A STUDY OF BEARING CAPACITY OF PILE FOUNDATION

by

JOHN IN-CHUNG YEN

Diploma, Taipei Institute of Technology, Taiwan, China, 1965

A MASTER'S REPORT

submitted in partial fulfillment of the requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1969

Approved by:

[Signature]
Wayne W. Williams
Major Professor
CONTENTS

INTRODUCTION 1

PURPOSE OF THE STUDY 2

SCOPE OF THE STUDY 2

REVIEW OF THE LITERATURE 3

DYNAMIC FORMULAS 3

DERIVATION OF GENERAL FORMULA 7

COMPARISON OF RESULTS USING DIFFERENT DYNAMIC FORMULAS 13

STATIC FORMULA 27

EVALUATION AND COMPARISON OF END BEARING AND FRICTIONAL RESISTANCES OF FRICION PILE 32

CONCLUSION 39

RECOMMENDATION FOR FUTURE WORK 40

APPENDIX 41

BIBLIOGRAPHY 44
ACKNOWLEDGEMENTS

The writer wishes to express his gratitude to a number of people for their help in the completion of this report. In particular, he is very grateful to Professor Wayne W. Williams and Dr. J. B. Blackburn who have given invaluable suggestions for the report; Mr. C. H. Chu for assistance in typing the manuscript; and, Mrs. Linda Bailey for typing the final copy.
INTRODUCTION

In the design of pile foundation the most important consideration is the evaluation of the bearing capacity of piles. Once the bearing capacity of a single pile is determined, either by dynamic formula or by static loading test, the engineer can then decide the number and length of piles needed for the foundation of structure and the effects of group reaction.

The evaluation of bearing capacity based on dynamic formulas has an advantage of economy both in time and in cost, but it has its disadvantage of giving unreliable result in most soil except in the non-cohesive sands and gravels. In these materials, the results from the formulas usually do not vary greatly from the results of full-size static loading test.

In the case of driving piles in plastic materials, such as soft clay or fine-grained silt, the relationship between the temporary resistance to driving and the permanent resistance to the working load on the pile varies greatly.

An exact criteria for the choice of dynamic formulas or static load testing is difficult to determine except by an understanding of soil characteristics and the different reactions of the pile under load.
PURPOSE OF THE STUDY

The purpose of this report was to investigate the bearing capacity of friction pile based on comparisons of results obtained from dynamic pile driving formulas and static pile loading tests.

SCOPE OF THE STUDY

The scope of the study included:

(a) A thorough research of the literature including a review of pile driving formulas and a study of the development and derivation of these formulas.

(b) A comparison of the results obtained from several selected formulas with assumed data to show variations of results.

(c) A general discussion of the static loading test.

(d) A suggestion for the evaluation of $Q_f$ vs. $Q_p$ by model pile tests.

(e) The principal consideration directed toward friction piles.
REVIEW OF THE LITERATURE

DYNAMIC FORMULAS

The earliest work on the bearing capacity of pile foundations indicated that the resistance of soil against the rapid penetration of a pile under the impact of the falling ram of the pile driver is theoretically proportional to the bearing capacity of the pile.

In about 1820, based on this relationship, Eytelwein proposed the first practical dynamic formula for pile driving:

\[ Q = \frac{W_h \cdot H}{S(1 + \frac{W_h}{W_p})} \]  \hspace{1cm} (1)

in which:

- \( Q \) = the bearing capacity of the pile,
- \( S \) = average penetration per blow,
- \( W_h \) = the weight of hammer,
- \( W_p \) = the weight of pile,
- \( H \) = height of fall of the hammer.

In 1851, Sander developed his formula based on the same concept that the energy from the falling hammer equals the work accomplished or

\[ E = \text{Work} = Q \cdot S \]

\( E \) is the energy from the falling hammer.
Since \[ E = (\text{weight of hammer}) \times (\text{Height of hammer falling}) \]
\[ = W_h \times H \]
we have thus the Sander's formula for dynamic driving of pile as:
\[ Q = \frac{W_h \times H}{S} \]  
(2)

The formula from Eytelwein and from Sander are used for the ideal condition in which there is no loss of energy during driving. In actuality an amount of energy is lost due to

1) Elastic deformation of pile,
2) Plastic deformation of pile,
3) Elastic deformation of soil,
4) Plastic deformation of soil,
5) The efficiency of the driving equipment.

In considering the loss of energy in driving of pile, Sander modified his formula to

\[ Q = \frac{W_h \times H}{S + C} \]  
(3)

C is a constant from experimental data for energy loss.

In 1859, Redtenbacher derived a formula based on Newton's Impact Theory and set some assumptions which neglected some of the driving energy loss. His formula is

\[ Q = \frac{SAE}{L} \left[ \frac{1 + \frac{2W_h^2}{W_h + W_p}}{S \cdot A \cdot E \cdot (W_p + W_h)} - 1 \right] \]  
(4)
In 1888, Wellington determined the value of $C$ for Sandor's formula by gathering experimental field data determining that $C = 1.0$ for a simple drop hammer and $C = 0.1$ for single acting hammer. In the formula, $S$ and $H$ are in terms of inches.

By comparing the results found by his formula with the results from actual static loading test, Wellington proposed the most commonly used "Engineering News Formula" by setting a safety factor $1/6$ in (3), which gives:

$$Q = (1/6) \cdot \frac{W \cdot H}{S+1}$$

for a drop hammer, and

$$Q = (1/6) \cdot \frac{W \cdot H}{S+0.1}$$

for single acting hammer.

In the early 1900's, this Engineering News Formula was widely used in the United States and in Europe.

