PROBABILITY-BASED LOAD FACTORS
AND LOAD COMBINATION FACTORS
IN STRUCTURAL DESIGN /

by

AMIR JAFARI

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Approved by:

[Signature]
Major Professor
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Abstract

Probability-based load factors and load combinations are presented which are compatible with the loads specified in ANSI A58 Standards. The load effects considered are due to dead, occupancy live, snow, wind and earthquake loads. The load criteria which are presented were developed by numerous investigators on the basis of probability of failure calculated by simplified theory, using statistical data on structural loads and resistances. The report presents the rationale and methodology used by the investigators for selecting the load factors which are intended to apply to all types of structural materials used in building construction.
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I. Introduction

Structural safety standards are developed to satisfy the ultimate limit states (for human safety) followed by checks of the serviceability limit states. Generally, any design specification must list the limit states to be considered and must present load and resistance factors for use in checking these limit states. The existence of safety factors is due to uncertainties associated with the calculations of resistance and loads. Examples of uncertainties associated with the resistance of materials are: 1) variability in material properties such as strength of steel; 2) variability in member geometry such as cross-sectional area; and 3) variability in the ratio of strength used in calculations to the actual member strength represented by available test data. Examples of uncertainties related to the loads are variations of live loads with time and variations caused by idealization of loads and their locations. In addition load and resistance factors account for uncertainties in structural analysis and modeling such as the assumption that connections are rigid, and the use of a rectangular stress block in concrete design. These uncertainties in resistances and loads indicate that they should be considered as random variables in statistical studies. As a result a single-value factor of safety is unacceptable because it implies that both loads and resistances can be obtained with the same degree of uncertainty.
in different loading conditions. Therefore a statistical approach to the safety criteria is considered in this report because of the necessity of dealing with random variables. Some of the advantages of probabilistic limit states design are; 1) more consistent reliability is attained for different design situations because the different variabilities of the strengths and loads are considered independently; 2) the reliability level can be chosen to reflect the consequences of failure; 3) it gives a better understanding of the fundamental structural requirements; 4) it simplifies the understanding of different design processes by encouraging the same philosophy and procedure to all materials of construction.
2. Structural Reliability

2.1 Normal Distribution

To determine the reliability of a structure, it is assumed that the mean, variance, and the type of distribution function of random variables (loads and resistance) are known. A mathematical model is first derived which relates the resistance and load variables for the limit state of interest,

\( g(x_1, x_2, \ldots, x_n) = 0 \). The terms \( x_i \) are the resistance or load variables, and failure occurs when \( g < 0 \) for any state of interest. Safety is assured by assigning a small probability \( P_f \) to the event that the limit state will be reached, \( g < 0 \) [Ref. 3],

\[
P_f = \int_{x_1}^{x_2} \cdots \int_{x_n}^{x_2} f(x_1, x_2, \ldots, x_n) \, dx_1 \, dx_2 \cdots \, dx_n
\]

which in this context \( f(x_1, x_2, \ldots, x_n) \) is the joint probability density function for \( x_1, x_2, \ldots, x_n \) and the integration is performed over the region where \( g < 0 \). To perform the reliability analysis, the mathematical model must be linearized at some point. This point is called the checking point and the linearized form of the model is

\[
z = g(x_1^*, x_2^*, \ldots, x_n^*) + \sum (x_i - x_i^*) (\partial g / \partial x_i) x_i^*
\]

where \( (x_1^*, x_2^*, \ldots, x_n^*) \) is the checking point. In this equation the partial derivatives are evaluated at the checking point \( (x_1^*, x_2^*, \ldots, x_n^*) \). By assuming the mean values of variables as the checking point, the reliability index, \( \beta \),
can be approximated as shown below,

\[ Z = \beta(x_1, x_2, \ldots, x_n) \]  

\[ \sigma_Z^2 = \left( \frac{\partial g}{\partial x_1} \right)^2 \sigma_{x_1}^2 + \left( \frac{\partial g}{\partial x_2} \right)^2 \sigma_{x_2}^2 + \ldots \]  

\[ \sigma_Z = \left[ \sum \left( \frac{\partial g}{\partial x_i} \right)^2 \sigma_{x_i}^2 \right]^{1/2} \]  

\[ \beta = \frac{Z}{\sigma_Z} \]  

in which \( Z \) and \( \sigma_Z \) are the mean and standard deviation of the linearized form, respectively, and \( \sigma(x_i) \) is the standard deviation of \( x_i \). It should be noted here that by linearizing the mathematical model, \( g( ) \), big errors can be obtained at greater distances from \( x_i^* \). Figure 1 shows the density functions of \( Z \) for a two-variable problem \( g(R,Q)=0 \). The mathematical model for this example is \( g=R-Q \), in which \( R \) is the resistance and \( Q \) is the load effect and failure occurs when \( g<0 \). Here the probability of failure is computed as,

\[ P_f = P(R<Q) = \int_0^\infty F_R(x) \cdot f_Q(x) \, dx \]  

where \( F_R \) is the cumulative probability distribution function (c.d.f.) of \( R \) and \( f_Q \) is the probability density function of \( Q \). If it is assumed that both \( R \) and \( Q \) are normally distributed and statistically independent, then \( R-Q \) is normal with mean \( R-Q \) and variance \( \sigma_R^2 + \sigma_Q^2 \) [Refs.1,6]. Therefore the probability of failure for a normal distribution is

\[ P_f = P(R-Q<0) = \left[ \frac{1}{2\pi^{1/2}\sigma_{R-Q}} \right] \int_{-\infty}^{0} \exp \left( -1/2 \left( \frac{x-(R-Q)}{\sigma_{R-Q}} \right)^2 \right) dx \]  

So the relationship between \( P_f \) and \( \beta \) is as shown below [Ref.3],

\[ \beta = \text{mean/standard deviation} \]  

\[ \beta = \frac{(R-Q)}{[\sigma_R^2 + \sigma_Q^2]^{1/2}} \]
Figure 1 - Illustration of the Reliability Index Concept
(Ref. 4)
\[ P_f = \Phi \left[ -\frac{(X-E)}{\left(\sigma_r^2 + \sigma_q^2\right)^{1/2}} \right] \]  
\[ P_f = \Phi(-\beta) \]  
\[ \beta = \Phi^{-1}(1-P_f) \]

in which \( \Phi(\ ) \) is the cumulative distribution function for the standard normal variate and \( \Phi^{-1}(\ ) \) is a statistical symbol. Its function is opposite to that of the cumulative distribution function; in other words for a given probability of occurrence it calculates the value of \( X \). Some of the numerical values of \( P_f \) and \( \beta \) for a normal distribution are shown below [Ref.2].

<table>
<thead>
<tr>
<th>( P_f )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>2.33</td>
</tr>
<tr>
<td>0.001</td>
<td>3.09</td>
</tr>
<tr>
<td>0.0001</td>
<td>3.72</td>
</tr>
<tr>
<td>0.00001</td>
<td>4.26</td>
</tr>
</tbody>
</table>

There are some shortcomings in linearizing the \( g(\ ) \) function at the mean values of \( X \)-variables. First, by linearizing \( g(\ ) \) at the mean values significant errors may be introduced by neglecting higher order terms of the Taylor series expansion used to linearize the model, at increasing distances from the linearized point. Second, \( \beta \) depends on how the limit state is formulated; and by linearizing at the mean value points, \( \beta \) becomes variant to the different mechanically equivalent formulations of the same problem. The function \( g(\ ) \) and its partial derivatives are independent of how the problem is formulated only on the surface where \( g(\ ) = 0 \). So to avoid this problem, \( g(\ ) \) must be linearized
at some point on the failure surface. This procedure is explained as follows. The variables, first, are transformed to reduced variables with zero mean and unit standard deviation by

\[ x_i = (X_i - \mu_i) / \sigma(X_i) \]  \hspace{1cm} (12)

and the limit state is \( g_1(x_1, x_2, \ldots, x_n) = 0 \) with failure occurring when \( g_1 < 0 \). Then the reliability index is defined as the shortest distance between the surface \( g_1(\cdot) = 0 \) and the origin. See Figure 2. The checking point is computed as shown below [Refs. 3, 4],

\[ \alpha_i = (\partial g_1 / \partial x_i) / \left( \sum (\partial g_1 / \partial x_i)^2 \right)^{1/2} \]  \hspace{1cm} (13)

in which \( \alpha_i \) is the direction cosine between the gradient vector associated with the surface and the \( i \)th coordinate axis. See Appendix A for the derivation of direction cosines for the general case. Then,

\[ x_i^* = -\alpha_i \cdot \beta \]  \hspace{1cm} (14)

\[ g_1(x_1^*, x_2^*, \ldots, x_n^*) = 0 \]

\( \beta \), the reliability index, is obtained by searching for the direction cosines \( \alpha_i \) to minimize the distance from the origin to surface.

2.2 Approximate Method For Non-Normal Distributions

The procedure described prior to this gives values of \( \beta \), the reliability index, which are related to the probability of failure if the distribution functions are normal and the \( g(\cdot) \)
Figure 2 - Formulation of Safety Analysis in Original and Reduced Variable Coordinates

(Ref. 4): $x_1^*$ is the checking point corresponding to the shortest distance from the origin, on $g_1(\cdot) = 0$. 
function is linearized. In other cases it gives an approximate relationship as explained before. For non-normal variables, the idea is to transfer them to equivalent normal variables. This transformation may be done by approximating the actual distribution by a normal distribution at a point on the failure surface. The justification for this is that if normalization takes place at the point of failure (minimum $\beta$), the probability of failure obtained by the approximate method should match the actual probability of failure very closely. The mean and standard deviation of the equivalent normal variable are calculated [Refs. 3, 14] such that at the checking point ($X_i$), the cumulative probability, and the probability density functions of the actual and the equivalent normal are equal. For example, if the actual distribution curve of a variable is log-normal, at the checking point where $g_i(\cdot)=0$ the probability of $x_i$ being less than $x_i^*$ is the same as for the equivalent normal distribution. See Figure 3. In this figure the shaded areas must be equal. These guidelines suggest that the equivalent mean and standard deviation must be as shown below [Refs. 14].

$$
\bar{X}_i = X_i^* - \Phi^{-1}(F_i(X_i^*)) \cdot \sigma_i^N
$$

$$
\sigma_i^N = \phi(\phi^{-1}[F_i(X_i^*)]) / \bar{f}_i(x)
$$

where $F_i$ and $f_i$ are non-normal cumulative distribution and density functions of $X_i$, and $\Phi$ represents the standard normal distribution function.
Figure 3 - Estimate of Equivalent Normal Distribution by Non-Normal Distribution
The following summarizes the procedure that must be taken to calculate $\beta$.

