A REINFORCED CONCRETE BRIDGE DESIGNED BY ULTIMATE AND ELASTIC THEORIES

by 566

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PURPOSE

The purpose of this report is to make a comparative study of two methods of designing a reinforced concrete structure: (i) ultimate strength design; (ii) working-stress design; and, to assess the advantage of ultimate strength design over working-stress design from the points of view of convenience, economy, and time required.

HISTORICAL BACKGROUND OF ULTIMATE STRENGTH DESIGN

A pronounced interest in the ultimate strength of structural members dates back only one or two decades but its origin may be found far back in engineering records, farther back, in fact, than the concept of linear elasticity and working-stress (1). The original ultimate strength design formulas were empirical, being based on the failure loads of typical elements as found by experiments (17).

Around 1900, A. N. Talbot and other early pioneers in this field pointed out that a curved stress-strain relationship must be used for an accurate determination of the ultimate strength of reinforced concrete members. Several early theories for predicting the strength of reinforced concrete members such as Thullie's flexural theory in 1897 (1), and Ritter's introduction of the parabolic distribution of concrete stresses in 1899 (1), were ultimate load theories. However, straight line theory was generally accepted because it was mathematically simple and the resulting safety factors with respect to ultimate load observed in tests were sufficiently controlled to satisfy the requirements of that time (3). Straight line theory was adopted by the Joint

Committee on Standard Specifications for concrete in 1909, with an allowable concrete compressive stress equal to 0.325 times the ultimate compressive strength of the concrete; therefore, the safety factor against compressive failure was nearly three (4).

There are two assumptions which form the basis for the formulas used in straight line theory. 1. Plane sections before bending remain plane after bending; this implies that the unit deformation of the material at any given point is proportional to the distance from the neutral axis. 2. Stress is proportional to strain, that is, unit stress at a point is proportional to its distance from the neutral axis (10).

In 1921, McMillan's study of column test data showed that building columns under load develop steel stresses due to creep of the concrete considerably higher than those predicted by straight line theory. In 1930, Whitney stated that the average stress in the concrete at ultimate load is $0.85f_c'$ and that at a stress of approximately $f_c'/2$ stresses and strains are no longer proportional (26). He also suggested the use of a simplified rectangular stress block and thereby greatly simplified the ultimate strength design equations (6).

In 1955, for the first time the Joint Committee of ACI-ASCE, allowed the use of ultimate strength design for simple structures (1).

Mattock, Kriz and Hognestad in their 1961 paper have also recommended the use of an equivalent stress distribution in the concrete (17).

This theory has been widely used in building frames all over the world. In 1960, a paper was published by Jain on plastic theory applied to two-hinged arches. The purpose of his paper was to present a method of calculating the actual ultimate strength of two hinged arches (22). The theoretical results have been verified by experiment and complete agreement is found to exist. From this paper it can be seen that in each case, elastic theory underestimated the ultimate strength by about 40 per cent, while the proposed theory (ultimate) gives results which agree closely with the test observations.

It is concluded, therefore, that the method of ultimate strength design permits prediction with sufficient accuracy of the ultimate strength in bending, in compression, and in combinations of the two, of all types of structural concrete sections likely to be encountered in practice (19).

The tables and curves presented in Wang's paper in 1962 are believed to be adequate for ordinary building frames (21). However, for more complicated members such as circular columns and irregular sections these curves are not applicable. Columns with biaxial bending are solved by adopting simplified and approximate formulas and using curves for uniaxial bending. The author has adopted the ACI-Code assumptions (23). With the aid of tables and curves, ultimate design of concrete structures will be made more appealing to practicing engineers, as evidenced by the examples shown (21).

ULTIMATE STRENGTH DESIGN THEORY

Ultimate strength design means the design of reinforced concrete structures by ultimate strength theory to resist shears, moments and thrusts which have been determined from elastic analysis of the structure. The assumed design loads are multiplied by specified load-factors to obtain ultimate shears, moments and thrusts. It should be noted that ultimate theory is a method of proportioning sections based on their actual strength as confirmed by tests. When combined with the use of load factors it provides a method of obtaining uniform factors of safety.

The assumptions on which ultimate strength design theory is based are as follows:

1. Strain in the concrete shall be assumed directly proportional to the distance from the neutral axis (5).
2. The maximum unit strain at the extreme compressive fibre at ultimate strength shall be assumed equal to 0.003 inch per inch (18).
3. Plane sections normal to the axis remain plane after bending (4).
4. Tensile strength in concrete is neglected in sections subjected to bending (5).
5. Maximum compressive fibre stress in concrete does not exceed $0.85f'_c$ (4).
6. The diagram for compressive stress distribution is assumed rectangular for all sections (14).

7. Stress in tensile and compressive reinforcement at ultimate load shall not be assumed greater than the yield point or 75,000 psi, whichever is smaller (23).

Ultimate strength theory considers only the stress distribution in the member at the ultimate load. This distribution as observed from standard test cylinders under standard loading and sustained loading takes the shape shown in Plate I. The stress-strain diagram of an actual structure more nearly approximates the stress distribution under a sustained loading which shows more strain for the same stress when compared with the standard rate of loading (9). This is caused by plastic flow or creep in the concrete. Ultimate failure of the concrete occurs under a sustained load at about 85 per cent of the maximum ordinate in the standard loading diagram (15).

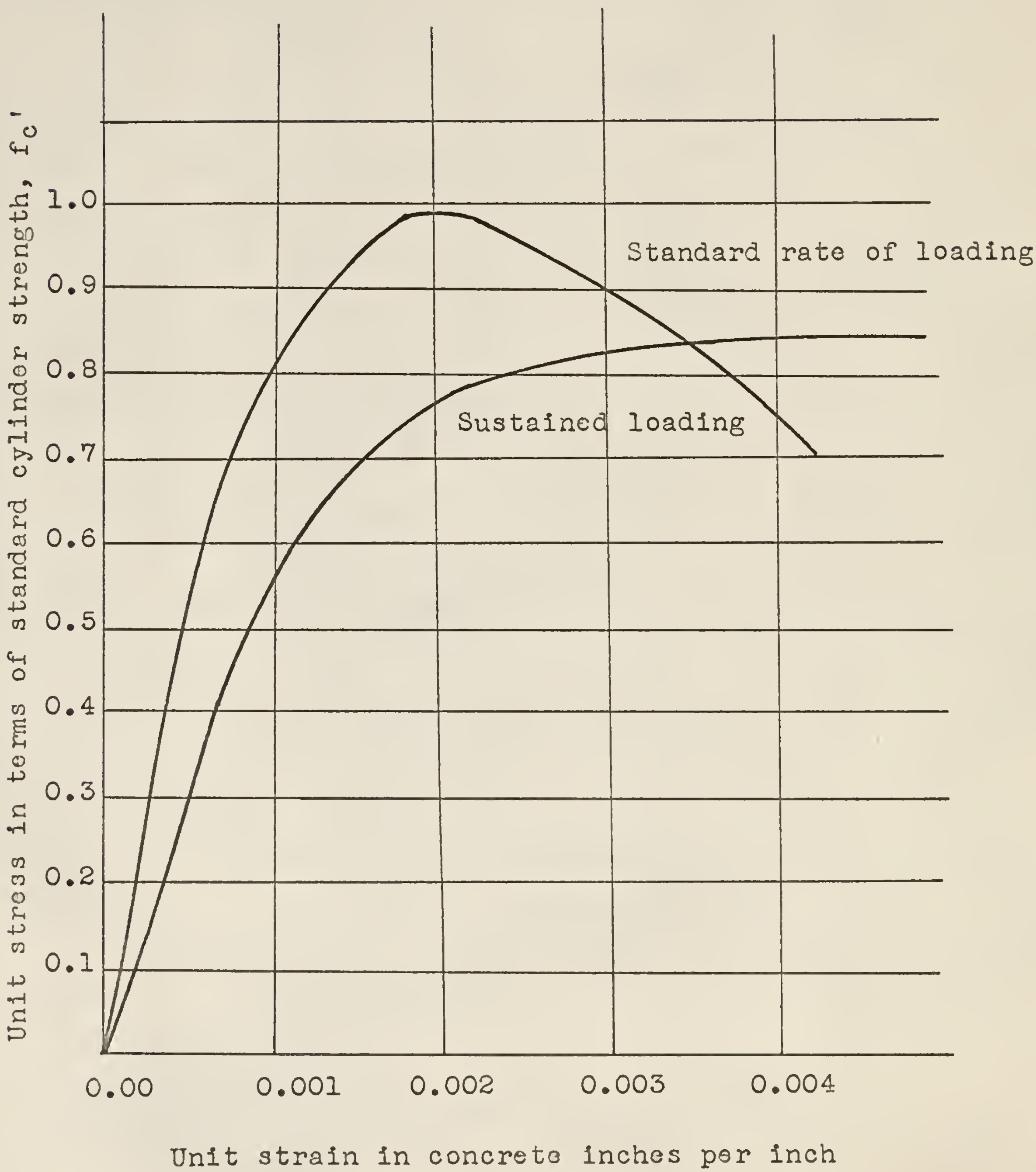
General requirements are as follows:

1. "The Building Code Requirements for Reinforced Concrete" by the ACI apply to the design of members (23).
2. Analysis of indeterminate structures shall be based on the assumption of elastic behavior.
3. Attention should be given to the deflection of members, including the effect of creep whenever the net ratio, p, of reinforcement in any section of a flexural member exceeds $0.18f'_c/f_y$ (1).

The advantages of this theory are:

1. As ultimate load is approached stress and strain are not proportional; therefore, the straight line theory does not

PLATE I


Unit stress in terms of standard cylinder strength, f_c'
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Standard rate of loading
Sustained loading
0.00
0.001
0.002
0.003
0.004
Unit strain in concrete inches per inch

give a reliable prediction of the ultimate strength of a section. It follows that the actual factor of safety cannot be determined by straight line theory (1).

2. Dead load is a determinate quantity and generally remains unchanged during the life of the structure but actual live load is less predictable. Therefore, it is unreasonable to apply the same load factors to dead load and live load. This deficiency is eliminated by this theory (1).

3. The design of beams under flexure by the method based on working stresses and assumed straight line variation of stress in the concrete can give approximately correct results for highly reinforced members but it grossly underestimates the compressive flexural strength of concrete. The use of ultimate strength theory permits smaller, tougher beams, more heavily reinforced for tension with reduction of compressive reinforcement (6).

4. A better evaluation of the critical moment-thrust ratio for members subject to combined bending and axial load is obtained by the ultimate strength design procedure. In structures like arches and multiple-story frames, the thrust may be due largely to dead load while moment is created by live load (1).

5. Ultimate strength design permits smaller, tougher sections, by reducing the size of the members and thus reducing the

rigidity of the structures. The stresses caused by volumetric changes are thus minimized (16).

6. The actual ultimate strength of two-hinged arches of uniform section for various pattern of loading is 50 to 100 per cent greater than the ultimate strength predicted by elastic theory (22).

