ANALYSIS OF AN ONLINE PLACEMENT EXAM FOR CALCULUS

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Abstract

An online mathematics placement exam was administered to new freshmen enrolled at Kansas State University for the Fall of 2009. The purpose of this exam is to help determine which students are prepared for a college Calculus I or Calculus II course. Problems on the exam were analyzed and grouped together using different techniques including expert analysis and item response theory to determine which problems were similar or even relevant to placement. Student scores on the exam were compared to their performance on the final exam at the end of the course as well as ACT data. This showed how well the placement exam indicated which students were prepared. A model was created using ACT information and the new information from the placement exam that improved prediction of success in a college calculus course. The new model offers a significant improvement upon what the ACT data provides to advisors. Suggestions for improvements to the test and methodology are made based upon the analysis

Table of Contents

List of Figures iv

List of Tables vi

Chapter 1 - Introduction 1

  Background 1

  Tools for Placement 2

  Developing the Placement Exam 4

  Sample Calculus Placement Exam 5

  Research Questions 9

  Item Response Theory 10

  Improving Placement Exams and Predicting Success 11

Chapter 2 - Analysis of the Placement Exam 14

Chapter 3 - Results 27

  Comparison with success in Calculus 27

  Noncompliance 36

Chapter 4 - Conclusions 41

  Recommendations for the Future 42

References 44

List of Figures

Figure 1.1 Kansas State University Math Course Prerequisites [13].. 2

Figure 1.2 Sample Section 1 of the Calculus Placement Exam... 5

Figure 1.3 Sample Section 2 of the Calculus Placement Exam... 6

Figure 1.4 Sample Section 3 of the Calculus Placement Exam... 7

Figure 1.5 Sample Section 4 of the Calculus Placement Exam... 8

Figure 2.1 Item Response Curve of Section 3, Problem 3 (grade as latent trait).......................... 15

Figure 2.2 Item Response Curve of Section 4, Problem 3 (grade as latent trait).......................... 15

Figure 2.3 Item Response Curve for Section 1, Problem 1 16

Figure 2.4 Item Response Curve for Section 1, Problem 2 17

Figure 2.5 Item Response Curve for Section 1, Problem 3 17

Figure 2.6 Item Response Curve for Section 1, Problem 4 18

Figure 2.7 Item Response Curve for Section 2, Problem 1 18

Figure 2.8 Item Response Curve for Section 2, Problem 2 19

Figure 2.9 Item Response Curve for Section 2, Problem 3 19

Figure 2.10 Item Response Curve for Section 2, Problem 4 20

Figure 2.11 Item Response Curve for Section 3, Problem 1 20

Figure 2.12 Item Response Curve for Section 3, Problem 2 21

Figure 2.13 Item Response Curve for Section 3, Problem 3 21

Figure 2.14 Item Response Curve for Section 3, Problem 4 22

Figure 2.15 Item Response Curve for Section 3, Problem 5 22

Figure 2.16 Item Response Curve for Section 4, Problem 1 23

Figure 2.17 Item Response Curve for Section 4, Problem 2 23

Figure 2.18 Item Response Curve for Section 4, Problem 3 24

Figure 2.19 Item Response Curve for Section 4, Problem 4 24

Figure 2.20 Item Response Curve for Section 4, Problem 5 25

Figure 2.21 Item Response Curve Coefficients 26

Figure 3.1Venn Diagram of Variance.. 34

Figure 3.2 Z-score versus Percent C or Better 35

Figure 3.3 Pie Chart of Freshmen in Calculus who took the Placement Exam 38

Figure 3.4Pie Chart of Freshmen in Calculus who Did NOT take the Placement Exam 38

Figure 3.5 Pie Chart of Freshmen in Calculus II who took the Placement Exam 40

Figure 3.6 Pie Chart of Freshmen in Calculus II who took the Placement Exam 40

List of Tables

Table 2.1 Levels of Difficulties and Discrimination (by ascending difficutly) 25

Table 3.1 Placement exam Z-scores and Model 35

Table 3.2 Percent of Grades C or better in Calculus I 37

Table 3.3 Percent of Grades C or better in Calculus II 39

Chapter 1 - Introduction

Background

Calculus is a required course for many majors at Kansas State University, especially in the sciences. To name a few, students pursuing a degree in Engineering, Chemistry, Biology, and engineering must successfully complete a course in Calculus and some must complete Calculus II. We would like to see a higher level of success in freshmen taking Calculus classes. Questions were raised about whether students were being properly placed into Calculus and Calculus II. Many institutions at the post-secondary level use a variety of methods in an attempt at placing students in the correct mathematics course for which they are prepared. Tools available to counselors and advisors include the ACT, SAT, high school data, and placement examinations. They develop a method of placement based upon the school's curriculum, methodology, student population, and accessibility to the placement exams. A placement exam would provide additional information to help place students into the appropriate math course for which they possess the skills necessary to succeed. Ninety percent of post secondary institutions use some form of placement test [7]. Until 2009, Kansas State University did not have a placement test for Mathematics. We believe that using ACT data and information from a placement exam will increase students' chance for success by improving the ability of advisers to place students into the correct mathematics course.

Predicting overall success of college students has long been a topic of interest to college admissions. It has been found that the best predictor for overall college success and retention is grade point average from the first year of college [9]. Furthermore, first year grades are "the single most revealing indicator of successful adjustment to the intellectual demands of a particular college's course of study"[3]. Placing students in the correct classes should improve grades in the first year. And thus proper placement plays an important role in increasing retention and overall success in college.

Tools for Placement

Advisers at Kansas State University currently place students into what they believe is the appropriate math course based upon the ACT college entrance test and high school data such as Advanced Placement testing and dual credit. The figure below lists the current prerequisites for the lower-level mathematics courses at Kansas State University.

Figure 1.1 Kansas State University Math Course Prerequisites [13]

MATH COURSES

MATH COURSE PREREQUISITES

MATH 010. Intermediate Algebra. (3) Pr.: Two units of mathematics in grades 9-12 and a College Algebra PROB >= C of 43 or more on the ACT assessment by K-State; or a score of at least 7 on the mathematics placement test; or a score of at least 26 on the arithmetic placement test.

MATH 100. College Algebra. (3) Pr.: B or better in MATH 010; or two years of high school algebra and a College Algebra PROB >= C of 60 or more on the ACT assessment by K-State; or a score of at least 18 on the mathematics placement test.

MATH 150. Plane Trigonometry. (3) Pr.: C or better in MATH 100; or two years of high school algebra and a score of 25 or more on Enhanced ACT mathematics; or a score of at least 20 on the mathematics placement exam.

MATH 220. Analytic Geometry and Calculus I. (4) I, II, S. Analytic geometry, differential and integral calculus of algebraic and trigonometric functions. Pr.: B or better in MATH 100 and C or better in MATH 150; or three years of college preparatory mathematics including trigonometry and Calculus I PROB >= C of 55 or more on the ACT assessment by K-State; or a score of at least 26 on the mathematics placement test.

- Find information about these and other undergraduate math courses in the K-State Undergraduate Catalog

The ACT is "designed to assess students' general educational development and their ability to complete college-level work…The tests emphasize reasoning, analysis, problem solving, and the integration of learning from various sources, as well as the application of these proficiencies to the kinds of tasks college students are expected to perform… The Mathematics Test is based on six content areas: pre-algebra, elementary algebra, intermediate algebra, coordinate geometry, plane geometry, and trigonometry" [11].

Students may have been exposed to a high school course in higher mathematics or enrolled for dual credit from a community college. Dual credit, also known as dual enrollment, is a course that is taken by a high school student that counts toward both high school credit and college credit. It is generally taught during the normal school day by a high school teacher in that school. Some students may have taken an Advanced Placement course in calculus during high school. "AP courses in calculus consist of a full high school academic year of work and are comparable to calculus courses in colleges and universities. It is expected that students who take an AP course in calculus will seek college credit, college placement or both from institutions of higher learning"[12]. A study on the effect of high school course work on lower-level undergraduate success in math has found that "taking more higher level math courses in high school is an accurate predictor of scoring well on aptitude tests commonly required for admission into four-year baccalaureate institutions."[4] Exposure to the material, despite the grade earned in the class, served as an advantage for students taking placement exams.

The ACT does provide information for placement into calculus courses and below, but not Calculus II. Some high school calculus courses do adequately prepare their students for entering in to a Calculus I or Calculus II course, but this is not always the case. The "Factors Influencing College Success in Mathematics (FICS-Math)" project at Harvard University is currently conducting a 3-year study to determine what background factors best prepare students for calculus in college. Anecdotally, exit interviews have suggested many students struggling in Calculus II at Kansas State have been exposed to calculus in high school. The ACT does not cover calculus, and previous exposure to calculus in high school does not guarantee retention or proficiency in the skills necessary to succeed in a college level calculus course. Students choose to place themselves in either Calculus I or Calculus II because we have no validating information to decide which class to place them. Placement exams aim to screen possible failure, not to guarantee success. Being placed in the correct mathematics course would enable a higher success rate and hopefully increase retention in college enrollment.

Developing the Placement Exam

To improve our ability to properly place students into the correct mathematics course the mathematics department developed a placement exam. In discussions with New Student Services about how to offer the exam to all students, the department was informed that there was insufficient time available to offer the exams during student orientation. It was decided to offer the exam online so all students could have access to it prior to their arrival for registration in June. The exam was built on the framework of the department's current online homework system. Most problems were not multiple-choice but required students to type in numbers or formulas. Students are given one chance to fix errors on any problems they missed, allowing them to correct simple computational or typographical errors. The system randomly generates different but similar problems for each student every time they sign in. Students are allowed to try multiple times if they felt their initial score was not reflective of their ability. The exam was split into two sections: Algebra and Calculus. Questions on Trigonometry are included in the Calculus exam. The problems on the calculus exam were written by faculty members of the Kansas State University Department of Mathematics. These problems demonstrate what they perceive to be the skills necessary to succeed in a calculus course at Kansas State University and representative of the course curriculum. The exams were administered online and at home, and the students were on the honor system.

Sample Calculus Placement Exam

Each student receives a randomly generated exam so we can only show one example to indicate the type of problems asked. The specific values in the equations vary for each attempt on the exam.

Section 1 of the Placement Exam covers basic trigonometry and includes problems on angles and right triangles.

Figure 1.2 Sample Section 1 of the Calculus Placement Exam

KSU Calculus Placement Exam
Section 1
Attempt 7

- **You are permitted to use a calculator on this exam.**
- You will get one chance to correct any errors before moving on. You receive full credit for problems where you are able to find and correct your errors (this also gives you a chance to correct typos in entering your answer without penalty).
- It is not assumed that you will know how to do all the problems. You may leave blank any problem you don't know how to do.
- If you need to leave the exam in the middle, it will restore you to where you left when you sign back in. If you leave an exam for over 24 hours however, it will be terminated and the next time you sign in you will have to start over.

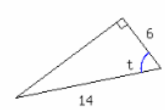
1. An angle θ has radian measure $-7\pi/18$ (`-7*pi/18`). Express the measure of the angle θ in degrees:
[ ]°

2. Find the quadrant (1, 2, 3, or 4) containing the points on the unit circle satisfying the given conditions.
cot(t) > 0
sec(t) < 0
Quadrant: [ ]

3.

