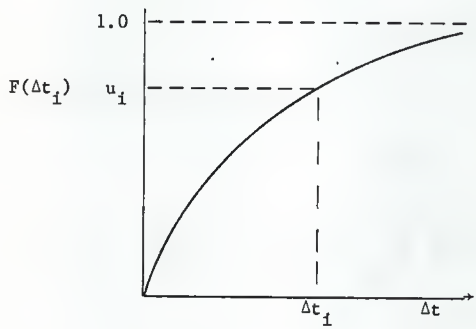

FIG. 2.5. A sample Δt CUD for the Δt_i's.

Thus, the time interval Δt between failures can be simulated by substituting a random number for $F(\Delta t)$ in Eq. (2.50) making

$$\Delta t_i = \left[-\ln(1-u_i)\right]/n\lambda \ , \tag{2.52}$$

where n is now the total number of components in the system that are operating at the beginning of the interval with a common failure rate of

λ. Successive operational intervals are analyzed in the same manner until the sum of the Δt_i's are greater than the maximum simulation time t_k.

In this manner a complete system history over the lifetime of the spare-component pool is obtained. To obtain the average or expected characteristics of a particular pool scenario, many such simulated histories are generated and the desired characteristic averaged over all these histories.

The development of all simulation models for this project was based on the components failing with constant failure rates, thereby randomly distributing the failures according to a Poisson process. The times between failures for a Poisson process with an infinite calling source are distributed negative exponentially characterized by a single constant failure rate (17). In contrast, the systems considered in this project consisted of a small number of components. The times between failures are distributed negatively exponentially but each interval is characterized by a specific failure rate depending on the number of components operational at the beginning of that interval. Thus, the time intervals between failures have different negative exponential distributions.

The cumulative distribution function F(t) is defined as the probability a failure will occur in the interval (0,t). Thus,

$$F(t) = \text{Prob}\left\{\begin{matrix}\text{a single failure in} \\ \text{the interval } (0,t)\end{matrix}\right\} = \int_0^t dt'\, f(t'), \qquad (2.53)$$

where $f(t')dt'$ is the associated distribution density function. This density function is the probability of a component failing in dt' about t'.

An important property of the Poisson distribution is the "forgetfulness property". Consider a plant component that is Poisson with mean rate λ of two failures per day. If the component has experienced no failures for the previous t_p hours of operation, what is the probability it will fail in the next t hours, i.e. what is

$$\mathrm{Prob}\{T > t_p + t \mid T > t_p\} \tag{2.54}$$

where T is the time from t_p to the time of the next failure. Recall that the conditional probability of A given that B has occurred is (20)

$$\mathrm{Prob}(A|B) = \frac{P(A \text{ and } B)}{P(B)} . \tag{2.55}$$

Thus, Eq. (2.54) can be written in the form

$$\mathrm{Prob}\{T > t_p + t \mid t > t_p\} = \frac{\mathrm{Prob}\{T > t_p + t \text{ and } T > t_p\}}{\mathrm{Prob}\{T > t_p\}} . \tag{2.56}$$

A closer analysis of $\mathrm{Prob}\{T > t_p + t \text{ and } T > t_p\}$ shows that $\mathrm{Prob}\{T > t_p + t\}$ necessarily includes $\mathrm{Prob}\{T > t_p\}$, therefore, Eq. (2.56) becomes

$$\mathrm{Prob}\{T > t_p + t \mid T > t_p\} = \frac{\mathrm{Prob}\{T > t_p + t\}}{\mathrm{Prob}\{T > t_p\}} . \tag{2.57}$$

Recalling that the intervals between failures are negative exponentially distributed, yields for Eq. (2.57)

$$\mathrm{Prob}\{T > t_p \mid T > t_p\} = \frac{e^{-\lambda(t_p+t)}}{e^{-\lambda t_p}} = e^{-\lambda t} . \tag{2.58}$$

The results of Eq. (2.58) show that waiting time t_p has no effect on the probability of a failure in the next t interval. This independence between two intervals of time is the so-called "forgetfulness property". Thus, for each operation interval simulated, the only concern need be the number of components operational at the beginning of that interval and no consideration need be given the previous failure history. Thus,

$$\text{Prob}\left\{\begin{matrix}\text{of a failure} \\ \text{in } (T_1, T_1+t)\end{matrix} \middle| \begin{matrix}\text{a failure occurred} \\ \text{at } T_1\end{matrix}\right\} = \text{Prob}\left\{\begin{matrix}\text{of a failure} \\ \text{in } (0,t)\end{matrix}\right\}.$$

2.5.1 Case of a System Without a Spare

Under Plan A, one must estimate the probability of a failure following the first failure in a system where the repair/replacement time for the failed component is a fixed time T_{repl} (see Fig. 2.6).

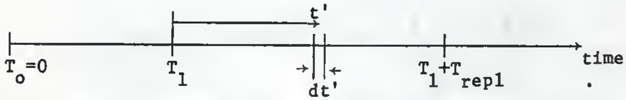

FIG. 2.6. A graphical representation of a failure in dt' about t' following the first failure at T_1 in a system with a fixed repair time.

For this discussion assume at time T_o there are N identical components operating with identical failure rates λ. The system characteristic failure rate, denoted by rate $\hat{\lambda}$, is defined as the system failure rate in a very small period of time dt' and is the sum of all the individual component failure rates operating at the beginning of the interval dt'. Thus, $\hat{\lambda}$ for the time intervals following T_1 is

$$\hat{\lambda} = \begin{cases} (N-1)\lambda & , \quad 0 < t' \leq T_{repl} \\ N\lambda & , \quad T_{repl} < t' \leq \infty , \end{cases} \tag{2.59}$$

where n is the number of plants operating at the beginning of the interval dt'. Thus,

$$\text{Prob}\{\text{of a failure in } dt' \text{ about } t'\} = \hat{\lambda}\, dt' = N\lambda\, dt' \ . \tag{2.60}$$

The density function for the time between failures can be written as

$$f(t')dt' = \text{Prob}\left\{\begin{matrix}\text{of a failure in } dt' \\ \text{about } t'\end{matrix} \;\middle|\; \begin{matrix}\text{a failure} \\ \text{occurred at } T_1\end{matrix}\right\} =$$

$$\sum_{i=1}^{N} \text{Prob}\left\{\begin{matrix}\text{of a failure of the i-th} \\ \text{component in } dt' \text{ about } t'\end{matrix} \;\middle|\; \begin{matrix}\text{the i-th component} \\ \text{survives until} \\ \text{at least } t'\end{matrix}\right\} \tag{2.61}$$

The probability a component survives to time t' (measured from T_1) is $e^{-\lambda t'}$. Thus, the above equation can be rewritten as

$$f(t')dt' = \{N\lambda\, dt'\}\{e^{-\lambda_1 t'} e^{-\lambda_2 t'} \ldots e^{-\lambda_N t'}\} \tag{2.62}$$

where N is the number of operating elements at t'. The density function for the time between failures is defined for two values of $\hat{\lambda}$ thereby resulting in the density function defined for two intervals of interest.

<u>Case 1</u>: $0 < t' \leq T_{repl}$

$$f(t')dt' = \{(N-1)\lambda\, dt'\}\{e^{-\lambda_1 t'} e^{-\lambda_2 t'} \ldots e^{-\lambda_{N-1} t'}\} =$$

$$(N-1)\lambda\, e^{-(N-1)\lambda\, t'}\, dt' \ . \tag{2.63}$$

<u>Case 2</u>: $T_{repl} < t' \leq \infty$

The N-th plant becomes operational only at time T_{repl} after T_1 and, therefore, only has to survive $(t' - T_{repl})$ making

$$f(t')dt' = \{N\lambda\, dt'\}\{e^{-\lambda_1 t'} e^{-\lambda_2 t'} \ldots e^{-\lambda_{N-1} t'} e^{-\lambda_N (t'-T_{repl})}\}$$

$$= N\lambda\, e^{-(N-1)\lambda\, t'}\, e^{-\lambda(t'-T_{repl})}$$

$$= N\lambda\, e^{-N\lambda\, t'}\, e^{\lambda T_{repl}}. \tag{2.64}$$

Thus, the density function for the time between the first and second failure is defined for the two ranges of t' as

$$f(t')dt' = \begin{cases} (N-1)\lambda\, e^{-(N-1)\lambda t'}\, dt' & , \quad 0 < t' \leq T_{repl} \\ N\lambda\, e^{-N\lambda t'}\, e^{\lambda\, T_{repl}}\, dt' & , \quad T_{repl} < t' \leq \infty\ . \end{cases} \tag{2.65}$$

If a substitution of variables is made with $\beta = \lambda T_{repl}$, $\tau = \lambda t$, and $d\tau' = \lambda dt'$, the above can be written as

$$f(\tau')\, d\tau' = \begin{cases} (N-1)\ e^{-(N-1)\tau'}\, d\tau' & 0 < \tau \leq \beta \\ N\ e^{\beta}\ e^{-N\tau'}\ d\tau' & \beta < \tau \leq \infty \end{cases} \tag{2.66}$$

The cumulative distribution function for this density function is

$$F(\Delta\tau) = \int_0^{\Delta\tau} d\tau'\ f(\tau') = \begin{cases} 1 - e^{-(N-1)\Delta\tau} & , \quad 0 < \Delta\tau \leq \beta \\ (1 - e^{\beta}\ e^{-N\Delta\tau}) & , \quad \beta < \Delta\tau \leq \infty\ . \end{cases} \tag{2.67}$$

The results for $F(\Delta\tau)$ satisfies the usual condition $F(0)=0$, $F(\infty) = 1$ and is continuous at $\Delta\tau = \beta$.

2.5.2 General Description of Simulation Models

A similar procedure for finding $f(\tau')\, d\tau'$ and $F(\tau)$ is followed for the situation in which a spare is added to the system (Plan B). The density function to be derived is a function of the number of operational components at the beginning of each time interval of interest. The time interval of interest is the time interval between the last simulated failure and the next failure to be simulated.

The terminology of the "k-th" plant was selected to mean the plant for which the probability of spare availability or fractional downtime is to be estimated. In addition, the repair/replacement time, denoted by β, is a standard dimensionless time unit. The interaction between the other (N-1) plants and k-th plant is simulated to obtain an estimate of the probability of spare availability and fractional downtime for the k-th plant. To determine this interaction three models were developed to simulate plant failure histories.

Model 1 is the simplest simulation performed. The major assumptions are:

1) The failed component, spare, and repaired/replacement component are removed or installed instantaneously.
2) The spare's failure rate is zero (i.e. a perfect spare).
3) The spare is immediately installed in a failed plant when it becomes available.

Because the spare has a zero failure rate, the system appears to have only as many components as there are plants. Also, the spare is available if all plants are operating at the time of the k-th plant failure. The assumptions used in this model are the same assumptions used in the renewal theory approximation to the spare-component pool problem. As a result, the probability of spare availability estimated by this model should agree with the probability of spare availability estimated by renewal theory. However, the fractional downtime estimates will not agree because renewal theory does not consider the possibility that the spare may become available prior to the end of the repair/replacement interval.

In Model 2 the major assumptions are:

1) The failed component, spare, and repair/replacement component are removed and installed instantaneously.

2) The spare's failure rate is zero (i.e., perfect spare).

3) If the spare is not immediately available at the time of failure it is <u>never</u> used to cover any portion of the downtime for that failure.

Once again, because the spare has no failure rate, the system appears to have only as many components as there are plants.

The availability of the spare to the k-th plant upon failure is determined by the last time the spare was used. The spare is available if there were no failures within β (where $\beta = \lambda T_{repl}$) of the k-th plant's failure. (see Fig. 2.7).

FIG. 2.7. Representation of a failure sequence where τ_L is time of last failure before the k-th plant failure.

If the spare is not used by any of the plants that fail in the interval, the spare is available. The spare is also available if the last time the spare is used is outside of the interval $(\tau_k - \beta, \tau_k)$ and the difference between the failures that occur in the previous interval are within β of the last time the spare was used (see Fig. 2.8).

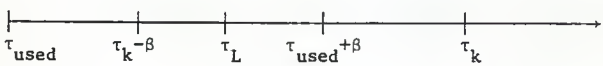

FIG. 2.8. Representation of a failure sequence where τ_L is the time of the last failure and τ_{used} is the last time the spare was used prior to the k-th plant failure.

Unlike Model 1, the spare may be available even when failures occur in the interval $(\tau_k - \beta, \tau_k)$. However, for the spare to be available τ_{used} cannot fail in the interval $(\tau_k - \beta, \tau_k)$. The results from this model should show a larger probability of spare availability than Model 1 or renewal theory because the spare may be in storage at τ_k with all except one plant failed.

For Model 3 the major assumptions are:

1) The failed component, spare, and repair/replacement component are removed and installed instantaneously.
2) The spare's failure rate is equal to the failure rate of the other similar components when installed in a plant system.
3) The spare is immediately installed in a failed plant when it becomes available.

Model 3 is identical to Model 1 except the spare component can fail and the fraction downtime can now assume any value over the interval [0,1]; that is,the spare contributes to the total number of components that can fail when installed in the system. As a result, failures can now occur in the interval $(\tau_k - \beta, \tau_k)$ and the spare still be available for use by the k-th plant.

A system with a finite spare failure probability should have a larger fractional downtime. In contrast, a system that uses the spare when available should have a smaller fractional downtime. Unlike the previous models, Model 3 is limited to evaluating the fractional downtime and not the probability of spare availablility.

2.5.3 Perfect Spare Used When Available, Model 1

In this section a mathematical model is derived to simulate the time between component failures for simulation Model 1. In this model the perfect spare case is considered with instantaneous removal and installation times. The spare is installed in a failed plant when it becomes available, even if for a very short period of time.

Two Plant Problem

Consider first a system of two plants and one spare component. The system characteristic failure rate, denoted by $\hat{\lambda}$, is defined as the sum of the individual component failure rates of the plants operational at the beginning of the time interval. The number of plants in the pool, denoted by N (here equal to 2), will remain constant throughout the pool lifetime.

The time between the first failure and the simulation starting time is a waiting time. If all plants are operational at time 0, the number of plants operating at the beginning of the interval is N. The density function for the first interval can be expressed by

$$f(\tau')d\tau' = N\, d\tau'(e^{-\tau_1'} \; . \; . \; . \; e^{-\tau_N'}) = N\, d\tau'\, e^{-N\tau'} \; . \qquad (2.68)$$

The cumulative distribution function $F(\Delta\tau)$ is

$$F(\Delta\tau) = \int_0^{\Delta\tau} d\tau'\, f(\tau') = \int_0^{\Delta\tau} N\, d\tau'\, e^{-N\tau'} = 1 - e^{-N\Delta\tau} \; . \qquad (2.69)$$

Thus, the random number u_1 is substituted for $F(\Delta\tau)$ and the first simulated time interval is

$$\Delta\tau_1 = -\ell n(1-u_1)/N \quad , \quad 0 < \Delta\tau_1 \leq \infty \; . \qquad (2.70)$$

The second failure time is dependent upon the first failure. For a period of time, β, following the first failure the number of operational components will be decreased by one, or

$$\hat{\lambda} = \begin{cases} (N - 1) & , \quad 0 < \tau \leq \beta \\ N & , \quad \beta < \tau \leq \infty \end{cases} \tag{2.71}$$

and the density function is

$$f(\tau')\, d\tau' = \begin{cases} (N-1) d\tau' \, e^{-(N-1)\tau'} & , \quad 0 < \tau \leq \beta \\ N \, d\tau' \, e^{-N\tau'} \, e^{\beta} & , \quad \beta < \tau \leq \infty \end{cases} \tag{2.72}$$

Thus,

$$F(\Delta\tau) = \begin{cases} 1 - e^{-(N-1)\Delta\tau} & , \quad 0 < \Delta\tau \leq \beta \\ 1 - e^{\beta} \, e^{-N\Delta\tau} & , \quad \beta < \Delta\tau \leq \infty \, . \end{cases} \tag{2.73}$$

The value of u_2 can now be compared to the probability of a single failure in the interval $(\tau', \tau' + \beta)$ to determine which form of $F(\Delta\tau)$ to invert to obtain the simulated time interval. The result is two potential simulation equations. The simulation equation is determined by comparing u_2 to the CDF evaluated at increments of β. There are two cases to be considered.

<u>Case 1</u>: $u_2 < F(\beta)$

The second time interval for this case is given by

$$\Delta\tau_2 = -[\ell n(1-u_2)]/(N-1) \tag{2.74}$$

and

$$\tau_2 = \tau_1 + \Delta\tau_2 = \tau_1 - [\ell n(1-u_2)]/(N-1) \, . \tag{2.75}$$

<u>Case 2</u>: $F(\beta) \leq u_2 < 1.0$

The second time interval for this case is given by

$$\Delta\tau_2 = [\beta - \ell n(1-u_2)]/N \tag{2.76}$$

and

$$\tau_2 = \tau_1 + \Delta\tau_2 = \tau_1 + [\beta - \ell n\ (1-u_2)]/N\ . \tag{2.77}$$

A summary of the results for the second time interval is,

$$\tau_2 = \begin{cases} \tau_1 - [\ell n(1-u_2)]/(N-1) & ,\quad 0 \le u_2 < F(\beta) \\ \tau_1 + [\beta - \ell n(1-u_2)]/N & ,\quad F(\beta) \le u_2 < 1.0\ . \end{cases} \tag{2.78}$$

The third failure is also dependent upon the first two failures (see Fig. 2.9). The value of δ_i is defined as

$$\delta_i \equiv \beta - (\tau_i - \tau_{i-1}) = \beta - \Delta\tau_{i-1} \tag{2.79}$$

where $i = 2,3,\ldots(N-1)$ and $\delta_1 = \beta$.

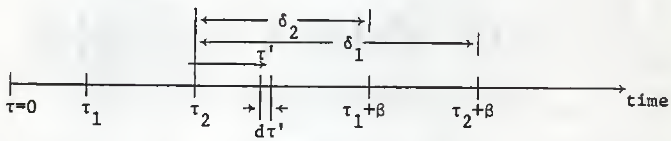

FIG. 2.9. A time line illustrating a possible configuration to be considered.

The limit on i is (N-1) because at least one plant must be operational for any failure to occur. If there are no operational plants then a period of time must elapse until at least one plant returns to operation. Thus,

$$\delta_1 = \beta$$

$$\delta_2 = \beta - (\tau_2 - \tau_1) = \beta - \Delta\tau_2\ . \tag{2.80}$$

For the case of $\delta_2 > 0$, a failure cannot occur in the system until at least one component has returned to service at time $\tau_1 + \beta$ (i.e., a repair/ replacement component is installed). The waiting time, denoted by τ_w, is given by δ_2. The next failure is then simulated. Starting from time $(\tau_1 + \beta)$, follow the same technique as outlined for the second failure, and compare u_3 to $F(\beta)$.

For the case that $\delta_2 \leq 0$, the plant that failed at τ_1 has returned to service before τ_2 occurred. The density function for the third interval is the same form as given in Eq. (2.72) where now

$$f(\tau')\, d\tau' = \begin{cases} (N-1)d\tau'\ e^{-(N-1)\tau'} & , \quad 0 < \tau \leq {}_1\delta \\ N\, d\tau'\ e^{-N\tau'} & , \quad \delta_1 < \tau \leq \infty \end{cases} \tag{2.81}$$

and

$$F(\Delta\tau) = \begin{cases} 1 - e^{-(N-1)\Delta\tau} & , \quad 0 < \Delta\tau \leq \delta_1 \\ 1 - e^{\delta_1}\, e^{-N\Delta\tau} & , \quad \delta_1 < \Delta\tau \leq \infty\ . \end{cases} \tag{2.82}$$

The value of u_3 is then compared to $F(\tau)$ to obtain the third time interval

$$\Delta\tau_3 = \begin{cases} -\ \ell n\ (1-u_3)/(N-1) & , \quad 0 \leq u_3 < F(\delta_1) \\ [\delta_1 - \ell n(1-u_3)]/N & , \quad F(\delta_1) \leq u_3 < 1.0\ . \end{cases} \tag{2.83}$$

Thus,

$$\tau_3 = \begin{cases} \tau_2 - \ell n(1-u_3)/(N-1) & , \quad 0 \leq u_3 < F(\delta_1) \\ \tau_2 + [\delta_1 - \ell n(1-u_3)]/N & , \quad F(\delta_1) \leq u_3 < 1.0\ . \end{cases} \tag{2.84}$$

<u>General N-plant Problem</u>

The time of the i-th failure can now be determined. For the general case of N plants in the spare pool and P previous failures in a time interval equal to β just prior to the failure of the k-th plant component, the density function for the j-th time interval is

$$f_j(\tau')\, d\tau' = \alpha \exp(-\alpha\, \tau') \exp\left\{\sum_{m=1}^{j} \delta_{P+3-m}\right\}$$

$$, \delta_{P+3-j} < \tau' \leq \delta_{P-j+2} , \qquad (2.85)$$

where $\delta_\ell = \beta - \sum_{i=1}^{\ell} \Delta\tau_i$, $\alpha = N-P-2+j$, $\delta_{P+2} = 0$, $\delta_1 = \beta$, and $\delta_0 = \infty$. Thus, the general form of the CDF is

$$F_j(\Delta\tau_i) = 1 - \exp(-\alpha\, \Delta\tau_j) \exp\left(\sum_{m=1}^{j} \delta_{P+3-m}\right), \ \delta_{P+3-j} < \Delta\tau_i \leq \delta_{P-j+2}. \qquad (2.86)$$

Because of the finite number of plants in the pool the maximum number of prior failures in the interval β is (N-2). If there are (N-1) failures, a waiting time must be evaluated by subtracting δ_{P+2} from the δ values in Eqs. (2.85) and (2.86). If there are no prior failures (e.g., P=0), the CDF is

$$F_j(\Delta\tau_i) = 1 - e^{-N\Delta\tau_i} . \qquad (2.87)$$

The $\Delta\tau_i$ for the i-th interval is now found by

$$\Delta\tau_i = \begin{cases} -\ell n(1-u_j)/N & , \quad P = 0 \\ \left\{\dfrac{\sum_{m-1}^{K} \delta_{P+3-m} - \ell n[1-u_j]}{[N-P-2+K]}\right\} & , \quad P > 0 \end{cases} \qquad (2.88)$$

such that $F_j(\delta_{P-K+3}) \leq u_j < F_j(\delta_{P-K+2})$ where $F(\delta_{P+2}) = 1$.

Because the spare is installed when it becomes available, the downtime for the k-th plant depends only upon the time of the last failure. Consider the case where τ_k is the time at which the evaluation is made. If the time of the previous failure is greater than β prior to

τ then the downtime is 0. If the last failure has occurred within β of τ_k, say τ_{LF}, then the downtime is $\beta-(\tau_k - \tau_{LF})$. Thus, the downtime S1 is given by

$$S1 = \begin{cases} 0 & , \quad \tau_K - \tau_{LF} \geq \beta \\ \beta - (\tau_K - \tau_{LF}) & , \quad \tau_K - \tau_{LF} < \beta \,. \end{cases} \tag{2.89}$$

At the end of M simulated pool histories the fractional downtime, denoted by f_{DT}, is determined by comparing the total simulated downtime to the maximum downtime $M\beta$, or

$$f_{DT} = \frac{\sum_{j=1}^{M} S1_j}{M\beta} \,. \tag{2.90}$$

The probability of spare availability estimated by this model can be compared to the renewal theory results. The spare is available at τ_k if the last failure occurred outside the interval $(\tau_k - \beta, \tau_k)$. A value of 1 is assigned to a counter when the last simulated failure prior to τ_k occurs in the interval $(\tau_k - \beta, \tau_k)$. A value of 0 is assigned if the last simulated failure occurs outside the interval $(\tau_k - \beta, \tau_k)$. The counters are then summed and divided by the number of simulation. This yields an estimate for the probability of spare availability at time τ_k, denoted by $P_{avail}(\tau_k)$ and represented as

$$P_{avail}(\tau_k) = 1 - \frac{1}{M} \sum_{j=1}^{M} (\text{counter})_j \,. \tag{2.91}$$

2.5.4 Perfect Spare Which is Used Only if Immediately Available, Model 2

In this section a mathematical model is derived to simulate the time between component failures for Model 2. The same assumptions as for Model 1 are used, except that if the spare is not immediately available at the time of failure it is <u>never</u> used to cover any portion of the downtime for that failure.

Because the spare has no failure rate, the spare does not contribute to the system failure rate. Thus, the time intervals and failure times can be simulated using the method outlined for Model 1. In addition to the failure times, the times the spare is used must be estimated. The first failure simulated would use the spare during the fixed downtime β. The remaining failures up to the maximum evaluation time (pool lifetime) would use the spare only if it were available at the time of failure. Thus, at each simulated failure, the previous time interval of length β is analyzed to determine if any other failures have occurred in the time interval. If no other failures have occurred in the time interval, the spare is used at the newly simulated failure.

In contrast to Model 1, the spare can be available even if failures occur in the interval $(\tau_k - \beta, \tau_k)$. A counter, denoted by S2, is used to determine the number of times τ_{used} occurs in the interval $(\tau_k - \beta, \tau_k)$. If τ_{used} occurs in the interval $(\tau_k - \beta, \tau_k)$ the spare is not available at τ_k. The probability that the spare is available at τ_k is given by

$$P_{avail}(\tau_k) = 1 - \frac{1}{M} \sum_{j=1}^{M} S2_j \; , \tag{2.92}$$

where M is the total number of simulations run. Because the spare is either used if immediately available or never used, the spare is either used the entire repair/replacement time or not at all. Thus, the fractional downtime is

$$f_{DT} = \frac{M\beta - \left(\sum_{j=1}^{M} S2_j\right)\beta}{M\beta} = 1 - \frac{1}{M}\sum_{j=1}^{M} S2_j \; . \tag{2.93}$$

Thus, the Model 2 probability of spare availability and fractional downtime are equal.

2.5.5 Imperfect Spare Which is Used when Available, Model 3

In this section a mathematical model is derived to simulate the time between component failures for simulation Model 3. Unlike Model 1, the spare failure rate is now equal to the failure rate of other similar components when installed in a plant.

As in Models 1 and 2, the system characteristic failure rate is the sum of the individual component failure rates for the plants operational at the beginning of the time interval. However, in this case the spare component contributes to the system's failure rate when it is installed in a plant.

<u>Two-plant Problem</u>

Consider a system of two plants and a spare component. The time between the first failure and the simulation starting time is a "waiting time". If all plants are operating at time 0, the density function for the first time interval can be expressed as

$$f(\tau')\,d\tau' = N\,e^{-N\tau'}d\tau' \; . \tag{2.94}$$

Thus,

$$F(\Delta\tau) = \int_0^{\Delta\tau} d\tau'\,f(\tau') = \int_0^{\Delta\tau} N\,e^{-N\tau'}d\tau' = 1 - e^{-N\Delta\tau} \; . \tag{2.95}$$

The probability of a single failure in the interval $(0,\infty)$ is 1. The interval simulation equation is found by substituting u_1 for $F(\Delta\tau)$ and solving for $\Delta\tau$ in Eq. (2.95)

$$\tau_1 = \Delta\tau_1 = -\ \ell n(1-u_1)/N, \qquad 0 < \Delta\tau_1 \leq \infty\ . \tag{2.96}$$

Because the spare now has a failure rate, the number of components operating at the beginning of the second failure interval is still N (see Fig. 2.10). The density function for the second time interval is identical to that of the first time interval, making

$$F(\Delta\tau_2) = 1 - e^{-N\Delta\tau_2} \qquad 0 < \Delta\tau_2 \leq \infty \tag{2.97}$$

and

$$\Delta\tau_2 = -\ \ell n(1-u_2)/N\ . \tag{2.98}$$

The value of τ_2 is

$$\tau_2 = \tau_1 + \Delta\tau_2 = \tau_1 - [\ell n(1-u_2)]/N\ . \tag{2.99}$$

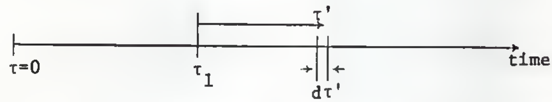

FIG. 2.10. A time line illustrating a possible configuration for the second failure.