In 1930, A. Hiley published through the Institute of Civil Engineering a pile driving formula which considered every loss possible and is known as the "Complete" pile driving formula

$$Q = \frac{(S+S_f)AE}{CL} \left[ \sqrt{1 + \frac{2 \cdot C \cdot L \cdot e_1 \cdot W \cdot H}{(S+S_f)^2 AE} \cdot \frac{W_h \cdot H + n^2 W_p}{W_h + W_p}} - 1 \right]$$

He simplified it by assuming $C = 1$
\[ Q = \frac{(S+S_f)AE}{L} \left[ \sqrt{1 + \frac{2 Le}{(S+S_f)AE} \cdot \frac{W_h}{W_h + W_p} \cdot \frac{W + n^2 W_p}{W_h + W_p} - 1} \right] \] (68)

Referring to equation (4), it is obvious that Redtenbacher's formula is the same as Hiley's by the simplifying assumptions that

\[ S_f = 0, \]

\[ e_1 = 1, \text{ and} \]

\[ n = 0. \]
DERIVATION OF THE GENERAL FORMULA

The simplest form of the driving formulas is the original Hiley's dynamic formula which is based on:

Driving energy = Work of pile penetration + energy losses

We know that the energy supplied by the hammer is

\[ E_0 = W_h \cdot H \] (7)

\( W_h \) = weight of hammer
\( H \) = falling height of hammer.

It is reasonable to expect some energy lost by the falling of hammer. If \( e_1 \) is the efficiency of the hammer fall then the energy reaching the pile is

\[ E_1 = e_1 \cdot E_0 = e_1 \cdot W_h \cdot H = e_1 \cdot W_h \cdot \frac{v_h^2}{2g} \] (8)

\( V_h \) = the velocity of hammer prior to striking.

According to Newton's impact relationship, the efficiency of impact is:

\[ e_2 = \frac{W_h \cdot v_h^2 + W_p \cdot v_p^2}{W_h \cdot \frac{v_h^2}{2g} + W_p \cdot \frac{v_p^2}{2g}} \] (9)

for which

\( V_p \) = velocity of pile prior to being struck,
\( v_p = \text{velocity of pile after being struck,} \)
\( v_h = \text{velocity of hammer after striking pile.} \)

The denominator is the energy before striking and the numerator is the energy after striking.

The impulses of the hammer and pile are equal during contact. The momentum change, which is equal to the impulse, is the product of mass and velocity change. Assuming both the hammer and the pile are free, massive bodies (which is not strictly true for the pile) we have

\[
\frac{W_h}{g}(v_h - v_p) = \frac{W_p}{g}(v_p - v_p) \tag{10}
\]

Another important coefficient that is needed is "Newton's coefficient of elastic restitution", defined as

\[
n = \frac{v_p - v_h}{v_h - v_p} \tag{11}
\]

From experimental data, constant values for \( n \) have been discovered which vary according to the material of the piles, but are usually in the range of 0.25 to 0.55. Reference for the \( n \) value are shown in Appendix (B).

From Eq. (10) and (11), since \( v_p = 0 \), we have

\[
W_h(v_h - v_p) = W_p(-v_p) \tag{10a}
\]

and

\[
nv_h - nv_p + v_h = 0 \tag{11a}
\]
From these two equations, \( v_p \) and \( v_h \) can be expressed in terms of \( n \) and \( v_h \),

\[
v_p = \frac{(1 + n) \, w_h}{n \, w_h - w_p} \cdot v_h
\]

\[
v_h = \frac{w_h + w_p}{n w_h - w_p} \cdot n \cdot v_h
\]

Substituting (12) into (9),

\[
e_2 = \frac{w_h \cdot v_h^2 + w_p \cdot v_p^2}{w_h \cdot v_h^2}
\]

\[
e_2 = \frac{w_h \cdot v_h^2 \cdot n^2 \left( \frac{w_h + w_p}{n w_h - w_p} \right)^2 + w_p \cdot v_p^2 \left( \frac{1 + n \, w_h}{n w_h - w_p} \right)^2}{w_h \cdot v_h^2}
\]

which can be simplified to:

\[
e_2 = \frac{w_h + n^2 \, w_p}{w_h + w_p}
\]

(13)

By the definition of \( e_2 \), the energy available after impact with the pile is

\[
E_2 = e_2 \, E_1 = e_1 \, e_2 \, E_0 = e_1 \, E_0 \left( \frac{w_h + n^2 \, w_p}{w_h + w_p} \right)
\]

\[
= e_1 \, w_h \, H \left( \frac{w_h + n^2 \, w_p}{w_h + w_p} \right)
\]

The amount of energy \( E_2 \) is used up in the useful work of forcing the pile into the ground \( (Q \cdot S) \), in losses due to crushing of material at the pile head \( (Q \cdot S_f) \), and in losses
due to the elastic compression of the soil-pile system
\((Q \cdot S_e)\). Thus

\[E_2 = Q(S + S_f + S_e)\] (15)

In considering the elastic compression of pile, there are two different types of compression forces to balance the loading from the top of the pile. The first type is the end bearing force and the other is the side frictional force. For frictional pile, the end bearing is small compared to the side frictional force and can be neglected. When we evaluate the elastic compression of friction pile the variance of frictional stress is based on the assumption that static earth pressure is increasing with depth and the magnitude of frictional stress is in proportion to the normal pressure on the pile surface which is actually static lateral earth pressure from the surrounding soil.

If we let \(\sigma\) be the maximum frictional stress which reasonably exists at point \(B\). For every element of the pile with length \(dx\) will have a compression \(d(S_e)\), while

\[d(S_e) = (L - x)\left(\frac{\sigma + \frac{x\sigma}{L^2}}{2}\right) \frac{\pi d \cdot dx}{AE}\]

\[(S_e)l = \int (L - x)\left(\frac{\sigma + \frac{x\sigma}{L^2}}{2}\right) \frac{\pi d \cdot dx}{AE}\]
Fig. 4 Friction pile under load.

\[(S_e)_{1} = \frac{\pi d \sigma}{2AE} \int_{0}^{L} (L - x)(L + x) \, dx\]

\[= \frac{\pi d \sigma}{2AE} \int_{0}^{L} (L^2 - x^2) \, dx\]

\[= \frac{\pi d \sigma}{2AE} \left[ L^2 x - \frac{x^3}{3} \right]_{0}^{L}\]

\[= \frac{\pi d \sigma}{2AE} \cdot \frac{2L^3}{3}\]

\[= \frac{2}{3} \cdot \frac{P \cdot L}{A \cdot E}
\]

P = \frac{\pi d L}{2}

More elementary than the above case, is an end bearing pile loaded by axial loading P with a reaction force P at the pile tip and no frictional force along the pile surface. When we calculate the elastic compression of pile for this case:
\[(S_e)_1 = \frac{P \cdot L}{A \cdot E} = (1) \frac{P \cdot L}{A \cdot E}\]

In actuality, there are no pile which takes loading only by side friction or by end bearing but most are a combination of both. If we let:

\[(S_e) = C \frac{P \cdot L}{A \cdot E}\]  \hspace{1cm} (16)

It is obvious that \(C\) should be between 0.67 to 1.00, for absolute end bearing pile \(C = 1.00\) and for absolute friction pile \(C = 0.67\).

