CONFIDENCE INTERVALS ON SEVERAL FUNCTIONS OF THE COMPONENTS OF VARIANCE IN A ONE-WAY RANDOM EFFECTS EXPERIMENT

by

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ABSTRACT

Variability is inherent in most data and often it is useful to study the variability so scientists are able to make more accurate statements about their data. One of the most popular ways of analyzing variance in data is by making use of a one-way ANOVA which consists of partitioning the variability among observations into components of variability corresponding to between groups and within groups. One then has $\sigma_Y^2 = \sigma_A^2 + \sigma_e^2$. Thus there are two variance components. In certain situations, in addition to estimating these components of variance, it is important to estimate functions of the variance components. This report is devoted to methods for constructing confidence intervals for three particular functions of variance components in the unbalanced One-way random effects models. In order to compare the performance of the methods, simulations were conducted using SAS® and the results were compared across several scenarios based on the number of groups, the number of observations within each group, and the value of σ_A^2.

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Chapter 1 Introduction

Most modern scientific claims are based on statements of statistical significance and probability. Whether one is comparing the likelihood of lung cancer in smokers to that of non-smokers (ACS, 2004) or observing that first-born male children exhibit IQ test scores that are 2.82 points higher than second-born males, a difference that is significant at the 95% confidence level (Kristensen & Bjerkedal, 2007), these examples tempt one to conclude what might seem obvious. It might sound reasonable to conclude that smoking causes lung cancer and that the older male siblings are smarter than the younger ones. However, both of these conclusions fail to accurately reflect the data. For example, not all smokers die from lung cancer - some smokers decide to quit, thus reducing their risk, some smokers may die prematurely from cardiovascular disease or from diseases other than lung cancer, and some smokers may simply never contract the disease. Inherently, data exhibits variability, and one of the roles that the science of statistics plays is to quantify this variability and allow scientists to make more accurate statements about their data.

Analysis of variance tables are often used to illustrate variability that is observed in data sets. When studying variability in data, one is often interested in partitioning the variability into various known categories (Searle, Casella & McCulloch, 1992). It is useful to study how the different levels of a factor may affect a variable being analyzed. The size of such effects is referred to as the effect of a level of a factor on that variable. When the focus of inference is on means then fixed effects are ones that are attributed to a finite set of levels of a factor occurring in the data that are of interest to the researcher and have been purposely selected by the researcher.

On the other hand, when the effects are due to an (usually) infinite set of factor levels, from which only a random sample is available in the data, the effects are known as random effects. A lot of attention has been focused on the analysis of fixed effect factors; however, one is often interested in studying the variability that can be partitioned among a set of random effect factors.

The simplest model that involves random effects is the one-way random effects model. The One–way random effects model is defined by

$$Y_{ij} = \mu + a_i + e_{ij}, \qquad i=1,\ldots,k,\ j=1,\ldots n_i, \qquad N = \sum_{i=1}^{k} n_i \tag{1.1}$$

where μ is the grand mean of all observations in the population, the group effects $\{a_i\}$ are independently normally distributed with mean 0 and variance σ_A^2, the residual effects $\{e_{ij}\}$ are independently normally distributed with mean 0 and variance σ_e^2, and where the $\{a_i\}$ and $\{e_{ij}\}$ are independent of each other.

In model (1.1) the total variability of the observed data, i.e. the variance of Y_{ij}, consists of variance corresponding to error and variance among groups and is given by $\sigma_Y^2 = \sigma_A^2 + \sigma_e^2$. Note that σ_A^2 and σ_e^2 are called variance components. In certain situations, in addition to estimating the individual variance components, it is important to estimate several functions of the variance components. It follows that one is also often interested in constructing confidence intervals for those functions of the variance components. One function that is of interest is the intraclass correlation coefficient, $\rho = \dfrac{\sigma_A^2}{\sigma_A^2 + \sigma_e^2}$, which is defined as the correlation between two observations in the same group. The intraclass correlation is commonly used in epidemiologic research. It measures the degree of familial resemblance with respect to environmental or biological characteristics. In genetics it plays an important role in the estimation of the heritability of traits in animal and plant populations. A third field of research is psychology, where the intraclass correlation plays a primary role in reliability theory and where the collection of observations is from one or more sets of raters or judges. Another area of application is in sensitivity analysis, where the intraclass correlation coefficient could be used as a measure of the effectiveness of an experimental treatment.

This report is devoted to comparing various methods for constructing confidence intervals for σ_A^2 and for three particular functions of the variance components in unbalanced one-way random effects models. The functions that will be considered are: $\rho = \dfrac{\sigma_A^2}{\sigma_A^2 + \sigma_e^2}$, $\dfrac{\sigma_A^2}{\sigma_e^2}$, and $\sigma_A^2 + \sigma_e^2$. Methods for constructing confidence intervals for σ_A^2 will be also presented. Confidence intervals for σ_e^2 are not considered since a uniformly most accurate method exists for σ_e^2. The main objective is to investigate the actual confidence levels of the different methods and to determine the most suitable technique that might be used for a given combination of n and k where n is the planned number of observations to be taken for each group and k is the number of groups. In other words, the aim is to determine the most suitable techniques for constructing confidence intervals that will give the narrowest interval width while providing the stated confidence level. That is, acceptable techniques will be those for which the true confidence levels are close to the specified confidence level. From the pool of the acceptable techniques of constructing confidence intervals with respect to a confidence level, the method(s) for which the average width of the confidence intervals is the smallest would be recommended. Unbalanced experiments are obtained by randomly deleting approximately 10% of observations.

In chapter 2 of this report, methods for constructing confidence intervals for the above mentioned functions are described. Chapter 3 consists of results and subsequent interpretations obtained from the simulations conducted as part of this research.

Chapter 2. Methods of Constructing Confidence Intervals

This chapter describes methods for constructing confidence intervals for four particular functions of variance components in an unbalanced One-Way random effects model. These functions are:-

i) $\rho = \dfrac{\sigma_A^2}{\sigma_A^2 + \sigma_e^2}$

ii) $\dfrac{\sigma_A^2}{\sigma_e^2}$

iii) $\sigma_A^2 + \sigma_e^2$

iv) σ_A^2

2.1 Methods for constructing confidence intervals for the intraclass correlation coefficient.

The Analysis of variance (ANOVA) table for the one-way random effects model defined in (1.1) is shown in Table 1, where

$$n_0 = [N - \sum_{i=1}^{k} n_i^2 / N] / (k-1) = \bar{n} - \sum_{i=1}^{k} (n_i - \bar{n}) / [(k-1)(N)], \quad (2.1)$$

and where $\bar{n} = N/k$ is the average group size. There are five columns in the Table 1. The first column identifies the source of variation. The second column gives degrees of freedom corresponding to each source of variation. The third column contains the sum of squares corresponding to each source of variation, followed by the fourth column containing the observed mean squares, and the expected mean squares are given in the last column. Unbiased estimators of σ_A^2 and σ_e^2 are given by $\hat{\sigma}_A^2=(MSA\text{-}MSE)/n_0$ and $\hat{\sigma}_e^2=MSE$, respectively. The intraclass correlation coefficient is defined as $\rho = \sigma_A^2 \big/ (\sigma_A^2 + \sigma_e^2)$. An estimator of ρ is then

$$r_A = \hat{\sigma}_A^2 \Big/ (\hat{\sigma}_A^2 + \hat{\sigma}_e^2) = (MSA\text{-}MSE)/[MSA+(n_0-1)MSE] \equiv (F-1)/(F+n_0-1) \quad (2.2)$$

where $F=MSA/MSE$.

Some methods provide exact confidence intervals for a function of variance components. However, most of the time approximate confidence intervals are obtained. Next, seven procedures that provide approximate two-sided confidence interval for the intraclass correlation coefficient are described.

Table 1. Analysis of variance for the unbalanced one-way random-effects model.

Source of variation	Degrees of freedom	Sum of squares	Mean square	Expected mean square
Among groups	$k-1$	$SSA=\sum_{i=1}^{k} n_i(\bar{Y}_{i.}-\bar{Y}_{..})^2$	$MSA=\dfrac{SSA}{k-1}$	$\sigma_e^2+n_0\sigma_A^2$
Within groups	$N-k$	$SSE=\sum_{i=1}^{k}\sum_{j=1}^{n_i}(Y_{ij}-\bar{Y}_{i.})^2$	$MSE=\dfrac{SSE}{N-k}$	σ_e^2
Total	$N-1$	$SST=\sum_{i=1}^{k}\sum_{j=1}^{n_i}(Y_{ij}-\bar{Y}_{..})^2$		

An exact (Searle, Casella & McCulloch, 1992) $(1-\alpha)100\%$ confidence interval in case of balanced designs, i.e. when $n_i=n$ for $i=1,\ldots,k$, is given by the following

$$\left\{\frac{F/F_U-1}{n+F/F_U-1},\frac{F/F_L-1}{n+F/F_L-1}\right\}, \tag{2.3}$$

where $F = MSA/MSE$ and $\Pr\{F_L \le F \le F_U\}=1-\alpha$. That is, F_L and F_U are the lower and upper $\alpha/2$ critical points of an F distribution with k-1 degrees of freedom for the numerator and N-k degrees of freedom for the denominator.

Method 1 (BAL).

The first method (Donner and Wells, 1986) is based on a modification to the solution for the balanced case. It has been shown that the procedure for the balanced case with a small modification may be appropriate for unbalanced experiments. An approximate $(1-\alpha)100\%$ confidence interval for ρ is given by

$$\left\{\frac{F/F_U-1}{n_0+F/F_U-1},\frac{F/F_L-1}{n_0+F/F_L-1}\right\}, \tag{2.4}$$

where $\Pr\{F_L \le F \le F_U\} = 1-\alpha$. Here F_L and F_U are the lower and upper critical points of an F distribution, with k-1 degrees of freedom for the numerator and N-k degrees of freedom for the denominator, respectively, where $N = \sum_{i=1}^{k} n_i$ and n_0 is defined in (2.1).

Method 2 (TH).

The second method was suggested by Thomas and Hultquist (1978) and is also based on an approximation to the F distribution. An approximate $(1-\alpha)100\%$ confidence interval for ρ is given by

$$\left\{ \frac{F^*/F_U - 1}{\hat{n} + F^*/F_U - 1}, \frac{F^*/F_L - 1}{\hat{n} + F^*/F_L - 1} \right\}, \tag{2.5}$$

where $F^* = \hat{n}\left[\sum_{i=1}^{k} \overline{Y}_{i.}^{2} - \frac{1}{k}\left(\sum_{i=1}^{k} \overline{Y}_{i.}\right)^2\right] \Big/ \left[(k-1)MSE\right]$ and $\hat{n} = k \Big/ \sum_{i=1}^{k} (1/n_i)$ is the harmonic mean of the group sizes. Thus, the only difference between method BAL and TH is that $\hat{n}$ replaces n_0 and F^* replaces F in (2.4). In case of balanced designs, these two methods are equivalent.

Method 3 (Fisher's method).

The third method is based on Fisher's Transformation. Weinberg and Patel (1981) have shown that $Z_F = \frac{1}{2}\log_e\left[\frac{1+(n_o-1)r_A}{1-r_A}\right] = \frac{1}{2}\log_e F$ is approximately normally distributed with a mean equal to $\frac{1}{2}\log_e\left[\frac{1+(n_0-1)\rho}{1-\rho}\right]$ and variance $V_Z = \frac{1}{2}\left[(k-1)^{-1} + (N-k)^{-1}\right]$.

Denote the inverse of this transformation by

$I(Z_F) = [\exp(2Z_F) - 1]/[\exp(2Z_F) + n_0 - 1]$.

An approximate $(1-\alpha)100\%$ confidence interval for ρ is given by

$$\{I(Z_F - Z_{\alpha/2}\sqrt{V_Z}), I(Z_F + Z_{\alpha/2}\sqrt{V_Z})\}, \tag{2.6}$$

where $Z_{\alpha/2}$ denotes $(1-\alpha)100\%$ two-sided critical value of the standard normal distribution.

Method 4 (Smith's method).

Smith (1956) derived the large-sample variance of r_A (2.2) as a function of ρ,

$$V(r_A) = \left[2(1-\rho)^2 / n_0^2\right](\{[1+\rho(n_0-1)]^2/(N-k)\} + \{(k-1)(1-\rho)[1+\rho(2n_0-1)] +$$

$$+ \rho^2[\sum n_i^2 - 2N^{-1}\sum n_i^3 + N^{-2}(\sum n_i^2)^2]\}/(k-1)^2).$$

It is known that in large samples r_A, an estimator of intraclass correlation, has an approximate normal distribution, and therefore an approximate (1−α)100% confidence interval for ρ is given by

$$\{r_A - Z_{\alpha/2}\sqrt{V(r_A)}, r_A + Z_{\alpha/2}\sqrt{V(r_A)}\} \qquad (2.7)$$

where $Z_{\alpha/2}$ denotes the (1−α)100% two-sided critical value of the standard normal distribution.

Method 5 (Swiger's method).

This method is a modification of the previous procedure. Swiger et al. (1964) derived an approximation to large-sample variance of an estimator of intraclass correlation, r_A. It has been shown (Swiger et al., 1964) that this method is accurate when the variation in the group sizes is small and the intraclass correlation is less than or equal to 0.10. The variance of r_A as a function of ρ is given by

$$V^*(r_A) = \frac{2(N-1)(1-\rho)^2\left[1+(n_0-1)\rho\right]^2}{n_0^2(N-k)(k-1)}.$$

An approximate (1−α)100% confidence interval for ρ is given by

$$\{r_A - Z_0\sqrt{V^*(r_A)}, r_A + Z_0\sqrt{V^*(r_A)}\} \qquad (2.8)$$

where Z_0 denotes the (1−α)100% two-sided critical value of the standard normal distribution.

In each of the last two methods ρ is replaced by its estimator, r_A, in the computations of confidence limits.

Method 6 (Delta Method).

The last procedure for constructing approximate confidence interval is based on the delta method (Miliken and Johnson, 2009). Let $\boldsymbol{\sigma}$ $=[\hat{\sigma}_e^2 \;\; \hat{\sigma}_A^2]$ denote the vector of estimated variance components, and let

$\hat{V}(\hat{\sigma}) = \begin{pmatrix} \hat{\sigma}^2_{\hat{\sigma}^2_A} & \hat{\sigma}^2_{\hat{\sigma}^2_A \hat{\sigma}^2_e} \\ \hat{\sigma}^2_{\hat{\sigma}^2_A \hat{\sigma}^2_e} & \hat{\sigma}^2_{\hat{\sigma}^2_e} \end{pmatrix}$ denote the estimated variance-covariance matrix corresponding to the two variance component estimators, and let $\varphi(\sigma) = \dfrac{\sigma_A^2}{\sigma_A^2 + \sigma_e^2}$. Then let

$\hat{f}' = \left(\dfrac{\partial\varphi(\sigma)}{\partial\sigma_A^2}, \dfrac{\partial\varphi(\sigma)}{\partial\sigma_e^2} \right) |_{\sigma=\hat{\sigma}} = \left[\dfrac{\hat{\sigma}_e^2}{(\hat{\sigma}_e^2 + \hat{\sigma}_A^2)^2}, \dfrac{-\hat{\sigma}_A^2}{(\hat{\sigma}_e^2 + \hat{\sigma}_A^2)^2} \right]$. The approximate variance of $\varphi(\hat{\sigma})$ is $\hat{\sigma}^2_{\varphi(\hat{\sigma})} = \hat{f}' \hat{V}(\hat{\sigma}) \hat{f}$. The Satterthwaite approximate degrees of freedom are $\hat{r} = \dfrac{2(\varphi(\hat{\sigma}))^2}{\hat{\sigma}^2_{\varphi(\hat{\sigma})}}$. An approximate (1−α)100% confidence interval for ρ is given by

$$\left(\frac{\hat{r}\hat{\sigma}^2_{\varphi(\hat{\sigma})}}{\chi^2_{\alpha/2,\hat{r}}}, \frac{\hat{r}\hat{\sigma}^2_{\varphi(\hat{\sigma})}}{\chi^2_{1-\alpha/2,\hat{r}}} \right) \tag{2.9}$$

where $\chi^2_{\alpha/2,\hat{r}}$ and $\chi^2_{1-\alpha/2,\hat{r}}$ are the upper and lower critical points of the chi-square distribution with $\hat{r}$ degrees of freedom.

2.2 Methods for constructing confidence intervals for $\dfrac{\sigma_A^2}{\sigma_e^2}$.

Ratio 1 method.

