A PATTERN RECOGNITION APPROACH TO GRAIN, SAMPLE ANALYSIS

by

ANIL HARIKANT VYAS

B. E., M. S. University, Baroda, India, 1970

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Electrical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1973

Approved by:

[Signature]
Major Professor
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>1.1</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>1</td>
</tr>
<tr>
<td>1.3</td>
<td>3</td>
</tr>
<tr>
<td>II</td>
<td>6</td>
</tr>
<tr>
<td>2.1</td>
<td>6</td>
</tr>
<tr>
<td>2.2</td>
<td>8</td>
</tr>
<tr>
<td>2.3</td>
<td>9</td>
</tr>
<tr>
<td>2.4</td>
<td>11</td>
</tr>
<tr>
<td>III</td>
<td>15</td>
</tr>
<tr>
<td>3.1</td>
<td>15</td>
</tr>
<tr>
<td>3.2</td>
<td>19</td>
</tr>
<tr>
<td>3.3</td>
<td>32</td>
</tr>
<tr>
<td>IV</td>
<td>36</td>
</tr>
<tr>
<td>4.1</td>
<td>36</td>
</tr>
<tr>
<td>4.2</td>
<td>38</td>
</tr>
<tr>
<td>4.3</td>
<td>38</td>
</tr>
<tr>
<td>4.4</td>
<td>39</td>
</tr>
<tr>
<td>4.5</td>
<td>42</td>
</tr>
<tr>
<td>V</td>
<td>50</td>
</tr>
<tr>
<td>5.1</td>
<td>50</td>
</tr>
</tbody>
</table>
CHAPTER I
INTRODUCTION

1.1. Introductory Remarks

This report concerns an initial feasibility study of grain sample analysis. All grain sample analysis includes a determination of the amount of material in the sample which is not the grain being analyzed. The methods which are available at present make use of appropriate sieves, followed by a manual hand picking of the remainder. Since this hand picking is both tedious and time consuming, it is performed separately. Clearly, it would be desirable to have a device that can replace the hand picking part of the analysis. Such a device should be capable of distinguishing between long grains such as wheat, barley, oats, rye, etc. with perhaps the greatest challenge being to distinguish between wheat and rye.

The approach entertained in this study is based on pattern recognition techniques. Such techniques may be able to do equally well on separating other types of grain such as corn and soybeans in addition to the long grains mentioned above. Thus, a device which is based on pattern recognition techniques could conceivably be placed ahead of the sieves, except perhaps the sand sieve. To this end, the main objective of this study is to consider some basic aspects pertaining to the feasibility of a pattern recognition system for grain identification. The types of grain used for the study are: (1) corn, (2) wheat, (3) barley, (4) oats and (5) milo.

1.2. General Remarks Pertaining to Pattern Recognition Problems

The basic ideas associated with pattern recognition problems are best introduced by referring to Figure 1-1.
THIS BOOK CONTAINS NUMEROUS PAGES WITH DIAGRAMS THAT ARE CROOKED COMPARED TO THE REST OF THE INFORMATION ON THE PAGE. THIS IS AS RECEIVED FROM CUSTOMER.
Figure 1-1. Block diagram representation of a pattern recognition problem.

As the name implies, the signal acquisition stage acquires a set of signals from each of the categories or classes which are to be classified. The signals acquired may be one-dimensional or multi-dimensional and hence the complexity of the signal acquisition stage varies from one pattern recognition problem to another.

The output of the signal acquisition stage is denoted by $C_i$, $i=1, 2, \ldots, k$, where $C_i$ represents the $i$th category or class, each of which consists of $N_i$ signals. The $j$th signal belonging to class $i$ is denoted by $x_{ij}$. Thus for each $i$, $j$ varies from 1 to $N_i$.

Consider a typical signal $x_{ij}$ which is fed into a signal representation stage is shown in Figure 1-1. The role of the signal representation stage is to seek some "measurements", "features" or "attributes" which will help discriminate a signal $x_{ij}$ from another signal $x_{mj}$, where, $i \neq m$. The output of this stage corresponding to the input $x_{ij}$ is denoted by $p_{ij}$ which may be
in the form of a finite n-dimensional vector or a finite multi-dimensional array. Thus \( P_{ij} \) is generally referred to as a pattern corresponding to the signal \( x_{ij} \).

The classification stage in Figure 1-1 is in essence a device which is "trained" to recognize a set of patterns \( \{ P_{ij} \} \). Consequently the set of patterns \( \{ P_{ij} \} \) whose classification is a known a-priori is referred to as the training set. Once the classifier has been trained using the training set, it is conceivable that it will make errors while classifying patterns not belonging to the training set. Therefore the game is to train the classifier in such a way that it makes as few errors as possible. There is a large number of training procedures available in the literature. The procedure which is best suited for a specific application is generally dictated by the nature of the signal representation stage.

In conclusion it is remarked that although the signal representation stage in Figure 1-1 plays a crucial role with respect to the overall system complexity and performance, very little theory is available which enables one to select the "best" measurements or features to represent the set of signals \( \{ x_{ij} \} \). Feature selection techniques vary from one pattern recognition problem to another. Some aspects of signal representation for the problem at hand are briefly considered in what follows.

1.3. Signal Representation for Grain Classification

The signal representation technique used in this study is simple in that it concerns only the size and shape of a given kernel. Consider a black

---

and white image of a grain kernel as shown in Figure 1-2.

![Figure 1-2. Black and White Image of a Kernel](image)

The image frame in Figure 1-2 is now divided into an (32x32) array. Again, each element of this array which contains the kernel is represented by a "1" while that which does not is represented by a "0". An example of the coded image which results by this process is shown in Figure 1-3. Information pertaining to the size and shape of this image frame is extracted by means of a two-dimensional spectral analysis. The transform used for the spectral analysis is analogous to the familiar discrete Fourier transform and is called the Walsh Hadamard or BIFORE (Binary Fourier Representation) transform. A brief introduction to the two-dimensional BIFORE transform is provided in Chapter II.
CHAPTER II
THE TWO-DIMENSIONAL BIFORE TRANSFORM

2.1. Definition

The (32x32) array of zeros and ones of a typical coded image as shown in Figure 3 can be represented by a matrix as follows:

\[
\begin{bmatrix}
  f(0,0) & f(0,1) & \cdots & f(0,N_2-1) \\
  f(1,0) & f(1,1) & \cdots & f(1,N_2-1) \\
    \vdots & \vdots & \ddots & \vdots \\
  f(N_1-1,0) & f(N_1-1,1) & \cdots & f(N_1-1,N_2-1)
\end{bmatrix}
\]

(2-1)

where \( N_1 = N_2 = 32 \) and each of the \( f(i,j) \) is a zero or a one. Then the two dimensional BIFORE transform (2-BT) of the data matrix \( f(x_1,x_2) \) in (2-1) is defined as

\[
\begin{bmatrix}
  F(u_1,u_2)
\end{bmatrix} = \frac{1}{N_1N_2} \sum_{x_1=0}^{N_1-1} \sum_{x_2=0}^{N_2-1} f(x_1,x_2) (-1)^{x_1}u_1 u_2
\]

(2-2)

where

- \( F(u_1,u_2) \) is a transform coefficient
- \( f(x_1,x_2) \) is an input data point
- \( u_1 = 0,1,\ldots,(N_1-1); i=1,2 \)
- \( n_1 = n_2 \)
- \( \langle x_1,u_1 \rangle = \sum_{m=0}^{n_1-1} u_{1}(m)x_{1}(m) \)

1. Note that \( N_1 \) need not be equal to \( N_2 \)
2. In general \( f(i,j) \) can be any finite real number
\[ \langle x, u \rangle = \langle x_1, u_1 \rangle + \langle x_2, u_2 \rangle \]

and

\[ n_i = \log_2 N_i, \quad i = 1, 2. \]

The terms \( u_i(m) \) and \( x_i(m) \) in (2-3) are the binary representations of \( u_i \) and \( x_i \) respectively. For example,

\[
\begin{bmatrix}
  u_{1,\text{decimal}} \\
  \end{bmatrix}
= \begin{bmatrix}
  u_i(k_i-1), u_i(k_i-2), \ldots, u_i(1), u_i(0) \\
  \end{bmatrix}_{\text{binary}}
\] (2-4)

where \( u_i(\cdot) \in \{0, 1\} \).

Alternately, (2-2) can be written in the form of a matrix to obtain

\[
\begin{bmatrix}
  E(u_1, u_2) \\
\end{bmatrix} = \frac{1}{N_1 N_2} \begin{bmatrix}
  H(n_1) \\
  \end{bmatrix} \begin{bmatrix}
  f(x_1, x_2) \\
  \end{bmatrix} \begin{bmatrix}
  H(n_2) \\
  \end{bmatrix}
\] (2-5)

where

\[
\begin{bmatrix}
  E(u_1, u_2) \\
\end{bmatrix}
\]

is a \((N_1 \times N_2)\) transform matrix corresponding to the data matrix \( \begin{bmatrix}
  f(x_1, x_2) \\
\end{bmatrix} \)

\[
\begin{bmatrix}
  H(n_1) \\
  \end{bmatrix}
\]

and \( \begin{bmatrix}
  H(n_2) \\
  \end{bmatrix} \) are \((N_1 \times N_1)\) and \((N_2 \times N_2)\) Hadamard matrices with \( n_i = \log N_i, \quad i = 1, 2. \)

The Hadamard matrices in (2-5) are defined by the recurrence relation

\[
\begin{bmatrix}
  H(k+1) \\
\end{bmatrix} = \begin{bmatrix}
  H(k) & \cdots & \cdots & H(k) \\
  \cdots & \cdots & \cdots & \cdots \\
  \cdots & \cdots & \cdots & \cdots \\
  H(k) & \cdots & \cdots & - H(k) \\
\end{bmatrix}
\] \quad \text{for } k = 0, 1, \ldots, n_1 (2-6)

\[
\begin{bmatrix}
  H(0) \\
\end{bmatrix} = \begin{bmatrix}
  1 \\
\end{bmatrix}
\]

From (2-6) it follows that the elements of a Hadamard matrix are either +1 or -1.

