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ACCELERATED LIFE TESTING AND RELIABILITY PREDICTION

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INTRODUCTION

During the last two decades, the trend towards automation and precision products in industry and military defense systems has led to ever-increasing complexities in equipment design and the consequent difficulty in the prediction of equipment reliability. The over-all reliability of a complex system with no redundant parts is the product of the reliability of its components, as given by the formula

$$R_{over\text{-}all} = \prod_{i=1}^{n} R_i \qquad (1)$$

where R_i is the reliability of the i-th component. The equation indicates that each individual component reduces the over-all reliability of such a system by its own reliability factor. A simple system consisting of, say, 100 components, each having a 99 per cent reliability, will have an over-all reliability of only 36.5 per cent. A more complex system consisting of 400 components, with 99 per cent individual component reliability, would have an over-all reliability of only 3 per cent.

However, a modern commercial airliner, with all its complexities, is known to be more reliable than the above discussion would suggest. This is so because only a few of its components, most of them structural, are really vital in the sense that failure of any one of them will cause a total loss of the aircraft. Many other components, particularly the electronic components, are not vital since even in the event of their failure, the aircraft can still be operated safely. On the other hand, each component of the guided missile system is vital since failure of any one of them will cause the complete missile system to fail. Hence, the components used in a missile must be made perhaps four orders of magnitude (or ten thousand times) more reliable than commercial components. In other words, the achievement of near absolute reliability of components used in missile would be a desirable goal to be specified in their design and manufacture. In order to specify that a component has a reliability of 99.99 per cent per 1000 hours, with 99 per cent confidence, it would be necessary to run a life test of a representative sample for more than 46,000,000 unit-hours until the first failure occurred.

It is clearly seen that in the case of life testing under simulated normal operating conditions, a decrease in the duration of the tests is accompanied by a corresponding increase in the number of components tested. However, it is possible to cause a small sample to fail in a short length of time by increasing the environmental stress level to many times that under normal operating conditions. Thus, the failure-rate distribution curves under accelerated life-testing conditions are obtained with considerable saving in time and cost.

The question then arises whether it is possible to predict the reliability of the component under normal operating conditions from the results of accelerated life-tests. In those cases where it is possible to establish a statistical correlation between the failure-rate distributions under the two different sets of environmental conditions, the component reliability under normal operating conditions can be predicted from the accelerated life-test data. The results of accelerated life-tests must be applied with great care since the accelerated conditions are far removed from the normal operating conditions. Despite the drawbacks encountered, accelerated life-testing is becoming increasingly important since it alone offers a solution to the dilemma of testing highly reliable components used in military defense systems.

A complex missile system costs millions of dollars, and hence, the simulated and accelerated life-testing of complete missiles is not monetarily feasible. Besides, a missile, once fired, cannot be retrieved and hence, in the event that a missile fails to hit its target, the causes of failure cannot be analyzed and corrective procedures applied. This then sets up a new problem which is the prediction of reliability of a complex system from the data on accelerated life-testing of its components.

An analytical treatment of various reliability models for a complex system, namely, the exponential model, the Weibull model, the Markovian model and the worst-case design model, is presented. It is shown that each of these models can explain certain aspects of system reliability, but because of the various assumptions involved, none of these can full explain the anomalies sometimes observed when the reliability predictions are subjected to operational verification. The reliability of a simple amplifier circuit is predicted using the exponential model which assumes the system failure-rate is the sum of the individual component failure-rates, provided the latter remain constant over the useful life of the system.

Finally, the technique of correlating the accelerated failure-rate data to the failure-rate under normal operating conditions is discussed, wherein it is assumed that the failure modes of the system are the same for various stress level combinations of different environments. A more general correlation technique for correlating the data in case of a non-exponential behavior of failure-rate with environment is also presented. The latter technique can be applied even in cases where the condition of same failure modes under the two sets of environmental conditions may not hold, provided the acceleration curve begins to take up its general shape within the region of accelerated environmental stress levels.

DEFINITIONS

Reliability. It is the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered. Adequate performance implies that a prespecified criteria is satisfied by the device under test. The period of time and the operating conditions affect the system performance and they are included as part of reliability specifications.

System. A combination of subsystems which is capable of operation by itself; for example, a guided missile.

Subsystem. A group of components, combined and packaged in one housing to perform a particular set of operations; for example, a radio transmitter, a servo amplifier, etc.

Component. An item, such as a resistor, capacitor, electron tube, gyro, etc. which is not normally subject to further disassembly.

Mean-Time-to-Failure. The ratio of the total test time of a system or a component to the total number of failures. For a life-test experiment in which n components are put on test and the test is terminated after the first r failures, the mean-time-to-failure of the components is

$$\theta = \frac{\sum_{i=1}^{r} t_i + (n - r)t_r}{r} \qquad (2)$$

where t_i is the time counted from the beginning of the test to the i-th failure. Equation (2) assumes that the components that fail are not replaced by fresh components. In case of testing with replacement the mean-time-to-failure of the components is given by

$$\theta = \frac{nt_r}{r} \tag{3}$$

If the testing is discontinued after a certain fixed time T, regardless of the number of components that failed, then

$$\theta = \frac{T}{r_T} \tag{4}$$

where r_T is the total number of components failing in time T.

Failure-Rate. It is the reciprocal of mean-time-to-failure. It is denoted by λ and is often expressed as failures for 1000 hours or as per cent failure-rate per 1000 hours.

$$\lambda = \frac{1}{\theta} \tag{5}$$

Failure-Modes. It is the physical state of the component at the instant of failure. For example, two of the possible failure modes of a resistor are "open" and "short" states. For a capacitor, dielectric breakdown is one of the failure modes.

Catastrophic Failures. These are failures which cause a normally operating system to suddenly become completely inoperative, for example, a random "open" occurring in a wire resistor after several hundred hours of operation. This type of failure is usually caused by chance and therefore cannot be predicted in advance for a particular time and a specific part, without the knowledge of the failure-rate distribution.

Degradation (Drift) Failures. These are the type of failures which occur gradually due to the change of a parameter with time, for example a change in the ohmic value of a resistor may cause excessive hum or frequency shift in a radio receiver.

Wear-Out Failures. These are failures which can be predicted on the basis of a known wearout characteristic, for example, the wear of the brushes of an electric motor.

Secondary Failures. A secondary failure occurs as a result of a primary failure. If a resistor in an electronic circuit is shorted, causing an excessive drain on a tube; the tube failure is considered as secondary failure because it was due to the primary resistor failure.

LIFE-TESTING TECHNIQUES

In recent years, considerable interest has been stimulated about the analysis of life-test data. A life-test is conducted by subjecting one or more identical components to a given set of operating conditions and noting the number of hours of satisfactory performance, that is, the time-to-failure of each component. Failure is defined as the state in which the component no longer performs satisfactorily. The techniques discussed by Zelen (1959) involve the analysis of failure-rate data for a range of environmental conditions. Oftentimes, life-testing procedures involve the acceleration of the failure of components by testing at over-stress environmental conditions. The accelerated life-test data are then extrapolated to normal operating stress levels. The various criteria, which must be satisfied if such extrapolation are to be valid and meaningful, have been outlined.

It is assumed that the time-to-failure follows an exponential distribution having the probability density function, given below

$$f(t) = \begin{cases} \frac{1}{\theta} \exp \left[- \frac{1}{\theta} (t-\gamma) \right] & t \geq \gamma \\ 0 & t < \gamma \end{cases} \tag{6}$$

where θ is the mean-time-to-failure or mean life of the components. The reciprocal of mean life is the mean failure-rate. The above distribution is called a two-parameter exponential distribution in (θ, γ), where the parameter γ has the property that no failures occur before time γ expires.

The discussion is limited to two different environments E_1 and E_2 with p and q stress levels, respectively. There are pq different combinations of environments and for each combination, n components are simultaneously placed on test. The experiments are terminated when exactly r of the components on test have failed. The r failure times, for each treatment combination, are denoted by $t_1 \leq t_2 \leq \ldots \leq t_r$.

A particular treatment combination is denoted by the symbol ij where i and j refer to the stress levels of E_1 and E_2, respectively. The effects of various treatment combinations on the mean-time-to-failures, θ_{ij} of the components, irrespective of the value of γ_{ij} are analyzed. Let θ_{ij} be defined as

$$\theta_{ij} = m p_i q_j k_{ij} \tag{7}$$

where p_i, q_j are positive constants representing the contributions from the stress level i of environment E_1 and the level j of E_2, respectively. The quantity k_{ij} is a constant which depends on both environments, and m is a constant common to all treatment combinations.

If, for all treatment combinations, $p_i q_j k_{ij} = 1$ then, $\theta_{ij} = m$ and θ_{ij} is independent of the stress levels of the environments of E_1 and E_2.

If $p_i k_{ij} = 1$ (or $q_j k_{ij} = 1$) then, $\theta_{ij} = mq_j$ (or $\theta_{ij} = mp_i$) and θ_{ij} depends on the stress levels of E_2 alone (or stress levels of E_1 alone). If $k_{ij} = 1$, $\theta_{ij} = mp_i q_j$ and hence, the effects of environment E_1 can be evaluated independently of the effects of environments E_2 and vice-versa.

Thus, when $\theta_{ij} = mp_i q_j$, it is seen that the effects of the two environments are completely multiplicative. With such a model, accelerated life-testing is feasible because the stress level of any one environment can be chosen unusually high in order to increase the component failure-rate, keeping the stress level of the other environment at the normally operating value.