LOAD FACTORS

In reality, the actual strength of a structure can fall below its calculated value for various reasons: inaccuracies and imperfections in erection; substandard steel or concrete; assumptions and approximations made in analysis; and, the actual load may exceed the assumed design loads. Blast pressure, fire, or other emergencies may cause unforeseen impact or excess load. To ensure the safety of the structure from these possibilities, the design strength should exceed the design load by a sufficient margin to accommodate these variations. This is achieved by multiplying the design loads by a load factor. The ACI-Code (23) recommends that members should be proportioned so that: (i) they will be capable of carrying without failure the critical load combination given below, thereby insuring an ample factor of safety against an increase in live load beyond that assumed in design; (ii) the strains under working loads should not be so large as to cause excessive cracking. These criteria are satisfied by the following formula.

For those structures in which effects of wind and earthquake loading are neglected,

$U = 1.5\ B + 1.8\ L.$

A large margin of safety is applied to live loads because they are much more uncertain than dead loads which are subject to very little change during the life of the structure.

U = ultimate strength of section.

B = effect of basic load consisting of dead load plus volume change due to plastic and elastic actions, shrinkage and temperature.

L = effect of live load plus impact.

THE DESIGN OF A RIGID FRAME CONCRETE BRIDGE BY ULTIMATE AND ELASTIC THEORIES

The Design Problem

Rigid frame bridges have been extensively used for intersecting highways and over numerous streams and in locations where it is necessary to meet conditions imposed by restricted headroom. This type of bridge has proved economical for spans of one hundred feet and more (24). This type of bridge was introduced in the U. S. A. in 1922, by Arthur G. Hayden, Design Engineer, Westchester County, New York, Park Commission.

The bridge has its abutment and deck cast as a unit. A great deal of benefit is derived from this continuity. From the theory of indeterminate structures it can be shown that the moments are small in the sections near the center of the deck of the rigid frame bridge as compared with the corresponding moments in a simply supported deck of the same span, hence the section of the deck can be reduced at the center as required. Arthur G. Hayden (29) stated that the reinforced concrete rigid frame bridge requires only about 60 per cent of the material which would be required for a constant section frame. The depth of the deck at the center of the bridge is commonly a fortieth of the clear span. This fact also provides one more advantage in that the height of the frame can be reduced.

As stated by Hayden no complex mathematical analysis is necessary for these structures (24).

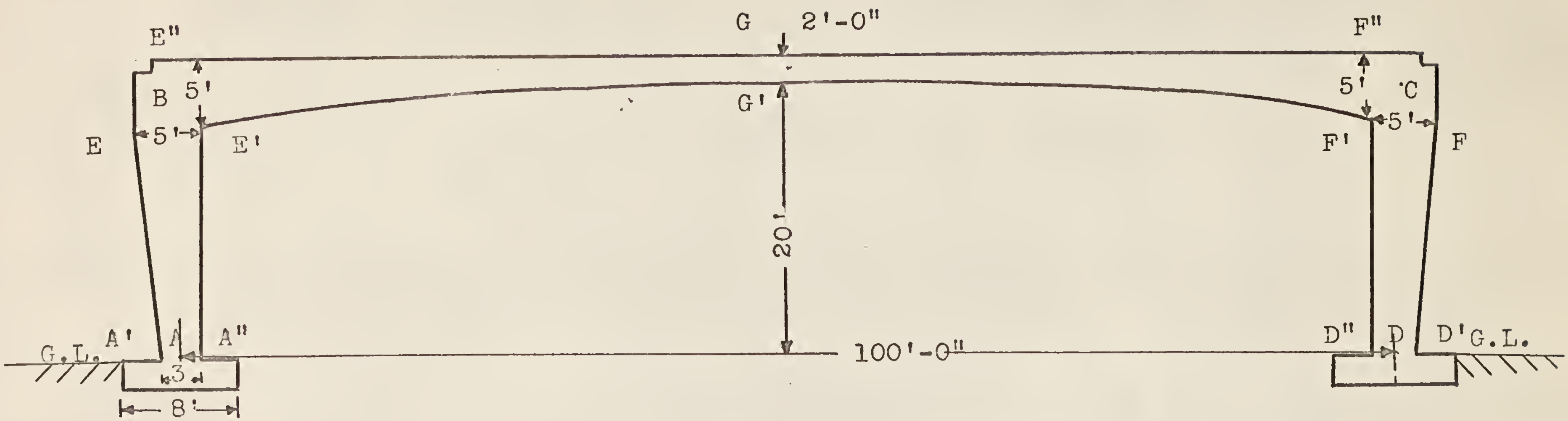
A rigid frame bridge may be widened without any major alterations in existing structures, and even the normal traffic is not interrupted. Traffic moves under and over these bridges with great safety.

Rigid frame concrete bridges with spans up to 175 feet have been built in the U. S. A., but it is realized that for heavy highway loading these are economical only up to a span of about 70 feet.

A common location for such a bridge is at an intersection of a divided highway and a secondary highway (30). The divided highways are usually 4 lane with a median of 12 to 20 feet and the secondary highway normally is 2 lane with sidewalks and curbs. The clearance at the center generally is taken to be 15 to 20 feet.

A booklet (24) published by the P. C. A. gives some empirical proportions for a rigid frame bridge. These empirical rules give approximate sections which should be checked for stresses for a particular loading in a detailed analysis.

The assumed details for the bridge design are as listed below and shown in Fig. 1.



Front Elevation of Bridge

Fig. 1

a. The top of the deck is assumed flat.

b. The clear span for the bridge is 100 feet.

c. The depth at the center of the deck can be taken approximately one fortieth of the span. Hence, select 2 feet for the depth of this section of the deck.

d. Assume EE' and EE" about L/18 or 5 feet.

e. Soffit curve is taken as a parabola.

f. Select a clearance of 20 feet for level of AD to G'.

g. Assume A'A" is equal to 3 feet.

h. Let A"E' be a straight line.

The footing of a rigid frame bridge could be hinged or fixed. Horizontal thrust and vertical load, both act on the footings. In the hinged support there is no restraining moment at the base of the column and thus it is free to rotate. Actually a support is almost always partially restrained.

These bridges should be designed to withstand the usual loads; dead load, live load and earth pressure. The dead and live loads are calculated in the same fashion as for other bridges with the exception of the earth pressure on the end walls. Earth pressure on abutments for a simple span bridge is usually active pressure, produced by the backfill moving towards the abutment. In the rigid frame bridge, it is possible to develop some passive earth pressure by a movement of the end wall against the backfill. It has been found experimentally that there is a little passive pressure on the end wall which may ordinarily be disregarded.

Another set of forces is caused by the relative displacement of foundations and volume changes due to temperature variations and shrinkage of concrete.

Analysis of the Structure

Calculation of Frame Constants

1. Deck Coefficients (Fig. 2)

$$r_c = r_b = \frac{5-2}{2} = 3/2 = 1.5$$

Using Table 20, (28)

$S = 17.0 \qquad S_{bc} = S_{cb} = 17.0$

The stiffness of the deck at B, or the moment at B necessary to give BC a unit rotation at B when C is fixed, is $S \times I_o/L$, or proportional to $17 \times \frac{(2)^3}{100} = 1.37$, say 1.40. The carry-over factor, C, equals

$$C_{bc} = 0.74$$

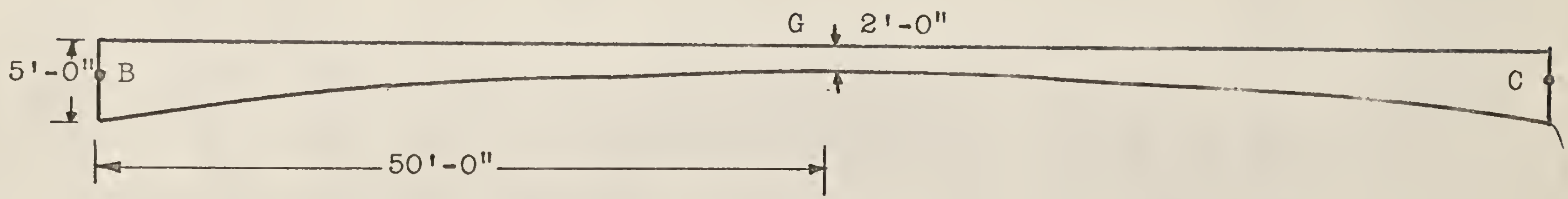
2. End Wall Coefficients (Fig. 3)

The end wall element in Fig. 3 is trapezoidal with height of 19.5' and defined by two straight lines 3' apart at A and 5' apart at B.

Using Table 26, (28)

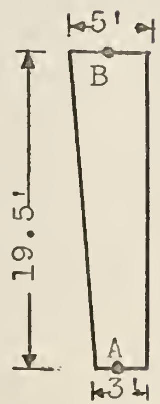
$r_b = \frac{5-3}{3} = \frac{2}{3} = 0.67 \qquad r_a = 0 \qquad a_b = 1 \qquad a_a = 0$

$S_{ba} = 10.5$



Deck Section

Fig. 2



Wall Section

Fig. 3

$S_{ab} = 4.95$

$C_{a_b}S_{a_b} = C_{b_a}S_{b_a} = 3.90$

The stiffness at B when A is fixed, is given by

$S_{b_a} I/L = 10.5(3)^3/19.5 = 14.5$

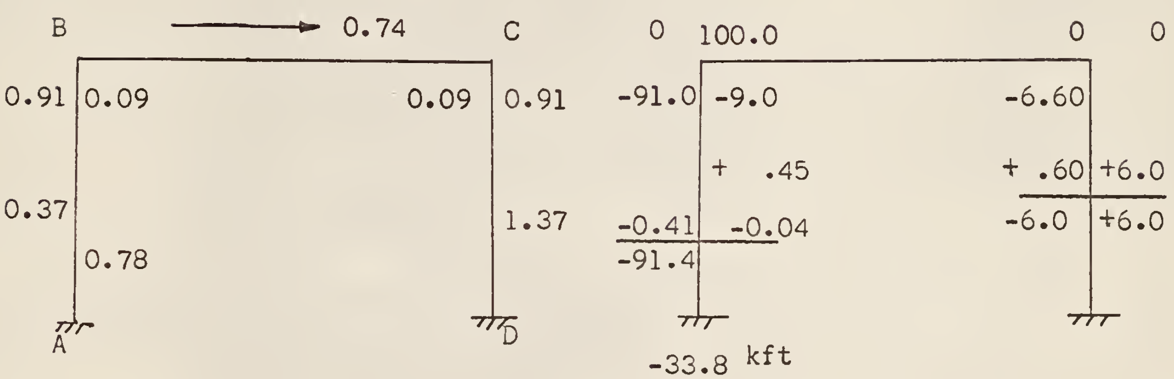
$C_{ab} = 0.78$

$C_{ba} = 0.35$

3. Distribution Factors

$$\text{At B, } D_{BC} = \frac{1.37}{1.39 + 14.5} = 0.09$$

$$D_{BA} = \frac{14.1}{1.37 + 14.1} = 0.91$$



Carry-over and distribution factors

(a)

Final distributed moments

(b)