Using the fact that the 2-BT is an orthogonal transform, it can be shown that (2-7) the corresponding inverse transform (2-IBT) is defined as
\[
\hat{f}(x_1, x_2) = \sum_{u_1=0}^{N_1-1} \sum_{u_2=0}^{N_2-1} F(u_1, u_2) (-1)^{<x, u>} \quad (2-7)
\]

or alternately as
\[
\begin{bmatrix}
\hat{f}(x_1, x_2)
\end{bmatrix} =
\begin{bmatrix}
H(n_1)
\end{bmatrix}
\begin{bmatrix}
F(u_1, u_2)
\end{bmatrix}
\begin{bmatrix}
H(n_2)
\end{bmatrix}
\quad (2-8)
\]

2.2. The 2-BT Power Spectrum

A BIFORE power spectrum which is analogous to a two-dimensional Fourier power spectrum can be defined as

\[
P(z_1, z_2) = \sum_{u_1=\left\lfloor 2^{z_1-1} \right\rfloor}^{2^{z_1}-1} \sum_{u_2=\left\lfloor 2^{z_2-1} \right\rfloor}^{2^{z_2}-1} F^2(u_1, u_2) \quad (2-9)
\]

where

\[
z_i = 0, 1, \ldots, k_i; \quad n_i = \log_2 N_i, \quad i=1, 2.
\]

and

\[
\left\lfloor 2^{z_i-1} \right\rfloor \text{is the integer part of } 2^{z_i-1}.
\]

From (2-9) it follows that the 2-BT power spectrum can be expressed in matrix form to obtain

\[
\begin{bmatrix}
P(k_1, k_2)
\end{bmatrix} =
\begin{bmatrix}
P(0,0) & P(0,1) & \ldots & P(0,n_2) \\
P(1,0) & P(1,1) & \ldots & P(1,n_2) \\
\vdots & \vdots & \ddots & \vdots \\
P(n_1,0) & P(n_1,1) & \ldots & P(n_1,n_2)
\end{bmatrix}
\quad (2-10)
\]

Inspection of (2-10) reveals that the number of 2-BT power spectrum points is given by

\[
\sum \quad = (1+n_1)(1+n_2); \quad n_i = \log_2 N_i, \quad i=1, 2 \quad (2-11)
\]
2.3 A Numerical Example

Before proceeding further, it is instructive to consider a simple numerical example which helps clarify the material discussed above. Suppose the 2-BT and 2-BT power spectrum of the pattern shown in Figure 2-1 are desired.

\[
\begin{array}{cccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

Figure 2-1. An (8x8) pattern

From Figure 2-1 it follows that \( N_1 = N_2 = 8 \). Thus (2-5) yields

\[
\left[ f(u_1, u_2) \right] = \frac{1}{64} \left[ H(3) \right] \left[ f(x_1, x_2) \right] \left[ H(3) \right] \]

(2-12)
Using (2-6) to evaluate \( [H(3)] \) and subsequently substituting in (2-12) results in the following matrix equation.

\[
E(u_1, u_2) = \frac{1}{64} [H(3)] \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} [H(3)]
\] (2-13)

where

\[
[H(3)] = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & 1 \\
1 & 1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & -1 \\
\end{bmatrix}
\] (2-14)

Evaluation of (2-13) yields the transform matrix
\[
\begin{bmatrix}
16 & 0 & 0 & 4 & 0 & -4 & -16 & 0 \\
0 & 0 & 0 & -4 & 0 & 4 & 0 & 0 \\
0 & 0 & 0 & 4 & 0 & -4 & 0 & 0 \\
0 & 0 & 0 & -4 & 0 & 4 & 0 & 0 \\
4 & 0 & 0 & 0 & 0 & 0 & -4 & 0 \\
-4 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
12 & 0 & 0 & 0 & 0 & 0 & -12 & 0 \\
-4 & 0 & 0 & 0 & 0 & 0 & 4 & 0 
\end{bmatrix}
\]

Substitution of the values of \( F(u_1, u_2) \) obtained from (2-15) into (2-9) results in the following 2-BT power spectrum in matrix form.

\[
P(k_1, k_2) = \frac{1}{256} \begin{bmatrix}
16 & 0 & 1 & 17 \\
0 & 0 & 1 & 1 \\
0 & 0 & 2 & 2 \\
12 & 0 & 0 & 12 
\end{bmatrix}
\]

(2-16)

2.4. Motivation for Using the 2-BT Power Spectrum.

In this study, we attempt to characterize the size and shape of grain kernels by the \((1+\log_2 N_1)(1+\log_2 N_2)\) power spectrum points given by the matrix \( P(k_1, k_2) \) in (2-10). There are basically three reasons which provide the motivation for doing so. These are discussed in what follows.

(1). The power spectrum represents the distribution of power in a given two-dimensional pattern. This is best illustrated by a simple example with \( N_1=2, N_2=4 \) and

\[
\begin{bmatrix}
3 & 0 & 3 & 4 \\
3 & 8 & 7 & 8 
\end{bmatrix}
\]

(2-17)
Applying (2-5) and (2-9) to (2-12) results in the following 2-BT power spectrum

\[ P(0,0) = 81/4, \quad P(0,1) = 1/4, \quad P(0,2) = 1 \]
\[ P(1,0) = 4, \quad P(1,1) = 1, \quad P(1,2) = 1. \quad (2-18) \]

Now, it can be shown that \[ \mathbf{f}(x_1, x_2) \] in (2-17) can be decomposed into the following mutually orthogonal sub patterns:

\[
\begin{bmatrix}
\mathbf{f}_{00}(x_1, x_2)
\end{bmatrix} =
\begin{bmatrix}
4.5 & 4.5 & 4.5 & 4.5 \\
4.5 & 4.5 & 4.5 & 4.5
\end{bmatrix}
\]

\[
\begin{bmatrix}
\mathbf{f}_{01}(x_1, x_2)
\end{bmatrix} =
\begin{bmatrix}
0.5 & 0.5 & -0.5 & 0.5 \\
-0.5 & 0.5 & -0.5 & 0.5
\end{bmatrix}
\]

\[
\begin{bmatrix}
\mathbf{f}_{02}(x_1, x_2)
\end{bmatrix} =
\begin{bmatrix}
-1 & -1 & 1 & 1 \\
-1 & -1 & 1 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
\mathbf{f}_{10}(x_1, x_2)
\end{bmatrix} =
\begin{bmatrix}
-2 & -2 & -2 & -2 \\
-2 & -2 & -2 & -2
\end{bmatrix}
\]

\[
\begin{bmatrix}
\mathbf{f}_{11}(x_1, x_2)
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & -1 & -1 \\
-1 & 1 & -1 & 1
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
\mathbf{f}_{12}(x_1, x_2)
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & -1 & -1 \\
-1 & 1 & 1 & 1
\end{bmatrix} \quad (2-19)
\]

It is straightforward to verify that

\[
\begin{bmatrix}
\mathbf{f}(x_1, x_2)
\end{bmatrix} = \sum_{i=0}^{2} \begin{bmatrix}
\mathbf{f}_{0i}(x_1, x_2)
\end{bmatrix} + \sum_{i=0}^{2} \begin{bmatrix}
\mathbf{f}_{1i}(x_1, x_2)
\end{bmatrix} \quad (2-20)
\]
Inspection of the sub-patterns in (2-19) reveals that \( f_{ij}(x_1,x_2) \) is 
\(^{2^1}\) periodic with respect to the first dimension (i.e., the columns) and 
\(^{2^1}\) periodic in the second dimension (i.e., the rows). If 
\[ \sum_{i,j} \left[ f_{ij}(x_1,x_2) \right]^2 \]
denotes the sum of the squares of the elements in the
sub-pattern \( f_{ij}(x_1,x_2) \), divided by \( N_1 N_2 \), then it can be verified that
\[ \sum_{i,j} \left[ f_{00}(x_1,x_2) \right]^2 = 81/4 \]
\[ \sum_{i,j} \left[ f_{01}(x_1,x_2) \right]^2 = 1/4 \]
\[ \sum_{i,j} \left[ f_{02}(x_1,x_2) \right]^2 = 1 \]
\[ \sum_{i,j} \left[ f_{10}(x_1,x_2) \right]^2 = 4 \]
\[ \sum_{i,j} \left[ f_{11}(x_1,x_2) \right]^2 = 1 \]
\[ \sum_{i,j} \left[ f_{12}(x_1,x_2) \right]^2 = 1 \]  \hspace{1cm} (2-21)

Comparing (2-18) and (2-21) it is clear that each of the 2-BT power
spectrum points represents the average power in one of the mutually
orthogonal sub-patterns \( f_{ij}(x_1,x_2) \). Thus the 2-BT power spectrum
represents the distribution of power in a given two-dimensional pattern.

In the problem at hand such two-dimensional patterns are coded images
(see Figure 1-3) which characterize the sizes and shapes of grain kernels.

(2). From the periodicities present in the sub-patterns \( f_{ij}(x_1,x_2) \),
the analogy between the 2-BT power spectrum and the familiar discrete
Fourier power spectrum is apparent. Again, the property that the Fourier
power spectrum is invariant with respect to cyclic shifts without rotation
of a pattern as illustrated in Figure 2-2 is also valid for the BIFORE
spectrum. However, the 2-BT power spectrum yields considerable data
compression since it consists of \( (1+\log_2 N_1)(1+\log_2 N_2) \) spectrum points in
contrast to \( (N_1/2+1)(N_2/2+1) \) independent discrete Fourier spectrum points.
Figure 2-2 Illustration of the shift-invariance property.

(3). The 2-BT power spectrum can be computed rapidly using an algorithm called the fast BIFORE transform (FBT). The corresponding computer program is included in Appendix 3-2. Since only real arithmetic operations are involved, the corresponding implementation is simpler relative to that of the Fourier spectrum which requires complex arithmetic operations.
CHAPTER III
EXPERIMENTAL RESULTS

3-1. Data Collection

A block diagram of the set up used to obtain the data is shown in Figure 3-1. Each kernel was placed at the base of a microfilm reader and projected on its screen to which a 32x32 grid was attached. Each square of the grid which was occupied by the kernel was coded by a "1" and by a "0" otherwise. A typical output obtained from this stage of the data processing is shown in Figure 3-2. Before such data was recorded, the reader was calibrated using a standard circular pattern which is shown below in Figure 3-3. The image coded pattern corresponding to this

Figure 3-3. Calibration pattern for microfilm reader calibration pattern is shown in Figure 3-4.