The density function for the third time interval is dependent upon the value of $\Delta\tau_2$. If $\Delta\tau_2$ is greater than β, the spare is available at τ_2, otherwise the spare is not available at the start of $\Delta\tau_3$. Thus, there are two possible cases to be determined by the value of δ_1, where (see Fig. 2.11).

$$\delta_1 = \tau_1 + \beta - \tau_2 = \beta - \Delta\tau_2\ . \tag{2.100}$$

Case 1: $\delta_1 \leq 0$

For this case the density function for the third time interval and CDF are the same as for the first and second failure, that is,

$$F(\Delta\tau_3) = 1 - e^{-N\Delta\tau_3} \qquad 0 < \Delta\tau_3 \leq \infty \,. \tag{2.101}$$

Case 2: $\delta_1 > 0$

Unlike the previous models, the spare now has a finite failure rate. Therefore, one component is operating at the beginning of the interval and the density function for the third time interval is now defined over two intervals of τ'. Thus, the density function for the third time interval is given by

$$f(\tau')\,d\tau' = \begin{cases} (N-1)\, e^{-(N-1)\tau'}\, d\tau' & , \; 0 < \Delta\tau_3 \leq \delta_1 \\ N\, e^{-N\tau'}\, e^{\delta_1}\, d\tau' & , \; \delta_1 < \Delta\tau_3 \leq \infty \,. \end{cases} \tag{2.102}$$

Thus,

$$F(\Delta\tau_3) = \begin{cases} 1 - e^{-(N-1)\Delta\tau_3} & 0 < \Delta\tau_3 \leq \delta_1 \\ 1 - e^{\delta_1}\, e^{-N\Delta\tau_3} & \delta_1 < \Delta\tau_3 \leq \infty \,. \end{cases} \tag{2.103}$$

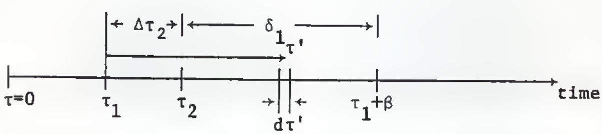

FIG. 2.11. A time line illustrating a possible configuration for the third failure.

The value of u_3 is then compared to $F(\delta_1)$ to find the appropriate expression for $F(\Delta\tau_3)$. The interval simulation equation is found by substituting u_3 for $F(\Delta\tau_3)$ and solving for $\Delta\tau_3$. Thus,

$$\Delta\tau_3 = \begin{cases} -\ln(1-u_3)/(N-1) & , \quad 0 < u_3 \le F(\delta_1) \\ [\delta_1 - \ln(1-u_3)]/N & , \quad F(\delta_1) < u_3 \le 1.0 \; . \end{cases} \tag{2.104}$$

The value of τ_3 is determined by δ_1 which can be expressed for two possible cases.

<u>Case 1</u>: $\delta_1 \le 0$

$$\tau_3 = \tau_2 - \ln(1-u_3)/N \tag{2.105}$$

<u>Case 2</u>: $\delta_1 > 0$

$$\tau_3 = \begin{cases} \tau_2 - \ln(1-u_3)/(N-1) & , \quad 0 < u_3 \le F(\delta_1) \\ \tau_2 + [\delta_1 - \ln(1-u_3)]/N & , \quad F(\delta_1) < u_3 \le 1.0 \end{cases} \tag{2.106}$$

The density function for $\Delta\tau_3$ is dependent upon $\Delta\tau_3$ and $\Delta\tau_2$ (see Fig. 2.12). If $\Delta\tau_2$ is greater than β and $\Delta\tau_3$ is less than β, the interval is analyzed in the same manner as described for $\Delta\tau_3$. If $\Delta\tau_3$ is greater than β, the interval is determined as described for $\Delta\tau_2$ or $\Delta\tau_1$. A different case must be analyzed when $\Delta\tau_3 + \Delta\tau_2 < \beta$.

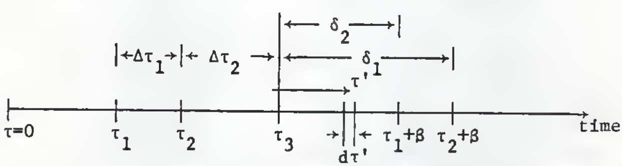

FIG. 2.12. A time line illustrating a possible configuration for the fourth failure.

The δ values for the fourth failure are

$$\begin{aligned} \delta_1 &= \tau_2 + \beta - \tau_3 = \beta - \Delta\tau_3 \\ \delta_2 &= \tau_1 + \beta - \tau_3 = \beta - \Delta\tau_3 - \Delta\tau_2 \; . \end{aligned} \tag{2.107}$$

The density function for the fourth time interval is now defined over three intervals of τ'. There are three possible combinations.

1. $\underline{\delta_2 \leq 0 \text{ and } \delta_1 \leq 0}$

For this case

$$F(\Delta\tau_4) = 1 - e^{N\Delta\tau_4} \qquad 0 < \Delta\tau_4 \leq \infty \tag{2.108}$$

and

$$\Delta\tau_4 = -\ \ell n(1-u_4)/N\ . \tag{2.109}$$

2. $\underline{\delta_2 \leq 0 \text{ and } \delta_1 > 0}$

This case is similar to the $\Delta\tau_3$ interval where the density function for the fourth time interval is

$$f(\tau')\ \ d\tau' = \begin{cases} (N-1)\ e^{-(N-1)\tau'} d\tau'\ , & 0 < \Delta\tau_4 \leq \delta_1 \\ N\ e^{-N\tau'}\ e^{\delta_1}\ d\tau'\ , & \delta_1 < \Delta\tau_4 \leq \infty \end{cases} \tag{2.110}$$

and

$$F(\Delta\tau_4) = \begin{cases} 1 - e^{-(N-1)\Delta\tau_4}\ , & 0 < \Delta\tau_4 \leq \delta_1 \\ 1 - e^{\delta_1}\ e^{-N\Delta\tau_4}\ , & \delta_1 < \Delta\tau_4 \leq \infty\ . \end{cases} \tag{2.111}$$

The value of u_4 is then compared to $F(\delta_1)$. The interval simulation equation is found by substituting u_4 for $F(\Delta\tau_4)$ and solving for $\Delta\tau_4$, thus

$$\Delta\tau_4 = \begin{cases} -\ \ell n(1-u_4)/(N-1)\ , & 0 < u_4 \leq F(\delta_1) \\ [\delta_1 - \ell n(1-u_4)]/N\ , & F(\delta_1 < u_4 \leq 1.0 \end{cases} . \tag{2.112}$$

3. $\delta_2 > 0$ and $\delta_1 > 0$

For this case all system components and the spare have failed. As in previous models, a waiting time is incurred until a plant component is operational. The waiting time is given by

$$\tau_w = \tau_1 + \beta - \tau_3 \ . \tag{2.113}$$

Therefore, the next simulation is started at time $\hat{\tau}_3 = \tau_3 + \tau_w = \tau_1 + \beta$. The next simulated time interval uses the procedure outlined above when $\delta_2 \leq 0$ and $\delta_1 > 0$.

The density function for $\Delta\tau_i$ is dependent upon the previous $\Delta\tau$'s for a period of τ_{i-1} to τ_{i-N}, where N is the number of plants in the pool. If $\sum_{i=1}^{N} \delta_i < \beta$, then all components in the system including the spare have failed by τ_{i-1}. The system must then wait until $\tau_{i-1} + \delta_n$ for at least one component to return to operation before the next failure time can be simulated. The sum of the individual δ's must, therefore, be determined at each τ prior to τ_{i-1} with the smallest τ being τ_{i-N}. The τ value at which the sum exceeds β is the starting index for determining the number of components operating.

The time of the i-th failure can now be determined. For the general case of N plants in the spare pool and for P previous failures ($P \geq 1$) in an interval of time equal to β just prior to the failure of the k-th plant component, the density function for the j-th time interval is

$$f_j(\tau')d\tau' = \alpha \exp(-\alpha\tau') \exp\left\{\sum_{m=1}^{j} \delta_{P+2-m}\right\} ,$$

$$\delta_{P+2-j} \leq \tau' \leq \delta_{P+1-j} \ , \tag{2.114}$$

where $\delta_{\ell} = \beta - \sum_{i=1}^{\ell} \Delta\tau_i$, $\alpha = (N-P+j-1)$, $\delta_{P+1} = 0$, and $\delta_0 = \infty$.

Thus, the general form of the CDF is

$$F_j(\Delta\tau_i) = 1 - \exp(-\alpha\tau)\ \exp\left\{\sum_{k=1}^{\ell} \delta_{P-K+2}\right\}, \quad \delta_{P-\ell+2} < \Delta\tau_i \leq \delta_{P-\ell+1} . \tag{2.115}$$

Because of the finite number of plants in the pool the maximum number of prior failures in the interval β is $(N-1)$. In addition, because the spare component contributes to the system characteristic failure rate, the general form for the density function and CDF apply only to interval equations where at least two prior failures have occurred in the interval $(\tau_{i-1} - \beta_1, \tau_{i-1})$. If there are no prior failures (e.g., $P = 0$), the CDF is

$$F_j(\Delta\tau_i) = 1 - e^{-N\Delta\tau_i} . \tag{2.116}$$

The value of $\Delta\tau_i$ is found by substituting u_j for $F_j(\Delta\tau_i)$ in either Eq. (2.116) or (2.115) and solving for $\Delta\tau_i$ giving

$$\Delta\tau_i = \begin{cases} -\ln(1-u_j)/N & P = 0,1 \\[2ex] \dfrac{\sum_{m=1}^{K} \delta_{P-2+m} - \ln(1-u_j)}{N - P - 1 + K}, & F_j(\delta_{P-K+2}) < u_j \leq F_j(\delta_{P-K+1}) \text{ and } P>1. \end{cases} \tag{2.117}$$

Because the spare is installed when it becomes available, the downtime for the k-th plant depends only upon the time of the last failure. Therefore, the fractional downtime and probability of spare availability are calculated in the same manner as used by Model 1.

2.5.6 Error Estimates for Probability of Spare Availability

In this section a method is derived to evaluate the error associated with the probability of spare availability estimates for Models 1, 2, and 3. Thus, the significance of the difference between the probability of spare availability estimated by the different simulation models can be determined. In addition, the significance of any difference between Model 1 and renewal theory estimates for probability of spare availability can be determined.

Consider a random variable X whose density function is given by $f(X|\theta)$. The likelihood function of n random variables $X_1, X_2 \ldots X_n$, denoted by $L(X_1, \ldots, X_n|\theta)$, has the joint density function $f_{x_1}(\ldots, x_n)$ (20). The likelihood is the value of the density function, which for discrete random variables is a probability.

For the case where θ is unknown and the joint density of n random variables is $f_{x_1,\ldots X_n}(X_1, \ldots, X_n|\theta)$, the maximum-likelihood estimator of θ, denoted by $\hat{\theta}$, is the value of θ which maximizes the likelihood function $L(X_1', \ldots, X_n'|\theta)$. Thus, for many cases the maximum likelihood estimator is the solution to (20)

$$\frac{\partial L(\theta)}{\partial \theta} = 0 \ . \tag{2.118}$$

Both $L(\theta)$ and $\ell nL(\theta)$ have a maxima at the same value of θ.

The probability mass function for the number of failures in a random sample of size n taken from a binomial distribution is given by

$$f(x|p) = p^x \, q^{1-x} \, \mathcal{F}_{\{0,1\}}(x) \ , \tag{2.119}$$

where

$$F_{\{0,1\}}(x) = \begin{cases} 1 & , \quad 0 \le x \le 1 \\ 0 & , \quad \text{otherwise} \end{cases} ,$$

$0 \le p \le 1$, and

$q = 1 - p$.

The likelihood function can be written as

$$L(p) = \prod_{i=1}^{n} p^{x_i} q^{1-x_i} = p^{\Sigma x_i} q^{n-\Sigma x_i} . \qquad (2.120)$$

The natural log of the above equation is

$$\ell n\,[L(p)] = \Sigma\, x_i\ \ell n(p) + (n - \Sigma\, x_i)\ \ell n(q) . \qquad (2.121)$$

The above equation differentiated with respect to p gives

$$\frac{\partial \left[\ell n[L(p)]\right]}{\partial p} = \frac{\Sigma x_i}{p} - \frac{n - \Sigma x_i}{q} . \qquad (2.122)$$

The maximum likelihood estimator $\hat{p}$ is obtained by setting the above expression equal to 0 and solving for p. Thus,

$$\hat{p} = \frac{\Sigma x_i}{n} = \bar{x} . \qquad (2.123)$$

The mean of x, denoted by μ_x, is found by taking the first moment of the binomial distribution (21)

$$\mu_x = \sum_{x=0}^{n} x\, f(x) , \qquad (2.124)$$

where X is a discrete random variable and $f(x_i)$ is its density function. The mean of X can now be written as

$$\mu_x = \sum_{x=0}^{n} x\binom{n}{x} p^x(1-p)^{n-x} = \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)!} p^x(1-p)^{n-x}$$

$$= np \sum_{x=1}^{n} \binom{n-1}{x-1} p^{x-1}(1-p)^{n-x} . \tag{2.125}$$

If $a = x - 1$ and $b = n - 1$, then

$$\mu_x = np \sum_{a=0}^{b} \binom{b}{a} p^a(1-p)^{b-a} , \tag{2.126}$$

where

$$\sum_{a=0}^{b} \binom{b}{a} p^a(1-p)^{b-a} = 1. \tag{2.127}$$

Thus, the mean of X is

$$\mu_x = np \quad . \tag{2.128}$$

The variance σ^2 is found by taking the second moment about the origin (21)

$$\sigma^2 = \sum_{x=0}^{n} x(x-1) \binom{n}{x} p^x(1-p)^{n-x} \tag{2.129}$$

Following a similar procedure as outlined for the mean of x, we find the variance is

$$\sigma^2 = np(1-p) . \tag{2.130}$$

The maximum likelihood estimator and variance can now be used to calculate confidence interval estimates for the probability of spare availability. Let p be the proportion of simulations when the spare is available at some time τ_k. If repeated samples of size n are taken, the number of simulations for which the spare is available will be a random variable, say X. For fairly large values of n the distribution of X will be approximately normal (20). If the distribution of X is normal, we say the random variable X is $N(\mu,\sigma^2)$ and

$$Z = \frac{X - \mu_x}{\sigma} \sim N(0,1) \ . \tag{2.131}$$

As was previously shown $\mu_x = np$ and $\sigma = \sqrt{np(1-p)}$, therefore, Eq. (2.131) can be written as

$$Z = \frac{X - np}{\sqrt{np(1-p)}} = \frac{\frac{X}{n} - p}{\sqrt{\frac{p(1-p)}{n}}} \ . \tag{2.132}$$

The standard deviation for the probability of spare availability at τ_k, denoted by $\hat{\sigma}(\tau_k)$, can be estimated by setting $\bar{x} = P_{avail}\ (\tau_k)$ giving

$$\hat{\sigma}(\tau_k) = \sqrt{\frac{P_{avail}\ (\tau_k)\ [1-P_{avail}\ (\tau_k)]}{M}} \ , \tag{2.133}$$

where M is the number of simulations run. The $(1-\alpha)$ confidence interval is given by

$$\text{Prob}\left\{- Z_{\alpha/2} < \frac{X - \mu_x}{\sigma} < Z_{\alpha/2}\right\} = 1 - \alpha \ , \tag{2.134}$$

or

$$\text{Prob}\left\{- Z_{\alpha/2} < \frac{\frac{X}{M} - p}{\sqrt{\frac{p(1-p)}{M}}} < Z_{\alpha/2}\right\} = 1 - \alpha \ . \tag{2.135}$$

Thus, the $100(1-\alpha)\%$ confidence interval for p is given by

$$\frac{X}{n} \pm Z_{\alpha/2} \sqrt{\frac{p(1-p)}{M}} \ . \tag{2.136}$$

Recall that the maximum-likelihood estimator for p is $P_{avail}(\tau_k)$ therefore the $100(1-\alpha)\%$ confidence interval for p is given by

$$P_{avail}(\tau_k) \pm Z_{\alpha/2} \sqrt{\frac{P_{avail}(\tau_k)[1-P_{avail}(\tau_k)]}{M}} \ . \tag{2.137}$$

where $P_{avail}\ (\tau_k)$ is the average probability of spare availability determined by the computer code.

2.6 SIMULATION Computer Code

The computer code, SIMULATION, was written to analyze Models 1, 2, and 3 (see Appendix A). The computer code is used first to simulate a random number over the interval [0,1], using a modified version of the IMSL subroutine GGUBFS (22). The most recent repair/replacement interval is then analyzed to determine the δ_i's by computing the time difference between the most recent failure and any failures prior to that using Eq. (2.79).

After the δ_i values have been computed, the value of δ_i's distribution function is computed and compared to the random number. If the value of the random number is greater than the numerical value of the distribution function, the distribution function associated with δ_2 is computed. The first distribution function value which is less than or equal to the random number is used for the simulation equation.

The simulation equation is the distribution function, where the random number is substituted for $F(\Delta\tau_i)$ and solved for $\Delta\tau_i$, e.g., in

Eq. (2.70). Thus, the value of the present failure time interval is computed.

The next random number is created and the above procedure is repeated until the sum of the failure time intervals exceeds the pool lifetime. At this point, a "history" of simulated plant failures has been compiled. The failure time just prior to the failure time that exceeded the pool lifetime is then compared to the pool lifetime. The difference between the two times is used to estimate the probability of spare availability for Model 1 and fractional downtime for Models 1 and 3.

In addition, the last time the spare was used is compared to the pool lifetime and the difference is used to estimate the probability of spare availability and fractional downtime for Model 2. Estimates of the probability of spare availability and fractional downtime are obtained by averaging the probability of spare availability and fractional downtime over a large number of histories (e.g., for this study 10,000 simulation histories were considered sufficient).

2.7 Sample Problem For a Five Component System

A sample problem was developed to compare the simulation model results to renewal theory results. The first version of the example problem considered a system with five plants in the spare pool and a $\beta = .02$ (see Fig. 2.13). A study of the Fig. (2.13) indicates that the Model 1 results for probability of spare availability were within $\pm\sigma$ of the renewal theory prediction for all but two data points. In addition, the Model 3 results were not different from the Model 1 results. Thus, for small values of β the numerical solution to the renewal equation can

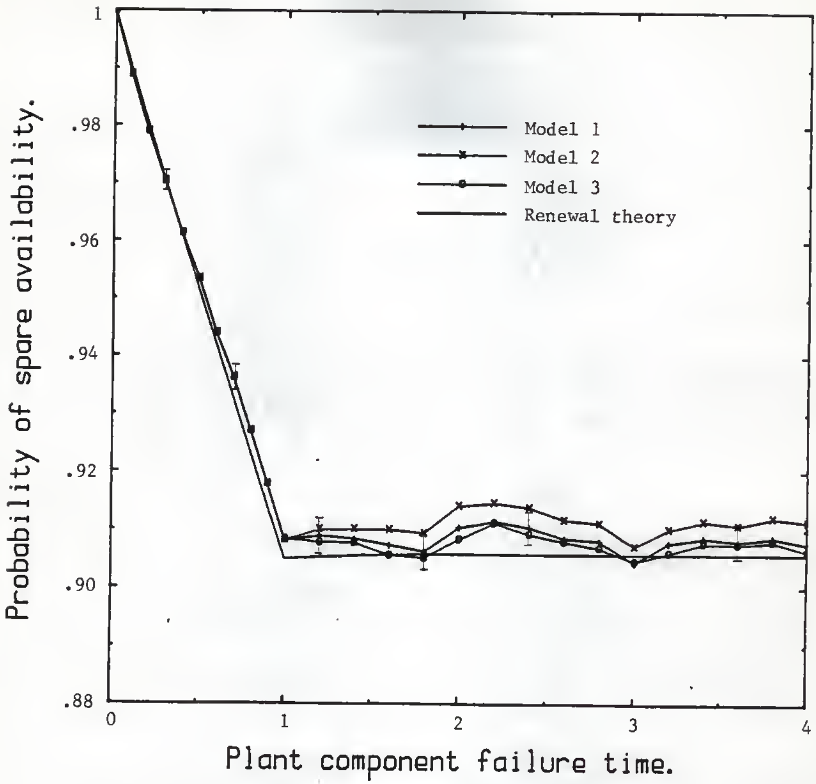

FIG. 2.13. Probability of spare availability for a five component system. The Model 1, 2, and 3 simulation results are compared to the renewal theory results with β = .02 and 10,000 simulations computed. The error bars indicate $\pm\sigma$.

be used to estimate the probability of spare availability for the Model 1 and Model 3 assumptions.

The second variation of the example problem considered the same system as before but by using simulation Model 1. The number of simulations was increased by a factor of ten (see Fig. 2.14). As expected, the Model 1 results were even closer to renewal theory prediction than the previous Model 1 results using fewer simulations.

The third variation of the example considered the same system with the initial seed value used by the random number subroutine changed to check for the "randomness" of the simulated failures (see Fig. 2.15). A study of the figure indicates that the fluctuations in the probability of spare availability occurred at different failure times. This indicated the fluctuations were due to the random numbers and not the logic of the computer code.

In the fourth and final variation of the above example problem considered a system with five plants in the spare pool and a β = .4 (see Fig. 2.16) was considered. Once again, the Model 1 results agreed with the renewal theory prediction. The Model 2 and Model 3 results were significantly different for this case. With the decision rule used in Model 2 (i.e., spare is used only if immediately available at the time of failure), failures can occur in the interval $(0, T_{repl})$ and the spare would still be available. The probability of spare availability was therefore substantially larger for times greater than β in the case of Model 2 when compared to Models 1 and 3. Because Model 3 assumes a finite spare failure rate, the probability of spare availability was different for Models 1 and 3.

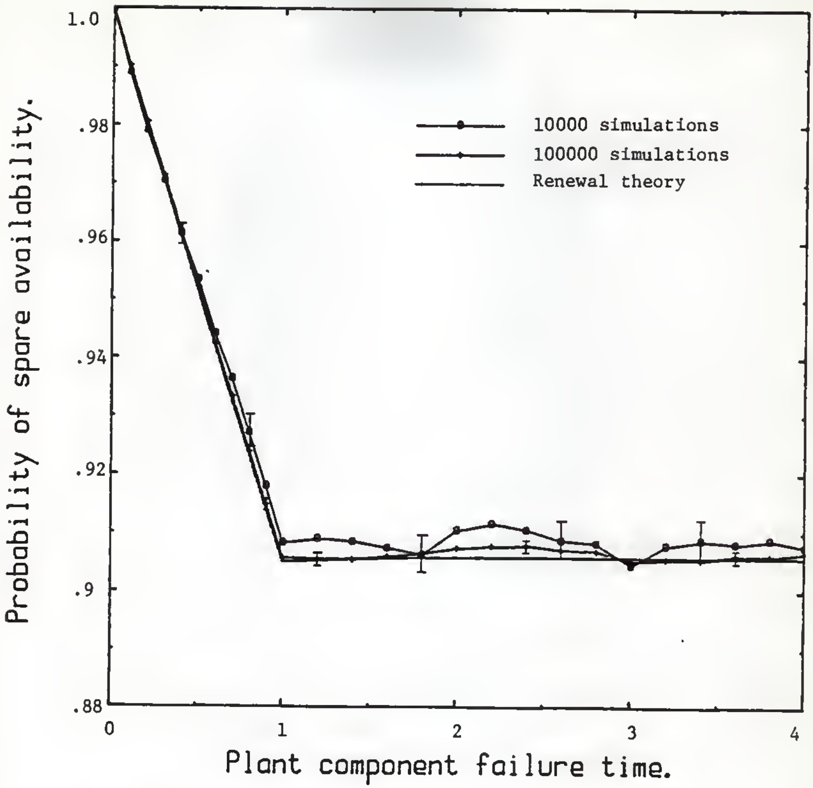

FIG. 2.14. Probability of spare availability for a five component system. The Model 1 simulation results are compared to the renewal theory results with $\beta = .02$ and two different numbers of simulations run. The error bars indicate $\pm\sigma$.

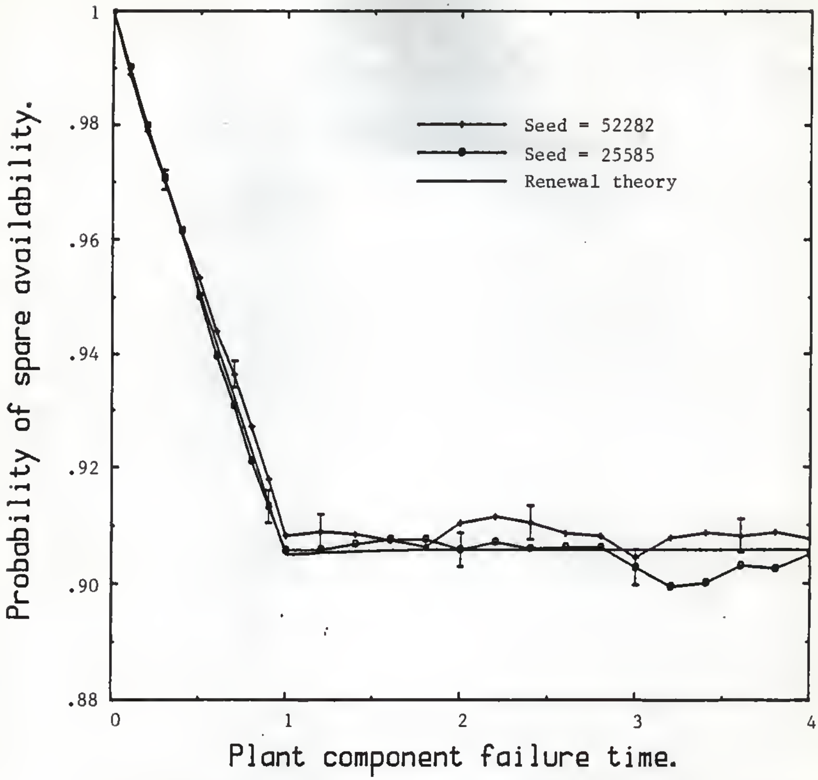

FIG. 2.15. Probability of spare availability for a five component system. The Model 1 simulation results are compared to the renewal theory results for two initial seed values for the random number generating subroutine with 10,000 simulations computed. The error bars indicate ±σ.

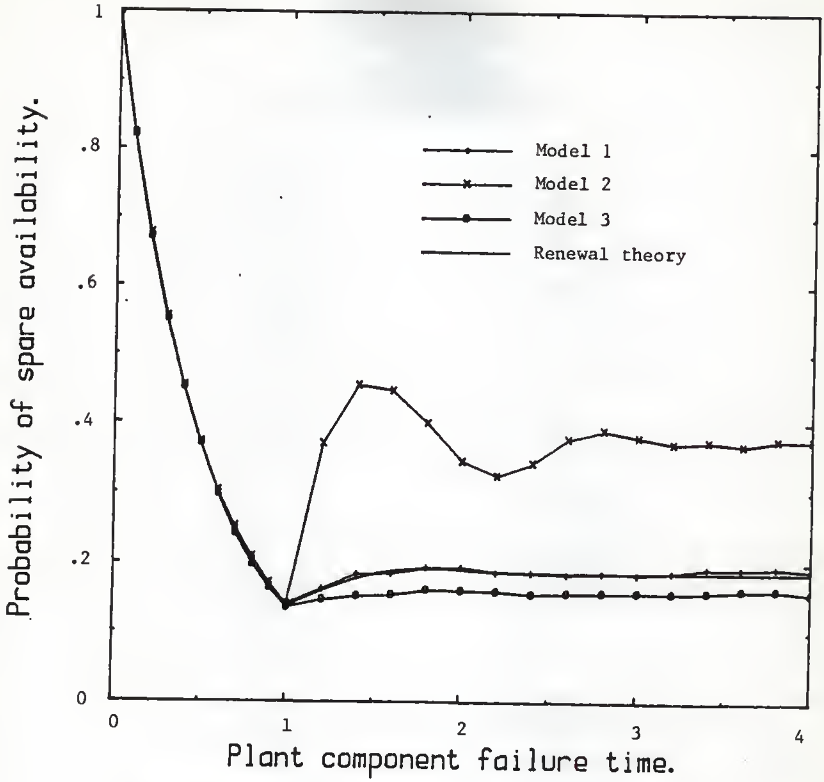

FIG. 2.16. Probability of spare availability for a five component system. The Model 1, 2, and 3 simulation results are compared to the renewal theory results with β = .4 and 10,000 simulations computed.

The simulation models and renewal theory estimate the same probability of spare availability for the interval $(0,\beta)$. This result supports the hypothesis that the spare-component pool (without any simplifying assumptions) is a "pseudo-renewal process." That is, the probability of all plants in the system operating at time τ is dependent upon their operational status in the interval $(\tau-\beta,\tau)$ for $\tau \geq \beta$.

Chapter 3

ECONOMIC MODEL FOR SPARE-COMPONENT POOL

3.1 Model Formulation

An economic model that accounts exactly for all possible costs and revenue requirements along with the various timing of these cash flows is a very complex problem. A simpler model utilizing the concept of a fixed charge rate to treat capital expenditures was used to analyze results for this study. To evaluate the attractiveness of either management plan a present worth cost of all future expenses for Plan A and Plan B are estimated and then compared. The plan with the lower present worth cost is chosen as the superior option.