Combining Eq. (14), (15) and (16) to obtain \(Q_u\):

\[Q_u = \frac{e_1 \cdot W \cdot H}{S + S_f + \frac{C \cdot Q_u \cdot L}{A \cdot E}} \cdot \frac{W_h + n^2 \cdot W_p}{W_h + W_p}\]

Rearranged as:

\[Q_u = \frac{(S + S_f) \cdot A \cdot E}{C \cdot L} \left[ \sqrt{1 + \frac{2CLE_1W \cdot H}{(S + S_f)^2 AE} \cdot \frac{W_h + n^2 \cdot W_p}{W_h + W_p}} - 1 \right]\]

This is the basic, general form of dynamic formula based on Newton's theory of impaction. Many simplified formulas were obtained by modifying this formula with various assumptions. The results from the original formula and the simplified formulas vary because of these assumptions. A numerical test for several formulas is given in the next section. A discussion concerning those assumptions is also included in the next section.
COMPARISON OF RESULTS USING DIFFERENT DYNAMIC FORMULAS

Three dynamic formulas (1) the general formula from Hiley, (2) that of Redtenbacher and (3) the Engineering News Formula, are considered in this comparison by substituting comparable numerical data into each formula.

Resulting curves of "bearing capacity Q" vs. "the average penetration per blow S" are plotted and the assumptions are discussed based on the results of calculated ultimate bearing capacity obtained from the three different formulas.

To show how the results differed for short, medium, and long piles, three different size of precast concrete piles were used; 50 ft with 10 inch diameter, 70 ft with 12 inch diameter, and 100 ft with 14 inch diameter. Three drop hammer of different weight were used for each pile; 2,500 lb for 50 ft pile, 3,500 lb for 70 ft pile, and 6,000 lb for 100 ft pile.

A computer program was prepared for these tests and three different sets of input data were made for the three piles. The data used are listed below:

\[ E = \text{modulus of elasticity} = 3 \times 10^6 \text{ psi} \]
\[ e_1 = \text{efficiency of hammer fall} = 0.75 \text{ for drop hammer} \]
\[ H = \text{height of hammer drop} = 15 \text{ ft}=180 \text{ inch} \]
\[ n = \text{Newton’s coefficient of elastic restitution} \]
\[ = 0.4 \text{ for concrete pile} \]
\[ C = \text{constant of the elastic deformation of pile under load} \]
\[ S_f = 0.67 \text{ for friction pile.} \]

\[ S_f = \text{non-elastic crushing of pile per blow} \]
\[ = (0.1) S \text{ assumed} \]

\[ S = \text{average penetration of pile per blow} \]
\[ = 0.1, 0.2, \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 3.9, 4.0. \]

\[ L = \text{length of the piles:} \]

\[ L_1 = 50 \text{ ft} = 600 \text{ inch}, \]
\[ L_2 = 70 \text{ ft} = 840 \text{ inch}, \]
\[ L_3 = 100 \text{ ft} = 1200 \text{ inch}. \]

\[ A = \text{average cross sectional area of piles:} \]

\[ A_1 = 78.5 \text{ sq in.}, \]
\[ A_2 = 113.0 \text{ sq in.}, \]
\[ A_3 = 158.0 \text{ sq in.} \]

\[ W_p = \text{weight of piles:} \]

\[ W_{p1} = 4,100 \text{ lb}, \]
\[ W_{p2} = 8,240 \text{ lb}, \]
\[ W_{p3} = 19,800 \text{ lb}. \]

\[ W_h = \text{weight of hammer:} \]

\[ W_{h1} = 2,500 \text{ lb}, \]
\[ W_{h2} = 3,500 \text{ lb}, \]
\[ W_{h3} = 6,000 \text{ lb}. \]

The formulas used are:

1) General form of dynamic formula from Hiley,

\[ C_h = \frac{(S+S_f) AE}{CL} \left[ \sqrt{1 + \frac{2Le_1\frac{W_h H}{(S+S_f)^2 AE} \cdot \frac{W_h n^2 W_p}{W_h + W_p}} - 1} \right] \]

2) Radtenbacker's formula,

\[ C_r = \frac{S AE}{L} \left[ \sqrt{1 + \frac{2L W_h^2 H} {S^2 AE (W_h + W_p)}} - 1 \right] \]
3) Engineering News Formula,

\[ Q_e = (1/6) \frac{W_h \cdot H}{S + 1} \]

Three tables and charts for 50 ft, 70 ft, and 100 ft piles are made when the data are substituted into the above formulas. From the charts it is obvious that the length of piles didn't influence the shape of the curves greatly. For all the three formulas, the curves are almost straight lines and shown linear relationship between \( Q \) and \( S \) for very soft soil. That is, when the average penetrations of piles are more than 2.5 inch, it will be found that \( Q = c_1 S \) and \( c_1 \) is a constant.

When \( s \) decreases from \( S = 2.5 \) inch, the curves turn slightly right and have higher bearing capacity for decreasing \( S \) values. The curves are more curved when \( s \) is about 1.5" to 0.5".

The curve from the general formula and that from Redtenbacker's are about the same shape and the values of \( Q_h \) are always higher than the values of \( Q_r \) for all kinds of soil conditions (from very hard to very soft).

If we let \( Q_h = X \cdot Q_r \), the curves shown \( X \) vs. \( s \) can be made as the chart on page 22.