Consider an experiment conducted over all possible treatment combinations of the stress levels of the environments E_1 and E_2, and let $t_1 \leq t_2 \leq \ldots \leq t_r$ be the time-to-failure of the first r components out of the initial n components put on test. The minimum variance unbiased estimate of θ_{ij} is

$$\hat{\theta}_{ij} = \frac{r(t - t_1) + (n - r)\ (t_r - t_1)}{(r - 1)} \tag{8}$$

where

$$t = \frac{1}{r} \sum_{i=1}^{r} t_i \tag{9}$$

The maximum likelihood estimators of the various parameters m, p_i, q_j and k_{ij} are

$$\left.\begin{aligned}
\hat{m} &= \left(\prod_{i=1}^{p} \prod_{j=1}^{q} \hat{\theta}_{ij}\right)^{1/pq} \\
\hat{p}_i &= \frac{1}{\hat{m}} \left(\prod_{j=1}^{q} \hat{\theta}_{ij}\right)^{1/q} \qquad i = 1,2,3 \ldots \\
\hat{q}_j &= \frac{1}{\hat{m}} \left(\prod_{i=1}^{p} \hat{\theta}_{ij}\right)^{1/p} \qquad j = 1,2,3 \ldots \\
\hat{k}_{ij} &= \frac{\hat{\theta}_{ij}}{\hat{m}\, \hat{p}_i\, \hat{q}_i}
\end{aligned}\right\} \quad (10)$$

Thus, if it is desired to predict the value of $\hat{\theta}_{ij}$ for $i = 3$ and $j = 2$, say; it is required to evaluate

$$\hat{m}\, \hat{p}_3\, \hat{q}_2\, \hat{k}_{32}$$

This should agree with the observed value of θ_{32}, except for rounding errors.

In some applications, where the parameter γ is taken to be equal to zero, the minimum variance unbiased estimate of θ_{ij} is modified to

$$\hat{\theta}_{ij} = \frac{\bar{t} -(n-r)t_r}{r} \tag{11}$$

The above techniques for predicting θ_{ij} were based on the assumption that the failure times have an exponential distribution. This leads to the question as to what procedures should be used if the underlying distribution of time-to-failure is other than exponential. Fortunately, all results for the exponential distribution can also be used for distributions having the probability density function of the form

$$f(t) = \begin{cases} h(t) \exp\left(-\int_o^t h(x)\, dx\right) & t \geq o \\ o & t < o \end{cases} \tag{12}$$

The function $h(t)$ is a non-negative function of the time-to-failure t. The exponential and Weibull distributions are special cases of the general family of distributions described by (12). If then, $H(t)$ follows an exponential distribution with mean-time-to-failure equal to θ. It is always possible to transform the time-to-failure to a quantity $H(t)$ which will follow an exponential distribution. Hence, all formulae based upon the exponential assumption can be used for the transformed variate $H(t)$.

For example, in the case of random failures, the probability of failure is independent of the time that the component has been performing satisfactorily. In such a case, the quantity $h(t)$, the probability that the component will fail in the next interval t and t +dt is independent of t, that is

$$h(t) = \frac{1}{\theta} \tag{14}$$

and hence

$$f(t) = \frac{1}{\theta} \ \exp\left(- \frac{t}{\theta} \right) \tag{15}$$

the exponential probability density function.

For the case of wearout failures, $h(t)$ is an increasing function of time, say

$$h(t) = \frac{\beta t^{\beta-1}}{\theta} \tag{16}$$

This yields the Weibull probability density function for which

$$H(t) = t^{\beta} \tag{17}$$

ACCELERATED LIFE-TESTING TECHNIQUES

It is shown, in Appendix I, that in order to predict with 99 per cent confidence that a component under test has a reliability of 99.99 per cent per 1000 hours, it is necessary to run a life-test of a statistically representative sample for more than 40,000,000 unit-hours until the first failure occurs. The present day life-testing techniques although theoretically feasible are not economical, and therefore, accelerated life-testing is resorted to. This section covers accelerated life-testing techniques and the reliability prediction at normal operating stresses from accelerated life-test data.

Accelerated life-testing is a misnomer because it is the component failure-rate, and not its life, which is accelerated. The environmental conditions, in such tests, are more severe than those under normal conditions. Such conditions are achieved by increasing the ambient temperature, the applied voltage, etc. or combinations of these factors. This invariably reduces the duration of the life test by causing early failures of the components on test.

The following three methods represent the commonly used techniques in the field of accelerated life-testing:

1. Accelerated life-testing by step-stress aging.
2. Accelerated life-testing by estimating acceleration factors.
3. Accelerated life-testing as a problem of modeling.

Each method yields significant information and each has its limitations.

Step-stress aging. An accelerated life-testing program is used to obtain failure-rate acceleration curves, as shown in Figure 1. For a particular environmental test condition, say S_1, a series of experiments are conducted to obtain component failure-rate data, using a fresh sample from the same statistical population each time. The failure-rate distribution curve for this particular environmental stress condition is obtained, as shown. The experiments are then repeated for different environmental stress levels, each more severe than the normal operating environmental stress level. The mean of the failure-rate distribution is a function of the environmental stress level and an acceleration curve is drawn through these points. If the failure-rate distributions have similar distributions at each stress level, it indicates that the failure modes for the components are the same for the different

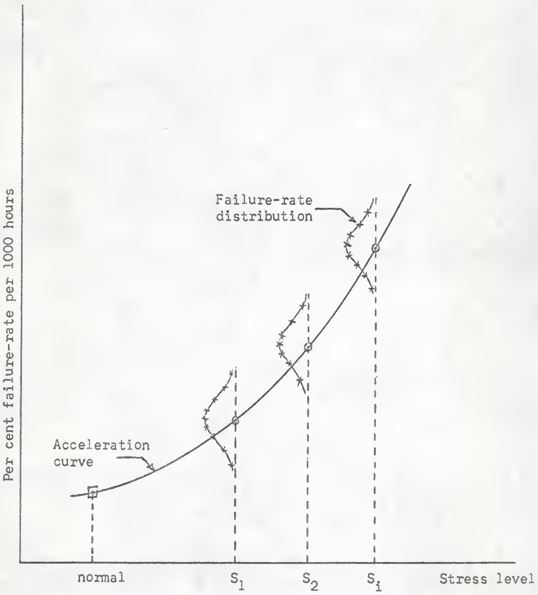

Figure 1: Accelerated failure-rate curve for various stress levels

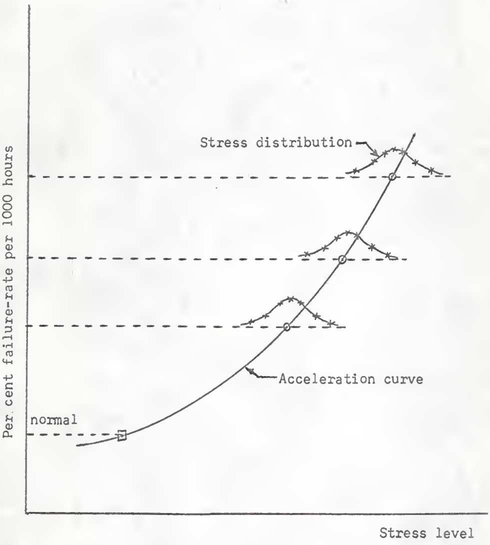

Figure 2: Accelerated stress curve for various failure-rates

environmental stress levels. In such a case, true acceleration has been achieved and it is possible to predict the failure-rate at normally operating stress levels by extrapolation of the accelerating curve.

Another method of obtaining the acceleration curve is to fix the failure-rate of the component and determine the amount of stress necessary to cause component failure, as shown in Figure 2. The above techniques of step-stress aging are particularly suitable for use in early stages of component development to compare small batches of components manufactured by different methods.

Estimating Acceleration Factors. This method requires the knowledge of analytical relationship between the number of components and their time-to-failures. For example, in case of the capacitors, the ratio of the mean lives of two samples of capacitors is inversely proportional to the p-th power of the applied voltages, all other environmental factors remaining the same; that is,

$$\frac{\theta_1}{\theta_2} = \left(\frac{v_2}{v_1}\right)^p \tag{18}$$

where θ_1 and θ_2 are mean lives of the two batches of capacitors when the applied voltages across their terminals are v_1 and v_2, respectively. The exponent p is called the acceleration factor for the applied voltage.

Accelerated testing is conducted on fresh batches of capacitors belonging to the same statistical population at different applied voltages. The mean life for a particular applied voltage can be estimated as

$$\hat{\theta} = \frac{t_1 + t_2 + \dots + t_r + (n-r)t_r}{r} \tag{19}$$

where $\hat{\theta}$ is the estimated mean life

t_i is the time-to-failure of the i-th component

r is the number of failures after which the experiment is terminated, and

n is the number of components initially on test.

The negative slope of the log-log plot of mean life versus applied voltage is the estimate of p. It is then possible to evaluate the mean life of the capacitor at rated applied voltage, with the use of equation (18).

The accuracy of this technique depends on the number of failures and on the assumption that the modes of component failure remain the same under different stress levels of the combination of environments.

A Problem of Modeling. It is assumed that the principles involved in the prediction of performance characteristics of a component from that of its model can also be applied to the prediction of component reliability from the tests associated with its model. The basic assumption in the use of accelerated testing is that the probability of failure is a function of test duration and stress. In this section, the measure of time is not the total number of operating hours, but instead, the actual time spent under high stress levels which cause signification degradation of the system.