The above output of the film reader which was in the form of a 32x32 array of ones and zeros was punched on IBM cards and subsequently fed to an IBM 360 computer. The computer program listed in Appendix 3-2 was used to compute the 2-BT power spectra of several grain kernels placed in various orientations for each kernel. The corresponding 2-BT power spectra that resulted are summarized in Appendix 3-1. For convenience, each spectrum point in this table has been multiplied by a scale factor of 10^3.
Figure 3-1. Block diagram of set up to gather data
Figure 3-2. Typical output from microfilm reader.

(Blank elements of the array consist of zeros).
Figure 3-3. Image of calibration pattern.

(Blank elements of the array consist of zeros)
APPENDIX 3-1

In this appendix, the 2-BT power spectra of various kernels of corn, wheat, barley, oats and milo are included. The spectrum points are denoted by $P(i,j), i,j=0,1,\ldots,5$. 
<table>
<thead>
<tr>
<th></th>
<th>P(0,1)</th>
<th>P(1,1)</th>
<th>P(2,1)</th>
<th>P(3,1)</th>
<th>P(4,1)</th>
<th>P(5,1)</th>
<th>P(0,2)</th>
<th>P(1,2)</th>
<th>P(2,2)</th>
<th>P(3,2)</th>
<th>P(4,2)</th>
<th>P(5,2)</th>
<th>P(0,3)</th>
<th>P(1,3)</th>
<th>P(2,3)</th>
<th>P(3,3)</th>
<th>P(4,3)</th>
<th>P(5,3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn 9</td>
<td>0.05</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td>0.14</td>
<td>0.17</td>
<td>0.12</td>
<td>0.3</td>
<td>0.05</td>
<td>0.11</td>
<td>0.44</td>
<td>1.01</td>
<td>0.32</td>
<td>0.04</td>
<td>0.08</td>
<td>0.69</td>
<td>2.11</td>
<td>3.97</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>0</td>
<td>0.02</td>
<td>0.07</td>
<td>0.06</td>
<td>0.21</td>
<td>0.04</td>
<td>0.06</td>
<td>0.04</td>
<td>0.2</td>
<td>0.34</td>
<td>0.55</td>
<td>0.43</td>
<td>0.01</td>
<td>0.14</td>
<td>0.76</td>
<td>1.10</td>
<td>3.17</td>
</tr>
<tr>
<td>13</td>
<td>0.01</td>
<td>0</td>
<td>0.03</td>
<td>0.04</td>
<td>0.07</td>
<td>0.23</td>
<td>0.05</td>
<td>0.02</td>
<td>0.16</td>
<td>0.21</td>
<td>0.44</td>
<td>0.64</td>
<td>0.10</td>
<td>0.03</td>
<td>0.10</td>
<td>0.56</td>
<td>1.19</td>
<td>1.89</td>
</tr>
<tr>
<td>14</td>
<td>0.01</td>
<td>0</td>
<td>0.04</td>
<td>0.02</td>
<td>0.08</td>
<td>0.12</td>
<td>0.02</td>
<td>0.03</td>
<td>0.11</td>
<td>0.29</td>
<td>0.58</td>
<td>0.77</td>
<td>0.03</td>
<td>0.07</td>
<td>0.44</td>
<td>0.81</td>
<td>2.14</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>0</td>
<td>0.04</td>
<td>0.03</td>
<td>0.14</td>
<td>0.15</td>
<td>0.02</td>
<td>0</td>
<td>0.14</td>
<td>0.24</td>
<td>0.55</td>
<td>0.16</td>
<td>0.24</td>
<td>0.05</td>
<td>0.34</td>
<td>1.10</td>
<td>2.44</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>0.05</td>
<td>0.03</td>
<td>0.04</td>
<td>0.12</td>
<td>0.23</td>
<td>0.05</td>
<td>0</td>
<td>0.03</td>
<td>0.11</td>
<td>0.56</td>
<td>0.52</td>
<td>0.19</td>
<td>0.19</td>
<td>0.26</td>
<td>1.16</td>
<td>0.94</td>
<td>2.75</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>0.01</td>
<td>0.08</td>
<td>0.1</td>
<td>0.05</td>
<td>0.11</td>
<td>0.20</td>
<td>0.01</td>
<td>0.05</td>
<td>0.10</td>
<td>0.26</td>
<td>0.70</td>
<td>0.36</td>
<td>0.03</td>
<td>0.21</td>
<td>0.53</td>
<td>0.64</td>
<td>1.77</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>0.11</td>
<td>0.01</td>
<td>0</td>
<td>0.02</td>
<td>0.21</td>
<td>0.11</td>
<td>0.21</td>
<td>0.03</td>
<td>0.03</td>
<td>0.08</td>
<td>0.17</td>
<td>1.01</td>
<td>1.15</td>
<td>0.04</td>
<td>0.18</td>
<td>0.26</td>
<td>1.13</td>
<td>2.49</td>
</tr>
<tr>
<td>19</td>
<td>0.03</td>
<td>0</td>
<td>0.01</td>
<td>0.02</td>
<td>0.09</td>
<td>0.15</td>
<td>0.20</td>
<td>0.04</td>
<td>0.05</td>
<td>0.21</td>
<td>0.46</td>
<td>0.98</td>
<td>2.49</td>
<td>0.02</td>
<td>0.15</td>
<td>0.27</td>
<td>0.85</td>
<td>3.30</td>
</tr>
<tr>
<td>20</td>
<td>0.02</td>
<td>0</td>
<td>0.02</td>
<td>0.08</td>
<td>0.18</td>
<td>0.06</td>
<td>0.06</td>
<td>0.07</td>
<td>0.11</td>
<td>0.61</td>
<td>0.79</td>
<td>0.48</td>
<td>0.05</td>
<td>0.27</td>
<td>0.52</td>
<td>1.34</td>
<td>2.93</td>
<td></td>
</tr>
<tr>
<td>Variable</td>
<td>P(0,0)</td>
<td>P(1,0)</td>
<td>P(2,0)</td>
<td>P(3,0)</td>
<td>P(4,0)</td>
<td>P(5,0)</td>
<td>P(0,1)</td>
<td>P(1,1)</td>
<td>P(2,1)</td>
<td>P(3,1)</td>
<td>P(4,1)</td>
<td>P(5,1)</td>
<td>P(0,2)</td>
<td>P(1,2)</td>
<td>P(2,2)</td>
<td>P(3,2)</td>
<td>P(4,2)</td>
<td>P(5,2)</td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>Wheat 1</td>
<td>0.01</td>
<td>0.08</td>
<td>0.64</td>
<td>18.62</td>
<td>37.25</td>
<td>0.25</td>
<td>0.07</td>
<td>0.26</td>
<td>1.07</td>
<td>1.65</td>
<td>3.30</td>
<td>11.25</td>
<td>0.14</td>
<td>0.27</td>
<td>0.67</td>
<td>12.33</td>
<td>24.66</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.19</td>
<td>0.24</td>
<td>10.41</td>
<td>28.47</td>
<td>0.18</td>
<td>0.03</td>
<td>0.09</td>
<td>0.31</td>
<td>2.81</td>
<td>3.42</td>
<td>14.32</td>
<td>0.15</td>
<td>0.67</td>
<td>2.08</td>
<td>15.38</td>
<td>33.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td>0.54</td>
<td>1.66</td>
<td>17.36</td>
<td>9.32</td>
<td>0.08</td>
<td>0.12</td>
<td>0.49</td>
<td>0.98</td>
<td>10.99</td>
<td>24.96</td>
<td>0.12</td>
<td>0.31</td>
<td>1.22</td>
<td>4.15</td>
<td>30.76</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>0.56</td>
<td>3.30</td>
<td>17.15</td>
<td>2.70</td>
<td>0.05</td>
<td>0.24</td>
<td>0.43</td>
<td>6.0</td>
<td>9.52</td>
<td>16.36</td>
<td>0.12</td>
<td>0.09</td>
<td>1.59</td>
<td>10.25</td>
<td>28.81</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td>0.56</td>
<td>4.91</td>
<td>19.68</td>
<td>2.56</td>
<td>0.06</td>
<td>0.31</td>
<td>0.37</td>
<td>4.52</td>
<td>7.81</td>
<td>17.15</td>
<td>0.12</td>
<td>0.03</td>
<td>1.59</td>
<td>10.50</td>
<td>29.79</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.02</td>
<td>0.59</td>
<td>3.88</td>
<td>20.68</td>
<td>1.89</td>
<td>0.09</td>
<td>0.34</td>
<td>0.49</td>
<td>5.79</td>
<td>8.54</td>
<td>18.34</td>
<td>0.15</td>
<td>0.55</td>
<td>1.95</td>
<td>10.74</td>
<td>31.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>0.31</td>
<td>5.03</td>
<td>20.22</td>
<td>0.80</td>
<td>0.07</td>
<td>0.32</td>
<td>0.21</td>
<td>6.29</td>
<td>7.69</td>
<td>15.88</td>
<td>0.14</td>
<td>0.52</td>
<td>1.16</td>
<td>12.82</td>
<td>30.52</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.03</td>
<td>0.22</td>
<td>1.05</td>
<td>17.43</td>
<td>10.65</td>
<td>0.15</td>
<td>0.55</td>
<td>0.61</td>
<td>11.69</td>
<td>27.16</td>
<td>0.31</td>
<td>1.10</td>
<td>3.17</td>
<td>31.74</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td>0.10</td>
<td>14.87</td>
<td>31.88</td>
<td>0.22</td>
<td>0.10</td>
<td>0.32</td>
<td>0.40</td>
<td>1.77</td>
<td>2.81</td>
<td>9.54</td>
<td>0.20</td>
<td>0.70</td>
<td>3.36</td>
<td>15.26</td>
<td>29.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.86</td>
<td>3.82</td>
<td>17.96</td>
<td>35.92</td>
<td>0.61</td>
<td>0.03</td>
<td>0.09</td>
<td>0.61</td>
<td>1.34</td>
<td>2.69</td>
<td>5.92</td>
<td>0.12</td>
<td>1.28</td>
<td>1.71</td>
<td>9.03</td>
<td>18.07</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.01</td>
<td>0.39</td>
<td>3.14</td>
<td>13.26</td>
<td>6.75</td>
<td>0.07</td>
<td>0.17</td>
<td>1.01</td>
<td>1.05</td>
<td>9.64</td>
<td>16.65</td>
<td>0.11</td>
<td>0.27</td>
<td>1.65</td>
<td>5.98</td>
<td>24.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td>0.21</td>
<td>4.44</td>
<td>14.59</td>
<td>4.33</td>
<td>0.03</td>
<td>0.21</td>
<td>0.67</td>
<td>2.81</td>
<td>8.06</td>
<td>14.56</td>
<td>0.09</td>
<td>0.49</td>
<td>1.71</td>
<td>8.06</td>
<td>24.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.06</td>
<td>0.02</td>
<td>6.70</td>
<td>16.33</td>
<td>1.65</td>
<td>0.06</td>
<td>0.12</td>
<td>0.49</td>
<td>3.78</td>
<td>6.10</td>
<td>11.54</td>
<td>0.12</td>
<td>0.31</td>
<td>0.98</td>
<td>11.47</td>
<td>24.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(P(0,0))</td>
<td>(P(1,0))</td>
<td>(P(2,0))</td>
<td>(P(3,0))</td>
<td>(P(4,0))</td>
<td>(P(5,0))</td>
<td>(P(0,1))</td>
<td>(P(1,1))</td>
<td>(P(2,1))</td>
<td>(P(3,1))</td>
<td>(P(4,1))</td>
<td>(P(5,1))</td>
<td>(P(0,2))</td>
<td>(P(1,2))</td>
<td>(P(2,2))</td>
<td>(P(3,2))</td>
<td>(P(4,2))</td>
<td>(P(5,2))</td>
</tr>
<tr>
<td>-------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
</tr>
<tr>
<td>wheat 6</td>
<td><strong>9.35</strong></td>
<td>0.02</td>
<td>0.30</td>
<td>1.74</td>
<td>11.41</td>
<td>22.81</td>
<td>0.11</td>
<td>0.08</td>
<td>0.26</td>
<td>0.46</td>
<td>0.92</td>
<td>1.83</td>
<td>8.87</td>
<td>0.14</td>
<td>0.70</td>
<td>1.65</td>