3.1.1 Revenue Requirements

The economic model was based on the concept of a fixed charge rate FCR(n,m) and a discount rate $i_d(n,m)$, both of which are generally functions of time (year m) and the plant of interest (plant n). In this study the assumption was made that the discount rate was the same for all plants in the pool. Thus, the discount for year M is denoted by $I_d(m)$. Let AR(n) be the levelized annual revenue requirement over m years and C(n,0) be an expenditure at the beginning of m years. The fixed charge rate is defined such that FCR(n,m)C(n,0) is the dollar amount that must be earned by the end of each year over a period of m years to capitalize the expense C(n,0) for plant n (23). Thus, AR(n) is given by

$$AR(n) = C(n,0)FCR(n,m). \tag{3.1}$$

The FCR(n,m) includes considerations for return on investment, return of investment, taxes, depreciation, etc. Thus, FCR(n,m) gives an annual levelized capital requirement over m years that is equivalent to the actual requirements obtained by a detailed analysis of the many factors influencing utility economics.

The total revenue required R(n,0) at time zero to cover expense C(n,0) without any further revenue is the present worth of AR(n), which is

$$R(n,0) = AR(n) \begin{bmatrix} \text{present} \\ \text{worth} \\ \text{factor} \end{bmatrix} , \tag{3.2}$$

where

$$\begin{bmatrix} \text{present} \\ \text{worth} \\ \text{factor} \end{bmatrix} = \left(\frac{1}{[1+i_d(1)]} + \frac{1}{[1+i_d(1)][1+i_d(2)]} \right.$$

$$+ \frac{1}{[1+i_d(1)][1+i_d(2)][1+i_d(3)]} +$$

$$\left. \ldots + \frac{1}{[1+i_d(1)][1+i_d(2)]\ldots[1+i_d(k-1)][1+i_d(k)]^{\gamma_k}} \right) .$$

γ_k is the fraction of the last year, k, over which the revenue must be discounted. For this specific case $\gamma_k = 1$, but, in general, for an expenditure that occurs somewhere between the beginning and the end of year k, γ_k will be a fractional value.

The notation in Eq. (3.2) can be simplified to give

$$R(n,0) = AR(n) \left\{ \sum_{j=1}^{k} \prod_{h=1}^{j} [1+i_d(h)]^{-1} \right\} . \tag{3.3}$$

The results of Eq. (3.1) and Eq. (3.3) are combined to get the relationship between initial expenditure and initial revenue requirement as

$$R(n,0) = C(n,0)\ FCR(n,k) \sum_{j=1}^{k} \prod_{h=1}^{j} [1+i_d(h)]^{-1}\ . \qquad (3.4)$$

Therefore, the FCR(n,k) divides the initial expenditure into equal payments over k years such that the sum of the discounted annual revenue requirement is equal to the initial revenue requirement.

For the case of a constant discount rate, the present worth of all the AR(n) is

$$R(n,0) = AR(n)\ (P/A)_k^{i_d}\ , \qquad (3.5)$$

where $(P/A)_k^{i_d}$ is the present worth of a uniform series expressed as

$$(P/A)_k^{i_d} = \frac{(1+i_d)^k - 1}{i_d\ (1+i_d)^k}\ . \qquad (3.6)$$

Thus, the initial revenue requirement for an initial expenditure of C(n,0) is

$$R(n,0) = C(n,0)\ FCR(n,k)(P/A)_k^{i_d} = C(n,0)\ FCR(n,k) \left(\frac{(1+i_d)^k - 1}{i_d(1+i_d)^k}\right)\ . \qquad (3.7)$$

3.1.2 Revenue Requirements and Total Cost Calculations For Management Plans

In this section a method is derived to evaluate the cost and revenue requirements for the two component management plans. The two plans are compared on the basis of the present worth of all future expenditures. Certain types of expenditures such as the spare component cost, replacement component and salvage values, are evaluated in terms of an equivalent revenue requirement. Other expenditures such as operation and maintenance costs, replacement energy, and reserve capacity are expensed in the year these costs are incurred.

The present worth of costs occurring during year m is given by Eq. (3.4) or (3.7), if the item is expensed. In contrast, items that are capitalized over k years have an associated FCR(k) and a present worth of the revenue requirement. For an item that is capitalized over m years, the revenue requirement in year m for an expenditure C(m) is

$$R(n,m) = REV(n,k)C(n,m) \ , \tag{3.8}$$

where REV(n,k) is the revenue-to-expense conversion factor for plant n.

The present worth of the revenue requirment at time zero is the revenue requirement in year k discounted over m years. Thus,

$$R(n,0) = REV(n,k)\ C(n,m) \prod_{j=1}^{m} [1+i_d(j)]^{-1} = REV(n,k)\ C(n,0) \ . \tag{3.9}$$

An expression for the revenue-to-expense conversion factor is found by equating Eq. (3.4) and Eq. (3.9) to give

$$REV(n,k) = FCR(n,k) \sum_{j=1}^{k} \prod_{h=1}^{j} [1+i_d(h)]^{-1}. \tag{3.10}$$

The revenue-to-expense conversion factor for plant n is, therefore, independent of the time at which the expense is incurred. Thus, the revenue requirement at time t' is given by

$$R(n,t') = REV(n,k)\ C(n,t') = FCR(n,k)\ C(n,t')$$

$$\cdot \sum_{j=1}^{k} \prod_{h=1}^{j} [1+i_d(h)]^{-1} \ . \tag{3.11}$$

For the remainder of this study the revenue-to-expense conversion factor dependence on the capitalization period will be assumed and denoted simply as REV(n).

3.1.3 Allocation of Plan B Spare Component Costs

Many of the costs considered in this project are shared by both Plan A and Plan B; however, costs associated with the spare component are specific to Plan B. These spare component costs include initial purchase cost $C_{Spr}(0)$, maintenance and storage costs C_{ms}, and the final salvage value C_{ssv}. Each plant must pay a share of these costs based on some allocation method.

The fraction of costs assigned to the n-th plant participating in the spare-component pool are defined by

$F(n,0)$ = fraction of initial spare component purchase cost at the beginning of the pool,

$F(n,m)$ = fraction of storage cost in year m, and

$F(n,T_{max})$ = fraction of spare-component salvage value after pool has been dissolved after T_{max} years.

The two allocation methods considered in this project are (1) allocation by protected capacity and (2) allocation by number of plants in the pool.

<u>Protected capacity method</u>

The protected capacity allocation method assigns a value to the fraction F equal to the ratio of the capacity protected by the spare component in plant n to the total capacity protected of all the plants in the pool. Thus if PMW(n) denotes the protected capacity in plant n and there are a total of N plants in the pool, the allocation fraction for the initial component purchase cost is

$$F(n,0) = \frac{PMW(n)}{\sum_{j=1}^{N} PMW(j)} . \tag{3.12}$$

The allocation fraction for storage costs would only be assigned to plants that were operating during the year; or

$$F(n,m) = \frac{PMW(n)}{\sum_{j=1}^{K} PMW(j)} , \tag{3.13}$$

where the summation is over only the K plants operating during year m. For this method, the salvage fraction $F(n,T_{max})$ is taken equal to $F(n,0)$.

<u>Number of plants method</u>

A simpler allocation method and the one used in the later examples is to assign costs on the basis of the number of plants in the pool. That is

$$F(n,0) = F(n,T_{max}) = \frac{1}{N} , \tag{3.14}$$

where N is the number of plants in the pool and

$$F(n,m) = \frac{1}{N'(m)} , \quad 0 < N'(m) \leq N , \tag{3.15}$$

where N'(m) is the number of plants in the pool that operate during year m. Regardless of which allocation method is used, no annual charge for pool expenses is assigned to non-operating plants. In this manner plants could be allowed to enter the pool after its formation as well as leave the pool before the pool's dissolution.

3.2 Classification of Cost Components

The types of costs associated with both management plans can be divided into four separate classifications: 1) failure-dependent variable costs, 2) plant-operation variable costs, 3) failure-dependent fixed costs, and 4) annual fixed costs (see Table 3.1).

Table 3.1 The four classifications of costs associated with both management plans and the costs in each category.

Types of Costs	Plan A	Plan B
Failure-dependent variable costs		
-replacement energy	X	X
Plant-operation variable costs		
-operation and maintenance	X	X
Failure-dependent fixed costs		
-component repair or replacement	X	X
-used component salvage value	X	X
-failed component salvage value	X	X
-spare used as temporary substitute		X
Annual fixed costs		
-reserve capacity	X	X
-spare purchase		X
-spare storage and maintenance		X
-spare salvage value		X

The following sections describe methods of estimating these costs for each classification. Costs classified as types 1, 2 and 3 are differential costs incurred in the repair/replacement interval and they are present valued to the time of failure, t'. These differential costs are then summed over the interval $(0,t)$ to estimate the present worth of the type costs. In contrast, type 4 costs are incurred at the end of each year and represent the value at that time. These costs are then summed over the interval $(0,k)$ to estimate the present worth of that type cost, where k is an integer value of years such that $k - 1 < t \leq k$ (e.g., if $t = 2.3$, then $k = 3$). Thus, the present worth costs are an estimate of the total costs for plant n over the interval $(0,t)$. In general, the value of t will be equal to the spare-pool lifetime.

3.3 Total Plan Costs

The total costs under the two managerial plans can now be estimated. The present worth of all expected costs at time t and at plant n under plan ℓ is

$$PWTOTAL_{\ell}(n,t) = PWFVC_{\ell}(n,t)+PWOM_{\ell}(n,t)+PWFFC_{\ell}(n,t)+PWAFC_{\ell}(n,t) \quad , \quad (3.16)$$

where PWFVC is the present worth of the failure-dependent variable costs, PWOM is the present worth of the plant-operation variable costs, PWFFC is the present worth of the failure-dependent fixed costs, and PWAFC is the present worth of the annual fixed costs.

3.4 Failure-dependent Variable Costs

The variable costs at the time of failure are dependent on the time of failure and the length of each shutdown period. In this section the general case is analyzed, followed by several special case solutions for the different management plan total variable costs. In general, numerical techniques must be used to evaluate the integrals in the expressions for these costs.

The total variable costs of a failure are dependent on the management plan the plant is operating under. The Plan A costs are the present worth of the variable costs that are paid by the k-th plant over the interval (0,t), and in general, are a function of the downtime following a component failure. The Plan B costs are the present worth of the variable costs that would be paid by the k-th plant over the interval (0,t) if the plant were a member of the spare pool. These costs are the weighted average of the variable costs for times when the spare is not available (Plan A costs) and for times the spare is available and used. In general, the Plan B costs incorporate some fractional portion of the Plan A costs and are, therefore, also a function of the downtime as well as the probability of spare availability at the time of failure.

3.4.1 General case

In this section a model is derived for estimating the Plan A and Plan B failure-dependent variable costs. The present worth of failure-dependent variable costs at plant n due to a failure at t' can be expressed as

$$fvc(n,t') = \text{Prob}\left\{\begin{matrix}\text{n-th plant fails}\\ \text{in unit time}\\ \text{about } t'\end{matrix}\;\middle|\;\begin{matrix}\text{n-th plant is}\\ \text{operating}\\ \text{at } t'\end{matrix}\right\}\cdot\left\{\begin{matrix}\text{fraction of the}\\ \text{repair/replacement}\\ \text{time the plant is}\\ \text{shutdown or derated}\end{matrix}\right\}\cdot\left\{\begin{matrix}\text{cost of}\\ \text{failure}\\ \text{at } t'\end{matrix}\right\}\left\{\begin{matrix}\text{present}\\ \text{worth}\\ \text{discount}\\ \text{factor}\end{matrix}\right\}.$$

Recall from renewal theory that g(t') is defined as

$$\text{Prob}\left\{\begin{matrix}\text{any plant is}\\ \text{operating (without}\\ \text{the spare) at } t'\end{matrix}\right\} = g(t'), \tag{3.17}$$

thus

$$\text{Prob}\left\{\begin{matrix}\text{a plant failure}\\ \text{in unit time}\\ \text{about } t'\end{matrix}\;\middle|\;\begin{matrix}\text{n-th plant}\\ \text{is operating}\\ \text{at } t'\end{matrix}\right\} = \text{Prob}\left\{\begin{matrix}\text{a plant}\\ \text{failure in}\\ \text{unit time}\\ \text{about } t'\end{matrix}\right\}\text{Prob}\left\{\begin{matrix}\text{n-th plant}\\ \text{is operating}\\ \text{at } t'\end{matrix}\right\}$$

$$= \lambda\, g(t'). \tag{3.18}$$

Define C(n,t') as the dollar rate per unit downtime at plant n and time t' making

$$\left\{\begin{matrix}\text{cost of a}\\ \text{failure at } t'\\ \text{in plant n}\end{matrix}\right\} = C(n,t')\, T_{repl}. \tag{3.19}$$

In general, the cost per unit downtime will increase over time by some yearly escalation rate making

$$C(n,t') = C(n,0) \left(\prod_{j=1}^{m-1} [1 + I(j)] \right) [1 + I(m)]^{\gamma_m}, \tag{3.20}$$

where m = INT(t') (i.e., m is the largest integer value $\leq$ t'), γ_m = t' - m, and I(m) is the escalation rate in year m. The present worth discount factor converts a future dollar amount into an equivalent present dollar amount. Let F(t') be the future value of P(0) present dollars making

$$P(0) = F(t') \left(\prod_{j=1}^{m-1} [1+i_d(j)]^{-1} \right) [1+i_d(m)]^{-\gamma_m}, \tag{3.21}$$

where m = INT(t'), γ_m = t'-m, and i_d is the discount rate in year m. For the case of a constant discount rate P(0) becomes

$$P(0) = F(t') \ (1+i_d)^{-t'}. \tag{3.22}$$

The present worth of the failure-dependent variable costs due to a failure at t' can now be written as

$$fvc(n,t') = \lambda \ g(t') \ f_{DT}(t') \ C(n,0) \ T_{repl} \ \hat{I}(t'), \tag{3.23}$$

where $f_{DT}(t')$ is defined as the "fractional downtime" defined as the effective fraction of the repair/replacement time that the plant is shutdown or derated while recovering from a component failure. The term $\hat{I}(t')$ is given by

$$\hat{I}(t') = \left(\prod_{j=1}^{m-1} \left(\frac{1+I(j)}{1+i_d(j)} \right) \right) \left(\frac{1+I(m)}{1+i_d(m)} \right)^{\gamma_m}, \tag{3.24}$$

where m = INT(t') and γ_m = t'-m, and represents the present worth factor.

The total present worth of the failure dependent variable costs is the sum of all the differential costs at t', or

$$PWFVC(n,t) = \int_0^t dt' \; fvc(n,t') = \int_0^t dt'\lambda \; g(t') \; f_{DT}(t') \; C(n,0) \; T_{repl} \; \hat{I}(t')$$

$$= \lambda \; C(n,0) \; T_{repl} \int_0^t dt' \; g(t') \; f_{DT}(t') \; \hat{I}(t'). \qquad (3.25)$$

For the case of constant escalation and discount rates, this result simplifies to

$$PWFVC(n,t) = \lambda \; C(n,0) \; T_{repl} \int_0^t dt' \; g(t') \; f_{DT}(t') \left(\frac{1+I}{1+i_d}\right)^{t'} . \qquad (3.26)$$

For Plan A, $f_{DT}(t') = 1$, so that

$$PWFVC_A(n,t) = \lambda \; C(n,0) \; T_{repl} \int_0^t dt' \; g(t') \; \hat{I}(t') \; , \qquad (3.27)$$

while for Plan B, $f_{DT}(t')$ is some function of time t'. In general, Eqs. (3.25) or (3.26) must be evaluated numerically because of the piecewise construction of g(t'), the variable discount and escalation rates, and the fractional downtime. Only for very simple situations in which analytical representations of the quantities are available is it possible to evaluate the integrals in Eqs. (3.27) or (3.28) analytically. In the following sections, PWFVC (n,t) is evaluated for two different system operating conditions and possible forms of g(t').

3.4.2 Case With Constant Plant Component Availability And Fraction Downtime

In this section the failure-dependent variable costs for Plan A and Plan B are evaluated under the following assumptions:

1) The annual escalation and discount rates (I and i_d) are constant.
2) The probability of spare availability and the probability of a plant operating without the spare at a given instant in time are both determined by the asymptotic renewal theory result.
3) The spare-component failure rate is zero.
4) The fractional downtime is a constant.

Under the above conditions, $g(t')$ becomes

$$g(t') = \frac{1}{1+\lambda\ T_{repl}} \tag{3.28}$$

so that the present worth of the Plan A variable costs are

$$fvc_A(n,t') = \lambda\ dt' \left(\frac{1}{1+\lambda\ T_{repl}}\right) a^{t'}\ C(n,0)\ T_{repl}\ , \tag{3.29}$$

where $a = (1+I)/(1+i_d)$.

The present worth of the Plan A variable costs for plant n over the interval $(0,t)$ becomes

$$PWFVC_A(n,t) = \int_0^t dt' fvc_A(n,t') = \int_0^t dt' \lambda \left(\frac{1}{1+\lambda\ T_{repl}}\right) a^{t'}\ C(n,0)\ T_{repl}. \tag{3.30}$$

Evaluation of the integral gives

$$PWFVC_A(n,t) = \frac{C(n,0)\ T_{repl}\ \lambda\ (a^t - 1)}{\ell n(a)(1 + \lambda\ T_{repl})}\ . \tag{3.31}$$

The present worth of the Plan B variable costs for plant n over the same interval (0,t) is

$$PWFVC_B\,(n,t) = \int_0^t dt' fvc_B(t')$$

$$= \int_0^t dt' \lambda \left(\frac{1}{1+\lambda T_{repl}}\right) a^{t'}\; C(n,0)\; T_{repl}\; f_{DT}(t'), \quad (3.32)$$

where $a = (1+I)/(1+i_d)$. This result reduces to

$$PWFVC_B(n,t) = \frac{C(n,0)\; T_{repl}\; \lambda\; (a^t - 1)}{\ell n(a)\; (1 + \lambda\, T_{repl})}\, f_{DT}. \quad (3.33)$$

3.4.3 Case with Constant Fraction Downtime

Now consider the system described by the following conditions:

1) The annual escalation and discount rates are constant.
2) The probability of a plant operating without the spare at a given instant in time is given by g(t') where

$$g(t') = \begin{cases} e^{-\lambda t'} & , \quad 0 < t' \leq T_{repl} \\ \left(\dfrac{1}{1+\lambda\; T_{repl}}\right) & , \quad T_{repl} < t \leq \infty\;. \end{cases} \quad (3.34)$$

3) The fractional downtime is a constant value equal to the asymptotic renewal theory result.

The present worth of the Plan A variable costs is

$$fvc_A(n,t') = \begin{cases} \lambda\; e^{-\lambda t'} a^{t'}\; C(n,0)\; T_{repl} & , \quad 0 < t' \leq T_{repl} \\ \lambda \left(\dfrac{1}{1+\lambda T_{repl}}\right) a^{t'} C(n,0)\; T_{repl} & , \quad T_{repl} < t' \leq \infty \end{cases} \quad (3.35)$$

where $a = (1+I)/(1+i_d)$. The present worth of the Plan A variable costs for plant n over the interval (0,t) is

$$
\mathrm{PWFVC}_A(n,t) = \begin{cases} \displaystyle\int_0^t dt'\ \lambda\ e^{-\lambda t'} a^{t'} C(n,0)\ T_{repl} , & 0<t\leq T_{repl} \\[2ex] \displaystyle\int_0^t dt'\ \lambda\ e^{-\lambda t'} a^{t'} C(n,0)\ T_{repl} \\ \displaystyle\quad + \int_0^t dt'\ \lambda \left(\frac{1}{1+\lambda T_{repl}}\right) a^{t'} C(n,0)\ T_{repl} , & T_{repl}<t\leq\infty. \end{cases} \tag{3.36}
$$

The above equation is solved using integration by parts resulting in

$$
\mathrm{PWFVC}_A(n,t) = \begin{cases} \dfrac{C(n,0)\ T_{repl}\ \lambda\ (a^t e^{-\lambda t}-1)\ e^{-\lambda t}}{\lambda + \ln(a)} , & 0 < t \leq T_{repl} \\[2ex] \dfrac{C(n,0)\ T_{repl}\ \lambda\ (a^t e^{-\lambda t}-1)\ e^{-\lambda t}}{\lambda + \ln(a)} \\[2ex] \quad + \dfrac{C(n)\cdot T_{repl}\ \lambda\ (a^t-1)}{\ln(a)(1+\lambda\ T_{repl})} , & T_{repl} < t \leq \infty . \end{cases} \tag{3.37}
$$

For a constant fractional downtime the present worth of the Plan B variable costs for plant n over the interval (0,t) is

$$
\mathrm{PWFVC}_B(n,t) = f_{DT}\ \mathrm{PWFVC}_A(n,t) \tag{3.38}
$$

making

$$
\mathrm{PWFVC}_B(n,t) = \begin{cases} \dfrac{C(n,0)\ T_{repl}\ \lambda\ (a^t e^{-\lambda t}-1)}{\lambda + \ln(a)}\ f_{DT} , & 0 < t \leq T_{repl} \\[2ex] \dfrac{C(n,0)\ T_{repl}\ \lambda\ (a^t e^{-\lambda t}-1)}{\lambda + \ln(a)} \\[2ex] \quad + \dfrac{C(n,0)\ T_{repl}\ \lambda\ (a^t-1)}{\ln(a)\ (1+\lambda\ T_{repl})}\ f_{DT} , & T_{repl} < t \leq \infty . \end{cases} \tag{3.39}
$$

3.4.4 Replacement Energy Costs

One important cost incurred when a plant is forced to be derated is the replacement energy costs. In general, the spare will not have the same rated output as the failed component it is replacing. If the spare is rated higher than the failed component, no replacement energy costs are incurred during the time in which the spare is installed. In contrast, if the spare is rated lower than the failed component the plant's output capacity is decreased from MW_{max} to $[MW_{max} - MW_{down}]$, where MW_{down} is the output capacity lost due to the component failure. In the case of Plan A the decrease in output capacity is MW_{max}.

Replacement power must be purchased from outside the plant to make up for the loss in output capacity as a result of the component failure. The system replacement energy costs $C_{en}(n,t)$ in the interval $(t_k, t_k + T_{repl})$ is given by

$$C_{en}(n,t) = f_{cap}(n,t)\, MW_{down}(n)\, \zeta_{en}(n,t) \tag{3.40}$$

where the three terms are defined by:

$f_{cap}(n,t)$ = the plant capacity factor averaged over the interval $(t_k, t_k + T_{repl})$ or the ratio of the scheduled plant output (prior to the failure) to the maximum possible plant output for time T_{repl}.

$MW_{down}(n)$ = the plant output which is lost because of the failed component. If the failed component is necessary for the plant to operate at any output then MW_{down} is the plant rated capacity. In general, MW_{down} is the failed component capacity.

$\zeta_{en}(n,t)$ = the differential cost of replacement energy (\$/down time) in the interval $(t_k, t_k + T_{repl})$. The average cost of replacement energy is the difference between the cost of purchasing power contracted from some outside source and the cost of producing power at the effected plant. An allowance is made for escalation by letting

$$\zeta_{en}(n,t) = \zeta_{en}(n,0) \left(\prod_{j=1}^{m-1} [1+I_{en}(j)]\right)[1+I_{en}(m)]^{\gamma_m}, \quad (3.41)$$

where $I_{en}(m)$ is the escalation rate in year m, m=INT(t) and $\gamma_m = t - m$.

For Plan A, the downtime is equal to the new component repair/replacement time and MW_{down} is equal to the full plant capacity, MW(n). For Plan B, the downtime is equal to MW_{down} which is defined by $\text{Max}(0, MW(n) - MW_{spr})$ where MW_{spr} is the spare capacity.

If the results of Eqs. (3.37) and (3.40) are combined, the present worth of the failure-dependent variable costs under Plan A for plant n over the interval (0,t) for the general case are

$$PWFVC_A(n,t) = T_{repl}\ MW(n)\ \zeta_{en}(n,0)\ \lambda \int_0^t dt'\ g(t')\ f_{cap}(n,t')\ \hat{I}_{en}(t'), \quad (3.42)$$

where

$$\hat{I}_{en}(t') = \left(\prod_{j=1}^{m-1} \frac{[1+I_{en}(j)]}{[1+i_d(j)]}\right) \left(\frac{1+I_{en}(m)}{1+i_d(m)}\right)^{\gamma_m}$$

and m = INT(t) such that $\gamma_m = t' - m$.

The above result can be simplified considerably by assuming a constant discount rate, constant escalation rate, and a constant capacity factor. For this case

$$PWFVC_A(n,t) = T_{repl}\, f_{cap}(n) MW(n) \zeta_{en}(n,0) \lambda \int_0^t dt'\; g(t') \left(\frac{1+I_{en}}{1+i_d}\right)^{t'} . \quad (3.43)$$

If the spare is not rated for full capacity output of the plant, the utility must purchase energy during the time the spare is installed. The models presented in Chapter 2 are used to determine the fraction of the repair/replacement interval for which replacement power must be purchased. Thus, the present worth of the Plan B total variable costs at the time of failure can be estimated by

$$PWFVC_B(n,t) = T_{repl}\; \zeta_{en}(n,0) \lambda \left\{ MW(n) \lambda \int_0^t dt'\; g(t') f_{cap}(n,t') f_{DT}(t') \hat{I}_{en}(t') \right.$$

$$\left. + MW_{down}\; \lambda \int_0^t dt'\; g(t')\; f_{cap}(n,t')\; [1 - f_{DT}(t')]\; \hat{I}_{en}(t') \right\} , \quad (3.44)$$

where

$$\hat{I}_{en}(t') = \prod_{j=1}^{m-1} \left(\frac{1+I_{en}(j)}{1+i_d(j)}\right) \left(\frac{1+I_{en}(m)}{1+i_d(m)}\right)^{\gamma_m}$$

and $m = INT(t)$ such that $\gamma_m = t' - m$.

Recall $MW_{down} = MW(n) - MW_{spr}$; thus, the Eq. (3.44) can be written as

$$PWFVC_B(n,t) = T_{repl}\; \zeta_{en}(n,0) \lambda \left\{ MW_{down} \int_0^t dt'\; g(t') f_{cap}(n,t') \hat{I}_{en}(t') \right.$$

$$\left. + MW_{spr} \int_0^t dt'\; g(t')\; f_{cap}(n,t')\; f_{DT}(t')\; \hat{I}_{en}(t') \right\} \quad (3.45)$$

Equation (3.45) can be simplified further by assuming a constant capacity factor, discount rate, and escalation rate so as to give

$$PWFVC_B(n,t) = T_{repl}\ \zeta_{en}(n,0)\lambda\ f_{cap}(n) \left\{ MW_{down} \int_0^t dt'\ g(t') \left(\frac{1+I_{en}}{1+i_d}\right)^{t'} \right.$$

$$\left. + MW_{spr} \int_0^t dt'\ g(t')\ f_{DT}(t') \left(\frac{1+I_{en}}{1+i_d}\right)^{t'} \right\} . \tag{3.46}$$

Equations (3.45) and (3.48) can be solved analytically in the case of constant $g(t')$ and $f_{DT}(t')$. The results are

$$PWFVC_A(n,t) = \kappa(n)\ \frac{(a^t-1)}{\ell n(a)}\ MW(n) \tag{3.47}$$

and

$$PWVC_B(n,t) = \kappa(n)\ \frac{(a^t-1)}{\ell n(a)}\ [MW(n) - (1-f_{DT}(n))\ MW_{spr}], \tag{3.48}$$

where $\kappa(n) = T_{repl}\ f_{cap}(n)\ \zeta_{en}(n,0)\ \lambda\ g$, and $a = \left(\frac{1+I_{en}}{1+i_d}\right)$.

3.5 Plant-operation Variable Costs

Expenditures during the time of plant operation include operation, maintenance, testing, and insurance costs. For the purposes of this study it is assumed these costs are the same (on a cost per unit operation time basis) for the original component, repair/replacement component, and spare component installed at a particular plant. The variable costs, denoted by vcom, are a function of the particular plant and the operation time.

