THE DESIGN PROCEDURES OF CONTINUOUS BRIDGES WITH PRECAST, PRESTRESSED CONCRETE GIRDERs

by

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INTRODUCTION

In recent years, continuous highway bridges with precast-prestressed concrete girders have been used in many countries. The primary advantages of this type of bridge are the elimination of maintenance costs of deck joints and deck drainage onto the substructure and the structural economy of continuous design. Continuity also can improve the appearance and riding qualities of a bridge.

There are many different kinds of construction methods available to achieve continuity of precast-prestressed concrete bridge girders or slabs. In this report, continuity is achieved for live load plus impact moment by using non-prestressed reinforcement in the deck slab and in the diaphragms over piers.

The effects of shrinkage and creep between the precast girder and the cast-in-place deck slab are more important than in other types of construction of continuous prestressed concrete bridges. The Research and Development Laboratory of the Portland Cement Association completed a research program on this kind of bridge during 1960-1961. Then the results of their research provided available information for designing a continuous precast-prestressed concrete bridge considering the effects of shrinkage and creep.

The design procedures of simple span prestressed concrete bridges are presented in most prestressed concrete textbooks. Designing continuous precast-prestressed concrete girders has
been mentioned by H. K. Preston\textsuperscript{11} and T. Y. Lin\textsuperscript{9}, while Clifford L. Freyermuth\textsuperscript{2} described the importance of the effects of shrinkage and creep in designing a continuous precast-prestressed concrete girder in his paper. But they did not state a proper and complete procedure for designing this kind of bridge. Therefore the main purpose of this report is to present design procedures for continuous precast-prestressed concrete bridges, including the influence of shrinkage and creep. A numerical example will be included to illustrate the procedures.

\begin{enumerate}
\item Freyermuth, C. L., "Design of Continuous Highway Bridges with Precast, Prestressed Girders." Journal of Prestressed Concrete Institute, April 1969.
\end{enumerate}
Design Procedure

Before designing, assume that the general conditions such as loading pattern, span length, provision for future wearing surface and the width of roadway are known. The specifications for material and the method of construction will be defined, because these vary from one locality to another.

(I). Shape of Cross Section

The cross section that has the capacity to sustain the desirable service loading with least cost and ease of erection will be selected.

The type of section which is most efficient and economical in prestressed concrete bridges for any specific structure is a function of the following:

(1) Length of span
(2) Live and impact loads which are to be carried by the bridge
(3) Allowable stresses
(4) Feasibility of various construction types and procedures as controlled by the requirements of the bridge site.

For economy in concrete and steel, the principle is that the section should have sufficient kern distance to produce the necessary internal resisting couple. The effective way is to put concrete near the extreme fibers as much as possible. There are many different kinds of sections such as I shape, T section, hollow slab, and box shape that accomplish these re-
quirements. These have their own advantages in particular cases.

To select a suitable section, the most important factors are the span length and the live load and impact. If the span is short, the dead load moment is small, which means that the \( \frac{M_d}{M_T} \) ratio is small, and the center of pressure at transfer may lie below the bottom kern point. Then tensile stress may occur in the top flange and high compression stress in the bottom flange. The section of a member must provide a relatively large bottom flange to resist the prestressing force during the period when live load is not being applied. In this case, solid slab, hollow slab, and beams with large top and bottom flanges will satisfy these conditions. In long spans, the dead load moment is a large percentage of the total moment, so that the \( \frac{M_d}{M_T} \) ratio is relatively large, the center of pressure at transfer will lie above the top kern point. Then the bottom flange is not required to resist the prestressing force during the period in which the moving live load is not applied. In this case, the T section is always used.

Another factor that should be pointed out is the simplicity of framework. In plant production more complicated shapes can be used. Only when the forms can be reused many times would the I shape and other complicated shapes be considered economical for cast-in-place beams.

---

(II) Properties of Sections

The shape of section to used can be decided from criteria outlined in the preceding paragraphs. Then assume the dimension of the cross section or pick a section from the available data, such as Standard AASHO-PCI prestressed concrete I beams or box beams for highways. Then use these dimensions of sections to calculate the properties of the sections.

First, calculate the properties of precast beams. The basic dimension and properties which must be determined are the area, the center of gravity and the moment of inertia about the centroid of the section. The other properties which are used to facilitate the computation of stresses can be determined from these basic properties.

Second, find the basic properties of composite section: the area, the center of gravity and the moment of inertia about the centroid of composite section. The concrete in the cast-in-place deck slab does not have the same elastic properties as the precast beam. The quality of the concrete used in deck slabs is not as good as that used in precast beams. For this reason, when computing the properties of composite sections one should use a transformed section. The transformed section to be used consists of the gross section of precast beam and a slab section having a depth equal to the actual slab and the effective width equal to the actual width of slab multiplied by the ratio of the elastic modulus of slab concrete to the elastic modulus of the concrete of the precast beam.
(III) Stresses due to dead loads

The dead load shall consist of the weight of the structure complete, including girder, cast-in-place deck slab, wearing surface, sidewalk, conduit, and diaphragms. In this kind of bridge, the construction sequence is to set in place the precast beam without shoring, and then the slab concrete and diaphragm cast over midspan, and then over the piers. For this reason, all the dead loads act on the precast girder as a simple beam action. These loads act uniformly along the beam, except the weight of diaphragm which acts as a concentrated load the beam.

The maximum moments at the center of the span, $M_G$, $M_S$, and $M_{Dia}$, due to the girder’s weight, deck slab and diaphragm respectively can be calculated. Then the total maximum moment at center of the span due to dead load, $M_{DL}$, is expressed as follows,

$$M_{DL} = M_G + M_S + M_{Dia}$$

(1)

Then use the flexure formula to find the fiber stress due to the dead load on precast girder,

$$f_{DL} = \frac{M_{DL}c}{I_p}$$

(2)

where, $f_{DL}$ = Extreme fiber stresses

$M_{DL}$ = The maximum bending moment due to dead load

$c$ = The distance from neutral axis to the extreme fiber of precast girder,

$I_p$ = The moment of inertia of the precast girder about its neutral axis
(IV) Stresses due to live load

The live load should consist of applied moving loads including vehicles, cars and pedestrians. From the general condition, the loading pattern acting on the bridge is known. In AASHO Sec. 1. 2. 5(A), there are two types of loading specified, the H loadings and HS loadings.

To aid in finding the maximum positive and negative moments due to live load, influence lines for them should be constructed. The writer assumes the reader is familiar with the technique of drawing influence lines and using them. In continuous girders, finding maximum moments at several critical points is necessary. These points are at the interior supports, the center of exterior spans. Therefore, the influence line for moment at these points should be completed. In some cases, the influence line for reactions at exterior supports and interior supports are needed.

After the influence lines are completed, determine the live load to be distributed on the girders from the AASHO Specification in Sec. 3, Division 1. Then according to the given loading pattern, the maximum positive and negative moments at these specified points can be computed by the arrangement of the loading condition stated in Sec. 1. 2. 8(C) AASHO.

In addition to the moment due to moving loads, the effects of dynamic loading, vibration and impact should be taken into consideration. In AASHO Specification, Sec. 1. 2. 12(C), the amount of this allowance or increment is expressed in the percentage of live load moment, and should be determined by the
formula,

\[ I = \frac{50}{L + 125} \]  \hspace{1cm} (3)

where \( I \) = Impact factor, (maximum 30 percent)

\( L \) = Length in feet of the portion of the span which is
loaded to produce the maximum moment in the member.

Then add the moment due to moving load and the effect of
impact to get the maximum live load moment at specified sections.
That moment can be expressed as follows;

\[ M_{LL} = M_{ML} (1 + I) \]  \hspace{1cm} (4)

Under live load, the composite section composed of precast
beam and deck slab acts as a continuous member. Therefore, the
\( I_p \) in the equation (2) should become \( I_c \), which is the moment of
inertia of the composite section about the neutral axis of the
transformed section. Then the stresses can be calculated by the
following formula;

\[ f_{LL}^b = \frac{M_{LLc}}{I_c} \]  \hspace{1cm} (5)

where \( f_{LL}^b \) = The extreme fiber stresses

\( M_{LL} \) = The maximum bending moment due to live load and
impact

\( I_c \) = The moment of inertia of the composite girder about
its neutral axis

(V) Designing the Prestressing Steel
In preliminary design of prestressed concrete sections for flexure is based on a knowledge of the internal C-T couple acting in the section. Then the required total effective prestress force \( F \) can be computed as following:

\[
F = \frac{M_T}{0.65 h}
\]  

(6)

where \( M_T \) = The total moment acts on the section

\( h \) = The depth of the section

The required prestressing force is governed by two controlling values of external moment; the total moment \( M_T \) acting on the section, which controls the stresses under the action of the working loads; and the girder load moment \( M_G \), which determine the location of the c.g.s. and the stresses at transfer. Therefore, design the prestress force to limit the fiber stresses of the section to be within the allowable at transfer and under working load.

The basic concept of designing the prestressing force is same as in noncomposite section. One additional concept introduced for composite sections is the reduction of the moments on the composite section to equivalent moments on the precast portion. This is accomplished by the use of the ratio of the section moduli of two sections. Then the required prestressing force can be determined by the following steps:

---

Step 1. Location of the center of gravity of reinforcing steel. For any given trial section, the c.g.s. should be located so that the precast girder will not be overstressed and has the optimum capacity in resisting the external moment. Therefore, the c.g.s. must be located as low as possible but not lower than given by the following value of eccentricity, Fig. (1),

\[ e_o = k_b + e_1 + e_2 \]  \hspace{1cm} (7)

where \( e_o \) = The distance between c.g.c. of precast girder and the c.g.s.

\[ k_b = \frac{I_p}{A_p c_t} \] , the lower kern distance

\[ e_1 = \frac{f_t I_p}{c_t F_o} \] , the eccentricity under the allowable tension stress on top fiber of precast girder at transfer

\[ e_2 = \frac{W_g}{F_o} \] , the eccentricity to resist optimum external moment

\( f_t' \) = allowable tension on top fiber of precast girder at transfer

\( A_p \) = area of precast girder

\( c_t, c_b \) = distance to top or bottom fiber from c.g.c. of precast girder

\( F_o = \frac{f_i}{f_e} F \), total prestressing force just after transfer

\( f_i \) = initial unit prestress in steel before transfer

\( f_e \) = effective unit prestress in steel after deducting losses
\( F = \text{total effective prestress after deducting losses}, \)

the value can be from Eq. (6).

Another way to locate the center of gravity of prestressing steel is to assume the eccentricity of prestress and then check it. This procedure can reduce some calculations.

Step 2. Compute the equivalent moment on the precast girder. There is a moment \( M_c \) acting on the composite section which will produce stresses on the precast girder as shown in Fig. (2) and computed as follows:

\[
\begin{align*}
    f^t &= \frac{M_c}{I_c c^t} \\
    f^b &= \frac{M_c}{I_c c^b}
\end{align*}
\]

where \( c^t \), \( c^b \) are distances to extreme fibers of the precast girder measured from c.g.c. of the composite section.

Let \( m_t = \frac{I_p}{I_c c^t} \), and \( m_b = \frac{I_p}{I_c c^b} \).

Then
\[
\begin{align*}
    f^t &= \frac{m_t M_c d_t}{I_p} = \frac{m_t M_c}{A_p k_t} \quad \text{(8a)} \\
    f^b &= \frac{m_b M_c c_b}{I_p} = \frac{m_t M_c}{A_p k_t} \quad \text{(8b)}
\end{align*}
\]

where \( k_t \) top kern distance of precast girder.

From Eq. (8a) and (8b) \( M_c \) can be modified by \( m_t \) and \( m_b \), so that it can be reduced to equivalent moments for computation based on the precast girder properties.