The first technique for constructing an approximate confidence interval for $\dfrac{\sigma_A^2}{\sigma_e^2}$ is an exact method (Searle, Casella, McCulloch, 2006) for balanced experiments. A (1−α)100% confidence limits for $\dfrac{\sigma_A^2}{\sigma_e^2}$ is given by

$$\left(\frac{F/F_U - 1}{n}, \frac{F/F_L - 1}{n}\right), \tag{2.10}$$

where Pr{ $F_L \le F \le F_U$ }=1−α. For unbalanced experiments, we replaced n by n_0 defined in (2.1).

Delta method.

The second method is a delta method described in Method 6 for constructing a CI for intraclass correlation coefficient. Let φ(σ) = $\frac{\sigma_A^2}{\sigma_e^2}$, then

$\hat{f}' = \left(\frac{\partial\varphi(\sigma)}{\partial\sigma_A^2}, \frac{\partial\varphi(\sigma)}{\partial\sigma_e^2}\right)\Big|_{\sigma=\hat{\sigma}} = \left[\frac{1}{(\hat{\sigma}_e^2)^2}, \frac{-\hat{\sigma}_A^2}{(\hat{\sigma}_e^2)^2}\right]$. The approximate variance of $\varphi(\hat{\sigma})$ is

$\hat{\sigma}^2_{\varphi(\hat{\sigma})} = \hat{f}'\hat{V}(\hat{\sigma})\hat{f}$. The Satterthwaite approximate degrees of freedom are $\hat{r} = \frac{2(\varphi(\hat{\sigma}))^2}{\hat{\sigma}^2_{\varphi(\hat{\sigma})}}$.

An approximate (1−α)100% confidence interval for $\frac{\sigma_A^2}{\sigma_e^2}$ is given by

$$\left(\frac{\hat{r}\hat{\sigma}^2_{\varphi(\hat{\sigma})}}{\chi^2_{\alpha/2,\hat{r}}}, \frac{\hat{r}\hat{\sigma}^2_{\varphi(\hat{\sigma})}}{\chi^2_{1-\alpha/2,\hat{r}}}\right) \tag{2.11}$$

where $\chi^2_{\alpha/2,\hat{r}}$ and $\chi^2_{1-\alpha/2,\hat{r}}$ are the upper and lower critical points of the chi-square distribution with $\hat{r}$ degrees of freedom.

2.3 Methods for constructing confidence intervals for $\sigma_A^2 + \sigma_e^2 = \sigma_Y^2$.

Method 1 (Delta Method).

The first technique for constructing a confidence interval for $\sigma_A^2 + \sigma_e^2 = \sigma_Y^2$ is a delta method. This method is described in Method 6 for constructing a CI for intraclass correlation coefficient. Let φ(σ) = $\sigma_A^2 + \sigma_e^2$, then

$\hat{f}' = \left(\frac{\partial\varphi(\sigma)}{\partial\sigma_A^2}, \frac{\partial\varphi(\sigma)}{\partial\sigma_e^2} \right)|_{\sigma=\hat{\sigma}}$ = [1 1]. The approximate variance of $\varphi(\hat{\sigma})$ is $\hat{\sigma}^2_{\varphi(\hat{\sigma})} = \hat{f}'\hat{V}(\hat{\sigma})\hat{f}$. The Satterthwaite approximate degrees of freedom are $\hat{r} = \frac{2(\varphi(\hat{\sigma}))^2}{\hat{\sigma}^2_{\varphi(\hat{\sigma})}}$.

An approximate (1−α)100% confidence interval for σ_Y^2 is given by

$$\left(\frac{\hat{r}\hat{\sigma}^2_{\varphi(\hat{\sigma})}}{\chi^2_{\alpha/2,\hat{r}}}, \frac{\hat{r}\hat{\sigma}^2_{\varphi(\hat{\sigma})}}{\chi^2_{1-\alpha/2,\hat{r}}} \right) \tag{2.12}$$

where $\chi^2_{\alpha/2,\hat{r}}$ and $\chi^2_{1-\alpha/2,\hat{r}}$ are the upper and lower critical points of the chi-square distribution with $\hat{r}$ degrees of freedom.

Method 2 (Satterthwaite method).

Let $Q = q_1 MSA + q_2 MSE$ where $q_1 = \frac{1}{n_0}$ and $q_2 = 1 - \frac{1}{n_0}$. Next let $r = \frac{(Q)^2}{\frac{(q_1 MSA)^2}{f_1} + \frac{(q_2 MSE)^2}{f_2}}$ where f_1 and f_2 are degrees of freedom corresponding to *MSA* and *MSE* respectively. Then $\frac{rQ}{E(Q)}$ is approximately distributed as a central chi-square random variable with r degrees of freedom. An approximate (1−α)100% confidence interval for $\sigma_A^2 + \sigma_e^2$ (Milliken and Johnson, 2009) is then given by

$$\left(\frac{\hat{v}Q}{\chi^2_{\alpha/2,\hat{v}}}, \frac{\hat{v}Q}{\chi^2_{1-\alpha/2,\hat{v}}} \right), \tag{2.13}$$

where $\hat{v}$ is estimated degrees of freedom, and $\chi^2_{\alpha/2,\hat{v}}$, $\chi^2_{1-\alpha/2,\hat{v}}$ are the upper and lower critical points of the chi-square distribution, respectively.

2.4 Methods for constructing confidence intervals for σ_A^2.

Method 1 (Delta Method).

The first technique for constructing a confidence interval for σ_A^2 is the delta method. The degrees of freedom are computed using Satterthwaite approximation. Let $\hat{r} = \dfrac{2(\hat{\sigma}_A^2)^2}{\hat{\sigma}^2_{\hat{\sigma}_A^2}}$ be the estimated degrees of freedom. An approximate (1−α)100% confidence interval for σ_A^2 is given by

$$\left(\frac{\hat{r}\hat{\sigma}_A^2}{\chi^2_{\alpha/2,\hat{r}}}, \frac{\hat{r}\hat{\sigma}_A^2}{\chi^2_{1-\alpha/2,\hat{r}}}\right) \tag{2.14}$$

where $\chi^2_{\alpha/2,\hat{r}}$ and $\chi^2_{1-\alpha/2,\hat{r}}$ are the upper and lower critical points of the chi-square distribution with $\hat{r}$ degrees of freedom.

Method 2 (Williams method).

Williams (1962) derived exact confidence limits for σ_A^2 for a balanced design. In our simulations, since unbalanced experiments are considered, n is substituted by n_0 given in (2.1). It follows that modified Williams method gives an approximate confidence interval in unbalanced experiments. An approximate (1−α)100% confidence interval for σ_A^2 is given by

$$\left(\frac{SSA(1-F_U/F)}{n\chi_{\alpha-1,U}}, \frac{SSA(1-F_L/F)}{n\chi_{\alpha-1,L}}\right) \tag{2.15}$$

where $\chi^2_{\alpha-1,U}$ and $\chi^2_{\alpha-1,L}$ are the upper and lower critical points of the chi-square distribution with U and L degrees of freedom respectively.

Method 3 (Johnson and Milliken - Simultaneous Confidence Interval).

Milliken and Johnson (2009) present a method for constructing a (1-α)100% simultaneous confidence region for (σ_e^2, σ_A^2). They conclude that a (1-α)100% confidence interval about σ_A^2 is

$$\left(\frac{u_2Q_2/\chi_{\tau/2,u_2} - u_1Q_1/\chi_{1-\tau/2,u_2}}{a}, \frac{u_2Q_2/\chi_{1-\tau/2,u_2} - u_1Q_1/\chi_{\tau/2,u_2}}{a}\right) \tag{2.16}$$

where Q_1=*MSE* and is based on u_1 degrees of freedom and Q_2 is the *MSA* which is based on $u_2=k-1$ degrees of freedom with expectation $\sigma_e^2 + a\sigma_a^2$, and $\tau = 1 - \sqrt{1-\alpha}$.

The next chapter summarizes the results of simulations conducted by making use of the above described methods.

Method 4 (Wald Z-scores).

The fourth method included in our analysis is a default method in SAS® (SAS® online documentation). An approximate (1-α)100% confidence interval for σ_A^2 is given by

$$\left(\hat{\sigma}_A^2 - Z_{\alpha/2}\sqrt{\hat{\sigma}^2_{\hat{\sigma}_A^2}}, \hat{\sigma}_A^2 + Z_{\alpha/2}\sqrt{\hat{\sigma}^2_{\hat{\sigma}_A^2}}\right) \tag{2.17}$$

where $Z_{\alpha/2}$ denotes (1−α)100% two-sided critical value of the standard normal distribution.

Chapter 3 Simulation Results

In order to compare the performance of the methods described in Chapter 2, one thousand datasets were simulated using SAS® for many different scenarios. The parameters used in the simulations included varying k, the number of groups, varying n, number of observations within each group and σ_A^2, the group variance component. The within group variance component, σ_e^2 was always taken to be equal to one without any loss of generality.

The values of k and corresponding values of n that were considered are listed in Table 2.

Table 2. The values of k and n considered in simulations.

values of k	values of n		
5	2	6	10
10	2	5	10
15	2	4	6
25	2	3	5
50	2	3	

For each of the combinations included in Table 2, σ_A^2 was taken to be σ_A^2 =1/8, 1/4, 1/2, 1, 2, 4, 8. In order to simulate missing observations in the data, approximately 10% of the simulated data points were replaced with missing values.

Estimated 95% confidence intervals for the intraclass correlation coefficient were obtained for each of the six methods described in Section 2.1 for each combination of k, n, and σ_A^2 considered. Also estimated 95% confidence intervals for the ratio $\frac{\sigma_A^2}{\sigma_e^2}$ were obtained for each of the two methods described in Section 2.2. In addition, estimated confidence intervals were obtained for $\sigma_A^2 + \sigma_e^2 = \sigma_Y^2$ for each of the two methods described in Section 2.3 and for σ_A^2 for each of the four methods described in Section 2.4. Cases where the estimated value of σ_A^2 was not greater than zero were omitted.

Based on the remaining number of simulations, the observed proportion of times that the estimated confidence interval level contained the true value of the function of the two variance components was obtained. For a delta method used to construct confidence intervals for intraclass correlation coefficient, for $\frac{\sigma_A^2}{\sigma_e^2}$, for $\sigma_A^2 + \sigma_e^2 = \sigma_Y^2$ and for σ_A^2, observed values of some of the estimated confidence intervals had to be controlled. Occasionally extremely large confidence interval widths or widths less then zero were observed to be due cases where there were very small estimated degrees of freedom and thus having very small chi-square critical values. In these cases the lower limit was restricted to be no less than 0, and upper limit to be no more than 100.

Appendix 1 contains tables with observed confidence levels and observed average confidence interval widths for each combination of *k, n,* and σ_A^2 considered. Appendix 2 contains figures that show plots of information contained in the tables in Appendix 1.

The following section illustrates the tables and figures in the Appendices, and interprets the results from one of the several combinations of *n* and *k* that were considered as part of the research.

Observed confidence levels for each value of σ_A^2 considered when k=10 and n=5 are given in Table 2 and the observed average confidence interval widths are given in Table 3. Those cases where the observed confidence level falls between $0.95 \pm 1.96\sqrt{\frac{(0.05)(0.95)}{1000}} = 0.95 \pm 0.0135$ are shown in bold. That is, if the observed confidence level is between 0.9365 and 0.9635 we would not be able to reject the hypothesis that the true confidence level is equal to 0.95.

An examination of Table 3 for those methods that give confidence intervals for the intraclass correlation coefficient reveals that the TH and BAL methods always provide coverage levels that are at least at the desired 95% level, and that these two methods give coverage levels that are very similar to one another. An examination of Table 4 reveals that the average confidence interval widths are exactly the same for both methods. The Swiger and Smith methods do not provide acceptable coverage levels when $\sigma_A^2 \geq 1/2$. However, for very small values of σ_A^2, the two methods provide coverage at a level greater than 0.95, and the two methods provide average confidence interval widths that are narrower than the widths given by the TH and Bal methods. The Delta method and Fisher's method cannot be recommended as they do not give acceptable confidence level coverage across the range of values of σ_A^2 that were considered. Summarizing, when k=10 and n=5, the two methods that can be recommended are the TH and Bal methods.

An examination of Table 3 for those methods that give confidence intervals for $\frac{\sigma_A^2}{\sigma_e^2}$ reveals that the Ratio1 method provides confidence intervals that have acceptable observed confidence levels for all values of σ_A^2 that were considered. Furthermore, the observed average widths of the confidence intervals for $\frac{\sigma_A^2}{\sigma_e^2}$ were narrower that all of the observed confidence interval widths for the Delta method. Thus when k=10 and n=5, the only method that can be recommended for finding a confidence interval for $\frac{\sigma_A^2}{\sigma_e^2}$ is the Ratio 1 method.

Next consider confidence intervals for $\sigma_A^2 + \sigma_e^2 = \sigma_Y^2$. An examination of Table 3 reveals that the Delta method and the Satterthwaite method provide comparable observed confidence levels for all values of σ_A^2 that were considered, and an examination of Table 4 reveals that the two methods also give comparable confidence interval widths. However, the observed confidence levels were not acceptable when σ_A^2 was equal to 1, 2, or 4. Better methods for finding a confidence interval for $\sigma_A^2 + \sigma_e^2 = \sigma_Y^2$ need to be discovered.

Finally, consider confidence intervals for σ_A^2. An examination of Tables 3 and 4 reveals that modified William's method is the only one that provides acceptable observed confidence levels for all values of σ_A^2 considered. Furthermore, the observed average confidence interval widths are smaller for William's method than for either the Delta method or the Satterthwaite method for all values of σ_A^2 considered. Wald's method outperforms William's method when σ_A^2 is much smaller than σ_e^2. Summarizing, when k=10 and n=5, William's method should be used.

Table 3. Observed confidence level for k=10 and n=5. Cases in which observed confidence level is greater than 0.9365 are shown in bold.

Configuration																
			OBSERVED CONFIDENCE LEVEL													
			$\sigma_A^2/(\sigma_A^2+\sigma_e^2)$						σ_A^2/σ_e^2		$\sigma_A^2+\sigma_e^2=\sigma_Y^2$		σ_A^2			
k	n	σ_A^2	Bal	TH	Fisher	Smith	Swiger	Delta Method	Ratio1	Delta Method	Delta Method	Satterthwaite	Delta Method	Williams	Milliken and Johnson	Wald
10	5	1/8	**96.3**	**96.4**	**97.3**	**98.0**	**98.0**	78.8	**93.8**	79.7	**95.2**	**95.2**	83.6	**94.4**	88.1	**99.9**
10	5	1/4	**96.6**	**96.6**	**98.0**	**97.9**	**97.9**	87.5	**94.5**	88.2	**95.9**	**95.9**	90.5	**95.3**	92.0	**96.9**
10	5	1/2	**97.3**	**96.9**	**96.4**	92.0	92.0	91.5	**94.6**	92.8	**94.8**	**94.7**	**94.4**	**94.6**	**94.0**	89.4
10	5	1	**95.0**	**95.5**	**94.3**	90.6	90.6	93.1	**94.1**	**94.6**	93.1	93.1	**96.3**	**94.6**	**95.1**	87.3
10	5	2	**94.7**	**95.0**	93.3	91.2	91.2	**93.9**	**94.5**	**95.5**	92.2	92.0	**97.1**	**95.1**	**96.2**	85.9
10	5	4	**95.1**	**95.0**	93.1	93.0	93.0	**94.1**	**95.1**	**96.1**	93.3	92.6	**97.5**	**94.9**	**97.0**	84.9
10	5	8	**95.0**	**94.9**	92.9	**94.0**	93.5	**94.3**	**95.0**	**95.6**	**94.6**	**94.1**	**96.1**	**94.9**	**96.6**	85.2

Table 4. Average Width for k=10 and n=5. Cases in which observed confidence level is greater than 0.9365 are shown in bold.