The probability of failure of a component is assumed to be dependent only on the cumulative effects of high stress levels over the entire test duration. Suppose that the ratio, A, of the time for which the component is subjected to excessive stresses to the total test time, is known. If the characteristic time T_o be defined as the number of operating hours required for the component to accumulate 1 hour of overstress degradation, then

$$T_o = \frac{1}{A} \tag{20}$$

By increasing the abuse ratio to A' for the model, the characteristic time T_o' for the model can be decreased. For a good analysis, the abuse ratio A' for the model should be unity, that is, the model is subjected to excessive stresses throughout the test duration. If T' is the total test time for the model, then, the corresponding time T for the component is

$$T = \left(\frac{T_o}{T_o'}\right) T' \quad = \quad \left(\frac{A'}{A}\right) \; T' \tag{21}$$

where $\frac{A'}{A}$ is called the time-scale factor. Equation (21) is valid only in the case of catastrophic failures.

Suppose that for a particular test conducted, the abuse ratio is found to be 0.001, that is, 1 hour of abuse is accumulated for every 1000 hours of operation. Further, suppose that 10 models are operated simultaneously, each at an abuse ratio of 0.9. The time-scale factor $\frac{A'}{A} = 900$, so that 1 hour of model operation is equivalent to 900 hours of component operation. If, for the models, the first failure occurs after 1000 hours of operation, this corresponds to 1 failure out of 10 components, operating at normal environmental conditions, at the end of 9×10^5 hours.

The preceding three methods of accelerated life-testing require that the tests should be relatively simple and inexpensive and that true acceleration must be achieved. The last requirement is somewhat difficult to achieve as the accelerated testing conditions are far removed from the operating conditions resulting in different modes of failure for the component. However, if the results are applied with care, accelerated life-testing techniques present the only feasible way out of the dilemma of reliability prediction of a complex system.

RELIABILITY MODELS OF A COMPLEX SYSTEM

A complex system, such as a missile control system, consists of thousands of components and costs millions of dollars and so it is not economically feasible to conduct any type of life tests, simulated or accelerated, on the complete system. In order to predict total system performance, various reliability models are used which employ life-test data on individual components comprising the system. In this section, four such reliability models, namely, the exponential model, the Weibull model, the Markovian model and the worst-case design model, are discussed and it is shown that each of these models, while explaining certain aspects of system reliability, cannot full explain the anamolies observed when reliability predictions are subjected to operational verification.

The Exponential Model. It assumes that the failure-rate distribution of any component can be approximated by three different segments, as shown in Figure 3. The early failure region, or "infant mortality" region, in which the component has a decreasing failure rate, followed by a long low stretch of chance failure region, in which the failure-rate is more or less constant, and then, the wear-out region during which the failure-rate increases.

The magnitude and length of the constant failure-rate region, also called the normally operating region, depends upon the different environmental stresses. Consider a component having a constant failure-rate, λ, for a particular combi-

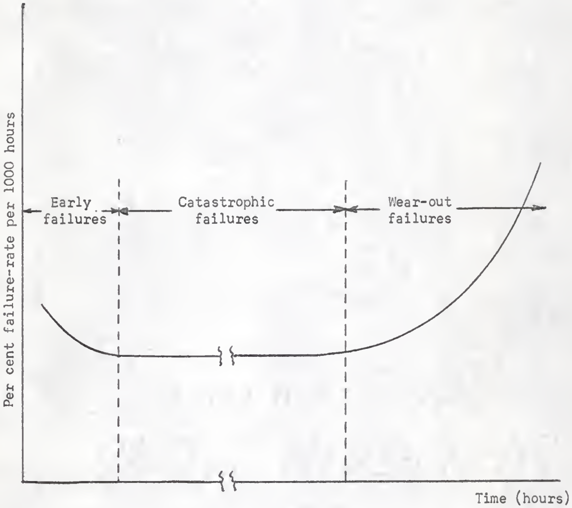

Figure 3: Failure-rate versus test duration for the exponential model

nation of environmental stresses, being subjected to life tests. In a small interval of time $(t, t + \Delta t)$, the probability of failure is $\lambda \Delta t$, independent of previous history and time because of the assumption of constant failure-rate. If $R(t)$ is the probability that the component survives t time units, that is, the reliability of the component, then

$$R(t + \Delta t) = R(t)\ (1 - \lambda\Delta t) + o(\Delta t) \tag{22}$$

which yields

$$\frac{R(t + \Delta t) - R(t)}{\Delta t} = -\lambda R(t) \tag{23}$$

and in the limit, the solution of the resulting differential equation is

$$R(t) = \exp(-\lambda t) \tag{24}$$

If T is the random variable denoting time to failure of the component, then

$$\Pr\ (T > t) = R(t) = \exp\ (-\lambda t) = 1\text{-}F(t) \tag{25}$$

where $F(t) = 1\text{-}\exp(-\lambda t)$ is the exponential distribution characterizing the constant failure rate, λ.

If it is assumed that the time-to-failure distributions for individual components are exponential, the best estimate of system failure-rate is obtained by adding the component failure-rates, and then

$$R_s(t) = \prod_{i=1}^{n} R_i(t) = \exp\left[-(\lambda_1+\lambda_2+ \ldots+\lambda_n)t \right] = \exp(-\lambda_s t) \tag{26}$$

where $R_s(t)$ is the system reliability

$R_i(t)$ is the reliability of the i-th component

λ_i is the failure-rate of the i-th component

λ_s is the failure-rate of the system

and n is the number of components in the system

Epstein (1960) enumerates two procedures for estimation from life test data when the underlying probability density function is exponential. In the first case, life-testing is discontinued after a fixed number, say r, of the components have failed. In the second case, the testing is discontinued after a fixed amount of total life T has elapsed. In both cases, the components under test may or may not be replaced in case of failure.

Equation (25) gives the exponential distribution, characterizing the constant failure-rate, as

$$F(t) = 1 - \exp(-\lambda t) \tag{27}$$

The probability density function for the above distribution is

$$f(t,\lambda) = \lambda \exp(-\lambda t) \qquad t > o,\ \lambda > o \tag{28}$$

which in terms of the mean life, θ, is

$$f(t;\theta) = \frac{1}{\theta} \exp\left(-\frac{t}{\theta}\right) \qquad t > o,\ \theta > o \tag{29}$$

Case I: If the number of components initially on test is n and testing is discontinued after r items fail, then the best estimate, $\theta_{r,n}$, of the mean life, θ, is

$$\theta_{r,n} = \frac{T_{r,n}}{r} \tag{30}$$

where $T_{r,n}$ is the accumulated test time until the r-th failure occurs. If the components were not replaced, in case of failure, then

$$T_{r,n} = \sum_{i=1}^{r} t_i + (n-r)t_r \tag{31}$$

where t_i $(\leq t_{i+1})$ is the observed time for the i-th failure. If the components were replaced in case of a failure, then

$$T_{r,n} = nt_r \tag{32}$$

The probability density function of $\theta_{r,n}$, in either the replacement or non-replacement case, is

$$f_r(y) = \begin{cases} \frac{1}{(r-1)!}\left(\frac{r}{\theta}\right)^r y^{r-1} \exp(-ry/\theta) & y > o \\ o & \text{elsewhere} \end{cases} \tag{33}$$

From quation (33), it follows that the quantity

$$\frac{2r\,\hat{\theta}_{r,n}}{\theta} = \frac{2T_{r,n}}{\theta} \tag{34}$$

is distributed as chi-square with 2r degrees of freedom, and hence the two-sided $100(1-\alpha)$ per cent confidence interval for θ is

$$\frac{2T_{r,n}}{\chi^2_{\frac{\alpha}{2}}(2r)} < \theta < \frac{2T_{r,n}}{\chi^2_{1-\frac{\alpha}{2}}(2r)} \tag{35}$$

A one-sided $100(1-\alpha)$ per cent confidence interval for θ is

$$\theta > \frac{2T_r}{\chi^2_{\alpha}(2r)} \tag{36}$$

It is also interesting to estimate another quantity, t_p, where t_p is that life such that

$$\Pr(T > t_p) = p \tag{37}$$

For the exponential distribution

$$t_p = \theta \ln(1/p) \tag{38}$$

and the maximum likelihood estimator of t_p is

$$\hat{t}_p = \hat{\theta}_{r,n} \ln(1/p) \tag{39}$$

The $100(1-\alpha)$ per cent confidence intervals for t_p are

$$\frac{2T_{r,n}\ln(1/p)}{\chi^2_{\frac{\alpha}{2}}(2r)} < t_p < \frac{2T_{r,n}\ln(1/p)}{\chi^2_{1-\frac{\alpha}{2}}(2r)} \tag{40}$$

for the two-sided case, and

$$t_p > \frac{2T_{r,n}\ln(1/p)}{\chi^2_{\alpha}(2r)} \tag{41}$$

for the one-sided case.

Case II: The number of components initially on test is n and the test is discontinued after a fixed amount of total life T has elapsed. If r be the number of items which fail in the interval (o,T), then, the two-sided $100(1-\alpha)$ per cent confidence interval for θ is

$$\frac{2T}{\chi^2_{\frac{\alpha}{2}}(2r+2)} < \theta < \frac{2T}{\chi^2_{1-\frac{\alpha}{2}}(2r+2)} \tag{42}$$

The one-sided $100(1-\alpha)$ per cent confidence interval for θ is

$$\theta > \frac{2T}{\chi^2_{\alpha}(2r+2)} \tag{43}$$

The one-sided 100(1-α) per cent confidence interval for t_p is

$$t_p > \frac{2T \ln (1/p)}{\chi^2_\alpha (2r+2)} \tag{44}$$

Zelen (1961) discusses some of the pitfalls associated with the use of the exponential model in life test. However, the exponential model has come to have a special significance in system reliability studies because of the ease and simplicity with which the prediction is achieved.