Step 3. Compute the amount of prestress required for
moments as follows. If \( M_p \) = the total moment acting the precast girder, and \( f'_b \) = allowable tensile stress at bottom fiber, so that:

\[
f'_b = -\frac{F}{A_p} \left( 1 + \frac{e_o}{k_t} \right) + \frac{M_p}{A_p k_t} + \frac{m_b M_c}{A_p k_t}
\]

Then, \( F = \frac{M_p + m_b M_c + f'_b k_t A_p}{e_o + k_t} \) \hspace{1cm} (9a)

If \( f'_b = 0 \),

then \( F = \frac{M_p + m_b M_c}{e_o + k_t} \) \hspace{1cm} (9b)

from which the required total prestress force \( F_o \) can be computed. If necessary, use this new value of \( F_o \) to revise the location of c.g.s.

Step 4. Compute the required prestressing steel area by the following formula:

\[
A_s = \frac{F_o}{f_e}
\]

(10)

Then the number of strands to be used can be selected.

Step 5. Check the top and bottom fiber stresses at the center of the span at transfer, then

\[
f^t = \frac{F_o}{A_p} - \frac{(F_o e_o - M_G)}{A_p k_b}
\]

(11a)

\[
f^b = \frac{F_o}{A_p} + \frac{(F_o e_o - M_G)}{A_p k_t}
\]

(11b)

Step 6. Check the top and bottom fiber stresses at the
* $f_b'$, $f_t'$ allowable tensile stress in bottom fiber stress under working load and top fiber stress at transfer respectively.

**FIG. 1.** PRECAST GIRDER, STRESS DISTRIBUTION AT TRANSFER

**FIG. 2.** COMPOSITE SECTION, STRESS DISTRIBUTION UNDER WORKING LOAD
center of the span under working load, since

\[ f^t = \frac{F}{A_p} + \frac{K_p + m_t N_c - Fc_0}{A_p k_b} \]  \hspace{1cm} (12a)

\[ f^b = \frac{F}{A_p} - \frac{K_p + m_b N_c - Fc_0}{A_p k_t} \]  \hspace{1cm} (12b)

(VI) Positive moment over piers due to creep and differential shrinkage

One of the unique features of the type of bridge under consideration is that positive moments are produced over piers due to creep in the precast prestressed girders as well as due to the effects of live loads in remote spans. These positive moments will be partially counteracted by negative moments resulting from differential shrinkage between the cast-in-place deck slab and the precast girders. The positive live load and impact moment over piers due to remote span can be calculated from previous steps. It is very difficult to measure these two effects, creep and shrinkage, separately in a structure, because the result measured is always the resultant of these two influences. For convenience in analysis, these two effects are considered separately.

(1) The effect of creep

Since the creep is time dependant, with the more rapid deformation occurring during the early stages of loading, the amount of positive restraint moment produced by the prestressing force depends on the time when the continuity connection is made.
It also depends on the inherent creep potential of concrete mix, the exposure conditions, and the volume to surface ratio of the precast girder. The deformation and restraint moments induced in a two span continuous girder by creep due to prestressing force are illustrated in Fig. (3).

The most accurate method of obtaining creep data is by extrapolation from a number of test samples prepared in advance from the actual mix to be used in the precast girders. In most

(a). Initial deformation

(b). Final deformation if left as two simple spans.

(c). Final deformation and restraint moment if spans are made continuous after prestressing

FIG. 3. DEFORMATIONS AND RESTRAINT MOMENTS IN A TWO-SPAN CONTINUOUS BEAM CAUSED UNDER PRESTRESS FORCE
Specific creep $\epsilon_c' \times 10^{-6}$ in./in./psi

**FIG. 4.** PREDICTION OF BASIC CREEP FROM ELASTIC MODULUS

**FIG. 5.** CREEP VS. AGE AT LOADING
design work, it is sufficient to rely on the available research information. Such research\(^1\)\(^6\) indicates that the basic creep value for loading at 28 days can be predicted from the elastic modulus from the curve in Fig. (4). For design purpose, the 20 year creep curve can be regarded as the ultimate creep. The ultimate creep value for loading at 28 days from Fig. (4) must be adjusted to account for the age when the girders are pre-stressed and for the volume/surface ratio of the girders. The correcting factor of creep due to the age at loading can be obtained from the curve in Fig. (5)\(^16\), and the correcting factor of creep due to volume/surface ratio is shown in Fig. (6)\(^5\).

Then ultimate creep can be determined as follows:

(a) Get the basic creep \(\varepsilon_c\) for loading at 28 days from Fig. (4).

(b) Find the correcting factor \(C_1\) due to age at loading

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16 "Recommendations for International Code of Practice for Reinforced Concrete," published by the American Concrete Institute and the Cement and Concrete Association.
FIG. 6. CREEP VS. VOLUME-TO-SURFACE RATIO

FIG. 7. PROPORTION OF FINAL SHRINKAGE OR CREEP VS. TIME
from Fig. (5), and the correcting factor \( C_2 \) due to volume/surface ratio from Fig. (6).

(c) Then the ultimate creep is \( \varepsilon_c \times C_1 \times C_2 \).

Then determine the percentage of ultimate creep which will occur after the continuity connection is made. This value can be obtained from Fig. (7). For example, if a connection were made at 28 days after casting the precast girder, about 40 percent of creep strains will have taken place. This can be found from Fig. (7). The remaining 60 percent is available to develop moments in the connection. These restraint moments due to creep under prestress and dead load, \( M_{\text{rp}} \) and \( M_{\text{rdl}} \) respectively, act on the continuous girders as shown in Fig. (3). These moments over each pier can be solved by any method of analysis of statically indeterminate structures, and the analyses are illustrated in Appendix IV. Therefore, the final restraint moment \( M_{\text{RC}} \) caused by the effect of creep on the precast girder after the creation of continuity will be expressed as follows:

\[
M_{\text{RC}} = (M_{\text{rp}} + M_{\text{rdl}})(1 - e^{-\Phi})
\]  

(13)

where \( M_{\text{RC}} \) = The final restraint moment over piers due to the effect of creep under prestress and dead load

\( M_{\text{rp}} \) = The restraint moment due to the effect of creep under prestress

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3 Hansen, J. A., "Prestress Loss as Affected by Type of Curing," Portland Cement Association Development Department, Bulletin D75.
$M_{rdl}$ = The restraint moment due to the effect of creep under dead load

$e$ = The base of Naperian logarithms, 2.7183

$\phi = \frac{\varepsilon_c}{\varepsilon}$, the ratio of creep strain per psi of stress to the elastic strain per psi of stress

If the value of $\phi$ is known, the value of $(1 - e^{-\phi})$ can be taken directly from Fig. (8).

(2) The effect of shrinkage

Shrinkage of the deck slab with respect to the precast girders causes negative restraint moments over piers that will reduce the creep restraint moment. The restraint moments due to differential shrinkage, $m_{rs}$, are applied to the continuous girder uniformly along its length, and can be expressed as follows: 11

$$m_{rs} = \varepsilon_s E_b A_b \left( e_2 + \frac{t_b}{2} \right)$$  (14)

where $\varepsilon_s$ = differential shrinkage strain (as described below)

$E_b$ = elastic modulus of the deck slab concrete

$A_b$ = cross-section area of deck slab

$(e_2 + t_b/2)$ = distance between the mid-depth of the slab and the centroid of composite section

$t_b$ = slab thickness

**FIG. 8.** VARIATIONS IN RESTRAINT MOMENT CREEP EFFECT FACTORS FOR BOTH CREEP AND SHRINKAGE

**FIG. 9.** SHRINKAGE HUMIDITY CORRECTION FACTOR C3

RELATIVE HUMIDITY - PERCENT
\[ \varepsilon_s = \frac{\varepsilon_{su} T^{16}}{N_s + T} \]  

\( \varepsilon_s \) = measured shrinkage strain at any time \( T \) in days  
\( \varepsilon_{su} \) = ultimate shrinkage strain at \( T = \infty \)  
\( N_s = 26.0 e^{0.36V/S} \)  
\( V/S \) = the volume/surface ratio of the member under consideration

In general, the way to get the value of \( \varepsilon_s \) in Eq. (14) is from equation 15. By testing specimens which were made from the mix to be used in the structure, the final value of shrinkage can be projected by assuming that the shrinkage-time relationship can be represented by Eq. (15). If the test data are not available, then \( \varepsilon_s \) can be obtained by the following steps:

(a) Assume the ultimate shrinkage \( \varepsilon_s(ult) \) under exposure at 50 percent relative humidity is \( 0.6 \times 10^{-3} \). 3, 13

(b) Find the shrinkage humidity correction \( C_3 \) from Fig. (9).

(c) Find the correction factor \( C_4 \) for the differential shrinkage between slab and girder for a given time lapse between the castings from Fig. (7).

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3 Hanson, J. A., "Prestress Loss as Affected by Type of Curing," Portland Cement Association Development Department, Bulletin D75.

13 "Design and Control of Concrete Mixtures," Portland Cement Association, Old Orchard Road, Skokie, Ill. 60076.

16 "Recommendations for International Code of Practice for Reinforced Concrete," Published by the American Concrete Institute and the Cement and Concrete Association.
(d) Then the value of $\varepsilon_s$ is $C_3 \times C_4 \times \varepsilon_s(ult)$, and can be substitute into Eq. (14) to calculate $m_{RS}$.

The restraint moments $m_{RS}$ due to differential shrinkage are known. Then any convenient method for the analysis of statically indeterminate structures can be applied to find the restraint moments. $M_{RS}$ can be computed by multiplying by the factor $(1 - e^{-\theta})/\theta$ for the effect of creep as given by the following,

$$M_{RS} = m_{RS}(1 - e^{-\theta})/\theta$$  \hspace{1cm} (16)

Then the total positive moments over piers due to creep and shrinkage are given as follows:

$$M_R = M_{RC} + M_{RS}$$  \hspace{1cm} (17)

(VII) Design positive moment connection

As determined in the P.C.A. Laboratories, the positive moment may develop at the interior support sections of continuous girders. If a positive moment continuity connection is not provided, cracking may occur at the bottom of the diaphragm at these sections. However, the cracking, if any, will affect only the behavior of the girder at service load level. The positive restraint moment due to creep and shrinkage will not affect the ultimate load carrying capacity of the girders. For this reason, it is recommended that design be based on the lower

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stress of 0.6 of the yield strength.

In the P.C.A. Development Department bulletin,\textsuperscript{10} there are two kinds of connection details to resist the positive restraint moment. The detail considered more practical is shown in Fig. (10).

When designing the connections, there are several details that should be considered. First, the length of embedment required for hooked connection bars. This can be calculated by considering the bond to develop uniformly from the face of the beam around the bend to the end of the bar.\textsuperscript{4} In Sec. 1.5. 1.(D), AASHO, the allowable bond stress is 0.1 $f'_c$ (Max. 350 psi).

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Second, the distance from the inside face of precast girder to the inside face of the hook should be at least 12 times the bar diameter.

Third, the removal of the end girder form after prestressing should be considered. It may be required that the connection bars be cast in the beam straight and then field bent after removal of the end form.