For the shortest pile, the values of \( X \) vary from 1.43 when \( S = 0.5" \) to 1.29" when \( S = 4.0" \). The difference of the maximum to minimum values of \( X \) is only 0.14 which is small when compared to the variance of \( S \).
Table 1. Bearing capacities of pile for variance of average penetration when L = 50 ft.

<table>
<thead>
<tr>
<th>C</th>
<th>C COMPUTER PROGRAM FOR PILE FORMULAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>QH</td>
</tr>
<tr>
<td>.10</td>
<td>735.35</td>
</tr>
<tr>
<td>.20</td>
<td>208.89</td>
</tr>
<tr>
<td>.30</td>
<td>186.00</td>
</tr>
<tr>
<td>.40</td>
<td>166.36</td>
</tr>
<tr>
<td>.50</td>
<td>147.56</td>
</tr>
<tr>
<td>.60</td>
<td>135.26</td>
</tr>
<tr>
<td>.70</td>
<td>122.92</td>
</tr>
<tr>
<td>.80</td>
<td>112.37</td>
</tr>
<tr>
<td>.90</td>
<td>103.27</td>
</tr>
<tr>
<td>1.00</td>
<td>95.37</td>
</tr>
<tr>
<td>1.10</td>
<td>88.49</td>
</tr>
<tr>
<td>1.20</td>
<td>82.45</td>
</tr>
<tr>
<td>1.30</td>
<td>77.12</td>
</tr>
<tr>
<td>1.40</td>
<td>72.40</td>
</tr>
<tr>
<td>1.50</td>
<td>68.16</td>
</tr>
<tr>
<td>1.60</td>
<td>64.41</td>
</tr>
<tr>
<td>1.70</td>
<td>61.01</td>
</tr>
<tr>
<td>1.80</td>
<td>57.93</td>
</tr>
<tr>
<td>1.90</td>
<td>55.14</td>
</tr>
<tr>
<td>2.00</td>
<td>52.60</td>
</tr>
<tr>
<td>2.10</td>
<td>50.27</td>
</tr>
<tr>
<td>2.20</td>
<td>48.13</td>
</tr>
<tr>
<td>2.30</td>
<td>46.17</td>
</tr>
<tr>
<td>2.40</td>
<td>44.35</td>
</tr>
<tr>
<td>2.50</td>
<td>42.67</td>
</tr>
<tr>
<td>2.60</td>
<td>41.10</td>
</tr>
<tr>
<td>2.70</td>
<td>39.65</td>
</tr>
<tr>
<td>2.80</td>
<td>38.29</td>
</tr>
<tr>
<td>2.90</td>
<td>37.02</td>
</tr>
<tr>
<td>3.00</td>
<td>35.83</td>
</tr>
<tr>
<td>3.10</td>
<td>34.72</td>
</tr>
<tr>
<td>3.20</td>
<td>33.67</td>
</tr>
<tr>
<td>3.30</td>
<td>32.68</td>
</tr>
<tr>
<td>3.40</td>
<td>31.74</td>
</tr>
<tr>
<td>3.50</td>
<td>30.86</td>
</tr>
<tr>
<td>3.60</td>
<td>30.02</td>
</tr>
<tr>
<td>3.70</td>
<td>29.23</td>
</tr>
<tr>
<td>3.80</td>
<td>28.48</td>
</tr>
<tr>
<td>3.90</td>
<td>27.77</td>
</tr>
<tr>
<td>4.00</td>
<td>27.09</td>
</tr>
</tbody>
</table>

0 STOP END OF PROGRAM AT STATEMENT 00006 + 00 LINES
Fig. 5. Curves of bearing capacity (Q) vs. average penetration (S) when L = 50 ft.
Table 2. Bearing capacities of pile for variance of average penetration when \( L = 70 \) ft.

<table>
<thead>
<tr>
<th>C</th>
<th>C</th>
<th>COMPUTER PROGRAM FOR PILE FORMULAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>263.97</td>
<td>175.54</td>
</tr>
<tr>
<td>0.20</td>
<td>236.33</td>
<td>196.46</td>
</tr>
<tr>
<td>0.30</td>
<td>212.14</td>
<td>143.34</td>
</tr>
<tr>
<td>0.40</td>
<td>191.11</td>
<td>130.03</td>
</tr>
<tr>
<td>0.50</td>
<td>172.90</td>
<td>118.38</td>
</tr>
<tr>
<td>0.60</td>
<td>157.15</td>
<td>108.18</td>
</tr>
<tr>
<td>0.70</td>
<td>143.53</td>
<td>99.27</td>
</tr>
<tr>
<td>0.80</td>
<td>131.72</td>
<td>91.47</td>
</tr>
<tr>
<td>0.90</td>
<td>121.45</td>
<td>84.63</td>
</tr>
<tr>
<td>1.00</td>
<td>112.47</td>
<td>78.60</td>
</tr>
<tr>
<td>1.10</td>
<td>104.59</td>
<td>73.28</td>
</tr>
<tr>
<td>1.20</td>
<td>97.65</td>
<td>68.55</td>
</tr>
<tr>
<td>1.30</td>
<td>91.49</td>
<td>64.35</td>
</tr>
<tr>
<td>1.40</td>
<td>86.00</td>
<td>60.58</td>
</tr>
<tr>
<td>1.50</td>
<td>81.09</td>
<td>57.20</td>
</tr>
<tr>
<td>1.60</td>
<td>76.68</td>
<td>54.15</td>
</tr>
<tr>
<td>1.70</td>
<td>72.70</td>
<td>51.39</td>
</tr>
<tr>
<td>1.80</td>
<td>69.09</td>
<td>48.88</td>
</tr>
<tr>
<td>1.90</td>
<td>65.80</td>
<td>46.59</td>
</tr>
<tr>
<td>2.00</td>
<td>62.81</td>
<td>44.50</td>
</tr>
<tr>
<td>2.10</td>
<td>60.06</td>
<td>42.58</td>
</tr>
<tr>
<td>2.20</td>
<td>57.53</td>
<td>40.81</td>
</tr>
<tr>
<td>2.30</td>
<td>55.20</td>
<td>39.18</td>
</tr>
<tr>
<td>2.40</td>
<td>53.05</td>
<td>37.66</td>
</tr>
<tr>
<td>2.50</td>
<td>51.05</td>
<td>36.26</td>
</tr>
<tr>
<td>2.60</td>
<td>49.20</td>
<td>34.95</td>
</tr>
<tr>
<td>2.70</td>
<td>47.47</td>
<td>33.74</td>
</tr>
<tr>
<td>2.80</td>
<td>45.85</td>
<td>32.60</td>
</tr>
<tr>
<td>2.90</td>
<td>44.34</td>
<td>31.53</td>
</tr>
<tr>
<td>3.00</td>
<td>42.93</td>
<td>30.53</td>
</tr>
<tr>
<td>3.10</td>
<td>41.60</td>
<td>29.59</td>
</tr>
<tr>
<td>3.20</td>
<td>40.35</td>
<td>28.71</td>
</tr>
<tr>
<td>3.30</td>
<td>39.17</td>
<td>27.87</td>
</tr>
<tr>
<td>3.40</td>
<td>38.05</td>
<td>27.09</td>
</tr>
<tr>
<td>3.50</td>
<td>37.00</td>
<td>26.34</td>
</tr>
<tr>
<td>3.60</td>
<td>36.00</td>
<td>25.63</td>
</tr>
<tr>
<td>3.70</td>
<td>35.06</td>
<td>24.96</td>
</tr>
<tr>
<td>3.80</td>
<td>34.16</td>
<td>24.33</td>
</tr>
<tr>
<td>3.90</td>
<td>33.30</td>
<td>23.72</td>
</tr>
<tr>
<td>4.00</td>
<td>32.49</td>
<td>23.15</td>
</tr>
</tbody>
</table>