The Weibull Model. In the development of the exponential model, it was stated that the failure-rate of mechanical and electrical components is distributed over time as shown in Figure 3. In practice, if a component or a system is unreliable in its first hours of life, it will be even less reliable in its hundredth or thousandth hour. The failure-rate curve of Figure 4 indicates that the failure-rate is a monotonically increasing function with time and that the failure-rate does not remain constant over any appreciable duration of time.

If the failure-rate is a function of the test duration or the operational age of the component, then, equation (23), in the limit, becomes

$$\frac{d}{dt} [R(t)] = -\lambda(t) R (t)dt \tag{45}$$

the solution of which is

$$R(t) = \exp [-H(t)] \tag{46}$$

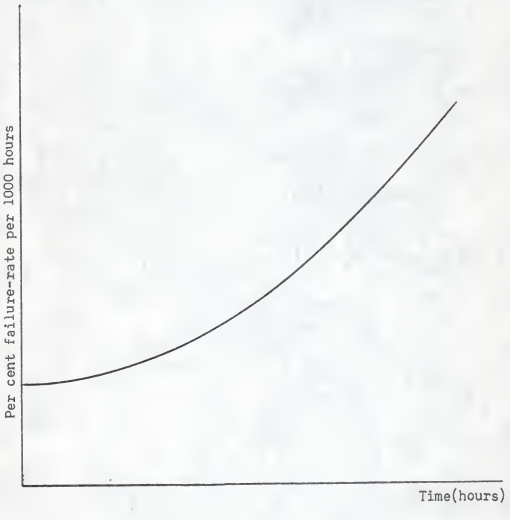

Figure 4: Failure-rate versus test duration for the Weibull model

where

$$H(t) = \int_o^t \lambda(\tau)d\tau \qquad (47)$$

The idea of a non-constant failure-rate function is associated with Weibull who showed that

$$H(t) = (kt)^\beta \qquad (48)$$

where k and β are positive real constants. Various values of these parameters result in distributions representing different types of failure phenomena. For example: if β is equal to 1, the resulting probability of survival, R(t), exhibits properties typical of random failure. If β is greater than 1, then, R(t) exhibits properties typical of wear-out failures.

A simple Weibull cumulative distribution function is defined as

$$F(t) = 1 - \exp\left[-(t-\gamma)^\beta/\alpha\right] \qquad t \geq \gamma,\ \alpha > o,\ \beta > o \qquad (49)$$

where α is the scale parameter

β is the shape parameter

γ is the location parameter

The Weibull probability density function is, then,

$$f(t) = \frac{\beta(t-\gamma)^{\beta-1}}{\alpha} \exp\left[-(t-\gamma)^\beta/\alpha\right] \qquad (50)$$

Electronic equipment failures can be classified as catastrophic or sudden failures and wear-out or delayed failures. For the catastrophic failures, a Weibull distribution with location parameter equal to zero and shape parameter less than one is an appropriate mathematical model. In case of wear-out failures, very few failures of this nature occur before a certain period has elapsed after which the failure-rate increases rapidly with time. In view of these considerations, a Weibull distribution with some positive location parameter and shape parameter greater than one would be a reasonable model. Kao (1959) has set up another useful model which is a combination of the above two models, thereby taking into account both the mechanical strength of the item and its wear-out quality.

The Weibull model for catastrophic failure is

$$F_1(t) = 1 - \exp\left(-t^{\beta_1}/\alpha_1\right) \qquad t > o,\ \alpha_1 > o,\ o < \beta_1 < 1 \tag{51}$$

The Weibull model for wear-out failures is

$$F_2(t) = 1 - \exp\left[-(t-\gamma_2)^{\beta_2}/\alpha_2\right] \quad t > \gamma_2,\ \alpha_2 > o,\ \beta_2 > 1 \tag{52}$$

Then, the combined Weibull model is

$$\begin{aligned} F(t) &= pF_1(t) + qF_2(t) \\ &= 1 - p\exp\left(-t^{\beta_1}/\alpha_1\right) - q\exp\left[-(t-\gamma_2)^{\beta_2}/\alpha_2\right] \end{aligned} \tag{53}$$

where p and q are the proportions of catastrophic and wear-out failures, respectively; and $p + q = 1$.

A graphical method, as presented by Kao (1959), for estimating the various parameters from life tests data is discussed below:

The experimental cumulative density function of the failure data is plotted on the Weibull probability paper* and a Weibull plot is fitted to this data, as shown in Figure 5. The tanget lines, drawn at each end of the Weibull plot, give the estimates $\widehat{pF_1}$ and $\widehat{qF_2}$ of $pF_1(t)$ and $qF_2(t)$, respectively. The intersection of $\widehat{qF_2}$ with the time scale gives $\hat{\gamma}_2$ which estimates γ_2. From the intersection of $\widehat{qF_2}$ with the upper borderline of the graph paper, drop a vertical line whose intersection with $\widehat{pF_1}$ as read on the per cent failure scale gives $\hat{p}$ which estimates p and hence, q(= 1-p). From these estimates of p and q, the combined Weibull population can be separated into the two sub-populations. The estimated number of components that belong to $F_1(t)$ and $F_2(t)$ are $n\hat{p}$ and $n\hat{q}$, respectively; and hence, the cumulative distribution functions $F_1(t)$ and $F_2(t)$ can be calculated. The y-intercept and the slope of the Weibull plots of the separated sub-populations $F_i(t)$ yield the estimated of $\ln\alpha_i$ and β_i, respectively (i = 1,2).

The combined Weibull model can be approximated by a composite model, in case of small p and large γ_2. The Weibull plot, then, consists of two distinct linear portions which are treated as two sections of a composite population, as shown.

$$F_3(t) = 1 - \exp\left(-t^{\beta_3}/\alpha_3\right) \qquad t \le \delta,\ \alpha_3 \ge o,\ o < \beta_3 < 1 \quad (54)$$

*Weibull probability paper contains ln versus ln-ln scales and other scales calibrated for life-testing use. It is available from Cornell University, Ithaca, New York.

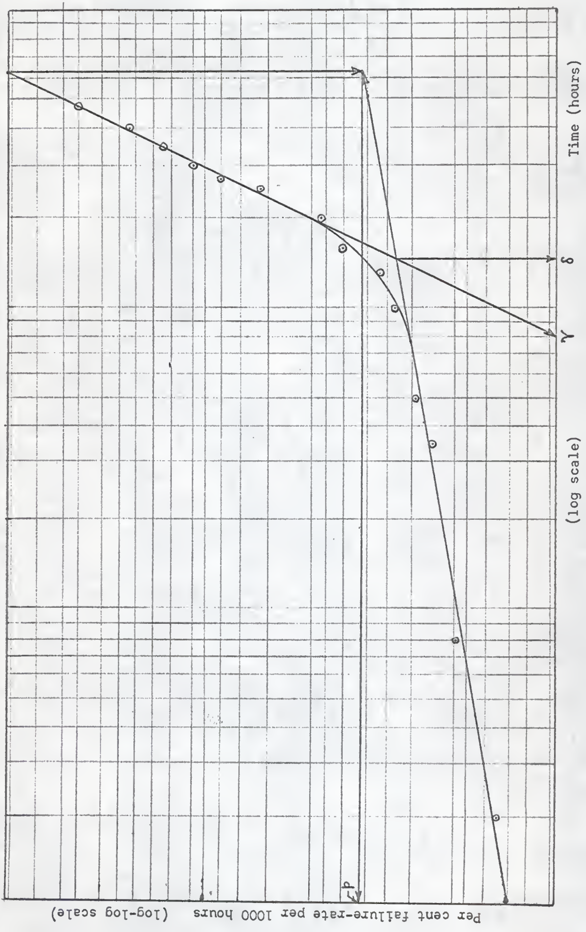

Figure 5: The Weibull plot of failure-rate versus test duration

$$F_4(t) = 1 - \exp\,(-t^{\beta_4}/\alpha_4) \qquad t \geq \delta,\ \alpha_4 > o,\ \beta_4 > 1 \tag{55}$$

where δ is the time at which the proportion of catastrophic failures is equal to that of wear-out failures, that is

$$1 - \exp\,(-\delta^{\beta_3}/\alpha_3) = 1 - \exp\,(-\delta^{\beta_4}/\alpha_4) \tag{56}$$

and hence

$$\delta = \left(\frac{\alpha_4}{\alpha_3}\right)^{1/(\beta_4 - \beta_3)} = \exp\left(\frac{\ln\alpha_4 - \ln\alpha_3}{\beta_4 - \beta_3}\right) \tag{57}$$

The estimation of the parameters in the composite model case is simplified, as discussed below:

The experimental cumulative density function is plotted on a Weibull probability paper and two straight lines are approximated among the points. The abscissa of the intersection of these two lines gives the estimate of δ. The y-intercept and slope of the lines yield the estimates of $\ln\alpha_i$ and β_i, respectively $(i = 3,4)$.

The combined Weibull model technique can be extended to the prediction of failure-rate of a complex system comprising of different components. The Weibull cumulative density function for the complex system is

$$F(t) = \sum_{i=1}^{n} p_i\, F_i\,(t) \qquad o \leq p_i \leq 1 \tag{58}$$

where

$$F_i(t) = 1 - \exp\left[- (t-\gamma_i)^{\beta_i}/\alpha_i\right] \tag{59}$$

is the i-th component cumulative density function form and p_i is the proportion of the i-th component in the system, so that $\sum_{i=1}^{n} p_i = 1$.

