ASSESSMENT OF THE NEW AASHTO DESIGN PROVISIONS FOR SHEAR AND COMBINED SHEAR/TORSION AND COMPARISON WITH THE EQUIVALENT ACI PROVISIONS

by

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B.Sc., Kabul University, 2007

A THESIS

submitted in partial fulfillment of the requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering
College of Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

2010

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Abstract

The shear and combined shear and torsion provisions of the AASHTO LRFD (2008) Bridge Design Specifications, as well as simplified AASHTO procedure for prestressed and non-prestressed reinforced concrete members were investigated and compared to their equivalent ACI 318-08 provisions. Response-2000, an analytical tool developed based on the Modified Compression Field Theory (MCFT), was first validated against the existing experimental data and then used to generate the required data for cases where no experimental data was available. Several normal and prestressed beams, either simply supported or continuous were used to evaluate the AASHTO and ACI shear design provisions

In addition, the AASHTO LRFD provisions for combined shear and torsion were investigated and their accuracy was validated against the available experimental data. These provisions were also compared to their equivalent ACI code requirements. The latest design procedures in both codes propose exact shear-torsion interaction equations that can directly be compared to the experimental results by considering all φ factors as one. In this comprehensive study, different over-reinforced, moderately-reinforced, and under-reinforced sections with high-strength and normal-strength concrete for both solid and hollow sections were analyzed. The main objectives of this study were to:

- Evaluate the shear and the shear-torsion procedures proposed by AASHTO LRFD (2008) and ACI 318-08
- Validate the code procedures against the experimental results by mapping the experimental points on the code-based exact interaction diagrams
- Develop a MathCAD program as a design tool for sections subjected to shear or combined shear and torsion

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Acknowledgements

I hereby thank my major professor Dr.Asad Esmaeily, my committee members Dr. Hayder Rasheed and Dr. Yacoub Najjar for their continuous support and contribution to this research, also the Kansas Department of Transportation (K-DOT) for their financial support.

Chapter 1 - Introduction

Overview

In this study the shear or combined shear and torsion provisions of AASHTO LRFD (2008) Bridge Design Specifications, simplified AASHTO procedure for prestressed and non-prestressed, and ACI 318-08 for reinforced concrete members are comparatively studied. Shear-critical beams were selected to evaluate the shear provisions for the mentioned codes. Because of the absence of experimental data for various beams considered for the analysis and loaded with shear, Response-2000 which is an analytical tool and is based on Modified Compression Field Theory (MCFT) was checked against the experimental data for cases where experimental data existed. Consequently, the shear capacity of simply supported beams was slightly under-estimated by Response-2000, while continuous beams were accurately quantified. To evaluate the corresponding shear provisions for AASHTO LRFD and ACI Code; a simply supported double-T beam with harped prestressed strands, continuous bulb-T beam with straight and harped prestressed strands, simply supported and continuous rectangular deep beams with and without longitudinal crack control reinforcement were selected for the analysis. The shear capacity using the aforementioned shear provisions has been calculated at various sections along the beam span and the results are plotted in chapter five of this document.

In addition, the AASHTO LRFD provisions for combined shear and torsion have been investigated and their accuracy has been validated against available experimental data. The provisions on combined shear and torsion have also been compared to the pertinent ACI code requirements for the behavior of reinforced concrete beams subjected to combined shear and torsion. The latest design procedures in both codes lend themselves to the development of exact shear-torsion interaction equations that can be directly compared to experimental results by considering all ϕ factors to be equal to one. In this comprehensive comparison, different sections with high-strength and normal-strength concrete as well as over-reinforced, moderately-reinforced, and under-reinforced sections for both solid and hollow sections were analyzed. The exact interaction diagrams drawn are also included in chapter five of this document.

1.1 Objectives

The objectives of the research are:

- Evaluate shear and shear-torsion procedures proposed by AASHTO LRFD (2008) and ACI 318-08.
- Develop a MathCAD program to design sections subjected to shear or shear and torsion.
- Validate the procedure with experimental results by drawing exact interaction diagrams and mapping experimental points on them

1.2 Scope

Chapter Two presents the experimental studies on shear or shear and torsion. In addition, design procedure for shear and combined shear and torsion using the AASHTO LRFD (2008) Bridge Design Specifications, and ACI 318-08 are also discussed in detail.

Chapter Three addresses the validity of Response-2000 for shear against available experimental data. Furthermore, the procedure to draw exact interaction diagrams using the AASHTO LRFD and ACI Code for beams under combined shear and torsion is discussed.

Chapter Four presents the flow chart for the developed MathCAD design tool for shear or shear and torsion.

Chapter Five presents the results and discussion with all the necessary plots for shear or shear and torsion.

Chapter Six presents the conclusions reached and provides suggestions or recommendations for future research.

Chapter 2 - Literature Review

Beams subjected to combined shear and bending, or combined shear, bending, and torsion frequently happens in practice. Often times one or two of the cases may control the design process while the other considered as secondary. In this study, structural concrete beams subjected to shear or combined shear and torsion are considered while the effects of bending moment are neglected. This chapter is devoted to the experimental studies and explaining the design procedures for the structural reinforced concrete beams with no controlling bending effects.

2.1 Experimental studies on reinforced concrete beams subjected to shear only

Even though the behavior of structural concrete beams subjected to shear has been studied for more than 100 years, there isn't enough agreement among researchers about how the concrete contributes into shear resistance of a concrete beam. This is mainly because of the many different mechanisms involved in shear transfer process of structural concrete members such as aggregate interlock or interface shear transfer across cracks, shear transfer in compression (un-cracked) zone, dowel action, and residual tensile stresses normal to cracks. However, there is a general agreement among researchers that aggregate interlock and compression zone are the key components of concrete contribution to shear resistance.

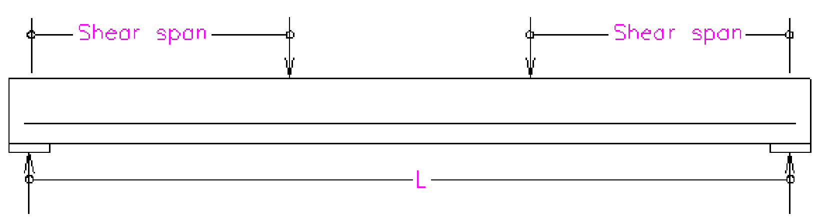


Figure 2.1 Traditional shear test set-up for concrete beams

Figure 2.1 shows the traditional shear test set-up for concrete beams. From the figure it is concluded that the region between the concentrated loads applied at the top of the beam is subjected to pure flexure whereas the shear spans are subjected to constant shear and linearly varying bending moment. It is very obvious that the results from such test could not be used to develop a general theory for shear behavior. Since it is almost impossible to design an experimental program where the beam is only subjected to pure shear, this in turn is one of the main reasons where the true shear behavior of beams has not been understood throughout the decades.

After conducting tests on reinforced concrete panels subjected to pure shear, pure axial load, and a combination of shear and axial load, a complex theory called Modified Compression Filed Theory (MCFT) was developed in 1980s from the Compression Field Theory (Vecchio and Collins, 1986). The MCFT was able to accurately predict the shear behavior of concrete members subjected to shear and axial loads. This theory was based on the fact that significant tensile stresses could exist in the concrete between the cracks even at very high values of average tensile strains. In addition, the value for angle θ of diagonal compressive stresses was considered as variable compared to the fixed value of 45˚ assumed by ACI Code.

To simplify the process of predicting the shear strength of a section using the MCFT, the shear stress is assumed to remain constant over the depth of the cross-section and the section is considered as a biaxial element in case any axial stresses are present. This in turn produces the basis of the sectional design model for shear where the AASHTO-LRFD Bridge Design Specifications have been based on (Bentz et,al 2006).

Even though the AASHTO LRFD procedure to predict the shear strength of a section was straight forward, yet the contribution of concrete to shear strength of a section was a function of β and varying angle θ for which their values were determined using the tables provided by AASHTO. The factor β indicated the ability of diagonally cracked concrete to transmit tension and shear. The modified compression field theory was even more simplified when simple equations were developed by Bentz et, al (2006) for β and θ . These equations were then used to predict the shear strengths of different concrete sections and the results compared to that obtained from MCFT. Consequently the shear strengths predicted by the simplified modified compression field theory and MCFT were compared with the experimental results.

It was found that the results for both simplified modified compression field theory and MCFT were almost exactly similar and both matched properly to the experimental results. In addition, the results were also compared with the ACI Code where it was pretty much inconsistent in particular for panels with no transverse reinforcements.

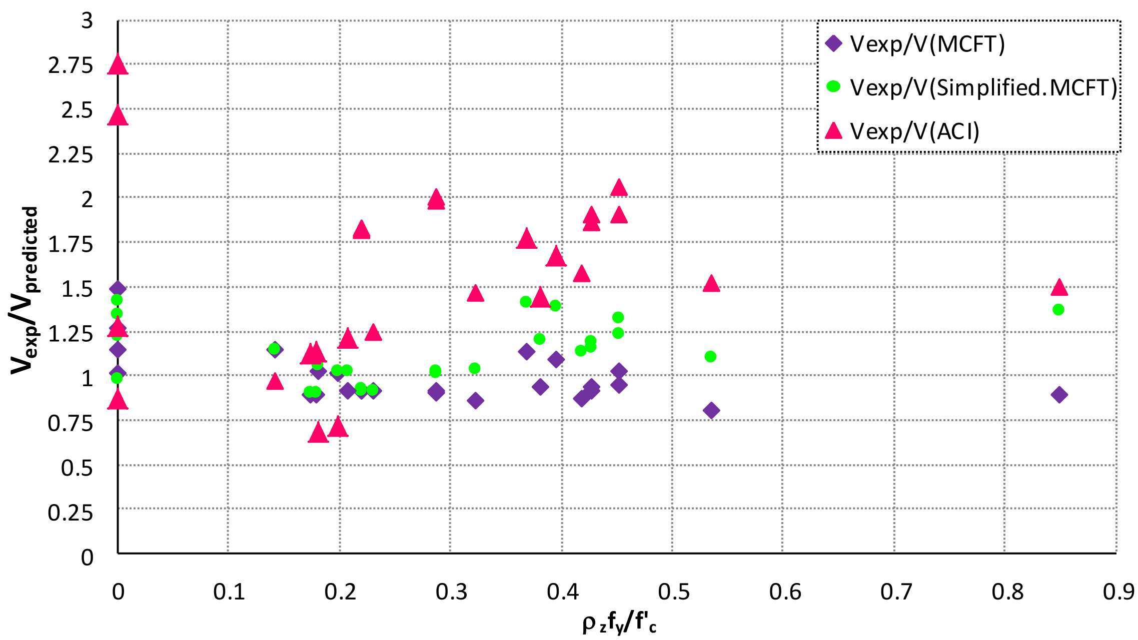


Figure 2.2 The ratio of experimental to predicted shear strengths Vs transverse reinforcement for the panels

The above figure shows that the ACI method to predict the shear strength of concrete sections subjected to pure shear or a combination of shear and axial load under-estimates the shear capacity of sections. However, the simplified modified compression field theory and MCFT give relatively accurate results. Note that the horizontal line where the ratio of experimental to predicted shear strength equals to one represent a case where the predicted and the experimental results are exactly equal to each other. On the other hand, points above and below that line simply means that the shear strength of a particular section is either under or over-estimated. Because the points corresponding to the shear strength predicted by simplified modified compression filed theory and MCFT are closer to the horizontal line with unit value, it is concluded that the MCFT can accurately predict the shear behavior of a section.

The details of the specimens corresponding to Figure **2.2** are tabulated below. The data provided below is taken from ref (2).

Table 2.1 Details of the cross-section and summary of the experimental results for the selected panels

Panel	f'_c, MPa	Reinforcement r_x, %	$^*f_{yx}$, Mpa	$^{**}S_x$, mm	$r_z f_y/f'_c$	Axial load $^{***}f_x/v$	V_{exp}/f'_c	$V_{exp}/V_{predicted}$ MCFT	Simplified MCFT	ACI
Yamaguchi et al, a_g=20mm										
S-21	19.0	4.28	378	150	0.849	0	0.34	0.89	1.37	1.50
S-31	30.2	4.28	378	150	0.535	0	0.28	0.80	1.10	1.52
S-32	30.8	3.38	381	150	0.418	0	0.28	0.87	1.14	1.58
S-33	31.4	2.58	392	150	0.323	0	0.26	0.86	1.04	1.46
S-34	34.6	1.91	418	150	0.230	0	0.21	0.91	0.92	1.25
S-35	34.6	1.33	370	150	0.142	0	0.163	1.15	1.15	0.97
S-41	38.7	4.28	409	150	0.452	0	0.31	0.95	1.23	1.91
S-42	38.7	4.28	409	150	0.452	0	0.33	1.02	1.32	2.06
S-43	41.0	4.28	409	150	0.427	0	0.29	0.91	1.16	1.86
S-44	41.0	4.28	409	150	0.427	0	0.30	0.94	1.19	1.91
S-61	60.7	4.28	409	150	0.288	0	0.25	0.90	1.01	1.98
S-62	60.7	4.28	409	150	0.288	0	0.26	0.91	1.03	2.01
S-81	79.7	4.28	409	150	0.220	0	0.20	0.92	0.92	1.82
S-82	79.7	4.28	409	150	0.220	0	0.20	0.92	0.93	1.83
Andre a_g=9mm; KP a_g=20mm										
TP1	22.1	2.04	450	45	0.208	0	0.26	0.92	1.02	1.21
TP1A	25.6	2.04	450	45	0.179	0	0.22	0.89	0.90	1.14
KP1	25.2	2.04	430	89	0.174	0	0.22	0.89	0.90	1.12
TP2	23.1	2.04	450	45	0.199	3	0.114	1.01	1.02	0.72
KP2	24.3	2.04	430	89	0.18	3	0.106	1.03	1.06	0.68
TP3	20.8	2.04	450	45	0	3	0.061	1.27	1.34	2.75
KP3	21	2.04	430	89	0	3	0.054	1.15	1.22	2.47
TP4	23.2	2.04	450	45	0.396	0	0.35	1.09	1.39	1.68
TP4A	24.9	2.04	450	45	0.369	0	0.35	1.14	1.41	1.77
KP4	23	2.04	430	89	0.381	0	0.30	0.94	1.20	1.44
TP5	20.9	2.04	450	45	0	0	0.093	1.49	1.42	1.28
KP5	20.9	2.04	430	89	0	0	0.063	1.01	0.98	0.87

$^*f_{yx}$ Yield stress of longitudinal reinforcement

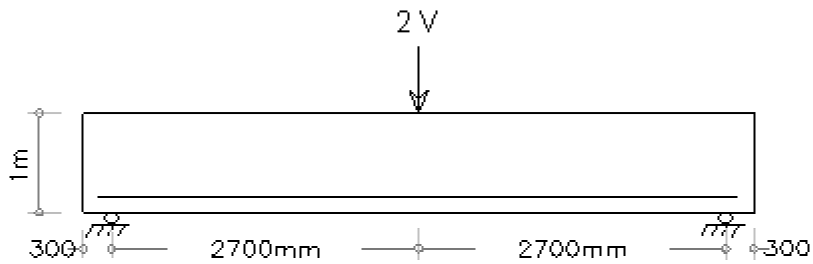
$^{**}S_x$ Vertical spacing between the bars aligned in the x-direction

$^{***}f_x/v$ Ratio of axial stress to shear stress

As stated earlier, the AASHTO LRFD Bridge Design Specifications for shear design are based on the sectional design model which in turn is based on MCFT. The current AASHTO LRFD (2008) bridge design specifications uses the simple equations for β and θ. These equations removed the need to use the table provided by AASHTO LRFD to find the values for β and θ. In addition, the equations enable the engineers to set up a spread sheet for the shear design calculations.

To evaluate the AASHTO LRFD (2008) shear design procedure for shear-critical sections, six prestressed and non-prestressed reinforced concrete beams were selected for the analysis. Among the total six beams considered, four of them were rectangular non-prestressed reinforced concrete beams which were tested by Collins and Kuchma (1999) and shown in Figure 2.3 and Figure 2.4. The remaining two beams were prestressed Double-T (8DT18) and Bulb-T (BT-72) with harped or a combination of harped and straight tendons shown in Figure 2.5 and Figure 2.6 respectively. Because the AASTHO LRFD shear design procedure takes into account the crack control characteristics of a section, two of the non-prestressed beams were selected to have crack control (skin) reinforcement. Furthermore, to check the AASHTO LRFD shear design provisions for different support conditions, three of the beams were purposefully selected as simply supported and the remaining three as continuous beams.

It is important to note that the experimental data existed for only four of the non-prestressed reinforced concrete beams (BM100, BM100D, SE100A-M-69, and SE100B-M-69) failed in shear at a certain location. Further, the shear strength of the beams at that particular location was also determined using the analytical tool called Response-2000 which is in turn based on MCFT. Since the intention was to evaluate the AASHTO LRFD shear design provisions for different combinations of moment and shear, the predicted shear strengths at different sections throughout the beam was calculated using AASHTO LRFD (2008) and compared to the results obtained from Response-2000. The validity of the results from Response-2000 is discussed in chapter three of this document. Note that Response-2000 was also used to verify the predicted shear strength for the prestressed beams (BT-72, 8DT18). In addition to the AASHTO LRFD (2008), the shear design provisions for the simplified AASHTO and ACI Code were also evaluated.



	No.30	No.20	No 15	No.10	# 3
Area (mm2)	700	300	200	100	71
fy (MPa)	550	475	483	522	508

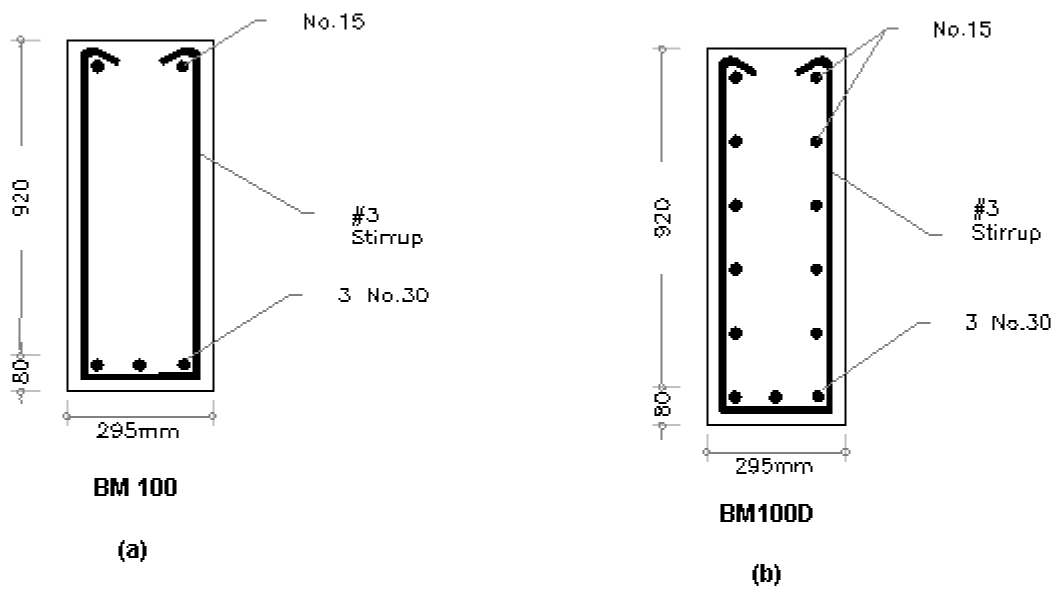


Figure 2.3 (a) Cross-section of normal strength, non-prestressed simply supported reinforced concrete beam (b) Cross-section with the crack control (skin) reinforcement.

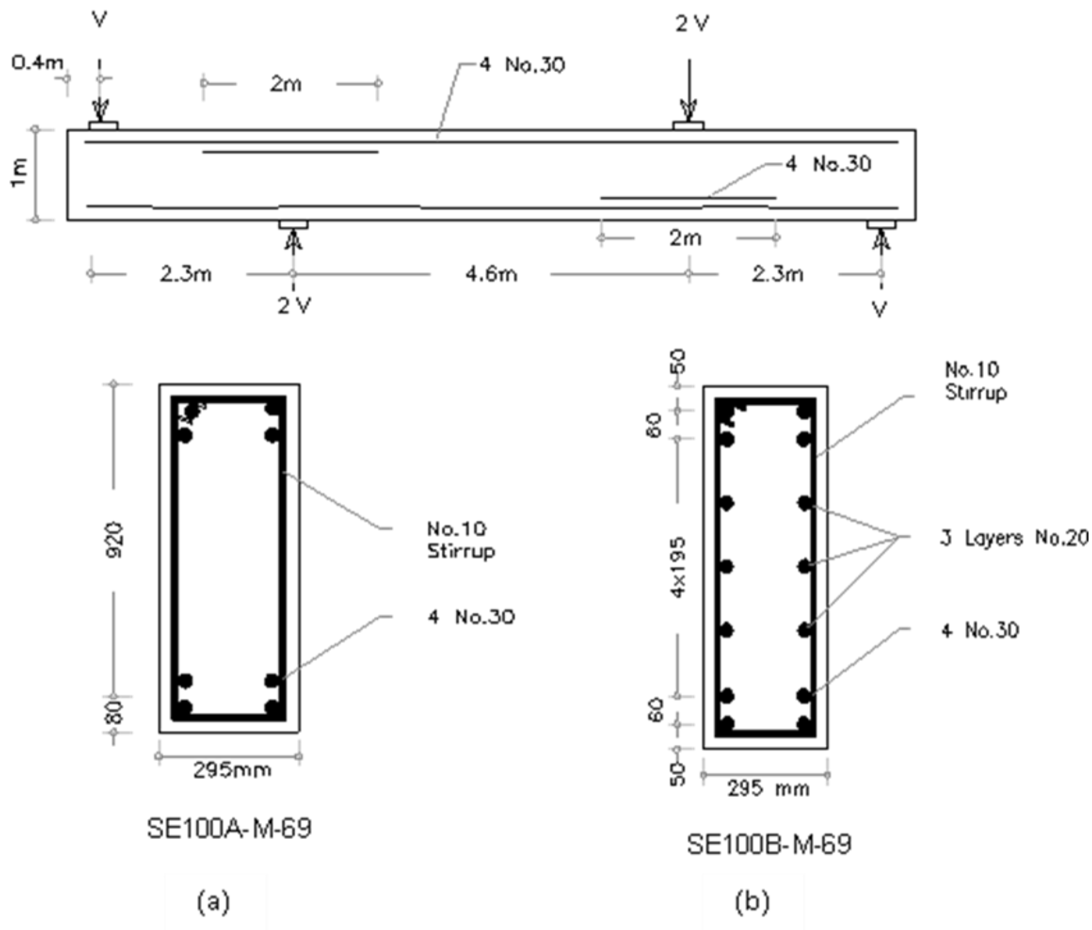


Figure 2.4 (a) Cross-section of high-strength, continuous non-prestressed reinforced concrete beam (b) Cross-section with the crack control (skin) reinforcement**.**

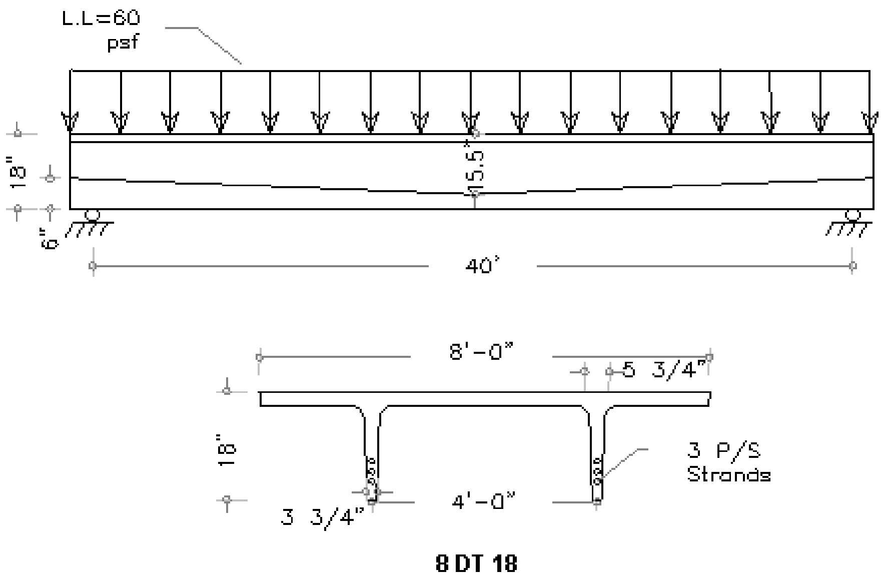


Figure 2.5 Profile and cross-section at mid-span of normal strength, simply supported, Double-T (8DT18) pre-stressed concrete member

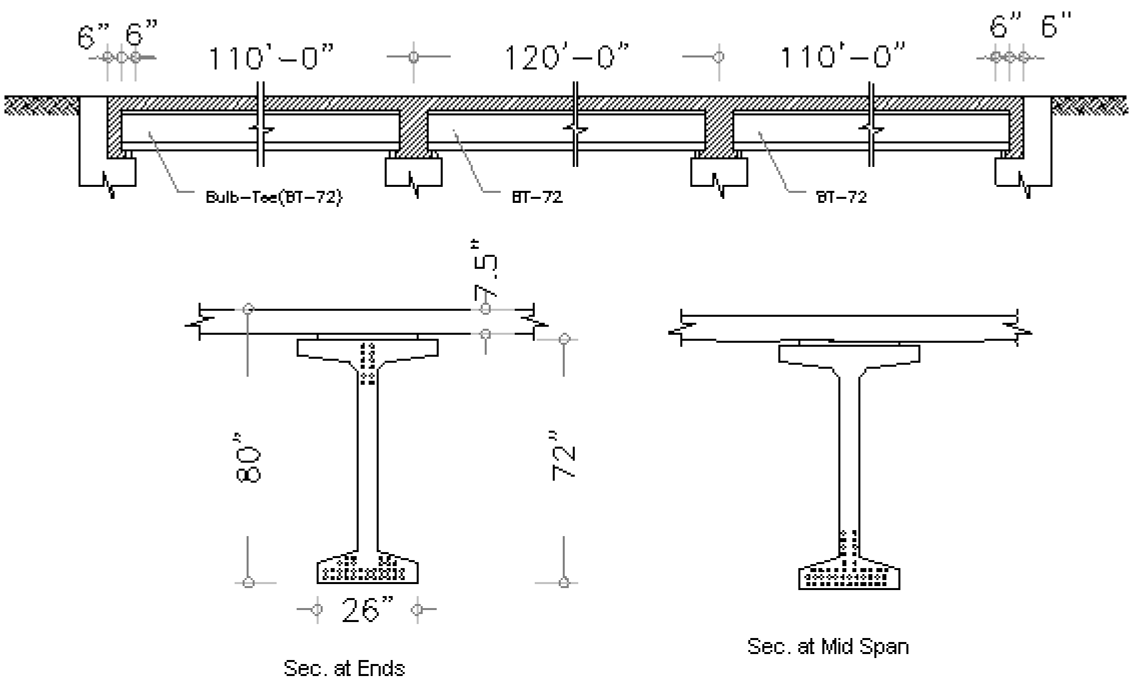


Figure 2.6 Profile and sections at mid-span and at end of high strength, continuous Bulb-T (BT-72) member

2.2 Experimental studies on reinforced concrete beams subjected to combined shear and torsion

The behavior of reinforced concrete beams subjected to any combination of torsional, bending, and shear stresses have been studied by many researchers and various formulas have been proposed to predict the behavior of these beams. Structural members subjected to combined shear force, bending moment, and torsion are fairly common. However, in some cases one of these actions (shear, bending, or torsion) may be considered as to have a secondary effect and may not be included in the design calculations.

Significant research has been conducted by different researchers to determine the behavior of reinforced concrete beams subjected to any combination of flexural shear, bending, and torsional stresses. Tests performed by Gesund et.al (1964) shown that bending stresses can increase the torsional capacity of reinforced concrete sections. Useful interaction equations for concrete beams subjected to combined shear and torsion have been proposed by Klus (1968).

Moreover, an interesting experimental program was developed by Rahal and Collins (1993) to determine the behavior of reinforced concrete beams under combined shear and torsion. Using similar experimental program, Fouad, et.al (2000) tested a wide range of beams covering normal strength and high strength under-reinforced and over-reinforced concrete beams subjected to pure torsion or combined shear and torsion. Consequently, interesting findings were reported about the contribution of concrete cover to the nominal strength of the beams, modes of failure, and cracking torsion for Normal Strength Concrete (NSC) and High Strength Concrete (HSC).

It is obvious that most of the construction codes practiced today consider in many different ways the effects of any of the combinations of flexural shear, bending, and torsional stresses. In other words, there are a variety of equations proposed by each code to predict the behavior of beams subjected to any possible combination of the stresses mentioned above.

In this study, the current AASHTO LRFD (2008) and ACI 318-08 shear and tosion provisions are evaluated against the available experimental data for beams under combined shear and torsion. In addition, Torsion-Shear (T-V) interaction diagrams are presented for AASTHO LRFD (2008) and ACI and the corresponding experimental data points shown on the plots.

Even though efforts have been made in the past to check the AASHTO LRFD and ACI shear and torsion provisions; in most of those cases it is limited to a certain range of concrete strengths or longitudinal reinforcement ratios ρ. As an example; Rahal and Collins (2003) have drawn the interaction diagrams using the AASHTO LRFD and ACI shear and torsion provisions for beam series RC2. This series was composed of four beams and subjected to pure shear or combined shear and torsion. The properties for the reinforcing bars and cross-sections for RC2 and other beams studied by the other are tabulated in Table 2.2 and Table 2.3.

The Torsion-Shear (T-V) interaction diagrams for AASHTO LRFD provided by Rahal and Collins have been drawn as linear. In fact, this is because of the absence of equations at that time for the factor β and θ which were calculated using discrete data from the Tables proposed by AASHTO. The factors β as defined earlier indicate the ability of diagonally cracked concrete to transmit tension and shear, while θ is the angle of diagonal compressive stresses.

Table 2.2 Properties of reinforcing bars

E.Fouad., et.al	Nominal Dia(mm)	Actual Area (mm^2)	Yield stress (Mpa)	Klus	Yield stress (Mpa)	Rahal and Collins	Yield stress (Mpa)
	8	50	275		265		-
	10	77	380		-		466
	12	108	399		-		-
	16	193	379		-		-
	18	245	386		429		-
	22	380	-		429		-
	25	468	370		-		480

Table 2.3 Cross-sectional properties of the beam studied

	Specimen*	Concrete Dimensions Width	Height	Cover	f'c	Longitudinal Reinforcement Top		Bottom		Stirrups	
		b_w (mm)	h (mm)	(mm)	(Mpa)	Type-1**	Type-2	Type-1**	Type-2	Dia (mm)	Spacing,s,(mm)
E.Fouad., et.al	NU1	200	400	20	27.5	2d12	-	2d12	-	8	66.7
	NU2	220	420	30	26.5	2d12	-	2d12	-	8	66.7
	NU3 (Box)	100	400	20	26	2d12	-	2d12	-	8	66.7
	NU4	200	400	20	28	2d16	3d16	2d16	3d16	8	66.7
	NU5	200	400	20	27	2d16	3d16	2d16	3d16	8	66.7
	NU6	200	400	20	26.9	2d16	3d16	2d16	3d16	8	66.7
	NO1	200	400	20	27.2	2d18	3d18	2d18	3d18	12	91
	NO2	200	400	20	26.7	2d18	3d18	2d18	3d18	12	91
	HU1	200	400	20	75.6	2d16	-	2d16	-	10	91
	HU2	220	420	30	74.9	2d16	-	2d16	-	10	91
	HU3 (Box)	100	400	20	73.5	2d16	-	2d16	-	10	91
	HU4	200	400	20	75.1	3d18	3d18	3d18	3d18	10	91
	HU5	200	400	20	76.4	3d18	3d18	3d18	3d18	10	91
	HU6	200	400	20	75	3d18	3d18	3d18	3d18	10	91
	HO1	200	400	20	74.6	2d25	2d25	2d25	2d25	12	77
	HO2	200	400	20	74	2d25	2d25	2d25	2d25	12	77
Klus	1	200	300	20***	21.5	2d18,1d22	-	2d18,1d22	-	8	100
	2	200	300	20	21.5	2d18,1d22	-	2d18,1d22	-	8	100
	3	200	300	20	21.5	2d18,1d22	-	2d18,1d22	-	8	100
	4	200	300	20	21.5	2d18,1d22	-	2d18,1d22	-	8	100
	5	200	300	20	21.5	2d18,1d22	-	2d18,1d22	-	8	100
	6	200	300	20	21.5	2d18,1d22	-	2d18,1d22	-	8	100
	7	200	300	20	21.5	2d18,1d22	-	2d18,1d22	-	8	100
	8	200	300	20	21.5	2d18,1d22	-	2d18,1d22	-	8	100
	9	200	300	20	21.5	2d18,1d22	-	2d18,1d22	-	8	100
	10	200	300	20	21.5	2d18,1d22	-	2d18,1d22	-	8	100
Rahal and Collins	RC2-1	340	640	42.5	53.9	5d25	-	5d25	5d25	10	125
	RC2-2	340	640	42.5	38.2	5d26	-	5d25	5d25	10	125
	RC2-3	340	640	42.5	42	5d27	-	5d25	5d25	10	125
	RC2-4	340	640	42.5	48.7	5d28	-	5d25	5d25	10	125

* HU=High strength Under reinforced, HO=High strength Over reinforced, NU= Normal strength Under reinforced, NO= Normal strength over reinforced

** Top layer of reinforcement at the top and lower layer of the bottom reinforcement

*** The cover was not given, it was assumed as being 200 mm.

During this study, exact Torsion-Shear (T-V) interaction diagrams were drawn using the AASHTO LRFD (2008) shear and torsion provisions. The word exact is used to indicate that the shear and torsion relationships are not assumed as linear. This is due to the fact that the proposed tables for β and θ have been replaced by the simple equations provided in the current AASHTO LRFD Bridge Design Specifications for shear and torsion.

For comprehensive evaluation of the AASHTO LRFD and ACI 318-08 shear and torsion equations for design, a wide range of specimens made of high-strength and normal strength concrete loaded with shear, torsion, or a combination of both were investigated in this study. The cases studied included under-reinforced, moderately-reinforced, and over-reinforced sections. Among the total 30 specimens studied, 22 were made of normal strength concrete while the remaining eight were specimens with high-strength concrete. Two hollow under-reinforced specimens, one made of high-strength and the other made of normal strength concrete were considered as well. The procedure for drawing the exact interaction diagrams are described in detail in chapter three of this document. Figures given below show some of the cross-sections for the specimens considered.

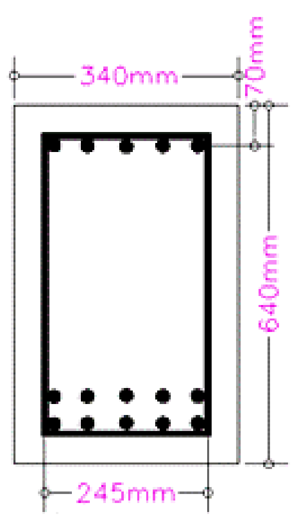


Figure 2.8 Typical beam section for RC2 series tested by Rahal and Collins (2003)

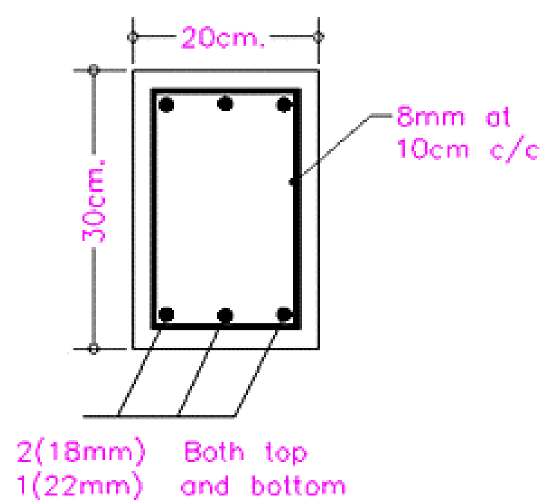


Figure 2.7 Typical beam section tested by Klus (1968)

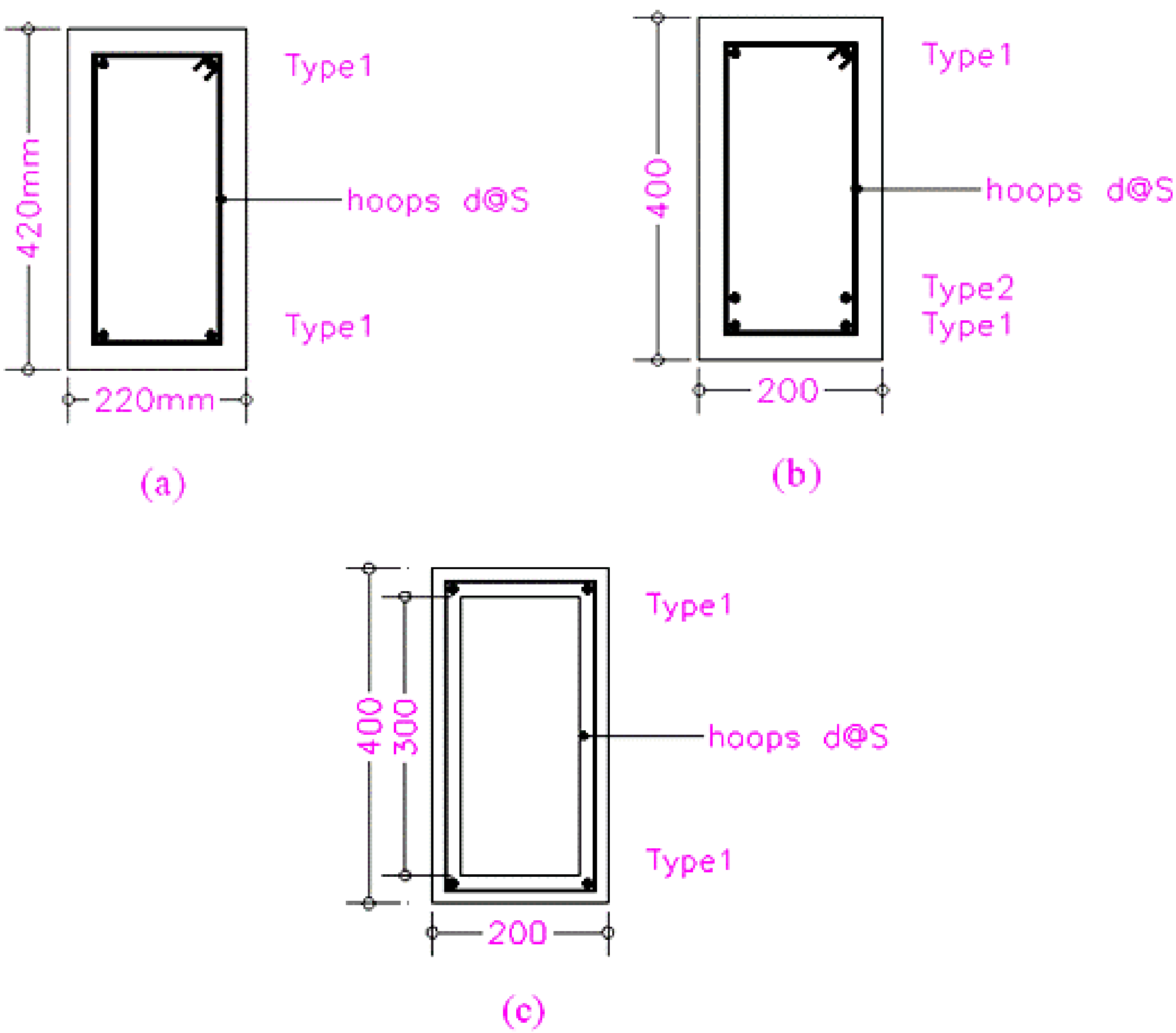


Figure 2.9 (a) NU2 & HU2 (b) For all other specimens (c) Hollow section NU3 & HU3 (Ref-13)

2.3 Procedure for Shear Design of a Concrete Section

The AASHTO LRFD Bridge Design Specifications (2008) proposes three methods to design a prestressed or non-prestressed concrete section for shear. It is important to understand that all requirements set by AASHTO to qualify a particular method have to meet prior to application of that method. In this document only two methods to design a section for shear i.e., the general procedure and the simplified procedure for prestressed and non-prestressed members are discussed in detail. In addition, the current ACI provisions for shear design of a concrete section are described briefly.

2.3.1 AASHTO LRFD General Procedure for Shear Design

The AASHTO LRFD general procedure to design or determine the shear strength of a section is based on the Modified Compression Field Theory (MCFT). As stated earlier, this theory has proved to be very accurate in predicting the shear capacity of a prestressed or non-prestressed concrete section. It is important to note that the current AASTHO LRFD provisions for the general method are based on the simplified MCFT.