Find the **exact** values of the six trigonometric functions of the angle t in the triangle shown to the right. Enter your answers as fractions. Use `sqrt()` to indicate a square root (e.g. `sqrt(3)` for $\sqrt{3}$).

sin(t) = [ ] cos(t) = [ ]
tan(t) = [ ] csc(t) = [ ]
sec(t) = [ ] cot(t) = [ ]

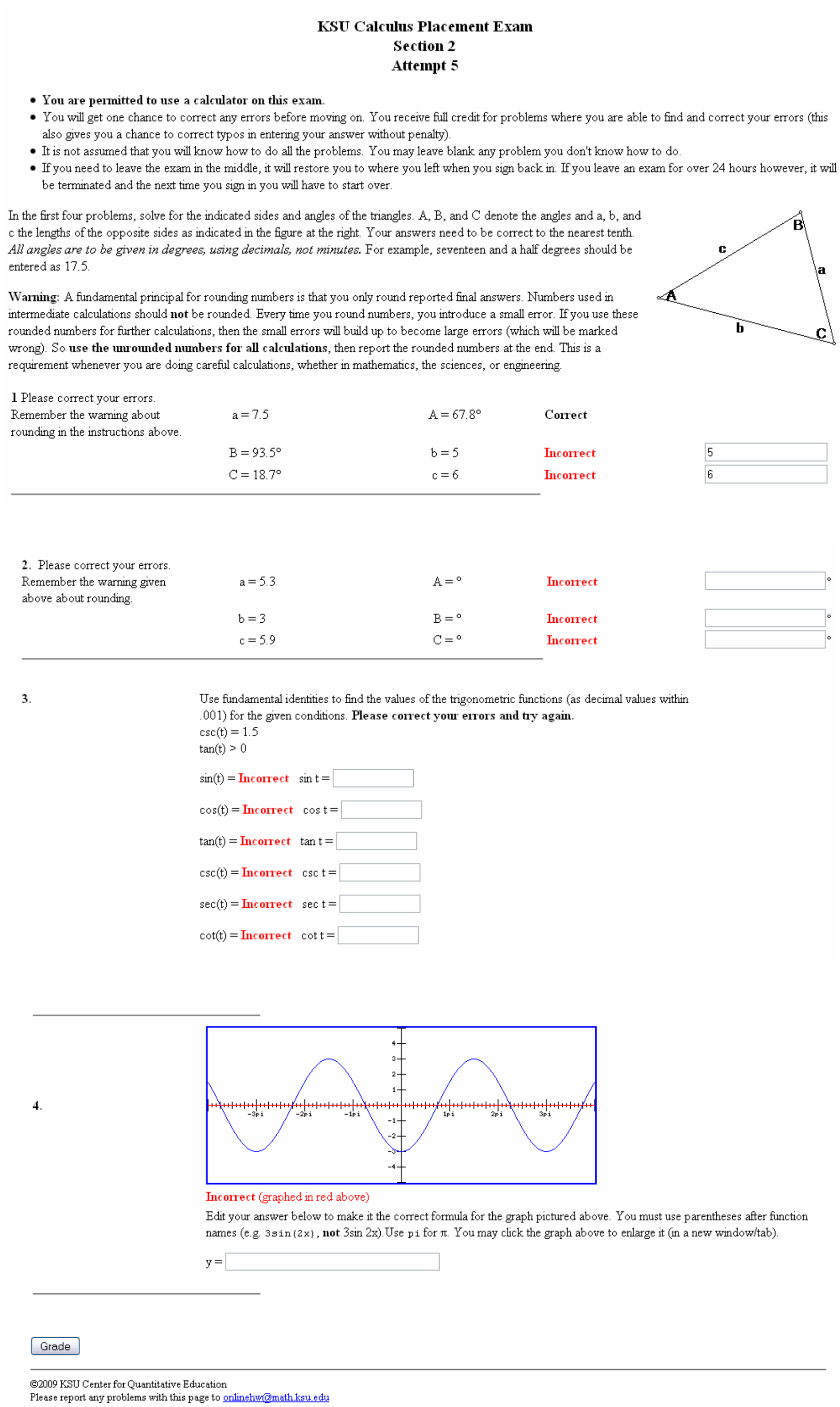


4. Find all values of θ in **radians** with `-6.28<θ<6.28` such that sin(θ) = 0.99. Enter your answers as decimal values correct to the nearest .001 with the values separated by commas.
[ ]

Grade

Section 2 of the Calculus Placement Exam covers more topics in trigonometry, including solving triangles, identities and functions.

Figure 1.3 Sample Section 2 of the Calculus Placement Exam



KSU Calculus Placement Exam
Section 2
Attempt 5

- **You are permitted to use a calculator on this exam.**
- You will get one chance to correct any errors before moving on. You receive full credit for problems where you are able to find and correct your errors (this also gives you a chance to correct typos in entering your answer without penalty).
- It is not assumed that you will know how to do all the problems. You may leave blank any problem you don't know how to do.
- If you need to leave the exam in the middle, it will restore you to where you left when you sign back in. If you leave an exam for over 24 hours however, it will be terminated and the next time you sign in you will have to start over.

In the first four problems, solve for the indicated sides and angles of the triangles. A, B, and C denote the angles and a, b, and c the lengths of the opposite sides as indicated in the figure at the right. Your answers need to be correct to the nearest tenth. *All angles are to be given in degrees, using decimals, not minutes.* For example, seventeen and a half degrees should be entered as 17.5.

Warning: A fundamental principal for rounding numbers is that you only round reported final answers. Numbers used in intermediate calculations should **not** be rounded. Every time you round numbers, you introduce a small error. If you use these rounded numbers for further calculations, then the small errors will build up to become large errors (which will be marked wrong). So **use the unrounded numbers for all calculations**, then report the rounded numbers at the end. This is a requirement whenever you are doing careful calculations, whether in mathematics, the sciences, or engineering.

1 Please correct your errors. Remember the warning about rounding in the instructions above.

	a = 7.5	A = 67.8°	**Correct**	
	B = 93.5°	b = 5	**Incorrect**	5
	C = 18.7°	c = 6	**Incorrect**	6

2. Please correct your errors. Remember the warning given above about rounding.

	a = 5.3	A = °	**Incorrect**	°
	b = 3	B = °	**Incorrect**	°
	c = 5.9	C = °	**Incorrect**	°

3. Use fundamental identities to find the values of the trigonometric functions (as decimal values within .001) for the given conditions. **Please correct your errors and try again.**
csc(t) = 1.5
tan(t) > 0

sin(t) = **Incorrect** sin t =

cos(t) = **Incorrect** cos t =

tan(t) = **Incorrect** tan t =

csc(t) = **Incorrect** csc t =

sec(t) = **Incorrect** sec t =

cot(t) = **Incorrect** cot t =

4.

Incorrect (graphed in red above)
Edit your answer below to make it the correct formula for the graph pictured above. You must use parentheses after function names (e.g. `3sin(2x)`, **not** 3sin 2x).Use `pi` for π. You may click the graph above to enlarge it (in a new window/tab).

y =

Grade

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Please report any problems with this page to onlinehw@math.ksu.edu

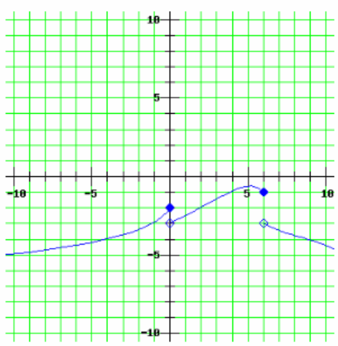
Section 3 on the exam has Calculus questions that require a numerical answer. This section covers a broad range of topics including continuity, limits, and integration.

Figure 1.4 Sample Section 3 of the Calculus Placement Exam

KSU Calculus Placement Exam
Section 3
Attempt 5

- **You are permitted to use a calculator on this exam.**
- You will get one chance to correct any errors before moving on. You receive full credit for problems where you are able to find and correct your errors (this also gives you a chance to correct typos in entering your answer without penalty).
- It is not assumed that you will know how to do all the problems. You may leave blank any problem you don't know how to do.
- If you need to leave the exam in the middle, it will restore you to where you left when you sign back in. If you leave an exam for over 24 hours however, it will be terminated and the next time you sign in you will have to start over.

1.



The function $y = f(x)$ is graphed above. Find the limits below.

Please enter your answer in the space provided. If the limit exists, enter the exact value of the limit (which will be an integer); if the limit does not exists, enter Und (for Undefined).

$\lim_{x \to 0^-} f(x) =$ ______ $\lim_{x \to 0^+} f(x) =$ ______

$\lim_{x \to 0} f(x) =$ ______ $f(0) =$ ______

2. Find the following limit:

$$\lim_{x \to 2} \frac{3x^2 - 3x - 18}{x - 3}$$

Your answer has to be exact. Please enter you answer as an integer or as a fraction (e.g. 1/3).

The limit is ______

3. For what value(s) of the constant a is the function f continuous on $(-\infty,\infty)$?

$$f(x) = \begin{cases} 4x^3 - 2x - 2a \text{ for } x \le 0 \\ ax - 1 \text{ for } x > 0 \end{cases}$$

Please enter all the value(s) of a that make(s) the function f continuous everywhere. You may enter your answer either as a fraction (e.g. 3/5) or as a decimal (e.g. .6). If you enter a decimal, it must be within .0005 of the exact value.

If the function f is continuous for all values of a, enter all; if there is no such value for a, enter none.

f is continuous everywhere for a= ______

4. Evaluate $\int_{-1}^{2} 4x^4 - 3x \, dx$. You may enter your answer either as a fraction (e.g. 3/5) or as a decimal (e.g. .6). If you enter a decimal, it must be within .0005 of the exact value.

______

5. Find the maximum and minimum values of the function $f(x) = 3x^4 - 24x^2 + 6$ on the interval $[-4, 3]$.

You may enter your answer either as a fraction (e.g. 3/5) or as a decimal (e.g. 0.6). If you enter a decimal, it must be within .0005 of the exact value.

minimim value = ______ maximum value = ______

Grade

Section 4 on the Calculus Placement Exam requires answers to be in the form of a function. Because these problems required answers to be entered as a function, they were placed on the same page so the prompt could offer instructions on how to enter the answers for those who have never seen this format.

Figure 1.5 Sample Section 4 of the Calculus Placement Exam

KSU Calculus Placement Exam
Section 4
Attempt 5

- You are permitted to use a calculator on this exam. **However, you may not use the symbolic differentiate and integrate functions on this page if you have a TI-89 or similar calculator.**
- You will get one chance to correct any errors before moving on. You receive full credit for problems where you are able to find and correct your errors (this also gives you a chance to correct typos in entering your answer without penalty).
- It is not assumed that you will know how to do all the problems. You may leave blank any problem you don't know how to do.
- If you need to leave the exam in the middle, it will restore you to where you left when you sign back in. If you leave an exam for over 24 hours however, it will be terminated and the next time you sign in you will have to start over.

You enter your answers below in a syntax similar to that of a TI graphing calculator. For example, you would enter

x^2+3x+2 as `x^2 + 3x + 2` and

cos(3x-π/3) as `cos(3x - pi/3)`

The parser will recognize numbers, the constants e and pi, the operations +, -, *, /, and ^ (for raising to a power), and the functions abs(x), ln(x), exp(x), sqrt(x), sin(x), cos(x), tan(x), asin(x), acos(x), atan(x), sinh(x), cosh(x), and tanh(x). You must use parenthesis in functions, e.g. `exp(2x)` **not** `exp 2x`. The parser understands that `2x` means `2` multiplied by `x`, **except** that if you write `xy` it will think you mean a variable with the two letter name "`xy`" instead of `x` times `y`. Write `x*y` if you want to multiply `x` and `y`. Click here for more details on the parser and entering functions.

1. Find the derivative of the function

$$f(x)=4x^4-3x^2+2x$$

$f'(x)=$

2. Differentiate $f(x)=5\sin(x)(2x-5)$.
$f'(x)=$

3.

Differentiate $f(x)=\dfrac{5\sin(x)}{x^2+3x}$

$f'(x)=$

4. Differentiate $f(x)=7\exp(3\sin(3x))$.
$f'(x)=$

5. Evaluate $\int -3x^3 * \sqrt{(5x^4+4)}\, dx$.

Use C for an arbitrary constant. If you have fractions as exponents, make sure to use parentheses.
For example, `(x^2+1)^(3/2) + C` means $(x^2+1)^{3/2}+C$, while `(x^2+1)^3/2 + C` means $[(x^2+1)^3]/2+C$.

Grade

A total score of 50 was possible on the exam with a minimum score of 10. The exam itself has 40 points possible. The iSIS system that the advisors use to view the scores does not allow 0 as a possible value for a placement exam. This is a common practice with such exams. Th ACT has a minimum possible score of 15. By adding 10 points to each score, we avoid the issue. This makes the exam worth 50 points. Along with the algebra placement exam (also worth 50 points), the total placement exam is worth 100 points. Because all students taking the Calculus exam were also asked to take the Algebra exam, the calculus pages are numbered 5 through 8.

Research Questions

The placement exam was administered typically during May and June of 2009 for incoming freshmen enrolled for the Fall of 2009. The exam was administered online for the students at home in an unproctored setting. We must now interpret the scores with the goal of improving the placement of students into a college mathematics course that is at the Calculus level or higher. This thesis will consider the following research questions:

- Will this exam provide us with more information than the methods already used in order to help us improve placement into Calculus I?
- Will this exam help us determine which students are prepared for Calculus II?
- How should we advise the students based on the results of the exam?
- How can the exam be improved to help meet these goals?

The first question is whether giving the exam actually provides any benefit to advising. It is not enough that we show performance on the placement exam is correlated to the performance in class. We need to show that this exam provides us with additional information beyond what is already available. The exam should not be continued if it is not providing any extra information. The exam is not perfected by any means. We must consider if the questions on the placement exam are useful in evaluating placement.

In the Fall of 2009, 145 out of 257 (about 60%) of students enrolled in Calculus II in the Fall were freshmen. This is typical for a Fall semester. Will this exam be successful in placing the correct students into Calculus II? If students have dual credit for Calculus, advisers will allow the students choose to place themselves into either Calculus I or Calculus II because we have no other indicators of preparedness. Assuming the exam does provide additional information, the third question asks how the placement exam scores should be interpreted. We could determine a minimum score on the placement exam required for students to be placed into Calculus I or Calculus II. Depending on the strength of correlation between performance on the placement exam and actual performance in the course, it might be more appropriate to provide an estimate of success and allow the student and advisor to make individual decisions based upon this estimate.

Item Response Theory

Item response theory, also known as Latent Trait Theory, Item Characteristic Curve Theory, Rasch model, 2PL Model, 3PL Model, and the Birnbaum Model, is the study of test and item scores based on the mathematical relationship between a hypothesized trait (the latent trait) and item responses [1]. In academics, the latent trait can be any measurable attribute of a test subject including IQ levels, academic ability, reading ability, or arithmetic ability. A test is developed with items that measure a different facet of interest. Each item possesses a corresponding Item Response Function that typically forms an S-shaped curve when plotting the latent trait versus the probability of getting a correct answer on the item. For the purpose of this report, we will only employ the one-dimensional (Rasch) model. We use this Item Response Theory in lieu of Classical Test Theory because it provides more information.