The Weibull probability density function for the system is

$$f(t) = \sum_{i=1}^{n} p_i f_i (t) \tag{60}$$

with the i-th component probability density function

$$f_i(t) = \frac{\beta_i (t-\gamma_i)^{\beta_i - 1}}{\alpha_i} \exp\left[-(t-\gamma_i)^{\beta_i}/\alpha_i\right] \tag{61}$$

The reliable life, θ_r, for any specified r, such that

$$r = \int_{\theta_r}^{\infty} f(t)\, dt \tag{62}$$

is given by

$$\theta_r = \gamma + \alpha^{1/\beta} (-\ln r)^{1/\beta} \tag{63}$$

The Markovian Model. The model developed by Tainiter (1963) takes into account failures occurring due to drifting of the individual circuit components from their nominal values and also failures due to the catastrophic failure of any single component. This model assumes that the state of a parameter of a component at some future time, given its state at the present time, is independent of the present age of the component, the number of previous states and the duration of these previous states. It is also assumed that a change in the value of a parameter of one component does not change the environment in which the remaining components are operating.

The state of the parameter of a component, at some given time of observation, is the number assigned to the set which contains the value of the parameter. It is assumed that the set of states of the system, which are formed from the component states, can be partitioned into operative and failure sets so that a known functional relationship exists between the component values and the output parameters of the system.

It is assumed that a transition of the system from one state to another is merely a transition of a single component parameter from one state to another and that the probability that two component parameters change state in a given time interval, Δt, is the order of $(\Delta t)^2$. From the theory of continuous-time Markov processes (Rosenblatt, 1962) it can be shown that

$$P(t) = \exp (\lambda t) \tag{64}$$

where $P(t)$ is a matrix whose elements $P_{ij}(t)$ are defined as the probability that the system is in state i at time zero and state j at time t; λ is a matrix whose elements λ_{ij} are defined so that $\lambda_{ij}\Delta t + o(\Delta t)$, $(i \neq j)$, is the probability that a system in state i at time zero enters state j by time Δt with $\lambda_{ii} = \sum_{i \neq j} \lambda_{ij}$. The matrices P and λ can be obtained from the matrices P_c and λ_c for the component parameters of the system.

Brender and Tainiter (1961) have derived a formula for computing the reliability of a system; namely

$$\bar{R}\ (t) = \exp(\lambda_t\ t).\ \bar{I} \tag{65}$$

where R(t) is a vector whose elements $R_i(t)$ are defined as the probability that the system has not entered the failure set of states in the time interval (o,t) given that the system was in state i at time zero. λ_t is obtained from λ by deleting those rows and columns of λ which correspond to failure states for the system. $\bar{I}$ is a unit column vector of conformable dimensions. If $\bar{P}(o)$ is the initial probability vector for the system, then, its elements $P_i(o)$ are the probabilities that the system is in state i initially, i ranging only over the operable states. The inner product of $\bar{P}(o)$ with $\bar{R}(t)$ will yield the unconditional system reliability.

Several problems become apparent when it is attempted to utilize this model to predict the reliability of an actual system. It is necessary to define as to what constitutes system failure. It must be possible to obtain life-test data for the time variations of each of the significant component parameters of the system measured at equal time intervals. The reliability prediction for the system is based upon the joint effect of the time variations of each component of the system as observed in life-test measurements of the behaviour of the component. Hence, the failure boundaries of the circuit must be determined in terms of the values of the component parameter.

The number of system states, and therefore, the numbers of states for each component parameter, must be kept as small as possible and it must be decided with a reasonably small error whether or not the system is operable, if the component parameters take any value within the ranges of the states.

The transition probabilities for each component parameter are estimated from the definition of component parameter states. Let n_{ij} denote the number of transitions a component parameter makes from state i at a given time point to state j at the next time point. Let n_i be the total number of transitions made by a component parameter out of state i. Then, the maximum likelihood estimates $\hat{P}_{ij}$ of P_{ij} are

$$\hat{P}_{ij} = \frac{n_{ij}}{n} \tag{66}$$

The matrix λ can be obtained from the knowledge that a transition from system state i to system state j is represented by a change of state for only a single component parameter. If the change in system from state i to j corresponds to a component transition k to l, then

$$\hat{\lambda}_{ij} = \hat{\lambda}_{kl}\ (y) \tag{67}$$

where $\hat{\lambda}_{kl}$ (y) is the k-l entry of the matrix λ_c (y) for the component y. Thus, the system matrix λ will have a large predetermined number of zeros, which also simplifies the reliability calculation. The estimate, $\hat{\lambda}_c$, of the component matrix λ_c is obtained from the estimated matrix $\hat{P}_c$ for the component from the following equation

$$\hat{\lambda}_c = \log \hat{P}_c \tag{68}$$

Equation (68) is useful only when $\log \hat{P}_c$ converges; for which P_{ij} must be greater than 0.5, for all i. This condition seems to be adequate, because for a reliable design, the probability that the component does not change state is greater than one-half. Having obtained $\hat{\lambda}_c$ and thus $\hat{\lambda}$, the system reliability iscomputed with the use of equation (65).

The Markovian model provides an analytic method of computing system reliability from component drift and failure data. The other models for computing system reliability define reliability as the probability of failure at a given time. The Markovian model goes a step further in considering reliability as the probability of failure in an interval of time. The model is not useful for highly complex systems for which the failure characteristics are described by equations whose solutions require an exorbitant amount of time even on a high speed computer.

The Worst-Case Design Model (Combs; 1963). A system is said to be worst-case designed if and only if it performs adequately with the component parameters assuming values at every combination of their combination limits. If the output, y, of a system be a function of its input and its component parameters, then

$$y = f(x_1, x_2, \ldots, x_n) \tag{69}$$

where x_i represents either an input or a component parameter. If $(x_i)_{max}$

and $(x_i)_{min}$ are the maximum and minimum tolerance limits, respectively, for the component x_i, the 2^n possible values of y from equation (69) also have a maximum and a minimum value. If

$$y_{max} \leq \text{maximum acceptable output}$$

and

$$y_{min} \geq \text{minimum acceptable output}$$

then, the system is worst-case designed.

Let λ be the failure-rate of a component having a cumulative failure distribution function of the form $1- \exp(-\lambda t)$. If p and (1-p) be the proportions of drift failure and catastrophic failures, respectively, then

$$\begin{aligned} p\lambda &= \text{drift failure-rate} \\ (1-p)\lambda &= \text{catastrophic failure-rate} \end{aligned} \tag{70}$$

If

$$\begin{aligned} &F_D(t) = 1 - \exp(-p\lambda t) \\ \text{and}\quad &F_c(t) = 1 - \exp[-(1-p)\lambda t] \end{aligned} \tag{71}$$

be the drift and catastrophic failure distributions, respectively, and if the drift and catastrophic failures be assumed to be independent events, then the component failure distribution is

$$F(t) = F_D(t) + F_c(t) - F_D(t)F_c(t)$$

$$= \left[1-e^{-p\lambda t}\right] + \left[1-e^{-(1-p)\lambda t}\right] - \left[1-e^{-p\lambda t}\right]\left[1-e^{-(1-p)\lambda t}\right] \quad (72)$$

For a system with n components, the failure distribution of the system is

$$F_n(t) = \prod_{i=1}^{n} \left[1-e^{-p_i\lambda_i t}\right] + \left[1- \prod_{i=1}^{n} e^{-(1-p_i)\lambda_i t}\right] -$$

$$\left[\prod_{i=1}^{n} (1-e^{-p_i\lambda_i t})\right] \left[1-\prod_{i=1}^{n} e^{-(1-p_i)\lambda_i t}\right] \quad (73)$$

Consider a component with the highest failure-rate λ_m for this system. If the system is assumed as being made up of n of these components, then, the failure distribution for a worst-case designed system is

$$F_n(t) = e^{-n(1-p)\lambda_m t}\left(1-e^{-p\lambda_m t}\right)^n - e^{-n(1-p)\lambda_m t} + 1 \quad (74)$$

$$= 1 + \sum_{k=1}^{n} (-1)^k \binom{n}{k} e^{-\alpha_k \lambda_m t} \quad (75)$$

where $\alpha_k = kp + n(1-p)$ and $\binom{n}{k}$ are binomial coefficients.

The probability density function for the system failure is

$$f_n(t) = \sum_{k=1}^{n} \left[\binom{n}{k} \quad (-1)^k \, e^{-\alpha_k \lambda_m t} \quad (-\alpha_k \lambda_m) \right] \tag{76}$$

The mean-time-to-failure for the system is

$$\theta = \int_o^\infty t \; f_n(t)dt = \sum_{k=1}^{n} (-1)^{k+1} \binom{n}{k} \frac{1}{\alpha_k \lambda_m} \tag{77}$$

For the special case of p=o, that is, no drift failures

$$\theta = \frac{1}{n\lambda_m} \tag{78}$$

and for p=1, that is, no catastrophic failures

$$\theta = \frac{1}{\lambda_m} \sum_{k=1}^{n} \frac{1}{k} \tag{79}$$

Thus, the simplified case of the worst-case design model yields equations (78) and (79) which appear frequently in the reliability literature as the equations of mean-time-to-failure of a series and parallel-redundant system of n components, respectively.

CORRELATION TECHNIQUES

In this section, two analytical treatments are discussed which indicate the existence of a definite correlation between the mean-time-to-failure under normal operating conditions and that under accelerated environmental conditions. The results of these two treatments prove the validity of our assumptions that true acceleration must be achieved and that the acceleration factor must not be too large so as to introduce a change in the mode of failure.