1. Define the limit state function.

2. Make the initial guess at $\beta$.

3. Set the initial checking point values $X^*_i = \bar{X}_i$, for all $i$.

4. Compute the mean and standard deviation of the normal for those variables that are non-normal.

5. Compute $\partial g/\partial X_i$, evaluated at $X^*_i$.

6. Compute $\alpha_i$ as $\alpha_i = [(\partial g/\partial X_i) \cdot \sigma_i^N] / (\sum (\partial g/\partial X_i) \cdot \sigma_i^N)^2)^{1/2}$

7. Compute new values of $X_i^*$ from $X_i^* = X_i^* - \alpha_i \cdot \beta \cdot \sigma_i^N$

8. Repeat step 4 to 7 until $\alpha_i$ stabilizes.

9. Compute the values of $\beta$ necessary for $g(X_1^*, X_2^*, \ldots, X_n^*) = 0$.

10. Repeat step 4 to 9 until the values of $\beta$ differ by some small amount.

See Appendix B for a numerical example.

In evaluating the load statistics, the sources of information were the load subcommittees within American National Standard Institute Committee A58.

In this study, the load effect $Q_i$ is considered to be related to structural loads as $Q_i = C_i B_i A_i$, in which $C_i$ is the influence coefficient; $B_i$ is the modeling parameter; and $A_i$ is the structural load.

**Dead Load** The dead load, $D$, is assumed to remain constant throughout the life of the structure. It appears that dead load has a normal distribution curve with the ratio of mean load to the calculated nominal load close to unity. The coefficient of variation, $V_d$, is 0.06-0.15, with a typical value of 0.10. In this study, it is assumed that $D/D_n = 1.05$ and $V_d = 0.10$ for all construction materials.

**Live Load** The total live load on a floor can be thought of as having a component relatively constant, referred to as "arbitrary point-in-time live load" and an extraordinary component which arises from infrequent clustering of people or activities such as remodeling. The arbitrary point-in-time live load appears to be fitted best by a Gamma probability distribution function. However, it appears that the maximum live load, $L$, fits best by an Extreme Value Type I distribution curve [Refs. 3, 7, 10, 11]. See Appendix C for a review on Extreme Value Type I distribution.
Wind Load: Statistical analysis of data has shown that the mean and coefficient of variation are dependent on geographical locations. However, the analysis has shown that the probability distribution of the annual extreme for extra-tropical winds is Extreme Value Type I [Refs. 3,7,10,11].

Snow Load: The snow load distribution is derived using climatological data and field studies. The U.S. Army Cold Regions Research and Engineering Laboratory analysis of current data indicates that the cumulative distribution function for annual extreme ground snow load is log-normal with parameters varying from site to site [Refs. 3,7,10,11].

Load Combinations: Most structural loads vary with time and more than one time-varying load will be acting on a structure at any given time. When more than one time-varying load acts, it is very unlikely that each load will reach its peak lifetime value at the same time. In this study Turkstra's rule has been adopted in dealing with load combinations [Ref. 4]. Turkstra's rule says that the maximum of a combination of load effects will occur when one of the loads is at its lifetime maximum value while others assume their instantaneous values. The minimum value of $\beta$, calculated from the load combinations would provide a lower bound on the reliability of the element. Table 1 represents the load parameters used to develop the proposed load factors and load combination factors [Ref. 3]. The data given in Table 1 is the best expected values of the variables presented. The available
<table>
<thead>
<tr>
<th>Load</th>
<th>$\bar{X}/X_n$</th>
<th>$\nu_X$</th>
<th>cdf</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>1.05</td>
<td>0.10</td>
<td>Normal</td>
</tr>
<tr>
<td>L</td>
<td>Eqs. 17 or 18</td>
<td>0.25</td>
<td>Type I</td>
</tr>
<tr>
<td>L_{apt}</td>
<td>Eq. 19</td>
<td>Varies</td>
<td>Gamma</td>
</tr>
<tr>
<td>W</td>
<td>0.78</td>
<td>0.37</td>
<td>Type I</td>
</tr>
<tr>
<td>W_{ann}</td>
<td>0.33</td>
<td>0.59</td>
<td>Type I</td>
</tr>
<tr>
<td>W_{apt}</td>
<td>*U = -0.021</td>
<td>*$\chi$ = 18.7</td>
<td>Type I</td>
</tr>
<tr>
<td>S</td>
<td>0.82</td>
<td>0.26</td>
<td>Type II</td>
</tr>
<tr>
<td>S_{ann}</td>
<td>0.20</td>
<td>0.73</td>
<td>Lognormal</td>
</tr>
<tr>
<td>E</td>
<td>(Site dependent)</td>
<td>*$\nu$ = 2.3</td>
<td>Type II</td>
</tr>
</tbody>
</table>

* Parameters used in the probability distribution functions of Type I or Type II

** Coefficient of variation

*** Cumulative distribution function
data to estimate the most expected values is provided by different agencies such as NASA, The U.S. Army Cold Regions Research and Engineering Laboratory Report, and The American Concrete Institute.

The nominal live load, Eq.17, is the value in ANSI A58.1-1972.

\[ L_n = \left[ 1 - \min \left( 0.0008 A_T, 0.6, 0.23 \left( 1 + \frac{D_n}{L_o} \right) \right) \right] L_o \] (17)

in which \( D_n \) = nominal dead load, \( L_o \) = the basic unreduced live load given in ANSI A58.1-1972, and \( A_T \) = the tributary area in square feet. This equation is used to determine the values of \( \beta \) corresponding to existing practice. Eq.18 presents the 1980 version of the A58 Standard.

\[ L_n = \left( 0.25 + \frac{15}{A_T} \right) L_o \] (18)

in which \( A_T \) is the influence area in square feet. Similarly, for the arbitrary point-in-time live load,

\[ L/L_n = 0.24/\text{Eq.17 or 18} \] (19)

The variability of the arbitrary point-in-time live load is dependent on the influence area (\( A_T \)). The larger the influence area, the smaller the variability gets.
4. Statistical Properties of Reinforced and Prestressed Concrete Members

Three major assumptions were made in determining the strength to be used in calculations [Ref. 3].

1. The variabilities of the material properties and dimensions correspond to average quality construction.
2. The strength of concrete in structures was based on the assumption of a slow rate of loading, corresponding to failure in a test lasting one hour. Because of rapid rates of loading, in the case of wind or earthquake loads the concrete and reinforcement strengths were assumed to increase by 5%.
3. Long time strength changes of the concrete and possible corrosion of the reinforcement were ignored.

Table 2 gives the tensile and compressive strengths, which are assumed to have a normal distribution curve [Refs. 3,13]. Table 3 shows the strength distribution of various reinforced and prestressed concrete members [Refs. 3,13]. For the purpose of this study, beams are assumed to have effective depths in excess of 10 inches and no moment re-distribution or strain hardening is considered.

The information given in Table 2 and Table 3 is based on the studies of North American construction practice and the data are provided by various agencies such as ACI and ASCE. Mean values given in Table 2 such as the dimension properties are the mean deviation from the nominal values tested.
Table 2 - Basic Variables (Refs. 3, 13)

<table>
<thead>
<tr>
<th>Property</th>
<th>Mean</th>
<th>( v^* )</th>
<th>( \sigma^{**} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concreter Normal Control</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compressive strength in structure loaded to failure in one hour.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f'_c = 3000 ) psi</td>
<td>2760</td>
<td>0.18</td>
<td></td>
</tr>
<tr>
<td>( f'_c = 4000 ) psi</td>
<td>3390</td>
<td>0.18</td>
<td></td>
</tr>
<tr>
<td>( f'_c = 5000 ) psi</td>
<td>4028</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>Tensile strength in structure, loaded to failure in one hour.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f'_c = 3000 ) psi</td>
<td>306</td>
<td>0.18</td>
<td></td>
</tr>
<tr>
<td>( f'_c = 4000 ) psi</td>
<td>339</td>
<td>0.18</td>
<td></td>
</tr>
<tr>
<td>( f'_c = 5000 ) psi</td>
<td>366</td>
<td>0.18</td>
<td></td>
</tr>
<tr>
<td>Reinforcement</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade 40, Static Yield</td>
<td>45.3</td>
<td>0.116</td>
<td>5.3 ksl</td>
</tr>
<tr>
<td>Grade 60, Static Yield</td>
<td>67.5</td>
<td>0.098</td>
<td>6.6 ksl</td>
</tr>
<tr>
<td>Grade 270 Prestressing Strand, Tensile Strength in Static Test</td>
<td>281</td>
<td>0.025</td>
<td>7.0 ksl</td>
</tr>
<tr>
<td>Dimensions###</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall depth - Nominal</td>
<td>+0.03 in</td>
<td>0.47 in</td>
<td></td>
</tr>
<tr>
<td>Slab (1696 Swedish Slabs)</td>
<td>+0.21 in</td>
<td>0.26 in</td>
<td></td>
</tr>
<tr>
<td>(99 Slabs)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beam (108 beams)</td>
<td>-0.12 in</td>
<td>0.25 in</td>
<td></td>
</tr>
<tr>
<td>(24 beams)</td>
<td>+0.81 in</td>
<td>0.55 in</td>
<td></td>
</tr>
<tr>
<td>Effective depth - Nominal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One-way Slab; Top Bars</td>
<td>-0.75 in</td>
<td>0.63 in</td>
<td></td>
</tr>
<tr>
<td>(1696 Swedish Slabs)</td>
<td>-0.04 in</td>
<td>0.37 in</td>
<td></td>
</tr>
<tr>
<td>(99 Slabs)</td>
<td>-0.40 in</td>
<td>0.50 in</td>
<td></td>
</tr>
<tr>
<td>Values Used</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One-way Slab; Bottom Bars</td>
<td>-0.13 in</td>
<td>0.34 in</td>
<td></td>
</tr>
<tr>
<td>(2805 Swedish Slabs)</td>
<td>-0.16 in</td>
<td>0.35 in</td>
<td></td>
</tr>
<tr>
<td>(96 Slabs)</td>
<td>-0.13 in</td>
<td>0.35 in</td>
<td></td>
</tr>
<tr>
<td>Values Used</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beam, Top Bars</td>
<td>-0.22 in</td>
<td>0.53 in</td>
<td></td>
</tr>
<tr>
<td>Beam Stem Width - Nominal Width</td>
<td>+0.10 in</td>
<td>0.15 in</td>
<td></td>
</tr>
<tr>
<td>Column width, breadth - Nominal Width</td>
<td>+0.06 in</td>
<td>0.25 in</td>
<td></td>
</tr>
<tr>
<td>Cover, bottom steel in beams</td>
<td>-0.06 in</td>
<td>0.45 in</td>
<td></td>
</tr>
<tr>
<td>1 psi = 6895 Pa ; 1 in = 25.4 mm</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Coefficient of variation