STOP END OF PROGRAM AT STATEMENT 6666 + 00 LINES
Fig. 6. Curves of bearing capacity \( Q \) vs. average penetration \( S \) when \( L = 70 \text{ ft} \).
Table 3. Bearing capacities of pile for variance
average penetration when L = 100 ft.

<table>
<thead>
<tr>
<th>C</th>
<th>C</th>
<th>COMPUTER PROGRAM FOR PILE FORMULAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>DH</td>
<td>OR</td>
</tr>
<tr>
<td>10</td>
<td>324.91</td>
<td>203.85</td>
</tr>
<tr>
<td>20</td>
<td>298.87</td>
<td>186.70</td>
</tr>
<tr>
<td>30</td>
<td>271.54</td>
<td>171.22</td>
</tr>
<tr>
<td>40</td>
<td>249.06</td>
<td>157.32</td>
</tr>
<tr>
<td>50</td>
<td>228.92</td>
<td>144.88</td>
</tr>
<tr>
<td>60</td>
<td>211.02</td>
<td>133.78</td>
</tr>
<tr>
<td>70</td>
<td>195.11</td>
<td>123.89</td>
</tr>
<tr>
<td>80</td>
<td>186.97</td>
<td>115.07</td>
</tr>
<tr>
<td>90</td>
<td>168.39</td>
<td>107.21</td>
</tr>
<tr>
<td>100</td>
<td>157.18</td>
<td>100.18</td>
</tr>
<tr>
<td>110</td>
<td>147.16</td>
<td>93.88</td>
</tr>
<tr>
<td>120</td>
<td>138.19</td>
<td>88.23</td>
</tr>
<tr>
<td>130</td>
<td>130.13</td>
<td>83.14</td>
</tr>
<tr>
<td>140</td>
<td>122.86</td>
<td>78.54</td>
</tr>
<tr>
<td>150</td>
<td>116.28</td>
<td>74.38</td>
</tr>
<tr>
<td>160</td>
<td>110.32</td>
<td>70.60</td>
</tr>
<tr>
<td>170</td>
<td>104.89</td>
<td>67.16</td>
</tr>
<tr>
<td>180</td>
<td>99.93</td>
<td>64.01</td>
</tr>
<tr>
<td>190</td>
<td>95.39</td>
<td>61.12</td>
</tr>
<tr>
<td>200</td>
<td>91.22</td>
<td>58.46</td>
</tr>
<tr>
<td>210</td>
<td>87.38</td>
<td>56.02</td>
</tr>
<tr>
<td>220</td>
<td>83.83</td>
<td>53.76</td>
</tr>
<tr>
<td>230</td>
<td>80.55</td>
<td>51.66</td>
</tr>
<tr>
<td>240</td>
<td>77.50</td>
<td>49.72</td>
</tr>
<tr>
<td>250</td>
<td>74.67</td>
<td>47.91</td>
</tr>
<tr>
<td>260</td>
<td>72.03</td>
<td>46.22</td>
</tr>
<tr>
<td>270</td>
<td>69.56</td>
<td>44.64</td>
</tr>
<tr>
<td>280</td>
<td>67.25</td>
<td>43.17</td>
</tr>
<tr>
<td>290</td>
<td>65.08</td>
<td>41.78</td>
</tr>
<tr>
<td>300</td>
<td>63.05</td>
<td>40.48</td>
</tr>
<tr>
<td>310</td>
<td>61.13</td>
<td>39.25</td>
</tr>
<tr>
<td>320</td>
<td>59.33</td>
<td>38.10</td>
</tr>
<tr>
<td>330</td>
<td>57.52</td>
<td>37.00</td>
</tr>
<tr>
<td>340</td>
<td>56.01</td>
<td>35.97</td>
</tr>
<tr>
<td>350</td>
<td>54.48</td>
<td>34.99</td>
</tr>
<tr>
<td>360</td>
<td>53.04</td>
<td>34.07</td>
</tr>
<tr>
<td>370</td>
<td>51.66</td>
<td>33.19</td>
</tr>
<tr>
<td>380</td>
<td>50.36</td>
<td>32.35</td>
</tr>
<tr>
<td>390</td>
<td>49.12</td>
<td>31.55</td>
</tr>
<tr>
<td>400</td>
<td>47.93</td>
<td>30.79</td>
</tr>
</tbody>
</table>

O  STOP  END OF PROGRAM AT STATEMENT CO06 + 00 LINES
Fig. 7. Curves of bearing capacity ($Q$) vs. average penetration ($S$) when $L = 100$ ft.
Fig. 8.
The data plotted in Figure 8 show that the value of $X$ is essentially independent of changes in $S$. It can therefore be assumed that

$$Q_h = \bar{X} Q_r$$

where

$\bar{X}$ is a constant and lies between 1.30 to 1.60.