We model this S-shaped curve with a logistic function of the form

$$P(\theta) = 1/(1+e^{-a(\theta-b)})$$

θ is a measurement of the latent trait. P(θ) is the probability of getting a correct answer at θ, a is the discrimination level, and b is the level of difficulty. The difficulty of the item is represented by the variable b. If an item is "easy" then the P(θ) of each θ will all be close to 1. If the item is hard, then all P(θ) will be close to 0. If the item has a medium level of difficulty, the lower values of θ will have corresponding low values of P(θ), and high values of θ will have high values of P(θ). The value of θ=b corresponds to when P(θ)=.5, or where the test taker's ability allows them a 50% chance of getting a correct answer. The slope of the curve at b is referred to as the "discrimination" of the curve. The steeper the curve, the higher the difference of P(θ) between different levels of ability and the easier it is to distinguish between the levels. [1]

Improving Placement Exams and Predicting Success

Classical test theories often utilize a raw score given by the total number of points received on a test. Item Response Theory allows us to look at each problem individually gives more information about the test and test taker. There has been evidence in recent studies that Item Response Theory is more successful in predicting success in students of all levels of education. Also analyzing an exam with Item Response Theory shows the underlying structure of the exam, including its strengths and weaknesses. This is why we chose to use Item Response Theory with our placement examination.

As stated before, a standard high school GPA and a battery of standardized tests are traditionally used as admission standards. However, an IRT-based high school GPA has been proven to be a more reliable predictor of college success. It takes into account "differences among courses both in the distribution of grades and the students enrolled" [10]. This method examines each course separately and creates statistical adjustment for things like the impact of course difficulty on GPA. The writers of the College Board Scholastic Aptitude Test (SAT) utilizes this IRT-based GPA along with SAT scores in its studies to predict success among college students [2].

A study using an IRT model to create the cut-off scales for a test taken by elementary school students was successful in identifying which students were in need of more assistants and which students mastered the material set forth by a state-mandated exam. [6]

A study looking to improve the reliability of test scores employed both Classical Test Theory and Item Response Theory to evaluate how well a test evaluates a student's academic standing. The No Child Left Behind Act of 2001 (U. S. Department of Education, 2002) requires statewide testing programs to report diagnostic information to examinees that allows parents, teachers, and principals to understand and address the specific academic needs of students [5]. Along with a raw point score, the report also contains subscores within each academic area. The subscores were generated by grouping together items considered to be alike using Item Response Theory. The study concludes that looking at the test as a whole and at items individually helps create a clearer picture of the student's academic ability [5]. This is a case in which utilizing Item Response theory in addition to Classical Test Theory provides an advantage in evaluation and prediction of success.

With an exam set into place, could improving the exam actually increase the amount of information it provides? In a study to improve the validity of testing mandated by The No Child Left Behind Act of 2001, instructors set out to create adjustments for disabled students that still covers the same amount of material. Carefully inspecting each item and student response to each question, they decided which questions were too hard, which created less of a struggle, and what materials required modifications to assist disabled students with understanding. This analysis employed Item Response Theory to enable the instructors to create a similar test with reduced difficulty that was still fair and covered the required curriculum for general mathematics. [7] This is a case in which adjusting an exam to fit its audience improved the amount of information provided by the exam.

Chapter 2 - Analysis of the Placement Exam

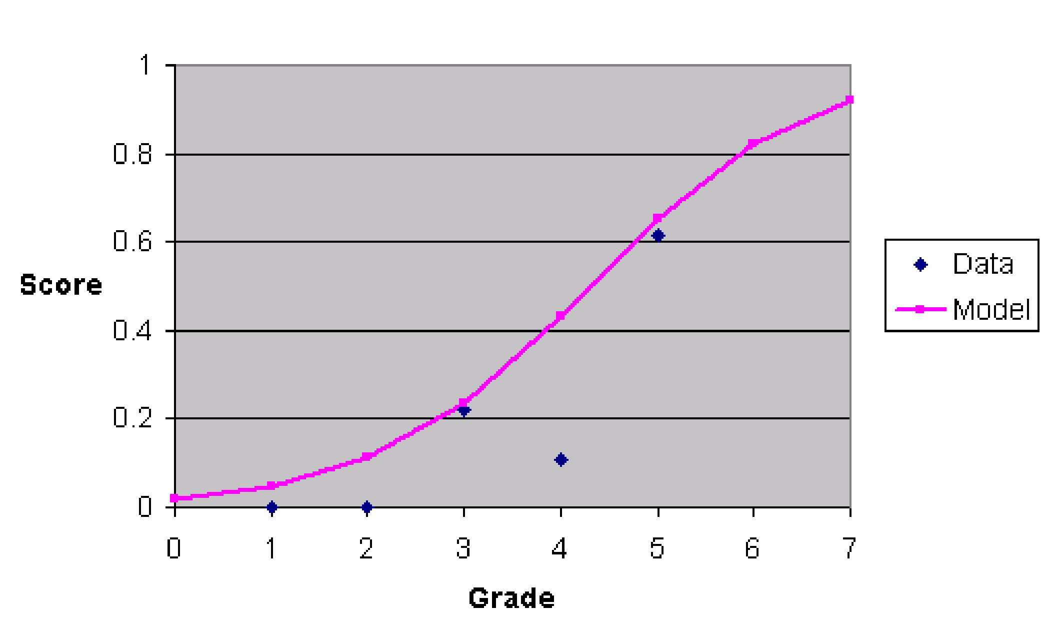
In the spring of 2009, all entering students planning to take Calculus I or II were asked to take an online placement exam covering algebra as well as calculus prior to enrollment in June. Problems were randomly generated and each student receives a different exam with the same types of problems but with different numbers.

The algebra exam was worth 50 points and consisted of 19 problems covering topics in basic, intermediate, and college algebra. The Calculus placement exam consisted of 18 problems divided into 4 sections. The first two sections consisted of topics in trigonometry. The third section covered limits. The fourth section covered derivatives and integrals of functions.

The exam is graded after each section, and the student is given automatic feedback on which ones were correct and incorrect. They are given one opportunity to edit and resubmit their answers for grading before moving on to the new section. If the student was unsatisfied with their score, they had the opportunity to try the exam again with different, newly generated problems. 2792 students took the algebra exam and 528 took the Calculus exam. Because the exams were available to anyone with a KSU eID and a WID, some people took the exams who were not students, for example advisers and faculty members.

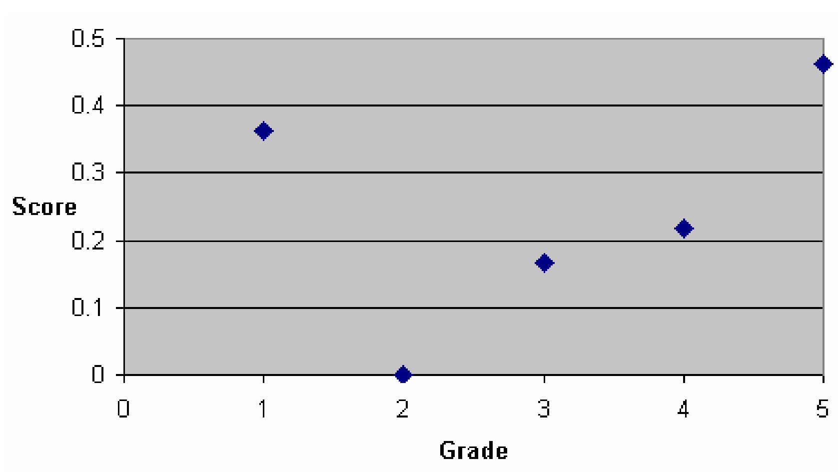
We had to decide on what to use as our latent trait. One options we considered was using the students' overall grades in the class (with A=5, B=4, etc.). In developing the Item Response Curves, some of the graphs attained that S-shaped curve such as the figure below.

Figure 2.1 Item Response Curve of Section 3, Problem 3 (grade as latent trait)



Some curves did not give good results. Among the 142 students who took the placement exam and completed the calculus course, only 11 received an F as a final grade. With such a small number, any small discrepancies in the data at that part of the graph would throw off the entire shape as shown below.

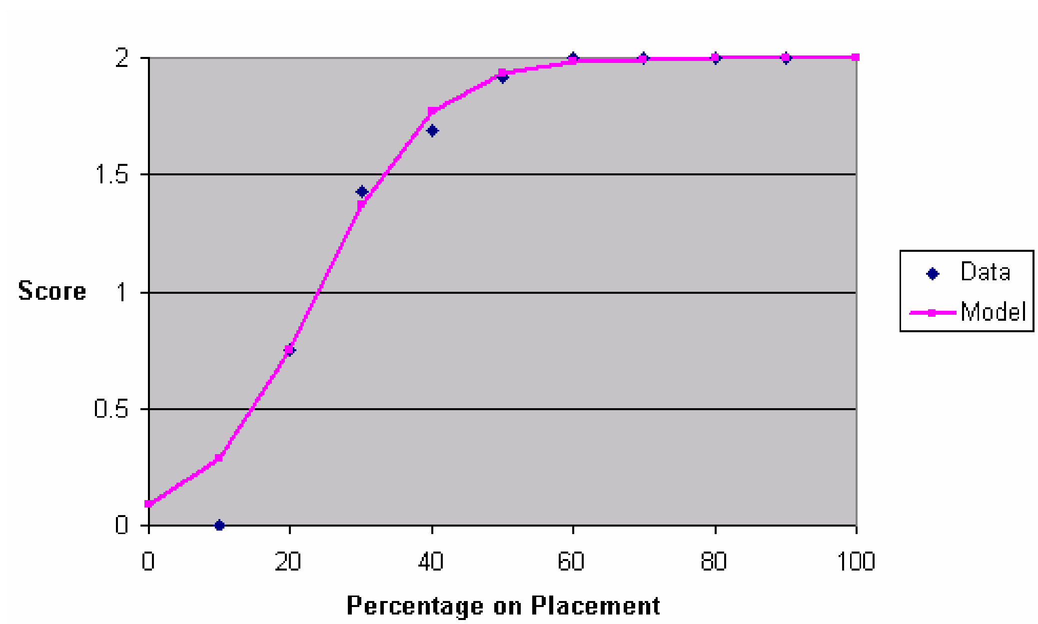
Figure 2.2 Item Response Curve of Section 4, Problem 3 (grade as latent trait)



We chose to develop item response curves for each problem by using how well the students did on the placement exam as the latent trait. The students were divided into groups representing ten percent intervals of the exam. Each curve plots the average number of points students in that percentile range earned. These graphs should be an S-shape as described by Item Response Theory. Failure to conform to this shape suggests that the problem is measuring something different from the overall exam. The Item Response Curve was fit to each set of data and the coefficients were plotted in order to detect a pattern.

An item response curve shows how likely the students were to get a certain problem correct against their overall score on the placement exam. The mean scores are labeled “data,” and the points on the fitted logistic curve are labeled “model”. Problem 2 on page 5 and problem 4 on page 7 were worth 1 point. Problem 3 on page 5 and problems 1, 2 and 3 on page 6 were worth 3 points. Problem 1 on page 7 was worth 4 points. The rest were worth 2 points each.

Figure 2.3 Item Response Curve for Section 1, Problem 1



Section 1, Problem 1 was considered “easy” because students in the top 75% of the class had a 50-50 chance of getting this answer correct. This curve also has a steep slope and therefore a higher degree of discrimination. A summary of difficulties and discrimination are in Table 2.1.