The present worth of the variable cost during the time of plant operation in unit time about t' is expressed by

$$\text{vcom}(n,t') = \text{Prob}\begin{Bmatrix}\text{plant is}\\ \text{operating}\\ \text{at } t'\end{Bmatrix}\cdot\begin{Bmatrix}\text{cost per unit}\\ \text{operating}\\ \text{time}\end{Bmatrix}\cdot\begin{Bmatrix}\text{present}\\ \text{worth}\\ \text{discount}\\ \text{factor}\end{Bmatrix} + \text{Prob}\begin{Bmatrix}\text{plant fails} \\ \text{in unit time} \\ \text{about } dt'\end{Bmatrix.\left|\begin{matrix}\text{plant is}\\ \text{operating}\\ \text{at } t'\end{matrix}\right.\Bigg\}$$

$$\cdot\begin{Bmatrix}\text{fraction of downtime}\\ \text{during which spare is}\\ \text{installed and operating}\end{Bmatrix}\cdot\begin{Bmatrix}\text{downtime}\\ \text{per failure}\end{Bmatrix}\cdot\begin{Bmatrix}\text{cost per unit}\\ \text{operating}\\ \text{time}\end{Bmatrix}\cdot\begin{Bmatrix}\text{present}\\ \text{worth}\\ \text{discount}\\ \text{factor}\end{Bmatrix} .$$

The present worth of the Plan A plant-operation variable costs for plant n in unit time about t' can now be written as

$$\text{vcom}_A(n,t') = g(t')\ \zeta_{om}(n,0)\ \hat{I}_{om}(t') \ , \tag{3.49}$$

where

$$\hat{I}_{om}(t') = \left\{\prod_{j=1}^{m-1} \frac{[1+I_{om}(j)]}{[1+i_d(j)]}\right\}\left\{\frac{1+I_{om}(m)}{1+i_d(m)}\right\}^{\gamma_m}$$

and m = INT(t') such that $\gamma_m = t' - m$. The present worth of all Plan A plant-operation variable costs for plant n over the interval (0,t) is

$$\text{PWOM}_A(n,t) = \int_0^t dt'\ \text{vcom}_A(n,t') = \zeta_{om}(n,0)\int_0^t dt'\ g(t')\ I_{om}(t') \ . \tag{3.50}$$

The present worth of the Plan B plant-operation variable cost for plant n in unit time about t' is

$$\begin{aligned}\text{vcom}_B(n,t') &= g(t')\ \zeta_{om}(n,0)\ \hat{I}_{om}(t') \\ &\quad + \lambda\ g(t')\ [1-f_{DT}(t')]\ T_{repl}\zeta_{om}(n,0)\ \hat{I}_{om}(t') \ .\end{aligned} \tag{3.51}$$

The present worth of all Plan B plant-operation variable costs for plant n over the interval (0,t) is

$$\text{PWOM}_B(n,t) = \int_0^t dt'\ \text{vcom}_B(n,t') = \zeta_{en}^{(n,0)}\left\{\int_0^t dt'\ g(t')\ \hat{I}_{om}(t')\right.$$

$$+ \lambda\, T_{repl} \int_0^t dt'\; g(t')\; [1-f_{DT}(t')]\; \hat{I}_{om}(t') \Bigg\}$$

$$= \zeta_{om}(n,0) \Bigg\{ (1+\lambda\, T_{repl}) \int_0^t dt'\; g(t')\; \hat{I}_{om}(t')$$

$$- \lambda\, T_{repl} \int_0^t dt'\; g(t')\; f_{DT}(t') \hat{I}_{om}(t') \Bigg\} . \qquad (3.52)$$

For the special case of a constant g(t), discount rate, escalation rate, and fractional downtime, the costs for the two plans become

$$PWOM_A(n,t) = \zeta_{om}(n,0)\; g \int_0^t dt'\; \hat{I}_{om}(t') = \zeta_{om}(n,0)\; g(t) \left(\frac{a^t - 1}{\ell n(a)}\right) \qquad (3.53)$$

and

$$PWOM_B(n,t) = \zeta_{om}(n,0)\; g \Bigg\{ (1+\lambda\, T_{repl} - f_{DT}\lambda\, T_{repl}) \int_0^t dt'\; \hat{I}_{om}(t') \Bigg\}$$

$$= \zeta_{om}(n,0)\; g\; (1 + \lambda\, T_{repl} - f_{DT}\lambda\, T_{repl}) \left(\frac{a^t - 1}{\ell n(a)}\right) , \qquad (3.54)$$

where $a = (1+I_{om})/(1+i_d)$.

3.6 Failure-Dependent Fixed Costs

Fixed costs occur at the time of a component failure and are independent of the length of time the plant is subsequently shut down or derated. These costs include the cost of repairing the failed component or purchasing a replacement component, $C_{comp}(t)$, and the salvage value of the failed component $C_{fsv}(t)$.

Plan B costs also include the cost of taking the spare component to the failed plant, installing it, and returning it to the pool when the repaired/replacement component arrives, denoted by $C_{temp}(t)$. A detailed description of the failure-dependent fixed costs is provided below.

3.6.1 Repaired/replacement Component Costs

The repair/replacement cost at time t for plant n includes the total repair/replacement and installation costs for the component such as capital cost, repair cost, shipping, removal of the failed component, etc. The utility has the option to repair the failed component instead of purchasing a new component in which case C_{comp} is the total restoration costs associated with returning the failed component to the "good as new" condition. An allowance is made for the escalation in the repair/replacement cost by

$$C_{comp}(n,t') = \zeta_{comp}(n,0) \left\{ \prod_{j=1}^{m-1} [1+I_{comp}(j)] \right\} [1+I_{comp}(m)]^{\gamma_m} , \quad (3.55)$$

where $I_{comp}(m)$ is the escalation rate in year m, m = INT(t'), and $\gamma_m = t'-m$.

The repair/replacement cost is assumed to be identical for both Plans A and B. In the event the spare's rated capacity is identical to that of the failed component, the affected plant may elect to keep the spare and place the repair/replacement component in the spare pool. As a result, the transportation costs may be different for Plan A and Plan B and a distinction must be made between the two plans for C_{comp}.

In this study the purchase cost was assumed to be capitalized by the utility. Thus, C_{comp} is multiplied by the revenue-to-expense conversion factor defined in Section 3.1. The value of REV(n) is determined for the affected plant and the year in which the purchase cost occurs.

3.6.2 Failed Component Salvage Value

The salvage value of the failed component is zero in the case that the failed component is repaired. An allowance is made for the escalation in the failed component salvage value by

$$C_{fsv}(n,t') = \zeta_{fsv}(n,0) \left(\prod_{j=1}^{m-1} [1+I_{fsv}(j)]\right) [1+I_{fsv}(m))]^{\gamma_m} , \quad (3.56)$$

where $I_{fsv}(m)$ is the escalation rate in year m, m = INT(t'), m and γ_m = t' - m. Similar to the repair/replacement cost, the failed component salvage value is a capitalized expense.

3.6.3 Used Plant Component Salvage Value

A used but operable plant component also has some salvage value at the end of the pool. In general, the salvage value will depend on the size and age of the component. Like the component repair/ replacement cost and the failed salvage value, the used salvage value is capitalized by using a revenue-to-expense factor. An allowance is also made for the escalation in the salvage value of the used component by

$$C_{pcsv}(n,t') = \zeta_{pcsv}(n,0) \left(\prod_{j=1}^{m-1} [1+I_{pcsv}(j)]\right) [1+I_{pcsv}(m)]^{\gamma_m}, \quad (3.57)$$

where $I_{pcsv}(m)$ is the escalation rate in year m, m = INT(t'), and γ_m = t' - m.

3.6.4 Cost of Installing a Spare Temporarily

For Plan B only, there is an additional cost associated with transporting the spare component to the failed plant, installing it and transporting it back to the pool storage location. If the spare capacity is identical to the failed component, then it is assumed that the spare remains installed in the plant and the spare transportation cost is neglected. Like the other fixed costs, the temporary installation costs escalate as

$$C_{temp}(n,t') = \zeta_{temp}(n,0) \left(\prod_{j=1}^{m-1} [1+I_{temp}(j)] \right) [1+I_{temp}(m)]^{\gamma_m}, \quad (3.58)$$

where $I_{temp}(m)$ is the escalation rate in year m, m = INT(t') and γ_m = t' - m.

3.6.5 Present Worth of Failure-Dependent Fixed Costs

The present worth at time t' of the failure costs for Plan A can be expressed as

$$ffc_A(n,t') = \text{Prob}\left\{\begin{matrix}\text{plant n fails}\\ \text{in unit time}\\ \text{about t'}\end{matrix} \middle| \begin{matrix}\text{n-th plant}\\ \text{is operating}\\ \text{at t'}\end{matrix}\right\} \cdot \left\{\begin{matrix}\text{cost of a}\\ \text{failure}\\ \text{at t'}\end{matrix}\right\} \cdot \left\{\begin{matrix}\text{present}\\ \text{worth}\\ \text{discount}\\ \text{factor}\end{matrix}\right\} \cdot \left\{\begin{matrix}\text{revenue-to-}\\ \text{expense con-}\\ \text{version factor}\end{matrix}\right\}$$

or

$$\begin{aligned} ffc_A(n,t') &= \lambda\, g(t')\, REV(n) \\ &\cdot [\zeta_{comp}(n,0)\, \hat{I}_{comp}(t') - \zeta_{fsv}\, \hat{I}_{fsv}(t')] , \end{aligned} \quad (3.59)$$

where

$$\hat{I}_{comp}(t') = \left[\prod_{j=1}^{m-1} \left(\frac{1+I_{comp}(j)}{1+i_d(j)} \right) \right] \left(\frac{1+I_{comp}(m)}{1+i_d(m)} \right)^{\gamma_m},$$

$$\hat{I}_{fsv}(t') = \left[\prod_{j=1}^{m-1} \left(\frac{1+I_{fsv}(j)}{1+i_d(j)} \right) \right] \left(\frac{1+I_{fsv}(m)}{1+i_d(m)} \right)^{\gamma_m},$$

m = INT(t'), and, γ_m = t'- m. The present worth over the interval (0,t) of the failure dependent fixed costs include the summation of all the failure costs less the revenue from the sale of the used component. Thus, the present worth of Plan A failure-dependent fixed costs for plant n over the interval (0,t) is

$$PWFFC_A(n,t) = \left[\int_0^t dt' \; ffc_A(n,t')\right] - \text{Prob}\begin{Bmatrix}\text{component is}\\ \text{operating}\\ \text{at } t\end{Bmatrix}\begin{Bmatrix}\text{revenue from}\\ \text{sale of used}\\ \text{component}\end{Bmatrix}\begin{Bmatrix}\text{present}\\ \text{worth}\\ \text{discount}\\ \text{factor}\end{Bmatrix}$$

or

$$PWFFC_A(n,t) = \left[\int_0^t dt' \; ffc_A(n,t')\right] - g(t)\;\zeta_{pcsv}(n,0)\;REV(n)\;\hat{I}_{pcsv}(t)$$

$$= REV(n)\left\{\lambda_{comp}(n,0)\int_0^t dt'\; g(t')\;\hat{I}_{comp}(t')\right.$$

$$- \lambda\;\zeta_{fsv}(n,0)\int_0^t dt'\; g(t')\;\hat{I}_{fsv}(t')$$

$$\left. - g(t)\;\zeta_{pcsv}(n,0)\;\hat{I}_{pcsv}(t)\right\}, \qquad (3.60)$$

where

$$\hat{I}_{pcsv}(t) = \left[\prod_{j=1}^{m-1}\left(\frac{1+I_{pcsv}(j)}{1+i_d(j)}\right)\right]\left(\frac{1+I_{pcsv}(m)}{1+i_d(m)}\right)^{\gamma_m}.$$

The present worth at time t' of the failure costs for Plan B are expressed in the same manner as for Plan A with an additional term that accounts for the temporary installation of the spare as

$$ffc_B(n,t') = \lambda\; g\left\{\zeta_{comp}(n,0)\;REV(n)\;\hat{I}_{comp}(t')\right.$$

$$- \zeta_{fsv}(n,t')\;REV(n)\;\hat{I}_{fsv}(t')$$

$$\left. + \rho(n)\;\zeta_{temp}(n,0)\;\hat{I}_{temp}(t')\right\}, \qquad (3.61)$$

where

$$\hat{I}_{temp}(t') = \left[\prod_{j=1}^{m-1}\left(\frac{1+I_{temp}(j)}{1+i_d(j)}\right)\right]\left(\frac{1+I_{temp}(m)}{1+i_d(m)}\right)^{\gamma_m},$$

$m = \mathrm{INT}(t')$, and $\gamma_m = t' - m$. The term $\rho(n)$ is defined by

$$\rho(n) = \begin{cases} 1 & MW(n) - MW_{SPR} \neq 0 \\ 0 & MW(n) - MW_{SPR} = 0 \,. \end{cases}$$

Thus, the present worth of the Plan B failure-dependent fixed costs over the interval $(0,t)$ is

$$\begin{aligned} PWFFC_B(n,t) = REV(n) \Bigg\{ & \lambda\zeta_{comp}(n,0) \int_0^t dt'\ g(t')\ \hat{I}_{comp}(t') \\ & - \lambda\zeta_{fsv}(n,0) \int_0^t dt'\ g(t')\ \hat{I}_{fsv}(t') \\ & - g(t)\ \zeta_{pcsv}(n,0)\ \hat{I}_{pcsv}(t) \Bigg\} \\ & + \lambda\ \rho(n)\ \zeta_{temp}(n,0) \int_0^t dt'\ g(t')\ \hat{I}_{temp}(t') \,. \end{aligned} \tag{3.62}$$

For the special case of constant operating probability (i.e., a constant $g(t)$), escalation rates, and discount rates the analytical evaluation of Eqs. (3.62) and (3.64) yields

$$PWFFC_A(n,t) = \kappa_1 \left(\frac{a^t - 1}{\ln(a)}\right) - \kappa_2 \left(\frac{b^t - 1}{\ln(b)}\right) - \kappa_3 \left(\frac{c^t - 1}{\ln(c)}\right) \tag{3.63}$$

and

$$PWFFC_B(n,t) = \kappa_1 \left(\frac{a^t - 1}{\ln(a)}\right) - \kappa_2 \left(\frac{b^t - 1}{\ln(b)}\right) - \kappa_3 \left(\frac{c^t - 1}{\ln(c)}\right) + \kappa_4 \left(\frac{d^t - 1}{\ln(d)}\right) , \tag{3.64}$$

where

$$\kappa_1 = \lambda\ \zeta_{comp}(n,0)\ REV(n)\ g,$$

$$\kappa_2 = \lambda\ \zeta_{fsv}(n,0)\ REV(n)\ g,$$

$$\kappa_3 = \zeta_{pcsv}(n,0)\ REV(n)\ g,$$

$$\kappa_4 = \rho(n)\ \zeta_{temp}(n,0)\ g,$$

$$a = \left(\frac{1+I_{comp}}{1+i_d}\right),$$

$$b = \left(\frac{1+I_{fsv}}{1+i_d}\right),$$

$$c = \left(\frac{1+I_{pcsv}}{1+i_d}\right), \text{ and}$$

$$d = \left(\frac{1+I_{temp}}{1+i_d}\right).$$

3.7 Annual Fixed Costs

Certain fixed costs are incurred by the generating station even if no failures occur. Such fixed costs are considered year-end expenses and must be estimated in annual increments. The assumption is made that the plants will contract for reserve margin capacity and pool membership on a yearly basis. Unlike the energy replacement cost, the plant reserve capacity cost is contracted and paid for on a yearly contract basis. Thus, the system reserve cost is a discrete function in time and can be evaluated only on a yearly basis. The system reserve capacity must, therefore, deal with averages over a yearly interval. The two types of annual fixed costs are described in further detail in the following sections.

3.7.1 Reserve Margin Requirement

A reserve capacity is necessary to ensure that adequate power is available in the event of an unscheduled outage at a plant. The types of components considered in this research contribute to the overall plant forced outage rate, denoted by FOR, and thus the component should be assigned some of the cost of the reserve margin for the plant. A method proposed by Garver (24) was adopted in this study.

The addition of plant n with a capacity MW and FOR to a utility system requires some fraction of MW to be placed in reserve status in order to maintain the same system reliability. Let RC(FOR) be the increase in reserve capacity necessary for the addition of MW. An estimate of RC(FOR) is

$$RC(FOR) = SRC \; \ell n[(1-FOR) + FOR \; e^{MW/SRC}] \; , \tag{3.65}$$

where SRC is the system risk characteristic of the utility's generating system. The system risk characteristic is a measure of the change in loss-of-load probability, denoted by LOLP, for a given change in reserve capacity and is a measure of the effect on LOLP by a change in reserve capacity. The SRC is approximated by plotting the natural logarithm of the LOLP for the system against the system reserve capacity. A straight line is drawn between the present system LOLP on this curve and a LOLP which is e times larger on the same curve, where e is 2.718... The negative of the slope of the line drawn is the inverse of the SRC. A typical value for SRC is between 0.2 to 0.4 of the unit generating capacity (12). An alternate expression used to estimate SRC is (24)

$$SRC = \sum_{i=1}^{N} MW(i) \; FOR(i) \; , \tag{3.66}$$

where N is the number of plants in the system. The above expression for SRC is only a "rough approximation" used to describe the SRC.

Each component in a plant contributes to the overall FOR. A portion of the reserve capacity can therefore be assigned to a particular component by

$$RC(FOR_c) \cong RC(FOR+FOR_c) - RC(FOR) \; , \tag{3.67}$$

where FOR_c is the forced outage rate attributed to the component. A method of determining FOR_c must be found to determine the reserve capacity associated with the component of interest.

The standard definitions of forced outage rate and plant availability factors are

$$FOR = \frac{FOH}{PH - SOH} \tag{3.68}$$

and

$$f_{avail} = \frac{PH - SOH - FOH}{PH} \; , \tag{3.69}$$

where PH is the number of hours in one year (8760 hrs), SOH is the number of scheduled outage hours in one year, and FOH is the number of forced outage hours in one year. Manipulating the above equation gives

$$PH - SOH = \frac{PH}{1-FOR} f_{avail} \; . \tag{3.70}$$

The amount of time the plant will be shutdown during year i due to a failure in a component of interest is

$$FOH_C(i) = f_{avail}(i) \left\{ \begin{matrix} \text{expected number} \\ \text{of failures} \\ \text{in year i} \end{matrix} \right\} \left\{ \begin{matrix} \text{average plant} \\ \text{downtime per} \\ \text{failure in year i} \end{matrix} \right\} \; . \tag{3.71}$$

The fractional downtime is the ratio of actual downtime to the maximum downtime, T_{repl}. Thus, the average plant downtime can be written in terms of the average fractional downtime as

$$\overline{TD}(i) = \bar{f}_{DT}(i)\ T_{repl} + [1-\bar{f}_{DT}(i)]\ T_{repl}\ \frac{MW_{down}}{MW(n)}\ . \tag{3.72}$$

In general, the average fractional downtime is a weighted average such that

$$\bar{f}_{DT}(i) = \frac{\int_i^{i+1} \lambda\ dt'\ g(t')\ f_{DT}(t')}{\int_i^{i+1} \lambda\ dt'\ g(t')} = \frac{\int_i^{i+1} dt'\ g(t')\ f_{DT}(t')}{\int_i^{i+1} dt'\ g(t')}\ , \tag{3.73}$$

where $\lambda\ dt'$ is the probability of failure in dt' about t' and $g(t')$ is the probability the plant is operating at t'.

The expected number of failures in year i is a very difficult quantity to estimate. The number of failures depends on the length of the downtime for each failure because during this time a failure cannot occur. For a Poisson process with instantaneous replacement, the expected number of failures in time t would be λt. As an upper bound for the FOH_C estimation, the value of $\{\lambda\}$ failures is used to approximate the expected number of failures in any year.

The forced outage hours for the component can now be written as

$$FOH_C(i) = f_{avail}(i)\ \{\lambda\}\ T_{repl}\left(\bar{f}_{DT}(i) + [1-\bar{f}_{DT}(i)]\ \frac{MW_{down}}{MW}\right)\ . \tag{3.74}$$

Combining the above with Eqs. (3.65), (3.66) and (3.67) gives the forced outage rate due to the component in year i as

$$FOR_c(i) = \frac{FOH_C(i)}{PH - SOH} = \{\lambda\}T_{repl}\left(\bar{f}_{DT}(i) + [1-\bar{f}_{DT}(i)]\ \frac{MW_{down}}{MW}\right)[1-FOR(i)]. \quad (3.75)$$

The reserve margin cost due to the component, denoted by RC, can now be estimated by

$$\begin{aligned} RC[FOR_c(i)] &= RC[FOR_c(i) + FOR(i)] \\ &= SRC\ \ell n\left(1 + \frac{FOR_c(i)(e^{MW/SRC}-1)}{1+FOR(i)(e^{MW/SRC}-1)}\right). \end{aligned} \quad (3.76)$$

For Plan A, the average downtime is the fixed repair/replacement time, T_{repl}. The forced outage rate due to the component for Plan A becomes

$$FOR_c^A(i) = \frac{MW\ \{\lambda\}\ T_{repl}[1-FOR(i)]}{MW\ PH} = \{\lambda\ T_{repl}\}\ [1-FOR(i)] \quad (3.77)$$

making the reserve margin cost due to the component

$$RC_A(FOR_c^A(i)) = SRC\ \ell n\left(1 + \frac{\{\lambda\ T_{repl}\}[1-FOR(i)](e^{MW/SRC}-1)}{1+FOR(i)(e^{MW/SRC}-1)}\right). \quad (3.78)$$

For Plan B, the average fractional downtime is found by numerically evaluating Eq. (3.75). For the case of a constant fractional downtime, and g(t) the forced outage rate due to the component is

$$FOR_c^B(i) = \{\lambda\ T_{repl}\}\ \left(\bar{f}_{DT}(i) + [1-\bar{f}_{DT}(i)]\ \frac{MW_{down}}{MW}\right)\ [1-FOR(i)] \quad (3.79)$$

making the reserve margin cost due to the component

$$RC_B(FOR_c^B(i)) = SRC\ \ell n\left(\frac{\{\lambda\ T_{repl}\}\left(\bar{f}_{DT}(i) + [1-\bar{f}_{DT}(i)]\ \frac{MW_{down}}{MW}\right)[1-FOR(i)](e^{MW/SRC}-1)}{[1+FOR(i)(e^{MW(n)/SRC}-1)]}\right). (3.80)$$

The reserve margin cost is an expenditure that must be converted to a revenue requirement using the fixed charge rate. Therefore, the reserve margin cost associated with the component at plant n in year m is

$$C_{rm}(n,m) = RC(n,m)\ FCR(n)\ \zeta_{rm}(n,m)\ , \tag{3.81}$$

where $\zeta_{rm}(n,m)$ is the cost per unit reserve margin for plant n in year m. An allowance is made for escalation in reserve margin per unit cost by

$$\zeta_{rm}(n,m) = \zeta_{rm}(n,0) \left(\prod_{j=1}^{m} [1+I_{rm}(j)] \right) , \tag{3.82}$$

where $I_{rm}(m)$ is the escalation rate in year m.

Because the reserve margin is contracted yearly, the plant must pay for the entire year. The annual reserve margin costs are therefore discounted for the entire year in which the total estimation is being performed.

3.7.2 Spare Purchase Cost

The spare purchase cost includes the capital cost of the component, transportation, initial storage, and initial administrative costs. The spare purchase cost is shared by all members of the pool and is a capitalized cost. The purchase cost for plant n is given by

$$C_{spr}(n,0) = F(n,0)\ \zeta_{spr}(0)\ REV(n)\ , \tag{3.83}$$

where $\zeta_{spr}(0)$ is the total initial purchase cost of the spare to be capitalized.

3.7.3 Spare Storage Cost

An additional cost associated with maintaining the spare component and managing the spare pool is a Plan B cost only. In general this cost will be very small but is included for completeness. A fraction of this cost is allocated to each member of the pool based on the plant allocation factor. The spare storage cost in year m for plant n is

$$C_{ss}(n,m) = F(n,m)\ \zeta_{ss}(n,0) \left\{ \prod_{j=1}^{m} [1+I_{ss}(j)] \right\} , \tag{3.84}$$

where $\zeta_{ss}(n,0)$ is the spare storage cost in year 0 for plant n and $I_{ss}(m)$ is the escalation rate in year m.

3.7.4 Spare Salvage Value

At the end of pool lifetime it is possible for the spare component to have some salvage value. The salvage value at the end of pool lifetime is capitalized by multiplying by the revenue-to-expense conversion factor. The spare salvage value is shared among the members in the pool and is a Plan B cost given by

$$C_{ssv}(n,M_{max}) = F(n,M_{max})\ \zeta_{ssv}(n,M_{max})\ REV(n) , \tag{3.85}$$

where $\zeta_{ssv}(n,M_{max})$ is the capitalized salvage value of the spare at the end of pool lifetime, denoted by M_{max}. An allowance is made for an escalation in cost of the spare salvage value by

$$\zeta_{ssv}(n,m) = \zeta_{ssv}(n,0) \left\{ \prod_{j=1}^{m} [1+I_{ssv}(j)] \right\} , \tag{3.86}$$

where $I_{ssv}(m)$ is the escalation rate in year m.

3.7.5 Present Worth of Annual Fixed Costs

Because the annual fixed costs are present worthed at the end of a year. The present worth of the annual fixed costs is estimated by summing over the interval (0,k) where k is an integer value, such that $k - 1 < t \leq k$. In general, the value of k will equal the pool lifetime. The present worth of the annual fixed costs for Plan A is obtained by summing all costs over the interval (0,t), as given by

$$PWAFC_A(n,t) = \sum_{m=1}^{k} \frac{C_{rm}^{A}(n,m)}{\prod_{j=1}^{m}[1+i_d(j)]}$$

$$= \sum_{m=1}^{k} \left[RC_A(n,m)\ FCR(n)\ \zeta_{rm}(n,0) \sum_{\ell=1}^{m} \hat{I}_{rm}(\ell) \right] , \qquad (3.87)$$

where $\hat{I}_{rm}(m) = \prod_{j=1}^{m} \left(\frac{1+I_{rm}(j)}{1+i_{d(j)}} \right)$ and $k = 1,2,3 \ldots$ such that $k-1 < t \leq k$.

The present worth of the Plan B annual fixed costs over the interval (0,t) and a pool lifetime of M_{max} is given by

$$PWAFC_B(n,t) = \sum_{m=1}^{k} \left[\frac{C_{rm}^{B}(n,m) + C_{ss}(n,m)}{\prod_{j=1}^{m}[1+i_d(j)]} \right] - \frac{C_{ssv}(n,M_{max})}{\prod_{j=1}^{M_{max}}[1+i_d(j)]} + C_{spr}\ (n,0)$$

$$= \sum_{m=1}^{k} \left\{ RC_B(n,t)\ FCR(n)\ \zeta_{rm}(n,0) \sum_{\ell=1}^{m} \hat{I}_{rm}(\ell) \right.$$

$$\left. + F(n,0)\ \zeta_{ss}(n,0) \sum_{\ell=1}^{m} \hat{I}_{ss}(\ell) \right\}$$

$$- F(n,0)\ \zeta_{ssv}(n,0)\ REV(n)\ \hat{I}_{ssv}(M_{max})$$

$$+ F(n,0)\ \zeta_{spr}(0)\ REV(n)\ , \tag{3.88}$$

where

$$\hat{I}_{ss}(m) = \prod_{j=1}^{m} \left(\frac{1+I_{ss}(j)}{1+i_d(j)} \right)$$

$$\hat{I}_{ssv}\ (M_{max}) = \prod_{j=1}^{M_{max}} \left(\frac{1+I_{ssv}(j)}{1+i_d(j)} \right) ,$$

and k = 1,2,3 ... such that $k-1 < t \leq k$.

The equations derived in Section 3.4, 3.5, 3.6, and 3.7 for the Plan A and Plan B present worth costs are used in the computer code KSUSPARE (see Appendix B). The total present worth costs for Plan A and Plan B are estimated by Eq. (3.16) given in Section 3.3. The estimated saving is found by taking the difference between the two plan costs.

3.8 Expected Saving and Benefit-Cost Ratio

The costs under the two managerial plans can now be compared. The present worth of all expected costs at time t and at plant n under the two plans is given by Eq. (3.18). If the plant is presently operating under Plan A then the expected saving of switching to Plan B for the time t is given by

$$SAV(n,t) = PWTOTAL_A(n,t) - PWTOTAL_B(n,t)\ . \tag{3.91}$$

To obtain the total expected saving for all the utility's plants in the pool, the utility manager would sum the expected saving for all his plants in the spare-component pool.

An industry such as the electrical utility industry, which is regulated by a government agency or commission, is usually required in an economic analysis to calculate an estimate of the benefits of a certain project compared to the cost of the project (16). To be acceptable economically, the economic benefits must exceed the cost of providing the benefits. Such a comparison is performed in a benefit-cost ratio, BCR. The expected economic benefits of the spare pool for plant n is the difference in total present worth costs less the purchase cost of the spare. The economic cost of the spare pool is the initial purchase cost of the spare. Thus the BCR for plant n for the time t can be expressed as

$$BCR(n,t) = \frac{PWTOTAL_A(n,t) - [PWTOTAL_B(n,t) - C_{spr}(n,0)]}{C_{spr}(n,0)} \qquad (3.90)$$

or

$$BCR(n,t) = 1 + \frac{SAV(n,t)}{C_{spr}(n,0)} , \qquad (3.91)$$

where C_{spr} is the initial capital cost of the spare for plant n. If the expected saving is negative, the benefit-cost ratio is less than 1 and Plan B should be rejected. In contrast, if the expected saving is positive, the benefit-cost ratio is greater than 1 and Plan B should be accepted.

