IMPROVING STUDENT ATTITUDES:
A STUDY OF A MATHEMATICS CURRICULUM INNOVATION

by

KARENA M. CURTIS

B.S., Pittsburg State University, 1992

M.S., Pittsburg State University, 1995

AN ABSTRACT OF A DISSERTATION

submitted in partial fulfillment of the requirements for the degree

DOCTOR OF PHILOSOPHY

Department of Curriculum and Instruction
College of Education

KANSAS STATE UNIVERSITY
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ABSTRACT

The purpose of this study was to assess the impact of student attitudes in a college algebra mathematics classroom when lessons are primarily composed of standards-based pedagogy. National reports advocate for a change in teaching K-12. Nowhere is this more needed than in community colleges where students are taught in traditional formats and rarely challenged to make connections between mathematics and their personal experiences. A thorough review of the literature shows the need for mathematics reform at every level, including the college mathematics classroom. There are several national reports, *Principles and Standards for School Mathematics*, *Adding it Up*, *How People Learn*, and *Undergraduate Programs and Courses in the Mathematical Sciences*, that have been published to address the need to change mathematics teaching and learning. They are advocates for the implementation of standards-based instruction into the mathematics classroom.

This study focused on students' perceptions about the nature of mathematics and learning mathematics, specifically, does such a learning environment impact students' perceptions of being a student of mathematics in the areas of confidence, anxiety, enjoyment, and motivation, and relevance of mathematics in personal and professional experiences. Over the course of one semester, two sections of college algebra students participated in the study. By using both qualitative and quantitative data collection methods, the study was able to see if there was an impact in student attitudes toward mathematics. The standards-based pedagogy used in this study was cooperative learning, problem solving, discourse, and the graphing calculator. Changes in attitude were determined by attitudinal surveys, student questionnaires, observations, and focus groups.

College algebra students had a statistically significant change in their enjoyment of mathematics. Although the other attitudes, confidence, motivation, and value did not have a statistically significant change, the qualitative data indicates a change in these attitudes did occur. This study identified that cooperative learning, problem-solving, discourse, and graphing calculators increased student confidence in doing mathematics because they felt more competent in working problems on exams. Students also found the class enjoyable, anxiety was reduced as students became more familiar with the instructional strategies, and students recognized the value of mathematics for job skills and personal business.

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CHAPTER 1

GENERAL NATURE AND PURPOSE OF THE STUDY

Introduction

Attitudes of students toward mathematics have been included in the literature about reform in math education over the past two decades. Middleton and Spanias (1999) state American children tend to enjoy mathematics in the primary grades but begin to show disinterest in the subject at the secondary level. As students get older, they start to perceive mathematics as a subject for the smart kids and a subject that you either pass or fail. Middleton et al. found that many students will agree that mathematics is important, but the interest in taking mathematic courses decreases as they progress through school. By the time students have reached college, they have already formed conclusions regarding their success in mathematics. Today, less than 1% of the college population has declared mathematics as a major (Gay, Bruening, & Bruce, 2000).

Why do students have such a drastic change in their attitudes toward mathematics? One possible explanation is that as students grow, they become more aware of their instructors' interest and enthusiasm for teaching mathematics. They will be less motivated to learn if they feel that their instructor is not happy teaching and does not enjoy being with them in the classroom (Jackson & Leffingwell, 1999). Instructors who care about students must realize that making a positive environment in which to teach and learn mathematics may reduce performance anxiety and encourage enjoyment in mathematics (Furner & Berman, 2003).

Another possible explanation for the change in student attitudes toward mathematics would be the nature of the classroom. College courses tend to be taught mostly by lecture rather than through the use of activities that encourage student participation. A change in pedagogy is needed to improve student attitudes toward mathematics. Changes at the college level are necessary to increase student's interest in mathematics. If students have good attitudes about learning math, they will be more likely to understand the concepts which will help them develop confidence in their ability to work mathematical operations (Furner & Berman, 2003). Teachers at the college level can promote a positive learning environment in the classroom by helping students develop the belief that math makes sense and showing them that students can be successful at working with mathematics (National Research Council [NRC], 2001). Findings commonly described as negatively affecting students' attitudes are teacher behaviors, an emphasis on correct procedures and answers, difficult content, testing, a lack of comprehension, perceived irrelevance of content, family attitudes, and peer attitudes (Ellsworth & Buss, 2000). More research on standards-based pedagogy at the college level could increase instructor awareness about the effectiveness of the use of activities on student attitudes toward mathematics.

Therefore, in recent years, numerous reports such as *Committee on the Undergraduate Program in Mathematics* (CUPM, 2004), *Adding it Up* (NRC, 2001), *How People Learn* (NRC, 2000), and *Principles and Standards for School Mathematics* (NCTM, 2000) have documented the need for mathematics reform. The research and the national reports address how teachers can motivate students with poor dispositions so that students are successful academically. While these reports support change in the mathematics classroom, the reports also acknowledge the problems teachers may face regarding student attitudes toward mathematics (Brown, Stein, & Forman, 1996; NRC, 2001; Riordan, & Noyce, 2001; Sunal, Hodges, Sunal, & Whitaker, 2001).

> "Most U.S. children enter school eager to learn and with positive attitudes toward mathematics. It is critical that they encounter good mathematics teaching in the early grades. Otherwise, those positive attitudes may turn sour as they come to see themselves as poor learners and mathematics as nonsensical, arbitrary, and impossible to learn except by rote memorization. Such views, once adopted, can be extremely difficult to change" (NRC, 2001, p. 132).

The evidence indicates that traditional curriculum and instructional methods are not serving our students well (Marshall, 2003; Hiebert, 1999). The teachers' role should change from demonstrating procedures to helping students build on their mathematical thinking by engaging them in a variety of problem-solving situations and encouraging them to talk about their mathematical thinking (Fennema et al., 1996).

There should be more of an emphasis on communication in the classroom where the students are encouraged to share their thinking processes and justify their answers out loud or in writing (Furner & Berman, 2003). Educators can use a variety of techniques, such as discussion, problem solving, discourse, and writing, to not only encourage communication, but to also reduce anxiety and increase positive attitudes. In undergraduate math courses, studies have shown that group work and standards-based pedagogies kept student interest and improved student attitudes toward math (Hill, Mabrouk, & Roberts, 2003; Wood & Craft, 2000; Elliott, Oty, Mcarthur, & Clark, 2000). Classrooms should help students build a foundation of skills while they use their 'intellectual abilities' to solve problems and develop positive attitudes toward the subject. Constructivism can help instructors develop this procedure (Brooks, 1990).

Learning based on constructivism uses lessons where exploration often precedes explanation. The teacher creates activities where students explore concepts through the use of technology, manipulatives, discussions, and drawings. When the teacher introduces new concepts, students ‘invent’ new procedures that are built from previous explorations (Brooks, 1990). Constructivism requires students ‘to work constantly, to think constantly, and to observe and reflect’ (Caprio, Powers, Kent, Harriman, Snelling, Harris, & Guy, 1998). Students are more comfortable with making mistakes since the environment encourages them to learn from their failures as well as their successes (Caprio, et al., 1998).

A classroom that incorporates hands-on activities, discussion, and small group work are manifestations of a constructivist learning theory (Crawford & Witte, 1999; Smith, 1999). In constructivist classrooms, teachers use the students’ energy to establish interest, confidence, and the need for mathematics by using manipulatives, problem-solving activities, and laboratory activities (Crawford & Witte, 1999).

Constructivist oriented classrooms can serve as a guide to improve student attitudes toward mathematics. Instead of the teachers telling the information to passive students, the students must be given the chance to ‘learn and do’ for themselves (Brooks, 1990). Students must be given the opportunity to construct accurate mathematical understandings. This approach is important for fostering a positive disposition toward mathematical learning and problem solving. The students will develop the ability to see mathematics as meaningful (Baroody & Ginsburg, 1990).

These opportunities are aligned with the reform that K-12 mathematics is undergoing (NCTM, 2000; NCTM, 1989). However, this reform has not had the impact on higher education. The Committee on the Undergraduate Program in Mathematics [CUPM] addresses the need to revamp course curriculum taken before calculus. College educators need to be aware of the knowledge and beliefs students bring with them because their confidence to work with mathematics shapes their attitudes. Classrooms at the college level need to be active with more discussions and time for reflection (CUPM, 2004). Studies at the college level need to be conducted to support the demands for reform in the college mathematics classroom.

There is a need for more research at the college level with respect to changing student attitudes toward mathematics. Research and national reports have advocated for a change in teaching methods for K-16 grade levels. However, while most literature concentrates on achievement, few studies have been conducted on mathematics reform and the affect on student attitudes at the college level. Emerging evidence that supports math reform shows promise for middle school and high school student attitudes toward mathematics. These studies should also be conducted for postsecondary levels since the challenge to change their way of thinking with regards to mathematics exists. This study is a response to this need. In particular, it addresses the question: How will implementation of standards-based teaching strategies, specifically cooperative learning, discourse, problem solving, and graphing calculators, impact the attitude of college students?

Statement of the Problem

Learning environments in the mathematics classroom must be designed to address negative attitudes toward mathematics. Students should be encouraged to make connections between the mathematics they learn in the classroom and their personal lives.

This can be done through cooperative learning, discourse, problem solving, and graphing calculator use. There is a need for research that investigates the attitudes of students immersed in these standards-based practices.

The purpose of the study was to analyze the change in attitudes of community college students in a college algebra course when instruction was standards-based and reflected pedagogical practices aligned with the constructivist learning theory. The components of the theory this study focuses on were cooperative learning, discourse, problem solving, and graphing calculator use.

Specifically, this research was designed to address the following questions:

1. What is the relationship between the use of standards-based practices and college algebra students' perceptions about the nature of mathematics and learning mathematics? Specifically, does such a learning environment impact students:

   a. attitudes, specifically in the areas of confidence, anxiety, enjoyment, and motivation,

   b. perceived value of mathematics in personal and professional experiences.

2. What specific instructional strategies do students believe most impact their attitudes about mathematics?

3. In what ways does the use of standards-based pedagogical strategies impact college algebra students' engagement in the learning process?

In order to address the questions of this study, the researcher conducted an action research study, using both qualitative and quantitative data collection techniques.

Qualitative studies offer the opportunity to explore reactions in the classroom with depth, while the quantitative analysis provides statistical information. This methodology was used so that the experiences of the students could be captured and described extensively using methods such as focus groups, surveys, and observations.

Significance of the Study

Available research on student attitudes predominantly reflects that of K-12 students. Therefore, research was needed at the college level to investigate the impact of standards-based practices as outlined in the Standards. This study offers new knowledge to the area of math reform at the college level because the principles of learning discussed in the Standards and their successful outcomes could also be applied to adult learners (NRC, 2000). Therefore, instructors at the college level need to design an environment that is more student centered and interactive (Panitz, 1999). Since standards-based teaching strategies are found to positively impact attitudes, it provides a rationale for college professors to incorporate such practices in their classrooms (NCTM, 1990).

Not only does this study address the impact on student attitudes, but it also looks at what strategies are perceived to make a difference. Extensive effort has been exerted in order to assess the use of the four standards-based pedagogies using triangulation for the data analysis. This study verifies the use of the four teaching strategies along with assessing their impact on student attitudes in the mathematics classroom.

Limitations of the Study

The participants were students enrolled in College Algebra courses at Labette Community College located in Southeast Kansas. Making generalizations of this study to other two-year or four-year institutions may be difficult for several reasons. First, the class sizes at community colleges tend to be smaller. Although this is comparable to other community colleges, it is not comparable to a four-year university where College Algebra is taught in lecture halls with many students. Second, the backgrounds and goals of college students are diverse. Community colleges have a larger population of older students who might be taking a class for job improvement rather than younger students seeking a four year degree. Third, the teaching methods used in this research may not be practical for larger classes at the four-year university. Another limitation is the inability to research math courses other than College Algebra at the community college. The majority of math credit hours are generated by students who are enrolled in College Algebra. It is a subject that is offered at more than one time slot, which allows the researcher to observe and survey more classrooms compared to upper level mathematics courses such as Trigonometry and Calculus that have less than 15 students and are only offered once a year. A potential limitation could be that the student population in this course changed due to adding or dropping the course within the first two weeks of the semester.

In preparing surveys and questionnaires, students had complete anonymity. However, it is possible that students worried that the instructor could identify handwriting. They might have felt compelled to write what they believed the researcher wanted to hear instead of their true reactions to the situation. However, most students simply circled the attitude related to the teaching strategy without expanding on why they chose that attitude for the week.

The researcher believes the questionnaires the students completed every other week could have addressed attitudes differently. Students filled out the questionnaires according to the current, or previous, week's tasks. It did not address how they felt as the semester progressed. Although students still wrote about improvement, the questionnaires were not designed to chart progress of student attitudes.

The final limitation is the instructor's rapport with her students. The students found the instructor to be caring, approachable, and interested in student success. The student interactions and their perception of the learning environment could have been impacted by the instructor's compassionate personality.

Definitions of Terms

Attitude - Students can describe their attitude toward a course by expressing like or dislike with regards to the subject. Attitude, for this study, is defined as the confidence, anxiety, value students place on the usefulness of mathematics in their life now and in the future, enjoyment, and motivation (Tapia & Marsh, 2004).

Conceptual Understanding - Refers to an integrated and functional grasp of mathematical ideas.

Constructivist Learning Theory – Students move from experience to knowledge by constructing their own meaning, building new learning from prior knowledge, and developing their learning through active tasks. The students' learning is enhanced through social interaction (Cooperstein & Kovevar-Weidinger, 2004).

Cooperative Learning - Cooperative learning is where students are grouped for instruction, assigned roles in the group, work together on a task, are assessed on their participation, and held accountable for each member having a significant part in the end result.

Discourse - Students are actively engaged in doing mathematics through a process where they realize that mathematics is about questioning, conjecturing, and trial and error (Nickson, 1992).

Standards-based Pedagogy – Inquiry implies involvement that leads to understanding. Involvement in learning implies possessing skills and attitudes that encourage students to solve problems while the students construct new knowledge (Exline, 2005).

Mathematics Reform - The challenge to make students succeed in math education despite diversity in racial and social upbringing. It calls for teachers to apply recommendations outlined in Principles and Standards for School Mathematics (NCTM, 2000).

Productive Disposition - The tendency to make sense of mathematics, to find it useful and worthwhile, to believe that steady effort in learning mathematics is beneficial to one's endeavors, and to see oneself as an effective learner and doer of mathematics (NRC, 2001).

Problem-Solving - Problem solving shifts the focus from memorization of facts and procedures to students investigating material and making conclusions or developing questions.

Standards - Refers to the recommendations by the National Council of Teachers of Mathematics that curriculum should be focused on problem solving, making connections amongst math topics, communicating about mathematics using appropriate language, and equity in learning for all students. These recommendations are outlined in the Principles and Standards for School Mathematics (NCTM, 2000).

Technology – Instructors use learning tools such as the internet, overhead projectors, videos, and graphing calculators to aid in the instruction of mathematics.

Traditional Classroom - Instruction is primarily lectures with little opportunity for student interaction. The instructor encourages memorization and recitation with no time for student reflection.

Summary

Poor dispositions among college students in mathematics classrooms are a common occurrence. It generates negative learning environments and creates hostility toward the subject. Too often students become irritable with themselves because they cannot grasp the concepts being taught. In turn, students refuse to complete homework or ask questions for fear of asking about a problem they think everybody else understands. National reports advocate a change in teaching K-12. Nowhere is this more needed than in community colleges where students are taught in traditional formats and rarely challenged to make connections between mathematics and their personal experiences. Research needed to address how using a standards-based approach, which incorporates cooperative learning, discourse, problem-solving, and technology, in community colleges impacts students' attitudes toward mathematics. Community college students have 13 years of prior experience with mathematics and have well-established beliefs about the value of mathematics. These beliefs are defined to be that math is useless, not worth learning, and difficult. This study investigated whether one community college-level mathematics course, taught with a standards-based pedagogy, could alter community college students' perceptions about mathematics.

CHAPTER 2

Literature Review

Students who are successful in mathematics have a set of attitudes and beliefs that direct their learning. They see mathematics as a meaningful, interesting, and worthwhile subject. These students feel confident in working with mathematics and are motivated to work at becoming better learners (NRC, 2001). Children enter school eager to learn mathematics because they view it as important and they feel that they can learn the material. However, by the time they reach middle school and high school, their level of enjoyment for the subject has fallen drastically (Middleton & Spanias, 1999; McLeod, 1992). They still believe that mathematics is important, but they don’t want to take more math classes. By the time they reach college, students have generally formed stable attributions regarding their successes in mathematics (Middleton & Spanias, 1999).

The decrease in positive feelings about mathematics can be paralleled to a decrease in enrollment of mathematics courses. University faculty consistently discourage unprepared students, or those perceived as lazy, from taking math and science courses. Those students already enrolled in math courses choose to leave because of the atmosphere of intimidation and the obvious discouragement for student participation (Daempfle, 2003/2004).

A major focus of this chapter is the literature on student attitudes in mathematics courses. Constructivism and the reform in math education has resulted in an effort to change mathematics teaching practices in a way that will enhance the learning experience for the student in order to improve their disposition. Disposition is defined as the attitude of students toward mathematics and their beliefs about mathematics (Royster, Harris, &

Schoeps, 1999). The call for reform provides the backdrop for the focus of this study. The focus was to look at a change in traditional teaching methods to those that use curriculum which are influenced by the Standards, developed by the National Council of Teachers of Mathematics [NCTM] to see if student attitudes toward mathematics improved. Several studies at the secondary level can give insight into how different teaching methods can positively contribute to a change in attitude. As this review will document, there is a need for further research at the college level to understand the impact of using curricula aligned with standards-based methods of teaching on college student attitudes. The teaching strategies in this study were identified to be standards-based methods, which include cooperative learning, discourse, problem solving, and the use of graphing calculators.

Constructivism

Creating an ideal learning environment begins with the teacher understanding students as learners. It is not only important for a teacher to have content knowledge, but also to develop awareness of how individual students learn. Teachers must make appropriate choices with regard to pedagogy to provide learning opportunities such that students are able to construct their mathematical knowledge (NRC, 2000).

Teachers can create environments where knowledge is constructed by the student. This environment enables students to build their mathematical knowledge and understanding of the subject. The classroom is defined as a constructivist classroom (Pirie & Kieren, 1992; Capraro, 2001).

A ‘mild’ version of constructivism, originating in the work of Jean Piaget, claims that knowledge is actively constructed by the learner and not passively transmitted by the educator (Boudourides, 1998). The emphasis is placed on the activity of the individual and reflection of the result of the activity. Students use their current knowledge to construct new knowledge. What they know and believe at the moment affects how they interpret new information (NRC, 2000). Students use a process of assimilation and accommodation. Assimilation is where new knowledge is created by building on, or reflecting on knowledge gained previously. Accommodation is when old knowledge beliefs are reshaped to accommodate new experiences (Gadanidis, 1994).

The constructivist learning theory states that students move from experience to knowledge by constructing their own meaning, building new learning from prior knowledge, and developing their learning through active tasks. The students' learning is enhanced through social interaction (Cooperstein & Kovevar-Weidinger, 2004).

> "It is reasonable to expect that when the dialogic function is dominant in classroom discourse, pupils will treat their utterances and those of others as thinking devices. Instead of accepting them as information to be received, encoded, and stored, they will take an active stance toward them by questioning and extending them, by incorporating them into their own external and internal utterances" (Steffe, 1990, p. 492).

A classroom reflecting a constructivist learning theory will establish a learning environment that provides students the opportunity to explore constructive processes where the teacher can understand the reasoning of the student (McCarty & Schwandt, 2000). It only makes sense that to enhance teaching and learning, teachers must consider the presence of others, and how interactions among students can promote learning (Tobin, 2000). Changes occur not only through reflection, but also through speech and communication (Boudourides, 1998). Social interactions are seen as a critical part of knowledge construction because that is where the construction takes place. Students should not be passive absorbers of information, but rather have an active part of acquiring knowledge (Koehler & Grouws, 1992).

Principles and Standards for School Mathematics [PSSM] encourages the constructivist approach which would enhance the students' experience through tasks and activities that enable students to construct their own knowledge through investigation and discourse (Capraro, 2001). Students in this environment should be exposed to hands-on activities, discussion, and group work (Crawford & Witte, 1999; Smith, 1999). Students actively create, interpret, and reorganize knowledge.

Variations are expected, but the goal for instruction can be stated as:

> "An instructor should promote and encourage the development for each individual within his/her class a repertoire for powerful mathematical constructions for posing, constructing, exploring, solving, and justifying mathematical problems and concepts and should seek to develop in students the capacity to reflect on and evaluate the quality of their constructions" (Confrey, 1990, p. 112).

The mathematics constructed in the classroom should have applications that can be adapted to real world scenarios. Knowledge grows when it can be applied to the outer world. If it is not viable in the students' world, then it is abandoned (Matthews, 2000). To test viability, reflection must be a big part of the environment (Tobin, 2000). Ernst von Glasersfeld emphasized the importance of environments where students have an opportunity to connect their everyday experience and practice.

This opportunity for students can occur in a constructivist learning environment where benefits for the students can be found. First, it can be a motivational tool for students. Making sense of their environment drives their curiosity. They are naturally drawn to search out patterns and relationships. If a task captures their attention, they will spend considerable time and effort working at it and reflecting upon it (Baroody &

Ginsburg, 1990). Second, a constructivist learning environment can raise student achievement. According to The International Academy of Education, when teachers are aware of how students construct knowledge, student achievement and understanding are significantly improved (Grouws & Cepulla, 2000). Third, in a constructivist-oriented environment, the opportunity for student contributions exists (McCarty & Schwandt, 2000). This is different from a traditional classroom where teachers lecture with minimal interruptions from the students.

The positive affect of constructivist teaching is seen in a study by Lord (1999) who compared traditional teaching and constructivist teaching in an undergraduate science course. Ninety-one students participated in the research. The groups were separated into traditional teaching methods and constructivist teaching methods. Lord found that the undergraduate students in the constructivist learning environment scored higher on unit exams. More than 80% of the students found the class interesting and enjoyable. He analyzed written statements and found that students felt they had mastered the material better and given them deeper insights. In a different study, Maypole and Davies (2001) offered 24 students in a college history course the opportunity to be a part of the constructivist pedagogy. They used students' prior knowledge to guide the curriculum. To obtain the content students already knew the classes began with discussion and questions about a certain event. During the semester, students wrote in journals, answered open-ended surveys, and participated in group discussions. The study found that students reacted positively to constructivist learning. The journals and surveys reported greater understanding of material and enjoyment for the class.

Designing a classroom that reflects the constructivist learning theory replaces the traditional teaching methods, which ignore students' personal construction of mathematical concepts and procedures. Constructivism aligns nicely with approaches that encourage inquiry and reflection on content and personal experiences (Scheurman, 1998). In constructivist classrooms, students establish interest, confidence, and a need for mathematics (Crawford & Witte, 1999).

National Calls for Change

Constructivism, as well as other factors such as dissatisfaction with student achievement and negative dispositions of the students toward mathematics, has prompted a call for reform in mathematics education. There are several national reports that have been published to address the need to change mathematics teaching and learning. One common theme is a challenge to instructors to utilize more effective methods of teaching to enhance the curriculum. Teachers with high involvement in the learning environment model a respect and interest in mathematics for their students. The following reports articulate ways that mathematics teaching and learning can be more effective.

Principles and Standards for School Mathematics

Principles and Standards for School Mathematics [PSSM] (NCTM, 2000), addresses the importance of students to know mathematics for life, as part of a cultural heritage, in the workplace, and in the scientific and technical communities. PSSM calls attention to the importance of creating a community of learners where students' ideas are the foundation of knowledge (Joyner & Reys, 2000). The need for students to see mathematics as an effective tool in their everyday lives is stressed.

> "We live in a time of extraordinary and accelerating change. New knowledge, tools, and ways of doing and communicating mathematics

> continue to emerge and evolve. Calculators, too expensive for common use in the early eighties, now are not only commonplace and inexpensive but also vastly more powerful. Quantitative information available to limited numbers of people a few years ago is now widely disseminated through popular media outlets.
>
> The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater and will continue to increase" (NCTM, 2000, p. 3).

PSSM argues that the learning environment a teacher establishes is crucial to the development of student knowledge. This learning environment boosts students' confidence in their ability to tackle difficult problems, excites them about discovering things on their own, gives them the desire to try more than one solution method while exploring math, and a willingness to persevere. When students work hard to solve or understand a problem, they feel successful and find themselves drawn to extend their work with mathematics (NCTM, 2000). When designed properly, these environments can positively influence both the way students approach the subject, and the outcomes they achieve. The Teaching Principle says:

> "Teachers establish and nurture an environment conducive to learning mathematics through the decisions they make, the conversations they orchestrate, and the physical setting they create" (NCTM, 2000, p. 18).

In 2002, Green and Gredler wrote that the movement of constructivism has played a large role in the development of *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). When PSSM was written in 2000, many of the examples in the document were drawn from research grounded in constructivist perspectives (Ferrini-Mundy & Martin, 2003). PSSM infers that mathematical programs should develop the following five processes: problem solving, reasoning and proof, communication, connections, and representation (Goldsmith & Mark, 1999; NCTM, 2000). These processes can be developed through learning environments that encourage students to recognize and value their own ability for mathematical thinking. Not only should the students understand the why of mathematical processes, but also the how of mathematics. PSSM recommends that in order to support students' construction and understanding of mathematical concepts, the students should interact with various materials which represent problem situations, work with groups as well as individually, and discuss mathematical ideas (Goldsmith & Mark, 1999). Instructors can address these issues by using cooperative learning, which engages students in group work. Discussions encourage communication among students while graphing calculators afford the opportunity for students to explore mathematical concepts visually. Problem solving shows students the applications of mathematics to their own experiences through exploration, investigation, and verification. These four practices can be used to construct student understanding of mathematical concepts. Cooperative learning, discussions, graphing calculator use, and problem solving are consistently mentioned in research that supports the teaching methods discussed in PSSM.

Adding it Up

While PSSM is focused on curriculum and instruction, and improving teaching methods, another recent document developed by the National Research Council [NRC], *Adding it Up* (NRC, 2001), addresses student learning. The purpose of this report is to answer questions regarding the teaching methods which can be used to improve learning in mathematics. In *Adding it Up*, the researchers identify how students can become mathematically proficient.

Mathematical proficiency should enable students to work with mathematics throughout life and help them successfully continue the study of mathematics. In order to be mathematically proficient on a particular topic, a student would have the following knowledge: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and a productive disposition. Conceptual understanding encourages students to build knowledge from previous experiences. If students have conceptual understanding, they will be able to represent mathematical situations in different ways and know which one is most useful for different situations. It is followed by procedural fluency where students should be able to efficiently and accurately perform basic operations, have knowledge of procedures as well as when to use them, and use the procedures appropriately. Strategic competence refers to the students' ability to form mathematical problems, represent them, and solve them. It is similar to problem solving. When students think logically about the concepts and situations, they are using adaptive reasoning. They should be able to justify their conclusions. The final one is productive disposition, where students should see sense in mathematics and find it useful and worthwhile. Although they are all woven together, productive disposition is a crucial factor.

> "Developing a productive disposition requires frequent opportunities to make sense of mathematics, to recognize the benefits of perseverance, and to experience the rewards of sense making in mathematics…As students build strategic competence in solving nonroutine problems, their attitudes and beliefs about themselves as mathematics learners become more positive. The more mathematical concepts they understand, the more sensible mathematics becomes" (NRC, 2001, p. 131).