Figure 2.4 Item Response Curve for Section 1, Problem 2

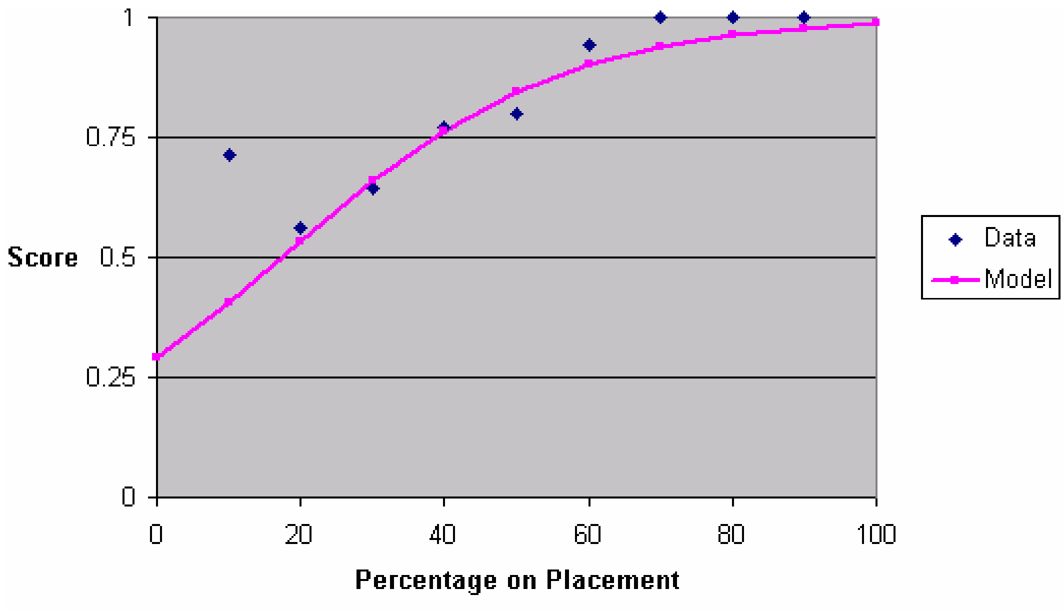


Figure 2.5 Item Response Curve for Section 1, Problem 3

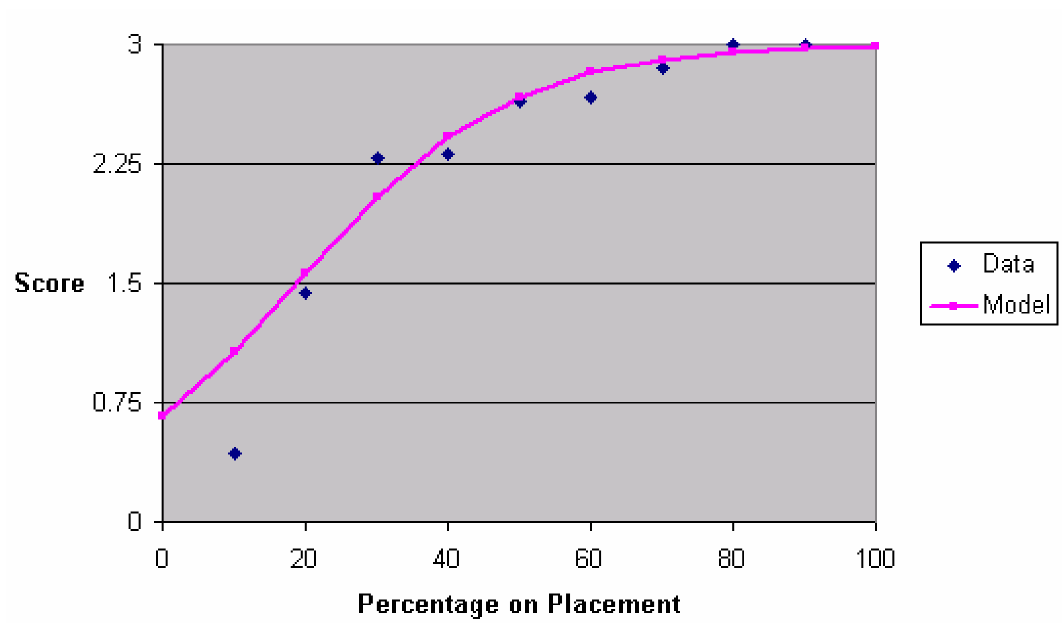


Figure 2.6 Item Response Curve for Section 1, Problem 4

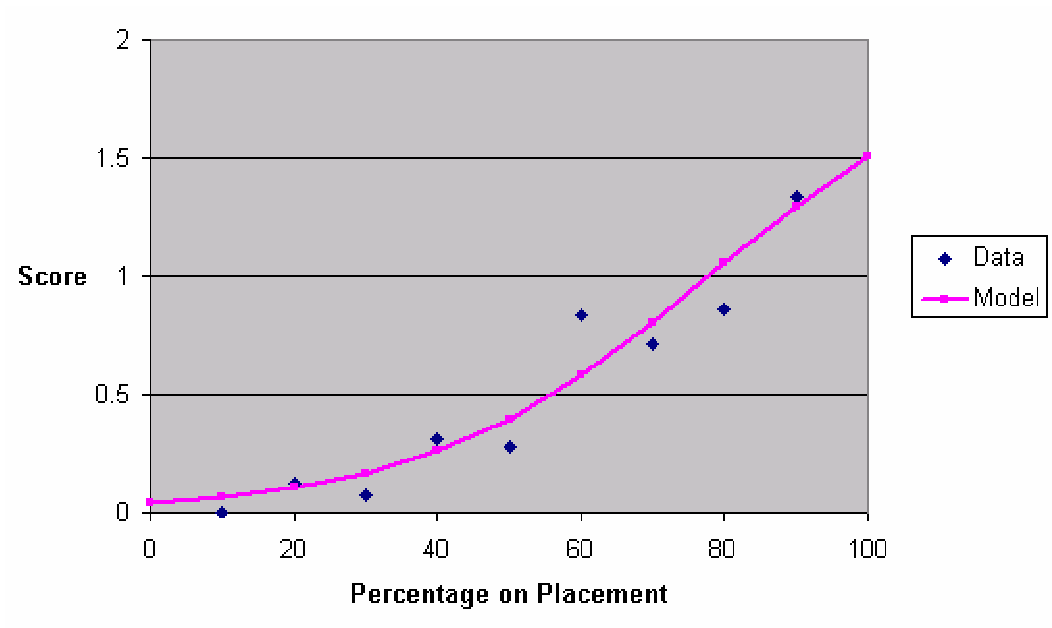


Figure 2.7 Item Response Curve for Section 2, Problem 1

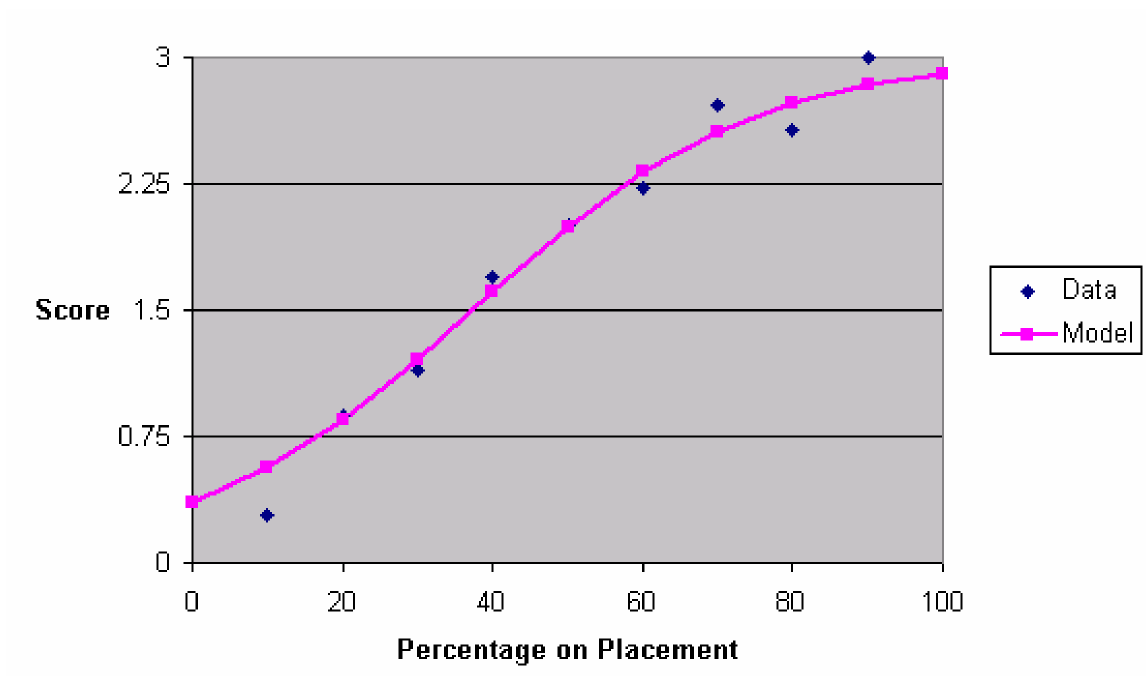


Figure 2.8 Item Response Curve for Section 2, Problem 2

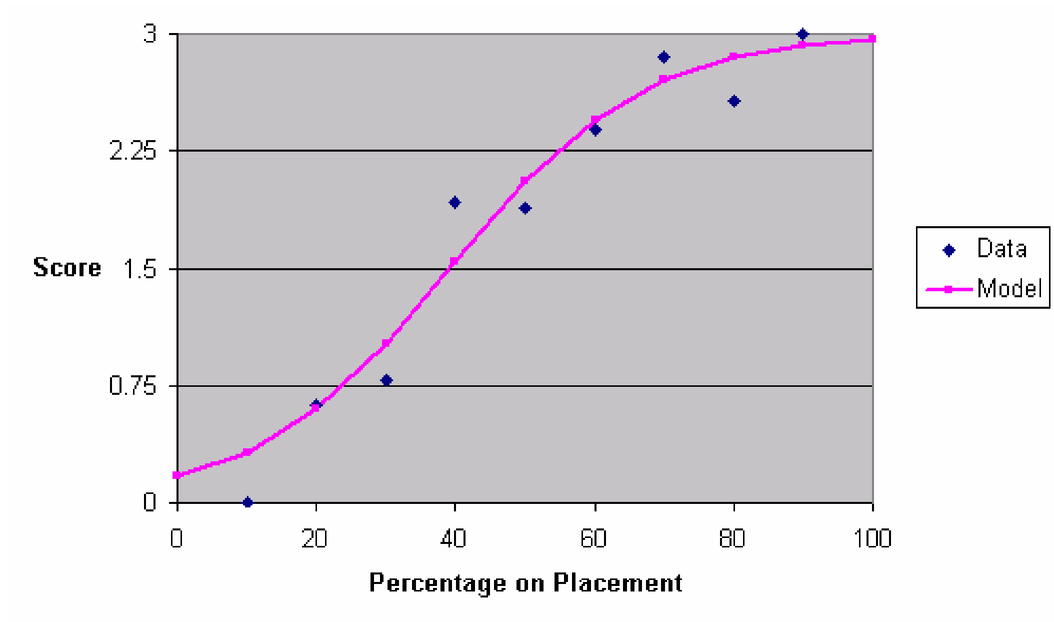


Figure 2.9 Item Response Curve for Section 2, Problem 3

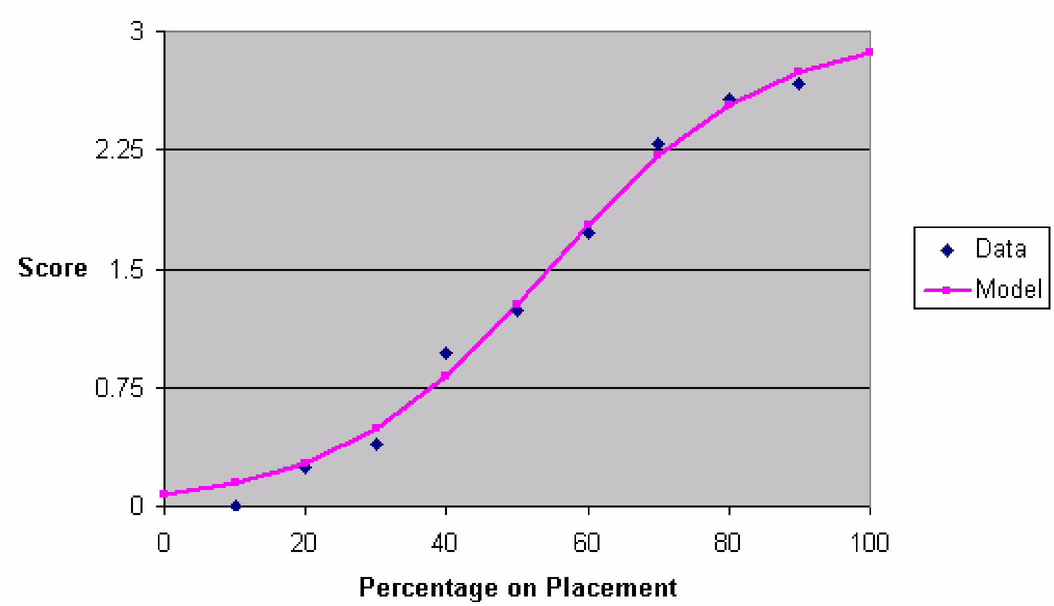


Figure 2.10 Item Response Curve for Section 2, Problem 4

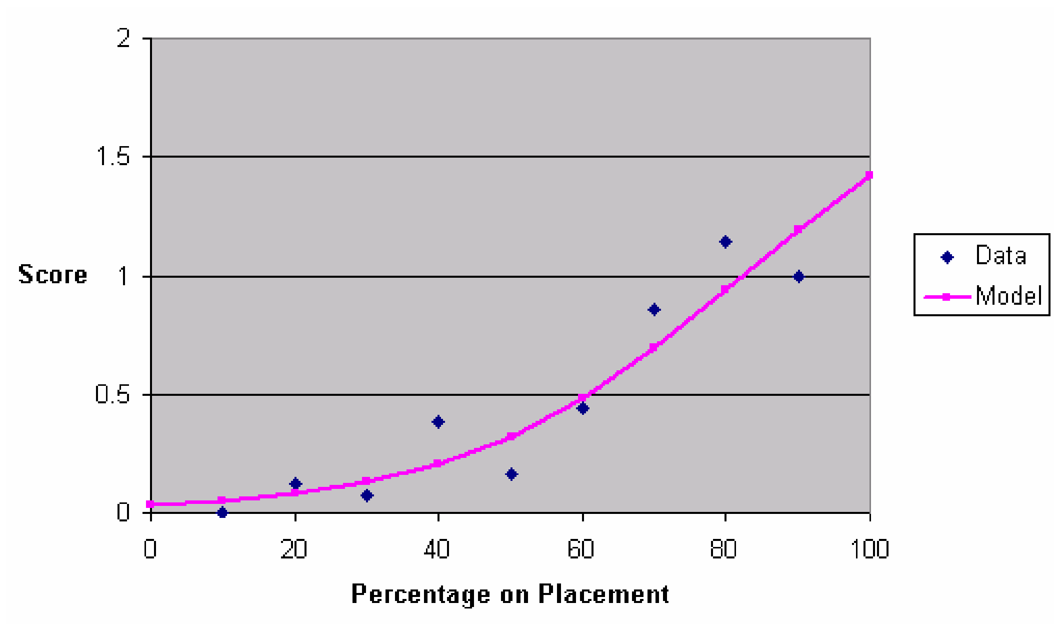


Figure 2.11 Item Response Curve for Section 3, Problem 1

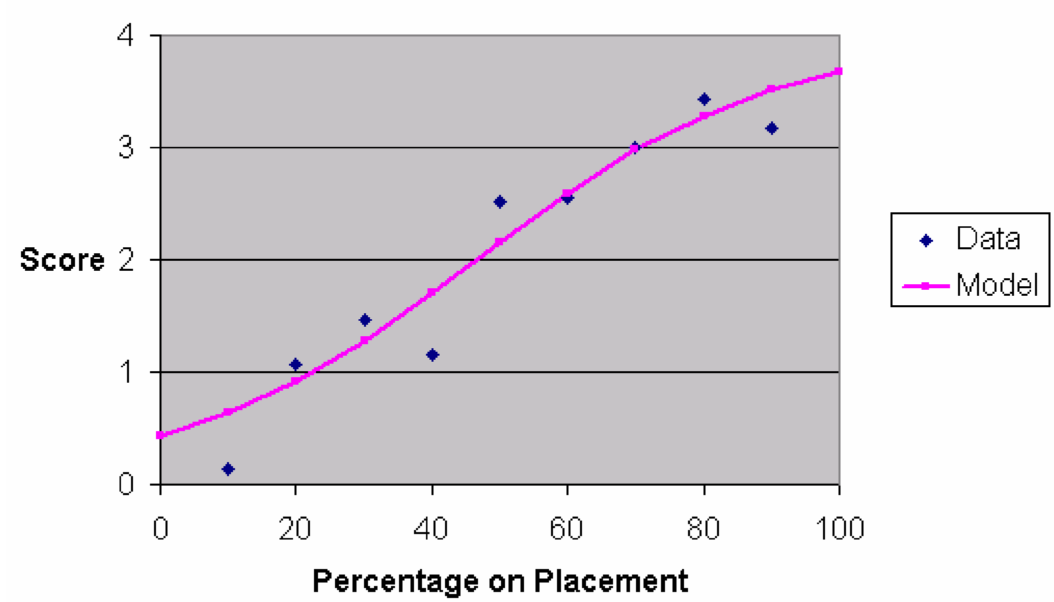


Figure 2.12 Item Response Curve for Section 3, Problem 2

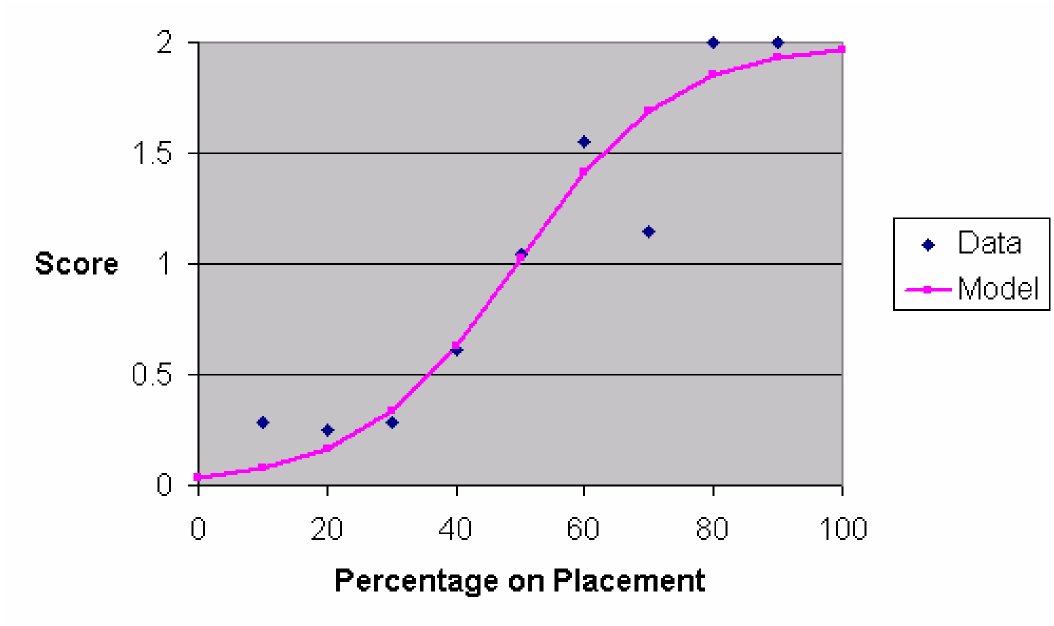


Figure 2.13 Item Response Curve for Section 3, Problem 3

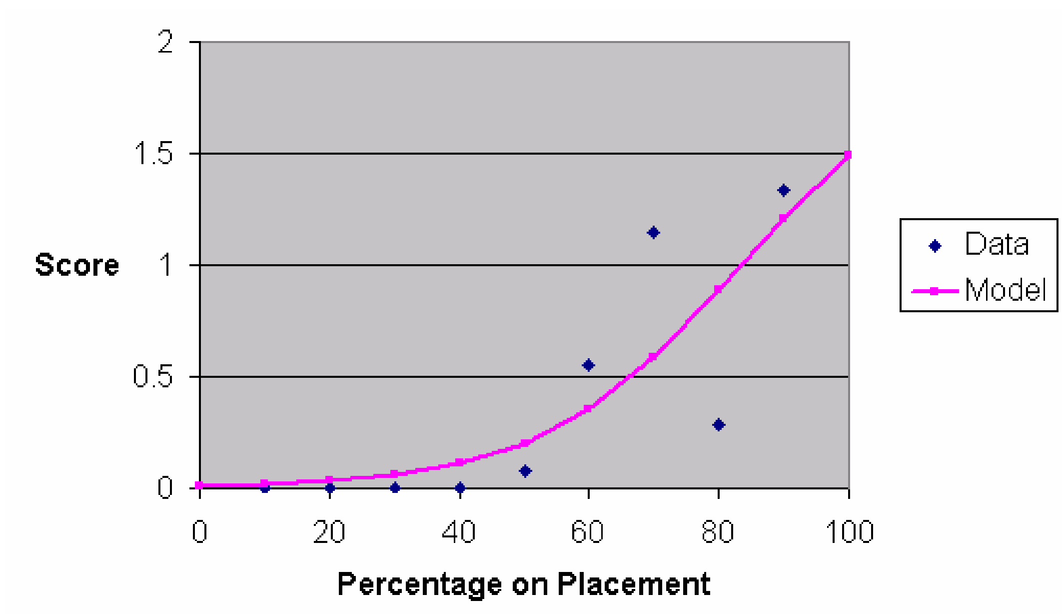


Figure 2.14 Item Response Curve for Section 3, Problem 4

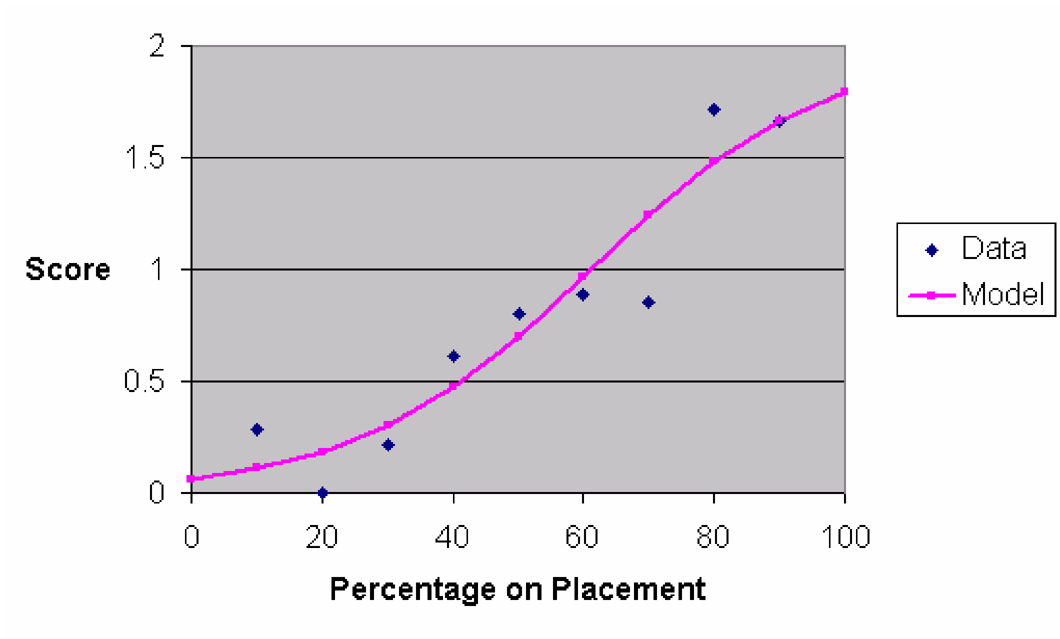


Figure 2.15 Item Response Curve for Section 3, Problem 5

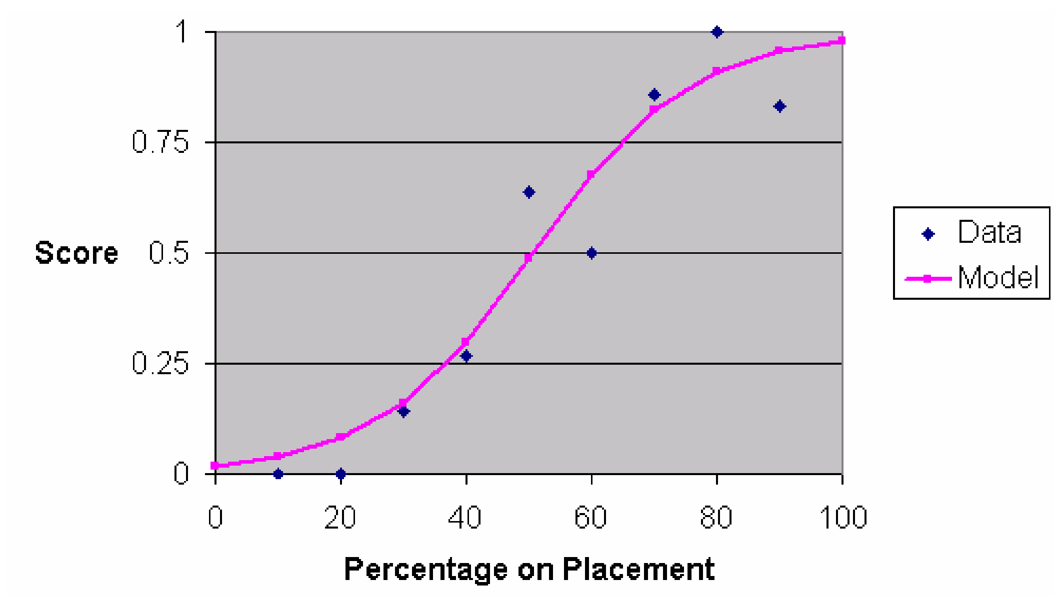


Figure 2.16 Item Response Curve for Section 4, Problem 1