Configuration																
			AVERAGE WIDTH													
			$\sigma_A^2/(\sigma_A^2+\sigma_e^2)$						σ_A^2/σ_e^2		$\sigma_A^2+\sigma_e^2=\sigma_Y^2$		σ_A^2			
k	n	σ_A^2	Bal	TH	Fisher	Smith	Swiger	Delta Method	Ratio1	Delta Method	Delta Method	Satterthwaite	Delta Method	Williams	Milliken and Johnson	Wald
10	5	1/8	**0.53**	**0.53**	**0.47**	**0.45**	**0.45**	0.89	**1.17**	27.15	**1.11**	**1.11**	21.63	**1.05**	3.46	**0.59**
10	5	1/4	**0.57**	**0.57**	**0.52**	**0.51**	**0.51**	0.88	**1.59**	20.47	**1.32**	**1.32**	15.44	**1.41**	3.82	**0.79**
10	5	1/2	**0.59**	**0.59**	**0.55**	0.58	0.57	0.83	**2.43**	13.90	**1.80**	**1.80**	**11.26**	**2.12**	**4.53**	1.22
10	5	1	**0.57**	**0.57**	**0.56**	0.59	0.59	0.72	**4.10**	**8.76**	2.92	2.92	**6.46**	**3.55**	**5.96**	2.16
10	5	2	**0.49**	**0.49**	0.49	0.51	0.51	**0.56**	**7.42**	**10.47**	5.40	5.40	**7.55**	**6.40**	**8.81**	4.05
10	5	4	**0.36**	**0.36**	0.38	0.38	0.37	**0.39**	**14.07**	**17.47**	10.73	10.74	**12.86**	**12.08**	**14.49**	7.78
10	5	8	**0.24**	**0.24**	0.25	**0.24**	0.24	**0.24**	**27.35**	**32.63**	**21.79**	**21.83**	**24.02**	**23.45**	**25.86**	15.20

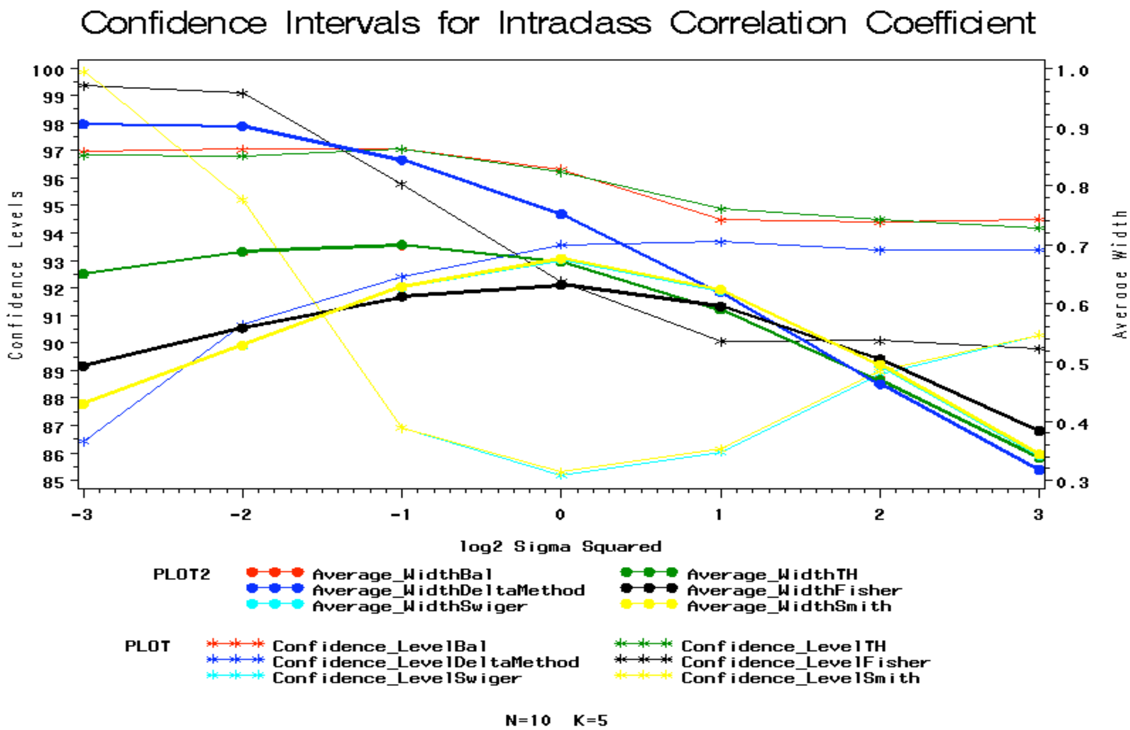
For confidence intervals on the intraclass correlation coefficient, the BAL method is acceptable for most of the cases considered and it is never worse than any of the other five methods.

For confidence intervals on the ratio of two variance components, the Ratio 1 method gives confidence intervals with acceptable confidence levels and gives much smaller average width intervals than the delta method.

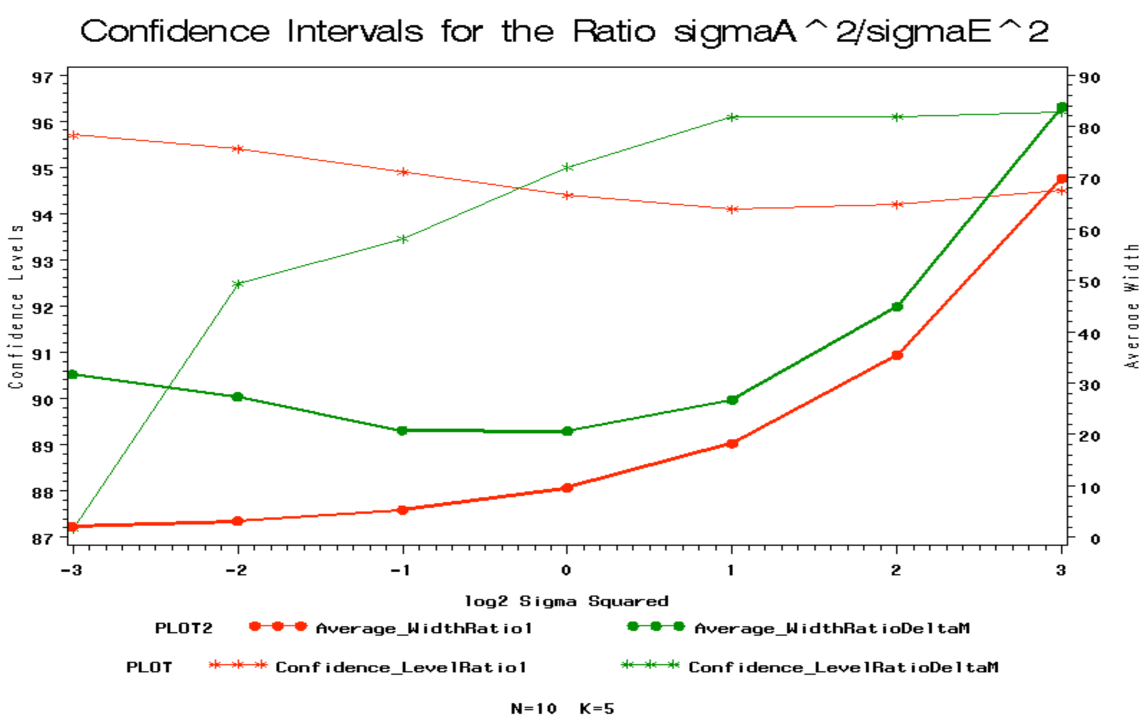
For confidence intervals on the sum of two variance components, the delta method and the Satterthwaite method provide confidence intervals with similar average widths. However, for k=5, the two methods only give acceptable coverage rates when $\sigma_A^2 \leq 1/2$; for k=10, the coverage rates are only acceptable when n=2 or $\sigma_A^2 \leq 1/2$; for k=15, the coverage rates are unacceptable when n=2 and σ_A^2=2 or 4; for k=25, the coverage rates are acceptable for all n except when σ_A^2>1; and when n=50, the coverage rates are acceptable for all k except when σ_A^2 is near 1.

For confidence intervals on σ_A^2, Williams' method provides acceptable coverage rates for all cases considered and provides average widths that are no wider than the widths of any of the other three methods.

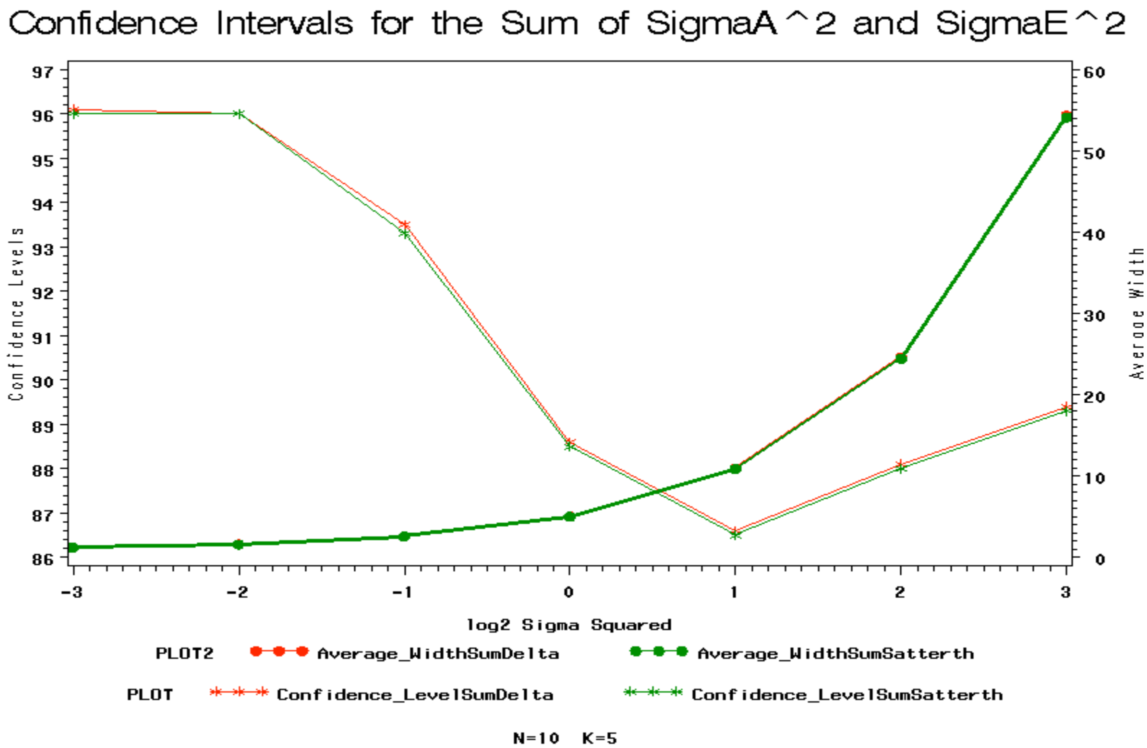
Graph 1. Confidence Intervals for the intraclass correlation coefficient, k=5 n=10.



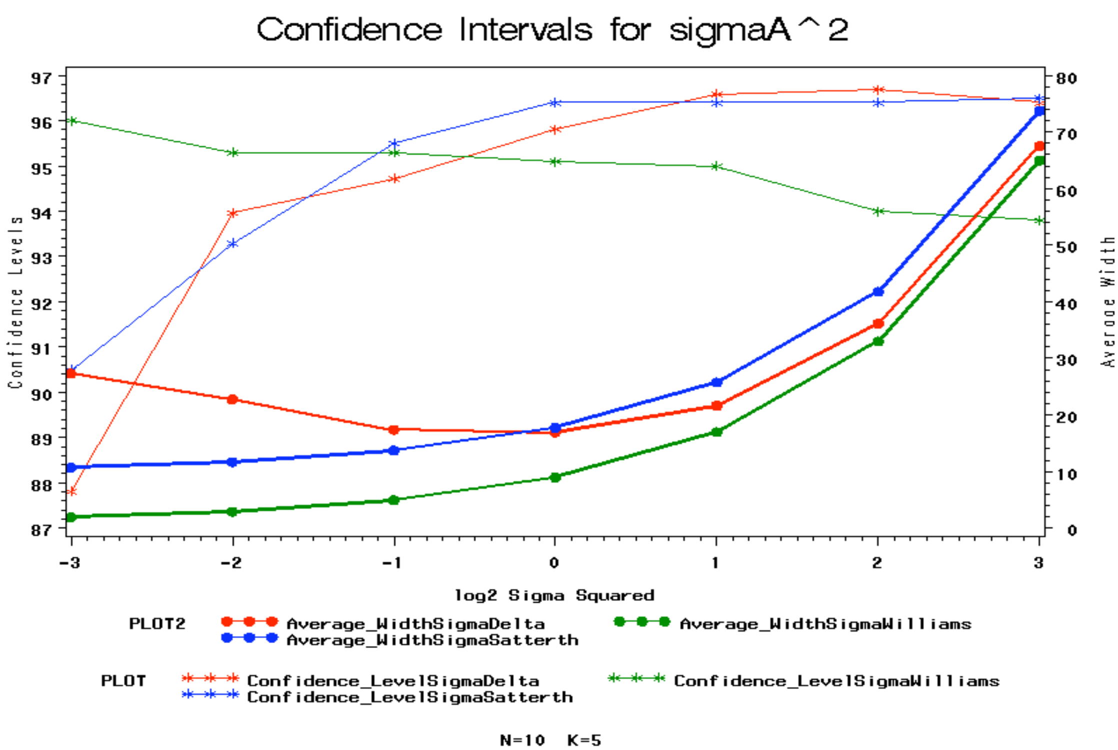
Graph 2. Confidence Intervals for σ_A^2 / σ_e^2, k=5 n=10.



Graph 3. Confidence Intervals for $\sigma_A^2 + \sigma_e^2 = \sigma_Y^2$, k=5 n=10.



Graph 4. Confidence Intervals for σ_A^2, k=5 n=10.



Conclusions

This report is devoted to comparing various methods for constructing confidence intervals for σ_A^2 and for three particular functions of the variance components in unbalanced one-way random effects models. The functions that were considered are intraclass correlation coefficient and both the ratio of σ_A^2 and σ_e^2 and sum of σ_A^2 and σ_e^2. Six methods (Delta Method, Fisher, TH, BAL, Swiger and Smith) to construct intraclass correlation coefficient were considered. The hypothesis is that the true confidence level is 95%. Appendix 1 summarizes the comparisons of these above mentioned methods for the combinations (for k=5 n=2, 6, 10; for k=10 n=2, 5, 10; for k=15 n=2, 4, 6; k=25 n=2, 3, 5; for k=50 n=2, 3) and σ_A^2 (1/8, 1/4, 1/2, 1, 2, 4, 8) where k represents number of groups and n represents the number of observations within each group. For instance when k=5 and n=10, the TH method failed to reject the null hypothesis for all σ_A^2 but 1/2, 1/4 and 1/8. The BAL method performed the best for every σ_A^2. At the same time, the worst method was Swiger. The null hypothesis is rejected for all values of σ_A^2.

In general, regardless of k, n and σ_A^2 the recommended method for constructing confidence intervals on the intraclass correlation is BAL. When constructing confidence intervals of $\frac{\sigma_A^2}{\sigma_e^2}$ is of interest, the recommended method is Ratio 1. There was not much difference in the performance of the delta method and the Satterthwaite method in the case of constructing confidence intervals on $\sigma_A^2 + \sigma_e^2 = \sigma_Y^2$. For constructing confidence intervals on σ_A^2 the method that performed the best is the modified Williams method.

Future research work might involve incorporating methods such as exact confidence interval for infraclass correlation coefficient and ML for the purpose of building comparison tables indicating the best method for a given combination of k and n. Another possible way to enhance this work is to expand the combinations of n and k beyond the current considerations.

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Appendix 1.