Fixed supports

Fig. 4

4. Hinged Support Condition

The stiffness at B, when A is fixed, is

$$S_{b_a} I/L = 10.5 \times \frac{3^3}{19.5} = 14.5$$

When A is hinged, it is given by

$$S_{b_a} I_o/L \;(1 - C_{a_b} C_{b_a}) = 10.5(1 - C_{a_b} C_{b_a}) \;;$$

$$C_{ab} = 0.78$$

$$C_{ba} = 0.35$$

$$= 10.5\left(1 - \frac{3.90^2}{10.5 \times 5}\right)$$

$$= 7.5\ I_o/L$$

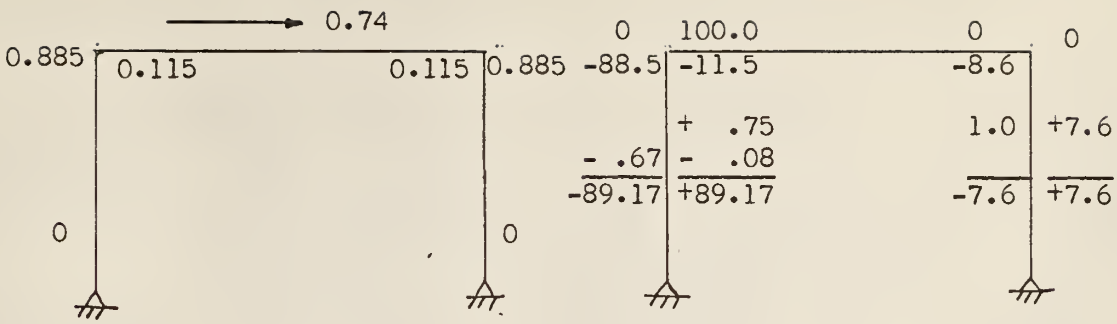
$$= 7.5 \times \frac{3^3}{19.6} = 10.4$$

The relative stiffness in per cent at B is then

$$D_{BA} = \frac{1.37}{1.37 + 10.4} = 11.5 \text{ per cent or } 0.115$$

$$D_{BC} = \frac{10.4}{1.37 + 10.4} = 88.5 \text{ per cent or } 0.885$$

In the fixed support condition the moment at the support is quite high. Moreover, there is not much difference at the haunch moment between fixed and hinged support conditions.



Carry-over and distribution factors

(a)

Final distributed moments

(b)

Hinged supports

Fig. 5

5. Dead Load

The frame carries its own dead load in addition to which a concrete thickness of 1/2" will be allowed for an integral roadway wearing surface.

The weight of the end walls is carried directly down to the footings and creates no moments. Effect of eccentricity is neglected.

The longitudinal section through the deck is divided into an area 100 ft. long with a constant depth of 2' - 1/2" weighing 290 psf. The remaining area is divided as shown in Fig. 6.

Fixed-end moments per foot of width:

Uniform load = $290 \times 100^2 \times 0.106 = {}^{*}-307{,}500$ ft. lbs.

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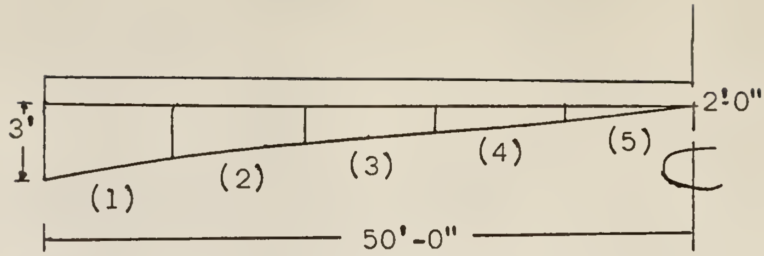
* 

Compression in bottom fibre.



Half portion of deck

Fig. 6

Equivalent concentrated load:

(1) = 3900 x 100 x 0.05 = 19500.0 ft. lbs.

(2) = 3024 x 100 x 0.13 = 39312.0 ft. lbs.

(3) = 2160 x 100 x 0.19 = 41000.0 ft. lbs.

(4) = 1300 x 100 x 0.22 = 28600.0 ft. lbs.

(5) = 430 x 100 x 0.20 = 8,600 ft. lbs.

(6) = 430 x 100 x 0.14 = 6,000 ft. lbs.

(7) = 1300 x 100 x 0.085 = 11,000 ft. lbs.

(8) - 2160 x 100 x 0.040 = 8,640 ft. lbs.

(9) = 3020 x 100 x 0.012 = 3,620 ft. lbs.

(10) = 3900 x 100 x 0.002 = 780 ft. lbs.

Total = 479,550 ft. lbs.

Say *480,0 kft

Using the values 89.17 and 7.6 per cent determined in Fig. 5a, the numerical values of the corner moments at B are

480(0.8917 + 0.076) = 470.0 kft

Reduction for curvature correction is about 2 per cent (24).

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*  Compression in top fibre.

Therefore, final moment at B = 460.0 kft

The total positive moment assuming a simply supported deck is,

U.D. = $290 \times 100^2 \times 0.125 = +362{,}000$ ft. lbs.

(1) = $3900 \times 0.05 \times 100 = +19{,}500$ ft. lbs.

(2) = $3024 \times 0.15 \times 100 = +44{,}300$ ft. lbs.

(3) = $2160 \times 0.25 \times 100 = +54{,}000$ ft. lbs.

(4) = $1300 \times 0.35 \times 100 = +45{,}500$ ft. lbs.

(5) = $430 \times 0.45 \times 100 = +20{,}350$ ft. lbs.

Total = +536,800 ft. lbs.

Say $\overset{*}{+}$537.0 kft

Reduction for curvature is about 2 per cent (24)

= 526.0 kft

The difference between this moment and negative corner moment

=+66.0 kft at crown

6. Live Load

Standard Specifications for Highway Bridges adopted by the American Association of State Highway Officials provide that a truck-train loading, or an equivalent lane loading consisting of a uniform load and a single concentrated load, be used for the design of bridges.

This bridge is designed for the heaviest loading, i.e., H20-S16 as shown in Fig. 7.

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* 

Compression in top fibre.

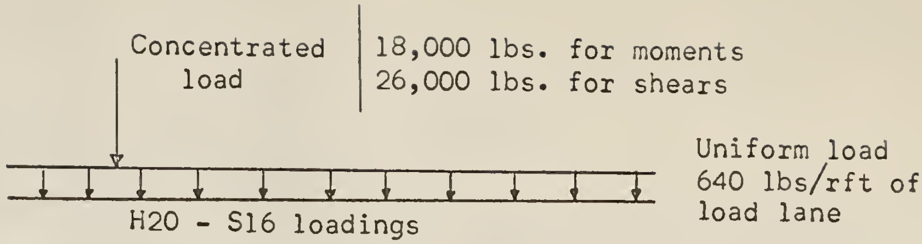


Fig. 7

Load per foot of width for moments:

concentrated load = 18000/10 = 1800 lbs.

distributed load = 640/10 = 64 lbs.

Including a 20 per cent impact allowance, the frame carries a concentrated live load of 2200 lbs. and a uniform live load of 80 psf per foot of width.

Influence lines for live load:

To find the influence lines for moments and shears at various points, matrix formulation of the slope deflection method is used.

Sign conventions:

Counterclockwise moments at the ends of member are considered positive; clockwise loads and rotations at joints are considered positive.

Writing the matrix formulation of slope deflection equations

$$\left[S_{on}\right] = \left[k\right] \left[V_{on}\right]$$ force-displacement equations

$$\left[V_{on}\right] = \left[a'\right] \left[r\right]$$ displacement transformation equations

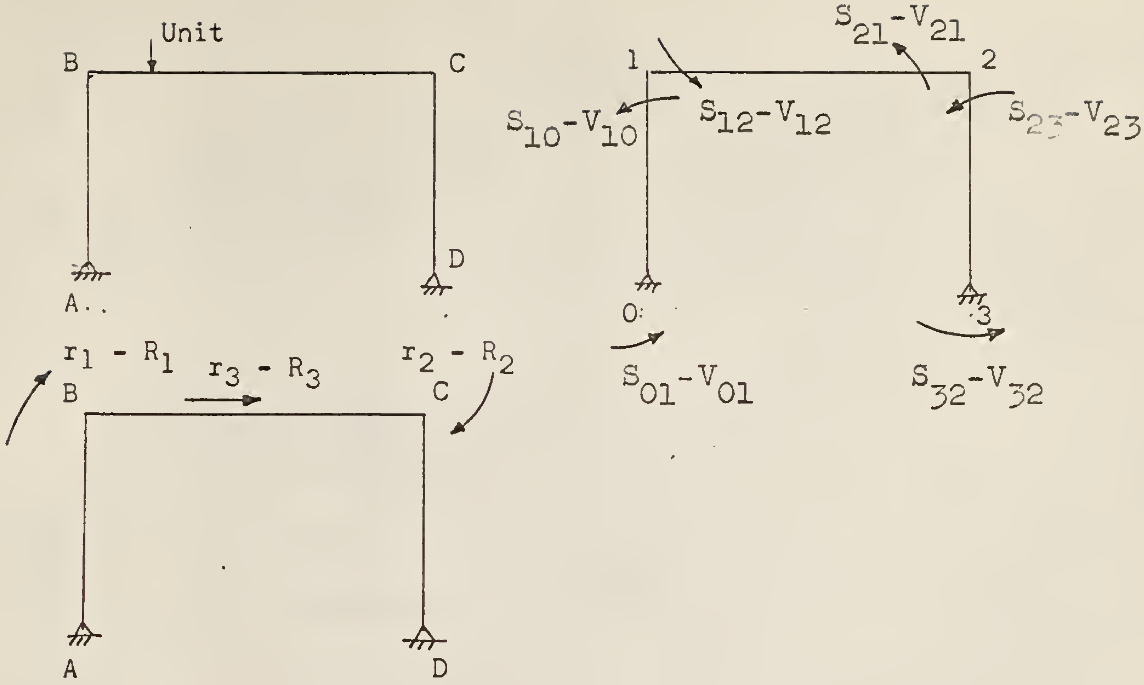


Fig. 8

$$\begin{bmatrix} S_{01} \\ S_{10} \\ S_{12} \\ S_{21} \\ S_{23} \\ S_{32} \end{bmatrix} = \begin{bmatrix} 0.64 & 0.23 & 0 & 0 & 0 & 0 \\ 0.23 & 0.64 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.17 & 0.13 & 0 & 0 \\ 0 & 0 & 0.13 & 0.17 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.64 & 0.23 \\ 0 & 0 & 0 & 0 & 0.23 & 0.04 \end{bmatrix} \begin{bmatrix} V_{01} \\ V_{10} \\ V_{12} \\ V_{21} \\ V_{23} \\ V_{32} \end{bmatrix}$$

$[V] = [a'](r)$

$$\begin{bmatrix} V_{01} \\ V_{10} \\ V_{12} \\ V_{21} \\ V_{23} \\ V_{32} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/19.5 \\ -1 & 0 & 1/19.5 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1/19.5 \\ 0 & 0 & 1/19.5 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}$$

Load matrix:

	R_1	R_2	R_3
Load 10' from left support	= 9.34	-0.5	0
Load 20' from left support	=15.5	-3.2	0
Load 30' from left support	=21.1	-6.0	0
Load 40' from left support	=19.3	-12.0	0
Load at crown	=17.4	-17.4	0