The nominal shear strength of a section for all three methods is equal to

$$V_n = V_c + V_s + V_P \qquad (Eq - 2.3.1)$$

where:

$V_n =$ nominal shear strength

$V_c =$ nominal shear strength provided by concrete

$V_s =$ nominal shear strength provided by shear reinforcement

$V_p =$ component in the direction of the applied shear of the effective prestressing force

V_c is a function of a factor β which shows the ability of diagonally cracked concrete to transmit tension and shear. The factor β is inversely proportional to the strain in longitudinal tension reinforcement, ε_s, of the section. For sections containing at least the minimum amount of transverse reinforcement , the value of β is determined as

$$\beta = \frac{4.8}{(1 + 750\varepsilon_s)} \qquad (Eq - 2.3.2)$$

When sections do not contain at least the minimum amount of shear reinforcement, the value of β is determined as follow

$$\beta = \frac{4.8}{(1+750\varepsilon_s)}\frac{51}{(39+s_{xe})} \qquad (Eq-2.3.3)$$

The above equations are valid only if the concrete strength f'_c is in psi and s_{xe} in inches. If the concrete strength f'_c is in MPa and s_{xe} in mm, then 4.8 in Eq-2.3 becomes 0.4 while 51 and 39 in Eq 2.3.3 become 1300 and 1000 respectively.

s_{xe} is called the crack spacing parameter which can be estimated as

$$s_{xe} = s_x \frac{1.38}{a_g + 0.63} \qquad (Eq-2.3.4)$$

s_x is the vertical distance between horizontal layers of longitudinal crack control (skin) reinforcement) and a_g is the maximum aggregate size in inches and has to equal zero when $f'_c \geq 10$ ksi. Note that if the concrete strength is in MPa and s_{xe} in mm, the 1.38 and 0.63 in Eq-2.3.4 should be replaced by 35 and 16 respectively.

The nominal shear strength provided by the concrete V_c for the general procedure is equal to $\beta\sqrt{f'_c}\, b_v d_v$ when the concrete strength is in MPa. However, $V_c = 0.0316\beta\sqrt{f'_c} b_v d_v$ in case f'_c is in ksi. The coefficient 0.0316 is $\frac{1}{1000}$ and is used to convert the V_c from psi to ksi.

The nominal shear strength provided by the shear reinforcement can be estimated as

$$V_s = \frac{A_v f_y d_v cot\theta}{s} \qquad (Eq-2.3.5)$$

where:

A_v = area of shear reinforcement within a distance s (in.^{2})

f_y = yield stress of shear (transverse) reinforcement in ksi or psi depending on the case.

d_v = effective shear depth (in.) and is equal to ($d_v = \frac{A_{ps} f_{ps} d_p + A_s f_y d_s}{A_{ps} f_{ps} + A_s f_y}$). Note that $\mathrm{d_v} \geq$ Max(0.9d, 0.72h)

b_v = effective web width (in.)

s = spacing of stirrups (in.)

θ = angle of inclination of diagonal compressive stresses (˚) as determined below

$$\theta = 29(\text{degree}) + 3500\varepsilon_s \qquad (Eq - 2.3.6)$$

The above equation is independent of which units are used for f'_c or s_{xe}. The strain in longitudinal tension reinforcement ε_s is calculated using the following equation

$$\varepsilon_s = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps}f_{po)}\right)}{E_sA_s + E_pA_{ps}} \qquad (Eq - 2.3.7)$$

where:

M_u = factored moment, not to be taken less than $(V_u - V_p)d_v$ (kip-in.)

N_u = factored axial force, taken as positive if tensile and negative if compressive (kip)

V_u = factored shear force (kip)

A_{ps} = area of prestressing steel on the flexural tension side of the member (in.^{2})

f_{po} = 0.7 times the specified tensile strength of pre-stressing steel, f_{pu} (ksi)

E_s = modulus of elasticity of non-prestressed steel on flexural tension side of the section

E_p = modulus of elasticity of pre-stressing steel on the flexural tension side of the section

A_s = area of non-prestressed steel on the flexural tension side of the section (in.^{2})

To make sure that the concrete section is large enough to support the applied shear, it is required that $V_c + V_s$ should not exceed $0.25f'_cb_vd_v$. Otherwise, enlarge the section.

2.3.1.1 Minimum Transverse Reinforcement

If the applied factored shear V_u is greater than the value of $0.5\phi(V_c + V_p)$; shear reinforcement is required. The amount of minimum transverse reinforcement can be estimated as

$$A_v \geq 0.0316\sqrt{f'_c}\frac{b_v\, s}{f_y} \qquad (Eq - 2.3.8)$$

2.3.1.2 Maximum Spacing of Transverse Reinforcement

According to the AASHTO LRFD Bridge Design Specifications, the spacing of the transverse reinforcement shall not exceed the maximum permitted spacing, s_{maz} determined as

- If $v_u(ksi) < 0.125\, f'_c$, then $s_{max} = 0.8d_v \leq 24$ in.

- If $v_u(ksi) \geq 0.125f'_c$, then $s_{max} = 0.4d_v \leq 12.0$ in.

Where v_u is calculated as

$$v_u = \frac{|V_u - \phi V_p|}{b_v d_v} \qquad (Eq - 2.3.9)$$

2.3.2 Simplified Procedure for Shear Design of Pre-stressed and Non-prestressed Concrete Beams

The nominal shear strength provided by the concrete V_c for perstressed and non-prestressed beams not subject to significant axial tension and containing at least the minimum amount of transverse reinforcement (specified in section 2.3.1.1 of this document) can be determined as the minimum of V_{ci} or V_{cw}.

$$V_{ci} = 0.02\sqrt{f'_c}b_v d_v + V_d + \frac{V_i M_{cre}}{M_{max}} \geq 0.06\sqrt{f'_c}b_v d_v \qquad (Eq - 2.3.10)$$

where:

V_{ci} = nominal shear resistance provided by concrete when inclined cracking results from combined shear and moment (kip)

V_d = shear force at section due to unfactored dead load and include both concentrated and distributed dead loads

V_i = factored shear force at section due to externally applied loads occurring simultaneously with M_{max} (kip)

M_{cre} = moment causing flexural cracking at section due to externally applied loads (kip-in)

M_{max} = maximum factored moment at section due to externally applied loads (kip-in)

$$M_{cre} = S_c\left(f_r + f_{cpe} - \frac{M_{dnc}}{S_{nc}}\right) \qquad (Eq - 2.3.11)$$

where:

S_c = section modulus for the extreme fiber of the composite section where tensile stress is caused by externally applied loads (in.^{3})

f_r = rupture modulus (ksi)

f_{cpe} = compressive stress in concrete due to effective prestress forces only (after allowance for all pre-stress losses) at extreme fiber of section where tensile stress is caused by externally applied loads (ksi)

M_{dnc} =total unfactored dead load moment acting on the monolithic or noncomposite section (kip-in.)

The web shear cracking capacity of the section can be estimated as

$$V_{cw} = \left(0.06\sqrt{f'_c} + 0.30f_{pc}\right)b_v d_v + V_p \qquad (Eq - 2.3.12)$$

where:

$V_{cw} =$ nominal shear resistance provided by concrete when inclined cracking results from excessive principal tensions in web (kip)

f_{pc} = compressive stress in concrete (after allowance for all prestress losses) at centroid of cross-section resisting externally applied loads or at junction of web and flange when the centroid lies within the flange (ksi). In a composite member, f_{pc} is the resultant compressive stress at the centroid of the composite section, or at junction of web and flange, due to both pre-stress and moments resisted by precast member acting alone.

After calculating the flexural shear cracking and web shear cracking capacities of the section, i.e., V_{ci} and V_{cw}; the minimum of the two values is selected as the nominal shear strength provided by concrete. The nominal shear strength provided by the shear reinforcement is calculated exactly the same as in Eq-2.3.5 with the only difference that $cot\theta$ is calculated as following

- If $V_{ci} < V_{cw}$; $cot\theta = 1$
- If $V_{ci} > V_{cw}$; $cot\theta = 1.0 + 3\left(\frac{f_{pc}}{\sqrt{f'_c}}\right) \leq 1.8$ (Eq-2.3.13)

To make sure that the concrete section is large enough to support the applied shear, it is required that $V_c + V_s$ should not exceed $0.25f'_c b_v d_v$. Otherwise, enlarge the section. This is condition is exactly similar to the AASHTO general procedure explained above. Note that the amount of minimum transverse reinforcement and the maximum spacing for stirrups is calculated the same as in sections 2.3.1.1 and 2.3.1.2 of this document. More importantly, the amount of longitudinal reinforcement should also be checked at all sections considered. This is true for both general and simplified procedures described above. AASHTO LRFD (2008) proposes the following equation to check the capacity of longitudinal reinforcement

$$A_{ps}f_{ps} + A_s f_y \geq \frac{|M_u|}{d_v \phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5V_s\right)cot\theta \qquad (Eq - 2.3.14)$$

where:

$\phi_f \phi_v \phi_c =$ resistance factors taken from Article 5.5.4.2 of AASHTO LRFD (2008) as appropriate for moment, shear and axial resistance.

For the general procedure, the value for θ in degree is calculated using Eq-2.3.4. However, the value for $cot\theta$ is directly calculated from Eq-2.3.13 for the simplified procedure for prestressed and non-prestressed beams.

2.3.3 ACI Code Procedure for Shear Design of Pre-stressed and Non-prestressed Reinforced Concrete Beams

ACI Code 318-08 presents a set of equations to predict the nominal shear strength of a reinforced concrete section. Experiments have shown that the ACI provisions for shear underestimate the shear capacity of a given section and are uneconomical. However, it was recognized that ACI equations for shear over-estimates the shear capacity for large lightly reinforced concrete beams without transverse reinforcement Shioya, et.al (1989).

As stated earlier, the nominal shear strength of a concrete section is the summation of the nominal shear strengths provided by the concrete V_c and the transverse reinforcement V_s. The value of V_c for a non-prestressed concrete section subjected only to shear and flexure can be estimated as

$$V_c = 2\lambda\sqrt{f'_c}b_w d \qquad (Eq - 2.3.15)$$

In addition to the equation above, ACI 318-08 proposes a detailed equation in which the effects of bending moment present at the section is also considered.

$$V_c = \left(1.9\lambda\sqrt{f'_c} + 2500\rho_w\frac{V_u d}{M_u}\right)b_w d \qquad (Eq - 2.3.16)$$

Whereas the shear strength provided by the concrete for pre-stressed members can be estimated using the following equations

$$V_{ci} = 0.6\lambda\sqrt{f'_c}b_w d_p + V_d + \frac{V_i M_{cre}}{M_{max}} \geq 1.7\lambda\sqrt{f'_c}b_w d \qquad (Eq - 2.3.17)$$

or

$$V_{cw} = \left(3.5\lambda\sqrt{f'_c} + 0.3f_{pc}\right)b_w d_p + V_p \qquad (Eq - 2.3.18)$$

where d_p need not be taken less than $0.80h$ for both equations. The value of moment causing flexural cracking due to externally applied loads, M_{cre} at a certain section in (lb.in) is

$$M_{cre} = \frac{I}{y_t}\left(6\lambda\sqrt{f'_c} + f_{pe} - f_d\right) \qquad (Eq - 2.3.19)$$

where:

f_{pe} = compressive stress in concrete due to effective pre-stress forces only (after allowance for all pre-stress losses) at extreme fiber of section where tensile stress is caused by externally applied loads (psi).

After calculating the values for V_{ci} and V_{cw}, the nominal shear strength provided by the concrete V_c is assumed as the minimum of V_{ci} or V_{cw}. It is important to note that the inclination angle θ for the diagonal compressive stress is assumed as 45˚ in the shear provisions of the ACI Code. Hence to determine V_s which is the nominal shear strength provided by the shear reinforcement, Eq-2.3.5 is modified to

$$V_s = \frac{A_v f_{yt} d_v}{s} \qquad (Eq - 2.3.20)$$

2.3.3.1 Minimum Transverse Reinforcement

According to section 11.4.6.1 of the ACI Code, a minimum area of shear reinforcement $A_{v,min}$ shall be provided in all reinforced concrete flexural members (prestressed and non-prestressed) where V_u exceeds $0.5\phi V_c$, except in members satisfying the cases specified by the code.

$$A_{v,min} = 0.75\sqrt{f'_c}\frac{b_w s}{f_{yt}} \qquad (Eq - 2.3.21)$$

but shall not be less than $\frac{50 b_w s}{f_{yt}}$. Also the concrete strength f'_c should be in psi.

According to section 11.4.6.4 of ACI Code, for pre-stressed members with an effective pre-stress force not less than 40 percent of the tensile strength of the flexural reinforcement, $A_{v,min}$ shall not be less than the smaller value of (Eq-2.3.21) and (Eq-2.3.22).

$$A_{v,min} = \frac{A_{ps} f_{pu}\, s}{80 f_{yt} d} \sqrt{\frac{d}{b_w}} \qquad (Eq - 2.3.22)$$

The above explanation can be written explicitly as

$$A_{v,min} = Min\left\{Max\left(0.75\sqrt{f'_c}\frac{b_w s}{f_{yt}}, \frac{50 b_w s}{f_{yt}}\right), \frac{A_{ps} f_{pu} s}{80 f_{yt} d} \sqrt{\frac{d}{b_w}}\right\} \qquad (Eq - 2.3.23)$$

2.3.3.2 Maximum Spacing of Transverse Reinforcement

According to section 11.4.5.1 of the ACI Code, spacing of shear reinforcement placed perpendicular to axis of member shall not exceed $d/2$ for non-prestressed members or $0.75h$ for prestressed members, nor 24 in. The maximum spacing shall be reduced by one-half if V_s exceeds $4\sqrt{f'_c} b_w d$. Furthermore, if the value for V_s exceed $8\sqrt{f'_c} b_w d$, the concrete at the section may crush. To avoid crushing of the concrete, a larger section should be selected.

2.4 Design Procedure for Sections under Combined Shear and Torsion

Section 5.8.3.6 of the AASTHO LRFD Bridge Design Specifications (2008) provides pertinent equations to design a concrete section under combined shear and torsion. The procedure is mainly based on the general method for shear discussed earlier. No details have been provided in the code about how to design a section for combined shear and torsion if the simplified approach is used for the shear part. Hence, only the design procedure which is in the code is discussed here. At the end, the ACI procedure to design a section under combined shear and torsion is explained.

2.4.1 AASHTO LRFD Design Procedure for Sections Subjected to Combined Shear and Torsion

As stated earlier, the AASHTO LRFD general procedure is used to design a section under combined shear and torsion. The section is primarily designed for bending. The geometry and the external loads applied on the section are then used to check the shear-torsion strength of the section. Since design is an iterative process, the cross-sectional properties and the reinforcement both longitudinal and transverse are provided different values until the desired shear-torsion strength is achieved.

Below are the necessary steps to design a section for shear and torsion:

1. Determine the external loads applied on the section considered. To do this, the beam has to be analyzed for the external loads using the load combination that provide the maximum load effects. The section is then designed for bending and the cross-sectional dimensions and the amount of longitudinal reinforcement are roughly determined.
2. Having the external load effects (axial force, shear, and bending moment) at the section, the strain in the longitudinal tension reinforcement ε_s is calculated using Eq-2.3.7 provided above. It is required to substitute V_u in Eq-2.3.7 with the equivalent shear $V_{u,eq}$.

   For solid sections:

$$V_{u,eq} = \sqrt{{V_u}^2 + \left(\frac{0.9P_hT_u}{2A_0}\right)^2} \qquad (Eq - 2.4.1)$$

   For box sections:

$$V_{u,eq} = V_u + \frac{T_ud_s}{2A_0} \qquad (Eq - 2.4.2)$$

3. To determine the nominal shear strength of a section provided by concrete,V_c, the value of ε_s from step 2 is substituted into Eq-2.3.2 to determine the value for β. If the concrete strength f'_c is provided in ksi, then $V_c = 0.0316\beta\sqrt{f'_c}b_vd_v$. Otherwise $V_c = \beta\sqrt{f'_c}b_vd_v$ if f'_c is given in MPa units.
4. Substitute the value of ε_s obtained from step 2 into Eq-2.3.6 to determine the modified angle of inclination of diagonal compressive stresses θ (in degrees).
5. Is shear reinforcement required? no shear reinforcement is required if $V_u < 0.5\phi(V_c + V_p)$
6. If $V_u > 0.5\phi\left(V_c + V_p\right)$, solve Eq-2.3.5 for $\frac{A_v}{s}$ after substituting the value for θ obtained in step 4. Note that $V_s = \frac{V_u}{\phi} - V_c - V_p$

7. Calculate the torsional cracking moment for the section considered using the given equation: $T_{cr} = 0.125\sqrt{f'_c}\frac{A_{cp}{}^2}{P_c}\sqrt{1+\frac{f_{pc}}{0.125\sqrt{f'_c}}}$ $(Eq - 2.4.3)$

   where:

   T_u = factored torsional moment (kip-in.)

   T_{cr} = torsional cracking moment (kip-in.)

   A_{cp} = total area enclosed by outside perimeter of concrete cross-section (in.^{2})

   P_c = length of the outside perimeter of the concrete section (in.)

   f_{pc} = compressive stress in concrete after prestress losses have occurred either at the centroid of the cross-section resisting transient loads or at the junction of the web and flange where the centroid lies in the flange (ksi).

   $\phi = 0.9$ (specified in Article 5.5.4.3 of the AASHTO LRFD (2008)

8. Should torsion be considered? If the external factored torsional moment T_u applied on the section is such that $T_u > 0.25\phi T_{cr}$, torsion must be considered. Otherwise, ignore the torsion.

$$T_n = \frac{2A_0 A_t f_{yt} cot\theta}{s} \qquad (Eq - 2.4.4)$$

   where:

   A_0 = area enclosed by the shear flow path, including any area of holes therein (in.^{2}). It is permitted to take A_0 as 85% of the area enclosed by the centerline of stirrups.

   A_t = area of one leg of closed transverse torsion reinforcement in solid members (in.^{2})

   θ = angle of crack as determined in accordance with Eq-2.3.6 using the modified strain ε_s calculated in step 2.

9. Solve Eq-2.4.4 for $\frac{2A_t}{s}$ and sum it with the output of step 5.

$$\frac{A_{v+t}}{s} = \frac{A_v}{s} + \frac{2A_t}{s} \qquad (Eq - 2.4.5)$$

10. The amount of transverse reinforcement obtained from step 8 should be equal to or greater than the amount given by the equation below

$$A_{v,min} \geq 0.0316\sqrt{f'_c}\frac{b_v\, s}{f_{yt}} \qquad (Eq - 2.4.6)$$

11. According to the AASHTO LRFD, the spacing of transverse reinforcement shall not exceed the maximum permitted spacing, s_{max} , determined as:
    - If $v_u(ksi) < 0.125\, f'_c$, then $s_{max} = 0.8 d_v \leq 24$ in.
    - If $v_u(ksi) \geq 0.125 f'_c$, then $s_{max} = 0.4 d_v \leq 12.0$ in.

    Note that v_u given in Eq-2.3.9 is modified for torsion using $V_{u,eq}$ provided by Eq's-2.4.1 and 2.4.2
12. Is the cross-section large enough? If $V_c + V_s < 0.25 f'_c b_v d_v$, the section is large enough, otherwise enlarge the section.
13. As a last step, the longitudinal reinforcement in solid sections shall be proportioned to satisfy

$$A_{ps} f_{ps} + A_s f_y \geq \frac{|M_u|}{\phi d_v} + \frac{0.5 N_u}{\phi} + cot\theta \sqrt{\left(\left|\frac{V_u}{\phi} - V_p\right| - o.5 V_s\right)^2 + \left(\frac{0.45 P_h T_u}{2 A_0 \phi}\right)^2} \qquad (Eq - 2.4.7)$$

while for box sections the longitudinal reinforcement for torsion, in addition to that required for flexure, shall not be less than

$$A_l = \frac{T_n P_h}{2 A_0 f_y} \qquad (Eq - 2.4.8)$$

2.4.2 ACI 318-08 Design Procedure for Sections Subjected to Combined Shear and Torsion

To design a pre-stressed or non-prestressed member under combined shear and torsion loading using the ACI 318-08 provisions, the following steps can be followed:

1. Should torsion be considered? If the applied torsion on a section (pre-stressed or non-prestressed) is greater than the corresponding value given by Eq-2.4.9, the section has to be designed accordingly. Otherwise, torsion is not a concern and could be ignored. For non-prestressed members:

$$T_{th} = \phi \lambda \sqrt{f'_c} \left(\frac{A^2{}_{cp}}{P_{cp}}\right) \qquad (Eq - 2.4.9a)$$

For pre-stressed members:

$$T_{th} = \phi \lambda \sqrt{f'_c} \left(\frac{A^2{}_{cp}}{P_{cp}}\right) \sqrt{1 + \frac{f_{pc}}{4\lambda \sqrt{f'_c}}} \qquad (Eq - 2.4.9b)$$

P_{cp} is the outside perimeter of concrete cross-section and is equal to P_c defined earlier.

ϕ is the resistance factor which is equal to 0.75. Note that T_{th} is the threshold torsion.

2. Equilibrium or compatibility torsion? According to section 11.5.2.1 of ACI Code, if the applied factored torsion, T_u in a member is required to maintain equilibrium and is greater than the value given by Eq-2.4.9 depending on whether the member is prestressed or non-prestressed, the member shall be designed to carry T_u. However, in a statically indeterminate structure where significant reduction in T_u may occur upon cracking, the maximum T_u is permitted to be reduced to the values given by Eq-2.4.10.

   For non-prestressed members:

$$T_u = \phi 4\lambda\sqrt{f'_c}\left(\frac{A^2{}_{cp}}{P_{cp}}\right) \qquad (Eq-2.4.9a)$$

   For pre-stressed members:

$$T_u = \phi 4\lambda\sqrt{f'_c}\left(\frac{A^2{}_{cp}}{P_{cp}}\right)\sqrt{1+\frac{f_{pc}}{4\lambda\sqrt{f'_c}}} \qquad (Eq-2.4.9b)$$

3. Is the section large enough to resist the applied torsion? To avoid crushing of the surface concrete due to inclined compressive stresses, the section shall have enough cross-sectional area. The surface concrete in hollow members may crush soon on the side where the flexural shear and torsional shear stresses are added.

   For solid sections:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2+\left(\frac{T_u P_h}{1.7A^2{}_{oh}}\right)^2} \le \phi\left(\frac{V_c}{b_w d}+8\sqrt{f'_c}\right) \qquad (Eq-2.4.10a)$$

   For hollow sections:

$$\left(\frac{V_u}{b_w d}\right)+\left(\frac{T_u P_h}{1.7A^2{}_{oh}}\right) \le \phi\left(\frac{V_c}{b_w d}+8\sqrt{f'_c}\right) \qquad (Eq-2.4.10b)$$

   Note that the above equations can be used both for pre-stressed and non-prestressed members. For pre-stressed members, the depth $\boldsymbol{d}$ in the above equations is taken as the distance from extreme compression fiber to centroid of the prestresses and non-prestressed longitudinal tension reinforcement but need not be taken less than $0.80h$.

4. The stirrups area required for the torsion is calculated using Eq-2.4.4. This area is then added to the stirrups area required by shear calculated based on Eq-2.3.20. The angle θ in Eq-2.4.4 is assumed as 45˚ for non-prestressed and 37.5˚ for prestressed members.

5. The minimum area of transverse reinforcement required for both torsion and shear shall not be less than

$$\frac{A_v + 2A_t}{s} \geq 0.75\sqrt{f'_c}\frac{b_w s}{f_{yt}} \qquad (Eq - 2.4.11)$$

   note that the spacing for transverse torsion reinforcement shall not exceed the smaller of $P_h/8$ or 12 in.

6. he longitudinal reinforcement required for torsion can be calculated using the following equation

$$A_l = \left(\frac{A_t}{s}\right) P_h \left(\frac{f_{yt}}{f_y}\right) cot^2\theta \qquad (Eq - 2.4.12)$$

   The required longitudinal reinforcement for torsion should not be less than the minimum reinforcement proposed by ACI and given below

$$A_{l,min} = \frac{5\sqrt{f'_c}A_{cp}}{f_y} - \left(\frac{A_t}{s}\right) P_h \frac{f_{yt}}{f_y} \qquad (Eq - 2.4.13)$$

Chapter 3 - Formulation for Evaluating Ultimate Capacity

The purpose of this chapter is to evaluate the analytical tool used to determine the shear capacity of a concrete section and develop exact interaction diagrams for concrete members subjected to combined shear and torsion. In chapter two of this document, necessary information about Modified Compression Field Theory (MCFT) and its application to determine the shear or combined shear and torsion capacity of a section were provided. Research performed by Bentz et.al (2006) showed that the MCFT and its simplified version give almost exactly the same results and conforms well to the experimental results. In this chapter, output from an analytical tool called Response-2000 which is based on modified compression field theory is evaluated. In addition, exact interaction diagram for the general procedure of AASHOT LRFD are drawn.

3.1 Evaluation of Response-2000

Response-2000 was developed by Bentz and Collins (2000). This windows program is based on MCFT which can analyze moment-shear, shear-axial load, and moment-axial load responses of a concrete section. Response-2000 is designed to obtain response of a section using the initial input data. The input data depends on the desired response of a section i.e., moment-shear, shear-axial load, moment-axial load.

Knowing the fact that Response-2000 is based on MCFT, the output values may shift slightly compared to AASHTO-LRFD (2008) general procedure for shear which is based on simplified MCFT.

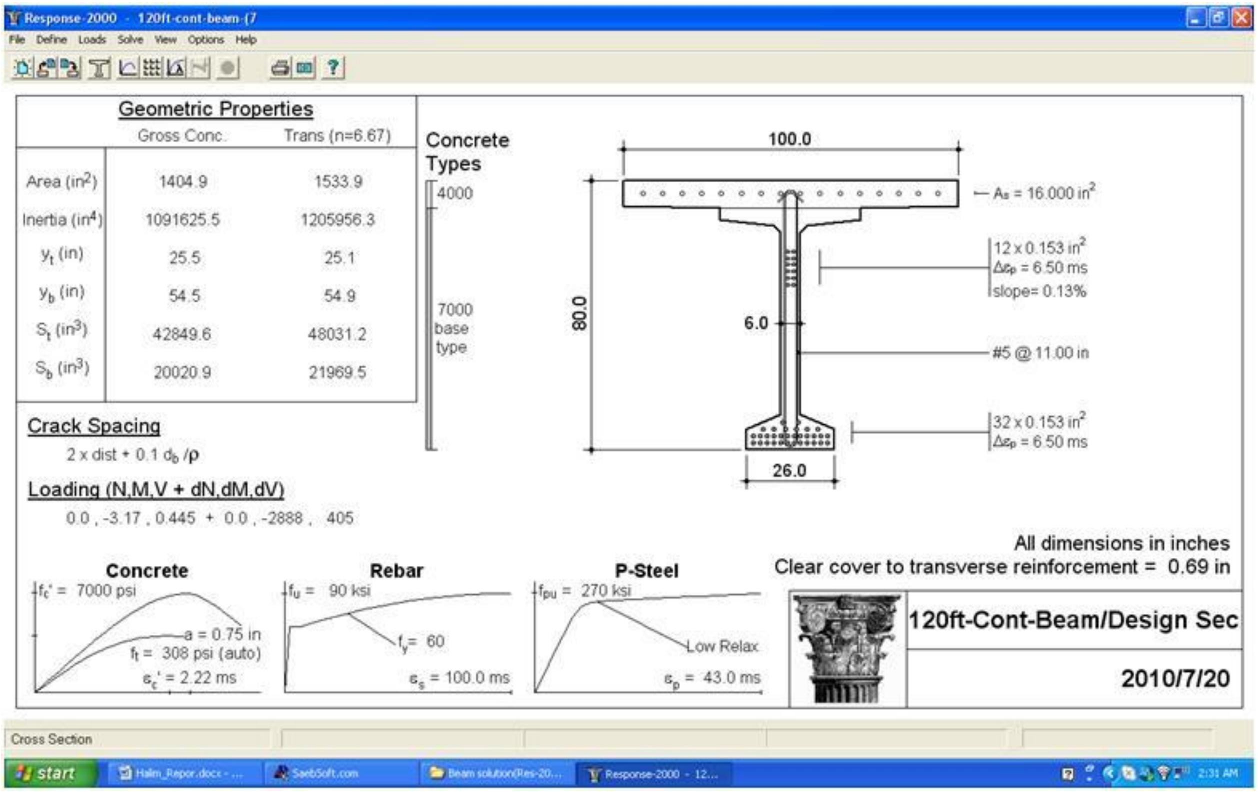


Figure 3.1 Typical Response-2000 interface

3.1.1 Review of Experimental Data Examined and Validity of Response-2000 to Determine the Shear Strength of a Concrete Section

The purpose of this section is to show how close Response-2000 can approximate the shear capacity of a member at a particular section. To study the shear behavior of concrete members, often times simply supported rectangular reinforced concrete beams without shear reinforcement are tested in research laboratories. These beams often have a depth of 15 in. or less and loaded with point loads over short shear spans (NCHRP-549). Unfortunately these tests can not represent real cases such as deep continuous bridge girders supporting distributed loads and have shear reinforcement. To address this deficiency in available experimental data and generate experimental data for cases similar to real-world situations for which no experimental data exists, the output from Response-2000 was evaluated for 34 beams. The experimental shear strengths for these beams were taken from ref (1).

Among the 34 beams selected, 22 beams were simply supported (Figure 2.3) with an overall depth, d, ranging between 125mm to 1000mm. These beams had a constant cross-sectional width, b_w, of 300mm, longitudinal reinforcement ratio, ρ_l of 0.5% to1.31%, and varying compressive strength, f'_c of 36 MPa to 98.8 MPa. The yield strength of longitudinal and shear reinforcement varied from 475 MPa to 550 MPa. In addition, two beams had shear reinforcement of #3 bars spaced 660 mm apart while the remaining twenty beams didn't have any shear reinforcement.

Twelve beams from the total 34 beams selected for the analysis were continuous (Figure **2.4**) with an overall depth, d, and cross-sectional width,b_w each ranging between 500 mm to 1000 mm and 169mm to 295 mm respectively. The longitudinal reinforcement ratio, ρ_l**,** varied between 1.03 to 1.36% while the concrete compressive strength, f'_c varied between 50 MPa and 91 MPa. The yield strength for the longitudinal and shear reinforcements varied between 475 MPa and 593 MPa. Four beams from the total twelve beams studied had shear reinforcement of $D4$ with spacing ranging between 276 mm to 440 mm.

All of the beams were shear critical in the sense that the member had enough capacity to support the associated bending moment. The longitudinal reinforcements for the simply supported beams were continued up to the ends. However, the longitudinal reinforcements for continuous beams were cut-off where bending moment had lower values. The critical section for the simply supported beam was assumed to be at the middle of the beam. This is due to the fact that the bending moment is a maximum at the middle and reduces from the full shear capacity of the section while the critical section for the continuous beam was located 1.2 m from the right support. The critical section is not necessarily where shear is a maximum; rather it is a section along the beam where the beam tends to fail in shear. For continuous beams, the critical section was located where some of the longitudinal bars on the flexural tension side of the section were not continued further. This in turn helped the strain $\boldsymbol{\varepsilon_s}$ to increase. Because the provided shear reinforcement was not enough, the cross-section was assumed to fail at that location.

To make sure that the beam exactly fails at this location, the shear-moment capacity along the length of the beam was determined using Respons-2000 and the location so called the critical-section provided the lowest moment-shear capacity. The experimental shear and moment capacity and the capacity determined using Response-2000 at shear-critical sections is tabulated in Table 3.1 .

Table 3.1 Experimental and Response-2000 shear and moment results at shear-critical section of the beams considered

		Simply Supported and Continuous* Beams						
		Beam Type	Beam Depth (mm)	Exp.Shear Force(KN)	Exp.Moment (KN.m)	Response 2000 (KN)	Resp-2000 Moment (KN.m)	$V_{exp}/V_{resp2000}$
Normal strength concrete	w/o crack control reinf.	B100	925	225	633.26	176	495.80	1.28
		BN100	925	192	544.16	175.3	496.90	1.10
		BN50	450	132	181.42	100.5	137.90	1.31
		BN25	225	73	49.68	57.6	39.50	1.27
		BN12	110	40	13.01	32.3	10.50	1.24
		B100L	925	223	627.86	158.9	447.70	1.40
		B100B	925	204	576.56	165.3	465.80	1.23
		BM100(w/stirrups)	925	342	949.16	317.5	874.90	1.08
		SE100A-45	920	201	274.49	221.06	299.20	0.909
		SE50A-45	459	69	43.78	79.91	51.00	0.863
	With crack control reinf.	B100D	925	320	889.76	213.9	595.20	1.50
		BND100	925	258	722.36	201.1	570.30	1.28
		BND50	450	163	223.27	108	147.90	1.51
		BND25	225	112	76.00	64.2	43.60	1.74
		BM100D (w/stirrups)	925	461	1270.46	308.8	851.20	1.49
		SE100B-45	920	281	370.46	261.56	346.30	1.074
		SE50B-45	459	87	54.57	88.82	56.40	0.980
High strength concrete	w/o crack control reinf.	B100H	925	193	546.86	222.7	627.20	0.87
		B100HE	925	217	611.66	222.7	627.20	0.97
		BH100	925	193	546.86	215.5	610.90	0.90
		BH50	450	132	181.42	123.5	169.30	1.07
		BH25	225	85	57.78	68.6	46.80	1.24
		BRL100	925	163	465.86	167.6	479.40	0.97
		SE100A-83	920	303	396.86	256.36	340.40	1.182
		SE100A-M-69 (w/stirrups)	920	516	652.39	521.56	658.50	0.989
		SE50A-83	459	93	58.17	92.31	57.90	1.007
		SE50A-M-69 (w/stirrups)	459	139	85.77	142.31	88.20	0.977
	With crack control reinf.	BHD100	925	278	776.36	252.8	706.00	1.10
		BHD50	450	193	263.77	134.4	184.70	1.44
		BHD25	225	111	75.33	81.9	55.30	1.36
		SE100B-83	920	365	471.24	297.16	388.60	1.228
		SE100B-M-69 (w/stirrups)	920	583	732.77	637.57	797.90	0.914
		SE50B-83	459	101	62.97	101.32	63.50	0.997
		SE50B-M-69 (w/stirrups)	459	152	93.56	154.21	95.40	0.986

* Data for continuous beams are shown shaded.

To generate data using Response-2000, the experimental shear and moment at shear critical sections and the necessary properties of the section such as $f'_c, f_y, b_w, h,$ and reinforcement configuration were used as the initial input values. From Figure 2.3 and Figure **2.4**, it is known that the shear is constant along the beams and is equal to V, which is the external applied load. To find the exact shear and moment applied at the critical section, the shear and moment from self-weight of the beams were also added. Refer to ref (1) for further details about the cross-sectional properties of the beams. From the data generated by Response-2000 and the experimental results tabulated in Table 3.1, it is concluded:

1. For normal strength concrete, simply supported beams w/o crack control reinforcement; Response-2000 underestimated the shear capacity by 24% (average) for the eight beams studied. However, for the two continuous beams, it overestimated the shear capacity by 11%.
2. For the five normal strength concrete, simply supported beams with crack control reinforcement, Response-2000 significantly under-estimated the shear capacity (51%), while for the two continuous beams, it underestimated the shear capacity by 2.7%.
3. For high-strength concrete, simply supported beams w/o crack control reinforcement, Response-2000 overestimated the shear capacity by 7.3% for the four beams out of six beams considered while it underestimated the shear strength by 15% for the remaining two beams. Nevertheless, on average, it can reasonably predict the shear capacity for all six beams. Meanwhile, Response-2000 underestimated the shear capacity by 3.9% for the four continuous beams.
4. For high strength concrete, simply supported beams with crack control reinforcement, Response-2000 considerably underestimated the shear capacity by 30% average for the three beams studied, while for the four continuous beams the shear capacity is underestimated roughly by 3.1%.

Based on the above explanation and Table 3.1, it is concluded that Response-2000 can accurately predict the shear strength for normal and high strength concrete continuous beams with and w/o crack control reinforcement. However, for the simply supported beams the shear strength predicted by Response-2000 differs from case to case as explained above. Nevertheless, according to NCHRP Web-Only-Document (78), the statistical data for the shear strength of 64 reinforced concrete members proved that Response-2000 gives the best shear strength predictions for the members. Figure 3.2 is provided to graphically present the above conclusions for the two general categories of beams (simply supported, and continuous).

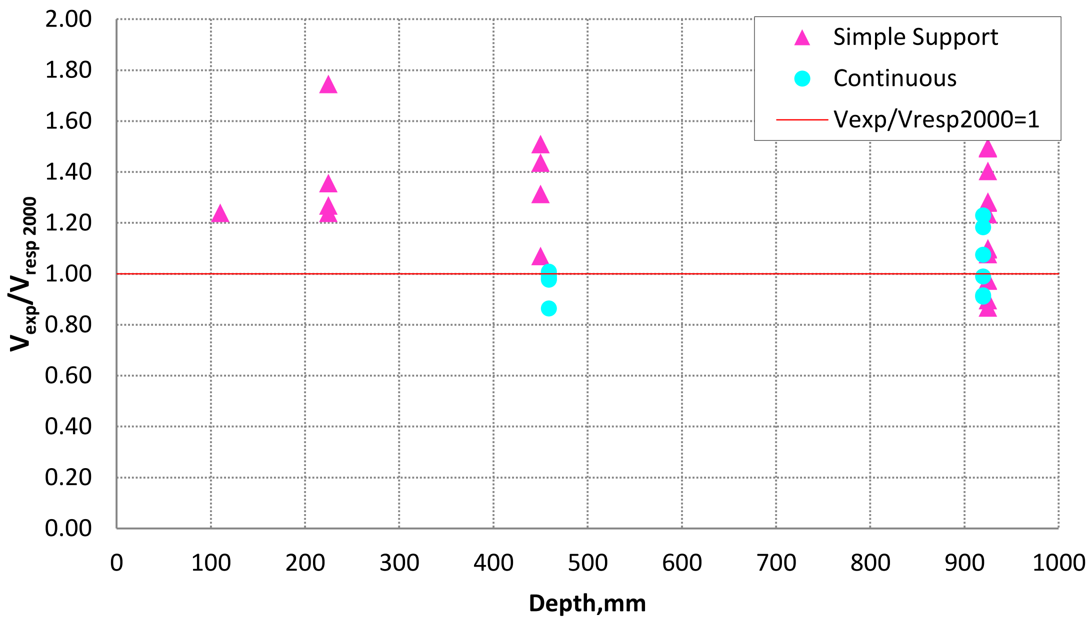


Figure 3.2 (Vexp/VResp-2000-Depth) Relationship for 34 reinforced concrete sections

The above figure shows the ratio of experimental shear and shear obtained from Response-2000 for all 34 beams considered. As stated earlier, it is observed that the ratio of $\frac{V_{exp}}{V_{Res2000}}$ is close to 1.0 for continuous beams while the values are considerably higher for simply supported beams. The line drawn at the middle shows the boundary where the experimental shear strength is equal to that obtained from Response-2000. The data points lower than the line show cases where Response-2000 over-estimates the shear capacity at the critical sections, while the values above the line show cases where Response-2000 under-estimate the shear strength of the sections.

3.2 Plotting Exact AASHTO LRFD Interaction Diagrams for Combined Shear and Torsion

Shear-torsion interaction diagram for a section provides the ultimate capacity of a section under various combinations of shear and torsion. Depending on the equations used for the combined shear and torsion response of a section, the interaction diagram could either be linear, a quarter of a circle, an ellipse, or composed of several broken lines. In the following section, the procedure to plot exact shear-torsion interaction diagrams using the corresponding provisions of AASHTO LRFD (2008) is presented.

To determine the nominal torsional capacity of a section (Eq-2.4.4), section 11.5.3.6 of the ACI Code permits to give θ values from 30˚ to 45˚ while it is always assumed 45˚ for shear. For the purpose of comparison, the ACI shear-torsion interaction diagrams for θ equal to 30˚ and 45˚ are also plotted.

3.2.1 Exact Shear-Torsion Interaction Diagrams Based on AASHTO LRFD (2008) Provisions

Knowing that the transverse reinforcement required for shear and torsion for a section shall be added together, this fact provides the basic equation to plot $T - V$ interaction diagrams. From Eq's-2.3.5 and 2.4.4, the amount of transverse reinforcement required to resist shear and torsion can be found as

$$\frac{A_t f_{yt}}{s} = \frac{T_n}{2A_0 cot\theta} + \frac{V_n - V_c}{2d_v cot\theta} \qquad (Eq - 3.2.1)$$

The nominal shear strength provided by the concrete V_c can be substituted with $0.0316\beta\sqrt{f'_c}b_v d_v$ when f'_c is given in ksi. However V_c is equal to $\beta\sqrt{f'_c}b_v d_v$ when the concrete strength is given in MPa. The factor β in Eq-2.3.2 is given in terms of longitudinal strain ε_s . Depending on the case, the value for ε_s in Eq-2.3.7 shall be modified. Furthermore, assuming the section is subjected to combined shear and torsion, the value for shear in Eq-2.3.7 should also be modified using the equivalent shear given in Eq-2.4.1 and 2.4.2 for solid and box sections respectively. The modified expression for ε_s is then substituted into Eq-2.3.2 as a result of which an expression for β would be obtained in terms of V and T. In addition, the modified expression for strain ε_s is also substituted into Eq-2.3.6 to determine an expression for θ. If the section is subjected to combined shear, torsion, and bending moment; the bending moment could either be written in terms of shear or a fixed value shall be provided. Consequently V_c and θ are substituted into above Eq-3.2.1. Knowing the reinforcement and cross-sectional properties of the section, Eq-3.2.1 would yield an equation containing V and T as the only variables. For a certain range of values for V provided it does not exceed the pure shear capacity of the section, the corresponding torsion is easily determined using "Excel Goal Seek" function or any other computer program.