** Standard deviation

### Mean deviation from actual
Table 3 - Resistance Statistics (Refs. 3,13)

<table>
<thead>
<tr>
<th>Action</th>
<th>Type of Member</th>
<th>Details</th>
<th>( \bar{V}/R_n )</th>
<th>( v_R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexure</td>
<td>Continuous one-way slabs</td>
<td>5 in. thick, Grade 40</td>
<td>1.22</td>
<td>0.16</td>
</tr>
<tr>
<td>Reinforced Concrete</td>
<td>5 in. thick, Grade 60</td>
<td></td>
<td>1.21</td>
<td>0.15</td>
</tr>
<tr>
<td>Concrete</td>
<td>Two-way slabs</td>
<td>5 in. thick, Grade 60</td>
<td>1.16</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>7 in. thick, Grade 60</td>
<td></td>
<td>1.12</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>One-way panel joints</td>
<td>13 in. overall depth, Grade 60</td>
<td>1.13</td>
<td>0.133</td>
</tr>
<tr>
<td></td>
<td>Beams, Grade 40, ( f'_c = 3 ) ksi</td>
<td>( \phi = 0.005 ) = 0.09 ( \phi )</td>
<td>1.13</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>Beams, Grade 60, ( f'_c = 5 ) ksi</td>
<td>( \phi = 0.019 ) = 0.35 ( \phi )</td>
<td>1.14</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \phi = 0.006 ) = 0.14 ( \phi )</td>
<td>1.04</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \phi = 0.015 ) = 0.31 ( \phi )</td>
<td>1.09</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \phi = 0.027 ) = 0.37 ( \phi )</td>
<td>1.05</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \phi = 0.034 ) = 0.73 ( \phi )</td>
<td>1.01</td>
<td>0.12</td>
</tr>
</tbody>
</table>

| Flexure, Reinforced Concrete - Overall Values | 1.05 | 0.11 |

| Flexure               | Plant Precast Pretensioned          | \( \phi = 0.054 \)                         | 1.06              | 0.057   |
| Pre-stressed Concrete | \( \phi = 0.122 \)                  |                                              | 1.05              | 0.061   |
| Cast-in-Place Post-tensioned | \( \phi = 0.223 \) | 1.06              | 0.083   |
|                        | \( \phi = 0.293 \)                  |                                              | 1.04              | 0.097   |
|                        | \( \phi = 0.054 \)                  |                                              | 1.02              | 0.061   |
|                        | \( \phi = 0.122 \)                  |                                              | 1.03              | 0.083   |
|                        | \( \phi = 0.223 \)                  |                                              | 1.05              | 0.111   |
|                        | \( \phi = 0.293 \)                  |                                              | 1.05              | 0.144   |

| Flexure, Plant Precast Pretensioned, Overall Value | 1.06 | 0.08 |
| Cast-in-Place Post-tensioned, Overall Value       | 1.04 | 0.395 |

| Axial Load          | Short Columns, Compression Failures | \( f'_c = 3 \) ksi                         | 1.05              | 0.16    |
|                     | Short Columns, Tension Failures     | \( f'_c = 5 \) ksi                         | 0.95              | 0.14    |
|                     | Slender Columns, \( k_d/h = 20 \), Compression Failures | \( f'_c = 5 \) ksi | 1.03 | 0.12 |
|                     | Slender Columns, \( k_d/h = 20 \), Tension Failures | \( f'_c = 5 \) ksi | 1.10 | 0.17 |
| Shear               | Beams with \( s/d > 2.5 \), \( \phi_y = 0.008 \) | No stirrups | 0.93 | 0.21 |
|                     |                                      | Min stirrups \( \phi_y = 150 \) psi         | 1.00              | 0.19    |

1 ksi = 6.9 N/mm²; 1 in = 25.4 mm

\( \phi = \text{steel ratio}, \phi = \text{balanced steel ratio}, \omega = A_{ps}/bdf, \text{partial reinforcing index}, f'_c = \text{concrete strength}, \phi_y = A_{ps}/bd \) web steel ratio. \( R/R_n \) is the ratio of mean strength to nominal strength.
5. Selection of Target Reliability

or Existing Reliability

For the new approach in determining the load and combination load factors, the reliability indices can be estimated by reviewing the reliabilities in the existing standards which have resulted in satisfactory performance.

Considering the gravity loads, dead plus live load on floors or dead plus snow load on roofs, the factors are formulated in current design standards as shown below.

In the allowable stress design specification,

\[ \frac{R_n}{F.S.} = D_n + L_n \]  \hspace{1cm} (20)

In the plastic design of steel structures,

\[ R_n = 1.7(D_n + L_n) \]  \hspace{1cm} (21)

In reinforced concrete structures,

\[ \phi R_n = 1.4D_n + 1.7L_n \]  \hspace{1cm} (22)

The reliability indices associated with reinforced concrete and steel beams are given in Fig. 4 for \( L_o/D_n \) and \( S_n/D_n \) [Ref. 5]. \( L_o \) is the basic live load tabulated in building codes. For given live and dead loads, in this figure, nominal resistance is calculated according to the design equations given above and the reliability index is determined as shown in the example in the appendix and Fig. 2. In Fig. 4, in each case \( \beta \) decreases as \( L/D_n \) or \( S_n/D_n \) increases. When viewing the similarity, it should be noted that reinforced concrete beams have practical ranges of
Figure 4—Reliability Index for Steel and Reinforced Concrete Beams
Conforming to Current Criteria — Gravity Loads (100 ft² = 9.3 m²)
(Ref. 5)
L_o/D_n or S_n/D_n of 0.5 to 1.5, while for steel beams this range is from 1 to 2. So the analysis of current standards gives values of \( \beta \), the reliability index, of 2.8 and 3.1 for concrete beams for the D+L and D+S combination.

Looking at gravity and environmental loads, the major load combinations are D+L+W and D+L+E. The variation of \( \beta \) with various L_o/D_n and W_n/D_n ratios is shown in Fig.5 for reinforced concrete beams [Ref. 5]. In each design situation, \( R_n \) is determined as below.

In the allowable stress design,

\[
R_n = F.S.(D_n + L_n + W_n)(3/4)
\]

In the plastic design of steel,

\[
R_n = 1.3(D_n + L_n + W_n)
\]

In reinforced concrete design,

\[
\phi R_n = 0.75(1.4D_n + 1.7L_n + 1.7W_n)
\]

The effect of the rate of loading has been considered by multiplying \( R/R_n \) by 1.05 for reinforced concrete members. In Fig.5, \( \beta \) decreases as \( W_n/D_n \) increases, and \( \beta \) increases as \( L/D_n \) increases. It can also be seen that \( \beta \) for wind approaches a value of 2 where wind is the major load component. With greater live and dead loads, \( \beta \) increases to that of the D+L case. It is possible that the reliability under wind is less because of factors such as load-sharing and load re-distribution among members.
Figure 5 - Reliability Index for Reinforced Concrete Beams Conforming to Current Criteria - Gravity Plus Wind Load - $A_p = 400 \text{ ft}^2 (37 \text{ m}^2)$ (Ref. 5)
Figure 6 - Reliability Index for Steel and Reinforced Concrete Beams - Gravity Plus Earthquake Load (100 ft² = 9.3 m²) (Refs. 5, 3)
Values of $\beta$ for the earthquake loading case D+L+E, are given in Fig. 6 for Boston and Los Angeles [Refs. 5, 3]. Due to the high variability of the earthquake loads as compared to dead loads, the $\beta$ Vs. $E_n/D_n$ curves flatten out rapidly to values which reflect essentially only the contribution of the earthquake load effect. When the effects of wind or earthquake counteract the effect of gravity loads, the reliability tends to be lower than when the loads are additive, as indicated in Fig. 7 [Ref. 5]. The descending branch of the curves in Fig. 7 is a result of the minimum strength that the member has even if it is not designed to resist counteracting load effects.