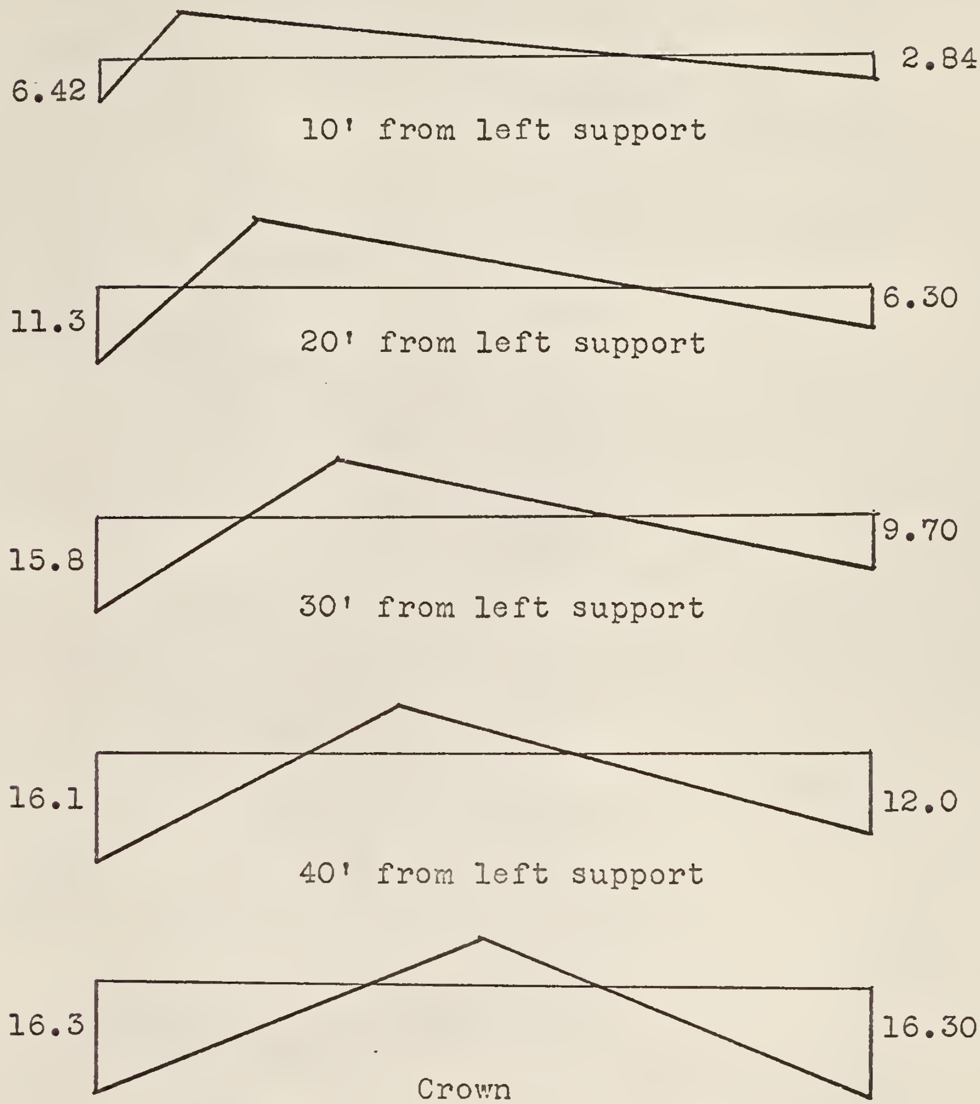
$$\begin{array}{c|ccccc} & 10' & 20' & 30' & 40' & 50' \\ \begin{matrix} R_1 = \\ R_2 = \\ R_3 = \end{matrix} & \begin{bmatrix} 9.34 \\ -0.50 \\ 0 \end{bmatrix} & \begin{bmatrix} 15.5 \\ -3.2 \\ 0 \end{bmatrix} & \begin{bmatrix} 21.1 \\ -6.0 \\ 0 \end{bmatrix} & \begin{bmatrix} 19.30 \\ -12.0 \\ 0 \end{bmatrix} & \begin{bmatrix} 17.4 \\ -17.4 \\ 0 \end{bmatrix} \end{array}$$

7. Earth Pressure

The frame is subjected to active earth pressure on the end walls due to the backfill. Moreover, the wall is also subjected to active earth pressure due to the live load as it moves across the backfill approaching the structures.

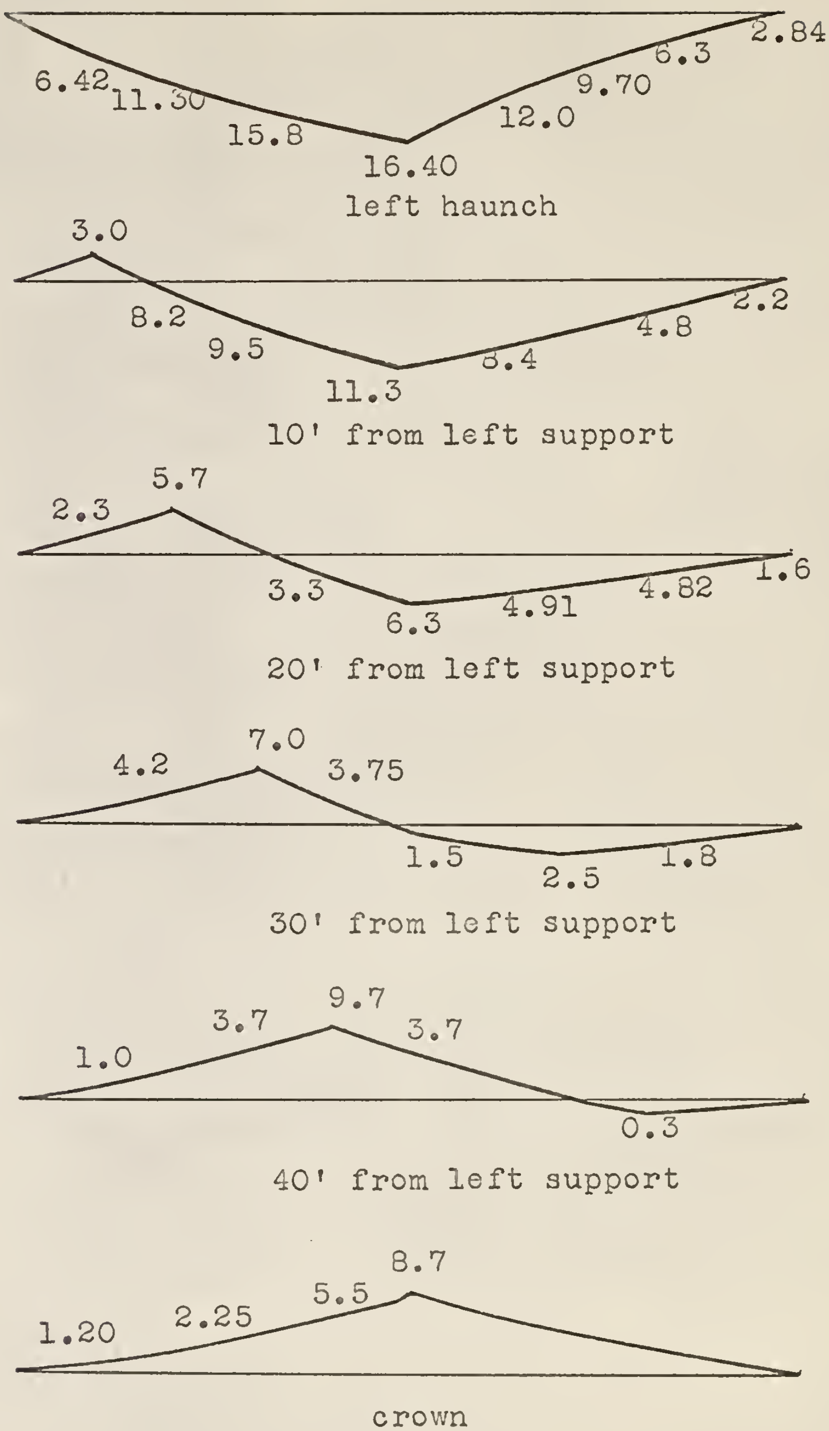
Coulomb's graphical construction is used to determine the active pressure as shown in Fig. 13. From Fig. 13, it can be seen

The final end moments on the following moment diagrams were calculated by an IBM 1620 computer.



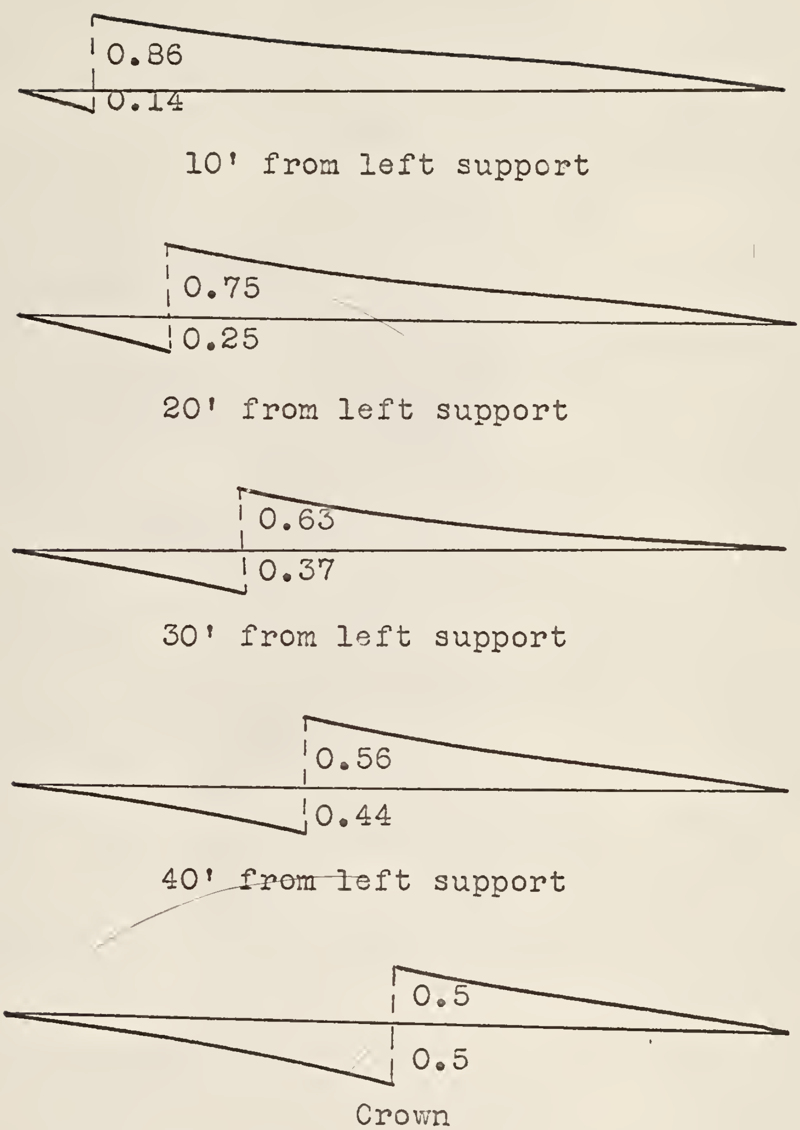
Deck Moment Diagrams for Unit Load at Each Load Point

Fig. 9



Influence Lines for Moment in the Deck Section

Fig. 10



Influence Lines for Shear in the Deck Section

Fig. 11

that when the concentrated load is near the wall, it exerts maximum pressure on the wall.