To determine the maximum torsion that a section can resist corresponding to the shear values provided, the shear stress in Eq-2.3.9 is set equal to the maximum allowable value of $0.25f'_c$ and the shear V_u modified using Eq-2.4.1 or 2.4.2. For a given value of shear, the related value for torsion is then determined by solving Eq-2.3.9.

On the other hand, Eq-2.4.7 is used to determine torsion that causes the longitudinal reinforcement to yield. To solve Eq-2.4.7, the same shear values as in the previous stages are substituted into the equation. Meanwhile the expression given as Eq-2.3.5 for V_s is also substituted. Note that the equation may further be modified depending on the case considered i.e., A_{ps}, f_{ps} and V_p for non-prestressed members and other terms not satisfying for a certain case shall be set to zero. It is extremely important to remember that V shall not be modified because it is already modified in Eq-2.4.7. Finally the equation is solved for T using "Excel Goal Seek" function.

For a particular value of shear, the corresponding minimum value for torsion is selected from the three analyses explained. Note that all resistance factors are assumed as 1.0 because the strength of a section that has already been designed is evaluated. Six $T - V$ interaction diagrams representing 20 beams are included in chapter five of this document.

3.2.2 Exact Shear-Torsion Interaction Diagrams Based on ACI 318-08 Provisions

The procedure to draw $T - V$ interaction diagrams using the corresponding ACI provisions is simple compared to AASHTO LRFD (2008). The main equations used to plot the interaction diagrams are the equations based on the fact that the shear and torsion transverse reinforcement are added together and that the shear stress in concrete should not exceed beyond the maximum allowable limit of$10\sqrt{f'_c}$.

$$\frac{A_t f_{yt}}{s} = \frac{T_n}{2A_0 cot\theta} + \frac{V_n - V_c}{2d} \qquad (Eq - 3.2.2)$$

Having $V_c = 2\sqrt{f'_c} b_w d$ when the concrete strength is given in psi and θ equal to 30° and 45°; the above equation is solved for T by providing different values for V. Making sure that V does not exceed the pure shear capacity of the section.

Eq-2.4.10a or 2.4.10b is solved for T depending on whether the section is solid or hollow to determine the maximum torsion that a section can support corresponding to a certain value of shear. The maximum torsion means that the concrete at section may crush if slightly larger torsion is applied on the section. Note that the resistance factor is set equal to 1.0.

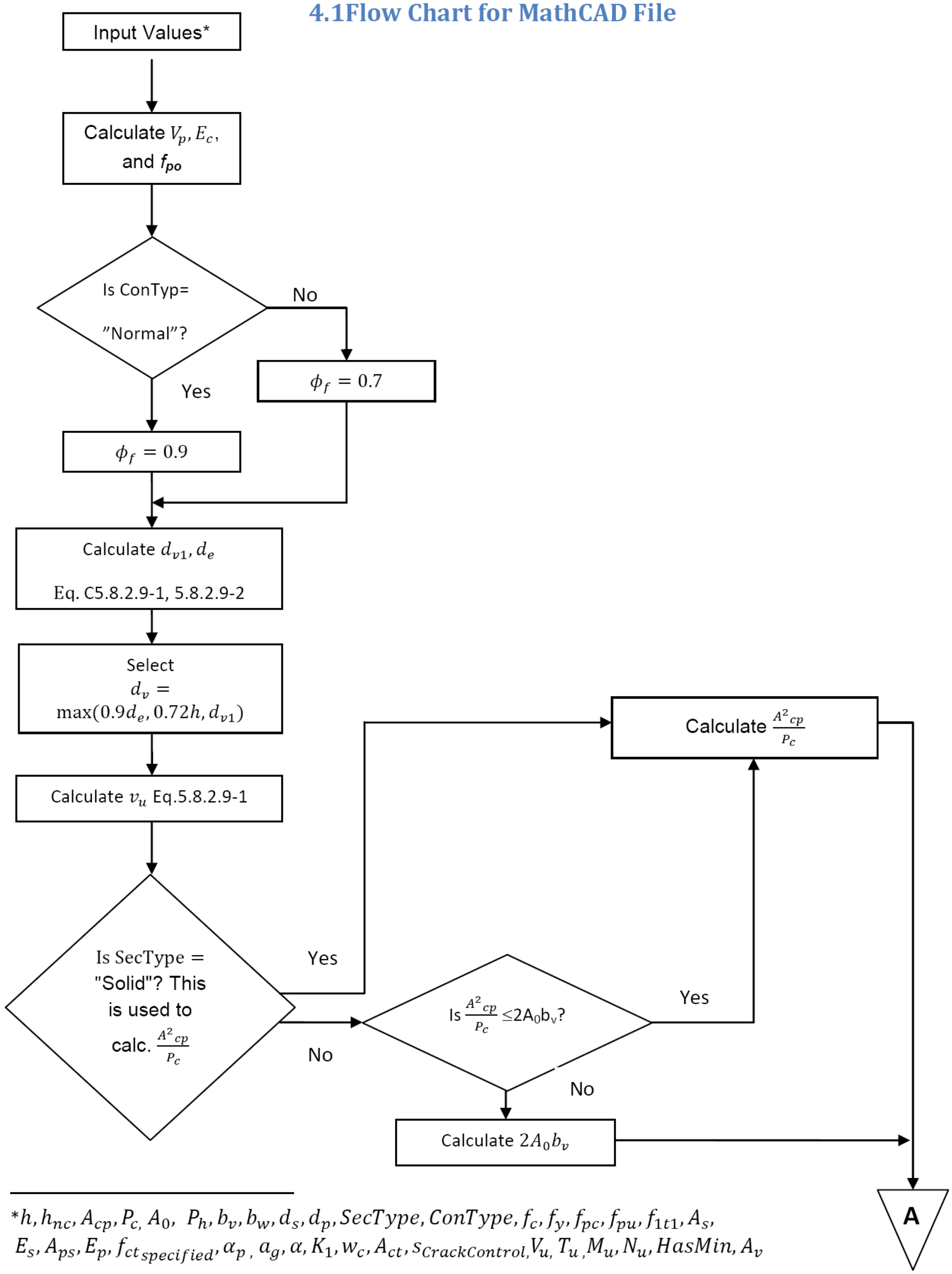
The smaller values for T is selected for a particular value of shear and the same process is followed for other points on $T - V$ interaction diagrams. The ACI interaction diagrams both for θ equal to 30˚ and 45˚ are included in chapter five of this document.

Chapter 4 - Development of AASHTO Based Shear-Torsion Design Tool (MathCAD)

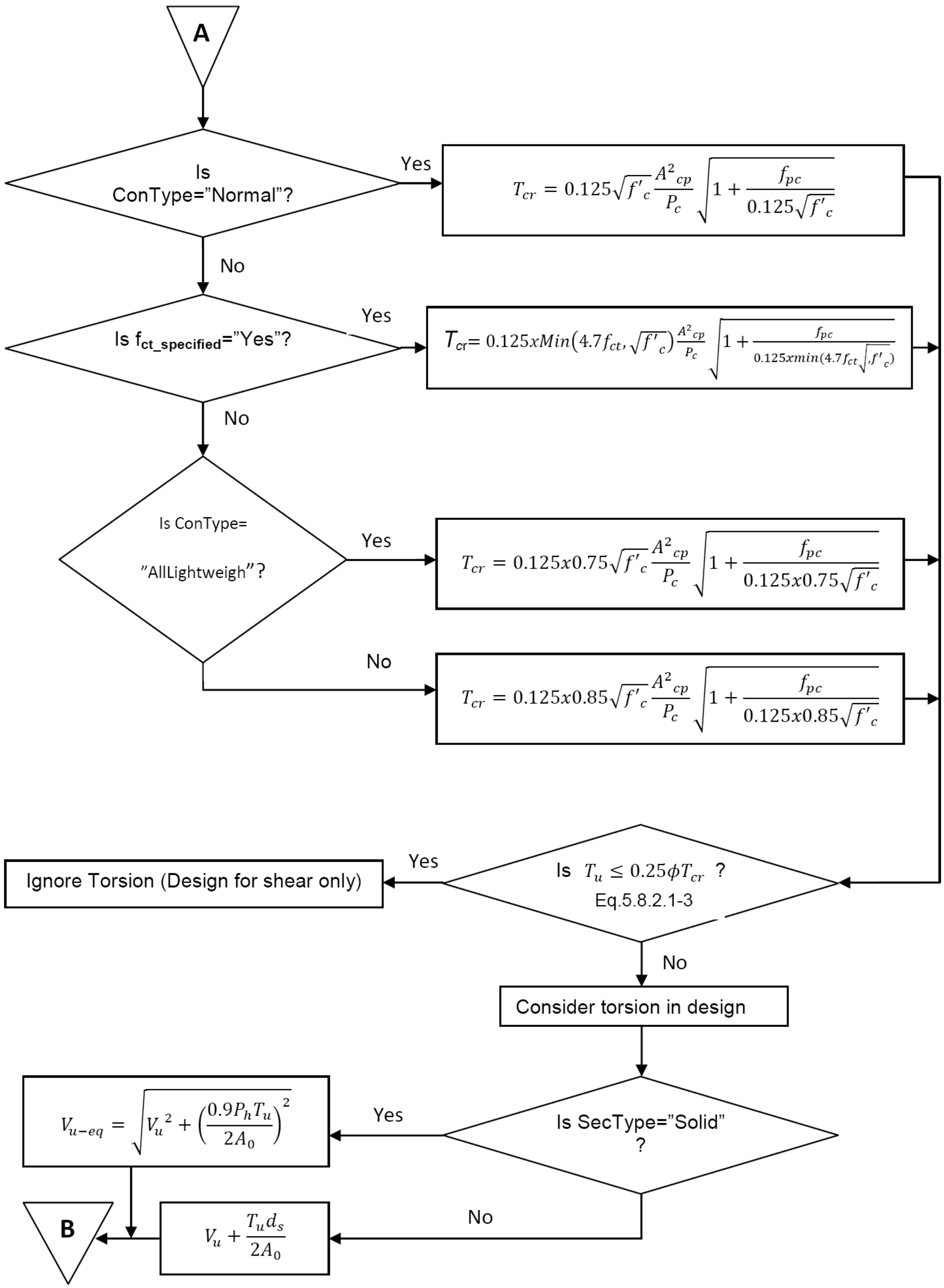
A MathCAD design tool was developed to design sections subjected to combined shear and torsion using the corresponding AASHTO LRFD provisions. However, sections under shear and torsion where torsion is negligible can also be designed using the developed design tool. The program is developed for kip-in units and the initial input values shall be entered in the highlighted yellow fields. In addition, the address of each equation used is also provided in the AASHTO LRFD (2008) code. This may help to locate the equation in the code.

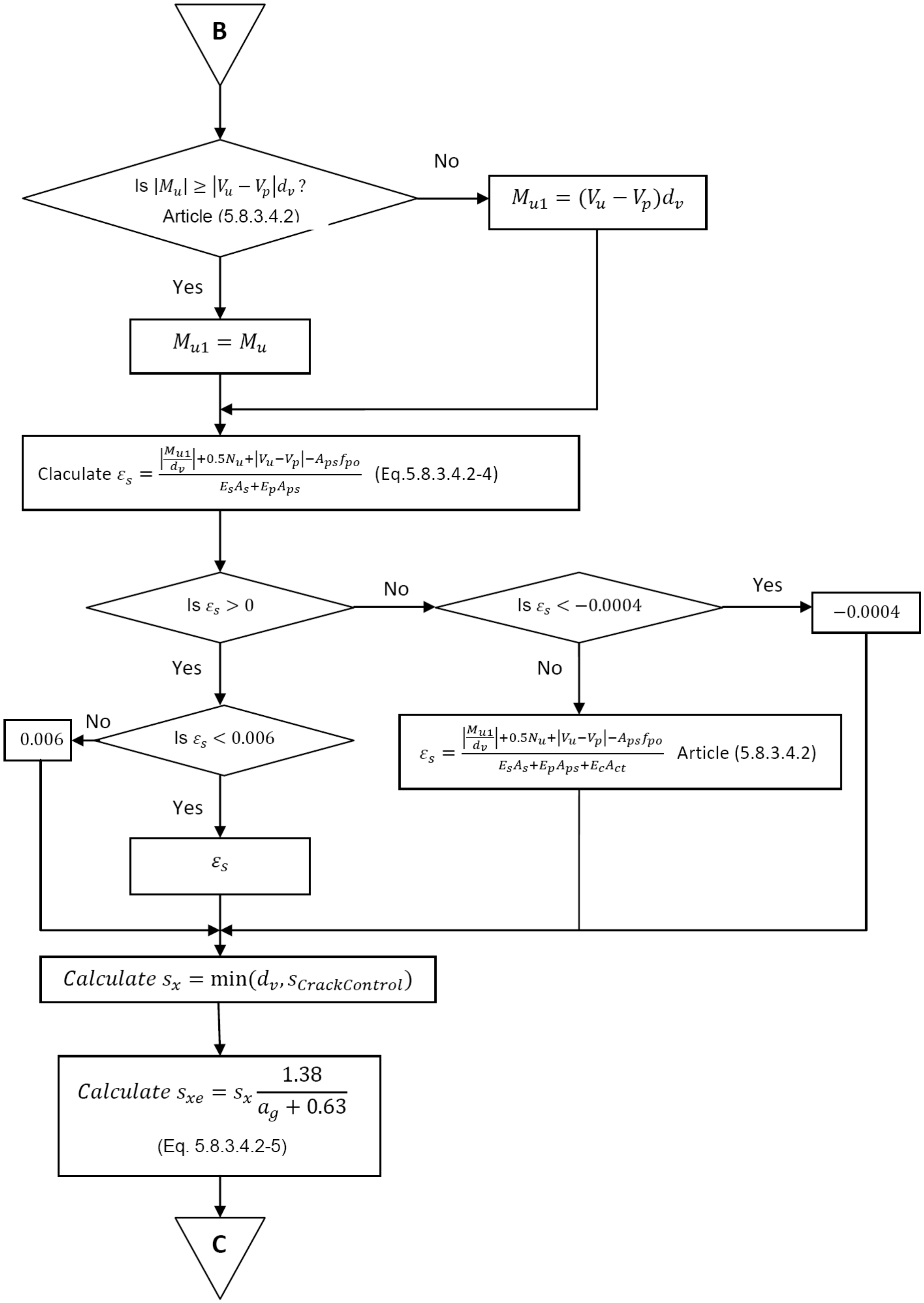
Brief description where ever needed has been provided in the program to help understand different variables used. It is essential to enter the required initial input with proper units as written in the program. Below is the flow chart for the MathCAD design tool to show how the program functions. Furthermore, an example solved using the developed file has been added in Appendix-C.

4.1Flow Chart for MathCAD File

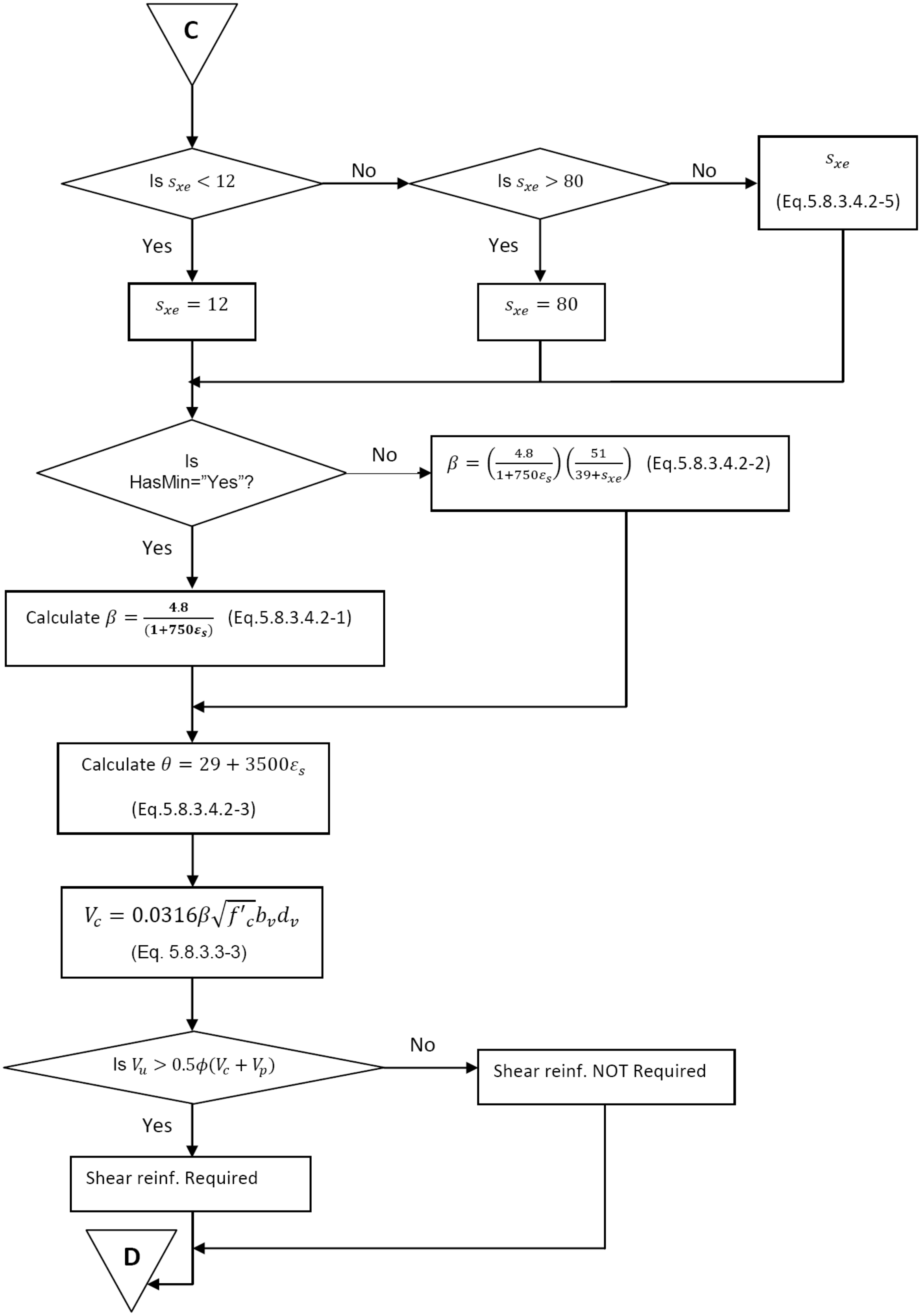


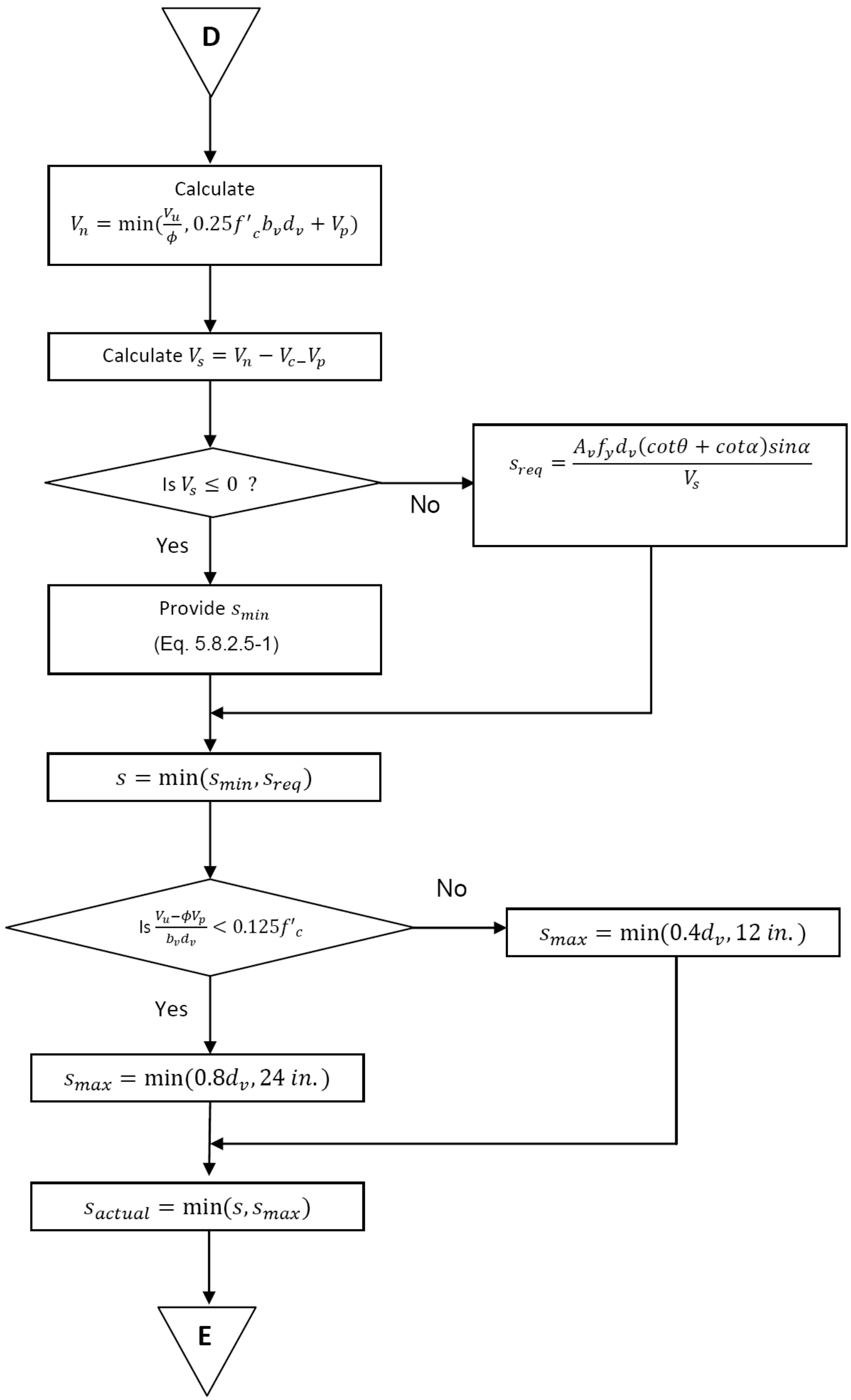
*$h, h_{nc}, A_{cp}, P_c, A_0, P_h, b_v, b_w, d_s, d_p, SecType, ConType, f_c, f_y, f_{pc}, f_{pu}, f_{1t1}, A_s, E_s, A_{ps}, E_p, f_{ct_{specified}}, \alpha_p, a_g, \alpha, K_1, w_c, A_{ct}, s_{CrackControl}, V_u, T_u, M_u, N_u, HasMin, A_v$

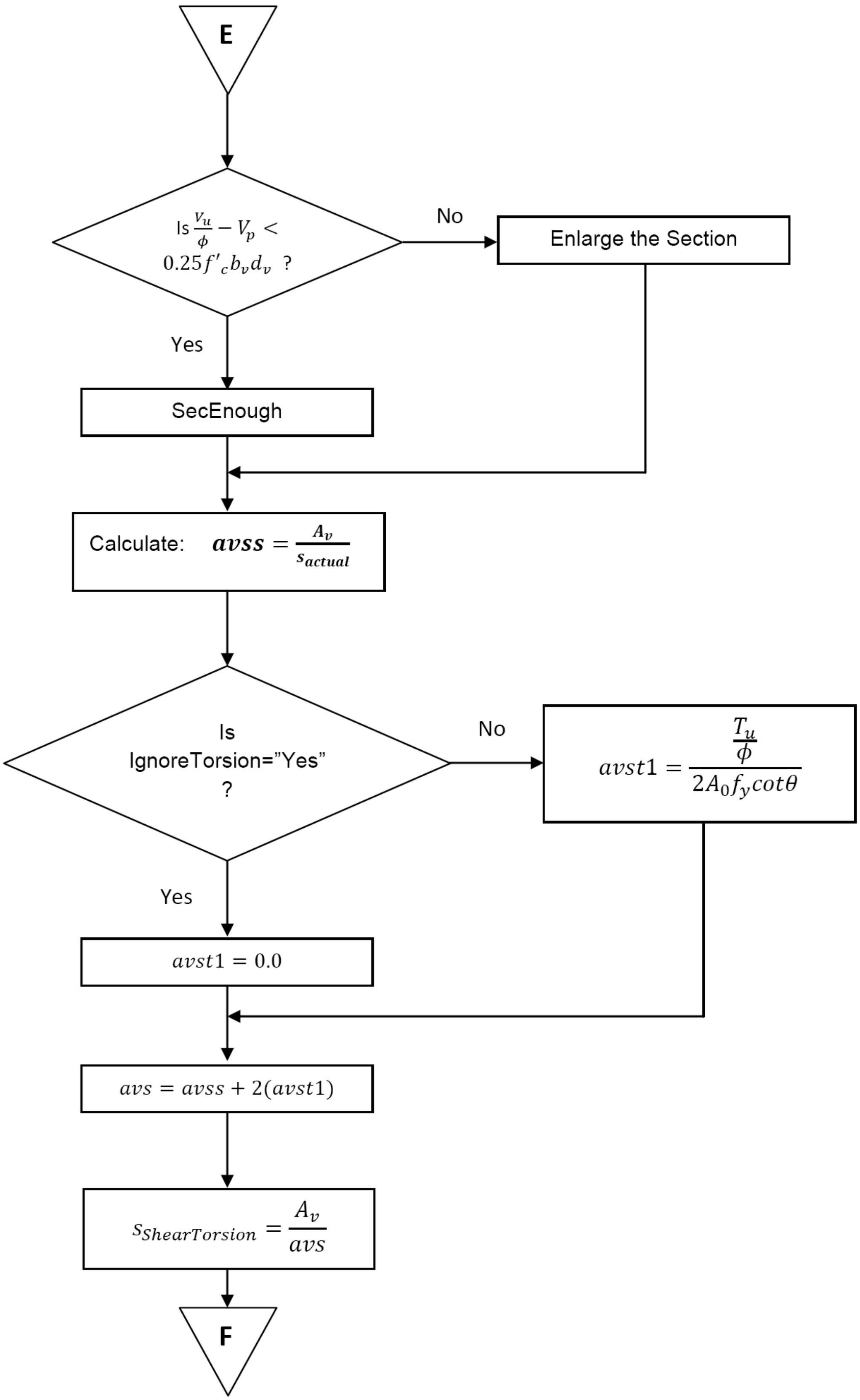

A
Is ConType="Normal"?
Yes
$T_{cr} = 0.125\sqrt{f'_c}\frac{A^2{}_{cp}}{P_c}\sqrt{1+\frac{f_{pc}}{0.125\sqrt{f'_c}}}$
No
Is $f_{ct\_specified}$="Yes"?
Yes
$T_{cr} = 0.125xMin(4.7f_{ct}, \sqrt{f'_c})\frac{A^2{}_{cp}}{P_c}\sqrt{1+\frac{f_{pc}}{0.125xmin(4.7f_{ct}, \sqrt{f'_c})}}$
No
Is ConType= "AllLightweigh"?
Yes
$T_{cr} = 0.125x0.75\sqrt{f'_c}\frac{A^2{}_{cp}}{P_c}\sqrt{1+\frac{f_{pc}}{0.125x0.75\sqrt{f'_c}}}$
No
$T_{cr} = 0.125x0.85\sqrt{f'_c}\frac{A^2{}_{cp}}{P_c}\sqrt{1+\frac{f_{pc}}{0.125x0.85\sqrt{f'_c}}}$
Is $T_u \le 0.25\phi T_{cr}$?
Eq.5.8.2.1-3
Yes
Ignore Torsion (Design for shear only)
No
Consider torsion in design
Is SecType="Solid" ?
Yes
$V_{u-eq} = \sqrt{V_u{}^2 + \left(\frac{0.9P_hT_u}{2A_0}\right)^2}$
No
$V_u + \frac{T_ud_s}{2A_0}$
B

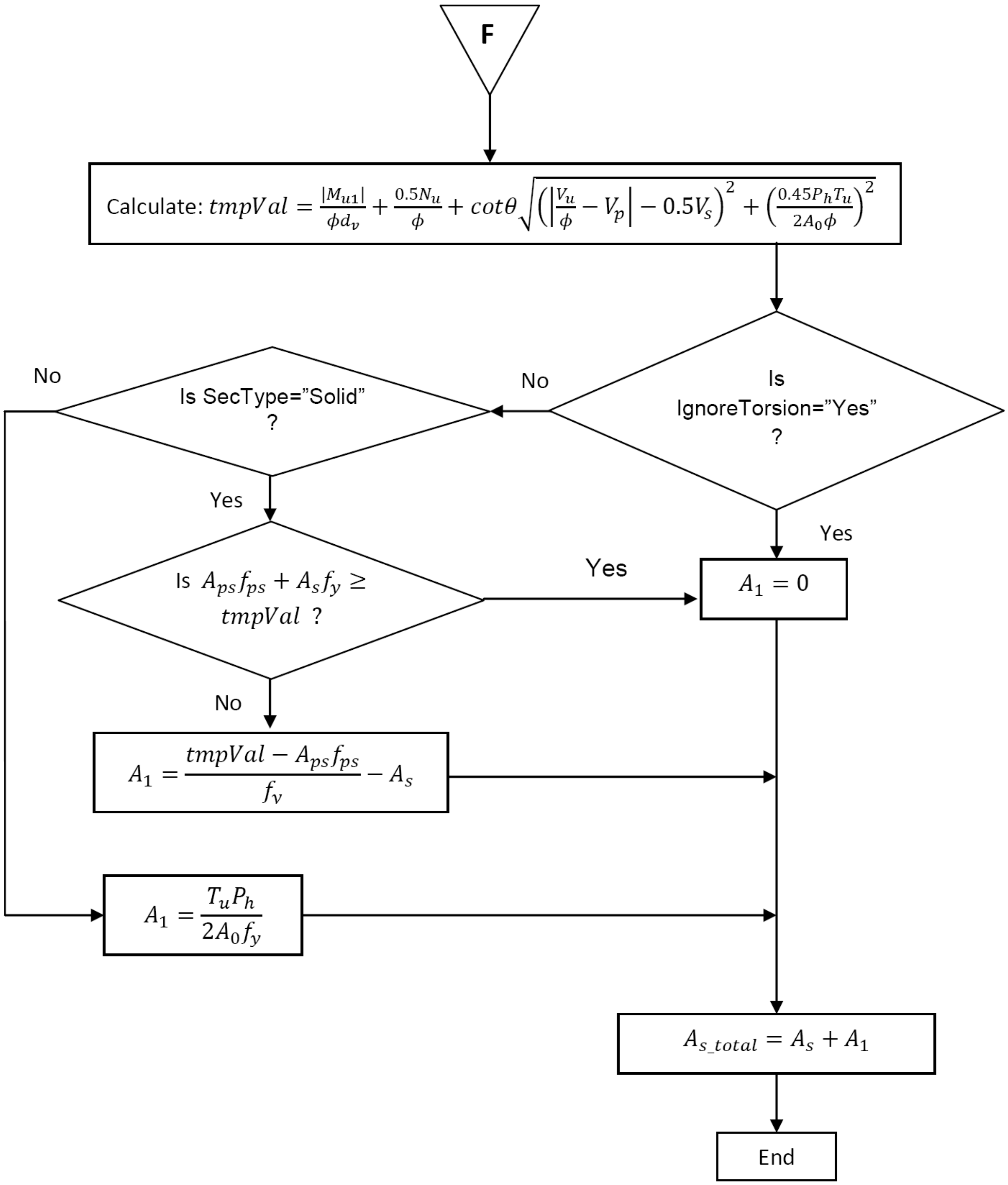


B
Is $|M_u| \geq |V_u - V_p|d_v$?
Article (5.8.3.4.2)
No
$M_{u1} = (V_u - V_p)d_v$
Yes
$M_{u1} = M_u$
Claculate $\varepsilon_s = \frac{\left|\frac{M_{u1}}{d_v}\right| + 0.5N_u + |V_u - V_p| - A_{ps}f_{po}}{E_sA_s + E_pA_{ps}}$ (Eq.5.8.3.4.2-4)
Is $\varepsilon_s > 0$
No
Is $\varepsilon_s < -0.0004$
Yes
-0.0004
Yes
No
Is $\varepsilon_s < 0.006$
0.006
No
$\varepsilon_s = \frac{\left|\frac{M_{u1}}{d_v}\right| + 0.5N_u + |V_u - V_p| - A_{ps}f_{po}}{E_sA_s + E_pA_{ps} + E_cA_{ct}}$ Article (5.8.3.4.2)
Yes
ε_s
$Calculate\ s_x = \min(d_v, s_{CrackControl})$
$Calculate\ s_{xe} = s_x \frac{1.38}{a_g + 0.63}$
(Eq. 5.8.3.4.2-5)
C


C
Is $s_{xe} < 12$
No
Is $s_{xe} > 80$
No
s_{xe}
(Eq.5.8.3.4.2-5)
Yes
Yes
$s_{xe} = 12$
$s_{xe} = 80$
Is HasMin="Yes"?
No
$\beta = \left(\frac{4.8}{1+750\varepsilon_s}\right)\left(\frac{51}{39+s_{xe}}\right)$ (Eq.5.8.3.4.2-2)
Yes
Calculate $\beta = \frac{\mathbf{4.8}}{(\mathbf{1+750}\boldsymbol{\varepsilon_s})}$ (Eq.5.8.3.4.2-1)
Calculate $\theta = 29 + 3500\varepsilon_s$
(Eq.5.8.3.4.2-3)
$V_c = 0.0316\beta\sqrt{f'_c}b_v d_v$
(Eq. 5.8.3.3-3)
Is $V_u > 0.5\phi(V_c + V_p)$
No
Shear reinf. NOT Required
Yes
Shear reinf. Required
D


D
Calculate
$V_n = \min(\frac{V_u}{\phi}, 0.25f'_c b_v d_v + V_p)$
Calculate $V_s = V_n - V_{c-}V_p$
Is $V_s \leq 0$?
No
$s_{req} = \frac{A_v f_y d_v (cot\theta + cot\alpha) sin\alpha}{V_s}$
Yes
Provide s_{min}
(Eq. 5.8.2.5-1)
$s = \min(s_{min}, s_{req})$
No
Is $\frac{V_u - \phi V_p}{b_v d_v} < 0.125f'_c$
$s_{max} = \min(0.4d_v, 12\ in.)$
Yes
$s_{max} = \min(0.8d_v, 24\ in.)$
$s_{actual} = \min(s, s_{max})$
E


E
Is $\frac{V_u}{\phi} - V_p <$ $0.25 f'_c b_v d_v$?
No
Enlarge the Section
Yes
SecEnough
Calculate: $\boldsymbol{avss} = \frac{A_v}{s_{actual}}$
Is IgnoreTorsion="Yes" ?
No
$avst1 = \frac{\frac{T_u}{\phi}}{2A_0 f_y cot\theta}$
Yes
$avst1 = 0.0$
$avs = avss + 2(avst1)$
$s_{ShearTorsion} = \frac{A_v}{avs}$
F


F
Calculate: $tmpVal = \frac{|M_{u1}|}{\phi d_v} + \frac{0.5N_u}{\phi} + cot\theta\sqrt{\left(\left|\frac{V_u}{\phi} - V_p\right| - 0.5V_s\right)^2 + \left(\frac{0.45P_hT_u}{2A_0\phi}\right)^2}$
Is IgnoreTorsion="Yes" ?
No
Yes
Is SecType="Solid" ?
No
Yes
Is $A_{ps}f_{ps} + A_sf_y \geq tmpVal$?
Yes
No
$A_1 = 0$
$A_1 = \frac{tmpVal - A_{ps}f_{ps}}{f_y} - A_s$
$A_1 = \frac{T_uP_h}{2A_0f_y}$
$A_{s\_total} = A_s + A_1$
End

Chapter 5 - Results and Discussion

5.1 Analysis for Shear Only:

In Figure 5.1 the predicted shear strength at different sections along the span for BM100 using AASHTO LRFD general procedure, simplified AASHTO procedure for prestressed and non-prestressed concrete members, ACI 318-08, and Response-2000 are plotted. For ACI 318-08, the nominal shear strength provided by concrete was calculated both using ACI Eq (11-3) and ACI Eq (11-5). Knowing that Response-2000 underestimated the shear strength by 24% for normal strength concrete simply supported beams without crack control reinforcement, it can be concluded that the results obtained using the general AASHTO procedure are reasonably accurate.

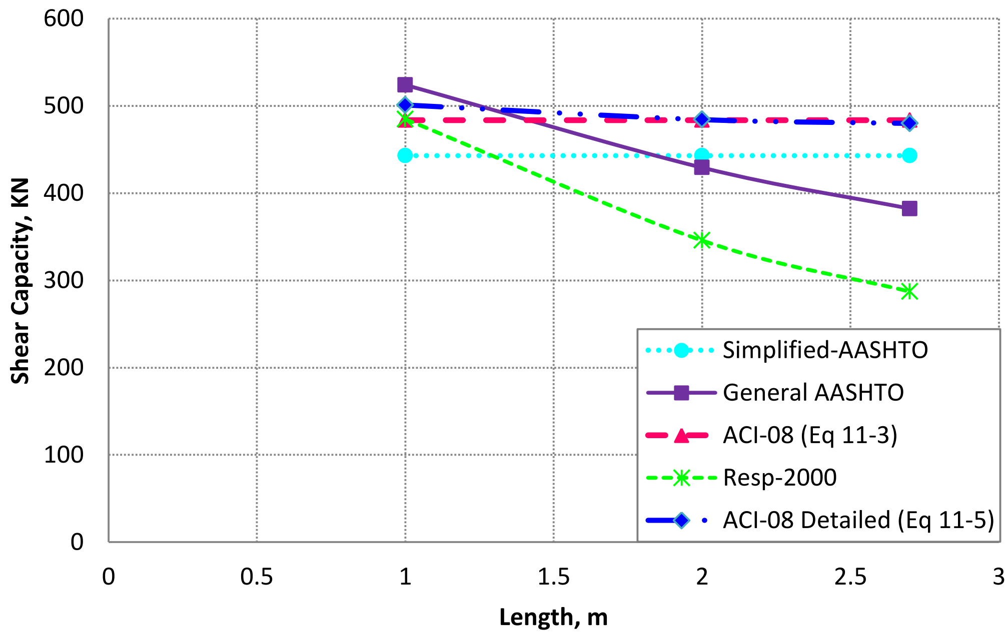


Figure 5.1 Predicted shear strength along the length of BM100, normal strength, non-prestressed simply supported reinforced concrete beam

On the other hand, both simplified AASHTO and ACI 318-08 seem to slightly overestimate the shear capacity. As shown in the figure, both ACI Eq (11-3) and (11-5) used to predict V_c led to almost the same overall shear capacity of sections. However, using ACI Eq (11-3) the shear strength at different sections along the beam is constant because the beam is prismatic and has the same spacing 16 in. (406 mm) for transverse reinforcement throughout the span while the shear strength using ACI Eq (11-5) follows decreasing trend because of the increasing moment towards center of the beam. Note that ACI Eq (11-3) and (11-5) are numbered here as Eq-2.3.15 and 2.3.16.

The shear strength for the general AASHTO procedure and Response-2000 varies along the beam span because of the varying longitudinal strain ε_x which is one-half of the strain in non-prestressed longitudinal tension reinforcement given in Eq-2.3.7.