Then the required steel area of the connection can be calculated by the ordinary reinforced concrete formula:

\[ A'_s = \frac{M_R}{f'_s j d} \]  \hspace{1cm} (18)

where \( A'_s \) = area of conventional tensile steel
\( f'_s \) = allowable tensile stress in conventional steel
\( j \) = ratio of distance between centroid of compression and centroid of tension to the depth \( d \)
\( d \) = distance from extreme compressive fiber to centroid of tensile force

(VIII) Design Negative Moment Reinforcement

Since most of the dead loads are carried by the precast beam acting as a simple beam, the negative moments over piers due to live load plus impact are sustained by the composite section. Although this negative moment is computed by the elastic theory, the reinforcement can be designed to meet the ultimate strength requirements with the specified minimum load factors, which are 2.5 for live load and impact and 1.5 for dead load. The reason is that the effect of initial precom-
pression due to prestress in the precast girders can be neglected in the negative moment calculation of ultimate strength if the maximum precompression stress is less than $0.4f'_c$ and the continuity reinforcement is less than 1.5 percent.\footnote{Kaar, P. H., Kriz, L. B. and Hognestad, E., "Precast-Prestressed Concrete Bridges I. Pilot Test of Continuous Beams," Portland Cement Association Development Department, Bulletin D34.}

It will be found that the depth of compression block will be less than the thickness of bottom flange of the precast girder. For this reason, the required negative moment reinforcement can be determined by assuming the beam as a rectangular section with a width equal to the width of the bottom flange of the girder. Therefore, the flexure formula for ultimate strength design from Sec. 1601 ACI Building Code 1963 may be used to compute the required steel area as follows;

$$M_u = a \left[ b d^2 f'_c q \left( 1 - 0.59q \right) \right] = a A_g f'_y (d - \frac{a}{2})$$

(19)

where $M_u =$ ultimate resisting moment

$a =$ capacity reduction factor, equals 0.9

$b =$ width of compression face of flexure member

$f'_c =$ compressive strength of concrete at 28 days

$q = \frac{p f'_y}{f'_c}$

$p' = \frac{A_g}{b d}$

$a = \frac{A_g f'_y}{0.85 f'_c b}$
The reinforcement ratio, $p'$, shall not exceed 0.75 of the ratio, $p_b$, which produces balanced conditions at ultimate strength given by:

$$p_b = \frac{0.85k_1f'_c}{f'} \frac{87,000}{37,000+f'_y} \tag{20}$$

where $k_1$ = a factor which shall be taken as 0.85 for strength $f'_c$, up to 4000 psi and shall be reduced continuously at a rate of 0.05 for each 1000 psi of strength in excess of 4000 psi.

(IX) Ultimate Strength

After completing the girder flexural design, the ultimate flexural capacity of the girder should be computed at the critical sections. First, consider the section of midspan of the girder. From Sec. 1.6.10. AASHO, if the neutral axis lies within the flange, then the ultimate flexural strength will be assumed as:

$$M_u' = A_s f_{su} (1 - 0.6 \frac{p f_{su}}{f'_c}) \tag{21}$$

where $A_s$ = area of main prestressing tensile steel

$f_{su}$ = average stress in prestressing steel at ultimate load

$p = \frac{A_s}{bd}$, ratio of prestressing steel

$M_u'$ = ultimate flexural strength

If the neutral axis falls outside flange, then the ultimate strength will be assumed as:
\[ M_u' = A_{sr}f_{su}d \left(1 - 0.6 \frac{A_{sr}f_{su}}{b'df_c}\right) + 0.85f'_c(b-b')t(d-0.5t) \] (22)

where \( A_{sr} = A_s - A_{sf} \), the steel area required to develop the ultimate compressive strength of the web of a flange section

\[ A_{sf} = 0.85f'_c(b-b')t/f_{su}, \text{ steel area required to develop the ultimate compressive of the overhanging portion of the flange} \]

\( t = \text{the thickness of the flange} \)

\( b = \text{width of flange of flanged member or width of rectangular member} \)

\( b' = \text{width of web of a flanged member} \)

Second, consider the sections over piers. Since these sections are designed by the reinforced concrete theory, Eq. (19) can be used without the capacity reduction factor, \( \alpha \), to calculate the ultimate flexural strength at these section. This moment can be expressed as:

\[ M_u' = bd^2 f'_cq \left(1-0.59q\right) = A_{sf}f'_y(d-\frac{a}{2}) \] (23)

(X) Compute the reaction over piers and design shear steel

Since the precast girders are acting as simple beams under dead load, then the reaction caused by dead load can be found easily by statics. The computation of reactions due to live load and impact that can make use of influence lines by arranging the load condition to get the maximum reaction over piers. The ultimate shear will be governed by the following
formula by using the load factor from AASHO Sec. 1. 6. 6.;

\[ V_{ult} = 1.5 \, V_{DL} + 2.5 \, V_{LL} \]  \hspace{1cm} (24)

where \( V_{ult} \) = ultimate shear due to dead load and live load
  including impact
\( V_{DL} \) = shear due to dead load
\( V_{LL} \) = shear due to live load and impact

Then the effective ultimate shear is the ultimate shear
minus the shear carried by the strand at the end, which can be
expressed as follows;

\[ V_u = V_{ult} - V_s \]  \hspace{1cm} (25)

where \( V_u \) = effective shear due to specified ultimate load and
  effect of prestressing
\( V_s \) = shear carried by the strands

Then the required web reinforcement can be calculated by
the formula from Sec. 1. 6. 13, AASHO.

\[ A_V = \frac{(V_u - V_c)s}{2f_y'jd} \]  \hspace{1cm} (26)

where \( A_V \) = area of web reinforcement at spacing \( s \), placed
  perpendicular to the axis of the member, which
shall not be less than 0.0025b's
\( V_c = 0.06 \, f_c' b' jd \), but not more than 180b'jd (assuming
  \( j = 7/8 \))
\( s \) = longitudinal spacing of web reinforcement

The spacing of web reinforcement shall not exceed three-
fourths the depth of the member and shall provide transverse
reinforcement across the bottom flanges.

(XI) Compute the camber or deflection

From Sec. 202.2 of Tentative Recommendations for Prestressed Concrete,\textsuperscript{14} camber and deflection may be design limitations and should be investigated for both short and long time effects. In fact, it is very difficult to compute a precise deflection or camber, because there are two difficulties that cannot be overcome. One is the determination of the value of $E_c$, flexural modulus of elasticity of concrete, the other is the estimation of the effect of creep on deflections.

According to the Sec. 203.3 in Tentative Recommendations for Prestressed Concrete, for the deflection or camber under short time loading the value of $E_c$ can be obtained by assuming $E_c$ is 1,800,000 plus 500 times the cylinder strength at the age considered. Also, for deflection associated with dead load, prestress and live loads sustained for a long time may be computed on the assumption that the corresponding concrete strains are increased as a result of creep.

By following the above-referenced specification, the deflection or camber can be computed by the conjugate-beam method at various loading stages. First, the deflections due to girder's weight, then the camber due to prestressing force, and then the

\textsuperscript{14} ACI-ASCE Joint Committee 323, "Tentative Recommendations for Prestressed Concrete," Journal of the American Concrete Institute, Vol.29, Jan. 1958.
additional deflection due to the weight of slab and diaphragm and finally the live load and impact. The final camber equals the summation of these deflections.
DESIGN EXAMPLE

In this design example, the interior girder of a four-span continuous precast-prestressed concrete bridge will be considered. Before designing the general information of the bridge and design data will be introduced as follows:

a. The cross section and elevation of the bridge are as shown in Fig. (11) and Fig. (12) respectively.

FIG. 11. CROSS SECTION OF BRIDGE

FIG. 12. ELEVATION OF BRIDGE
b. Loading:
Live load: AASHO Loading H20-40
Weight of concrete: 150 lb/cu.ft.
Wearing surface: 2 in. thickness of concrete

c. Specifications:
The structural design is essentially based AASHO Specifications for Highway Bridges, 1969, 10th edition, and use several portions of ACI-ASCE Joint Committee 323 recommendations for Prestressed Concrete, 1958, and ACI Building Code Requirement for Reinforced Concrete (ACI 318-63).

d. Prestressing steel:
Use 7/16 in. 7-wire strands, with $A_s = 0.1089$ in.$^2$ per strand, $f_i = 175,000$ psi, $f_e = 140,000$ psi, $f_s = 250,000$ psi.
e. Reinforcing steel; intermediate grade $f_{y'} = 40,000$ psi.
f. Precast concrete; $f_{ci} = 5,000$ psi, $f_c' = 6,000$ psi.
g. In-place slab concrete; $f_c' = 4,000$ psi.

All the design procedures will be follow the procedures described before, and will illustrate every step in the follows:

(I). Shape of cross section;

In this example, the type II of I beam of AASHO-PCI Standard will be used. The thickness of slab is assumed 6.5 in. and the dimensions of the section are shown in Fig. (13).

(II). Properties of section

Precast section $A_p = 369$ in.$^2$
$c_b = 15.83$ in., $c_t = 20.17$ in.
$I_p = 50,980$ in.$^4$
a. SECTION OF PRECAST GIRDER

b. SECTION OF COMPOSITE SECTION

FIG. 13. CROSS SECTION OF GIRDER
Then the sectional moduli are:

\[ Z_t = \frac{50,980}{20.17} = 2,530 \text{ in.}^3 \]

\[ Z_b = \frac{50,980}{15.83} = 3,220 \text{ in.}^3 \]

weight per unit foot = \( 369 \times 150 / 144 = 384 \text{ lb/ft} \)

Composite section

Because the modulus of elasticity in precast girder and slab are different, the modulus of elasticity for precast and slab should be computed separately according to Sec. 203.2

Tentative Recommendations for Prestressed Concrete as follows:

For slab \( E_b = 1,800,000 + 500 \times 4,000 = 3,800,000 \text{ psi} \)

For precast girder

\[ E_c = 1,800,000 + 500 \times 6,000 = 4,800,000 \text{ psi} \]

\[ \text{Ratio} = \frac{3,800,000}{4,800,000} = 0.792 \]

Then taking moments about the axis through the top fiber of the slab, to find the section properties:

<table>
<thead>
<tr>
<th>Section</th>
<th>A</th>
<th>y</th>
<th>Ay</th>
<th>Ay^2</th>
<th>I_o</th>
</tr>
</thead>
<tbody>
<tr>
<td>66 x 6.5 x 0.792</td>
<td>339</td>
<td>3.25</td>
<td>1,101.</td>
<td>3,580.</td>
<td>1,195.</td>
</tr>
<tr>
<td>precast girder</td>
<td>369</td>
<td>26.67</td>
<td>9,850.</td>
<td>263,580</td>
<td>50,980.</td>
</tr>
<tr>
<td></td>
<td>708</td>
<td></td>
<td>10,951.</td>
<td></td>
<td>52,175.</td>
</tr>
</tbody>
</table>

\[ I = 266,580 - 52,175 = 318,755 \text{ in.}^4 \]

\[ c_t'' = \frac{10,953}{707} = 15.48 \text{ in.} \quad c_b'' = 42.5 - 15.48 = 27.02 \text{ in.} \]

\[ I_c = 318,755 - 708(15.48)^2 = 149,155 \text{ in.}^4 \]

Then the sectional moduli are:
\[ Z_t = \frac{149,155}{15.48} = 9,650 \text{ in.}^3 \]
\[ Z_b = \frac{149,155}{27.02} = 5,745 \text{ in.}^3 \]

(III) Stresses due to dead loads;

Compute the moments at center of the span due to girder's weight,

Ext. span \[ M_G = \frac{1}{8} \times 384 \times (40)^2 = 86,500 \text{ ft-lb} \]

Int. span \[ M_G = \frac{1}{8} \times 384 \times (70)^2 = 235,200 \text{ ft-lb} \]

Compute the moment at center of the span due to the weight of slab and wearing surface,

weight of slab and wearing surface is:

\[ (6.5 + 2) \times 66 \times \frac{150}{144} = 584.3 \text{ lb/ft} \]

Ext. span \[ M_S = \frac{1}{8} \times 584.3 \times (40)^2 = 116,900 \text{ ft-lb} \]

Int. span \[ M_S = \frac{1}{8} \times 584.3 \times (70)^2 = 358,000 \text{ ft-lb} \]

