

INTERIM REPORT OF A VIBRATIONAL
ANALYSIS OF A 3-AXLE, FULLY
ARTICULATED, HIGHWAY VEHICLE

by 589

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NOMENCLATURE

- L horizontal distance between front tractor axle and tractor center of gravity (CG)
- N horizontal distance between tractor CG and tractor rear axle
- P horizontal distance between fifth wheel kingpin and trailer CG
- Q horizontal distance between trailer CG and trailer axle
- R horizontal distance between tractor CG and fifth wheel kingpin
- S vertical distance between tractor CG and fifth wheel kingpin
- T vertical distance between trailer CG and fifth wheel kingpin
- M_1 sprung mass of the tractor
- M_2 sprung mass of the trailer
- M_3 mass of the tractor front axle set
- M_4 mass of the tractor rear axle set
- M_5 mass of the trailer axle set
- J_1 mass moment of inertia of the sprung tractor mass about its CG
- J_2 mass moment of inertia of the sprung trailer mass about its CG
- K_1 combined equivalent spring constant of the tractor front tires
- K_2 combined spring constant of the tractor front axle springs
- K_3 combined equivalent spring constant of the tractor rear tires
- K_4 combined spring constant of the tractor rear axle
- K_5 combined equivalent spring constant of the trailer tires
- K_6 combined equivalent spring constant of the trailer springs
- C_1 combined equivalent damping constant of the tractor tires
- C_2 combined damping constant of the tractor front axle dampers

- C_3 combined equivalent damping constant of the tractor rear tires
 C_4 combined damping constant of the tractor rear axle dampers
 C_5 combined equivalent damping constant of the trailer tires
 C_6 combined damping constant of the trailer axle dampers
 X_1 vertical motion coordinate of the CG of the sprung tractor mass
 X_2 vertical motion coordinate of the CG of the sprung trailer mass
 X_3 vertical motion coordinate of the tractor front axle
 X_4 vertical motion coordinate of the tractor rear axle
 X_5 vertical motion coordinate of the trailer axle
 X_6 horizontal motion coordinate of the CG of the sprung tractor mass
 X_7 horizontal motion coordinate of the CG of the sprung trailer mass
 θ_1 rotational coordinate about the CG of the sprung tractor mass
 θ_2 rotational coordinate about the CG of the sprung trailer mass
 t time
 τ_1 time necessary for the truck to travel the distance of the tractor wheelbase ($L + R$)
 τ_2 time necessary for the truck to travel a distance measured from the tractor front axle to the trailer axle ($R + P + Q + L$)
 $G(t)$ displacement function of the road contour applied to the tractor front axle
 $G(t-\tau_1)$ displacement function of the road contour applied to the tractor rear axle
 $G(t-\tau_2)$ displacement function of the road contour applied to the trailer axle

- \bar{s}_A displacement vector of point A
- $\bar{s}_{A/G}$ displacement vector of point A with respect to point G
- H mean point of attachment of the tractor front axle spring to the chassis
- I mean point of attachment of the tractor rear axle spring to the chassis
- J mean point of spring attachment of the trailer axle spring to the chassis
- T' kinetic energy of the system due to vibrations
- D energy dissipation function due to the damping of the tires and shock absorbers
- V potential energy function due to the springing of the tires and suspension springs
- λ classical eigenvalue
- β eigenvalue solutions to the characteristic equation
- {q} transient solution vector corresponding to a value of β
- c exponential decay rate due to damping
- ω_d damped natural circular vibrational frequency, radians/second
- e 2.7183 ...
- $\mu_{\alpha\gamma}$ ratio of the γ 'th element of the {q} vector corresponding to the α 'th β value to the first element of that {q} vector
- {A} complementary solution vector of constant amplitudes of $\sin \omega t$
- {B} complementary solution vector of constant amplitudes of $\cos \omega t$
- L angle