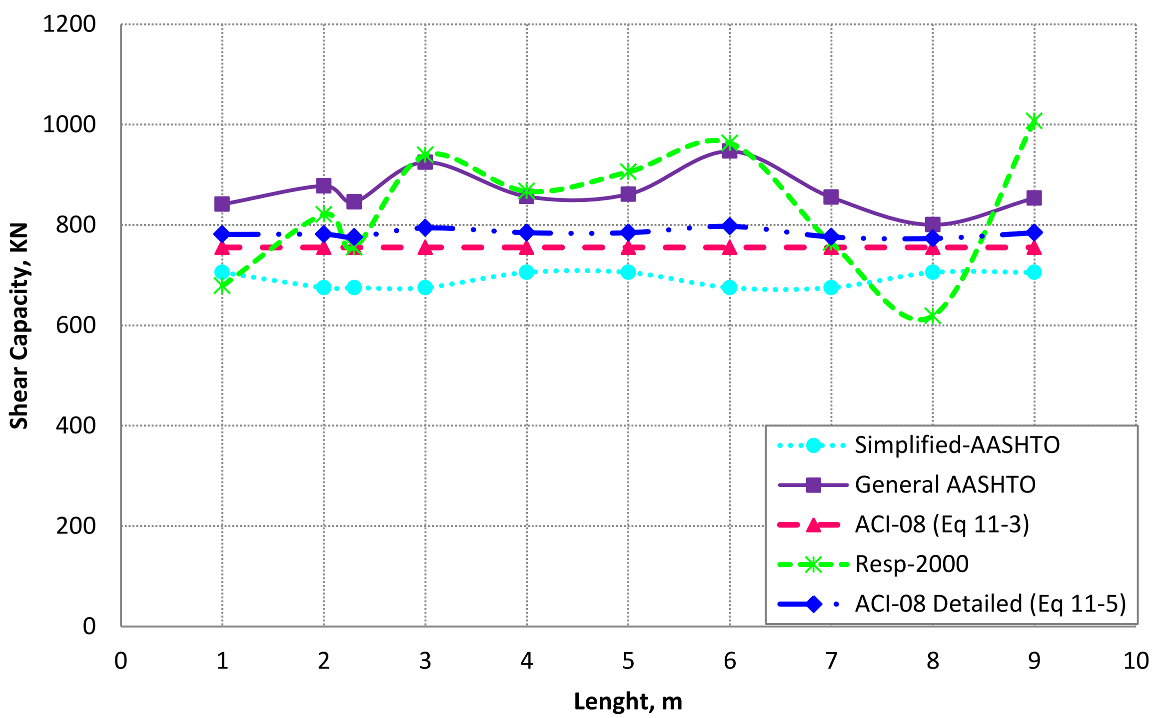


Figure 5.2 Predicted shear strength along the length of SE100A-M-69, high strength continuous non-prestressed reinforced concrete beam

The above figure shows the shear strength predictions for SE100A-M-69. From the previous evaluation of Response-2000, it was obtained that Response-2000 underestimates the shear strength by 3.9% (average) for high strength concrete continuous beams without crack control reinforcement. In the above figure, it is seen that the shear strength predictions using the general AASHTO procedure closely follow Respnse-2000 predictions for most of the sections along the span. Note that Response-2000 highly underestimates the shear strength for sections subjected to large moment and relatively less longitudinal reinforcement. Such locations happen to be at 1m and 8 m from the left. Accordingly, Response-2000 highly overestimates the shear strength for locations with approximately zero moment and enough longitudinal reinforcement. An example for such location would be a section at 9 m from left along the beam (Figure **2.4**). As shown in the figure, the simplified AASHTO and ACI 318-08 where V_c is calculated using ACI Eq (11-3) give conservative results while ACI Eq (11-5) is better in this regard. Overall, the general AASHTO procedure gives convincing results for this case. Meanwhile, the shear strength is influenced by the variations in moment and longitudinal tensile reinforcement both for simplified AASHTO and a case where V_c is calculated using ACI Eq (11-5).

Figure 5.3 shows the shear strength predictions using the aforementioned procedures for continuous prestressed high strength concrete girder (BT-72). The beam as depicted in Figure 2.6 has a span of 120 ft (36.6 m) and a total number of 44, half-inch (12.7 mm) diameter, seven wire, 270 ksi (1861 MPa) low relaxation prestress strands. The beam had a combination of draped and straight strands such that twelve of the strands were draped and the remaining 32 were straight. Noting the fact that Response-2000 was not validated for prestressed concrete beams, the shear strength results for all the methods are reasonably close to each other. In contrast to the previous cases, the shear strength for the entire methods follow decreasing trend as it goes far from the support. This is due to the fact that the detailed ACI Eq's. (11-10) and (11-12) takes into account the bending moment effects present at the section.

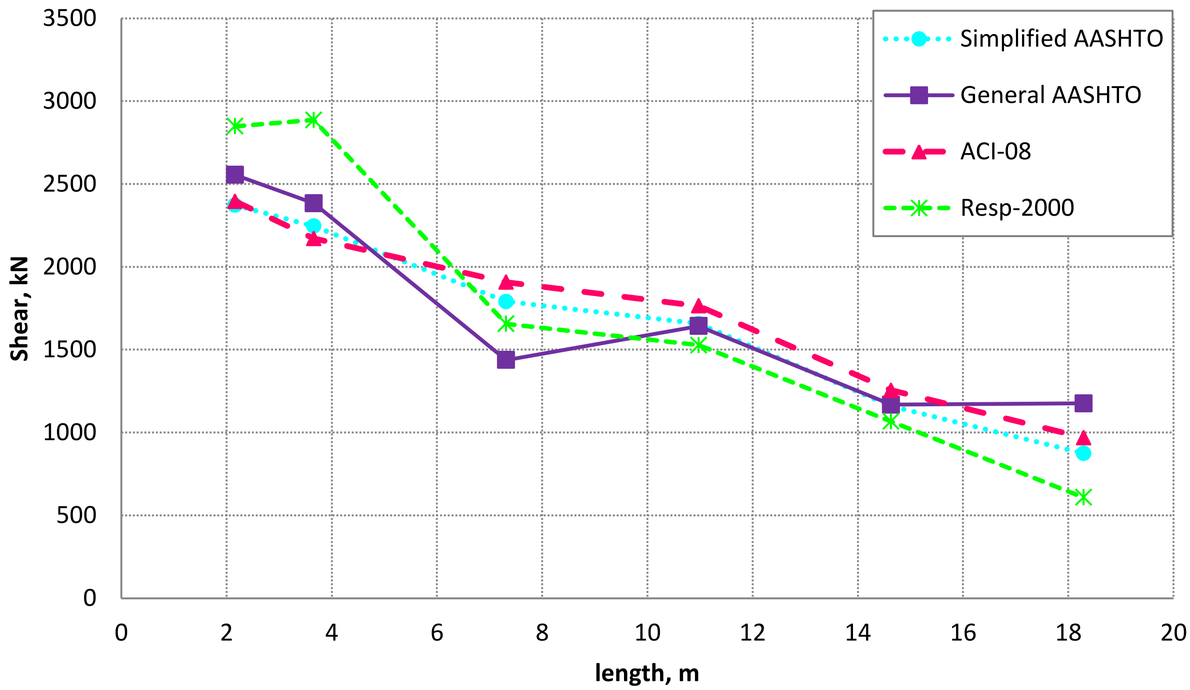


Figure 5.3 Predicted shear strength for Bulb-T (BT-72), high strength continuous pre-stressed concrete member

In Figure 5.4 the results for (8DT18) simply supported double-T prestressed beam with harped strands are plotted. The beam did not have any transverse reinforcement and the whole nominal shear strength for the section was provided by the concrete and the P/S effects. In other words, the results plotted show the nominal shear strength provided by the concrete V_c. As stated earlier, Response-2000 gives higher shear strength at section 1.5 ft (458 mm) from the support because of small bending moment and underestimates the shear strength at 16 ft (4.88 m) from the support where the moment is almost reaching its peak value. For cases other than this, both ACI 318-08 and simplified AASHTO give consistent results, however the general AASHTO procedure highly overestimates the shear strength or V_c in this case. To verify which method gives reliable results, more experimental work has to be done.

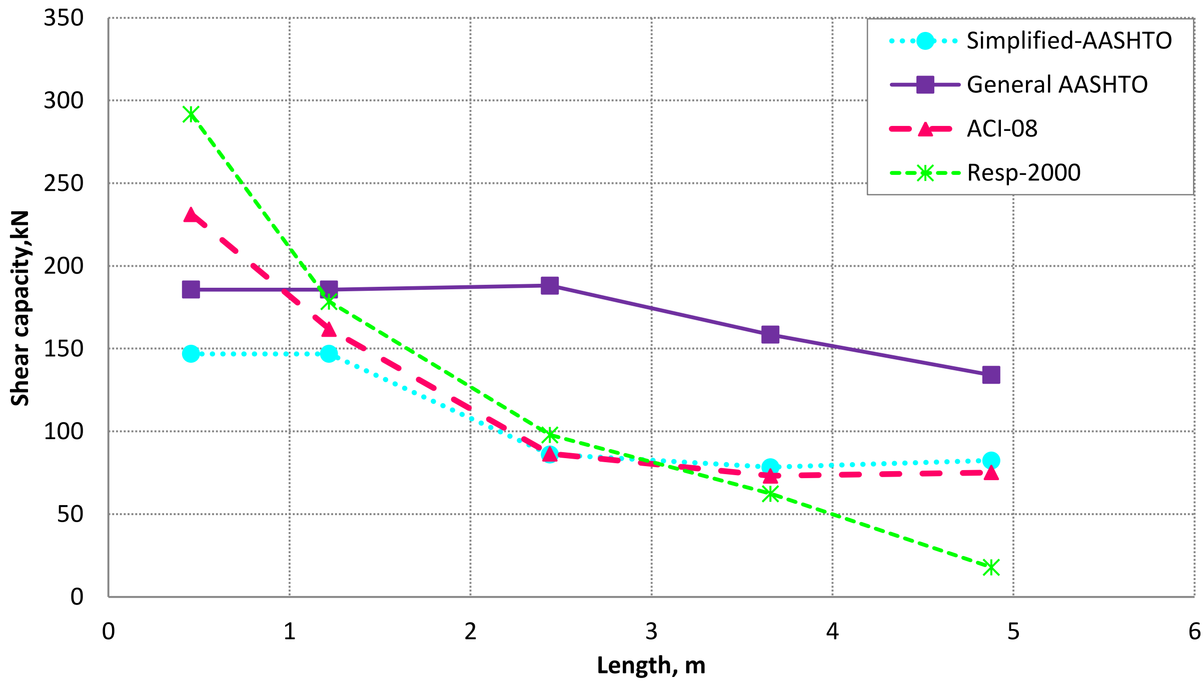


Figure 5.4 Predicted shear strength along the length of Double-T (8DT18), normal strength simply supported pre-stressed reinforced concrete beam

The results for BM100D are plotted in Figure 5.5. From the previous knowledge about Response-2000, it was found that Response-2000 underestimated the shear strength by 51% (average) for normal strength concrete simply supported beams with crack control reinforcement. As shown in the figure, the simplified AASHTO procedure highly underestimates the shear strength while the general AASHTO procedure gives reasonable results. The results for ACI are almost exactly the same as for BM100 (without crack control reinforcement). The only difference is that the predicted shear strength by the general AASHTO procedure increases and makes ACI results relatively accurate for BM100D.

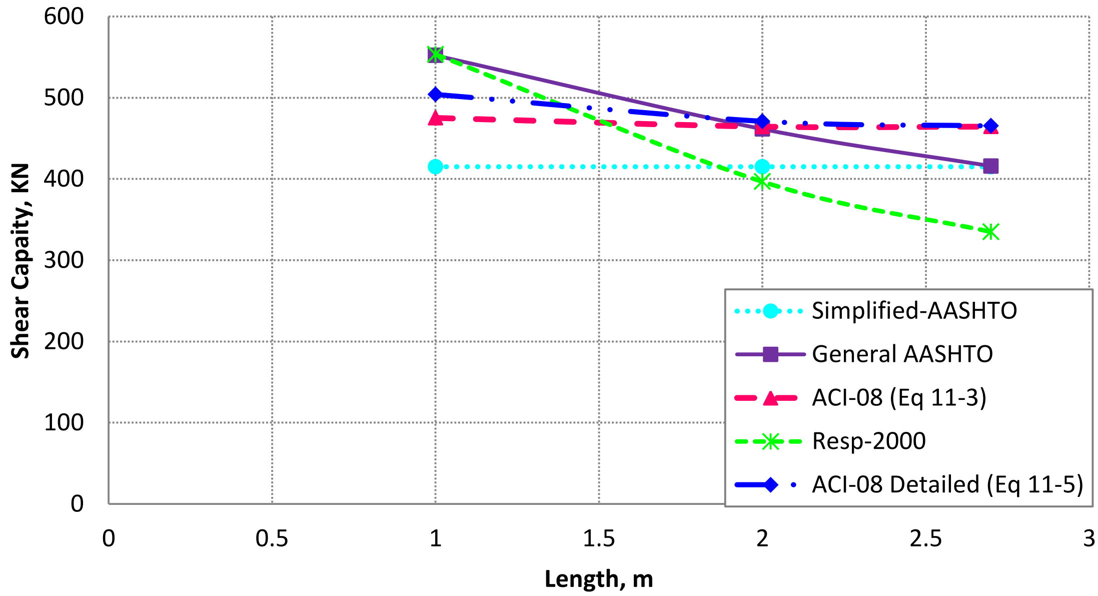


Figure 5.5 Predicted shear strength along the length of BM100-D, normal strength simply supported non-prestressed reinforced concrete beam with longitudinal crack control reinforcement

Figure 5.6 presents results for SE100B-M-69. Both Response-2000 and the general AASHTO procedure give very close results except for the critical locations as mentioned earlier. This is in total conformance with the results showing 3.1% (average) difference obtained from qualifying Response-2000 against experimental results tabulated in Table 3.1. The shear strength predicted using the general AASHTO procedure and Response-2000 show considerable increase, while it remains unchanged for ACI and simplified AASHTO. In other words, ACI and simplified AASHTO fail to encounter the effect of crack control reinforcement on the nominal shear strength of a section.

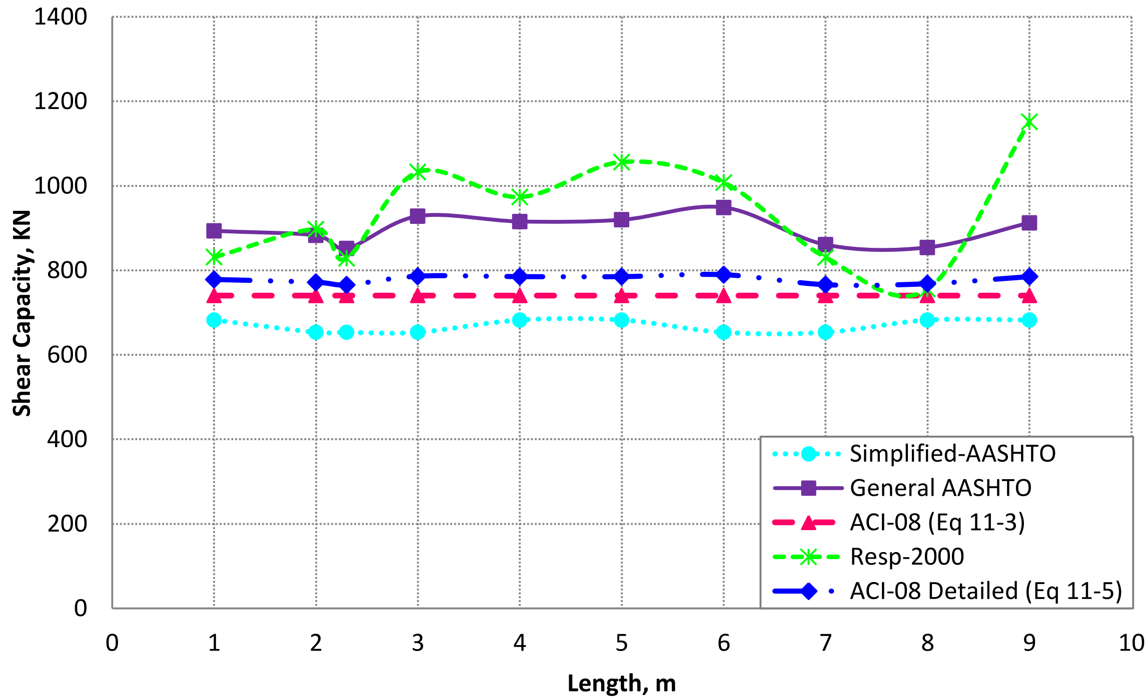


Figure 5.6 Predicted shear strength along the length of SE100B-M-69, high strength continuous non-prestressed reinforced concrete member with longitudinal crack control reinforcement

5.2 Analysis for Shear and Torsion

Figure 5.7 shows the T-V interaction diagrams for AASHTO LRFD (2008) and ACI Code. Details of the reinforcement for these beams tested by (Klus 1968) are tabulated in Table 2.2 and Table 2.3. Having the related properties of the section, the torsion obtained from Eq-3.2.1 controlled. This means that the section will neither fail due to yielding of the longitudinal tension reinforcement nor the concrete crushing. For the pure shear case, the predicted shear capacity is the same for ACI when θ is equal to 45˚ and 30˚. This is due to the fact that the angle θ in Eq-3b is only used in the term that includes torsion which in turn equals zero for the pure shear case. The equation for nominal shear capacity provided by shear reinforcement, V_s, of a section is independent of θ for ACI. The value for θ is inherently assumed as 45˚.

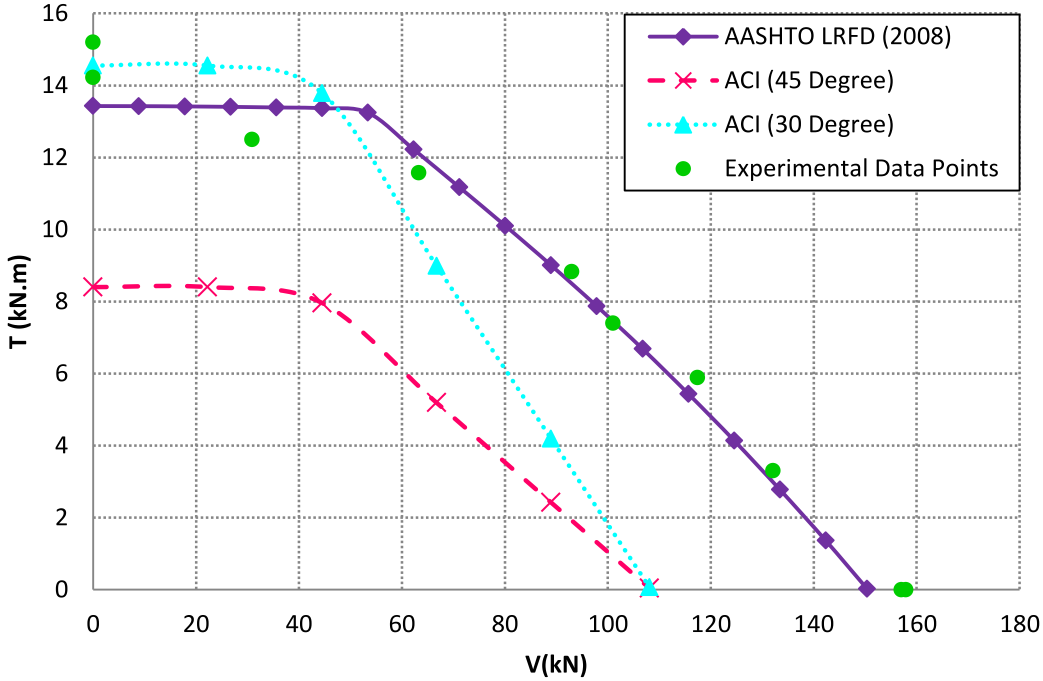


Figure 5.7 Shear-torsion interaction diagrams along with experimental data for specimens tested by Klus (1968)

The flat plateau at the top of the graphs is due to the fact that the applied shear force is less than the nominal shear strength provided by concrete, V_c. Hence, the total transverse reinforcement is used to resist the applied torsion. In other words, the applied shear does not alleviate from the full nominal torsional capacity of the section. This is because of the fact that for $V < V_c$, the applied shear V is resisted by the concrete and not the shear reinforcement. This situation will continue until the applied shear, V, is greater than V_c.

In Figure 5.8, the AASHTO LRFD shear-torsion interaction curve for (RC2 series) is flat approximately up to a shear force of 269 kN; while for the curve based on the ACI it is horizontal up to a shear force of 222 kN. This is due to the fact that the value of V_c for AASHTO LRFD is calculated to be 279 kN while it is equal to 218 kN for ACI. After the section is subjected to greater shear force and torsion, the curve follows a decreasing trend as shown in the figure.

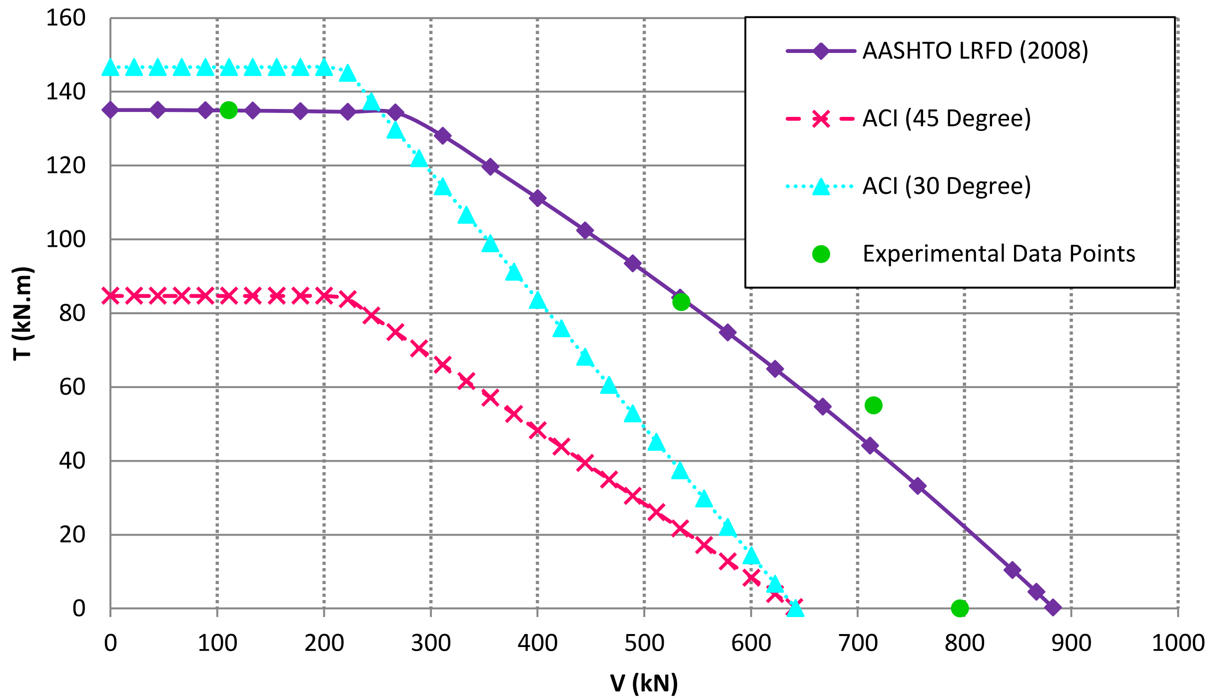


Figure 5.8 Shear-torsion interaction diagrams for RC2 series

From Figure 5.7 and Figure 5.8, it is evident that the experimental data is perfectly matching the AASHTO LRFD curve. On the other hand, the corresponding ACI curves for 30˚ and 45˚ are consistent in both figures. In a sense the ACI provisions for combined shear and torsion are very conservative and uneconomical when θ is equal to 45˚. However, these provisions seem to be slightly un-conservative when θ is equal to 30˚ and the shear force is less than V_c.

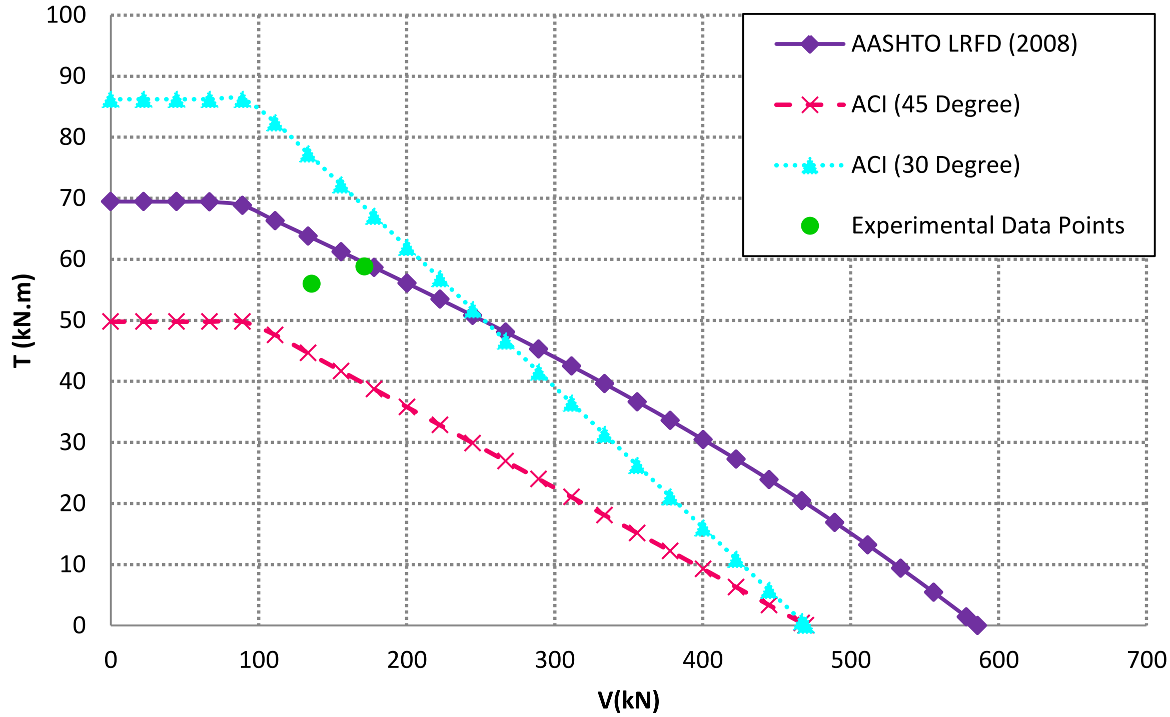


Figure 5.9 Shear-torsion interaction diagrams for High-Strength over-reinforced specimens HO-1, and HO-2

The figure shown above shows the T-V interaction diagram for HO-1 and HO-2 specimens based on AASHTO LRFD and ACI 318-08. As stated earlier, the AASHTO LRFD provisions closely approximate the torsion-shear strength of HO-1 and HO-2 sections. The ACI procedure over-estimates the shear-torsion strength for θ equal to 30˚ up to a shear value as great as 250 kN.

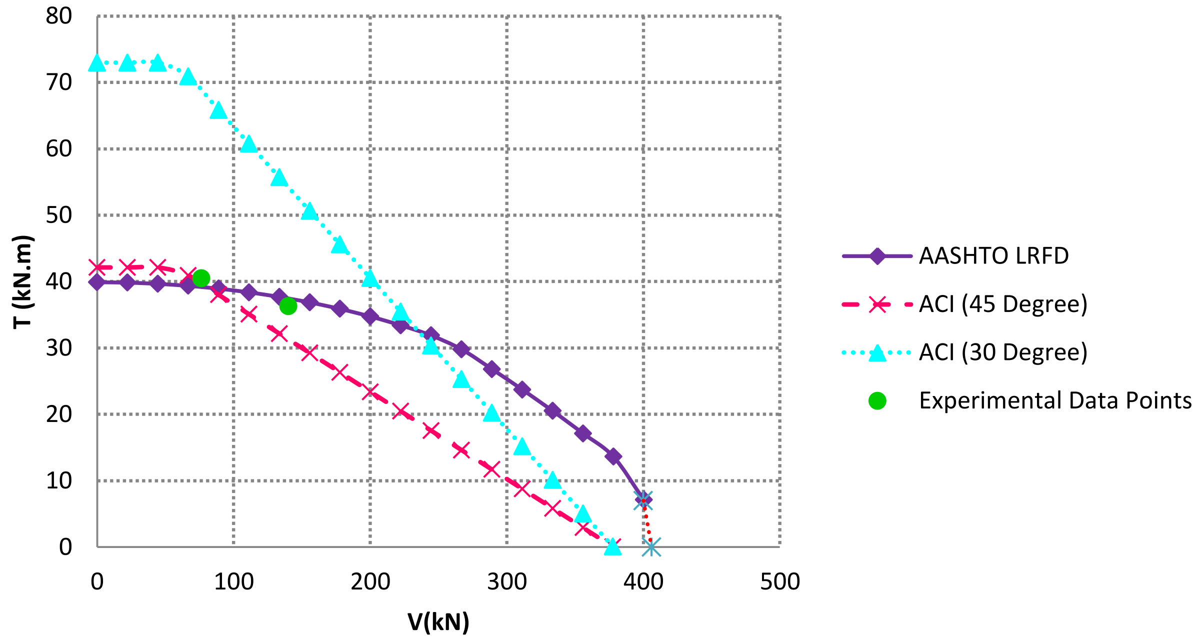


Figure 5.10 Shear-torsion interaction diagram for NO-1 and NO-2

As shown in Figure 5.10, again the AASHTO LRFD provisions closely approximate the combined shear and torsion capacity for NO-1 and NO-2 specimens. However, when the combined shear force and torsion reaches 400 kN and 7 kN.m respectively, the equation produced from substituting the shear stress v_u with $0.25f'_c$ in Eq-2.3.9 and substituting V_u with V_{u-eq} yields negative number under the square root. This means that the concrete crushes and no torsion would be obtained from the corresponding equation for applied shear force greater than 400 kN. To obtain the pure shear capacity of the section, T was set equal to zero and the pure shear capacity of the section was found to be $0.25f'_c b_v d_v$. The estimated capacity of the specimens where the equation yields negative number under the square root is shown as dotted line on the AASHTO LRFD curve.

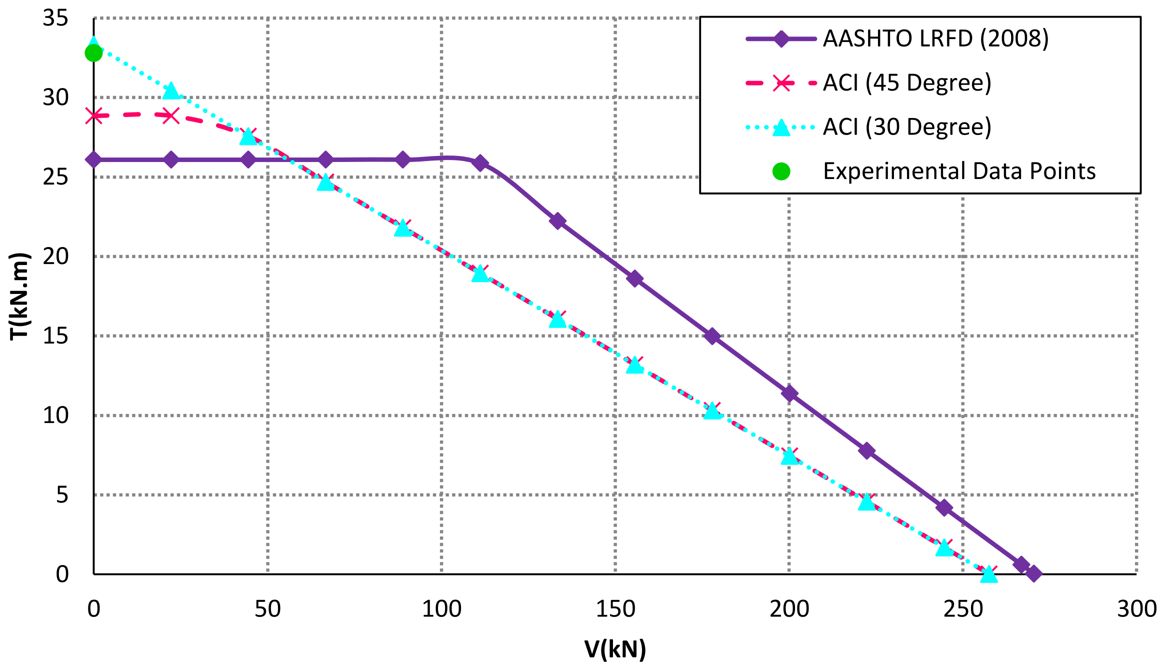


Figure 5.11 Shear-torsion Interaction diagram for High-Strength box section HU-3

In the above figure, the predicted shear-torsion capacity for the box section HU-3 subjected only to combined shear force and torsion is shown. According to ref (13) the section is slightly under-reinforced. Using the ACI provisions, the torsion is controlled by Eq-2.4.10b when the angle θ is equal to 30˚. This imply that the concrete crushes if shear-torsion greater than that shown in Figure 5.11 are applied on the section. However, the torsion is controlled by Eq-3.2.2 when θ is equal to 45˚ and the shear force is lower than 22 kN. For shear force greater than 22 kN, the maximum torsion that the section can resist is controlled by Eq-2.4.10b. This simply means that the concrete may crush before the reinforcement yields if a larger torsion is applied. Since Eq-2.4.10b is independent of the angleθ, both curves for ACI (30˚ and 45˚) give exact similar results after the curve for θ equal to 45˚ bifurcates. The experimental result for pure torsion is exactly the same as predicted by ACI when θ is 30˚. The results for NU-3 which is not included here were also consistent with that shown in Figure 5.11. The only difference was that the experimental pure torsion strength was slightly greater compared to the strength predicted by ACI using 30,

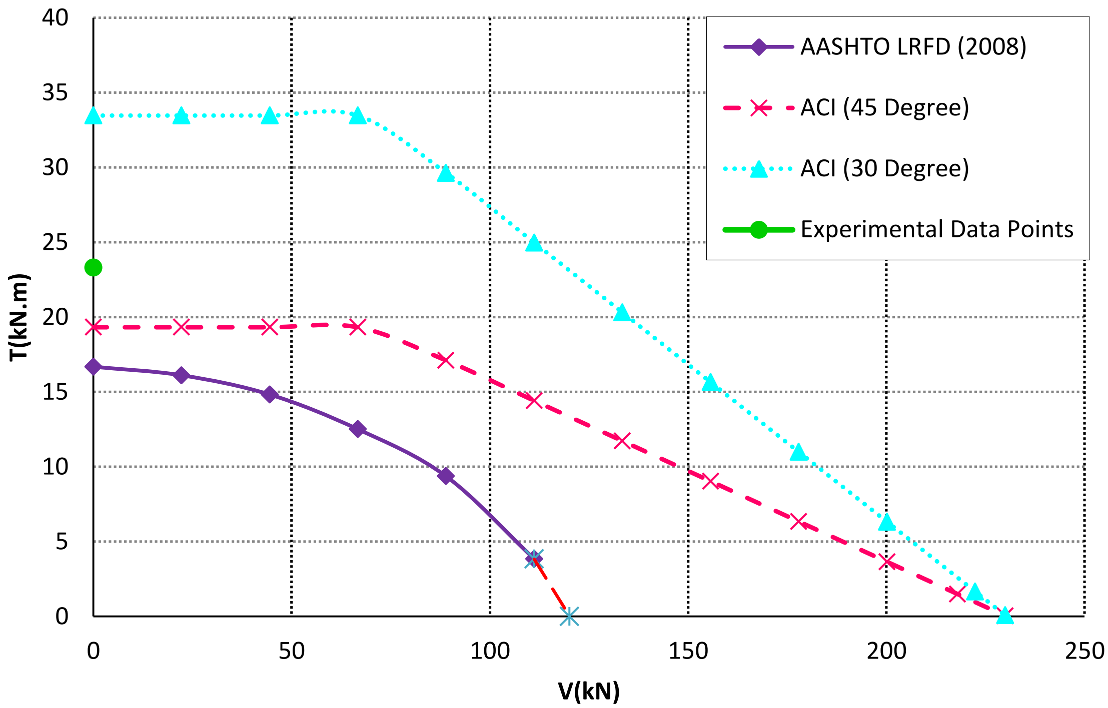


Figure 5.12 Shear-torsion interaction diagram for NU-2

From the above figure, it is obvious that the ACI provisions for θ equal to 30˚ are extremely un-conservative for NU-2 which is an under-reinforced specimen made of normal strength concrete. The AASHTO LRFD curve seems to be conservative for most of the cases studied. However, when the shear and torsion reaches 111 kN and 3.85 kN.m respectively; the longitudinal reinforcement starts yielding. As a value for shear force greater than 111 kN is substituted in Eq-2.4.7 knowing that V_p,$N_{u,}$ A_{ps}, and M_u are zero, a negative number under the square root is produced and the equation remains unsolved. To determine the pure shear capacity of the section, T_n in Eq-2.4.7 was set equal to zero and the equation was solved forV_n. The portion of the curve where the reinforcement yields is shown by hidden lines in Figure 5.12. The same responses were observed for NU-1, HU-1, and HU-2 where ACI Code for θ equal to 30˚ gave extremely un-conservative results.

Chapter 6 - Conclusions and Recommendations

Summary:

In this study, shear and combined shear and torsion provisions for the general AASHTO LRFD procedure, simplified AASHTO LRFD procedure for prestressed and non-prestressed concrete members, and ACI 318-08 were comparatively studied. The results from the aforementioned procedures were compared with those obtained from Modified Compressions Field Theory (MCFT) using Response-2000. Note that the AASHTO LRFD provisions for shear and combined shear and torsion are based on simplified MCFT. In addition, exact shear-torsion interaction curves were drawn for the three methods for sections subjected to combined shear and torsion.

6.1 Conclusions:

6.1.1 Members Subjected to Shear Only

The general AASHTO procedure proved to behave more realistically and in some cases economically. This was in particularly true for transversely reinforced simply supported and continuous beams with crack control reinforcement, while ACI 318-08 and simplified AASHTO were inaccurate in this regard. In addition, ACI 318-08 Eq (11-5) used to predict V_c has led to convincing results compared to ACI Eq (11-3). This was due to the fact that ACI Eq (11-5) takes into account the existing bending moment effects at section.

The simplified AASHTO procedure for prestressed and non-prestressed concrete members almost underestimated the shear strength for all of the beams considered in particularly for SE100A-M-69, SE100B-M-69, and BM100D.

The analytical program used in this study (Response-2000) which is based on Modified Compression Field Theory (MCFT) highly overestimated the moment effects on the overall nominal shear strength, V_n, of a section. This has caused V_n to be highly overestimated at sections with small bending moment and significantly underestimated at sections with large moment.

To analyze the shear strength of a concrete beam, it is extremely important to know that a shear-critical section may not be limited to locations where the section is subjected to maximum shear; rather a section with less flexural reinforcement and relatively smaller shear could fail in shear. Because the higher the longitudinal strain,ε_x, the lesser the overall nominal shear capacity of the section. However, ACI 318-08 doesn't take this phenomenon into account.

The reason why Response-2000 and the general AASHTO procedure do not match well is because Response-2000 is based on MCFT while the general AASHTO procedure for shear is based on simplified MCFT. Furthermore, the limitations imposed by the AASHTO in regards to the effective depth, d, and M_u also has their rule to play.

6.1.2 Members Subjected to Combined Shear and Torsion

A research program was conducted to explore the accuracy and validity of the AASHTO LRFD 2008 provisions for combined shear and torsion design, validating against 30 experimental data from different sections. These sections covered a wide range of specimens from over-reinforced to under-reinforced and made from normal tor high strength concrete. Solid or hollow sections were among the specimens for which the experimental data was used for comparison.AASHTO LRFD (2008) provisions were also compared to the ACI 318-08 provisions for combined shear and torsion design. AASHTO LRFD 2008 provisions consistently were more accurate and the predictions, while conservative in majority of the cases, were much closer to the experimental data for close to all of the specimens. This included over-reinforced and under-reinforced sections made of high strength and normal strength concrete.

During this study it was found that the AASHTO LRFD (2008) provisions to analyze a section under combined shear and torsion may not be able to predict the complete T-V interaction curve for cases leading to negative terms under the square root in the derivation process. This particularly happens for over-reinforced or under-reinforced sections made of high strength or normal strength concrete. The analytical reason is the limitation dictated by the AASHTO LRFD Equation 5.8.3.6.3-1 related to the amount of longitudinal steel and equations 5.8.3.3-2 and 5.8.2.1-6 or 5.8.2.1-7 related to the maximum sustainable shear stress by concrete which implicitly affects the level of the combined shear and torsion. However, it should be noted that the maximum shear stress limit of 0.25f'c dictated by the AASHRO LRFD 2008, was accurate in prediction of the behavior of sections experiencing relatively high levels of shear stress. This was especially true for over-reinforced sections.

On the other hand, the results by the ACI are frequently un-conservative when the angle θ is equal to 30 degrees. This is especially true for the under-reinforced sections. However, when the angle θ is considered to be 45 degrees, the results are conservative for close to all of the specimens. An important point for the ACI code is that the angle θ is always considered as 45 degrees for shear even if the angle for torsion is used as 30 degrees. This is a discrepancy in the ACI code, while AASHTO is consistent from this perspective.

Compared to the ACI code, AASHTO LRFD 2008 provides a more detailed process to assess the shear/torsion capacity of a section. As a result, the capacities evaluated by the AASHTO LRFD 2008 were found to be closer to the experimental data, compared to those predicted by the ACI code. It should be noted that the strain compatibility is not directly considered in the ACI code, while it plays a critical role in derivation of the AASHTO LRFD 2008 design equations. This in turn has added more value to the AASHTO process in accurate assessment of the shear-torisonal capacity of a section.

6.2 Recommendations and possible modifications to AASHTO LRFD Bridge Design Specifications (2008)

The AASTHO LRFD Bridge Design Specifications needs to address the following items:

- The AASHTO LRFD is not clear where to substitute V_u with $V_{u-eq}$for section subjected to combined shear and torsion. C.5.8.2.1 on Page 5-61 of the AASHTO LRFD (2008) states " V_u in Eqs. 5.8.3.4.2-1, 5.8.3.4.2.-2, and 5.8.3.4.2-3 for ε_s, and in Eq 5.8.2.9-1 for v_u, are not modified for torsion. In other words, the values used to select β, θ in Tables 5.8.3.4.2-1 and 5.8.3.4.2-2 have not been modified for torsion." This could be inferred either to modify V_u for the applied torsion or simply use V_u without any modification to it. However, section 5.8.3.6.2 on page 5-84 of the AASHTO LRFD Bridge Design Specifications (2008) defines θ as angle of crack with the modifications for v and V_u.
- The strain ε_s in the longitudinal tensile reinforcement can be determined using Eq. 5.8.3.4.2-4 of the AASHTO LRFD (2008) or Eq-2.3.7 of this document. In the code, it is not explicitly stated whether the whole longitudinal reinforcement A_s should be used for sections subjected

to pure torsion or only the positive longitudinal reinforcement as defined on page 5-73 of the code shall be used in the corresponding equations for ε_s.

