A STUDY OF THE MEMRISTOR, THE FOURTH CIRCUIT ELEMENT

by

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Abstract

Every person with an electronics background will be familiar with the three fundamental circuit elements — the resistor, the capacitor, and the inductor. These three elements are defined by the relation between two of the four fundamental circuit variables — current, voltage, charge and flux. In 1971, Leon Chua reasoned on the grounds of symmetry that there should be a fourth fundamental circuit element which gives the relationship between flux and charge. He named this circuit element the memristor, which is short for "memory resistor." In May 2008, researchers at HP Labs published a paper announcing a model for the physical realization of the memristor.

This report focuses on the memristor and reviews its properties. The HP model for the memristor is also discussed, and its behavior is studied through simulations. A few of the potential applications of the memristor are presented.

Acknowledgments

Many minds and hands have contributed to the successful completion of this research. This research gave me an opportunity to learn from some of the best minds.

First, I would like to thank my major advisor, Dr. Stephen A. Dyer for acting as a guiding light throughout the course of this research. Dr. Dyer was there to help me whenever I needed it in spite of his busy schedule. I would also like to thank him for suggesting such an interesting topic for my research.

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I would also like to take this opportunity to thank my parents, Mrs. Rashmi Kerur and Mr. Ram Kerur, and my friends for their invaluable support and motivation.

Dedication

Dedicated to my family and friends

for their unconditional love and support.

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Chapter 1: Introduction

In circuit theory, the three basic two-terminal devices — namely the resistor, the capacitor and the inductor — are well understood. These elements are defined in terms of the relation between two of the four fundamental circuit variables, namely, current i, voltage v, charge q and flux φ. The current i is defined as the time derivative of the charge q. According to Faraday's law, the voltage v is defined as the time derivative of the flux φ. A resistor is defined by the relationship between voltage and current ($dv = Rdi$), the capacitor is defined by the relationship between charge and voltage ($dq = Cdv$) and the inductor is defined by the relationship between flux and current($d\varphi = Ldi$). Out of the six possible combinations of the four fundamental circuit variables, five are defined. In 1971, Prof. Leon Chua proposed that there should be a fourth fundamental circuit element to set up the relation between charge and magnetic flux and complete the symmetry as shown in Fig. 1.

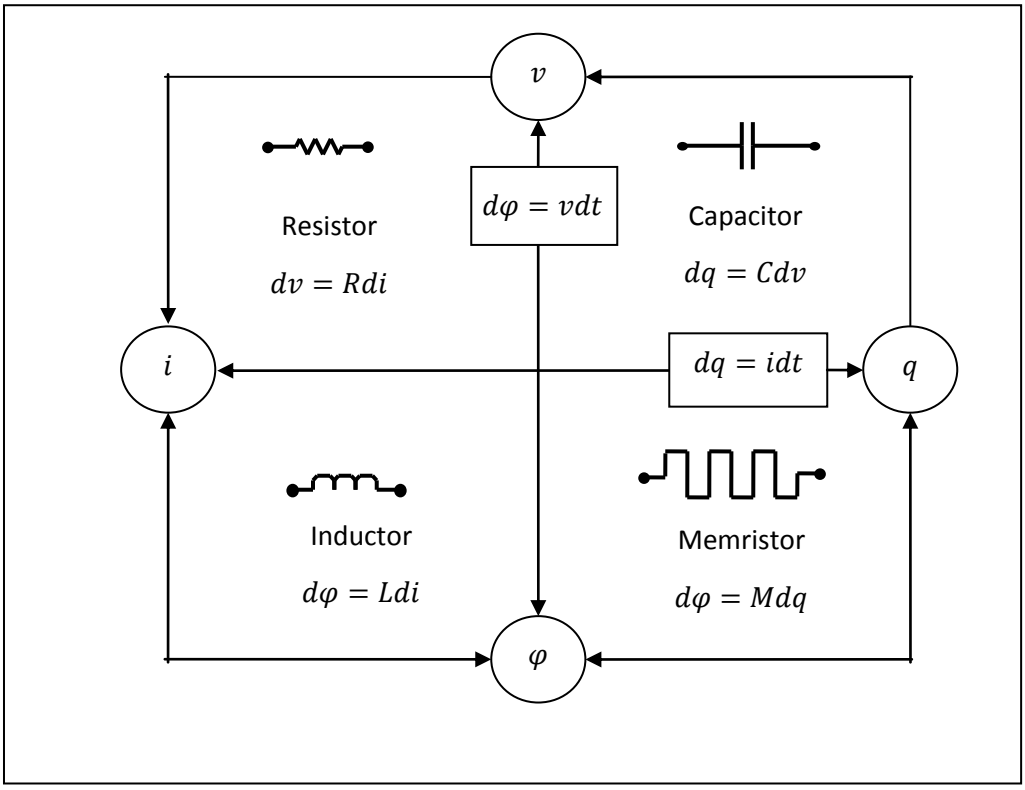

Fig. 1. Four basic circuit elements (Adapted from [1]).

Prof. Leon Chua named this the memristor, a short for "memory resistor." The memristor has a memristance M and provides a functional relation between charge and flux, $d\varphi = Mdq$. In 2008, Stanley Williams, et al., at Hewlett Packard, announced the first fabricated memristor.

This report focuses on the memristor and reviews its properties and applications. The model of the memristor from HP Labs and simulations showing the behavior of this memristor model for different input voltages are included in this report. The simulations are carried out using PSPICE and MATLAB.

1.1 Structure of the Report

Chapter 2 gives a brief description of the historical background of the memristor. Chapter 3 presents the theory and properties of the memristor. The HP model for the memristor is described in Chapter 4. Chapter 5 gives the results of the simulations. The potential applications of the memristor are described in Chapter 6. Finally, Chapter 7 provides a conclusion and a brief insight regarding possible future work.

Chapter 2: Historical Background

The concept of a resistor with memory existed even before Leon Chua's publication on the memristor in 1971. In 1960, Prof. Bernard Widrow of Stanford University developed a new circuit element named the "memistor." The memistor was a three-terminal device for which the conductance between two of the terminals was controlled by the time-integral of the current into the third terminal. Thus, the resistance of the memistor was controlled by charge. Memistors formed the basic components of the neural-network architecture called ADALINE (ADAptive LInear NEuron).

In 1968, F. Argall published a paper, "Switching phenomena in titanium oxide thin films," which shows results similar to that of the memristor model proposed by Stanley Williams and his team.

In 1971, Leon Chua mathematically predicted [2] that there is a fourth fundamental circuit element characterized by a relationship between charge and flux linkage.

In 1976, Leon Chua and Sung Mo Kang published a paper entitled "Memristive devices and systems" [3], generalizing the theory of memristors and memristive systems.

In 1990, S.Thakoor, et al., [4] demonstrated a tungsten-oxide variable- resistance device that is electrically reprogrammable. It is not clear whether the memistor device described has any relation with Chua's memristor [5].

Four years later, in 1994, Buot and Rajgopal published an article entitled "Binary information storage at zero bias in quantum-well diodes" [6]. The article described current–voltage characteristics similar to that of the memristor in AlAs/GaAs/AlAs quantum-well diodes. The analysis showed no direct connection to Chua's memristor [5].

In 2000, Beck, et al., of IBM's Zurich Research Laboratory, described reproducible resistance switching effects in thin oxide films [7]. The hysteretic features of these switches are similar to those of the memristor.

In 2001, Liu, et al., researchers in the Space Vacuum Epitaxy Center of the University of Houston, presented results [8] during a non-volatile memory conference held in San Diego, California, showing the importance of oxide bilayers to achieve high-to-low resistance ratio.

Apart from the devices mentioned above, it is interesting to note that between 1994 and 2008 there were many other devices developed with behavior similar to that of the memristor, but only the HP scientists were successful in finding a link between their work and the memristor postulated by Chua [9].

In 2008, thirty-seven years after Leon Chua's proposal, the memristor in device form was developed by Stanley Williams and his group in the Information and Quantum Systems (IQS) Lab at HP. Dmitri Strukov, Gregory Snider, Duncan Stewart, and Stanley Williams, of HP Labs, published an article [10] identifying a link between the two-terminal resistance switching behavior found in nanoscale systems and Leon Chua's memristor. The model proposed by them is described in detail in Chapter 4 of this report.

Victor Erokhin and M. P. Fontana claimed to have developed a polymeric memristor [11] before the titanium-dioxide memristor developed by Stanley Williams' group.

Since the announcement of the break through by Stanley Williams' group, numerous papers with the aim to analyze the elementary attributes of the memristor and memristor applications in various areas of circuit design have appeared.

Later in 2008, J. Joshua Yang, Matthew D. Pickett, Xuema Li, Douglas A. A. Ohlberg, Duncan R. Stewart and R. Stanley Williams published an article [12] demonstrating the memristive switching behavior and mechanism in nanodevices.

In October 2008, Yu V. Pershin, S. La Fontaine, M. Di Ventra published an article [13] identifying memristive behavior in amoeba's learning.

In January 2009, Sung Hyun Jo, Kuk-Hwan Kim, and Wei Lu of the University of Michigan published an article [14] describing an amorphous-silicon–based memristive material capable of being integrated with CMOS devices.

In June 2009, scientists at NIST [15] reported that they had fabricated nonvolatile memory using a flexible memristor that is both inexpensive and low-power.

Chapter 3: Memristor Theory

3.1 Origin of the Memristor

There are four fundamental circuit variables in circuit theory. They are current i, voltage v, charge q and flux φ. There are six possible combinations of the four fundamental circuit variables. We have a good understanding of five of the possible six combinations. The three basic two-terminal devices of circuit theory—namely, the resistor, the capacitor and the inductor—are defined in terms of the relation between two of the four fundamental circuit variables. A resistor is defined by the relationship between voltage and current ($dv = Rdi$), the capacitor is defined by the relationship between charge and voltage ($dq = Cdv$), and the inductor is defined by the relationship between flux and current ($d\varphi = Ldi$). In addition, the current i is defined as the time derivative of the charge q, and according to Faraday's law, the voltage v is defined as the time derivative of the flux φ. These relations are shown in Fig. 2.

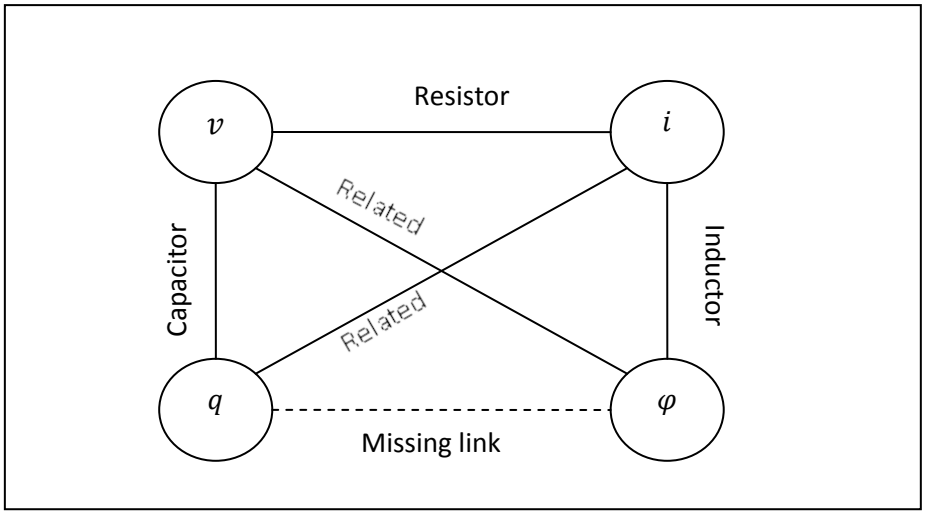

Fig. 2. The three circuit elements defined as a relation between four circuit variables (Adapted from [16]).

Leon Chua compared this way of defining circuit elements to Aristotle's theory of matter [16]. According to this theory, all matter consisted of the following four elements:

- Earth
- Water
- Air
- Fire

Each of these elements exhibited two of the four fundamental properties — moistness, dryness, coldness and hotness.