The analytical treatment is restricted to the combination of only two types of environments, E_1 and E_2, say. Let $E_{1,i}$ and $E_{2,j}$ represent the i-th and j-th stress levels of the environments E_1 and E_2, respectively. Let $\theta_{i,j}$ denote the mean-time-to-failure of a system under a particular combination of the i-th stress level of E_1 and the j-th stress level of E_2.

If it is assumed that the mean-time-to-failure is a negative exponential function of the environmental stress level, then

$$\theta_{i,j} = a_j \exp(-b_j E_{1,i}) \qquad i = 1, 2, \ldots \tag{80}$$

for a particular value of $E_{2,j}$ ($j = 1, 2, \ldots$). The constants a_j and b_j are the intercept on the mean-time-to-failure axis and the slope, respectively, of the semi-logarithmic plot of the mean-time-to-failure curves. A set of these curves can be obtained by varying the stress level of the environment E_1 over its entire range, for a particular level of E_2; and then repeating this for all other values of E_2.

For two different values of j, as shown in Figure 6, the correlation coefficient between the two mean-time-to-failure curves is

$$\rho(j, j') = \frac{1}{\delta E} \int_{E_{1,i}}^{E_{1,i'}} (a_j e^{-b_j E_{1,i}}) \left[a_{j'} e^{-b_{j'}(E_{1,i} + \delta E)}\right] dE_{1,i} \tag{81}$$

where

$$\delta E = \int_{E_{1,i}}^{E_{1,i'}} dE_{1,i} = E_{1,i'} - E_{1,i} \tag{82}$$

is the separation parameter which is related to the acceleration factor. Equation (81) yields

$$\rho(j, j', \delta E) = K(j, j') \frac{e^{-b_{j'}\delta E}}{\delta E} \tag{83}$$

The graph of normalized correlation coefficient versus the separation parameter, see Figure 7, indicates that for reasonable correlation to exist, the separation parameter and therefore, the acceleration factor should not be too large.

If the mean-time-to-failure is not a simple negative exponential function of the environmental stress level, as shown in Figure 8, it can be divided into different sections, such that, each section has its own characteristic exponential behaviour, that is

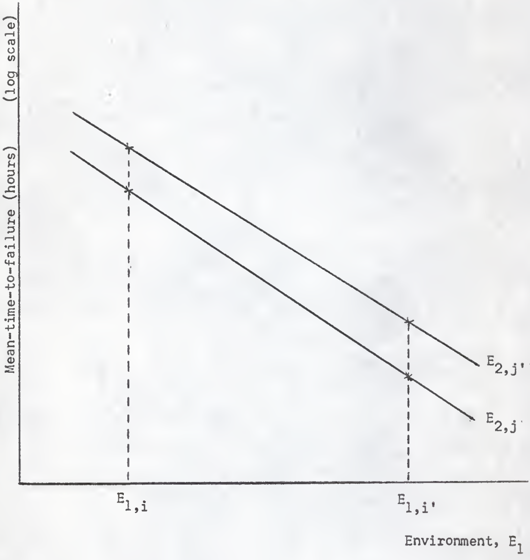

Figure 6: Mean-time-to-failure versus environment

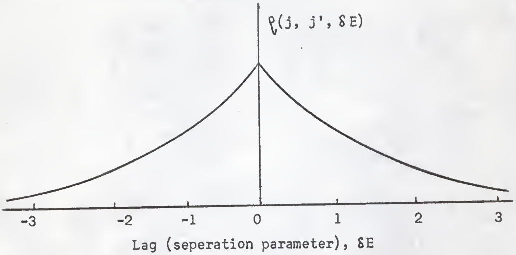

Figure 7: Auto-correlation function versus lag

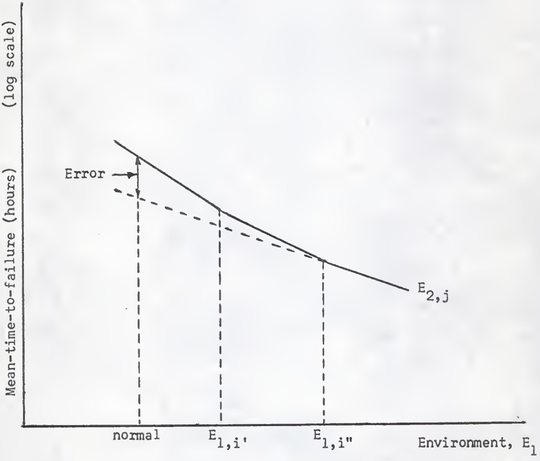

Figure 8: Semi-logarithmic plot of mean-time-to-failure

$$\theta_{ij} = \begin{cases} a_j \exp(-b_j E_{1,i}) & o \le i \le i' \\ a'_j \exp(-b'_j E_{1,i}) & i' \le i \le i'' \\ a''_j \exp(-b''_j E_{1,i}) & i \ge i'' \end{cases} \qquad (84)$$

for a particular $E_{2,j}$. However, in order to achieve any useful prediction of the system mean life under normal environmental conditions from accelerated life-test data, it is essential that the points of discontinuities of the slope of the curve be known. If the accelerated life-tests yield data in the region beyond the stress level, $E_{1,i''}$, then, the extrapolation of the curve assuming the same exponential variation with environment would yield an error in the prediction of mean-time-to-failure under normal environmental conditions, as shown in Figure 8.

The anomalies brought out in the above discussion could be irradicated if the variation of the failure-rate of the system with environmental stress level is assumed to be of the form

$$\lambda_{i,j} = 1 - \exp\left[-(E_{1,i})^{\beta_j}/\alpha_j\right] \qquad i = 1, 2, 3, \ldots \qquad (85)$$

for a particular $E_{2,j}$ ($j = 1, 2, \ldots$). Equation (85) is similar to the Weibull probability density function and the parameters α_j and β_j can be estimated in the same way as discussed in the section on the Weibull model.

This technique yields good results in cases where the acceleration curve is smooth and well defined over a range of environmental stress levels and no sharp discontinuities appear at or around the stress level at which the failure-rate is to be predicted.

It has been discussed in the sections on the life-testing techniques and the exponential model that the estimation of system failure-rate is subject to variation within the confidence interval (dotted lines), in Figure 9. The width of these confidence interval depends on the degree of confidence placed in the estimation. A $1-\alpha$ per cent confidence interval, $CI_{1-\alpha}$, implies that the true percentage of the observations lie within the confidence interval states, subject to α per cent error.

The system failure-rate for a set of environmental conditions is, say, normally distributed about its mean value. The $1-\alpha$ per cent confidence interval is then obtained by the use of statistical procedures. The confidence interval for the associated correlation of the data under two different sets of environmental conditions may also be similarly calculated. The latter part of this section is devoted to the development of the correlation coefficient between two random variables, namely, the failure-rates under different sets of environmental conditions.

The degree of variation in the failure-rate of the system corresponding to a given stress level of environment E_1, measures the closeness of the correlation. In the present discussion, it is assumed that for the correlation between the system failure-rate and the stress levels of the environment E_1, E_2 can be maintained constant at its j-th stress level. Usually there is no means of varying the environment E_1 quickly, while E_2 remains constant, in order to observe the effects upon the system failure-rate. In the latter case, the problem is one of multiple or partial correlation. However, the present discussion deals only with the correlation of two variables, λ_s and

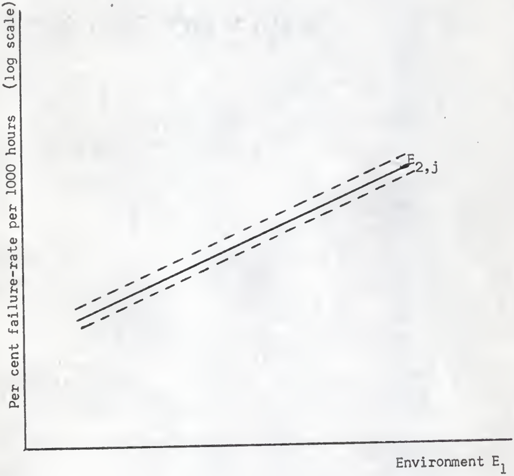

Figure 9: Confidence limits on failure-rate prediction

E_1. Let

$$\lambda_s = a_j \exp(b_j E_{1,i}) \qquad i = 1, 2, \ldots \tag{86}$$

for a particular $E_{2,j}$ ($j = 1, 2, \ldots\ldots$). Hence

$$\ln \lambda_s = \ln a_j + b_j E_{1,i} \tag{87}$$

Replacing $\ln \lambda_s$ by Y and $E_{1,i}$ by X

$$Y = \ln a_j + b_j X \tag{88}$$

Using Pearson's formula (Chaddock, 1925) the coefficient of correlation is

$$\rho = \frac{\Sigma xy}{N\sigma_X\sigma_Y} \tag{89}$$

where x and y are the deviations of each X and Y from their respective means, σ_X and σ_Y are the standard deviations of the entire X and Y distributions, respectively, and N is the total number of related pairs of observations.

If a very large number of similar random samples of the same system are examined and related, ρ is shown to be itself a variable. The significance of ρ evidently depends upon the amount of this probable variation which is ue to the uncontrolled conditions of sampling.

$$\Pr\ (\text{error in } \rho) = 0.6745 \quad \frac{1 - \rho^2}{\sqrt{N}} \tag{90}$$

Conservative statistical practice in interpreting ρ requires that the size of the coefficient should be at least four times its probability of error before it becomes indicative of any significant degree of correlation. It has been strongly emphasized that a ρ of low value to be significant must be based upon many more cases than one of high value.