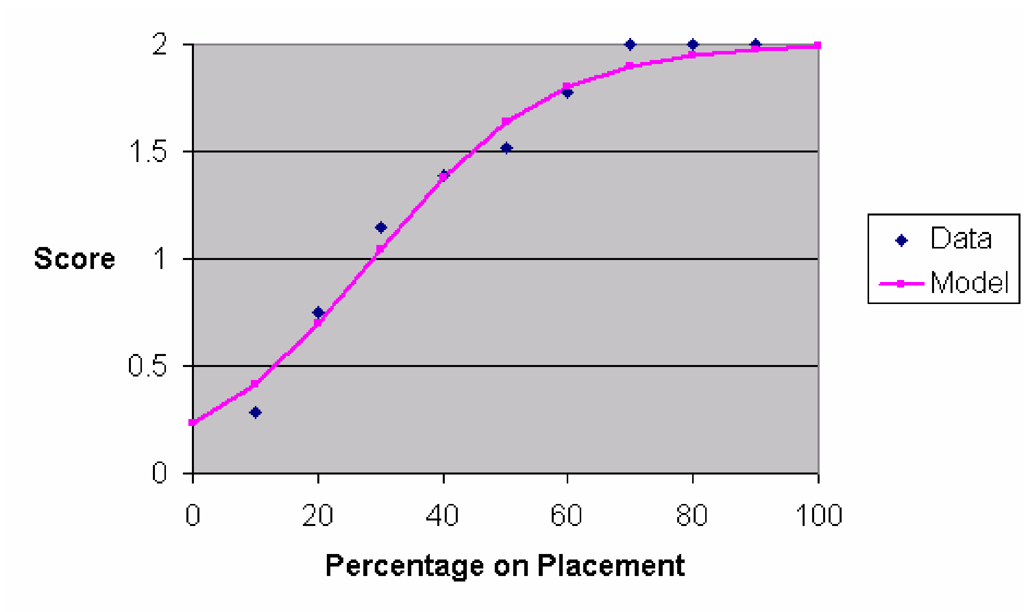


Figure 2.17 Item Response Curve for Section 4, Problem 2

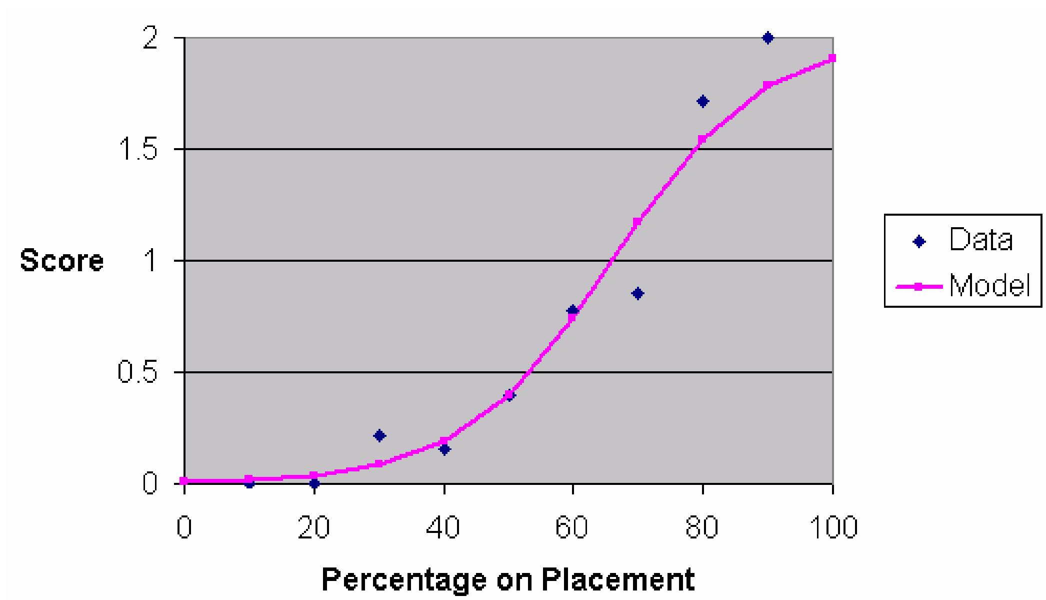


Figure 2.18 Item Response Curve for Section 4, Problem 3

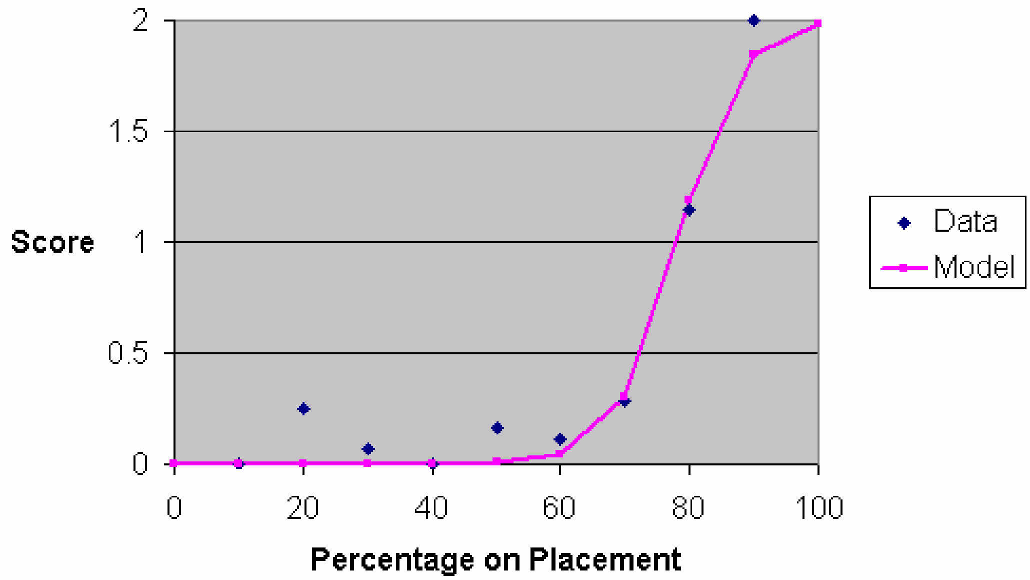


Figure 2.19 Item Response Curve for Section 4, Problem 4

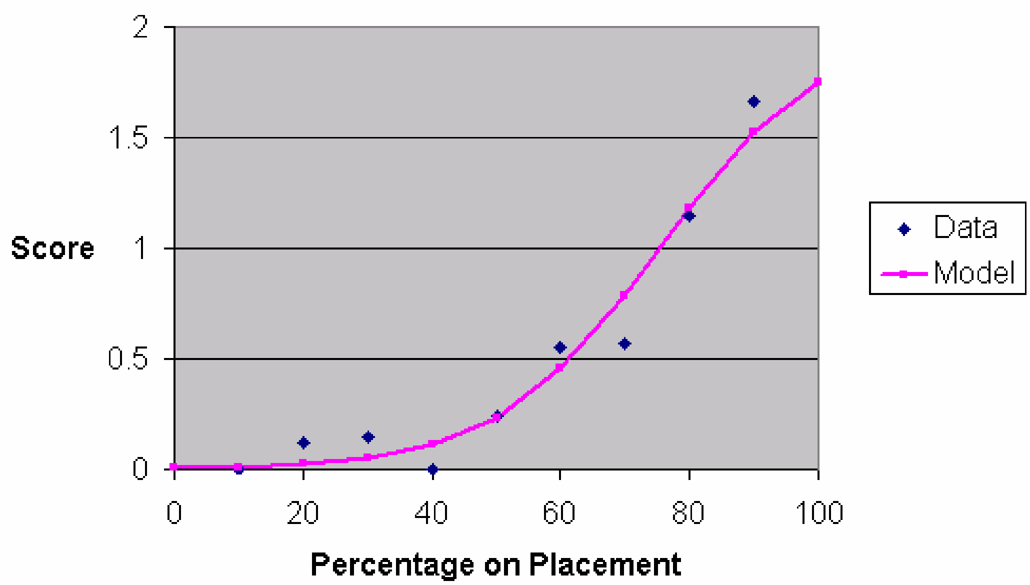


Figure 2.20 Item Response Curve for Section 4, Problem 5

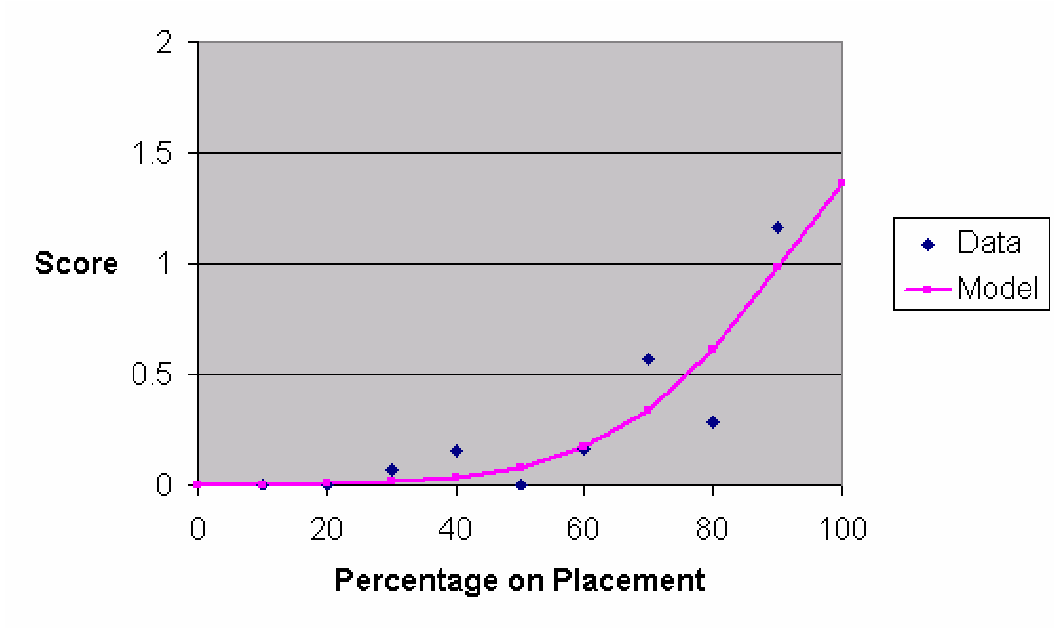
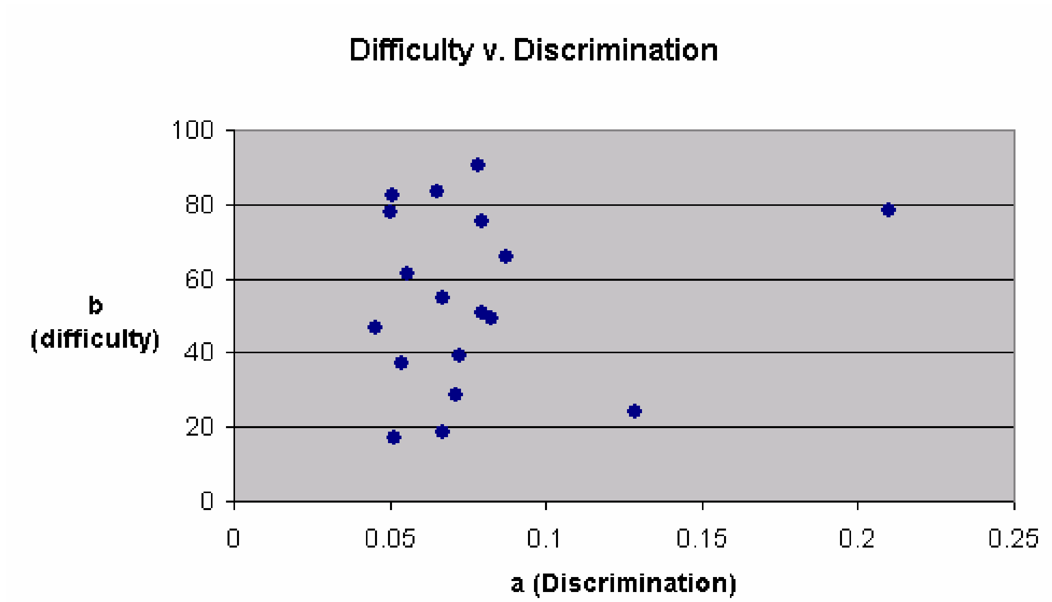


Table 2.1 Levels of Difficulties and Discrimination (by ascending difficulty)

Section #	Problem #	a *Discrimination*	b *Difficulty*	Topic ** required a functional answer*
1	2	0.051696962	17.30442524	find the quadrant given the sign of a trig functions
1	3	0.067204356	18.75560464	solving a right triangle
1	1	0.128475028	23.91610955	converting radians to degrees
4	1	0.071005479	28.7654147	find derivative of polynomial*
2	1	0.053871468	37.2870001	Law of Cosines
2	2	0.072189609	39.24792601	Law of Sines
3	1	0.045537483	46.59005773	find limits given a graph
3	2	0.082323475	49.34954327	find limit of a function
3	5	0.079513071	50.73547047	find global max/min
2	3	0.06694292	54.53027171	fundamental trig identities
3	4	0.055628811	61.11399687	evaluate a finite integral
4	2	0.087297502	65.95800463	differentiate using product rule *
4	4	0.079683567	75.41228262	differentiate using chain rule *
1	4	0.050482008	77.80555836	inverse trig functions
4	3	0.210036529	78.14780827	differentiate using quotient rule *
2	4	0.051106924	82.28565251	find the formula of a trig function given a graph *
3	3	0.065473365	83.4800715	find values of an unknown that make a function cont.
4	5	0.078350081	90.38932958	evaluate an indefinite integral *

We plot the coefficients of each Item Response Curve and visually inspect the graph for groups of similar problems.

Figure 2.21 Item Response Curve Coefficients



There are two points that immediately stand out. These correspond to Page 5 Problem 1 and Page 8 problem 3. Looking at the table above, Page 5 Problem 1 is the only problem classified as an easy problem with a very high level of discrimination. The Item Response Curve for Page 8 Problem 3 does not provide a very good fit to the data. The high level of discrimination and high level of difficulty shows that only the very top of the class correctly answered the question. We can determine three groups by visual inspection: the easy problems with difficulty levels between 0 and 39, easy problems with difficulty levels between 40 and 65, and the hard problems with difficulty levels between 66 and 100.