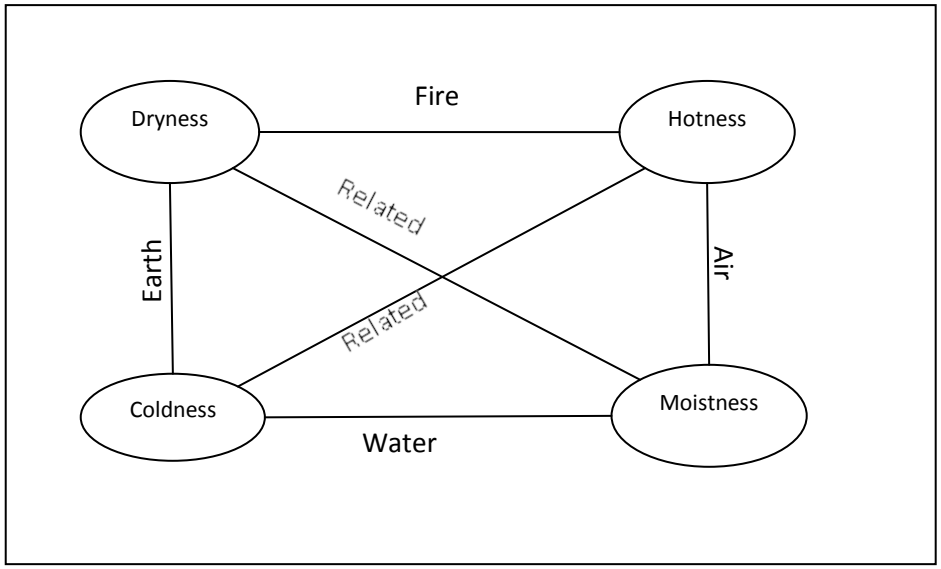

Fig. 3. Aristotle's theory of matter (Adapted from [16]).

Leon Chua saw a striking resemblance between the relation among elements and the relation among various circuit variables. He reasoned that there should be a fourth fundamental circuit element to complete the symmetry. He called this element, which sets up a relation between flux and charge, the *memristor*.

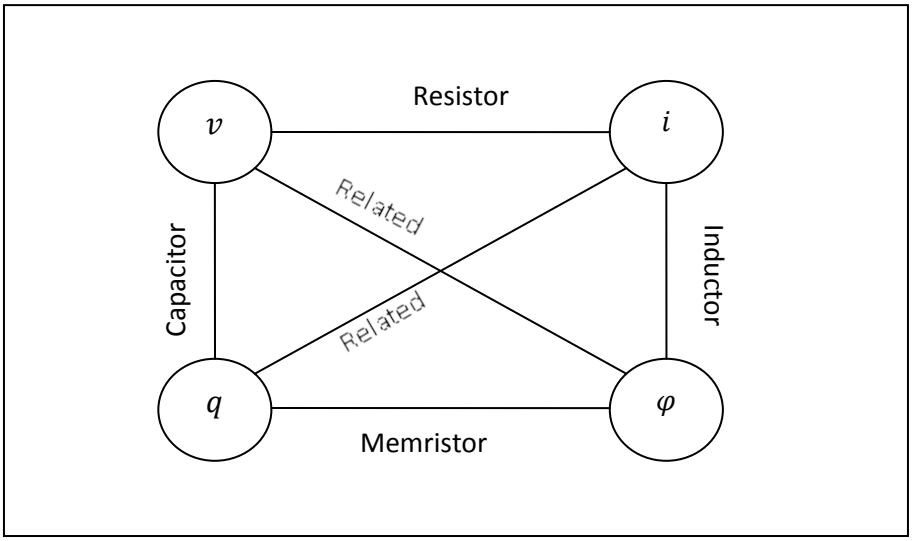

Fig. 4. The four fundamental two-terminal circuit elements (Adapted from [16]).

3.2 Definition of a Memristor

Memristor, the contraction of "memory resistor," is a passive device that provides a functional relation between charge and flux. "It is defined as a two-terminal circuit element in which the flux φ between the two terminals is a function of the amount of electric charge q that has passed through the device" [5]. Memristor is not an energy- storage element. Fig. 5 shows the symbol for a memristor.

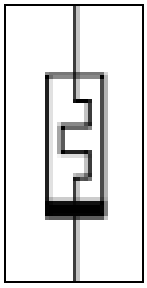

Fig. 5. Symbol of the memristor (Adapted from [5]).

A memristor is said to be charge-controlled if the relation between flux and charge is expressed as a function of electric charge q and it is said to be flux-controlled if the relation between flux and charge is expressed as a function of the flux linkage φ [2].

For a charge-controlled memristor,

$$\varphi = f(q) \tag{1}$$

Differentiating equation (1) yields

$$\frac{d\varphi}{dt} = \frac{df(q)}{dq}\,\frac{dq}{dt}, \tag{2}$$

giving the voltage $v(t) = \frac{d\varphi}{dt}$ as

$$v(t) = M(q)i(t), \tag{3}$$

where

$$M(q) = \frac{df(q)}{dq}. \tag{4}$$

$M(q)$ is called as memristance, and it has the units of resistance. Memristance defines a linear relationship between current and voltage, as long as the charge does not vary. Thus if M is constant, a memristor behaves as a resistor.

For a flux-controlled memristor,

$$q = f(\varphi). \tag{5}$$

Differentiating equation (5) yields

$$\frac{dq}{dt} = \frac{df(\varphi)}{d\varphi}\frac{d\varphi}{dt}, \tag{6}$$

giving the current $i(t) = \frac{dq}{dt}$ as

$$i(t) = W(\varphi)v(t), \tag{7}$$

where

$$W(\varphi) = \frac{df(\varphi)}{d\varphi}. \tag{8}$$

$W(\varphi)$ is called as memductance and it has the units of conductance.

3.3 What is Memristance?

Memristance is a property of the memristor [17]. When the charge flows in one direction through a circuit, the resistance of the memristor increases, and its resistance decreases when the charge flows in the opposite direction in the circuit. If the applied voltage is turned off, thus stopping the flow of charge, the memristor "remembers" the last resistance that it had. When the flow of charge is started again, the resistance of the circuit will be what it was when it was last active.

3.4 Analogy of a Memristor

An analogy of a memristor is described in [17]. A resistor is analogous to a pipe through which water flows. The pressure of water at the input of the pipe is analogous to the voltage, and water is analogous to electric charge. The rate of flow of water through the pipe is similar to electric current. If the pipe has a larger diameter, the flow of water through the pipe is faster, just like more current flows through resistor with a small value of resistance. An analogy for a memristor is a different kind of pipe—whose diameter expands or shrinks depending on the direction of the

water flow through it. The diameter of the pipe increases when the water flows in one direction, enabling water to flow faster, and the diameter of the pipe decreases when the water flows in the opposite direction, thus slowing down the water flow. If no water is let into the pipe, the pipe will retain its most recent diameter until the water is turned back on. Thus, the pipe remembers the amount of the water that has flowed through it.

3.5 Properties of a Memristor

3.5.1 φ–q Curve of a Memristor

The φ–q curve of a memristor is a monotonically increasing [16]. The memristance $M(q)$ is the slope of the φ–q curve. According to the memristor passivity condition, a memristor is passive if and only if memristance $M(q)$ is non-negative [2]. If $M(q) \geq 0$, then the instantaneous power dissipated by the memristor, $p(i) = M(q)\big(i(t)\big)^2$, is always positive, and so the memristor is a passive device [9]. The memristor is purely dissipative, like a resistor. Thus, the φ–q curve of a memristor is always a monotonically increasing function. Fig. 6 shows some examples of typical φ–q curves of memristors.

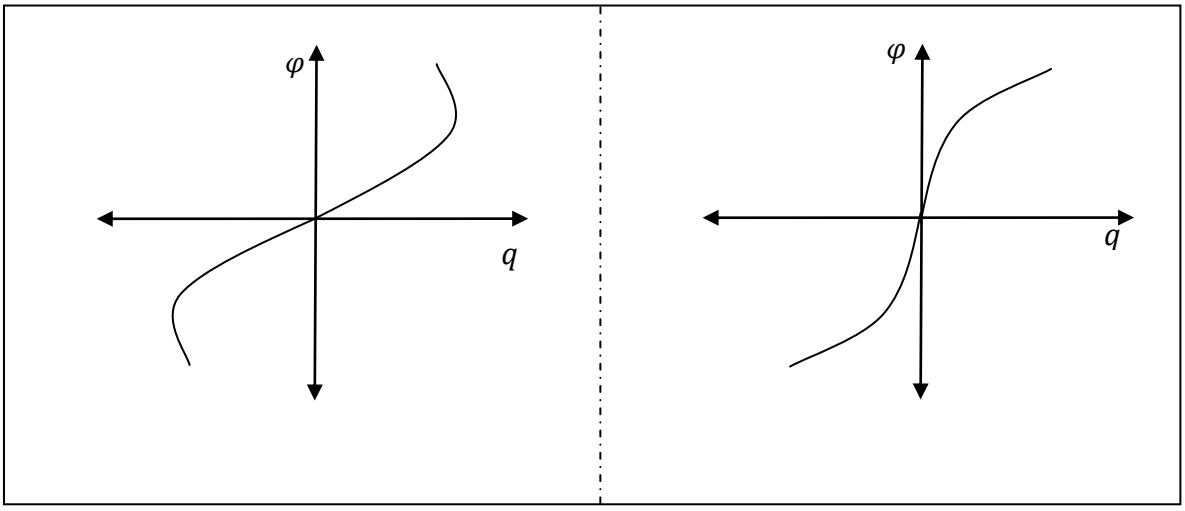

Fig. 6. Typical φ–q curves of memristors(Adapted from [16]).

3.5.2 Current–Voltage Curve of a Memristor

An important fingerprint of a memristor is the "pinched hysteresis loop" current—voltage characteristic [16]. For a memristor excited by a periodic signal, when the voltage $v(t)$ is zero, the current $i(t)$ is also zero and vice versa. Thus, both voltage $v(t)$ and current $i(t)$ have

identical zero-crossing. If any device has a current–voltage hysteresis curve, then it is either a memristor or a memristive device [16]. Another signature of the memristor is that the "pinched hysteresis loop" shrinks with the increase in the excitation frequency [16]. Figure 7 shows the "pinched hysteresis loop" and an example of the loop shrinking with the increase in frequency. In fact, when the excitation frequency increases towards infinity, the memristor behaves as a normal resistor.

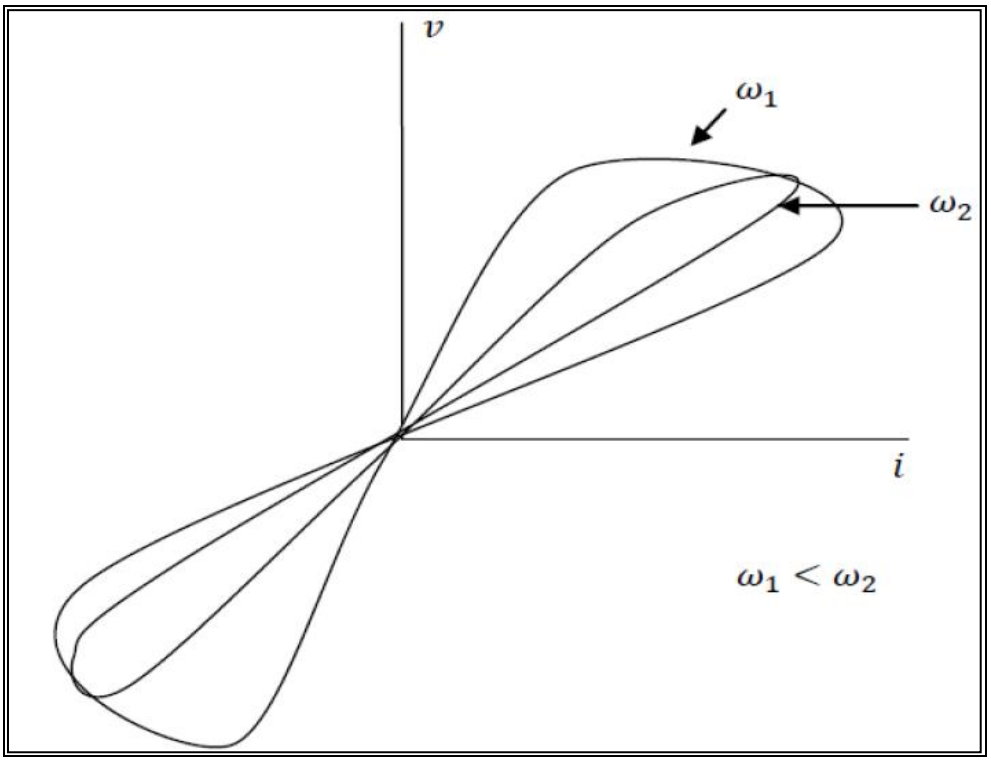

Fig. 7. The pinched hysteresis loop and the loop shrinking with the increase in frequency (Adapted from [18]).

3.6 Memristive Systems

In 1976, five years after Chua's paper on the memristor, he and Kang [3] published a paper defining a much broader class of systems, named memristive systems. The memristive systems are described by

$$v = M(w, i)i \tag{9}$$

and $$\frac{dw}{dt} = f(w, i), \quad (10)$$

where w is a set of state variables, M and f can be functions of time, and v and i are the voltage and current, respectively.