This productive disposition can be attained in a learning environment that promotes exploration and discussion while students are engaged in problem solving. It is also beneficial when the problems are of interest to the students because they are more likely to be motivated to participate in the mathematical task (Goldsmith & Mark, 1999).

The ultimate outcome is for the student to make the connection between math used in the classroom and their own personal experiences.

How People Learn

A third influential report is *How People Learn* (NRC, 2000). By reviewing all areas of research on how people learn, the report offers recommendations on what research indicates will make the most productive learners. Learning is enhanced when teachers pay attention to the beliefs students bring to the course, use this knowledge as a place to start, and watch the students as the instruction continues. The report links this type of learning to standards-based practices where students reflect their own assessments and review those of other students. This edition is an expansion of the first volume that focused on research about the science of learning. The second edition connects the science of learning to practice in the classroom. Unlike PSSM and *Adding it Up*, this research is not restricted to mathematics students. However, the research does identify mathematics as one of the key subjects where it would like to see an improvement of student knowledge. The design of the learning environment should address being learner-centered, knowledge-centered, assessment-centered, and community-centered.

Learner-centered refers to being aware of the 'knowledge, skills, attitudes, and beliefs' that students bring with them (NRC, 2000). Teachers in this environment are aware of what students know and can do as well as their interests. There is often discourse as it provides an opportunity for students to share what knowledge they have about the topic. Doing mathematics involves abstracting, inventing, and proving. Cooperative learning provides the activities that encourages this way of learning. The knowledge-centered environment identifies the kinds of activities that will develop students’ understanding of a topic. These activities should support the idea that student learning should lead to a better understanding and transfer of the subject material. In mathematics, the presentation and solving of the problems could be less formal in order to get away from the drill and practice tasks previously used. Assessment-centered classrooms do not limit assessments to pencil and paper. It is an ongoing process that is completed individually and also in groups. Assessment is most helpful when it provides feedback, which allows students to reflect on their response and revise their thinking. The final factor is a community-centered environment. This allows for students to make connections between the subject and personal experiences. It encourages teachers to bring the broader community into the classroom through examples of businesses, states, and the nation. Technology can be used to bring real-world problems to the classroom. Calculators and computer-based technology allow students to explore simulations of real world problems.

Each piece enhances the other; therefore, all four should be a contributing factor to the classroom. Classrooms that use these four factors model constructivism through the social interactions. Teachers are encouraged to use students’ previous knowledge and build a community of learners by incorporating teaching practices such as cooperative learning, discourse, graphing calculator use, and problem solving. The social opportunities offered through standards-based practices increase the motivation to learn (NRC, 2000).

Committee on the Undergraduate Program in Mathematics

Since so many reports focus on K-12 grade levels, The Mathematical

Association of America's Committee on the Under-graduate Program in Mathematics [CUPM] was established to set recommendations for mathematics departments of undergraduate students. CUPM also sees a need to enlighten students on the necessity of mathematics for success in the workplace.

> "At the start of the twenty-first century, the undergraduate study of mathematics can and should be a vital and engaging part of preparation for many careers and for well-informed citizenship" (CUPM, 2004, p. 3).

The committee chose the following six recommendations for mathematics departments. First, departments should understand the student population and evaluate courses and programs; second, develop mathematical thinking and communication skills; third, communicate the breadth and interconnections of the mathematical sciences; fourth, promote interdisciplinary cooperation; fifth, use computer technology to support problem solving and to promote understanding; and sixth, provide faculty support for curricular and instructional improvement. The committee provides insight and suggestions for a successful mathematics curriculum, including understanding the strengths and weaknesses students bring with them to the classroom. It also states that the environment should incorporate activities that develop analytical, critical reasoning, problem solving, and communication skills. For students to feel successful, they encourage the representation of concepts using more than one model. Mathematics classes based primarily on lecture may discourage student interest and their motivation to learn (CUPM, 2004).

There are common themes among these reports that provide a vision for how to teach mathematics effectively. Although the terminology differs among the documents, they all convey the same ideas. First, they are consistent in wanting students to see how mathematics is a part of their personal experiences. "The challenge, therefore, is to provide mathematical experiences that are true to the spirit of mathematics yet also relevant to students' futures in other fields. The question then is not *whether* they need mathematics, but *what* mathematics is needed and in what context" (CUPM, 2004, p. 1). Helping students see the connection in mathematics between the classroom and personal experience can be difficult because media and parents consistently convey the message that success in mathematics is not essential (PSSM, 2000). Therefore, the ideas should be introduced so that the students see the reason for its use (NRC, 2000).

The second common theme combines student motivation with interesting teaching strategies. Not only should the topics be useful, but also by making the mathematics relevant to them, students will stay interested in the topic and become motivated to learn about mathematics (CUPM, 2004). This motivation will come from a change in a positive learning environment created by the teacher. Successful teachers not only expect their students to succeed, but also see themselves as motivational instructors who can teach effectively (NRC, 2001). Effective teachers recognize that the decisions they make shape students' attitudes. They also recognize that they must create ideal settings for learning. Their actions encourage students to think, question, solve problems, and discuss their ideas, strategies, and solutions. If tasks are chosen appropriately, students' curiosity can be heightened and they will be drawn into mathematics.

> "Students learn mathematics through the experiences that teachers provide. Thus, students' understanding of mathematics, their ability

> to use it to solve problems, and their confidence in, and disposition toward, mathematics are all shaped by the teaching they encounter in school. The improvement of mathematics education for all students requires effective mathematics teaching in all classrooms" (NCTM, 2000, p. 17).

There is a report at each grade level which states that a change in math education is needed. The students need an improvement in their attitudes about mathematics and its uses. Students should be making connections between what they know and the new content so that the information can be transferred into other courses. If students are to have better attitudes and see these connections, tasks in the classroom should encourage students to discover concepts and ask questions. Students should be encouraged to be responsible for their learning by engaging in ‘constructing, symbolizing, applying, and generalizing mathematical ideas’ (NCTM, 1989, p. 128). The challenge is to motivate students to take responsibility of their own learning. They are more likely to work with mathematics if the teacher can show it to be useful and interesting (Deitte & Howe, 2003).

The Mathematics Classroom

A traditional classroom, where lecture is the primary method of instruction with minimal student interaction, does not give students an opportunity to explore mathematical concepts. *PSSM, Adding it Up, How People Learn,* and *CUPM* address the current interest in changing the traditional mathematics classroom. Research has been done where traditional classrooms were compared with constructivist classrooms, and standards-based teaching methods were incorporated into the curriculum to see an impact on student attitude and achievement. The instructor in a traditional classroom encourages memorization and recitation with no time for student reflection. This section will look at studies that compare traditional classrooms with learning environments that use standards-based methods, the importance of student centered learning, and instructional strategies that support an active student learning approach. Researchers question the value of traditional teaching methods (Hiebert, 1999; Marshall, 2003; & Latterell & Copes, 2003). Hiebert (1999) urges educators to take under serious consideration the research findings that are accumulating with regards to alternative teaching methods.

Changing the learning environment to encourage student exploration and discovery could be helpful. When students experience a different learning environment from the traditional teaching, it can have a positive impact on student attitudes. A study by Lord, Travis, Magill, and King, (2000) found that student satisfaction with the course increased in a student-centered learning environment when they were able to interact with other students. Two undergraduate biology sections were used for this study. One section was taught in a traditional manner while the other section was taught with student-centered learning. The nontraditional class reported higher exam grades, and higher interest and appreciation for the subject. In a larger study, Boaler (1998) analyzed the impact of different teaching practices in two schools over 3 years. Using lesson observations and focus groups, a comprehensive understanding of the students' experiences was developed. The traditional students found that they could not apply their mathematical knowledge outside the realm of the textbook. The students in the nontraditional atmosphere saw a difference between school mathematics and the mathematics needed outside school. They were encouraged to interpret situations, and adapt mathematical procedures that gave them a scenario closer to an experience in the real world. The students who learned mathematics in this environment acquired a conceptual understanding that allowed them to use their mathematics in a diverse amount of situations. The study also found that students in the nontraditional environment developed more positive views about mathematics. Research supports the need to use student-centered curricula in the classroom in lieu of traditional methods. Student attitudes and motivation for learning will be greatly improved (Smith, 1997; Boaler, 1998; Elliott, et al., 2000; Cooper & Robinson, 2002; Burrowes, 2003).

Instructors should discard the traditional teaching methods for standards-based pedagogies which are aligned with the constructivist teaching method. Students will become better learners of mathematics by using previous knowledge to develop new concepts. Caprio, Powers, Kent, Harriman, et al. (1998) took the step toward constructivist classrooms when they changed the structure of their undergraduate science course. The four courses became student-centered with collaborative learning strategies. Lecturing was held to a minimum and students were encouraged through hands-on activities and guidance to construct understanding of the subject. At first students resisted the change because they were used to the traditional teaching methods where lectures were the only way to acquire information about the subject. By the end of the term, students began to see the advantages of the constructivist method of teaching. The information students discovered themselves became more valuable to them. Students enjoyed the mutual respect between themselves and the teachers. The students also stated that although they had more work than their friends in the traditional courses, the students in the constructivist class retained more information because they had explored and discovered the concepts. Burrowes (2003) investigated the effects of the constructivist teaching method on her undergraduate biology course. She designated one section of

General Biology, approximately 100 students, as her control group. They were taught in a traditional manner, where instruction was primarily lecturing with little opportunity for student interaction. A second course, approximately 104 students, was the experimental group. They were taught using a constructivist learning method defined as student-centered with cooperative groups. The traditional method encouraged memorization and recitation with no time for student reflection while the constructivist method followed an engage, explore, explain, elaborate, and evaluate process. A questionnaire given at the beginning, and again at the end, of the course showed that students in the cooperative learning environment were more motivated to learn biology than the students in the passive, traditional course. Students in the experimental group were also more competent when speaking about the subject. Although the constructivist method creates more work for the teacher, the change in student attitudes toward the subject is worth the effort (Burrowes, 2003).

Influenced by methodologies and finding of research in mathematics education, the Principles and Standards for School Mathematics [PSSM], encourage teachers to create more student-centered classrooms with less emphasis on memorization of facts, or drill and practice, and more emphasis on communication and problem solving (Ferrini-Mundy & Martin, 2003). The documents, like CUPM, (CUPM, 2004), PSSM (NCTM, 2000), *Adding it Up* (NRC, 2001), and *How People Learn* (NRC, 2000), challenge teachers to discard the traditional teaching methods for new strategies, which promote learning and engage students in the study of mathematics.

The Standards-Based Classroom

Through National Science Foundation [NSF] funding, mathematics educators, mathematicians, and teachers developed standards-based curriculum. Standards-based teaching looks to shift a classroom from the traditional methods to one where

> "Reform teachers (a) view classrooms as mathematical communities rather than collections of individuals; (b) use logic and mathematical evidence to verify results rather than relying on the teachers as the authority; (c) emphasize mathematical reasoning rather than memorizing procedures; (d) focus on conjecture, inventing, and problem solving rather than mechanical answer finding; and (e) make connections among the ideas and applications of mathematics rather than seeing them as isolated concepts and procedures (McCaffrey et al., 2001, p. 494).

In a standards-based mathematics curriculum, students' experiences are varied. They no longer do repetitive computations because they are involved with exploration and discovery. An important part of standards-based curriculum is to understand how students learn and react to this environment (Bay, Reys, & Reys, 1999).

Several studies have documented that standards-based classrooms effectively improve student learning and change student attitudes toward mathematics. One study that analyzed standards-based classrooms was conducted by NCTM and involved seventeen sites, which included schools and school districts (Johnson, 1998). The purpose of the study was to investigate the effects of implementing the NCTM standards. They found that most students enjoyed working with their peers in group projects and discussions. The importance of communication became evident for these teachers as they watched their students become more confident with their mathematical ability. By using small-group activity, the students were able to share their experiences with one another. Researchers found evidence that a shift in pedagogy offered more interesting mathematical experiences to students, which could be a contributing factor to positive student attitudes in mathematics (Johnson, 1998). Gay, Bruening, and Bruce (2000), through The Teacher Development Coalition, have written a report that summarizes the positive impact classrooms which model standards-based teaching have on student attitudes toward mathematics. The report is a comprehensive literature review of various studies and journal articles. The collection of research is designed to provide guidance so teachers can improve instruction. The contents are arranged to address mathematics, number and computation, algebra, geometry, data, and support for learning such as problem solving, motivation, and cooperative learning. The common themes were that students learn mathematics only when they construct their own mathematical understanding. They found that this happens best when students are working in groups, engaging in discussion, and taking charge of their own learning (Gay et al., 2000).

Instructors need to establish an environment that encourages active participation from the students. Hill, Mabrouk, & Roberts (2003) used various projects and group work in undergraduate math courses. They found that the projects kept the students' interest while they were learning basic concepts. By using standards-based practices, which included strategies such as cooperative learning, discussion, and problem solving, they remained consistent with the reform in undergraduate mathematics education. In another study, Wood and Craft (2000) used integrated problem-based courses. The researchers reported a dramatic improvement in student attitude and motivation. This study was conducted in an engineering program with approximately 100 students. The students who completed the problem-based courses showed strong problem solving ability when compared with those students who were taught with a traditional approach of lectures and note-taking. Courses with critical thinking and problem solving contributed to improved attitudes toward mathematics in a study conducted by Elliott, et al. (2000). Eight college algebra sections were used in the study. Four of the sections were the experimental groups while the other four were the control group for a total of 211 students. The experimental group incorporated more applications, critical thinking, and problem solving skills while the control group was a traditional college algebra class. At the end of the semester, students in the experimental group thought math was important for life and had more positive attitudes toward math than the traditionally taught students.

Standards-based pedagogies intend to help students become mathematically literate, encourage exploration, reason logically, and discover the use of various methods to solve problems (Reys, Robinson, Sconiers, & Mark, 1999). Although this pedagogy has primarily been encourage in K-12 classrooms, the practices are now being studied in undergraduate classrooms. The research shows college students have an improved value of themselves as mathematics students as well as acquiring a deeper conceptual understanding of mathematical topics. More studies are needed at the collegiate level to increase awareness of the positive implications standards-based pedagogy, which includes cooperative learning, problem solving, graphing calculator use, and discourse, has on college students.

Teaching Methods

The teaching strategies used in this study were limited to cooperative learning, discourse, problem solving, and the use of graphing calculators. These strategies are aligned with PSSM recommendations and encourage an active student learning environment.

Cooperative Learning. In a cooperative learning environment, students work together on a given task while still being held accountable on an individual basis. Individualistic learning situations and competitive situations create negative interdependence because they rely on only one person succeeding and that success is independent of the class' performance. Cooperative learning gives students a common goal where the group will be rewards for its efforts (Johnson & Johnson, 1988). Cooperative learning helps students learn how to work with each other, build up confidence in their fellow peers, and learn from each other (NRC, 2001; Walmsley & Muniz, 2003). Cooperative learning does not stress individual achievements or create a competitive classroom.

There are five essential elements that must be placed into group learning situations so that cooperative learning efforts are implemented for long-term success. The first and most important element is positive interdependence. Group members must know that their individual success is linked to the success of the group. In other words, the group will sink or swim together. The second element is to promote interaction. In order for the students to have positive outcomes, the students must encourage, support, help, and praise each other's efforts. They promote each other through face-to-face interactions conducted within the group setting. The third basic element of cooperative learning is individual and group accountability. Not only is the group accountable for achieving the goal, but also the individual must have a portion of the task to be accountable for. Individual accountability works best when the efforts of the individual have a direct effect on the group's score or end result. The fourth element improves individual competency so that the student can share with the group individual attempts, and possibly failures, in order to understand how to reach the goal. The fifth element is teaching students the required interpersonal and small group skills. Social skills must be taught in order for students to learn how to get along, especially in the case of conflicts. These skills include leadership, decision-making, trust-building, and communication (Johnson, Johnson, & Smith, 1998). The five elements must be used so that cooperative learning environments promote proper student engagement and student attainment of the skills they need to work together as a team.

Considerable research indicates that small group work has positive effects on student learning (Deeds, Wood, Callen, & Allen, 1999; Panitz, 1999; Rumsey, 1999; Grouws & Cebulla, 2000; Hill, Mabrouk, & Roberts, 2003). For example, an investigation was conducted in order to learn about junior high school students' perceptions of cooperative learning, Gillies (2003) looked at 220 eighth-grade students from six different schools. Some of the schools had structured cooperative learning, meaning there teachers had been to workshops for cooperative learning. Students participated in cooperative learning activities in more than one subject and at least once a week. The other schools' cooperative learning methods were labeled unstructured. These classrooms did not regularly offer cooperative learning tasks. Through a follow-up Learning Outcomes Questionnaire, the researcher found that the students in the structured cooperative learning groups showed a higher tendency to want to help promote the groups' learning. These students were more involved in asking questions, providing explanations, clarifying points, and participating in discussions than the unstructured classrooms. Brodie (2000) examined a high school teacher in a ninth-grade mathematics classroom. The researcher observed teacher facilitation of student group work. The class was made up of five groups, with between three and five students in each group. During this study, the students used geoboards to find area and perimeter of various shapes. At the end of each class period, the groups were required to share their findings with the entire class. If the group finds were insufficient, the instructor did not simply state the group was wrong. Instead, she offered challenges to their theories with counter-examples in the hopes that students would use this doubt to test their own conjectures and come up with new theories. The groups had to convince the other pupils, and the instructor, that their ideas were correct and do work. Cooperative learning provides a learning environment that encourages students to continually test their theories. Students are engaged in exploration and discussion during the tasks. Throughout the activity, students come to understand the thought processes of discovering mathematical ideas.

When students understand the mathematical concepts, their knowledge of the material improves. Therefore, it only stands to reason that cooperative learning can also have a positive impact on achievement. Johnson, Johnson, and Stanne (2000) did a meta-analysis of 164 studies on eight cooperative learning methods. They found a significant positive impact on student achievement for all eight methods. There was an increased enthusiasm and motivation for math. The students spent more time on task and found advantages to small group work such as help from peers and the fostering of social skills. In another meta-analysis, Bowen (2000) compiled 37 studies to identify how cooperative learning affects high school and college science, mathematics, engineering, and technology (SMET) courses. The researcher used effect sizes between control and treatment groups to report the findings of the meta-analysis. The effect size shows how far the mean outcome variable of one treatment group is above or below the mean outcome variable of another group in terms of standard deviations. The analysis indicated that cooperative learning has a significant and positive effect on achievement outcomes for SMET courses. The outcomes also showed that students had a positive attitude toward SMET courses that used cooperative learning. Cooperative learning also improved achievement for students in a study conducted by Duncan and Dick (2000). In order to decrease failure rates at a university, the mathematics department implemented cooperative learning groups in the hopes that student grades would increase. In four college level mathematics courses, Math Excel labs were added to the curriculum. Students met outside regular class time in computer labs for the Excel workshops. Students worked together in cooperative groups on problem sets. The problems were either related to a topic already discussed in course or a topic not covered yet. Groups were consistently changed so that all students worked together at some point during the semester. The instructors did not give direct instruction, but the instructors did alert students to possible errors for the group to discuss. By comparing the grades of the students in the Math Excel program versus those students who did not participate, the researchers concluded that students in the Math Excel program attained higher grades. The difference was over half a grade point. Instructors should be aware of the positive implications cooperative learning has on student achievement. This could be very beneficial in college level courses where students take courses more than once due to failing grades.

Higher achievement is not the only positive outcome from cooperative learning. Student attitudes toward mathematics have also been shown to change in a positive manner when exposed to cooperative learning. Walmsley and Muniz (2003) conducted a study of two geometry classes. They wanted to compare grades and attitudes of students in cooperative learning groups (experimental group) versus students in a lecture course (control group). After nine weeks, they compared the grades and found that the students who were taught using cooperative learning had increased their grades more than the control group. They concluded, through post surveys, that the experimental group had gained a more positive attitude toward mathematics. The students enjoyed working with each other and also felt cooperative learning could help them understand and learn mathematical ideas. An undergraduate course was used in a study of 141 participants to measure student attitudes toward mathematics. Using a pre-posttest mathematics self-concept scale, a mathematics anxiety scale, and open-ended comments, math attitudes were examined in a cooperative learning program. The program used substantial collaborative small-group work where tutors were directed to engage the students in the principles of cooperative learning, including positive interdependence, face-to-face interaction, and individual accountability. The groups focused on conceptual understanding and problem solving approaches. Part of the time was also devoted to whole-class discussions and direct instruction. The analysis showed that more positive attitudes were reflected at the end of the semester. Overall, the majority of students reported either gaining a positive attitude toward mathematics or maintaining their positive attitudes (Townsend & Wilton, 2003). Students able to work together and explore solutions through various avenues improve their confidence and attitudes toward mathematics (Cooper & Robinson, 2002). The cooperative learning encouraged student interaction, leadership, and participation in classroom discussions. The learning environment gave students an opportunity to clarify their responses, which made the students more responsible for the content of the course. The current findings reinforce the effectiveness of cooperative learning to improve attitudes toward mathematics.

The use of small groups has become popular and there are several sound reasons for students to become part of cooperative learning groups. Since students usually reflect on talk that occurs in groups, they begin to challenge themselves, ask for reasons, and watch their own work more closely while listening to the conversations (Noddings, 1990). Students become part of a group effort rather than an individual effort. Our society is built to be competitive and centered around the individual. The college classroom is designed to map society's belief. Cooperative learning gives students an opportunity to learn to work with others. The use of cooperative learning in college courses will benefit the students and prepare them better for society when positive interdepence, promotive interaction, individual accountability, interpersonal skills, and group processing are incorporated into the cooperative learning pedagogy (Johnson, Johnson, & Smith, 1998). There is not enough research done at the college level on cooperative learning, but there are indications in similar studies of the positive impact of having students working with peers in small groups (Springer, Stanne, & Donovan, 1999).

Discourse. One aid to gaining mathematical understanding is through classroom discourse. Discourse is a way that knowledge is constructed and shared in the classroom. Students engage in inquiry, explore ideas and concepts, and negotiate the meanings and connections of math concepts with fellow classmates (Manouchehri & Enderson, 1999). Students are actively engaged in doing mathematics through a process where they realize that mathematics is about questioning, conjecturing, and trial and error (Nickson, 1992). NCTM calls for students to have the opportunity to reflect upon and defend their thinking. They should be able to express their ideas both orally and in written form. Since reading is an important tool for sound oral discussion, students should be encouraged to read the textbook and other relevant materials. The use of small groups to promote discourse can be very effective (NCTM, 1989).

In an effort to look at the relation between instructional practices and students' motivation, Turner, Meyer, Midgley, and Patrick (2003) examined teacher and student discourse in a middle school mathematics classroom. Thirty-four students were observed over two semesters. The students were also asked to complete surveys that showed students had high support for discourse. The classroom supported the students' needs to correct their errors and develop the knowledge for understanding mathematics. Tanner and Casados (1998) researched a class of 17 high school students for implementation of discourse in the curriculum. The students were given guideline questions regarding the math content. The more discussions the students were involved with, the less they relied on the teacher for direction. The study found that students enjoyed the discussions and learned from them. The researchers also noticed an improvement in students using proper vocabulary when speaking about mathematics. The students began talking through their ideas and feeling confident about the subject. Discourse provided a community of learners willing to engage in discussing mathematical topics. Students became comfortable with mathematical language and confident with their own understanding of the material.

Students working in small groups work effectively to enhance discourse amongst the members. The evidence shows the thought processes students used facilitated the building of ideas. They were able to discuss and develop problem solving methods while resolving misconceptions. Curcio and Artzt (1998) studied a mathematics course where fifth-graders were placed in small groups and encouraged to discuss mathematical tasks. The students had to explore options to solve the problem, implement their method, verify if the method worked, and listen to each other as the group determined if they had a correct solution. A coding chart was used to categorize the student behavior. When looking at the timelines, students did not use much communication in the first two minutes, but progressed steadily as the class continued. The researchers began to see the students first read directions in order to understand the task. Then, there was exploration which led to a plan. The students revised the plan as they sought to verify the solution. Throughout the discourse, the group was enhanced by individual and group contributions. In a middle school classroom, Li and Adams (1995) observed students individually and in small work groups as the teacher sought to bring discourse to the classroom. The students in this study were members of a seventh- and eighth-grade mathematics course. The teacher led whole-class discussions and the groups were required to give problem solving demonstrations. In order to make a conclusion about the teaching methods, the researchers used videotaping, field notes, student materials, interviews, and surveys. Over the course of the study, the teacher became less of a dominant speaker in the course because the students began to become more observant and participate in dialogues taking place in the classroom. Students used these discussions to organize their thoughts about the topics. Interviews and observations verified that students could use the small introduction of a topic from the teacher and incorporate it with previous material to produce the new concept or rule. Symbols became more meaningful and students found them easier to manipulate. Although these students demonstrate the impact discourse can have on student understanding, discourse will not work effectively unless members listen as well as talk within the groups or amongst the entire class.

Discourse can also be a motivational tool for students. When given the opportunity, students can talk about mathematics in order to understand concepts or their own theories. In turn, they develop better dispositions toward mathematics because of their confidence with mathematical topics. Fennema et al., (1996) studied both student and teacher learning in classrooms rich in communication. 21 teachers and their students participated. In this study, students were asked to explain procedures when they did not understand. This improved the students' fluency in reporting their procedures to both observers and teachers. Students knew that peers and teachers valued their thinking. Eventually, the students were inventing their own strategies for solving problems using inquiry methods developed through continuing discussion formats. As the students gained a better understanding, teachers saw more positive attitudes toward mathematics (Fennema et al., 1996). In a high school classroom, Fonzi and Smith (1998) explored student reactions to mathematical discourse. The students were required to read several essays from a book, *The Mathematical Experience*, and then share their thoughts about the essay with the rest of the class. The essays were chosen because they referred to the history of mathematics and also on various approaches to doing mathematics. The instructor hoped to broaden the thinking of the students with regards to mathematics. First, the students shared their understanding of the essay with the class. The teacher hoped that students would find a way to convey their own understanding of the material with the rest of the class identifying which information would be most useful. Then, the student sharing the essay expanded on what they had learned. This gave the other students an opportunity to question the interpretation given and encourage the speaker to find ways to support theories. Throughout this process, the teacher is a facilitator and does not intervene to say if the speaker is right or wrong. However, the teacher does encourage the students to analyze the theories in order to determine what remained unclear. In this manner, the classroom is now a community of learners working on a common effort to understand the essay and relate it to their own way of thinking. The researchers state that if instructors want students to become better learners of mathematics, then mathematic classrooms must encourage this way of communication. The students in this study found that they were better thinkers and doers of mathematics.

Discourse can have a positive impact on student attitudes. Through inquiry and communication with other students, an understanding of math concepts is attained. The language of mathematics is no longer a barrier to the students' skill of understanding concepts. If discourse is continued in college courses, students' mathematical understandings will flourish as they progress through math courses. Although PSSM does not elaborate on standards-based learning at the college level, research in the college classroom seems to be the next step. Since college students will graduate to the workplace where discourse is key to most professions, these classrooms will give students an opportunity to experience sharing of ideas, re-evaluating those ideas based on other peoples' thoughts or beliefs, and supporting their own theories through testing. More studies are needed at the college level to give evidence to the positive impact discourse has on students dispositions toward mathematics.