- AASHTO LRFD (2008) proposes Eq.5.8.3.6.3-1 to check the longitudinal reinforcement for solid sections subjected to combined shear and torsion. The resistance factor ϕ used in this equation is the same for axial load, shear force, and moment. However, the resistance factor varies for axial load, shear force, and moment in Eq. 5.8.3.5-1 which is used to check the longitudinal reinforcement for solid sections subjected to shear not torsion.
- Also the resistance factor in the denominator of Eq. 5.8.2.9-1 shall be removed.
- To check the AASHTO LRFD (2008) provisions for sections subjected shear or combined shear and torsion, a wide range of beams such as pre-stressed or non-prestressed over-reinforced, moderately reinforced, and under-reinforced concrete sections made of high-strength or normal strength concrete shall be tested. To get even more realistic results, it is highly recommended that the test specimens shall be designed using the applicable provisions of the AASHTO LRFD (2008).

6.3 Suggestions for Future Research

More experimental work is needed to investigate the behavior of normal strength and high strength, prestressed shear-critical simply supported and continuous concrete beams designed based on the AASHTO LRFD general and simplified procedures.

In addition, the simplified AASHTO provisions for shear have to be modified to accurately take into account the effects of crack-control reinforcement on the overall nominal shear strength of a section.

As stated earlier, in this study only exact shear-torsion interaction curves were drawn neglecting the effects of bending moment. However, in most close to all of the real world situations, the effects of bending moment exist, hence shear-bending moment- torsion ($V - M - T$) exact interaction curves are required to be developed for design.

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Appendix A - AASHTO LRFD Simplified Procedure for Shear of Pre-Stressed and Non-Prestressed Members

Example -1:

Determine the shear strength of the middle girder at 7.1 ft from the left support or/ first interior support from the left shown in Figure 2.4**Error! Reference source not found.**, using the simplified AASHTO LRFD method for pre-stressed and non-prestressed members, AASHTO LRFD general procedure, and ACI 318-08. The girder is simply supported while the deck on the top of it is continuous. This girder has a combination of draped (harped) and straight pre-stressing strands. This example is based on example 9.6 of the PCI Bridge Design Manual.

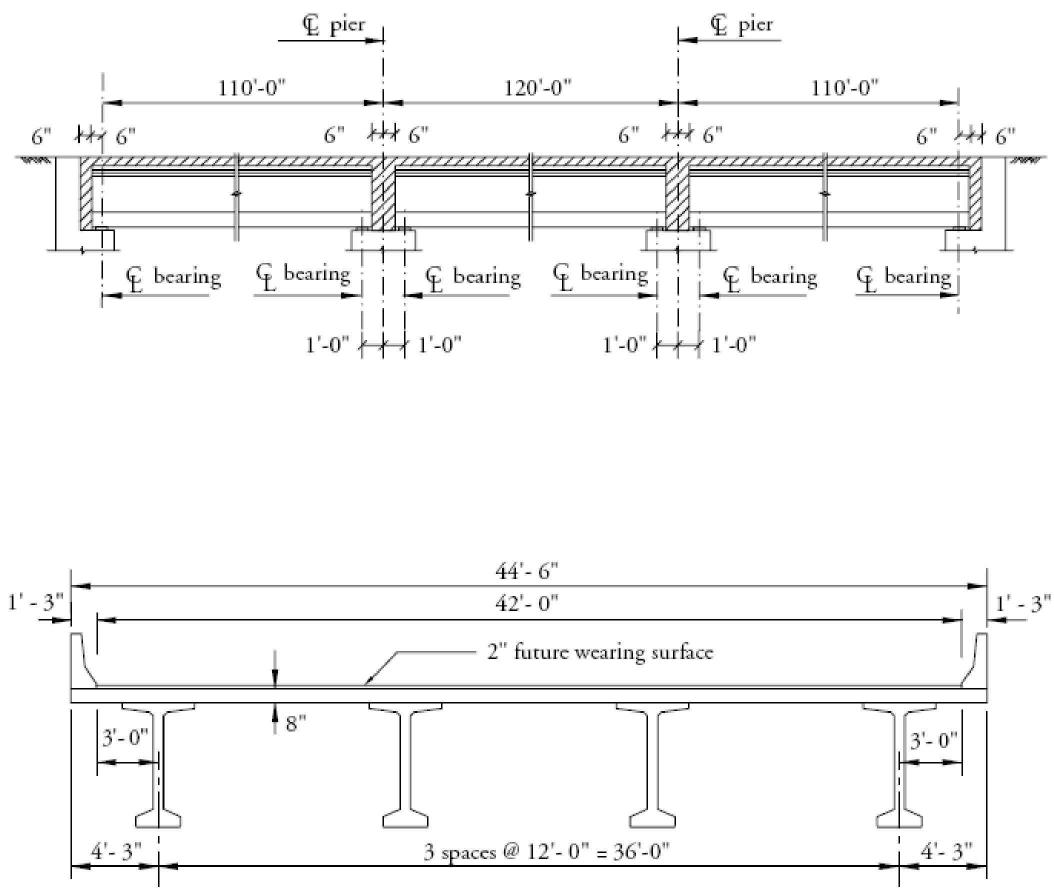


Figure A. 1 profile and section of the bridge girder (BT-72)

1). ε_x is calculated at the section and all reinforcement should be on the tension side at that particular section.
2).d_p ?
3).f_{pe} for composite section where the moment is negative equals zero.
4).for calculating Mcr in negative moment region, f'c for topping should be considered.Also more importantly, if the applied moment is positive, substitute the correct values for "y or c" in flexure formula.
5). The eccentricity, e, used to calculate fpc is the distance between the centroid of P/S and centroid of NON-composite section.

Legend:	
	Input Values
	Final Answers for important parameters
	Not used in this or/other files

Concrete Properties:			* *Note* : In case the topping and girder both have the same f'c, make sure that you enter f'c$_{\text{, deck, topping}}$ equal to that of f'c$_{\text{,girder}}$
fc'$_{\text{,girder}}$	7	ksi	
fc'$_{\text{,deck/topping}}$	4	ksi	
n(modular ratio)	0.756		
$\gamma_{concrete}$	0.15	kcf	
$E_{c,slab,topping}$	3834.253513	ksi	
$E_{c, beam}$	5072.240629	ksi	
Prestressing Strands:			
A_{ps}	0.153	in^2	(0.5 in. dia., seven-wire, low-relaxation)
f_{pu}	270	ksi	
f_{py}	243	ksi	
f_{po}	189	ksi	(f_{po}= A parameter for P/S)
f_{se}	152.9	ksi	(f_{se}= Effective prestress after all loses)
E_p	28500	ksi	
Reinforcing Bars			
As	15.53	in^2	
f_y	60	ksi	
E	29500	ksi	

Over All Geometry and Sectional Properties:		
Non-Composite Section		
Span Length, **L**	120	ft
Over All Depth of Girder, **h**	72	in.
Width of Web, b_v	6	in.
Area of Cross-Section of Girder, $\mathbf{A_g}$	767	in^2
Moment of Inertia, $\mathbf{I_g}$	545894	in^4
Dis. From centroid to ext. bottom fiber, $\mathbf{y_b}$	36.6	in.
Dis. From centroid to ext. top fiber, $\mathbf{y_t}$	35.4	in.
Sec.modulus, ext.bottom fiber, $\mathbf{S_b}$	14915	in^3
Sec.modulus, ext.top fiber, $\mathbf{S_t}$	15421	in^3
Weigh of Beam	0.799	k/ft
Composite Section		
Over all depth of the composite section, $\mathbf{h_c}$	80	in.
Slab thickness, $\mathbf{t_s}$	8	in.
Total Area (transformed)of composite sect., $\mathbf{A_c}$	1412	in^2
Moment of Inertia of the composite sec. $\mathbf{I_c}$	1097252	in^4
Dis. From centroid of composite section to extreme bottom fiber, $\mathbf{y_{bc}}$	54.67	in.
Dis. From centroid of composite section to extreme top fiber of beam, $\mathbf{y_{tg}}$	17.33	in.
Dis. From centroid of composite section to extreme top fiber of slab, $\mathbf{y_{tc}}$	25.33	in.
Composite section modulus for the extreme bottom fiber of beam, $\mathbf{S_{bc}}$	20070.45912	in^3
Composite section modulus for the extreme top fiber of beam, $\mathbf{S_{tg}}$	63315.176	in^3
Composite section modulus for the extreme top fiber of slab, $\mathbf{S_{tc}}$ = **1/n*($\mathbf{I_c/y_{tc}}$)** *Critical in case n=0*	57304.69636	in^3
Total # of P/S strands	44	
Area of P/S tension reinforcement	6.732	in^2

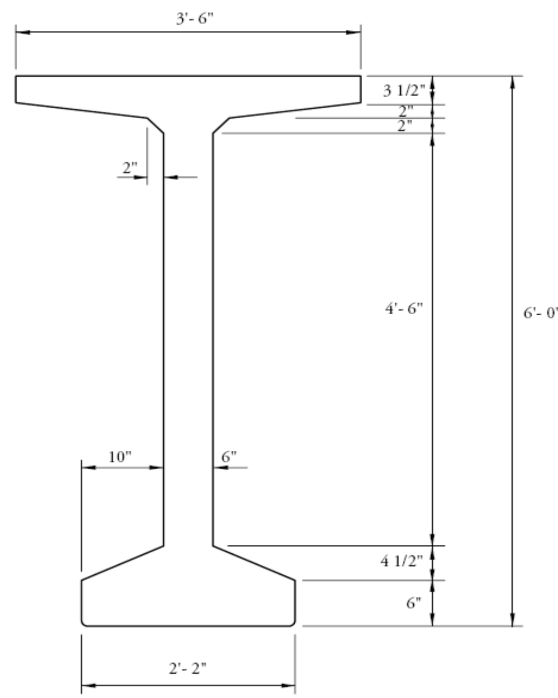
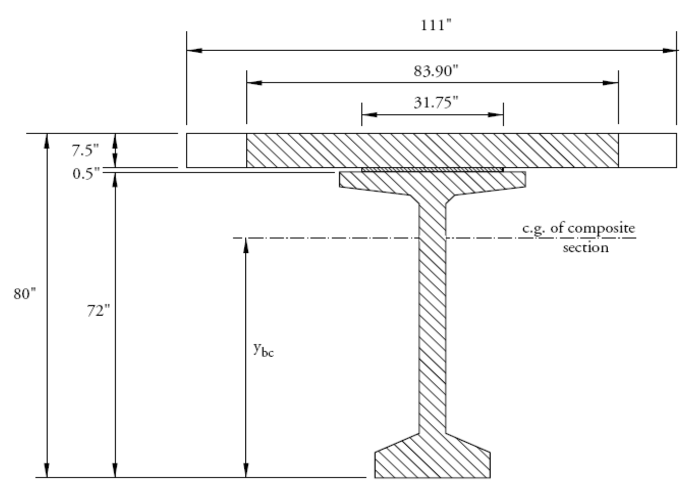


Figure A. 2 Cross-section (BT-72)



*Note: If modulus of Elasticity for topping, slab, is different than modulus of elasticity for girder, determin **$n=E_{c,slab}/E_{c,beam}$** called modular ratio and multiply it by the area of slab(b_{eff} x slab thickness). Add the given value to Area of Girder for total area of composite section.

Figure A. 3 Composite Section(BT-72)

Sectional Forces at Design Section		
Axial Load		
N_u	0	kips
Shear Forces (D.L, L.L)		
Unfactored shear force due to beam weight, $V_{d,girder}$	42.3	kips
Unfactored shear force due to deck slab, $V_{d,slab}$	64.6	kips
Unfactored shear force due to barrier weight, $V_{d,barrier}$	7.8	kips
Unfact. shear force due to future wearing surface, $V_{d,wearing}$	14.2	kips
Unfactored shear force due to ***TOTAL*** D.L, V_d	128.9	kips
Unfactored shear forces due to live load, V_{LL}	137.3	kips
FACTORED SHEAR FORCE, V_u	404.95	kips
Moments (D.L,L.L)		
Unfactored moment due to beam weight, $M_{d,girder}$	272.7	kips-ft
Unfactored moment due to deck slab, $M_{d,slab}$	417.1	kips-ft
Unfactored moment due barrier, $M_{d,barrier}$	-139.6	kips-ft
Unfact. moment due to future wearing surface, $M_{d,wearing}$	-244.4	kips-ft
Unfactored moment due to ***TOTAL*** D.L, M_d	305.8	kips-ft
Unfactored moment due to live load, M_{LL}	-1717.8	kips-ft
FACTORED MOMENT, M_u	-2877.57	kips-ft

*Note: The factored shear and moment is calculated using the following combinations:
$V_u=0.9(V_{d,girder}+V_{d,slab}+V_{d,bearing})+1.50(V_{d,wearing})+1.75(V_{LL})$
$V_u=1.25(V_{d,girder}+V_{d,slab}+V_{d,bearing})+1.50(V_{d,wearing})+1.75(V_{LL})$
Select the MAX shear from above
$M_u=0.9(M_{d,girder}+M_{d,slab}+M_{d,bearing})+1.50(M_{d,wearing})+1.75(M_{LL})$
$M_u=1.25(M_{d,girder}+M_{d,slab}+M_{d,bearing})+1.50(M_{d,wearing})+1.75(M_{LL})$
Select the MAX moment from above
It is conservative to select the Max moment rather than the moment corresponding to Max shear. (*check the formula for M_u for abs.value*)

SOLUTION:

Calculation of Effective Depth, d_v:

d_e	76.25	in.
a (depth of compression)	6.02	in.
$d_v=d_e-a/2$	73.24	in.
$d_v=0.9d_e$	68.625	in.
$d_v=0.72h$	57.6	in.
Max d_v (*controls*)	73.24	in.

*d_e=Effective depth from extreme compression fiber to the centroid of the tensile force in the tensile reinforcement

**Note*: the value of "a"depends on location of cross-section.
$a=(A_s \text{ or } A_{ps})*f_y/(0.85*f'c*b)$ because in this particular case it is intended to determine "a" at critical section 7.1 ft from support for continuous beam, only non-prestrestressed reinforcement at the deck is considered in the calculation.

a).Evaluation of Web-Shear Cracking Strength:

$V_{cw}=(0.06*\sqrt{f'c}+0.3f_{pc})b_vd_v+V_p$

$f_{pc}=P_{se}/A_g-P_{se}*e*(y_{bc}-y_b)/I_g+(M_{dg}+M_{ds})*(y_{bc}-y_b)/I_g$

p_{se}= # of Strands*A_{strand}*f_{se}

V_p	35.2	kips
e(eccentricity)	18.79	in.
P_{se}	1029.3228	kips
fpc	0.975795333	ksi
V_{cw}	233.5999877	kips

b).Evaluation of Flexure-Shear Cracking Strength:

$V_{ci}=0.02*\sqrt{f'c}\ b_v d_v+V_d+V_i M_{cr}/M_{max}$ >=$0.06\sqrt{f'c}\ b_v d_v$			
$V_i=V_u-V_d$			
$M_{max}=M_u-M_d$			
$M_{cr}=(I_c/y_{tc})*(0.2*\sqrt{f'c}+f_{pe}-f_d)$ *where **f'c** for section with neg.moment is that for topping.*			
$f_d=M_{dw}y_{tc}/I_c$			
V_i	276.05	kips	*Make sure that you put right f'c, y_{tc} and f_{pe} while calculating Mcr
M_{max}	-3183.37	kips-ft	
$f_{pe}=A_{ps}*No.P/S*f_{se}/A_g+A_{ps}*No.P/S*f_{se}*e*c/I_g$	0	ksi	
f_d , *put the right* y_{tc}	0.084712507	ksi	
M_{cr}	1138.142624	kips-ft	
V_{ci}	250.8484721	kips	

c).Evaluation of Concrete Contribution:

V_c=**Min**(V_{ci},V_{cw})		
Vc	233.5999877	kips

Explanation for Parts, “a” and “b":

*f_{pc} = compressive stress in concrete (after allowance for all pretension losses) at centroid of cross-section resisiting externally applied loads. ***f_{pc} = compressive stress in concrete after all prestress losses have occurred either at the centroid of the cross-section resisting live load or at the junction of the web and flange when the centroid lies in the flange. In a *composite section* , f_{pc} is the resultant compressive stress at the centroid of the composite section, or at the junction of the web and flange when the centroid lies within the flange, due to both prestress and to the bending moments resisted by the precast member acting alone. ---

*V_p = vertical component of effective pretension force at section.(*strands which are not straight or draped or harped)*

*e= eccentricity of P/S strands from the *centroid of the non-composite section* of cross-section.

*I_g = moment of inertia of *non-composite* section.

*f_{pe} = compressive stress in concrete due to effective pretension forces ONLY (after allowance for all pretension losses) at extreme fiber of section where tensile stress is caused by externally applied loads. In this particular case, where the beam at this section is under net negative moment, hence the top portion of deck slab is in tension where prestressing doesn't affect because P/S is limited to non-composite section. fpe=0 *(Always satisfy for composite section under negative moment)* . When calculating fpe using flexure formula, consider Ic or comp.moment of inertia .

*f_d was calculated using $f_d=M_{dw}y_{tc}/I_c$. Because this section is under net negative moment Mdw was evaluated conservatively by considering the DL negative moment as that resulting from the DL acting on a continuous span. ***$V_{ci} > 0.06\sqrt{f'c}b_v d_v$**

d).Evaluation of Required Transverse Reinforcement:

$V_s=A_v*f_y*d_v*\cot\theta/s$			
CHECK:	**Transverse Reinforcement Required**		
$\cot\theta$	1		
$Vs_{(Req'd)}$	216.34	kips	
$A_{v,min}/S$	0.0084	$in^2/in.$	
$A_v/s_{(Req'd)}$	0.049231853	$in^2/in.$	
Assume:			
s	12	in. c/c	**OK**
Area (#5 stirrups)	0.306796158	in^2	**OK**
A_v/s (Provided)	0.051132693	in^2	
V_s(Provided)	224.6975058	kips	

*** $\cot\theta=1$ if $M_u>M_{cr}$ else $\cot\theta=1+3f_{pc}/\sqrt{f'c}$**

**Note*: the assumed spacing and selected bars are valid for CSA approach also.

e).Checks:

Maximum Spacing Limit of Transverse Reinforcement:		
$v_u/f'c$	0.134828888	
S_{max}	12	in. c/c

Minimum Reinforcement Requirement:		
$A_{v,min}\geq 0.0316\sqrt{f'c}b_vS/fy$		
$A_{v,min}$	0.10032689	in^2

Maximum Nominal Shear Resistance
$V_n\leq 0.25f'cb_vd_v+V_p$
OK

AASHTO LRFD General Procedure (Modified Compression Field Theory):

a).Evaluation of ε_x

$\varepsilon_x=(M_u/dv+0.5N_u+ \lvert V_u-V_p \rvert -A_{ps}*f_{po})/(2*(E_sA_s+E_p*A_{ps}))$			
# of A_{ps}	12		If thereis no tension reinforcement in conjunction with P/S, EsAs=0 in the
Aps	1.836	in^2	
ε_x	0.000484093	in/in	

Note : The parameters for caculating εx is quite dependent on the location of cross-section for the support,such that εx is the tensile stress at cross-sec caused by external and internal loads.

*A_{ps} , A_s= P/S and non-prestressed reinforcement respectively at tensile zone NOT all cross-section.

*M_u = Absolute value of total factored moment at the cross-section.

*N_u = Factored axial force, taken as positive if tensile and negative if compressive (kips)

b).Evaluation of β and θ

$\theta=29+7000*\varepsilon_x$			
θ	32.4	Degrees	
β	2.78		

* For Calculating **β**, it is assumed that at least minimum amount of shear reinforcement is provided.

c).Evaluation of Concrete Contribution

$Vc=0.0316\beta\sqrt{f'c}\, b_vd_v$		
Vc	102.164744	kips

d).Evaluation of Required Transverse Reinforcement

$V_s=A_v*f_y*d_v*\cot\theta/s$		
Check	**Transverse Reinforcement Required**	
$V_{s\ (Req'd)}$	312.6	kips
$A_v/s_{\ Req'd)}$	0.0451	$in^2/in.$
Spacing, S	11.0	in.
A_v/s (Porvided)	0.05578112	$in^2/in.$
V_s (Provided)	386.4236566	kips

e).Checks:

Maximum Spacing Limit of Transverse Reinforcement:		
$v_u/f'c$	0.135	
S_{max}	12	in.

Minimum Reinforcement Requirement:		
$A_{v,min} \geq 0.0316\sqrt{f'c}b_vS/fy$		
$A_{v,min}$	0.10032689	in^2

Maximum Nominal Shear Resistance
$V_n \leq 0.25f'cb_vd_v+V_p$
OK

ACI 318-08 approach for shear design (BT-72)

Concrete Properties:		
$fc'_{,girder}$	7000	psi
$fc'_{,deck/topping}$	4000	psi
Prestressing Strands:		
A_{ps}	0.153	in^2
f_{pu}	270000	psi
f_{py}	243000	ksi
f_{po}	189000	ksi
f_{se}	152900	psi
E_p	28500000	psi
Total # P/S	44	
Total Area P/S	6.732	in^2
Slope of Draped P/S	7	Degree
Centroid of Straight P/S (Frm-Bottom)	3.875	in.
Centroid of Draped P/S (From-Bottom)	54.54	in.
No.of Straight P/S strands	32	
No.of Draped P/S strands	12	
V_p	34211.6983	lb
General centroid of P/S strands from Bottom	17.6927273	in.
e (eccentricity)	18.9072727	in.
P_{se}	1029322.8	lb
Reinforcing Bars		
As	15.53	in^2
f_y	60000	psi
E	29500000	psi

* *Note* : In case the topping and girder both have the same f'c, make sure that you enter $f'c_{,\ deck,\ topping}$ equal to that of $f'c_{,girder}$

(0.5 in. dia., seven-wire, low-relaxation)

(f_{po}= A parameter for P/S)

(f_{se}= Effective prestress after all loses)

Overall Geometry and Sectional Properties:		
Non-Composite Section		
Over All Depth of Girder, **h**	72	in.
Width of Web, b_v	6	in.
Area of Cross-Section of Girder, $\mathbf{A_g}$	767	in^2
Moment of Inertia, $\mathbf{I_g}$	545894	in^4
Dis. From centroid to ext. bottom fiber, $\mathbf{y_b}$	36.6	in.
Dis. From centroid to ext. top fiber, $\mathbf{y_t}$	35.4	in.
Composite Section		
Over all depth of the composite section, $\mathbf{h_c}$	80	in.
Slab thickness, $\mathbf{t_s}$	8	in.
Total Area (transformed)of composite sect.,$\mathbf{A_c}$	1412	in^2
Moment of Inertia of the composite sec. $\mathbf{I_c}$	1097252	in^4
Dis. From centroid of composite section to extreme bottom fiber, $\mathbf{y_{bc}}$	54.67	in.
Dis. From centroid of composite section to extreme top fiber of beam, $\mathbf{y_{tg}}$	17.33	in.
Dis. From centroid of composite section to extreme top fiber of slab, $\mathbf{y_{tc}}$	25.33	in.
Overall depth, d	76	in
distance from the top fiber to the centroid of P/S tendons, $\mathbf{d_p >= 0.8*h}$	64	in.
Sectional Forces at Design Section		
Axial Load		
$\mathbf{N_u}$	0	lb
Shear Forces (D.L, L.L)		
Unfactored shear force due to beam weight, $\mathbf{V_{d,girder}}$	42300	lb
Unfactored shear force due to deck slab, $\mathbf{V_{d,slab}}$	64600	lb
Unfactored shear force due to barrier weight, $\mathbf{V_{d,barrier}}$	7800	lb
Unfact. shear force due to future wearing surface, $\mathbf{V_{d,wearing}}$	14200	lb
Unfactored shear force due to ***TOTAL*** D.L, $\mathbf{V_d}$	128900	lb
Unfactored shear forces due to live load, $\mathbf{V_{LL}}$	137300	lb
FACTORED SHEAR FORCE, $\mathbf{V_u}$	404950	lb
Moments (D.L,L.L)		
Unfactored moment due to beam weight, $\mathbf{M_{d,girder}}$	3272400	lb.in
Unfactored moment due to deck slab, $\mathbf{M_{d,slab}}$	5005200	lb.in
Unfactored moment due barrier, $\mathbf{M_{d,barrier}}$	-1675200	lb.in
Unfact. moment due to future wearing surface, $\mathbf{M_{d,wearing}}$	-2932800	lb.in
Unfactored moment due to ***TOTAL*** D.L, $\mathbf{M_d}$	3669600	lb.in
Unfactored moment due to live load, $\mathbf{M_{LL}}$	-20613600	lb.in
FACTORED MOMENT, $\mathbf{M_u}$	-34530840	lb.in

SOLUTION:

Detailed ACI Approach:

$V_{ci}=0.6\sqrt{f'c}\, b_w d_p+v_d+V_i M_{cre}/M_{max}>=1.7\sqrt{f'c} b_w d$		
$V_i=1.6*V_{L.L}$		
$M_{cre}=I_c/Y_{tension,Comp}(6\sqrt{f'c}+f_{pe}-f_d)$		
$M_{max}=1.6M_{L.L}$		
V_i	219680	lb
M_{max}	-32981760	lb
f_{pe}	0	psi
f_d	84.7125072	psi
M_{cre} , put the right **f'c**	12768531	lb.in
Vci	233223.379	lb
$V_{cw}=(3.5\sqrt{f'c}+0.3f_{pc})*b_w d_p+V_p$		
$f_{pc}=P_{se}/A_g-P_{se}*e*(y_{bc}-y_b)/I_g+(M_{dg}+M_{ds})*(y_{bc}-y_b)/I_g$		
fpc	971.799582	psi
V_{cw}	258610.118	lb
Evaluation of Concrete Contribution:		
V_c=**Min**(V_{ci},V_{cw})		
Vc	233223.379	lb
Transverse Reinforcement		
if Vu> ϕVc (Transverse Reinforcement Req'd)		
if ϕVc>Vu>ϕVc/2 (Provide Minimum Trans.Reinforcement)		
if Vu< ϕVc/2(No Trans.Reinforcement Req'd)		
$V_s=V_u/\phi-V_c \quad < \quad 8\sqrt{f'c}\, b_w d$		
ϕVc	174917.534	lb
Vs	305213.578	lb
Trans. Reinforcement Req'd		
$A_v/S=Vs/f_y d$		
A_v/S (Req'd)	0.0669328	$in^2/in.$

Assume:		
s	9	in. c/c
Area (#5 stirrups)	0.30679616	in^2
A_v/s (Provided)	0.06817692	in^2
V_s(Provided)	305213.578	lb

Minimum Shear Reinforcement			
1 $A_{v,min}=50b_w s/f_y$			Select the larger
2 $A_{v,min}=0.75\sqrt{f'c}\, b_w s/f_y$			
3 $A_{v,min}=A_{ps}f_{pu}S/80f_y d\sqrt{(d/b_w)}$			
$A_{v,min}$=Min(3,max(1,2)) , *check the formula*			
$A_{v,min}/S$	0.005	in^2/in.	
$A_{v,min}/S$	0.00627495	in^2/in.	
$A_{v,min}/S$	0.01773308	in^2/in.	
Selected $A_{v,min}/S$	0.00627495	in^2/in.	

Maximum Spacing Allowed			
Max.Spacing= 0.75*h or 24in.			
If Vs > $4\sqrt{f'c}\, b_w d$ Provide Max.Spacing/2			
Max.Spacing	24	in.	
Applicabe or Selected S_{max}	12	in.	

Capacity Predictions: (V=Vc+Vs)			
Simplified Method (Kips)	AASHTO LRFD (Kips)	ACI (Kips)	Response 2000 (Kips)
458.2974935	488.588401	538.437	576.5555556

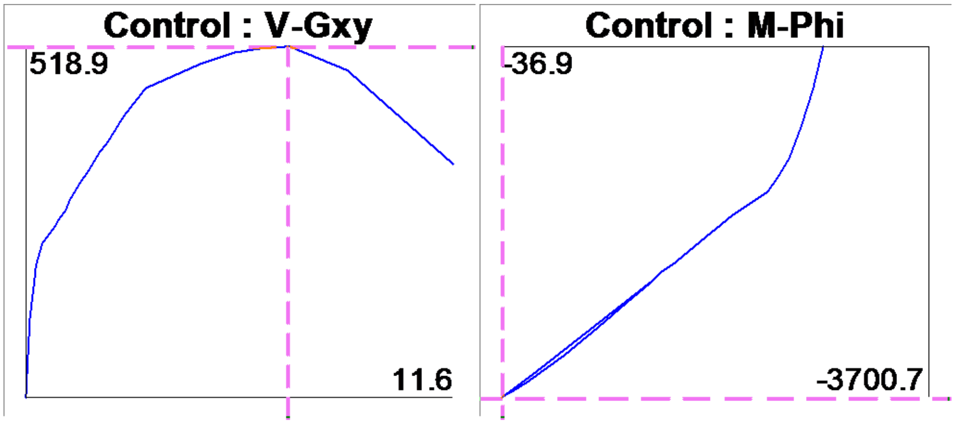


Figure A. 4 Response-2000 output (V_u, M_u)

Development of Interaction Diagram for Combined Shear and Torsion using AASHTO LRFD General Procedure and Corresponding ACI 318-08 Provisions

AASHTO LRFD General Procedure:

Information required:		
bv	8	in
h	12	in
dv	9.27	in
f'c	3.12	ksi
Aoh	59.4	in^2
Ph	31.8	in
At	0.077	in^2
fyt	38.4	ksi
s	4	in
Es	29000	ksi
As	1.342	in^2
fy	62.3	ksi

AASHTO-LRFD APPROACH:	
$V_n=V_c+V_S$	Eq-1
$T_n=2*A_o*A_t*f_{yt}*\cot\theta/s$	Eq-2
$V_s=A_v*f_{yt}*d*\cot\theta/s$	Eq-3
$V_c=0.0316\beta(f'c)^{0.5}b_Vd_V$	Eq-4
$A_o=0.85A_{oh}$	Eq-5
$A_t*f_{yt}/s=T_n/(2A_o*\cot\theta)+(V_n-V_c)/(2d\cot\theta)$	Eq-6
$\varepsilon_s=[V^2+\{0.9P_h*T/(1.7A_{oh})\}^2]^{0.5}/E_sA_s$	Eq-7
$\beta=4.8/(1+750\varepsilon_s)$	Eq-8
$\theta=29+3500\varepsilon_s$	Eq-9

Perform Goal Seek function such that the value in column "L,M,N" is equal to zero by changing the values in column "C". Also the columns with pink color also require Goal Seek such that the value of column "F" has to equal zero by changing the value of column "E". It is very IMPORTAN to note that the goal seek for the columns with yellow backgraound has to be performed first.

	V, kip	T, kip.in (from Eq-6, above)	$0.25fcb_vd_v=[V^2+\{0.9*Ph*T/(1.7*A_{oh})\}^2]^{0.5}$ T, kip.in	T, kip.in	T, satisfying long.reinf. AASHTO-LRFD (5.8.3.6.3-1)	Controlling, T (kip.in)	ε_s	θ(Degree)	β	Vc	$A_t*f_{yt}/s-T_n/(2A_o*\cot\theta)-(V_n-V_c)/(2d\cot\theta)=0$
Klus	0	118.84	204.09	366.62	0.00	118.84	0.00	32.03	2.91	12.05	0.00
	2	118.82	203.97	365.80	0.00	118.82	0.00	32.03	2.91	12.04	0.00
	4	118.75	203.61	365.00	0.00	118.75	0.00	32.05	2.90	12.02	0.00
	6	118.64	202.99	364.20	0.00	118.64	0.00	32.07	2.89	11.98	0.00
	8	118.49	202.13	363.42	0.00	118.49	0.00	32.10	2.88	11.93	0.00
	10	118.30	201.02	362.64	0.00	118.30	0.00	32.15	2.87	11.87	0.00
	12	117.21	199.65	361.55	0.00	117.21	0.00	32.18	2.86	11.82	0.00
	14	108.19	198.03	357.49	0.00	108.19	0.00	32.03	2.91	12.04	0.00
	16	98.87	196.13	353.47	0.00	98.87	0.00	31.90	2.96	12.25	0.00
	18	89.43	193.96	349.63	0.00	89.43	0.00	31.80	3.00	12.43	0.00
	20	79.73	191.51	346.00	0.00	79.73	0.00	31.71	3.04	12.56	0.00
	22	69.67	188.76	342.62	0.00	69.67	0.00	31.66	3.06	12.66	0.00
	24	59.16	185.70	339.57	0.00	59.16	0.00	31.63	3.07	12.70	0.00
	26	48.11	182.32	336.93	0.00	48.11	0.00	31.64	3.07	12.69	0.00
	28	36.63	178.59	334.80	0.00	36.63	0.00	31.69	3.05	12.61	0.00
	30	24.57	174.50	333.23	0.00	24.57	0.00	31.77	3.01	12.47	0.00
	32	12.06	170.02	332.27	0.00	12.06	0.00	31.89	2.96	12.26	0.00
	33.8	0.25	165.63	331.94	0.00	0.25	0.00	32.04	2.91	12.03	0.00

EXPERIMENTAL DATA FROM KHALDOUN. N. RAHAL, 2003:

Speci men	Experimental			
	T (kN.m)	T (kip.in)	V (kN)	V (kip)
1		0		35.5
2		0		35.3
3		134.52		0
4		125.88		0
5		78.12		20.9
6		52.08		26.4
7		102.42		14.22
8		110.64		6.94
9		65.52		22.7
10		29.256		29.7

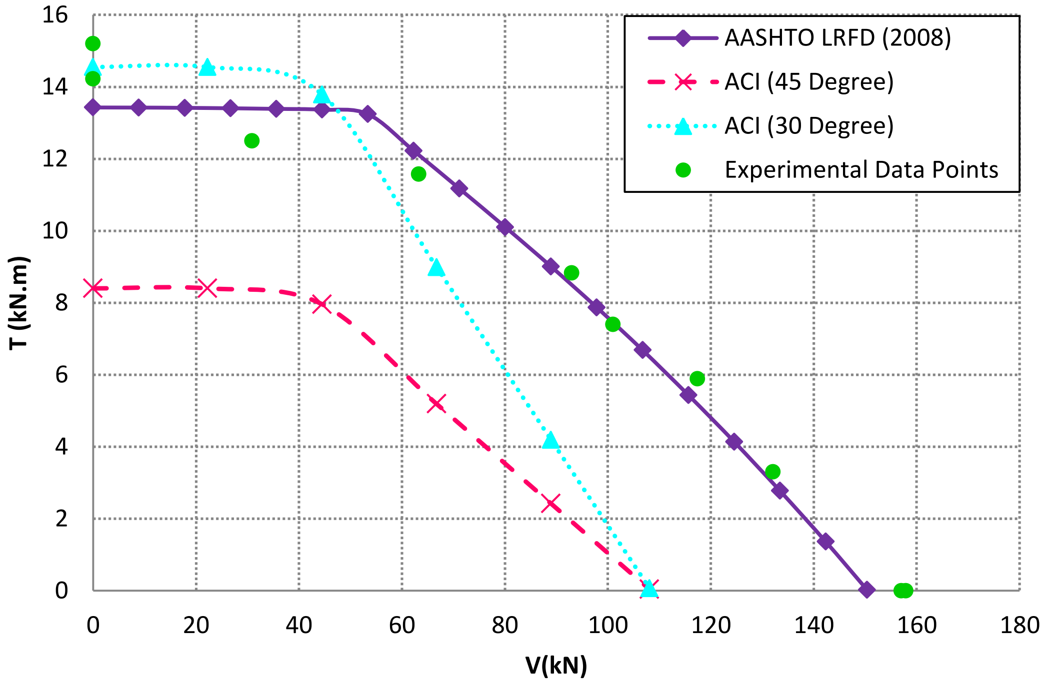


Figure A. 5 Shear-torsion interaction diagrams along with experimental data for specimens tested by Klus (1968)

ACI 318-08 assuming $\theta = 45°$

For Solid Sections:		
bw	8	in
d	10.3	in
f'c	3.12	ksi
Aoh	59.4	in^2
Ph	31.8	in
At	0.0767	in^2
fyt	38.4	ksi
s	4	in
θ	45	Degree

ATTN:
In Eq-4, the value for f'c has to be in psi.
Modify Eq-7 for hollow sections, ACI-08, Eq(11-19)

ACI APPROACH θ=45 Deg:		
$V_n=V_c+V_S$		
$T_n=2*A_o*A_t*f_{yt}*\cot\theta/s$		
$V_s=A_v*f_{yt}*d*\cot\theta/s$		
$V_c=2(f'c)^{0.5}b_wd$		
$A_o=0.85A_{oh}$		
$A_t*f_{yt}/s=T_n/(2A_o*\cot\theta)+(V_n-V_c)/(2d\cot\theta)$		
Note: The value of f'c for determining Vc is in (psi)		
Vc	9.205227	kips

		***According To ACI-08 Equation (11-18):
		$[(V/(b_wd))^2+(T*Ph/(1.7A_{oh}^2))^2]^{0.5} \leq (10(f'c)^{0.5}$
V (kips)	**T (kip.in)**	**T (kip.in), ACI(11-18)**
0	74.3535936	3331.746793
5	74.3535936	3331.727133
10	70.45766283	3331.668154
15	45.94795409	3331.569853
20	21.43824536	3331.432226
24.3	0.359895842	3331.282411

ACI Assuming $\theta = 30°$

Solid Sections:		
bw	8	in
d	10.3	in
f'c	3.12	ksi
Aoh	59.4	in^2
Ph	31.8	in
At	0.0767	in^2
fyt	38.4	ksi
s	4	in
θ	30	Degree

ATTN:
In Eq-4, the value for f'c has to be in psi.
Modify Eq-7 for hollow sections, ACI-08, Eq(11-19)

ACI APPROACH:		
$V_n=V_c+V_S$		Eq-1
$T_n=2*A_o*A_t*f_{yt}*\cot\theta/s$		Eq-2
$V_s=A_v*f_{yt}*d*\cot\theta/s$		Eq-3
$V_c=2(f'c)^{0.5}b_wd$		Eq-4
$A_o=0.85A_{oh}$		Eq-5
$A_t*f_{yt}/s=T_n/(2A_o*\cot\theta)+(V_n-V_c)/(2d)$		Eq-6
Note: The value of f'c for determining Vc is in (psi)		
Vc	9.205227	kips

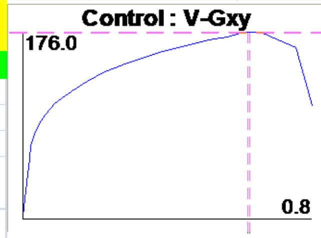
		***According To ACI-08 Equation (11-18): $[(V/(b_wd))^2+(T*Ph/(1.7A_{oh}^2))^2]^{0.5} \leq (10(f'c)^{0.5}$
V (kips)	**T (kip.in)**	**T (kip.in), ACI(11-18)**
0	128.7842018	3331.746793
5	128.7842018	3331.727133
10	122.0362518	3331.668154
15	79.584191	3331.569853
20	37.13213018	3331.432226
24.3	0.623357883	3331.282411

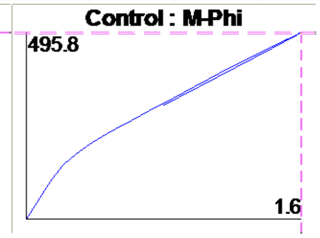
Appendix B - Verification of Response-2000

(Simply Supported Beams)

B100 SIMPLY SUPPORTEDC CASE:

Member Properties:		
Total Spans Length	5.4	m
width, w	0.3	m
height, h	1	m
critical section from right support, L	2.7	m
$\gamma_{concrete}$	23.56	KN/m^3
$W_{self\text{-}wt}$	7.068	KN/m
$V_{experimental}$	225	KN
Shear Force at Critical Section, L.L and D.L		
$V_{crit,D.L}$	0	KN
$V_{crit,L.L}$	225	KN
Vu	225	KN
Moment at Critical Section, LL and D.L		
$M_{cirt,D.L}$	25.76286	KN.m
$M_{cirt,L.L}$	607.5	KN.m
Mu	633.26286	KN.m



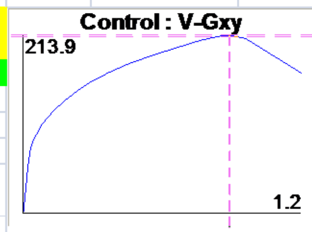


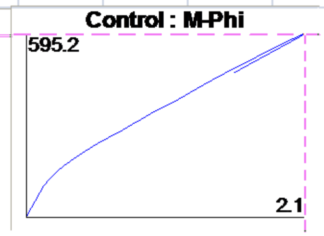
Note : Vexperimental is the shear where it causes shear failure at critical section of the member.