<td>11.35</td>
<td>22.71</td>
</tr>
<tr>
<td></td>
<td><strong>10.31</strong></td>
<td>0</td>
<td>0.02</td>
<td>0.02</td>
<td>8.32</td>
<td>18.68</td>
<td>0.87</td>
<td>0.02</td>
<td>0.09</td>
<td>0.49</td>
<td>3.17</td>
<td>4.64</td>
<td>11.51</td>
<td>0.03</td>
<td>0.18</td>
<td>0.98</td>
<td>12.70</td>
<td>25.39</td>
</tr>
<tr>
<td>8</td>
<td><strong>17.38</strong></td>
<td>0</td>
<td>0.35</td>
<td>2.06</td>
<td>19.78</td>
<td>39.57</td>
<td>0.3</td>
<td>0.05</td>
<td>0.35</td>
<td>1.68</td>
<td>2.38</td>
<td>4.76</td>
<td>8.47</td>
<td>0.11</td>
<td>0.76</td>
<td>0.79</td>
<td>10.13</td>
<td>20.26</td>
</tr>
<tr>
<td>*19.50</td>
<td>0</td>
<td>0</td>
<td>0.22</td>
<td>0.98</td>
<td>20.71</td>
<td>10.19</td>
<td>0.08</td>
<td>0.14</td>
<td>0.27</td>
<td>0.67</td>
<td>11.35</td>
<td>30.26</td>
<td>0.11</td>
<td>0.21</td>
<td>1.04</td>
<td>3.3</td>
<td>34.91</td>
<td></td>
</tr>
<tr>
<td>*14.43</td>
<td>0.01</td>
<td>0.07</td>
<td>0.24</td>
<td>1.50</td>
<td>16.25</td>
<td>7.68</td>
<td>0.05</td>
<td>0.11</td>
<td>0.75</td>
<td>2.99</td>
<td>11.60</td>
<td>22.54</td>
<td>0.08</td>
<td>0.27</td>
<td>1.65</td>
<td>5.49</td>
<td>30.03</td>
<td></td>
</tr>
<tr>
<td>18.96</td>
<td>0</td>
<td>0.01</td>
<td>0.74</td>
<td>6.58</td>
<td>26.29</td>
<td>0.25</td>
<td>0.10</td>
<td>0.41</td>
<td>0.52</td>
<td>4.46</td>
<td>5.74</td>
<td>16.62</td>
<td>0.14</td>
<td>0.64</td>
<td>1.53</td>
<td>15.50</td>
<td>34.42</td>
<td></td>
</tr>
<tr>
<td>19.50</td>
<td>0.02</td>
<td>0.02</td>
<td>0.54</td>
<td>5.81</td>
<td>25.89</td>
<td>0.36</td>
<td>0.05</td>
<td>0.23</td>
<td>0.64</td>
<td>5.43</td>
<td>6.71</td>
<td>17.96</td>
<td>0.14</td>
<td>0.64</td>
<td>1.10</td>
<td>14.77</td>
<td>34.91</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>8.43</td>
<td>0</td>
<td>0.10</td>
<td>0.16</td>
<td>5.51</td>
<td>14.19</td>
<td>1.5</td>
<td>0.08</td>
<td>0.15</td>
<td>0.79</td>
<td>4.76</td>
<td>6.84</td>
<td>9.64</td>
<td>0.12</td>
<td>0.37</td>
<td>1.34</td>
<td>11.47</td>
<td>22.95</td>
</tr>
<tr>
<td>7.9</td>
<td>0</td>
<td>0</td>
<td>0.23</td>
<td>2.63</td>
<td>10.76</td>
<td>6.8</td>
<td>0.02</td>
<td>0.11</td>
<td>0.58</td>
<td>2.14</td>
<td>9.64</td>
<td>14.88</td>
<td>0.05</td>
<td>0.27</td>
<td>1.28</td>
<td>5.74</td>
<td>22.22</td>
<td></td>
</tr>
<tr>
<td><strong>8.25</strong></td>
<td>0.02</td>
<td>0.25</td>
<td>2.86</td>
<td>11.39</td>
<td>22.78</td>
<td>0.02</td>
<td>0.02</td>
<td>0.47</td>
<td>0.40</td>
<td>0.92</td>
<td>1.83</td>
<td>6.94</td>
<td>0.05</td>
<td>0.89</td>
<td>2.01</td>
<td>9.89</td>
<td>19.78</td>
<td></td>
</tr>
<tr>
<td><strong>8.43</strong></td>
<td>0</td>
<td>0.05</td>
<td>0.08</td>
<td>6.81</td>
<td>15.38</td>
<td>0.93</td>
<td>0.05</td>
<td>0.31</td>
<td>0.79</td>
<td>3.74</td>
<td>5.86</td>
<td>9.55</td>
<td>0.09</td>
<td>0.49</td>
<td>1.34</td>
<td>11.47</td>
<td>22.95</td>
<td></td>
</tr>
<tr>
<td>8.25</td>
<td>0</td>
<td>0</td>
<td>0.16</td>
<td>2.53</td>
<td>10.94</td>
<td>5.42</td>
<td>0.02</td>
<td>0.26</td>
<td>0.56</td>
<td>3.6</td>
<td>9.89</td>
<td>13.87</td>
<td>0.08</td>
<td>0.40</td>
<td>1.40</td>
<td>6.96</td>
<td>22.71</td>
<td></td>
</tr>
<tr>
<td>wheat 1</td>
<td>P(0,1)</td>
<td>P(1,1)</td>
<td>P(2,1)</td>
<td>P(3,1)</td>
<td>P(4,1)</td>
<td>P(5,1)</td>
<td>P(0,2)</td>
<td>P(1,2)</td>
<td>P(2,2)</td>
<td>P(3,2)</td>
<td>P(4,2)</td>
<td>P(5,2)</td>
<td>P(0,3)</td>
<td>P(1,3)</td>
<td>P(2,3)</td>
<td>P(3,3)</td>
<td>P(4,3)</td>
<td>P(5,3)</td>
</tr>
<tr>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>**</td>
<td>0</td>
<td>0.01</td>
<td>0.02</td>
<td>0.01</td>
<td>0.04</td>
<td>0.08</td>
<td>0</td>
<td>0</td>
<td>0.03</td>
<td>0.07</td>
<td>0.11</td>
<td>0.21</td>
<td>0.17</td>
<td>0.05</td>
<td>0.13</td>
<td>0.35</td>
<td>0.70</td>
<td>1.40</td>
</tr>
<tr>
<td>**</td>
<td>0</td>
<td>0</td>
<td>0.02</td>
<td>0.05</td>
<td>0.09</td>
<td>0.04</td>
<td>0</td>
<td>0</td>
<td>0.01</td>
<td>0.05</td>
<td>0.15</td>
<td>0.24</td>
<td>0.05</td>
<td>0.05</td>
<td>0.14</td>
<td>0.24</td>
<td>0.49</td>
<td>0.98</td>
</tr>
<tr>
<td>*</td>
<td>0</td>
<td>0</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
<td>0.05</td>
<td>0.03</td>
<td>0.34</td>
<td>0.67</td>
<td>0.25</td>
<td>0.04</td>
<td>0.11</td>
<td>0.15</td>
<td>1.16</td>
<td>0.71</td>
</tr>
<tr>
<td>*</td>
<td>0</td>
<td>0</td>
<td>0.04</td>
<td>0.02</td>
<td>0.06</td>
<td>0.12</td>
<td>0.04</td>
<td>0.01</td>
<td>0.05</td>
<td>0.15</td>
<td>0.18</td>
<td>0.43</td>
<td>0.34</td>
<td>0.06</td>
<td>0.15</td>
<td>0.43</td>
<td>0.61</td>
<td>0.59</td>
</tr>
<tr>
<td>*</td>
<td>0</td>
<td>0.02</td>
<td>0</td>
<td>0.01</td>
<td>0.06</td>
<td>0.08</td>
<td>0.1</td>
<td>0.02</td>
<td>0.02</td>
<td>0.08</td>
<td>0.15</td>
<td>0.37</td>
<td>0.29</td>
<td>0.03</td>
<td>0.08</td>
<td>0.58</td>
<td>0.85</td>
<td>1.83</td>
</tr>
<tr>
<td>5</td>
<td>*0.02</td>
<td>0</td>
<td>0</td>
<td>0.04</td>
<td>0.03</td>
<td>0.09</td>
<td>0.05</td>
<td>0</td>
<td>0.02</td>
<td>0.05</td>
<td>0.18</td>
<td>0.31</td>
<td>0.27</td>
<td>0.05</td>
<td>0.17</td>
<td>0.79</td>
<td>0.92</td>
<td>2.20</td>
</tr>
<tr>
<td>*0.0</td>
<td>0</td>
<td>0.01</td>
<td>0.02</td>
<td>0.05</td>
<td>0.05</td>
<td>0.14</td>
<td>0.03</td>
<td>0.05</td>
<td>0.03</td>
<td>0.08</td>
<td>0.20</td>
<td>0.40</td>
<td>0.15</td>
<td>0.01</td>
<td>0.13</td>
<td>0.49</td>
<td>1.25</td>
<td>2.01</td>
</tr>
<tr>
<td>*</td>
<td>0</td>
<td>0</td>
<td>0.02</td>
<td>0.05</td>
<td>0.02</td>
<td>0.09</td>
<td>0.22</td>
<td>0</td>
<td>0.05</td>
<td>0.06</td>
<td>0.34</td>
<td>0.67</td>
<td>0.17</td>
<td>0</td>
<td>0.05</td>
<td>0.21</td>
<td>1.16</td>
<td>1.59</td>
</tr>
<tr>
<td>**</td>
<td>0</td>
<td>0</td>
<td>0.01</td>
<td>0.03</td>
<td>0.04</td>
<td>0.08</td>
<td>0.02</td>
<td>0.05</td>
<td>0.10</td>
<td>0.04</td>
<td>0.14</td>
<td>0.34</td>
<td>0.12</td>
<td>0.03</td>
<td>0.14</td>
<td>0.11</td>
<td>0.40</td>
<td>0.79</td>
</tr>
<tr>
<td>**</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
<td>0.01</td>
<td>0</td>
<td>0.04</td>
<td>0.05</td>
<td>0.09</td>
<td>0.16</td>
<td>0.14</td>
<td>0.03</td>
<td>0.05</td>
<td>0.15</td>
<td>0.37</td>
<td>0.73</td>
</tr>
<tr>
<td>6</td>
<td>*</td>
<td>0</td>
<td>0</td>
<td>0.02</td>
<td>0.05</td>
<td>0.08</td>
<td>0.12</td>
<td>0.02</td>
<td>0.04</td>
<td>0.07</td>
<td>0.26</td>
<td>0.52</td>
<td>0.04</td>
<td>0.01</td>
<td>0.05</td>
<td>0.07</td>
<td>0.26</td>
<td>0.52</td>
</tr>
<tr>
<td>*</td>
<td>0</td>
<td>0</td>
<td>0.02</td>
<td>0.02</td>
<td>0.08</td>
<td>0.12</td>
<td>0.02</td>
<td>0</td>
<td>0.05</td>
<td>0.23</td>
<td>0.12</td>
<td>0.43</td>
<td>0.28</td>
<td>0.05</td>
<td>0.18</td>
<td>0.58</td>
<td>0.61</td>
<td>0.71</td>
</tr>
<tr>
<td>**</td>
<td>0</td>
<td>0.03</td>
<td>0</td>
<td>0.01</td>
<td>0.05</td>
<td>0.09</td>
<td>0.02</td>
<td>0.01</td>
<td>0.04</td>
<td>0.03</td>
<td>0.21</td>
<td>0.31</td>
<td>0.34</td>
<td>0.02</td>
<td>0.08</td>
<td>0.43</td>
<td>0.73</td>
<td>0.59</td>
</tr>
<tr>
<td>( (s',r)d )</td>
<td>( (r',s) )</td>
<td>( (s',r) )</td>
<td>( (r',s) )</td>
<td>( (s',r) )</td>
<td>( (r',s) )</td>
<td>( (s',r) )</td>
<td>( (r',s) )</td>
<td>( (s',r) )</td>
<td>( (r',s) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.09</td>
<td>0.04</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.04</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.09</td>
<td>0.04</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.04</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.09</td>
<td>0.04</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.04</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.09</td>
<td>0.04</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.04</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.09</td>
<td>0.04</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.04</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.09</td>
<td>0.04</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.04</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.09</td>
<td>0.04</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.04</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.09</td>
<td>0.04</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.04</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.09</td>
<td>0.04</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.04</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.09</td>
<td>0.04</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.04</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* wheat \( b \) = 8, \( \sigma \) = 10, \( \beta \) = 0.02