Graphing Calculators. There are many technological tools that can enhance the learning experience in mathematics. The graphing calculator was the primary source of technology used in this research. Although it has been argued that calculators diminish the learning of basic skills, research does not support this. It has been shown that calculators enhance conceptual understanding, strategic competence, and disposition toward mathematics. Students using calculators were also found to have a better attitude toward mathematics (NRC, 2001; Grouws & Cebulla, 2000; Hiebert, 1999). For example, Hembree and Dessart (1986) completed a comprehensive meta-analysis of 79 reports that dealt with the use of calculators in the classroom. In all of the studies, one group of students was permitted to use calculators while the second group received the same instruction, but without the use of calculators. The meta-analysis showed that students with the calculators showed an improvement in problem solving. The studies also helped the researchers to discover that if calculators are used appropriately, they could be beneficial on tests. Most importantly, studies consistently showed students who used calculators had a better attitude toward mathematics. Using graphing calculators as a tool can facilitate learning. Students are accustomed to technology and should expect to see it in the classroom. Students will find they have a better understanding of the material and a new value for mathematics.

More recent research supports the fact that student use of calculators improves their attitude toward mathematics (Glasgow & Reys, 1998; Kennedy, 2002; Edwards, 2003; Dick, Dion, & Wright, 2004). Calculators enable students to visualize concepts often left to appear abstract. Students can make connections to personal experiences through simulation tasks. Using the graphing calculators to improve student understanding can impact student performance. Santos-Trigo (2002) investigated using various representations to promote and enhance mathematical understanding, and thus, student performance. The study was conducted in a university calculus course with 25 students. The students were placed individually and in groups to work on tasks using graphing calculators. Data analysis was conducted using student interviews and written reports. Throughout the semester, ten tasks were executed. The researcher used student work shown from group and individual work. Whole class discussion was also conducted to demonstrate these results. The study showed that students looked for patterns, described procedures, made conclusions, and had a high level of communication amongst themselves. The researcher found that student performances increased in their ability to explain processes and look at topics from different perspectives. Graphing calculators enhance student educational experiences in various ways. They allow students to explore, visualize, and make connections to real-world experiences. Graphing calculators enable students to become more proficient at mathematics, which ultimately improves their performance.

Using graphing calculators to visualize problem solving can also impact student attitudes and conceptual understanding along with mathematical knowledge. Smith (1997) introduced the graphing calculator to 38 college algebra students in order to measure the impact the graphing calculator had on student attitudes and understanding of mathematics. There were also 45 students in two courses where calculators were not used. Pre and post-surveys were given to measure student attitudes. Students found that they had increased confidence in their ability to do mathematics. The calculator helped them visualize concepts. Only three students responded with less than favorable attitudes toward the use of calculators. O'Callaghan (1998) initiated the use of graphing calculators in six college algebra courses at the university level. Three more sections were identified as the control group because they were taught in a traditional manner without the use of calculators. The study attempted to evaluate the impact of the graphing calculator on student attitudes and success. Two attitude scales were administered at the beginning, and again at the end, of the semester. The results revealed that while students taught in the traditional section showed no change in attitudes, the students who used calculators had an overall improvement in their attitude toward mathematics, their opinion of the teacher, their self-concept relative to mathematics, and their enjoyment of mathematics. The calculators allowed students to explore and solve more realistic applications. In a study by Hollar and Norwood (1999), 90 college students enrolled in intermediate algebra were broken into two groups, an experimental group and a control group. The experimental group had access to graphing calculators for homework and exams, except the final. The control group did not have any access to calculators. The study concluded that the experimental group not only had a better understanding of the concept, but also expressed more positive attitudes toward mathematics and their ability to work mathematics improved more than the control group. Graphing calculators significantly impact student attitude and conceptual understanding of mathematical topics. Instructors should be encouraged to use graphing calculators in the classroom to improve student conceptualization with mathematics.

Through the use of graphing calculators, a learning environment can be found where students explore and develop mathematical concepts. Students gain confidence when they struggle less with difficult tasks. More research needs to be done at the college level to discover the positive role graphing calculators can have for mathematics students. If students find mathematics more interesting and more personal, their attitudes will improve and they will find more value in mathematics. Graphing calculators combine with cooperative learning, discourse, and problem solving to make an active student learning environment that encourages positive attitudes toward mathematics.

Problem solving. It is through teaching via problem solving that students acquire various methods of solving mathematical problems. Teaching via problem solving is where students move from "the concrete (a real-world problem…) to the abstract (a symbolic representation…)" (Schroeder & Lester, 1989, p. 33). The way students represent and connect pieces of knowledge is an important part in whether they will understand it deeply and can use it in problem solving (NRC, 2001). The goal is not only for students to solve the problem, but to also use various concepts to do it (Bay, 2000). Problem solving shifts the focus from memorization of facts and procedures to students investigating material and making conclusions or developing questions. The most important role is that student understanding increases.

Research supports the use of problem solving to improve student understanding of mathematical topics and create positive attitudes toward mathematics. Problem solving is used throughout K-16 grade levels to develop student confidence in solving problems. Capraro (2001) was interested in the problem solving skills of young learners. The participants were 76 students from 18 fourth and fifth grade mathematics classes. She found that students of teachers with constructivist beliefs performed better in problem solving than students placed with teachers who had low constructivist beliefs. Capraro concluded that teacher beliefs play a crucial role in developing students' confidence with problem solving. Stanley (2002) examined problem solving in a university calculus course. One purpose of the intervention was to encourage students to mature mathematically. Students should develop their conceptual understanding and move beyond memorization of a set procedure to solve problems. Activities began with examples that students used to find a solution or hypothesis. These tasks helped students develop a conceptual understanding rather than learning through memorization. Students said they benefited from the activities and enjoyed working real world problems. It gave the students an opportunity to see how mathematics applied to everyday life. In another study, Elliott et al., (2000) redesigned an undergraduate algebra course to change students' critical thinking processes in their problem solving skills. The researchers also wanted to evaluate student attitudes toward math after the course design was changed. A traditionally taught college algebra course was used as the control group. Altogether, 211 students were used for this study. To measure problem solving skills, answers on the course final were compared between the two groups. To measure attitudes toward mathematics, statements were taken from survey administered at the end of the semester. Although the researchers did not find a significant difference in the problem solving skills between the groups of students, the students in the treatment group showed improved critical thinking. The treatment group also had significantly more positive attitudes toward mathematics than the control group. These studies support the research which states that problem solving in classrooms benefits the students and enhances the students' thinking processes.

Problem solving has also been shown to increase student retention and student advancement in math courses. By bringing real-world problems into the curriculum, one instructor helped underachieving students pass a remedial high school math course where some of these same students had remained for the past few semesters. Students were placed into small groups and given tasks where they had to use problem solving skills such as exploration and discussion to find a solution. When students used skills to investigate concepts, the instructor found through student surveys that they had a lasting understanding of mathematics (Jarrett, 2000). At the college level, four technical colleges in South Carolina developed a common problem-based course for engineering technology majors. The goal was to retain students by creating an environment that improved the students' ability to work in teams and to solve problems. Pre and post-course surveys showed students had better attitudes and increased motivation in the engineering program. The college graduates showed an improvement in their problem solving ability (Wood & Craft, 2000). Problem solving will benefit math departments that struggle with retention by creating environments where students can investigate real-world problems which connect personal experience to classroom topics. Students will have an increased value for mathematics as they see the application of mathematics to workplace scenarios.

Problem solving instruction increases student connections in mathematics by allowing them to explore solutions. Students reflect on concepts they have learned and use the procedures to solve applications. Students have the opportunity to see how mathematics is a part of their personal experiences. College level research in problem solving is necessary because students leave college classrooms for the workplace. Students will utilize the exploration and investigation skills attained in the classroom to find solutions to problems in their personal experiences. They can take the concrete application and transform it to the abstract representation in order to find the solution.

Cooperative learning, discourse, graphing calculators, and problem solving are teaching practices consistently mentioned in mathematics reform research. All of these practices have been shown to enhance student understanding of mathematical topics and improve student attitudes toward mathematics. The reform in K-12 has produced considerable studies with all four methods. Therefore, instructors should be encouraged to use these practices in college math courses. There is a need for more research at the college level to show the benefits of cooperative learning, discourse, graphing calculators, and problem solving for student learning of mathematics. These practices can improve student attitudes and also give students an opportunity to discover the value of mathematics in their personal experiences.

Student Attitudes

The everyday notion of attitude refers to someone's basic like or dislike of a topic or idea. Attitude is a behavior that is measured by various evaluative processes. For example, the Minnesota Research and Evaluation Project identifies the following factors pertaining to attitude: attitude toward mathematics, anxiety toward mathematics, self-concept in mathematics, motivation to increase mathematical knowledge, perception of mathematics teachers, and the value of mathematics in society (Ellington, 2003). The Attitudes Toward Mathematics Inventory (Tapia & Marsh, 2004) investigates other areas of attitude toward mathematics. Specifically, it assesses confidence, anxiety, value, enjoyment, and motivation. Confidence measures students' confidence and how they perceive their performance in mathematics. Anxiety measures feelings of anxiety and the outcomes of those feelings. The value of mathematics refers to the students' beliefs on the usefulness, relevance, and worth of mathematics in their personal and future professional lives. Enjoyment of mathematics measures how much students enjoy working with mathematics and attending mathematics classes. Motivation measures the interest a student has in mathematics and the desire to take more mathematical courses (Tapia & Marsh, 2004).

Attitude can also be seen as an emotional disposition toward mathematics. This definition has four components:

> "1) the emotions the student experiences during mathematics related activities; 2) the emotions that the student automatically associates with the concept 'mathematics'; 3) evaluations of situations that the student expects to follow as a consequence of doing mathematics; and 4) the value of mathematics-related goals in the student's global goal structure" (Hannula, 2002, p. 26).

This section discusses some of the factors that influence the attitudes of mathematical students. Research shows that attitudes can be changed when these factors contributing to the negative attitudes are addressed. However, the influence of instructors on students to create positive attitudes cannot stop at the high school level. Instructors at the college level must be encouraged to constructively change student attitudes by changing teaching practices.

Factors Establishing Negative Attitudes

Students who will express like or dislike of mathematics have experiences that have controlled their emotions, expectations, and values with regard to the subject (Hannula, 2002). There are several factors that affect student attitudes or beliefs about themselves as learners. Previous experiences in mathematics courses influence their actions. Confidence in their ability to learn mathematics, their belief about the usefulness of mathematics, and their feelings about being able to 'discover' mathematics all influence student actions (Koehler & Grouws, 1992). Actions are defined to be student reactions to subject topics, student willingness to participate in group activities, student contributions to successful achievement, or student disposition in the classroom. There are several causes that contribute to a negative disposition or perception of mathematics.

Math Anxiety. Math anxiety is often stated as a factor that causes students to have a negative attitude toward mathematics. Mathematics anxiety is defined as an “irrational dread of mathematics that interferes with manipulating numbers and solving mathematical problems within a variety of everyday life and academic situations” (Furner & Berman, 2003, p. 170). In an overview of literature on math anxiety, Furner & Berman found that two thirds of American adults loathe and fear mathematics. Math anxiety usually develops from a lack of confidence when working in mathematical situations (Perina, 2002). When students are not comfortable with mathematics, the cultural attitude, which is where society presents mathematics as difficult and useless because of technology, discourages the students from finding the relevance and making sense of the mathematics. This idea has diminished the importance of mathematics. Many people view it as an abstract and difficult subject reserved for a select few (Turner et al., 2002).

Research has found that specific instructional strategies produce more anxiety in students. For example, Hoyles (1981) reports that students recall more bad experiences in mathematics than in other subjects. Students were satisfied when they perceived themselves successful at their work. They blamed their dissatisfaction on their teachers. The dissatisfaction was a result of not being able to complete tasks successfully or failure to understand math. These bad experiences contributed to the students’ anxiety, feelings of inadequacy, and shame. Stuart (2000) implemented various teaching methods into her fifth-grade class to reduce math anxiety. Over the year, she distributed surveys to see how the students felt about the different strategies. She found that when she used cooperative learning her students felt more comfortable discussing problems with their peers. The journals allowed students to verbalize frustrations. Three-fourths of her students felt manipulatives were helpful. Stuart discovered that her students learned to share and accept more than one way to solve a problem. Their trust in each other developed and they were more comfortable in taking risks when solving problems. She concluded through their comments that their mathematics confidence increased. In a different study, Jackson and Leffingwell (1999) surveyed 157 students about their experiences with mathematics from kindergarten through college. They found that the most common explanation for developing math anxiety was lectures delivered too rapidly. They found that 27 percent of their respondents said their freshman year in college was when they first felt math anxiety. Another common explanation was that the language of mathematics commonly created barriers between the student and teacher. Uncaring instructors who asked them to leave class if they did not understand the material also disgruntled them. When students went for help, they were turned away because the instructor did not have time for them. The students should be given an opportunity in the class to ask questions in order to alleviate barriers in knowledge growth. Allowing students to ask questions will encourage them to become more comfortable with the subject and less anxious about discussing topics they do not understand.

Delivery of Mathematics Instruction. Not only is math anxiety a problem for students, but also disdain for the subject. It is different from math anxiety in that it is influenced more by the presentation of the material than the subject itself (Fiore, 1999; McLeod, 1992).

Studies show that when students become comfortable within the class, their expectations and outcomes for the course change. Students become easily disgruntled by courses taught solely through textbooks and memorization. Cornell (1999) distributed several surveys to graduate students to evaluate their feelings about mathematics. He found that students become disinterested with instruction that is highly focused on rote memorization rather than on the study of concepts. Textbook exercises, workbooks, and worksheets are rarely stimulating for students. They typically concentrate on the calculations of multiple problems rather than on conceptual understanding (Cornell, 1999). In a similar study, Hannula (2002) studied the psychological emotions of mathematics among lower secondary level students (grades 7 to 9). Student behavior was documented through interviews and observation. Secondary level students expressed unpleasant emotions toward mathematics because of previous experiences that had given them a feeling of failure. These previous experiences included not understanding word problems, feeling academically inferior to other students in the course, and failure to make use of the mathematics outside the classroom. Over the course of the study, the students became active members of their group and began to show positive attitudes toward mathematics. Students who perceive their learning environments to model good teaching report greater satisfaction with the course. Lizzio, Wilson, & Simons (2002) surveyed 5000 students within one university to obtain information about their perceptions of the learning environment. The students found more satisfaction in the courses which were less packed with drill and practice assignments and allowed time for analytic, problem solving, and interactive learning opportunities. When teachers emphasize understanding of mathematical concepts and provide interactive classroom environments, students tend to be more receptive and less anxious with regard to mathematical activities than when teachers stress rote activities and are perceived to be authoritarian (Middleton & Spanias, 1999). Students are aware of their emotions and reflect on them to control a situation. Negative experiences with learning environments affect students' willingness to learn. If students feel that they are in a caring environment that nurtures them and allows them to freely ask questions, negative attitudes can be changed.

Lack of Confidence. The final factor addressed here is the lack of confidence students feel when working with mathematical problems. When students are not confident with their work, they will be less likely to complete the tasks and avoid the work. Turner, et al. (2002) studied 1,197 middle-school students in 65 classrooms to investigate the learning environment and avoidance strategies. They found that avoidance strategies are one way that students deal with their lack of confidence in mathematics. They are fearful of making mistakes in front of peers or looking incompetent by asking for help. To protect their pride, students who are uncertain about their ability to be successful at mathematics may develop strategies that take away the attention from their ability. Examples of avoidance strategies are avoiding seeking help, resisting simple approaches to work, and intentionally not exerting effort to make themselves appear math illiterate. These are used by students to discourage negative judgments made by others with regards to their knowledge of mathematics. Students begin to feel inferior to their peers and turn away from the subject that is intimidating.

Attitudes in Standards-Based Classrooms

Research about attitudes has shown that when the emphasis is on learning, understanding, and intellectual development, students are less likely to feel threatened and may not see a need for these avoidance strategies (Turner, et al., 2002). DePree (1998) placed three hundred eighty-six algebra students from a community college into small-group instruction. The research design was a quasi-experimental design because existing classes were used for the group work classes (experimental) and the lecture classes(control). The two groups covered the same material but received different instruction. The experimental group used cooperative small-group work at least once a week. Logs were used to record the use of group work and any comments. Instructors in the control group used the typical lecture method. A student survey, conducted at the end of the semester, used a Likert-type scale so students could rate how often lecture and small-group methods were used in the respective courses. The Fennema-Sherman Mathematics Attitudes Scales were used as pre- and posttest instruments to measure algebraic achievement and problem solving abilities. The experimental group also completed an open-ended questionnaire regarding likes and dislikes about group work and suggestions for improvement. While the study did not find a significant different in achievement, the researcher found that the students enjoyed working in groups. By working with their peers, they stated they had a better understanding of mathematical concepts. In another study, Panitz (1999) used cooperative learning in his college course to study the interactive learning environment. Two weeks prior to the start of class, the instructor sent a letter to each student defining cooperative learning and assigned the first chapter. When the course began, students were placed in groups and completed worksheets daily over the current topic using cooperative activities and group discussions. Quizzes were given within the groups, but exams were administered individually. The cooperative learning techniques were used to encourage students to take responsibility for their own learning. At the end of the semester, each student completed a self evaluation to identify their thoughts on progress in class or any changes the students had made about learning mathematics. The researcher found that the motivation levels increased as students began working more with their peers. The students get to know each other better which helped to form lasting relationships among peers. They worked together outside of class and took follow-up mathematics classes together. This motivation helped achievement levels increase which improved their self-esteem in working with mathematics. These studies provide evidence that cooperative small-group methods develop more mathematical confidence in students than using the traditional lecture method of instruction. Student motivation increases when student interaction is a part of the classroom.

In a study that incorporated more than one new teaching method, Smith and Moore (1991) looked to compare attitudes of students in a traditional classroom and those in a reformed calculus course named Project CALC (PC). In the PC courses, it was recommended that teachers lecture less and have students do more group activities. The course included technology, cooperative learning, group projects, writing, and real world problems. The PC course met in a classroom three times a week and they also met in a computer lab two hours a week. Lecturing was limited to brief introductions of new topics and further explanation of information when demanded. The in-class computer use was for active involvement of the students. Active involvement was also part of noncomputer tasks such as exploration of numerical approximations, studying the normal distribution, and comparing integral calculations. The activities were completed in teams, which students noted in writing reports made them nervous. Over the semester, students completed writing reports about activities and completed an evaluation at the end of the course. Students in the PC course had a statistically significant attitude difference over the traditional course. The PC students found calculus to be useful and they found that they understood mathematics. The PC students were working hard, learning more, and excited about learning. They had acquired a positive attitude toward the learning of mathematics.

Math becomes more meaningful when students can make a connection between what is learned in the classroom and their personal experiences. Standards-based mathematics was developed so the student could be encouraged to see this connection (Trafton, Reys, & Wasman, 2001). Students in a standards-based program create mathematics through their own investigations of situations set up by the teacher who steps aside so that students can learn (Trafton, et. al., 2001).

An emphasis on process, with less of an emphasis on right or wrong answers, can help alleviate students' anxiety about math (Furner & Berman, 2003). The learning environment for mathematics, described in How People Learn (NRC, 2000) differs from many of today's classrooms. However, as students move through the grade levels, lecturing is the most prevalent method of teaching. Instructors need to find ways to promote active learning in higher education (DePree, 1998).

> "Overall, learner-centered environments include teachers who are aware that learners construct their own meanings, beginning with the beliefs, understandings, and cultural practices they bring to the classroom….The teachers attempt to get a sense of what students know and can do as well as their

> interests and passions—what each student knows, cares about, is able to do, and wants to do” (NRC, 2000, p. 136).

Lizzio, Wilson, and Simons (2002) found that university students’ perceptions of their learning environment influenced their approach to studying. The students’ attitude toward the positive or negative learning environment had a direct correlation with learning outcomes. Professors at the college level may have a disadvantage because by the time students come to college, they are discouraged or apprehensive about mathematics (Gilroy, 2002). Students fail to see a connection between math and science and everyday life. Instructors must accept the challenge to motivate students. If college students can see mathematics is interesting and useful, then these students might become more motivated to learn math (Deitte & Howe, 2003).

These studies support the idea that student attitudes can improve in nontraditional classroom settings. Students enjoyed working with their peers, solving real-world problems, and finding value in mathematics. However, more research should be done that focuses on attitudes in the college mathematics courses when the pedagogy is standards-based. College math departments might find higher retention rates and improved student achievement.

College Mathematics

College mathematics can be conducive to the math reform found in PSSM. By lecturing less and incorporating activities that encourage student participation, a change in college student attitudes is possible (Smith & Moore, 1991). Making change in college has many barriers at the institutional, administrative, and policy level; however, few barriers exist at the instructional and classroom level. Therefore, the place to make the change in higher education is where personal control is possible, which is at the course level (Sunal, Hodges, Sunal, & Whitaker, 2001).

Faculty members have control over the methods of instruction. Typically, these methods are lectures and note-taking. College students would benefit from a change in teaching pedagogy that shifts from traditional methods to standards-based pedagogies. Hiebert (1999) states that there is sufficient evidence to indicate that the traditional curriculum and instructional methods are not serving our students well. Traditional curriculum is defined by its emphasis on teaching procedures, especially computational procedures. The traditional methods show serious deficiencies; therefore, math departments should consider the evidence that shows alternative programs such as standards-based teaching are beneficial to students. Alternative programs can be designed so students achieve additional goals of conceptual understanding and skill proficiency. When implemented correctly, students have been shown to learn more and learn more deeply than in traditional programs (Hiebert, 1999).

Several studies have analyzed traditional teaching methods versus using more activities to encourage student participation. The courses were re-designed to incorporate real-world problems students could relate to. The studies sought to improve student interest and attitudes toward mathematics. To move from the traditional program, Smith and Moore (1991) developed a different calculus course because they discovered that their students were not able to apply calculus properties to other subjects. They explored real-world problems using calculators and computers. Students were given more activities to do in class as opposed to listening to a lecture. The pedagogy also included cooperative learning, group projects, and writing. The attitude difference favored the students in the new calculus course. Although they had complained about the project Calculus two years prior, they showed statistically different attitudes and appreciated the usefulness of calculus more than their traditionally taught peers (Bookman & Friedman, 1998). In another study, Stanley (2002) designed a precalculus course in which students would learn from others and become more persistent problem solvers. The researcher wanted the students to be able to make connections between different mathematical ideas. Stanley used problem-based learning to develop their decision-making skills and to help the students gain confidence at integrating different mathematical procedures and concepts to arrive at a solution. Course evaluations, completed by the students, reported that the students benefited from the collaborative tasks of the course and that they enjoyed learning mathematics using real world problems. By incorporating problem solving activities, Stanley concluded that over half of the students experienced improvement in their ability to solve problems. The students enjoyed working on problems and participating in the learning process when given appropriate support.

Working on real-world problems enables students to see the value of mathematics in their personal experiences. Deitte and Howe (2003) used various methods of teaching to convey the importance of mathematics. The researchers found the various teaching methods improved students' attitudes toward mathematics and helped increase their motivation to learn the subject. The activities used in the course included writing papers on mathematical topics, interviewing individuals off campus who share their area of interest to discover the use of math in those fields, and cooperative learning. At the end of the semester the researchers distributed a survey which asked the students to rate the statement, 'My motivation to learn mathematics was increased.' Although students were not excited about all of the projects, the researchers found that, overall, their motivation to learn mathematics increased. Activities related to applying mathematics to job applications received approval ratings of almost 70 percent. However, writing a paper on a mathematician only received an approval rating of 40 percent. In the conclusion, Deitte and Howe state that the students seemed to have been enlightened by the process of using various activities or projects to learn mathematics (Deitte & Howe, 2003).

These studies indicate that when courses include teaching strategies such as problem solving, cooperative learning, technology, and discussions, that college students can improve their experiences, attitudes, and motivation. Further research needs to address if students will continue to take mathematical courses since their attitudes toward mathematics have improved.

Student Attitudes and Dropout Rates

If courses are poorly taught, some students might not only drop the course, but also change career paths (Ellis, 1997). This could be harmful for an area that continually seeks qualified students. Retaining students in colleges and universities remains a high priority. Only 50% of those who enter higher education actually earn a bachelors degree (Center for the Study of College Student Retention, 2004). The access for higher education is readily available for many people; however, for many the desire to earn the degree or complete a goal falls short.

Math and science departments continue to see this decline in enrollment and retention rates. Retention may be difficult because there is a belief that the only reason to take mathematics is because it is a requirement for graduation. The common thought is that a person is never really going to use it (Gilroy, 2002). CUPM (2004) attributes a decline in the annual number of bachelor's degrees in mathematics to an increase in students majoring in education. Students are interested in mathematics at the introductory level courses, but lose interest with the advanced math courses. CUPM (2004) also reported an increase in engineering and statistical departments offering their own advanced math courses for their majors, which would show a decrease in enrollment for mathematics departments.

When students do enroll in math and science courses, teacher attitudes affect the attrition rate. Daempfle (2003/2004) completed an analysis of attrition rates for undergraduate math and science majors. The researcher was concerned that there were not enough math and science majors to meet the future needs in this area. There were 335 students from seven university campuses. Students compared their high school experiences, where dialogue was consistent and they also engaged in cooperative learning strategies, to the college courses which contained more lectures. In the interviews, students stated faculty members at colleges were intimidating, sarcastic, and disinterested in teaching. Students were turned away by the cold climate of the college classroom where faculty members were more concerned about their research than teaching the materials. He found that students were negatively influenced by one-way lectures in the college courses. In order to improve retention rates, college faculty members should switch from basic lectures to strategies that create opportunities for students to actively participate in the lectures. College professors need to find a way to create the positive experiences these students found at the high school level.

Standards-based Teaching at the College Level

College instructors need to be aware that large numbers of students are arriving on college campuses after having been through NCTM standards-based courses in the high schools (Committee on the Undergraduate Program in Mathematics [CUPM], 2004). These students have all been exposed to a conceptual approach to mathematics. They are accustomed to learning environments that embrace communication, technology, and understanding.

There is a current movement to reform college algebra courses. The belief is that introductory math courses should be designed to enhance students' thinking more clearly and effectively in both the classroom and their personal lives. Students should understand that math is not merely learning a set of rules and techniques. Being active learners and active participants in constructing the ideas of the course will enhance the students learning and retention of ideas (CUPM, 2004).

Improving attitudes can also occur when students' confidence in their ability is increased. To research mathematical ability, DePree (1998) conducted a quasi-experimental study to look at the impact of small-group work of algebra students at a community college. She found that teaching strategies that promoted cooperative environments, rather than competitive, helped students to make sense of mathematics. The students showed an increase in confidence of their mathematical ability. In another study about ability, Thompson (2001) conducted a study of 5,276 random full and part-time community college students hoping to identify what had a positive influence on their academic performance. He distributed a survey asking about external influences, gender issues, interaction with faculty, effort in science courses, and educational gains in science and mathematics. Quantifying the results, he found an increase in the amount of informal interaction with the faculty caused a higher positive attitude toward the development of learning in science and math. Those faculty members who encouraged standards-based practices and small work groups also positively affected students.

This research supports what has been discovered at the K-12 level. Students have varying attitudes toward mathematics, but with the proper teaching strategies, students can be motivated to learn mathematics, and see that it is enjoyable. This research needs to extend to the college level where young adults have been shown to drop out of mathematics programs due to lack of interest. Studies have shown that standards-based teaching can be put into the curriculum at the college level. If it is going to become a part of the curriculum, there should be a demand for research that shows if standards-based teaching will also improve their attitudes toward mathematics like it has done for students in the K-12 level.