From the AASHO-PCI Standards the diaphragms are at the center of the span. The dimensions of diaphragms are 8 in x 2 ft-6 in x 5 ft-6 in, therefore the weight of the diaphragm is

\[ 2.5 \times 5.5 \times 2/3 \times 150 = 1,375 \text{ lb} \]

Computing the moments at the center of the span,

Ext. span \[ M_{\text{Dia.}} = \frac{1}{4} \times 1,375 \times 40 = 1,375 \text{ ft-lb} \]

Int. span \[ M_{\text{Dia.}} = \frac{1}{4} \times 1,375 \times 70 = 24,100 \text{ ft-lb} \]

Then the total moments and stresses at the center of the span of the precast girder due to dead load as follows;

Ext. span \[ M_{DL} = 86,500 + 116,900 + 13,750 = 217,150 \text{ ft-lb} \]
\[ f_{DL}^b = 217,150 \times 12 / 3220 = 809 \text{ psi} \]
\[ f_{DL}^t = 217,150 \times 12 / 2,530 = 1,030 \text{ psi} \]

\[ M_{DL} = 235,200 + 358,000 + 24,100 = 617,100 \text{ ft-lb} \]
\[ f_{DL}^b = 617,100 \times 12 / 3,220 = 2,300 \text{ psi} \]
\[ f_{DL}^t = 617,100 \times 12 / 2,530 = 2,925 \text{ psi} \]

(IV). Moments due to live load

From Sec. 1.3.1. (B) AASHO, the distribution factor of live load is \( S/5.5 \),

\[ \text{D.F.} = 5.5 / (5.5 \times 2) = 0.5 \]

Then compute the impact factor by Eq. (3),

Ext. span; \( +M \), \( I = 50/125+40 = 0.303 \), use 0.30
\[ -M, I = 50/125+55.5 = 0.27 \]

Int. span \( +M \), \( I = 50/125+70 = 0.256 \)
\[ -M, I = 50/125+70 = 0.256 \]

By using the influence lines, the maximum live load moments per lane are as follows;

Ext. span,
\( +M \) near midspan
\[ M = 99.856966 \times 2 \times 0.64 + 8.505965 \times 18 = 281 \text{ ft-k} \]
\( -M \) over pier
\[ M = -198.334 \times 2 \times 0.64 - 3.0597802 \times 18 - 7.1451 \times 18 \]
\[ = -437.5 \text{ ft-k} \]
Int. span
+M near midspan
\[ M = 170.0631 \times 2 \times 0.64 + 11.02275 \times 18 = 415.8 \text{ ft-k} \]
-M over pier
\[ M = -248.1423 \times 2 \times 0.64 - 5.793148 \times 18 \times 2 = -526.5 \text{ ft-k} \]
Total moments due to live load and impact are as follows;
Ext. span +M_{LL} = 281 \times 0.5 \times 1.30 = 182.5 \text{ ft-k}
-M_{LL} = -437.5 \times 0.5 \times 1.27 = -278 \text{ ft-k}
Int. span +M_{LL} = 415.8 \times 0.5 \times 1.256 = 261 \text{ ft-k}
-M_{LL} = -526.5 \times 0.5 \times 1.256 = -331 \text{ ft-k}

(V) Design the prestressing steel
Ext. span, assuming the c.g.s. is 4 in. above the bottom fiber of precast girder, and from the previous steps, the following data can be computed:
\[ M_p = 217.15 \text{ ft-k}, M_G = 86.5 \text{ ft-k}, M_C = 182.5 \text{ ft-k} \]
\[ m_t = \frac{50980}{20.17} / \frac{149155}{15.48} = 0.263 \]
\[ m_b = \frac{50980}{15.83} / \frac{149155}{27.02} = 0.583 \]
\[ k_b = \frac{50980}{369 \times 20.17} = 6.86 \text{ in.} \]
\[ k_t = \frac{50980}{369 \times 15.83} = 8.72 \text{ in.} \]
Using Eq. (9b), to find F.
\[ F = \frac{(217.15 - 0.583 \times 182.5) \times 12}{11.83 + 8.72} = 189 \text{ k} \]
FIG. 14. ARRANGEMENT OF RPESTRESSING STEEL OF SPAN AB

FIG. 15. PATH OF C.G.S. OF SPAN AB
\[ F_o = \frac{175}{140} \times 189 = 236.3 \text{ k} \]
\[ A_S = \frac{189}{140} = 1.35 \text{ in.}^2 \text{ Min. steel } 0.003 \times 369 = 1.107 \text{ in.}^2 \]

Use 12 strands

The arrangement of the prestressing steel is shown in Fig. (14). Check the eccentricity of prestress, \( e_o \).

From Fig. (14), \( e_o = 3 \text{ in.} \), therefore the magnitude of \( F \) and the required prestressing steel area should be recomputed as followings:

\[ F = \frac{323.45 \times 12}{21.55} = 180 \text{ k} \]
\[ F_o = \frac{175 \times 180}{140} = 225 \text{ k} \]
\[ A_S = \frac{180}{140} = 1.285 \text{ in.}^2 \], use 12 strands.

Find the center of gravity of prestressing steel at the end of the girder and draw the path of the c.g.s. of the girder as shown in Fig. (15). Taking moments about the bottom fiber,

\[ 4 \times 33 = 132 \]
\[ 8 \times 3 = 24 \]
\[ \frac{156}{12} = 13 \text{ in. above the bottom fiber} \]

Check the stresses at the end of the girder,

\[
\begin{align*}
\sigma^t &= \frac{-225}{369} + \frac{225 \times 2.58}{2530} = -0.61 + 0.23 = -0.38 \text{ ksi} \\
\sigma^b &= \frac{-225}{369} - \frac{225 \times 2.58}{3220} = -0.61 - 0.18 = -0.79 \text{ ksi}
\end{align*}
\]

Check the stresses at transfer at the center of the span,

\[
\begin{align*}
\sigma^t &= \frac{-225}{369} + \frac{225 \times 12.83 - 86.5 \times 12}{369 \times 6.86} = -0.61 - \frac{1853}{369 \times 6.86}
\end{align*}
\]
\[-0.61 + 0.731 = +0.121 \text{ ksi} < 6\sqrt{\frac{f_{ci}'}{f_{ci}}} = 424 \text{ psi (tension)}\]

\[f_b^t = -\frac{225}{369} - \frac{1853}{369 \times 8.72} = -0.61 - 0.576 = -1.186 \text{ ksi} \]
\[< 0.6 f_{ci}' = 3.0 \text{ ksi}\]

Check the stresses under working load at the center of the span by Eq. (12a) and Eq. (12b),

\[f_t^t = -\frac{180}{369} - \frac{217.15 \times 12 - 0.263 \times 12 - 180 \times 12.83}{369 \times 6.86} \]
\[= -0.488 - \frac{870}{369 \times 6.86} = -0.488 - 0.344 = -0.832 \text{ ksi} \]
\[< 0.4 f_{ci}' = 2.4 \text{ ksi}\]

\[f_b^b = -\frac{180}{369} + \frac{870}{369 \times 8.72} = -0.488 + 0.27 = -0.218 \text{ ksi}\]

Compression O.K.

Int. span, assuming the o.g.s. is 4 in. above the bottom fiber of precast girder, and from previous steps, the following data can be computed;

\[M_p = 617.1 \text{ ft-k}, \quad M_G = 235.2 \text{ ft-k}, \quad M_c = 261 \text{ ft-k}\]
\[m_b = 0.583, \quad m_t = 0.263\]
\[k_b = 6.86 \text{ in.}, \quad k_t = 8.72 \text{ in.}\]

Using Eq. (9b) to find F

\[F = \frac{(617.1 - 0.583 \times 261) \times 12}{11.83 + 8.72} = 769.1 \times 12 / 20.55 = 449 \text{ k}\]

\[F_o = 175 \times 449 / 140 = 560.5 \text{ k}\]
\[ A_s = \frac{449}{140} = 3.205 \text{ in.}^2 \quad 3.205/0.1089 = 29.4 \]

Use 30 strands.

The arrangement of the prestressing steel is shown in Fig. (16). Check the eccentricity of prestress, \( e_o \).

From Fig. (16), \( e_o = 4.027 \) in. above the bottom fiber, it's close enough, so no revision is needed. Then find center of gravity of prestressing steel at the end of the girder and draw the path of c.g.s. of the girder as shown in Fig. (17).

Taking moments about the bottom fiber to locate the c.g.s. at the end of the girder,

\[ 12 \times 29 = 348 \]
\[ 18 \times 4 = 72 \]
\[ 420 \div 30 = 14 \text{ in. above the bottom fiber} \]

Check the stresses at ends at transfer,

\[ f^t = -\frac{560.5}{369} + \frac{560.5 \times 1.83}{2530} = -1.52 + 0.406 = -1.014 \text{ ksi} \]

\[ f^b = -\frac{560.5}{369} + \frac{560.5 \times 1.83}{3220} = -1.52 - 0.319 = -1.839 \text{ ksi} \]

Check the stresses at transfer at midspan by Eq. (11a) and Eq. (11b),

\[ f^t = -\frac{560.5}{369} + \frac{560.5 \times 11.83 - 235.2 \times 12}{369 \times 6.86} \]
\[ = -1.52 + \frac{6,640 - 2,820}{369 \times 6.86} = -1.52 + \frac{3,820}{369 \times 6.86} \]
FIG. 16. ARRANGEMENT OF PRESTRESSING STEEL OF SPAN BC

FIG. 17. PATH OF C.G.S. OF SPAN BC
\[ r^b = -\frac{560.5 - 3,820}{369} = -1.52 - 1.188 = -2.708 \text{ ksi} \]

\[ r^b < 3.0 \text{ ksi o.k.} \]

Check the stresses under working load at midspan by Eq. (12a) and (12b);

\[ f_t = -\frac{\frac{449}{369} x 12 - 0.267 x 261 x 12 - 449 x 12}{369 x 6.86} \]

\[ = -12.15 - \frac{2,834}{369 x 6.86} = -1.215 - 1.12 = -2.335 \text{ ksi} \]

\[ f^b = -\frac{449}{369} + \frac{2,834}{369 x 8.72} = -1.215 + 0.881 = -0.334 \text{ ksi compression o.k.} \]

(VII). The moments over piers due to creep and shrinkage

a. due to creep

The final restraint moments \( M_{rc} \) without adjustment for creep effect at supports, can be known from appendix IV p. 76, and p. 80, as follows;

At support B, \( M_{rc} = -327.15 + 507 = 175.85 \text{ ft-k} \)

\( C, M_{rc} = -452.93 + 822 = 369.07 \text{ ft-k} \)

Evaluating the creep factors as followings;

(1). The modulus of elasticity at strand release = \( 4.3 \times 10^6 \text{ psi} \), from Fig. (4) the specific creep at 20 years is \( 0.3 \times 10^{-6} \text{ in/in/psi} \).