** \( \gamma \) = 0.01

*** \( \delta \) = 0.01
<p>|       | P(0,0) | P(1,0) | P(2,0) | P(3,0) | P(4,0) | P(5,0) | P(0,1) | P(1,1) | P(2,1) | P(3,1) | P(4,1) | P(5,1) | P(0,2) | P(1,2) | P(2,2) | P(3,2) | P(4,2) | P(5,2) | P(0,3) | P(1,3) | P(2,3) | P(3,3) | P(4,3) | P(5,3) | P(0,4) | P(1,4) | P(2,4) | P(3,4) | P(4,4) | P(5,4) | P(0,5) | P(1,5) | P(2,5) | P(3,5) | P(4,5) | P(5,5) |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| oats1 |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |
| *30.56| 0.01   | 0.09   | 0.62   | 1.06   | 21.65  | 13.70  | 0.02   | 0.2    | 0.7    | 2.75   | 8.67   | 44.91  | 0.05   | 0.4    | 1.89   | 5.74   | 34.42  |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |
| *35.2 | 0      | 0.02   | 0.08   | 2.72   | 29.2   | 10.21  | 0.02   | 0.37   | 0.88   | 1.75   | 7.27   | 27.30  | 0.05   | 0.38   | 2.72   | 8.71   | 27.20  |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |
| *29.3 | 0      | 0.02   | 0.57   | 12.32  | 39.2   | 0.55   | 0.2    | 0.37   | 2.19   | 3.27   | 5.32   | 20.21  | 0.15   | 0.21   | 3.14   | 8.73   | 35.24  |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |
| *30.56| 0      | 0.02   | 0.27   | 0.87   | 8.29   | 27.32  | 0.01   | 0.21   | 0.58   | 2.31   | 8.82   | 58.33  | 0.82   | 0.20   | 0.87   | 3.27   | 30.31  |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |
| *28.21| 0.03   | 0.55   | 2.56   | 31.36  | 62.71  | 0.72   | 0.05   | 0.40   | 1.04   | 2.20   | 4.39   | 5.22   | 0.34   | 0.15   | 2.08   | 7.81   | 75.63  |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |
| 2     | *20.33 | 0.06   | 0.23   | 2.45   | 23.07  | 46.14  | 2.32   | 0.08   | 0.21   | 0.92   | 2.08   | 4.15   | 6.10   | 0.18   | 1.04   | 2.44   | 9.77   | 19.53  |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |
| *29.40| 0.02   | 0.32   | 3.12   | 29.70  | 58.32  | 0.57   | 0.05   | 0.32   | 0.87   | 2.13   | 7.15   | 9.27   | 0.20   | 1.22   | 3.40   | 5.28   | 25.37  |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |
| *37.13| 0.00   | 0.82   | 3.21   | 3.37   | 28.2   | 0.59   | 0.01   | 0.20   | 0.83   | 3.12   | 9.23   | 7.37   | 0.20   | 1.02   | 1.28   | 8.27   | 23.57  |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |
| *26.83| 0      | 0.02   | 0.07   | 0.88   | 35.3   | 13.23  | 0.02   | 0.33   | 0.63   | 2.20   | 10.22  | 34.73  | 0.02   | 0.53   | 2.28   | 9.72   | 39.83  |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |
| *27.89| 0      | 0.05   | 0.09   | 0.47   | 24.70  | 11.30  | 0.01   | 0.23   | 0.52   | 1.69   | 9.64   | 40.02  | 0.05   | 0.52   | 1.28   | 4.27   | 37.35  |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |</p>
<table>
<thead>
<tr>
<th></th>
<th>P(0,1)</th>
<th>P(1,1)</th>
<th>P(2,1)</th>
<th>P(3,1)</th>
<th>P(4,1)</th>
<th>P(5,1)</th>
<th>P(0,2)</th>
<th>P(1,2)</th>
<th>P(2,2)</th>
<th>P(3,2)</th>
<th>P(4,2)</th>
<th>P(5,2)</th>
<th>P(0,3)</th>
<th>P(1,3)</th>
<th>P(2,3)</th>
<th>P(3,3)</th>
<th>P(4,3)</th>
<th>P(5,3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>oats 1</td>
<td>* 0</td>
<td>0</td>
<td>0.02</td>
<td>0.03</td>
<td>0.05</td>
<td>0.05</td>
<td>0.02</td>
<td>0.01</td>
<td>0.03</td>
<td>0.13</td>
<td>0.14</td>
<td>0.82</td>
<td>0.54</td>
<td>0</td>
<td>0.05</td>
<td>0.41</td>
<td>1.74</td>
<td>3.23</td>
</tr>
<tr>
<td></td>
<td>* 0</td>
<td>0</td>
<td>0</td>
<td>0.03</td>
<td>0.02</td>
<td>0.05</td>
<td>0.01</td>
<td>0</td>
<td>0.02</td>
<td>0.27</td>
<td>0.29</td>
<td>0.52</td>
<td>0.57</td>
<td>0.21</td>
<td>2.1</td>
<td>0.27</td>
<td>2.22</td>
<td>4.72</td>
</tr>
<tr>
<td></td>
<td>* 0</td>
<td>0.01</td>
<td>0.01</td>
<td>0.03</td>
<td>0.04</td>
<td>0.09</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
<td>0.03</td>
<td>0.52</td>
<td>0.87</td>
<td>0.08</td>
<td>0.12</td>
<td>0.21</td>
<td>0.37</td>
<td>0.42</td>
<td>2.10</td>
</tr>
<tr>
<td></td>
<td>* 0</td>
<td>0</td>
<td>0</td>
<td>0.01</td>
<td>0.03</td>
<td>0.04</td>
<td>0.58</td>
<td>0</td>
<td>0.02</td>
<td>0.06</td>
<td>0.38</td>
<td>0.93</td>
<td>3.12</td>
<td>0.01</td>
<td>0.05</td>
<td>0.21</td>
<td>3.12</td>
<td>2.13</td>
</tr>
<tr>
<td></td>
<td>* 0</td>
<td>0</td>
<td>0</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
<td>0.02</td>
<td>0</td>
<td>0.04</td>
<td>0.03</td>
<td>0.12</td>
<td>0.24</td>
<td>0.31</td>
<td>0.01</td>
<td>0.05</td>
<td>0.12</td>
<td>0.49</td>
<td>0.98</td>
</tr>
<tr>
<td>2</td>
<td>* 0</td>
<td>0</td>
<td>0</td>
<td>0.02</td>
<td>0.03</td>
<td>0.06</td>
<td>0.01</td>
<td>0</td>
<td>0.02</td>
<td>0.12</td>
<td>0.15</td>
<td>0.31</td>
<td>0.03</td>
<td>0.05</td>
<td>0.08</td>
<td>0.46</td>
<td>0.55</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>* 0</td>
<td>0</td>
<td>0</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td>0.05</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td>0.24</td>
<td>0.28</td>
<td>0.59</td>
<td>0.03</td>
<td>0.05</td>
<td>0.08</td>
<td>0.27</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>* 0</td>
<td>0</td>
<td>0</td>
<td>0.02</td>
<td>0.07</td>
<td>0.09</td>
<td>0.01</td>
<td>0</td>
<td>0.02</td>
<td>0.21</td>
<td>0.28</td>
<td>0.52</td>
<td>0.03</td>
<td>0.03</td>
<td>0.08</td>
<td>0.37</td>
<td>0.47</td>