CHAPTER 4

EXAMPLE RESULTS AND COMPARISON TO PC-SPARE

4.1 Comparison of KSUSPARE With PC-SPARE

In this section the difference between the methodology used in the KSUSPARE computer code and the PC-SPARE computer code is examined. The reasons for differences between the two code results can be classified into two different types.

The type I difference refers to the difference in the treatment of how model costs are incurred and how they are brought to a present value. Total present worth costs in the KSUSPARE code are determined by integrating differential present worth costs over an interval of time. In contrast, the total present worth costs in the PC-SPARE code are determined by summing annual present worth costs for each year over the time interval. The KSUSPARE code allows payment of all failure related costs at any time after the failure occurs, while the PC-SPARE code requires payment of all failure related costs along with yearly costs at the end of each year.

The type II difference refers to the difference due to the fundamental model differences between the KSUSPARE methodology versus the PC-SPARE methodology. Three major reasons account for the type II differences. The KSUSPARE methodology includes the probability that a plant be operational before a failure can occur in a differential time interval while the PC-SPARE methodology assumes all plants are operational at the beginning of each yearly interval. The KSUSPARE methodology incorporates the fractional downtime to estimate how long the spare was used during the repair/replacement time interval and the reserve margin requirement while the PC-SPARE methodology incorporates the product of the probability of spare availability and the repair/replacement time for the same estimates. Although not evaluated in this section, another factor contributing to a type II difference is the use of a variable spare availability $g_s(t)$. The exponential behavior of $g_s(t)$ over the first repair/replacement time interval increases the probability a plant is operating compared to values at later times. Thus, the likelihood of a failure in the first repair/replacement time increases as does the fractional downtime. The end result is an increase in both plans' total costs. The difference caused by a variable $g_s(t)$ is significant only when evaluating the spare-pool costs for short pool lifetimes (e.g., less than five repair/replacement time intervals).

The following sections describe how the type I difference is estimated along with the type II difference estimates for: 1) operation and maintenance costs, 2) reserve capacity costs, 3) replacement energy costs, and 4) used component salvage value (see Table 4.1). To analyze the differences between the computer codes certain simplifying assumptions were made: (i) constant component availability (i.e., constant $g(t)$), (ii) constant fractional downtime, and (iii) constant plant capacity factors. For these assumptions the spare component purchase cost, spare final salvage value, and the spare storage costs are identical for both studies and are, therefore, neglected in this analysis.

Table 4.1 A grouping of cost categories which constitute the type I and type II differences.

Cost Category	Type I difference	Type II difference
Operation and maintenance	X	X
Replacement energy	X	X
Reserve capacity		X
Used component salvage value		X

The concept of equivalence factors is introduced to quantify the type I and type II differences. The equivalence factor is defined such that if the i-th cost estimated by KSUSPARE is multiplied by the i-th type I equivalence factor, denoted by $\hat{X}_i$, and the product is then subtracted from the i-th cost estimated by PC-SPARE, the result would equal the type II difference. Thus, the type I equivalence factor for a particular cost i is the ratio of the PC-SPARE estimate of the cost to that which would be predicted by KSUSPARE with type II differences removed (i.e., using the same model except for the continuous time involved in taking the present worth). The type II equivalence factor, denoted by $\hat{Y}_i$, is the ratio of cost i estimated by PC-SPARE to that predicted by KSUSPARE using the same economic method for obtaining the present worth (i.e. with type I differences removed). Expressions for these equivalence factors are derived in the sections below.

With these equivalence factors one can now quantify the numerical differences between PC-SPARE and KSUSPARE. To obtain the i-th cost difference for plant n, the term, PWi(n,K,PC-SPARE), is used to denote the present worth of the i-th cost over K years estimated by the PC-SPARE code and the term, D(n,K,j), represents the j-th difference between KSUSPARE and PC-SPARE after an integral number of years of spare-pool operation. Thus, the difference between KSUSPARE and PC-SPARE is given by

$$D(n,K,I) = PWi(n,K,PC\text{-}SPARE) - \hat{Y}_i(n)\ PWi(n,K,KSUSPARE) \tag{4.1}$$

and

$$D(n,K,II) = PWi(n,K,PC\text{-}SPARE) - \hat{X}_i(n)\ PWi(n,K,KSUSPARE). \tag{4.2}$$

Later in Section 4.4 an example is given and explicit values for differences are shown for certain costs.

4.2 Equivalence Factors for Type I Differences

The present worth of the i-th cost estimated by KSUSPARE for Plan A over the interval (0,K) is expressed as

$$PWi_A(n,K,KSUSPARE) = \zeta_i(n,0)\ g \int_0^K dt'\ \hat{I}_i(t'), \tag{4.3}$$

where

$$\hat{I}_i(t') = \left\{ \prod_{j=1}^{m-1} \left(\frac{1+I_i(j)}{1+i_d(j)}\right) \right\} \left\{ \frac{1+I_i(m)}{1+i_d(m)} \right\}^{\gamma_m}$$

and m = INT(t') such that $\gamma_m = t' - m$. Because the escalation and discount rates are constant, Eq. (4.3) can be analytically evaluated as

$$PWOM_A(n,K,KSUSPARE) = \zeta_{om}(n,0) g \int_0^K dt'\ a^{t'} = \zeta_{om}(n,0)\ g\ \frac{a^K-1}{\ell n(a)}, \tag{4.4}$$

where $a = (1+I_i)/(1+i_d)$.

By contrast, the PC-SPARE present worth of the i-th cost estimated by PC-SPARE for Plan A over the interval (0,K) is expressed as

$$PWi_A(n,K,PC\text{-}SPARE) = \zeta_i(n,0)(1-P_f T_{repl}) \sum_{j=1}^{K} \hat{I}_i(j)\ , \tag{4.5}$$

where P_f is the probability of a single failure in a one year interval equal to $\lambda e^{-\lambda}$. Equation (4.5) can be rewritten for constant escalation and discount rates as

$$PWi_A(n,K,PC\text{-}SPARE) = \zeta_i(n,0)(1-\lambda e^{-\lambda}T_{repl}) \sum_{j=1}^{M} a^j$$

$$= \zeta_i(n,0)(1-\lambda e^{-\lambda}T_{repl})\ \frac{1-a^K}{a^K(1-a)}\ . \tag{4.6}$$

Thus, the type I difference is given by

$$\hat{X}_i(n) = \frac{\int_0^K dt' \; \hat{I}_i(t')}{\sum_{j=1}^{K} \hat{I}_i(j)} , \tag{4.7}$$

where for constant escalation and discount rates

$$X = \frac{(a-1)\ a\ (1-a)}{\ln(a)\ (1-a)} = \frac{(a-1)\ a}{\ln(a)} . \tag{4.8}$$

4.3 Equivalence Factors for Type II Differences

Type II differences between PC-SPARE and KSUSPARE result because of the different formulations of the individual costs in the two computer modeling techniques. Thus, the type II equivalence factor must be evaluated separately for each individual cost. Similar to the type I equivalence factors, the type II equivalence factors will be evaluated over the time interval (0,K). The type II differences for Plan A and Plan B are evaluated for the 1) operation and maintenance costs, 2) reserve capacity costs, 3) replacement energy costs, and 4) used component salvage value.

4.3.1 Operation and Maintenance Costs

The expression for Plan A operation and maintenance costs used by PC-SPARE is

$$PWOM_A(n,K,\text{PC-SPARE}) = \zeta_{om}(n,0)[1-\lambda \exp(-\lambda\ T_{repl})] \sum_{j=1}^{K} \hat{I}_{om}(j) . \tag{4.9}$$

The expression for Plan A operation and maintenance costs used by KSUSPARE is

$$\text{PWOM}_A(n,K,\text{KSUSPARE}) = \zeta_{om}(n,0)\ g \int_0^K dt'\ \hat{I}_{om}(t')\ . \tag{4.10}$$

Thus, the Plan A type II equivalence factor for operation and maintenance at plant n is given by

$$\hat{Y}^A_{om}(n) = \frac{1-\lambda\exp(-\lambda\ T_{repl})}{g} = (1+\lambda\ T_{repl})[1-\lambda\exp(-\lambda\ T_{repl})]. \tag{4.11}$$

The expression for Plan B operation and maintenance costs used by PC-SPARE is

$$\text{PWOM}_B(n,K,\text{PC-SPARE}) = \zeta_{om}(n,0)[1-\lambda\exp(-\lambda\ T_{repl})$$

$$+ \lambda\exp(-\lambda\ T_{repl})] \sum_{j=1}^{K} \hat{I}_{om}(j)\ . \tag{4.12}$$

The expression for Plan B operation and maintenance costs used by KSUSPARE is

$$\text{PWOM}_B(n,K,\text{KSUSPARE}) = \zeta_{om}(n,0)\ g(1+\lambda T_{repl}-f_{DT}\ \lambda T_{repl}) \int_0^K dt'\ \hat{I}_{om}(t'). \tag{4.13}$$

Thus, the Plan B type II equivalence factor for operation and maintenance is given by

$$\hat{Y}^B_{om}(n) = \frac{[1-\lambda\exp(\lambda\ T_{repl}) + \lambda\exp(-\lambda\ T_{temp})]}{g(1+\lambda\ T_{repl}-f_{DT}\lambda\ T_{repl})}$$

$$= \frac{(1+\lambda\ T_{repl})[1-\lambda\exp(-\lambda\ T_{repl}) + \lambda\exp(-\lambda\ T_{temp})]}{(1+\lambda\ T_{repl} - f_{DT}\lambda\ T_{repl})}\ . \tag{4.14}$$

4.3.2 Reserve Capacity Costs

The expression describing the forced outage rate due to the component for Plan A used by PC-SPARE is

$$FOR_C^A(n,\text{PC-SPARE}) = \lambda \exp(-\lambda\ T_{repl})(1-FOR(n)), \tag{4.15}$$

where FOR(n) is the forced outage rate of the plant. In contrast, the relationship describing the forced outage rate due to the component for Plan A used by KSUSPARE is

$$FOR_C^A(n,\text{KSUSPARE}) = \lambda T_{repl}(1-FOR(n)) \ . \tag{4.16}$$

The expression describing the reserve margin costs used by both codes is

$$PWRM(n,K) = SRC(n)\,\ell n\left\{1 + \frac{FOR_c(n)[e^{MW(n)/SRC(n)} - 1]}{1+FOR(n)[e^{MW(n)/SRC(n)}-1]}\right\}$$

$$\cdot\ \zeta_{rm}(n,0)FCR(n) \cdot \sum_{j=1}^{K} \hat{I}_{rm}(j) \ , \tag{4.17}$$

where $\hat{I}_{rm}(j) = \prod_{i=1}^{j} \left(\frac{1+I_{rm}(i)}{1+i_d(i)}\right)$.

Thus, the Plan A type II equivalence factor for reserve capacity is given by

$$\hat{Y}_{rm}^A(n) = \frac{\ell n\left\{1 + \frac{\lambda \exp(-\lambda\ T_{repl})[1-FOR(n)][e^{MW(n)/SRC(n)}]}{1+FOR[e^{MW(n)/SRC(n)}-1]}\right\}}{\ell n\left\{1 + \frac{\lambda T_{repl}[1-FOR(n)][e^{MW(n)/SRC(n)}-1]}{1 + FOR[e^{MW(n)/SRC(n)} - 1]}\right\}} \ . \tag{4.18}$$

The expression describing the forced outage rate due to the component for Plan B used by PC-SPARE is

$$FOR_c^B(n,PCSPARE) = \lambda\ e^{-\lambda(T_{repl} - P_{avail}\ T_{temp} + P_{avail}\ \frac{MW_{down}}{MW(n)}\ T_{temp})}[1-FOR(n)] \ , \tag{4.19}$$

where P_{avail} is the probability of spare availability estimated by the PC-SPARE code using binomial theory. In contrast, the relationship describing the forced outage rate due to the component for Plan B used by KSUSPARE is

$$FOR_c^B(n,KSUSPARE) = \lambda\ T_{repl}\left(\bar{f}_{DT} + (1-\bar{f}_{DT})\ \frac{MW_{down}}{MW(n)}\right)[1-FOR(n)]\ , \qquad (4.20)$$

where $\bar{f}^B_{DT}$ is the average fractional downtime and MW_{down} is the derated capacity for plant n during the time the spare is installed.

Thus, the Plan B type II equivalence factor for reserve capacity at plant n is given by

$$\hat{Y}^B_{rm}(n) = \frac{\ln\left\{1 + \dfrac{\lambda\ e^{-\lambda}[T_{repl} - P_{avail}\ T_{temp} + P_{avail}\ \dfrac{MW_{down}}{MW(n)}\ T_{temp}][1-FOR(n)](e^{MW(n)/SRC(n)} - 1)}{1 + FOR(n)\ (e^{MW(n)/SRC(n)} - 1)}\right\}}{\ln\left\{1 + \dfrac{\lambda\ T_{repl}\ [\bar{f}_{DT} + (1-\bar{f}_{DT})\ \dfrac{MW_{down}}{MW(n)}]\ [1-FOR(n)][e^{MW(n)/SRC(n)} - 1]}{1 + FOR(n)\ [e^{MW(n)/SRC(n)} - 1]}\right\}}\ . \qquad (4.21)$$

4.3.3 Replacement Energy Costs

The expression describing the replacement energy costs for Plan A used by PC-SPARE is

$$PWEN_A(n,K,PC\text{-}SPARE) = \lambda\exp(-\lambda T_{repl})f_{cap}(n)MW(n)\zeta_{en}(n,0)\ \sum_{j=1}^{K}\ \hat{I}_{en}(j), \qquad (4.22)$$

where $$\hat{I}_{en}(j) = \prod_{i=1}^{j}\left(\frac{1+I_{en}(i)}{1+i_D(i)}\right)\ .$$

The expression used by KSUSPARE is

$$PWEN_A(n,K,KSUSPARE) = \lambda T_{repl}\ f_{cap}(n)MW(n)\zeta_{en}(n,0)\ g\int_0^K dt'\ \hat{I}_{en}(t'). \qquad (4.23)$$

Thus, the Plan A type II equivalence factor for replacement energy at plant n is given by

$$\hat{Y}^{A}_{en}(n) = \frac{e^{-\lambda}}{g} = (1 + \lambda\, T_{repl})\, e^{-\lambda} \ . \qquad (4.24)$$

The expression describing the replacement energy for Plan B used by PC-SPARE is

$$PWEN_B(n,K,\text{PC-SPARE}) = \lambda\, e^{-\lambda}\, f_{cap}(n)\, MW(n)\, \zeta_{en}(n,0) \Big(T_{repl} - P_{avail}\, T_{temp} + P_{avail} \frac{MW_{down}}{MW(n)}\, T_{temp} \Big) \sum_{j=1}^{K} \hat{I}_{en}(j) \ . \qquad (4.25)$$

The expression used by KSUSPARE is

$$PWEN_B(n,K,\text{KSUSPARE}) = \lambda\, T_{repl}\, f_{cap}(n)\, \zeta_{en}(n,0)\, g\, [MW_{down} + MW_{spr} f_{DT}] \cdot \int_0^K dt'\, \hat{I}_{en}(t') \ , \qquad (4.26)$$

where MW_{spr} is the spare capacity.

Thus, the Plan B type II equivalence factor for replacement energy is given by

$$\hat{Y}^{B}_{en}(n) = \frac{(1+\lambda T_{repl})\, e^{-\lambda}\, MW(n)\, [T_{repl} - P_{avail}\, T_{temp} + P_{avail} \frac{MW_{down}}{MW(n)}\, T_{temp}]}{T_{repl}\, [MW_{down} + MW_{spr}\, f_{DT}]} \ . \qquad (4.27)$$

4.3.4 Used Component Salvage Value

The expression describing the used component salvage value used by PC-SPARE is

$$PWUSV(n,K,\text{PC-SPARE}) = \zeta_{usv}(n)\, REV(n) \sum_{j=1}^{K} \hat{I}_{usv}(j) \ , \qquad (4.28)$$

where $\hat{I}_{usv}(j) = \prod_{i=1}^{m} \left(\frac{1+I_{usv}(i)}{1+i_d(i)}\right)$.

The expression for the used component salvage value used by KSUSPARE is

$$PWUSV(n,K,KSUSPARE) = \zeta_{usv}(n,0)\ REV(n)\ g \sum_{j=1}^{K} \hat{I}_{usv}(j) \ . \qquad (4.29)$$

Thus, the equivalence factor for used component salvage value at plant n is given by

$$\hat{Y}_{usv}(n) = \frac{1}{g} = (1 + \lambda\ T_{repl}) \ . \qquad (4.30)$$

4.4 Example Problem Comparing KSUSPARE and PC-SPARE

An example problem was designed to compare the KSUSPARE code results to the PC-SPARE code results. The Plan A and Plan B costs, expected savings, and benefit-cost ratios were estimated for one plant and a 15 year pool lifetime. Table 4.2 provides a listing of the problem input data with all input dollar values listed as present worth values. The fractional downtimes estimated by the three simulation failure models as well as renewal theory were used. In addition, the asymptotic and time-dependent g(t) were used to estimate the total plans' costs. Table 4.3 includes a listing of the computer output for both KSUSPARE and PC-SPARE. A close examination of this table indicates that the Plan A costs estimated by KSUSPARE are larger than the Plan A costs estimated by PC-SPARE. The Plan B costs estimated by KSUSPARE, except for the renewal theory estimates, are smaller than the Plan B costs estimated by PC-SPARE.

Table 4.2 Input data for sample calculation.

Plant data				
discount rate				.13
fixed charge rate				.174
time period for fixed charge rate (yr)				25
cost of reserve capacity (k\$[1]/MW)				186
system risk characteristic (MW)				1000
Plant component data				
component capacity (MW)				800
cost of replacement component (k\$)				2770
differential cost for replacement energy (k\$/MWY[2])				74.898
shipping and installation cost for using spare (k\$)				40
annual operation and maintenance cost (k\$)				200
component failure rate (yr^{-1})				.01
salvage value of used component (k\$)				400
salvage value of failed component (k\$)				40
repair/replacement time for failed component (yr)				.7397(270d)
maximum time spare can be used as temporary substitute (yr)				.6164(225d)
Spare component data				
spare capacity (MW)				500
initial cost of spare component (k\$)				2200
final salvage value at end of pool life (k\$)				1000
annual maintenance and storage (k\$)				20
spare failure rate (yr^{-1})				.01
spare pool lifetime (yr)				15
Annual plant rates				
plant capacity factor				0.390
plant forced outage rate				0.220
Annual escalation rates		Years		
	0 - 5	6 - 10	11 - 15	
repaired/replacement component	0.050	0.070	0.070	
replacement energy	0.040	0.040	0.040	
reserve margin	0.050	0.050	0.050	
operation and maintenance	0.030	0.030	0.030	
spare storage	0.030	0.030	0.030	
spare shipping and installation	0.035	0.035	0.035	
salvage values	0.055	0.055	0.055	

[1]k\$ = \$1000

[2]MWY = mega-watt year

Table 4.3 Results from KSUSPARE and PC-SPARE. The fractional downtime was estimated by simulation Models 1, 2, and 3 and from renewal theory. The costs were calculated by KSUSPARE for asymptotic g(t) time-dependent g(t), and time-dependent fractional downtime. The time-dependent g(t) and fractional downtime were numerically evaluated at each interval using the trapezoid rule with five sub-intervals.

Cost Categories (k$[1])	PC-SPARE		Simulation Model 1[2]				Simulation Model 2[2]			
g(t)			Asymptotic		Time-dependent		Asymptotic		Time-dependent	
Plan	A	8	A	B	A	B	A	8	A	B
Operation & maintenance	1535.5	1545.0	1608.9	1620.3	1609.4	1620.8	1608.7	1619.6	1609.4	1620.4
Reserve capacity	1559.4	783.1	1575.0	632.1	1575.0	632.1	1575.0	669.6	1575.0	669.6
Replacement energy	1408.1	706.2	1472.1	591.6	1472.6	589.8	1472.1	627.0	1472.6	623.7
Repair/replacement component	318.2	318.2	347.7	347.7	347.8	347.8	347.7	347.7	347.8	347.8
Used component salvage	-182.1	-182.1	-180.8	-180.8	-180.8	-180.8	-180.8	-180.8	-180.8	-180.8
Spare component purchase		215.8		215.8		215.8		215.8		215.8
Spare final salvage		-35.0		-35.0		-35.0		-35.0		-35.0
Spare installation & removal		3.0		3.3		3.3		3.3		3.3
Spare storage		11.9		11.9		11.9		11.9		11.9
Total	4639.1	3366.1	4822.9	3206.9	4824.0	3205.7	4822.7	3279.1	4824.0	3276.7
Savings	1273.0		1616.0		1618.3		1543.6		1547.3	
Benefit-cost ratio	6.90		8.49		8.50		8.15		8.17	

[1] k$ = $1000

[2] Method used to estimate fractional downtime.

Table 4.3 (Cont.)

Cost Categories (k$[1])	PC-SPARE		Simulation Model 3				Renewal theory[2]			
g(t)			Asymptotic		Time-dependent		Asymptotic		Time-dependent	
Plan	A	B	A	B	A	B	A	B	A	B
Operation & maintenance	1535.5	1545.0	1608.7	1620.1	1609.4	1620.8	1608.7	1617.8	1609.4	1618.5
Reserve capacity	1559.4	783.1	1575.0	633.2	1575.0	633.2	1575.0	825.0	1575.0	825.0
Replacement energy	1408.1	706.2	1472.1	591.7	1472.6	589.9	1472.1	770.6	1472.6	767.3
Repair/Replacement component	318.2	318.2	347.7	347.7	347.8	347.8	347.7	347.7	347.8	347.8
Used component salvage	-182.1	-182.1	-180.8	-180.8	-180.8	-180.8	-180.8	-180.8	-180.8	-180.8
Spare component purchase		215.8		215.8		215.8		215.8		215.8
Spare final salvage		-35.0		-35.0		-35.0		-35.0		-35.0
Spare installation & removal		3.0		3.3		3.3		3.3		3.3
Spare storage		11.9		11.9		11.9		11.9		11.9
Total	4639.1	3366.1	4822.7	3207.9	4824.0	3206.9	4822.7	3576.3	4824.0	3573.8
Savings	1273.0		1614.8		1617.1		1246.4		1250.2	
Benefit-cost ratio	6.90		8.48		8.49		6.77		6.79	

[1] k$ = $1000

[2] Method used to estimate fractional downtime.

4.4.1 Type I and type II Differences

One reason for the difference in cost estimates can be explained by the type I and type II differences derived previously in Sections 4.1.1 and 4.1.2. The type I and type II differences associated with the operation and maintenance costs, reserve capacity costs, replacement energy costs, and used component salvage were estimated for the renewal theory cost estimates (see Table 4.4). The renewal theory cost estimates were multiplied by the type I equivalence factors evaluated for the different costs (see Table 4.5). The percentage difference (now only due to type II differences) between the renewal theory cost estimates and the PC-SPARE cost estimates again were calculated (see Table 4.4). A study of Table 4.4 indicates the following results:

1. The major portion of Plan A and Plan B operation and maintenance costs, and Plan A replacement energy costs are due to type I differences.

2. The Plan B replacement energy cost difference is due to equal contributions from type I and II differences.

3. The Plan A and Plan B reserve capacity cost and used salvage value differences are due to only type II differences.

4.4.2 Fractional downtime estimate

Another reason for the smaller Plan B costs using the simulation models' estimates of the fractional downtimes is because the component removal and installation times are neglected. Therefore, the fractional downtimes are underestimated by the simulation models except when the component removal and installation times are very small compared to the component repair/replacement times.

Table 4.4. Percentage differences between PC-SPARE results and KSUSPARE results with the PC-SPARE results used as the benchmark. The renewal theory estimate for fractional downtime was used in KSUSPARE calculations for the case of asymptotic g(t) and constant fractional downtime.

	KSUSPARE and PC-SPARE percent differences		KSUSPARE multiplied by type I equivalence factors and PC-SPARE percent differences	
Cost categories (k$[1])	Plan A	Plan B	Plan A	Plan B
Operation and maintenance	+4.8	+4.7	0.0	0.1
Reserve capacity	+1.0	+5.4	+1.0	+5.4
Replacement energy	+4.6	+9.0	+0.3	+4.7
Used component salvage value	-0.7	-0.7	-0.7	-.07

[1]k$ = $1000

Table 4.5. Type I equivalence factors calculated for the sample problem.

Equivalence factor	Value
$\hat{X}_{om}(n)$	0.95438
$\hat{X}_{rm}(n)$	1.00000
$\hat{X}_{en}(n)$	0.95908
$\hat{X}_{usv}(n)$	1.00000

4.5 Time of Payment

In Chapter 3 the assumption was made that the costs incurred due to a failure are paid at the time of failure. Another possible scenario is that the costs incurred due to a failure are paid at the end of the repair/replacement time. The effect of such a payment schedule was evaluted using the input data from the previously described example problem (see Table 4.2). Equations for the present worth of the operation and maintenance costs, replacement energy costs, and the used component salvage value were modified by changing the value of t' to t' + T.

The percentage difference between the "paid at the time of failure" costs and the "paid at the end of the repair/replacement time" costs for the renewal theory estimates are listed in Table 4.6. A study of this table reveals that both the Plan A and Plan B "paid at the end of the repair/replacement time" costs are lower than the "paid at the time of failure" costs. This result may have occurred because the discount rate is larger than any of the escalation rates. In contrast, if the escalation rates were larger than the discount rate, the "paid at the end of the repair/replacement time" costs would be higher.

Table 4.6. Percentage difference between the "paid at the time of failure" costs and the "paid at the end of the repair/replacement time" costs with the latter results used as the benchmark. The KSUSPARE calculations are for the case of constant g(t) and fractional downtime.

Cost categories (k$[1])	Percentage differences Plan A	Plan B
Operation and maintenance	-6.6	-6.6
Replacement energy	-5.9	-5.9
Used component salvage value	-5.0	-5.0

[1]k$ = $1000

4.6 Optimal Number of Spares

Throughout the present study only the case of one spare in the spare-component pool has been considered. As the failure rate of the component increases and/or more plants join the spare pool, it may be necessary to purchase and store more than one spare. The question to be answered would be how many spares should the spare pool have for the optimal benefit-cost ratio for difference component failure rates and number of plants in the spare pool. The input data from the previous example problem (see Table 4.2) were used to estimate the benefit-cost ratios as a function of time and component failure rates using the KSUSPARE code (see Figure 4.1). A study of the figure indicates that indeed some optimal number of spares exists.

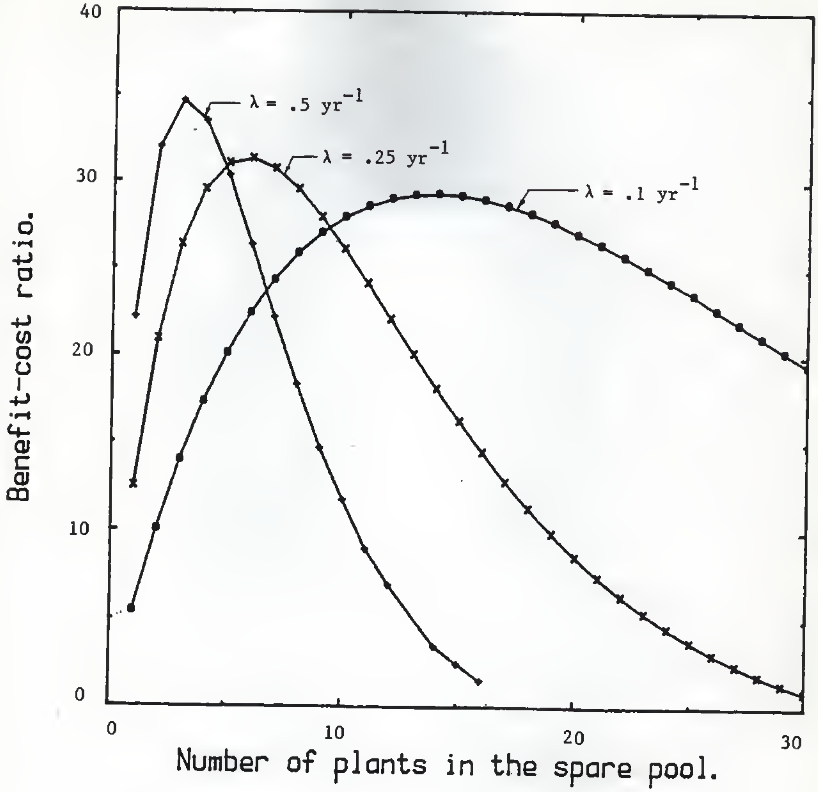

FIG. 4.1. Benefit-cost ratio for different numbers of plants in the spare pool and different values of component failure rates. The calculation was performed using renewal theory to estimate the fractional downtime with the input data from the sample problem in Chapter 4.