(2). For release of the prestress force at 2 days, the
value of specific creep is adjusted by Fig. (5) to:

\[ 1.66 \times 0.30 \times 10^{-6} = 0.498 \times 10^{-6} \text{ in/in/psi} \]

(3). The volume/surface ratio of Type II I-beam is 3.4 from AASHO-PCI Standards data, and from Fig. (6), the creep volume/surface ratio correction factor is 1.25. So the specific creep is adjusted as follows:

\[ 0.498 \times 10^{-6} \times 1.25 = 0.623 \times 10^{-6} \text{ in/in/psi} \]

(4). From Fig. (7), at 28 days 0.4 of the creep has occurred, leaving 0.6 to produce restraint moment, that is,

\[ 0.623 \times 10^{-6} \times 0.6 = 0.3735 \times 10^{-6} \text{ in/in/psi} \]

(5). The value of ϕ for the structure with a connection made at 28 days would be:

\[ ϕ = 0.3735 \times 10^{-6} \times 4.8 \times 10^{6} = 1.79 \]

(6). ϕ = 1.79 is known, then from Fig. (8) can get;

\[ (1 - e^{-ϕ}) = 0.85 \]

\[ \frac{1 - e^{-ϕ}}{ϕ} = 0.47 \]

Therefore the final moment \( M_{RC} \) at supports can be computed as follows;

At support B, \( M_{RC} = 175.85 \times 0.85 = 149.5 \text{ ft-k} \)
\[ C, M_{RC} = 369.07 \times 0.85 = 314 \text{ ft-k} \]

b. due to shrinkage

From appendix V p. 83, the final restraint moments \( M_{RS} \) at supports are:

At support B, \( M_{RS} = 1.216 \, m_{RS} \left( \frac{1-e^{-\varphi}}{\varphi} \right) \)

At support C, \( M_{RS} = 0.892 \, m_{RS} \left( \frac{1-e^{-\varphi}}{\varphi} \right) \)

where \( m_{RS} = 0.0006 \times 0.4 \times 1.0 \times 3.8 \times 10^6 \times 66 \times 6.5 \times 12.23 \)

\[ = 4,795,000 \text{ in-lb} = 399 \text{ ft-k} \]

Assume the condition of the continuity connections made 28 days after prestress release, exposure at 50 percent relative humidity and ultimate shrinkage is 0.0006 in/in. Substituting the value of \( m_{RS} \) and \( \frac{1-e^{-\varphi}}{\varphi} \) into the expression of \( M_{RS} \), then

At support B, \( M_{RS} = 1.216 \times 399 \times 0.47 = 228 \text{ ft-k} \)

At support C, \( M_{RS} = 0.892 \times 399 \times 0.47 = 167.4 \text{ ft-k} \)

(VIII). The positive moments over piers and design the positive connection

By summing the restraint moments due to creep and shrinkage in previous the step and the live load and impact from appendix VI p. 90, then the max, positive moments at supports are as follows:

At support B, \( M = 149.5 - 228 - (39.456 \times 2 \times 0.64 + \)

\[ 1.842 \times 18) \times 0.5 \times 1.3 \]

\[ = 149.5 - 228 - 54.4 \times 0.5 \times 1.3 = -23.5 \text{ ft-k} \]
\[ C, M = 314 - 167.4 - (21.587 \times 2 \times 0.64 + 0.831 \times 18) \]
\[ \times 0.5 \times 1.256 = 314 - 167.4 - 26.3 = 172.9 \text{ ft-k} \]

In this particular case, there is no positive moment at support B, therefore design the positive moment connection for support C only. Use intermediate grade reinforcement with an allowable stress \( f'_{y} = 24 \text{ ksi} \), then required area of steel is;

\[ A_{s}^{''} = 172.9 \times 12/24 \times 7/8 \times 39.5 = 2.5 \text{ in}^2 \]

Use 6-#6 bars, provided \( A_{s}^{''} = 2.64 \text{ in}^2 \).
To find the stress in the steel, first compute the ratio of moduli of elasticity \( n \),

\[ n = \frac{2,900,000}{w^{3.53} \sqrt{T_{C}} = \frac{2,900,000}{(150)^{1.53} \sqrt{6,000}} = 6.49} \]

\[ p' = \frac{2.64}{66} \times 39.5 = 0.001025, \quad p'n = 0.00665 \]

\[ k = \sqrt{2pn - (pn)^2} = \frac{\sqrt{0.0133 - (0.00665)^2}}{0.1155 - 0.0133} = 0.10885 \]

\[ j = 1 - \frac{k}{3} = 0.96705 \]

\[ f_{s} = 172.9 \times 12/2.64 \times 0.96705 \times 39.5 = 20.6 \text{ ksi} < 24 \text{ ksi} \text{ o.k.} \]

Designing the embedment length for the allowable bond stress = 0.1 \( f'_{c} = 400 \text{ psi} > 350 \text{ psi} \), so use 350 psi.

Assume the distance from the end face of the precast girder to the inside face of the hook = 12 bar diameter = 12 \times 6/8 = 9 \text{ in.}
Min. hook radius = 3x6/8 = 2.25 in.
Req. embedment length = 206,000 x 0.6/350 = 2.36 = 15 in.
1/4 bend = 3.54 in.
15 - 3.54 - 6.75 = 4.71 in.

Use 7.71 in. extension, gives 18 in. embedment.

To ensure that bars extend into the part of the beam where the prestress force is effective, and to avoid termination all connection bars at one point, extend 2 bars 2 ft. into the beam, and extend the other 4 bars 3 ft. 6 in. into the beam.

(IX) Design of negative reinforcement

From the previous step, the moments over the piers due to live load can be determined.

At support B, \( M = (-198.334 \times 0.64 - 3.0597802 \times 18 \)
\(-7.145 \times 18) \times 0.5 \times 1.27
\=-437.5 \times 0.5 \times 1.27 = -278 \text{ ft-k} \)

C, \( M = (248.147 \times 2 \times 0.64 - 5.793 \times 18 \times 2) \)
\times 0.5 \times 1.256 = -526.5 \times 0.5 \times 1.256
\=-331 \text{ ft-k} \)
Design the negative moment reinforcement at support C; the ultimate negative moment is

\[ M_u = 2.5 \times 331 = 828 \text{ ft-k} \]

Then use the ordinary ultimate design procedures.

Assume \( a = 4.9 \text{ in.} \)

\[ A_s^t = 828 \times 12/0.9 \times 40 \times (39.25 - 2.45) = 7.5 \text{ in.}^2 \]

then, \( a = 7.5 \times 40/0.85 \times 4 \times 18 = 4.91 \text{ in.} \) (it's close enough.)

Use 13 - #7 bars, providing \( A_s^t = 7.8 \text{ in.}^2 \)

Check the steel percentage,

\[ P_b = \frac{0.85 \times 0.85 \times 4.0}{40} \frac{87}{87 + 40} = 0.0537 \]

\( p^i = 7.8/18 \times 39.25 = 0.0105 < 0.0537 \times 0.75 = 0.04025 < P_b \text{ o.k.} \)

(X) Ultimate Strength

Ext. span, the ultimate strength of midspan;

Use Eq. (21),
\[ A_s = 12 \times 0.1089 = 1.308 \text{ in.}^2, \; f_{su} = 250 \text{ ksi} \]
\[ p = 1.308/66 \times 0.792 \times 39.5 = 0.0063 \]
\[ M_u^r = 1.308 \times 250 \times 39.5 \times (1 - 0.6 \times 0.0063 \times 250/6) \]
\[ = 1.308 \times 250 \times 39.5 \times (1 - 0.16) = 10.860 \text{ in.-k} = 905 \text{ ft-k} \]
\[ > 217.15 \times 1.5 + 182.5 \times 2.5 = 325 + 456 = 781 \text{ ft-k} \]

Int. span, the ultimate strength of midspan:
\[ A_s = 30 \times 0.1089 = 3.27 \text{ in.}^2 \]
\[ p = 3.485/66 \times 0.792 \times 38.5 = 0.01625 \]
\[ M_u^r = 3.27 \times 250 \times 38.5 \times (1 - 0.6 \times 0.001625 \times 250/6) \]
\[ = 3.27 \times 250 \times 38.5 \times (1 - 0.04) = 30.200 \text{ in.-k} = 2,515 \text{ ft-k} \]
\[ > 617.1 \times 1.5 + 261 \times 2.5 = 925 + 652.5 = 1,577.5 \text{ ft-k} \]

The ultimate strength over piers, use following equations:
\[ M_u^r = A_s^r f_y (d - \frac{a}{2}) \]
\[ A_s^r = 7.8 \text{ in.}^2, \; f_y = 40 \text{ ksi}, \; d = 39.25 \text{ in.} \]
\[ a = 7.8 \times 40 / 0.85 \times 4 \times 18 = 4.9 \text{ in.} \]
\[ M_u^r = 7.8 \times 40 \times (39.25 - 2.45) = 1.148 \text{ in.-k} = 955 \text{ in.-k} \]
\[ > 2.5 \times 331 = 828 \text{ ft-k} \quad \text{o.k.} \]

(X) Design of shear steel

Ext. span;

Reaction due to dead load:
- girder's weight 0.38420 = 7.68
- slab 0.584320 = 11.686
- diaphragm 1.3751.5 = 2.063
\[ \frac{21.429 \text{ k}}{} \]
The max. reactions due to live load and impact, can be obtained under the loading condition shown in Fig. (18) and Fig. (19) for support A and B respectively:

Reaction at support A, \( 40 \times 1.27 \times 0.5 = 25.4 \) k

\( B, 38 \times 1.27 \times 0.5 = 24.1 \) k

Then shear diagram for span AB will be shown in Fig. (20). To calculate the ultimate shear at support B to design the shear steel,

\[ V_{ult} = 24.1 \times 2.5 + 20.054 \times 1.5 = 90.40 \text{ k} \]

Shear carried by strand can be computed by followings;

\[ \sqrt{30^2 + (13 \times 12)^2} = \sqrt{25,200} = 158.8 \text{ in.} \]

\[ V_s = 60 \times 30 / 158.8 = 11.31 \text{ k} \]
FIG. 19. MAX. REACTION AT SUPPORT B.

(a). DUE TO DEAD LOAD

(b). DUE TO LIVE LOAD AND IMPACT FROM FIG. (18) LOADING CONDITION
(Loadsx1.27x0.5)

(c). DUE TO LIVE LOAD AND IMPACT FROM FIG. (19) LOADING CONDITION
(Loadsx1.27x0.5)

FIG. 20. SHEAR DIAGRAM OF SPAN AB
The effective ultimate shear \( V_u = 90.40 - 11.31 = 79.09 \) k

use \#4 bars, \( A_v = 0.4 \) in.\(^2\)

\[ V_c = 180 \times b' \times jd = 180 \times 6 \times 7/8 \times 39.5 = 37.3 \] k

\[ jd = 7/8 \times 39.5 = 34.6 \text{ in.} \]

\[ s = 2 \times 0.4 \times 40 \times 34.6/79.09 = 24.4 \text{ in.} \]

Min. spacing \( 3/4 \times 39.5 = 29.6 \text{ in.} \)

or \( 0.4/0.0025 = 26.7 \text{ in.} \)

use \#4 bars at 24 in. c.c. spacing

Check the horizontal shearing stress at top fiber of precast girder between slab and girder under live load and impact by the following equation;

\[ v = \frac{VQ}{I_c b} \]

\[ Q = 66 \times 6.5 \times (15.48 - 3.25) \times 0.792 = 4,160 \text{ in.}^3 \]

\[ v = 60.25 \times 4.160 \times 1000/149,155 \times 12 = 140 \text{ psi} < 225 \text{ psi} \]

According Sec. 212.3.3. Tentative Recommendations for Prestressed Concrete, the allowable bond stress is 225 psi for using additional steel ties. Therefore, \#4 ties should be placed at 12 in. spacing in outer quarter of the span.

Int. span;

Reaction due to dead load;

Girder's weight \( 0.384 \times 35 = 13.43 \)

slab \( 0.5843 \times 35 = 20.41 \)

diaphragm \( 1.375 \times 1.5 = 2.063 \)

\[ 35.903 \text{ k} \]
The max. reactions due to live load and impact can be obtained under the loading condition shown in Fig. (19) and Fig. (21) for supports B and C respectively;

Reaction at support B, \((25 + 26) \times 1.256 \times 0.5 = 32\, k\)

\(C, 46.5 \times 1.256 \times 0.5 = 29.8\, k\)

\(Or, (27.15 + 26) \times 1.256 \times 0.5 = 33.4\, k\)

The shear diagram for span BC shown in Fig. (22).