Response2000		
Vresp2000	176	KN
Vexp	225	KN
Vexp/Vresp2000	1.278409	

B100D SIMPLY SUPPORTEDC CASE:

Member Properties:		
Total Spans Length	5.4	m
width, w	0.3	m
height, h	1	m
critical section from right support, L	2.7	m
$\gamma_{concrete}$	23.56	KN/m^3
$W_{self\text{-}wt}$	7.068	KN/m
$V_{experimental}$	320	KN
Shear Force at Critical Section, L.L and D.L		
$V_{crit,D.L}$	0	KN
$V_{crit,L.L}$	320	KN
Vu	320	KN
Moment at Critical Section, LL and D.L		
$M_{cirt,D.L}$	25.76286	KN.m
$M_{cirt,L.L}$	864	KN.m
Mu	889.76286	KN.m



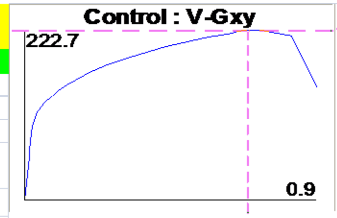


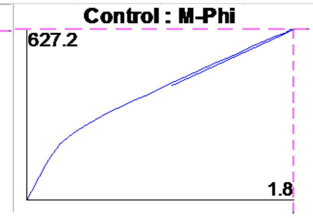
Note : Vexperimental is the shear where it causes shear failure at critical section of the member.

Response2000		
Vresp2000	213.9	KN
Vexp	320	KN
Vexp/Vresp2000	1.496026	

B100H SIMPLY SUPPORTEDC CASE:

Member Properties:		
Total Spans Length	5.4	m
width, w	0.3	m
height, h	1	m
critical section from right support, L	2.7	m
$\gamma_{concrete}$	23.56	KN/m^3
$W_{self\text{-}wt}$	7.068	KN/m
$V_{experimental}$	193	KN
Shear Force at Critical Section, L.L and D.L		
$V_{crit,D.L}$	0	KN
$V_{crit,L.L}$	193	KN
Vu	193	KN
Moment at Critical Section, LL and D.L		
$M_{cirt,D.L}$	25.76286	KN.m
$M_{cirt,L.L}$	521.1	KN.m
Mu	546.86286	KN.m



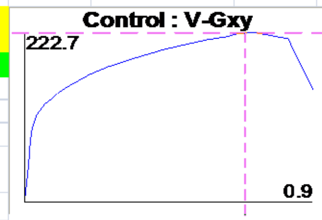


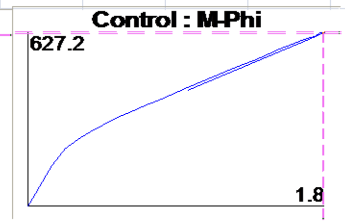
Note : Vexperimental is the shear where it causes shear failure at critical section of the member.

Response2000		
Vresp2000	222.7	KN
Vexp	193	KN
Vexp/Vresp2000	0.866637	

B100HE SIMPLY SUPPORTEDC CASE:

Member Properties:		
Total Spans Length	5.4	m
width, w	0.3	m
height, h	1	m
critical section from right support, L	2.7	m
$\gamma_{concrete}$	23.56	KN/m^3
$W_{self\text{-}wt}$	7.068	KN/m
$V_{experimental}$	217	KN
Shear Force at Critical Section, L.L and D.L		
$V_{crit,D.L}$	0	KN
$V_{crit,L.L}$	217	KN
Vu	217	KN
Moment at Critical Section, LL and D.L		
$M_{cirt,D.L}$	25.76286	KN.m
$M_{cirt,L.L}$	585.9	KN.m
Mu	611.66286	KN.m



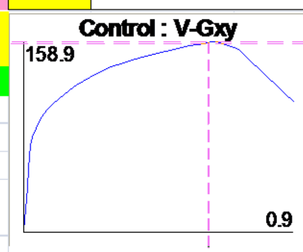


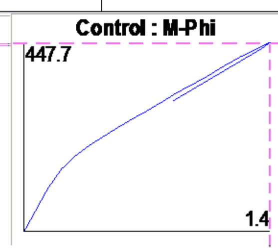
Note : Vexperimental is the shear where it causes shear failure at critical section of the member.

Response2000		
Vresp2000	222.7	KN
Vexp	217	KN
Vexp/Vresp2000	0.974405	

B100L SIMPLY SUPPORTEDC CASE:

Member Properties:		
Total Spans Length	5.4	m
width, w	0.3	m
height, h	1	m
critical section from right support, L	2.7	m
$\gamma_{concrete}$	23.56	KN/m^3
$W_{self\text{-}wt}$	7.068	KN/m
$V_{experimental}$	223	KN
Shear Force at Critical Section, L.L and D.L		
$V_{crit,D.L}$	0	KN
$V_{crit,L.L}$	223	KN
Vu	223	KN
Moment at Critical Section, LL and D.L		
$M_{cirt,D.L}$	25.76286	KN.m
$M_{cirt,L.L}$	602.1	KN.m
Mu	627.86286	KN.m



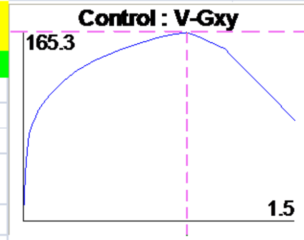


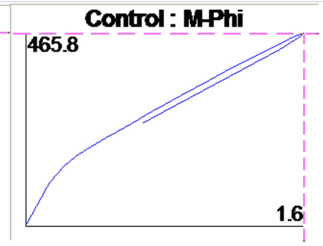
Note : Vexperimental is the shear where it causes shear failure at critical section of the member.

Response2000		
Vresp2000	158.9	KN
Vexp	223	KN
Vexp/Vresp2000	1.403398	

B100B SIMPLY SUPPORTEDC CASE:

Member Properties:		
Total Spans Length	5.4	m
width, w	0.3	m
height, h	1	m
critical section from right support, L	2.7	m
$\gamma_{concrete}$	23.56	KN/m^3
$W_{self\text{-}wt}$	7.068	KN/m
$V_{experimental}$	204	KN
Shear Force at Critical Section, LL and D.L		
$V_{crit,D.L}$	0	KN
$V_{crit,L.L}$	204	KN
Vu	204	KN
Moment at Critical Section, LL and D.L		
$M_{cirt,D.L}$	25.76286	KN.m
$M_{cirt,L.L}$	550.8	KN.m
Mu	576.56286	KN.m



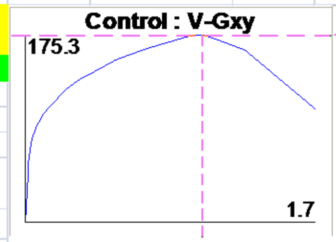


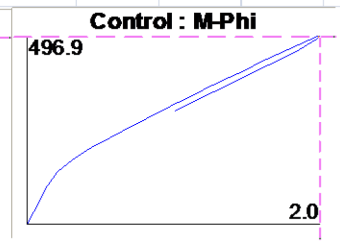
Note: Vexperimental is the shear where it causes shear failure at critical section of the member.

Response2000		
Vresp2000	165.3	KN
Vexp	204	KN
Vexp/Vresp2000	1.23412	

BN100 SIMPLY SUPPORTEDC CASE:

Member Properties:		
Total Spans Length	5.4	m
width, w	0.3	m
height, h	1	m
critical section from right support, L	2.7	m
$\gamma_{concrete}$	23.56	KN/m^3
$W_{self\text{-}wt}$	7.068	KN/m
$V_{experimental}$	192	KN
Shear Force at Critical Section, LL and D.L		
$V_{crit,D.L}$	0	KN
$V_{crit,L.L}$	192	KN
Vu	192	KN
Moment at Critical Section, LL and D.L		
$M_{cirt,D.L}$	25.76286	KN.m
$M_{cirt,L.L}$	518.4	KN.m
Mu	544.16286	KN.m



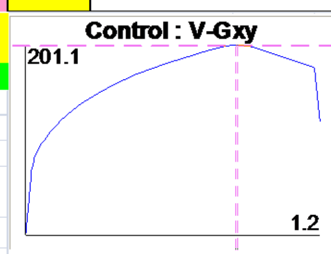


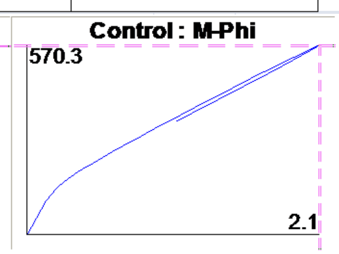
Note: Vexperimental is the shear where it causes shear failure at critical section of the member.

Response2000		
Vresp2000	175.3	KN
Vexp	192	KN
Vexp/Vresp2000	1.095265	

BND100 SIMPLY SUPPORTEDC CASE:

Member Properties:		
Total Spans Length	5.4	m
width, w	0.3	m
height, h	1	m
critical section from right support, L	2.7	m
$\gamma_{concrete}$	23.56	KN/m^3
$W_{self\text{-}wt}$	7.068	KN/m
$V_{experimental}$	258	KN
Shear Force at Critical Section, LL and D.L		
$V_{crit,D.L}$	0	KN
$V_{crit,L.L}$	258	KN
Vu	258	KN
Moment at Critical Section, LL and D.L		
$M_{cirt,D.L}$	25.76286	KN.m
$M_{cirt,L.L}$	696.6	KN.m
Mu	722.36286	KN.m



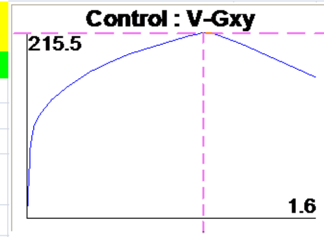


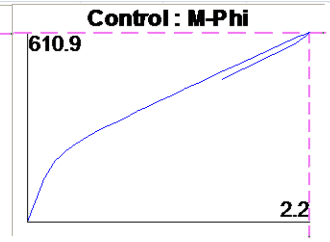
Note: Vexperimental is the shear where it causes shear failure at critical section of the member.

Response2000		
Vresp2000	201.1	KN
Vexp	258	KN
Vexp/Vresp2000	1.282944	

BH100 SIMPLY SUPPORTEDC CASE:

Member Properties:		
Total Spans Length	5.4	m
width, w	0.3	m
height, h	1	m
critical section from right support, L	2.7	m
$\gamma_{concrete}$	23.56	KN/m^3
$W_{self\text{-}wt}$	7.068	KN/m
$V_{experimental}$	193	KN
Shear Force at Critical Section, L.L and D.L		
$V_{crit,D.L}$	0	KN
$V_{crit,L.L}$	193	KN
Vu	193	KN
Moment at Critical Section, LL and D.L		
$M_{cirt,D.L}$	25.76286	KN.m
$M_{cirt,L.L}$	521.1	KN.m
Mu	546.86286	KN.m



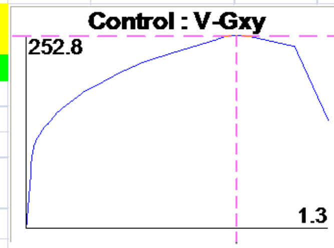


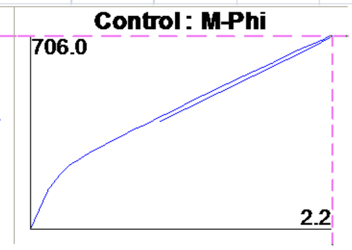
Note: Vexperimental is the shear where it causes shear failure at critical section of the member.

Response2000		
Vresp2000	215.5	KN
Vexp	193	KN
Vexp/Vresp2000	0.895592	

BHD100 SIMPLY SUPPORTEDC CASE:

Member Properties:		
Total Spans Length	5.4	m
width, w	0.3	m
height, h	1	m
critical section from right support, L	2.7	m
$\gamma_{concrete}$	23.56	KN/m^3
$W_{self\text{-}wt}$	7.068	KN/m
$V_{experimental}$	278	KN
Shear Force at Critical Section, L.L and D.L		
$V_{crit,D.L}$	0	KN
$V_{crit,L.L}$	278	KN
Vu	278	KN
Moment at Critical Section, LL and D.L		
$M_{cirt,D.L}$	25.76286	KN.m
$M_{cirt,L.L}$	750.6	KN.m
Mu	776.36286	KN.m



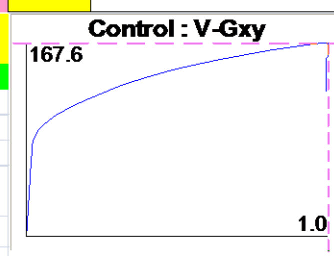


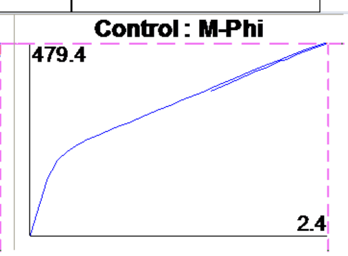
Note: Vexperimental is the shear where it causes shear failure at critical section of the member.

Response2000		
Vresp2000	252.8	KN
Vexp	278	KN
Vexp/Vresp2000	1.099684	

BRL100 SIMPLY SUPPORTEDC CASE:

Member Properties:		
Total Spans Length	5.4	m
width, w	0.3	m
height, h	1	m
critical section from right support, L	2.7	m
$\gamma_{concrete}$	23.56	KN/m^3
$W_{self\text{-}wt}$	7.068	KN/m
$V_{experimental}$	163	KN
Shear Force at Critical Section, L.L and D.L		
$V_{crit,D.L}$	0	KN
$V_{crit,L.L}$	163	KN
Vu	163	KN
Moment at Critical Section, LL and D.L		
$M_{cirt,D.L}$	25.76286	KN.m
$M_{cirt,L.L}$	440.1	KN.m
Mu	465.86286	KN.m

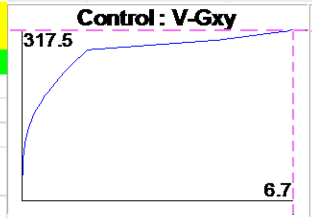


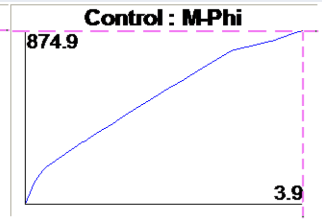


Note: Vexperimental is the shear where it causes shear failure at critical section of the member.

Response2000		
Vresp2000	167.6	KN
Vexp	163	KN
Vexp/Vresp2000	0.972554	

BM100 SIMPLY SUPPORTEDC CASE:		
Member Properties:		
Total Spans Length	5.4	m
width, w	0.3	m
height, h	1	m
critical section from right support, L	2.7	m
$\gamma_{concrete}$	23.56	KN/m^3
$W_{self\text{-}wt}$	7.068	KN/m
$V_{experimental}$	342	KN
Shear Force at Critical Section, L.L and D.L		
$V_{crit,D.L}$	0	KN
$V_{crit,L.L}$	342	KN
Vu	342	KN
Moment at Critical Section, LL and D.L		
$M_{cirt,D.L}$	25.76286	KN.m
$M_{cirt,L.L}$	923.4	KN.m
Mu	949.16286	KN.m

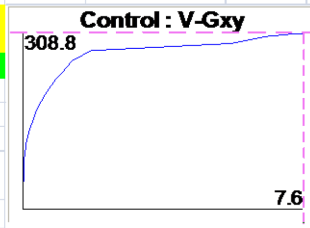


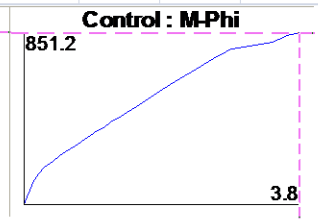


Note : Vexperimental is the shear where it causes shear failure at critical section of the member.

Response2000		
Vresp2000	256.6	KN
Vexp	342	KN
Vexp/Vresp2000	1.332814	

BM100D SIMPLY SUPPORTEDC CASE:		
Member Properties:		
Total Spans Length	5.4	m
width, w	0.3	m
height, h	1	m
critical section from right support, L	2.7	m
$\gamma_{concrete}$	23.56	KN/m^3
$W_{self\text{-}wt}$	7.068	KN/m
$V_{experimental}$	461	KN
Shear Force at Critical Section, L.L and D.L		
$V_{crit,D.L}$	0	KN
$V_{crit,L.L}$	461	KN
Vu	461	KN
Moment at Critical Section, LL and D.L		
$M_{cirt,D.L}$	25.76286	KN.m
$M_{cirt,L.L}$	1244.7	KN.m
Mu	1270.46286	KN.m





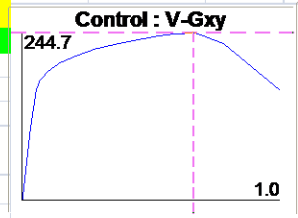
Note : Vexperimental is the shear where it causes shear failure at critical section of the member.

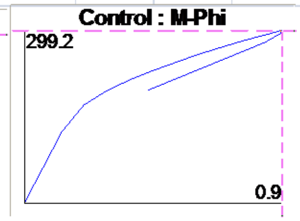
Response2000		
Vresp2000	308.8	KN
Vexp	461	KN
Vexp/Vresp2000	1.492876	

(Continuous Beam)

SE100A-45 CONTINUOUS CASE:

Member Properties:		
Total Span lengths	9.2	m
width, w	0.295	m
height, h	1	m
critical section from right support, L	1.19968	m
$\gamma_{concrete}$	23.56	KN/m^3
$W_{self\text{-}wt}$	6.9502	KN/m
$V_{experimental}$	201	KN
Shear Force at Critical Section, L.L and D.L		
$V_{crit,D.L}$	23.63290406	KN
$V_{crit,L.L}$	201	KN
Vu	224.6329041	KN
Moment at Critical Section, LL and D.L		
$M_{cirt,D.L}$	33.35339783	KN.m
$M_{cirt,L.L}$	241.13568	KN.m
Mu	274.4890778	KN.m





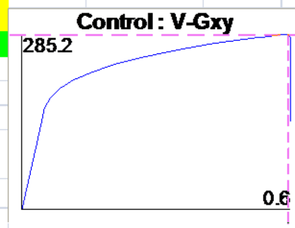
Note: Vexperimental is the shear where it causes shear failure at critical section of the member.

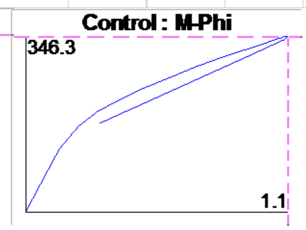
*$V_{resp2000} = V_{resp2000,Actual} - V_{crit,D.L}$

Response 2000		
Vresp2000	221.0671	KN
Vexp	201	KN
Vexp/Vresp200	0.909226	

SE100B-45 CONTINUOUS CASE:

Member Properties:		
Total Span lengths	9.2	m
width, w	0.295	m
height, h	1	m
critical section from right support, L	1.19968	m
$\gamma_{concrete}$	23.56	KN/m^3
$W_{self\text{-}wt}$	6.9502	KN/m
$V_{experimental}$	281	KN
Shear Force at Critical Section, L.L and D.L		
$V_{crit,D.L}$	23.63290406	KN
$V_{crit,L.L}$	281	KN
Vu	304.6329041	KN
Moment at Critical Section, LL and D.L		
$M_{cirt,D.L}$	33.35339783	KN.m
$M_{cirt,L.L}$	337.11008	KN.m
Mu	370.4634778	KN.m



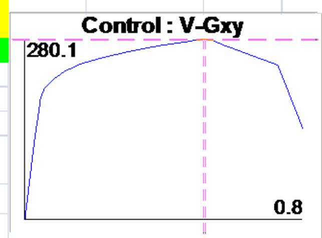


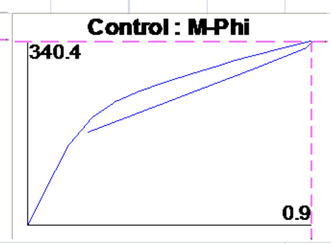
Note: Vexperimental is the shear where it causes shear failure at critical section of the member.

Response 2000		
Vresp2000	261.5671	KN
Vexp	281	KN
Vexp/Vresp200	1.074294	

SE100A-83 CONTINUOUS CASE:

Member Properties:		
Total Span lengths	9.2	m
width, w	0.295	m
height, h	1	m
critical section from right support, L	1.19968	m
$\gamma_{concrete}$	23.56	KN/m^3
$W_{self\text{-}wt}$	6.9502	KN/m
$V_{experimental}$	303	KN
Shear Force at Critical Section, L.L and D.L		
$V_{crit,D.L}$	23.63290406	KN
$V_{crit,L.L}$	303	KN
Vu	326.6329041	KN
Moment at Critical Section, LL and D.L		
$M_{cirt,D.L}$	33.35339783	KN.m
$M_{cirt,L.L}$	363.50304	KN.m
Mu	396.8564378	KN.m



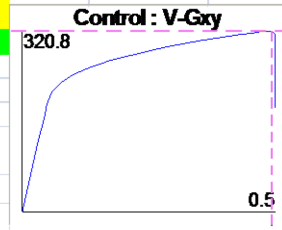


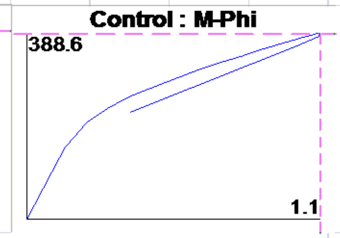
Note: Vexperimental is the shear where it causes shear failure at critical section of the member.

Response 2000		
Vresp2000	256.3671	KN
Vexp	303	KN
Vexp/Vresp200	1.181899	

SE100B-83 CONTINUOUS CASE:

Member Properties:		
Total Span lengths	9.2	m
width, w	0.295	m
height, h	1	m
critical section from right support, L	1.19968	m
$\gamma_{concrete}$	23.56	KN/m^3
$W_{self-wt}$	6.9502	KN/m
$V_{experimental}$	365	KN
Shear Force at Critical Section, L.L and D.L		
$V_{crit,D.L}$	23.63290406	KN
$V_{crit,L.L}$	365	KN
Vu	388.6329041	KN
Moment at Critical Section, LL and D.L		
$M_{crit,D.L}$	33.35339783	KN.m
$M_{crit,L.L}$	437.8832	KN.m
Mu	471.2365978	KN.m



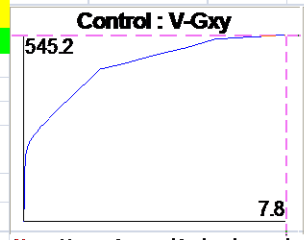


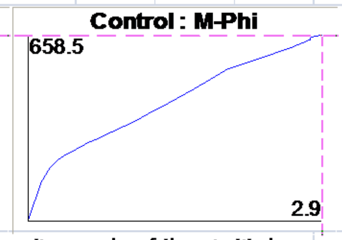
Note : Vexperimental is the shear where it causes shear failure at critical section of the member.

Response 2000		
Vresp2000	297.1671	KN
Vexp	365	KN
Vexp/Vresp200	1.228265	

SE100A-M-69 CONTINUOUS CASE:

Member Properties:		
Total Span lengths	9.2	m
width, w	0.295	m
height, h	1	m
critical section from right support, L	1.19968	m
$\gamma_{concrete}$	23.56	KN/m^3
$W_{self-wt}$	6.9502	KN/m
$V_{experimental}$	516	KN
Shear Force at Critical Section, L.L and D.L		
$V_{crit,D.L}$	23.63290406	KN
$V_{crit,L.L}$	516	KN
Vu	539.6329041	KN
Moment at Critical Section, LL and D.L		
$M_{crit,D.L}$	33.35339783	KN.m
$M_{crit,L.L}$	619.03488	KN.m
Mu	652.3882778	KN.m



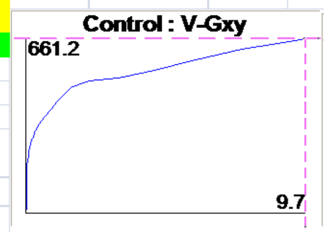


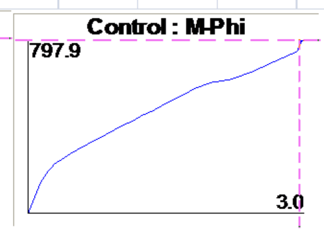
Note : Vexperimental is the shear where it causes shear failure at critical section of the member.

Response 2000		
Vresp2000	521.5671	KN
Vexp	516	KN
Vexp/Vresp200	0.989326	

SE100B-M-69 CONTINUOUS CASE:

Member Properties:		
Total Span lengths	9.2	m
width, w	0.295	m
height, h	1	m
critical section from right support, L	1.19968	m
$\gamma_{concrete}$	23.56	KN/m^3
$W_{self-wt}$	6.9502	KN/m
$V_{experimental}$	583	KN
Shear Force at Critical Section, L.L and D.L		
$V_{crit,D.L}$	23.63290406	KN
$V_{crit,L.L}$	583	KN
Vu	606.6329041	KN
Moment at Critical Section, LL and D.L		
$M_{crit,D.L}$	33.35339783	KN.m
$M_{crit,L.L}$	699.41344	KN.m
Mu	732.7668378	KN.m





Note : Vexperimental is the shear where it causes shear failure at critical section of the member.

Response 2000		
Vresp2000	637.5671	KN
Vexp	583	KN
Vexp/Vresp200	0.914414	

Appendix C - Solved Examples using the Developed MathCAD Design Tool

Example 1:

This example calculates the shear / Torsion reinforcement for a section of the pre-stressed girder shown in Figure C. **1**. Geometry and reinforcement arrangement is shown in Figure C. **2**.

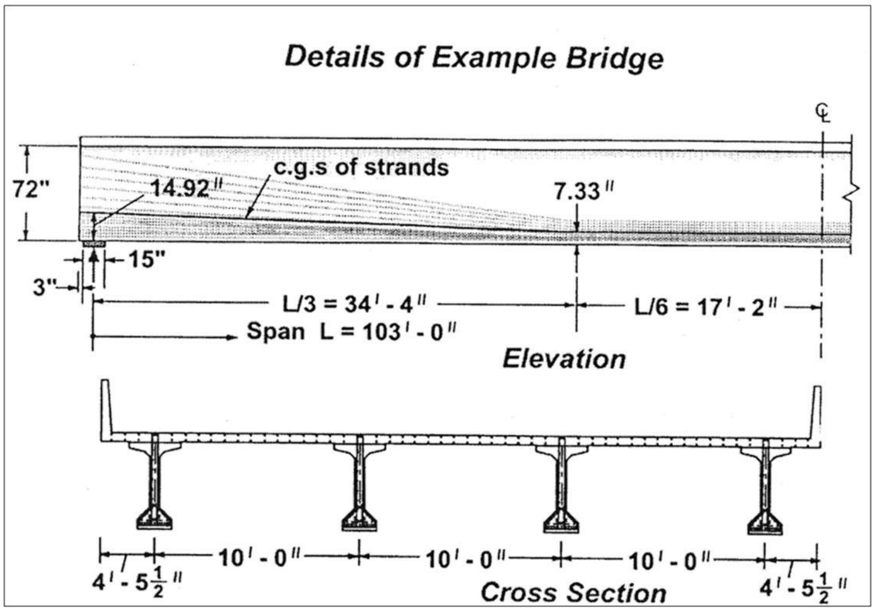


Figure C. 1 the girder used in this example, and a cross section of the bridge

The cross-section geometry and reinforcement is as follows:

Girder height	=72 in.
f'_c of the girder concrete	=8000 psi
Deck thickness	=7.5 in.
Deck thickness on the girder flange	=8 in.
f'_c of the Deck concrete	=4000 psi
Total height (including the Deck part)	=80 in.
Effective width (b_{eff})	=98 in.
Width of the web	=8 in.

Width of the lower flange =28 in.

Pre-stressing strands =54-1/2” strands

Pre-stressing stress =270 ksi

Area on flexural tensile side, A_c =580 in^2

Centroid of strand on support =14.92 in. from lower edge of the girder

Area of the regular (non-pre-stressing)
steel on the section: A_s =0.0 in^2

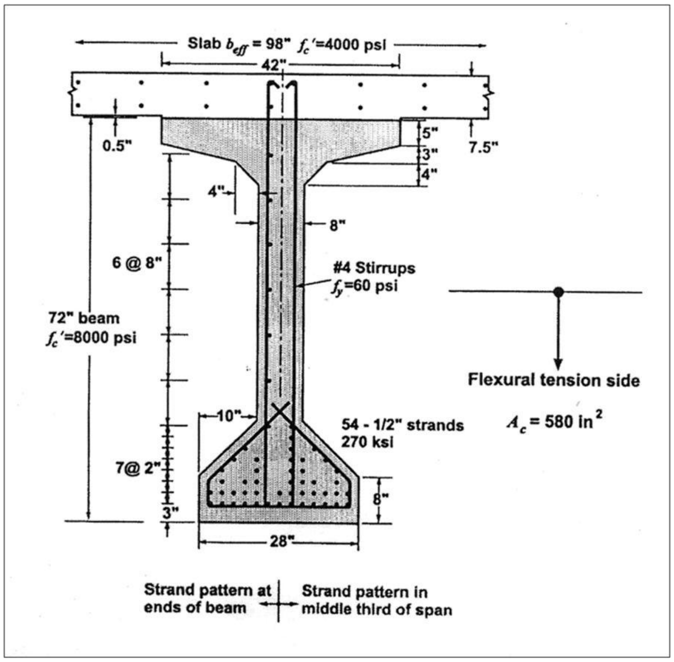


Figure C. 2 Beam cross section showing general geometry and reinforcement

Demanded shear and moment values at various locations along the beam are shown in Figure C. **3**. These values have been evaluated using the AASHTO loading standards and the envelope is provided at tenth points. Note that there was no torsion in this bridge, which was selected as a sample to be used to test the program. However, for demonstration purpose, various levels of torsion will be considered to check the program performance.

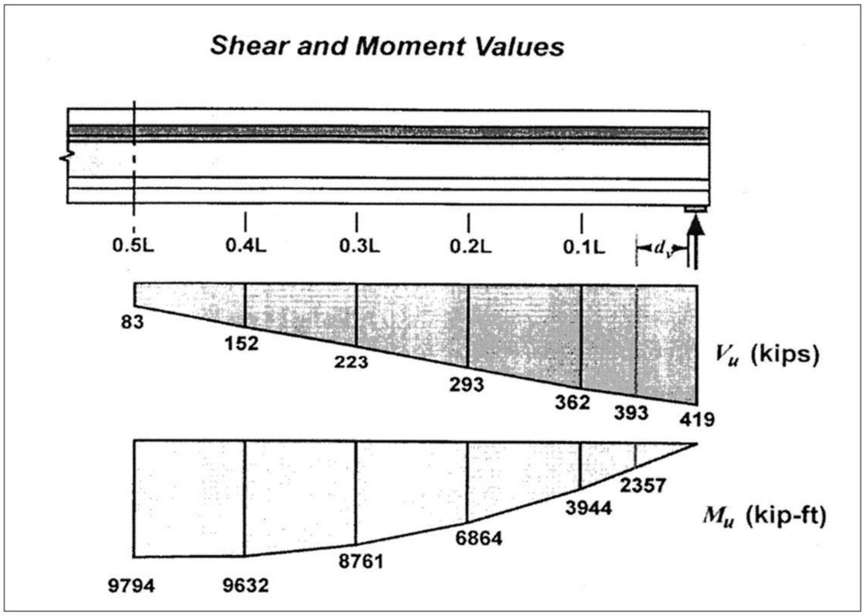


Figure C. 3 Demanded shear and moment values at various locations along the beam

Preparing the input data:

Before calculating the required input data, it should be noted that the values of the input data for various parameters depend on factors that are beyond the scope of this project. So, it is assumed that for each case, the user, will calculate the necessary values and provides them as the input data. The "designed" shear and torsion reinforcement depends on the accuracy of the input data.

On the other hand, depending of the case (girder geometry, demanded load level, reinforcement arrangement, and location of the section) the Strut-and-Tie Model should be used in lieu of the process implemented the program. Decrease in pre-stressing force based on the bonding situation is another factor that needs to be addressed by the user.

Section:

The section located at a distance equal to 20% of the girder length (span) is selected for design in this example. The distance between the supports, center to center, is equal to 103 feet. So, the distance between the support and the center of the span, is 51' 6".

The demanded values at this section, located at 0.2*103'=20' 7.2" from the support are:

M_u=6864 kip-ft

V_u=293 kips

N_u=0.0 kips

T_u=988 kip-in

The profile of the pre-stressing strands is shown in Figure C. **1**. The slope of the pre-stressing strands starts at a distance 34' 4" (34.33') away from the support, where the centroid of the pr-stressing strands is located at a height of 7.33" from the lower edge of the section. This centroid is at a height of 14.92" on the support. So, the tangent of the angle of the pre-stressing force and the horizontal direction will be:

$$\tan\gamma = \frac{14.92 - 7.33}{34.33 \times 12} = 0.0184 \qquad \text{(Eq. C. 1)}$$

Therefore

$$\tan\alpha_p = \frac{1}{0.0184} = 54.34 \qquad \text{(Eq. C. 2)}$$

So, the angle:

$$\alpha_p = 88.95\ deg \qquad \text{(Eq. C. 3)}$$

Note that design of the girder includes flexural design as well, and it is, in turn, related to the shear/torsion. As example, the amount of longitudinal reinforcement comes not only from flexure but also the amount needed for torsion. Following, shows the brief calculations for evaluation of the flexural strength at the mid-span, as compared to the demanded value.

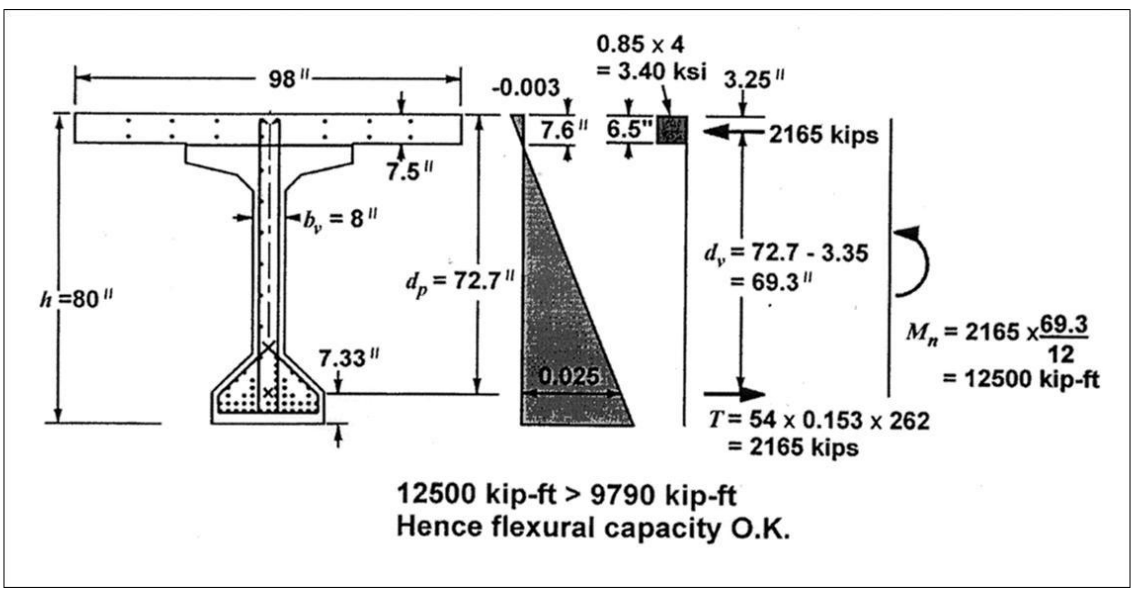


Figure C. 4 Calculation of the flexural strength at the mid-span

Area of the pre-stressing steel on the tensile side of the section:

Number of strands: 54

Area of each strand: 0.153 in^2

A_{ps}=54×0.153 in^2=8.262 in^2

A_s=0.0 in^2 (Area of regular steel)

The length of the outside perimeter of the concrete section

P_c=$2\times(14+8+\sqrt{2\times 64}+44+\sqrt{2\times 64}+\sqrt{9+13^2}+5+21)$

P_c=249.31 in or P_c=250 in

Perimeter of the centerline of the closed transverse torsion reinforcement (in.)

P_h=$2\times(76+5)$ $\quad$ P_h =162 in

Total area enclosed by outside perimeter of concrete cross-section (in2):

A_{cp}=$2\times((5+8)\times\frac{13}{2}+(8+12)\times\frac{4}{2}+4\times 72+)+(8+16)\times\frac{10}{2}$

A_{cp}=1065 in^2 $\quad$ A_{ct}=580 in^2

Width of web adjusted for the presence of ducts (in.); width of the interface (in.)

b_v=8 in. $\quad$ b_w=8 in.

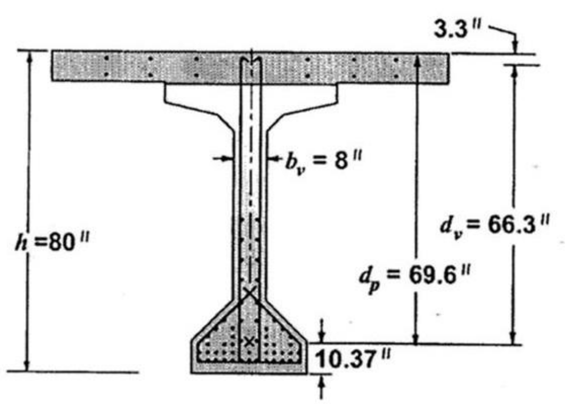


Figure C. 5 Calculating the section properties at 0.2L

Area enclosed by the shear flow path, including any area of holes therein (in2)

A_o=5×76 $\quad$ A_o=380 in^2

Distance from extreme compression fiber to the centroid of the pre-stressing tendons (in.):

d_p=80-10.73 $\quad$ d_p=69.6 in**.**

Example-1 MathCAD Solution:

LRFD Shear and Torsion, using Sectional Model
(SECTION 5, Interim 2008, Revised based on 2010)

IMPORTANT to Note: changing the unit system will lead to inaccuracy of the results, since most of the factors are based on teh US system

Example 1 (Please see the WORD File for detailed information)

1) Please see the ***WORD file for graphical definition of the parameters***.
2) The ***PDF file with links*** can help you with jumping to a desired location
3) Some parameters such as $\mathbf{A_0}$, $\mathbf{P_h}$, etc., need to be evaluated beforehand due to possible irregularities associated with various sections. Also, demanded loads are closely related to the bridge configuration and related load combination. This file, ONLY addresses shear/torsion part of design.

If the **distance between zero-shear point and the face of support is less than 2d** (d is the distance between the compression side and centriod of tensile reniforcement) OR a **concentrated load causing more than 1/2 of the shear at a support is closer than 2d from the face of support**, this method is not applicable. You need to use **Stut-and-Tie Model** (Section 5.6.3).
IN GENATRAL: Where the "plane sections assumption of felxural theory" is NOT valid, strut-and-Tie Model should be used.

Enter your data in the highlighted (yellow) fields

INPUT DATA:

Section Geometry:

$h := 80in$ Overall section depth (in)

$h_{nc} := 75in$ Depth of the non-composite section (in)

$A_{cp} := 1160in^2$ total area enclosed by outside perimeter of concrete cross-section (in^2). Having the section, this value needs to be calculated

$p_c := 220in$ the length of the outside perimeter of the concrete section (in.). Having the section, this value needs to be calculated

$A_o := 380in^2$ area enclosed by the shear flow path, including any area of holes therein (in^2). Having the section, this value needs to be calculated

$p_h := 162in$ perimeter of the centerline of the closed transverse torsion reinforcement (in.). Having the section, this value needs to be calculated

$b_v := 8in$ width of web adjusted for the presence of ducts (in.); width of the interface (in.)

OR: effective web width taken as the minimum web width, measured parallel to the neutral axis, between the resultants of the tensile and compressive forces due to flexure, or for circular sections, the diameter of the section, modified for the presence of ducts where applicable (in.)

$b_w := 8in$ width of web (in.)

$d_s := 72in$ distance from extreme compression fiber to the centroid of the nonprestressed tensile reinforcement (in.)

$d_p := 69in$ distance from extreme compression fiber to the centroid of the prestressing tendons (in.)

SecType := "Solid" Type of the cross section to design

$d_t := 74in$ distance from extreme compression fiber to the centroid of the extreme tensile reinforcement on teh section (it is different from d_s)

$c_{comp} := 20in$ Depth of the compression zone (you need to calculate it depending on the geometry of the section, amount of the reinforcement on tensile and compression side and material properties)

The section will be specified depending on the input data for regular and/or prestressing steel to check if the girder is nonprestressed, prestressed, or partially prestressed.