The fundamental memristive system theorem states that "every two-terminal device which exhibits a pinched hysteresis loop in the current-versus-voltage plane when driven by a dc and/or sinusoidal signal of any frequency is a memristive system"[16].

Chapter 4: Model of the Memristor from HP Labs

In 2008, thirty-seven years after Chua proposed the memristor, Stanley Williams and his group at HP Labs realized the memristor in device form. The HP model of the memristor is described in [9]. To realize a memristor, they used a very thin film of titanium dioxide (TiO_2). The thin film is sandwiched between two platinum (Pt) contacts and one side of TiO_2 is doped with oxygen vacancies. The oxygen vacancies are positively charged ions. Thus, there is a TiO_2 junction where one side is doped and the other side is undoped. The device established by HP is shown in Fig. 8.

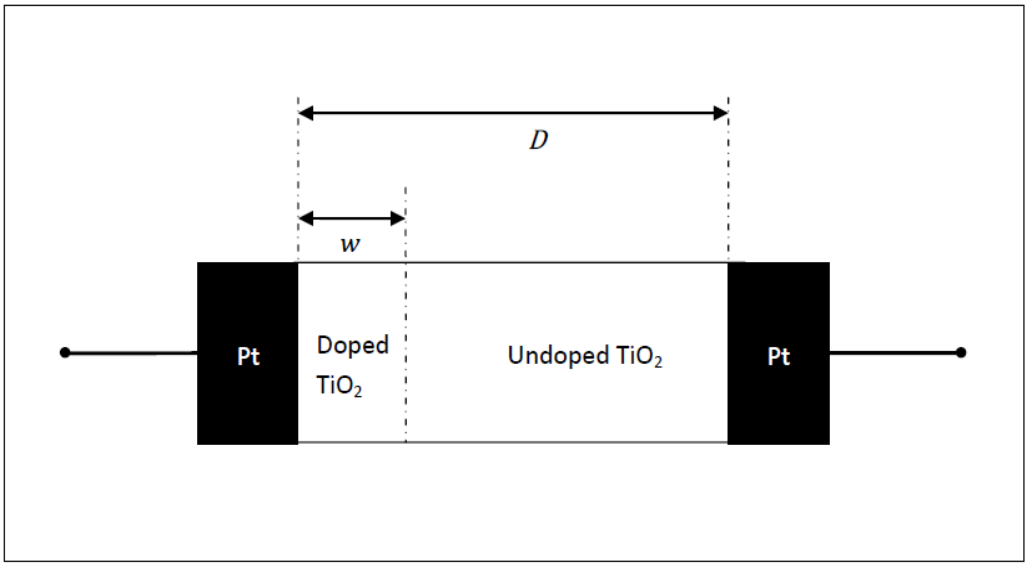

Fig. 8. Schematic of HP memristor (Adapted from [9]).

In Fig.8, *D* is the device length and *w* is the length of the doped region. Pure TiO_2 is a semiconductor and has high resistivity. The doped oxygen vacancies make the TiO_{2-x} material conductive. The working of the memristor established by HP is described in [19]. When a positive voltage is applied, the positively charged oxygen vacancies in the TiO_{2-x} layer are repelled, moving them towards the undoped TiO_2 layer. As a result, the boundary between the two materials moves, causing an increase in the percentage of the conducting TiO_{2-x} layer. This increases the conductivity of the whole device. When a negative voltage is applied, the positively charged oxygen vacancies are attracted, pulling them out of TiO_2 layer. This increases the amount of insulating TiO_2, thus increasing the resistivity of the whole device. When the voltage is turned off, the oxygen vacancies do not move. The boundary between the two titanium dioxide

layers is frozen. This is how the memristor 'remembers' the voltage last applied. Figure 9 explains the behavior of the memristor model when positive and negative voltage is applied. Fig. 9(a) shows the thin film of titanium-dioxide where one side is doped with positive oxygen vacancies and the other side is the undoped. Fig. 9(b) shows the behavior when a positive voltage is applied. The positive oxygen vacancies are repelled and they move towards the undoped TiO_2 layer, reducing the percentage of the insulating TiO_2, thus decreasing the resistivity. Fig. 9(c) shows the behavior when a negative voltage is applied. The positive oxygen vacancies are attracted and they move towards the doped TiO_{2-x} layer, increasing the percentage of the insulating TiO_2, thus increasing the resistivity.

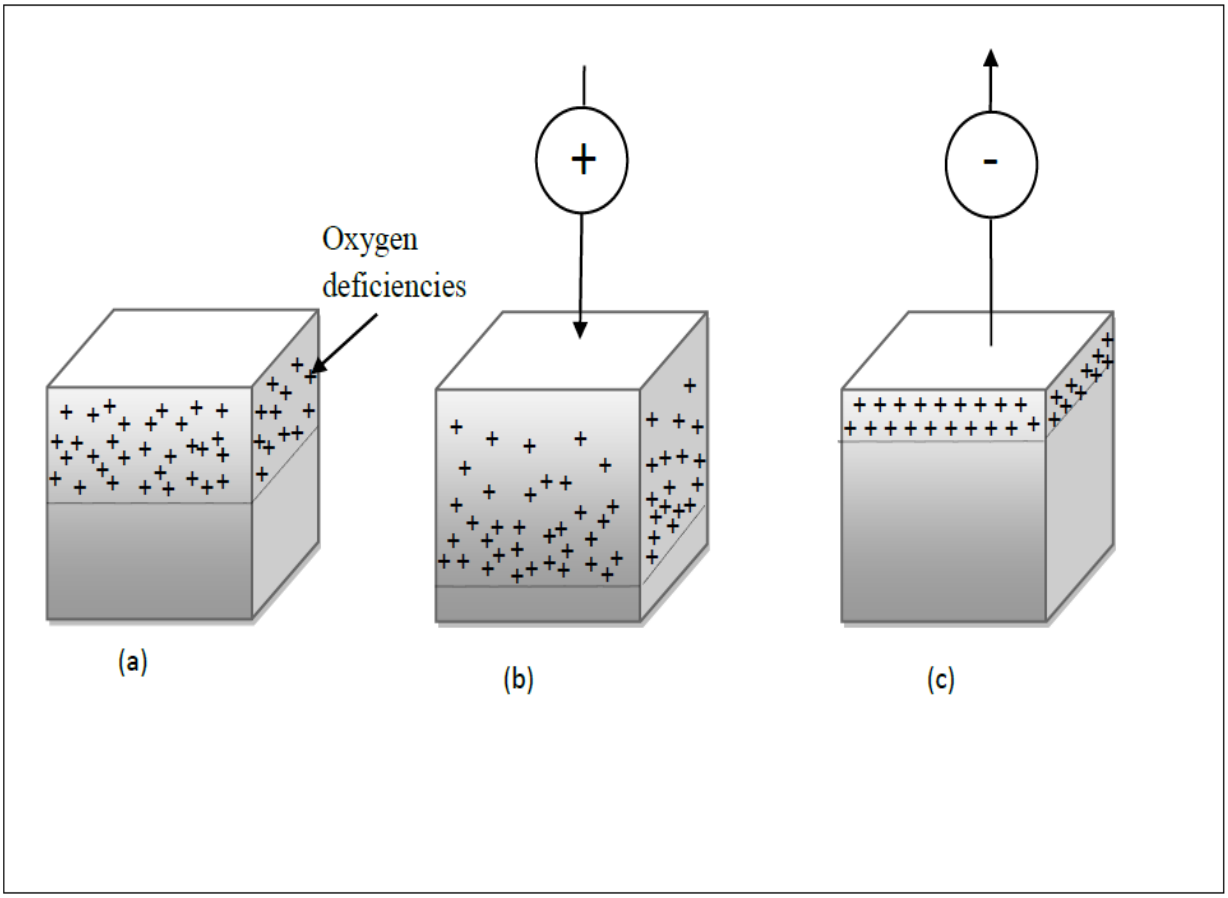

Fig. 9. Behavior of HP memristor when positive and negative voltages are applied (Adapted from [20]).

The simple mathematical model [9], [10] of the HP memristor is given by

$$M(q) = R_{OFF}(1 - \frac{R_{ON}}{\beta} q(t)) \tag{11}$$

where $\beta = \frac{D^2}{\mu_D}$ has the dimensions of magnetic flux. μ_D is the average drift velocity and has the units cm^2/sV; D is the thickness of titanium-dioxide film; R_{OFF} and R_{ON} are 'on-state' and 'off-state' resistances; and $q(t)$ is the total charge passing through the memristor device.

4.1 Linear Drift Model [9]

Let us assume a uniform electric field across the device. Therefore, there is a linear relationship between drift-diffusion velocity and the net electric field. The state equation can be written as

$$\frac{1}{D}\frac{dw(t)}{dt} = \frac{R_{ON}}{\beta} i(t). \tag{12}$$

Integrating (12) gives

$$\frac{w(t)}{D} = \frac{w(t_0)}{D} + \frac{R_{ON}}{\beta} q(t) \tag{13}$$

where $w(t_0)$ is the initial length of w. The speed of drift under a uniform electric field across the device is then given by

$$v_D = \frac{dw(t)}{dt}. \tag{14}$$

In an uniform field $D = v_D t$. In this case, $Q_D = it$ defines the amount of charge required to move the boundary from $w(t_0)$, where $w \to 0$, to distance $w(t_D)$, where $w \to D$. Therefore, $Q_D = \frac{\beta}{R_{ON}}$. Thus,

$$\frac{w(t)}{D} = \frac{w(t_0)}{D} + \frac{q(t)}{Q_D} \tag{15}$$

If $x(t) = \frac{w(t)}{D}$, then equation 15 can be written as

$$x(t) = x(t_0) + \frac{q(t)}{Q_D} \tag{16}$$

The amount of charge that is passed through the channel over the required charge for a conductive channel is given as $\frac{q(t)}{Q_D}$. Using the equations in [10] , we have

$$v(t) = \left(R_{ON} \frac{w(t)}{D} + R_{OFF} \left(1 - \frac{w(t)}{D} \right) \right) i(t) \tag{17}$$

Substituting $x(t) = \frac{w(t)}{D}$ in Eq. 17, we get

$$v(t) = (R_{ON} x(t) + R_{OFF}\big(1 - x(t)\big) i(t). \tag{18}$$

If we assume that the initial charge $q(t_0) = 0$, then $w(t) = w(t_0) \neq 0$, and

$$M_0 = R_{ON}(x(t_0) + r\big(1 - x(t_0)\big), \tag{19}$$

where $r = (\frac{R_{OFF}}{R_{ON}})$ and M_0 is the memristance value at t_0. Thus the memristance at time t is given by

$$M(q) = M_0 - \Delta R \left(\frac{q(t)}{Q_D} \right), \tag{20}$$

where $\Delta R = R_{OFF} - R_{ON}$. When $R_{OFF} \gg R_{ON}$, $M_0 \approx R_{OFF}$.

Substituting equation 20 into $v(t) = M(q) i(t)$, when $i(t) = \frac{dq(t)}{dt}$ we get,

$$v(t) = \left(M_0 - \Delta R \left(\frac{q(t)}{Q_D} \right) \right) \frac{dq(t)}{dt} \tag{21}$$

Since $M(q) = \frac{d\varphi(q)}{dq}$, the solution is

$$q(t) = \frac{Q_D M_0}{\Delta R} \left(1 \pm \sqrt{1 - \frac{2\Delta R}{Q_D {M_0}^2} \varphi(t)} \right) \tag{22}$$

For $\Delta R \approx M_0 \approx R_{OFF}$, Eq. 22 becomes

$$q(t) = Q_D \left(1 - \sqrt{1 - \frac{2}{Q_D R_{OFF}} \varphi(t)} \right). \quad (23)$$

If $Q_D = \frac{D^2}{\mu_D \, R_{ON}}$, then the internal state of the memristor is

$$x(t) = 1 - \left(\sqrt{1 - \frac{2\mu_D}{rD^2} \varphi(t)} \right) \quad (24)$$

The current-voltage relationship in this case is

$$i(t) = \frac{v(t)}{R_{OFF} \left(\sqrt{1 - \frac{1\mu_D}{rD^2} \varphi(t)} \right)} \quad (25)$$

Eq. 25 shows the inverse-square relation between memristance and TiO_2 thickness, D. Thus, for smaller values of D, the memristance shows improved characteristics. Nowadays, memristance becomes more important for understanding as the dimensions of electronic devices are shrinking to nanometer scale.

Chapter 5: Results and Simulations

5.1 SPICE Model [21]

Biolek, et al., provided in [21] a useful SPICE model of a memristor, the structure of which is shown in Fig. 10.