Table A.1

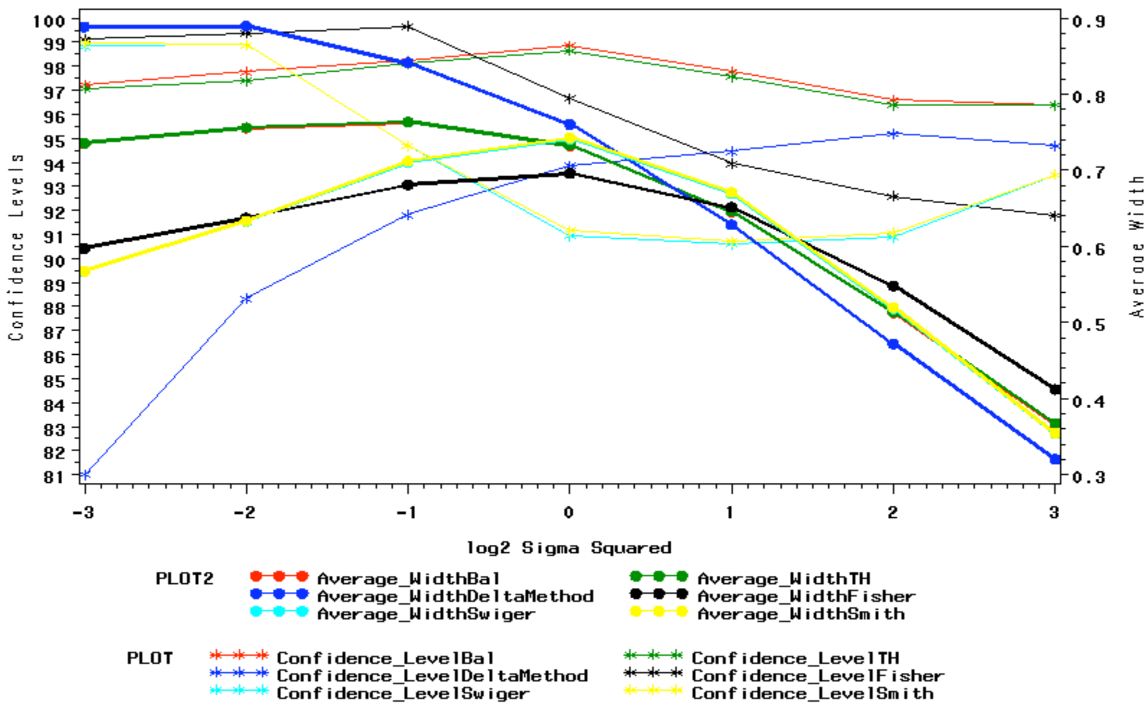
Configuration				OBSERVED CONFIDENCE LEVEL													
				$\sigma_A^2/(\sigma_A^2+\sigma_e^2)$					σ_A^2/σ_e^2		$\sigma_A^2+\sigma_e^2=\sigma_Y^2$		σ_A^2				
k	n	sigma2	TH	Bal	Fisher	Smith	Swiger	Delta Method	Ratio1	Delta Method	Delta Method	Satterthwaite	Delta Method	Williams	Milliken and Johnson	Wald	
5	2	1/8	91.4	94.8	90.6	79.2	78.7	46.2	95.1	51.5	95.4	95.7	59.0	95.6	98.3	99.8	
5	2	1/4	92.4	95.5	92.2	81.0	80.7	57.7	94.8	60.8	95.5	95.7	73.6	96.0	98.3	99.4	
5	2	1/2	93.0	96.0	93.0	82.8	82.8	67.6	94.6	70.8	95.7	95.9	82.5	96.3	97.7	98.1	
5	2	1	94.5	96.5	93.7	82.7	82.5	75.9	94.5	78.6	94.6	94.8	91.5	95.8	97.3	93.9	
5	2	2	95.4	96.6	94.7	83.3	83.3	79.1	93.9	83.2	93.8	93.8	93.1	95.5	96.8	88.7	
5	2	4	96.1	96.9	93.9	85.0	84.9	82.4	94.5	86.4	93.7	93.6	95.5	96.0	96.9	84.1	
5	2	8	95.6	95.6	91.5	85.1	85.1	83.5	94.3	87.9	93.6	93.5	96.3	94.8	96.3	81.1	
5	6	1/8	97.1	97.2	99.1	99.0	98.8	81.0	95.8	82.2	96.8	96.8	83.8	95.9	95.5	100.0	
5	6	1/4	97.4	97.8	99.4	98.9	98.9	88.3	95.3	90.2	96.0	95.8	91.7	95.3	94.7	97.6	
5	6	1/2	98.1	98.2	99.7	94.7	94.7	91.8	96.2	92.5	95.3	95.2	94.8	95.9	96.4	89.2	
5	6	1	98.6	98.9	96.7	91.2	91.0	93.9	97.0	95.1	92.4	92.5	96.9	96.5	97.0	83.0	
5	6	2	97.6	97.8	93.9	90.7	90.6	94.5	96.9	95.9	91.4	91.2	97.4	96.1	97.2	80.6	
5	6	4	96.4	96.6	92.6	91.1	90.9	95.2	96.4	96.8	91.0	91.5	98.2	96.5	97.6	81.1	
5	6	8	96.4	96.4	91.8	93.5	93.5	94.7	96.3	97.1	91.9	92.0	98.6	96.1	97.2	80.3	
5	10	1/8	96.8	97.0	99.4	99.9	99.9	86.4	95.7	87.2	96.1	96.0	87.8	96.0	90.5	99.4	
5	10	1/4	96.8	97.0	99.1	95.2	95.2	90.7	95.4	92.5	96.0	96.0	94.0	95.3	93.3	92.8	
5	10	1/2	97.0	97.0	95.8	86.9	86.9	92.4	94.9	93.5	93.5	93.3	94.7	95.3	95.5	83.8	
5	10	1	96.2	96.3	92.2	85.3	85.2	93.6	94.4	95.0	88.6	88.5	95.8	95.1	96.4	80.8	
5	10	2	94.9	94.5	90.1	86.1	86.0	93.7	94.1	96.1	86.6	86.5	96.6	95.0	96.4	78.6	
5	10	4	94.5	94.4	90.1	89.0	88.9	93.4	94.2	96.1	88.1	88.0	96.7	94.0	96.4	77.9	
5	10	8	94.2	94.5	89.8	90.3	90.3	93.4	94.5	96.2	89.4	89.3	96.4	93.8	96.5	77.7	
10	2	1/8	86.4	95.5	93.0	86.0	86.0	61.4	94.6	62.1	95.9	96.0	66.2	95.5	98.2	99.7	
10	2	1/4	87.3	96.1	93.7	87.3	87.2	70.0	94.6	70.6	96.4	96.4	78.1	96.7	98.1	99.9	
10	2	1/2	89.4	96.5	94.6	87.7	87.6	77.4	94.8	79.1	96.1	95.8	88.7	96.4	98.1	99.3	
10	2	1	91.8	97.0	95.3	87.7	87.7	81.0	94.8	83.9	94.9	95.0	92.6	96.2	98.2	95.3	
10	2	2	94.1	96.5	94.6	87.6	87.2	83.8	94.8	87.6	94.2	94.1	95.3	95.8	97.3	89.5	
10	2	4	94.4	94.9	93.8	86.2	85.8	84.8	94.5	88.7	94.4	93.9	95.7	95.4	96.4	86.4	
10	2	8	94.8	94.8	92.6	86.1	86.1	84.9	94.8	89.5	93.6	93.1	95.4	95.1	96.0	85.1	
10	5	1/8	96.4	96.3	97.3	98.0	98.0	78.8	93.8	79.7	95.2	95.2	83.6	94.4	88.1	99.9	
10	5	1/4	96.6	96.6	98.0	97.9	97.9	87.5	94.5	88.2	95.9	95.9	90.5	95.3	92.0	96.9	
10	5	1/2	96.9	97.3	96.4	92.0	92.0	91.5	94.6	92.8	94.8	94.7	94.4	94.6	94.0	89.4	
10	5	1	95.5	95.0	94.3	90.6	90.6	93.1	94.1	94.6	93.1	93.1	96.3	94.6	95.1	87.3	
10	5	2	95.0	94.7	93.3	91.2	91.2	93.9	94.5	95.5	92.2	92.0	97.1	95.1	96.2	85.9	
10	5	4	95.0	95.1	93.1	93.0	93.0	94.1	95.1	96.1	93.3	92.6	97.5	94.9	97.0	84.9	
10	5	8	94.9	95.0	92.9	94.0	93.5	94.3	95.0	95.6	94.6	94.1	96.1	94.9	96.6	85.2	
10	10	1/8	96.9	97.0	98.8	99.0	99.0	90.5	95.4	90.7	94.7	94.6	92.6	96.2	67.1	96.9	
10	10	1/4	97.5	97.3	96.1	92.1	92.0	93.1	95.1	93.8	94.3	94.4	94.5	94.9	81.2	90.2	
10	10	1/2	95.1	94.5	93.5	90.3	90.2	93.9	94.0	95.1	93.7	93.8	95.4	94.0	90.1	87.7	
10	10	1	94.0	93.9	92.9	88.8	88.8	94.4	93.9	95.4	92.2	91.9	96.1	93.6	93.8	86.3	
10	10	2	94.1	93.8	92.4	90.4	90.4	94.4	93.8	95.7	90.3	90.4	95.3	93.4	95.4	85.9	
10	10	4	94.3	94.3	92.4	92.0	91.7	94.2	94.3	94.8	91.0	90.8	94.4	93.4	96.5	85.3	
10	10	8	94.0	94.1	91.9	93.5	93.5	94.1	94.1	94.6	91.7	91.8	94.2	93.4	96.1	84.6	
15	2	1/8	83.8	95.7	94.3	90.9	90.8	62.6	93.9	63.8	95.1	95.3	68.3	94.3	98.4	99.3	
15	2	1/4	85.4	96.1	95.3	90.8	90.8	73.8	93.9	75.3	95.2	95.1	80.9	94.8	98.1	99.7	
15	2	1/2	86.7	96.6	95.8	90.4	90.4	81.5	94.1	83.5	94.4	94.4	88.8	95.3	97.9	98.7	
15	2	1	90.5	96.5	95.7	89.6	89.5	85.3	94.1	86.6	94.2	93.5	93.2	95.6	97.8	94.0	
15	2	2	91.6	94.8	93.5	89.5	89.5	86.8	94.2	89.5	93.0	92.6	95.4	95.7	97.3	90.6	
15	2	4	93.3	94.3	92.8	89.2	89.1	87.6	94.2	90.6	93.5	93.3	96.1	95.4	96.7	89.4	
15	2	8	94.5	94.4	92.9	89.2	89.1	87.8	94.4	90.8	93.8	93.6	95.6	95.3	95.8	87.9	
15	4	1/8	97.1	97.1	97.9	97.9	97.8	79.4	95.9	80.3	94.5	94.5	82.1	96.2	89.7	100.0	
15	4	1/4	97.6	97.7	98.1	97.9	97.9	87.2	95.9	87.9	95.1	95.2	89.7	95.9	93.1	97.1	
15	4	1/2	97.1	97.4	96.6	93.5	93.4	91.1	96.0	92.5	94.9	95.1	94.1	95.6	95.1	91.8	
15	4	1	95.9	95.8	95.0	92.9	92.8	92.5	95.2	94.3	94.4	94.2	96.0	94.6	96.2	90.3	
15	4	2	95.1	94.9	93.9	93.0	92.7	92.3	94.9	95.1	94.2	94.0	96.5	94.3	96.3	89.3	
15	4	4	95.0	94.7	93.9	93.5	93.2	93.1	94.7	95.6	94.1	93.5	96.1	94.5	96.8	88.3	
15	4	8	94.9	94.6	93.7	94.4	94.1	93.5	94.6	95.0	94.2	94.4	95.0	94.9	96.6	88.3	
15	6	1/8	96.7	96.5	97.8	99.2	99.2	86.2	95.1	86.5	95.5	95.4	88.2	95.4	69.2	99.8	
15	6	1/4	97.0	96.7	97.5	94.9	94.9	91.8	95.3	92.5	95.2	95.1	94.0	95.8	81.7	92.6	
15	6	1/2	95.4	95.9	94.5	92.7	92.7	93.8	95.7	94.6	94.7	94.6	96.2	95.9	89.9	90.1	
15	6	1	96.0	96.1	94.5	93.3	93.3	94.8	96.0	95.9	94.6	93.8	97.2	95.8	94.6	90.2	
15	6	2	96.3	95.9	94.4	94.0	93.9	94.9	95.9	97.2	93.3	93.2	96.9	95.7	96.6	89.6	
15	6	4	96.2	96.3	94.4	95.2	94.9	95.4	96.3	97.0	93.9	93.7	96.1	95.7	97.0	89.4	
15	6	8	96.4	96.5	94.5	95.7	95.5	95.1	96.5	96.4	94.2	94.3	96.0	95.4	96.9	89.0	
25	2	1/8	77.8	96.2	95.7	93.4	93.2	68.4	94.3	68.8	94.9	94.9	71.5	95.9	98.6	98.7	
25	2	1/4	78.6	96.8	96.4	93.2	93.2	80.0	94.4	81.5	95.2	95.4	84.6	96.5	98.7	99.1	
25	2	1/2	83.0	97.7	96.9	93.5	93.3	86.9	95.3	88.5	95.5	95.7	92.2	96.9	98.6	98.4	
25	2	1	87.5	95.8	95.3	92.1	92.0	89.8	95.1	91.3	95.0	94.9	95.6	97.0	98.8	93.5	
25	2	2	91.2	94.4	93.6	92.1	92.0	91.0	94.4	92.8	94.7	94.3	96.8	97.1	97.8	91.0	
25	2	4	93.7	94.3	93.8	91.8	91.5	91.0	94.3	93.4	94.8	94.1	96.6	95.8	96.8	90.1	
25	2	8	94.7	94.3	93.5	91.0	90.7	90.9	94.3	93.3	94.1	93.1	95.1	95.1	95.3	89.6	
25	3	1/8	93.2	96.5	97.5	96.5	96.5	80.1	94.9	81.1	94.0	93.9	82.9	96.0	94.6	99.9	
25	3	1/4	94.2	97.6	97.8	97.0	96.9	88.0	94.9	89.1	94.5	94.4	90.1	95.3	95.2	99.1	
25	3	1/2	93.8	96.5	96.1	94.8	94.7	91.4	94.7	92.9	94.8	94.8	94.3	95.4	95.6	93.9	
25	3	1	93.1	94.8	93.7	93.2	93.1	93.4	94.7	95.2	94.7	94.7	96.2	95.4	96.1	90.9	
25	3	2	93.9	94.2	93.6	93.7	93.4	94.5	94.2	95.9	94.4	94.4	96.6	95.6	96.3	89.8	
25	3	4	93.7	93.3	93.0	94.2	93.8	94.9	93.3	96.2	94.4	94.1	96.2	94.5	95.6	90.5	
25	3	8	94.4	93.5	93.0	94.8	94.8	95.2	93.5	95.3	94.6	93.9	95.6	94.7	95.5	90.0	
25	5	1/8	97.6	97.6	98.5	98.6	98.6	85.8	96.2	86.9	95.4	95.3	88.0	96.3	58.1	99.5	
25	5	1/4	96.8	96.5	96.4	94.3	94.1	91.2	95.1	92.2	94.8	94.8	92.6	95.6	73.1	92.5	
25	5	1/2	94.5	94.9	94.3	92.7	92.7	93.9	94.9	95.2	94.2	94.0	95.6	95.5	84.4	91.3	
25	5	1	94.5	94.5	93.7	92.4	92.3	94.8	94.5	95.9	93.6	93.8	96.4	94.5	91.9	91.6	
25	5	2	94.5	94.4	93.5	93.5	93.5	95.3	94.4	95.1	93.6	93.1	94.7	94.0	94.5	91.9	
25	5	4	94.0	94.5	94.0	94.1	93.9	94.9	94.5	95.2	93.5	93.2	94.0	93.6	95.8	91.8	
25	5	8	94.6	94.5	94.0	94.3	94.1	95.0	94.5	95.1	93.5	93.4	94.1	93.9	96.2	91.2	
50	2	1/8	60.7	97.4	97.3	96.2	96.2	77.6	94.7	78.5	95.1	95.1	80.0	95.5	98.2	98.6	
50	2	1/4	65.4	98.1	97.9	96.9	96.9	86.1	94.8	87.1	95.0	94.9	87.8	95.9	98.1	98.8	
50	2	1/2	71.3	96.6	96.5	96.1	96.0	90.5	94.9	92.0	93.5	93.5	92.9	96.5	97.9	95.8	
50	2	1	81.8	95.0	94.8	94.3	94.3	92.8	94.9	94.1	93.5	92.8	95.6	96.1	97.5	93.2	
50	2	2	90.4	94.4	94.4	94.2	93.8	93.2	94.4	95.0	93.7	92.9	96.0	95.9	96.9	92.7	
50	2	4	93.4	94.1	93.9	94.2	94.1	93.8	94.1	95.2	94.1	93.4	94.5	95.7	96.3	91.8	
50	2	8	94.2	94.1	93.8	94.5	94.4	93.7	94.1	94.9	94.0	93.5	94.4	94.8	94.9	91.4	
50	3	1/8	91.6	97.7	98.4	98.2	98.2	85.9	95.5	86.6	95.6	95.6	86.9	96.2	87.4	99.4	
50	3	1/4	91.8	97.9	97.7	97.0	96.9	90.9	95.7	92.1	95.3	95.4	92.3	96.2	90.3	96.9	
50	3	1/2	90.2	94.9	94.6	93.8	93.8	93.9	94.7	94.6	95.8	95.8	95.7	96.0	92.6	93.8	
50	3	1	91.2	94.3	94.1	93.4	93.4	95.0	94.3	95.6	95.1	95.1	97.3	95.6	94.5	92.8	
50	3	2	93.2	94.2	94.0	93.8	93.8	95.6	94.2	95.6	94.2	93.5	95.9	94.9	95.4	92.5	
50	3	4	94.1	94.4	94.1	94.6	94.5	95.5	94.4	94.8	94.2	93.9	94.9	94.4	95.8	92.5	

Table A.2.