> "Adults learn best when they are not threatened and when they are treated as responsible individuals. Learning should take place in a pleasant environment that encourages change. An adult who is dissatisfied with the learning process is unlikely to continue taking part in it" (Reed, 1993, p.20).

Summary

Constructivism has prompted a change in classroom practice. The theory is grounded in the notion that students construct their own knowledge through previous experiences and interactions with others. Knowledge is not simply attained by transmitting information from the teacher to the student. Classrooms based on constructivism try to optimize opportunities for students to reflect upon previous held conceptions and challenge their own theories. Reports such as *Principles and Standards for School Mathematics*, *Adding it Up*, *How People Learn*, and *Undergraduate Programs and Courses in the Mathematical Sciences* advocate for this type of classroom. Constructivist classrooms have been compared to standards-based classrooms. Characteristics of what are referred to as a standards-based classroom include cooperative learning, problem solving, classroom discourse, and the use of graphing calculators. Research on each of these has found improved achievement, increased understanding, and better attitudes. In addition, research that focuses more holistically on standards-based classrooms is emerging. Findings from studies that focus on standards-based classrooms in K-12 have found that students enjoy working with their peers, gain insight into communicating about mathematics, have interesting mathematical experiences, and show improvements in achievement and attitudes.

At the college level, studies report that students view mathematics as useless, become easily intimidated by inhospitable instructors, and are bored with drill and practice. Dropout rates are higher because the students do not become a part of the learning environment. They do not have the opportunity to ask questions, explore new concepts, and discover possible solutions. Research on college students shows that a change in a teachers' approach can change attitudes. Yet not enough is known as to what specific strategies impact student attitudes. The number of studies regarding the science and mathematics field at the higher education level are limited (Cooper & Robinson, 2002). This study added to the research by examining college algebra students' reactions as they completed a semester of a course that used strategies aligned with standards-based methods of teaching. It analyzed their perceptions of how they learned the content and the impact these teaching strategies had on their attitudes toward mathematics.

CHAPTER 3

METHODOLOGY

Introduction

The purpose of this study was to assess potential benefits of using components of the standards-based pedagogy with community college students. The components of standards-based pedagogy used in this study were cooperative learning, problem solving, discourse, and graphing calculators. Since this study focused more on instruction than curriculum, the components selected addressed classroom practices. More specifically, this study analyzed the following questions:

1. What is the relationship between the use of standards-based pedagogy and college algebra students' perceptions about the nature of mathematics and learning mathematics? Specifically, does such a learning environment impact students:

   a. attitudes, specifically in the areas of confidence, anxiety, enjoyment, and motivation,

   b. perceived value of mathematics in personal and professional experiences.

2. What specific instructional strategies do students believe most impact their attitudes about mathematics?

3. In what ways does the use of standards-based pedagogical strategies impact college algebra students' engagement in the learning process?

The instructional strategies used in this study reflected standards-based teaching. Standards-based instruction encourages students to be active participants in their own learning in order to develop cognitive skills and processes. The NCTM reports that students' understanding of mathematics, their motivation to learn mathematics, their confidence in themselves to succeed are all shaped by the teaching in the classroom. Effective teaching of mathematics involves implementing worthwhile mathematical tasks, using appropriate tools, including technology, encouraging students to reflect upon work completed, and discussing solutions and strategies for solving problems (NCTM, 2000). Using this form of instruction is beneficial because standards-based teaching can motivate students and enhance their learning experience (Trafton, et.al., 2001).

The independent variables, the teaching strategies, and the dependent variables, student attitudes toward mathematics, were validated through the data collection methods The components of standards-based teaching used in this study included cooperative learning, problem solving, discourse, and graphing calculators. The dependent variables were Cooperative learning entailed grouping the students into clusters of three or four to work on a specific task together, usually without teacher assistance. The goal is that students will collectively work together and use each other to expand their own knowledge about the content (NRC, 2001). In this study, cooperative learning tasks required students to work together in groups of three or four. Each individual in the group was assigned a role for the activity. Roles included the group leader, the person in charge of materials, a motivator or coach, and a record keeper. If each student had a significant contribution to make for the completion of the task, then everyone was responsible for their own grade.

The Principles and Standards for School Mathematics (NCTM, 2000) state that problem solving is where students are actively participating in tasks where they do not know the solution procedures prior to the task. They must use knowledge and concepts acquired in previous class sessions. In this study, problem solving involved tasks designed for the students to discover specific concepts such as special product trinomials and slope. These activities were designed such that the mathematical concept was not directly given to them. In previous class sessions, topics were discussed which were the foundation for the next concept. Students completed problems with a goal of finding a pattern emerging from their work. This pattern set the student on track to identify a rule, which ultimately, was the new concept.

Problem solving can lead to discourse which is an important part of the mathematical classroom because it encourages the students and teachers to discuss strategies and solutions. Discourse also improves student understanding and use of the mathematical language. Discourse occurred in this study when students engaged in communication to solve assigned tasks while in individual and group work settings. Students were expected to defend their solutions by showing they had a solid foundation of the concept when they were asked to present their solution to the entire class. While defending a position, students have the opportunity to engage in discussions with other students about other solution methods that will work (NRC, 2001). Students in the class are expected to verify solutions given through questioning and exploration. When discourse occurs while participating in group work, the students verbally explain why they choose a certain procedure amongst their group and then the group is expected to share the solution with the entire class. The smaller groups give some students a better opportunity to receive new ideas and also ask questions to their peers. The students may be intimidated by the larger class and more willing to express thoughts to a few students.

In some instances, the smaller group will appoint a spokesperson that will share the group's conclusions with the entire class. This offers another opportunity for students to discuss strategies for solving problems.

Solutions and strategies for solving problems can also be found using tools of instruction such as graphing calculators. The instruction of mathematical content can also be accomplished with graphing calculators. When used appropriately, they can enhance mathematics learning because they allow students to quickly visualize what they see on paper. In the college algebra courses for this study, students were required to purchase a graphing calculator. The students used graphing calculators to graph equations that produced lines, circles, and parabolas. The students also used the graphing calculators to see how transformations of functions made graphing functions simpler. Students used the table feature to solve for multiple values of a single function. The calculators were also beneficial when working with linear, quadratic, and exponential regressions. Matrices were worked out using the graphing calculators. Students were shown how the concepts work without a calculator so that they could recognize how the technology enhanced their learning experience. Worksheets were given to students in small-group work in order for the students to help each other effectively learn the benefits of the graphing calculator. The instructor used a TI-83 viewing screen to allow students to follow calculator procedures during a task.

Assessing potential benefits of using components of the standards-based pedagogy with community college students was completed using a mixed method research. This researcher sought to determine if there was a relationship between student attitudes and the four teaching practices, which were cooperative learning, discussion, the use of graphing calculators, and problem solving.

Research Methods

The mixed method research design chosen for this study was action research. Action research uses both qualitative and quantitative data, but the focus is more applied in action research (Creswell, 1998). The purpose of action research is to assist teachers, through the gathering of qualitative and quantitative data, in improving their classroom structures, techniques in teaching, or how well their students learn. More specifically, this study focused on practical action research, which is used in specific school situations to improve practices (Creswell, 1998). Some characteristics of action research include a focus on practical issues, the study of the educator's own practices, collaboration between the participants and the researcher, and the process of reflection, data collection, and action.

Sampling

This section describes the research site, the research setting, the research participants, and the role of the participants. The researcher will also discuss the environment of the classroom in the study. The participants were chosen through purposeful sampling because students selected teachers and class times when they enrolled.

Research Site

To conduct a sound qualitative study, a realistic site must be acquired. Marshall and Rossman (1999) define a realistic site where entry is possible, the researcher is likely to be able to build trusting relations with the participants in the study, and the data quality and credibility of the study are reasonably assured. The participants for this study were selected from students enrolled at Labette Community College in Parsons, Kansas. Access to classrooms, programs, and students was easily accomplished. There was also ample opportunity to develop trusting relationships with the participants.

Labette Community College is in Labette County located in Southeastern Kansas. The town is a rural community located in a county with a population of 22,483 persons (U.S. Census, 2000) with 89.3% White, 4.7% Black, 1.9% American Indian, and 4.1% Other. Labette Community College has a yearly enrollment of approximately 2,598 students with an average age of 26. The population of students on the Labette Community College campus is 4.1% Black, 86.9% White, 1.5% American Indian, and 3.1% Hispanic. 64.3% of the students are female while 34.8% are male. Of this student population, 37% are full-time students.

Research Setting

Students participating in this study were enrolled in a college algebra course taught by the researcher. Three sections of college algebra, of which two were taught by the researcher, were offered during the spring semester. This course is the lowest college level course that fulfills the mathematics requirement for many of the social science, education, humanities, and other liberal arts majors. In order to complete a degree, 68.5% of the students need college algebra. To enroll in mathematics courses, students must have a passing grade in Intermediate Algebra or an appropriate score on the mathematics portion of the Compass placement test. Based on their score, they are placed into the mathematics course that is given on the placement chart designed by the mathematics department.

COMPASS RESULTS – MATH PLACEMENT

Placement Subtest	Score Range	Placement Recommendation
Algebra	0 – 33	Takes student to Pre-Algebra Subtest
	34 – 56	Intermediate Algebra
	57 - 100	College Algebra Mathematics for Education
College Algebra	0 – 43	College Algebra Mathematics for Education
	44 - 100	College Algebra, Mathematics for Education, Statistics, Trigonometry, Business Calculus
Trigonometry	0 – 43	College Algebra, Mathematic for Education, Statistics, Trigonometry, Business Calculus
	44 - 100	All of the above and Calculus I

Each day students see different teaching methods to support the learning of mathematics. The distinguishing characteristics of the proposed college algebra course included a delivery mechanism with minimal lecture, support of students in their efforts to become better participants, and an environment to build up their confidence to show their work to their peers in group work and in front of the entire class.

This course was designed to model the standards-based practices and generally included the four components of standards-based teaching described earlier. Each day began with questions students had regarding content previously covered. They might have had a problem on the homework or inquired about why a particular method was used to find an answer. Often times, a student was asked to share how they solved the problem by using the chalkboard. This gave students an opportunity to orally describe what steps they used and reinforce their confidence in mathematics. Other students could question steps or procedures used, and if they chose, show their solution on the board as another method to solve the problem. Approximately 15 minutes of a 50-minute class was used to discuss questions from prior lessons. The remaining time period was spent in different ways. For example, if a cooperative learning task had been chosen, the remaining class period was used to complete the task. Roles were assigned to the students, who must complete and discuss the assignment prior to leaving. Another example was creating concept maps, which are completed in two parts. First, students discuss in their groups the pieces of the map. Second, each group shares with the entire class, through a spokesperson, how they designed their map. For technological activities, some time was spent explaining how the calculator would be used to enhance the students' understanding of the topic. The view screen was used to demonstrate steps and then the students were given an activity to complete that related to the content being taught.

Research Participants

The participants for this study were enrolled in one of two college algebra sections with a typical class size of thirty students, taught by the same instructor, the researcher. Students pre-enrolled in the sections of college algebra that best fit their schedule. The courses were listed in the school schedule by time, place, and instructor. Of the four sections available, the researcher taught two and the students in these sections were asked to participate in the study.

Some of the students who took college algebra wanted to develop their foundation for future math courses while others were simply completing a requirement for their major. The course contained both traditional and nontraditional students who might experience low self-esteem, high math anxiety, low interest in the subject, and negative attitudes toward mathematics.

Traditional students are defined as students who have been continuously enrolled in post secondary school since graduating from high school. They vary in their mathematical background. These students might have recently graduated from high school where their last math course might have been their senior year or the one required course taken their freshman year. The traditional students who have been in college for a few semesters may have had developmental math courses, such as Foundations of Math, Beginning Algebra, and Intermediate Algebra, or their last math course has been at least five years ago. The researcher's experience with traditional students suggests they have varied feelings about taking a math course. Some of the students are over confident while others struggle with the anxiety of being in the mathematics classroom. Many of these students become disinterested during lectures. Their note taking skills are poor, and they need an incentive to complete homework. Their mathematical abilities to complete homework and understand concepts vary from very strong to relatively weak.

The nontraditional students are older students who have come back to college after several years away from the classroom. They may have started college years ago, but left school to raise families. Or the nontraditional student may have never even had a chance to begin college. Many of them return looking for degrees and better job opportunities. Nontraditional students, like the traditional students, vary in the length of time they have had a mathematics course. However, if college algebra is their first mathematics course in college, it is possible that they have spent ten years or more away from mathematics. The researchers experience with nontraditional math students suggests that these students typically are excited to learn and do the course work without complaining. Many times they do more than what is required because they want the practice.

Role of Participants

All of the students completed the Attitudes Toward Mathematics Inventory (see Appendix C), designed by Martha Tapia (1996), at the beginning and end of the semester. Throughout the semester, students were involved with completing a course evaluation (Appendix D) about their thoughts on the teaching methods used in the classroom, cooperative learning tasks, problem solving assignments, group and individual discussions, and the use of graphing calculators. Approximately 15 students were involved with focus group dialogue for the purpose of validating their responses to questionnaires.

Data Collection

To assess potential benefits of using components of the standards-based pedagogy with community college students and how the pedagogies impact students' attitudes toward mathematics, six data collection methods were used. The instruments used in this study included Attitudes Toward Mathematics Inventory, focus groups, questionnaires, personal observations, and a course evaluation on the teaching methods used in the classroom. The researcher also used a modified version of the University of Wisconsin Observation Scale (Appendix A) to measure the appropriate use of active student learning, developed at the University of Wisconsin (University of Wisconsin, 1998). Data was collected throughout the semester in the two college algebra courses.

Sources of Data

This study sought to assess potential benefits of using components of the standards-based pedagogy with community college students and how the pedagogies impact students' attitudes toward mathematics. The Attitudes Toward Mathematics Inventory (Appendix C) provided information on how students viewed themselves as students of mathematics prior to enrolling in the college algebra course. Questionnaires (see Appendix B) routinely checked how the students perceived their attitudes toward mathematics and how what was happening in the classroom impacted this perception. In order to document student behaviors and attitudes, naturalistic observations occurred through incident sampling, observing individuals while they participated in a task, and time sampling, monitoring time on task.

Focus groups and questionnaires gave students an opportunity to explain their reactions to activities in the classroom. A course evaluation questionnaire allowed students to rate their experiences in the classroom. These sources provided the data necessary to determine the potential benefits of implementing standards-based pedagogies, and which strategies had an impact on student attitudes.

Attitudes Toward Mathematics Inventory. An instrument to measure mathematics attitudes was administered to all students participating in the study at the beginning and, again, at the end of the semester. This instrument was developed by Martha Tapia and George Marsh (1996) to address mathematics attitudes with regards to confidence, value, motivation, anxiety or enjoyment of the subject matter. It was administered to 545 students enrolled in high school mathematics courses. The Attitudes Toward Mathematics Inventory (ATMI) was originally a 49-item scale with a Likert-scale format. It has since been shortened by the original developers to 40 items. To estimate internal consistency of the scores, Cronbach alpha coefficient was calculated. For scores on the 40 items, the alpha was .96, indicating a high degree of internal consistency for group analyses. The item-to-total correlation had a high of 0.82 and never fell below 0.50.

This inventory covers the domain of attitudes toward mathematics by looking at confidence, anxiety, value, enjoyment, and motivation, which all contribute to determining mathematical attitudes. These factors were important to this study as the researcher looked at students' interest in the area of mathematics and their motivation to participate in class activities.

Focus Groups. The purpose of the focus groups was to collect data on student thoughts and reactions to the tasks assigned throughout the semester. Focus groups have been around since Merton and Kendall’s work in 1946. The guidelines, developed by the two researchers, indicates that the participants share an experience or opinion of the topic, there is interaction within the group related to questions given by the researcher, and an interview guide is used (Gibbs, 1997). Focus groups allow the researcher to gather information in a comfortable environment with a limited number of individuals (Lewis, 1995).

For this study, nine students were chosen from the first class and six students were selected from the second class. Initially, the researcher had targeted for three students to represent different attitudes toward mathematics, negative, positive, or neutral. However, the second class only included two students representing each of the three attitudes toward mathematics. To determine which attitudinal level students were at, a preliminary survey was distributed to both classes (see Appendix H). Focus group discussions were used to identify which strategies have the strongest impact on student attitudes toward mathematics, their feelings regarding being a student of mathematics and the applications of mathematics in their personal and professional experiences.

The focus groups were conducted three times during the semester in the researcher's office. The focus groups took about one hour and were audio taped for transcription purposes. The discussion began with an opportunity for the students to expand on attitudes toward tasks completed in the classroom. Further questions followed pertaining to conclusions the researcher made about questionnaires or observations. The discussions also served as a member check for the researcher.

Observations. Documenting student participation through video-taped observations occurred twice during the semester. Personal observation occurred frequently over the semester. Incident sampling infers that the researcher will choose particular incidents to observe. These incidents included their participation in varying tasks to see how the tasks impacted student engagement. Time sampling is the amount of time the students stay on task. Calculating time on task benefited the researcher in determining student actions in the classroom with active student learning.

The nature of the instruction can also be explored using observations. It was necessary to document that the instructor was using active student learning. The tracking of instructional strategies was done through a matrix (Appendix D) which showed the amount of time, in each class period, that a particular instructional strategy was used. The Wisconsin Observation Scale (Appendix A), developed at the University of

Wisconsin, provided a protocol for the researcher to document the use of certain active learning strategies.

Questionnaires. The purpose of the questionnaires was to allow students the opportunity to give written documentation of reactions to tasks and teaching methods used in the classroom (see Appendix B). Students, who signed consent forms from both sections, completed the questionnaires eight times over the semester. The students identified different techniques used in the classroom. Students circled the attitude (confidence, anxiety, value, enjoyment, motivation) associated with each technique they experienced. They had an opportunity to write about the different ways topics were presented, times they felt encouraged to participate, if the activities made them feel more confident about their ability with mathematics, and if they have found applications of the concepts to their personal or professional life.

The researcher considered the scenario where participants would write what they believe the instructor wanted to read instead of writing how they honestly felt about the experience. Confidentiality of the responses was assumed by the researcher with regards to the students' responses. Each student was assigned a random number to be placed on his or her questionnaire. It was the same number they were given for their inventories. The numbers were used during the analysis of the data for comments and common themes.

As an incentive for the students to complete the questionnaire, points were given for each entry, which was a part of their homework grade. Points were given on completion and not quality because the researcher did not have access to the names and numbers. A list of the student names with their respective number was given to a work-study helper who was responsible for marking a spreadsheet with a checkmark so that the students received credit for their responses.

Course Evaluation Questionnaire. The questionnaire was designed by the researcher to confirm that active learning instructional strategies were used in the college algebra courses (Appendix D). It provided an opportunity for students to evaluate, through a Likert-scale model, the techniques used in the classroom. This questionnaire was administered one time at the end of the course to all students.

Wisconsin Observation Scale. The Wisconsin Observation Scale (Appendix A) was used as a tool to monitor the appropriate use of the four instructional strategies in the classroom. This protocol was developed by the University of Wisconsin (1998). It is an open-ended survey filled in by the researcher to identify techniques used by the instructor.

Data Analysis

The data collection matrix (Appendix G) developed by the researcher was established to show that the independent variables, the teaching strategies, and the dependent variables, student attitudes toward mathematics, could be validated through the data collection methods discussed earlier. The surveys and observation scale looked for significant differences in student attitudes between the beginning and the end of the semester. The transcripts and questionnaires were used to detail why or why not a change in attitude occurred through coding for common themes.

There are steps taken in analyzing qualitative research. The analysis begins with analysis of the data through reduction. Data reduction refers to the process of selecting, simplifying, and transforming the data that will appear in handwritten notes about the observations, transcriptions of the focus group discussions, questionnaires, attitude inventories, course evaluation completed by the students, and the observation scale protocol. Information given to the researcher that did not address the specific questions outlined in the study was not used. The data that directly dealt with attitudes was further used for writing summaries, coding behavior of students by type of instruction and type of attitude related to the instructional strategy, looking for themes, making clusters of similar ideas, and writing memos. Data reduction was used to sort out unnecessary and necessary statements, to find common themes, to discard useless information, and, ultimately, to organize the data so final conclusions could be made (Miles & Huberman, 1994).

This meant that after the data reduction was completed, analysis continued by looking for descriptive conclusions in the data. This was done by looking for common themes among student responses or comments and their actions in the observations. The conclusions that could be drawn from these common themes were used to describe what was happening with the students in the classroom. Descriptions were used to make the phenomena understandable and clear to show how all of the data fit together. The main themes lead to impressions which set up summary statements.

There was also quantitative data analysis in this study. Match pair t-distributions were used to determine the outcome of the Attitude Inventory. This analysis revealed statistically significant differences between the first and second distributions of the inventory for enjoyment. Although the quantitative data did not show statistically significant differences for the remaining attitudes, the qualitative data revealed attitude changes regarding confidence, motivation, anxiety, value, and enjoyment.

Data analysis is more successful if triangulation is a key part of the qualitative research. Triangulation of each research question required the researcher to use at least three methods of data collection so that the findings were confirmed by more than one source. This study used the triangulation of observations, focus group discussions, an attitudinal inventory survey, observation scale, course evaluation, and questionnaires.

Validity and Reliability

The results emerging from the data have to be tested for their validity. It is not enough to say that data collection and analysis carried out properly will result in valid conclusions. There are some standard that could help us judge the quality of the results. From tradition, Miles & Huberman (1994) identify four issues that should be addressed in a qualitative study. The five issues are internal validity, external validity, reliability, and objectivity.

Internal validity refers to the credibility and authenticity of the study. The researchers would ask if the findings of the study make sense. Internal validity is supported through triangulation. This study had triangulation through surveys, observations, questionnaires, and focus groups. The triangulation provided numerous methods for a conclusion. The study was conducted over an extended period of time so that data collection could be conducted to establish patterns. Triangulation of the data in this study validated the findings because the data sources supported each other. With regards to what instructional strategies impact attitudes, triangulation occurred through focus groups, the questionnaire, Attitudes Toward Mathematics Inventories, course evaluation, and personal observations. When looking at feelings regarding being a student of mathematic and the relevance students find in math for personal and professional experiences, triangulation happened through focus groups, Attitudes Toward Mathematics Inventory, the questionnaire, and personal observations. The triangulation of focus groups, Attitudes Toward Mathematics Inventory, and personal observations determined the impact of standards-based practices on student engagement in the classroom.

In research, it is necessary to see if the findings can be applied to other settings. In qualitative research this is described as external validity or generalizability. In qualitative research, this is described as transferability. Because this is an action research project using primarily qualitative data, it is not appropriate to generalize. Instead the researcher sought to increase transferability by providing rich thick description of the research to enable the reader to determine if findings are transferable to his or her situation.

The third issue for qualitative studies involves reliability. Reliability or dependability of the study is based on consistency and stability over time without regard to researcher or methods. The findings should be the same regardless of the time and conditions under which the data is collected. The findings for this research are consistent because the sample and setting are the same each semester. This study discussed students' individual perceptions and opinions, which are important elements because they should remain consistent without researcher influence. The prolonged contact the researcher had with the participants enhances the validity and reliability of the study. The researcher had repeated contact, was able to report information firsthand, and observe the behavior.

The last test for validity is objectivity. Will the findings of the study be confirmed by processes other than just the researcher? Objectivity is enhanced by putting aside biases and personal perspectives during analysis by allowing participants many ways to express their perspectives, and by member checks. Questionnaires, focus groups, and surveys provided participants multiple opportunities to reflect and share thoughts about the teaching pedagogies and how these pedagogies impacted their attitudes. Tabulating the frequency of these themes enhanced objectivity for this study. Also, member checks were conducted with the participants to confirm researcher interpretations of responses. By using member checks and work-study helpers, consistency of data entry and analysis was insured. The overall impact of these steps assured accurate reporting of results and interpretation of findings. Not only did the study assess the impact on student attitudes, but it also validated the use of the four teaching standards-based pedagogies in the learning environment. Verifying the use of the teaching strategies also enhances objectivity. The study used the observation scale, course evaluation, personal observations, and comments from student questionnaires and focus groups to verify that all four teaching strategies were used. From field notes to transcriptions, this study could be followed by peers to corroborate the conclusions.

Summary

By using a mix-designed study, the researcher assessed potential benefits of using standards-based pedagogical strategies in a college mathematics course. The standards-based pedagogy was defined as using components of the standards-based practices endorsed by the National Council of Teachers of Mathematics, specifically cooperative learning, discourse, problem solving, and graphing calculators.

The participants in the study were students enrolled in college algebra courses taught by the researcher. Once the semester began, the students took an Attitude in

Mathematics Inventory to establish their current beliefs in their own success as a student of mathematics. A simple survey established students as having negative, neutral, or positive attitudes toward mathematics. A portion of these students were used in focus group discussions in order to allow them to expand on their feelings and experiences in the mathematics classroom. Other data collection methods included questionnaires and student evaluations of the classroom techniques. In order to insure that the researcher was using standards-based teaching practices, a Wisconsin Observation Scale, developed by the University of Wisconsin, was used to document that the correct protocol was being following in the classroom.

This study sought to discover if student attitudes and actions were affected by the four teaching methods, cooperative learning, discourse, problem solving, and graphing calculators in the college algebra course presented. The qualitative measures used in this study offered the opportunity for students to give detailed statements on their evaluation of the course while the quantitative data provided statistics which supported the qualitative data that a change in attitude occurred. This will provide valuable information for instructors who wish to use standards-based pedagogical strategies.

CHAPTER 4

RESULTS

Introduction

The purpose of this study was to assess potential benefits of using components of the standards-based pedagogy with community college students and how the pedagogies impact students' attitudes toward mathematics. Results from the analysis of data obtained in this study are reported in this chapter in tabular and narrative form. The impact of the four aspects of standards-based instruction on student attitudes toward mathematics is analyzed using the methods and procedures described in Chapter Three. The chapter begins with an overview of four standards-based practices used in the study, cooperative learning, discourse, problem solving, and graphing calculators. Each practice is defined, verification of the use of the practice is given, and examples of the practice being used in the classroom are described. The second part of the chapter describes the data sources and results for the dependent variables, which are student attitudes, perceived relevance of mathematics, student perceived impact of instructional strategies, and student engagement. The chapter ends with an explanation of the triangulation used in the study to answer the research questions. The descriptions will be followed by discussion of common themes and patterns in the data analysis related to the dependent variables, the student attitudes.

Practices in Classroom

The independent variables in this study were four teaching practices used in the College Algebra courses. The practices were cooperative learning, problem solving, discussion, and graphing calculator use. Using the data collected in the study, descriptions will be given on how the practices were implemented in the College Algebra class sessions.

Cooperative Learning

In a cooperative learning environment, students work together on a given task while still being held accountable on an individual basis. Cooperative learning gives students a common goal where the group will be rewarded for its efforts (Johnson & Johnson, 1988). In this study, a lesson was considered to include cooperative learning if the following conditions were present: (1) The students had to be placed in groups of at least three people, (2) students were assigned individual tasks to complete the project, and (3) students were graded on the group's outcome of the assignment. Examples of lessons that were considered cooperative learning include "plotting points," "balloon task," and "discovering transformations."