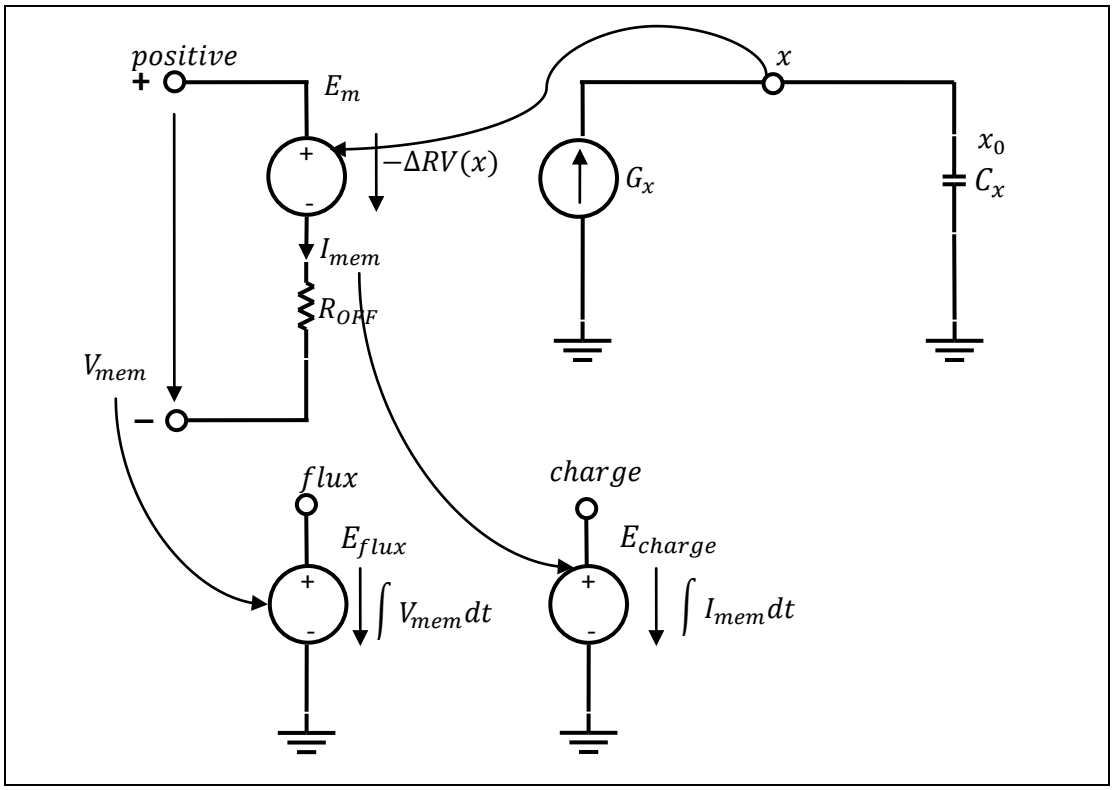

Fig. 10. Structure of the SPICE model (Adapted from [6]).

In the above circuit, V_{mem} is the input voltage and I_{mem} is modeled to be the current through the memristor. The flux is calculated by integrating the voltage V_{mem} and the charge is calculated by integrating the current I_{mem}. E_m is the voltage source whose terminal voltage is controlled according to the formula "$-x\Delta R$." G_x is a current source whose current is controlled according to the equation "$I_{mem}f(V(x))$" where $V(x)$ is the voltage across the capacitor C_x and it models the normalized width of the doped layer.

The relation between memristor current and voltage is modeled as on the basis of Eq. 26

$$R_{MEM}\,(x) = R_{OFF} - x\Delta R, \tag{26}$$

where

$$\Delta R = R_{OFF} - R_{ON}. \tag{27}$$

R_{OFF} is the resistor in series voltage source whose terminal voltage is controlled by the formula, $-x\Delta R$. The voltage $V(x)$ across the capacitor C_x models the normalized width x of the doped layer. The initial state of x is modeled by the initial voltage of the capacitor. The flux is calculated by the time-integral of voltage, and the charge is calculated by the time-integral of current. The listing for the SPICE model is included in the appendix.

5.2 Simulation Results— Using SPICE model

5.2.1 Simulations for a Memristor Driven by Voltage $v_0 sin(\omega_0 t)$ using PSPICE

For this simulation, the width D of the TiO_2 film is considered to be 10 nm and the dopant mobility μ_D= 10^{-10} cm^2 s^{-1} V^{-1}. The values assumed are-R_{ON}=1KΩ, R_{OFF}=100KΩ and the initial resistance R_{INIT} required to model the initial conditions of the capacitor is assumed to be 80KΩ. The simulation results are shown below in Figs. 11, 12, 13, 14, 15, 16 and 17.

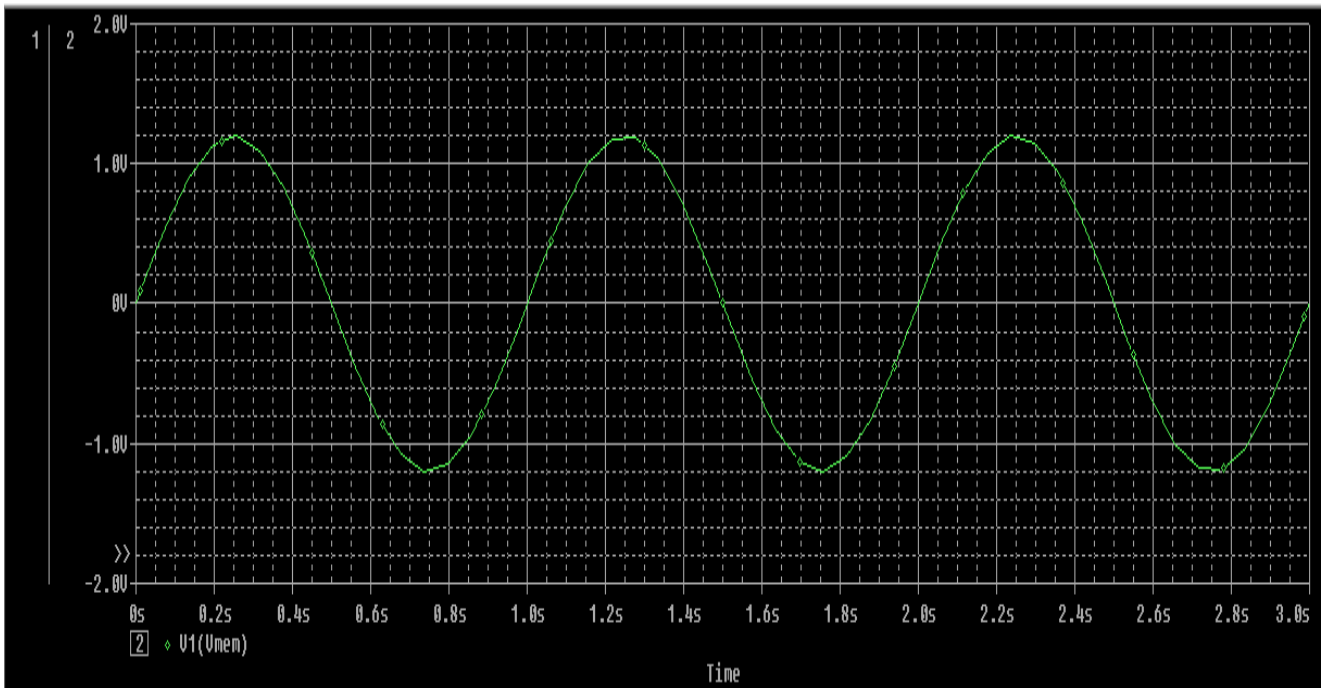

Fig. 11. An input voltage $v_0 \sin \omega_0 t$ applied to the memristor.

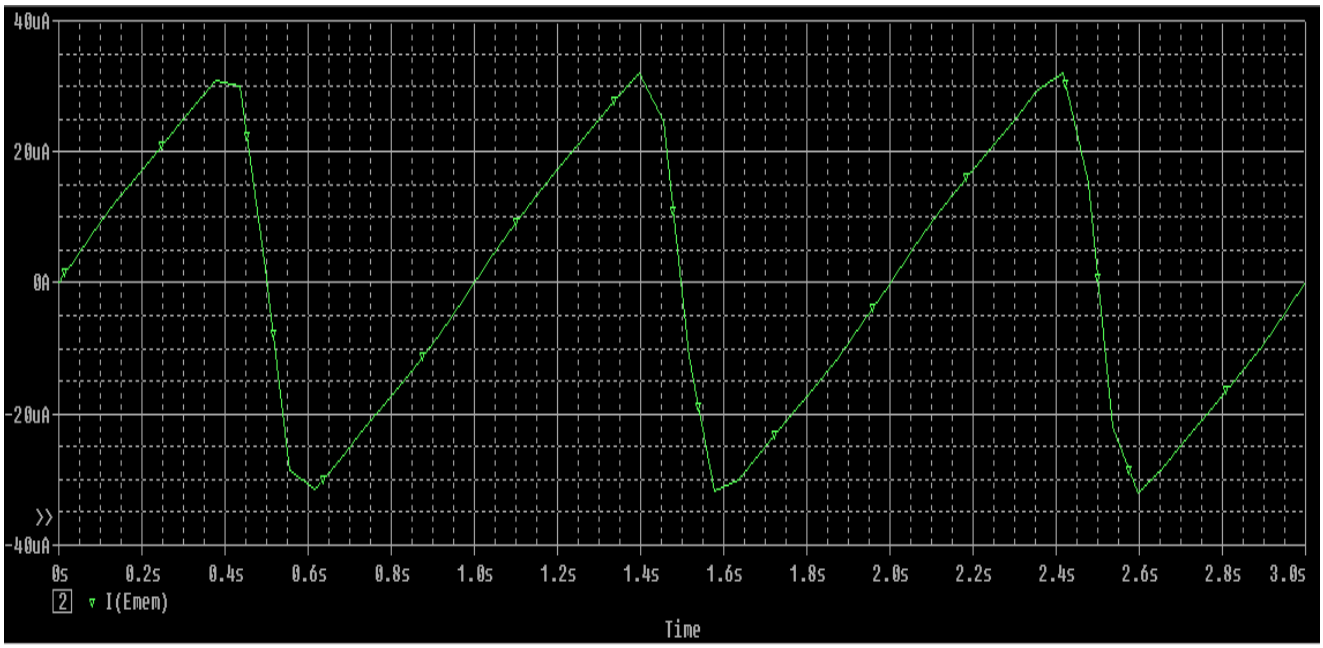

Fig. 12. Waveform of the current I_{mem} through the memristor.

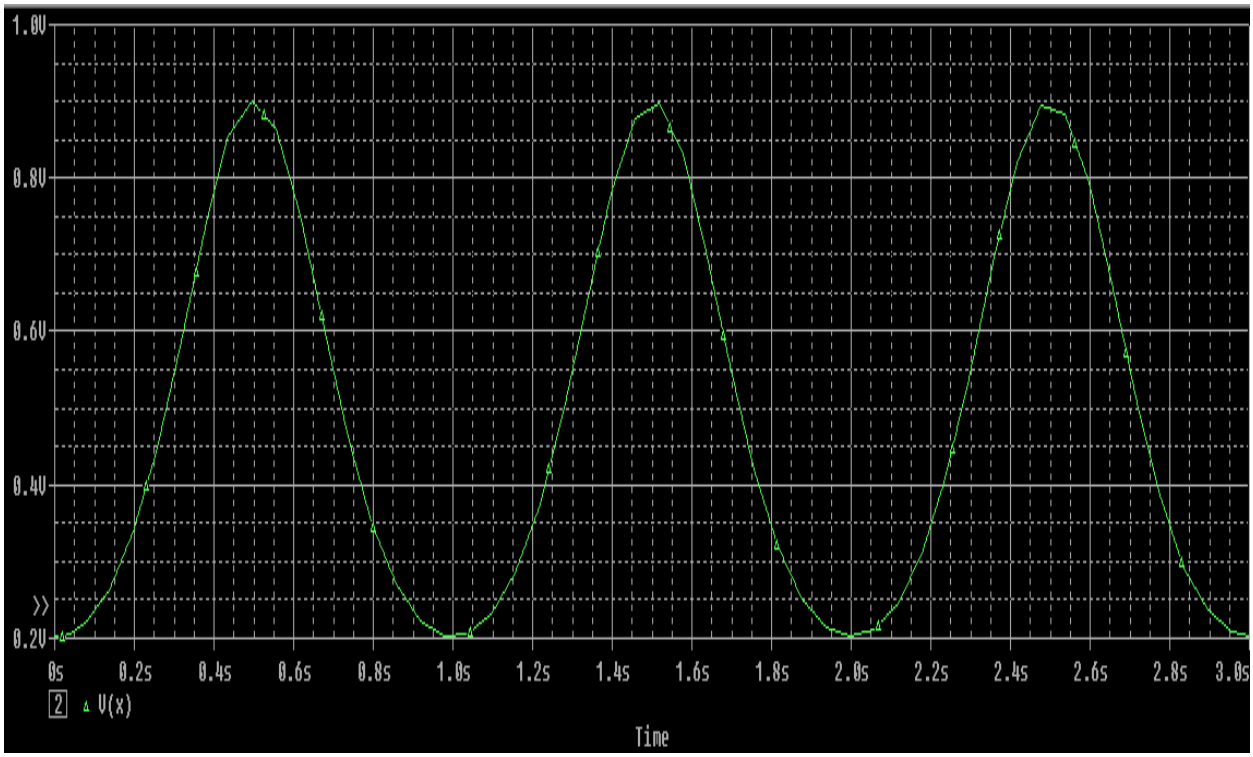

Fig. 13. Voltage across the capacitor C_x which gives the internal state of the device.