To calculate the ultimate shear at support B to design the shear steel,

\[v_{ult} = 34.528 \times 1.5 + 29.8 \times 2.5 = 51.7 = 74.5 = 126.2\, k\]

Shearing force carried by the strand can be computed as follows;

\[\sqrt{(25)^2 + (22.5\, 12)^2} = \sqrt{625 + 72,900} = \sqrt{73,525} = 271\, \text{in.}\]

\[V_s = 179.5 \times 25 / 271 = 16.5\, k\]

The effective ultimate shear \(V_u = 126.2 - 16.5 = 109.7\, k\)

use #4 bars, \(A_v = 0.4\, \text{in.}^2\)

\[V_c = 180 \times 6 \times 7 / 8 \times 38.5 = 36.4\, k\]

\[jd = 7/8 \times 38.5 = 33.7\, \text{in.}\]

\[s = 2 \times 0.4 \times 40 \times 33.7 / (109.7 - 36.4) = 14.74\, \text{in.}\]

Use #4 bars at 12 in. c.c. spacing

Check the horizontal shearing stress at top fiber of precast girder between slab and girder under live load and impact by following formula;

\[v = \frac{VQ}{I_{cb}}\]

\[Q = 66 \times 6.5 \times (15.48 - 3.25) \times 0.792 = 4,160\, \text{in.}^3\]
FIG. (21). MAX. REACTION AT SUPPORT C.

(a) DUE TO DEAD LOAD

(b) DUE TO LIVE LOAD AND IMPACT FROM FIG. (19) LOADING CONDITION (Loadsx1.256x0.5)

(c) DUE TO LIVE LOAD AND IMPACT FROM FIG. (21) LOADING CONDITION (Loadsx1.256x0.5)

FIG. (22). SHEAR DIAGRAM OF SPAN BC
\[ v = 75.4 \times 4,160 \times 1,000 / 149,155 \times 12 = 173 \text{ psi} < 225 \text{ psi} \]

So, use \#4 bars at 12 in. spacing will be satisfied.

(XI) Compute the camber

a. due to dead load of the girder.

At the time prestressing, \( f_{ci}^* = 4,000 \text{ psi} \), so \( E_c = 3.8 \times 10^6 \text{ psi} \), then the deflection can be found as follows:

Ext. span

\[
A_G = \frac{5wl^4}{384EI_p} = 5 \times 384 \times (40)^4 \times (12)^3 / 384 \times 3.8 \times 10^6 \times 50,980
\]

\[ = 0.114 \text{ in. (downward)} \]

Int. span;

\[
A_G = 5 \times 384 \times (70)^4 \times (12)^3 / 384 \times 3.8 \times 10^6 \times 50,980 = 1.07 \text{ in. (downward)}
\]

b. due to eccentricity of the prestressing force

Ext. span

Moment due to prestress at end is

\[ M = 2.83 \times 225 = 637,000 \text{ in-lb} \]

at center is \( M = 12.83 \times 225 = 2,890,000 \text{ in-lb} \)

The moment diagram constructed as shown in Fig. (23), and

\( M/EI \) diagram also in Fig. (23). Then use conjugate beam method

\| to find the deflection at the center of the span;

<table>
<thead>
<tr>
<th>part</th>
<th>A</th>
<th>X</th>
<th>AX</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>15.3 \times 10^7</td>
<td>120</td>
<td>18.35 \times 10^9</td>
</tr>
<tr>
<td>II</td>
<td>27.0 \times 10^7</td>
<td>136</td>
<td>30.65 \times 10^9</td>
</tr>
<tr>
<td>III</td>
<td>18.9 \times 10^7</td>
<td>42</td>
<td>7.95 \times 10^9</td>
</tr>
<tr>
<td></td>
<td>61.1 \times 10^7</td>
<td></td>
<td>56.95 \times 10^9</td>
</tr>
</tbody>
</table>
\[ \Delta_p = \frac{(61.1 \times 10^7 \times 20 \times 12 - 56.95 \times 10^9)}{3.8 \times 10^6 \times 50,980} = 0.465 \text{ in. (upward)} \]

Net camber = 0.465 - 0.114 = 0.351 in. (upward)

Int. span

Moment due prestress at end is

\[ M = 1.83 \times 560.5 \times 10^3 = 1,010,000 \text{ in-lb} \]

at center is \[ M = 11.83 \times 56.5 \times 10^3 = 6,640,000 \text{ in-k} \]

The moment diagram constructed as shown in Fig. (24), and M/EI diagram is also in Fig. (24). Then use conjugate beam method to find deflection at the center of the span;

**FIG. 23. M/EI DIAGRAM OF SPAN AB DUE TO PRESTRESS**

**FIG. 24. M/EI DIAGRAM OF SPAN BC DUE TO PRESTRESS**
<table>
<thead>
<tr>
<th>Part</th>
<th>A</th>
<th>X</th>
<th>AX</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$235 \times 10^6$</td>
<td>210</td>
<td>$494 \times 10^8$</td>
</tr>
<tr>
<td>II</td>
<td>$760 \times 10^6$</td>
<td>240</td>
<td>$1,825 \times 10^8$</td>
</tr>
<tr>
<td>III</td>
<td>$844 \times 10^6$</td>
<td>75</td>
<td>$632.5 \times 10^8$</td>
</tr>
<tr>
<td></td>
<td>$1,839 \times 10^6$</td>
<td></td>
<td>$2,951.5 \times 10^8$</td>
</tr>
</tbody>
</table>

Then deflection at center is,

$$\Delta_p = \frac{(1,839 \times 10^6 \times 35 \times 12 - 2,951.5 \times 10^8)}{3.8 \times 10^6} = 2.46 \text{ in. (upward)}$$

Net camber $= 2.46 - 1.07 = 1.39 \text{ in. (upward)}$

c. due to the weight of slab and diaphragm

When the slab and diaphragm are poured, the strength of the concrete in the girder will 6,000 psi, and $E_c = 4.8 \times 10^6$ psi.

Then compute the deflection at the center due to weight of slab are;

Ext. span;

$$\Delta_{ss} = 5 \times 584.3 \times (40)^4 \times (12)^3 / 384 \times 4.8 \times 10^6 \times 50,980$$

$$= 0.1375 \text{ in. (downward)}$$

due to diaphragm

$$\Delta_{\text{dia.}} = 1,375 \times (40 \times 12)^3 / 48 \times 4.8 \times 10^6 \times 50,980$$

$$= 0.013 \text{ in. (downward)}$$

Net camber $= 0.351 - 0.1375 - 0.013 = 0.213 \text{ in. (upward)}$

Int. span;

$$\Delta_{ss} = 5 \times 584.3 \times (70)^4 \times (12)^3 / 384 \times 4.8 \times 10^6 \times 50,980$$
\[ \Lambda_{\text{dia.}} = 1.375 \times (70 \times 12)^3 / 48 \times 4.8 \times 50,980 = 0.07 \text{ in.} \] 
\[ \text{Net camber} = 1.39 - 1.29 - 0.07 = 0.03 \text{ in. (upward)} \]

After the slab is poured, the net camber in the girder are 0.213 in. and 0.03 in. for exterior and interior span respectively. Since all the stresses in the beam at this time will be sustained permanently, the camber will gradually increase owing to creep in the concrete. If the beam were acting independently, camber could be expected to increase 100 to 300 percent from Sec. 203.3 Tentative recommendations for Prestressed Concrete. Actually the beam and slab now form a composite structure in which the stress in the slab is zero. The tendency of the beam to increase its camber is resisted by the composite section, so that the resultant increase is lower than would be expected if the girder were acting alone. We shall assume that the camber growth beyond the point would be 150 per cent if the beam were acting independently. Therefore, the camber growth will computed by followings:

**Ext. span**

\[ 1.5 \times 0.213 \times 50,980 / 149,155 = 0.1093 \text{ in.} \]

\[ : \text{Total final camber} = 0.213 - 0.1093 = 0.3223 \text{ in. (upward)} \]

**Int. span**

\[ 1.5 \times 0.03 \times 50,980 / 149,155 = 0.0153 \text{ in.} \]

\[ : \text{Total final camber} = 0.03 + 0.0153 = 0.0453 \text{ in. (upward)} \]
d. due to live load and impact

For this is a continuous girder, the max. positive moment is not exactly at center of the span, but it is near the mid-span. So in computing the max. camber or deflection, for convenience in computation, the deflection at center of the span will be assumed the max.

Ext. span

Consider the loading condition as shown in Fig. (25), the moments at supports are computed,

\[ M_B = -158.8 \text{ ft-k}, \quad M_C = -149.8 \text{ ft-k}, \quad M_D = -203.3 \text{ ft-k} \]

Then the \( M/\text{EI} \) diagram of span AB can be constructed as in Fig. (26) and use conjugate beam method to find the deflection at midspan by parts as in the following:

\[ \delta_1 = \frac{5 \times 640 \times (40)^4}{3 \times 384 \times 4.8 \times 10^6 \times 149,155} \]
\[ = 0.00516 \text{ in.} \text{ (downward, due to uniform load)} \]

\[ \delta_2 = \frac{18,000 \times (40 \times 12)^3}{4 \times 4.8 \times 10^6 \times 149,155} \]
\[ = 0.058 \text{ in.} \text{ (downward, due to concentrate load)} \]

\[ \text{FIG. 25. MAX. DEFLECTION AT MIDSPAN OF SPAN AB} \]
\[ \delta_3 = 1,210 \times 1,000 \ (40 \times 12)^2 \div 16 \times 4.8 \times 149.155 = 0.0244 \text{ in.} \]
(downward, due to end moment)

Total deflection at the center of span AB is

\[ \Delta_{LL} = 0.0516 - 0.058 - 0.0244 = 0.134 \text{ in.} \] (downward)

Net camber = 0.3223 - 0.134 = 0.1883 in. (upward)

(a). DUE TO UNIFORM LOAD

(b). DUE TO CONCENTRATE LOAD

(c). DUE TO END MOMENT

FIG. 26. M/EI DIAGRAM DUE LIVE LOAD IN SPAN AB

Int. span

Consider the loading condition as shown in Fig. (27), the moments at supports are computed,

\[ M_B = -321.83 \text{ ft-k}, \ M_C = -236.2 \text{ ft-k}, \ M_D = 27.83 \text{ ft-k} \]

Then the M/EI diagram of span BC can be constructed as
in Fig. (28) and the conjugate beam method can be used to find the deflection at midspan by parts as in the following:

\[ \delta_1 = \frac{5 \times 640 \times (70)^4 \times (12)^3}{384 \times 4.8 \times 10^6 \times 149,155} \]
\[ = 0.484 \text{ in. (downward, due to uniform load)} \]

\[ \delta_2 = \frac{18,000 \times (70 \times 12)^3}{48 \times 4.8 \times 10^6 \times 149,155} \]
\[ = 0.312 \text{ in. (downward, due to concentrate load)} \]

**Fig. 27.** Max. Deflection at Midspan of Span BC

**Fig. 28.** M/EI Diagram Due to Live Load of Span BC

(a). Due to Uniform Live Load

(b). Due to Concentrate Load

(c). Due to End Moments
\[ s_3 = (2,835,000 - 386,000) \frac{(70\times12)^2}{16\times4.8\times10^6\times149,155} = 0.412 \text{ in. (upward, due end moment)} \]

Total deflection at the center of span BC is

\[ \Delta_{LL} = 0.484 - 0.312 - 0.412 = 0.3687 \text{ in. (downward)} \]

Net camber = 0.0453 - 0.384 = -0.3387 in. (downward)
CONCLUSION

1. The effects of creep and differential shrinkage are very important to this type of bridge. The positive restraint moments over piers do not reduce the ultimate load carrying capacity, but will cause cracks over piers without the connection.