By investigating the assumptions that simplified the General formula to Redtenbacker's formula, it seems that the general formula will result in more accurate values of $Q$ for the following reasons:

(a) The real value of $C$ is between 1.00 and 0.67 depending on soil condition of the driving location. It can not be assumed that $C = 1.00$ since generally, for friction piles, the point resistance is not effective. In addition to this, the area of pile head is small because in practice the area of the cross section of a pile decreases from the top to the bottom. Since we used different values of $C$ for both formulas, it appears that the $Q_r$ value from Redtenbacker's formula is about 30% less than $Q_h$ from Hiley's. From the stand point of safety, Redtenbacker's formula is more conservative.

(b) $e$ is the efficiency of hammer fall. Many experimental values of $e_1$ for different kinds of hammer were found and are listed in Appendix B of the report. These efficiencies are for drop hammer actuated by rope
and friction winch but this figure may decrease when the drop is small and increase somewhat if the drop is very large. For the assumption, although \( e_1 = 100\% \) is high, using \( e_1 = 75\% \) does not greatly affect \( Q \) and it is clear that \( e_1 = 100\% \) is a more reasonable assumption than \( C = 1 \).

(c) \( n \), Newton's coefficient of elastic restitution, varies from 0.25 to 0.55 according to the materials used in the pile for example, concrete, wood, steel etc. For this test, \( n = 0.40 - 0.45 \) for concrete pile without cushion on pile top. Since \( n^2 = 0.16 - 0.22 \) and for most cases \( \frac{W_p}{W_h} \), the value \( n^2 \cdot W_p \) is significant and it seems wise to keep it in the formula rather than letting \( n = 0 \).

(d) The value of non-elastic crushing of the pile per blow, \( (S_f) \), is a typical experimental figure and is very hard to determine from experimental tables. The only way suggested is to measure this value in the field after each blow of hammer. In concrete pile, for example, we found \( S_f \) is almost equal to zero when there is a cushion block upon the pile top and it still be very small when there is no cushioning. Redtenbacher's assumption to let \( S_f = 0 \) is reasonable.

The comparison chart on page 22 shows \( Q_r \) is about 30% lower than \( Q_h \). It is wise to use Redtenbacher's formula when safety design is wanted.
Actually the general formula may not be more accurate than the simplified one since the general form itself is derived from theory under ideal soil condition which is not true for the practical soil conditions.

Recalling the charts of bearing capacity vs. average penetration per blow, we found $Q_e$ after the Engineering News formula is very low compared to $Q_h$ and $Q_r$. The tables and charts shown that $Q_e$ is about 25-30% of $Q_r$ for soft soil and is about 30-50% of $Q_r$ for hard soil. If the bearing capacity after Redtenbacher's formula is reliable it is obvious that the factor of 1/6 in Engineering News formula is too conservative.

Going back to equations (14) and (15) on pages 9 and 10, the first type of Engineering News formula is formed from the relation function

$$E_2 = Q_u (S + S_f + S_e)$$

$$= e_1 \cdot e_2 \cdot W_h \cdot H$$

Energy = Energy

By assuming $e_1 = 1$, $e_2 = 1$, and $S_e + S_f = 1$, it gives

$$Q_e = \frac{W_h \cdot H}{S + 1}$$

for drop hammer.

We can understand that bearing capacity from this formula is high because it neglects energy loss due to impact by
assuming $e_2 = 1$ and neglects energy loss due to the hammer dropping by assuming $e_1 = 1$.

A factor of safety $1/6$ was used to justify bearing capacity from the original type of Engineering News formula. Once again, that factor is low and it results in an over safe design.
STATIC FORMULA

The evaluation of bearing capacity of pile by static formula (static loading test) is recognized to be the most accurate method for determining safe working loads. Karl Terzaghi understood that any dynamic formula used to estimate bearing capacity of piles, even the most elaborate one, would not be accurate because every dynamic formula involved various arbitrary assumptions with unknown practical implications. Most important of all, the soil conditions in nature vary greatly. Terzaghi concluded that the reliable way to determine the bearing capacity of a pile was to make load tests on "full-size" test piles in the field. The only limitation of the single pile load test was its application to pile groups.

The basic form of static formula is

\[ Q_u = Q_f + Q_p \]

\( Q_u \) = ultimate bearing capacity of pile,
\( Q_f \) = the part of bearing capacity from skin friction,
\( Q_p \) = the part from end point resistance.

The value of \( Q_u \) can be obtained by the testing results, from static load test but the determination of the ratio between \( Q_f \) and \( Q_p \) is impossible to determine. \( Q_f \) is based on the value of skin friction and

\[ Q_p = Q_u - Q_f \]
Friction value between the pile and soil depend on:

1) Type of soil,
2) Depth of embedment,
3) Degree of natural consolidation and saturation,
4) Shape of the pile,
5) Amount of compaction by pile,
6) The time interval between driving and testing.

To study the factors affecting friction between piles and soil, the first essential is complete boring logs and test data including the unit weight of the soil, cohesion, and friction of the soil.

For simplification, the skin friction for different kinds of soil will be discussed separately.

Sand and Gravel

The unit value of skin friction and point bearing for a pile in sand increases with increasing depth. In dense sand, driving resistance may reach refusal in a very few feet. In loose sand, piles may be driven long distances with little or no resistance.

Of the total amount of friction that may develop in sand and gravel, except in quick conditions, according to suggestion by Hiley (6), one-half of the total amount may be taken as being operative during driving, and consequently included in the driving resistance.

It has been observed that if piles are driven in a saturated coarse-grained previous soil as sand and gravel, they
may lose, upon redriving, up to 40 to 50 percent of their resistance in 24 hr. This is probably a consequence of the compaction of the sand during driving, after which the sand has a chance to absorb water and readjust itself. An example has been reported by Perrin in which 40 ft wood piles driven 22 ft into wet sand gave a good immediate test; but on the next day all resistance had disappeared and it was necessary to drive another 10 ft.