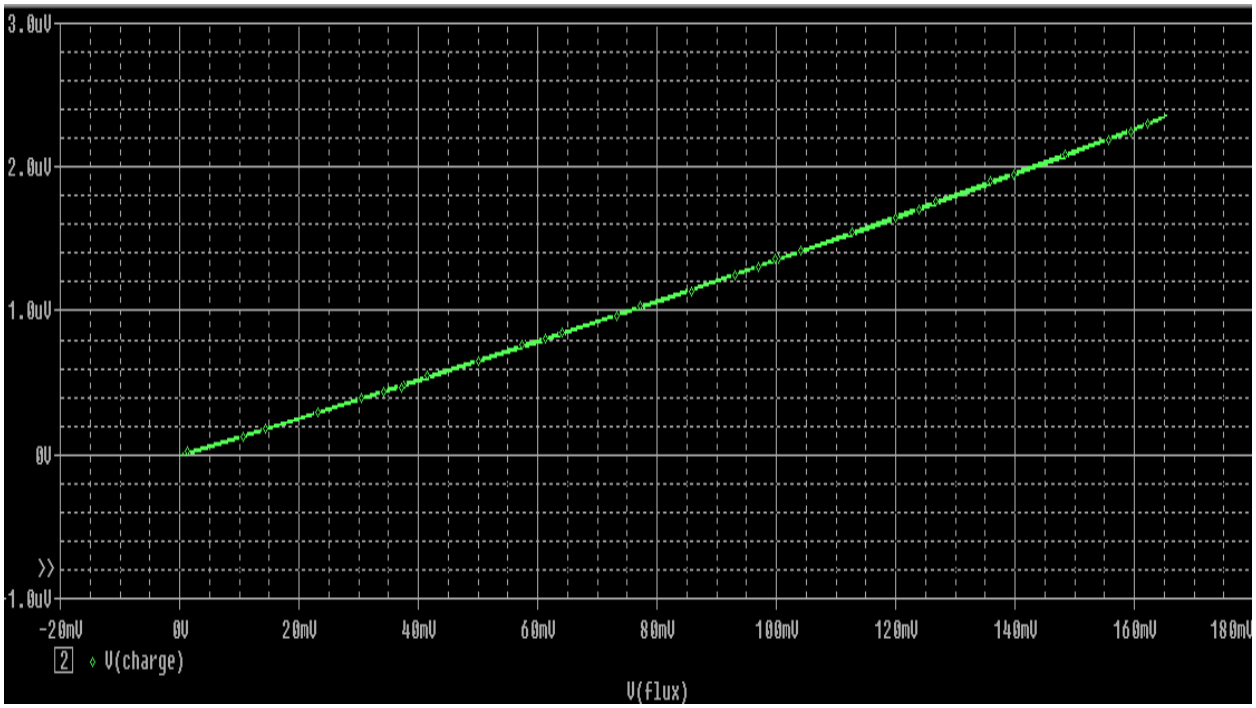

Fig. 14. Charge-versus-flux curve for memristor.

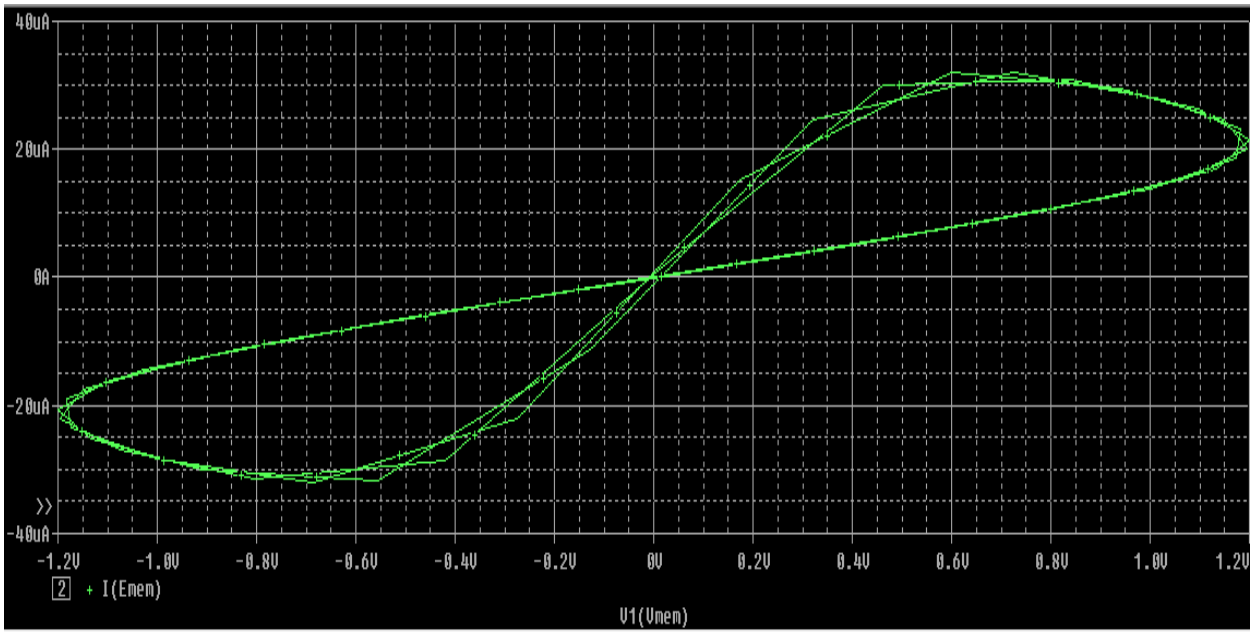

Fig. 15. Current-versus-voltage curve for input frequency of 1 Hz.

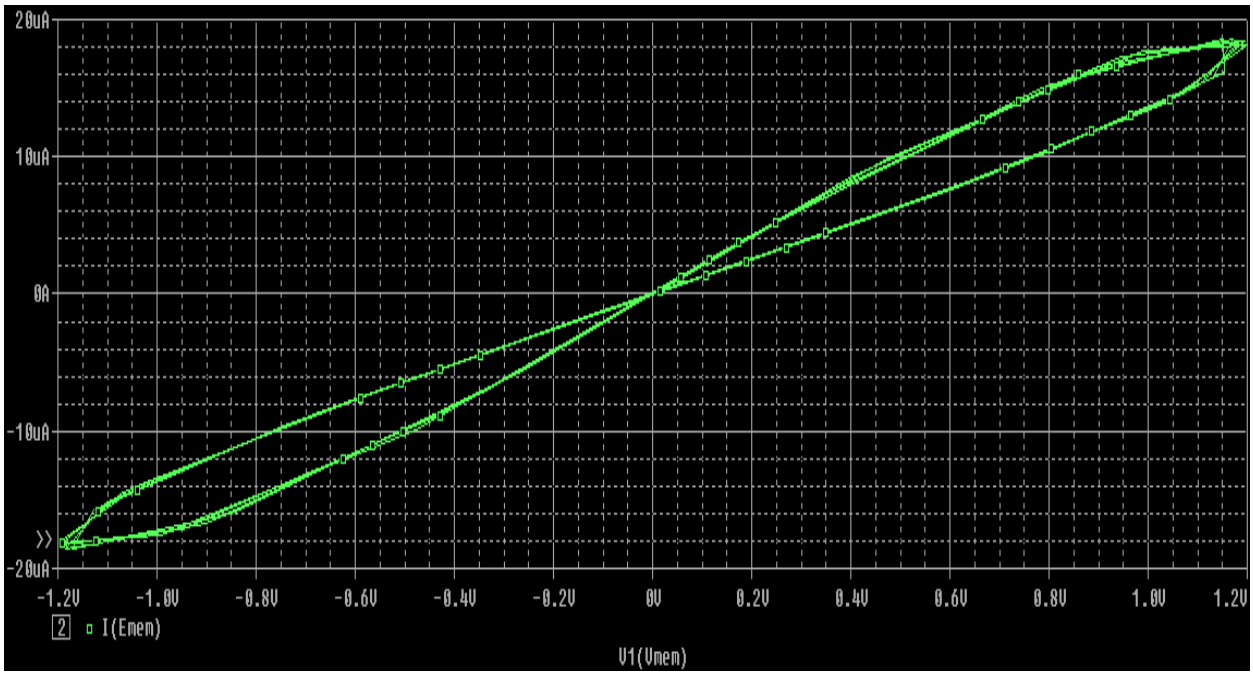

Fig. 16. Current-versus-voltage curve for input frequency of 1.5 Hz.

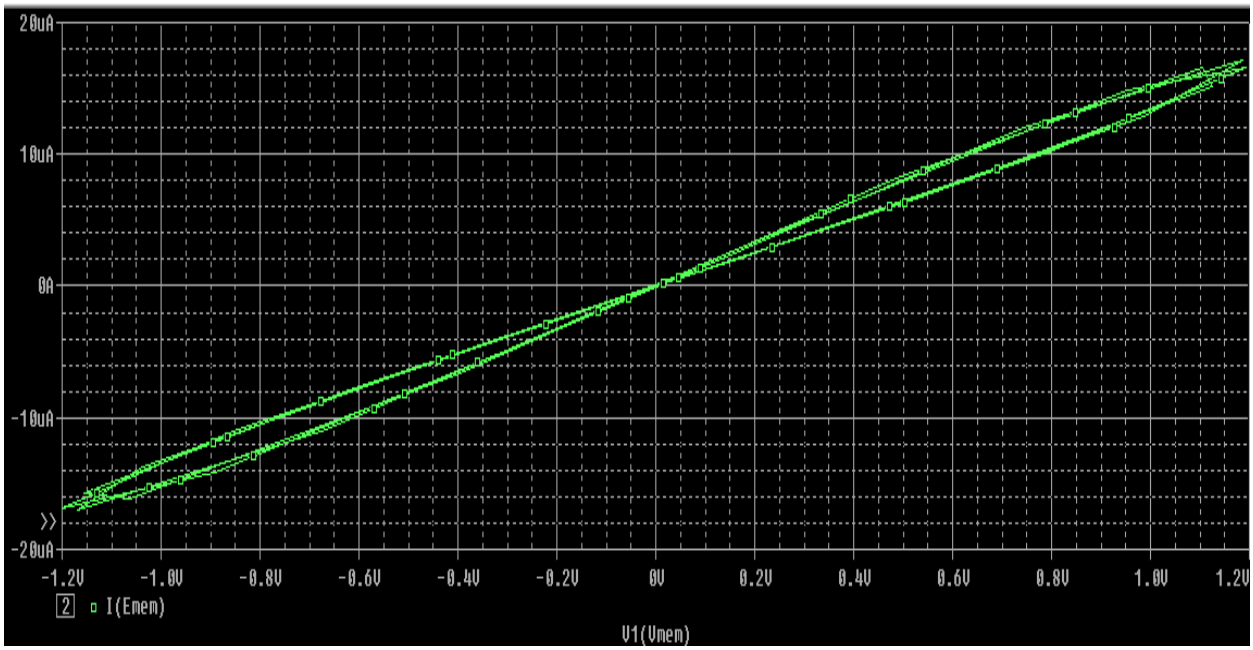

Fig. 17.Current-versus-voltage curve for input frequency of 2 Hz.

From Fig. 14, we can see that the flux—charge curve is a monotonically increasing curve. We can also observe the current—voltage “pinched hysteresis loop” and the hysteresis loop shrinking

with the increase in the input frequency in Fig. 15, Fig. 16 and Fig. 17. Thus, all the properties that are said to be a signature of a memristor are satisfied.

5.3 Simulation Results— Using MATLAB

5.3.1 Simulations for a Memristor Driven by Voltage $v_0 sin(w_0 t)$ using MATLAB

Using the equations (23), (24) and (25), studied in Chapter 4, a memristor is simulated using MATLAB. For this simulation, the width D of the TiO_2 film is considered to be 10 nm and the dopant mobility μ_D= 10^{-10} cm^2 s^{-1} V^{-1}. The values assumed are —R_{ON}=100Ω , R_{OFF}=16KΩ. The simulation results are shown below. The MATLAB code is included in the appendix.