The coefficient of correlation is a pure number indexing the degree of correlation between two variables. It does not indicate how much variation on the average may be expected in one variable when the amount of variation in another related variable is known. However, if the regression equation

$$Y - \overline{Y} = \rho \frac{\sigma_Y}{\sigma_X} (X - \overline{X}) \tag{91}$$

is considered, it gives a more complete description of the relationship than the coefficient of correlation.

The predicted single value of Y, from equation (91), corresponding to a given value of X is subject to a range of error determined by the scatter about the straight line of average relationship, as shown in Figure 10. The less this scatter, the more closely it approaches the condition where there would be only one value of Y for a given value of X. The amount of scatter, S_Y, in the Y variable is given by

$$S_Y = \sigma_Y \sqrt{1 - \rho^2} \tag{92}$$

If the distribution of Y is assumed to be normal, then, in about 68 per cent of the cases, the actual values of Y will not differ more than $\pm S_Y$ from the

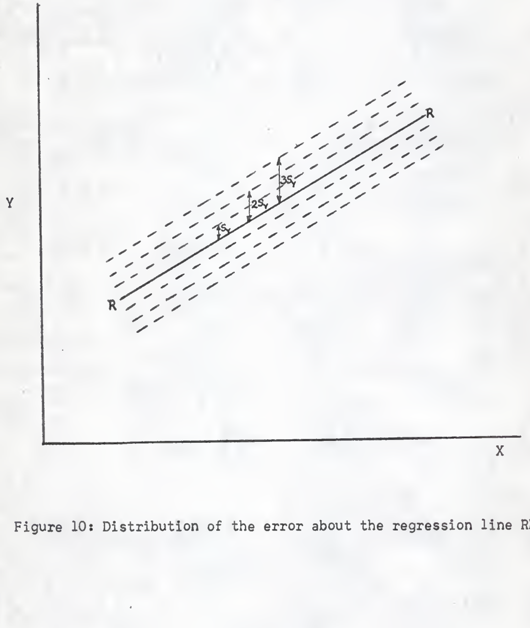

Figure 10: Distribution of the error about the regression line RR

predicted values.

Pearson's product-deviation method of measuring the coefficient of correlation ρ between two variables is based upon the hypothesis that a straight line fits most closely the means of Y and X in a correlation table. But sometimes, the means conform more closely to some other form of curve. When the line of the means is non-linear, a low of value of ρ, as calculated from the straight line fit, does not necessarily indicate that the degree of correlation is really small or that the two variables are unrelated.

For measuring non-linear relationships Pearson has proposed a new constant, η, the correlation ratio.

$$\eta_{YX} = \frac{\sigma \text{ of the means of Y for a particular X}}{\sigma \text{ of the entire Y distribution of all X}}$$

that is

$$\eta_{YX} = \frac{\sqrt{\dfrac{\Sigma n_X(\bar{Y}_X - \bar{Y})^2}{N}}}{\sigma_Y} \tag{93}$$

where n_X is the total frequencies for a single value of X

$\bar{Y}_X$ is the mean of Y corresponding to a particular value of X

$\bar{Y}$ is the mean of the entire Y distribution of all X

N is the total number of related items in table.

and σ_Y is the standard deviation of the entire Y distribution.

The correlation ratio is a measure of the degree of correlation applicable to both linear and non-linear relationships. It is limited in that it is not applicable to ungrouped data and in that it does not enable one to estimate values of the related variable from known values of the given variable, as in the case of regression equations of the straight lines. Its value lies in givirgan index of maximum correlation. It furnishes a means of detecting divergence from linear relationship, and its use prevents errors in conclusions due to the wrong assumption.

RELIABILITY PREDICTIONS FOR A SIMPLE AMPLIFIER

Consider a simple amplifier circuit, as shown in Figure 11. The failure-rates of the resistors, capacitors and tubes over a range of ambient temperature and for a particular stress ratio of applied to rated wattage are obtained from Military Standardization Handbook (MIL-HDBK-217). With the help of techniques discussed in the handbook, the component failure-rates are calculated and listed as λ_R, λ_c and λ_T for the resistor, capacitor and tube, respectively, in Table I.

Assuming that the failure-rate of each omponent remains constant over the operating life of the amplifier, the system (complete amplifier circuit) failure-rate, using the exponential model is obtained as the sum of the component failure-rates, that is

$$\lambda_s = 3\lambda_R + 2\lambda_c + \lambda_T \tag{94}$$

neglecting the failure-rates of the power source and connector leads. A semi-logarithmic plot of the system failure-rate versus temperature does not yield a straight line over the entire range of temperature. Hence, any effort to

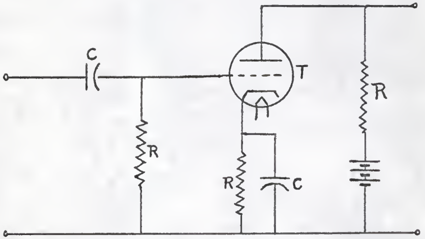

Figure 11: A simple amplifier circuit

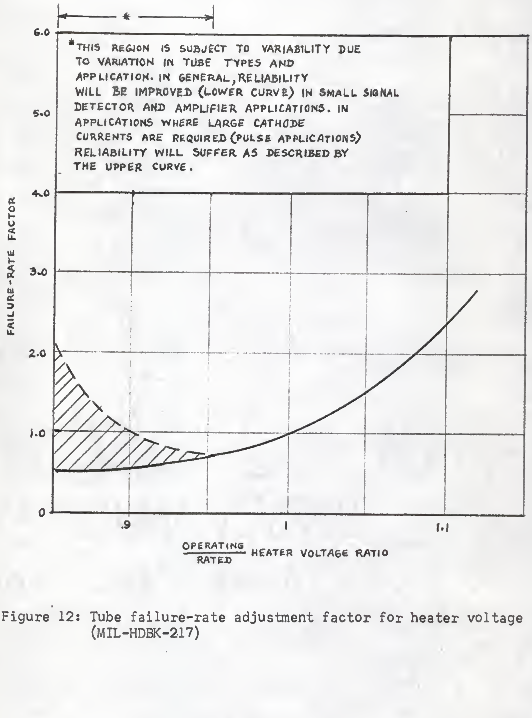

Figure 12: Tube failure-rate adjustment factor for heater voltage (MIL-HDBK-217)

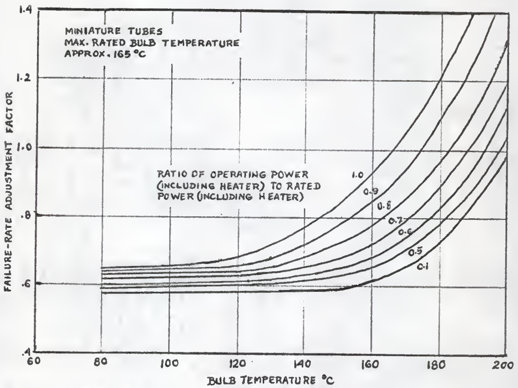

Figure 13: Adjustment factor for temperature/dissipation effects (MIL-HDBK-217)

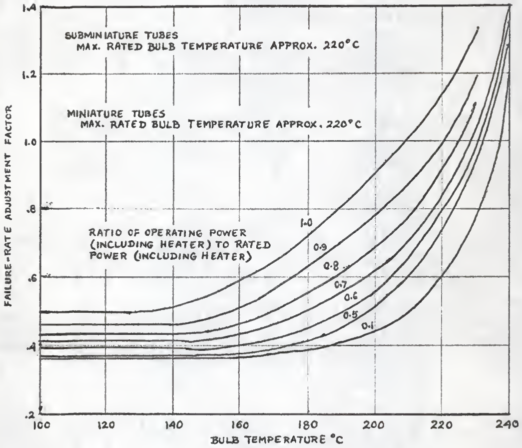

Figure 14: Adjustment factor for temperature/dissipation effects (MIL-HDBK-217)

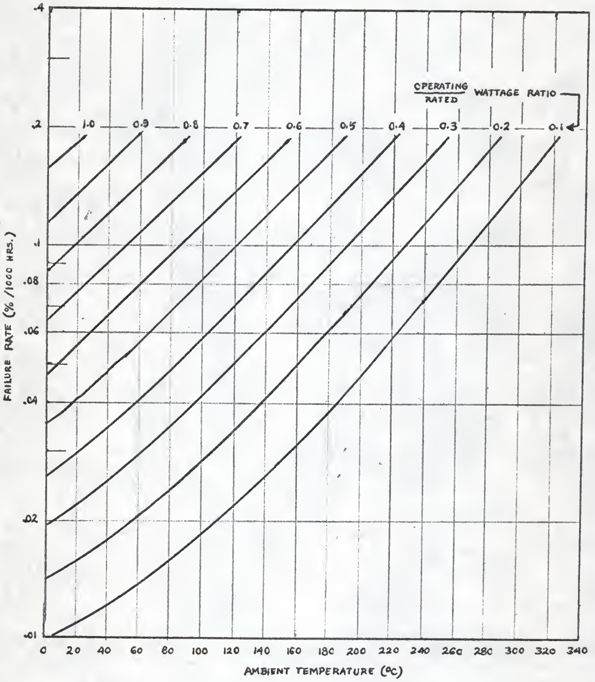

Figure 15: Failure-rates for MIL-R-26C quality power wirewound resistors, temperature range V and Y (relative humidity less than 60%) (MIL-HDBK-217)

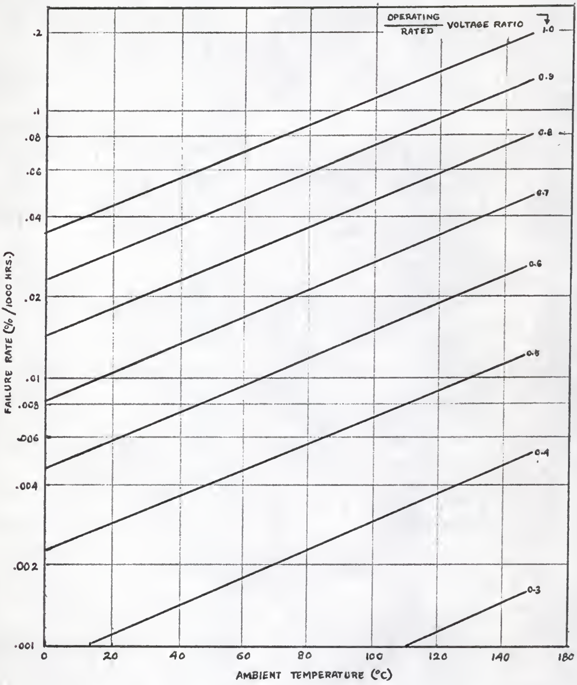

Figure 16: Failure-rates for MIL-C-5B quality mica capacitor (molded type), temperature range P (relative humidity less than 60%) (MIL-HDBK-217)

Table 1. System failure-rate for the amplifier circuit.