Chapter 3 - Results

Comparison with success in Calculus

To evaluate the effectiveness of the placement exam, we compare the total placement scores to the performance of the students in the Calculus course. The performance of the students was based upon three semester exams, a final exam, final grade, and final point total in the course. Student performance was analyzed against their score on the final exam. Because ACT is historically used for placement, we also included this information.

- actm is the math score on the ACT.
- actc is the composite score on the ACT.
- final is the student's point total on the final.
- Atotal is the student's point total on the algebra portion of the placement exam.
- Cptotal is the student's point total on the calculus placement exam.
- PageX is the score on the placement exam on all of page X.
- easy is the student's score on what were considered easy problems as defined on the previous page.
- med is the student's score on what were considered medium problems as defined on the previous page.
- hard is the student's score on what were considered hard problems as defined on the previous page.

Each student taking the Calculus placement exam was also required to take the College Algebra exam. Pages 1 through 4 of the placement exam were the College Algebra portion. Pages 5 through 8 correspond to Sections 1 through 4 respectively on the Calculus Placement exam. Linear regression models for different measures of student performance (exam scores, overall class grade, etc.) as a function of the data above were run to find correlation between student performance and the placement exam.

If we use more variables, the model will be a better fit. However, there is a question of whether it is actually modeling the data or trying to just fitting the noise created by the data. More variables do not always mean more information. We proceed by grouping certain problems together by specific properties (difficulty level, page number, subject, etc.)

When separating the College Algebra placement exam by subject matter, it was successful in predicting initial student success in the first exam (Ostapyuk, 2009). We decided to see if this was also true with the Calculus students. We ran a linear regression of the student's score on the first exam given in the Calculus course as a function of the students' scores on the Algebra Placement Exam, Calculus Placement Exam, and their score on the Mathematics portion on the ACT.   The Analysis of Variance (ANOVA) output is given below:

```
Call:
lm(formula = x1 ~ Atotal + Cptotal + actm)

Residuals:
    Min      1Q  Median      3Q     Max
-46.569  -8.498   2.509  10.233  33.134

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  14.0973    10.7304   1.314  0.19109
Atotal        0.4798     0.3049   1.574  0.11780
Cptotal       0.5515     0.1981   2.784  0.00612 **
actm          1.2235     0.4234   2.890  0.00447 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 16.21 on 139 degrees of freedom
Multiple R-squared: 0.2635,     Adjusted R-squared: 0.2476
F-statistic: 16.58 on 3 and 139 DF,  p-value: 2.907e-09
```

A linear regression model with the first exam as a function of the ACT math score and Calculus Placement exam only account for 26% of the variation. This is not significant enough to draw any conclusions. Similar conclusions were drawn when comparing ACT data and Calculus

Placement scores with exams 2 and 3. Notice that Atotal is not statistically significant in the linear regression model. We remove Atotal and run the regression again, this time separating the Calculus Placement exam into its Trigonometry (Pages 5 and 6) and Calculus (Pages 7 and 8) components. This time we compare it to the total number of points the student earned in the class. The ANOVA output is given below:

```
Call:
lm(formula = grand ~ I(P5 + P6) + I(P7 + P8) + actm)

Residuals:
    Min      1Q  Median      3Q     Max 
-372.56  -68.59   15.87   75.86  223.68 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   20.735     72.382   0.286  0.77494    
I(P5 + P6)     6.444      2.173   2.966  0.00355 ** 
I(P7 + P8)     5.393      2.158   2.499  0.01363 *  
actm          12.663      2.793   4.534 1.24e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 115.3 on 139 degrees of freedom
Multiple R-squared: 0.3366,     Adjusted R-squared: 0.3223 
F-statistic: 23.51 on 3 and 139 DF,  p-value: 2.28e-12
```

Only 33.66% of the variation in the data is explained by this model, and the ACT math score is more significant than the rest of the data. We try a few other options.

Based on the analysis of the exam in Chapter 2, the problems were grouped together based on difficulty level. We ran a linear regression model of combinations of exams, final scores and final grades as a function of the ACT math score and the Calculus Placement exam separated by difficulty level of the problems. We find that no matter what we run the regression against, splitting the placement exam by problem difficulty adds almost no extra information. We received the best results with the final exam score as a function of the Calculus Placement Exam. The ANOVA table for a linear model of final exam grade as a function of the total Calculus Placement Exam score is given below:

```
Call:
lm(formula = final ~ Cptotal)

Residuals:
      Min        1Q    Median        3Q       Max
-118.2961  -23.2083    0.3175   32.4147   78.1606

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  57.5935     7.2784   7.913 6.68e-13 ***
Cptotal       3.0351     0.4062   7.473 7.54e-12 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 39.67 on 141 degrees of freedom
Multiple R-squared: 0.2837,     Adjusted R-squared: 0.2786
F-statistic: 55.84 on 1 and 141 DF,  p-value: 7.537e-12
```

28.37 percent of the variation is explained by this model. Now we compare this to a model that considers each level of difficulty as its own variable. The ANOVA table is given below:

```
Call:
lm(formula = final ~ easy + med + hard)

Residuals:
       Min         1Q     Median         3Q        Max
-122.11334  -23.80288   -0.05077   31.69228   76.16066

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   57.575      8.761   6.572 9.26e-10 ***
easy           2.812      0.974   2.887  0.00451 **
med            3.707      1.119   3.312  0.00118 **
hard           2.486      1.356   1.834  0.06886 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 39.89 on 139 degrees of freedom
Multiple R-squared: 0.2859,     Adjusted R-squared: 0.2705
F-statistic: 18.55 on 3 and 139 DF,  p-value: 3.54e-10
```

28.59 percent of the variation is explained by this model. This model adds less than 0.3 percent more information and two extra variables. Also notice the "hard" problems are not very statistically significant.

Since we received the best results comparing the final exam to the total Calculus placement score, we now consider adding in the ACT mathematics score. A linear regression model for the final exam score as function of the Calculus Placement total was generated using the R statistical language. The output ANOVA table is given below:

```
Call:
lm(formula = final ~ Atotal + Cptotal + actm)

Residuals:
     Min       1Q   Median       3Q      Max 
-128.133  -21.259   -1.379   27.365   77.773 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -66.2991    23.9477  -2.768   0.0064 ** 
Atotal        0.4454     0.6804   0.655   0.5138    
Cptotal       2.0009     0.4421   4.526 1.28e-05 ***
actm          4.5787     0.9449   4.846 3.33e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 36.17 on 139 degrees of freedom
Multiple R-squared: 0.4128,     Adjusted R-squared: 0.4001 
F-statistic: 32.57 on 3 and 139 DF,  p-value: 5.222e-16 
```

The placement scores from the Algebra portion of the exam show to be statistically insignificant in the linear model. We omit the Algebra score and create another linear model. The ANOVA table is given below:

```
Call:
lm(formula = final ~ Cptotal + actm)

Residuals:
     Min       1Q   Median       3Q      Max 
-130.245  -20.570   -1.506   27.326   78.298 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -61.1676    22.5823  -2.709   0.0076 ** 
Cptotal       2.1147     0.4057   5.212 6.55e-07 ***
actm          4.8103     0.8744   5.501 1.74e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 36.1 on 140 degrees of freedom
Multiple R-squared: 0.411,      Adjusted R-squared: 0.4026 
F-statistic: 48.85 on 2 and 140 DF,  p-value: < 2.2e-16 
```

The output ANOVA table for the final as a function of ACT data only:

```
Call:
lm(formula = final ~ actm)

Residuals:
     Min       1Q   Median       3Q      Max
-127.808  -20.633    3.743   30.917   78.642

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -79.5798    24.2859  -3.277  0.00132 **
actm          6.6899     0.8674   7.713 2.03e-12 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 39.31 on 141 degrees of freedom
Multiple R-squared: 0.2967,     Adjusted R-squared: 0.2917
F-statistic: 59.49 on 1 and 141 DF,  p-value: 2.025e-12
```

The output ANOVA table for the final as a function of placement data:

```
Call:
lm(formula = final ~ Cptotal)

Residuals:
      Min        1Q    Median        3Q       Max
-118.2961  -23.2083    0.3175   32.4147   78.1606

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  57.5935     7.2784   7.913 6.68e-13 ***
Cptotal       3.0351     0.4062   7.473 7.54e-12 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 39.67 on 141 degrees of freedom
Multiple R-squared: 0.2837,     Adjusted R-squared: 0.2786
F-statistic: 55.84 on 1 and 141 DF,  p-value: 7.537e-12
```

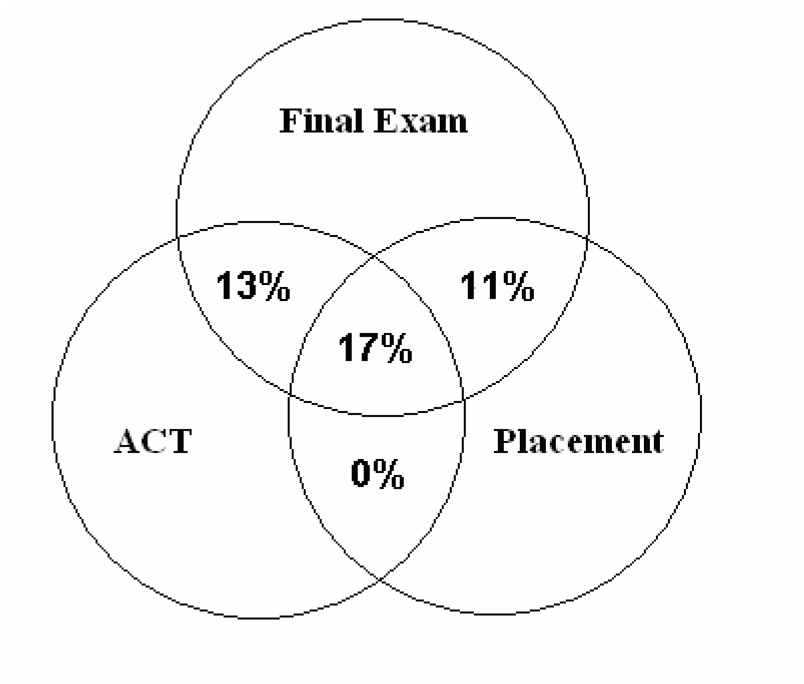
The output ANOVA table for the placement exam score as a function of ACT data:

```
Call:
lm(formula = Cptotal ~ actm)

Residuals:
    Min      1Q  Median      3Q     Max 
-16.181  -5.292  -0.403   4.541  23.486 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  -8.7068     4.6297  -1.881   0.0621 .  
actm          0.8889     0.1654   5.375 3.09e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 7.493 on 141 degrees of freedom
Multiple R-squared: 0.1701,     Adjusted R-squared: 0.1642 
F-statistic:  28.9 on 1 and 141 DF,  p-value: 3.087e-07
```

We have a model in which all variables are statistically significant and explain more than 41% of the variation in student test scores. ACT alone only explains 30% of the variation and placement scores alone only explain 28% of the variation. Adding the test scores provides explanation of only 17% of the variation.

There is an overlap between the information provided by ACT data and the placement exam. We analyze the overlap and obtain the following Venn Diagram below showing how much information is obtained from each test. There is no overlap in the information provided by placement and ACT scores outside of the information about performance on the final. There was an overlap between variation explained by the ACT and Placement exam, but the placement exam does offer an explanation of an additional 11% of the variation not explained by the ACT.

Figure 3.1Venn Diagram of Variance



After running these analyses on the final exam scores as a function of ACT scores and Calculus Placement scores, we observe that the estimate for the ACT math score is 4.8103 with a standard error of 0.4057 and the estimate for the Placement scores is 2.1147 with a standard error of 0.8744. A simplified formula is derived to ease advising. The coefficients 7 and 3 are chosen to be proportional to the estimates. The placement score is now defined as

7*actm + 3*Cptotal

We compute the standardized z-scores for student placement scores. These scores are normalized to have their mean score at 0 and standard deviation of 1. Plotting the z-scores against the probability of a student obtaining a grade of C or higher in the class, we obtain the graph below. A logistic curve is fit to the data, and those points are plotted with squares.