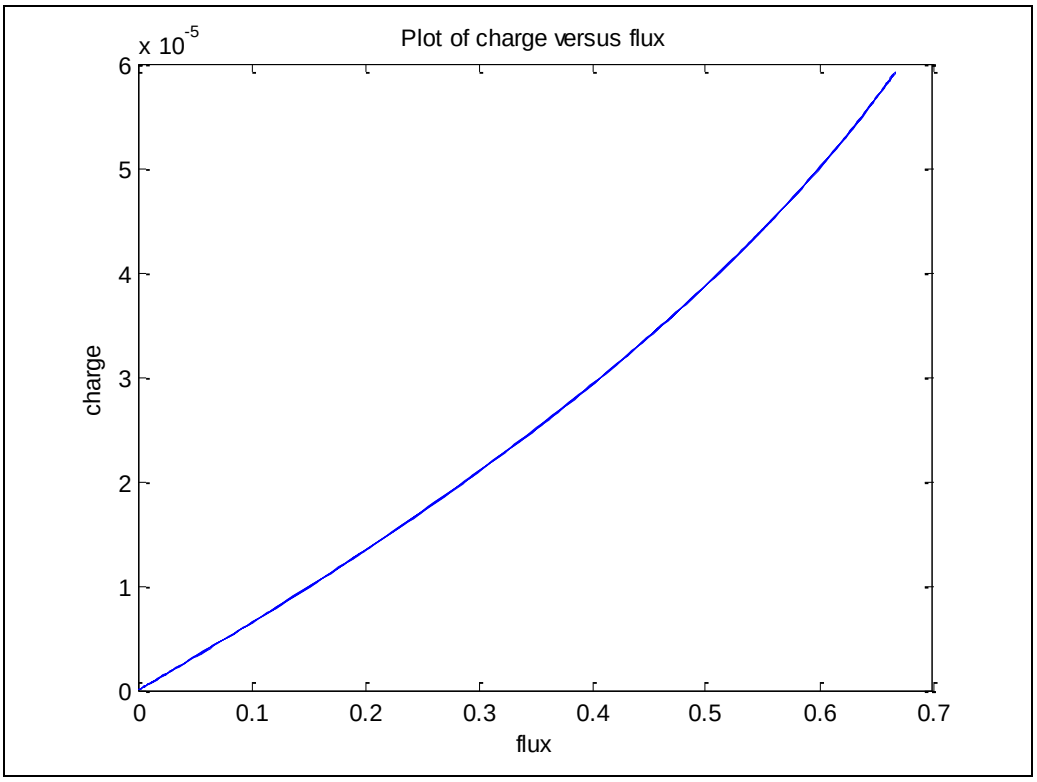

Fig. 18. Plot of flux versus charge.

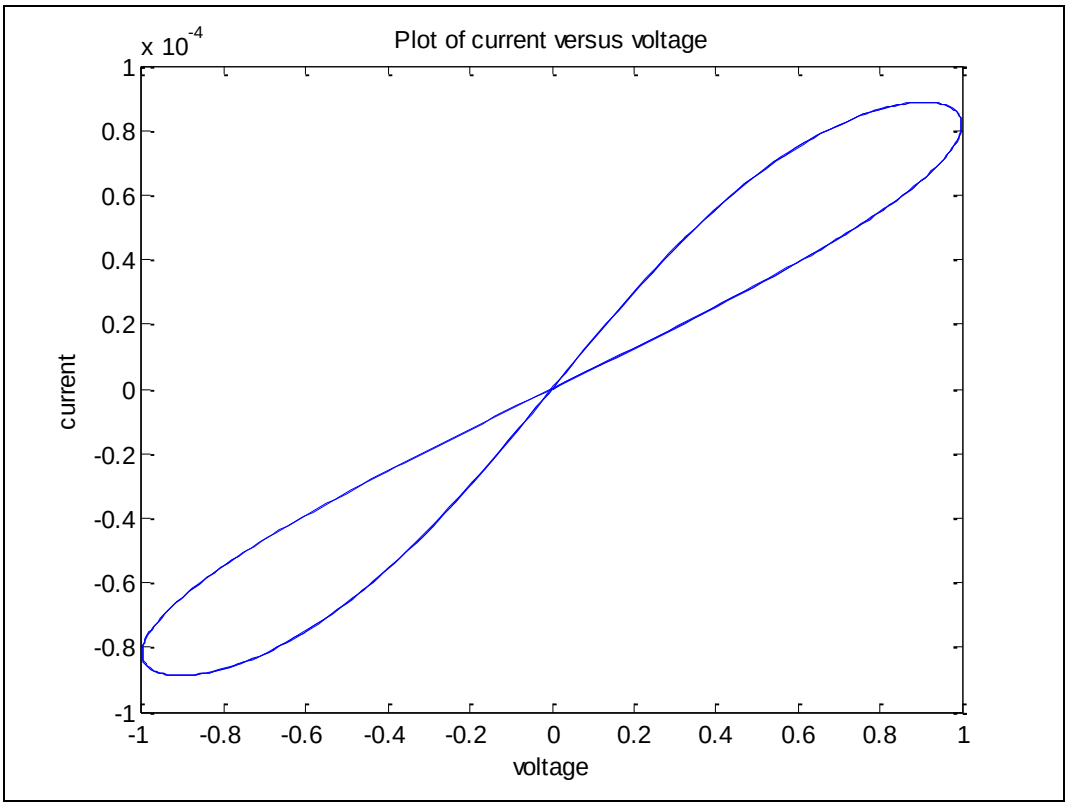

Fig. 19. Plot of current versus voltage for $\omega = 3$ rad/s.

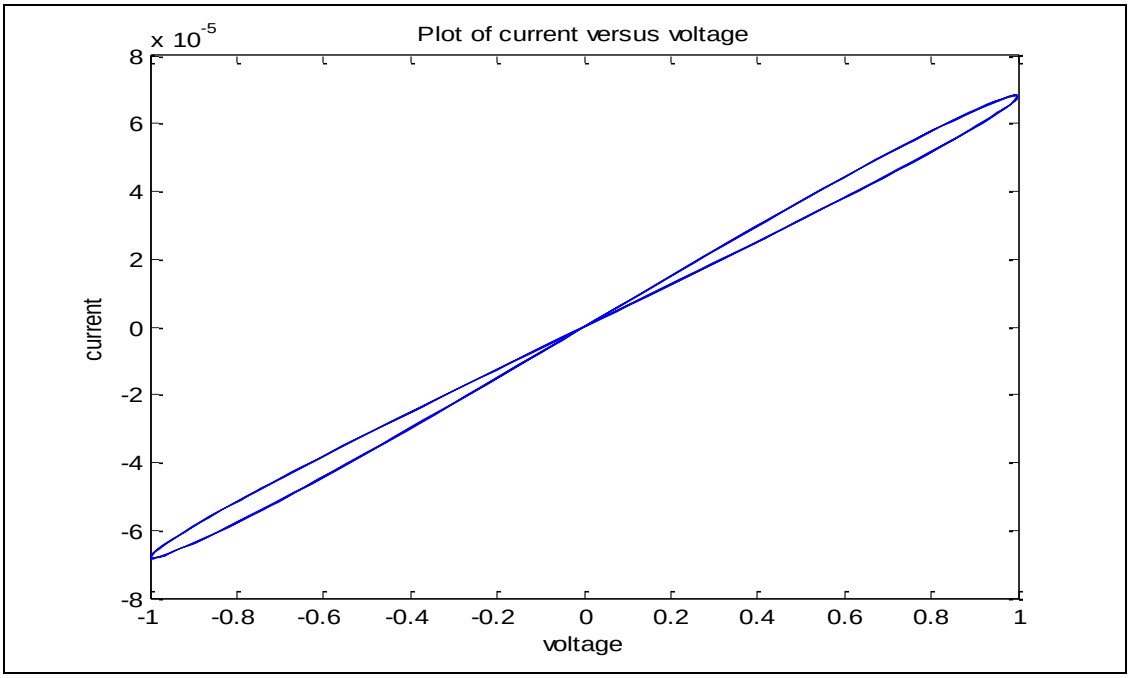

Fig. 20. Plot of current versus voltage for $\omega = 8$ rad/s.

From Fig. 18, we can see that the flux–charge curve is a monotonically increasing curve. Fig. 19 and Fig. 20 show the current–voltage “pinched hysteresis loop” and the hysteresis loop is shrinking with the increase in the input frequency. Thus, all the properties that are said to be a signature of a memristor are satisfied.

Chapter 6: Potential Applications of Memristor

6.1 Two-state Charge-controlled Memristor [16]

The φ–q curve of a two-state charge-controlled memristor is shown in Fig. 21. The slope of the φ–q curve gives the memristance M. The two values of the memristance can be considered as two different states which can be used as binary states. The memristor holds logical values as impedance state and not as voltages. The resistance can be changed from one state to another by applying appropriate voltage.

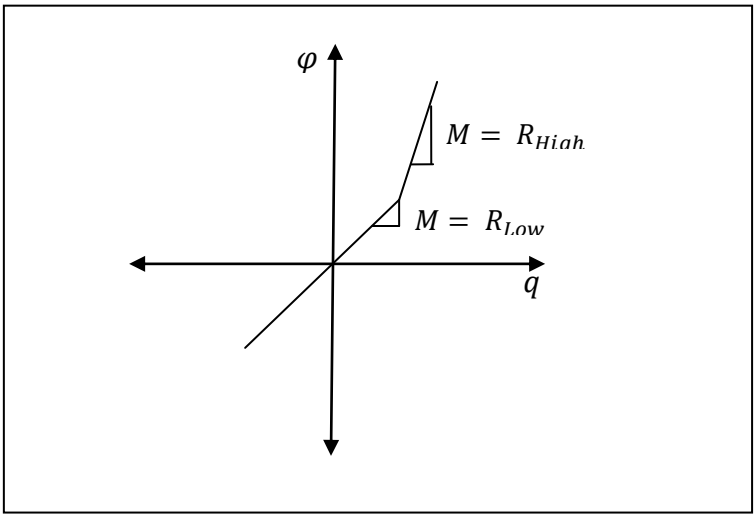

Fig. 21. $\varphi - q$ curve of a two-state charge-controlled memristor

6.2 Digital Logic Implementation Using Memristor

6.2.1 Memristor as a Logical State Element [22]

Consider the memristor shown in Fig. 22. One of the terminals of the memristor is connected to *control* and the other is connected to either *input* or *output*.

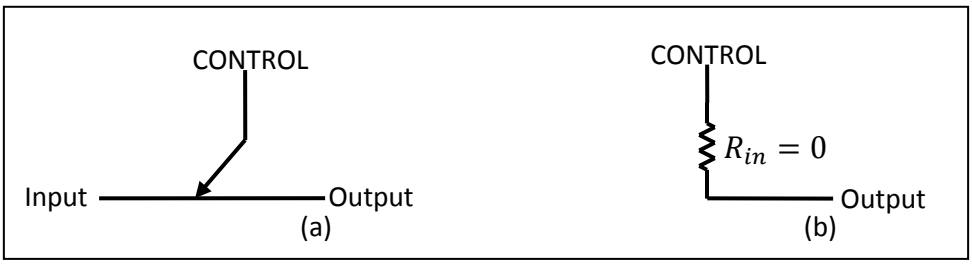

Fig. 22. Memristor as a state element (Adapted from [22]).

When the memristor is closed as shown in Fig. 22(a), it has zero impedance across *input* and *control* and hence represents logic ’0’. The logic ‘0’ can be read out by applying a positive voltage at *control* and reading the voltage at *output*. A high voltage at *output* implies logic ’0’ since the impedance R_{in} is zero. The equivalent circuit during readout is shown in Fig. 22(b). An open memristor contains high impedance and designates logic ‘1’.

6.2.2 Inverting Configuration [22]

Figure 23 shows two memristors connected in an inverting configuration. Mem1 is the driving memristor and its value is not changed. The state of Mem2 is changed to create the logical computation. The logical computation can be achieved in three stages.