On the other hand, piles may drive very easily in loose submerged fine uniform sand strata, which however, are firm under the steady pressure of the working load and are capable of sustaining large quiescent load both in end bearing and friction.

**Silt and Clay**

The unit value of skin friction in soft silt is low during driving, because of liquefaction, but within a few hours or days the silt apparently regains its original strength by dissipation of the excess pore pressure in the soil. The unit value of skin friction for a pile in soft clay depends upon the properties of the clay such as density, moisture content and unit cohesion. Point resistance is considered to be negligible in soft clay. While driving resistance may remain small and be fairly constant with depth, skin friction that will develop varies, in general, with depth but greatly increases as time passes after driving.
The unit value of skin friction for a pile in medium hard and hard clay may vary widely for the same clay, depending on the method used in placing the pile. Driving may have remolded the soil to such an extent that the original structure is broken down and the clay loses strength by remolding as compared with a pile cast in a bored hole. Excess hydrostatic pressure may prevent bond between the pile and the soil to develop over very long period of time.

In hard brittle clay driving causes fracturing which causes a definite loss of strength with time due to a softening of the broken and porous clay by the ground water.

Because a certain type of pile has been tested and found a given friction value in a particular soil, it does not follow that a pile placed by some other method would sustain an equal load, even if the elapsed time period is the same. The time factor is important because bearing capacity is likely to increase with time as the water dissipates or decrease in the hard brittle clays.

Therefore in the firm clays load tests should be made at as long an interval of time as possible after driving but can be made immediately after a cast concrete pile has gained the necessary strength.

From the above understanding of friction value, both in sand and in clay, it can be shown by the following graph of \( Q \) vs. \( T \) where \( T \) is the time interval between driving and load test, \( Q \) is the bearing capacity.
A: Saturated clay and silt.
B: Saturated sand

Fig. 9
EVALUATION AND COMPARISON OF END BEARING AND FRICIONAL
RESISTANCES OF FRICITION PILE

It was shown in a previous section that for a static loading test,

Ultimate bearing capacity \( (Q_u) \) = Point bearing resistance \( (Q_p) \) + Skin frictional resistance \( (Q_f) \).

Among the three quantities, the variance of \( Q_f \) has been discussed in the previous section but the actual value is very difficult to determine. For the purpose of analyzing the variance of \( Q_f/Q_p \) for a pile under load testing, a paper by B. B. Broms and L. Hellman (1) is worthy of discussion.

Broms and Hellman worked out their report by using a special device that can be used to separate the point resistance from the total applied load by measuring the compression of the lower part of the test pile. Since 1964, that device has been used extensively in Sweden during load tests on driven precast reinforced concrete piles. This was also used by Van Doren for cast-in-place pile in Wichita, Kansas for Kansas Highway Commission and the technique used may be obtained from them.

The measuring device is shown in Fig. 10 and Fig. 11. It consists of a 3/4" standard pipe with an expander unit at its lower end, a central rod, and a locking device. After
driving the test pile, the measuring unit is inserted into a thin walled seamless steel pipe which is cast in the pile and is 1.65" inside diameter. The device is shown in Fig. 11.

When the measuring unit with its expander has been lowered to approximately 10 ft to 15 ft above the pile point, the expander is released through the center rod. The expander locks the 3/4 inch pipe to the thin-walled pipe. The central rod is therefore lowered to the bottom of the pile and tapped lightly by a hammer in order to seat the rod on the pile point. The displacement of pile point and of the point where the 3/4 inch pipe is attached to the pile can be measured by dial indicator attached to the head of the pile. It is thus possible, with this arrangement, to measure the compression of a known length of the lower part of the pile.

The load $Q_p$ transferred to the bottom section of the pile, can then be calculated from the displacement difference $\Delta L$, between the two points and from the elastic properties of the pile materials by the formula

$$Q_p = \frac{L \cdot \Lambda \cdot E_{eq}}{L}$$  \hspace{1cm} (17)

where $\Lambda$ = area of the pile section.

$L$ = length of the pile between the expander and the pile point.

$E_{eq}$ = equivalent modulus of elasticity defined by the equation $E_{eq} = (A_s E_s + A_c E_c) / \Lambda$ which
Fig. 10. Measuring device in test pile

Fig. 11. Expander
\( A_s \) is the area of the reinforcement, \( A_c \) is the area of the concrete, \( E_s \) and \( E_c \) are the modulus of elasticity of the reinforced steel and of the concrete, respectively.

Another way of determining \( E_{eq} \) is by a load test on a test specimen cut, with a diamond saw from the test pile. The specimen length should be about four pile diameters. After the specimen has been cured in wet storage it is loaded axially in a high capacity testing machine and by measuring the stress vs. strain relationships the modulus of the pile can be determined.

Once \( E_{eq} \) is determined, \( Q_p \) can be computed by Eq. (1) since \( \Delta L, L \) and \( A \) are knowns.

The test results from this special device are of interest and variance of \( Q_f \) and \( Q_p \) due to increasing of displacement of pile point is presented in Fig. 12.

The curve of \( Q_p \) is almost a straight line and shows that \( Q_p \) increases as settlement \( (S) \) increases, indicating that the pile tip resistance becomes the major part of the pile bearing capacity as frictional resistance is lost by shear during settlement.

The curve of \( Q_f \) shows that the pile tip carries no load until the load reaches a certain point. That is as the displacement of pile point is small friction resistance is the major part of the bearing resistance. The skin friction reaches its maximum value just before the occurrence of shear
failure between the pile and the surrounding soil. After that point the pile moved downward and point resistance increased rapidly while skin friction decreased.

To find the shape of the curves as in Fig. 12, for different types of soil the actual values of $S$ and $Q_f, Q_p$ are not necessary, may be studied by a model test such as the following.