Reinforcement:

$A_s := 0.0in^2$ area of the non-prestressing tensile reinforcement on the section

$E_s := 29000ksi$ modulus of elasticity of reinforcing bars (ksi)

$A_{ps} := 8.262in^2$ area of the pre-stressing steel on tensile side of the member as in Fig 1 (Word) file or PDF file page 5-73

$A_b := 0.31in^2$ area of the cross section of the lateral reinforcement (here #5, change accordingly)

$n_{leg} := 2$ Number of the legs of the lateral reinforcement

$A_v := A_b \cdot n_{leg}$ Area of the shear reinforcement (used for torsion as well)

HasMin := "yes" Put "Yes" if section Contains minimum or more lateral reinforcement

Material Properties:

ConType := "Normal" put one of the: "Normal", "AllLightweight", "SandLightweight"

$f_c := 6ksi$ compressive stregth of concrete

$f_y := 60ksi$ yield strength of the non-prestressing tensile steel

$f_{py} := 60ksi$ yield strength of prestressing steel (ksi)

$f_{pc} := 1ksi$ (**can be calculated**) compressive stress in concrete after prestress losses have occurred either at the centroid of the cross-section resisting transient loads or at the junction of the web and flange where the centroid lies in the flange (ksi)

$f_{pu} := 270ksi$ specified tensile strength of prestressing steel (ksi)

$f_{ltl} := 32ksi$ Assumed value for long-term losses of prestressing steel (ksi)

$f_{pi} := 0.7 \cdot f_{pu}$ You may change the value if there are explicit information

$\boxed{f_{pi} = 189 \cdot ksi}$

$E_p := 28600ksi$ Modulus of elasticity of prestressing tendons (ksi)

$f_{ct\_specified} := \text{"No"}$

$f_{ct} := 0.2ksi$ (If you have a lightweight concrete, put "Yes" if f_{ct} available and enter the value, otherwise, put "No" and ignore f_{ct}) average splitting tensile strength of lightweight concrete

$\alpha_p := 88.95deg$ This is the angle between the prestressing force and postive direction of shear force (demand).

$a_g := 0.25in$ maximum aggregate size

$\alpha := 90deg$ angle of inclination of transverse reinforcement to longitudinal axis (°)

$K_1 := 1.0$ correction factor for source of aggregate to be taken as 1.0 unless determined by physical test, and as approved by the authority of jurisdiction (see **5.4.2.4**)

$w_c := 0.145 \frac{kip}{ft^3}$ unit weight of concrete (kcf); refer to **Table 3.5.1-1** or **Article C5.4.2.4**

$A_{ct} := 580in^2$ A_{ct} is the area of concrete on the flexural tension side of the member as shown in Figure 1 (in.^{2}). It is calculated as teh area bellow the centroid of the non-composite section. It can also be calculated as the area of the non-composite section, divided by the height of the non-composite section, then multiplied by half of the overall height of the section.

$s_{CrackControl} := 2in$ maximum distance between layers of longitudinal crack control reinforcement, where the area of the reinforcement in each layer is not less than **0.003*b*$_v$*s*$_x$,** as shown in Figure 3 (in.)

*Demanded **Shear** and **Torsion, Moment, Axial Force**:*

The induced internal shear and torsion in the section by the applied factored loads

$V_u := 293kip$ $T_u := 988kip \cdot in$ $M_u := 82368kip \cdot in$ $N_u := 0kip$ positive is tensile

Calculations and Design:

average stress in prestressing steel at the time for which the nominal resistance of member is required (ksi)

$f_{ps} := f_{pi} - f_{ltl}$ $\boxed{f_{ps} = 157 \cdot ksi}$

V_p is the component in the direction of the applied shear of the effective prestressing force; positive if resisting the applied shear (kip) (C5.8.2.3) (can be calculated if the angle is known)

$$V_p := A_{ps} \cdot f_{ps} \cdot \cos(\alpha_p)$$

$$V_p = 23.77 \cdot kip$$

$$E_c := 33000 K_1 \cdot w_c^{1.5} \cdot \left(\frac{ft^3}{kip}\right)^{1.5} \cdot \sqrt{f_c \cdot ksi}$$

$E_c = 4.463 \times 10^3 \cdot ksi$ Modulus of elasticity of concrete (ksi) (**5.4.2.4**)

$$PPR := \frac{A_{ps} \cdot f_{py}}{A_{ps} \cdot f_{py} + A_s \cdot f_y}$$

$$PreStress := \begin{cases} \text{"Prestressed"} & \text{if } PPR = 1.0 \\ \text{otherwise} & \\ \quad \begin{cases} \text{"Nonprestressed"} & \text{if } PPR = 0.0 \\ \text{"Partial prestressed"} & \text{otherwise} \end{cases} \end{cases} = \text{"Prestressed"}$$

The string variable "Prestress" shows the prestressing condition of the section.

Evaluation of stregth reduction factors ϕ, for flexure, shear, and torsion:

The following algorithm addresses **Article 4.5.4.2.1**, and **Equations 5.5.4.2.1-1 and 5.5.4.2.1-2**

(See: Article 5.5.4.2), p 5-25

$\varepsilon_y := \frac{f_y}{E_s}$ $\varepsilon_y = 2.069 \times 10^{-3}$

$$ebsY := \begin{cases} \varepsilon_y & \text{if } PreStress = \text{"Nonprestressed"} \\ 0.002 & \text{otherwise} \end{cases}$$

$ebsY = 2 \times 10^{-3}$

Evaluation of the flexural strength reduction factor:

$phiT := 0.9 + 0.1 \cdot PPR \qquad phiT = 1$

This temporal variable is for implementation of prestressing condition (normal, partial, or full)

C5.5.4.2.1 states that "For sections subjected to axial load with flexure, factored resistances are determined by multiplying both P_n and M_n by the appropriate single value of ϕ

$$\phi_f := \begin{cases} phiT & \text{if } c_{comp} \le 0.375 \cdot d_t \\ \text{otherwise} & \\ \quad \begin{cases} phiT & \text{if } c_{comp} \ge \dfrac{0.003}{0.003 + ebsY} \cdot d_t \\ phiT + \dfrac{0.75 - phiT}{\dfrac{0.003}{0.003 + ebsY} \cdot d_t - 0.375 \cdot d_t} \cdot \left(c_{comp} - 0.375 \cdot d_t\right) & \text{otherwise} \end{cases} \end{cases}$$

$\phi_f = 1 \qquad \phi_c := \phi_f$ As per C5.5.4.2.1

Evaluation of the shear and torsional strength reduction factors based on **5.5.4.2**

$$\phi_v := \begin{cases} 0.9 & \text{if } ConType = \text{"Normal"} \\ 0.7 & \text{otherwise} \end{cases} \qquad \phi_t := \begin{cases} 0.9 & \text{if } ConType = \text{"Normal"} \\ 0.7 & \text{otherwise} \end{cases}$$

$\boxed{\phi_f = 1} \quad \boxed{\phi_v = 0.9} \quad \boxed{\phi_t = 0.9} \quad \boxed{\phi_c = 1}$

$$d_e := \frac{A_s \cdot f_y \cdot d_s + A_{ps} \cdot f_{ps} \cdot d_p}{A_s \cdot f_y + A_{ps} \cdot f_{ps}}$$

(Equation 5.8.2.9-2) AASHTO 2010

Calculating the value of d_v:

effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure; it need not be taken to be less than the greater of 0.9 d_e or 0.72*h* (in.)

To calculate d_v, using the commentary C5.8.2.9-1, **assuming M_n=M_u/φ,** we have:

$$d_{v1} := \frac{\left(\dfrac{M_u}{\phi_f}\right)}{A_s \cdot f_y + A_{ps} \cdot f_{ps}}$$

(C 5.8.2.9-1)

$d_{v1} = 63.5 \cdot in$

$$d_v := \max\left(0.9d_e, 0.72\cdot h, d_{v1}\right)$$ **(Art. 5.8.2.9)**

$$d_v = 63.5\cdot in$$

Calculate the Shear Stress:

$$v_u := \frac{\left|V_u - \phi_v\cdot V_p\right|}{\phi_v\cdot b_v\cdot d_v}$$

(Eq 5.8.2.9-1)

AASHTO 2010

Induced demanded shear stress

$$v_u = 0.594\cdot ksi$$

Note that this should be v_n (equivalent to v_u/ϕ) and not v_u or we need to remove the strength reduction fator. It is the same in 2010 as well.

Calculating $\mathbf{S_{max}}$ (maximum permissible spacing of lateral reinforcement) considering the demanded shear stress ***(Eqs. 5.8.2.7-1 and 5.8.2.7-2)*** will be done when the shear reinorcement, if needed, is designed

Following is evaluation of the $\boldsymbol{A_{cp}^2/p_c}$ so that we address a cellular case, ***per 5.8.2.1-5***

$$Val := \begin{cases} \dfrac{A_{cp}^{\;2}}{p_c} & \text{if } SecType = \text{"Solid"} \\ \text{otherwise} & \\ \quad \begin{cases} \dfrac{A_{cp}^{\;2}}{p_c} & \text{if } \left(\dfrac{A_{cp}^{\;2}}{p_c}\right) \le 2A_o\cdot b_v \\ 2A_o\cdot b_v & \text{otherwise} \end{cases} \end{cases}$$

(Equation 5.8.2.1-5)

<u>Calculate Cracking Toresion (no segmental):</u> ***(Equation 5.8.2.1-4)***

$$T_{cr} := \begin{cases} 0.125\cdot\sqrt{f_c\cdot ksi}\cdot Val\cdot\sqrt{1 + \dfrac{f_{pc}}{0.125\cdot\sqrt{f_c\cdot ksi}}} & \text{if } ConType = \text{"Normal"} \\ \text{otherwise} & \\ \quad \begin{cases} 0.125\cdot\min\left(4.7\cdot f_{ct}, \sqrt{f_c\cdot ksi}\right)\cdot Val\cdot\sqrt{1 + \dfrac{f_{pc}}{0.125\cdot\min\left(4.7\cdot f_{ct}, \sqrt{f_c\cdot ksi}\right)}} & \text{if } f_{ct\_specified} = \text{"Yes"} \\ \text{otherwise} & \\ \quad \begin{cases} 0.125\cdot 0.75\sqrt{f_c\cdot ksi}\cdot Val\cdot\sqrt{1 + \dfrac{f_{pc}}{0.125\cdot 0.75\sqrt{f_c\cdot ksi}}} & \text{if } ConType = \text{"AllLightweight"} \\ 0.125\cdot 0.85\sqrt{f_c\cdot ksi}\cdot Val\cdot\sqrt{1 + \dfrac{f_{pc}}{0.125\cdot 0.85\sqrt{f_c\cdot ksi}}} & \text{otherwise} \end{cases} \end{cases} \end{cases}$$

$$T_{cr} = 3.868\times 10^{3}\cdot kip\cdot in$$

Check if the torsion can be ignored:

$$\text{IgnoreTorsion} := \begin{cases} \text{"Yes"} & \text{if } T_u \le 0.25 \cdot \phi_t \cdot T_{cr} \\ \text{"No"} & \text{otherwise} \end{cases}$$

(Equation 5.8.2.1-3)

$\text{IgnoreTorsion} = \text{"No"}$

Calculating the demanded equivalent shear force:

Note that here, since the equivalent shear can be used for checking the section, we have to use the **T_u** regardless of being less that 0.25 times the cracking torsion or not.

$$V_{u\_eq} := \begin{cases} \sqrt{V_u^{\,2} + \left(\dfrac{0.9 p_h \cdot T_u}{2 \cdot A_o}\right)^2} & \text{if SecType} = \text{"Solid"} \\ V_u + \dfrac{T_u \cdot d_s}{2 \cdot A_o} & \text{otherwise} \end{cases}$$

(Equation 5.8.2.1-6 and 5.8.2.1-7)

$V_{u\_eq} = 348.962 \cdot \text{kip}$

This equivalent V_{u_eq} is just for checking the adequecy of the section, when needed, otherwise the shear and torsional steel need to be evaluated as needed and then added up (shear as per 5.8.3.3 and Torsion as per 5.8.3.6.2)

Calculating the shear strength provided by concrete, V_c:

$f_{po} := 0.7 f_{pu}$

a parameter taken as modulus of elasticity of prestressing tendons multiplied by the lockedin difference in strain between the prestressing tendons and the surrounding concrete (ksi). For the usual levels of prestressing, a value of **0.7 f_{pu}** will be appropriate for both pretensioned and post-tensioned members

*Calculating ε_s using equation **5.8.3.4.2-4***

IMPORTANT Note: Deduct a portion of the area of the bars and tendons terminated less than their development length from the section that you are designing for, with the same proportion as their lack of full length (Here, the development length of a bar can be evaluated, and then having the actual length, the area can be reduced proportional to the ratio of available length to development length.

Addressing the requirement to have **M_u** used not to be less than **$(V_u-V_p)d_v$**, we use **M_{u1}** as follows:

$$M_{u1} := \begin{cases} M_u & \text{if } |M_u| \ge \left|(V_u - V_p) \cdot d_v\right| \\ (V_u - V_p) \cdot d_v & \text{otherwise} \end{cases}$$

(Article 5.8.3.4.2)

$M_{u1} = 8.237 \times 10^4 \cdot \text{kip} \cdot \text{in}$

Note that 1) $\mathbf{A_s}$ and $\mathbf{A_{ps}}$ should be reduced proportionally if a lack of full development length to the section under design 2) If closer than $\mathbf{d_v}$ to the face of support, use ε_s at distance $\mathbf{d_v}$ to evaluate β and θ

$$\varepsilon_{s1} := \frac{\dfrac{|M_{u1}|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps} \cdot f_{po}}{E_s \cdot A_s + E_p \cdot A_{ps}}$$

(Equation 5.8.3.4.2-4)

$$\varepsilon_{s1} = 2.051 \times 10^{-5}$$

(Article 5.8.3.4.2) Page 5-72 (2008), Page 5-71 (2010)

$$\varepsilon_{s2} := \begin{cases} \dfrac{\dfrac{|M_{u1}|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps} \cdot f_{po}}{E_s \cdot A_s + E_p \cdot A_{ps} + E_c \cdot A_{ct}} & \text{if } \varepsilon_{s1} < 0.0 \\ \varepsilon_{s1} & \text{otherwise} \end{cases}$$

This and the follwing are evaluates ε_s, presisely and does not use zero.

$$\varepsilon_{s2} = 2.051 \times 10^{-5}$$

For pretensioned members, f_{po} can be taken as the stress in the strands when the concrete is cast around them, i.e., approximately equal to the jacking stress. For post-tensioned members, f_{po} can be conservatively taken as the average stress in the tendons when the posttensioning is completed.

$$\varepsilon_s := \begin{cases} \varepsilon_s \leftarrow -0.0004 & \text{if } \varepsilon_{s2} < -4.0 \times 10^{-4} \\ \varepsilon_s \leftarrow 0.006 & \text{if } \varepsilon_{s2} > 0.006 \\ \varepsilon_{s2} & \text{otherwise} \end{cases}$$

Taking care of the last condition on evaluation of ε_s

$$\varepsilon_s = 2.051 \times 10^{-5}$$

(Article 5.8.3.4.2) Page 5-72 to 74 (2008), Page 5-71 to 73 (2010)

NOTE

For sections closer than $\mathbf{d_v}$ to the face of support the value for ε_s calculated at $\mathbf{d_v}$ from the face of support may be used to evaluate β and θ

If the axial tension is large enough to crack the flexural compression face of the section, the value calculated should be doubled.

$$s_x := \min(d_v, s_{CrackControl})$$

Reinforcement in each layer not less than $\mathbf{0.003 b_v s_x}$

$$s_x = 2 \cdot in$$

$$s_{xe1} := s_x \cdot \frac{1.38}{a_g + 0.63in}$$ **(Equation 5.8.3.4.2-5)**

$$s_{xe} := \begin{cases} 12 \text{ if } s_{xe1} < 12 \\ \text{otherwise} \\ \quad \begin{cases} 80 \text{ if } s_{xe1} > 80 \\ s_{xe1} \text{ otherwise} \end{cases} \end{cases}$$

$$s_{xe} = 12$$

$$\beta := \begin{cases} \dfrac{4.8}{1 + 750 \cdot \varepsilon_s} \text{ if HasMin} = \text{"Yes"} \\ \left(\dfrac{4.8}{1 + 750\varepsilon_s}\right) \cdot \left(\dfrac{51}{39 + s_{xe}}\right) \text{ otherwise} \end{cases}$$

(Equations 5.8.3.4.2-1 and 2)

$$\beta = 4.727$$

$$\theta := 29deg + 3500deg\,\varepsilon_s$$ **(Equation 5.8.3.4.2-3)**

Here we use deg to get the angle in deg for further calculations

$$\theta = 29.072 \cdot deg$$

$$V_c := 0.0316 \cdot \beta \cdot \sqrt{f_c \cdot ksi} \cdot b_v \cdot d_v$$ **(Equation 5.8.3.3-3)**

$$V_c = 185.882 \cdot kip$$

Designing the transverse reinforcement for **SHEAR** (Note that the A_v/s will be evaluated. Then when we have Torsion as well, we can add them appropriately depending on the type of section)

Check if we need shear reinforcement per 5.8.2.4-1

$$NeedShear := \begin{cases} \text{"Yes" if } V_u > 0.5 \cdot \phi_v \cdot (V_c + V_p) \\ \text{"No" otherwise} \end{cases}$$ **(Equation 5.8.2.4-1)**

$$NeedShear = \text{"Yes"}$$

Here we need to check for mininum reinforcement as per **5.8.2.5** (AASHTO 2010)

$$A_v = 0.62 \cdot in^2$$

$$V_n := \min\left(\frac{V_u}{\phi_v}, 0.25 f_c \cdot b_v \cdot d_v + V_p\right) \quad V_n = 325.556 \cdot kip$$ **Based on Eq. 5.8.3.3-1 and 2**

$$V_s := V_n - V_c - V_p$$

$$V_s = 115.903 \cdot kip$$

Here S_{min} actually means the "s" based on the minimum requirement, otherwise, this is the limit for spacing and the specing should be less or at most equal to this

$$s_{max} := \frac{A_v \cdot f_y}{0.0316 \cdot b_v \cdot \sqrt{f_c \cdot ksi}} \quad s_{max} = 60.075 \cdot in$$ ***Eq. 5.8.2.5-1***

$$s_{req} := \text{if}\left[V_s \le 0, s_{max}, \frac{A_v \cdot f_y \cdot d_v \cdot (\cot(\theta) + \cot(\alpha)) \cdot \sin(\alpha)}{V_s}\right]$$ **(Equation 5.8.3.3-4**

$s_{req} = 36.66 \cdot in$

$s := \min(s_{max}, s_{req})$

$s = 36.66 \cdot in$

Based on **5.8.2.7** and **5.8.2.9,** we need to evaluate the final spacing by finding the othe maximum limits as follows:

$s_{max2} := \text{if}\left(v_u < 0.125 \cdot f_c, \min\left(0.8 \cdot d_v, 24in\right), \min\left(0.4 \cdot d_v, 12in\right)\right)$ ***(Eqs. 5.8.2.7-1 and 5.8.2.7-2)***

$s_{max2} = 24 \cdot in$

The spacing of the transversal reinforcement is as follows:

$s_{actual} := \min(s, s_{max2})$

$\boxed{s_{actual} = 24 \cdot in}$

To address 5.8.3.3-2 , The upper limit of V_n, given by Eq. 2, is intended to ensure that the concrete in the web of the beam will not crush prior to yield of the transverse reinforcement.

$0.25 \cdot f_c \cdot b_v \cdot d_v = 762 \cdot kip$

Note that $\mathbf{V_u/\varphi}$ **-Vp** should be less than $\mathbf{0.25\ f_c b_v d_v}$ otherwise section is not enough and concrete crushes due to local shear demand.

Check if the section is enough:

$$\text{SecEnough} := \begin{cases} \text{"Yes"} & \text{if } \dfrac{V_u}{\phi_v} - V_p < 0.25 \cdot f_c \cdot b_v \cdot d_v \\ \text{"STOP and Change Section"} & \text{otherwise} \end{cases}$$ ***(Equation 5.8.3.3-2)***

$\boxed{\text{SecEnough} = \text{"Yes"}}$

Also we need to specify where we have to switch to **strut-and-tie model**

Note : $\mathbf{V_p}$ is the vertical component of the prestressing force (as has already been evaluated by using the angle $\boldsymbol{\alpha_p}$)

Finding the ratio of **avss=A_v/s** *for shear:*

$$\text{avss} := \frac{A_v}{s_{actual}}$$

$\boxed{\text{avss} = 0.026 \cdot in}$

Design for Torsional lateral reinforcement:

Note that if we have a torsion that cannot be ignored, we design the lateral reinforcement for that and we call that avst. later we add the shear and torsion reinforcement properly. Corresponding longitudinal steel will be calculated as well.

$$avst1 := \begin{vmatrix} \dfrac{\frac{T_u}{\phi_t}}{2 \cdot A_o \cdot f_y \cdot \cot(\theta)} \ \text{ if IgnoreTorsion} = \text{"No"} \\ 0.0 \ \text{ otherwise} \end{vmatrix}$$

(Equation 5.8.3.6.2-1)

$avst1 = 0.013 \cdot in$ This is for one leg

$avst := 2 \cdot avst1$ $\qquad avst = 0.027 \cdot in$

$avs := avss + avst$ This is the total lateral reinforcement needed for shear and torsion

$$s_{ShearTorsion} := \text{floor}\left(\frac{A_v}{avs \cdot in}\right) \cdot in$$

$\boxed{s_{ShearTorsion} = 11 \cdot in}$

Design for Torsional longitudinal reinforcement: (Article 5.8.3.6.3 Longitudinal Reinforcement) [Also consider Eq. 5.8.3.5-1]

$$tmpVal := \frac{|M_{u1}|}{\phi_f \cdot d_v} + \frac{0.5 \cdot N_u}{\phi_c} + \cot(\theta) \cdot \sqrt{\left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5 \cdot V_s\right)^2 + \left(\frac{0.45 \cdot p_h \cdot T_u}{2 \cdot A_o \cdot \phi_t}\right)^2}$$

$tmpVal = 1.775 \times 10^3 \cdot kip$

$$A_l := \begin{vmatrix} 0.0 \ \text{ if IgnoreTorsion} = \text{"Yes"} \\ \text{otherwise} \\ \quad \begin{vmatrix} \text{if SecType} = \text{"Solid"} \\ \quad \begin{vmatrix} 0.0 \ \text{ if } A_{ps} \cdot f_{ps} + A_s \cdot f_y \ge tmpVal \\ \dfrac{tmpVal - A_{ps} \cdot f_{ps}}{f_y} - A_s \ \text{ otherwise} \end{vmatrix} \\ \dfrac{T_u \cdot p_h}{\phi_t \cdot 2 \cdot A_o \cdot f_y} \ \text{ otherwise} \end{vmatrix} \end{vmatrix}$$

Note that $\mathbf{A_l}$ is the additional steel needed due to torsion

Eq. 5.8.3.6.3-1

Eq. 5.8.3.6.3-2

$A_l = 7.962 \cdot in^2$

$A_{s\_total} := A_s + A_l$ Note that distribution of $\mathbf{A_l}$ should be evenly around the section

$\boxed{A_{s\_total} = 7.962 \cdot in^2}$

Example-2

Solution using the Developed MathCAD Design Program

Example-2 MathCAD Solution:

LRFD Shear and Torsion, using Sectional Model
(SECTION 5, Interim 2008, Revised based on 2010)

IMPORTANT to Note: changing the unit system will lead to inaccuracy of the results, since most of the factors are based on teh US system

Example 2 (Using some numbers for an assumed section to check the functionality)

1) Please see the ***WORD file for graphical definition of the parameters***.
2) The ***PDF file with links*** can help you with jumping to a desired location
3) Some parameters such as $\mathbf{A_0}$, $\mathbf{P_h}$, etc., need to be evaluated beforehand due to possible irregularities associated with various sections. Also, demanded loads are closely related to the bridge configuration and related load combination. This file, ONLY addresses shear/torsion part of design.

If the **distance between zero-shear point and the face of support is less than 2d** (d is the distance between the compression side and centriod of tensile reniforcement) OR a **concentrated load causing more than 1/2 of the shear at a support is closer than 2d from the face of support**, this method is not applicable. You need to use **Stut-and-Tie Model** (Section 5.6.3).
IN GENATRAL: Where the "plane sections assumption of felxural theory" is NOT valid, strut-and-Tie Model should be used.

Enter your data in the highlighted (yellow) fields

INPUT DATA:

Section Geometry:

$h := 54in$ Overall section depth (in)

$h_{nc} := 54in$ Depth of the non-composite section (in)

$A_{cp} := (2923in)^2$ total area enclosed by outside perimeter of concrete cross-section (in^2). Having the section, this value needs to be calculated

$p_c := 216in$ the length of the outside perimeter of the concrete section (in.). Having the section, this value needs to be calculated

$A_o := 360in^2$ area enclosed by the shear flow path, including any area of holes therein (in^2). Having the section, this value needs to be calculated

$p_h := 312in$ perimeter of the centerline of the closed transverse torsion reinforcement (in.). Having the section, this value needs to be calculated

$b_v := 8in$ width of web adjusted for the presence of ducts (in.); width of the interface (in.)

OR: effective web width taken as the minimum web width, measured parallel to the neutral axis, between the resultants of the tensile and compressive forces due to flexure, or for circular sections, the diameter of the section, modified for the presence of ducts where applicable (in.)

$b_w := 8in$ width of web (in.)

$d_s := 50in$ distance from extreme compression fiber to the centroid of the nonprestressed tensile reinforcement (in.)

$d_p := 47in$ distance from extreme compression fiber to the centroid of the prestressing tendons (in.)

SecType := "Solid" Type of the cross section to design

$d_t := 52in$ distance from extreme compression fiber to the centroid of the extreme tensile reinforcement on teh section (it is different from d_s)

$c_{comp} := 14in$ Depth of the compression zone (you need to calculate it depending on the geometry of the section, amount of the reinforcement on tensile and compression side and material properties)

The section will be specified depending on the input data for regular and/or prestressing steel to check if the girder is nonprestressed, prestressed, or partially prestressed.

Reinforcement:

$A_s := 012.7in^2$ area of the non-prestressing tensile reinforcement on the section

$E_s := 29000ksi$ modulus of elasticity of reinforcing bars (ksi)

$A_{ps} := 5.5in^2$ area of the pre-stressing steel on tensile side of the member as in Fig 1 (Word) file or PDF file page 5-73

$A_b := 0.31in^2$ area of the cross section of the lateral reinforcement (here #5, change accordingly)

$n_{leg} := 2$ Number of the legs of the lateral reinforcement

$A_v := A_b \cdot n_{leg}$ Area of the shear reinforcement (used for torsion as well)

HasMin := "yes" Put "Yes" if section Contains minimum or more lateral reinforcement

Material Properties:

ConType := "Normal" put one of the: "Normal", "AllLightweight", "SandLightweight"

$f_c := 4ksi$ compressive stregth of concrete

$f_y := 60ksi$ yield strength of the non-prestressing tensile steel

$f_{py} := 300ksi$ yield strength of prestressing steel (ksi)

$f_{pc} := 1ksi$ (**can be calculated**) compressive stress in concrete after prestress losses have occurred either at the centroid of the cross-section resisting transient loads or at the junction of the web and flange where the centroid lies in the flange (ksi)

$f_{pu} := 270\text{ksi}$ specified tensile strength of prestressing steel (ksi)

$f_{ltl} := 32\text{ksi}$ Assumed value for long-term losses of prestressing steel (ksi)

$f_{pi} := 0.7 \cdot f_{pu}$ You may change the value if there are explicit information

$\boxed{f_{pi} = 189 \cdot \text{ksi}}$

$E_p := 28500\text{ksi}$ Modulus of elasticity of prestressing tendons (ksi)

$f_{ct\_specified} := \text{"No"}$

$f_{ct} := 0.6\text{ksi}$ (If you have a lightweight concrete, put "Yes" if f_{ct} available and enter the value, otherwise, put "No" and ignore f_{ct}) average splitting tensile strength of lightweight concrete

$\alpha_p := 90\text{deg}$ This is the angle between the prestressing force and postive direction of shear force (demand).

$a_g := 0.25\text{in}$ maximum aggregate size

$\alpha := 90\text{deg}$ angle of inclination of transverse reinforcement to longitudinal axis (°)

$K_1 := 1.0$ correction factor for source of aggregate to be taken as 1.0 unless determined by physical test, and as approved by the authority of jurisdiction (see **5.4.2.4**)

$w_c := 0.145 \frac{\text{kip}}{\text{ft}^3}$ unit weight of concrete (kcf); refer to **Table 3.5.1-1** or **Article C5.4.2.4**

$A_{ct} := 460.2\text{in}^2$ A_{ct} is the area of concrete on the flexural tension side of the member as shown in Figure 1 (in.^{2}). It is calculated as teh area bellow the centroid of the non-composite section. It can also be calculated as the area of the non-composite section, divided by the height of the non-composite section, then multiplied by half of the overall height of the section.

$s_{CrackControl} := 2\text{in}$ maximum distance between layers of longitudinal crack control reinforcement, where the area of the reinforcement in each layer is not less than $\mathbf{0.003} b_v s_x$, as shown in Figure 3 (in.)

Demanded ***Shear*** *and* ***Torsion, Moment, Axial Force****:*

The induced internal shear and torsion in the section by the applied factored loads

$V_u := 211\text{kip}$ $T_u := 5450\text{kip} \cdot \text{in}$ $M_u := 12130\text{kip} \cdot \text{in}$ $N_u := 10\text{kip}$ positive is tensile

Calculations and Design:

average stress in prestressing steel at the time for which the nominal resistance of member is required (ksi)

$f_{ps} := f_{pi} - f_{ltl}$ $\boxed{f_{ps} = 157 \cdot ksi}$

$\mathbf{V_p}$ is the component in the direction of the applied shear of the effective prestressing force; positive if resisting the applied shear (kip) (C5.8.2.3) (can be calculated if the angle is known)

$$V_p := A_{ps} \cdot f_{ps} \cdot \cos(\alpha_p)$$

$$V_p = 5.287 \times 10^{-14} \cdot kip$$

$$E_c := 33000 K_1 \cdot w_c^{1.5} \cdot \left(\frac{ft^3}{kip}\right)^{1.5} \cdot \sqrt{f_c \cdot ksi}$$

$E_c = 3.644 \times 10^3 \cdot ksi$ Modulus of elasticity of concrete (ksi) (**5.4.2.4**)

$$PPR := \frac{A_{ps} \cdot f_{py}}{A_{ps} \cdot f_{py} + A_s \cdot f_y}$$

$$PreStress := \begin{cases} \text{"Prestressed"} & \text{if } PPR = 1.0 \\ \text{otherwise} & \\ \quad \begin{cases} \text{"Nonprestressed"} & \text{if } PPR = 0.0 \\ \text{"Partial prestressed"} & \text{otherwise} \end{cases} \end{cases} = \text{"Partial prestressed"}$$

The string variable "Prestress" shows the prestressing condition of the section.

Evaluation of stregth reduction factors ϕ, for flexure, shear, and torsion:

The following algorithm addresses **Article 4.5.4.2.1**, and **Equations 5.5.4.2.1-1 and 5.5.4.2.1-2**

(See: Article 5.5.4.2), p 5-25

$\varepsilon_y := \dfrac{f_y}{E_s}$ $\varepsilon_y = 2.069 \times 10^{-3}$

$$ebsY := \begin{cases} \varepsilon_y & \text{if } PreStress = \text{"Nonprestressed"} \\ 0.002 & \text{otherwise} \end{cases}$$

$ebsY = 2 \times 10^{-3}$

Evaluation of the flexural strength reduction factor:

$phiT := 0.9 + 0.1 \cdot PPR$ $phiT = 0.968$ This temporal variable is for implementation of prestressing condition (normal, partial, or full)

C5.5.4.2.1 states that "For sections subjected to axial load with flexure, factored resistances are determined by multiplying both P_n and M_n by the appropriate single value of ϕ

$$\phi_f := \begin{cases} phiT & \text{if } c_{comp} \le 0.375 \cdot d_t \\ \text{otherwise} & \begin{cases} phiT & \text{if } c_{comp} \ge \dfrac{0.003}{0.003 + ebsY} \cdot d_t \\ phiT + \dfrac{0.75 - phiT}{\dfrac{0.003}{0.003 + ebsY} \cdot d_t - 0.375 \cdot d_t} \cdot \left(c_{comp} - 0.375 \cdot d_t\right) & \text{otherwise} \end{cases} \end{cases}$$

$\phi_f = 0.968$ $\phi_c := \phi_f$ As per C5.5.4.2.1

Evaluation of the shear and torsional strength reduction factors based on **5.5.4.2**

$$\phi_v := \begin{cases} 0.9 & \text{if } ConType = \text{"Normal"} \\ 0.7 & \text{otherwise} \end{cases} \qquad \phi_t := \begin{cases} 0.9 & \text{if } ConType = \text{"Normal"} \\ 0.7 & \text{otherwise} \end{cases}$$

$\phi_f = 0.968$ $\phi_v = 0.9$ $\phi_t = 0.9$ $\phi_c = 0.968$

$$d_e := \frac{A_s \cdot f_y \cdot d_s + A_{ps} \cdot f_{ps} \cdot d_p}{A_s \cdot f_y + A_{ps} \cdot f_{ps}}$$

(Equation 5.8.2.9-2) AASHTO 2010

Calculating the value of d_v:

effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure; it need not be taken to be less than the greater of 0.9 d_e or 0.72*h* (in.)

To calculate d_v, using the commentary C5.8.2.9-1, **assuming $M_n=M_u/\varphi$,** we have:

$$d_{v1} := \frac{\left(\dfrac{M_u}{\phi_f}\right)}{A_s \cdot f_y + A_{ps} \cdot f_{ps}}$$

(C 5.8.2.9-1)

$d_{v1} = 7.706 \cdot in$

$d_v := \max(0.9d_e, 0.72 \cdot h, d_{v1})$ **(Art. 5.8.2.9)**

$d_v = 43.566 \cdot in$

Calculate the Shear Stress:

$$v_u := \frac{|V_u - \phi_v \cdot V_p|}{\phi_v \cdot b_v \cdot d_v}$$

(Eq 5.8.2.9-1)

AASHTO 2010

Induced demanded shear stress

$v_u = 0.673 \cdot ksi$

Note that this should be v_n (equivalent to v_u/ϕ) and not v_u or we need to remove the strength reduction fator. It is the same in 2010 as well.

Calculating $\mathbf{S_{max}}$ (maximum permissible spacing of lateral reinforcement) considering the demanded shear stress ***(Eqs. 5.8.2.7-1 and 5.8.2.7-2)*** will be done when the shear reinorcement, if needed, is designed

Following is evaluation of the $\boldsymbol{A_{cp}^2/p_c}$ *so that we address a cellular case,* ***per 5.8.2.1-5***

$$Val := \begin{cases} \dfrac{A_{cp}^2}{p_c} & \text{if } SecType = \text{"Solid"} \\ \text{otherwise} & \\ \quad \begin{cases} \dfrac{A_{cp}^2}{p_c} & \text{if } \left(\dfrac{A_{cp}^2}{p_c}\right) \le 2A_o \cdot b_v \\ 2A_o \cdot b_v & \text{otherwise} \end{cases} \end{cases}$$

(Equation 5.8.2.1-5)

Calculate Cracking Toresion (no segmental): ***(Equation 5.8.2.1-4)***

$$T_{cr} := \begin{cases} 0.125 \cdot \sqrt{f_c \cdot ksi} \cdot Val \cdot \sqrt{1 + \dfrac{f_{pc}}{0.125 \cdot \sqrt{f_c \cdot ksi}}} & \text{if } ConType = \text{"Normal"} \\ \text{otherwise} & \\ \quad \begin{cases} 0.125 \cdot \min\left(4.7 \cdot f_{ct}, \sqrt{f_c \cdot ksi}\right) \cdot Val \cdot \sqrt{1 + \dfrac{f_{pc}}{0.125 \cdot \min\left(4.7 \cdot f_{ct}, \sqrt{f_c \cdot ksi}\right)}} & \text{if } f_{ct\_specified} = \text{"Yes"} \\ \text{otherwise} & \\ \quad \begin{cases} 0.125 \cdot 0.75 \sqrt{f_c \cdot ksi} \cdot Val \cdot \sqrt{1 + \dfrac{f_{pc}}{0.125 \cdot 0.75 \sqrt{f_c \cdot ksi}}} & \text{if } ConType = \text{"AllLightweight"} \\ 0.125 \cdot 0.85 \sqrt{f_c \cdot ksi} \cdot Val \cdot \sqrt{1 + \dfrac{f_{pc}}{0.125 \cdot 0.85 \sqrt{f_c \cdot ksi}}} & \text{otherwise} \end{cases} \end{cases} \end{cases}$$

$T_{cr} = 1.889 \times 10^{11} \cdot kip \cdot in$

Check if the torsion can be ignored:

$$IgnoreTorsion := \begin{cases} \text{"Yes"} & \text{if } T_u \le 0.25 \cdot \phi_t \cdot T_{cr} \\ \text{"No"} & \text{otherwise} \end{cases}$$

(Equation 5.8.2.1-3)

$IgnoreTorsion = \text{"Yes"}$

Calculating the demanded equivalent shear force:

Note that here, since the equivalent shear can be used for checking the section, we have to use the **T_u** regardless of being less that 0.25 times the cracking torsion or not.

$$V_{u\_eq} := \begin{cases} \sqrt{V_u^2 + \left(\dfrac{0.9p_h \cdot T_u}{2 \cdot A_o}\right)^2} & \text{if } SecType = \text{"Solid"} \\ V_u + \dfrac{T_u \cdot d_s}{2 \cdot A_o} & \text{otherwise} \end{cases}$$

(Equation 5.8.2.1-6 and 5.8.2.1-7)

$V_{u\_eq} = 2.136 \times 10^3 \cdot kip$

This equivalent V_{u_eq} is just for checking the adequecy of the section, when needed, otherwise the shear and torsional steel need to be evaluated as needed and then added up (shear as per 5.8.3.3 and Torsion as per 5.8.3.6.2)

Calculating the shear strength provided by concrete, V_c:

$f_{po} := 0.7f_{pu}$

a parameter taken as modulus of elasticity of prestressing tendons multiplied by the lockedin difference in strain between the prestressing tendons and the surrounding concrete (ksi). For the usual levels of prestressing, a value of **0.7** ***f_{pu}*** will be appropriate for both pretensioned and post-tensioned members

*Calculating ε_s using equation **5.8.3.4.2-4***

IMPORTANT Note: Deduct a portion of the area of the bars and tendons terminated less than their development length from the section that you are designing for, with the same proportion as their lack of full length (Here, the development length of a bar can be evaluated, and then having the actual length, the area can be reduced proportional to the ratio of available length to development length.