Chapter 5

SUMMARY AND CONCLUSION

In this chapter a summary and suggestions of areas for future research are presented.

5.1 Failure Models

One of the objectives of this study was to develop a more refined failure model to predict the economic benefit of a pooled inventory management system. This study incorporated some techniques used in previous studies in addition to developing new techniques which included using computer simulation and renewal theory to predict spare availability and plant operational probability.

5.1.1 Renewal Theory

In this study the probability of a plant operating at a given time was characterized by a system described by renewal theory. The plant being analyzed was not considered a potential candidate for failure up to the time the plant failed and the economic analysis was performed.

The simulation of random Poisson failures with a constant repair/replacement time agreed with renewal prediction. The asymptotic renewal theory result was also a good approximation for the probability of spare availability (or the probability a plant was operational) for small repair/replacement times (i.e., one-half year or less).

Near the beginning of the spare pool lifetime, renewal theory showed that the probability of spare availability and fractional downtime varied rapidly and showed several oscillations before stabilizing at an asymptotic value at time $\geq 4\ T_{repl}$. The maximum difference between the asymptotic renewal equation result and the minimum undershoot value at T_{repl} was highly dependent on the component failure rate (i.e., the larger component failure rates had larger maximum differences). In contrast, the maximum difference between the asymptotic renewal equation result and the minimum undershoot value at T_{repl} was independent of the number of plants in the spare pool.

The transient renewal theory results are particularly important when considering short pool lifetimes (of the order of a few repair/replacement times). For long pool lifetimes the asymptotic renewal theory result can be used to estimate the probability that a plant is operating. In both cases the renewal theory results are easily determined and incorporated in the economic analysis. Because renewal theory cannot model the use of the spare for less than the entire repair/replacement time, the fractional downtime calculated using renewal theory is the maximum expected fractional downtime.

5.1.2 Computer Simulation

In this study computer simulation was used to investigate the effects of the spare with and without a failure rate and the decision rule for determining if and for how long the spare was used by a plant during a failure time. The decision rule was considered to be one of two alternatives: (i) the spare is used when it becomes available even for very short periods of time and (ii) the spare is only used if immediately available at the time of failure and used for the entire repair/replacement interval.

All simulation models as well as renewal theory results for probability of spare availability were identical through the time interval $(0, T_{repl})$. This was expected because any failure occurring in this interval affected the spare availability. Not until some time past the first repair/replacement time did the spare failure rate or decision rule need to be considered.

An analysis of the simulation results indicated there was very little difference between a spare with or without a failure rate in a system of five or more plants. The difference was negligible in ranges of component failure rates considered by this study. However, for systems with larger component failure rates the probability of spare availability was somewhat lower and the fractional downtime a slightly larger for a spare-component pool containing a spare with a failure rate equal to the other system components versus a spare with zero failure rate.

The decision rule used resulted in a significant effect on both the spare availability and fractional downtime. For small component failure rates the decision rule had little effect on the probability of spare availability, however, for large component failure rates the "use the spare only if immediately available" showed a much larger probability of spare availability. As expected, the difference between the probability of spare availability for "use the spare when available" and "use the spare only if immediately available" was much greater for larger component failure rates.

The fractional downtimes were significantly different for all values of component failure rates. The combination of the plants in the spare pool (except the plant that fails at the analysis time) operating under "use the spare when available", a non-zero spare failure rate, and the plant failing at the analysis time operating under "use the spare only if immediately available" resulted in the largest fractional downtime and the largest Plan B costs. The increase in the fractional downtime due to the non-zero spare failure rate was less than the decrease in the fractional downtime due to the plant operating under "use the spare only if immediately available". The fractional downtime calculated using renewal theory to estimate the probability of spare availability ia therefore considered to be an estimate of the maximum expected fractional downtime.

5.2 Economic Analysis

Another objective of this study was to develop an extension of the economic models used in earlier studies. The refined probability failure models were incorporated into the generalized economic model to obtain the general cost of the two component management plans. The economic costs were also estimated on a continuous-time basis instead of the typical year-end basis.

5.2.1 Comparing KSUSPARE to PC-SPARE

The failure dependent variable costs (i.e., operation and maintenance coats) were a significantly larger proportion of the total plan costs for KSUSPARE while the difference between the Plan A and Plan B costs remained approximately the same as the PC-SPARE results. Using the simulation results for fractional downtime resulted in significantly smaller Plan B costs because the simulation models did not include any allowance for removal or installation times. As expected, the fractional downtime estimated by renewal theory provided a maximum Plan B coat and the smallest benefit-cost ratio.

The major reasons for the differences in costs between KSUSPARE results and PC-SPARE results were due to the method used to estimate the fraction of the repair/replacement time the failed plant would be shutdown or derating and the treatment of how model costs were incurred and how they were brought to a present value. In addition, a comparison of the differences for the case of payment at the end of the repair/replacement time after a failure versus payment at the beginning of the repair/replacement time showed a significant decrease in some costs.

Evaluating the renewal equation numerically or assuming an asymptotic value resulted in no significant difference in the estimated savings or benefit-cost ratios for long pool lifetimes. The total Plan A and Plan B costs were different but the savings between Plan B and Plan A were approximately the same. For this reason the extra expense and time necessary to evaluate the renewal equation numerically is not justified unless either very short pool lifetimes or very large repair/replacement times are being considered.

5.2.2 Advantages of KSUSPARE

A major improvement of KSUSPARE is the ability to handle repair/ replacement times of greater than one year. In addition, costs incurred due to a failure can be paid at any time in the future after the failure instead of at the end of the year in which the failure occurs. Another improvement is the ability to estimate costs as a continuous function of time instead of at the end of each year.

5.3 Suggestions for Further Study

A logical step would be to determine the optimal number of spares in the pool. As the failure rate of the component increases and/or more plants join the pool it may be necessary to purchase and store more than one spare to maintain a positive savings of Plan B over Plan A. Because the benefit-cost is a function of the spare availability, an optimal number of spares is necessary to ensure that the spare is not always being used when needed. Another area of study might be an attempt to quantify the variance associated with the benefit-cost ratio. Of concern in such probabilistic studies is not only the expected cost but the variation in the expected cost. The analysis of such a variation would be difficult to do with renewal theory and analytical models but easy to incorporate in simulation models using Monte Carlo techniques.

In further studies increased realism could be added to the analysis by incorporating a yearly load following curve. The addition of seasonal variations in demand for electricity would be more realistic than the yearly capacity factors presently used. The escalation and discount rates could also be described as continuous functions of time. Increased realism could also be added by including the interdependence of a failure at one plant upon the forced outage rates and resulting reserve capacity requirements for other plants in the pool.

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Appendix A: SIMULATION Computer Code

The computer code, SIMULATION, was written in FORTRAN and consists of a main program and 10 subroutines. A large number of variables were placed in "common" instead of being "passed" when a particular subroutine was called. This variable transfer method resulted in substantial computer run time savings.

The main program calls a number of subroutines that perform the actual analysis. A subroutine to read in the code parameters is first called. The next subroutine called simulates the time intervals by following the method outlined in Section 2.5 which requires simulating failures by calling the random number generating subroutine RANDOM. Subroutine RANDOM returns a random number on the interval (0,1) based on an input seed value. The seed value is multiplied by a constant followed by a modulus operation with a large number A. The result is then divided by another large number (B which is A+1) and set equal to the random number. The next seed value used is the result obtained from the modulus operation.

The probability of failure in the first interval (0, TREPL) and the system characteristic failure rate are calculated next. The failure times are then simulated by calling either TIMES1 or TIMES3. After each entire simulation is completed one of the CHECK subroutines is called to determine the sums used in subroutine RESULT to estimate the probability of spare availability and fractional downtime. After all the simulations have been run then the results are calculated.

Subroutine TIMES1 simulates failures using the process outlined for Model 1 and Model 2. Failures are simulated until the last failure simulated exceeds the value of TMAX. The entire simulated time scheme is then transferred to the CHECK1 subroutine for analysis.

Subroutine TIMES3 simulates failures using the process outlined for Model 3. Like in subroutine TIMES1, the entire time scheme is simulated and then the results are transferred to subroutine CHECK 3.

The total number of times a failure occurs in the interval (τ_k - β, τ_k) for each value of τ_k is determined for use in the Model 1 probability of spare availability and fractional downtime estimations made in subroutine RESULT. In addition, the total number of times and difference in time between τ_k and the last time the spare was used for each value of τ_k is determined for use in the Model 2 probability of spare availability and fractional downtime estimations made in subroutine RESULT.

The total number of times a failure occurs in the interval (τ_k - β, τ_k) and difference between the repair/replacement time and the actual downtime is determined for each value of τ_k for use in the Model 3 probability of spare availability and fractional downtime estimations made in subroutine RESULT. The last subroutine, PRINT3, prints out the probability of spare availability and its associated standard deviation, and the fraction downtime.

Input Data

TREPL = the failed component procurement time.

NP = the number of plants in the pool.

LAMBDA = the component failure rate.

MAXIT = the number of simulations the code is run for.

FSEED = the initial seed value for the random number subroutine.

METHOD = determines the mode which the code simulates. An input value of 1 simulates models 1 and 2; an input value of 2 simulates Model 3.

TMAX = the maximum evaluation time at which probability of spare availability and fraction downtime are estimated.

NIBTR = the number of time intervals between 0 and TREPL that determine the evaluation times.

NTMAX = the number of time intervals between TREPL and TMAX that determine the evaluation times.

NREAD = determines the evaluation times. An input value of other than 999 results in the evaluation times being read in as input data. An input value of 999 calls subroutine GPROG.

Code Listing

The following is a listing of the computer code SIMULATION.

```
C****************************SIMULATION************************************
C*                                                                        *
C*                                                                        *
C*    THIS PROGRAM ESTIMATES THE PROBABILITY OF BEING IN PLAN B OR        *
C*    THE FRACTION DOWN TIME BY SIMULATING THE TIME INTERVALS BETWEEN     *
C*    FAILURES USING NEGATIVE EXPONENTIAL DISTRIBUTIONS. THE KEY          *
C*    ASSUMPTION FOR MODELS 1, 2, AND, 3 IS THE COMPONENTS AND THE        *
C*    SPARE ARE REMOVED AND INSTALLED INSTANTANEOUSLY. THE PROB-          *
C*    ABILITY OF BEING IN PLAN B AND THE FRACTION DOWN TIME ARE           *
C*    ESTIMATED IN EACH MODEL. ALL TIMES ARE DIMENSIONLESS (I.E.,         *
C*    REAL TIME * COMPONENT FAILURE RATE).                                *
C*                                                                        *
C*    THE PROGRAM USES THREE MODELS:
C*                                                                        *
C*      MODEL 1                                                           *
C*      -------                                                           *
C*      THE FAILURE RATE OF THE SPARE IS EQUAL TO ZERO. PLANTS IN THE     *
C*      POOL AND THE FAILED PLANT USE THE SPARE WHEN IT BECOMES AVAIL-    *
C*      ABLE, EVEN FOR VERY SHORT PERIODS OF TIME.                        *
C*                                                                        *
C*      MODEL 2                                                           *
C*      -------                                                           *
C*      THE FAILURE RATE OF THE SPARE IS EQUAL TO ZERO. IF THE SPARE      *
C*      IS NOT AVAILABLE AT THE TIME OF A PLANT FAILURE, THE PLANT        *
C*      REMAINS SHUTDOWN UNTIL THE REPAIR/REPLACEMENT COMPONENT IS        *
C*      INSTALLED. SIMILARLY, IF THE SPARE IS NOT AVAILABLE AT THE        *
C*      TIME OF THE PLANT FAILURE IT WILL NOT BE INSTALLED AT A LATER     *
C*      DATE.                                                             *
C*                                                                        *
C*      MODEL 3                                                           *
C*      -------                                                           *
C*      THE SPARE HAS A FAILURE RATE EQUAL TO THE FAILURE RATE OF THE     *
C*      OTHER SIMILAR COMPONENTS IN THE POOL. PLANTS IN THE POOL AND      *
C*      THE FAILED PLANT USE THE SPARE WHEN IT BECOMES AVAILABLE, EVEN    *
C*      FOR VERY SHORT PERIODS OF TIME.                                   *
C*                                                                        *
C*                                                                        *
C**************************************************************************
C
C
C
C
C
C                           **************
C                           *MAIN PROGRAM*
C                           **************
      COMMON T(100),SUM2(100),RATIO2(100),TIME(100),TUSED(100),DOWN1(100
     1),DOWN2(100),S1(100),RATIO1(100),SUM1(100),PFTR,BETA,I,NP,MAXIT,MO
     1DEL,NTMAX,NTMAX2
      COMMON/RNG/FSEED,U
      DOUBLE PRECISION FSEED,U
      CALL READ
      CALL ITER
```

```
      CALL PRINT
      STOP
      END
C
C
C
C
C##############################################################################
C#                            START OF SUBROUTINES                            #
C##############################################################################
C
C
C
C
C                              #################
C                              #SUBROUTINE READ#
C                              #################
      SUBROUTINE READ
      COMMON T(100),SUM2(100),RATIO2(100),TIME(100),TUSED(100),DOWN1(100
     1),DOWN2(100),S1(100),RATIO1(100),SUM1(100),PFTR,BETA,I,NP,MAXIT,MO
     1DEL,NTMAX,NTMAX2
      COMMON/MAXT/TMAX,MAXTAU,NREAD,NIBTR
      COMMON/RNG/FSEED,U
      COMMON/REPAIR/DEV
      DOUBLE PRECISION FSEED,U
C     ------------------------------------------------------------------
C     READ IN REPLACEMENT TIME, NUMBER OF PLANTS, COMPONENT FAILURE
C     RATE, TEST VALUE, MAXIMUM NUMBER OF ITERATIONS, AND INITIAL SEED.
C     ------------------------------------------------------------------
      READ (5,10) BETA,NP,MAXIT,FSEED
   10 FORMAT (/,F10.6,I5,I10,D15.8)
C     ---------------------------------------
C     READ IN THE SIMULATION MODEL TO BE USED.
C     ---------------------------------------
      READ (5,20) MODEL
   20 FORMAT (/,I5)
C     ----------------------------------------------------------------
C     READ IN THE MAXIMUM TIME TO EVALUATE THE PROBABILITY AND OR THE
C     FRACTION DOWN TIME AT.
C     ----------------------------------------------------------------
      READ (5,20) MAXTAU
C     ----------------------------------------------------------
C     READ IN THE NUMBER OF TIME INTERVALS BETWEEN 0 AND BETA.
C     ----------------------------------------------------------
      READ (5,20) NIBTR
C     -------------------------------------------------------------
C     READ IN THE TOTAL NUMBER OF TIME VALUES BETWEEN 0 AND TMAX.
C     -------------------------------------------------------------
      READ (5,20) NTMAX
C     -----------------------------------------------------------------
C     READ IN THE NUMBER OF STANDARD DEVIATIONS USED TO CALCULATE THE
C     ERROR ESTIMATE.
C     -----------------------------------------------------------------
      READ (5,50) DEV
```

```
   50 FORMAT (/,F10.3)
C     ------------------------------------------------------------------
C     DETERMINE IF THE TIME VALUES TO EVALUATE THE PROBABILITY ARE READ
C     IN OR CALCULATED WITH A GEOMETRIC PROGRESSION METHOD. IF THE VALUE
C     IS 999 THEN THE PROGRESSION METHOD IS USED OTHERWISE THE TIME
C     VALUES MUST BE READ IN.
C     ------------------------------------------------------------------
      READ (5,20) NREAD
      IF (NREAD.NE.993) GOTO 80
C     --------------------------------------------------------------
C     READ IN THE TIME VALUES TO EVALUATE THE FRACTION DOWN TIME AT.
C     --------------------------------------------------------------
      DO 30 J=1,NTMAX,1
         READ (5,40) TIME(J)
   40    FORMAT (F10.4)
         S1(J)=0.0
         SUM1(J)=0.0
         SUM2(J)=0.0
   30 CONTINUE
      NTMAX2=NTMAX
      GOTO 70
   80 CALL GPROG
   70 RETURN
      END
C
C
C
C
C                          ##################
C                          #SUBROUTINE GPROG#
C                          ##################
      SUBROUTINE GPROG
      COMMON T(100),SUM2(100),RATIO2(100),TIME(100),TUSED(100),DOWN1(100
     1),DOWN2(100),S1(100),RATIO1(100),SUM1(100),PFTR,BETA,I,NP,MAXIT,MO
     1DEL,NTMAX,NTMAX2
      COMMON/MAXT/TMAX,MAXTAU,NREAD,NIBTR
      TMAX=MAXTAU*BETA
      NTMAX2=NTMAX+1
C     -----------------------------------------------------------------
C     THE FIRST TIME INTERVAL IS GIVEN THE VALUE .01 TIMES THE VALUE OF
C     THE FIRST EQUALLY SPACED VALUE. THE NEXT NIBTR-2 INTERVALS ARE
C     EQUALLY SPACED. THE NIBTR TIME POINT IS CHOSEN AS EQUAL TO BETA.
C     -----------------------------------------------------------------
      TINT=BETA/NIBTR
      TIME(1)=.01*TINT
      NBT=NIBTR+1
      DO 20 K=2,NBT,1
         TIME(K)=(K-1)*TINT
   20 CONTINUE
      TINT2=(TMAX-BETA)/(NTMAX-NIBTR)
C     ------------------------------------------------------------
C     THE EVALUATION TIMES BETWEEN TMAX AND BETA ARE EITHER EQUALLY
C     SPACED OR DIVIDED BY A GEOMETRIC PROGRESSION TECHNIQUE.
C     ------------------------------------------------------------
```

```
      IF (NREAD.EQ.991) THEN
C         ----------------------------------
C         TECHNIQUE FOR EQUALLY SPACED TIMES.
C         ----------------------------------
          DO 30 K= NBT,NTMAX,1
              TIME(K+1)=TIME(K)+TINT2
   30     CONTINUE
      ELSE
C         -------------------------------------------------------------
C         THE TIME INTERVALS BETWEEN BETA AND TMAX ARE DETERMINED BY A
C         GEOMETRIC PROGRESSION METHOD WHERE THE COMMON RATIO R IS THE
C         RATIO OF INTERVAL WIDTH TO THE PREVIOUS INTERVAL WIDTH. THE
C         COMMON RATIO MUST BE A VALUE GREATER THAN 1 FOR THIS METHOD.
C         -------------------------------------------------------------
          R=1.1
          A1=(TMAX-BETA)*(1-R)/(1-R**(NTMAX-NIBTR))
          TIME(NIBTR+2)=A1+TIME(NIBTR+1)
          NBR=NIBTR+3
          DO 10 J=NBR,NTMAX2,1
              TIME(J)=TIME(J-1)+A1*R**(J-NBR+1)
   10     CONTINUE
      END IF
C     -----------------------------------------------------------------
C     ALL SUMS ARE ZEROED IN THIS SUBROUTINE FOR THE CALCULATED TIMES.
C     -----------------------------------------------------------------
      DO 40 J=1,NTMAX2,1
          S1(J)=0.0
          SUM1(J)=0.0
          SUM2(J)=0.0
   40 CONTINUE
      RETURN
      END
C
C
C
C
C                           #################
C                           #SUBROUTINE ITER#
C                           #################
C
      SUBROUTINE ITER
      COMMON T(100),SUM2(100),RATIO2(100),TIME(100),TUSED(100),DOWN1(100
     1),DOWN2(100),S1(100),RATIO1(100),SUM1(100),PFTR,BETA,I,NP,MAXIT,MO
     1DEL,NTMAX,NTMAX2
      COMMON/ERROR/R1(100),R2(100),R3(100),R4(100),SA1(100),SA2(100),SA3
     1(100),SA4(100)
      COMMON/RNG/FSEED,U
      COMMON/REPAIR/DEV
      INTEGER Z,K,H,NP,MAXIT
      DOUBLE PRECISION FSEED,U,X
C     ---------------------------------------------------------------
C     CALCULATE THE CONSTANT TO DETERMINE IF A FAILURE OCCURS IN THE
C     INTERVAL OF LENGTH EQUAL TO THE REPLACEMENT TIME BETA FOR USE
C     IN THE SUBROUTINE TIMES1.
```

```
C     ------------------------------------------------------------------
      PFTR=1-EXP(-(NP-1)*BETA)
C     ---------------------------
C     THE SIMULATION STARTS HERE.
C     ---------------------------
      DO 10 NIT= 1,MAXIT,1
          IF (MODEL.EQ.3) GOTO 3
          CALL TIMES1
          CALL CHECK1
          GOTO 10
    3     CALL TIMES3
          CALL CHECK3
   10 CONTINUE
      CALL RESULT
      RETURN
      END
C
C
C
C
C                         ###################
C                         #SUBROUTINE TIMES1#
C                         ###################
      SUBROUTINE TIMES1
      COMMON T(100),SUM2(100),RATIO2(100),TIME(100),TUSED(100),DOWN1(100
     1),DOWN2(100),S1(100),RATIO1(100),SUM1(100),PFTR,BETA,I,NP,MAXIT,MO
     1DEL,NTMAX,NTMAX2
      COMMON/RNG/FSEED,U
      DIMENSION DELTA(100),NFAIL(100),DT(100)
      DOUBLE PRECISION FSEED,U,X
      INTEGER Z,K,H,NP,MAXIT
C     ---------------------------------
C     DEFINE FREQUENTLY USED FUNCTIONS.
C     ---------------------------------
      FUNC1(X,J)=-DLOG(1.00000000D0-X)/J
      FUNC3(Y,X,J)=(Y-DLOG(1.00000000D0-X))/J
      FUNC4(Y,X,C)=(Y-DLOG(1.00000000D0-X))/C
      FUNC5(V,Y,C)=1.0-(EXP(-V*Y)*C)
      FUNC6(Y)=EXP(Y)
C     -------------------------------------------------------------------
C     INITIALIZE SUMS AND VECTORS USED IN THE INNER LOOPS. THE VECTORS
C     MUST BE REZEROED OVER THE RANGE GIVEN IN THE DIMENSION STATEMENT.
C     -------------------------------------------------------------------
      I=0
      DO 76 M= 1,100,1
          DELTA(M)=0.0
          T(M)=0.0
          DT(M)=0.0
          TUSED(M)=0.0
   76 CONTINUE
C     -----------------------------------------------------------------
C     INCREMENT COUNTER TO KEEP TRACK OF THE NUMBER OF TIME INTERVALS
C     BEFORE TMAX IS EXCEEDED FOR EACH SIMULATION.
C     -----------------------------------------------------------------
```

```
    5 I=I+1
C     ----------------------------------------------------------------
C     SET THE FIRST DELTA VALUE EQUAL TO THE REPLACEMENT TIME, BETA.
C     ----------------------------------------------------------------
      DELTA(1)=BETA
C     ------------------------------------------
C     INITIALIZE SOME COUNTERS AND TIME KEEPERS.
C     ------------------------------------------
      TUSED(I+1)=TUSED(I)
      TNEW=TUSED(I)+BETA
      F=0.0
      DSUM=0.0
      XF=0.0
      NFAIL(I)=0
C     -------------------------------------------------
C     GENERATE RANDOM NUMBER BY CALLING ROUTINE RANDOM.
C     -------------------------------------------------
      CALL RANDOM
C     -------------------------------------------------------------------
C     IF IT IS THE FIRST INTERVAL IN THE SIMULATION TRIAL THE FOLLOWING
C     STEPS ARE BYPASSED.
C     -------------------------------------------------------------------
  110 IF (I.EQ.1) THEN
          DT(I)=FUNC1(U,NP)
          T(I)=DT(I)
          TUSED(I+1)=T(I)
C         --------------------------------------------------------
C         CHECK PRESENT TIME AGAINST THE MAXIMUM EVALUATION TIME.
C         --------------------------------------------------------
          IF (T(I)-TIME(NTMAX2)) 23,77,77
C         ----------------------------------------------------------
C         THE LENGTH OF THE INTERVAL FOR THE FIRST FAILURE HAS BEEN
C         DETERMINED. CONTINUE IN THE PRESENT SIMULATION.
C         ----------------------------------------------------------
   23     GOTO 5
      ELSE
      END IF
C     -------------------------------------------------------------------
C     IF THE LENGTH OF THE PREVIOUS INTERVAL IS GREATER THAN BETA, THEN
C     (NP-1) PLANTS ARE OPERATING AT THE START OF THE INTERVAL.
C     -------------------------------------------------------------------
      IF (DT(I-1).GE.BETA.OR.I.EQ.2) THEN
C         -------------------------------------------------------------
C         THE VALUE OF THE RANDOM NUMBER IS COMPARED TO THE PROBABILITY
C         OF FAILURE IN THE INTERVAL BETA.
C         -------------------------------------------------------------
          IF (U.LT.PFTR) THEN
              DT(I)=FUNC1(U,(NP-1))
          ELSE
              DT(I)=FUNC3(BETA,U,NP)
          END IF
          T(I)=DT(I)+T(I-1)
          IF (T(I).GE.TNEW) TUSED(I+1)=T(I)
C         -------------------------------------------------------
```

```
C          CHECK PRESENT TIME AGAINST THE MAXIMUM EVALUATION TIME.
C          -----------------------------------------------------
           IF (T(I)-TIME(NTMAX2)) 24,77,77
C          ------------------------------------------------------------
C          THE LENGTH OF THE INTERVAL FOR THE CASE OF DT(I-1)|BETA HAS
C          BEEN DETERMINED. CONTINUE IN THE PRESENT SIMULATION.
C          ------------------------------------------------------------
   24      GOTO 5
       ELSE
           ICOUNT=0
           II=I-2
           IF (II.GE.(NP-1)) II=(NP-1)
           DO 14 J= 1,II,1
               DELTA(J+1)=BETA-T(I-1)+T(I-J-1)
               ICOUNT=ICOUNT+1
               IF (DELTA(J+1)) 13,13,14
   13          DELTA(J+1)=0.0
               ICOUNT=ICOUNT-1
               GOTO 17
   14      CONTINUE
C          -------------------------------------------------------
C          THE NEXT LOOP DETERMINES WHICH F(T) FUNCTION TO USE.
C          -------------------------------------------------------
   17      NFAIL(I)=ICOUNT
           IF (ICOUNT.GE.(NP-1)) THEN
               DO 11 J=1,II,1
                   DELTA(J)=DELTA(J)-DELTA(II+1)
   11          CONTINUE
               TWAIT=DELTA(II+1)
               H=NFAIL(I)+1
               DELTA(II+1)=0.0
           ELSE
               TWAIT=0.0
               H=NFAIL(I)+2
           END IF
           DO 75 K= 1,H,1
               ALPHA=NP-H+K
               DSUM=DSUM+DELTA(H+1-K)
               EP=FUNC6(DSUM)
               IF (K.EQ.H) THEN
                   F=1.0
                   GOTO 29
               ELSE
                   F=FUNC5(ALPHA,DELTA(H-K),EP)
               END IF
   29          XF=FUNC4(DSUM,U,ALPHA)
               IF (U.LE.F) THEN
                   DT(I)=XF
                   T(I)=T(I-1)+DT(I)+TWAIT
                   IF (T(I).GE.TNEW) TUSED(I+1)=T(I)
                   IF (T(I)-TIME(NTMAX2)) 25,77,77
   25              GOTO 5
               ELSE
               END IF
```

```
 75     CONTINUE
      END IF
C     --------------------------------------------------------------
C     THE LENGTH OF THE INTERVAL HAS NOW BEEN DETERMINED FOR THE CASE
C     DT(I-1)[BETA. CONTINUE IN THE PRESENT SIMULATION.
C     --------------------------------------------------------------
      GOTO 5
 77   RETURN
      END
C
C
C
C
C                            ###################
C                            #SUBROUTINE TIMES3#
C                            ###################
      SUBROUTINE TIMES3
      COMMON T(100),SUM2(100),RATIO2(100),TIME(100),TUSED(100),DOWN1(100
     1),DOWN2(100),S1(100),RATIO1(100),SUM1(100),PFTR,BETA,I,NP,MAXIT,MO
     1DEL,NTMAX,NTMAX2
      COMMON/RNG/FSEED,U
      DIMENSION DELTA(100),NFAIL(100),DT(100)
      DOUBLE PRECISION FSEED,U,X
      INTEGER Z,K,H,NP,MAXIT
C     -------------------------------
C     DEFINE FREQUENTLY USED FUNCTIONS.
C     -------------------------------
      FUNC1(X,J)=-DLOG(1.00000000D0-X)/J
      FUNC3(Y,X,J)=(Y-DLOG(1.00000000D0-X))/J
      FUNC4(Y,X,C)=(Y-DLOG(1.00000000D0-X))/C
      FUNC5(V,Y,C)=1.0-(EXP(-V*Y)*C)
      FUNC6(Y)=EXP(Y)
C     -----------------------------------------------------------------
C     INITIALIZE SUMS AND VECTORS USED IN THE INNER LOOPS. THE VECTORS
C     MUST BE REZEROED OVER THE RANGE GIVEN IN THE DIMENSION STATEMENT.
C     -----------------------------------------------------------------
      I=0
      DO 76 M= 1,100,1
          DELTA(M)=0.0
          T(M)=0.0
          DT(M)=0.0
 76   CONTINUE
C     -------------------------
C     SET LOGICAL VARIABLE VALUES.
C     -------------------------
      IFLAG=0
      IM=1
C     --------------------------------------------------------------
C     INCREMENT COUNTER TO KEEP TRACK OF THE NUMBER OF TIME INTERVALS
C     BEFORE TMAX IS EXCEEDED FOR EACH SIMULATION.
C     --------------------------------------------------------------
  5   I=I+1
C     ------------------------------------
C     INITIALIZE SOME COUNTERS AND TIME KEEPERS.
```

```
C      ----------------------------------------
       F=0.0
       DSUM=0.0
       XF=0.0
       NFAIL(I)=0
C      ----------------------------------------------
C      GENERATE RANDOM NUMBER BY CALLING ROUTINE RANDOM.
C      ----------------------------------------------
       CALL RANDOM
C      --------------------------------------------------------------
C      IF IT IS THE FIRST OR SECOND INTERVAL IN THE SIMULATION TRIAL THE
C      FOLLOWING STEPS ARE BYPASSED.
C      --------------------------------------------------------------
   110 IF (IM.LT.3) THEN
          DT(I)=FUNC1(U,NP)
          IF (I.EQ.1) THEN
             T(I)=DT(I)
          ELSE
             T(I)=DT(I)+T(I-1)
          END IF
C         ------------------------------------------------------
C         CHECK PRESENT TIME AGAINST THE MAXIMUM EVALUATION TIME.
C         ------------------------------------------------------
          IF (T(I)-TIME(NTMAX2)) 23,77,77
C         ---------------------------------------------------------
C         THE LENGTH OF THE INTERVAL HAS BEEN DETERMINED. THE COUNTER
C         IS NOW INCREASED AND THE SIMULATION IS CONTINUED.
C         ---------------------------------------------------------
    23    IM=IM+1
          GOTO 5
       ELSE
       END IF
C      --------------------------------------------------------------
C      IF THE LENGTH OF THE PREVIOUS INTERVAL IS GREATER THAN BETA, THEN
C      NP PLANTS ARE OPERATING AT THE START OF THE INTERVAL.
C      --------------------------------------------------------------
       IF (DT(I-1).GE.BETA) THEN
          DT(I)=FUNC1(U,NP)
          T(I)=DT(I)+T(I-1)
          IFLAG=0
          IF (T(I)-TIME(NTMAX2)) 24,77,77
    24    GOTO 5
       ELSE
          IF (NP.LE.1) THEN
             I=I+1
             DT(I)=BETA+T(I-NP)-T(I-1)
             T(I)=DT(I)+T(I-1)
             IF (T(I)-TIME(NTMAX2)) 36,77,77
    36       GOTO 5
           ELSE
           END IF
       END IF
       ICOUNT=0
       II=I-2
```

```
      IF (II.GE.NP) II=NP
      DO 14 J= 1,II,1
          DELTA(J)=BETA-T(I-1)+T(I-J-1)
          ICOUNT=ICOUNT+1
          IF (DELTA(J)) 13,13,14
   13     DELTA(J)=0.0
          ICOUNT=ICOUNT-1
          GOTO 17
   14 CONTINUE
C     ----------------------------------------------------
C     THE NEXT LOOP DETERMINES WHICH F(T) FUNCTION TO USE.
C     ----------------------------------------------------
   17 NFAIL(I)=ICOUNT
      H=NFAIL(I)+1
      IF (IFLAG.EQ.0) THEN
          NN=0
      ELSE
          NN=1
      END IF
      DO 75 K= 1,H,1
          ALPHA=NP-H+K+NN
          DSUM=DSUM+DELTA(H+1-K)
          EP=FUNC6(DSUM)
          IF (K.EQ.H.OR.ALPHA.GE.NP) THEN
              F=1.0
              GOTO 29
          ELSE
              F=FUNC5(ALPHA,DELTA(H-K),EP)
          END IF
   29     XF=FUNC4(DSUM,U,ALPHA)
          IF (U.LE.F) THEN
              DT(I)=XF
              T(I)=T(I-1)+DT(I)
              IF (T(I)-TIME(NTMAX2)) 25,77,77
C             ----------------------------------------------------
C             IF A FAILURE OCCURS IN THE INTERVAL WHERE THERE IS
C             ONLY ONE PLANT OPERATING THEN THE SYSTEM CANNOT FAIL
C             UNTIL AT LEAST ONE PLANT RETURNS TO SERVICE.
C             ----------------------------------------------------
   25         IF (ALPHA.EQ.1) THEN
                  I=I+1
                  DT(I)=BETA+T(I-NP)-T(I-1)
                  T(I)=DT(I)+T(I-1)
                  IF (T(I)-TIME(NTMAX2)) 26,77,77
   26             IFLAG=1
                  GOTO 5
              ELSE
              END IF
              GOTO 5
          ELSE
          END IF
   75 CONTINUE
C     ----------------------------------------------------------------
C     THE LENGTH OF THE INTERVAL HAS NOW BEEN DETERMINED FOR THE CASE
```

```
C     DT(I-1)[BETA. CONTINUE IN THE PRESENT SIMULATION.
C     ---------------------------------------------------------------
      GOTO 5
   77 RETURN
      END
C
C
C
C
C                           ###################
C                           #SUBROUTINE RESULT#
C                           ###################
      SUBROUTINE RESULT
      COMMON T(100),SUM2(100),RATIO2(100),TIME(100),TUSED(100),DOWN1(100
     1),DOWN2(100),S1(100),RATIO1(100),SUM1(100),PFTR,BETA,I,NP,MAXIT,MO
     1DEL,NTMAX,NTMAX2
      COMMON/ERROR/R1(100),R2(100),R3(100),R4(100),SA1(100),SA2(100),SA3
     1(100),SA4(100)
      COMMON/REPAIR/DEV
C     ------------------------------------------------------------
C     CALCULATE THE PROBABILITY OF PLAN B AT THE FAILURE TIMES FOR
C     METHOD 1 OR THE FRACTION DOWN TIME FOR METHOD 2.
C     ------------------------------------------------------------
      DO 20 J=1,NTMAX2,1
          TIME(J)=TIME(J)/BETA
          IF (MODEL.EQ.3) GOTO 3
C         ------------------------------------------------------------
C         THE RESULTS FOR MODEL 1 AND 2 ARE CALCULATED SIMULTANEOUSLY.
C         ------------------------------------------------------------
          RATIO1(J)=(MAXIT-SUM1(J))/MAXIT
          RSIGMA=SQRT(RATIO1(J)*(1-RATIO1(J))/MAXIT)*DEV
          R1(J)=RATIO1(J)+RSIGMA
          R2(J)=RATIO1(J)-RSIGMA
          SS=(MAXIT*BETA+S1(J))/MAXIT/BETA
          SSIGMA=SQRT(SS*(1-SS)/MAXIT)*DEV
          DOWN1(J)=1.00-SS
          SA1(J)=1.00-SS+SSIGMA
          SA2(J)=1.00-SS-SSIGMA
          RATIO2(J)=(MAXIT-SUM2(J))/MAXIT
          RSIGMA=SQRT(RATIO2(J)*(1-RATIO2(J))/MAXIT)*DEV
          R3(J)=RATIO2(J)+RSIGMA
          R4(J)=RATIO2(J)-RSIGMA
          SS=RATIO2(J)
          DOWN2(J)=1-SS
          SSIGMA=RSIGMA
          SA3(J)=1.00-SS+SSIGMA
          SA4(J)=1.00-SS-SSIGMA
          GOTO 20
C         ------------------------------------
C         THE RESULTS FOR MODEL 3 ARE CALCULATED.
C         ------------------------------------
    3     RATIO1(J)=(MAXIT-SUM1(J))/MAXIT
          RSIGMA=SQRT(RATIO1(J)*(1-RATIO1(J))/MAXIT)*DEV
          R1(J)=RATIO1(J)+RSIGMA
```

```
        R2(J)=RATIO1(J)-RSIGMA
        SS=(MAXIT*BETA+S1(J))/MAXIT/BETA
        SSIGMA=SQRT(SS*(1-SS)/MAXIT)*DEV
        DOWN1(J)=1.00-SS
        SA1(J)=1.00-SS+SSIGMA
        SA2(J)=1.00-SS-SSIGMA
   20 CONTINUE
      RETURN
      END
C
C
C
C
C                        ##################
C                        #SUBROUTINE PRINT#
C                        ##################
      SUBROUTINE PRINT
      COMMON T(100),SUM2(100),RATIO2(100),TIME(100),TUSED(100),DOWN1(100
     1),DOWN2(100),S1(100),RATIO1(100),SUM1(100),PFTR,BETA,I,NP,MAXIT,MO
     1DEL,NTMAX,NTMAX2
      COMMON/ERROR/R1(100),R2(100),R3(100),R4(100),SA1(100),SA2(100),SA3
     1(100),SA4(100)
      COMMON/MAXT/TMAX,MAXTAU,NREAD,NIBTR
      COMMON/REPAIR/DEV
      NFLAG=0
   10 WRITE (6,20)
   20 FORMAT ('1','##########OUTPUT FROM SIMULATION FORTRAN##########')
      WRITE (6,30) BETA
   30 FORMAT ('0','REPAIR/REPLACEMENT TIME  = ',F10.6)
      WRITE (6,50) NP
   50 FORMAT (' ','NUMBER OF PLANTS IN THE POOL =',I5)
      WRITE (6,60) MAXTAU
   60 FORMAT (' ','MAXIMUM TIME VALUE (IN UNITS OF BETA) =',I5)
      WRITE (6,70) NIBTR
   70 FORMAT (' ','NUMBER OF INTERVALS BEFORE BETA =',I4)
      WRITE (6,80) NTMAX2
   80 FORMAT (' ','TOTAL NUMBER OF INTERVALS =',I4)
      WRITE (6,90) MAXIT
   90 FORMAT (' ','TOTAL NUMBER SIMULATIONS RUN =',I10)
      WRITE (6,100) MODEL
  100 FORMAT (' ','MODEL',I2,' WAS USED')
      WRITE (6,110) DEV,DEV,DEV,DEV
  110 FORMAT ('0',' TIME PROB.AVAIL  +',F4.2,' SIGMA  -',F4.2,' SIGMA '
     1,'FRAC. DOWN  +',F4.2,' SIGMA  -',F4.2,' SIGMA')
      WRITE (6,120)
  120 FORMAT (' ',' --------------------------------------------------'
     1,'----------------------------')
      DO 140 J=1,NTMAX2,1
      IF (MODEL.EQ.3) GOTO 3
      IF (MODEL.EQ.2) GOTO 2
      WRITE (6,130) TIME(J),RATIO1(J),R1(J),R2(J),DOWN1(J),SA1(J),SA2(J)
  130 FORMAT(' ',F6.3,'   ',F7.4,'     ',F9.6,'     ',F9.6,'    ',F8.6,'  '
     1,'  ',F9.6,'     ',F9.6)
      GOTO 140
```

```
    2 WRITE (6,130) TIME(J),RATIO2(J),R3(J),R4(J),DOWN2(J),SA3(J),SA4(J)
      GOTO 140
    3 WRITE (6,130) TIME(J),RATIO1(J),R1(J),R2(J),DOWN1(J),SA1(J),SA2(J)
  140 CONTINUE
      IF (NFLAG.EQ.1) GOTO 150
      IF (MODEL.EQ.1) THEN
          MODEL=2
          NFLAG=1
          GOTO 10
      ELSE
      END IF
      IF (MODEL.EQ.2) THEN
          MODEL=1
          NFLAG=1
          GOTO 10
      ELSE
      END IF
  150 RETURN
      END
C
C
C
C
C                            ###################
C                            #SUBROUTINE RANDOM#
C                            ###################
C     -----------------------------------------------------------------
C     SUBROUTINE ROUTINE THAT OUTPUTS A RANDOM NUMBER FROM 0 TO 1 FOR A
C     GIVEN INPUT SEED VALUE.
C     -----------------------------------------------------------------
      SUBROUTINE RANDOM
      COMMON/RNG/DSEED,U
      DOUBLE PRECISION DSEED,U
      DSEED=DMOD(16807.DO*DSEED,2147483647.DO)
      U=DSEED/2147483648.DO
      RETURN
      END
C
C
C
C
C                             ###################
C                             #SUBROUTINE CHECK1#
C                             ###################
C     -------------------------------------------------------------
C     THE SUBROUTINE CHECK DETERMINES IF THE PRESENT VALUE OF TIME
C     EXCEEDS ANY OF THE TIME VALUES AT WHICH THE K-TH PLANT FAIL-
C     URE OCCURS.
C     -------------------------------------------------------------
      SUBROUTINE CHECK1
      COMMON T(100),SUM2(100),RATIO2(100),TIME(100),TUSED(100),DOWN1(100
     1),DOWN2(100),S1(100),RATIO1(100),SUM1(100),PFTR,BETA,I,NP,MAXIT,MO
     1DEL,NTMAX,NTMAX2
      M=1
```

```
      DO 90 J=1,I,1
          DO 80 L=M,NTMAX2,1
               IF (T(J)-TIME(L)) 85,10,10
   10          IF (TUSED(J)) 70,70,30
   30          IF (TIME(L)-TUSED(J)-BETA) 20,70,70
   20          SUM2(L)=SUM2(L)+1.0
   70          IF (J.EQ.1) GOTO 80
   40          TNEW=TIME(L)-T(J-1)-BETA
               IF (TNEW) 60,80,80
   60          SUM1(L)=SUM1(L)+1.0
               S1(L)=S1(L)+TNEW
   80     CONTINUE
   85     M=L
   90 CONTINUE
      RETURN
      END
C
C
C
C
C                             ###################
C                             #SUBROUTINE CHECK3#
C                             ###################
C     ------------------------------------------------------------
C     THE SUBROUTINE CHECK DETERMINES IF THE PRESENT VALUE OF TIME
C     EXCEEDS ANY OF THE TIME VALUES AT WHICH THE K-TH PLANT FAIL-
C     URE OCCURS.
C     ------------------------------------------------------------
      SUBROUTINE CHECK3
      COMMON T(100),SUM2(100),RATIO2(100),TIME(100),TUSED(100),DOWN1(100
     1),DOWN2(100),S1(100),RATIO1(100),SUM1(100),PFTR,BETA,I,NP,MAXIT,MO
     1DEL,NTMAX,NTMAX2
      M=1
      DO 90 J=1,I,1
          DO 80 L=M,NTMAX2,1
               IF (T(J)-TIME(L)) 85,10,10
   10          IF (J.EQ.1) GOTO 80
               TNEW=TIME(L)-T(J-1)-BETA
               IF (TNEW) 60,80,80
   60          SUM1(L)=SUM1(L)+1.0
               S1(L)=S1(L)+TNEW
   80     CONTINUE
   85     M=L
   90 CONTINUE
      RETURN
      END
```