It has been shown that the $\beta$ values for many flexural and compression members fall within the range 2.5 to 3.0 for the D+L, D+S, and D+L+W load combinations. Therefore, the target $\beta$ the reliability index, for D+L and D+S is 3.0; for D+L+W, $\beta$ is 2.5; and for D+L+E, $\beta$ is 1.75. These values are slightly more conservative than indicated by the current practice when the time-dependent load $(L_n, W_n, S_n, E_n)$ is large compared with the permanent load and less conservative when the permanent load is a major component.
Figure 7 - Current Reliability Index for Steel and Reinforced Concrete Members - Counteracting Loads
(Ref. 5)
6. Derivation of Load And Resistance Factors

6.1 Basic Theory

This section presents the bases for the derivation of load and resistance factors for use in design. As explained before, the probability of failure can be expressed as:

\[ P_f = P[R/Q < 1.0] \]  \hspace{1cm} (26)

or,

\[ P_f = P[\ln(R/Q) < 0] \]  \hspace{1cm} (27)

Both of these are true regardless of the actual frequency distributions of R and Q. If we assume \( \ln(R/Q) \) is normally distributed, \( (R/Q) \) will be log-normally distributed. A log-normal distribution of \( (R/Q) \) has been assumed because Q tends to be skewed; and because theoretically a log-normal distribution better represents the products of random variables \( (R/Q) \).

Assuming \( Y = \ln(R/Q) \), the mean and standard deviation of \( Y \) are:

\[ \bar{Y} = \ln(\bar{R}/\bar{Q}) \]  \hspace{1cm} (28)

\[ \sigma_Y^2 = \sigma^2(\ln(R/Q)) = \sigma^2(\ln R) + \sigma^2(\ln Q) \]  \hspace{1cm} (29)

The function \( Y \) is the same as function \( g \) in Fig.1. As was explained before, the safety criteria is as shown below.

\[ \ln(\bar{R}/\bar{Q}) \gg \beta \sigma(\ln R/Q) \]  \hspace{1cm} (30)

or,

\[ \ln(\bar{R}/\bar{Q}) \gg \beta [\sigma^2(\ln R) + \sigma^2(\ln Q)]^{1/2} \]  \hspace{1cm} (31)

For a log-normal distribution the coefficient of variation is [Ref. 11],

\[ V_R = \left[ (\exp(\sigma^2(\ln R)) - 1) \right]^{1/2} \]  \hspace{1cm} (32)
For $V_R < 0.5$, $V_R$ can be approximated as [Ref. 12];

$$V_R^2 = \sigma^2 (\ln R)$$  \hfill (33)

The error in this approximation is less than 2% for $V_R = 0.3$

rising to about 10% for $V_R = 0.6$. So we can approximate,

$$\ln(R/Q) \approx \beta[\left(\frac{V_R^2 + V_Q^2}{2}\right)]^{1/2}$$  \hfill (34)

The right side of this equation can be further simplified by the

use of the separation function as shown below.

$$\left[\frac{V_R^2 + V_Q^2}{2}\right]^{1/2} = \alpha V_R + \alpha V_Q$$  \hfill (35)

in which $\alpha$ is a "separation function" having values between 0.707

and 1.0. For $V_R/V_Q$ between 1/3 and 3, $\alpha = 0.75 \pm 0.06$. So from

above,

$$\ln(R/Q) \approx \beta \alpha V_R + \beta \alpha V_Q$$  \hfill (36)

or,

$$R/Q \geq \exp[\beta \alpha V_R + \beta \alpha V_Q]$$  \hfill (37)

or,

$$R(\exp[-\beta \alpha V_R])  \geq Q(\exp[\beta \alpha V_Q])$$  \hfill (38)

This equation is the same as ACI (1971) format. If $\gamma_r$ and $\gamma_q$

are taken as;

$$\gamma = \gamma_r \cdot R$$  \hfill (39)

$$Q = \gamma_q \cdot Q$$  \hfill (40)

then the safety criteria $\gamma R \gamma Q$ can be written as;

$$\gamma_r R(\exp[-\beta \alpha V_R])  \geq Q(\exp[\beta \alpha V_Q])$$  \hfill (41)

in which $\gamma$ is a resistance factor and $\lambda$ is a load factor. From

the above equations, it can be seen that load and resistance

factors are defined as;

$$\gamma = \gamma_r \cdot \exp[-\beta \alpha V_R]$$  \hfill (42)

$$\lambda = \gamma_q \cdot \exp[\beta \alpha V_Q]$$  \hfill (43)
6.2 Example Calculation For Flexure of a Reinforced Concrete Beam

![Diagram of a reinforced concrete beam]

The properties of the beam in the calculations are assumed according to ACI 1971, and are listed in Table 4 [Ref.12]. The beam strength is calculated as shown below.

\[ R = M_n = A_s \cdot f_y \cdot (d - a/2) \]
\[ a/2 = (A_s \cdot f_y) / (2)(0.85)(f'_c \cdot b) \]

in which \( A_s \) is the area and \( f_y \) is the yield strength of steel; \( d \) is the depth of reinforcing bars and \( a \) is the width of the stress block.

\[ R = M_n = (3.24)(60)(18 - [(3.24)(60)/(2)(0.85)(4)(12)])/12 \]
\[ R = M_n = 253 \text{ ft-k} \]

Now, using the mean strength and dimensions, we can calculate the mean strength.

\[ \bar{M}_n = \bar{A}_s \cdot \bar{f}_y \cdot (d - \bar{a}/2) \]
\[ \bar{a}/2 = (3.24)(62)/(2)(0.85)(3.8)(12.05) = 2.58 \text{ in.} \]
\[ \bar{M}_n = (3.24)(62)(17.85 - 2.58)/12 = 256 \text{ ft-k} \]

Since the average of measured to calculated strength, \( P \), is given in Table 4 as 1.06,

\[ R = \bar{M}_n \cdot P = (256)(1.06) = 271 \text{ ft-k} \]
\[ \bar{y} = R / R = 1.071 \]
Here, it is necessary to indicate the effect of the accuracy of the design equation by calculating $V_R$, the coefficient of variation of strength. The coefficient of variation will be computed in a number of stages:

$V(a/2)$: Since $(a/2)$ is the product of a number of variables,

$$V(a/2) = [\frac{V(A_s)^2}{2}(f_y^2) + V(f_c)^2 + V(b)^2]^{1/2}$$

$$V(a/2) = [0.06^2 + 0.07^2 + 0.18^2 + 0.023^2]^{1/2} = 0.204$$

$V(d-a/2)$: Since $(d-a/2)$ is the sum of two variables the below equation holds,

$$\sigma(d-a/2) = [\sigma^2(d) + \sigma^2(a/2)]^{1/2}.$$ And since,

$$\sigma(a/2) = V(a/2). \bar{a}/2 = (2.58)(0.204) = 0.526 \text{ in.}$$

$$\sigma(d-a/2) = [0.45^2 + 0.526^2]^{1/2} = 0.69 \text{ in.}$$

$$V(d-a/2) = \sigma(d-a/2)/(d-a/2) = 0.69/15.27 = 0.045$$

$V(M_n)$: Since $M_n$ is calculated as the product of $A_s$, $f_y$, and $(d-a/2)$,

$$V(M_n) = [\frac{V(A_s)^2}{2}(f_y^2) + V(d-a/2)]^{1/2}$$

$$V(M_n) = [0.06^2 + 0.07^2 + 0.045^2]^{1/2} = 0.103$$

$V_R$: Finally, $R$ is the product of $M_n$ and $P$,

$$V_R = [0.103^2 + 0.04^2]^{1/2} = 0.11$$

So in conclusion, $R = 253 \text{ ft-K}$, $R = 271 \text{ ft-K}$, $V_R = 1.071$, and $V_R = 0.11$.

Now, the resistance factor $\phi$ can be calculated as described in the earlier part of this chapter.
<table>
<thead>
<tr>
<th>Material Strengths, $M$</th>
<th>Specified</th>
<th>Mean in situ</th>
<th>$\mu$ Mean Specified $\sigma$</th>
<th>$V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete strength</td>
<td>4000 psi</td>
<td>3800 psi</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Concrete tensile strength</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Yield strength</td>
<td>60 ksi</td>
<td>62 ksi</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td><strong>Dimensions, $F$</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$-beam, column (in.)</td>
<td>12</td>
<td>12.05</td>
<td>1.004</td>
<td>0.3</td>
</tr>
<tr>
<td>$d$-beam (in.)</td>
<td>18</td>
<td>17.85</td>
<td>0.992</td>
<td>0.45</td>
</tr>
<tr>
<td>$h$-column (in.)</td>
<td>12</td>
<td>12.05</td>
<td>1.004</td>
<td>0.3</td>
</tr>
<tr>
<td>$s$-stirrups (in.)</td>
<td>9</td>
<td>9.0</td>
<td>1.00</td>
<td>0.25</td>
</tr>
<tr>
<td>$A_v$-beam (in.$^2$)</td>
<td>3.24</td>
<td>—</td>
<td>1.00</td>
<td>—</td>
</tr>
<tr>
<td>$A_v$-column (in.$^2$)</td>
<td>2.16</td>
<td>—</td>
<td>1.00</td>
<td>—</td>
</tr>
<tr>
<td>$A_v$-stirrups (in.$^2$)</td>
<td>0.22</td>
<td>0.22</td>
<td>1.00</td>
<td>—</td>
</tr>
<tr>
<td><strong>Accuracy of code equations, $P$</strong></td>
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<td></td>
<td></td>
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<tr>
<td>$M_u$-under-reinforced beams</td>
<td>—</td>
<td>—</td>
<td>1.06</td>
<td>—</td>
</tr>
<tr>
<td>$P_u$-axially loaded columns</td>
<td>—</td>
<td>—</td>
<td>0.98</td>
<td>—</td>
</tr>
<tr>
<td>$V_s$-shear carried by concrete</td>
<td>—</td>
<td>—</td>
<td>1.10</td>
<td>—</td>
</tr>
<tr>
<td>$V_s$-shear carried by stirrups</td>
<td>—</td>
<td>—</td>
<td>1.20</td>
<td>—</td>
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<tr>
<td><strong>Loadings, $S$</strong></td>
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<td></td>
</tr>
<tr>
<td>Dead load</td>
<td>—</td>
<td>—</td>
<td>1.0</td>
<td>—</td>
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<tr>
<td>Maximum floor load in 30 year life</td>
<td>—</td>
<td>—</td>
<td>0.7</td>
<td>—</td>
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<td><strong>Structural analysis, $E$</strong></td>
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<td></td>
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<tr>
<td>Dead load effects</td>
<td>—</td>
<td>—</td>
<td>1.0</td>
<td>—</td>
</tr>
<tr>
<td>Live load effects</td>
<td>—</td>
<td>—</td>
<td>1.0</td>
<td>—</td>
</tr>
</tbody>
</table>