Configuration				*AVERAGE WIDTH*												
				$\sigma_A^2/(\sigma_A^2+\sigma_e^2)$					σ_A^2/σ_e^2		$\sigma_A^2+\sigma_e^2=\sigma_Y^2$		σ_A^2			
k	n	sigma2	Bal	TH	Fisher	Smith	Swiger	Delta Method	Ratio1	Delta Method	Delta Method	Sattert h waite	Delta Method	Williams	Milliken and Johnson	Wald
5	2	1/8	0.90	0.88	0.85	0.90	0.90	0.77	23.43	1009.12	4.63	4.75	96.28	8.04	10.11	1.87
5	2	1/4	0.90	0.88	0.85	0.89	0.89	0.75	27.27	989.60	5.34	5.46	104.83	9.06	11.14	2.15
5	2	1/2	0.90	0.88	0.85	0.86	0.86	0.72	35.14	1020.74	6.87	6.98	105.81	11.11	13.18	2.72
5	2	1	0.89	0.87	0.84	0.80	0.80	0.68	51.14	1093.31	10.12	10.31	119.39	15.20	17.27	3.87
5	2	2	0.86	0.85	0.80	0.71	0.71	0.58	83.51	1337.06	17.24	17.50	119.83	23.38	25.45	6.15
5	2	4	0.80	0.79	0.72	0.56	0.55	0.46	148.75	1895.35	32.24	32.78	106.49	39.73	41.81	10.84
5	2	8	0.67	0.66	0.57	0.39	0.38	0.33	279.97	3109.55	63.34	64.49	115.63	72.44	74.52	20.39
5	6	1/8	0.74	0.74	0.60	0.57	0.57	0.89	2.81	58.34	1.65	1.65	43.25	2.50	10.42	0.78
5	6	1/4	0.76	0.76	0.64	0.63	0.63	0.89	3.88	57.22	2.04	2.04	43.20	3.45	11.36	1.04
5	6	1/2	0.76	0.76	0.68	0.71	0.71	0.84	6.04	49.23	2.99	2.99	37.12	5.34	13.26	1.56
5	6	1	0.73	0.73	0.70	0.74	0.74	0.76	10.35	42.34	5.33	5.32	29.97	9.14	17.05	2.67
5	6	2	0.65	0.65	0.65	0.67	0.67	0.63	19.00	40.07	10.97	10.97	27.28	16.73	24.64	4.92
5	6	4	0.51	0.51	0.55	0.52	0.52	0.47	36.32	57.45	23.92	23.94	38.37	31.92	39.83	9.50
5	6	8	0.37	0.37	0.41	0.35	0.35	0.32	71.00	99.10	52.15	52.23	66.55	62.31	70.22	18.72
5	10	1/8	0.65	0.65	0.49	0.43	0.43	0.90	2.07	31.70	1.23	1.23	27.33	1.93	10.71	0.58
5	10	1/4	0.69	0.69	0.56	0.53	0.53	0.90	3.15	27.36	1.62	1.61	22.75	2.94	11.72	0.88
5	10	1/2	0.70	0.70	0.61	0.63	0.63	0.84	5.32	20.82	2.59	2.58	17.43	4.96	13.74	1.46
5	10	1	0.67	0.67	0.63	0.68	0.68	0.75	9.65	20.67	5.03	5.00	16.97	8.98	17.76	2.65
5	10	2	0.59	0.59	0.60	0.62	0.62	0.62	18.27	26.77	10.94	10.88	21.65	17.00	25.78	5.04
5	10	4	0.47	0.47	0.51	0.50	0.50	0.46	35.49	44.90	24.57	24.46	36.15	33.02	41.80	9.88
5	10	8	0.34	0.34	0.38	0.34	0.34	0.32	69.90	83.81	54.32	54.12	67.55	65.02	73.80	19.54
10	2	1/8	0.75	0.72	0.73	0.85	0.85	0.82	4.48	139.06	2.03	2.04	54.13	2.68	3.39	1.20
10	2	1/4	0.76	0.73	0.74	0.86	0.85	0.82	5.26	142.07	2.31	2.32	59.52	3.04	3.75	1.38
10	2	1/2	0.78	0.75	0.75	0.85	0.85	0.79	6.88	152.99	2.90	2.91	57.80	3.77	4.48	1.82
10	2	1	0.78	0.75	0.75	0.80	0.80	0.70	10.18	120.76	4.16	4.18	40.75	5.22	5.93	2.73
10	2	2	0.73	0.71	0.69	0.67	0.66	0.57	16.86	116.14	6.86	6.88	32.49	8.14	8.85	4.54
10	2	4	0.59	0.58	0.54	0.48	0.47	0.41	30.36	133.75	12.47	12.53	22.20	13.99	14.70	8.29
10	2	8	0.41	0.40	0.37	0.29	0.29	0.26	57.54	212.05	23.90	24.07	27.93	25.69	26.40	15.83
10	5	1/8	0.53	0.53	0.47	0.45	0.45	0.89	1.17	27.15	1.11	1.11	21.63	1.05	3.46	0.59
10	5	1/4	0.57	0.57	0.52	0.51	0.51	0.88	1.59	20.47	1.32	1.32	15.44	1.41	3.82	0.79
10	5	1/2	0.59	0.59	0.55	0.58	0.57	0.83	2.43	13.90	1.80	1.80	11.26	2.12	4.53	1.22
10	5	1	0.57	0.57	0.56	0.59	0.59	0.72	4.10	8.76	2.92	2.92	6.46	3.55	5.96	2.16
10	5	2	0.49	0.49	0.49	0.51	0.51	0.56	7.42	10.47	5.40	5.40	7.55	6.40	8.81	4.05
10	5	4	0.36	0.36	0.38	0.38	0.37	0.39	14.07	17.47	10.73	10.74	12.86	12.08	14.49	7.78
10	5	8	0.24	0.24	0.25	0.24	0.24	0.24	27.35	32.63	21.79	21.83	24.02	23.45	25.86	15.20
10	10	1/8	0.40	0.40	0.34	0.30	0.30	0.92	0.74	9.31	0.76	0.76	8.51	0.70	3.51	0.38
10	10	1/4	0.46	0.46	0.40	0.40	0.39	0.87	1.13	4.98	0.96	0.96	4.53	1.06	3.87	0.61
10	10	1/2	0.50	0.50	0.46	0.48	0.48	0.76	1.91	3.56	1.44	1.44	3.18	1.80	4.61	1.10
10	10	1	0.49	0.49	0.48	0.51	0.51	0.64	3.48	4.32	2.57	2.57	3.75	3.26	6.07	2.07
10	10	2	0.43	0.43	0.43	0.45	0.45	0.50	6.60	7.47	5.09	5.10	6.52	6.19	9.00	3.99
10	10	4	0.32	0.32	0.34	0.34	0.34	0.35	12.86	14.09	10.55	10.56	12.34	12.04	14.85	7.80
10	10	8	0.21	0.21	0.23	0.22	0.22	0.22	25.37	27.39	21.91	21.94	24.01	23.75	26.56	15.40
15	2	1/8	0.67	0.64	0.66	0.76	0.75	0.84	2.39	79.20	1.48	1.48	42.49	1.84	2.29	0.97
15	2	1/4	0.69	0.66	0.67	0.77	0.76	0.84	2.83	79.60	1.67	1.68	44.78	2.09	2.54	1.16
15	2	1/2	0.70	0.67	0.69	0.77	0.76	0.80	3.71	66.88	2.09	2.10	37.11	2.59	3.04	1.51
15	2	1	0.70	0.67	0.68	0.73	0.73	0.71	5.47	46.03	2.98	2.99	23.95	3.59	4.04	2.29
15	2	2	0.63	0.60	0.60	0.60	0.60	0.56	9.00	32.43	4.86	4.88	13.76	5.60	6.04	3.86
15	2	4	0.47	0.45	0.44	0.42	0.41	0.38	16.05	30.34	8.74	8.78	11.64	9.60	10.05	6.94
15	2	8	0.30	0.29	0.28	0.25	0.25	0.23	30.18	49.33	16.62	16.72	17.97	17.61	18.06	12.99
15	4	1/8	0.47	0.48	0.44	0.43	0.43	0.90	0.95	24.29	0.97	0.97	19.80	0.87	2.31	0.55
15	4	1/4	0.50	0.51	0.47	0.48	0.48	0.88	1.25	17.20	1.13	1.13	14.43	1.11	2.56	0.72
15	4	1/2	0.53	0.54	0.51	0.53	0.53	0.82	1.83	10.13	1.50	1.49	8.25	1.60	3.04	1.10
15	4	1	0.52	0.52	0.51	0.53	0.53	0.67	3.00	5.47	2.32	2.30	3.98	2.57	4.02	1.90
15	4	2	0.43	0.43	0.43	0.44	0.44	0.49	5.34	7.14	4.10	4.08	5.08	4.52	5.96	3.40
15	4	4	0.31	0.31	0.32	0.31	0.31	0.32	10.00	12.07	7.84	7.80	8.76	8.40	9.84	6.35
15	4	8	0.20	0.20	0.20	0.20	0.19	0.20	19.32	22.59	15.51	15.45	16.50	16.16	17.60	12.25
15	6	1/8	0.38	0.39	0.35	0.33	0.33	0.89	0.68	11.89	0.78	0.78	10.21	0.64	2.31	0.41
15	6	1/4	0.43	0.43	0.40	0.40	0.39	0.84	0.96	7.61	0.92	0.92	6.23	0.88	2.55	0.60
15	6	1/2	0.47	0.47	0.45	0.47	0.46	0.71	1.51	3.46	1.28	1.27	2.77	1.37	3.04	1.00
15	6	1	0.45	0.46	0.45	0.47	0.47	0.56	2.61	3.31	2.08	2.08	2.68	2.35	4.02	1.77
15	6	2	0.38	0.38	0.39	0.40	0.40	0.43	4.80	5.57	3.86	3.86	4.56	4.31	5.98	3.27
15	6	4	0.28	0.28	0.29	0.29	0.28	0.29	9.20	10.26	7.62	7.61	8.45	8.23	9.90	6.25
15	6	8	0.18	0.18	0.19	0.18	0.18	0.18	18.00	19.73	15.33	15.33	16.28	16.08	17.74	12.22
25	2	1/8	0.56	0.54	0.56	0.62	0.61	0.84	1.43	44.26	1.05	1.05	28.92	1.25	1.54	0.75
25	2	1/4	0.58	0.56	0.58	0.63	0.63	0.86	1.70	42.54	1.18	1.19	29.06	1.42	1.70	0.89
25	2	1/2	0.61	0.58	0.60	0.65	0.65	0.82	2.24	32.27	1.47	1.47	22.92	1.76	2.04	1.20
25	2	1	0.60	0.57	0.59	0.61	0.61	0.70	3.31	15.64	2.08	2.09	10.10	2.44	2.72	1.82
25	2	2	0.50	0.48	0.48	0.48	0.48	0.52	5.46	9.77	3.37	3.38	4.63	3.79	4.08	3.01
25	2	4	0.35	0.33	0.33	0.32	0.32	0.33	9.76	13.68	6.01	6.04	6.58	6.50	6.79	5.30
25	2	8	0.21	0.21	0.20	0.19	0.19	0.19	18.37	24.11	11.34	11.42	11.83	11.93	12.21	9.89
25	3	1/8	0.43	0.43	0.41	0.42	0.42	0.89	0.83	22.41	0.83	0.83	17.24	0.76	1.53	0.51
25	3	1/4	0.46	0.46	0.45	0.46	0.46	0.87	1.04	16.23	0.95	0.95	13.33	0.93	1.70	0.66
25	3	1/2	0.50	0.50	0.49	0.50	0.50	0.77	1.45	8.86	1.21	1.21	6.87	1.26	2.02	0.97
25	3	1	0.47	0.47	0.47	0.48	0.48	0.59	2.29	4.12	1.79	1.78	2.86	1.91	2.68	1.57
25	3	2	0.38	0.38	0.38	0.38	0.38	0.41	3.97	5.01	3.03	3.02	3.43	3.23	4.00	2.70
25	3	4	0.26	0.26	0.27	0.26	0.26	0.27	7.34	8.66	5.60	5.59	6.00	5.87	6.63	4.95
25	3	8	0.16	0.16	0.16	0.16	0.16	0.16	14.07	16.13	10.83	10.82	11.24	11.14	11.91	9.46
25	5	1/8	0.33	0.33	0.31	0.30	0.30	0.82	0.53	7.61	0.64	0.64	6.73	0.50	1.55	0.37
25	5	1/4	0.37	0.37	0.35	0.35	0.35	0.69	0.72	4.82	0.75	0.75	4.35	0.67	1.72	0.52
25	5	1/2	0.39	0.40	0.39	0.40	0.40	0.53	1.11	1.93	1.01	1.01	1.59	1.01	2.05	0.84
25	5	1	0.38	0.38	0.37	0.39	0.38	0.42	1.87	2.21	1.59	1.59	1.81	1.68	2.72	1.42
25	5	2	0.31	0.31	0.31	0.32	0.32	0.33	3.40	3.77	2.84	2.84	3.11	3.02	4.06	2.57
25	5	4	0.22	0.22	0.22	0.22	0.22	0.22	6.44	6.96	5.45	5.45	5.77	5.70	6.75	4.86
25	5	8	0.14	0.14	0.14	0.14	0.14	0.14	12.54	13.37	10.74	10.76	11.11	11.06	12.10	9.46
50	2	1/8	0.44	0.43	0.44	0.46	0.46	0.88	0.89	23.80	0.70	0.70	17.48	0.81	0.98	0.53
50	2	1/4	0.47	0.45	0.47	0.49	0.49	0.85	1.06	18.94	0.79	0.79	13.87	0.92	1.09	0.66
50	2	1/2	0.49	0.46	0.49	0.51	0.51	0.74	1.39	8.45	0.98	0.98	6.63	1.14	1.31	0.90
50	2	1	0.46	0.42	0.45	0.46	0.45	0.54	2.07	3.61	1.38	1.38	2.25	1.58	1.75	1.33
50	2	2	0.35	0.32	0.34	0.34	0.34	0.36	3.41	4.26	2.22	2.23	2.45	2.46	2.63	2.12
50	2	4	0.23	0.22	0.22	0.22	0.22	0.22	6.10	7.14	3.94	3.96	4.11	4.22	4.38	3.72
50	2	8	0.14	0.13	0.13	0.13	0.13	0.13	11.48	13.04	7.40	7.46	7.55	7.73	7.90	6.94
50	3	1/8	0.32	0.32	0.31	0.31	0.31	0.83	0.53	11.40	0.57	0.57	9.91	0.50	0.98	0.37
50	3	1/4	0.36	0.36	0.35	0.35	0.35	0.71	0.67	6.16	0.65	0.65	5.39	0.61	1.09	0.50
50	3	1/2	0.37	0.38	0.37	0.38	0.38	0.50	0.93	1.89	0.82	0.82	1.46	0.82	1.30	0.72
50	3	1	0.34	0.34	0.34	0.34	0.34	0.37	1.47	1.77	1.21	1.21	1.32	1.25	1.73	1.12
50	3	2	0.27	0.27	0.27	0.27	0.27	0.27	2.55	2.85	2.03	2.02	2.13	2.11	2.59	1.91
50	3	4	0.18	0.18	0.18	0.18	0.18	0.18	4.70	5.09	3.72	3.71	3.83	3.83	4.31	3.50
50	3	8	0.11	0.11	0.11	0.11	0.11	0.11	9.00	9.62	7.13	7.12	7.25	7.26	7.74	6.68

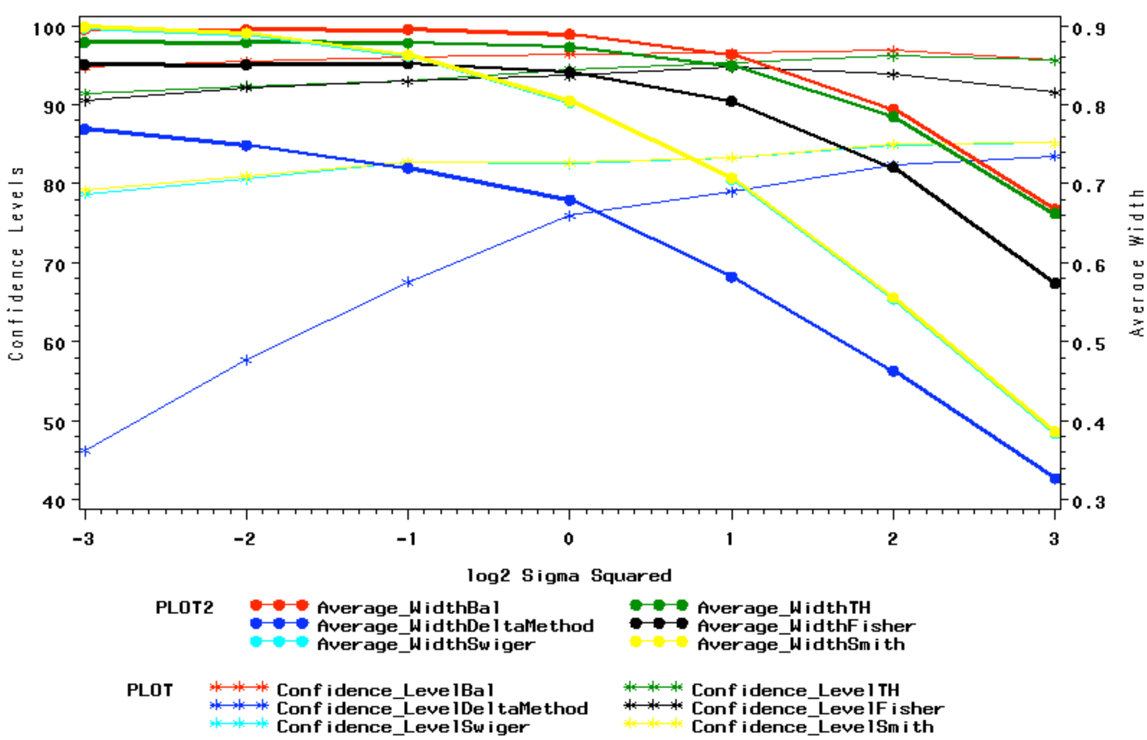
Appendix 2.