Examples of Cooperative Learning

In order to illustrate the way in which cooperative learning was used in this study, three lessons are described in detail. For this study, cooperative learning tasks were designed by placing students into groups that remained intact throughout the semester. Within each group, students were assigned a duty, which helped the group complete the task. Examples of these duties are things such as gopher, or material retriever, records keeper, timekeeper, coach or leader, and spokesperson. For some tasks, a student may be responsible for more than one duty. If a student fails to complete their portion, the student loses participation points for the day.

For most of the cooperative learning activities, the students are given a handout that describes the goal of the task, identifies the materials students will need, outlines the instructions, and asks for feedback from their results. The handout enables the students to choose their duties for the day and how they will accomplish the projected outcome. While students are working within their groups, the instructor rotates among the groups as a facilitator and guide. If students have questions, they are expected to sort it out within their group before they ask the instructor. Upon completion of the task, students share their results with the remaining groups. Often times they are expected to have charts or other visual aids available that the students can follow.

One lesson with cooperative learning involved identifying *x*- and *y*-intercepts. The groups were given graphs and asked to identify the points where the graph crossed the *x*- and *y*-axis'. Each person in the group was given a different equation where they had to identify the intercepts. Once a person in the group determined the intercepts, the information was shared among the rest of the group. The students worked quietly within the groups. Once all groups had completed the first task, they were instructed to write a process for how to find *x*- and *y*-intercepts. The groups were expected to use their experiences in the first part of the task to help them. Students were given ten minutes to write the rule on large sheets of paper. The spokesperson for the group shared the rule with the rest of the class. Once the rules were placed around the room, the class discussed similarities and differences among them. The students were also expected to pick out flaws in any of the posted rules. Although this was one of the first tasks for the semester, student comments at the end of the week reflected their positive attitudes toward the learning environment. Statements from students included, 'The group work helped me understand the process better.' 'As a returning student, it was helpful to review with other students.'

The balloon activity, “Up, Up, and Away”, afforded another opportunity to use cooperative learning and to also incorporate graphing calculators, and problem solving. After students moved to their groups, members were assigned duties to complete the task. Students had to blow up the balloon, measure the circumference of the balloon using string and a ruler, and then record the time it took for the balloon to deflate. The process was repeated five times. The gopher retrieved the materials needed for the group. The gopher was also responsible for measuring the balloon once the coach had blown it up. The timekeeper gave the seconds it took the balloon to deflate. The recorder wrote down the times stated by the timekeeper. Students were encouraged to make the balloon different sizes to see the effect of small balloons versus large balloons. The task was designed to encourage students to investigate slope, linear regression, and use the linear regression formula to predict values. The instructor provided the students with graph paper so that once the data was recorded, the students could graph the information to predict how long it would take a balloon of *n* size to deflate. Before the students began on the last part of the activity, the instructor discussed with the class the importance of the correlation coefficient. Once the groups found the value of r, each group was asked to explain what type of relationship they had. Most groups stated they had a strong relationship, but others had values of r closer to 0.5. It was helpful that the groups had such variation in the correlation coefficient. This enabled the groups to talk about steps they took during the experiment that could have affected the relationship. Were there things the students could have changed, or improved, to change their results? The students were expected to follow the directions from the instructional sheet for input of their data, drawing a scatter plot on the grapher, and finding the linear regression equation. A couple of groups were able to go through the steps without too much input from the instructor, while the remaining groups consistently asked questions and wanted reassurance that they were doing the activity correctly. Once the groups had the regression equation, they had to predict deflation times for various sized balloons using the regression equation. By the end of the class, students were expected to write a statement about the relationship they had found between the two variables to validate their understanding of linear regression and correlation. The groups also had to turn in a paper copy of the scatter plot they had constructed on the graphing calculator.

Cooperative learning, as well as problem solving, and graphing calculators, were also used after a brief introduction of graphing functions. The topic for the day was transformations of functions. Each group received a marker, large piece of graph paper, and one of the six common functions. The instructor provided them with the original function and two simple vertical and horizontal transformations. For example, one problem was $f(x) = x^2$, a second problem was $f(x) = (x - 2)^2$, and a third problem was $f(x) = x^2 - 2$. Then students were instructed to find common formulas that would designate when the graph moved up, down, left, or right. Using their graphing calculators, groups sketched the equations onto the bigger paper so they could better examine the functions and variations of the same equation. The groups had to describe what was happening to $f(x) = x^2$ and give rules which could be applied to any function, not just the squared function. Next, the groups were given two more graphs so they could investigate reflections, vertical stretches, and vertical shrinks. Again, the groups were expected to find rules so that functions could easily be graphed without tables. The final component required groups to write their own transformation for the function they were working with. Groups were instructed not to graph the function they had created. Another group would graph the transformation using the rules they had just developed. The groups rotated around the room from table to table graphing other groups' functions to gain experience with recognizing the formulas. The groups had developed formulas and tested them while practicing graphing techniques.

Verification

Various data methods were used to verify the use of cooperative learning in the classroom. This section will describe the data sources and the findings.

University of Wisconsin Observation Scale. The observation scale (Appendix A) gives statements about what happened in the classroom. Students were then expected to choose from a group of statements the one that best reflected the original statement. Of the 12 questions, one statement was directly related to collaborative working relationships.

Cooperative learning received high ratings from the students who felt that everyone seemed to stay on task, working hard to complete the problem (69.6%, n = 16). Table 1 shows that 65.2% (n = 15) of the students found themselves involved with classmates in solving problems and working with others to either learn or assist another student in learning material. None of the students felt that the group work was individualized. They all stated that at some level, more than one person was working on the task.

Table 1
Student Responses for the University of Wisconsin Observation Scale by Percentages
Question 11. Interactions among students reflected collaborative working relationships.

Percentage of Responses	Number of Responses	Statements students could choose under Question 11
0.0	0	None of the students were working together in small groups or in a large-group setting. If students were working in small groups, then one student typically gave answers to other members of group without explanation of why certain procedures were used.
0.0	0	Few students were sharing ideas or discussing how a problem should be solved in small groups or in a large-group setting. Although students physically sat together, there was little exchange of ideas or assistance. Many of the students in a group were working on different problems and at different paces.
34.8	8	Some students were exchanging ideas, or providing assistance to their classmates; however, a few students relied on other members of the group to solve problems. Contributions to solving problems were not made equally by all students.
65.2	15	Most students were involved with their classmates in solving problems and made sure that other group members were caught up and understood the problems before moving on to the next problem.

<u>Course Evaluation.</u> The second survey was an assessment completed by the students using a Likert-scale instrument designed by the researcher (Appendix D). This survey directly relates to how effectively the instructor used standards-based teaching pedagogies in the classroom. The evaluation was given once at the end of the semester (n = 21). Table 2 lists the percentages for student responses to each statement that pertained to cooperative learning. These statements are referenced by Qn, where *n* represents the question number on the evaluation.

Students were asked to evaluate how well cooperative learning was integrated into the coursework and the relevance of the activity to the current topic. The percentage shown reflects the number of students who chose to strongly agree, agree, be neutral, disagree, or strongly disagree with the given statement. Over half of the students reported that cooperative learning was used to relate the topics being taught (57.1%, n = 12), there was integration of concepts between chapters (47.6%, n = 10), connections were made between material previously learned and current content (33.3%, n = 7), and almost half of the students were given the opportunity to explain and defend their solutions (42.9%, n = 9).

Table 2
Evaluating the Course Responses for Cooperative Learning by Percentage of Students who Took the Survey

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
Q1	57.1 n = 12	42.9 n = 9	0 n = 0	0 n = 0	0 n = 0
Q2	47.6 n = 10	33.3 n = 7	19.1 n = 4	0 n = 0	0 n = 0
Q14	42.9 n = 9	42.9 n = 9	14.3 n = 3	0 n = 0	0 n = 0

Student Questionnaires. Every other week, students were asked to complete a questionnaire (Appendix B) that encouraged them to reflect on the learning environment over that period of time. The questionnaire was designed such that students would circle an appropriate attitude, or attitudes, that were impacted by the standards-based instruction. Although there was ample room for them to explain how that attitude was impacted, few students wrote descriptions as the directions had instructed. The data in this analysis are related to those students who wrote thoughts and reactions on the questionnaires. In order to code the data from the questionnaires, the researcher completed a contact summary form for each questionnaire that was completed. The form was used to record main issues or themes, summarizations of information written, and interesting or illuminating comments from the contact. As the data was analyzed, common themes regarding cooperative learning started to emerge. The data was coded first according to the teaching pedagogy it referenced and then, secondly, the impact the pedagogy had on that attitude. Once these themes were identified, the researcher went through the questionnaires again to tabulate the frequency of each theme. Table 3 illustrates the analysis of the questionnaires with respect to cooperative learning. The themes included things such as attitudes toward working with others, the impact of working with others, and anxiety about exams. Student comments could be distributed under the appropriate theme for easier analysis. For the code, attitudes toward working with others, the researcher found that seventeen students specifically stated they like working with others, while one student was not comfortable working with groups. While this theme had only two subgroups, the code, impact of working with others, had several. Working with others improved the understanding of material (n = 14), sharing ideas allowed for the viewing of multiple ways to complete a problem (n = 11), cooperative learning made the work easier (n = 6), cooperative learning allowed students to help each other (n = 5), working with others increased the enjoyment of math (n = 7), and cooperative learning made class enjoyable (n = 4). The last theme, anxiety about exams, had only one subgroup. A few students (n = 3) wrote that cooperative learning decreased their anxiety when taking exams. Often times, students only wrote ‘anxiety about tests’ or ‘class is more fun’. However, some students did elaborate. These written comments, coded under the impact of cooperative learning, included:

“Cooperative learning increased confidence because there was an opportunity to learn from others, which increased knowledge.”

"Togetherness [cooperative learning] forms unity which will help with future job skills."

Table 3
Coding for Student Questionnaires – Cooperative Learning

Code	Comment	Frequency
AWO (Attitudes working with others)		
	Liked working with others	7
	Not comfortable working with others	1
IWO (Impact working with others)		
	Improved understanding of material	14
	Sharing ideas gives multiple ways to do problems	11
	CL made the work easier	6
	CL allowed students to help each other	5
	Increased enjoyment of math	7
	Made class enjoyable	4
AE (Anxiety about exams)		
	Decreased anxiety for test	3

Time on Task Matrix. A daily tabulation was maintained in order to obtain the frequency that each teaching practice was administered. The matrix (Appendix F) shows that over the semester, a teaching practice, other than lecture, was implemented 20 times out of a total of 37 class periods. Although this may appear low, nine days were used for exams. Of the 20 times, cooperative learning was used individually or in conjunction with another task 40% (n = 8) of the classroom periods where an alternative teaching strategy was implemented. Table 4 identifies the tasks associated with cooperative learning. Some of these tasks were combined with problem solving, discourse, or graphing calculators when implemented. Appendix F details when classroom tasks involved more than one teaching practice.

Table 4
Frequency of Usage for Cooperative Learning

Date	Activity
February 7	Plotting Points
February 9	Rules for Intercepts
February 18	Balloon Task
February 25	Rubber Bands and Linear Regression
March 18	Discussion on Math Education
April 27	Converting from Systems to Matrices
May 2	Linear Programming

Based on the data analysis, cooperative learning was an integral part of the curriculum for this course. Students recognized when it was used in the classroom as well as verified the use of cooperative learning in their responses on the Student Questionnaires, The University of Wisconsin Observation Scale, and the Course Evaluation. The researcher was also able to verify the use of cooperative learning through a time on task matrix.

Discourse

Discourse is a way that knowledge is constructed and shared in the classroom. Students engage in standards, explore ideas and concepts, and negotiate the meanings and connections of math concepts with fellow classmates (Manouchehri & Enderson, 1999). Students are actively engaged in doing mathematics through a process where they realize that mathematics is about questioning, conjecturing, and trial and error (Nickson, 1992). NCTM calls for students to have the opportunity to reflect upon and defend their thinking. They should be able to express their ideas both orally and in written form. Since reading is an important tool for sound oral discussion, students should be encouraged to read the textbook and other relevant materials. The use of small groups to promote discussion can be very effective (NCTM, 1989). For this study, discourse was considered to occur when students were individually, or in groups, reflecting on a mathematical problem that had been completed, discussing procedures to solve a new problem, or conversing about math education topics.

Examples of Discourse

This section describes three days in the classroom where discourse surrounding a mathematical task occurred. The discourse was conducted in an open floor format where students randomly gave their input with little encouragement from the instructor. When the discussion focused on mathematical procedures, students were asked to write down their rules or actions to share with the class.

One examples of a lesson coded as including discourse was on the topic of logarithmic functions. The class session previously had been on exponential functions, and students were engaged in making connections between exponential and logarithmic functions. In order for students to understand the connection between these two functions, the students were placed into their groups. They were given worksheets, dry erase grid boards, and the graphing calculators. Students were instructed to follow the directions on the worksheets. The ultimate goal for the class period was for students to find the properties for inverse functions. First, students had to graph an exponential function. Then, they were given a logarithmic function to graph on the same grid. The groups had to discuss the similarities and differences between the two graphs amongst each other.

The instructor asked if the groups could see a relationship between the domain and range. When the groups did not answer, they were prompted to make tables of (x,y) coordinates for each graph. The groups had to draw conclusions about the domain and range of each function. The instructor walked around the room listening to the student discourse. Finally, the discourse was extended through whole class discussion of the problem. The instructor asked for properties of the inverse functions. The spokesperson from each group was asked to give the summary from their group. This dialogue is from the first class period.

> Group #1: We noticed that they were symmetrical. We can fold our graph, like with the functions in Chapter 3, and they're on top of each other.
>
> Instructor: Very good. Does another group have something to add?
>
> Group #2: We thought our graphs looked the same. We drew the line $y = x$ and saw that our graph was symmetrical to that line. It's like what we did with the other functions.
>
> Group #1: Yeah, like that. We also saw that the same rule about domain and range applied here. The numbers were reversed and…
>
> Group #3: Exactly. When we made the tables, all we had to do was switch the columns and the numbers were the same.
>
> Instructor: Group 4, can you make a list of the properties these groups have found and incorporate them into what you found?
>
> Group #4: Sure. We said that the graph should reflect the $y = x$ line and that the domain and range get swapped. We couldn't find any others.

Students also explored natural exponentials and natural logarithms in the same manner. The second half of the lesson, working with natural exponential and natural logarithmic functions, was quicker than the first because the rules did not change much and the students quickly discovered that aspect of the task. Students reflected on mathematical concepts in order to develop properties of inverse functions. They had to discuss their findings with the entire group in order for students to agree on the properties.

In addition to discourse that was focused on a mathematics task, the students were asked to engage in discussions about mathematics education, based on readings. Students read five articles chosen by the researcher related to teaching mathematics, the history of mathematics reform, and uses of mathematics in real life. Students had approximately two months to read the articles and write a reflection paper. One class day was devoted to student discussion of the articles. The students were placed in their groups and then expected to discuss common themes among the articles. While in their groups, they were also told to discuss the past and future of mathematics education. After 25 minutes, the groups took turns presenting their ideas to the class. Members from other groups not only questioned but also contributed to the ideas presented. The following is an excerpt from one discussion:

> Group #1: The future of math education is to go more in depth at younger ages.
>
> Group #2: I don't think I would have so many problems with math now if my teachers had taught it to me like we learn in this class or in the articles.
>
> Group #1: Yes, we all agree that younger students would like the games.

In a whole group discussion, the following topics were mentioned and discussed: group work and hands on activities are beneficial, connections between math and personal experiences are important for instructors to make because math improves job skills. One group of students also talked with amazement how math was not required until after World War II. They thought that mathematics had always been an important part of school. This discussion of math education was continued in focus groups where one student felt that he had finally made the connection between mathematics and our personal lives. He was surprised that math was so connected to other things. Because this lesson included discussion of mathematical topics amongst small groups followed by whole class discussion, it was coded as a "lesson with discourse".

The third example of a lesson including discourse focused on relationships between independent and dependent variables. Discourse rarely occurred without the presence of one of the other three practices identified in this study (problem solving, graphing calculator use, and cooperative learning). However, in order for students to distinguish between independent and dependent variables, it was the only teaching pedagogy used for the lesson. Students were placed into their groups, where each group was given a line graph, histogram, or pie chart. The groups had to identify what information the graph displayed and how the information was being measured. In a previous chapter, students looked at independent and dependent variables. Therefore, the groups were asked to label the independent and dependent variables in their graphs. Each group had to present their graph, identify the variables, and discuss the relationship between the two. Once all of the graphs were presented, the instructor asked the students if they saw any commonalities among the dependent variables. Students responded to the question, "Would they be able to pick the dependent variable from any graph?" In one class, the discussion began with a few students trying to determine which axis represented the dependent variable. These students spoke confidently that the horizontal axis was the dependent variable. Other students were quick to argue that the dependent variable was on the vertical axis. "The dependent variable is usually a measurement." "You [the instructor] told us we choose the independent variable. That's what is on the horizontal axis." The few students who struggled with this definition asked for more explanation. Then each group was asked once again to talk about their graphs explaining the characteristics of each variable. Once each group was finished, the struggling students were asked, “Does it make sense now?” One student spoke up, “Yes. Looking at the graphs again helped.” A few other students agreed. This lesson was consumed with discussion as students attempted to help each other understand independent variables and dependent variables.

Verification

This section describes the data sources used to verify the use of discourse in the classroom. Two surveys were distributed to allow students to rate the effectiveness of the discourse in the course. Along with the surveys, student questionnaires were also administered to give students the opportunity to write about the attitude discourse impacted. However, few students wrote comments and simply circled the attitude they felt for that week. The final data source was a matrix created by the instructor to keep track of how many times discourse was used in the course, alone or with another teaching strategy.

University of Wisconsin Observation Scale. The observation scale (Appendix A) gives statements about what happened in the classroom. Students were then expected to choose from a group of statements the one that best reflected the original statement. Of the 12 questions, six statements were related to discourse in the classroom.

According to the students, discourse was present in the classroom. The percentage of students who marked the statement is given in the results. Table 5 represents a question that is related to student statements in the classroom, the teacher valued students’ statements about mathematics and used them to build discussion or work toward shared understanding for the class. Most of the students found there were stimulated discussions, which led to discourse about the lesson (78.3%, n = 18). In other questions related to discussion and engagement, students not only marked that the teacher opened up discussion by using student responses, but they also agreed that the teacher encouraged student inquiry in order to investigate mathematical topics (87.0%, n = 20). When students reflected on their solutions, discussion ensued regarding the reasonableness of the answer (65.2%, n = 15). If there were questions, students felt that they could ask a classmate how to solve a particular problem and discuss alternative strategies (56.5%, n = 13).

Table 5
Student Responses for the University of Wisconsin Observation Scale by Percentages
Question 7. The teacher valued students' statements about mathematics and used them to build discussion or work toward shared understanding for the class.

Percentage of Responses	Number of Responses	Statements students could choose under Question 7
0.0	0	The teacher was interested only in correct answers. The majority of the teacher's remarks about student responses were neutral short comments such as "Okay," "All right," or "Fine." No attempt was made to use students' responses to further discussions.
21.7	5	The teacher established a dialogue with the student by asking probing questions in an attempt to elicit a student's thinking processes or solution strategies.
78.3	18	The teacher valued students' statements about mathematics by using them to stimulate discussion or to relate them to the lesson in some way. The teacher opened up discussion about the student response by asking other students questions such as: "Does everyone agree with this?" or "Would anyone like to comment on this response?"

Course Evaluation. The second survey was an assessment completed by the students using a Likert-scale instrument designed by the researcher (Appendix D). This survey directly relates to how effectively the instructor used standards-based teaching pedagogies in the classroom. The evaluation was given once at the end of the semester (n = 21). Table 6 lists the percentages for student responses to each statement that pertained to discourse. These statements are referenced by Qn, where *n* represents the question number on the evaluation.

Students were asked to evaluate how well discourse was integrated into the coursework and the relevance of the activity to the current topic. The percentage shown reflects the number of students who chose to strongly agree, agree, be neutral, disagree, or strongly disagree with the given statement. Students reported that discourse was used to relate the topics being taught (61.90%, n = 13), there was integration of concepts between chapters (42.86%, n = 9), connections were made between material previously learned and current content (57.14%, n = 12), there was opportunity for you to reflect on ideas openly in the classroom and discuss your thoughts with other students (42.86%, n = 9), and students were given the opportunity to explain and defend their solutions (47.62%, n = 10).

Table 6
Evaluating the Course Responses for Discourse by Percentage of Students who Took the Survey

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
Q3	42.9 n = 9	57.1 n = 12	0 n = 0	0 n = 0	0 n = 0
Q9	42.9 n = 9	42.9 n = 9	14.3 n = 3	0 n = 0	0 n = 0
Q10	47.6 n = 10	33.3 n = 7	19.1 n = 4	0 n = 0	0 n = 0
Q11	61.9 n = 13	38.1 n = 8	0 n = 0	0 n = 0	0 n = 0
Q15	57.1 n = 12	38.1 n = 8	4.8 n = 1	0 n = 0	0 n = 0

Student Questionnaires. Every other week, students were asked to complete a questionnaire (Appendix B) that encouraged them to reflect on the learning environment over that period of time. As the data was analyzed, common themes regarding discourse started to emerge. The students were asked to circle which attitude, or attitudes, were impacted by the teaching pedagogy. They were also asked to write, with description, how the attitude was impacted. Not every student wrote out the descriptions. However, for the questionnaires with explanation, the themes were reactions to talking in front of others, the impact of discourse on learning, and how discourse affected the desire to learn math beyond the college algebra course. Table 7 illustrates the coding of the student comments from the questionnaires with regards to discourse.

Table 7
Coding for Student Questionnaires - Discourse

Code	Comment	Frequency
BAO (Talking in front of others		
	Anxiety over talking in front of others	4
	Improved confidence when talking with others	10
ID (Impact of Discourse)		
	Review helps (more understandable)	15
	Don't mind getting wrong answer, helps to learn	2
	Reassures thoughts	4
	Didn't understand other students	1
	Good explanations	8
	Anxious when discussing unfamiliar topics	1
	Starting to become familiar with discourse	6
MMD (Math beyond this course - discourse)		
	Makes student want to learn more about math	3
	See applications beyond course	1

Some of the students were not comfortable, in the beginning, with discussion periods, but did like how this minimized lecture time. Toward the end of the semester, more students stated they were confident with talking in front of others (n = 10). Some of the discussions served as review for a few students (n = 15) while others found that they wanted to learn more mathematics because of the discussions (n = 4). Many students felt that the discussions were an opportunity to participate in good explanations of the material (n = 8). A small number of students also stated they found value with mathematics when discussing topics with other students (n = 6).

<u>Time on Task Matrix</u>. A daily tabulation was maintained in order to obtain the frequency that each teaching practice was administered. The matrix (Appendix F) shows that over the semester, a teaching practice, other than lecture, was implemented 20 times out of a total of 37 class periods. Although this may appear low, nine days were used for exams. Of the 20 times, discourse was used individually or in conjunction with another task 25% (n = 5) of the classroom periods where an alternative teaching strategy was implemented. Table 8 identifies the tasks associated with discourse. Some of these tasks were combined with problem solving, graphing calculators, or cooperative learning when implemented. Appendix F details when classroom tasks involved more than one teaching practice.

Table 8
Frequency of Usage for Discourse

Date	Activity
February 9	Rules for Intercepts
February 14	Parallel and Perpendicular Lines
March 11	Independent versus Dependent Variables
March 18	Discussion on Math Education
April 15	Defining Logs Using Inverse Functions
May 2	Linear Programming

Based on the data analysis, discourse was a part of the curriculum for this course. Students recognized when it was used in the classroom as well as verified the use of discourse in their responses on the Student Questionnaires, The University of Wisconsin Observation Scale, and the Course Evaluation. The researcher was able to verify the use of discourse through a time on task matrix.

Problem Solving

It is through teaching via problem-solving that students acquire various methods of solving mathematical problems. Teaching via problem-solving is where students move from “the concrete (a real-world problem…) to the abstract (a symbolic representation…)” (Schroeder & Lester, 1989, p. 33). The way students represent and connect pieces of knowledge is an important part in whether they will understand it deeply and can use it in problem-solving (NRC, 2001). The goal is not only for students to solve the problem, but to also use various concepts to do it (Bay, 2000). Problem-solving shifts the focus from memorization of facts and procedures to students investigating material and making conclusions or developing questions. The most important role is that student understanding increases. For this study, problem solving tasks were identified as a task solved by a student or group using previous knowledge and concepts they had already acquired in the college algebra course, without additional input from the instructor. Students were expected to solve problems by making connections between current and previous topics. The goal for the task is for students to develop procedures or rules to solve the application.

Examples of Problem Solving

Various lessons were designed to incorporate problem solving. This section describes a few of these lessons to illustrate how problem solving was used.

Problem solving was used on a lesson involving slope as a rate of change. Students were given a problem that asked them to write an equation that would identify the total cost of a reception with x number of guests. The activity had two different scenarios and the groups had to identify which scenario was the better deal for the cost of the reception. This was coded as a problem solving lesson because the groups were expected to use their knowledge of slope and equations of lines and apply it to a real world scenario. The goal was for students to see slope as a rate of change. Once the groups wrote the equation, the problem prompted the groups to identify what happened to the cost of the reception as x number of people were added. The instructor walked around the room listening to the communication between group members. Several students saw that as the number of people increased, the cost of the reception increased the amount of the coefficient on x, although they did not make the connection between this value and the slope of the line. The next part of the problem told the students to graph their cost equation. When the groups did this, then the students saw the connection between the coefficient and the slope of the equation. The final part of the problem asked the students to describe what information the coefficient on x gave them. When the first group presented their answer, that the coefficient was the increase in cost per person, the other groups nodded their heads in agreement. In one of the classes, a student commented on how the application helped to view the slope as a rate of change rather than just rise over run. When discussing problem solving later in a focus group interview, two students expressed that it was fun because the class used tools to help them solve the problem.

Several problem solving activities were implemented in the unit on functions. One task that involved problem solving and graphing calculators focused on rational functions. Students were given a worksheet that was designed to have students explore what happens to the value of $f(x)$ as x approaches positive and negative infinity. Students also had to investigate the behavior of the graph as x approached the value that made the denominator undefined. This less was coded as problem solving because it encouraged the students to examine the behavior of a function and explain that behavior. The students were using their knowledge of graphing functions and the properties of denominators to produce properties for rational functions. Each group had a different rational function to graph on the graphing calculator. They had to copy the graph from the calculator to a grid board. Drawing on the board would allow the group to view a bigger picture of the function enabling them to identify key elements of the graph. The groups completed tables to look for patterns in these scenarios. Then the table of values was compared to the graphs to look for relationships. Different functions allowed the groups to identify the three ways to name horizontal asymptotes. Each group presented their function to the class and then the class discussed how to identify the horizontal asymptote by looking at the rational function. It was not necessary to discuss vertical asymptotes in depth because many of the students quickly figured out how to identify the vertical asymptote. One group gave a simple explanation on how they used the graph to determine vertical asymptotes. This value was the same number that made the denominator equal to zero. The class concluded by making rules for finding the vertical and horizontal asymptotes.

Verification

In order to verify the use of problem solving in the classroom, two surveys were distributed to allow students to rate the effectiveness of the problem solving in the course. The student questionnaires gave students the opportunity to write about the attitudes problem solving impacted and how. The second data source was a matrix created by the instructor to keep track of how many times problem solving was used in the lessons either alone or with another teaching strategy.