$1$ Ksi $= 6.9 \text{ N/mm}^2$
If it is assumed that \( \beta = 3.5 \) and \( \alpha = 0.75 \) for an under-reinforced or ductile concrete beam,
\[
\phi = y_r \cdot \left\{ \exp\left[ -\beta \cdot \alpha \cdot V_R \right] \right\}
\]
\[
\phi = 1.071 \cdot \left\{ \exp\left[ (-3.5)(0.75)(0.11) \right] \right\}
\]
\[
\phi = 0.802
\]
The corresponding load factors are calculated for live plus dead loads.
\[
\lambda = y_r \cdot \left\{ \exp\left[ \beta \cdot \alpha \cdot V_Q \right] \right\}
\]
But, \( Q = D + L \) and since the coefficient of variation of \( D \) is much smaller than that of \( L \), it is desired to separate these variables. Therefore
\[
\lambda_Q = \overline{Q} \left\{ \exp\left[ \beta \cdot \alpha \cdot V_Q \right] \right\}
\]
using the first two terms of the expansion for \( \exp(x) \), and also taking \( \overline{V_Q} = V_Q \) gives,
\[
\overline{Q} \left\{ \exp\left[ \beta \cdot \alpha \cdot V_Q \right] \right\} = \overline{Q}[1 + \beta \cdot \alpha \cdot \overline{V}_Q]
\]
Since \( \overline{V_Q} = \sigma_Q / \overline{Q} \)
\[
\overline{Q} \left\{ \exp\left[ \beta \cdot \alpha \cdot V_Q \right] \right\} = \left( \overline{D} + \overline{L} \right) \left[ 1 + \left( \beta \cdot \alpha \cdot \sigma_Q / \left( \overline{D} + \overline{L} \right) \right) \right]
\]
or,
\[
\overline{Q} \left\{ \exp\left[ \beta \cdot \alpha \cdot V_Q \right] \right\} = \left( \overline{D} + \overline{L} \right) \left[ 1 + \left( \beta \cdot \alpha \cdot \overline{V}_D + \beta \cdot \alpha \cdot \overline{L} \cdot \overline{V}_L / \left( \overline{D} + \overline{L} \right) \right) \right]
\]
Using the separation function,
\[
\overline{Q} \left\{ \exp\left[ \beta \cdot \alpha \cdot V_Q \right] \right\} = \overline{D} \left[ 1 + \left( \beta \cdot \alpha \cdot \overline{V}_D \right) \right] + \overline{L} \left[ 1 + \left( \beta \cdot \alpha \cdot \overline{V}_L \right) \right]
\]
From this equation the load factors for \( D \) and \( L \) are:
\[
\lambda(D) = y_d \cdot \exp\left[ \beta \cdot \alpha^2 \cdot V_D \right]
\]
\[
\lambda(L) = y_L \cdot \exp\left[ \beta \cdot \alpha^2 \cdot V_L \right]
\]
(44)
(45)
Now, taking \( \beta = 3.5 \) for a ductile member; \( V_D \) is affected by
variations in the load, $V_{sd}$, and variations due to the structural analysis, $V_{sd}$, (see Table 4).

\[
V_D^2 = V_{sd}^2 + V_{ed}^2 \\
V_D^2 = 0.07^2 + 0.08^2 \\
V_D = 0.106
\]

Thus, from Eq. 44; \( \lambda(D) = 1.23 \)

Also $V_L$ is affected by the same variables as $V_D$. See Table 4.

\[
V_L^2 = V_{sl}^2 + V_{el}^2 \\
V_L^2 = 0.3^2 + 0.2^2 = 0.36^2 \\
V_L = 0.361
\]

Thus, from Eq. 45; \( \lambda(L) = 1.42 \)

In summary;

For a ductile member, with $\beta = 3.5$;

\[
\Phi R \geq 1.23D + 1.42L
\]

For a brittle member, with $\beta = 4.0$, similar calculations show

\[
\Phi R \geq 1.27D + 1.58L
\]
7. Probability-Based Design

It can be seen from Figs. 8, 9, and 10 that the resistance factor ($\phi$) is insensitive to the time-varying load in the combination, when that load is very small. These figures were obtained for load and resistance factors corresponding to $\beta$ of 3 for the gravity loads, and $\beta$ of 2.5 for D+L+W combination [Refs. 4, 5]. For a given limit state and $\beta$, the factored loads and factored resistances can be obtained according to Chapter 2 of this report, describing the reliability index. The load and resistance factors are then calculated for a given set of nominal loads and resistances as shown below.

$$\psi_i = \frac{x_i}{x_{ni}}$$

in which $x_i$ = the checking point value of load or resistance corresponding to the given $\beta$, and $x_{ni}$ = nominal value of that load or resistance specified in the appropriate specification. From these figures, the load factor for dead load is seen to be very low due to its small variability compared to other load variabilities. A comparison of these figures shows that $\phi$ is in the same range for D+L and D+L+W combinations. From these figures it appears that $\psi_D$ is independent of the magnitude of the time-varying loads in the equation. So, taking $\psi_D$ as constant in the load combination will not significantly change the target reliability. However, the load factors on the time-varying load in the combination increases as that load increases because its
Figure 8 - Load and Resistance Factors for Flexure in Reinforced Concrete Beams (Live Loads) (Refs. 4, 5)
Figure 9 - Load and Resistance Factors for Flexure in Reinforced Concrete Beams (Snow Loads) (Refs. 4,5)
Figure 10 - Load and Resistance Factors for Flexure in Reinforced Concrete Beams (Gravity plus Wind Load) (Refs. 4, 5)
higher variability becomes increasingly more important in determining the total load effect. So it can be concluded that if load factors for the time-varying loads were taken constant as in the current design procedure, there would be some deviation from the ideal constant reliability for certain load situations. It should be noted here that while the resistance factor will depend on the material and limit state of interest, the load factors will be independent of these conditions. To determine the optimal set of load factors to be applied to all situations, and a set of resistance factors that only depend on the material and limit state the following procedure must be met. (1) Define some function which measures the difference between the target reliability and the reliability associated with constant load and resistance set. (2) Select the load factors so as to minimize this function. To do this, first the resistance $R_{ni}^{II}$ was calculated with a given set of nominal loads and associated target reliability. Then, the nominal resistance $R_{ni}^{I}$ was calculated with a design equation which prescribed a set of load factors that were constant for all load ratios. To simplify the procedure, the difference in nominal resistance is being minimized by selecting a set of $Y_i$ and $\theta$ factors as shown below [Refs. 3,5],

$$I(\theta, Y_i) = \sum [R_{ni}^{II} - R_{ni}^{I}]^2 P_i$$  \hspace{1cm} (46)$$

over a predefined set of combinations of dead, live, snow, wind, and earthquake loads; where $P_i$ is the relative weight assigned
Table 5a - Weights for D+S - Values of $P_d$ (Refs. 2, 3)

<table>
<thead>
<tr>
<th>Material</th>
<th>$S_n/D_n$</th>
<th>0.25</th>
<th>0.50</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>3.0</th>
<th>5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td></td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>25</td>
<td>35</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>R/C</td>
<td></td>
<td>30</td>
<td>40</td>
<td>20</td>
<td>5</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Light Gage &amp; Aluminum</td>
<td></td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>17</td>
<td>22</td>
<td>33</td>
<td>22</td>
</tr>
<tr>
<td>Glulam</td>
<td></td>
<td>0</td>
<td>2</td>
<td>16</td>
<td>32</td>
<td>32</td>
<td>18</td>
<td>0</td>
</tr>
<tr>
<td>Masonry</td>
<td></td>
<td>36</td>
<td>36</td>
<td>20</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5b - Weights for D+L - Values of $P_L$ (Refs. 2, 3)

<table>
<thead>
<tr>
<th>Material</th>
<th>$L_n/D_n$</th>
<th>0.25</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>3.0</th>
<th>5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td></td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>25</td>
<td>35</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>R/C</td>
<td></td>
<td>10</td>
<td>45</td>
<td>30</td>
<td>10</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Light Gage &amp; Aluminum</td>
<td></td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>17</td>
<td>22</td>
<td>33</td>
<td>22</td>
</tr>
<tr>
<td>Glulam</td>
<td></td>
<td>0</td>
<td>5</td>
<td>26</td>
<td>26</td>
<td>26</td>
<td>12</td>
<td>5</td>
</tr>
<tr>
<td>Masonry</td>
<td></td>
<td>36</td>
<td>36</td>
<td>20</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
to the ith load combination. Table 5 gives the values of $P_i$ for D+L and D+S load combinations [Refs. 2, 3]. Values of $P_i$ are the best estimates for the relative likelihood of these load situations. The justification for Eq. 46 is that for a constant optimal set of load factors over all likely combination of loads, the resistance factors will depend only on the material and the limit state of interest. So this is the resistance factor that measures the limit state or the reliability index. Also in Eq. 46, the reason for minimizing the square of the difference between $R_n^H$ and $R_n^I$ is to penalize equally deviations from target reliability which are conservative and those which are unconservative. Studies of the sensitivity of the optimal safety factors to various assumptions showed that they were considerably more sensitive to the range of $L_n/D_n$, etc., than to the distribution of $P_i$ within that range [Refs. 2, 3].
8. Combination Load Factors

The selection of load factors can not be made independently of the resistance side of the design equation. In this report, it was assumed that the resistance factor for ordinary steel and reinforced concrete lies in the range of 0.80-0.90. The justification for this range is that when \( \phi \) is more than 0.90 for ordinary member, there is little margin for further adjustment to reflect improvements in fabrication or quality control that tend to reduce variability. Here first as a simple measure of comparison only one load at a time is considered. Setting \( \phi = 0.80 \) or 0.85, Table 6 presents the required \( Y_Q \) for the prescribed \( \beta \) [Ref. 5]. This analysis suggests that the load factors values are approximately for \( Y_D = 1.2-1.3 \), \( Y_L = 1.6-1.7 \), \( Y_W = 1.3-1.4 \), and \( Y_E = 1.4-1.5 \).