Confidence Intervals for Intraclass Correlation Coefficient

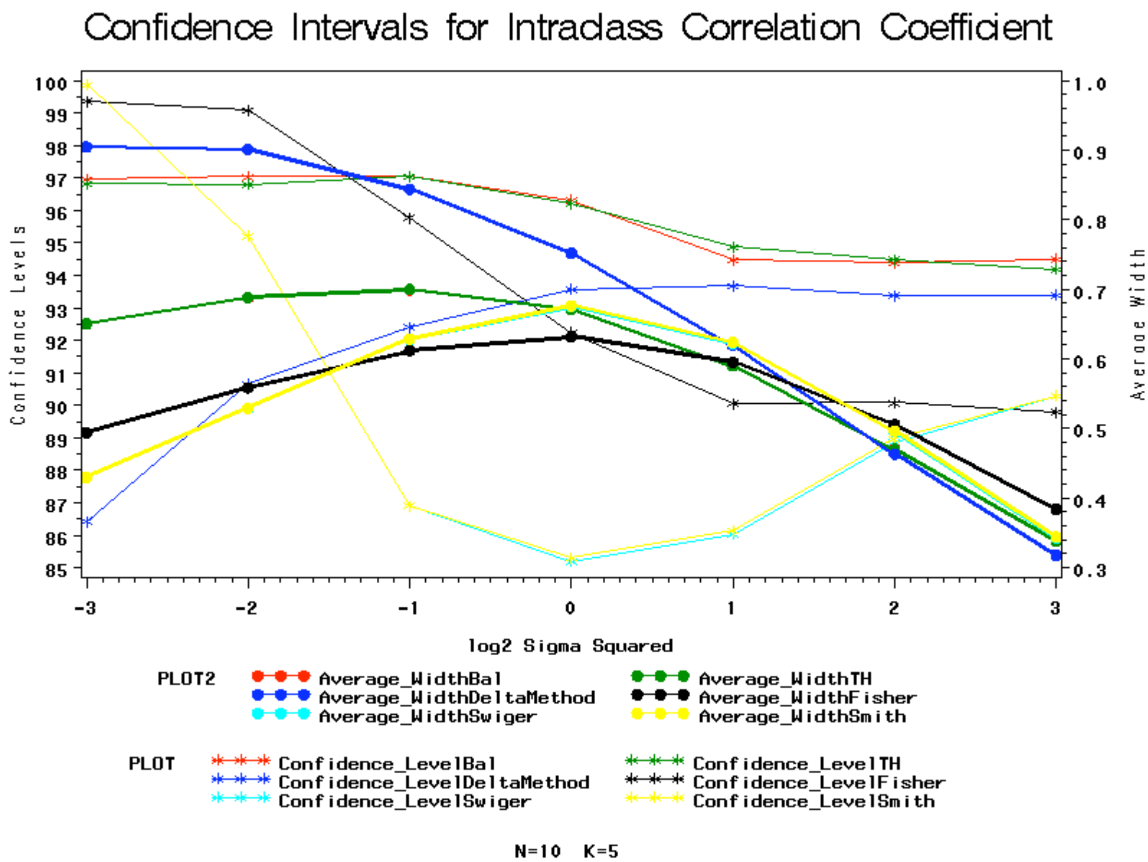


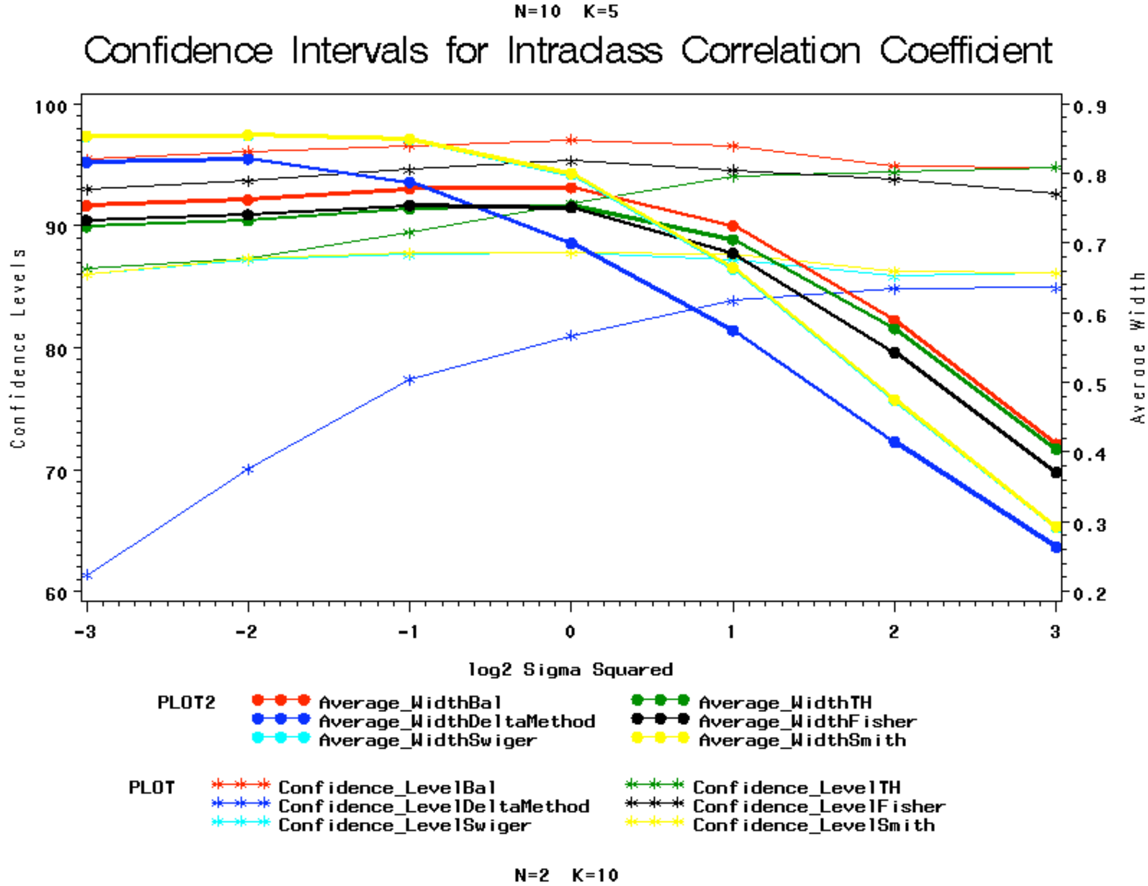
N=6 K=5

Confidence Intervals for Intraclass Correlation Coefficient

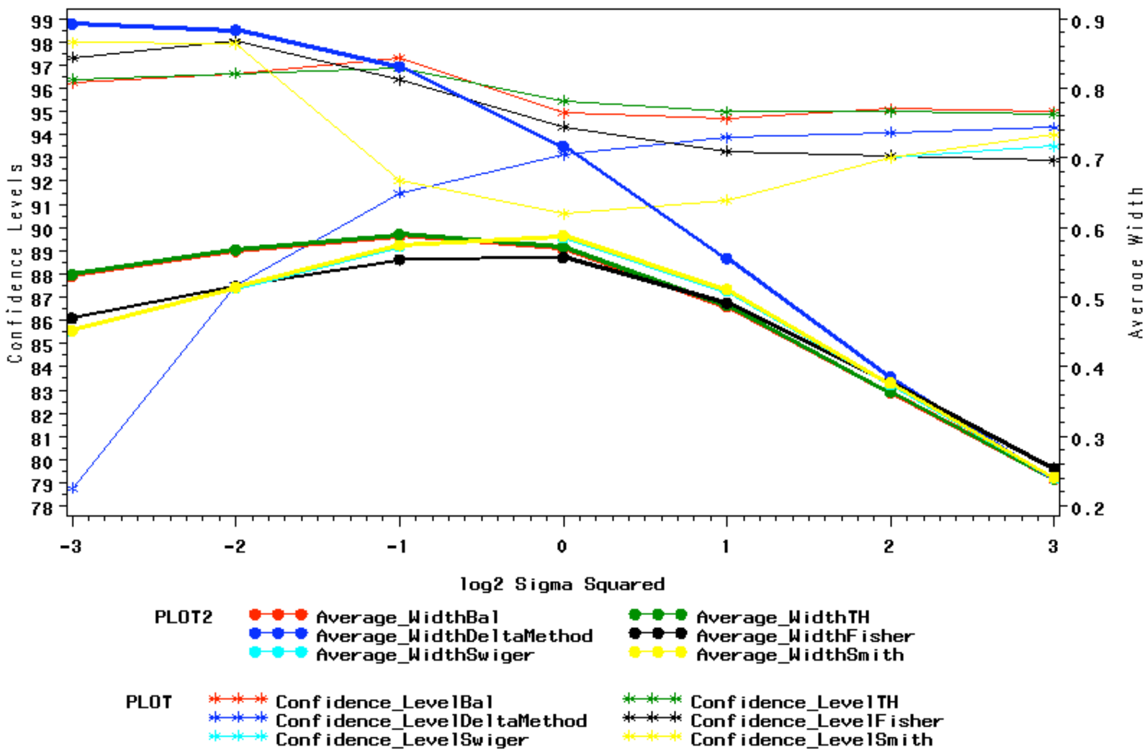


N=2 K=5


Confidence Intervals for Intraclass Correlation Coefficient
Confidence Levels
Average Width
log2 Sigma Squared
PLOT2 Average_WidthBal Average_WidthDeltaMethod Average_WidthSwiger Average_WidthTH Average_WidthFisher Average_WidthSmith
PLOT Confidence_LevelBal Confidence_LevelDeltaMethod Confidence_LevelSwiger Confidence_LevelTH Confidence_LevelFisher Confidence_LevelSmith
N=10 K=5

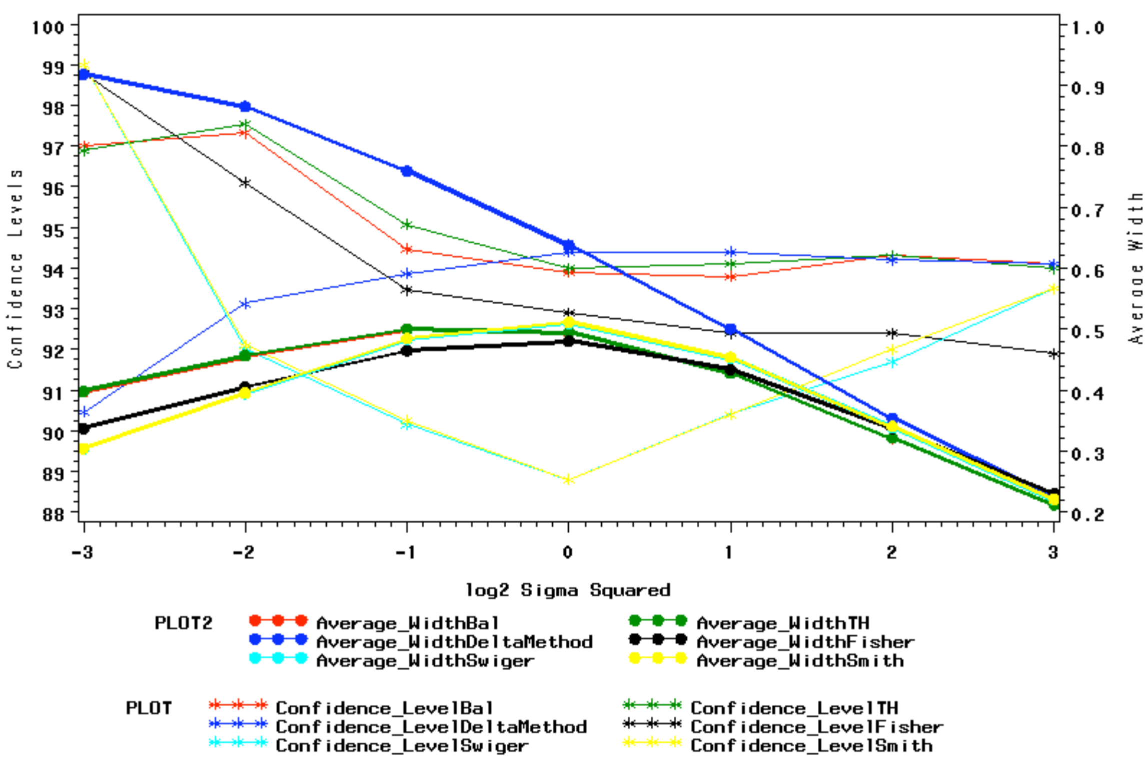

Confidence Intervals for Intraclass Correlation Coefficient
Confidence Levels
Average Width
log2 Sigma Squared
PLOT2 Average_WidthBal Average_WidthDeltaMethod Average_WidthSwiger Average_WidthTH Average_WidthFisher Average_WidthSmith
PLOT Confidence_LevelBal Confidence_LevelDeltaMethod Confidence_LevelSwiger Confidence_LevelTH Confidence_LevelFisher Confidence_LevelSmith
N=2 K=10