Temperature ^{o}C	ratio of applied to rated wattage = 0.5			
	λ_R	λ_c	λ_T	λ_s
10	0.038	0.041	0.130	0.326
20	0.041	0.044	0.130	0.341
30	0.045	0.048	0.130	0.361
40	0.048	0.053	0.130	0.380
50	0.053	0.058	0.130	0.405
60	0.057	0.064	0.130	0.429
70	0.063	0.071	0.130	0.461
80	0.068	0.079	0.130	0.492
90	0.075	0.088	0.130	0.539
100	0.082	0.099	0.130	0.574
110	0.090	0.112	0.130	0.624
120	0.099	0.126	0.130	0.679
130	0.108	0.145	0.132	0.746
140	0.119	0.165	0.134	0.821
150	0.131	0.190	0.136	0.909

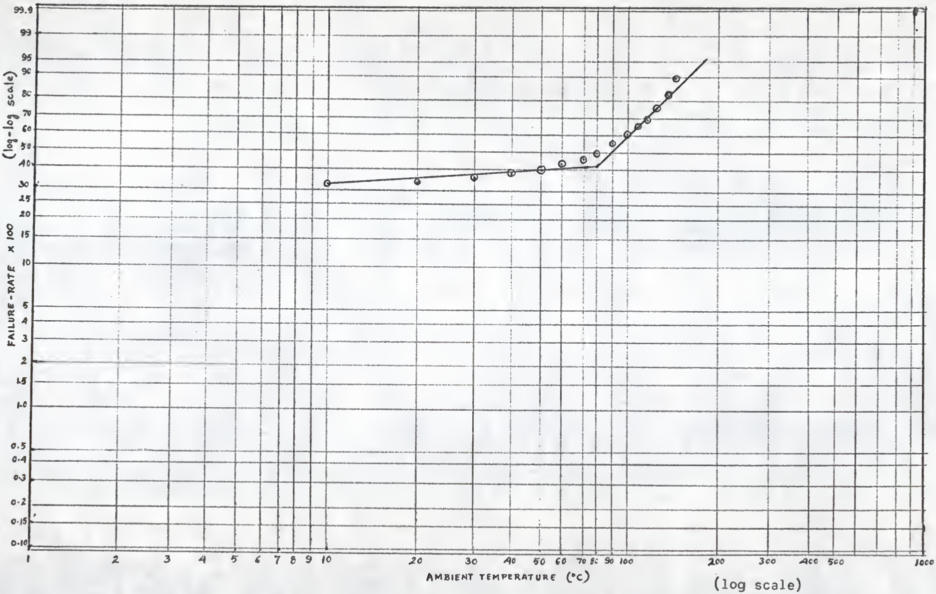

Figure 17: Variation of failure-rate with ambient temperature of the simple amplifier

predict the system failure-rate at normal temperature, of say 30° C, from a range failure-rate data over a range of accelerated temperatures, say 100° C to 150° C by means of this semi-logarithmic plot would involve considerable error, similar to that indicated in Figure 8.

However, if the system failure-rate is plotted versus temperature on a Weibull probability paper it yields two distinct straight lines over the entire temperature range. The application of Berrettoni's (1964) technique for evaluating the constants α_j and β_j, using the characteristics of the Weibull paper yields

$$\lambda_{i,j} = \begin{cases} 1 - \exp\left(-E_{1,i}{}^{0.15} / 3.67\right) & E_{1,i} \leq 83^{\circ}\ \mathrm{C} \\ 1 - \exp\left(-E_{1,i}{}^{2.05} / 6 \times 10^{4}\right) & E_{1,i} \geq 83^{\circ}\ \mathrm{C} \end{cases} \qquad (95)$$

for a particular $E_{2,j}$ (= 0.5). (Refer to Figure 17).

Using equation (95) to predict the system failure-rate at a temperature of 30° C, a value of 0.364 failures per 1000 hours is obtained, which agrees satisfactorily with the observed system failure-rate of 0.361 failures per 1000 hours from Table I.

CONCLUSIONS

The rapid developments in military and space systems of today, and the stringent need for adhering to specifications provide only one feasible way of predicting the reliability of these systems and that is, through accelerated life-testing. The testing of a component under severe environmental conditions accelerates its failure-rate and enables prediction in a short period of time from a small statistical population, thereby enabling considerable saving in time and cost.

The accelerated failure-rate curves for the system are obtained from failure-rate curves of its components by means of various reliability models. Among the four models discussed, the exponential and the Weibull models afford simplicity in the evaluation of the system failure-rate. The Markovian model yields very satisfactory results provided the probability matrix of component parameter drift from one state to another is known. The worst-case design model is a mere extension of the exponential model.

The correlation techniques which are discussed enable the evaluation of system failure-rate under normal accelerated environmental conditions. These techniques yield satisfactory correlation provided the accelerated environmental conditions are within the limits of system specifications, and provided the failure modes of the system remain same over all sets of environmental combinations.

The limited data available in military specification handbook have been used to evaluate the failure-rate of an amplifier circuit. The failure-rate under normal environmental conditions as predicted from the failure-rate data under accelerated environment are seen to be in close agreement with the observed data.

APPENDIX I

Let θ be the mean life of the component, in hours;

T be the mean life estimated from r failures in a sample of n tested;

and λ be the failure-rate, in per cent per 1000 hours.

The required reliability is 99.99 per cent per 1000 hours, in other words, the failure-rate is 0.0001 per 1000 hours, so that

$$\lambda = \frac{0.0001}{1000} \quad 10^{-7} \text{ failures per hour}$$

and the mean life is

$$\theta = \frac{1}{\lambda} = 10^{7} \text{ hours}$$

If the confidence coefficient is

$$1 - \alpha = 0.99$$

that is

$$\alpha = 0.01$$

then, for a two-tailed chisquare prediction and r = 1

$$\frac{2r}{\chi^2_{\frac{\alpha}{2}}} = \frac{2}{\chi^2_{\frac{0.01}{2}}} = \frac{2}{9.21}$$

and, hence, for 99 per cent confidence

$$T = \frac{10^{7} \times 9.21}{2} \approx 4.6 \times 10^{7} \text{ hours}$$

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ACCELERATED LIFE TESTING AND RELIABILITY PREDICTION

by

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ABSTRACT

Accelerated life-testing provides the only economically feasible way of predicting the reliability of a complex missile control system comprising of thousands of components, wherein each component must be highly reliable. If the accelerated failure-rate curves of the individual components are known, then, the exponential reliability model can be used to obtain the system failure-rate as

$$\lambda_s = \sum_{i=1}^{n} \lambda_i \tag{1}$$

where λ_i is the failure-rate of the i-th component which is assumed to be constant over the useful operating life of the system. The reliability of the complex system is, then

$$R_s\ (t) = \exp\ (-\lambda_s t) \tag{2}$$

For a system subject to various combined stress levels of two environments, the correlation between failure-rate data under accelerated and under normal operating environmental conditions is obtained by assuming that

$$\theta_{i,j} = a_j \exp\ (-b_j E_{1,i}) \qquad i = 1,2,\ldots, k_1 \tag{3}$$

for a particular $E_{2,j}$ ($j = 1,2,\ldots, k_2$). $\theta_{i,j}$ is the mean-time-to-failure (reciprocal of the failure-rate) of the system under a combination of the i-th stress level of environment E_1 and the j-th stress level of environment E_2.

This correlation technique assumes that a_j and b_j are constant for the entire range of the stress levels of the environment, E_1; or in other words, the failure modes of the system are the same under combined environmental stress levels ($E_{1,i}$, $E_{2,j}$) for a given j and any i within the specified range of E_1. In general, the quantities a_j and b_j are functions of the combination environmental stress levels and represent the intercept on the mean-time-to-failure axis and the slope, respectively, of the mean-time-to-failure versus environmental stress level when plotted on a semilog paper. If the points of discontinuities of the slope of such a plot are not known, a considerable error will be introduced in the prediction of system failure-rate under normal operating conditions from accelerated life-test data.

A more definite prediction can be achieved if it is assumed that

$$\lambda_{i,j} = 1 - \exp\left\{ -(E_{1,i})^{\beta_j} / \alpha_j \right\} \qquad i = 1,2,\ldots, k_1 \qquad (4)$$

for a particular $E_{2,j}$ ($j = 1,2,\ldots, k_2$), where α_j and β_j are the scale and shape parameters, respectively, and can be obtained graphically. The failure-rate under normal operation conditions can, then, be evaluated from equation (4).