<td>2.10</td>
</tr>
<tr>
<td></td>
<td>* 0</td>
<td>0</td>
<td>0.02</td>
<td>0.05</td>
<td>0.02</td>
<td>0.09</td>
<td>0.01</td>
<td>0.01</td>
<td>0.03</td>
<td>0.05</td>
<td>0.07</td>
<td>0.18</td>
<td>0.82</td>
<td>0.01</td>
<td>0.27</td>
<td>0.58</td>
<td>2.32</td>
<td>10.83</td>
</tr>
<tr>
<td></td>
<td>* 0</td>
<td>0.01</td>
<td>0.05</td>
<td>0.01</td>
<td>0.04</td>
<td>0.11</td>
<td>0.02</td>
<td>0.01</td>
<td>0.03</td>
<td>0.19</td>
<td>0.14</td>
<td>0.27</td>
<td>0.72</td>
<td>0.02</td>
<td>0.16</td>
<td>0.47</td>
<td>1.74</td>
<td>2.62</td>
</tr>
<tr>
<td></td>
<td>$P(0,1)$</td>
<td>$P(1,1)$</td>
<td>$P(2,1)$</td>
<td>$P(3,1)$</td>
<td>$P(4,1)$</td>
<td>$P(5,1)$</td>
<td>$P(0,2)$</td>
<td>$P(1,2)$</td>
<td>$P(2,2)$</td>
<td>$P(3,2)$</td>
<td>$P(4,2)$</td>
<td>$P(5,2)$</td>
<td>$P(0,3)$</td>
<td>$P(1,3)$</td>
<td>$P(2,3)$</td>
<td>$P(3,3)$</td>
<td>$P(4,3)$</td>
<td>$P(5,3)$</td>
</tr>
<tr>
<td>-----</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
</tr>
<tr>
<td><strong>barley 1</strong></td>
<td>*</td>
<td>0</td>
<td>0.02</td>
<td>0.04</td>
<td>0.09</td>
<td>0.14</td>
<td>0</td>
<td>0.01</td>
<td>0.03</td>
<td>0.18</td>
<td>0.37</td>
<td>0.40</td>
<td>0.02</td>
<td>0.06</td>
<td>0.12</td>
<td>0.61</td>
<td>1.22</td>
<td></td>
</tr>
<tr>
<td>*</td>
<td>0</td>
<td>0</td>
<td>0.04</td>
<td>0.03</td>
<td>0.62</td>
<td>0.38</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td>0.27</td>
<td>0.93</td>
<td>0.48</td>
<td>0.02</td>
<td>0.05</td>
<td>0.18</td>
<td>0.68</td>
<td>1.77</td>
<td></td>
</tr>
<tr>
<td>*0.16</td>
<td>0</td>
<td>0.01</td>
<td>0</td>
<td>0.02</td>
<td>0.38</td>
<td>0.39</td>
<td>0.02</td>
<td>0.01</td>
<td>0.05</td>
<td>0.11</td>
<td>0.52</td>
<td>0.57</td>
<td>0.03</td>
<td>0.02</td>
<td>0.08</td>
<td>0.21</td>
<td>1.89</td>
<td></td>
</tr>
<tr>
<td>*0.02</td>
<td>0</td>
<td>0.02</td>
<td>0.02</td>
<td>0.19</td>
<td>0.19</td>
<td>0.03</td>
<td>0.04</td>
<td>0.07</td>
<td>0.2</td>
<td>0.21</td>
<td>0.43</td>
<td>0.27</td>
<td>0.11</td>
<td>0.08</td>
<td>1.50</td>
<td>0.34</td>
<td>3.30</td>
<td></td>
</tr>
<tr>
<td>*0.02</td>
<td>0</td>
<td>0.03</td>
<td>0.02</td>
<td>0.05</td>
<td>0.12</td>
<td>0</td>
<td>0.06</td>
<td>0.02</td>
<td>0.06</td>
<td>0.27</td>
<td>0.43</td>
<td>0.08</td>
<td>0.01</td>
<td>0.24</td>
<td>0.15</td>
<td>0.85</td>
<td>1.34</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>*</td>
<td>0</td>
<td>0.02</td>
<td>0.01</td>
<td>0.04</td>
<td>0.08</td>
<td>0.6</td>
<td>0.01</td>
<td>0.08</td>
<td>0.05</td>
<td>0.14</td>
<td>1.50</td>
<td>0.79</td>
<td>0.01</td>
<td>0.04</td>
<td>0.17</td>
<td>0.40</td>
<td>3.85</td>
</tr>
<tr>
<td>*</td>
<td>0</td>
<td>0</td>
<td>0.03</td>
<td>0.06</td>
<td>0.09</td>
<td>0.01</td>
<td>0.04</td>
<td>0.11</td>
<td>0.09</td>
<td>0.24</td>
<td>0.49</td>
<td>0.15</td>
<td>0.01</td>
<td>0.08</td>
<td>0.70</td>
<td>0.61</td>
<td>1.59</td>
<td></td>
</tr>
<tr>
<td>*</td>
<td>0</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td>0.09</td>
<td>0.7</td>
<td>0.01</td>
<td>0.01</td>
<td>0.14</td>
<td>0.20</td>
<td>0.70</td>
<td>0.86</td>
<td>0</td>
<td>0.13</td>
<td>0.44</td>
<td>0.52</td>
<td>2.50</td>
</tr>
<tr>
<td>*0.03</td>
<td>0.02</td>
<td>0.01</td>
<td>0.07</td>
<td>0.05</td>
<td>0.18</td>
<td>0.65</td>
<td>0.01</td>
<td>0.03</td>
<td>0.05</td>
<td>0.12</td>
<td>0.61</td>
<td>2.56</td>
<td>0.02</td>
<td>0.05</td>
<td>0.12</td>
<td>0.31</td>
<td>2.56</td>
<td></td>
</tr>
<tr>
<td>*0.01</td>
<td>0.01</td>
<td>0.03</td>
<td>0.02</td>
<td>0.07</td>
<td>0.08</td>
<td>0.01</td>
<td>0.03</td>
<td>0.32</td>
<td>0.08</td>
<td>0.50</td>
<td>0.95</td>
<td>0.05</td>
<td>0.03</td>
<td>0.11</td>
<td>1.24</td>
<td>0.82</td>
<td>1.77</td>
<td></td>
</tr>
<tr>
<td><strong>milo 1</strong></td>
<td>*</td>
<td>0</td>
<td>0</td>
<td>0.01</td>
<td>0.05</td>
<td>0.06</td>
<td>0</td>
<td>0.02</td>
<td>0.04</td>
<td>0.06</td>
<td>0.18</td>
<td>0.25</td>
<td>0.04</td>
<td>0.20</td>
<td>0.37</td>
<td>0.73</td>
<td>1.59</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>*</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
<td>0.05</td>
<td>0</td>
<td>0.01</td>
<td>0.07</td>
<td>0.14</td>
<td>0.17</td>
<td>0.40</td>
<td>0.36</td>
<td>0.03</td>
<td>0.11</td>
<td>0.14</td>
<td>0.64</td>
<td>1.28</td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX 3-2

A listing of the computer program used to compute the 2-BT power spectra listed in Appendix 3-1 is presented in this appendix. For the purposes of illustration, the computation of the 2-BT power spectrum for an oats kernel is included in the listing which follows.
ILLEGIBLE DOCUMENT

THE FOLLOWING DOCUMENT(S) IS OF POOR LEGIBILITY IN THE ORIGINAL

THIS IS THE BEST COPY AVAILABLE
**WARNING**  F90 IS UNREFERENCED

**WARNING**  F90 IS UNREFERENCED
57  31  I1=I1+F7/2
58  IF (I1.EQ.E-0.0) GO TO 32
59  I1=I1+1
60  GO TO 11
61  32  CONTINUE
62  GO TO 11,17
63  IF (N.EQ.1) JUMP=JUMP+1
64  I9=I1+1 NUM
65  GO 46  MP1=1 MNUM
66  MNUM4=I9+1/2
67  GO MP2=1,NUM
68  Y(IK1)+X(IK1)+X(MNUM)+19
69  I12=I16+MP2
70  Y(IK1)+X(IK1)+X(MNUM+21)
71  Y(IK1)+X(IK1)+X(MNUM+21)
72  CONTINUE
73  GO TO 1-1,NUM
74  70 X(I1)=Y(I1)
75  40 CONTINUE
76  GO 120 I1=NUN
77  120 Y(I1)=Y(I1)/NUN
80  RETURN
82  END