2. To date, the research on this subject is limited to studies conducted by P.C.A. Development and Research Laboratory on a two-span precast prestressed concrete bridge. Much research is still to be done on this type of bridge for controlling the influence of shrinkage and creep.

3. In this report, the design of end bearing and other details such as end block, pick up hook, etc. are not mentioned, because these details are different from one job to another. In practical design, however, these should be taken into account.

4. In recent years, the digital computer has been widely used, this kind of design work can be written into a program to eliminate the tedious calculations.
ACKNOWLEDGMENT

The writer wishes to express his sincere gratitude to Professor Vernon H. Rosebraugh, for his advice, suggestions and encouragement in the preparation of this report.
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APPENDIX II - NOTATIONS

\( A_b \) = cross-section area of deck slab

\( A_p \) = area of precast girder

\( A_s \) = area of main prestressing tensile steel

\( A_s' \) = area of conventional tensile steel

\( A_{sf} \) = area of steel required to develop the ultimate compressive strength of the overhanging portion of the flange

\( A_{sr} \) = the steel area required to develop the ultimate compressive strength of the web of a flange section

\( A_v \) = area of web reinforcement at spacing \( s \), placed perpendicular to the axis of the member, which shall not be less than \( 0.0025 b \)'s

\( b \) = width of compression force of the flexure member

\( b' \) = width of web of flanged member

\( c \) = the distance from neutral axis to the extreme fiber of precast girder

\( c_t, c_b \) = distance to top or bottom fiber from c.g.c. of precast girder

\( c_t', c_b' \) = distance to top or bottom fiber from c.g.c. of precast girder

\( d \) = distance from extreme compressive fiber to centroid of tensile force

\( E_b, E_c \) = elastic modulus of the deck slab or precast girder concrete
\[ e = \text{the base of Naperian logarithms, } 2.7183 \]

\[ e_0 = \text{the distance between c.g.c. of precast girder and the c.g.s.} \]

\[ e_1 = \text{the eccentricity under the allowable tension stress on top fiber of precast girder at transfer} \]

\[ e_2 = \text{the eccentricity to resist optimum external moment} \]

\[ (e_2' + t_b/2) = \text{distance between the mid-depth of the slab and centroid of composite section} \]

\[ F = \text{required total effective prestress force after deducting losses} \]

\[ F_o = \text{total prestressing force just after transfer} \]

\[ f^b, f^t, f^b, f^t = \text{extreme fiber stresses} \]

\[ f'_b = \text{allowable tensile stress at bottom fiber} \]

\[ f'_c = \text{compressive strength of concrete at 28 days} \]

\[ f'_{ci} = \text{ultimate unit stress in concrete, at time of transfer} \]

\[ f_e = \text{effective unit prestress after deducting losses} \]

\[ f_i = \text{initial unit prestress in steel before transfer} \]

\[ f_s = \text{ultimate unit stress in prestressing steel} \]

\[ f'_s = \text{allowable tensile stress in conventional steel} \]

\[ f_{su} = \text{average stress in prestressing steel at ultimate load} \]

\[ f'_t = \text{allowable tension on top fiber precast girder at transfer} \]

\[ f'_y = \text{yield strength of conventional steel} \]

\[ h = \text{the depth of the section} \]
\( I = \) impact factor (max. 30 percent)

\( I_c = \) the moment of inertia of the composite section about its neutral axis

\( I_p = \) the moment of inertia of the precast girder about its neutral axis

\( j = \) ratio of distance between centroid of compression and centroid of tension to the depth \( d \)

\( k_b, k_t = \) the top or bottom distance

\( k_l = \) a factor which shall be taken as 0.85 for strength \( f'_c \), up to 4,000 psi and shall be reduce continuously at a rate of 0.05 for each 1,000 psi of strength in excess of 4,000 psi

\( L = \) Length in feet of the portion of the span which is loaded to produce the max. moment in the member

\( M_c = \) any moment acting on the composite section

\( M_{DL} = \) the max. bending moment due to dead load

\( M_{Dia} = \) the max. moment at the center of the span due to diaphragm

\( M_G = \) the max. moment at the center of span due to girder's weight

\( M_{LL} = \) the max. bending moment due to live load and impact

\( M_{ML} = \) the max. bending moment due to live load

\( M_p = \) the total moment acting the precast girder

\( M_R = \) the total positive moments over piers due to creep and shrinkage

\( M_{RC} = \) the final restraint moment over piers due to the
effect of creep under prestress and dead load

\[ M_{rdl} = \text{the restraint moment due to the effect of creep under dead load} \]

\[ M_{rp} = \text{the restraint moment due to the effect of creep under prestress} \]

\[ M_{RS} = \text{the total restraint moment due to shrinkage} \]

\[ m_{rs} = \text{restraint moment due to differential shrinkage} \]

\[ M_T = \text{the total moment acts on the section} \]

\[ M_u = \text{ultimate resisting moment} \]

\[ M_u' = \text{ultimate flexural strength} \]

\[ n = \text{ratio of modulus of elasticity} \]

\[ p = \text{ratio of prestressing steel} \]

\[ p' = \text{ratio of non-prestressing steel} \]

\[ s = \text{longitudinal spacing of web reinforcement} \]

\[ t = \text{the thickness of the flange} \]

\[ t_b = \text{slab thickness} \]

\[ V_c = 0.06f_c'b'jd, \text{ but not more than } 180b'j\text{d (assuming } j = 7/3) \]

\[ V_{DL} = \text{shear due to dead load} \]

\[ V_{LL} = \text{shear due to live load and impact} \]

\[ V_s = \text{shear carried by the strands} \]

\[ V_u = \text{effective shear due to specified ultimate load and effect of prestressing} \]

\[ V_{ult} = \text{ultimate shear due to dead load and live load including impact} \]

\[ V/S = \text{the volume/surface ratio of the member under} \]
consideration

$Z_b, Z_t$ = sectional modulus

$\alpha$ = capacity reduction factor, equals 0.9

$\gamma, \gamma'$ = angle of rotation

$\Delta$ = deflection of beam

$\epsilon_o$ = the ratio of creep strain per psi of stress to the elastic strain per psi of stress

$\epsilon_c$ = basic creep strain

$\epsilon_s$ = differential shrinkage strain

$\epsilon_{su}$ = ultimate shrinkage strain at $T = \text{infinity}$
# APPENDIX IV TABLE OF FIGURES

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<td>21</td>
</tr>
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<td>9. Shrinkage humidity correction details</td>
<td>21</td>
</tr>
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<td>10. Positive restraint moment connection details</td>
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<td>11. Cross section of bridge</td>
<td>32</td>
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<td>16. Arrangement of prestressing steel of span BC</td>
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<td>17. Path of c.g.s. of span BC</td>
<td>43</td>
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<td>18. Max. reaction at support A</td>
<td>51</td>
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<td>19. Max. reaction at support B</td>
<td>52</td>
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<td>20. Shear diagram of span AB</td>
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</tr>
<tr>
<td>21</td>
<td>Max. reaction at support C</td>
</tr>
<tr>
<td>22</td>
<td>Shear diagram of span BC</td>
</tr>
<tr>
<td>23</td>
<td>M/EI diagram of span AB due to prestress</td>
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<tr>
<td>24</td>
<td>M/EI diagram of span BC due to prestress</td>
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<td>25</td>
<td>Max. deflection at midspan of span AB</td>
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<tr>
<td>26</td>
<td>M/EI diagram due to live load in span AB</td>
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<td>27</td>
<td>Max. deflection at midspan of span BC</td>
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<td>28</td>
<td>M/EI diagram due to live of span BC</td>
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<td>Influence line for moment at support B, MB</td>
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<td>30</td>
<td>Influence line for moment at support C, MC</td>
</tr>
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<td>Influence line for positive moment at center of span AB, M1</td>
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<tr>
<td>32</td>
<td>Influence line for positive moment at center of span BC, M2</td>
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APPENDIX IV

Derivation of general equation for creep restraint moment

1. Dead load creep restraint moment

Using the conjugate beam method to find the slope at ends,

\[ \theta_1 = \frac{M_{DL1} L_1}{3EI} \]

span BC,
\[ \theta_2 = \frac{M_{DL2} L_2}{3EI} \]

where \( M_{DL1} = w_1 L_1^2/8 \), and \( M_{DL2} = w_2 L_2^2/8 \), and assume EI is constant.

Apply moments \( m_{dl} \) to return beam ends to horizontal as shown in the Figure below:

Ext. span
\[ \theta_1 = \theta_3 \]
\[ -m_{dlB} L_1/3EI = M_{DL1} L_1/3EI \]
\[ m_{dlB} = -M_{DL1} \]

Int. span
\[ \theta_2 = \theta_4 \]
\[ +\frac{M_{DL2}L^2}{3EI} = -\frac{m_{dB}L^2}{3EI} - \frac{m_{dC}L^2}{6EI} \]
\[ m_{dB} = m_{dC} \]
\[ m_{dB} = m_{dC} = -\frac{2M_{DL2}}{3} \]

Distribute the restraint moments \( m_{dl} \) by moment distribution, then compute the final dead load restraint moments \( M_{rdl} \) due to creep (not reduced by the creep effects factor).

For example,

\[ M_{DL1} = 217.15 \text{ ft-k}, \quad m_{dB} = 217.15 \text{ ft-k} \]
\[ M_{DL2} = 617.1 \text{ ft-k}, \quad m_{dC} = -411 \text{ ft-k} \]

Then use moment distribution method;

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<th>C</th>
<th>D</th>
<th>E</th>
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<td>C.O.</td>
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<td>( \equiv M ) 0.</td>
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<td>-327.15</td>
<td>-327.15</td>
<td>-327.15</td>
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</table>

The final restraint moments are;

\[ M_{rLB} = -327.15 \text{ ft-k}, \quad M_{rLC} = -452.93 \text{ ft-k} \]

2. Prestress creep restraint moment
The creep under the prestress force depends on the location of the various forces with respect to the center of gravity of the composite section. The tendons will be replaced in the calculations by the equivalent forces acted on the member.

The moment diagram by parts can be shown as follows;

Strictly speaking, the moments should vary from zero to full value along the transfer length (dotted lines). For convenience, the rectangular moment diagram will be used in calculation;

Using the conjugate beam method, the slopes due to prestress at the ends are as follows;

Ext. span
\[ \theta_1 = \frac{P_1d_1L_1}{2EI} - \frac{P_2d_2L_1}{2EI} - \frac{P_3L_1^2}{2EI} - \frac{P_3L_1L_2}{2EI} \]

Int. span

\[ \theta_2 = \frac{P_1d_1L_2}{2EI} - \frac{P_2d_2L_2}{2EI} - \frac{P_3L_1^2}{2EI} - \frac{P_3L_1L_2}{2EI} \]

Moments \( m_{rp} \) to return beam ends to horizontal are as shown in the figure below:

Ext. span

\[ \theta_1 = \theta_3 \]
\[ \theta_1 = \frac{m_{rpB}L_1}{3EI} \]
\[ m_{rpB} = -\frac{3EI\theta_1}{2L_1} \]

Int. span

\[ \theta_4 = \frac{m_{rpB}L_2}{3EI} - \frac{m_{rpC}L_2}{6EI} \]
\[ m_{rpB}L_2/2EI \]
\[ \theta_2 = \theta_4 \]
\[ m_{rpB} = m_{rpC} = -\frac{2EI\theta_2}{L_2} \]

Distribute the restraint moments \( m_{rp} \) by moment distribution, then the final prestress restraint moments \( M_{rp} \) due to creep (not reduced by the creep effects factor) can be computed.