The assumed properties of the backfill are shown in Fig. 13. This is a medium sand. The natural water table is assumed to be well below the ground line. Therefore, water will not accumulate behind the wall, so constant water pressure is not taken as one of the loads on the end wall. As sand is a free-draining soil, rain water will be drained without exerting any significant transient water pressure on the wall.

The total earth pressure, P_A, is 11.85^k. (From Fig. 13)

$$\Delta P_A = 1.0^k$$

$$P_A \cos S = 1/2\ p_A H \qquad S = 15^\circ$$

$$\text{Therefore } p_A \doteq 1.0^k \qquad H = 21'$$

To calculate the fixed-end moments, the triangular load is split up into several concentrated loads as shown in Fig. 12.

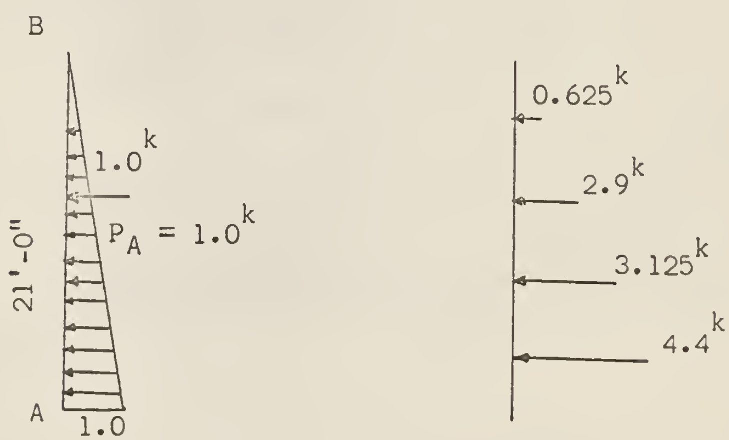
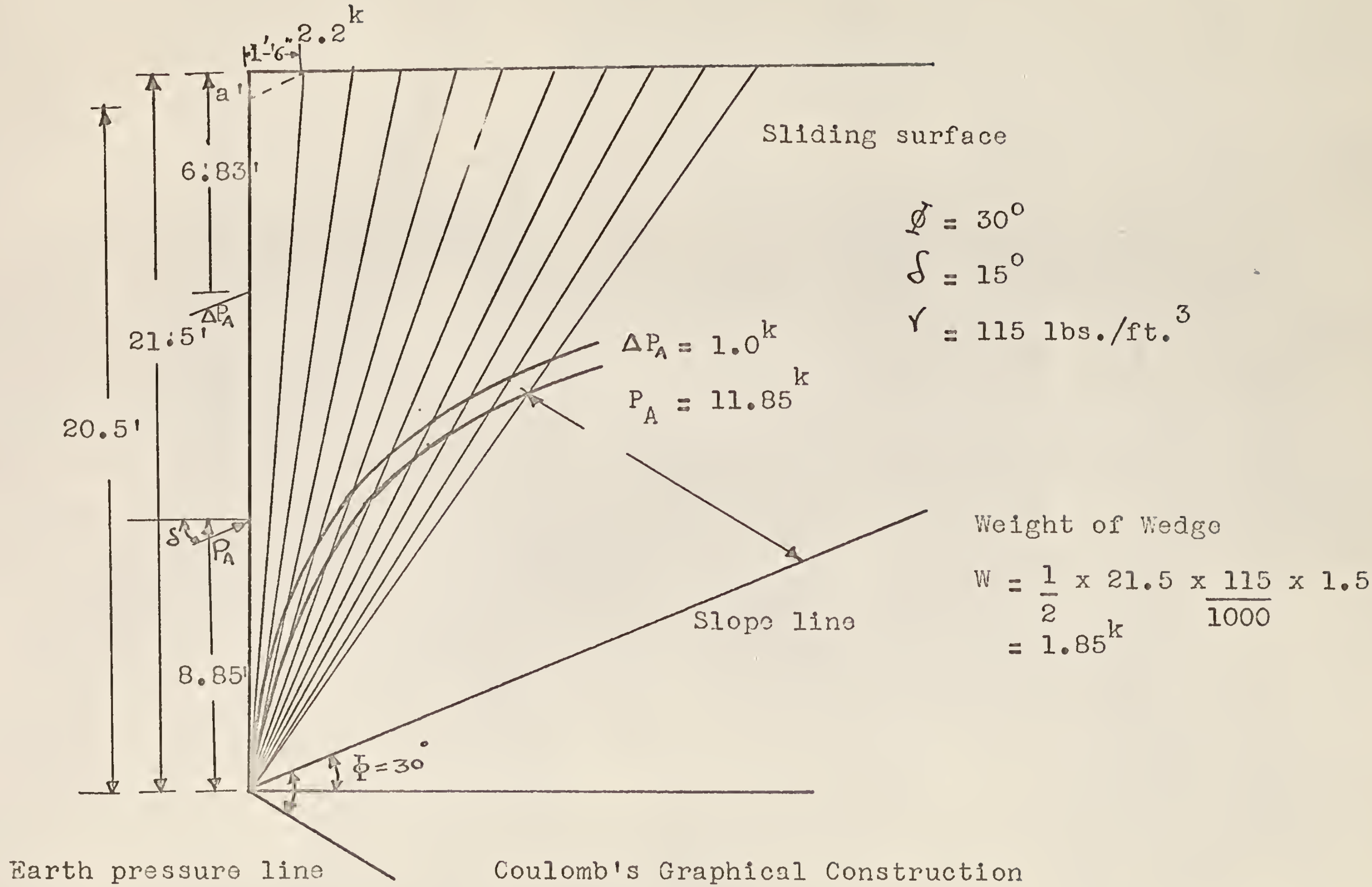


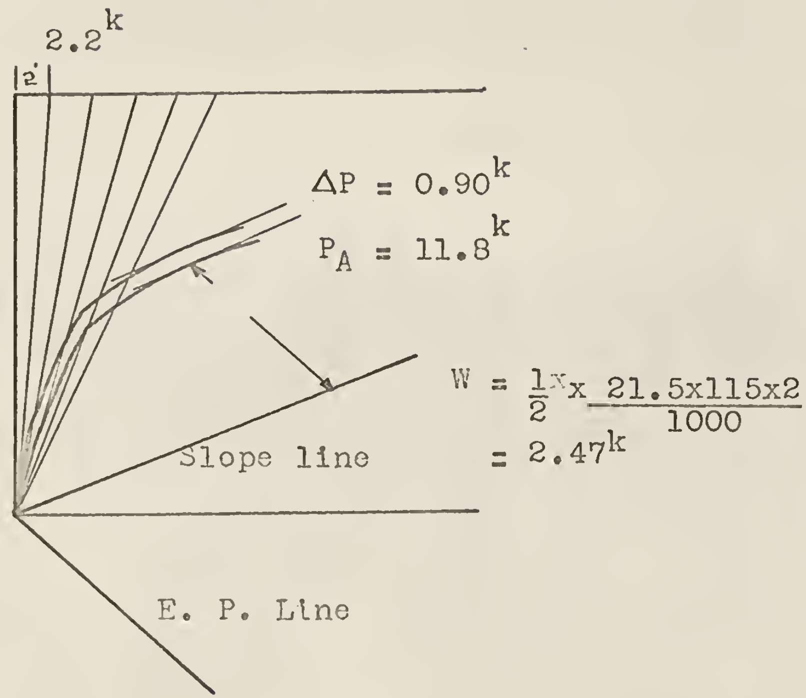
Fig. 12. Earth Pressure Diagram



Coulomb's Graphical Construction

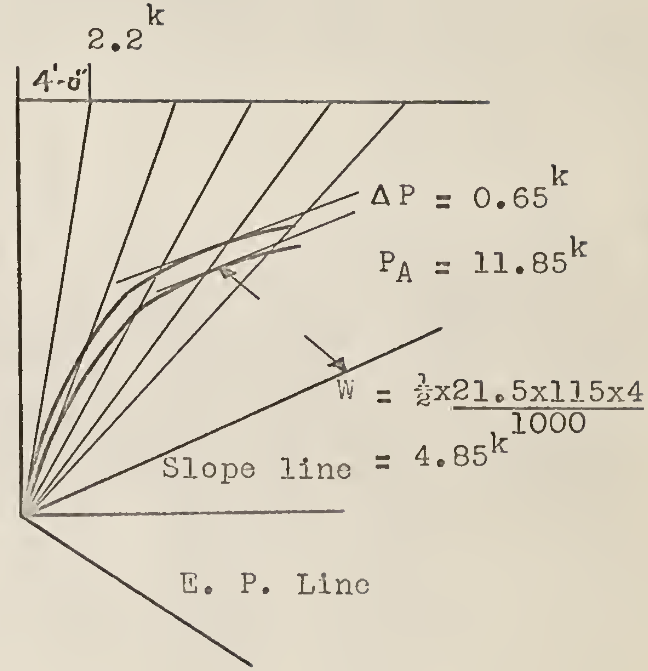
Live load is 1'-6" from the wall. The width of the wedge is 1'-6".

Fig. 13A



Live load is 2'-0" from the wall.
The width of the wedge is 2'-0".

Fig. 13B



Live load is 4'-0" from the wall.
The width of the wedge is 4'-0".

Fig. 13C

Frame Constants:

$r_b = 0.67, \quad a_b = 1, \quad r_a = 0, \quad a_a = 0$

Fixed-end moment coefficients:

Distance		2.5'	7.5'	12.5'	17.5'
Load		0.625^k	2.9^k	3.125^k	4.4^k
Coefficient	M_{AB}	0.077	0.105	0.065	0.004
	M_{BA}	0.016	0.144	0.184	0.09

$$M_{FAB} = (0.077 \times 0.625 + 2.9 \times 0.105 + 3.12 \times 0.065 + 4.4 \times 0.004)$$
$$21.5 = 0.55 \times 21.5 = 12.0^{kft}$$

$$M_{FBA} = (0.016 \times 0.625 + 2.9 \times 0.144 + 3.12 \times 0.184 + 4.4 \times$$
$$0.09)21.5 = 1.4 \times 21.5 = 30.0^{kft}$$

	A		B			C	D
		BA	BC		CB	CD	
Distribution Factors		0.885	0.115	0.74	0.115	0.885	
Carry-over	0.77						0.77
Fixed-End Moments	+11.8 -11.8	-30.0 -9.1	0		0	+30.0 + 9.1	-11.8 +11.8
Distribution	0	-39.1 +34.6	4.5		-4.5	+39.1 -34.6	
Carry-over			-3.8		3.8		
Distribution		+3.4	+.4 -0.3		-0.4 +0.3	-3.4	
Carry-over		0.27	+0.03		-0.03	-0.27	
Final moments	0	-0.83	+0.83		-0.83	0.83	0

8. Live Load Moments

Calculation of live load moments at the points of interest through the use of the influence lines in Fig. 10:

Haunch moments:

Negative moment:

$$= 2.5 \times 16.40^{**} + (2/3 \times 16.40^{*} \times 50 + 1/2 \times 16.40 \times 50) \times 0.08$$

$$= 41 + 76 = -117.0 \text{ k ft.}$$

At 0.1L or 10' from left support:

Negative moment:

$$= 2.5^{**} \times 11.30 + (1/2 \times 11.30^{*} \times 50 + 35 \times 3/4 \times 11.30) \times 0.08$$

$$= 28.2 + 46.2 = (-74.4) \text{ k ft.}$$

Positive moment:

$$= 3 \times 2.5^{**} + \left(1/2 \times 15^{*} \times 3\right) \times 0.08$$

$$= 7.5 + 1.8 = 9.30 \text{ k ft.}$$

At 0.2L or 20' from left support:

Negative moment:

$$= 6.30^{**} \times 2.50 + (1/2 \times 20^{*} \times 6.30 + 2/3 \times 6.30 \times 50)0.08$$

$$= 15.8 + 22 = -37.8 \text{ k ft.}$$

Positive moment:

$$= 2.5^{**} \times 5.7 + (1/2 \times 5.70 \times^{*} 20 + 1/3 \times 5.7 \times 10)0.08$$

$$= 14.2 + 6 = +20.2 \text{ k ft.}$$

At 0.3L or 30' from left support:

Negative moment:

$$= 2.5^{**} \times 25 + (2/3 \times^{*} 54 \times 2.5)0.08$$

$$= 6.25 + 7.2 = -13.4 \text{ k ft.}$$

---

* = due to distributed load; ** = due to concentrated load

Positive moment:

$$= 7 \times {}^{**}2.5 + (1/2 \times 46 \times {}^{*}7) \times 0.08$$

$$= 17.5 + 13 = +30.5 \text{ k ft.}$$

At 0.4L or 40' from left support:

Positive moment:

$$= 9.5 \times {}^{**}2.5 + (1/3 \times {}^{*}9.5 \times 70)0.08$$

$$= 23.8 + 18 = 41.8 \text{ k ft.}$$

Negative moment:

$$= 2.5 \times {}^{**}.3 + (2.3 \times 30 \times {}^{*}3 \times)\,0.08$$

$$= .75 + .48 = 1.25 \text{ k ft.}$$

At crown:

Positive moment:

$$= 8.70 \times {}^{**}2.5 + \left(1/2 \times {}^{*}8.70 \times 100\right) \times 0.08$$

$$= 22.8 + 34.8 = +57.6 \text{ k ft.}$$

Shears:

Dead load shear at sections of interest:

haunch = 22.0 k

0.1L or 10' = 15.2 k

0.2L or 20' = 9.3 k

0.3L or 30' = 4.3 k

0.4L or 40' = 0.1 k

at crown = 0

9. Live Load Shears

Live load shears calculated at sections of interest through the use of the influence lines in Fig. 11.

---

* = due to distributed load; ** = due to concentrated load

Haunch = 6.4 k

10' from support = 5.9 k

20' from support = 4.85 k

30' from support = 3.8 k

40' from support = 3.7 k

Crown = 2.6 k

Top of footings:

Dead load shear (thrust) = 23k

Live load shear (thrust) = 6k

Earth pressure = 6.2k

= 35.2k

Reaction at footings:

Weight of leg itself = 10.2k ⊙

Dead load reaction = 22.0

Live load reaction = 6.4

= 38.6k

⊙ = weight of leg itself 10.2k

TABLE 1.-ULTIMATE MOMENTS, SHEARS, AND THRUSTS

Distance	0.1L	0.2L	0.3L	0.4L	Crown	Haunch
Moments kft.						
Dead Load	-361.5	-139.5	+15.6	+87.0	+96.0	-690.0
Live Load	-113.4	- 68.0	-24.0	-	-	-222.0
	+ 16.7	+ 36.4	+55.0	+76.5	+84.6	-
Earth Pressure	- 1.25	- 1.25	- 1.25	- 1.25	- 1.25	- 1.25
Thrusts kips						
Dead Load	34.2	34.2	34.2	34.2	34.2	34.2
Live Load	10.8	10.8	10.8	10.8	10.8	10.8
Earth Pressure	5.2	5.2	5.2	5.2	5.2	5.2
Shears kips						
Dear Load	22.8	14.0	6.45	0.15	-	33.0
Live Load	10.8	8.72	6.84	5.70	4.70	10.5
Earth Pressure	-	-	-	-	-	-

Negative moments compression bottom fibre.

Positive moments compression top fibre.

Ultimate Strength Design

Cross-sections in rigid frame bridges are subjected to shear, bending and axial thrust, and all deck sections have tensile reinforcement. Compressive reinforcement in the deck is practically eliminated in ultimate strength design.

Design equations for beams reinforced in tension:

From equilibrium of internal and external forces as shown in Fig. 14:

$$0.85\, f_c'\, ba - A_s\, f_y = P_u \tag{1}$$

From equilibrium of internal and external moments about the tensile reinforcement:

$$M_u = P_u\, e' = 0.85\, f_c'\, ab(d - a/2) \tag{2}$$

These equations are modified by capacity reduction factors,

$$P_u = \phi(0.85 f_c' ba - A_s f_y) \tag{1a}$$

$$M_u = P_u e' = \phi(0.85 f_c' ba(d - a/2)) \tag{2a}$$

From equation (1a)

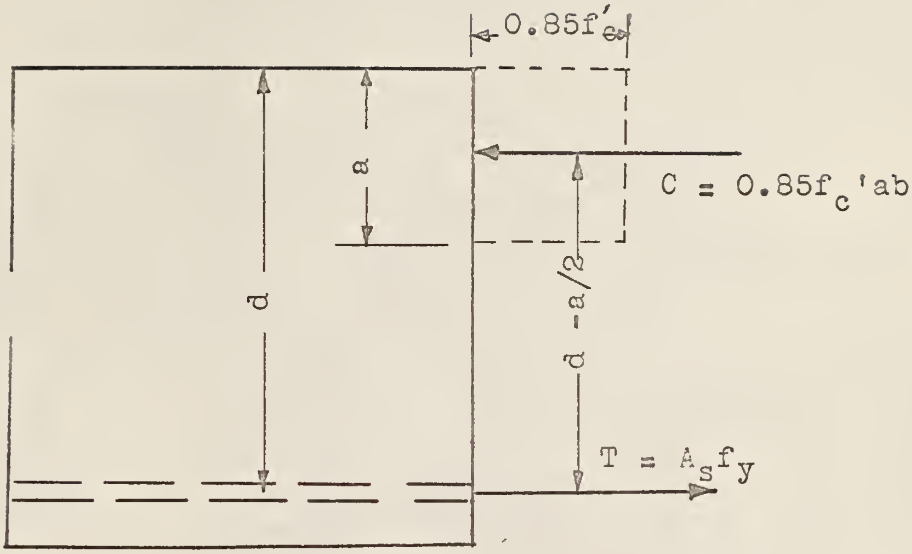
$$a = \frac{P_u/\phi + A_s f_y}{0.85 f_c' b}$$

Some design conditions are specified in ACI-Code (23):

1. The reinforcement ratio, p, shall not exceed 0.75 of the ratio, p_b, where p_b is given by $(0.85 k_1 f_c'/f_y)(87000/87000 + f_y)$.

2. Deflection Control:

   Deflection shall always be checked whenever the required net reinforcement ratio p in any section of a flexural member exceeds $0.18 f_c'/f_y$.



Simplified Rectangular Stress Block Assumed by Whitney

Fig. 14

3. Check for Shear:

Ultimate shear stress $v_u = V_u/bd$ should be less than

$v_c = 3.5\ \phi\sqrt{f_c'}$

4. Check for Bond:

Ultimate bond stress $u_u = V_u/\phi\Sigma ojd$ should be less than

u_a where $u_a = 4.2\sqrt{f_c'}$, for bars in tension.

Section at Haunch

Design for moment and thrust.

$M_u = 1.5B + 1.8L$

$= 1.5 \times 460 + 1.8 \times 124.0 = 910.0^{kft}$

$P_u = 22.0 \times 1.5 + 6.4 \times 1.8 = 50.0^{k}$

$d = 44"$, Equation (1a) gives

$A_s = 5.0$ sq in

$$a = \frac{P_u/\phi + A_s f_y}{0.85 f_c' b} = \frac{50/0.9 + 5 \times 50}{0.85 \times 4 \times 12} = 7.8"$$

$M_u = P_u e' = \phi(0.85 f_c' ba(d - a/2))$

$= 0.9(0.85 \times 4 \times 12 \times 7.5 \times 40/12)$

$= 924^{kft} > 910$

$P_u = \phi(0.85 f_c' ba - A_s f_y)$

$= 0.9(0.85 \times 4 \times 12 \times 7.8 - 5 \times 50)$

$= 50.0^{k}$

Check for Deflection.

$p = A_s/bd = 5/12 \times 44 = 0.0095$

$p_d = 0.18\ f_c'/f_y = 0.014$

$p < p_d$

Check for Shear.

$$v_u = V_u/bd = \frac{43.5 \times 1000}{12 \times 44} = 82.0 \text{ psi}$$

$$v_c = 3.5 \; \phi\sqrt{f_c'}$$

$$= 3.5 \times 0.85 \times \sqrt{4000}$$

$$= 188 \text{ psi}$$

$$v_u < v_c$$

Check for Bond.

$$u_u = \frac{V_u}{\phi \sum ojd} = \frac{43.5 \times 1000}{0.85 \times 4 \times 5 \times 0.87 \times 44} = 67 \text{ psi}$$

$$u_a = 42\sqrt{f_c'} = 4.2\sqrt{4000} = 266 \text{ psi}$$

The perimeter of the bars is sufficient so that the calculated bond stress at ultimate load is less than the allowable.

Section at 10' from Support

Design for Moment and Thrust.

$$M_u = 1.5 \text{ B} + 1.8 \text{ L}$$

$$= 15 \times 241 + 1.8 \times 74.5 = 475.0^{kft}$$

$$P_u = 50.0$$

$$d = 30''$$

$$A_s = 3.5 \text{ sq. in.}$$

Equation (1a) gives

$$a = \frac{P_u/\phi + A_s f_y}{0.85 f_c' b} = \frac{67 + 175}{0.85 \times 4 \times 12} = 6.0''$$

$$M_u = P_u e' = \phi((0.85 \times 4 \times 12 \times 6.0 \times 27/12)) = 495 > 475$$

Check for Deflection.

$p = A_s/bd = 3.5/12 \times 30 = 0.0097$

$p_d = 0.18\ f_c'/f_y = 0.014$

$p < p_d$

Check for Shear.

$v_u = V_u/bd = \dfrac{33.6 \times 1000}{12 \times 30} = 140$ psi

$v_c = 3.5\ \phi\sqrt{f_c'} = 3.5 \times 0.85 \times \sqrt{4000} = 188$ psi

Check for Bond.

$u_u = \dfrac{V_u}{\phi \sum o j d} = \dfrac{33.6 \times 1000}{0.85 \times 3.5 \times 4 \times 0.87 \times 30} = 110$ psi

$u_a = 4.2\sqrt{f_c'} = 4.2\sqrt{4000} = 266$ psi

$u_u < u_a$

The perimeter of the bars is sufficient so that the calculated bond stress at ultimate load is less than the allowable.

Section at 20' from Support

Design for Moment and Thrust.

$M_u = 207.5^{kft}$

$P_u = 50.0^{\ k}$

$d = 18''$

$A_s = 2.5$ sq. in.