Figure 3.2 Z-score versus Percent C or Better

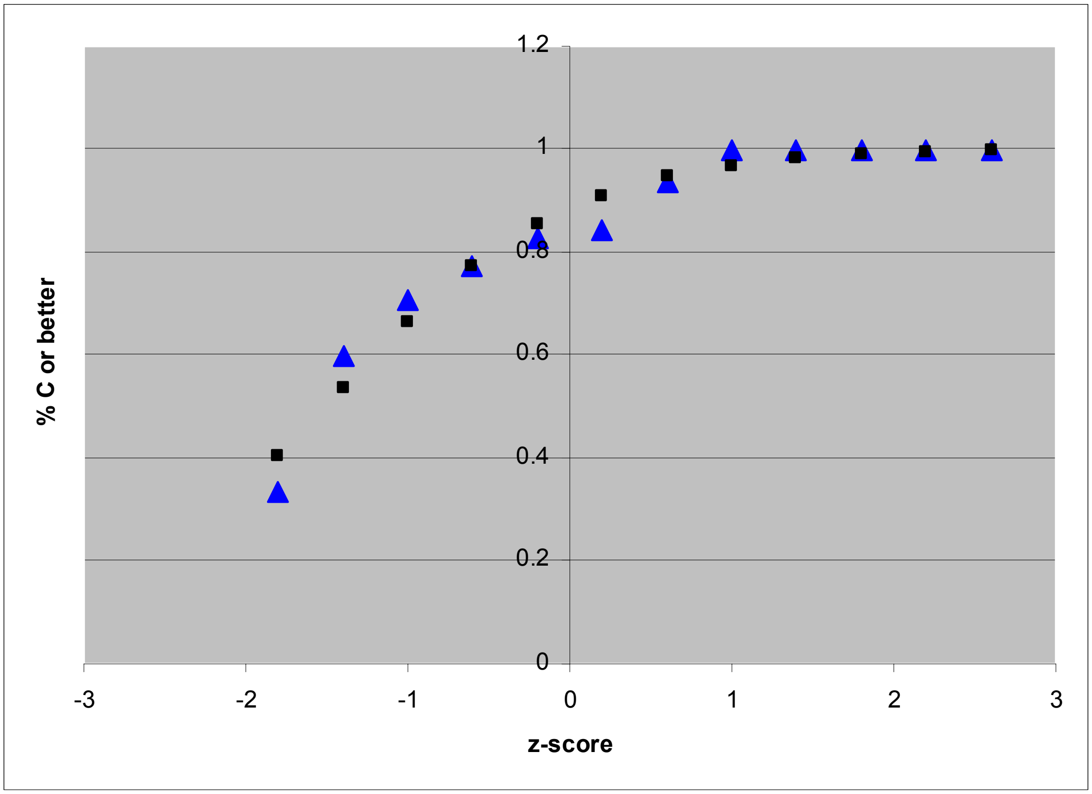


Table 3.1 Placement exam Z-scores and Model

z-score	#	# C or better	% C or better	model	error	a	b
-1.8	6	2	0.333333	0.4020	0.0047	1.358557	-1.5076
-1.4	5	3	0.6	0.5365	0.0040		
-1	17	12	0.705882	0.6659	0.0016		
-0.6	22	17	0.772727	0.7743	0.0000		
-0.2	29	24	0.827586	0.8553	0.0008		
0.2	19	16	0.842105	0.9105	0.0047		
0.6	16	15	0.9375	0.9460	0.0001		
1	12	12	1	0.9679	0.0010		
1.4	5	5	1	0.9811	0.0004		
1.8	7	7	1	0.9889	0.0001		
2.2	3	3	1	0.9935	0.0000		
2.6	2	2	1	0.9962	0.0000		
				SSE:	0.0174		

Using this table of data, we find that the logistic model is $y=1/(1+e^{-a(x-b)})$ where a=1.358557 and b=-1.5076. This means that a person with a z-score of -1.5076 has a 50% chance of getting a C or better in Calculus.

Noncompliance

All students were asked to take the placement exam prior to arriving for student orientation in June, but some refused. We decided to look at what proportion of students took the placement exam that completed Calculus I in the Fall of 2009. We compared the grades of those students that took the exam to those who did not take the exam.

Table 3.2 Percent of Grades C or better in Calculus I

With Placement			
A	26	18.31%	
B	55	38.73%	
C	36	25.35%	C or better
D	14	9.86%	82.39%
F	11	7.75%	
Total	142		
Freshmen w/o placement			
A	32	11.72%	
B	70	25.64%	
C	90	32.97%	C or better
D	38	13.92%	70.33%
F	43	15.75%	
Total	273		
"Upperclassmen"			
A	15	12.93%	
B	25	21.55%	
C	32	27.59%	C or better
D	16	13.79%	62.07%
F	28	24.14%	
Total	116		
Total			
A	73	13.75%	
B	150	28.25%	
C	158	29.76%	C or better
D	68	12.81%	71.75%
F	82	15.44%	
Total	531		

Regardless of how well they performed on the placement exam, there is a clear difference between the freshmen who took the placement exam and freshmen who did not. Figures 3.3 and 3.4 show the proportion of freshmen who received a C or better in the class based on if they did or did not take the placement exam, respectively. However, those who opted out of the placement exam are on par with the rest of the class.

Figure 3.3 Pie Chart of Freshmen in Calculus who took the Placement Exam

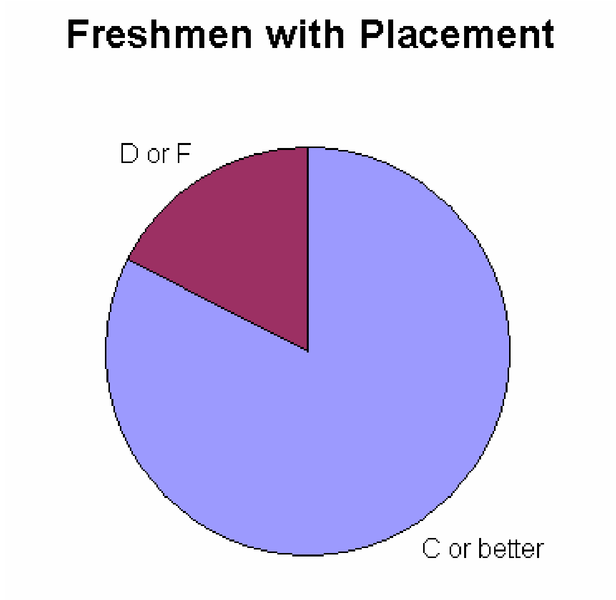
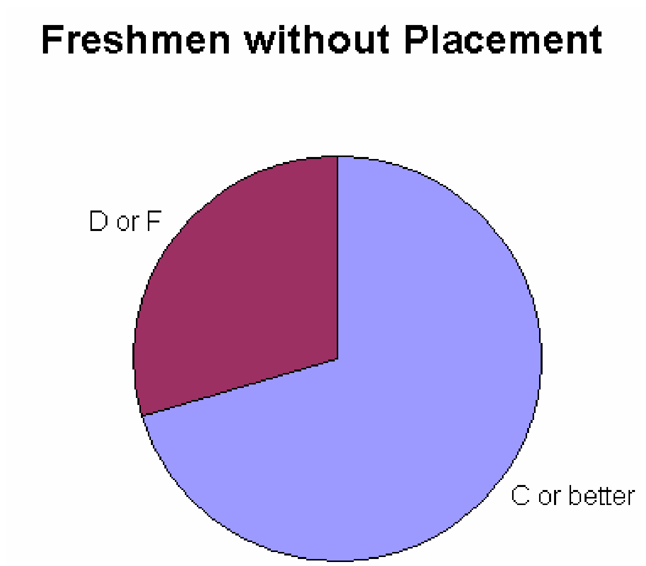


Figure 3.4Pie Chart of Freshmen in Calculus who Did NOT take the Placement Exam



We also looked at what proportion of students took the placement exam that completed Calculus II in the Fall of 2009. We compared the grades of those students that took the exam to those who did not take the exam, as well as the rest of the class.

Table 3.3 Percent of Grades C or better in Calculus II

With Placement			
A	40	42.55%	
B	24	25.53%	
C	26	27.66%	C or better
D	3	3.19%	95.74%
F	1	1.06%	
Total	94		

Freshmen w/o placement			
A	7	13.73%	
B	9	17.65%	
C	9	17.65%	C or better
D	16	31.37%	49.02%
F	10	19.61%	
Total	51		

"Upperclassmen"			
A	27	24.11%	
B	12	10.71%	
C	26	23.21%	C or better
D	20	17.86%	58.04%
F	27	24.11%	
Total	112		

Total			
A	74	28.79%	
B	45	17.51%	
C	61	23.74%	C or better
D	39	15.18%	70.04%
F	38	14.79%	
Total	257		

The numbers for Calculus II are more dramatic. 95.74% of the students entering into Calculus II who took the placement exam, regardless of score, passed with a grade of C or better compared to 49% of the students who chose not to take the exam.

Figure 3.5 Pie Chart of Freshmen in Calculus II who took the Placement Exam

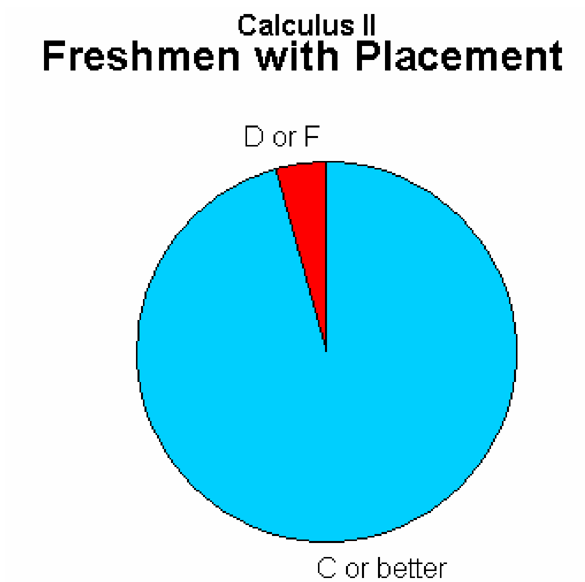
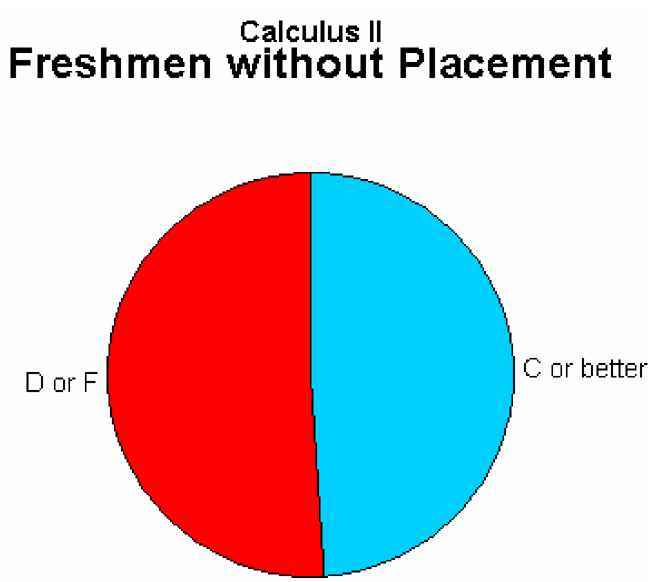


Figure 3.6 Pie Chart of Freshmen in Calculus II who took the Placement Exam



Chapter 4 - Conclusions

We asked whether or not administering a placement exam would increase the ability to place students in the correct mathematical course that matched their level of preparedness. Based upon our analysis of the administered test, we conclude that a placement exam does provide additional information that would aide advisors in helping a student decide what math course they should take. We were able to provide 11% more information that the ACT alone did not provide.

We may conclude that the exam does a fairly decent job in predicting chances of success in Calculus I students. Looking at table 3.2, there is a higher percentage of students who succeed that take the placement exam. However, the students who do not take the exam are on par with the rest of the class. This is a different story for Calculus II freshmen. The data for Calculus II students shows a clear self-selection bias. Many of the students who chose not to take the placement exam had skill deficiencies that would probably have been indicated by a placement exam. Because of the level of noncompliance among these students, we did not have enough data to determine whether or not the placement exam would accomplish the goal of placing students into Calculus II over Calculus I.

We now have to determine how these scores will be used to advise students. Because only 41% of the overall variation could be explained by ACT data and placement exam scores, the cutoff scores should be for advising purposes only and placement of students into classes should not be solely based on placement exam scores. Other factors such as previous high school experience, GPA, and AP credit should be taken into account.

Using item response theory to analyze a test, we want to see an exam that has item response curves of varying difficulties and levels of discrimination. The test that was administered does for the most part meet this goal. With exception of the two outliers, all the problems on the exam varied in difficulty from easy to hard and with low to high levels of discrimination. I do not think the two outliers are unreasonable problems. Page 5 Problem 1 asks the student to convert radians to degrees. This is a necessary skill required of every Calculus student. With a low level of difficulty and a high level of discrimination, we conclude that students either knew exactly how to perform the operation or they had no idea. Nearly 80% of students had a 50-50 chance of getting it correct. Problem 3 on page 8 of the exam covers the quotient rule. Though many students have been exposed to calculus before taking this exam, some have not. The Quotient Rule is a skill covered during the Calculus I course at Kansas State University. This might be a good question for discerning the difference between students ready for Calculus I versus Calculus II. But again, we do not have enough data to analyze the validity and usefulness of this problem due to the self-selection and noncompliance.

Recommendations for the Future

The exam is by no means perfected. The test could be improved by changing a few of the problems. The test does a great job in covering problems of different levels of difficulty. What is missing from the exam are problems with high levels of discrimination. Other than the two outliers, all of the problems have relatively equal levels of discrimination ranging from 0.05 to 0.08. The outliers have levels of .12 and .21. A bank of possible problems should be developed and tested to determine the levels of difficulty and discrimination.

As evidenced by the self-selection of the Calculus II students, this exam should be in place as a mandatory requirement for new freshmen. As stated before, the exam results should be considered for advising purposes only, but we believe that it will catch some students that are not prepared for a Calculus II course and help correctly place them in Calculus I.

We believe the addition of new problems will increase the reliability of the predictions of these exams. Hopefully it will lead to a reliable, mandated placement exam at Kansas State University.

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