Confidence Intervals for Intraclass Correlation Coefficient

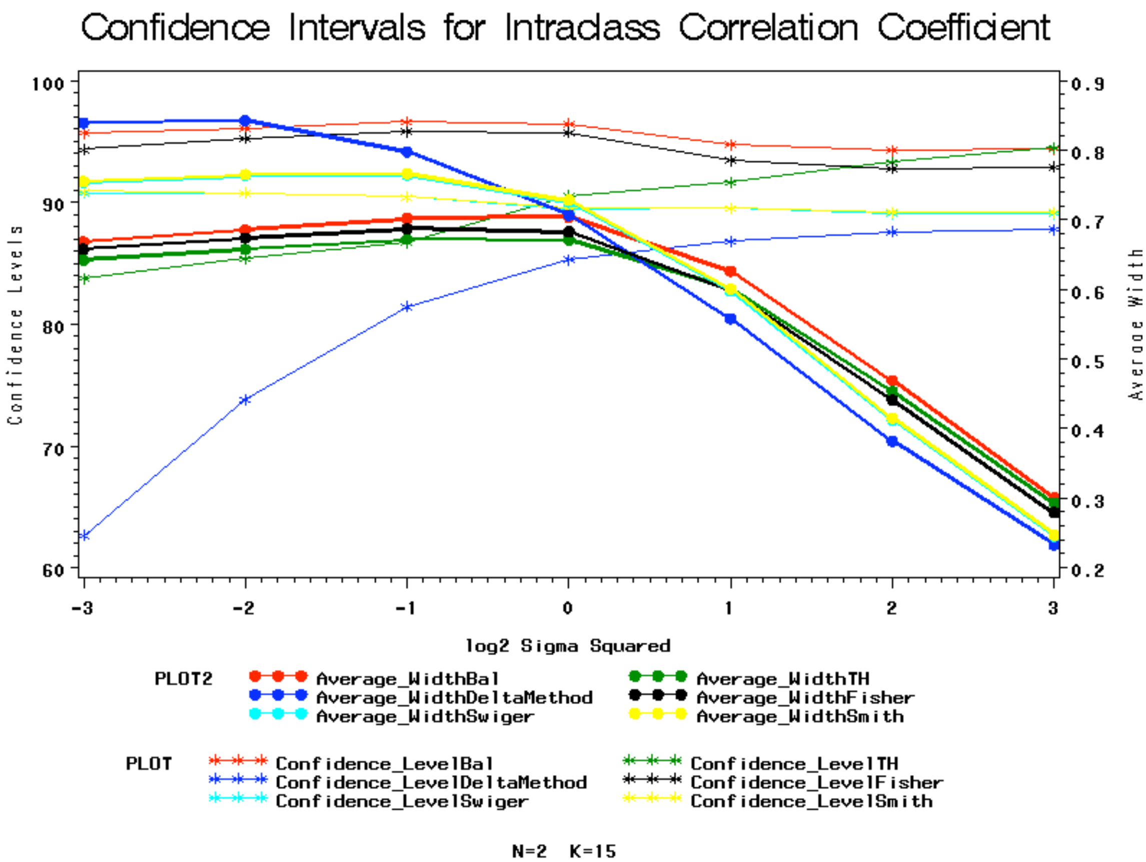


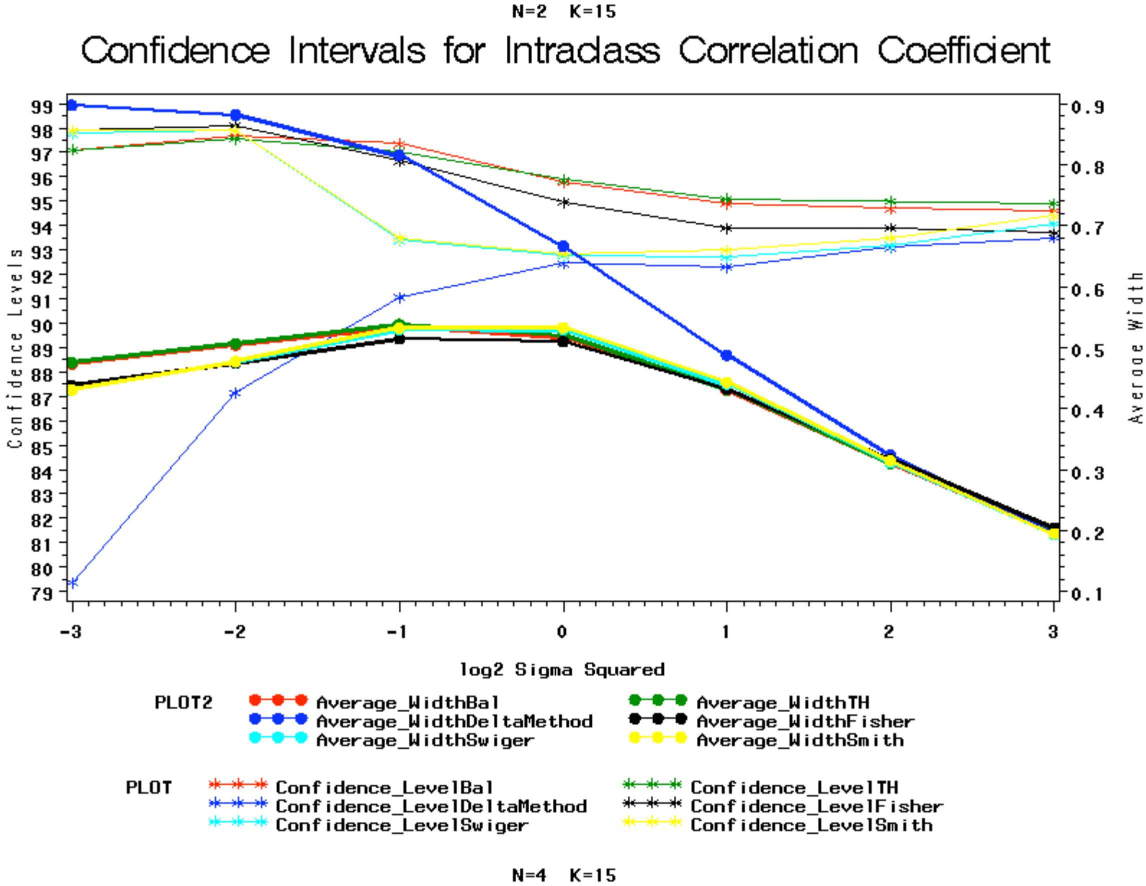
N=5 K=10

Confidence Intervals for Intraclass Correlation Coefficient

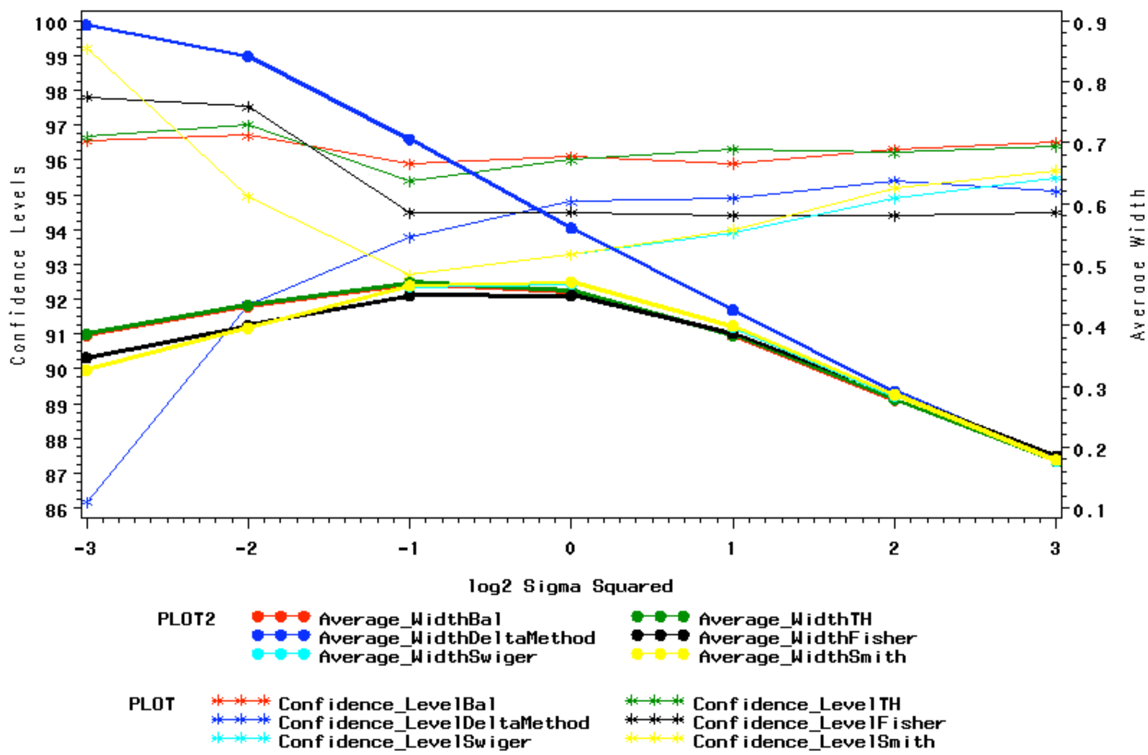


N=10 K=10


Confidence Intervals for Intraclass Correlation Coefficient
Confidence Levels
Average Width
100
90
80
70
60
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
-3
-2
-1
0
1
2
3
log2 Sigma Squared
PLOT2
Average_WidthBal
Average_WidthDeltaMethod
Average_WidthSwiger
Average_WidthTH
Average_WidthFisher
Average_WidthSmith
PLOT
Confidence_LevelBal
Confidence_LevelDeltaMethod
Confidence_LevelSwiger
Confidence_LevelTH
Confidence_LevelFisher
Confidence_LevelSmith
N=2 K=15

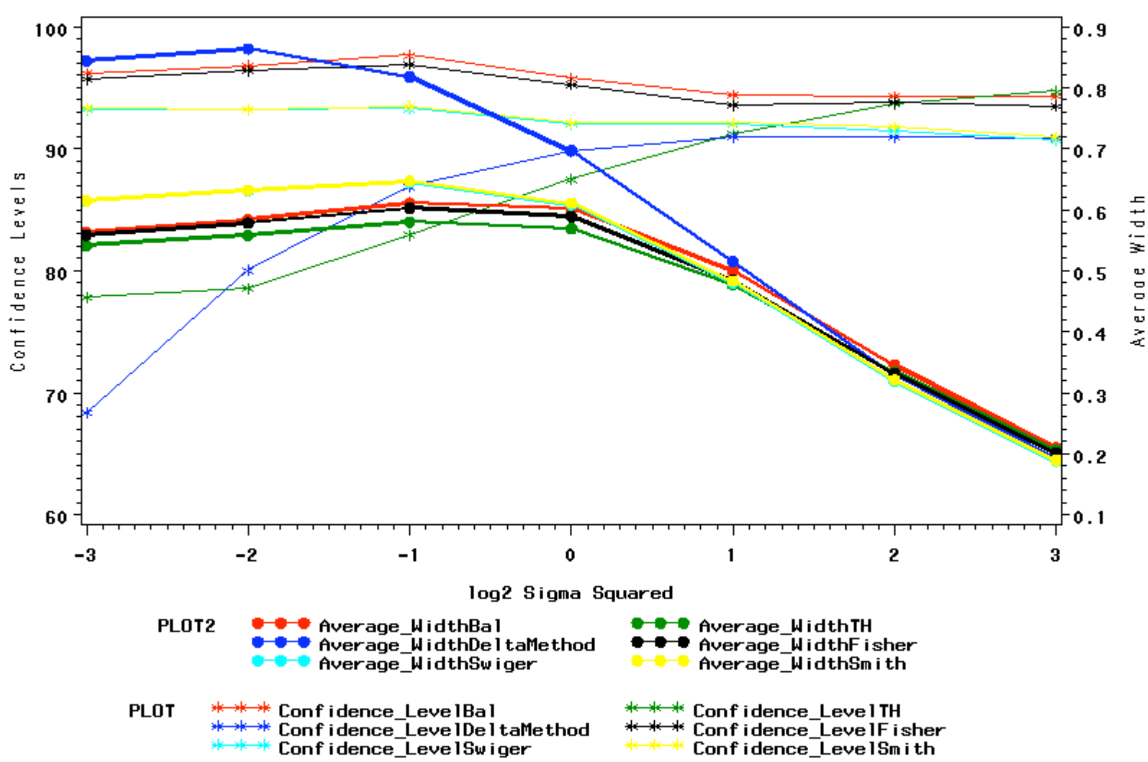

Confidence Intervals for Intraclass Correlation Coefficient
Confidence Levels
Average Width
99
98
97
96
95
94
93
92
91
90
89
88
87
86
85
84
83
82
81
80
79
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
-3
-2
-1
0
1
2
3
log2 Sigma Squared
PLOT2
Average_WidthBal
Average_WidthDeltaMethod
Average_WidthSwiger
Average_WidthTH
Average_WidthFisher
Average_WidthSmith
PLOT
Confidence_LevelBal
Confidence_LevelDeltaMethod
Confidence_LevelSwiger
Confidence_LevelTH
Confidence_LevelFisher
Confidence_LevelSmith
N=4 K=15