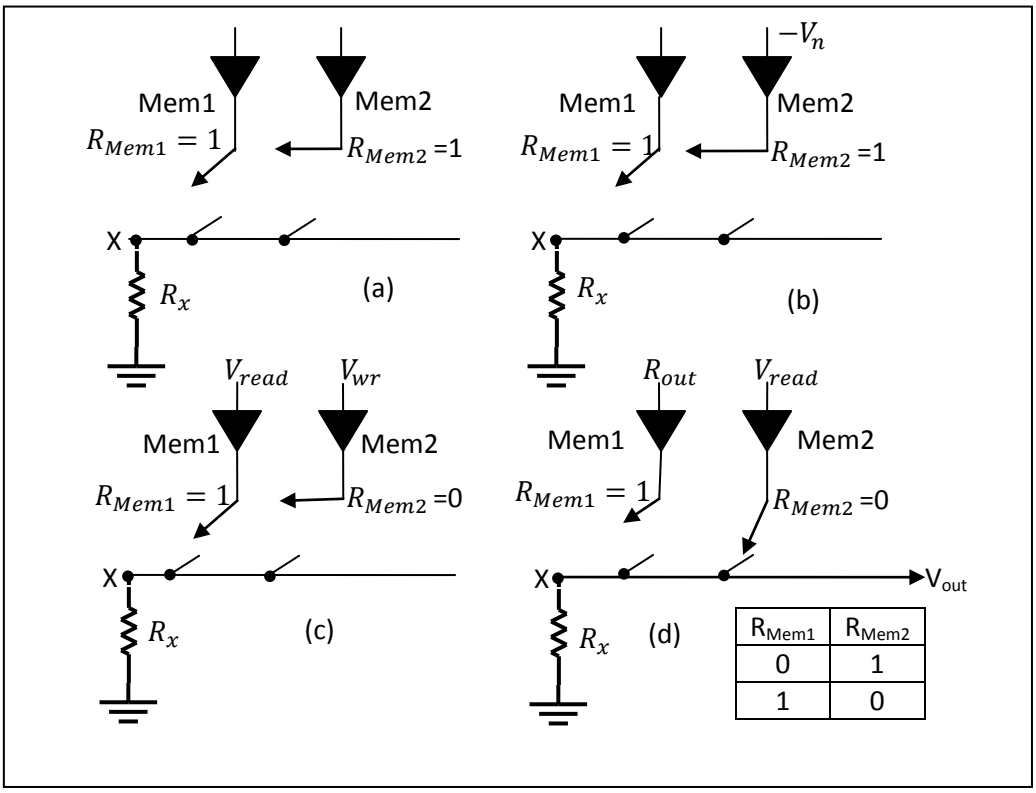

Fig. 23. Inverting configuration of two memristors (Adapted from [22]).

The first stage is called ‘unconditional open.’ In this stage, the memristor Mem2 is preset to the open state by forcing high impedance on the *control* of Mem1 and by applying a high negative voltage $–V_n$ at the *control* of Mem2. The voltage $–V_n$ is greater than the threshold required to open the memristor and hence the state of Mem2 is set to logic ‘1’ irrespective of its previous state. This case is shown in Fig. 23(b).

The second stage is called the 'conditional close' and this stage is shown in Fig. 23(c). The memristor Mem1 is placed in the read out mode by applying a voltage V_{read} at its *control* terminal. A voltage V_{wr} is applied at the *control* of Mem2. If memristor Mem1 is closed, the impedance R_{Mem1}=0. This makes the voltage at the intermediate node X close to V_{read}. The voltage across Mem2 is (V_{wr}-V_{read}) which is not enough to close Mem2 and it remains in the open state. If Mem1 is open, the voltage on node X is close to zero and the voltage across Mem2 is V_{wr}. The voltage drop is over the threshold of Mem2 and it closes memristor Mem2. Thus, logical operation of inversion takes place. The truth table is shown in Fig. 23.

The third stage is 'read out.' The *control* node of Mem1 is placed in high impedance so that it does not have an effect to node X. A voltage V_{read} is applied at the control terminal of Mem2. This will result a voltage at *output*, depending on the state of Mem2.

6.2.3 NAND Operation [22]

Consider the set of memristors as shown in Fig.24. The memristor Mem1 of inverting configuration is replaced by set of memristors Mem1-Mem3, which are connected in parallel. The *control* terminals of Mem1-Mem3 are connected. The memristor Mem4 is unconditionally open by applying a high negative voltage $-V_n$ at the *control* terminal. Then a voltage V_{read} is applied at the common *control* terminal and V_{wr} is applied to the *control* of Mem4. In the scenario where memristors Mem1-Mem3 are open, the voltage at the terminal X is close to 0. The voltage drop across Mem4 is V_{wr}, which is enough to close Mem4. In the scenario where one memristor Mem1 is closed and Mem2 and Mem3 are open, the intermediate node settles close to V_{read} and the voltage across Mem4 is not enough to close the memristor. Similar results occur when Mem2 or Mem3 are open. Hence, the logical computation can be treated as Mem4 = (Mem1.Mem2.Mem3)` which is NAND operation. This configuration is referred to as 'wired-AND' as various inputs are wired together to produce result.

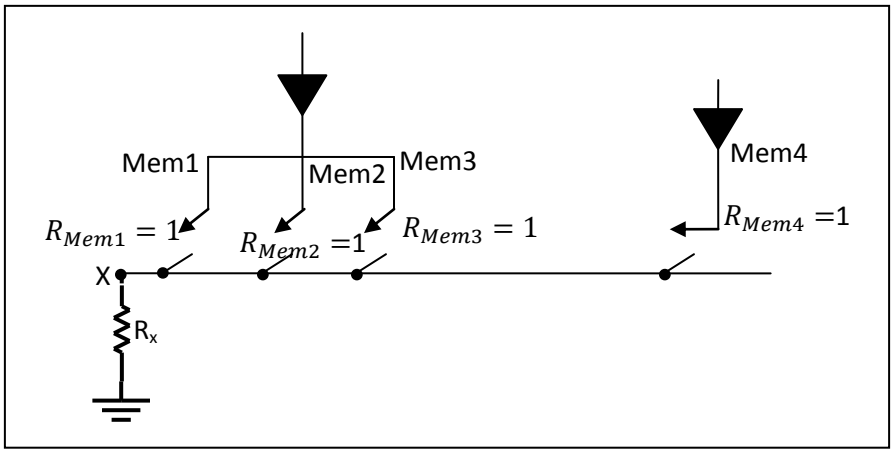

Fig. 24. Wired- AND logical implementation (Adapted from [22]).

6.3 Arithmetic Processing Using Memristors

6.3.1 Memristor Crossbar

Crossbars are formed from a first array of vertical conductive wires crossing with second array of horizontal conductive wires. Memristance material is placed between the two arrays so that any particular wire in the vertical array can be connected to a wire in the horizontal array by switching the resistance of a particular intersection to a low state. The intersection is called as a crosspoint. Fig. 25 shows an example of a memristor based crossbar array.

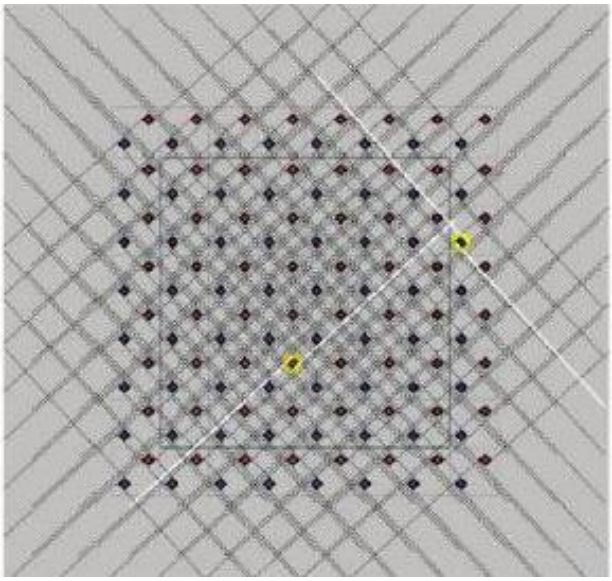

Fig. 25. Memristor Crossbar Array (Adapted from [23]).

6.3.2 Arithmetic Processing [24]

The memristor crossbar array can be used for arithmetic processing. Fig. 26 shows example of a memristor crossbar array which includes a horizontal wire intersected by eight vertical wires and the memristor is sandwiched between the horizontal wire and the vertical wires. An input voltage below the threshold, required for changing the resistance of the memristor is applied to the vertical wires. Assuming that the memristor may be approximated as a fuse (i.e. high resistance is approximately an open circuit and low resistance is a short circuit), the output current in the horizontal wire can be calculated based on the ratio of the input voltage and the parallel combination of the number of low resistances. Thus if one low resistance state produces a current of I, two low resistance states will produce a current of 2I, three low resistance states will produce a current of 3I and so on.

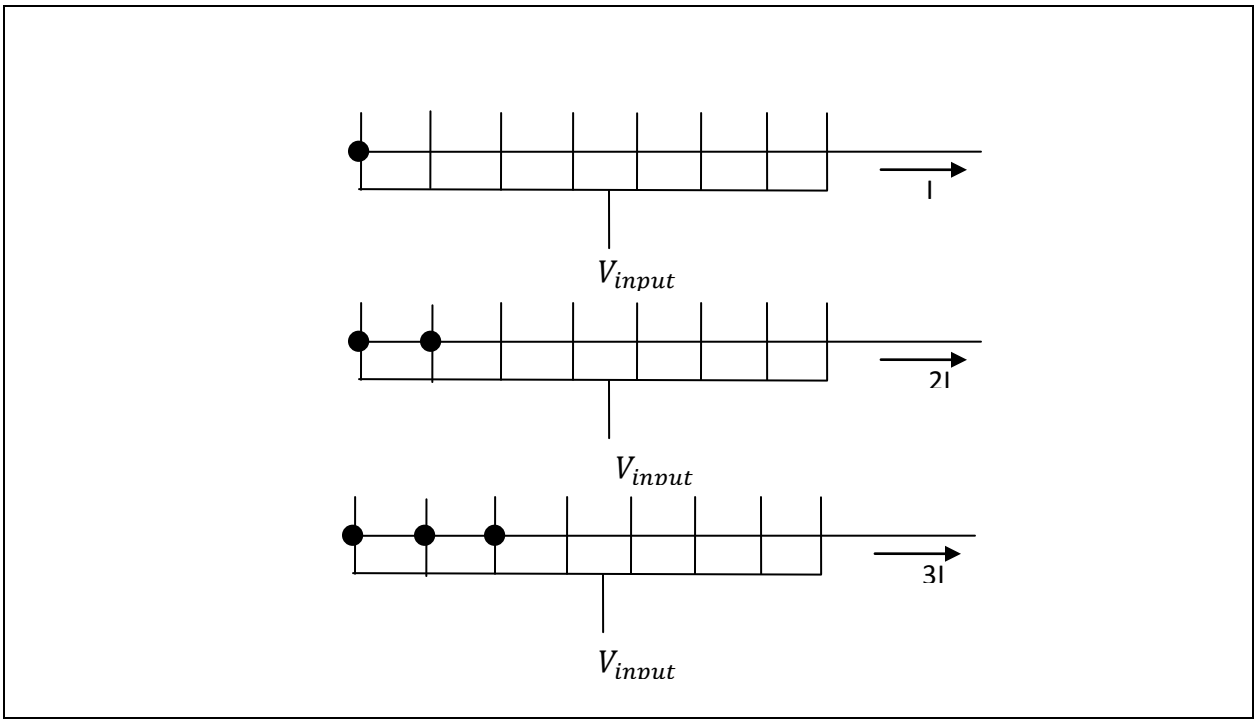

Fig. 26. Memristor crossbar array which includes a horizontal wire intersected by eight vertical wires (Adapted from [24]).

In the system illustrated in Fig. 27 each column of the crossbar is configured to store the equivalent of a binary numerical value where low resistance states are indicated as a closed connection and high resistance states are open connections. Each row wire includes a weighting resistor set to be sufficiently larger than the low-resistance state of the memristor so that each

row has an associated bit significance ranging from a least significant bit row (uppermost row) to a most significant bit row (lowermost row). By selecting particular columns (i.e. applying a positive voltage V_{input} less than the threshold necessary to alter the resistance of the memristor material) the binary numerical values of these columns may be added together. In the example of Fig. 27, the second, third, and sixth column values are summed. In the second column, only the second-row crosspoint is in a low-resistance state, so this contributes a current of approximately ($V_{input}/(\frac{R}{2})$). In the third column, the first and second row crosspoints are in the low-resistance states which contribute ($V_{input}/R + V_{input}/(\frac{R}{2}) = 3V_{input}/R$) to the current. In the sixth column, the second and third row crosspoints are in the low-resistance states, which contribute ($V_{input}/(\frac{R}{2}) + V_{input}/(\frac{R}{4}) = 6V_{input}/R$) to the current. The overall current is thus ($V_{input}/(\frac{R}{2}) + 3V_{input}/R + 6V_{input}/R = 11V_{input}/R$). Using an analog-to-digital converter with a resolution set to (V_{input}/R), the output is converted to 1011, which is the expected sum (0010 + 0011 + 0110).