APPENDIX B: KSUSPARE Computer Code

The computer code KSUSPARE was developed to determine the present worth of Plan A and Plan B costs. The computer code, written in FORTRAN, consists of a main program and 13 subroutines.

Input Data

The main program calls subroutines to read in and print out the problem data and to perform the economic analysis. The program is currently set up to estimate the Plan A and Plan B costs, the expected savings, and the benefit-cost ratio for one plant in the spare pool over the spare pool lifetime at variable time steps.

LAMBDA = component failure rate.

TREPL = repair/replacement time.

SPRMW = spare component capacity (MW).

N = number of plants in the pool.

NI = number of intervals used for numerical integration in all but the subroutine GOT (i.e., the subroutine to estimate g(t)).

NIG = number of intervals used for numerical integration in the subroutine GOT.

METHOD = the values of g(t) and $f_{DT}(t)$ are constant when a value of 1 is read in, otherwise these values are determined using numerical integration.

IT = the maximum evaluation time as an integer value.

TI = the maximum evaluation time as a real value.

TLOOP = the spare pool lifetime.

QSPR = present worth cost of the spare.

QSSV = present worth salvage value of the spare.

QSTOR = present worth storage cost per year for the spare.

MODEL = selects the method by which the fraction downtime for each time interval is assigned. An input value of 4 causes the fraction downtime to be calculated numerically using renewal theory; otherwise, the fraction downtime is read in by data statements.

The remaining input parameters are plant-specific costs and data.

Subroutines

The subroutine PWORTH calls other subroutines to calculate the Plan A and Plan B present worth costs for each plant. The subroutine COST1 calculates the failure-dependent variable and fixed costs and the variable costs during plant operation. The subroutine COST2 calculates the annual fixed costs. The subroutine GOT calculates the numerical solution to the renewal equation and returns the value of g(t) for each time interval. The subroutine DCOUNT and ESCLAT calculate the discount and escalation rates for each time interval. The subroutine CPCITY initializes the interval values for plant capacity factors and fraction downtime for each time interval. The fraction downtime is calculated using renewal theory by subroutine DTIME if Model 4 is selected.

The escalation and discount rates are initialized in their respective subroutines by DATA statements. Thus, for each simulation model the data statements must be modified and a separate version compiled. The remaining data are read in from an external data file.