Addressing the requirement to have **M_u** used not to be less than **$(V_u-V_p)d_v$**, we use **M_{u1}** as follows:

$$M_{u1} := \begin{cases} M_u & \text{if } |M_u| \ge \left|(V_u - V_p) \cdot d_v\right| \\ (V_u - V_p) \cdot d_v & \text{otherwise} \end{cases}$$

(Article 5.8.3.4.2)

$M_{u1} = 1.213 \times 10^4 \cdot kip \cdot in$

Note that 1) $\mathbf{A_s}$ and $\mathbf{A_{ps}}$ should be reduced proportionally if a lack of full development length to the section under design 2) If closer than $\mathbf{d_v}$ to the face of support, use ε_s at distance $\mathbf{d_v}$ to evaluate β and θ

$$\varepsilon_{s1} := \frac{\frac{|M_{u1}|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps} \cdot f_{po}}{E_s \cdot A_s + E_p \cdot A_{ps}}$$

(Equation 5.8.3.4.2-4)

$$\varepsilon_{s1} = -1.038 \times 10^{-3}$$

(Article 5.8.3.4.2) <u>Page 5-72 (2008), Page 5-71 (2010)</u>

$$\varepsilon_{s2} := \left| \begin{array}{ll} \dfrac{\frac{|M_{u1}|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps} \cdot f_{po}}{E_s \cdot A_s + E_p \cdot A_{ps} + E_c \cdot A_{ct}} & \text{if } \varepsilon_{s1} < 0.0 \\ \varepsilon_{s1} & \text{otherwise} \end{array} \right.$$

This and the follwing are evaluates ε_s, presisely and does not use zero.

$$\varepsilon_{s2} = -2.475 \times 10^{-4}$$

For pretensioned members, f_{po} can be taken as the stress in the strands when the concrete is cast around them, i.e., approximately equal to the jacking stress. For post-tensioned members, f_{po} can be conservatively taken as the average stress in the tendons when the posttensioning is completed.

$$\varepsilon_s := \left| \begin{array}{l} \varepsilon_s \leftarrow -0.0004 \ \text{ if } \varepsilon_{s2} < -4.0 \times 10^{-4} \\ \varepsilon_s \leftarrow 0.006 \ \text{ if } \varepsilon_{s2} > 0.006 \\ \varepsilon_{s2} \ \text{ otherwise} \end{array} \right.$$

Taking care of the last condition on evaluation of ε_s

$$\varepsilon_s = -2.475 \times 10^{-4}$$

(Article 5.8.3.4.2) <u>Page 5-72 to 74 (2008), Page 5-71 to 73 (2010)</u>

<u>NOTE</u>

For sections closer than $\mathbf{d_y}$ to the face of support the value for ε_s calculated at $\mathbf{d_y}$ from the face of support may be used to evaluate β and θ

If the axial tension is large enough to crack the flexural compression face of the section, the value calculated should be doubled.

$$s_x := \min(d_v, s_{CrackControl})$$

Reinforcement in each layer not less than $\mathbf{0.003b_v s_x}$

$$s_x = 2 \cdot in$$

$$s_{xe1} := s_x \cdot \frac{1.38}{a_g + 0.63in}$$

(Equation 5.8.3.4.2-5)

$$s_{xe} := \begin{cases} 12 & \text{if } s_{xe1} < 12 \\ \text{otherwise} & \begin{cases} 80 & \text{if } s_{xe1} > 80 \\ s_{xe1} & \text{otherwise} \end{cases} \end{cases}$$

$s_{xe} = 12$

$$\beta := \begin{cases} \dfrac{4.8}{1 + 750 \cdot \varepsilon_s} & \text{if HasMin} = \text{"Yes"} \\ \left(\dfrac{4.8}{1 + 750\varepsilon_s}\right) \cdot \left(\dfrac{51}{39 + s_{xe}}\right) & \text{otherwise} \end{cases}$$

(Equations 5.8.3.4.2-1 and 2)

$\beta = 5.894$

$$\theta := 29deg + 3500deg\,\varepsilon_s$$

(Equation 5.8.3.4.2-3) Here we use deg to get the angle in deg for further calculations

$\theta = 28.134 \cdot deg$

$$V_c := 0.0316 \cdot \beta \cdot \sqrt{f_c \cdot ksi} \cdot b_v \cdot d_v$$

(Equation 5.8.3.3-3)

$V_c = 129.831 \cdot kip$

Designing the transverse reinforcement for **SHEAR** (Note that the A_v/s will be evaluated. Then when we have Torsion as well, we can add them appropriately depending on the type of section)

Check if we need shear reinforcement per 5.8.2.4-1

$$NeedShear := \begin{cases} \text{"Yes"} & \text{if } V_u > 0.5 \cdot \phi_v \cdot (V_c + V_p) \\ \text{"No"} & \text{otherwise} \end{cases}$$

(Equation 5.8.2.4-1)

NeedShear = "Yes"

Here we need to check for mininum reinforcement as per **5.8.2.5** (AASHTO 2010)

$A_v = 0.62 \cdot in^2$

$$V_n := \min\left(\frac{V_u}{\phi_v}, 0.25f_c \cdot b_v \cdot d_v + V_p\right) \qquad V_n = 234.444 \cdot kip$$

Based on Eq. 5.8.3.3-1 and 2

$$V_s := V_n - V_c - V_p$$

$V_s = 104.613 \cdot kip$

Here S_{min} actually means the "s" based on the minimum requirement, otherwise, this is the limit for spacing and the specing should be less or at most equal to this

$$s_{max} := \frac{A_v \cdot f_y}{0.0316 \cdot b_v \cdot \sqrt{f_c \cdot ksi}} \qquad s_{max} = 73.576 \cdot in$$

Eq. 5.8.2.5-1

$$s_{req} := \text{if}\left[V_s \le 0, s_{max}, \frac{A_v \cdot f_y \cdot d_v \cdot (\cot(\theta) + \cot(\alpha)) \cdot \sin(\alpha)}{V_s}\right]$$

(Equation 5.8.3.3-4

$$s_{req} = 28.972 \cdot in$$

$$s := \min(s_{max}, s_{req})$$

$$s = 28.972 \cdot in$$

Based on **5.8.2.7** and **5.8.2.9,** we need to evaluate the final spacing by finding the othe maximum limits as follows:

$$s_{max2} := \text{if}(v_u < 0.125 \cdot f_c, \min(0.8 \cdot d_v, 24in), \min(0.4 \cdot d_v, 12in))$$

(Eqs. 5.8.2.7-1 and 5.8.2.7-2)

$$s_{max2} = 12 \cdot in$$

The spacing of the transversal reinforcement is as follows:

$$s_{actual} := \min(s, s_{max2})$$

$$\boxed{s_{actual} = 12 \cdot in}$$

To address 5.8.3.3-2 , The upper limit of V_n, given by Eq. 2, is intended to ensure that the concrete in the web of the beam will not crush prior to yield of the transverse reinforcement.

$$0.25 \cdot f_c \cdot b_v \cdot d_v = 348.526 \cdot kip$$

Note that $\mathbf{V_u/\varphi}$ **-Vp** should be less than $\mathbf{0.25\ f_c b_v d_v}$ otherwise section is not enough and concrete crushes due to local shear demand.

Check if the section is enough:

$$SecEnough := \begin{cases} \text{"Yes"} & \text{if } \dfrac{V_u}{\phi_v} - V_p < 0.25 \cdot f_c \cdot b_v \cdot d_v \\ \text{"STOP and Change Section"} & \text{otherwise} \end{cases}$$

(Equation 5.8.3.3-2)

$$\boxed{SecEnough = \text{"Yes"}}$$

Also we need to specify where we have to switch to **strut-and-tie model**

Note : $\mathbf{V_p}$ is the vertical component of the prestressing force (as has already been evaluated by using the angle $\boldsymbol{\alpha_p}$)

Finding the ratio of ***avss=A$_v$/s*** *for shear:*

$$avss := \frac{A_v}{s_{actual}}$$

$$\boxed{avss = 0.052 \cdot in}$$

Design for Torsional lateral reinforcement:

Note that if we have a torsion that cannot be ignored, we design the lateral reinforcement for that and we call that avst. later we add the shear and torsion reinforcement properly. Corresponding longitudinal steel will be calculated as well.

$$avst1 := \begin{cases} \dfrac{\frac{T_u}{\phi_t}}{2 \cdot A_o \cdot f_y \cdot \cot(\theta)} & \text{if } IgnoreTorsion = \text{"No"} \\ 0.0 & \text{otherwise} \end{cases}$$

(Equation 5.8.3.6.2-1)

$avst1 = 0 \cdot in$ This is for one leg

$avst := 2 \cdot avst1$ $avst = 0 \cdot in$

$avs := avss + avst$ This is the total lateral reinforcement needed for shear and torsion

$$s_{ShearTorsion} := \frac{A_v}{avs}$$

$\boxed{s_{ShearTorsion} = 12 \cdot in}$

Design for Torsional longitudinal reinforcement: (Article 5.8.3.6.3 Longitudinal Reinforcement) [Also consider Eq. 5.8.3.5-1]

$$tmpVal := \frac{|M_{u1}|}{\phi_f \cdot d_v} + \frac{0.5 \cdot N_u}{\phi_c} + \cot(\theta) \cdot \sqrt{\left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5 \cdot V_s\right)^2 + \left(\frac{0.45 \cdot p_h \cdot T_u}{2 \cdot A_o \cdot \phi_t}\right)^2}$$

$tmpVal = 2.527 \times 10^3 \cdot kip$

$$A_l := \begin{cases} 0.0 & \text{if } IgnoreTorsion = \text{"Yes"} \\ \text{otherwise} & \\ \quad \begin{cases} \text{if } SecType = \text{"Solid"} & \\ \quad \begin{cases} 0.0 & \text{if } A_{ps} \cdot f_{ps} + A_s \cdot f_y \ge tmpVal \\ \dfrac{tmpVal - A_{ps} \cdot f_{ps}}{f_y} - A_s & \text{otherwise} \end{cases} & \\ \dfrac{T_u \cdot p_h}{\phi_t \cdot 2 \cdot A_o \cdot f_y} & \text{otherwise} \end{cases} \end{cases}$$

Note that $\mathbf{A_l}$ is the additional steel needed due to torsion

Eq. 5.8.3.6.3-1

Eq. 5.8.3.6.3-2

$A_l = 0 \cdot in^2$

$A_{s\_total} := A_s + A_l$ Note that distribution of $\mathbf{A_l}$ should be evenly around the section

$\boxed{A_{s\_total} = 12.7 \cdot in^2}$

Example-3

Solution using the Developed MathCAD Design Program

Example 3: (No pre-stressing steel)

Design a precast, non-pre-stressed normal-weight concrete spandrel beam for combined shear and torsion. Roof members are simply supported on spandrel ledge. Spandrel beams are connected to columns to transfer torsion. Continuity between spandrel beams is not provided.

Design Criteria:

Live load = 30 lb/ft^2

Dead load = 90 lb/ft^2 (double tee + topping + insulation + roofing)

f'_c = 5000 psi (w_c = 150 pcf)

f_y = 60,000 psi

Roof members are 10 ft wide double tee units, 30 in. deep with 2 in. topping. Design of these units is not included in this design example. For lateral support, alternate ends of roof members are fixed to supporting beams.

Figure C. 6 Partial plan of precast roof system

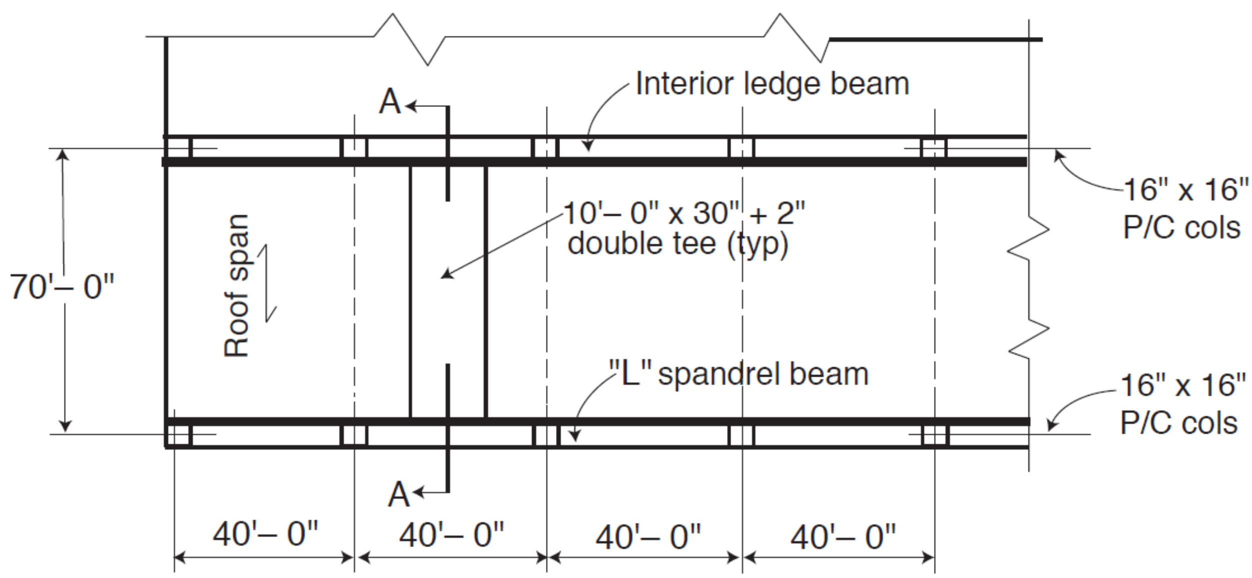
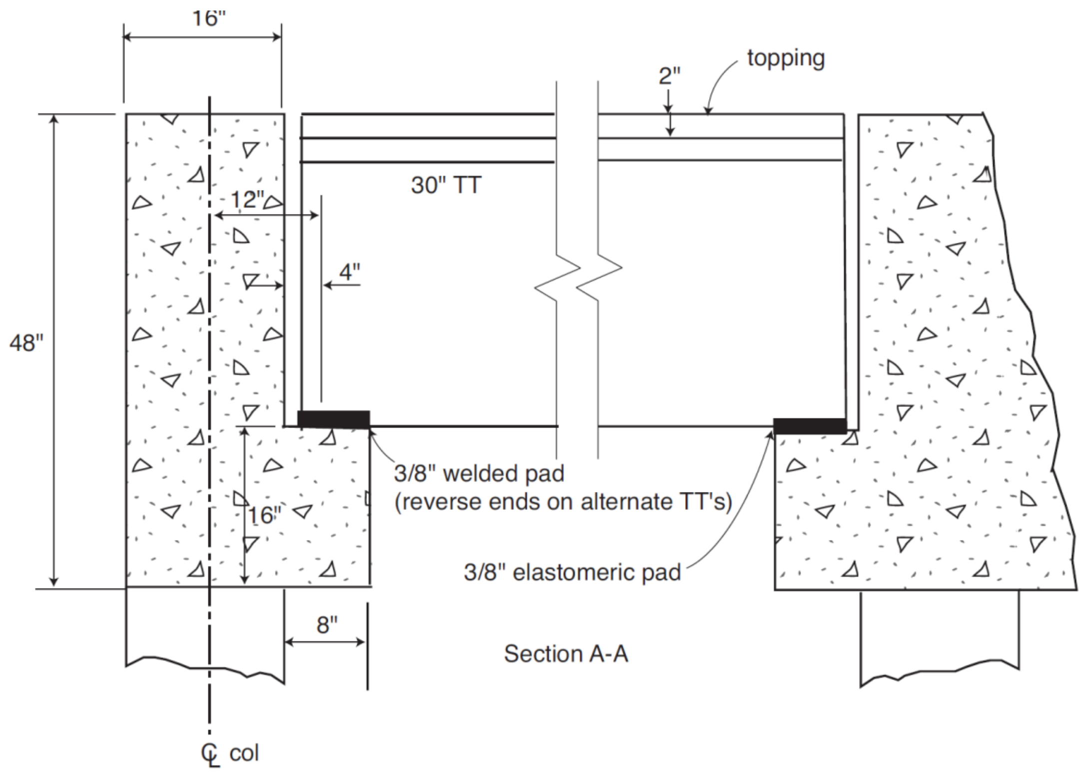


Figure C. 7 Spandrel beam cross section



Spandrel beam height	=48 in.
f'$_c$ of the beam concrete	=5000 psi
Effective width (b_{eff})	=16 in.
Width of the web	=16 in.
Area of the regular (non-pre-stressing)	
steel on the section: A_s	=3.0 in^2

Preparing the input data:

Before calculating the required input data, it should be noted that the values of the input data for various parameters depend on factors that are beyond the scope of this project. So, it is assumed that for each case, the user, will calculate the necessary values and provides them as the input data. The "designed" shear and torsion reinforcement depends on the accuracy of the input data.

On the other hand, depending of the case (beam geometry, demanded load level, reinforcement arrangement, and location of the section) the Strut-and-Tie Model should be used in lieu of the process implemented in this program.

Section:

The section located at a distance equal to (d=45.5") of the girder length (span) is selected for design in this example. The distance between the supports, center to center, is equal to 40 feet. So, the distance between the support and the center of the span, is 20'.

The demanded values at the critical section, located at 45.5" from the support are:

M_u=451.7 kip-ft (at the critical section), [M_u=1316 kip-ft (at the center)]

V_u=127.2 kips (at the critical section)

N_u=0.0 kips

T_u=1348.8 kip-in (at the critical section)

Calculate the tensile flexural steel:

$$M_u = 0.85 \times f_c' \times b \times a \times (d - \frac{a}{2})$$

$$1316 \times 12 = 0.85 \times 5 \times 16 \times a \times (45.5 - \frac{a}{2}) \Rightarrow a = 5.428 \text{in.}$$

$$A_s = \frac{0.85 \times f_c' \times b \times a}{f_y} \Rightarrow A_s = \frac{0.85 \times 5 \times 16 \times 5.428}{60}$$

A_s=6.15 in^2 (Area of regular steel needed)

A_s=7.8 in^2 (Area of regular steel provided)

The length of the outside perimeter of the concrete section

P_c=2 (16 + 48) + 2 (8) = 144 in.

P_c=144 in

Perimeter of the centerline of the closed transverse torsion reinforcement (in.)

P_h=2×(76 + 5)

P_h =162 in

Total area enclosed by outside perimeter of concrete cross-section (in2):

A_{cp}= (16) (48) + (16) (8) = 768 + 128 = 896 in^2.

Width of web adjusted for the presence of ducts (in.); width of the interface (in.)

b_v=16 in.

b_w=16 in.

Area enclosed by the shear flow path, including any area of holes therein (in2)

A_{oh}=(13) (45) + (8) (13) = 689 in^2.

A_o= 0.85 (689) = 585.6 in^2.

A_o=585.6 in^2

Note: For the same section and demanded values, the spacing is 5 in. using ACI code.

LRFD Shear and Torsion, using Sectional Model
(SECTION 5, Interim 2008, Revised based on 2010)

IMPORTANT to Note: changing the unit system will lead to inaccuracy of the results, since most of the factors are based on teh US system

Example 3 (Please see the WORD File for detailed information)

1) Please see the ***WORD file for graphical definition of the parameters***.
2) The ***PDF file with links*** can help you with jumping to a desired location
3) Some parameters such as $\mathbf{A_0}$, $\mathbf{P_h}$, etc., need to be evaluated beforehand due to possible irregularities associated with various sections. Also, demanded loads are closely related to the bridge configuration and related load combination. This file, ONLY addresses shear/torsion part of design.

If the **distance between zero-shear point and the face of support is less than 2d** (d is the distance between the compression side and centriod of tensile reniforcement) OR a **concentrated load causing more than 1/2 of the shear at a support is closer than 2d from the face of support**, this method is not applicable. You need to use **Stut-and-Tie Model** (Section 5.6.3).
IN GENATRAL: Where the "plane sections assumption of felxural theory" is NOT valid, strut-and-Tie Model should be used.

Enter your data in the highlighted (yellow) fields

INPUT DATA:

Section Geometry:

$h := 48in$ Overall section depth (in)

$h_{nc} := 48in$ Depth of the non-composite section (in)

$A_{cp} := 896in^2$ total area enclosed by outside perimeter of concrete cross-section (in^2). Having the section, this value needs to be calculated

$p_c := 114in$ the length of the outside perimeter of the concrete section (in.). Having the section, this value needs to be calculated

$A_o := 585in^2$ area enclosed by the shear flow path, including any area of holes therein (in^2). Having the section, this value needs to be calculated

$p_h := 162in$ perimeter of the centerline of the closed transverse torsion reinforcement (in.). Having the section, this value needs to be calculated

$b_v := 16in$ width of web adjusted for the presence of ducts (in.); width of the interface (in.)

OR: effective web width taken as the minimum web width, measured parallel to the neutral axis, between the resultants of the tensile and compressive forces due to flexure, or for circular sections, the diameter of the section, modified for the presence of ducts where applicable (in.)

$b_w := 16in$ width of web (in.)

$d_s := 45.5in$ distance from extreme compression fiber to the centroid of the nonprestressed tensile reinforcement (in.)

$d_p := 0.0in$ distance from extreme compression fiber to the centroid of the prestressing tendons (in.)

SecType := "Solid" Type of the cross section to design

$d_t := 45.5in$ distance from extreme compression fiber to the centroid of the extreme tensile reinforcement on teh section (it is different from d_s)

$c_{comp} := 6.785in$ Depth of the compression zone (you need to calculate it depending on the geometry of the section, amount of the reinforcement on tensile and compression side and material properties)

The section will be specified depending on the input data for regular and/or prestressing steel to check if the girder is nonprestressed, prestressed, or partially prestressed.

Reinforcement:

$A_s := 7.8in^2$ area of the non-prestressing tensile reinforcement on the section

$E_s := 29000ksi$ modulus of elasticity of reinforcing bars (ksi)

$A_{ps} := 0.0in^2$ area of the pre-stressing steel on tensile side of the member as in Fig 1 (Word) file or PDF file page 5-73

$A_b := 0.2in^2$ area of the cross section of the lateral reinforcement (here #5, change accordingly)

$n_{leg} := 2$ Number of the legs of the lateral reinforcement

$A_v := A_b \cdot n_{leg}$ Area of the shear reinforcement (used for torsion as well)

HasMin := "yes" Put "Yes" if section Contains minimum or more lateral reinforcement

Material Properties:

ConType := "Normal" put one of the: "Normal", "AllLightweight", "SandLightweight"

$f_c := 5ksi$ compressive stregth of concrete

$f_y := 60ksi$ yield strength of the non-prestressing tensile steel

$f_{py} := 300ksi$ yield strength of prestressing steel (ksi)

$f_{pc} := 0.0ksi$ (**can be calculated**) compressive stress in concrete after prestress losses have occurred either at the centroid of the cross-section resisting transient loads or at the junction of the web and flange where the centroid lies in the flange (ksi)

$f_{pu} := 270ksi$ specified tensile strength of prestressing steel (ksi)

$f_{ltl} := 32ksi$ Assumed value for long-term losses of prestressing steel (ksi)

$f_{pi} := 0.7 \cdot f_{pu}$ You may change the value if there are explicit information

$\boxed{f_{pi} = 189 \cdot ksi}$

$E_p := 28600ksi$ Modulus of elasticity of prestressing tendons (ksi)

$f_{ct\_specified} :=$ "No"

$f_{ct} := 0.2ksi$ (If you have a lightweight concrete, put "Yes" if f_{ct} available and enter the value, otherwise, put "No" and ignore f_{ct}) average splitting tensile strength of lightweight concrete

$\alpha_p := 9deg$ This is the angle between the prestressing force and postive direction of shear force (demand).

$a_g := 0.25in$ maximum aggregate size

$\alpha := 90deg$ angle of inclination of transverse reinforcement to longitudinal axis (°)

$K_1 := 1.0$ correction factor for source of aggregate to be taken as 1.0 unless determined by physical test, and as approved by the authority of jurisdiction (see **5.4.2.4**)

$w_c := 0.150 \frac{kip}{ft^3}$ unit weight of concrete (kcf); refer to **Table 3.5.1-1** or **Article C5.4.2.4**

$A_{ct} := (0.0in)^2$ A_{ct} is the area of concrete on the flexural tension side of the member as shown in Figure 1 (in.^{2}). It is calculated as the area bellow the centroid of the non-composite section. It can also be calculated as the area of the non-composite section, divided by the height of the non-composite section, then multiplied by half of the overall height of the section.

$s_{CrackControl} := 2in$ maximum distance between layers of longitudinal crack control reinforcement, where the area of the reinforcement in each layer is not less than **0.003*b*$_v$*s*$_x$**, as shown in Figure 3 (in.)

*Demanded **Shear** and **Torsion, Moment, Axial Force**:*

The induced internal shear and torsion in the section by the applied factored loads

$V_u := 127.2kip$ $T_u := 1348.8kip \cdot in$ $M_u := 5420.4kip \cdot in$ $N_u := 0kip$ positive is tensile

Calculations and Design:

average stress in prestressing steel at the time for which the nominal resistance of member is required (ksi)

$f_{ps} := f_{pi} - f_{ltl}$ $\quad f_{ps} = 157 \cdot ksi$

V$_p$ is the component in the direction of the applied shear of the effective prestressing force; positive if resisting the applied shear (kip) (C5.8.2.3) (can be calculated if the angle is known)

$$V_p := A_{ps} \cdot f_{ps} \cdot \cos(\alpha_p)$$

$$V_p = 0 \cdot kip$$

$$E_c := 33000 K_1 \cdot w_c^{1.5} \cdot \left(\frac{ft^3}{kip}\right)^{1.5} \cdot \sqrt{f_c \cdot ksi}$$

$E_c = 4.287 \times 10^3 \cdot ksi$ Modulus of elasticity of concrete (ksi) (**5.4.2.4**)

$$PPR := \frac{A_{ps} \cdot f_{py}}{A_{ps} \cdot f_{py} + A_s \cdot f_y}$$

$$PreStress := \begin{cases} \text{"Prestressed"} & \text{if } PPR = 1.0 \\ \text{otherwise} & \\ \quad \begin{cases} \text{"Nonprestressed"} & \text{if } PPR = 0.0 \\ \text{"Partial prestressed"} & \text{otherwise} \end{cases} \end{cases} \quad = \text{"Nonprestressed"}$$

The string variable "Prestress" shows the prestressing condition of the section.

Evaluation of stregth reduction factors ϕ, for flexure, shear, and torsion:

The following algorithm addresses **Article 4.5.4.2.1**, and **Equations 5.5.4.2.1-1 and 5.5.4.2.1-2**

(See: Article 5.5.4.2), p 5-25

$\varepsilon_y := \frac{f_y}{E_s} \qquad \varepsilon_y = 2.069 \times 10^{-3}$

$$ebsY := \begin{cases} \varepsilon_y & \text{if } PreStress = \text{"Nonprestressed"} \\ 0.002 & \text{otherwise} \end{cases}$$

$ebsY = 2.069 \times 10^{-3}$

Evaluation of the flexural strength reduction factor:

$phiT := 0.9 + 0.1 \cdot PPR \qquad phiT = 0.9$

This temporal variable is for implementation of prestressing condition (normal, partial, or full)

C5.5.4.2.1 states that "For sections subjected to axial load with flexure, factored resistances are determined by multiplying both P_n and M_n by the appropriate single value of ϕ

$$\phi_f := \begin{cases} phiT & \text{if } c_{comp} \le 0.375 \cdot d_t \\ \text{otherwise} & \begin{cases} phiT & \text{if } c_{comp} \ge \dfrac{0.003}{0.003 + ebsY} \cdot d_t \\ phiT + \dfrac{0.75 - phiT}{\dfrac{0.003}{0.003 + ebsY} \cdot d_t - 0.375 \cdot d_t} \cdot \left(c_{comp} - 0.375 \cdot d_t\right) & \text{otherwise} \end{cases} \end{cases}$$

$\phi_f = 0.9 \qquad \phi_c := \phi_f$ As per C5.5.4.2.1

Evaluation of the shear and torsional strength reduction factors based on **5.5.4.2**

$$\phi_v := \begin{cases} 0.9 & \text{if } ConType = \text{"Normal"} \\ 0.7 & \text{otherwise} \end{cases} \qquad \phi_t := \begin{cases} 0.9 & \text{if } ConType = \text{"Normal"} \\ 0.7 & \text{otherwise} \end{cases}$$

$\boxed{\phi_f = 0.9} \quad \boxed{\phi_v = 0.9} \quad \boxed{\phi_t = 0.9} \quad \boxed{\phi_c = 0.9}$

$$d_e := \frac{A_s \cdot f_y \cdot d_s + A_{ps} \cdot f_{ps} \cdot d_p}{A_s \cdot f_y + A_{ps} \cdot f_{ps}}$$

(Equation 5.8.2.9-2) AASHTO 2010

Calculating the value of d_v:

effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure; it need not be taken to be less than the greater of 0.9 d_e or 0.72*h* (in.)

To calculate d_v, using the commentary C5.8.2.9-1, **assuming M_n=M_u/φ**, we have:

$$d_{v1} := \frac{\left(\dfrac{M_u}{\phi_f}\right)}{A_s \cdot f_y + A_{ps} \cdot f_{ps}}$$

(C 5.8.2.9-1)

$d_{v1} = 12.869 \cdot in$

$d_v := \max(0.9 d_e, 0.72 \cdot h, d_{v1})$ **(Art. 5.8.2.9)**

$d_v = 40.95 \cdot in$

Calculate the Shear Stress:

$$v_u := \frac{|V_u - \phi_v \cdot V_p|}{\phi_v \cdot b_v \cdot d_v}$$

(Eq 5.8.2.9-1)

AASHTO 2010

Induced demanded shear stress

$v_u = 0.216 \cdot ksi$

Note that this should be v_n (equivalent to v_u/ϕ) and not v_u or we need to remove the strength reduction fator. It is the same in 2010 as well.

Calculating $\mathbf{S_{max}}$ (maximum permissible spacing of lateral reinforcement) considering the demanded shear stress ***(Eqs. 5.8.2.7-1 and 5.8.2.7-2)*** will be done when the shear reinorcement, if needed, is designed

Following is evaluation of the $\mathbf{A_{cp}^2/p_c}$ *so that we address a cellular case,* ***per 5.8.2.1-5***

$$Val := \begin{cases} \dfrac{A_{cp}^2}{p_c} & \text{if } SecType = \text{"Solid"} \\ \text{otherwise} & \begin{cases} \dfrac{A_{cp}^2}{p_c} & \text{if } \left(\dfrac{A_{cp}^2}{p_c}\right) \le 2A_o \cdot b_v \\ 2A_o \cdot b_v & \text{otherwise} \end{cases} \end{cases}$$

(Equation 5.8.2.1-5)

Calculate Cracking Toresion (no segmental): ***(Equation 5.8.2.1-4)***

$$T_{cr} := \begin{cases} 0.125 \cdot \sqrt{f_c \cdot ksi} \cdot Val \cdot \sqrt{1 + \dfrac{f_{pc}}{0.125 \cdot \sqrt{f_c \cdot ksi}}} & \text{if } ConType = \text{"Normal"} \\ \text{otherwise} & \begin{cases} 0.125 \cdot \min(4.7 \cdot f_{ct}, \sqrt{f_c \cdot ksi}) \cdot Val \cdot \sqrt{1 + \dfrac{f_{pc}}{0.125 \cdot \min(4.7 \cdot f_{ct}, \sqrt{f_c \cdot ksi})}} & \text{if } f_{ct\_specified} = \text{"Yes"} \\ \text{otherwise} & \begin{cases} 0.125 \cdot 0.75 \sqrt{f_c \cdot ksi} \cdot Val \cdot \sqrt{1 + \dfrac{f_{pc}}{0.125 \cdot 0.75 \sqrt{f_c \cdot ksi}}} & \text{if } ConType = \text{"AllLightweight"} \\ 0.125 \cdot 0.85 \sqrt{f_c \cdot ksi} \cdot Val \cdot \sqrt{1 + \dfrac{f_{pc}}{0.125 \cdot 0.85 \sqrt{f_c \cdot ksi}}} & \text{otherwise} \end{cases} \end{cases} \end{cases}$$

$T_{cr} = 1.968 \times 10^3 \cdot kip \cdot in$

Check if the torsion can be ignored:

$$IgnoreTorsion := \begin{cases} \text{"Yes"} & \text{if } T_u \le 0.25 \cdot \phi_t \cdot T_{cr} \\ \text{"No"} & \text{otherwise} \end{cases}$$

(Equation 5.8.2.1-3)

$IgnoreTorsion = \text{"No"}$

Calculating the demanded equivalent shear force:

Note that here, since the equivalent shear can be used for checking the section, we have to use the $\mathbf{T_u}$ regardless of being less that 0.25 times the cracking torsion or not.

$$V_{u\_eq} := \begin{cases} \sqrt{V_u^{\,2} + \left(\dfrac{0.9p_h \cdot T_u}{2 \cdot A_o}\right)^2} & \text{if } SecType = \text{"Solid"} \\ V_u + \dfrac{T_u \cdot d_s}{2 \cdot A_o} & \text{otherwise} \end{cases}$$

(Equation 5.8.2.1-6 and 5.8.2.1-7)

$V_{u\_eq} = 210.787 \cdot kip$

This equivalent V_{u_eq} is just for checking the adequecy of the section, when needed, otherwise the shear and torsional steel need to be evaluated as needed and then added up (shear as per 5.8.3.3 and Torsion as per 5.8.3.6.2)

Calculating the shear strength provided by concrete, V_c:

$f_{po} := 0.7 f_{pu}$

a parameter taken as modulus of elasticity of prestressing tendons multiplied by the lockedin difference in strain between the prestressing tendons and the surrounding concrete (ksi). For the usual levels of prestressing, a value of **0.7** f_{pu} will be appropriate for both pretensioned and post-tensioned members

Calculating ε_s using equation ***5.8.3.4.2-4***

IMPORTANT Note: Deduct a portion of the area of the bars and tendons terminated less than their development length from the section that you are designing for, with the same proportion as their lack of full length (Here, the development length of a bar can be evaluated, and then having the actual length, the area can be reduced proportional to the ratio of available length to development length.

Addressing the requirement to have $\mathbf{M_u}$ used not to be less than $\mathbf{(V_u - V_p)d_v}$, we use $\mathbf{M_{u1}}$ as follows:

$$M_{u1} := \begin{cases} M_u & \text{if } |M_u| \ge \left|\left(V_u - V_p\right) \cdot d_v\right| \\ \left(V_u - V_p\right) \cdot d_v & \text{otherwise} \end{cases}$$

(Article 5.8.3.4.2)

$M_{u1} = 5.42 \times 10^3 \cdot kip \cdot in$

Note that 1) $\mathbf{A_s}$ and $\mathbf{A_{ps}}$ should be reduced proportionally if a lack of full development length to the section under design 2) If closer than $\mathbf{d_v}$ to the face of support, use ε_s at distance $\mathbf{d_v}$ to evaluate β and θ

$$\varepsilon_{s1} := \frac{\dfrac{|M_{u1}|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps} \cdot f_{po}}{E_s \cdot A_s + E_p \cdot A_{ps}}$$

(Equation 5.8.3.4.2-4)

$$\varepsilon_{s1} = 1.148 \times 10^{-3}$$

(Article 5.8.3.4.2) <u>Page 5-72 (2008), Page 5-71 (2010)</u>

$$\varepsilon_{s2} := \begin{cases} \dfrac{\dfrac{|M_{u1}|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps} \cdot f_{po}}{E_s \cdot A_s + E_p \cdot A_{ps} + E_c \cdot A_{ct}} & \text{if } \varepsilon_{s1} < 0.0 \\ \varepsilon_{s1} & \text{otherwise} \end{cases}$$

This and the follwing are evaluates ε_s, presisely and does not use zero.

$$\varepsilon_{s2} = 1.148 \times 10^{-3}$$

For pretensioned members, $\boldsymbol{f_{po}}$ can be taken as the stress in the strands when the concrete is cast around them, i.e., approximately equal to the jacking stress. For post-tensioned members, $\boldsymbol{f_{po}}$ can be conservatively taken as the average stress in the tendons when the posttensioning is completed.

$$\varepsilon_s := \begin{cases} \varepsilon_s \leftarrow -0.0004 & \text{if } \varepsilon_{s2} < -4.0 \times 10^{-4} \\ \varepsilon_s \leftarrow 0.006 & \text{if } \varepsilon_{s2} > 0.006 \\ \varepsilon_{s2} & \text{otherwise} \end{cases}$$

Taking care of the last condition on evaluation of ε_s

$$\varepsilon_s = 1.148 \times 10^{-3}$$

(Article 5.8.3.4.2) <u>Page 5-72 to 74 (2008), Page 5-71 to 73 (2010)</u>

<u>NOTE</u>

For sections closer than $\mathbf{d_y}$ to the face of support the value for ε_s calculated at $\mathbf{d_y}$ from the face of support may be used to evaluate β and θ

If the axial tension is large enough to crack the flexural compression face of the section, the value calculated should be doubled.

$$s_x := \min(d_v, s_{CrackControl})$$

Reinforcement in each layer not less than $\boldsymbol{0.003 b_v s_x}$

$$s_x = 2 \cdot in$$

$$s_{xe1} := s_x \cdot \frac{1.38}{a_g + 0.63in}$$

(Equation 5.8.3.4.2-5)

$$s_{xe} := \begin{cases} 12 & \text{if } s_{xe1} < 12 \\ \text{otherwise} & \begin{cases} 80 & \text{if } s_{xe1} > 80 \\ s_{xe1} & \text{otherwise} \end{cases} \end{cases}$$

$s_{xe} = 12$

$$\beta := \begin{cases} \dfrac{4.8}{1 + 750 \cdot \varepsilon_s} & \text{if HasMin} = \text{"Yes"} \\ \left(\dfrac{4.8}{1 + 750\varepsilon_s}\right) \cdot \left(\dfrac{51}{39 + s_{xe}}\right) & \text{otherwise} \end{cases}$$

(Equations 5.8.3.4.2-1 and 2)

$\beta = 2.58$

$$\theta := 29deg + 3500deg\, \varepsilon_s$$

(Equation 5.8.3.4.2-3) Here we use deg to get the angle in deg for further calculations

$\theta = 33.016 \cdot deg$

$$V_c := 0.0316 \cdot \beta \cdot \sqrt{f_c \cdot ksi} \cdot b_v \cdot d_v$$

(Equation 5.8.3.3-3)

$V_c = 119.434 \cdot kip$

Designing the transverse reinforcement for <u>**SHEAR**</u> (Note that the $\mathbf{A_v/s}$ will be evaluated. Then when we have Torsion as well, we can add them appropriately depending on the type of section)

<u>*Check if we need shear reinforcement per 5.8.2.4-1*</u>

$$\text{NeedShear} := \begin{cases} \text{"Yes"} & \text{if } V_u > 0.5 \cdot \phi_v \cdot (V_c + V_p) \\ \text{"No"} & \text{otherwise} \end{cases}$$

(Equation 5.8.2.4-1)

NeedShear = "Yes"

Here we need to check for mininum reinforcement as per **5.8.2.5** (AASHTO 2010)

$A_v = 0.4 \cdot in^2$

$$V_n := \min\left(\frac{V_u}{\phi_v}, 0.25 f_c \cdot b_v \cdot d_v + V_p\right) \qquad V_n = 141.333 \cdot kip$$

Based on Eq. 5.8.3.3-1 and 2

$$V_s := V_n - V_c - V_p$$

$V_s = 21.9 \cdot kip$

Here S_{min} actually means the "s" based on the minimum requirement, otherwise, this is the limit for spacing and the specing should be less or at most equal to this

$$s_{max} := \frac{A_v \cdot f_y}{0.0316 \cdot b_v \cdot \sqrt{f_c \cdot ksi}} \qquad s_{max} = 21.228 \cdot in$$

Eq. 5.8.2.5-1

$$s_{req} := \text{if}\left[V_s \le 0, s_{max}, \frac{A_v \cdot f_y \cdot d_v \cdot (\cot(\theta) + \cot(\alpha)) \cdot \sin(\alpha)}{V_s}\right]$$

(Equation 5.8.3.3-4

$s_{req} = 69.062 \cdot in$

$s := \min(s_{max}, s_{req})$

$s = 21.228 \cdot in$

Based on **5.8.2.7** and **5.8.2.9,** we need to evaluate the final spacing by finding the othe maximum limits as follows:

$s_{max2} := \text{if}\left(v_u < 0.125 \cdot f_c, \min(0.8 \cdot d_v, 24in), \min(0.4 \cdot d_v, 12in)\right)$ ***(Eqs. 5.8.2.7-1 and 5.8.2.7-2)***

$s_{max2} = 24 \cdot in$

The spacing of the transversal reinforcement is as follows:

$s_{actual} := \min(s, s_{max2})$

$\boxed{s_{actual} = 21.228 \cdot in}$

To address 5.8.3.3-2 , The upper limit of V_n, given by Eq. 2, is intended to ensure that the concrete in the web of the beam will not crush prior to yield of the transverse reinforcement.

$0.25 \cdot f_c \cdot b_v \cdot d_v = 819 \cdot kip$

Note that $\mathbf{V_u/\varphi}$ **-Vp** should be less than $\mathbf{0.25\ f_c b_v d_v}$ otherwise section is not enough and concrete crushes due to local shear demand.

Check if the section is enough:

$$SecEnough := \begin{cases} \text{"Yes"} & \text{if } \dfrac{V_u}{\phi_v} - V_p < 0.25 \cdot f_c \cdot b_v \cdot d_v \\ \text{"STOP and Change Section"} & \text{otherwise} \end{cases}$$

(Equation 5.8.3.3-2)

$\boxed{SecEnough = \text{"Yes"}}$

Also we need to specify where we have to switch to **strut-and-tie model**

Note : $\mathbf{V_p}$ is the vertical component of the prestressing force (as has already been evaluated by using the angle $\boldsymbol{\alpha_p}$)

Finding the ratio of ***avss=A$_v$/s*** *for shear:*

$$avss := \frac{A_v}{s_{actual}}$$

$\boxed{avss = 0.019 \cdot in}$

Design for Torsional lateral reinforcement:

Note that if we have a torsion that cannot be ignored, we design the lateral reinforcement for that and we call that avst. later we add the shear and torsion reinforcement properly. Corresponding longitudinal steel will be calculated as well.

$$avst1 := \begin{cases} \dfrac{\dfrac{T_u}{\phi_t}}{2 \cdot A_o \cdot f_y \cdot \cot(\theta)} & \text{if IgnoreTorsion} = \text{"No"} \\ 0.0 & \text{otherwise} \end{cases}$$

(Equation 5.8.3.6.2-1)

$avst1 = 0.014 \cdot in$ This is for one leg

$avst := 2 \cdot avst1$ $\quad avst = 0.028 \cdot in$

$avs := avss + avst$ This is the total lateral reinforcement needed for shear and torsion

$$s_{ShearTorsion} := \text{floor}\left(\frac{A_v}{avs \cdot in}\right) \cdot in$$

$s_{ShearTorsion} = 8 \cdot in$

Design for Torsional longitudinal reinforcement: (Article 5.8.3.6.3 Longitudinal Reinforcement) [Also consider Eq. 5.8.3.5-1]

$$tmpVal := \frac{|M_{u1}|}{\phi_f \cdot d_v} + \frac{0.5 \cdot N_u}{\phi_c} + \cot(\theta) \cdot \sqrt{\left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5 \cdot V_s\right)^2 + \left(\frac{0.45 \cdot p_h \cdot T_u}{2 \cdot A_o \cdot \phi_t}\right)^2}$$

$tmpVal = 393.872 \cdot kip$

$$A_l := \begin{cases} 0.0 & \text{if IgnoreTorsion} = \text{"Yes"} \\ \text{otherwise} & \\ \quad \begin{cases} \text{if SecType} = \text{"Solid"} & \\ \quad \begin{cases} 0.0 & \text{if } A_{ps} \cdot f_{ps} + A_s \cdot f_y \ge tmpVal \\ \dfrac{tmpVal - A_{ps} \cdot f_{ps}}{f_y} - A_s & \text{otherwise} \end{cases} & \\ \dfrac{T_u \cdot p_h}{\phi_t \cdot 2 \cdot A_o \cdot f_y} & \text{otherwise} \end{cases} \end{cases}$$

Note that $\mathbf{A_l}$ is the additional steel needed due to torsion

Eq. 5.8.3.6.3-1

Eq. 5.8.3.6.3-2

$A_l = 0 \cdot in^2$ $\quad A_{s\_total} := A_s + A_l$

$A_{s\_total} = 7.8 \cdot in^2$

Note that distribution of $\mathbf{A_l}$ should be evenly around the section