University of Wisconsin Observation Scale. The observation scale (Appendix A) gave statements about what happened in the classroom. Students were then expected to choose from a group of statements the one that best reflected the original statement. Of the 12 questions, five statements were related to problem solving in the classroom.

According to the results of the observation scale, problem solving was present in the classroom. The percentage of students who marked the statement is given in the results. Table 9 represents a question that is related to applications in the classroom, connections between mathematics and students' daily lives were apparent in the lesson. Over half (52.2%, n = 12) of the respondents agreed connections were made between mathematics and students' daily lives. A few students felt the course gave them the problem solving tools needed to expand their knowledge in mathematics for future opportunities (38.1%, n = 8). The results also revealed more than half of the participants felt that the problem solving tasks were related to the topics being taught (52.4%, n = 11).

Table 9
Student Responses for the University of Wisconsin Observation Scale by Percentages
Question 4. Connections between mathematics and students' daily lives were apparent in the lesson.

Percentage of Responses	Number of Responses	Statements students could choose under Question 4
4.4	1	Connections between mathematics and students' daily lives were not apparent in the lesson.
39.1	9	Connections between mathematics and students' daily lives were not apparent to the students, but would be reasonably clear if explained by the teacher.
52.2	12	Connections between mathematics and students' daily lives were clearly apparent in the lesson.

Course Evaluation. The second survey was an assessment completed by the students using a Liker-scale instrument designed by the researcher (Appendix D). This survey directly relates to how effectively the instructor used standards-based teaching pedagogies in the classroom. The evaluation was given once at the end of the semester (n = 21). Table 10 lists the percentages for student responses to each statement that pertained to problem solving. These statements are referenced by Qn, where *n* represents the question number on the evaluation.

Students were asked to evaluate how well problem solving was integrated into the coursework and the relevance of the activity to the current topic. The percentage shown reflects the number of students who chose to strongly agree, agree, be neutral, disagree, or strongly disagree with the given statement. Eleven of the twenty-one students (52.38%) reported that problem solving was used to relate the topics being taught. Other results of the evaluation found that the course gave problem solving tools needed to expand knowledge in mathematics with future opportunities (38.10%, n = 8), a few students marked they have a guide to solve problems with problem solving techniques (28.57%, n = 6), and a large majority strongly agreed that connections were made between material previously learned and current content (57.14%, n = 12).

Table 10
Evaluating the Course Responses for Problem Solving by Percentage of Students who Took the Survey

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
Q6	38.1 n = 8	47.6 n = 10	14.29 n = 3	0 n = 0	0 n = 0
Q8	28.6 n = 6	57.1 N = 12	14.3 n = 3	0 n = 0	0 n = 0
Q13	52.4 N = 11	42.9 n = 9	4.8 n = 1	0 n = 0	0 n = 0
Q17	57.1 n = 12	38.1 n = 8	4.8 n = 1	0 n = 0	0 n = 0

Student Questionnaires. Students were asked to complete a questionnaire (Appendix B) eight times over the semester. The questionnaire directed them to reflect on the most recent learning environment experienced in the classroom. The questionnaire was designed for students to circle the attitude impacted by the teaching pedagogy. Students were then expected to write in detail how the attitude was impacted. The analysis of written comments was coded under three common themes, the value of mathematics, direct feeling toward problem solving, and the impact of problem solving. Table 11 displays the student comments and how they were coded for the analysis.

Table 11
Coding for Student Questionnaires – Problem Solving

Code	Comment	Frequency
VM (Value of Mathematics)		
	Pursue other math topics	1
	Shows Applications	4
IPS (Impact of Problem Solving)		
	Not confident about test problems	2
	Confidence with problem solving improves by end	9
	Felt good about test problems	1
	Understand Problems	2
FPS (Feelings about Problem Solving		
	Enjoys figuring out problems, individual or group	22
	Anxiety at beginning, but decreased by end	7
	Doesn’t enjoy problem solving	4
	Anxiety over amount of work	1

At the beginning of the semester, students had anxiety with problem solving, but it decreased as the semester progressed. Toward the end of the semester, a few students stated they were confident with problem solving (n = 9). Many students enjoyed figuring out problems as groups or individuals (n = 22). One student wrote, ‘I can solve problems when I have someone else to bounce off ideas.’ A small number of students also wrote that problem solving showed them applications of the mathematics (n = 4).

<u>Time on Task Matrix</u>. A daily tabulation was maintained in order to obtain the frequency that each teaching practice was administered. The matrix (Appendix F) shows that over the semester, a teaching practice other than lecture, was implemented 20 times out of a total of 37 class periods. Although this may appear low, nine days were used for exams. Of the 20 times, problem solving was used individually or in conjunction with

another task 55% (n = 11) of the classroom periods where an alternative teaching strategy was implemented. Table 12 identifies the tasks associated with problem solving. Some of these tasks were combined with discourse, graphing calculators, or cooperative learning when implemented. Appendix F details when classroom tasks involved more than one teaching practice.

Table 12
Frequency of Usage for Problem Solving

Date	Activity
February 14	Parallel and Perpendicular Lines
February 16	Reception Problem
February 18	Balloon Task
February 25	Rubber Bands and Linear Regression
March 16	Quadratic Regression
March 30	Discovering Transformations
April 1	Rational Functions
April 15	Defining Logs Using Inverse Functions
April 20	Exponential Regression
April 27	Converting from Systems to Matrices
May 2	Linear Programming Exercise

Based on the data analysis, problem solving was a part of the curriculum for this course. Students recognized when it was used in the classroom as well as verified the use of problem solving in their responses on the Student Questionnaires, The University of

Wisconsin Observation Scale, and the Course Evaluation. The researcher was able to verify the use of problem solving through a time on task matrix.

Graphing Calculators

There are many technological tools that can enhance the learning experience in mathematics. The graphing calculator will be the primary source of technology used in this research. Although it has been argued that calculators diminish the learning of basic skills, research does not support this. It has been shown that calculators enhance conceptual understanding, strategic competence, and disposition toward mathematics. Students using calculators were also found to have a better attitude toward mathematics (NRC, 2001; Grouws & Cebulla, 2000; Hiebert, 1999). In this study, a lesson was coded for graphing calculators when graphing calculators were used as an integral component of the lesson. Graphing calculator tasks are defined by student use of the graphing calculator in order to complete the task. Students were expected to use the graphing calculator for exploration and discovery. Simple operational functions on the graphing calculator such as addition or subtraction were not considered as graphing calculator tasks.

Examples of Graphing Calculators

Several examples of the lessons with graphing calculators are discussed in this section. In order to verify how the technology was incorporated into the curriculum, three lessons were chosen.

When teaching students how to graph lines, the graphing calculator was the only teaching practice for this task. Students were placed in their groups so they could lean on each other for help. Each student received a worksheet with step-by-step instructions for inputting linear equations into their calculator. The second part of the worksheet had several linear equations for the students to practice graphing on their calculators. There were also Cartesian planes below the equations so the students could transfer the picture from the calculator to the paper. The class began with the instructor using the TI-83 View Screen to demonstrate some linear equations being graphed on the calculator. First, the students saw how to put the equation into the calculator, even learning how to isolate ‘y’ when necessary. Second, the class discussed the importance of the size of the display window. The groups looked at what happened to the graph if the x-axis and y-axis had both large values and small values. Once the window was set to the standard size of (-10, 10) on the x-axis and (-10, 10) on the y-axis, the students were instructed to hit the graph button. Some students were faster than others, so the instructor had to move slowly when using the view screen so students could follow the sequence more proficiently and not become frustrated. Once the linear equations were on the display windows, students also investigated the line using features such as Zoom, Trace, Calculate Value, Calculate Zero, and setting up a table of values. This lesson was coded as a graphing calculator lesson because the graphing calculator was necessary for students to complete the task. Students were expected to explore graphing lines using technology.

Graphing calculators are a great tool for exploring the behavior of graphs because multiple graphs can be placed on the same grid. When it was time for exponential and logarithmic functions, the graphing calculator was a large part of the instruction. The first day of exponentials students were given three functions resembling $y = b^x$ to graph. The students were asked to look for commonalities among the three graphs. They quickly noticed that all three graphs crossed the *y*-axis at the same point, (0,1). Since the course had already introduced students to horizontal asymptotes, a few of the students noticed how the graph never crossed the x-axis. Other students began to question if the graph would ever cross the axis so the instructor chose values for x that would get closer to negative infinity. Students investigated the behavior of the numbers and concluded that the negative exponent meant large numbers in the denominator. The exponential function could never equal zero. In each class, at least one student pointed out the linear appearance of the right side of the graph. In order for the students to see this as an exponential growth, the instructor gave them a few points to plot. Students were amazed at how fast the y-coordinate increased. The idea of exponential growth was beginning to make sense to the students. To enhance the students' understanding, they also looked at applications of exponential growth, such as population, and the corresponding graph. Since the students used graphing calculators for exploration of exponential functions, this lesson was coded under graphing calculator use.

Besides graphing functions, students used the graphing calculator to solve systems of equations. On the first day of this section, students are expected to solve systems of equations by hand. Although this procedure is tedious, working the problems out helps the students understand the steps the calculator is using to solve the system. To assist the students with use of the graphing calculator, the instructor provided them with instructional sheets for solving matrices. As a class, the students worked a problem together while the instructor was using the view screen. The instructor slowly went over the calculation procedures and walked around the room answering questions from students. The class completed a problem without instructor assistance for practice. Once the problem was completed, the instructor circled the room helping students who were having problems with the procedure. Students were then expected to solve ten systems of equations using the graphing calculator for practice. One class expanded on their feelings with the graphing calculator by saying the problems were easier to work with the graphing calculator. They expressed their desire to only learn using the graphing calculator without the long hand first. The graphing calculator was a key component in student learning of matrices; therefore, this lesson was coded as a graphing calculator lesson.

Verification

Various data methods, course evaluations, students questionnaires, and the time on task matrix, were used to verify the use of graphing calculators in the classroom. This section will describe those data sources and report the findings.

Course Evaluation. The evaluation survey was an assessment completed by the students using a Likert-scale instrument designed by the researcher (Appendix D). This survey directly relates to how effectively the instructor used standards-based teaching pedagogies in the classroom. The evaluation was given once at the end of the semester (n = 21). Table 13 lists the percentages for student responses to each statement that pertained to graphing calculators. These statements are referenced by Qn, where *n* represents the question number on the evaluation.

Students were asked to evaluate how well graphing calculators were integrated into the coursework and the relevance of the activity to the current topic. The percentage shown reflects the number of students who chose to strongly agree, agree, be neutral, disagree, or strongly disagree with the given statement. A large portion of students reported that the course gave technological tools needed to expand knowledge in mathematics with future opportunities (38.1%, n = 8), connections were made between material previously learned and current content (47.6%, n = 10), over half of the participants strongly agreed there was an integration of concepts from chapter to chapter using the graphing calculators (52.4%, n = 11), and graphing calculator tasks were directly related to the topics being taught (57.1%, n = 12).

Table 13
Evaluating the Course Responses for Graphing Calculators by Percentage of Students who Took the Survey

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
Q6	38.1 n = 8	47.6 n = 10	14.3 n = 3	0 n = 0	0 n = 0
Q7	47.6 n = 10	52.4 n = 11	0 n = 0	0 n = 0	0 n = 0
Q12	52.4 n = 11	42.7 n = 9	4.8 n = 1	0 n = 0	0 n = 0
Q16	57.1 n = 12	42.9 n = 9	0 n = 0	0 n = 0	0 n = 0

Student Questionnaires. Every other week, students were asked to complete a questionnaire (Appendix B) that required them to reflect on the learning environment of the classroom over the previous two weeks period. Students circled the attitude impacted by the practice used for the lessons. Some of the students wrote details of how the attitude was impacted. Comments related to graphing calculators were coded for common themes. The themes were attitudes toward calculators and the impact of calculator use. Table 14 shows how student comments were coded under the themes.

A few students reported confidence when working mathematics with graphing calculators (n = 6). Most students enjoyed working with graphing calculators (n = 15) and noted that the calculators lessened their workload (n = 9). At the beginning of the semester, one student consistently had anxiety over having the wrong type of calculator, but noted she had purchased the TI-83 and was now confident about her work.

Table 14
Coding for Student Questionnaires – Graphing Calculator

Code	Comment	Frequency
FGC (Feelings about Graphing Calculators)		
	Anxiety over calculator strokes	8
	Does not like calculators	2
	Enjoys working with calculators	15
	Anxiety over having wrong TI	3
IGC (Impact of Graphing Calculators)		
	Increased confidence with tests and work	6
	Not confident in beginning, but improves	11
	Lessons workload	9
	Understanding more	5

Time on Task Matrix. A daily tabulation was maintained in order to obtain the frequency that each teaching practice was administered. The matrix (Appendix F) shows that over the semester, a teaching practice, other than lecture, was implemented 20 times out of a total of 37 class periods. Although this may appear low, nine days were used for exams. Of the 20 times, graphing calculators were used individually or in conjunction with another task 52% (n = 13) of the classroom periods where an alternative teaching strategy was implemented. Table 15 identifies the tasks associated with graphing calculators. Some of these tasks were combined with problem solving, discourse, or cooperative learning when implemented. Appendix F details when classroom tasks involved more than one teaching practice.

Based on the data analysis, graphing calculators were an integral part of the curriculum for this course. Students recognized when it was used in the classroom as well as verified the use of graphing calculators in their responses on the Student

Questionnaires and the Course Evaluation. The researcher was also able to verify the use of graphing calculators through a time on task matrix.

Table 15
Frequency of Usage for Graphing Calculators

Date	Activity
February 11	Graphing Lines
February 23	Linear Regression
February 25	Rubber Bands and Linear Regression
March 16	Quadratic Regression
March 30	Discovering Transformations
April 1	Rational Functions
April 13	Looking at Exponentials
April 15	Defining Logs Using Inverse Functions
April 20	Exponential Regression
April 25	Introduction to Matrices
April 27	Converting from Systems to Matrices
April 29	Matrix Operations
May 2	Linear Programming Exercise

Attitudes

The dependent variables were measures of how a standards-based learning environment affected student attitudes, and more specifically, which instructional strategies impacted student attitudes, the feelings of the participants regarding being a student of mathematics, the student pursuit of additional math courses or career choices in mathematics, how instructional strategies impacted engagement, and the relevance of mathematics in the students' personal and professional experiences.

The everyday notion of attitude refers to someone's basic like or dislike of a topic or idea. This study uses the definition from the Minnesota Research and Evaluation Project which identifies the following factors pertaining to attitude: attitude toward mathematics, anxiety toward mathematics, self-concept in mathematics, motivation to increase mathematical knowledge, perception of mathematics teachers, and the value of mathematics in society (Ellington, 2003). The Attitudes Toward Mathematics Inventory (Tapia & Marsh, 2004), used in this study, investigates other areas of attitude toward mathematics. Specifically, it assesses confidence, anxiety, value, enjoyment, and motivation. The assessment of a change in attitude over the semester began by asking for students' previous experiences in the classroom.

Previous Experiences

This study began by inquiring about students' previous experiences in the mathematics classroom. On the first day of class, students were given an open-ended survey (Appendix H) prompting them for information about previous experiences in mathematics courses. The results were coded by teaching strategies such as hands-on activities, lectures, note-taking, technology use, group work, and discussions. Student responses were also separated by the impact of the teaching strategies, personal reactions to the strategies, personal relevance of material, amount of participation, and the impact on the students' attitudes toward mathematics being in that environment. The math courses students had been enrolled in ranged from high school algebra courses to college level Intermediate Algebra. The most common instructional strategy the 35 students had been exposed to was lecture (n = 19). The second highest teaching practice was teaching by showing examples on the board (n = 11). In some instances, these practices were combined, but 5 students stated they were taught through only lectures. The use of graphing calculators, group work, and discussions was only mentioned by 3 students. Problem-solving was never specifically stated. However, this could be a reflection of students not using the same definition for problem-solving as the researcher.

With respect to these teaching strategies, most students stated they felt better about mathematics when there were many examples shared by the teacher. For example, one student wrote, ‘The textbook was not helpful. I needed more examples from the teacher.’ Seven students liked different strategies they explored for solving problems. Although the majority of students (n = 19) had previous experiences with lecture, some of those students (n = 11) stated their classroom also had a caring instructor who helped them one on one and showed numerous examples. One student wrote, ‘Our teacher was caring, optimistic, and encouraging.’ Other students (n = 8) had lecture classrooms without caring instructors and wished their instructor had demonstrated a better attitude toward the students. One written comment stated, ‘Few applications were shown; our instructor didn’t help students understand.’

Students not only wrote about their personal reactions to the teaching strategies, but also the personal relevance they found to mathematics. A few students stated (n = 8) that one disadvantage to lecturing was that it did not show how mathematics would be used in personal experiences. However, a portion of other students (n = 9) who had combined lectures with other teaching strategies found mathematics useful for job skills, banking, budgets, baking, and building. According to some student responses (n = 15), math applications were discussed in group work and board work. One such written comment stated, ‘I learned more applications through games and relays.’

Overall, students had fewer negative attitudinal impacts (n = 9) toward mathematics than positive impacts (n = 11). Negative comments consistently referred to not understanding material due to unclear instructions or feeling rushed. These students were discouraged by lectures and courses that were set at a fast pace. On the other hand, student attitudes were positive toward mathematics when met with caring instructors who encouraged student participation and gave positive reinforcement. In general, it appeared that students had few prior experiences with standards-based pedagogies, but would adjust to the course as long as the teacher appeared interested in their understanding of the material.

Attitude Instruments

To measure if an attitude change occurred, several instruments were used. This section will describe each data collection method and the results of the data analysis.

Focus Groups

Focus groups were conducted outside of the class period. Nine students were chosen from the first class and six students were selected from the second class. Initially, the researcher had targeted for three students from each level of negative, positive, or neutral attitudes toward mathematics. However, the second class only allowed for two from each level. To determine which level students were at, a preliminary survey was distributed to both classes (Appendix H). Three focus groups were conducted over the semester, with each one lasting approximately 45 minutes. The focus groups were used to answer the research questions regarding the college algebra students’ perceptions about the nature of mathematics and learning mathematics when placed in a standards-based learning environment. Once the focus groups were completed, contact summary forms were filled out. The forms looked for common themes, interesting comments, and important statements. The data analysis from the focus groups produced themes relating to students' perceptions about being a student of mathematics, pursuing additional math courses or careers in mathematics, and the relevance of mathematics in their personal and professional experiences. Although the researcher consistently prompted the students for responses to the questions, the participants remained relatively quiet.

The open-ended surveys, which asked for previous experiences, were distributed the first day and provided the researcher some prompts for the first focus group interview, which was done three weeks after class started. The students were asked to expand on their thoughts from the survey given the first day of class. Their responses were coded as previous experiences. Within those experiences, lectures, note taking, board work, and hands-on activities were coded separately. Three students in the interview repeated that lecture was the common teaching practice in their previous math classrooms. Two students mentioned their experience with board work and two students had experienced group work in prior math courses. One student expanded on his thoughts by expressing that the lectures and long homework assignments inhibited his attitude toward mathematics.

Student #1: Lectures are boring. I don't like to do a lot of homework.

Student #2: I would agree. If I have to do a lot of homework, then I don't like the class.

Only one student had previous experience with hands on activities and that had been seen at the college level. The group from the second course had no previous experience with graphing calculators. After discussing previous learning experiences, the focus changed to the current college algebra course in which they were enrolled. Using questions previously written (Appendix E), students were asked to describe instructional strategies that enhanced their attitude toward mathematics. Three students stated they enjoyed cooperative learning because they were able to feed ideas off each other, there was a collaboration of ideas, and the students liked to work together. The cooperative learning tasks made class less stressful and more enjoyable. Another student added that the graphing calculator saved her time. In response, one student stated his dislike of the calculator. He found the calculators hard to learn and felt like he spent too much time learning how to work them. When one student mentioned he was not comfortable with discussion periods, two other students agreed. The researcher asked what made them uncomfortable. One student did not like to be under pressure to give the right answer. Another student claimed it was embarrassing to talk in front of others. However, one student did say discussions were good because they shortened lecture time. When speaking about problem-solving, two students expressed that it was enjoyable because the class used tools to help them solve the problem.

The second interview was conducted one month later. Two students stated they were starting to understand why cooperative learning and problem solving was necessary and that it could be fun:

> Student #1: Math is less intimidating with activities. Group work is a good tool. After coming back to school, it is helpful because other people are helping to explain.

Student #2: Everybody has to participate, so it makes the work easier.

A third student spoke about problems relating to her personal experiences:

Student #3: If going on, math is useful. I like the pizza problem because it showed a connection between function and what I'm familiar with.

Two other students still did not see the connection between mathematics and everyday life. A conversation began regarding mathematics and sports:

Student #4: Math isn't the foundation. Everything comes from an idea.

Instructor: What about basketball stats? [Note: This student is the basketball manager for the men's team.]

Student #4: Again, basketball was started by somebody who didn't know much.

Student #2: Math takes up space in the mind.

Instructor: So, you don't notice math on the court or on the baseball diamond?

Student #4: No, not really.

After this short exchange, the instructor encouraged these students to reflect on the applications in the textbook because several of them were sports related.

The final interview was conducted the last week of classes. The main topic the students discussed was calculators and how the calculators became a useful tool. Some students felt that their confidence level had increased. One student commented it also helped that the instructor made class enjoyable because he was not anxious about coming to class. Several students agreed and then referred to the instructor's positive attitude as encouraging. The class had also had a recent discussion on mathematics education. Four students said the discussion helped them make a connection between mathematics and their everyday lives.

Student #5: I see the connections between math and the real world. I can see where I will use this for budgets or basic skills to get by.

The focus groups were beneficial for this study because it enabled the students to expand on their reactions to the learning environment. The students spoke about the teacher's positive attitude, their like and dislike of various teaching strategies, and their attitudes toward mathematics and how it changed throughout the semester.

Questionnaires

Questionnaires (Appendix B) were distributed eight times over the semester. The questionnaires were designed so that each time it was used, students could choose an attitude (anxiety, confidence, enjoyment, motivation, and value) with regards to each of the instructional strategies, which were all listed on the questionnaire. Students were also given an opportunity to expand their thoughts on the attitudes they chose by using the boxes, big enough to write in, on the questionnaire. However, most students simply circled the attitude related to the teaching strategy without expanding on why they chose that attitude for the week. The questionnaires were used to answer the research questions pertaining to their perceptions about the nature of mathematics, the relevance of mathematics, and their perception of being a student of mathematics. The questionnaire was also analyzed in order to answer the research question regarding the strategies the students felt had an impact on their attitude toward mathematics and if the strategies affected their participation in the classroom. Table 16 details the frequency each attitude was marked throughout the semester with regards to the respective teaching strategy. This section will elaborate on these findings.

Table 16
Frequency of Attitudes Marked for each Teaching Strategy

	Cooperative Learning	Discussions	Problem Solving	Graphing Calculators
Anxiety	n = 7	n = 15	n = 23	n = 20
Confidence	n = 25	n = 27	n = 25	n = 27
Enjoyment	n = 30	n = 22	n = 22	n = 22
Motivation	n = 9	n = 7	n = 5	n = 2
Value	n = 3	n = 6	n = 5	n = 2

Anxiety was prevalent in the problem solving area for students. Over the eight weeks the questionnaire was distributed, anxiety was circled the most (n = 23) for problem solving. 'I have anxiety over the amount of work and my ability to work word problems.' As the semester progressed, the number of students who referenced anxiety toward the teaching pedagogies decreased (n = 7). For cooperative learning, anxiety was addressed seven times. These statements were more positive as a few students (n = 3) claimed that cooperative learning decreased their anxiety for tests. When addressing discussions, one student wrote, 'I have anxiety over talking in front of others.' The graphing calculators were a factor for anxiety because students did not know how to use them at the beginning of the semester. 'I don't have the same calculator as the instructor so I don't know how to use mine.'

Confidence was lacking at the beginning of the semester with regards to all of the teaching pedagogies. A common theme was students' lack of confidence in working in this standards-based learning environment. However, as the semester progressed, more students wrote that their 'confidence is improving' when they addressed this attitude in the questionnaire.

Enjoyment was marked for cooperative learning more than the other practices (n = 30). Almost half of the open response comments related to cooperative learning noted that working with each other was pleasing (n = 17), they liked sharing ideas with each other (n = 11), and they found that it improved their understanding of the material (n = 14). Although most students enjoyed cooperative learning, they also wrote that they 'did not enjoy problem solving' and 'didn't like discussions.' The dislike for these two teaching strategies did not seem to decrease as the semester progressed even though one student wrote, 'I enjoy figuring out problems by myself or in groups.'

Students seldom marked any of the four practices as motivating or of value and comments rarely addressed these areas. One student wrote, 'I see applications beyond this course,' when pertaining to discussions. As Table 15 shows, these two attitudes were not circled as much as the other attitudes. A vast number of students were motivated by the graphing calculators because it 'lessened the workload.'

The questionnaires gave the students an opportunity to write about their own attitude changes throughout the semester. Data analysis shows that most students felt their attitudes changed over the course of the semester with regards to anxiety and confidence. Few students (n = 5) reported a connection to the mathematics and experiences in their personal or professional life. Although the teaching strategies were enjoyable, no students mentioned that it motivated them to learn more mathematics.

Attitudes Toward Mathematics Inventory

The *Attitudes Toward Mathematics Inventory* (Appendix C) was given to 35 students at the beginning of the semester. The inventory was used to answer the research questions pertaining to the students' perceptions about the nature of mathematics, learning mathematics, being a student of mathematics, pursuing math careers, and the relevance of mathematics in their personal and professional experiences. The follow-up survey was administered during the last week of the semester. Due to withdrawals and students completing the course, 29 students completed the post-survey. The responses were analyzed using a two-sample t-test between the first time the survey was given and the second time. The t-test is used to test the differences of the mean for each question between the two separate times the survey was administered. The null hypothesis is that there is no difference in attitudes between the beginning of the semester and the end. The tables are broken into subscales of self-confidence, value of mathematics, enjoyment, and motivation for learning mathematics. Each question is denoted with Qn, where n denotes the question number, and references the results from the first time the survey was given (n = 35), while XQn references student responses on the second time the survey was administered (n = 29). The values in the table represent the percentages of the students who took the survey and chose that response.

Results on Self-Confidence. Questions 9 through 22, and 40 reflected student assessment regarding their confidence in mathematics. Table 17 is the result of the two-sample t-test. Taking the mean score from the second distribution of the survey and subtracting it from the mean score of the first distribution calculated the difference.