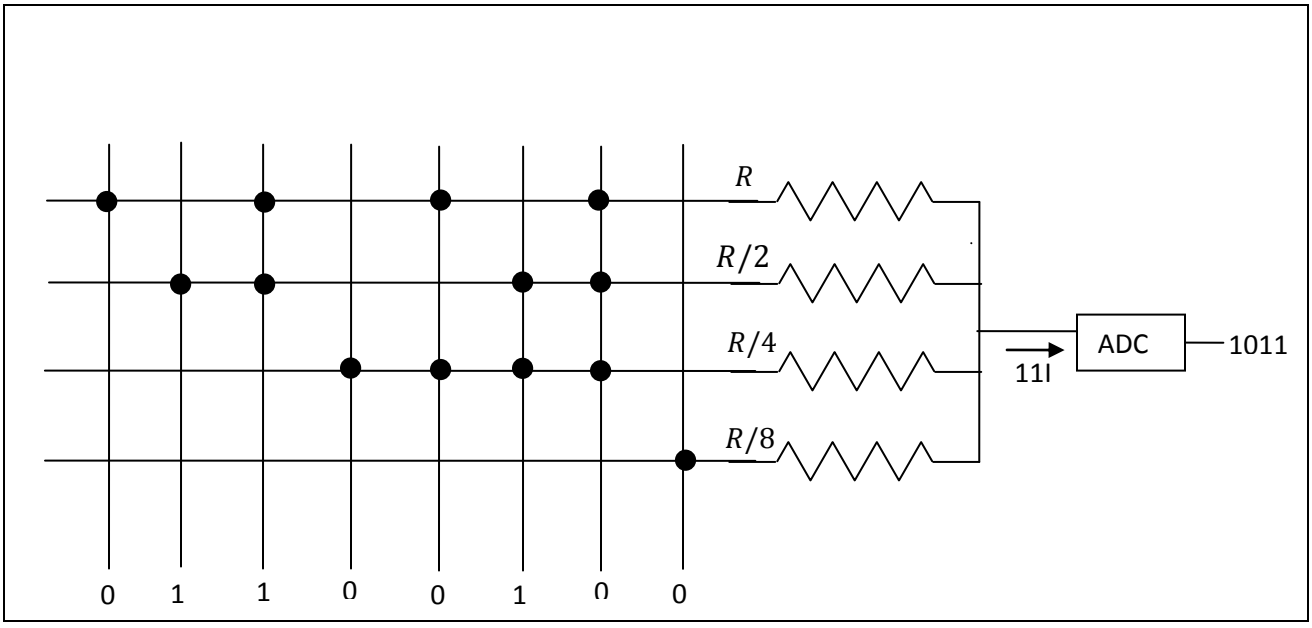

Fig. 27. Memristor crossbar array as an arithmetic processor (Adapted from [24]).

The above configuration has some disadvantages, one being that the analog circuitry can be more sensitive to noise than purely digital electronics, but it has the advantage of integrating memory with computation. Thus, the use of memristors enables calculation on the same chip where the data is stored.

6.4 Memristor Memory

Memristors can be used as non-volatile memory, allowing greater data density than hard drives [5]. The memristor based crossbar latch memory prototyped by HP can fit 100 gigabits within a square centimeter [5]. HP also claims that memristor memory can handle up to 1,000,000 read/write cycles before degradation, compared to flash at 100,000 cycles [25]. In addition, memristors also consume less power.

In memristor memories, the reading operation is performed by applying a voltage lesser than the threshold value. The memristor will conduct even at this voltage if it is 'on'. If it is 'off' then it will not conduct. To write one of the logic levels (0 or 1) a voltage greater than the threshold value is applied. To write the other logic level, a voltage of opposite polarity whose magnitude is greater than the threshold voltage is applied. This turns the memristor 'off'.

Memristors can 'remember' even when the power is turned off. Thus, the computers developed using memristors will have no boot up time [16]. The computer can be turned on, like turning on a light switch and it will instantly display all information that was there on it when it was turned off [26].

Chapter 7: Conclusion and Future Research

7.1 Conclusion

This report presents a detailed study of the memristor. The properties of the memristor and the model proposed by HP are discussed. This model is simulated by subjecting it to various input voltages and noting the results obtained. This report also presents a brief insight into the potential applications of the memristor.

Nanotechnology is fast emerging, and nanoscale devices automatically bring in memristive functions. Thus, memristors might revolutionize the 21st century as radically as the transistor in the 20th century. Memristor memories have already been developed and the researchers at HP believe that they can offer a product with a storage density of about 20 gigabytes per square centimeter by 2013.

Leon Chua rightly said "It's time to rewrite all the Electronics Engineering books." [16]

7.2 Future Research

Recently, researchers have defined two new memdevices- memcapacitor and meminductor, thus generalizing the concept of memory devices to capacitors and inductors. These devices also show 'pinched' hysteresis loops in two constitutive variables— charge—voltage for the memcapacitor and current—flux for meminductor. Figure 28 shows the symbols for the memcapacitor and the meminductor.

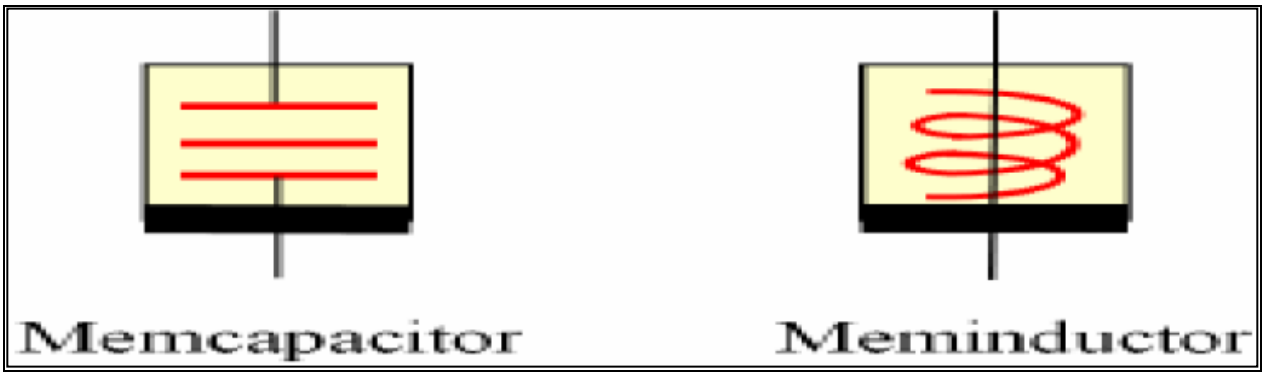

Fig. 28. Symbol of memcapacitor and meminductor (Adapted from [27]).

Memristors are not lossless devices. As non-volatile memories, memristors do not consume power when idle but they do dissipate energy when they are being read or written. Hence, there is a need to invent lossless non-volatile device. Memcapacitors and meminductors are good contenders as they are lossless devices.

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Appendix

MATLAB Code for Memristor Simulation

For this simulation, the width D of the TiO_2 film is considered to be 10 nm and the dopant mobility μ_D= 10^{-10} cm^2 s^{-1} V^{-1}. The values assumed are —R_{ON}=100Ω , R_{OFF}=16KΩ. The input frequency considered is 3 rad/s. The equations used for charge, the internal state of the memristor and current are

$$q(t) = Q_D\left(1 - \sqrt{1 - \frac{2}{Q_D R_{OFF}}\varphi(t)}\right),$$

$$x(t) = 1 - \left(\sqrt{1 - \frac{2\mu_D}{rD^2}\varphi(t)}\right),$$

and

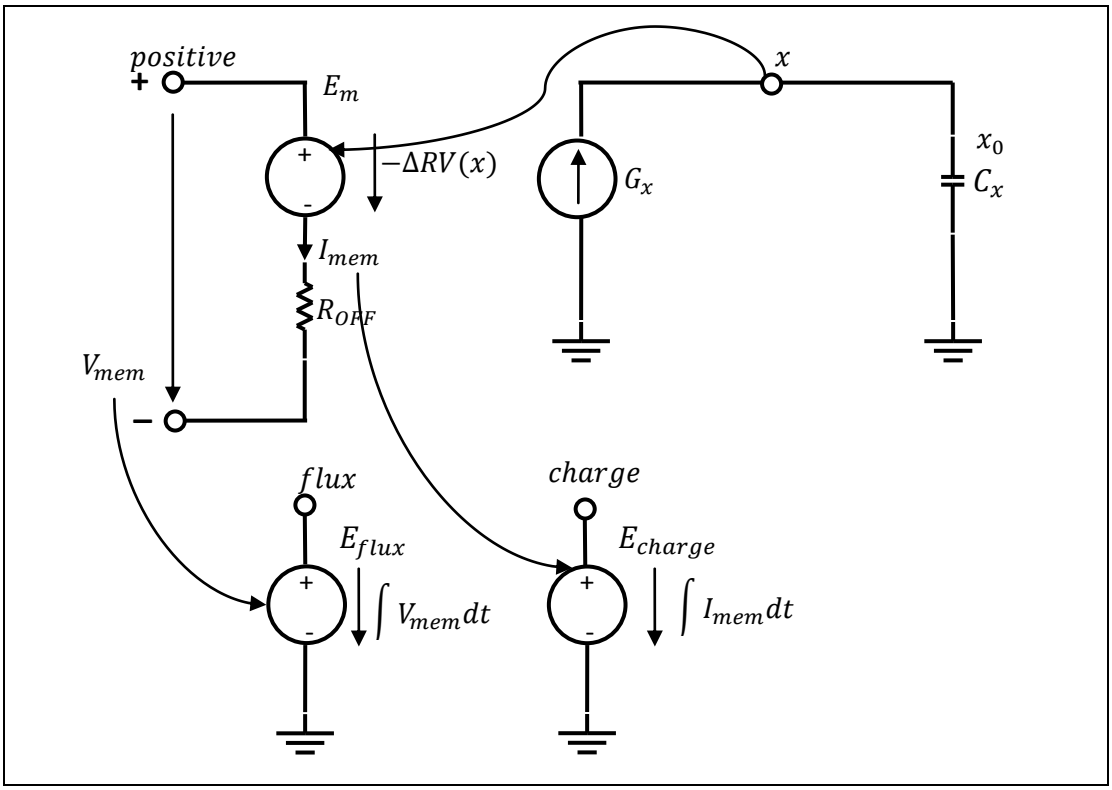
$$i(t) = \frac{v(t)}{R_{OFF}\left(\sqrt{1 - \frac{1\mu_D}{rD^2}\varphi(t)}\right)}$$

```
% MATLAB code for simulating a memristor
v0=1; % amplitude of the input wave
omega=3; % frequency (rad/s)
MD=1e-14;%drift velocity
ROFF=16e+3; % resistance of undoped region
RON=100;% resistance of the doped region
r=ROFF/RON;
D = 10e-9; % Width of Memristor
w0 = 1e-009; % Width of doped region
R0=(RON*(w0/D))+(ROFF*(1-(w0/D)));
q0=(D^2)/(MD*RON);
deltaR=ROFF-RON;
t=0:0.0095:6;
vt=v0*sin(omega*t);%input voltage
flux=v0*(1-cos(omega*t))/omega;%flux
for n=1:632;
i(n)=(vt(n))/(ROFF*sqrt(1-((2*MD*flux(n))/(r*(D^2))))); %Current
charge(n)=(q0*(1-sqrt(1-((2*flux(n))/(q0*ROFF)))));%charge
```

```
width(n)= 1-sqrt(1-((2*MD*flux(n))/(r*(D^2))));%internal state of memristor
end
figure(1)
plot(flux,charge);%Plot of charge versus flux
figure(2)
plot (vt,i);%plot of current versus voltage
```

SPICE Listing for Memristor Simulation [21]

The SPICE model used for the simulation is shown below.



The SPICE listing is as follows. The input voltage is a 1 Hz sine wave. For this simulation, the width D of the TiO_2 film is considered to be 10 nm and the dopant mobility μ_D= 10^{-10} cm^2 s^{-1} V^{-1}. The values assumed are R_{ON}=1KΩ, R_{OFF}=100KΩ and the initial resistance R_{INIT} =80KΩ .

```
Memristor1.cir-Memory resistor
Vmem positive 0 SIN(0 1.2 1Hz)
Gx 0 x value={I(Em)*10F*1K/10N^2*f(V(x),1)}
Cx x 0 1 IC={(100K-80K)/(100K-1K)}
```

```
Raux x 0 1T
Em positive aux value={-I(Em)*V(x)*(100K-1K)}
Roff aux 0 {100K}
Eflux flux 0 value={SDT(V(positive,0))}
Echarge charge 0 value={SDT(I(Em))}
.func f(x,p)={1-(2*x-1)^(2*p)}
* ANALYSIS
.TRAN        0S  3S
* VIEW RESULTS
.PLOT   TRAN V([positive],0)
.PRINT TRAN  V([positive],0)
.PROBE
.END
```