Code Listing

The following is a listing of the KSUSPARE computer code.

```
$JOB          VAS,TIME=(2,40),PAGES=25
C***************************KSUSPARE************************************
C*                                                                     *
C*    THIS PROGRAM ESTIMATES THE TOTAL PRESENT WORTH COSTS FOR PLAN A  *
C*    AND PLAN B. THE PROGRAM USES THE TRAPEZOID RULE WHEN NUMERICAL   *
C*    INTEGRATION IS PERFORMED.                                        *
C*                                                                     *
C***********************************************************************
C                            **************
C                            *MAIN PROGRAM*
C                            **************
      COMMON DTF(1000),CAP(1000),ENA,ENB,OMA,OMB,REV,SRC,FOR,TREV,TLOOP,
     1TOTALA,TOTALB,SAV,BCR,T,FCR,LAMBDA,TREPL,QEN,QOM,QCOMP,QSSV,QTEMP,
     1QRM,QSTOR,QFSV,QPCSV,QSPR,DELTA,SPRMW,MW,TI,STEP,FRAC,STOR,COMP,SS
     1V,TTEMP,RMA,RMB,FSV,PCSV,SPR,NI,NIG,IT,METHOD,MODEL,NP,N,ICOUNT,TE
     1MP
      REAL LAMBDA,MW
      CALL READ1
      CALL PRINT1
      CALL PWORTH
      STOP
      END
C
C
C
C
C                            ******************
C                            *SUBROUTINE READ1*
C                            ******************
      SUBROUTINE READ1
      COMMON DTF(1000),CAP(1000),ENA,ENB,OMA,OMB,REV,SRC,FOR,TREV,TLOOP,
     1TOTALA,TOTALB,SAV,BCR,T,FCR,LAMBDA,TREPL,QEN,QOM,QCOMP,QSSV,QTEMP,
     1QRM,QSTOR,QFSV,QPCSV,QSPR,DELTA,SPRMW,MW,TI,STEP,FRAC,STOR,COMP,SS
     1V,TTEMP,RMA,RMB,FSV,PCSV,SPR,NI,NIG,IT,METHOD,MODEL,NP,N,ICOUNT,TE
     1MP
      REAL LAMBDA,MW
      READ (5,10) LAMBDA,TREPL,TTEMP,SPRMW
   10 FORMAT (/,F10.3,2F10.6,F10.3)
      READ (5,20) N,NI,NIG,METHOD,MODEL
   20 FORMAT (/,5I5)
      READ (5,30) QSPR,QSSV,QSTOR
   30 FORMAT (/,3F10.3)
      READ (5,40) IT,TI,STEP,TLOOP
   40 FORMAT (/,I5,3F10.3)
      RETURN
      END
C
C
C
C
C                            *******************
C                            *SUBROUTINE PRINT1*
C                            *******************
```

```
      SUBROUTINE PRINT1
      COMMON DTF(1000),CAP(1000),ENA,ENB,OMA,OMB,REV,SRC,FOR,TREV,TLOOP,
     1TOTALA,TOTALB,SAV,BCR,T,FCR,LAMBDA,TREPL,QEN,QOM,QCOMP,QSSV,QTEMP,
     1QRM,QSTOR,QFSV,QPCSV,QSPR,DELTA,SPRMW,MW,TI,STEP,FRAC,STOR,COMP,SS
     1V,TTEMP,RMA,RMB,FSV,PCSV,SPR,NI,NIG,IT,METHOD,MODEL,NP,N,ICOUNT,TE
     1MP
      REAL LAMBDA,MW
      WRITE (6,5)
    5 FORMAT ('1','##################KSUSPARE#######################')
      WRITE (6,10) LAMBDA
   10 FORMAT (' ','COMPONENT FAILURE RATE (YR-1) =',F7.3)
      WRITE (6,20) TREPL
   20 FORMAT (' ','NEW COMPONENT REPLACEMENT TIME (YR) =',F10.6)
      WRITE (6,30) IT
   30 FORMAT (' ','MAXIMUM EVALUATION TIME (YR) =',I5)
      WRITE (6,40) STEP
   40 FORMAT (' ','EVALUATION TIME STEP (YR) =',F7.3)
      WRITE (6,50) TLOOP
   50 FORMAT (' ','SPARE POOL LIFETIME (YR) =',F10.3)
      WRITE (6,60) N
   60 FORMAT (' ','NUMBER OF PLANTS IN THE SPARE POOL =',I5)
      WRITE (6,70) NI
   70 FORMAT (' ','NUMBER OF INTEGRATION INTERVALS (EXCEPT GOT) =',I5)
      WRITE (6,80) NIG
   80 FORMAT (' ','NUMBER OF INTEGRATION INTERVALS FOR SUB. GOT =',I5)
      WRITE (6,90) METHOD,MODEL
   90 FORMAT (' ','G(T) METHOD NUMBER ',I1,' AND MODEL',I2,' WERE USED')
      WRITE (6,100) SPRMW
  100 FORMAT (' ','SPARE COMPONENT CAPACITY (MW) =',F10.3)
      WRITE (6,110) QSPR
  110 FORMAT (' ','SPARE COMPONENT COST (K$) =',F10.3)
      WRITE (6,120) QSSV
  120 FORMAT (' ','SPARE COMPONENT SALVAGE VALUE (K$) =',F10.3)
      RETURN
      END
C
C
C
C
C                            *******************
C                            *SUBROUTINE PWORTH*
C                            *******************
      SUBROUTINE PWORTH
      COMMON DTF(1000),CAP(1000),ENA,ENB,OMA,OMB,REV,SRC,FOR,TREV,TLOOP,
     1TOTALA,TOTALB,SAV,BCR,T,FCR,LAMBDA,TREPL,QEN,QOM,QCOMP,QSSV,QTEMP,
     1QRM,QSTOR,QFSV,QPCSV,QSPR,DELTA,SPRMW,MW,TI,STEP,FRAC,STOR,COMP,SS
     1V,TTEMP,RMA,RMB,FSV,PCSV,SPR,NI,NIG,IT,METHOD,MODEL,NP,N,ICOUNT,TE
     1MP
      REAL LAMBDA,MW
C     ------------------------------------------------------------
C     THE INTERVAL WIDTH FOR THE NUMERICAL INTEGRATION IS CALCULATED.
C     ------------------------------------------------------------
      Y=FLOAT(NI)
      DELTA=1/Y
```

```
C     ------------------------------------------------------------
C     FRACTION OF YEARLY FIXED COSTS PAID BY EACH PLANT IS CALCULATED.
C     ------------------------------------------------------------
      YN=FLOAT(N)
      FRAC=1/YN
      ICOUNT=0
      NP=1
C     ----------------------------------------
C     SPARE COMPONENT SALVAGE VALUE IS ESTIMATED.
C     ----------------------------------------
      CALL READ2
      CALL PRINT2
      CALL CPCITY
      CALL REVNUE
      CALL ESCLAT (TLOOP,EF1,EF2,EF3,EF4,EF5,EF6,EF7,EF8,EF9)
C     ----------------------------
C     ALL DATA HAS BEEN INITIALIZED.
C     ----------------------------
      ICOUNT=1
      CALL DCOUNT (TLOOP,DF)
      SSV=DF*FRAC*QSSV*REV*EF9
C     ----------------------------------------
C     SPARE COMPONENT PURCHASE COST IS ESTIMATED.
C     ----------------------------------------
      SPR=QSPR*FRAC*REV
C     --------------------------------------------------------------
C     THE PLANT DEPENDENT SUBROUTINES ARE NOW CALLED AFTER THE PLANT TO
C     BE ANAYLZED IS ESTABLISHED.
C     --------------------------------------------------------------
      T=0.0
C     ------------------------------
C     THE OUTSIDE LOOP IS SETUP HERE.
C     ------------------------------
      T=STEP
   10 CALL COST1
      CALL COST2
      CALL RESULT
      CALL PRINT3
      T=T+STEP
      IF (T.GT.TI) GOTO 20
      GOTO 10
   20 CONTINUE
      RETURN
      END
C
C
C
C
C                              ******************
C                              *SUBROUTINE READ2*
C                              ******************
      SUBROUTINE READ2
      COMMON DTF(1000),CAP(1000),ENA,ENB,OMA,OMB,REV,SRC,FOR,TREV,TLOOP,
     1TOTALA,TOTALB,SAV,BCR,T,FCR,LAMBDA,TREPL,QEN,QOM,QCOMP,QSSV,QTEMP,
```

```
      1QRM,QSTOR,QFSV,QPCSV,QSPR,DELTA,SPRMW,MW,TI,STEP,FRAC,STOR,COMP,SS
      1V,TTEMP,RMA,RMB,FSV,PCSV,SPR,NI,NIG,IT,METHOD,MODEL,NP,N,ICOUNT,TE
      1MP
       REAL LAMBDA,MW
       READ (5,10) QEN,QOM,QCOMP,QPCSV,QFSV,QTEMP,QRM
    10 FORMAT (/,7F10.3)
       READ (5,20) MW,FCR,FOR,SRC,TREV
    20 FORMAT (/,5F10.3)
       RETURN
       END
C
C
C
C
C                        ********************
C                        *SUBROUTINE PRINT2*
C                        ********************
       SUBROUTINE PRINT2
       COMMON DTF(1000),CAP(1000),ENA,ENB,OMA,OMB,REV,SRC,FOR,TREV,TLOOP,
      1TOTALA,TOTALB,SAV,BCR,T,FCR,LAMBDA,TREPL,QEN,QOM,QCOMP,QSSV,QTEMP,
      1QRM,QSTOR,QFSV,QPCSV,QSPR,DELTA,SPRMW,MW,TI,STEP,FRAC,STOR,COMP,SS
      1V,TTEMP,RMA,RMB,FSV,PCSV,SPR,NI,NIG,IT,METHOD,MODEL,NP,N,ICOUNT,TE
      1MP
       REAL LAMBDA,MW
       WRITE (6,10) NP
    10 FORMAT ('-','-----INPUT/OUTPUT FOR PLANT NUMBER ',I3,'-----')
       WRITE (6,20)
    20 FORMAT (' ','ENERGY-----O&M-----COMP.-COMP. SAL.-FAILED COMP. SAL'
      1,'.-REPLACEMENT-RES.MARGIN')
       WRITE (6,30) QEN,QOM,QCOMP,QPCSV,QFSV,QTEMP,QRM
    30 FORMAT (' ',9F10.3)
       WRITE (6,40) MW,FCR,TREV
    40 FORMAT (' ','CAPACITY (MW) =',F10.3,'  FCR =',F7.3,'  YEARS (REV)'
      1,' =',F7.3)
       WRITE (6,50) FOR,SRC
    50 FORMAT (' ','FORCED OUTAGE RATE =',F7.3,'  SYSTEM RISK CHARACTERI'
      1,'STIC =',F10.3)
       WRITE (6,60)
    60 FORMAT ('-',' TIME  TOTALA  TOTALB     SAV   BCR      ENERGY     '
      1,'    O&M          RESERVE      COMP   FSV    USV   SSV  TEMP  '
      1,'SPARE   STOR')
       WRITE (6,70)
    70 FORMAT (' ','-------------------------------------------------------'
      1,'----------------------------------------------------------'
      1,'------------')
       RETURN
       END
C
C
C
C
C                       *******************
C                       *SUBROUTINE COST1*
C                       *******************
```

```
      SUBROUTINE COST1
      COMMON DTF(1000),CAP(1000),ENA,ENB,OMA,OMB,REV,SRC,FOR,TREV,TLOOP,
     1TOTALA,TOTALB,SAV,BCR,T,FCR,LAMBDA,TREPL,QEN,QOM,QCOMP,QSSV,QTEMP,
     1QRM,QSTOR,QFSV,QPCSV,QSPR,DELTA,SPRMW,MW,TI,STEP,FRAC,STOR,COMP,SS
     1V,TTEMP,RMA,RMB,FSV,PCSV,SPR,NI,NIG,IT,METHOD,MODEL,NP,N,ICOUNT,TE
     1MP
      DIMENSION VCA(1000),VCB(1000),FCA(1000),FCB(1000),FCC(1000),VCC(10
     100),VCD(1000)
      REAL LAMBDA,MW
C     ----------------------------------------------
C     THE CONSTANT MULTIPLIERS ARE CALCULATED FIRST.
C     ----------------------------------------------
      C1A=TREPL*QEN*LAMBDA
      C1B=(MW-SPRMW)
      IF (C1B.LT.0.0) C1B=0.0
      C2=TREPL*QOM
      IF (METHOD.EQ.1) THEN
          C3=1/(1+LAMBDA*TREPL)
      ELSE
          C3=1.0
      END IF
C     ------------------------------------------------------------------
C     THE VALUES FOR THE FUNCTION TO BE NUMERICALY INTEGRATED ARE FIRST EVAL-
C     UATED AT THE TIME VALUES CALCULATED ABOVE.
C     ------------------------------------------------------------------
      ID=NI*T
      ID2=ID+1
      VCA(1)=CAP(1)
      VCB(1)=0.0
      DO 10 K=2,ID2,1
          J=K-1
          X=J*DELTA
          CALL GOT(X,GTX)
          CALL ESCLAT (X,EF1,EF2,EF3,EF4,EF5,EF6,EF7,EF8,EF9)
C         ----------------------------------------
C         THE LAST PIECE OF DATA HAS BEEN INITIALIZED.
C         ----------------------------------------
          ICOUNT=1
          CALL DCOUNT (X,DF)
          VCA(K)=GTX*DF*EF1*CAP(J)
          VCB(K)=VCA(K)*DTF(J)
          VCC(K)=DF*EF2*GTX
          VCD(K)=DF*EF2*GTX*(1-DTF(J))
          FCA(K)=GTX*EF3*DF
          FCB(K)=GTX*EF4*DF
          FCC(K)=GTX*EF5*DF
   10 CONTINUE
C     ----------------------------------------
C     THE NUMERICAL INTEGRATION IS NOW PERFORMED.
C     ----------------------------------------
      SUM1=0.0
      SUM2=0.0
      SUM3=0.0
      SUM5=0.0
```

```
      SUM6=0.0
      SUM7=0.0
      SUM8=0.0
      SUM9=0.0
      DO 20 K=2,ID,1
          SUM1=SUM1+VCA(K)
          SUM2=SUM2+VCB(K)
          SUM5=SUM5+FCA(K)
          SUM6=SUM6+FCB(K)
          SUM7=SUM7+FCC(K)
          SUM8=SUM8+VCC(K)
          SUM9=SUM9+VCD(K)
   20 CONTINUE
C     ------------------------------------------------------------------
C     THE INITIAL VALUES FOR THE F(0) TERMS USED IN THE NUMERICAL INTE-
C     GRATION ARE ESTABLISHED. IF THE CONSTANT METHOD IS USED THE F(0)
C     TERM IS SET EQUAL TO SOME CONSTANT VALUE.
C     ------------------------------------------------------------------
      IF (METHOD.EQ.1) THEN
          GT=1/(1+LAMBDA*TREPL)
          VCA(1)=CAP(1)
          VCB(1)=DTF(1)
          VCC(1)=GT
          VCD(1)=GT
          FCA(1)=GT
          FCB(1)=GT
          FCC(1)=GT
      ELSE
          VCA(1)=CAP(1)
          VCB(1)=0.0
          VCC(1)=1.0
          VCD(1)=1.0
          FCA(1)=1.0
          FCB(1)=1.0
          FCC(1)=1.0
      END IF
C     ------------------------------------
C     ENERGY REPLACEMENT COST IS ESTIMATED.
C     ------------------------------------
      AA=DELTA/2*(VCA(1)+VCA(ID2)+2*SUM1)
      BB=DELTA/2*(VCB(1)+VCB(ID2)+2*SUM2)
      ENA=MW*C1A*AA
      ENB=C1A*(C1B*AA+SPRMW*BB)
C     ---------------------------------------------
C     OPERATION AND MAINTENANCE COSTS ARE ESTIMATED.
C     ---------------------------------------------
      CC=DELTA/2*(VCC(1)+VCC(ID2)+2*SUM8)
      DD=DELTA/2*(VCD(1)+VCD(ID2)+2*SUM9)
      OMA=QOM*CC
      OMB=OMA+LAMBDA*TREPL*DD*QOM
C     ----------------------------------------
C     NEW COMPONENT PURCHASE COST IS ESTIMATED.
C     ----------------------------------------
      CC=DELTA/2*(FCA(1)+FCA(ID2)+2*SUM5)
```

```
      COMP=QCOMP*CC*REV*LAMBDA
C     ---------------------------------------------
C     SALVAGE OF THE FAILED COMPONENT IS ESTIMATED.
C     ---------------------------------------------
      DD=DELTA/2*(FCB(1)+FCB(ID2)+2*SUM6)
      FSV=QFSV*DD*REV*LAMBDA
C     ----------------------------------------
C     TEMPORARY REPLACEMENT COST IS ESTIMATED.
C     ----------------------------------------
      IF (MW-SPRMW.LE.0.0) THEN
          TEMP=0.0
      ELSE
          EE=DELTA/2*(FCC(1)+FCC(ID2)+2*SUM7)
          TEMP=LAMBDA*EE*QTEMP
      END IF
C     ------------------------------------------
C     PLANT COMPONENT SALVAGE VALUE IS ESTIMATED.
C     ------------------------------------------
      CALL DCOUNT (T,DF)
      CALL GOT (T,GTX)
      CALL ESCLAT (T,EF1,EF2,EF3,EF4,EF5,EF6,EF7,EF8,EF9)
      PCSV=QPCSV*EF8*DF*REV*GTX
      RETURN
      END
C
C
C
C
C                         ******************
C                         *SUBROUTINE COST2*
C                         ******************
      SUBROUTINE COST2
      COMMON DTF(1000),CAP(1000),ENA,ENB,OMA,OMB,REV,SRC,FOR,TREV,TLOOP,
     1TOTALA,TOTALB,SAV,BCR,T,FCR,LAMBDA,TREPL,QEN,QOM,QCOMP,QSSV,QTEMP,
     1QRM,QSTOR,QFSV,QPCSV,QSPR,DELTA,SPRMW,MW,TI,STEP,FRAC,STOR,COMP,SS
     1V,TTEMP,RMA,RMB,FSV,PCSV,SPR,NI,NIG,IT,METHOD,MODEL,NP,N,ICOUNT,TE
     1MP
      DIMENSION XX(1000),YY(1000)
      REAL LAMBDA,MW
C     ------------------------------------------------------------------
C     RESERVE CAPACITY REQUIRED CALCULATION IS FIRST. THE CONSTANTS USED
C     IN DETERMINING THE DISCOUNT/ESCALATION FACTOR AND IN NUMERICALY
C     EVALUATING THE AVERAGE DOWN TIME ARE ESTIMATED.
C     ------------------------------------------------------------------
      IF (T.LE.1.0) THEN
          CALL DCOUNT (1.0,DF)
          CALL ESCLAT (1.0,EF1,EF2,EF3,EF4,EF5,EF6,EF7,EF8,EF9)
          C1=DF*EF6
          C2=DF*EF7
          IF (METHOD.EQ.1) THEN
              FBAR=DTF(1)
              GOTO 40
          ELSE
          END IF
```

```
            BB=T*NI
            NT1=INT(BB)+1
            NB1=2
            NB2=NB1
            XX(1)=1.0
            YY(1)=1.0
            L=1
        ELSE
            X=T-INT(T)
            IF (X.EQ.0.0) THEN
                M=INT(T)
            ELSE
                M=INT(T)+1
            END IF
            NB1=(M-1)*NI+1
            NT1=M*NI+1
            NB2=NB1+1
            L=NB1
            SUM1=0.0
            SUM2=0.0
C           -----------------------------------------------
C           DISCOUNT/ESCALATION FACTOR FOR RESERVE MARGIN.
C           -----------------------------------------------
            DO 10 K=1,M,1
                X=FLOAT(K)
                CALL DCOUNT (X,DF)
                CALL ESCLAT (X,EF1,EF2,EF3,EF4,EF5,EF6,EF7,EF8,EF9)
                SUM1=SUM1+DF*EF6
                SUM2=SUM2+DF*EF7
   10       CONTINUE
            C1=SUM1
            C2=SUM2
        END IF
C       -----------------------------------------------------------------
C       THE AVERAGE DOWN TIME FRACTION FOR YEARLY INTERVALS IS NUMERICALY
C       EVALUATED.
C       -----------------------------------------------------------------
        DO 20 K=NB1,NT1,1
            J=K-1
            S=K*DELTA
            CALL GOT(S,GTS)
            XX(K)=GTS*DTF(J)
            YY(K)=GTS
   20   CONTINUE
        NT2=NT1-1
        SUM5=0.0
        SUM6=0.0
        DO 30 K=NB2,NT2,1
            SUM5=SUM5+XX(K)
            SUM6=SUM6+YY(K)
   30   CONTINUE
        FBAR=(XX(L)+XX(NT1)+2*SUM5)/(YY(L)+YY(NT1)+2*SUM6)
C       --------------------------------------------------------------
C       THE RESERVE MARGIN DUE TO THE COMPONENT AND ASSOCIATED COST IS
```

```
C     ESTIMATED.
C     ---------------------------------------------------------------
   40 RCA=LAMBDA*TREPL*(1-FOR)
      AA=EXP(MW/SRC)-1
      RATIO=(MW-SPRMW)/MW
      IF (RATIO.LT.0.0) RATIO=0.0
      RMA=SRC*ALOG(1+(RCA*AA/(1+FOR*AA)))*FCR*QRM*C1
      RCB=LAMBDA*TREPL*(FBAR+(1-FBAR)*RATIO)*(1-FOR)
      RMB=SRC*ALOG(1+(RCB*AA/(1+FOR*AA)))*FCR*QRM*C1
C     ---------------------------------------------
C     THE SPARE COMPONENT STORAGE COST IS ESTIMATED.
C     ---------------------------------------------
      STOR=QSTOR*FRAC*C2
      RETURN
      END
C
C
C
C
C                         ********************
C                         *SUBROUTINE RESULT*
C                         ********************
      SUBROUTINE RESULT
      COMMON DTF(1000),CAP(1000),ENA,ENB,OMA,OMB,REV,SRC,FOR,TREV,TLOOP,
     1TOTALA,TOTALB,SAV,BCR,T,FCR,LAMBDA,TREPL,QEN,QOM,QCOMP,QSSV,QTEMP,
     1QRM,QSTOR,QFSV,QPCSV,QSPR,DELTA,SPRMW,MW,TI,STEP,FRAC,STOR,COMP,SS
     1V,TTEMP,RMA,RMB,FSV,PCSV,SPR,NI,NIG,IT,METHOD,MODEL,NP,N,ICOUNT,TE
     1MP
      REAL LAMBDA,MW
C     ------------------------------------------------------------------
C     DETERMINE THE ESTIMATE OF THE TOTAL PRESENT WORTH COST FOR PLAN A.
C     ------------------------------------------------------------------
      TOTALA=ENA+OMA+COMP-FSV+RMA-PCSV
C     ------------------------------------------------------------------
C     DETERMINE THE ESTIMATE OF THE TOTAL PRESENT WORTH COST FOR PLAN B.
C     ------------------------------------------------------------------
      TOTALB=ENB+OMB+COMP-FSV+TEMP+RMB+STOR-PCSV-SSV+SPR
C     ---------------------------------------------------
C     THE SAVINGS IS THE DIFFERENCE BETWEEN THE TWO PLANS.
C     ---------------------------------------------------
      SAV=TOTALA-TOTALB
C     -----------------------------------------------------
C     THE FINAL STEP IS TO CALCULATE THE BENEFIT-COST RATIO.
C     -----------------------------------------------------
      BCR=1+(SAV/SPR)
      RETURN
      END
C
C
C
C
C                         ********************
C                         *SUBROUTINE PRINT3*
C                         ********************
```

```
      SUBROUTINE PRINT3
      COMMON DTF(1000),CAP(1000),ENA,ENB,OMA,OMB,REV,SRC,FOR,TREV,TLOOP,
     1TOTALA,TOTALB,SAV,BCR,T,FCR,LAMBDA,TREPL,QEN,QOM,QCOMP,QSSV,QTEMP,
     1QRM,QSTOR,QFSV,QPCSV,QSPR,DELTA,SPRMW,MW,TI,STEP,FRAC,STOR,COMP,SS
     1V,TTEMP,RMA,RMB,FSV,PCSV,SPR,NI,NIG,IT,METHOD,MODEL,NP,N,ICOUNT,TE
     1MP
      REAL LAMBDA,MW
      WRITE (6,30) T,TOTALA,TOTALB,SAV,BCR,ENA,ENB,OMA,OMB,RMA,RMB,COMP,
     1FSV,PCSV,SSV,TEMP,SPR,STOR
   30 FORMAT (' ',F5.2,3F8.2,F6.2,F8.2,F7.2,3F8.2,2F7.2,F6.2,F7.2,2F6.2,
     1F7.2,F6.2,F8.2,F6.2)
      RETURN
      END
C
C
C
C
C                             *****************
C                             *SUBROUTINE GOT*
C                             *****************
      SUBROUTINE GOT(Y,GTY)
      COMMON DTF(1000),CAP(1000),ENA,ENB,OMA,OMB,REV,SRC,FOR,TREV,TLOOP,
     1TOTALA,TOTALB,SAV,BCR,T,FCR,LAMBDA,TREPL,QEN,QOM,QCOMP,QSSV,QTEMP,
     1QRM,QSTOR,QFSV,QPCSV,QSPR,DELTA,SPRMW,MW,TI,STEP,FRAC,STOR,COMP,SS
     1V,TTEMP,RMA,RMB,FSV,PCSV,SPR,NI,NIG,IT,METHOD,MODEL,NP,N,ICOUNT,TE
     1MP
      DIMENSION GT(1000)
      REAL LAMBDA,MW
C     -----------------------------------------------------------------
C     THE VALUE OF G(T) CAN EITHER BE CONSTANT OR ESTIMATED BY NUMERICAL
C     INTEGRATION. IF THE METHOD IS 1 THEN G(T) IS CONSTANT.
C     -----------------------------------------------------------------
      IF (METHOD.EQ.1) THEN
          GTY=1/(1+LAMBDA*TREPL)
          GOTO 50
      ELSE
      END IF
      IF (Y.LE.TREPL) THEN
          GTY=EXP(-LAMBDA*Y)
      ELSE
C         ---------------------------------------------------------
C         CONSTANTS USED IN THE SUBROUTINE ARE CALCULATED FIRST.
C         ---------------------------------------------------------
          C1=EXP(TREPL)
          ALPHA=LAMBDA*EXP(LAMBDA*TREPL)
          DELTA2=TREPL/NI
          BETA=ALPHA*DELTA2/2
          ID3=Y/DELTA2+1
          NI1=NI+1
          DO 10 I=1,NI1,1
              X=(I-1)*DELTA2
C             ---------------------------------------
C             THE FIRST TREPL INTERVAL IS ESTIMATED.
C             ---------------------------------------
```

```
                GT(I)=EXP(-LAMBDA*X)
   10       CONTINUE
            NI2=NI1+1
            DO 40 I=NI2,ID3,1
                X=(I-1)*DELTA2
C               ----------------------------------
C               THE REMAINING TIMES ARE ESTIMATED.
C               ----------------------------------
                AA=EXP(-LAMBDA*X)
                SUM=0.0
                NEND=I-NI2
                IF (NEND.EQ.0) GOTO 30
                DO 20 J=1,NEND,1
                    SUM=SUM+EXP(LAMBDA*(J)*DELTA2)*GT(J+1)
   20           CONTINUE
   30           BB=EXP(LAMBDA*(X-TREPL))*GT(I-NI)
                GT(I)=AA*(1+BETA*(GT(1)+BB+2*SUM))
   40       CONTINUE
            GTY=GT(ID3)
      END IF
   50 CONTINUE
      RETURN
      END
C
C
C
C
C                             *******************
C                             *SUBROUTINE DCOUNT*
C                             *******************
      SUBROUTINE DCOUNT (Y,DF)
      COMMON DTF(1000),CAP(1000),ENA,ENB,OMA,OMB,REV,SRC,FOR,TREV,TLOOP,
     1TOTALA,TOTALB,SAV,BCR,T,FCR,LAMBDA,TREPL,QEN,QOM,QCOMP,QSSV,QTEMP,
     1QRM,QSTOR,QFSV,QPCSV,QSPR,DELTA,SPRMW,MW,TI,STEP,FRAC,STOR,COMP,SS
     1V,TTEMP,RMA,RMB,FSV,PCSV,SPR,NI,NIG,IT,METHOD,MODEL,NP,N,ICOUNT,TE
     1MP
      DIMENSION DR(100)
      REAL LAMBDA,MW
C     -----------------------------------------------------------------
C     DISCOUNT RATES MUST BE ASSIGNED FOR THE DIFFERENT PLANTS. IF THE
C     VALUES HAVE ALREADY BEEN INITIALIZED THEN THE DATA STATEMENT IS
C     BYPASSED.
C     -----------------------------------------------------------------
      IF (ICOUNT.EQ.1) GOTO 10
      DATA DR/100*0.13/
   10 M=INT(Y)
      IF (Y.LE.1.0) M=0
      DF=(1+DR(M+1))**(-Y)
      RETURN
      END
C
C
C
C
```

```
C                             ********************
C                             *SUBROUTINE ESCLAT*
C                             ********************
      SUBROUTINE ESCLAT(X,XEF1,XEF2,XEF3,XEF4,XEF5,XEF6,XEF7,XEF8,XEF9)
      COMMON DTF(1000),CAP(1000),ENA,ENB,OMA,OMB,REV,SRC,FOR,TREV,TLOOP,
     1TOTALA,TOTALB,SAV,BCR,T,FCR,LAMBDA,TREPL,QEN,QOM,QCOMP,QSSV,QTEMP,
     1QRM,QSTOR,QFSV,QPCSV,QSPR,DELTA,SPRMW,MW,TI,STEP,FRAC,STOR,COMP,SS
     1V,TTEMP,RMA,RMB,FSV,PCSV,SPR,NI,NIG,IT,METHOD,MODEL,NP,N,ICOUNT,TE
     1MP
      DIMENSION ER1(100),ER2(100),ER3(100),ER4(100),ER5(100),ER6(100),ER
     17(100),ER8(100),ER9(100)
      REAL LAMBDA,MW
      IF (ICOUNT.EQ.1) GOTO 10
C     -----------
C     ENERGY RATE
C     -----------
      DATA ER1/100*0.04/
C     ------------------------------
C     OPERATION AND MAINTENANCE RATE
C     ------------------------------
      DATA ER2/100*0.03/
C     ------------------
C     NEW COMPONENT RATE
C     ------------------
      DATA ER3/5*0.05,95*0.07/
C     -----------------------------
C     FAILED COMPONENT SALVAGE RATE
C     -----------------------------
      DATA ER4/100*0.055/
C     --------------------------
C     TEMPORARY REPLACEMENT RATE
C     --------------------------
      DATA ER5/100*0.035/
C     -------------------
C     RESERVE MARGIN RATE
C     -------------------
      DATA ER6/100*0.05/
C     ------------------
C     SPARE STORAGE RATE
C     ------------------
      DATA ER7/100*0.03/
C     ---------------------------------
C     USED COMPONENT SALVAGE VALUE RATE
C     ---------------------------------
      DATA ER8/100*0.055/
C     -----------------------------------
C     SPARE COMPONENT SALVAGE VALUE RATE.
C     -----------------------------------
      DATA ER9/100*0.055/
C     ------------------------------------------------------------------
C     IF THE TIME TO EVALUATE THE ESCALATION RATE IS LESS THAN ONE YEAR
C     THEN THE ESCALATION RATE IS BASED ON A FRACTION OF THE FIRST YEAR.
C     ------------------------------------------------------------------
   10 M=INT(X)
```

```
      IF (X.LE.1.0) M=0
      XEF1=(1+ER1(M+1))**X
      XEF2=(1+ER2(M+1))**X
      XEF3=(1+ER3(M+1))**X
      XEF4=(1+ER4(M+1))**X
      XEF5=(1+ER5(M+1))**X
      XEF6=(1+ER6(M+1))**X
      XEF7=(1+ER7(M+1))**X
      XEF8=(1+ER8(M+1))**X
      XEF9=(1+ER9(M+1))**X
      RETURN
      END
C
C
C
C
C                            *******************
C                            *SUBROUTINE REVNUE*
C                            *******************
      SUBROUTINE REVNUE
      COMMON DTF(1000),CAP(1000),ENA,ENB,OMA,OMB,REV,SRC,FOR,TREV,TLOOP,
     1TOTALA,TOTALB,SAV,BCR,T,FCR,LAMBDA,TREPL,QEN,QOM,QCOMP,QSSV,QTEMP,
     1QRM,QSTOR,QFSV,QPCSV,QSPR,DELTA,SPRMW,MW,TI,STEP,FRAC,STOR,COMP,SS
     1V,TTEMP,RMA,RMB,FSV,PCSV,SPR,NI,NIG,IT,METHOD,MODEL,NP,N,ICOUNT,TE
     1MP
      REAL LAMBDA,MW
      SUM2=0.0
      X=0.0
   10 X=X+1.0
      CALL DCOUNT (X,DF)
      SUM2=SUM2+DF
      IF (X.LT.TREV) GOTO 10
      REV=FCR*SUM2
      RETURN
      END
C
C
C
C
C                            *******************
C                            *SUBROUTINE CPCITY*
C                            *******************
      SUBROUTINE CPCITY
      COMMON DTF(1000),CAP(1000),ENA,ENB,OMA,OMB,REV,SRC,FOR,TREV,TLOOP,
     1TOTALA,TOTALB,SAV,BCR,T,FCR,LAMBDA,TREPL,QEN,QOM,QCOMP,QSSV,QTEMP,
     1QRM,QSTOR,QFSV,QPCSV,QSPR,DELTA,SPRMW,MW,TI,STEP,FRAC,STOR,COMP,SS
     1V,TTEMP,RMA,RMB,FSV,PCSV,SPR,NI,NIG,IT,METHOD,MODEL,NP,N,ICOUNT,TE
     1MP
      REAL LAMBDA,MW
      DIMENSION FDT(100),FCAP(100)
C     ------------------------------------------------------------------
C     ASSIGN CAPACITY FACTORS AND FRACTION DOWN TIMES FOR THE DIFFERENT
C     PLANTS. IF THE VALUES HAVE ALREADY BEEN ASSIGNED THEN THE DATA
C     STATEMENT IS BYPASSED.
```

```
C     ------------------------------------------------------------------
      IF (ICOUNT.EQ.1) GOTO 10
      DATA FCAP,FDT/100*0.390,0.0418,0.0429,0.0430,0.0427,0.044,0.043,0.
     10433,0.0444,0.0435,0.0417,0.0414,0.0419,0.043,0.0397,0.043,85*0.04
     13/
   10 ID=IT*NI
      DO 20 I=1,ID,1
          X=I*DELTA
          M=INT(X)
          IF (MODEL.EQ.4) THEN
               CALL DTIME (X,DTX)
               DTF(I)=DTX
               IF (METHOD.EQ.1) THEN
                   CAP(I)=FCAP(1)
               ELSE
                   R=M-IT
                   IF (R.EQ.0.0) M=M-1
                   CAP(I)=FCAP(M+1)
               END IF
          ELSE
C             ------------------------------------------------------------
C             IF THE THE EVALUATION TIME IS LESS THAN ONE YEAR THEN THE
C             CAPACITY FACTOR AND FRACTION DOWN TIME ARE BASED ON A
C             FRACTION OF THE FIRST YEAR.
C             ------------------------------------------------------------
              IF (X.LT.1.0) THEN
                  CAP(I)=FCAP(1)
                  IF (METHOD.EQ.1.OR.X.GE.TREPL) THEN
                      DTF(I)=FDT(5)
                  ELSE
                      IF (X.GE.TREPL) THEN
                          DTF(I)=FDT(1)
                      ELSE
                          DTF(I)=FDT(1)/2
                      END IF
                  END IF
              ELSE
                  R=M-IT
                  IF (R.EQ.0.0) M=M-1
                  CAP(I)=FCAP(M+1)
                  DTF(I)=(FDT(M)+FDT(M+1))/2
              END IF
          END IF
   20 CONTINUE
      RETURN
      END
C
C
C
C
C                         ******************
C                         *SUBROUTINE DTIME*
C                         ******************
      SUBROUTINE DTIME (Y,DTY)
```

```
      COMMON DTF(1000),CAP(1000),ENA,ENB,OMA,OMB,REV,SRC,FOR,TREV,TLOOP,
     1TOTALA,TOTALB,SAV,BCR,T,FCR,LAMBDA,TREPL,QEN,QOM,QCOMP,QSSV,QTEMP,
     1QRM,QSTOR,QFSV,QPCSV,QSPR,DELTA,SPRMW,MW,TI,STEP,FRAC,STOR,COMP,SS
     1V,TTEMP,RMA,RMB,FSV,PCSV,SPR,NI,NIG,IT,METHOD,MODEL,NP,N,ICOUNT,TE
     1MP
      REAL LAMBDA
      TEX=(TREPL-TTEMP)/2
      C1=TTEMP/TREPL
      C2=1-C1
      C3=(1/(1+LAMBDA*TREPL))**(N-1)
      IF (METHOD.EQ.1) THEN
          DTY=C2+(1-C3)*C1
      ELSE
          X=Y+TEX
          CALL GOT (X,GTX)
          AA=1-(GTX**(N-1))
          DTY=C2+AA*C1
      END IF
      RETURN
      END
C
C
C
C
C                                 ******
C                                 *DATA*
C                                 ******
$ENTRY
LAMBDA,REPAIR/REPLACE,TEMPORARY REPLACEMENT,SPARE MW CAPACITY
     0.010  0.739726  0.616438   500.000
#PLANTS,#INTERVALS(EXCEPT GOT),#INTERVALS(GOT),METHOD,MODEL
   13   10   10    1    4
SPARE COST,SPARE SALVAGE VALUE,SPARE STORAGE
  2200.000  1000.000   20.000
MAXIMUM EVALUATION TIME(MET,INTEGER),MET(REAL),TIME STEP,POOL LIFETIME
   15    15.000    15.000    15.000
ENERGY,O&M,NEW COMP,USED COMP SAL,FAILED COMP SAL,TEMPORARY,RESERVE MAR.
    74.898   200.000  2770.000   400.000    40.000    40.000    186.000
PLANT MW CAPACITY,FCR,FOR,SRC,REV(YRS)
   800.000     0.174     0.220  1000.000    25.000
```

A PROBABILISTIC AND ECONOMIC ANALYSIS OF A MAJOR COMPONENT SHARED AMONG ELECTRIC UTILITIES

by

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ABSTRACT

The purpose of this study was to refine techniques developed in a previous study on the economic feasibility of sharing a spare critical component among several electric generating stations. Total costs attributable to a particular plant component were estimated under two component-management plans: Plan A in which no preparations are made prior to a component failure, and Plan B in which a spare component is purchased and made available for temporary use should a plant component fail. The modeling techniques previously developed to estimate the difference between Plan A and Plan B costs over the lifetime of the spare-component pool were studied and refined in two major areas. The first refinement was to the failure model that describes both the probability of spare availability and the fraction of the component repair/replacement time that the spare is not used as a temporary substitute. A Monte Carlo simulation code was used to investigate three different stochastic models. A fourth failure model, based on a renewal theory description, was also studied. The second area of refinement was to the economic model so that costs could be estimated on a continuous-time basis rather than at year ends as was done in the earlier study.

A computer code was written that incorporated the continuous-time economic models and the four different stochastic failure models. This code was used to investigate the sensitivity of various assumptions about the spare-component pool to the benefits afforded by the pool. Results from the program were also compared to results obtained in the earlier study. From this investigation, it was found that for a system of five or more plants, consideration of the possibility of the spare

failing while used as a temporary substitute had little effect on the benefits of a spare component pool. Of greater importance was the choice of model to describe the spare availability and the duration that the spare is to be used as a temporary substitute. For long spare pool lifetimes, constant asymptotic failure models gave excellent results; however, for short pool lifetimes it is important to use a detailed description of spare availability.