ENTRY
2-hT power spectrum

CORE USAGE

OBJECT CODE = 3712 BYTES, ARRAY AREA = 4416 BYTES, TOTAL AREA AVAILABLE = 49280 BYTES

DIAGNOSTICS

NUMBER OF ERRORS = G, NUMBER OF WARNINGS = 1, NUMBER OF EXTENSIONS = 0

COMPILE TIME = 1.99 SEC, EXECUTION TIME = 19.69 SEC, WATFIV - VERSION 1 LEVEL 3 MARCH 1971

DATE = 72/037
CHAPTER IV

GRAIN CLASSIFICATION RESULTS

4.1. Elements of Pattern Classification

Some basic concepts of pattern classification are best introduced by referring to Figure 4-1. Let \( X_{ij} \) represent the \( j \)-th pattern belonging to class or category \( i, i=1,2,\ldots,k \), where \( k \) is the total number of classes. Generally \( X_{ij} \) is in the form of a vector - however it can also be in the form of a multi-dimensional array. The set of patterns \( X_{ij}, j=1,2,\ldots,N_i \) is denoted by \( \{X_{ij}\} \) and consists of a total of \( N \) samples, \( N_i \) of which belong to class \( i, i=1,2,\ldots,k \). This set of patterns \( \{X_{ij}\} \) is called the training set since they are used to "train" or "teach" the classifier to classify a pattern \( X \) (whose classification is unknown) as belonging to a particular class. That is, once a classifier has been trained, it is capable of classifying incoming patterns on its own. The overall performance of a classifier is generally measured in terms of the number of errors (i.e., misclassifications) it makes. Many different types of training algorithms are available [8]. The choice of a particular algorithm is generally dictated by the problem which has to be solved.
X: A pattern whose classification is not known.

Figure 4-1: A pattern classification scheme
4.2. The Training Algorithm

As mentioned earlier, there are various types of training algorithms [8]. The specific algorithm used in the present grain classification study uses the "least squares mapping" approach. The basic idea in this approach is to derive a linear transformation \( A \) which maps in the least-squares sense the training samples belonging to class \( i \), into the unit vector \( V_i \). We recall that \( V_i \) is a vector, whose elements are all zero, except for the \( i \)-th element which is unity. Once the classifier is trained, the transformation \( A \) is obtained in the form of a matrix. To classify an incoming pattern \( X \) whose classification is unknown, the following steps are used:

1. Compute \( Z = AX \). Then \( Z \) is the mapping of \( X \) into the unit vector space.

2. Find the distance of \( Z \) from the unit vectors \( V_i, i=1,2,\ldots,K \).
   If \( Z \) falls closest to \( V_{i0} \), then \( X \) is decided that \( X \) belongs to class \( i_0 \).

For a detailed discussion of the above training algorithm, the reader should consult references [4] and [5]. A listing of the computer program associated with this algorithm is included in Appendix 4-1.

4.3. Classification of Corn, Wheat, Barley, Oats and Milo.

The 2-NT power spectrum points tabulated in Appendix 3-1 are used to obtain the training set. The following 10 of the 36 power spectrum points are used.

\[
P(0,0), P(4,0), P(5,0), P(5,4), P(0,5) \\
P(4,5), P(5,5), P(3,3), P(4,3), P(5,3)
\]

Thus each \( X_{ij} \) in Figure 4-1 is a \((10 \times 1)\) vector in this application.
Examination of the table of 2-BT power spectrum points in Appendix 3-1 reveals that \( P(0,0) \) for each corn sample is much greater than that for any sample of wheat, barley, oats or milo. Thus the classification of corn becomes quite simple as shown in Figure 4-2. If a \( P(0,0) \) exceeds a convenient threshold such as 100, it is decided that the corresponding kernel is corn. On the other hand, if \( P(0,0) \) is less than 100, it is then decided whether the corresponding kernel is wheat, barley, oats or milo, (see Figure 4-2). The decision matrix referred to in Figure 4-2 is directly related to the transformation \( A \) referred to in Section 4.2. It is obtained using the training algorithm as evident from Appendix 4-1 (see page 49).

4.4. Classification Results

During the training process it was detected that the 10 components used for training purposes (see Section 4-3) formed a bi-modal distribution in the case of wheat. Thus, two separate classes of wheat were considered while training to obtain the decision matrix in Figure 4-2 (See page 49 of Appendix 4-1.) That is, class 1 and class 5 both represented wheat. The corresponding training samples used as belonging to classes 1 and 5 are denoted by the asterisk and double asterisk respectively.

After the decision matrix was obtained, the training samples were classified using the trained classifier as shown in Figure 4-2. The overall results may be summarized by means of a confusion matrix (see Appendix 4-1, page 49).
Figure 4-2 Summary of the classification of corn, wheat, barley, oats and milo.

* See Appendix 4-1, page 49
\[
\begin{bmatrix}
10 & 0 & 0 & 0 & 0 \\
1 & 7 & 1 & 0 & 1 \\
0 & 0 & 10 & 0 & 0 \\
0 & 0 & 0 & 5 & 0 \\
0 & 0 & 0 & 2 & 8 \\
\end{bmatrix}
\]

(4-1)

where, class 1 through class 5 are wheat, oats, barley, milo and wheat respectively.

With respect to F in (4-1) we make the following observations:

1. Of the 20 samples of wheat, 2 have been erroneously classified as being milo.
2. Of the 10 samples of oats a total of three are erroneously classified as being wheat, barley and milo.
3. All samples of barley and milo (10 and 5 respectively) are classified correctly.

The above observations lead to the following % correct classification scores:

<table>
<thead>
<tr>
<th>Type of grain</th>
<th>% correct classification score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat</td>
<td>90%</td>
</tr>
<tr>
<td>Oats</td>
<td>70%</td>
</tr>
<tr>
<td>Barley</td>
<td>100%</td>
</tr>
<tr>
<td>Milo</td>
<td>100%</td>
</tr>
</tbody>
</table>

We recall that corn can also be attributed with a correct recognition score of 100% since for each sample of corn, F(00) > 100 (see Appendix 4-1).
APPENDIX 4-1

This appendix provides a listing of a computer program for a training algorithm which uses the least-squares mapping principle. Details pertaining to the algorithm are available in references [4] and [5].
DATA NR., NI/K5, N/6
DIMENSION ISAMP(10), PROB(N10, 1, 1, 10). Y(10,10), P(120,10), Y(10,10)

WRITE(4, 2) NCMP, NCMP
2 FORMATT(IX, 'THIS PROBLEM HAS *15* CLASSES & EACH CLASS IS*15*,
1* DIMENSIONAL///)

WRITE(4, 3) NCMP
3 FORMATT(IX, 'CLASS', 15*, ' HAS', 15*, ' SAMPLES & ITS
PROBABILITY
115*, 'a10', 4)

DO 10 IC = 1, N
10 READ (XJ, 201)(I, J, I=1, LV)

WRITE(4, 20)

WRITE(4, 35)

FORMATT(IX, 'Y MATRIX///)

WRITE(4, 32)

READ(AK, 32)(K, I=1, L)

X(1, J) = 0

WRITE(4, 45)

FORMATT(IX, 'INCREMENT PATTERNS OF CLASS', 15///)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)

WRITE(4, 40)

WRITE(4, 41)
DO 55 J=1,NI
45 Y(I,I2)=Y(I,I2)*X(I,J)*X(I,J)
44 Y(I,I2)=Y(I,I2)/XI(I,12)/FI
IF(*.E1,11)XX(I/I2)=0.0
11 XX(I/I2)=XX(I/I2)*Y(I/I2)
DO 65 I=1,NI
DO 65 J=1,NJ
65 XI(I,J)=XI(I,J)*X(I,J)
60 DO 70 T=1,LV
DO 70 J=1,NJ
A(I,J)=A(I,J)*XX(I,J)/FI
70 VX(I,J)=VX(I,J)+A(I,J)
90 CONTINUE
CALL INVXX(XX,LV)
30 DO 120 I=1,LV
DO 120 J=1,NJ
A(I,J)=A(I,J)+VX(I,J)*XX(I,J)
30 WRITE(*,120)
120 WRITE(*,125)
125 FORMAT(1X,"TRANSFORMATION MATRIX A***")
126 DO 130 I=1,LV
127 WRITE(*,130)(A(I,J),J=1,NJ)
130 WRITE(*,135)(A(I,J),J=1,LV)
135 DO 140 J=1,NJ
136 WRITE(*,140)(A(I,J),I=1,LV)
140 FORMAT(35X,"DECISION" MATRIX***")
146 DO 150 I=1,N
147 WRITE(*,155)(E(I,J),J=1,N)
150 WRITE(*,150)(E(I,J),I=1,N)
155 IF(1,J)=0
156 DO 160 K=1,N
160 WRITE(*,160)(E(I,J),I=1,N)
165 WRITE(*,165)(E(I,J),J=1,N)
160 DO 165 J=1,N
167 WRITE(*,167)
165 LS=1
167 CONTINUE
165 C=0
170 GO TO 165
170 END
180 CONTINUE
107  180 ICFL(L $= ICFL(K,LS)+1
108  185 CONTINUE
109  190 WRITE(NW,10)
110  190 WRITE(NW,190)
111  190 WRITE(NW,190)
112  200 DO 195 I=1,N
113  205 WRITE(NW,205)ICF(I,J),J=1,N
114  210 FORMAT(10X,10I5)
115  215 STOP
116  220 END
117  225 SUBROUTINE INVERS(L,N)
118  230 DIMENSION U(L+1,1),V(L+1,1),Z(L+1,1)
119  235 DO 30 I=1,N
120  240 DO 30 J=1,N
121  245 U(I,J)=A(I,J)
122  250 IF (I.EQ.J) U(I,J)=1.0
123  255 L=L+1
124  260 K=L
125  265 G=ABS(U(K,K))
126  270 IF(C-.000001)120,20,30
127  275 20 IF(K,L)=0.0
128  280 K=K+1
129  285 IF(K,L)=111,90
130  290 DO 30 J=1,N
131  300 IF(K,J)=ABS(EK,J)
135  310 IF(K,J)=ABS(EK,J)
136  320 IF(K,J)=ABS(EK,J)
137  330 IF(K,J)=ABS(EK,J)
138  340 IF(K,J)=ABS(EK,J)
139  350 IF(K,J)=ABS(EK,J)
140  360 IF(K,J)=ABS(EK,J)
141  370 IF(K,J)=ABS(EK,J)
142  380 IF(K,J)=ABS(EK,J)
143  390 IF(K,J)=ABS(EK,J)
144  400 IF(K,J)=ABS(EK,J)
145  410 IF(K,J)=ABS(EK,J)
146  420 IF(K,J)=ABS(EK,J)
147  430 IF(K,J)=ABS(EK,J)
148  440 IF(K,J)=ABS(EK,J)
149  450 IF(K,J)=ABS(EK,J)
150  460 IF(K,J)=ABS(EK,J)
151  470 IF(K,J)=ABS(EK,J)
152  480 IF(K,J)=ABS(EK,J)
153  490 IF(K,J)=ABS(EK,J)
154  500 IF(K,J)=ABS(EK,J)
155  510 IF(K,J)=ABS(EK,J)
156  520 IF(K,J)=ABS(EK,J)
157  530 IF(K,J)=ABS(EK,J)
158  540 IF(K,J)=ABS(EK,J)
159  550 IF(K,J)=ABS(EK,J)
160  560 IF(K,J)=ABS(EK,J)
161  570 IF(K,J)=ABS(EK,J)
162  580 IF(K,J)=ABS(EK,J)
163  590 RETURN
164  600 END
This problem has 5 classes & each class is 10 dimensional.