Equation (1a) gives

$$a = \frac{P_u/\phi + A_s f_y}{0.85 f_c' b} = \frac{67 + 125}{41} = 4.70''$$

$$M_u = P_u\ e' = \phi(0.85\ f_c' ba(d - a/2)$$

$$= 0.9(0.85 \times 4 \times 12 \times 4.70 \times 15.65/12)$$

$$= 225^{kft} > 207.5$$

Check for Deflection.

$$p = A_s/bd = 2.5/12 \times 18 = 0.0114$$

$$p_d = 0.18\ f_c'/f_y = 0.014$$

$$p < p_d$$

Check for Shear.

$$v_u = V_u/bd = \frac{22.7 \times 1000}{12 \times 18} = 105.0 \text{ psi}$$

$$v_c = 3.5\ \phi\sqrt{f_c'} = 3.5 \times 0.85 \times \sqrt{4000} = 188 \text{ psi}$$

$$v_u < v_c$$

Check for Bond.

$$u_u = \frac{V_u}{\phi \Sigma ojd} = \frac{22.7 \times 1000}{0.85 \times 2.5 \times 4 \times 0.87 \times 18} = 170 \text{ psi}$$

$$u_a = 4.2\sqrt{f'c} = 4.2\sqrt{4000} = 266 \text{ psi}$$

$$u_u < u_a$$

The perimeter of the bars is sufficient so that the calculated bond stress at ultimate load is less than the allowable.

Section at Crown: Same section is provided for 30' and 40' distance from support.

Design for Moment and Thrust.

$M_u = 180.6^{kft}$

$P_u = 50.0^{k}$

$d = 18''$

$A_s = 2.0$ sq. in.

Equation (1a) gives

$$a = \frac{P_u/\phi + A_s f_y}{0.85 f_c' \ b} = 167/41 = 4.06''$$

$$M_u = P_u e' = \phi((0.85 \ f_c' \ ba(d - a/2))$$

$$= 0.9(0.85 \times 4 \times 12 \times 4.06 \times 16/12)$$

$$= 200.0^{kft}$$

$$= 200^{kft} > 180.6^{kft}$$

Check for Deflection.

$P = A_s/bd = 2/12 \times 18 = 0.0093$

$p_d = 0.18 \ f_c'/f_y = 0.014$

$p < p_d$

Check for Shear.

$$v_u = V_u/bd = \frac{4.7 \times 1000}{12 \times 18} = 22.0 \text{ psi}$$

$$v_c = 3.5\,\phi\sqrt{f_c'} = 3.5 \times 0.85\sqrt{4000} = 190 \text{ psi}$$

<u>Check for Bond.</u>

$$u_u = \frac{V_u}{\phi\,\Sigma ojd} = \frac{4.7 \times 1000}{0.85 \times 2 \times 4 \times 0.87 \times 18} = 44 \text{ psi}$$

$$u_a = 4.2\sqrt{f'_c} = 4.2\sqrt{4000} = 266 \text{ psi}$$

$$u_u < u_a$$

The perimeter of the bars is sufficient so that the calculated bond stress at ultimate load is less than the allowable.

Reinforcement for shrinkage and temperature stress normal to the principal reinforcement shall be provided in structural members where the principal reinforcement extends in one direction only. The ratio of reinforcement to concrete area shall be 0.0020.

TABLE 2.- MOMENTS, SHEARS, AND THRUSTS FOR WORKING-STRESS METHOD

Distance	0.1L	0.2L	0.3L	0.4L	Crown	Haunch
Moments kft						
Dead Load	-241.0	-93.0	+10.4	+58.0	+64.0	-460.0
Live Load	- 74.5	-37.8	-13.4	- 1.25	-	-123.4
	+ 9.30	+20.2	+30.5	+42.50	+47.0	0
Earth Pressure	- .83	- .83	- .83	- .83	- .83	- .83
Thrusts kips						
Dead Load	22.8	22.8	22.8	22.8	22.8	22.8
Live Load	6.0	6.0	6.0	6.0	6.0	6.0
Earth Pressure	3.5	3.5	3.5	3.5	3.5	3.5
Shears kips						
Dead Load	15.2	9.3	4.3	0.1	0	22.0
Live Load	5.9	4.85	3.8	3.17	2.6	6.4
Earth Pressure	0	0	0	0	0	0

Negative moments compression bottom fibre.

Positive moments compression top fibre.

Working-Stress Design

Cross sections in rigid frame bridges are subjected to shear, bending, and axial thrust. Most sections have both tensile and compressive reinforcement.

Method of Transformed Section:

In a homogeneous beam the neutral axis passes through the center of gravity of the cross section. A reinforced concrete beam can be treated as a homogeneous beam, if the steel is considered to be replaced by contrete. For beams of unusual stress distribution and reinforcement, this method is quite convenient as it allows the application of the simple familiar formulas for the design of homogeneous beams.

Estimate the depth, z, of the neutral axis of an equivalent section of a homogeneous beam; then proceed as follows (Fig. 15): compute the section constants with respect to the extreme concrete compressive fibre.

Area $A = bz + (n-1)A'_s + n \times A_s$

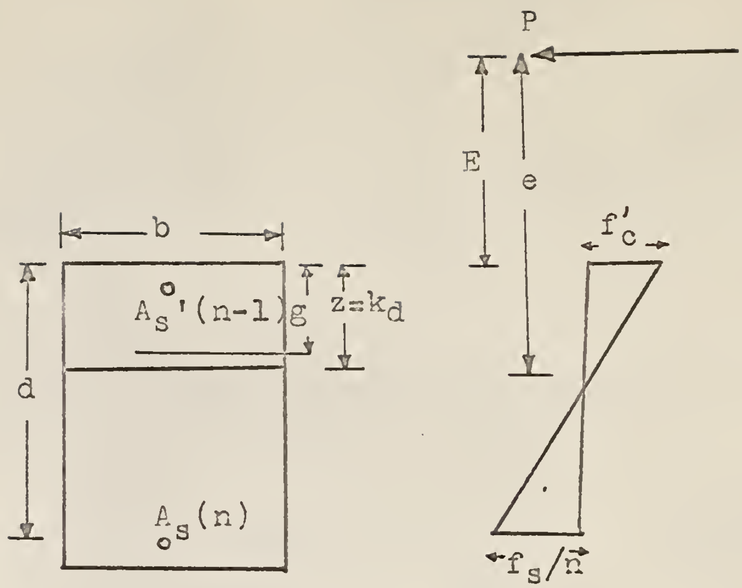
$$Q = 1/2\, bz^2 + (n-1)\, A'_s\, c + n \times A_s \times d$$

Then the depth of the center of gravity of the transformed area, $g = \frac{Q}{A}$. Moment of inertia about the center of gravity $I_g = 1/3\, bg^3 + (n-1)A'_s\,(g-c)^2 + n\, A_s(d-g)^2$.

Also determine E, the eccentricity with respect to the extreme concrete fibre, $e = E + g$.

Check for stresses:

$$f_c = \frac{P \times e \times z}{I_g}$$



Working-Stress Design

Fig. 15

$$f_s = \frac{P \times e \times (d-z)}{I_g}$$

Finally compute

$$z = \frac{f_c}{f_c + f'_s/n}$$

Section at Haunch:

	Moments	Thrusts
Dead Load	460.0	22.8
Live Load	123.4	6.0
Earth Pressure	0.83	3.5
	584.2	32.3

Eccentricity with respect to center line $= \frac{583.4}{32.3} = 18.0' = 216''$

If the axial thrust is disregarded, the following steel area is required in tension:

$$A_s = \frac{M}{f_s jd} = \frac{583.4}{20 \times 0.87 \times 57} = 7.0 \text{ sq. in.}$$

The depth of the neutral-axis in the concrete section equals

$$z = d\left[\sqrt{2\,pn + (pn)^2} - (pn)\right]$$

$$= 57\ (0.446 - 0.1) = 0.35 \times 57 = 20''$$

Estimated section coefficients:

$A = 12 \times 20 + 10 \times 7 = 310.0$ $\quad in^2$

$Q = 1/2 \times 12 \times 20^2 + 10 \times 7 \times 57 = 6380.0$ $\quad in^3$

$g = \frac{Q}{A} = \frac{6380}{310} = 20.6''$

$I_g = 1/3 \times 12 \times 20.6^3 + 70 \times 36.4^2$

$= 35000 + 92500 = 1,29500$ $\quad in^4$

$E = 216 - 0.5 \times 57 = 187.5$

$e = E + g = 187.5 + 20.6 = 208.0$

Check for stresses:

$$f_c = \frac{P \times e \times z}{I_g} = \frac{32.3 \times 208 \times 20}{1{,}27500} = 1.05 = 1050 \text{ psi}$$

$$f_s = \frac{P \times ex(d-z)}{I_g} \times n = \frac{32.3 \times 208 \times 37}{1{,}27500} \times 10 = 18.10 = 18100 \text{ psi}$$

$$\text{Check for } z = \frac{f_c}{f_c + f_s/n} = \frac{1050}{1050 + 1810} = 20''$$

<u>10' from Support:</u>

	Moments	Thrusts
Dead load	241.0	22.8
Live load	74.5	6.0
Earth Pressure	.8	3.5
	316.3	32.3

$$\text{Eccentricity with respect to center line} = \frac{316.3}{32.3} = 117''$$

If the axial thrust is disregarded, the following area is required in tension:

$$A_s = \frac{316.3}{20 \times 0.87 \times 44} \times 12 = 4.8 \text{ sq. in.}$$

Depth of neutral-axis

$$z = d\left[\sqrt{2\,pn + (pn)^2} - pn)\right]$$

$$= 44 \times 0.34 = 15''$$

Estimated section coefficients:

$$A = 12 \times 15 + 4.8 \times 10 = 228$$

$$Q = \frac{12}{2} \times 15^2 + 68 \times 46 = 3454$$

$$g = \frac{Q}{A} = \frac{3454}{228} = 15.2$$

$$I_g = 1/3 \times 12 \times 15.2^3 + 48 \times 28.8^2$$

$$= 14000 + 39800 = 5,3800.0$$

Check for stresses:

$$f_c = \frac{P \times e \times z}{I_g} = \frac{32.3 \times 108.4 \times 15}{513800} = 0.970 = 970 \text{ psi}$$

$$f_s = \frac{P \times ex(d-2)}{I_g} \times n = \frac{32.3 \times 108.4 \times 29}{5,3800} \times 10 = 18.8$$

$$= 18800 \text{ psi}$$

Check for shear:

$$v = \frac{V}{bd} = \frac{21.1 \times 1000}{12 \times 44} = 40.0 \text{ psi}$$

$$v_c = 1.75\sqrt{f'_c} = 1.75\sqrt{3000} = 1.75 \times 56.7 = 96.0 \text{ psi}$$

$$v < v_c$$

Check for bond:

$$u = \frac{V}{\Sigma\, ojd} = \frac{21.1 \times 1000}{4 \times 5 \times 0.87 \times 44} = 27.6 \text{ psi}$$

$$u_a = 3.4\sqrt{\frac{f'_c}{D}} = \frac{3.4 \times 56.7}{1} = 193.0 \text{ psi}$$

$$u < u_a$$

<u>20' from Support</u>

	Moments	Thrusts
Dead Load	93.0	22.8
Live Load	37.8	6.0
Earth Pressure	.8	3.5
	131.6	32.3