(a) A series of secondary tests as shown in Fig. 13 by pushing a model pile vertically into the surface of soil sample. A compressional force $Q_p$ is applied on the top of the model pile. The model pile will penetrate into soil sample for an amount $s$ which must be a low value to prevent
side friction. This test should be done for the variance value of \( Q_p \) until different values of \( s \) are obtained.

![Diagram of model pile and sample soil](image)

**Fig. 13. Secondary model test.**

(b) Primary tests as shown in Fig. 14 will be run for the varying \( Q_u \) which is the axial loading force on the top of the model pile with both point and side resistance. Settlement of the model pile for different \( Q_u \) is obtained from reading dial set on the pile top.
From the primary and secondary tests, we can get $Q_u$ and $Q_p$ for the corresponding value of $s$. Since $Q_u$ and $Q_p$ are known from the above tests, $Q_f$ is obtained by

$$Q_f = Q_u - Q_p$$

This test of the model pile will show the variance of $Q_p$ and $Q_f$ for different soils, but it will not show the real values of $Q_p$ and $Q_f$.

Two assumptions are made for this model test:

(a) Elastic deformation of model pile under load is neglected because it is very small.

(b) A homogeneous and isotropic soil sample is used for this test.
CONCLUSION

An effective design of a pile foundation requires an adequate safety factor for the bearing capacity and at the same time an economical foundation.

As it was pointed out in the introduction, this report was written to guide the analysis for estimating bearing capacity of friction piling in different kinds of soil. As an aid for wise engineering decision, the following procedures for various soil conditions should be considered.

(a) For cohesionless soil, sand for example, both dynamic formulas and static loading tests may be used. The former has the advantage of saving both time and cost. Test results show that the resistance of soil to the pile during driving does not differ greatly from the resistance of the pile after a period of time.

(b) For cohesive soil, applying the dynamic formulas the indicated load carrying capacity is usually below the real bearing capacity of the pile as determined from static loading test on the pile conducted following a period of time after driving has elapsed. The reasons have been explained in the report. It is wise to use static loading test for this soil.
RECOMMENDATION FOR FUTURE WORK

The model test suggested on page 36-36 could be helpful for the investigation of \( Q_f \) vs. \( Q_p \) for different kinds of soil. For that test, all the equipment can be found in the soil laboratory and the work can be done for the various typical soils or repeated for one kind of soil as water content varies. It should be very informative to determine the \( Q_f/Q_p \) curve for different settlement of the model pile.
APPENDIX A

Computer program for dynamic pile formulas testing.

```c

1 FORMAT(46H S QH QR QE X)
2 FORMAT(F8.0,F5.2,F4.0,F2.5)  
3 FORMAT(F6.0,F6.2)  
4 FORMAT(F5.2,F3F12.2,F6.2)  

PUNCH 1
READ 2*,E1,H,EN,C
READ 3*,PL,WP,WH,A
S=1

5 SF=.1*S
F1=(2.*PL*E1*WH*H)/((S+SF)**2*A*E)
F2=(WH+EN*E*WP)/(WH+WP)
F3=((S+SF)*A*E)/(C*PL)
QH=(F3*(SQRFT(1.+F1*F2)-1.))/2000.
F4=(S*A*E)/PL
F5=2.*PL*WH*WH*H
F6=S*S*A*E/(WH+WP)
QR=(F6*(SQRFT(1.+F5/F6)-1.))/2000.
X=QH/QR
QE=(WH*H)/(6.*((S+1.)*2000.))
PUNCH 4,S,QH,QR,QE,X
S=S+1
1F(S-4.0)5,5,6

6 STOP
END

3000000.  .75180.  .40  .67
```
(1) Efficiency of the hammer fall
\[ e_1 = 0.75 \] for drop hammer, rope actuated.
\[ e_1 = 1.00 \] for free drop hammer.
\[ e_1 = 0.75-0.85 \] for single-acting steam hammers.
\[ e_1 = 0.65-0.85 \] for double-acting steam hammers.
\[ e_1 = 1.00 \] for diesel hammers.

(2) Newton's coefficient of elastic restitution
\[ n = 0.80 \] for micarta cushion when driving Raymond piles. (This type withstands 400 to 500 blows of heavy driving and therefore gives more uniform results.)
\[ n = 0.55 \] for no cushion. Steel in steel when driving pipe piles.
\[ n = 0.50 \] for oak cap blocks when driving Raymond piles.
\[ n = 0.50 \] for well-compacted cushion when driving pipe piles.
\[ n = 0.50 \] for ram of double-acting hammers striking on steel anvil and driving steel piles or precast concrete piles.
\[ n = 0.40 \] for medium-compacted cushion when driving pipe piles.
\[ n = 0.40 \] for ram of double-acting hammers striking steel anvil and driving timber piles, also for striking steel helmet containing wood and driving steel piles.
= 0.40 for ram of single-acting or drop hammers striking directly on head of precast concrete piles not fitted with driving cap.

= 0.32 for ram of single-acting hammers striking on steel plate cover of wood cap or steel piles.

= 0.25 for fresh wood cushion when driving pipe piles.

= 0.25 for ram of single-acting or drop hammers striking on well-conditioned wood cap of driving cap in driving precast concrete piles or directly on wood pile heads.

= 0.0 for deteriorated condition of heads of timber piles or of wood cap and for excess packing in driving cap.
BIBLIOGRAPHY


A STUDY OF BEARING CAPACITY OF PILE FOUNDATION

by

JOHN IN-CHUNG YEN

Diploma, Taipei Institute of Technology, Taiwan, China, 1965

---

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1969
ABSTRACT

For the design of pile foundations, two conventional methods have been used for the determination of the bearing capacity of a single pile. The dynamic pile driving formulas have been used widely since it is an easier method while the method of static loading tests is uneconomical both in cost and in time.

Experience indicates that these driving formulas are not accurate in most types of soil giving an indicated load carrying capacity which results in overdesign in certain soils and underdesign in others.

This study compared the most commonly used formulas by assigning arbitrary but realistic soil and piling parameters with the aid of a computer program. Based on the results of the computer program, the assumptions which simplified the dynamic formulas were discussed.

A summary of the reactions of the various soil to driving piles and a suggested method for static and model tests were given in this report.