<table>
<thead>
<tr>
<th>Class</th>
<th>Has</th>
<th>Samples</th>
<th>Its Probability</th>
<th>Is</th>
<th>0.2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1</td>
<td>10</td>
<td>Sample</td>
<td>Its Probability</td>
<td>15</td>
<td>0.2000</td>
</tr>
<tr>
<td>Class 2</td>
<td>10</td>
<td>Sample</td>
<td>Its Probability</td>
<td>15</td>
<td>0.2000</td>
</tr>
<tr>
<td>Class 3</td>
<td>10</td>
<td>Sample</td>
<td>Its Probability</td>
<td>15</td>
<td>0.2000</td>
</tr>
<tr>
<td>Class 4</td>
<td>5</td>
<td>Sample</td>
<td>Its Probability</td>
<td>15</td>
<td>0.2000</td>
</tr>
<tr>
<td>Class 5</td>
<td>10</td>
<td>Sample</td>
<td>Its Probability</td>
<td>15</td>
<td>0.2000</td>
</tr>
</tbody>
</table>

V Matrix

<table>
<thead>
<tr>
<th></th>
<th>1.000</th>
<th>0.000</th>
<th>0.000</th>
<th>0.000</th>
<th>0.000</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
TRANSFORMATION MATRIX A

<table>
<thead>
<tr>
<th></th>
<th>0.241</th>
<th>-0.644</th>
<th>0.037</th>
<th>0.100</th>
<th>0.012</th>
<th>0.002</th>
<th>-0.000</th>
<th>-0.000</th>
<th>-0.000</th>
<th>-0.000</th>
<th>-0.000</th>
<th>-0.000</th>
<th>-0.000</th>
<th>-0.000</th>
<th>-0.000</th>
<th>0.228</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.628</td>
<td>0.329</td>
<td>-0.619</td>
<td>-0.012</td>
<td>-0.014</td>
<td>-0.043</td>
<td>0.007</td>
<td>0.329</td>
<td>0.108</td>
<td>0.308</td>
<td>-0.181</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.028</td>
<td>0.012</td>
<td>-0.406</td>
<td>0.030</td>
<td>0.011</td>
<td>0.035</td>
<td>-0.004</td>
<td>-0.242</td>
<td>-0.035</td>
<td>0.241</td>
<td>0.608</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.026</td>
<td>0.008</td>
<td>-0.002</td>
<td>0.020</td>
<td>-0.014</td>
<td>-0.005</td>
<td>-0.000</td>
<td>0.078</td>
<td>0.000</td>
<td>0.248</td>
<td>1.248</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.015</td>
<td>-0.006</td>
<td>0.008</td>
<td>0.069</td>
<td>0.010</td>
<td>0.012</td>
<td>-0.003</td>
<td>0.056</td>
<td>0.043</td>
<td>0.324</td>
<td>-0.543</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

DECISION MATRIX

<table>
<thead>
<tr>
<th></th>
<th>-0.041</th>
<th>-0.644</th>
<th>0.037</th>
<th>0.100</th>
<th>0.012</th>
<th>0.002</th>
<th>-0.000</th>
<th>-0.000</th>
<th>-0.000</th>
<th>-0.000</th>
<th>-0.000</th>
<th>-0.000</th>
<th>-0.000</th>
<th>-0.000</th>
<th>-0.000</th>
<th>0.228</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.054</td>
<td>0.039</td>
<td>-0.019</td>
<td>-0.012</td>
<td>-0.019</td>
<td>-0.003</td>
<td>0.007</td>
<td>0.329</td>
<td>0.108</td>
<td>0.308</td>
<td>-0.181</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.027</td>
<td>0.012</td>
<td>-0.006</td>
<td>0.020</td>
<td>0.011</td>
<td>0.035</td>
<td>-0.004</td>
<td>-0.242</td>
<td>-0.035</td>
<td>0.241</td>
<td>0.608</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.709</td>
<td>0.008</td>
<td>-0.020</td>
<td>0.020</td>
<td>-0.014</td>
<td>-0.005</td>
<td>-0.000</td>
<td>0.078</td>
<td>0.000</td>
<td>0.248</td>
<td>1.248</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.015</td>
<td>-0.006</td>
<td>0.008</td>
<td>0.069</td>
<td>0.010</td>
<td>0.012</td>
<td>-0.003</td>
<td>0.056</td>
<td>0.043</td>
<td>0.324</td>
<td>-0.543</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

CONFUSION MATRIX

<table>
<thead>
<tr>
<th></th>
<th>10</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

CODE USAGE

OBJECT CODE= 12440 BYTES ARRAY AREA= 15100 BYTES TOTAL AREA AVAILABLE= 49520 BYTES

DIAGNOSTICS

NUMBER OF ERRORS= 0 NUMBER OF WARNINGS= 0 NUMBER OF EXTENSIONS= 0

COMPILATION TIME= 4.93 SECONDS EXECUTION TIME= 21.19 SECONDS WATTIV - VERSION 1 LEVEL 3 MARCH 1971 DATE= 72/040
CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

5.1. Conclusions

The results of this initial study show that it is plausible that the two-dimensional BIFORE or Walsh-Hadamard transform power spectrum can be used to distinguish several types of grain on the basis of their shapes and sizes. The variation in the shapes and sizes of different types of kernels is reflected in a corresponding variation in the spectrum points \( P(0,0), P(4,0), P(5,0), P(5,4), P(0,5), P(4,5), P(5,5), P(3,3), P(4,3), \) and \( P(5,3) \). Subsequently, such variations in this set of spectrum points can be "learned" by means of training algorithms. One such algorithm which uses a least-squares mapping approach seems to be adequate as evident from the \% correct classification scores listed in Section 4-4. However, considering the small sample sizes, one must appreciate that these figures are merely estimates. Larger sample sites must be considered to come to some definite conclusions.
5.2. Recommendations For Future Work.

On the basis of the results obtained from the present study, the following recommendations are made for future work:

(1) Obtain % correct classification scores using a larger training set with the objective of realizing better estimates for the classification scores reported in this study.

(2) Incorporate soybeans and/or rye with the types of grain considered in this study and hence obtain the overall performance of the classification scheme initiated in this study.

(3) Compare the results obtained in (2) with those obtained by an alternate approach to the automatic grain classification problem \[6, 7\].
REFERENCES


ACKNOWLEDGEMENTS

The author wishes to express his deep gratitude and sincere appreciation to his advisor Dr. N. Ahmed for his advice, encouragement and patience during the course of this report.
A PATTERN RECOGNITION APPROACH TO GRAIN SAMPLE ANALYSIS

by

ANIL HARIKANT VYAS

B. E., M. S. University, Baroda, India, 1970

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the
requirements for the degree

MASTER OF SCIENCE

Department of Electrical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas
1973
ABSTRACT

This report concerns an initial feasibility study of grain sample analysis. Specifically, the problem of automatically separating various types of grains is considered. The approach entertained in this study is based on pattern recognition techniques.

The shape and size of a kernel are used as criteria to distinguish it from a kernel belonging to a different type of grain. Each kernel is coded in the form of a (32x32) array of zeros and ones. The portion of the array occupied by the kernel is represented by a "1" while that which is not is represented by a "0". The two-dimensional BFORE (Binary Fourier Representation) or Walsh-Hadamard transform is used to carry out a spectral analysis of the coded representations of several types of kernels in various orientations. The variations in the shapes and sizes of the kernels were thus obtained in terms of a set of BFORE or Walsh-Hadamard power spectrum points.

A subset of the above power spectrum points were used to train a specific pattern classifier which uses the least-squares mapping principle. The final percent correct classification scores for the types of grain considered were as follows: (1) corn, 100%, (2) wheat, 90%, (3) barley, 100%, (4) oats, 70%, and (5) milo, 100%.

The results of this initial study demonstrate that the two-dimensional BFORE power spectrum can be used as an effective tool to characterize the shape and size of given kernel. Since the classification results cited above were obtained from the analysis of a small number of grain samples, it is recommended that the techniques introduced in this study be applied
to a relatively larger number of grain samples. It is further recommended that grains such as rye and soybeans also be included in future studies since the same were not considered in this research effort.