Eccentricity with respect to center line $= \frac{131.6}{32.3} = 48.6''$

If the axial thrust is disregarded, the following steel area is required

$$A_T = \frac{131 \times 12}{20 \times 0.87 \times 26} = 3.5 \text{ sq. in.}$$

However, compressive steel must be provided for axial thrust as it is a significant portion of the total load. Therefore the steel area is divided as follows:

$$A_s = 2.4 \text{ sq. in.}$$

$$A'_s = 1.2 \text{ sq. in}$$

The depth of neutral axis

$$z = d\left[\sqrt{2n\left(p + \frac{2p'c}{d}\right) + n^2(p + 2p')^2} - n(p + 2p')\right]$$

$$= 26\left[\sqrt{0.205} - (0.15)\right]$$

$$= 26 \times 0.30 = 7.8''$$

Estimated section coefficients:

$$A = 12 \times 7.8 + 9 \times 1.2 + 10 \times 2.4 = 128.5$$

$$Q = 1/2 \times 12 \times 7.8^2 + 10.8 \times 6.3 + 24 \times 18.2 = 868.0$$

$$g = \frac{Q}{A} = \frac{868}{128.5} = 6.8''$$

$$E = 48.6 - 13 = 35.6''$$

$$e = E + g = 35.6 + 6.8 = 42.4''$$

$$I_g = 1/2 \times 12 \times 6.8^3 + 10.8 \times 5.3^2 + 24 \times 19.2^2 = 11474.0$$

Check for stresses:

$$f_c = \frac{P \times e \times z}{I_g} = \frac{32.3 \times 42.4 \times 7.8}{11474} = 0.91 = 910 \text{ psi}$$

$$f_s = \frac{P \times ex(d-z)}{I_g} \times n = \frac{32.3 \times 42.4 \times 18.2 \times 10}{11674} = 20.80$$

$$= 20800 \text{ psi}$$

$$f'_s = \frac{P \times ex(z - c)}{I_g} \times n = \frac{32.3 \times 42.4 \times 6.3 \times 10}{11474} = 7.5$$

$$= 7500.0 \text{ psi}$$

Check for shear:

$$v = \frac{V}{bd} = \frac{14.15}{12 \times 26} \times 1000 = 45.0 \text{ psi}$$

$$v_c = 1.75\sqrt{f'_c} = 1.75\sqrt{3000} = 96.0 \text{ psi}$$

$$v < v_c$$

Check for bond:

$$u = \frac{V}{\Sigma ojd} = \frac{21.1 \times 1000}{4 \times 3.6 \times 0.87 \times 26} = 65.0 \text{ psi}$$

$$u_a = \frac{3.4\sqrt{f'_c}}{D} = \frac{3.4 \times 56.7}{1} = 193.0 \text{ psi}$$

$$u < u_a$$

Section at Crown

	Moments	Thrusts
Dead Load	64.0	22.8
Live Load	47.0	6.0
Earth Pressure	0.80	3.5
	111.6	32.3

Eccentricity with respect to center line $= \frac{111.6}{32.3} = 41''$

The area of steel, if the axial thrust is disregarded

$$A_T = \frac{111.6 \times 12}{20 \times 0.87 \times 22} = 3.5 \text{ sq. in.}$$

However, compressive steel must be provided for axial thrust as it is a considerable portion of the total load. Therefore, the steel area is divided as follows:

$A_s = 2.4$

$A'_s = 1.2$

The depth of neutral axis

$$z = d\left[\sqrt{2n(p + \frac{2p'c}{d} + n^2(p + 2p')} - n(p + 2p')\right]$$

$$= 22\left[\sqrt{0.205} - (0.15)\right] = 22 \times 0.30 = 6.8$$

Estimated section coefficients:

$$A = 12 \times 6.8 + 9 \times 1.2 + 10 \times 2.4 = 114.3$$

$$Q = \frac{12}{2} \times 6.8^2 + 10.8 \times 5.3 + 24 \times 15.2 = 681.2$$

$$g = \frac{Q}{A} = \frac{681.2}{114.3} = 5.8$$

$$I_g = 1/3 \times 12 \times 5.8^3 + 10.8 \times 4.5^2 + 24 \times 16.2^2 = 7810 \quad in^4$$

$E = 41 - 12 = 29$ $\qquad$ $e = e + g = 34.8''$

Check for stresses:

$$f_c = \frac{P \times e \times z}{I_g} \times n = 32.3 \times \frac{34.8}{7810} \times 6.8 \times 10 = 975 \text{ psi}$$

$$f_s = \frac{P \times e \times (d-z)}{I_g} \times n = \frac{32.3 \times 34.8 \times 15.2}{7810} \times 10 = 21600 \text{ psi}$$

$$f'_s = \frac{P \times e \times (z-c)}{I_g} \times n = \frac{32.3 \times 34.8 \times 5.3}{7810} \times 10 = 7600 \text{ psi}$$

Check for shear:

$$v = \frac{V}{bd} = \frac{2.6}{12 \times 22} \times 1000 = 9.8 \text{ psi}$$

$$v_c = 1.75\sqrt{f'_c} = 96.0 \text{ psi}$$

Check for bond:

$$u = \frac{V}{\sum ojd} = \frac{2.6 \times 1000}{4 \times 3.5 \times 0.87 \times 22} = 9.7 \text{ psi}$$

$$u_a = 3.4\sqrt{f'_c} = 34 \times 56.7 = 193.0$$

$$u < u_a$$

TABLE 3.-COMPARISON OF REQUIRED CROSS SECTION FOR "ULTIMATE STRENGTH" AND "WORKING-STRESS" DESIGN PROCEDURES AT SELECTED LOCATIONS

Design Method	Section Properties	Section Location: Haunch	10'	20'	30'	40'	Crown
Ultimate Strength Design	d - in.	44	32	18	18	18	18
	A_s - sq. in.	5.0	3.5	2.5	2.0	2.0	2.0
Working-Stress Design	d - in.	57	44	26.0	22	22	22
	A_s - sq. in.	7.0	4.8	2.5	2.5	2.5	2.5
	A'_s - sq. in.	-	-	1.0	1.0	1.0	1.0

CONCLUSIONS

In this study the influence lines were calculated using matrix formulation of the slope deflection equations. This method shows many advantages over many other methods of calculating influence lines providing an electronic computer is readily available.

As has been shown on pages 32 to 50 and Table 3, the procedures for the design of reinforced concrete structures by ultimate strength design are not difficult and in most cases substantially simpler in ideas and arithmetic than working-stress design.

The adoption of ultimate strength design methods for the design of a reinforced concrete member provides a saving in time spent on design, in the amount of material used in the construction, and in total cost, and still maintains an adequate factor of safety.

From Table 3, it can be calculated that the total reduction in materials for the ultimate strength design as opposed to the working-stress design is approximately 40 per cent.

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NOTATION

A_s' = area of compressive reinforcement

A_s = area of tensile reinforcement

A_T = total area of steel

a = depth of rectangular stress block

$[a']$ = coefficients relating beam rotations to joint rotations

b = width of rectangular beam

c = distance from centroid of the compressive reinforcement to the extreme compressive fibre

C = carry-over factor

d = depth of deck at crown

d' = depth of deck at support

d = distance from the centroid of the tensile steel to the extreme compressive fibre

E = eccentricity of the load from the extreme compressive fibre

e = eccentricity of the load from the center of gravity of the transformed area

e' = eccentricity of the load from the center of tensile reinforcement

g = depth of the center of gravity of the transformed area from the extreme compressive fibre

I_g = moment of inertia of the transformed section

I_o = moment of inertia at center of deck

I = moment of inertia

f_c' = 28-day cylinder strength of concrete under standard loading condition

f_c = design strength of concrete

f_s = design strength of steel by an elastic theory

f_y = yield strength of steel

$[k]$ = stiffness coefficients for force-displacement equations

k_u = ratio of the depth of the compressive stress block to d

k_d = depth of neutral axis

k_1 = a fraction and shall be taken as 0.85 for strength up to 4000 psi

M_{F_A} = fixed-end moments

M = moment at section

P_u = ultimate load at section

P_A = active earth pressure

ΔP_A = active earth pressure due to live load

p_A = intensity of earth pressure

p_b = balanced steel ration $0.45f_c'/f_y$

p = A_s/bd

p_d = $0.18f_c'/f_y$ deflection control

p' = A_s'/bd

Q = static moment of inertia

Σo = sum of perimeter of the bars

r = ratio of increase in depth of the deck at the support to the depth at the crown

$[r]$ = rotations at the joints for matrix formulation of slope deflection equations; they are positive when clockwise at the joints

S_{on} = S_{01}, S_{02} = internal moments at the ends of members

V_{on} = V_{01}, V_{02} = rotations of the ends of members

S = stiffness of members

R = load at joints, considered positive when clockwise

V = shear at section

V_u = ultimate shear at section

v_u = ultimate shear stress

v_c = shear stress carried by concrete

u_u = ultimate bond stress

u_a = allowable bond stress

z = depth to neutral axis

γ = unit weight of soil

δ = angle of wall friction

ϕ = angle of internal friction

ϕ = capacity reduction factor

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A REINFORCED CONCRETE BRIDGE DESIGNED BY
ULTIMATE AND ELASTIC THEORIES

by

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Anand, India

AN ABSTRACT OF A MASTER'S REPORT

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It has long been recognized that the stress-strain relationship of concrete does not follow a straight-line under many conditions of loading. In the early stages of reinforced concrete design in this country, the straight-line relationship was adopted because of the apparent ease of manipulation of the formulas. After about twenty years of use, engineers found that this practice did not give results comparable to those found in tests; therefore, they proposed changes in the specifications. After ten years of changing specifications there was a completely inconsistent approach to the design of reinforced concrete structures.

Ultimate strength design is a method of proportioning reinforced concrete members based on calculations of their ultimate strength. Whitney has stated that the average ultimate stress equal to the thickness of the simplified rectangular stress block, is $0.85f_c'$, the width of the block is equal to the width of the member, and the depth of the block, defined as a, is calculated from statics. Assuming the ultimate strength of the member to be controlled by the steel in tension, the internal resisting moment is taken about the tensile steel and set equal to the external moment.

It was the purpose of this report to use ultimate strength design in a practical problem and to compare it with working-stress design. Rigid frame bridges have been used extensively for intersecting highways and in locations where it is necessary to meet conditions imposed by restricted headroom. This type bridge has the abutment and the deck cast as a unit, hence there is a continuity at their junction.

In this study the influence lines were calculated using matrix formulation of the slope deflection equations. This method shows many advantages over other methods of calculating influence lines providing an electronic computer is readily available.

The calculations in this report demonstrate that the procedures for the design of reinforced concrete structures by ultimate strength design are not difficult and in most cases substantially simpler in ideas and arithmetic than working-stress design.

The adoption of ultimate strength design for the design of a reinforced concrete member provides a saving in time spent on design, in the amount of material used in the construction, and in total cost, and still maintains an adequate factor of safety.

It was demonstrated in this report that the total reduction in materials for the ultimate strength design as opposed to the working-stress design of this structure is approximately 40 per cent.