8.1 Gravity Load Combinations

For different construction materials, the safety factors for D+L and D+S combinations can be determined by calculating \( R_n \) for \( \beta \) of 3.0 and minimizing Eq.46 to get the optimal \( \phi \), \( Y_L \), \( Y_S \). The term \( Y_D \) is taken as a constant equal to 1.2 from Table 6 and Figures 8, 9, and 10; because \( Y_D \) is very insensitive to the magnitude of other load factors in the limit state equation. As shown in Table 7, the next step is to select
<table>
<thead>
<tr>
<th>Member (1)</th>
<th>$\phi$ (2)</th>
<th>$\gamma_D$</th>
<th>$\gamma_L$</th>
<th>$\gamma_S$</th>
<th>$\gamma_W$</th>
<th>$\gamma_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot-rolled steel beam</td>
<td>0.80</td>
<td>1.18</td>
<td>1.28</td>
<td>1.52</td>
<td>1.78</td>
<td>1.78</td>
</tr>
<tr>
<td></td>
<td>0.85</td>
<td>1.25</td>
<td>1.35</td>
<td>1.61</td>
<td>1.89</td>
<td>1.89</td>
</tr>
<tr>
<td>Concrete beam—</td>
<td>0.80</td>
<td>1.18</td>
<td>1.33</td>
<td>1.44</td>
<td>1.70</td>
<td>1.66</td>
</tr>
<tr>
<td>Grade 40 reinforced</td>
<td>0.85</td>
<td>1.26</td>
<td>1.41</td>
<td>1.53</td>
<td>1.81</td>
<td>1.77</td>
</tr>
<tr>
<td>Concrete beam—</td>
<td>0.80</td>
<td>1.18</td>
<td>1.29</td>
<td>1.52</td>
<td>1.77</td>
<td>1.79</td>
</tr>
<tr>
<td>Grade 60 reinforced</td>
<td>0.85</td>
<td>1.26</td>
<td>1.37</td>
<td>1.61</td>
<td>1.88</td>
<td>1.90</td>
</tr>
</tbody>
</table>
Table 7 - Optimal Load and Resistance Factors for Gravity Loads (Ref. 3)

<table>
<thead>
<tr>
<th>Material</th>
<th>Combination</th>
<th>Optimum Values</th>
<th>Optimum $\phi$ for $\gamma_D = 1.2$, $\gamma_L = 1.6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Beam ($\beta_o = 3$)</td>
<td>D + L</td>
<td>0.96, 2.10</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>D + S</td>
<td>1.05, 2.32</td>
<td>0.79</td>
</tr>
<tr>
<td>R/C Beam, Gr. 60  ($\beta_o = 3$)</td>
<td>D + L</td>
<td>0.87, 1.83</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>D + S</td>
<td>0.93, 1.93</td>
<td>0.84</td>
</tr>
<tr>
<td>R/C Beam, Gr. 40  ($\beta_o = 3$)</td>
<td>D + L</td>
<td>0.82, 1.61</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>D + S</td>
<td>0.85, 1.56</td>
<td>0.86</td>
</tr>
<tr>
<td>Glulam Beam* ($\beta_o = 2.5$)</td>
<td>D + L</td>
<td>0.59, 1.38</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>D + S</td>
<td>0.59, 1.08</td>
<td>0.77</td>
</tr>
<tr>
<td>Brick Masonry Wall* ($\beta_o = 7.5$)</td>
<td>D + L</td>
<td>0.38, 4.10</td>
<td>0.22</td>
</tr>
<tr>
<td>Brick Masonry Wall* ($\beta_o = 5.0$)</td>
<td>D + L</td>
<td>0.52, 2.45</td>
<td>0.41</td>
</tr>
<tr>
<td>Concrete Masonry Wall* ($\beta_o = 6.5$)</td>
<td>D + L</td>
<td>0.41, 3.28</td>
<td>0.27</td>
</tr>
<tr>
<td>Concrete Masonry Wall* ($\beta_o = 5.0$)</td>
<td>D + L</td>
<td>0.49, 2.38</td>
<td>0.40</td>
</tr>
</tbody>
</table>

* $R/R_n$ assumed to equal to 1.0 for illustration.
one factor to be used with both live and snow loads so to constrain further the final load criteria. This factor should be very close to the $Y_L$ and $Y_S$ factors calculated earlier (Table 7). At the same time, $\phi$ should be close to the desirable range of 0.80-0.85 for flexure in reinforced concrete beams. The range of $\phi$ was selected as such to allow different quality control procedures. It was found that $Y_L=Y_S=1.6$ will fulfill the necessary requirement for $\phi$ regarding the values in Table 6. Again Eq. 46 can be used in calculating the optimal $\phi$ corresponding to $Y_D=1.2$ and $Y_L=Y_S=1.6$, as shown in Table 7. An additional condition is to place some restriction on the factored load in case the time-varying load approaches zero, according to Table 6. The gravity load combinations are shown below.

\begin{align}
U &= 1.2D_n + (1.6L_n \text{ or } 1.6S_n) \\
U &\geq 1.4D_n
\end{align}

(47) \hspace{1cm} (48)

8.2 Gravity Plus Wind Load Combinations

In this case the maximum of the checking equations given below govern the design.

\begin{align}
U &= (Y_D)D_n + (Y_L)L_n + (Y_W)W_n \\
U &= (Y_D)D_n + (Y_L)L_n + (Y_W)W_n
\end{align}

(49) \hspace{1cm} (50)

in which $(Y_L)L_n$ and $(Y_W)W_n$ are the factored arbitrary point-in-time live and wind loads, respectively. Again, optimal
load and resistance factors were determined by first calculating $R_n$ corresponding to $\beta$ of 2.5 for Eq. 49 and $\beta$ of 3.0 for Eq. 50 and minimizing Eq. 46. $\psi_D$ is taken as 1.2. The result for a reinforced concrete beam with grade 60 reinforcement and an influence area of 1000 sq. ft. is shown in Table 8. Keeping the limit state the same, the $\phi$-factor should stay the same as $D+L$ and $D+S$ combinations. But as Table 8 shows some reduction in $\psi_L$, $\psi_W$, and $\psi_W$ is necessary. Moreover, $\psi_L$ should approach 1.6 as the influence of $W_n$ approaches zero, corresponding to the $D+L$ case. If by taking $\psi_L=1.6$, $\psi_L=0.5$, $\psi_W=1.3$, and $\psi_W=0.10$, also regarding Table 6; the $\phi$-factor falls in the desired range (0.80-0.85), (see Table 8). The final load factors in the combination are shown below.

$$U = 1.2D_n + 0.5L_n + 1.3W_n$$  \hspace{1cm} (51a)  
$$U = 1.2D_n + 1.6L_n + 0.10W_n$$  \hspace{1cm} (51b)

### 8.3 Gravity and Earthquake Combinations

Similarly, an optimal set of load factors was determined by calculating $R_n$ corresponding to $\beta$ of 1.75 for reinforced concrete beams, for Boston and Los Angeles. The related influence area to find the arbitrary point-in-time live load is taken as 1000 sq. ft. To make $\psi_E$ site independent, the results of the $R_n$ calculation were combined to find optimal $\phi$, $\psi_L$, and $\psi_E$ for reinforced concrete beams. The term $\psi_D$ is again taken equal to
Table 8—Load and Resistance Factors for Gravity Plus Wind Loads (Ref. 3)

<table>
<thead>
<tr>
<th>Material</th>
<th>Eq.</th>
<th>( \phi )</th>
<th>( Y_L )</th>
<th>( Y_W )</th>
<th>( Y_{L_1} = 0.3 )</th>
<th>( Y_{L_1} = 0.4 )</th>
<th>( Y_{L_1} = 0.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Beam</td>
<td>51a</td>
<td>1.11</td>
<td>0.61</td>
<td>1.71</td>
<td>0.85</td>
<td>0.87</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>51b</td>
<td>0.93</td>
<td>1.97</td>
<td>0.08</td>
<td>-</td>
<td>0.81</td>
<td>-</td>
</tr>
<tr>
<td>Concrete Beam</td>
<td>51a</td>
<td>1.06</td>
<td>0.49</td>
<td>1.76</td>
<td>0.82</td>
<td>0.83</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>51b</td>
<td>0.86</td>
<td>1.63</td>
<td>0.14</td>
<td>-</td>
<td>0.81</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 9—Load Factors for Earthquake Loads (Ref. 3)