For example,
Ext. span

\[ p_1 = 60 \text{ k}, \quad p_2 = 120 \text{ k}, \quad p_3 = \frac{36.68 \times 60}{13 \times 12} = 14.1 \text{ k}, \]

\[
\theta_1 = \frac{14.1 \times (13 \times 12)^2}{2EI} - \frac{14.1 \times 13^2 \times 13 \times (12)^2}{2EI} - \frac{1}{2EI} (359,000 - 1,427,000 - 343,000 - 343,000)
\]

\[ = \frac{1,754,000}{2EI} \]

\[ m_{rpB} = -\frac{3 \times EI}{2 \times 40 \times 12} \frac{1,754,000}{2EI} = 2,760 \text{ in-k} \]

Int. span

\[ p_1 = 179.5 \text{ k}, \quad p_2 = 269.5 \text{ k}, \]

\[ p_3 = (31.5/22.5 \times 12) \times 179.5 = 21.4 \text{ k} \]
\[
\theta_1 = \frac{179.5 \times 8.48 \times 70 \times 12}{2EI} - \frac{269.5 \times 23.01 \times 70 \times 12}{2EI}
- \frac{21.4 \times (22.5 \times 12)^2}{2EI} + \frac{21.4 \times 22.5 \times (12)^2}{2EI}
\]
\[
- \frac{1}{2EI}(1,280,000 - 5,200,000 - 1,560,000 - 1,730,000)
= \frac{1}{2EI}(-7,210,000)
\]
\[
m_{rpC} = \frac{2EI \times 7,210,000}{70 \times 12} = 8,600 \text{ in-k}
\]

Then use moment distribution method:

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The final restraint moments are:

\[
M_{r_{LB}} = 6080 \text{ in-k} = 507 \text{ ft-k}
\]
\[
M_{r_{LC}} = 9860 \text{ in-k} = 822 \text{ ft-k}
\]

Summing up the \(M_{r_{DL}}\) and \(M_{r_{rp}}\), and multiplying the value of \((1 - e^\delta)\), then the final restraint moments due to creep \(M_{r_{RC}}\) can be computed.
APPENDIX V

Derivation of general equation for restraint moment due to shrinkage.

As previously discussed, the restraint moments due to differential shrinkage between the deck slab and the precast beam are uniform along the composite section. The moment is expressed as following:

$$m_{rs} = \epsilon_{s}E_{b}A_{b}\left(e_{2}^{' + \frac{t_{b}}{2}}\right)$$

Assuming the bridge were composed of four simple spans, these moments will cause bending as shown below;

Therefore the slope at the ends of the simple span beams, can be obtained by the conjugate beam method as follow:

$$\theta = \frac{m_{rs}L}{2EI}$$

Then apply moments $M_{rs}$, at both sides of support B, C and D to return the beam ends to horizontal;

Ext. span
\[ \theta_3 = \frac{M_{rs} L_1}{3EI} \]

\[ \theta_3 = 0 \]

\[ \frac{M_{rs} L_1}{3EI} = \frac{m_{rs} L_1}{2EI} \]

\[ M_{rs} = 3 \frac{m_{rs}}{2} \]

Int. span

\[ \theta_4 = \frac{M_{rs} L_2}{2EI} \]

\[ \theta_4 = 0 \]

\[ M_{rs} = m_{rs} \]

Then use the moment distribution method to distribute the restraint moments \( M_{rs} \) and the final shrinkage moment \( M_{RS} \) at supports B, C and D can be computed by multiplying by the value of \( \frac{1-e^{-\varphi}}{\varphi} \).

For example,

In exterior span \( M_{rs} \) is 1.5 \( m_{rs} \), interior is \( m_{rs} \).

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<td>1.216</td>
<td>1.216</td>
<td>1.216</td>
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</tbody>
</table>
Final moment at B and D are $1.216 \, m_{rs} \left( \frac{1-e^{-\varphi}}{\varphi} \right)$ and the final moments at C is $0.89 \, m_{rs} \left( \frac{1-e^{-\varphi}}{\varphi} \right)$
ILLEGIBLE DOCUMENT

THE FOLLOWING DOCUMENT(S) IS OF POOR LEGIBILITY IN THE ORIGINAL

THIS IS THE BEST COPY AVAILABLE
APPENDIX VI

C 83200. 1. INSTRUCTION TO FIND THE COORDINATE OF INFLUENCE
C LINE 3093 POSITIVE MOVEMENT AT G1 J=1 AT G, YG, AT EVERY TAB HEFT
C TIMES 24 SPAN(J)=FSM(0), O(00), 01(00), ..., 04(00), FSM(24), RL(24)
C SPAN(J)*2(J)
C T=1+SPAN
C 1 FORMAT (33)
C SPAX(J)=SPAN(J)+1,NSPAN
C 2 FORMAT (4(5.2))
C A=Z(NSPAN+1)
C AT=AT(J), J=1,NV)
C 3 FORMAT (1,7.4)
C SPAX(J)=J+7
C TL=...
C 15 CONTINUE
C DT=TL/SPMN
C AT
C UNIT=1.
C J=1
C SUM=SPAN(J)
C 13 LC 12 1=1,N
C 12 K=1(J)
C A=SPMN*UNIT
C 5 IF (A=SUM) 5,6,4
C 6 I=2
C =SUM=SPAN(J)-(SUM-A)
C FSM(I)=A*(SUM-A)**2/SPAN(J)**2
C FSM(I+1)=(FSM(I)*SUM-A)/SPAN(J)**2
C GO TO 11
C 4 J=J+1
C =SUM=SPAN(J)
C GO TO 5
C 10 LC 100 JJ=1,2
C 20 I=1,2
C 20 K=1(J)
C =SUM=(FSM(I)+FSM(I+1))*3E(I)
C GO TO 29
C 30 MET(1)=(SUM-0)^2*(SUM-0)^2
C GO TO 40
C 40 FSM(I+1)=FSM(I)+SUM(I)+SUM(I-1)*2.5
C IF (I=0)
C GO TO 50
C 50 FSM(I)=FSM(I)+SUM(I)+SUM(L)*2.5
C 100 CONTINUE
I = UNIT
K = J
     IF ((SG - A) - SPAN(J)/2. > 21,21,22
21 SL2(I) = (SG - A) / SPAN(J)
     GC TO 22
22 SL2(I) = 1. - (SG - A) / SPAN(J)
23 J1 = J
     GC 116 11 = 1, NSPAN
     PN(11) = .
115 PM(11) = .
     PP(J1) = SL2(I0) * SPAN(J) / 2.
     GC 117 J2 = 1, NSPAN
     J2 = J2
119 PM(J2) = (PM(1) - PM(I+1)) / 2.
     GC 118 K = 1, NSPAN
110 PM(K) = PM(K) + PM(K)
     PUNCH 7,L,PM(4),PM(6),PM(1),PM(2)
     FORMAT (13,4F14.7)
     IF (UNIT - PT) 51.52.52
51 UNIT = UNIT + 1.
     GC TO 13
52 CONTINUE
     STEP
     END
C PROGRAM 2
C USING THE OUTPUT OF PROGRAM 1 TO REARRANGE THE ORDINATES OF
C INFLUENCE LINES FOR AB AND BC AND FIND INFLUENCE LINES FOR
C POSITIVE MOMENTS AT MIDSPAN OF AB AND BC
DIMENSION X(120), R(120), C(120), D(120), E(120)
DO 10 I=2, 111
10 READ 1, (X(I), R(I), C(I), D(I), E(I))
1 FORMAT (15, 4F14.7)
K(I)=
R(I)=
C(I)=
D(I)=
E(I)=
PUNCH 3
2 FORMAT (37+THE ORDINATES OF INFLUENCE LINE OF MC)
PUNCH 2, (K(I), I=1, 111)
PUNCH 4
3 FORMAT (37+THE ORDINATES OF INFLUENCE LINE OF MC)
PUNCH 2, (R(I), I=1, 111)
PUNCH 5
4 FORMAT (37+THE ORDINATES OF POSITIVE MOMENT M1)
PUNCH 2, (C(I), I=1, 111)
PUNCH 6
5 FORMAT (37+THE ORDINATES OF POSITIVE MOMENT M2)
PUNCH 2, (D(I), I=1, 111)
SUPS=C.
DC 11 I=1, 56
11 SUPS=SU1$B$+B(I)
DO 50 I=72, 111
50 SUPS=SU1$B$+B(I)
SU1$C$=0.
DC 12 I=22, 91
12 SU1$C$=SU1$B$+C(I)
SU1$D$=0.
DC 13 I=1, 21
13 SU1$D$=SU1$B$+D(I)
DO 60 I=56, 91
60 SU1$D$=SU1$B$+D(I)
SU1$E$=0.
DC 14 I=22, 56
14 SU1$E$=SU1$B$+E(I)
DO 70 I=72, 111
70 SU1$E$=SU1$B$+E(I)
SU1$F$=0.
DC 20 I=56, 91
20 SU1$F$=SU1$B$+E(I)
30 SUNE=SUNE+C(I)
    SUHC=0.
30 40 I=91,111
31 SUMH=SUMH+C(I)
    SUMC=SUNE+SUMH
    PUNCH 7
7 FORMAT (10H THE MAXIMUM NEGATIVE MOMENTS DUE TO UNIFORM LOAD)
    PUNCH 6
8 FORMAT (22H MP MC//)
    PUNCH 9, SUMD,SUMC
9 FORMAT (2H 14.7)
    PUNCH 13
15 FORMAT (10H THE MAXIMUM POSITIVE MOMENTS DUE TO UNIFORM LOAD)
    PUNCH 16
16 FORMAT (22H MP N2//)
    PUNCH 17, SUMD,SUMC
17 FORMAT (2F14.7)
    PUNCH 18
18 FORMAT (10H POSITIVE MOMENTS OVER PIERS DUE TO UNIFORM LOAD)
    PUNCH 16
19 FORMAT (22H MP MCP//)
    PUNCH 21, SUMF,SUNE
21 FORMAT (2F14.7)
STOP
END
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SOME M  M2
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FIG. 29. INFLUENCE LINE FOR MOMENT AT SUPPORT B, MB.
FIG. 30. INFLUENCE LINE FOR MOMENT AT SUPPORT C, MC.
SCALES; SAME AS FIG. 29.

FIG. 31. INFLUENCE LINE FOR POSITIVE MOMENT AT CENTER OF SPAN AB, ML.
SCALES; SAME AS FIG. 29.

FIG. 32. INFLUENCE LINE FOR POSITIVE MOMENT AT CENTER OF SPAN BC, M2.
THE DESIGN PROCEDURES OF CONTINUOUS BRIDGE WITH PRECAST, PRESTRESSED CONCRETE GIRDER

by

MING-LEE CHANG

Diploma, Taipei Institute of Technology, 1963

A MASTER'S REPORT

submitted in partial fulfillment of the

requirement for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1970
In recent years, continuous highway bridges with precast-prestressed concrete girders have been used in many countries. The primary advantages of this type of bridge are the elimination of maintenance costs of deck joints and deck drainage onto the substructure and the structural economy of continuous design. Continuity also can improve the appearance and riding qualities of a bridge.

There are many different kinds of construction methods available to achieve continuity of precast-prestressed concrete bridge girders or slabs. In this report, continuity is achieved for live load plus impact moment by using nonprestressed reinforcement in the deck slab in the diaphragms over piers.

The effects of shrinkage and creep between the precast girder and the cast-in-place deck slab are more important than in other types of construction of continuous prestressed concrete bridges. The Research and Development Laboratory of the Portland Cement Association completed a research program on this kind of bridge during 1960-1961. Then the results of their research provided available information for designing a continuous precast-prestressed concrete bridge considering the effects of shrinkage and creep.

The main parts of this report are the design procedures for continuous precast-prestressed concrete bridges, including the influence of shrinkage and creep, and a design example of a four-span continuous bridge to illustrate the procedures.