Table 17
Two-Sample T-Test Results for Self-Confidence

	Number of Respondents	Mean	Standard Deviation	P-Value
Q9	35	2.69	1.45	
XQ9	29	2.97	1.43	0.441
Q10	35	3.14	1.12	
XQ10	29	2.97	1.24	0.553
Q11	34	3.15	1.21	
XQ11	28	3.25	1.11	0.728
Q12	35	3.43	1.14	
XQ12	29	3.34	1.04	0.761
Q13	35	3.17	1.27	
XQ13	29	3.38	1.24	0.511
Q14	34	3.09	1.33	
XQ14	29	3.59	1.15	0.117
Q15	35	3.34	1.19	
XQ15	29	3.45	1.12	0.717
Q16	35	3.00	1.06	
XQ16	29	2.83	1.10	0.529
Q17	35	2.97	1.10	
XQ17	29	2.931	0.998	0.878
Q18	35	3.057	0.938	
XQ18	29	2.83	1.07	0.371
Q19	33	2.818	0.882	
XQ19	29	2.655	0.857	0.464
Q20	34	3.471	0.992	
XQ20	29	3.448	0.827	0.923
Q21	35	3.11	1.08	
XQ21	29	3.10	1.05	0.968
Q22	35	2.829	0.954	
XQ22	29	2.90	1.08	0.793
Q40	32	2.84	1.02	
XQ40	29	2.62	1.08	0.412

With an alpha = 0.05, the results show the study did not have a statistically significant change in confidence, but based on the p-values, there was an increase in student confidence to work with mathematics. Student responses show the greatest changes where students did not dread the subject as much, $t(60) = -0.78$, $p < 0.441$, the students felt better about problem-solving, $t(56) = 0.90$, $p < 0.371$, and anticipated they would fair well in a mathematics class, $t(59) = 0.74$, $p < 0.464$.

Results for Value. Questions 1, 2, 4 through 8, 35, 36, and 39 related to student value for mathematics. Table 18 is the result of the two-sample t-test. Taking the mean score from the second distribution of the survey and subtracting it from the mean score of the first distribution calculated the difference.

Table18
Two-Sample T-Test Results for Value

	Number of Respondents	Mean	Standard Deviation	P-Value
Q1	35	2.029	0.891	
XQ1	29	2.00	1.04	0.907
Q2	35	1.943	0.725	
XQ2	28	2.107	0.832	0.414
Q4	35	2.029	0.664	
XQ4	29	1.897	0.724	0.454
Q5	33	2.091	0.805	
XQ5	29	2.103	0.900	0.954
Q6	35	2.457	0.886	
XQ6	29	2.379	0.979	0.742
Q7	35	2.486	0.702	
XQ7	29	2.138	0.789	0.070
Q8	35	2.200	0.719	
XQ8	29	2.138	0.789	0.746
Q35	30	2.700	0.837	
XQ35	29	2.55	1.06	0.553
Q36	32	2.375	0.707	
XQ36	29	2.069	0.704	0.096
Q39	32	2.344	0.902	
XQ39	29	2.241	0.872	0.654

With an alpha = 0.05, the results show that although the study did not have a statistically significant change in how students value mathematics, the students did increase their value for the subject. The students wanted to develop their mathematical skills, $t(53) = -0.82$, $p < 0.414$, saw that mathematics helps develop the mind and encourages thinking, $t(57) = 0.75$, $p < 0.454$, and believed studying advanced mathematics is useful, $t(53) = 0.60$, $p < 0.553$. Some results which were close to statistically significant were students believing studying math helps with problem-solving in other areas, $t(58) = 1.69$, $p < 0.096$, and students noting that math courses would be helpful no matter their course of study, $t(56) = 1.84$, $p < 0.07$.

Results for Enjoyment. Questions 3, 24 through 27, 29 through 31, 37 and 38 address student enjoyment for mathematics. Table 19 is the result of the two-sample t-test. Taking the mean score from the second distribution of the survey and subtracting it from the mean score of the first distribution calculated the difference. With an alpha = 0.05, the results show the study did have a statistically significant difference, $t(61) = 2.15$, $p < 0.036$ for the statement, 'Mathematics is a very interesting subject.' The difference moved toward students agreeing with this statement. Even though this was the only statistically significant difference, the results show that students received satisfaction out of solving a mathematics problem, $t(61) = 1.01$, $p < 0.318$, found mathematics less dull and boring, $t(61) = -0.80$, $p < 0.430$, chose mathematics over writing an essay, $t(55) = -1.04$, $p < 0.303$, and became comfortable expressing their own ideas in looking for solutions to mathematics problems, $t(57) = 1.35$, $p < 0.183$. Solving new problems in mathematics, $t(61) = 1.64$, $p < 0.106$, really liking mathematics, $t(61) = 1.53$, $p < 0.132$, being happier in mathematics class than in other classes, $t(61) = 1.55$, $p < 0.126$, and being comfortable answering questions in mathematics class, $t(58) = 1.70$, $p < 0.095$ had significantly more students agreeing with these statements at the end of the semester.

Table 19
Two-Sample T-Test Results for Enjoyment

	Number of Respondents	Mean	Standard Deviation	P-Value
Q3	35	2.54	1.04	
XQ3	29	2.310	0.806	0.318
Q24	35	3.00	1.26	
XQ24	29	2.83	1.10	0.562
Q25	35	3.371	0.942	
XQ25	29	3.552	0.870	0.430
Q26	35	3.086	0.919	
XQ26	29	2.724	0.841	0.106
Q27	35	2.46	1.17	
XQ27	29	2.79	1.37	0.303
Q29	35	3.029	0.923	
XQ29	29	2.690	0.850	0.132
Q30	35	3.54	1.09	
XQ30	29	3.138	0.990	0.126
Q31	35	2.886	0.932	
XQ31	29	2.414	0.825	0.036
Q37	31	2.94	1.06	
XQ37	29	2.586	0.946	0.183
Q38	32	3.03	1.06	
XQ38	29	2.586	0.983	0.095

Results for Motivation. Questions 23, 28, and 32 through 34 related to how motivated students were to doing and learning mathematics. Table 20 is the result of the two-sample t-test. Taking the mean score from the second distribution of the survey and subtracting it from the mean score of the first distribution calculated the difference. With an alpha = 0.05, the results show that although the study did not have any statistically significant changes, there was an increase in student motivation. Students became more confident they could learn advanced mathematics, $t(54) = 0.78$, $p < 0.440$, were less apt to avoid using mathematics in college, $t(61) = -0.64$, $p < 0.527$, planned to take as much mathematics as they could during their education, $t(61) = 0.86$, $p < 0.394$, and found the challenge of mathematics appealing, $t(61) = 1.02$, $p < 0.313$. Although it was not statistically significant, students willing to take more than the required amount of mathematics, t(58) = 1.69, $p < 0.096$, had a significant change.

Table 20
Two-Sample T-Test Results for Motivation

	Number of Respondents	Mean	Standard Deviation	P-Value
Q23	35	2.829	0.954	
XQ23	29	2.62	1.15	0.440
Q28	35	3.31	1.13	
XQ28	29	3.483	0.986	0.527
Q32	35	3.34	1.14	
XQ32	29	2.897	0.976	0.096
Q33	35	3.40	1.12	
XQ33	29	3.17	1.00	0.394
Q34	35	3.03	1.10	
XQ34	29	2.76	1.02	0.313

Triangulation

This section will verify that each research question was answered using at least three different data sources. Each research question will be addressed with the findings from each data source to follow the question. This section is a brief synopsis of the data findings for this study.

Research Question 1

The first research question was: *What is the relationship between the use of standards-based pedagogy and college algebra students' perceptions about the nature of mathematics and learning mathematics? Specifically, does such a learning environment impact students':*

a. attitudes, specifically in the areas of confidence, anxiety, enjoyment, and motivation,

b. perceived value of mathematics in personal and professional experiences.

Triangulation was attained for the first research question by using the focus groups, attitude survey, student questionnaires, and personal observations. Part (a) of this question was addressed through the attitude inventory and the student questionnaires. The relationship between the standards-based pedagogies and students' attitudes was measured by the Attitudes Toward Mathematics Inventory, a quantitative data collection method. The results from the two-sample t-test show that three attitudes, motivation, value for mathematics, and confidence, did not show statistically significant changes between the mean scores of the pre- and post-administration of the survey. The fourth attitude, enjoyment, had a statistically significant positive change for students finding mathematics interesting, $t(61) = 2.15$, $p < 0.036$. Although the quantitative data did not show much change, the analysis of the qualitative data methods indicates that student attitudes, with regards to confidence, anxiety, motivation, enjoyment, and value, did improve over the course of the semester.

Results of the student questionnaires indicate that the four practices resulted in improved attitudes. Common themes were that student confidence was improved through cooperative learning tasks where they spent time talking with other students about problems ($n = 11$), a small number of students stated they became better with the mathematics as they helped each other ($n = 5$). A student comment, regarding confidence with cooperative learning, less than two months after the start of the semester was, "I'm understanding the material." Another student circled confidence beside cooperative learning and wrote, "Because I feel like I'm improving." Themes regarding the graphing calculators included stating that the calculators boosted their confidence with mathematics because they could be used to verify answers and know the work was correct (n = 6). Students inferred that if they did not know how to use graphing calculators, they quickly became frustrated, which contributed to their low confidence. Also, students were also anxious with the calculators if they did not have the same calculator as the instructor. As the semester progressed, students who were unfamiliar with the graphing calculator began to enjoy them as they became more confident in operating the calculators. Tracking the comments of one student, the data shows that at the beginning of the semester she was not confident yet about the calculators. Two weeks later she still writes that she is not confident yet. And, finally, one month after her first comment, she writes that the graphing calculators boosted her confidence. Several students commented that the confidence with solving problems gradually improved over the semester as they worked with problem solving (n = 9).

Common themes directed toward motivation revealed that students felt that the motivation to learn mathematics was impacted mostly by cooperative learning and discourse. They included comments where some students became interested in learning more mathematics while solving problems (n = 1) or discussing mathematical procedures (n = 3). One student wrote that she was motivated to pursue more mathematics after only two months of problem solving. Another student commented that the problem solving motivated her to want to solve the problems.

The student questionnaires also indicate that students' enjoyment was impacted positively by cooperative learning, discourse, problem solving, and graphing calculators. The strongest evidence of impact on enjoyment was present in data related to graphing calculators and cooperative learning. These two teaching strategies made the workload easier (n = 9) and the mathematics more understandable (n = 19).

The initial use of the four teaching strategies made some students anxious about being a student in the mathematics course. The graphing calculators were unfamiliar ($n = 3$), students were anxious about talking in front of other students ($n = 4$), working in groups was intimidating ($n = 1$), and problem solving was difficult ($n = 7$). However, most students reported a decrease in anxiety about mathematics as the semester progressed ($n = 7$) and they become more familiar with the four teaching methods. For example, one student began the semester by saying she had anxiety about talking in front of others. At the end of the semester, she wrote, “I have less anxiety,” when referring to discourse.

Analysis of the observations shows that as the students progressed through the semester, becoming more accustomed to the teaching strategies, their anxiety weakened and their confidence improved. This could be seen in their improved roles as participants in the classroom and students’ willingness to discuss mathematical procedures with the entire class. Data from the focus groups also indicates students’ attitudes toward the mathematical classroom are more positive. The groups spoke about the relaxed atmosphere of the classroom which kept them coming to class. One of the nontraditional students commented on the confidence he got by discussing how to work problems within the groups in class.

The final part of the first research question addressed the relevance of mathematics in students’ personal and professional experiences. This question was assessed through the Attitudes Toward Mathematics Inventory, course evaluation, the University of Wisconsin Observation Scale, focus groups, and student questionnaires. In a statement from a student questionnaire, it is seen how cooperative learning encouraged students to place value on mathematics, “Togetherness [in cooperative learning] formed unity, which would help students with future job skills.” In focus groups, a few of the participants noted that discussion made an impact because they found value in mathematics when discussing topics with other students. In the classroom discussions, the groups talked about how society is growing fast and education must keep up. One of the themes the groups agreed on was the need of mathematics for job skills. Student comments in the questionnaires revealed that exposure to applications through problem-solving ($n = 4$) and discussions about mathematics in education and everyday life ($n = 1$) opened students’ eyes to mathematics in their own personal experiences. The course evaluation showed that graphing calculators were also beneficial to a large portion of students who reported that the course gave technological tools needed to expand knowledge in mathematics with future opportunities (38.1%, $n = 8$). The results of the University of Wisconsin Observation Scale report that problem solving gave over half of the students (52.2%, $n = 12$) the opportunity to see connections between mathematics and their everyday life. The results of the evaluation also found that the course provided problem solving tools needed to expand knowledge in mathematics with future opportunities (38.1%, $n = 8$). The Attitude Toward Mathematics Inventory results show that students felt the math courses would be helpful no matter their course of study, $t(56) = 1.84$, $p < 0.07$, which indicated students could see value for mathematics in other areas of interest.

Research Question 2

The second research question was: *What specific instructional strategies do students believe most impact their attitudes about mathematics?* The data collected for this question came from focus groups, student questionnaires, and personal observations.

Common themes throughout the data analysis point toward the suggestion that cooperative learning and graphing calculators impacted students' attitudes the most. Although problem solving and discourse were not seen to have a large impact on the students, the results show that student attitudes were still impacted by the two teaching strategies. Statements written by students in the questionnaires indicate cooperative learning provided students the opportunity to learn from others (n = 9), as well as talk with other students about solution strategies (n = 9). Students also indicated that learning from others boosted student confidence (n = 14). Some students wrote that the material was easier to understand because they were helping each other (n = 14). Students stated that cooperative learning reduced student anxiety for taking tests (n = 3). In the focus groups, three students stated they enjoyed cooperative learning because they were able to feed ideas off each other, there was a collaboration of ideas, and the students liked to work together.

According to student questionnaires, the graphing calculators were perceived to be a powerful tool for students because it lessened their workload (n = 9) and still helped them understand mathematical topics (n = 4). Students thought the graphing calculators were not only enjoyable (n = 15), but they also felt it improved their confidence with mathematical tests and work (n = 6).

Comments, in the student questionnaires, pertaining to problem solving reflected students' enjoyment at working in groups on problems because it was helpful since the students could look at different ways to solve problems (n = 22). As students worked with each other, discussed mathematics openly with fellow classmates, and solved problems, they indicated that their anxiety began to weaken (n = 7). Part way through the semester, two students stated during a focus group they were starting to understand why cooperative learning and problem solving was necessary and that it could be fun.

Discourse was also addressed in student questionnaires where students indicated an increase in motivation to learn more about mathematics when discussing topics in the classroom (n = 3). The questionnaires also show that students stated communicating in the small groups improved their ability to talk in front of fellow classmates (n = 10). An increase in student discussion could also be seen in personal observations when student participation increased. Students were more apt to offer advice or share procedures as the semester progressed.

Research Question 3

The third research question was: *In what ways does the use of standards-based pedagogical strategies impact college algebra students' engagement in the learning process?* To address students' engagement in the learning process, data was collected through focus groups, student questionnaires, University of Wisconsin Observation Scale, and personal observations. The results of the data analysis are presented in this section. They indicate that as students' confidence with their ability to solve problems and engage in mathematical conversations improved, they became more active participants in the classroom.

Through personal observations, it could be seen that as students became more comfortable with the instructional strategies, they were more apt to participate in the classroom. For example, at the beginning of the semester, in a group of four students, one student was very quiet and reserved. The other group members were athletes and were good at leading the group to conclusions. The fourth student rarely became involved, unless he was assigned a task. When the instructor asked him why he wasn't participating as much as the others, he said he didn't know how to ask the rest of the group questions. As the semester progressed, this student gradually became more involved. The other group members started to include him more and ask for his opinion on various problems. By the end of the semester, all four group members were highly engaged with each other discussing procedures and helping each other with problems. One student who was afraid of giving wrong answers during discussion stated in her questionnaire toward the end of the semester that she felt the contribution she made to the communication was more important as the semester progressed.

The results of the statements given in the observation scale show that discourse and cooperative learning had an impact on student interaction in the classroom. A large majority of the students found there were stimulated discussions, which encouraged discourse about the lesson (78.3%, n = 18). In other questions related to discussion and engagement, when over half of the students reflected on their solutions, discussion ensued regarding the reasonableness of the answer (65.2%, n = 15). If there were questions, most students felt that they could ask a classmate how to solve a particular problem and discuss alternative strategies (56.5%, n = 13). Over half, 65.2%, of the students found themselves involved with classmates in solving problems and working with others to either learn or assist another student in learning material.

The analysis of focus group comments and student questionnaires indicates that most students became more comfortable participating in the learning environment as the semester progressed. Students were less intimidated in talking in front of others. In the focus groups, the participants pointed to how they were more comfortable with each other, and, thus, more inclined to speak up in the groups and in the class.

Triangulation must be done in order for the results of the study to be conclusive. The combination of the data collection methods validated the researcher's findings and conclusions because more than one method indicated the same results. Triangulation was especially necessary for this study since students were not as active in writing their reactions as the researcher would have liked. The other data collection methods provided additional insight into answering the research questions for this study.

Summary of Findings

Cooperative learning, problem-solving, discourse, and graphing calculators were the primary instructional strategies for this study. Students reported use of these strategies through the ratings on the University of Wisconsin Observation Scale and the researcher constructed course evaluation. Students agreed that connections were made between chapters when using the four instructional strategies at various times. Observations indicated discussions were started by the instructor as well as other students. The discussions pertained to the current topic. Students felt they were encouraged to participate in discussions and share solution strategies. Questionnaires and discussions point out that students were able to make connections between math and their daily lives.

Students believed the cooperative learning pedagogy used in this study created an environment where they could ask questions and help each other. Students stated everyone seemed to stay on task when they were working in their groups. Students also indicated that graphing calculator tasks were directly related to the topics being taught. Students believed the calculators connected current content to previously taught material. Researcher observations and frequency of use data also verified the use of cooperative learning, problem solving, discourse, and graphing calculators in the classroom. Given the existence of these practices, the researcher was able to study the impact of their use on students' attitudes toward mathematics, students' value of mathematics, and students' engagement in the mathematical classroom.

The attitudes of enjoyment, motivation, value for mathematics, and confidence were measured using the Attitudes Toward Mathematics Inventory. The results from the two-sample t-test show that three attitudes, motivation, value for mathematics, and confidence, did not have statistically significant changes between the differences of means between the pre- and post-administration of the survey. The fourth attitude, enjoyment, had a statistically significant positive change for students finding mathematics interesting, $t(61) = 2.15$, $p < 0.036$.

Although the attitude inventory revealed no significant differences in motivation, value for mathematics, and confidence, student comments, through surveys and focus groups, indicate that the standards-based pedagogies had an impact on students' attitudes in all four areas. Cooperative learning was characterized as an approach that allowed students to help each other and, they believed this increased their confidence, decreased their anxiety for exams, and improved their understanding of material. According to many students, incorporating discourse into the curriculum reduced their anxiety of speaking in front of others over time, increased their interest in learning mathematics, and made content more understandable. Students related that problem solving increased their confidence in finding answers for word problems, made class enjoyable because they liked figuring out problems individually or in groups, and showed students applications of mathematics to real world experiences. According to comments on the questionnaires and focus groups, students felt that the fourth teaching pedagogy, graphing calculators, made the workload easier, decreased their anxiety as they became more familiar with the procedures for using the calculator, increased their confidence with tests and work, and increased their enjoyment of mathematics.

The study also looked at the specific instructional strategies that impacted students' attitudes the most. According to the frequency tabulation on student questionnaires, working in groups while engaged in cooperative learning helped students understand the material better. Students stated that the opportunity made mathematics more enjoyable for them. Students also indicated in the questionnaires that cooperative learning also made the mathematical work easier and most of the students were then less anxious about taking exams. According to the student comments in focus groups and student questionnaires, graphing calculators also made mathematics more enjoyable because the calculators lessened the workload. Most students stated that the calculators increased their confidence with tests and work. Students also indicated that discourse of various topics also motivated most students to learn more about mathematics and that problem solving gave them more awareness to the value of mathematics because they could relate it to personal or professional experiences. Finally, students indicated they were also less fearful of taking advanced mathematics classroom. They expressed confidence in their ability to succeed in upper level mathematics courses.

Focus groups gave students an opportunity to detail how instructional strategies impacted their attitudes. Students said cooperative learning gave them an opportunity to share ideas, feed off each other, and become confident with their own math skills. Although the graphing calculator was intimidating at first, the students in the focus group found it to make their work easier. Students liked the minimal lecture time and the less intense homework assignments. They were not eager to become math majors, but they felt that the classroom environment made math fun.

The final factor analyzed in this study was how the use of standards-based pedagogical strategies impacted students' engagement in the learning process. As the semester progressed, student questionnaires indicate that students became more comfortable with discussing various topics in front of their groups and the entire class. More students found that they were comfortable with not only asking other students for help, but also offering assistance without fear of being wrong. With cooperative learning, observations show reserved students in the first task only doing their part, but as the groups worked together more, all members were trying to do equal amounts of work. Through personal observations, it was seen that student engagement increased with quantity and quality over the semester.

The quantitative data, Attitudes Toward Mathematics Inventory, shows a statistical significant change in enjoyment of mathematics for students as the semester progressed. Although the quantitative data indicates no statistical significant difference in a change of student attitudes for confidence, motivation, and the value for mathematics, the qualitative data expresses that the four teaching pedagogies did have a positive impact on the attitudes of students. Students reported that cooperative learning, discourse, problem solving, and graphing calculators boosted their confidence, motivated them to want to learn more about mathematics, decreased their anxiety about working with mathematics, and made the classroom more enjoyable. The innovation in the mathematics curriculum which took place in this study had a positive impact on the students in the classroom. The researcher has learned that there is a relationship between standards-based pedagogies and how college algebra students view their perceptions of mathematics.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

Summary of the Problem

Background

Student attitudes toward mathematics have been the focus of literature and research for decades. When students are in younger grades, they find mathematics enjoyable. However, as students progress through grade levels, their interest in mathematics begins to decline. By the time they reach college, few students pursue a mathematics degree. Others take mathematics courses only because it is a requirement to graduate from college.

Students report that teacher interest with the subject and the learning environment in the classroom affects their attitudes negatively. To address this issues several national reports, *Principles and Standards for School Mathematics* [PSSM], *Adding it Up*, *How People Learn, and Committee on the Under-graduate Program in Mathematics* [CUPM], have called for mathematics reform. The reports encourage instructors to change from a traditional classroom, which is primarily lecture, to an environment that encourages student participation. The constructivist learning theory has been used as a guide for the improvement of instruction. The need for change is seen at all levels, especially at the college level where mathematics departments continue to see a decline in enrollment as well as an increase in student withdrawals.

Standards-Based Curriculum

Through National Science Foundation [NSF] funding, mathematics educators, mathematicians, and teachers collaboratively developed standards-based curriculum.

Standards-based teaching, as outlined in Principles & Standards, includes a shift in the classroom from teacher directed instruction to one where teachers nurture a mathematics community where students verify results through exploration, more emphasis is placed on mathematical reasoning, and connections are made among the ideas and applications of mathematics (McCaffrey et al., 2001). In a standards-based mathematics curriculum, students' experiences are varied. Rather than imitate procedures explained by the teacher, students are involved with exploration and discovery of important mathematics concepts.

Through the Teacher Development Coalition, Gay et al., (2000), have found that a classroom which model standards-based teaching have a positive impact on student attitudes toward mathematics. Students learn mathematics only when they construct their own mathematical understanding. They found that this happens best when students are working in groups, engaging in discussion, and taking charge of their own learning (Gay, Bruening, & Bruce, 2000).

Research on Standards-Based Instruction at the College Level

Studies at the college level have shown that by using various projects and group work in undergraduate math courses, students' interest is maintained while they learn basic concepts (Cooper & Robinson, 2002; Bookman & Friedman, 1998; Hollar & Norwood, 1999; Bowen, 2000; Burrowes, 2003). By using standards-based practices, which included strategies such as cooperative learning, discussion, and problem solving, the studies remained consistent with the reform in undergraduate mathematics education. The reform in undergraduate mathematics calls for instructors to teach content using discovery and exploration, group projects, and technology in order to enhance learning and encourage students to enroll in mathematics courses. In three college-level research studies, the researchers reported a dramatic improvement in student attitude and motivation. At the end of the semester, students in the experimental group thought math was important for life and had more positive attitudes toward math than the traditionally taught students (Hill et al., 2003; Wood & Craft, 2000; Elliott et al., 2000).

Standards-based curriculum is designed to help students become mathematically literate, encourage exploration, reason logically, and discover the use of various methods to solve problems (Reys et al., 1999). Although this pedagogy has primarily been encouraged in K-12 classrooms, the practices are now being studied in undergraduate classrooms. The research shows college students have an improved value of themselves as mathematics students as well as acquiring a deeper conceptual understanding of mathematical topics. More studies are needed at the collegiate level to increase awareness of the positive implications standards-based pedagogy, which includes cooperative learning, problem solving, graphing calculator use, and discourse, has on college students.

Purpose of the Study

The purpose of this study was to assess potential benefits of using components of the standards-based pedagogy with community college students and how the pedagogies impact students' attitudes toward mathematics. The standards-based pedagogy used in this study was cooperative learning, problem solving, discourse, and graphing calculators. Since this study focused more on instruction than curriculum, the components selected address classroom practices. More specifically, the research analyzed the following questions:

1. What is the relationship between the use of standards-based pedagogy and college algebra students' perceptions about the nature of mathematics and learning mathematics? Specifically, does such a learning environment impact students:

   a. attitudes, specifically in the areas of confidence, anxiety, enjoyment, and motivation,

   b. perceived value of mathematics in personal and professional experiences.

4. What specific instructional strategies do students believe most impact their attitudes about mathematics?

3. In what ways does the use of standards-based pedagogical strategies impact college algebra students' engagement in the learning process?

Methodology

In order to assess if student attitudes toward mathematics were affected by standards-based pedagogies, the researcher conducted an action research study, using both qualitative and quantitative data collection techniques. The study took place at a two-year community college. The participants were students enrolled in two college algebra courses taught by the researcher. The characteristics of the college algebra course included a delivery mechanism with minimal lecture, support of students in their efforts to become better participants, and an environment to build up their confidence to show their work to their peers in group work and in front of the entire class. This course was designed to model four standards-based instruction practices and generally included the four components of standards-based pedagogies, cooperative learning, problem solving, discourse, and graphing calculators.

Data was collected through pre- and post surveys, questionnaires, focus groups, observations, and course evaluations. In order to learn about previous student experiences in the mathematics classroom, the students completed an open-ended survey the first day of class. The students also completed a likert-scale survey, the Attitudes Toward Mathematics Inventory, on the first day of class to determine their attitudes toward mathematics at the beginning of the semester. This survey was also administered at the end of the semester. Students filled out open-ended questionnaires eight times during the semester to determine if their attitudes were influenced by the activities in the classroom. One hour focus groups were conducted at four different times during the semester. The focus groups were taped and transcribed. Two observations were conducted to view students in the learning environment. The observations were videotaped and coded for student interaction and participation in the lessons. At the end of the semester, students completed an observation scale and course evaluation that referred to if and how the activities were used in the classroom.

The data collection methods allowed for triangulation of the data. The initial survey was used to determine if a change in attitude occurred, positive or negative. The questionnaires, focus groups, and observations determined how that change took place. The observation scale and course evaluation verified that the instructor used the four standards-based instructional strategies described in this study.

Summary of Findings

The first part of the findings was designed to establish the extent to which the four standards-based practices were used in the course. Cooperative learning in this course was characterized as students in small groups, each assigned a specific duty to complete the task, assisting each other and exploring mathematics concepts. Students described the following aspects as important to cooperative learning, sharing ideas to understand the material, helping each other, making class enjoyable. Discourse was beneficial to students who found the classroom discussions helped them understand course material, decreased their anxiety related to speaking in front of others, and showed them applications beyond the course. Lessons were coded for problem solving when students used discovery methods to develop mathematical procedures either individually or in groups. According to student comments, problem solving built confidence with exams, showed applications of mathematics, and decreased the anxiety of solving problems. The final strategy, graphing calculators, was defined as student use of the graphing calculator in order to complete the task. Students were expected to use the graphing calculator for exploration and discovery. Students described the use of graphing calculators as enjoyable, improving their confidence with mathematics, and decreasing the workload.