<table>
<thead>
<tr>
<th>Material</th>
<th>Combinations</th>
<th>( \phi )</th>
<th>( Y_L )</th>
<th>( Y_E )</th>
<th>Optimal ( \phi ) when ( Y_E = 1.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( Y_{L_1} = 0.0 )</td>
</tr>
<tr>
<td>Steel Beam</td>
<td>D + L + E</td>
<td>1.25</td>
<td>0.39</td>
<td>2.31</td>
<td>0.82</td>
</tr>
<tr>
<td>R/C Beam</td>
<td>D + L + E</td>
<td>1.21</td>
<td>0.38</td>
<td>2.37</td>
<td>0.80</td>
</tr>
</tbody>
</table>
1.2. Table 9 shows the calculated load factors for earthquake loads. Since the $\theta$-factor was large, the $U_E$ factor was adjusted to bring down the $\theta$-factor. It was found that by making $U_{L_1}=0.2$, and $U_E=1.5$, according to Table 6; the corresponding $\theta$ will fall in the optimal range. However, $0.2L_n$ in many cases would be less than the arbitrary point-in-time live load. So $U_{L_1}$ was increased to 0.5 in order to be compatible with the arbitrary point-in-time live load. The load factor for earthquake load is higher than the factor for wind load ($U_w=1.3$) because of the big difference in the coefficient of variation of wind load (0.30–0.40) compared to the earthquake load (greater than 1.00). The analysis of D+S+E combination determined that the snow load factor was close to zero. However for conservatism in areas with heavy snow and earthquake loads $U_{S_1}$ is set equal to 0.2. The final load factors in the combination are shown below.

$$U = 1.2D_n + 1.5E_n + (0.5L_n \text{ or } 0.2S_n)$$  \hspace{1cm} (52)

8.4 Counteracting Loads

Common cases when loads counteract one another are the wind or earthquake loads opposing the gravity loads.

$$U = E - D$$  \hspace{1cm} (53)

$$U = W - D$$  \hspace{1cm} (54)

To simplify the minimizing problem, there should be some constraints placed on the safety factors. First, since $U_D=1.2$ or
1.1 for the additive case, it is reasonable to take $y_D = 0.9$ for the counteracting case. Further, the $\psi$-factor must stay the same (0.80-0.85) for both additive and counteracting cases. So, fixing $y_D = 0.90$ and $\psi = 0.85$, $y_E$ (or $y_w$) can be calculated by finding $R_n$ and then minimizing Eq. 46.

The optimal value of $y_w$ (or $y_E$) depends on $\psi$. In the case where $\psi = 0.85$ and $y_D = 0.9$, $y_w$ was found to be 1.22. To be consistent with the additive combinations, the load factors for counteractive case were chosen as below.

$$U = 0.9D_n - 1.3W_n$$

$$U = 0.9D_n - 1.5E_n$$

(55)  

(56)

Other counteracting load combinations may be treated similarly. Some of the cases which may be important to consider are;

$$D + L_{apt} + S$$  

$$D + W_{ann} + S$$

(57)  

(58)

in which $L_{apt}$ is the arbitrary point-in-time live load and $W_{ann}$ is the maximum annual wind load. In these cases, the load factors for $L_n$ and $W_n$ to give the desired $\psi(0.80-0.85)$ are $y_{L_1} = 0.5$ and $y_{W_1} = 0.80$.

In sum, the load combinations and load factors recommended for use by the individual material specification writers in their design specifications are:

$$1.4D_n$$

$$1.2D_n + 1.6L_n$$

$$1.2D_n + 1.6S_n + (0.5L_n \text{ or } 0.8W_n)$$
\[ 1.2D_n + 1.3W_n + 0.5L_n \]
\[ 1.2D_n + 1.5E_n + (0.5L_n \text{ or } 0.2S_n) \]
\[ 0.9D_n - (1.3W_n \text{ or } 1.5E_n) \]

For other loading combinations, the same methodology can be used to derive the related safety factors.
9. Determination of Resistance Factors

The first step in calculating the $\varphi$-factor for concrete members is selecting a target $\beta$. As was shown previously, the reliability indices associated with current practice are as below [Ref. 3].

a. Reinforced concrete beams in flexure, 2.6 to 3.2
b. Plant produced pretensioned beams in flexure, 3.2 to 4.0
c. Tied columns, compression failure, 3.0 to 3.5
d. Spiral columns, compression failure, 2.6 to 3.3
e. Shear, beams with stirrups, 1.9 to 2.4

In many practical instances, the design is governed by the D+L combination. Accordingly, Figure 11 (11a through 11e) presents curves relating the reliability $\beta$, $\bar{R}/R_n$, $V_R$, and $\varphi$ for the design criterion,

$$R_n > 1.2D_n + 1.6L_n$$

These curves are computed for an influence area of 1000 sq. ft.; and are presented in terms of the most common live load $L=50$ psf in offices. Table 10 presents the $\varphi$ values based on $\beta$ of 3.0 for ductile failure, and $\beta$ of 3.5 for brittle failure. Finally, in Fig. 12 a comparison of existing design and the future design is presented.
<table>
<thead>
<tr>
<th>Action</th>
<th>Type of Member</th>
<th>$\bar{R}/R_n$</th>
<th>$V_R$</th>
<th>Range of $\phi$ for $L_o/D_n = 0.25 - 2.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexure, Reinforced Concrete, $\beta = 3.0$</td>
<td>Beam, Grade 40, $\rho = 0.35p_b$</td>
<td>1.14</td>
<td>0.14</td>
<td>0.82 - 0.84</td>
</tr>
<tr>
<td></td>
<td>Beam, Grade 60, $\rho = 0.57p_b$</td>
<td>1.05</td>
<td>0.11</td>
<td>0.80 - 0.85</td>
</tr>
<tr>
<td></td>
<td>Beam, Grade 60, $\rho = 0.73p_b$</td>
<td>1.01</td>
<td>0.12</td>
<td>0.76 - 0.80</td>
</tr>
<tr>
<td></td>
<td>Two way slabs, Grade 60</td>
<td>1.16</td>
<td>0.15</td>
<td>0.83 - 0.86</td>
</tr>
<tr>
<td></td>
<td>Continuous, one-way slabs</td>
<td>1.22</td>
<td>0.16</td>
<td>0.85 - 0.88</td>
</tr>
<tr>
<td>Flexure, Plant Produced Pretensioned Concrete, $\beta = 3.0$</td>
<td>Double T $\omega_p = 0.054$</td>
<td>1.06</td>
<td>0.057</td>
<td>0.86 - 0.95</td>
</tr>
<tr>
<td></td>
<td>Beam $\omega_p = 0.228$</td>
<td>1.06</td>
<td>0.083</td>
<td>0.83 - 0.90</td>
</tr>
<tr>
<td></td>
<td>Beam $\omega_p = 0.295$</td>
<td>1.04</td>
<td>0.10</td>
<td>0.80 - 0.86</td>
</tr>
<tr>
<td>Flexure, Cast-in-Situ Post-Tensioned Concrete $\beta = 3.0$</td>
<td>$\omega_p = 0.228$</td>
<td>1.03</td>
<td>0.11</td>
<td>0.78 - 0.83</td>
</tr>
<tr>
<td></td>
<td>$\omega_p = 0.295$</td>
<td>1.05</td>
<td>0.14</td>
<td>0.76 - 0.79</td>
</tr>
<tr>
<td>Tied Columns, Compression Failures, $\beta = 3.5$</td>
<td>3000 psi Concrete, short</td>
<td>1.05</td>
<td>0.16</td>
<td>0.65 - 0.69</td>
</tr>
<tr>
<td></td>
<td>5000 psi Concrete, short</td>
<td>0.95</td>
<td>0.14</td>
<td>0.61 - 0.66</td>
</tr>
<tr>
<td></td>
<td>5000 psi Concrete, $l/h = 20$</td>
<td>1.10</td>
<td>0.17</td>
<td>0.66 - 0.70</td>
</tr>
<tr>
<td>Spiral Columns, Compression Failures, $\beta = 3.0$</td>
<td>3000 psi Concrete, short</td>
<td>1.05</td>
<td>0.16</td>
<td>0.74 - 0.76</td>
</tr>
<tr>
<td></td>
<td>5000 psi Concrete, short</td>
<td>0.95</td>
<td>0.14</td>
<td>0.69 - 0.72</td>
</tr>
<tr>
<td>Shear, $\beta = 3.5$</td>
<td>Beams without stirrups</td>
<td>0.93</td>
<td>0.21</td>
<td>0.50 - 0.52</td>
</tr>
<tr>
<td></td>
<td>Beams with minimum stirrups</td>
<td>1.00</td>
<td>0.19</td>
<td>0.60 - 0.64</td>
</tr>
<tr>
<td></td>
<td>Beams with $p_{fy} = 150$</td>
<td>1.09</td>
<td>0.17</td>
<td>0.66 - 0.70</td>
</tr>
</tbody>
</table>

Note: 1 psi = 6895 N/m²
Figure 11 - Aids to the Selection of Resistance Factors (Ref. 5)
Figure 11d
Figure 12 - Comparison of Designs Using Existing and Proposed Criteria for Reinforced Concrete Beams (100 ft$^2 = 9.3$ m$^2$)

(Ref. 3), $R_{nf}$ = future nominal resistance, $R_{nc}$ = current nominal resistance.
10. Summary and Conclusions

This report has described the development of a set of load and load combination factors for use with loads in the American National Standard A58. The scope of these load criteria is the same as of the A58 Standard, which covers dead, live, wind, snow, and earthquake loads. The given criterion does not apply to vehicle loads on bridges, nuclear reactor constructions, and other loads that are considered outside the A58 Standard.

The load factors were developed by first evaluating the reliabilities associated with the current practice. The procedure for this evaluation is described in Chapter 2. Then, the load factors were developed consistent with these reliabilities, taking the statistical and probabilistic information of various types of structural loads and structural capacities (resistances) into the consideration. The data for loads and resistances were obtained from the load subcommittees of American National Standard Institute Committee A58.

