THE APPLICATION OF DIFFERENTIAL EQUATIONS TO MATHEMATICAL SOCIOLOGY

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CHAPTER I

INTRODUCTION

Recent years have shown an increased interest in applying mathematics to the social sciences. The power of mathematics has been demonstrated in its application to many areas of natural science, especially physics. If a mathematical model can be built for a given physical or social phenomenon, the mathematical symbols ". . . can serve as a proxy for experimental manipulation of the objects themselves, so that behavior of an actual object may easily be predicted by the behavior of these symbols alone. . . ." (Coleman, 5). The very idea of experimenting with something so unpredictable as people demonstrates the value of an accurate mathematical model to predict social phenomena. In addition to this "prediction" process, the mathematization of a set of axioms may reveal hidden relations between variables.

The reason that mathematics is such a cumbersome tool for the behavorial scientist may be considered on a global and local level. At the global level the social scientist sees different rules governing different societies. This inhibits the type of generality that is conducive to mathematics. For example, for the physicist, it is the same gravity (in a mathematical sense) that pulls a ball to earth no matter where on the globe it is dropped. Hence, Newton's law of gravitation has an universal scope and the same situation exists for many axioms in physics. On the other hand, the sociologist's axioms may have a very narrow scope. For example, Hoffmann (7) states that his heirarchy of eleven theorems concerning the Pawnee marriage rules does not have "wide application".

That is, they apply only to one small portion of the world's population.

At the local level, the most obvious difficulty confronting the mathematical sociologist is one of measurement. The physicist can measure time, distance, mass and force with considerable precision. These measurements also provide a ready check for the equations used to describe the system. On the other hand, suppose the sociologist makes the observation that "the level of group friendliness will increase if the actual level of interaction is higher than that 'appropriate' to the existing level of friendliness" (Simon, 9). Using Simon's notation, let:

$I(t)$ = the intensity of interaction among members

$F(t)$ = the level of friendliness among the members

b, β = positive constants

($I(t)$ and $F(t)$ indicate that intensity and friendliness are functions of time.) Then, Simon (9) translated this axiom into the differential equation

$$\frac{dF(t)}{dt} = b\left[I(t) - \beta F(t)\right] . \qquad (1.0)$$

The value of such an equation seems doubtful and among the reasons for this is that fact that the sociologist has only a crude "friendliometer" to determine $F(t)$, the level of friendliness at time t. The situation is similiar for many sociological variables. However, this may not entirely discount the usefulness of equations like (1.0). Indeed, one may still be able to discover new relationships between $F(t)$ and $I(t)$ that are brought to light through symbolic representation. Even if this fails, all is not lost becuase the language of a proposition is more precise in symbolic form than in a verbal representation. As Coleman (5) stated, "verbal statements easily hide ambiguity, mathematical ones do not."

CHAPTER II

THE DETERMINISTIC MODEL

With this bit of meta-social verbosity behind us we proceed with the primary function of this paper: To demonstrate the progress that has been made in mathematical sociology by expounding on several concrete models applicable to social phenomena. The models to be considered fall into two classes, the deterministic (explicit) model and the stochastic (probabilistic) model. (For a comparison between the two procedures being applied to model the spread of an epidemic in a population, see Baily, 1.)

First, we consider the deterministic model. For each example, we shall: (1) state the assumptions made about the given process to be modeled in order to adequately "close" the process and obtain a workable model, (2) describe the process or phenomena to be modeled, (3) write the equations involved in the model and solve them, and finally, (4) point out the virtues and/or weaknesses of the given model.

Social Diffusion

Social diffusion lends itself well to the deterministic model. One of the simplest social diffusion theories stated verbally is the following: ". . . the rate of propagation of the attribute (i.e., piece of information, behavorial innovation, belief, style) is proportional to the number of people who already have it" (Coleman, 5). Coleman (5) models this verbal proposition in the following manner: Let t = time, let x = the number of people who have the attribute at time t, and let k = diffusion constant of