The first research question was: *What is the relationship between the use of standards-based pedagogy and college algebra students' perceptions about the nature of mathematics and learning mathematics? Specifically, does such a learning environment impact students': a. attitudes, specifically in the areas of confidence, anxiety, enjoyment, and motivation, b. perceived value of mathematics in personal and professional experiences.* The data sources used to answer the questions were the Attitudes Toward

Mathematics Inventory, student questionnaires, the University of Wisconsin Observation Scale, and focus groups.

The relationship between the standards-based pedagogies and students' attitudes was measured by the Attitudes Toward Mathematics Inventory. The results from the two-sample t-test, show that three attitudes, motivation, value for mathematics, and confidence, did not have statistically significant changes between the differences of means between the pre- and post-administration of the survey. The fourth attitude, enjoyment, had a statistically significant positive change for students find mathematics interesting, $t(61) = 2.15$, $p < 0.036$.

Although the quantitative analysis did not show great change in student attitudes, the qualitative data indicates that attitude changes did occur with regards to confidence, motivation, enjoyment, anxiety, and value. Common themes generated through the student questionnaires and focus groups were that student confidence in being a student of mathematics primarily improved with the use of cooperative learning and graphing calculators, the motivation to learn mathematics was impacted mostly by cooperative learning and discourse, and student enjoyment of mathematics was positively changed by cooperative learning and graphing calculators.

The second area the first question addressed was students' perceptions of the value of mathematics in their personal and professional experiences. Themes that came out in the analysis show that cooperative learning encouraged students to place value on mathematics, "Togetherness [in cooperative learning] formed unity, which would help students with future job skills," and discussion made an impact on their value for mathematics because they found value in mathematics when discussing topics with other students.

The data analysis revealed that the four standards-based pedagogies, cooperative learning, discourse, problem solving, and graphing calculators, were used in the study. Although the attitude inventory did not show a large change in students' attitudes, the qualitative data points toward the results that most students found at least one of the teaching strategies impacted their attitude of being a student of mathematics with regards to confidence, motivation, value, and enjoyment.

The second research question of this study was: *What specific instructional strategies do students believe most impact their attitudes about mathematics?* This research question was assessed through the student questionnaires, focus groups, and personal observations.

Themes that were found in the analysis of this data indicate that comments reflecting cooperative learning state students' attitudes were positively impacted because they were able to learn from others, increased student confidence, helping each other made material understandable, and students also perceived that cooperative learning decreased their test anxiety. Common themes regarding graphing calculators were that students felt the calculators lessened their workload, made mathematical topics more understandable, class was more enjoyable, and the students believed that the graphing calculators increased their confidence on exams. Although problem solving and discourse did not appear to have the same impact as cooperative learning and graphing calculators, some themes appeared in the analysis of student comments which indicate that students felt problem solving provided different ways to solve problems and also weakened their anxiety toward mathematics. Students perceived that discourse improved their ability to talk in front of others, and motivated them to want to learn more mathematics.

All four of the teaching pedagogies, cooperative learning, graphing calculators, discourse, and problem solving had an impact on the attitudes of students in mathematics. Each strategy played a role in making the learning environment in this classroom a better experience for the students. In one of the final focus groups, the main topic the students discussed was calculators and how the calculators became a useful tool. Some students felt that their confidence level had increased. One student commented it also helped that the instructor made class enjoyable because he was not anxious about coming to class.

The third research question was: *In what ways does the use of standards-based pedagogical strategies impact college algebra students' engagement in the learning process?* The personal observations, University of Wisconsin Observation Scale, and student questionnaires were the data sources for the third question.

The data analysis indicates that students' engagement increased over the course of the semester. Personal observations showed an increase in confidence was seen with a decrease in anxiety. A common theme that was found in the analysis of the qualitative data was that discourse and cooperative learning impacted student interaction in the classroom. As the semester progressed, student comments reflected their belief that the communications and discussions in the classroom was improving their confidence to speak to each other.

The data supports a gradual growth of student engagement in the classroom over the course of the semester. Students were more aware of the contributions they could make to the classroom as well as the knowledge they could gain from others.

Discussions of Findings

Students in this study indicated they felt more confident in their ability to do mathematics. Many of those attributed this to the interactions in their groups. The students frequently stated that by helping others and gaining ideas from their classmates improved their understanding of the material. Social interactions are seen as a critical part of knowledge construction because that is where the construction takes place. Koehler & Grouws (1992) advocated this when they said that students should not be passive absorbers of information, but rather have an active part of acquiring knowledge.

Social interactions occurred quite frequently with cooperative learning. When students were working with their peers, they stated that the process allowed them to not only learn from others, but to also teach what they knew to the other participants in the group. In the groups, it was perceived they were able to compare ideas and strategies to find solutions. Cooperative learning also encouraged individual thinking, which helped them understand the material and become more confident with working on mathematics. These findings are consistent with the research that indicates small group work has positive effects on student learning (Deeds, et al., 1999; Panitz, 1999; Rumsey, 1999; Grouws & Cebulla, 2000; Hill et al., 2003).

Cooperative learning was not the only strategy students referenced that increased confidence in their ability to do mathematics. Students attributed working with others, discovering multiple ways to do problems, discussion of mathematical topics and procedures, solving problems in groups, and becoming comfortable with the calculator increased their confidence. Exams and homework were no longer difficult to complete for many students. Although confidence did not have a statistical difference, the qualitative analysis shows that students frequently said their confidence was improved by graphing calculators, discourse, and problem solving. It is possible that students need more than four months in such a learning environment for the change in confidence to have statistical significance.

Students might also need more time with standards-based pedagogy to acquire a value for mathematics. Some students saw connections between the mathematics in the classroom and personal experiences; however, few students expanded on these connections. The questionnaires consistently had comments about each attitude, except for value and motivation. In only five occurrences, did a student report the connection between the mathematics they were learning and a personal experience. Perhaps students were too focused on the process instead of the application. And although the teaching strategies were enjoyable, no students mentioned that it motivated them to learn more mathematics. Only three students felt motivated to take more mathematics courses. In focus groups, most of the students were anxious to complete their final math course. They were appreciative of the varied teaching methods so that class was more fun, but they did not have a desire to take more math courses.

Reactions to the teaching methods, cooperative learning, discourse, problem solving, and graphing calculators varied throughout the semester. At the beginning of the semester, many students felt anxiety toward the methods of teaching in the classroom. A few students did not want to work in groups or speak in front of other students. Other comments reflected students' fears of problem solving and using graphing calculators. A small number of students were worried they did not have the appropriate calculator nor could they learn how to use the graphing calculator. As the course progressed, student reactions began to change. Although students marked ‘anxiety’ on the questionnaire, they elaborated by saying they were feeling less anxious about the course or the mathematics. Taking exams was easier because working and learning in groups had given them an opportunity to understand the material using various methods to complete the problems. The graphing calculators were becoming easier to operate and students felt that the calculators made the work easier and less time consuming. Even though a few students were anxious about speaking in front of others, the learning environment afforded them the chance to make a contribution to the discussion on mathematics. The discourse could also be seen during lessons coded as problem solving. Working individually, or in groups, most students felt they had become better problem solvers, which decreased their anxiety toward working with mathematics.

While working in small groups discourse was effective among the members. The evidence shows the thought processes students used facilitated the building of ideas. Discourse was expanded to give students an opportunity to discuss and develop problem solving methods while resolving misconceptions. Students in this study became more comfortable as the semester progressed at engaging in conversation about the procedures used in mathematics. In some cases, students felt that talking about the mathematics as a group helped them to understand the material. Discussions were also an avenue for students to verify their knowledge of the subject. This finding concurred with the idea that when given the opportunity, students can talk about mathematics in order to understand concepts or their own theories. In turn, they develop better dispositions toward mathematics because of their confidence with mathematical topics (Fennema et al., 1996).

Student confidence with mathematics and their enjoyment for mathematics was also enhanced by the use of the graphing calculators. Once the students became comfortable with using the calculators, they stated that their confidence with using the calculators to solve mathematical problems improved. Using the calculators lessened the workload, which made mathematics more enjoyable. These were the same findings in a study where students reported the graphing calculators made the work easier and the students felt they could accomplish more. The researcher in that study found that the graphing calculators visualized many algebra concepts, which made the work appear easier for the students (Santos-Trigo, 2002).

This study found that although confidence did not have a statistical significant change for student attitudes, students consistently mentioned, in student questionnaires and focus groups, their increase in confidence with mathematics because of the four teaching strategies. Most students felt that the group interaction improved their ability to solve mathematical problems. The group interaction also helped most students speak about mathematics in front of other people. Student reactions to the four teaching strategies varied throughout the semester, but interesting themes students indicated were the gradual appreciation for the graphing calculators, decreased anxiety as students became familiar with the learning environment, and an increase in communication amongst students regarding mathematical procedures. Although students felt more confident with mathematics, they had no desire to take more math courses.

Lessons Learned and Future Research

Lessons Learned

This study was chosen because the instructor wanted to see if incorporating a standards-based pedagogy into the curriculum would impact student attitudes toward mathematics positively. Over the course of one semester, the four teaching strategies, cooperative learning, problem solving, discourse, and the use of graphing calculators, were implemented into various tasks and projects. Although there were no statistically significant changes, the instructor did perceive through questionnaires, focus groups, and observations a more positive attitude toward mathematics by most of the students. Therefore, the instructor will continue to use these teaching strategies in the classroom in order to enhance the students' learning experiences with mathematics. The instructor identifies the following recommendations for future college algebra course taught:

1. An increased use of cooperative learning will encourage students to work together. Students' social skills and mathematical skills benefit from cooperative learning not only at the K-12 level, but also at the college level. Students will enjoy the tasks and become confident in their ability to work mathematics.
2. College algebra courses will incorporate an increased focus on mathematical discourse. Discourse nurtures quiet students in small groups and enhances students' ability to speak about mathematics correctly.
3. As in K-12, use of standards-based practice can improve student confidence and engagement in college level courses. This college mathematics instructor, therefore, will continue to be informed and engage in standards-based practices.

4. When students are posed with problem solving situations and they have an opportunity to work in groups, it decreases the anxiety of problem solving and encourages the students to stay on task. Concepts are linked and learned when students investigate and explore with minimal instructor interference. Problem-solving enhances students' understanding of mathematical topics, which decreases their anxiety about taking exams or working problems.
5. College algebra students will have access to graphing calculators because it took less time to do the homework. Students were more confident about their ability to complete mathematical problems when they used the graphing calculators. The calculators increased student enjoyment of the course and decreased their anxiety when taking exams.

The instructor is now aware of instructional strategies that can positively impact the classroom. Using cooperative learning, problem-solving, discourse, and graphing calculators was not too time consuming. The researcher spent a few days prior to the first class day choosing tasks that aligned with course content. As the semester progressed, activities were located on websites or obtained from colleagues. The instructor is not expected to make every lesson reflect active learning; however, by using these instructional strategies at least once a week can enhance student attitudes toward mathematics.

Recommendations for Future Studies

The researcher recommends that future studies about student attitudes at the college level should address the following:

1. Would student attitudes toward mathematics be impacted by standards-based pedagogies in advanced mathematics courses?
2. Is there a difference in the impact of student attitudes based on gender, age, or type of learner?
3. If student attitudes toward mathematics improve, how does it affect student achievement?

Future studies could also be done where each of the four pedagogies are separated. For this study, cooperative learning and graphing calculators both had a large impact on student attitudes. However, if a study was done with cooperative learning being the only teaching pedagogy, how would that affect the results of the study?

Conclusion

This study addressed the need to change student attitudes toward mathematics. Math reform is becoming a part of K-12 education. National reports call educator's attention to the lack of motivation students have for mathematics. However, math reform should not stop at high school. College instructors must also become aware of the attitudes students bring with them to the mathematics classroom. If college students continue to show a negative disposition for mathematics, our society will see fewer mathematics majors and a workforce with minimal mathematical skills. The purpose of this study was to assess potential benefits of implementing standards-based pedagogies at the college level using cooperative learning, problem solving, discourse, and graphing calculators.

The researcher used two self-taught college algebra courses for this study. The participants were a combination of traditional and nontraditional students with limited experience in an active learning environment. Using a combination of surveys, questionnaires, focus groups, observations, and evaluation tools, this study effectively shows that there are benefits to using standards-based pedagogies, such as cooperative learning, problem-solving, discourse, and graphing calculators, in the mathematics classroom. The analysis of this data shows common themes on the improvement of students' attitudes toward mathematics. The triangulation of data enhances the findings for this study.

This study identified that cooperative learning, problem solving, discourse, and graphing calculators improved student confidence in doing mathematics. Students felt less anxious in working problems on exams. Students also found the class enjoyable and were appreciative of the minimal lecture time. Anxiety was reduced as students became more familiar with the instructional strategies. This study also found that students recognized the value of mathematics for job skills and personal business, such as banking. The level of student engagement increased as the semester continued because students saw the benefits of sharing ideas, even if the idea was wrong.

This study recommends that college instructors become aware of student attitudes toward mathematics in their classrooms. The researcher suggests college instructors should become more informed on how standards-based pedagogies can be effectively implemented in their classroom. This study provides insight into the benefits and gives suggestions that can be placed into the classroom.

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APPENDIX A: WISCONSIN OBSERVATION SCALE

University of Wisconsin Observation Scale

Check the box that best describes what you observed in the course and how it relates to the initial statement.

1. The lesson provided opportunities for students to make conjectures about mathematical ideas.

______ No conjectures of any type were observed in the lesson.

______ Observed conjectures consisted mainly of making connections between a new problem and problems previously seen.

______ Observed conjectures consisted mainly of student investigations about the truthfulness of particular statements.

______ Students made generalizations about mathematical ideas.

2. The lesson fostered the development of conceptual understanding.

______ The lesson as presented did not promote conceptual understanding.

______ The lesson asked few questions that fostered students' conceptual development of mathematical ideas, or conceptual understanding was a small part of lesson design.

______ Some lesson questions fostered students' conceptual development of mathematical ideas, or some aspects of the lesson focused on conceptual understanding, but the main focus of the lesson was on building students' procedural understanding without meaning.

______ The continual focus of the lesson was on building connections between disparate pieces of information or linking procedural knowledge with conceptual knowledge.

3. Connections within mathematics were explored in the lesson.

______ The mathematical topic of the lesson was covered in ways that gave students only a surface treatment of its meaning. The mathematical topic was presented in isolation of other topics, and the teacher and students did not talk about connections between the topic of the lesson and other mathematical topics.

______ Connections among mathematical topics were present in the lesson. The teacher or students might have briefly mentioned that the topic was related to others, but these connections were not discussed in detail by the teacher or the students.

______ Connections among mathematical topics were discussed by teacher and students during the lesson, or connections were clearly explained by the teacher.

______ The mathematical topic of the lesson was explored in enough detail for students to think about relationships and connections among mathematical topics. During instruction, many students did at least one of the following: looked for and discussed relationships among mathematical ideas, expressed understanding of mathematical relationships, or provided explanations of their solution strategies for relatively complex problems in which two or more mathematical ideas were integrated.

4. Connections between mathematics and students' daily lives were apparent in the lesson.

______ Connections between mathematics and students' daily lives were not apparent in the lesson.

______ Connections between mathematics and students' daily lives were not apparent to the students, but would be reasonably clear if explained by the teacher.

______ Connections between mathematics and students' daily lives were clearly apparent in the lesson.

5. Students explained their responses or solution strategies.

______ Students simply stated answers to problems. They did not explain their responses or solution strategies orally or in written form.

______ Students explained how they arrived at an answer, but these explanations focused on the execution of procedures for solving problems rather than an elaboration on their thinking and solution path.

______ Students explained their responses or solution strategies. They elaborated on their solutions orally or in written form by justifying their approach to a problem, explaining their thinking, or supporting their results.

6. Multiple strategies were encouraged and valued.

______ Multiple strategies were not elicited from students.

______ Different problem-solving strategies were rarely elicited from students or only briefly mentioned by the teacher.

______ Students were asked if alternate strategies were used in solving particular problems, but this was not a primary goal of instruction.

______ Discussion of alternative strategies was frequent, substantive in nature, and an important element of classroom instruction.

7. The teacher valued students' statements about mathematics and used them to build discussion or work toward shared understanding for the class.

______ The teacher was interested only in correct answers. The majority of the teacher's remarks about student responses were neutral short comments such as "Okay," All right," or "Fine." No attempt was made to use students' responses to further discussion.

______ The teacher established a dialogue with the student by asking probing questions in an attempt to elicit a student's thinking processes or solution strategies.

______ The teacher valued students' statements about mathematics by using them to stimulate discussion or to relate them to the lesson in some way. The teacher opened up discussion about the student response by asking other students questions such as: "Does everyone agree with this?" or "Would anyone like to comment on this response?"

8. The teacher used student inquiries as a guide for instructional mathematics investigations or as a guide to shape the mathematical content of the lesson.

Yes No N/A

Circle Yes, if the teacher used students' inquiries as a guide for instructional mathematics investigations or as a guide to shape the mathematical content of the lesson.

Circle No, if a student's comment or question potentially could have led to such a discussion, but the teacher did not pursue it.

Circle N/A, if no such opportunities came about during the lesson.

9. The teacher encouraged students to reflect on the reasonableness of their responses.

______ The teacher rarely asked students whether their answers were reasonable. If a student gave an incorrect response, another student provided or was asked to provide a correct answer.

______ The teacher asked students if they checked whether their answers were reasonable but did not promote discussion that emphasized conceptual understanding.

______ The teacher encouraged students to reflect on the reasonableness of their answers, and the discussion involved emphasis on conceptual understanding.

10. Student exchanges with peers reflected substantive conversation of mathematical ideas.

______ There were no exchanges between peers in small groups or as a formal part of the general discourse within a large-group setting.

______ Student exchanges with peers reflected little or no substantive conversation of mathematical ideas.

______ Most students only asked one another for a clarification of directions given by the teacher or simply accepted someone's answer without an explanation of how it was found. Few students asked how a solution was found or asked for a clarification of another student's answer.

______ Most of the students asked their classmates for a description of how they solved a particular problem, discussed alternative strategies, and/or questioned how classmates arrived at a solution.

11. Interactions among students reflected collaborative working relationships.

______ None of the students were working together in small groups or in a large-group setting. If students were working in small groups, then one student typically gave answers to other members of group without explanation of why certain procedures were used.

______ Few students were sharing ideas or discussing how a problem should be solved in small groups or in a large-group setting. Although students physically sat together, there was little exchange of ideas or assistance. Many of the students in a group were working on different problems and at different paces.

______ Some students were exchanging ideas, or providing assistance to their classmates; however, a few students relied on other members of the group to solve problems. Contributions to solving problems were not made equally by all students.

______ Most students were involved with their classmates in solving problems and made sure that other group members were caught up and understood the problems before moving on to the next problem.

12. The overall level of student engagement throughout the lesson was serious.

______ Disruptive disengagement. Students were frequently off task, as evidenced by gross inattention or serious disruptions by many. This was the central characteristic during much of the class.

______ Passive disengagement. Students appeared lethargic and were only occasionally on task carrying out assigned activities. For substantial portions of time, many students were either clearly off task or nominally on task but not trying very hard.

______ Sporadic or episodic engagement. Most students, some of the time, were engaged in class activities, but this engagement was inconsistent, mildly enthusiastic, or dependent on frequent prodding from the teacher.

______ Widespread engagement. Most students, most of the time, were on task pursuing the substance of the lesson. Most students seemed to take the work seriously and were trying hard.

APPENDIX B: QUESTIONNAIRES

Date ______________________ Student #_____________

Which of the following, if any were you involved with this week? Circle your response. For each instructional strategy you experienced, describe in the space next to the strategy, its impact on your attitude toward math by writing beside the descriptors that were affected.

Cooperative Learning	Enjoyment
	Anxiety
	Confidence
	Motivation to pursue math
	Value of math for your personal or professional
Discourse	Enjoyment
	Anxiety
	Confidence
	Motivation to pursue math
	Value of math for your personal or professional
Problem Solving	Enjoyment
	Anxiety
	Confidence
	Motivation to pursue math
	Value of math for your personal or professional
Graphing Calculators	Enjoyment
	Anxiety
	Confidence
	Motivation to pursue math
	Value of math for your personal or professional

APPENDIX C: ATTITUDES TOWARD MATHEMATICS INVENTORY

ATTITUDES TOWARD MATHEMATICS INVENTORY

Directions: This inventory consists of statements about your attitude toward mathematics. There are no correct or incorrect responses. Read each item carefully. Please think about how you feel about each item. Place an X in the box that most closely corresponds to how the statements best describes your feelings. Use the following response scale to respond to each item.

PLEASE USE THESE RESPONSE CODES:

A – Strongly Disagree
B – Disagree
C – Neutral
D – Agree
E – Strongly Agree

	A	B	C	D	E
Mathematics is a very worthwhile and necessary subject.					
I want to develop my mathematical skills.					
I get a great deal of satisfaction out of solving a mathematics problem.					
Mathematics helps develop the mind and teaches a person to think.					
Mathematics is important in everyday life.					
Mathematics is one of the most important subjects for people to study.					
College math courses would be very helpful no matter what I decide to study.					
I can think of many ways that I use math outside of school.					
Mathematics is one of my most dreaded subjects.					
My mind goes blank and I am unable to think clearly when working with mathematics.					
Studying mathematics makes me feel nervous.					
Mathematics makes me feel uncomfortable.					
I am always under a terrible strain in a math class.					
When I hear the word mathematics, I have a feeling of dislike.					
It makes me nervous to even think about having to do a mathematics problem.					
Mathematics does not scare me at all.					
I have a lot of self-confidence when it comes to mathematics.					
I am able to solve mathematics problems without too much difficulty.					
I expect to do fairly well in any math class I take.					
I am always confused in my mathematics class.					
I feel a sense of insecurity when attempting mathematics.					

I learn mathematics easily.					
I am confident that I could learn advanced mathematics.					
I have usually enjoyed studying mathematics in school.					
Mathematics is dull and boring.					
I like to solve new problems in mathematics.					
I would prefer to do an assignment in math than to write an essay.					
I would like to avoid using mathematics in college.					
I really like mathematics.					
I am happier in a math class than in any other class.					
Mathematics is a very interesting subject.					
I am willing to take more than the required amount of mathematics.					
I plan to take as much mathematics as I can during my education.					
The challenge of math appeals to me.					
I think studying advanced mathematics is useful.					
I believe studying math helps me with problem solving in other areas.					
I am comfortable expressing my own ideas on how to look for solutions to a difficult problem in math.					
I am comfortable answering questions in math class.					
A strong math background could help me in my professional life.					
I believe I am good at solving math problems.					

APPENDIX D: COURSE EVALUATION

EVALUATING THE COURSE

Directions: This evaluation consists of statements about your thoughts toward this course. There are no correct or incorrect responses. Read each item carefully. Please think about how you feel about each item. Place an X in the box that most closely corresponds to how the statements best describes your feelings. Use the following response scale to respond to each item.

PLEASE USE THESE RESPONSE CODES:

1 – Strongly Disagree
2 – Disagree
3 – Neutral
4 – Agree
5 – Strongly Agree

	1	2	3	4	5
Group work was directly related to the topics being taught.					
There was an integration of concepts from chapter to chapter using group work.					
There was opportunity for you to reflect on ideas openly in the classroom and discuss your thoughts with other students.					
Connections were made between material previously learned and current content through group work.					
You were given the chance to verify your work with correct answers.					
The course gave you the problem solving and technological tools you will need to expand your knowledge in mathematics with future opportunities.					
Connections were made between material previously learned and current content through graphing calculators.					
You have a guide to solve problems with problem solving techniques.					
There was an integration of concepts from chapter to chapter using discussions.					
You were given the opportunity to explain and defend your solutions during discussions.					
Discussions were directly related to the topics being taught.					
There was an integration of concepts from chapter to chapter using the graphing calculators.					
Problem solving tasks were directly related to the topics being taught.					
You were given the opportunity to explain and defend your solutions during group work.					
Connections were made between material previously					

learned and current content through discussions.					
Graphing calculator tasks were directly related to the topics being taught.					
Connections were made between material previously learned and current content through problem solving.					

APPENDIX E: FOCUS GROUP PROMPTS

What instructional strategies (coop learning, discourse, etc) enhanced your attitude toward math (enjoyment, anxiety, etc)?

What instructional strategies (coop learning, discourse, etc) inhibited your attitude toward math (enjoyment, anxiety, etc)?

APPENDIX F: FREQUENCY OF USE OF INSTRUCTION PRACTICES

	Date	Technology	Discussion Forum	Problem Solving	Cooperative Learning
Plotting Points	F7				X
Rules for Intercepts	F9		X		X
Graphing Lines	F11	X			
Parallel/Perpendicular Lines	F14		X	X	
Reception Problem	F16			X	
Balloon Task	F18			X	X
Linear Regression	F23	X			
Rubber Bands and Linear Regression	F25	X		X	X
Independent vs. Dependent Variables	M11		X		
Quadratic Regression	M16	X		X	
Discussion on Math Education	M18		X		X
Discovering Transformations	M30	X		X	X
Rational Functions	A1	X		X	
Looking at Exponentials	A13	X			
Defining Logs Using Inverse Functions	A15	X	X	X	
Exponential Regression	A20	X		X	
Introduction to Matrices	A25	X			
Converting from Systems to Matrices	A27	X		X	X
Matrix Operations	A29	X	X		
Linear Programming Exercise	M2	X		X	X

APPENDIX G: DATA COLLECTION MATRIX

Dependent:	What instructional strategies impacted attitudes	Feelings regarding being a student of mathematics	Relevance of math in personal and professional experiences	What instructional strategies impacted engagement?
Focus Groups	X	X	X	X
Attitude Survey		X	X	
Questionnaires	X	X	X	X
Personal Observations	X	X	X	X
Researcher-Constructed Survey				
Wisconsin Observation Scale				

Independent:	Cooperative Learning	Problem Solving	Discussion	Tools
Focus Groups	X	X	X	X
Attitude Survey				
Questionnaires	X	X	X	X
Personal Observations				
Researcher-Constructed Survey	X	X	X	X
Wisconsin Observation Scale	X	X	X	X

APPENDIX H: RATING OF INDIVIDUAL ATTITUDES

Name ________________________

How would you describe your attitude toward mathematics? Attitude here deals with your confidence, anxiety, enjoyment, motivation to pursue mathematics, and how you value mathematics in your personal or professional life. Please circle one.

Positive attitude toward mathematics (enjoy working the problems, want a job that works with mathematics, don't mind homework)

Neutral attitude toward mathematics (math is something we all have to take so that's why I'm here, if I find a job that uses math I'll be okay)

Negative attitude toward mathematics (mathematics is useless, don't want a job that has any mathematics at all)

Name ________________________

How would you describe your attitude toward mathematics? Attitude here deals with your confidence, anxiety, enjoyment, motivation to pursue mathematics, and how you value mathematics in your personal or professional life. Please circle one.

Positive attitude toward mathematics (enjoy working the problems, want a job that works with mathematics, don't mind homework)

Neutral attitude toward mathematics (math is something we all have to take so that's why I'm here, if I find a job that uses math I'll be okay)

Negative attitude toward mathematics (mathematics is useless, don't want a job that has any mathematics at all)