The proposed load and load combination factors for A58 Standard are as follows:

\[ 1.4D_n \]
\[ 1.2D_n + 1.6L_n \]
\[ 1.2D_n + 1.6S_n + (0.5L_n \text{ or } 0.8W_n) \]
\[ 1.2D_n + 1.3W_n + 0.5L_n \]
\[ 1.2D_n + 1.5E_n + (0.5L_n \text{ or } 0.2S_n) \]
$0.9D_n - (1.3\bar{w}_n \text{ or } 1.5E_n)$

In conclusion, it should be mentioned that by setting the load and resistance factors constant over all range of nominal loads and resistance the reliability index does not stay constant. However by setting the load factors constant, the resistance factors can be changed to fulfill a relatively constant reliability index.
Acknowledgements

The guidance and assistance of my major professor Dr. Stuart E. Swartz is gratefully acknowledged with thanks. Special thanks are also extended to Dr. Sundheim, professor of statistics at Kansas State University, for his help in explaining some of the statistical analysis of data to me.
References


Direction Angles - $\alpha$, $\beta$, $\gamma$, angles between $\mathbf{A}$ and the standard $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ basis vectors.

So, the cosines of the three direction angles are,

$$
\cos \alpha = (\mathbf{A} \cdot \mathbf{i}) / |\mathbf{A}|
$$

$$
\cos \beta = (\mathbf{A} \cdot \mathbf{j}) / |\mathbf{A}|
$$

$$
\cos \gamma = (\mathbf{A} \cdot \mathbf{k}) / |\mathbf{A}|
$$

since;

$$
\mathbf{A} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}
$$

$$
a = \mathbf{A} \cdot \mathbf{i}
$$

$$
b = \mathbf{A} \cdot \mathbf{j}
$$

$$
c = \mathbf{A} \cdot \mathbf{k}$$
\[ |\overline{A}| = \left( a^2 + b^2 + c^2 \right)^{1/2} \]

Then:

\[ \cos \alpha = a / |\overline{A}| \]

\[ \cos \beta = b / |\overline{A}| \]

\[ \cos \gamma = c / |\overline{A}| \]
Appendix B - Example Reliability Calculations

In the following example the 10-step procedure described in Chapter 2.2 will be followed to determine the optimum $\beta$.

Suppose a beam, 16WF31 section with yield stress $F_y=36$ Ksi and plastic section modulus $Z=54$ in. supports a moment of 1140 in-Kip. Statistics of $F_y$ and $Z$ are:

$F_y$: Log-Normal $\overline{F}_y=38$ Ksi, $V(F_y)=0.10$

$Z$: Normal $\overline{Z}=54$ in. $V(Z)=0.05$

and the limit state, $g()=F_y.Z - 1140 = 0$

Step 1. Limit State, $g()=F_y.Z - 1140 = 0$

Step 2. Initial Guess, $\beta=3.0$

Step 3. $X_i^e=\overline{X}_i$, $F_i^e=38$, $Z_i^e=54$

Step 4. The mean and standard deviation of the equivalent normal distribution for the Log-Normal distribution are as below.

$$\overline{X}_i^e=\overline{X}_i - \beta^e [F_i(X_i^e)] \cdot \sigma^e$$

$$\sigma^e=\phi^e [F_i(X_i^e)]/f_i(x^e)$$

$F_y$: Log-Normal Distribution, $F_y=38$, $V(F_y)=0.10$

Because of the special properties of log-normal and normal distribution, we can take their coefficients of variation equal. So, $V(F_y)=0.10$ for the equivalent normal distribution. In a normal distribution the equation below holds.

$$V=\sigma/\mu$$

in which $V$=coefficient of variation, $\sigma$=standard deviation, and $\mu$=
Figure 13 Lognormal distribution, probability paper
(Ref. 8)
mean. Therefore, the equivalent standard deviation is,

\[ \sigma^- = \mu V \]

\[ \sigma^- = (38)(0.10) = 3.8 \]

To find the equivalent mean, we need to find the cumulative distribution value of the log-normal distribution at \( X_i^- \), \( F(X_i^-) \).

As shown in Fig.13 the median value, \( m \), corresponds to 50% probability of occurrence in a log-normal distribution [Ref. 8]. Further, \( m \exp(\sigma^-) \), corresponds to 84.1% and \( m \exp(-\sigma^-) \), corresponds to 15.9% probability of occurrence. The calculation below evaluates these values.

In a log-normal distribution, the coefficient of variation is;

\[ V = [\exp(\sigma^2) - 1]^{1/2} \]

so,

\[ V = 0.10 = [\exp(\sigma^2) - 1]^{1/2} \]

therefore,

\[ \sigma^- = 0.0997 \]

also,

\[ \text{mean} = m \exp(\sigma^2/2) \]

therefore,

\[ 38 = m \exp(0.0997^2/2) \]

\[ m = 37.811 \]

\[ m \exp(\sigma^-) = (37.811) \exp(0.0997) = 41.775 \]

\[ m \exp(-\sigma^-) = (37.811) \exp(-0.0997) = 34.223 \]

From Fig.13, and the above calculations, the probability of the variate being equal to or less than \( X_i^- = 38 \) can be obtained as shown below.

\[ \log(X_i^-) = \log(38) = 1.57978 \]

\[ \log(m) = \log(37.811) = 1.57761 \]

\[ \log(m \exp(\sigma^-)) = \log(41.775) = 1.62091 \]
\[ \log(m \exp(-\sigma^2)) = \log(34.223) = 1.53431 \]

So the probability \( F(38) \) computes according to the straight line in Figure 13.

\[
F(38) = 50 + \left[ (1.57978 - 1.57761) / (1.62091 - 1.57761) \right] (84.1 - 50)
\]

\[
F(38) = 51.709\%
\]

Now, assuming a standard normal distribution with this probability of occurrence, we can proceed by using the Standard Normal Distribution Table given in Ref. 6.

\[
\Phi^{-1}(0.51709)
\]

Or

\[
\Phi^{-1}(0.51709 - 0.5) = 0.04279
\]

And therefore from Chapter 2,

\[
\bar{X}_i = 38 - (0.04279)(3.8)
\]

\[
\bar{X}_i = 37.83
\]

Step 5.

\[
\frac{\partial g}{\partial F_y} \bigg|_{Z} = Z = 54
\]

\[
\frac{\partial g}{\partial Z} \bigg|_{F_y} = F_y = 38
\]

Step 6.

\[
\sigma_z = \text{mean} \times \text{coeff. of variation} = (54)(0.05) = 2.7
\]

\[
\alpha_{F_y} = \left[ (54)(3.8) / [(54)^2(3.8)^2 + (38)^2(2.7)^2]^{1/2} \right] = 0.894
\]

\[
\alpha_z = \left[ (38)(2.7) / [(54)^2(3.8)^2 + (38)^2(2.7)^2]^{1/2} \right] = 0.447
\]

Step 7. New values of \( \bar{X}_i^* \) are

\[
\bar{X}_i^* = 54 - (0.447)(3.0)(2.7) = 50.38
\]

\[
\bar{X}_y^* = 37.83 - (0.894)(3.0)(3.8) = 27.64
\]

Here, we repeat steps 4 through 7 until the estimates of \( \alpha_i \) stabilize. The results of these calculations and other steps taken in this problem are tabulated in Table II [Ref. 3].

\[
\bar{X}_i^* = \bar{X}_i^* - \alpha_i \cdot \beta \cdot \sigma_i^N
\]

is used to calculate \( \beta \) in the limit state equation.
Table 11—Illustration of Reliability Calculations-
Iterative Solution Steps (Ref. 3)

<table>
<thead>
<tr>
<th>Step 2</th>
<th>3</th>
<th>3.001</th>
<th>5.136</th>
</tr>
</thead>
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<td>38</td>
<td>27.64</td>
<td>29.02</td>
<td>23.95</td>
</tr>
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<td>54</td>
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<td>47.60</td>
</tr>
<tr>
<td>37.83</td>
<td>36.31</td>
<td>36.71</td>
<td>34.90</td>
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<td>3.80</td>
<td>2.76</td>
<td>2.90</td>
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</tr>
<tr>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
</tr>
<tr>
<td>2.7</td>
<td>2.7</td>
<td>2.7</td>
<td>2.7</td>
</tr>
<tr>
<td>54</td>
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<td>50.17</td>
<td>47.6</td>
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<tr>
<td>38</td>
<td>27.64</td>
<td>29.02</td>
<td>23.95</td>
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<tr>
<td>0.894</td>
<td>0.881</td>
<td>0.880</td>
<td>0.870</td>
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<td>0.493</td>
</tr>
<tr>
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<td>5.136</td>
<td>5.144</td>
<td></td>
</tr>
</tbody>
</table>
Appendix C - Extreme Value Type I Distribution

Type I distributions are sometimes called the log-Weibull distributions or they are sometimes called "double exponential" distributions. The function below is the general form of the Extreme Value Type I distribution.

\[ P(X \leq x) = \exp\left\{ -\exp\left[-\left(x-\xi\right)/\theta\right]\right\} \]

where \( \xi \), and \( \theta \) are parameters.

The name 'extreme value' is attached to these distributions because they can be obtained as limiting distributions (as \( n \to \infty \)) of the greatest value among \( n \) independent random variables each having the same continuous distribution. By replacing \( X \) by \(-X\), limiting distributions of least values are obtained.
PROBABILITY-BASED LOAD FACTORS
AND LOAD COMBINATION FACTORS
IN STRUCTURAL DESIGN

by

AMIR JAFARI
B.S., Kansas State University, 1980

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AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the
requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering
KANSAS STATE UNIVERSITY
Manhattan, Kansas

1983
Abstract

Probability-based load factors and load combinations are presented which are compatible with the loads specified in ANSI A58 Standards. The load effects considered are due to dead, occupancy live, snow, wind and earthquake loads. The load criteria which are presented were developed by numerous investigators on the basis of probability of failure calculated by simplified theory, using statistical data on structural loads and resistances. The report presents the rationale and methodology used by the investigators for selecting the load factors which are intended to apply to all types of structural materials used in building construction.