Confidence Intervals for Intraclass Correlation Coefficient



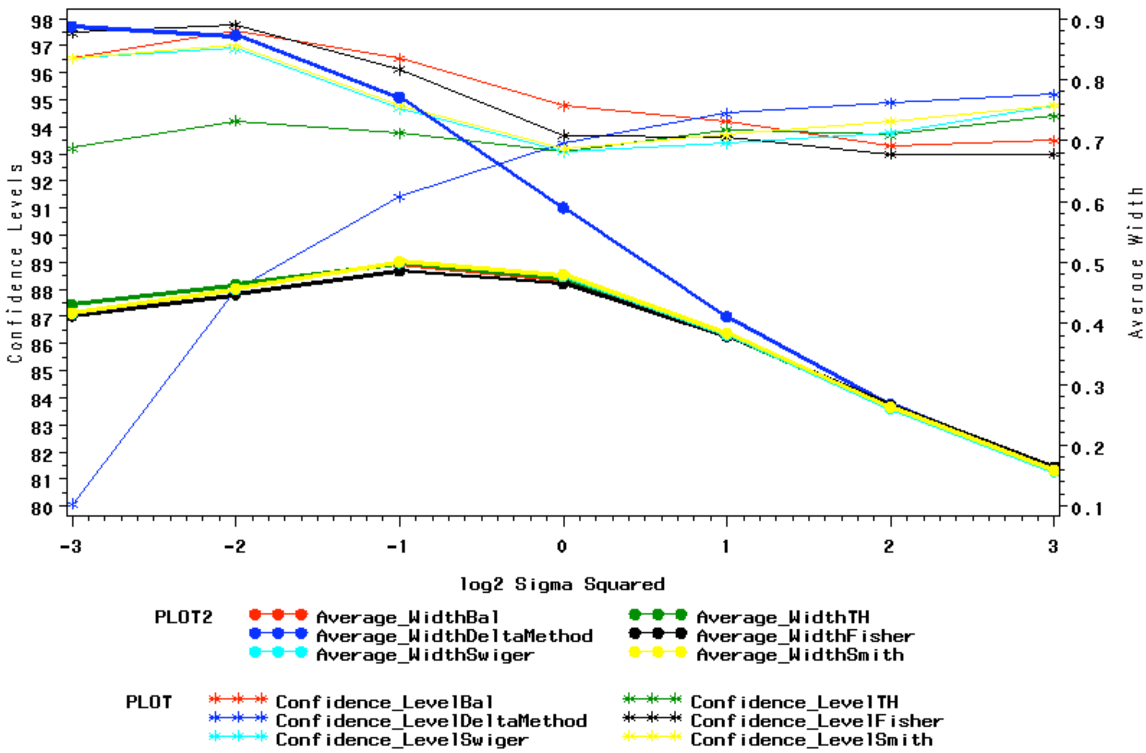
N=6 K=15

Confidence Intervals for Intraclass Correlation Coefficient



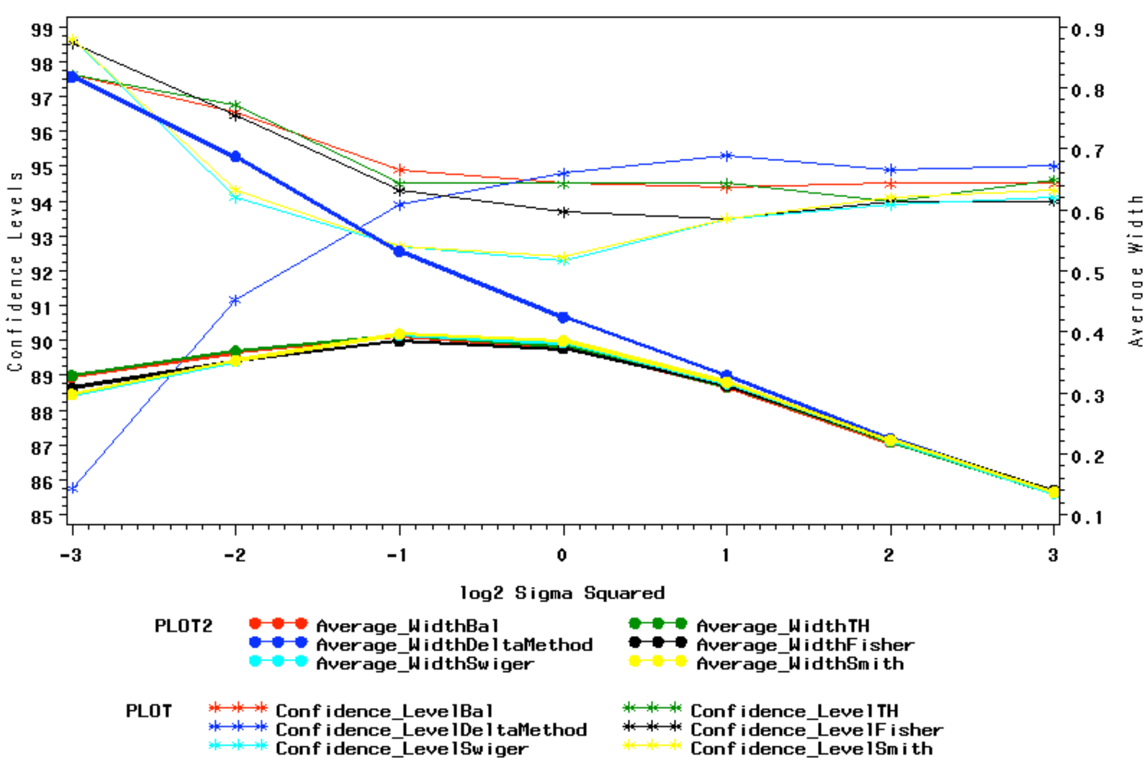
N=2 K=25

Confidence Intervals for Intraclass Correlation Coefficient

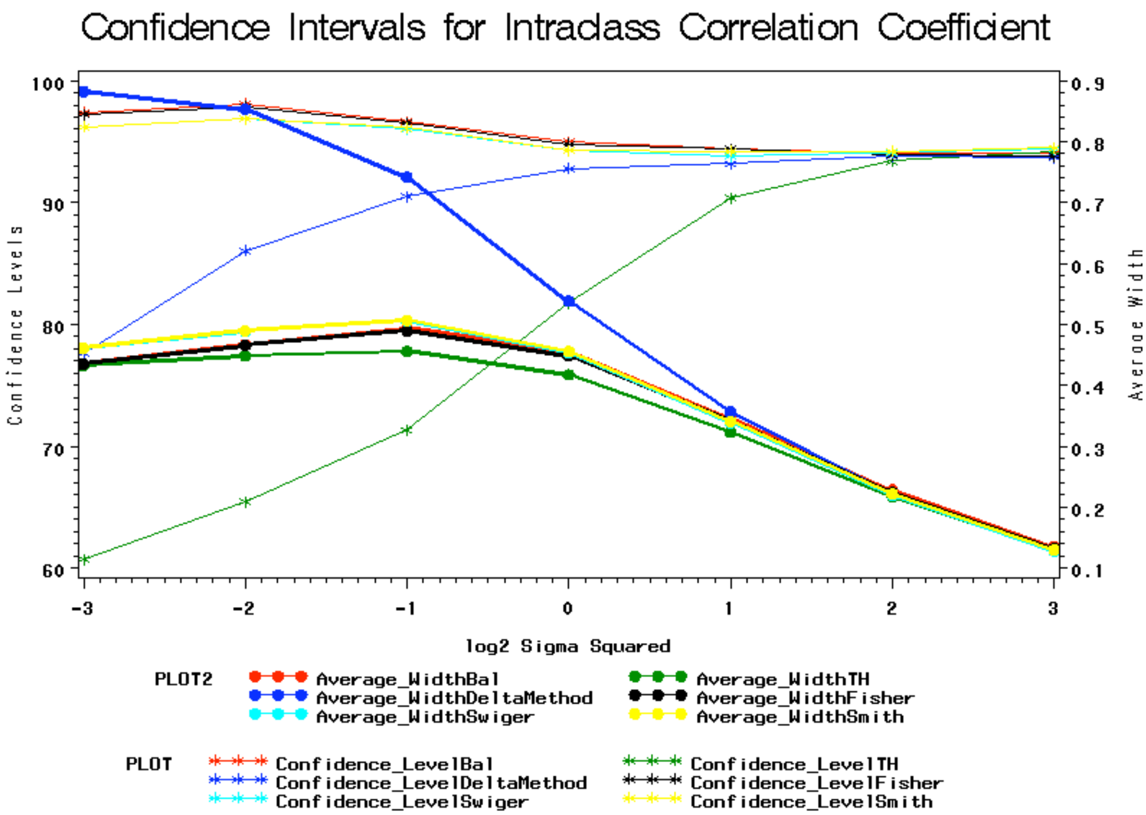


N=3 K=25

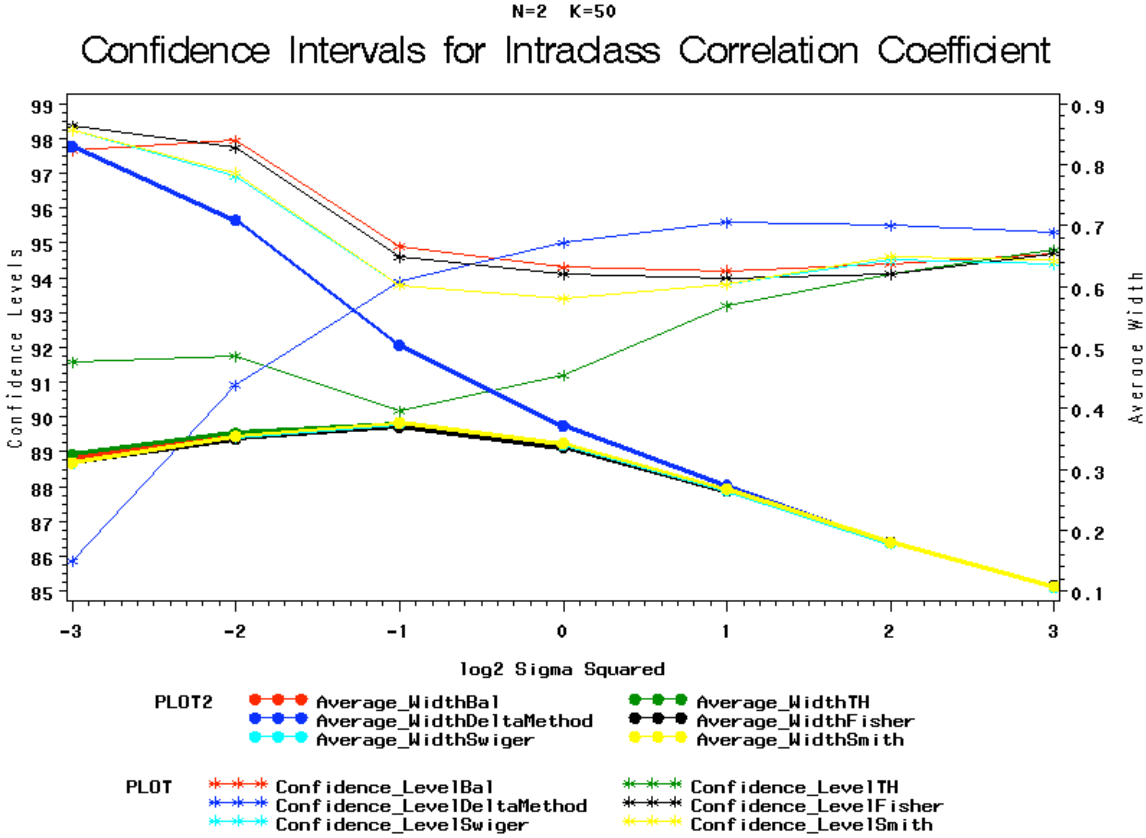
Confidence Intervals for Intraclass Correlation Coefficient



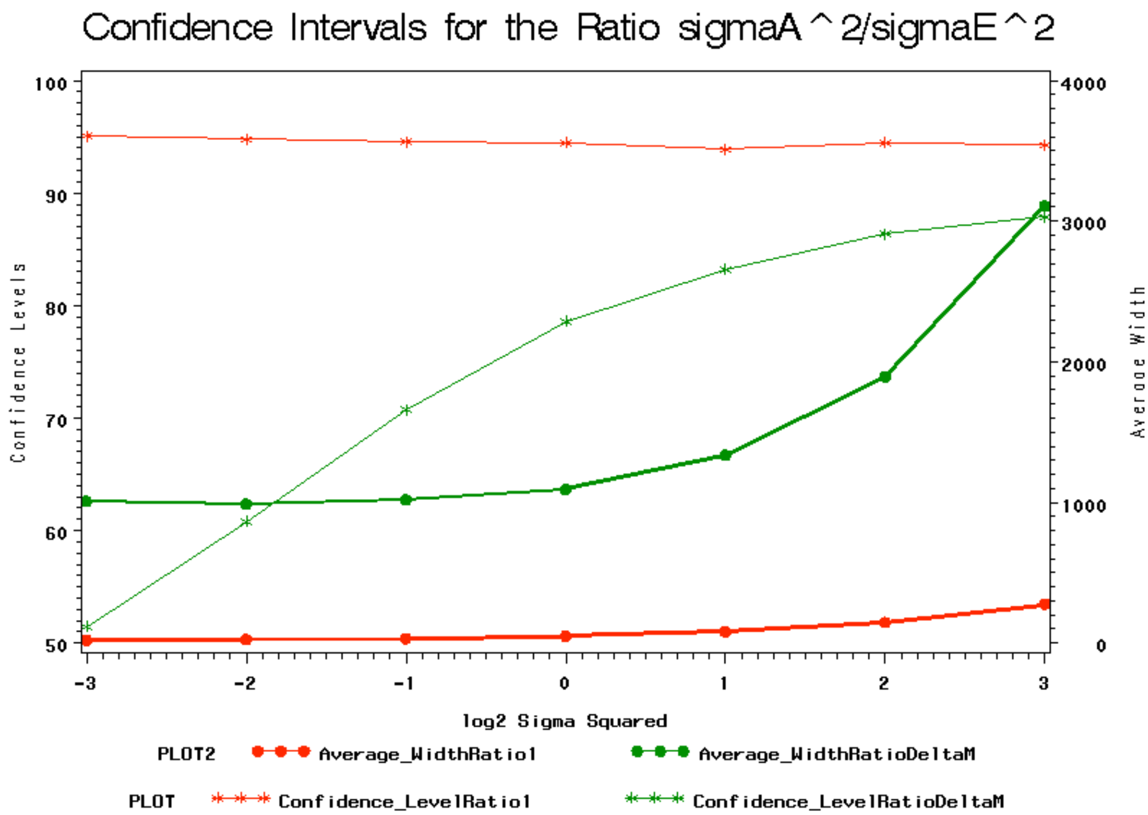
N=5 K=25


Confidence Intervals for Intraclass Correlation Coefficient
Confidence Levels
Average Width
log2 Sigma Squared
PLOT2
Average_WidthBal
Average_WidthDeltaMethod
Average_WidthSwiger
Average_WidthTH
Average_WidthFisher
Average_WidthSmith
PLOT
Confidence_LevelBal
Confidence_LevelDeltaMethod
Confidence_LevelSwiger
Confidence_LevelTH
Confidence_LevelFisher
Confidence_LevelSmith

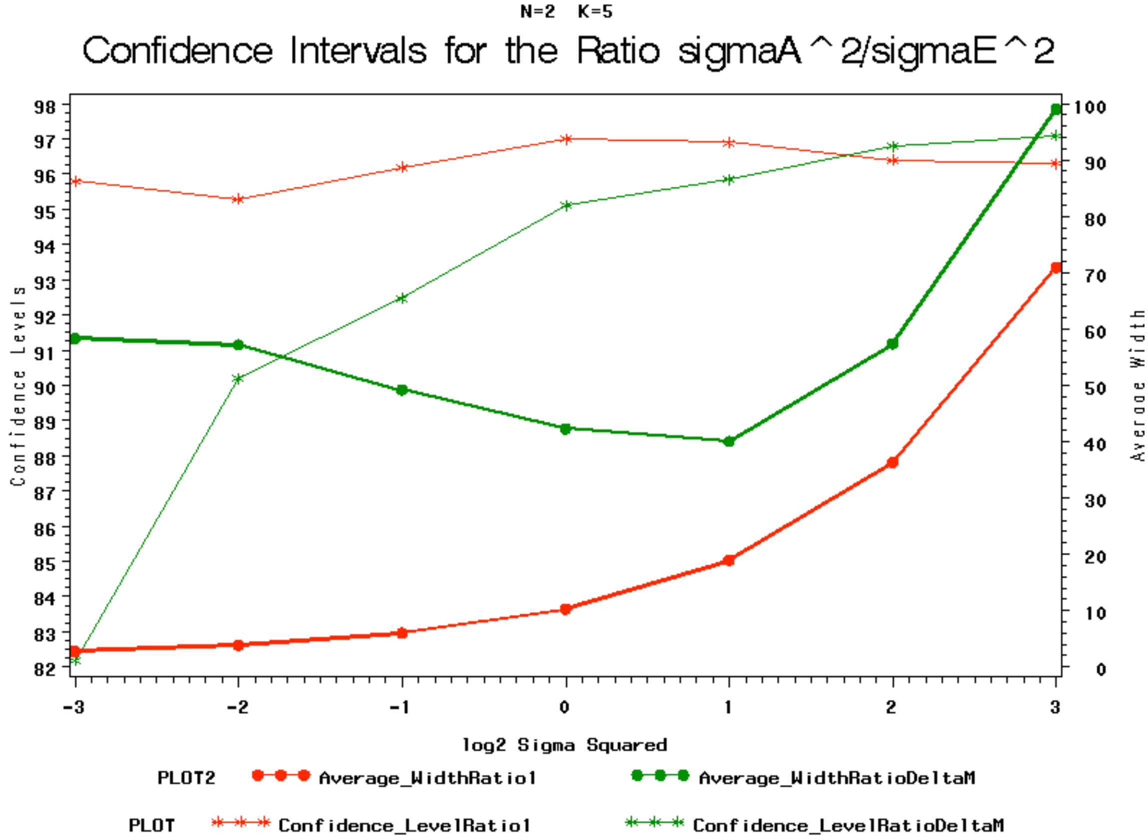
N=2 K=50


Confidence Intervals for Intraclass Correlation Coefficient
Confidence Levels
Average Width
log2 Sigma Squared
PLOT2
Average_WidthBal
Average_WidthDeltaMethod
Average_WidthSwiger
Average_WidthTH
Average_WidthFisher
Average_WidthSmith
PLOT
Confidence_LevelBal
Confidence_LevelDeltaMethod
Confidence_LevelSwiger
Confidence_LevelTH
Confidence_LevelFisher
Confidence_LevelSmith

N=3 K=50

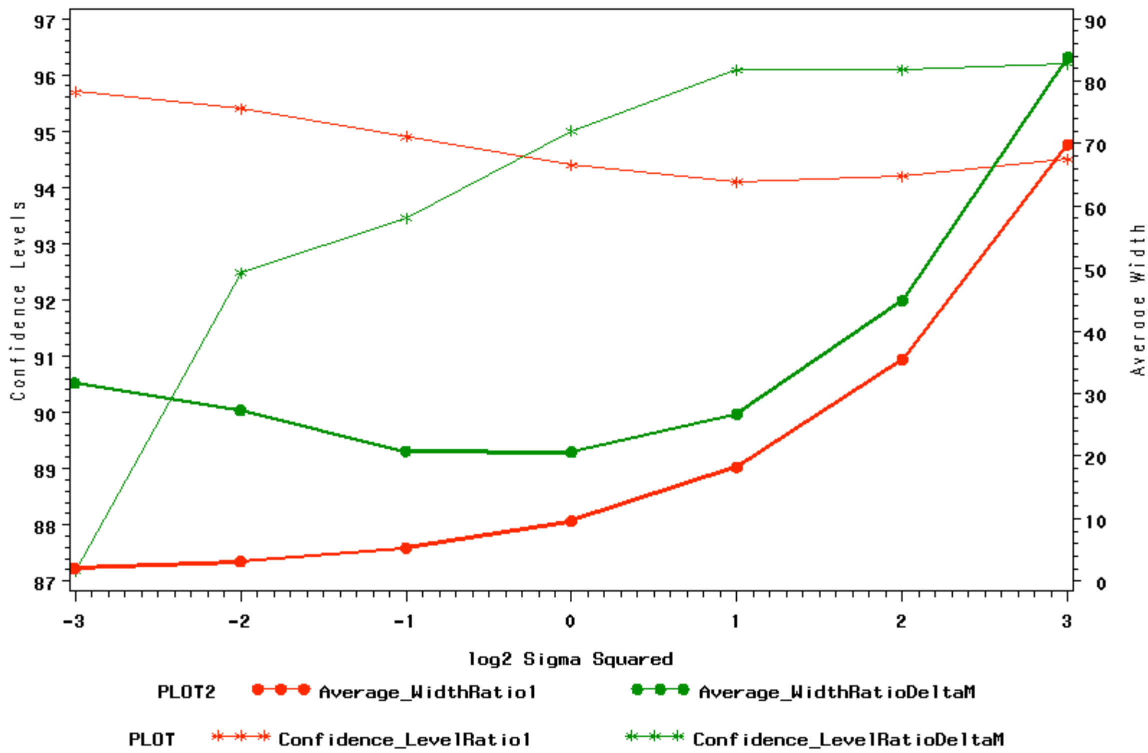

Confidence Intervals for the Ratio sigmaA^2/sigmaE^2
Confidence Levels
Average Width
log2 Sigma Squared
PLOT2 Average_WidthRatio1 Average_WidthRatioDeltaM
PLOT Confidence_LevelRatio1 Confidence_LevelRatioDeltaM

N=2 K=5


Confidence Intervals for the Ratio sigmaA^2/sigmaE^2
Confidence Levels
Average Width
log2 Sigma Squared
PLOT2 Average_WidthRatio1 Average_WidthRatioDeltaM
PLOT Confidence_LevelRatio1 Confidence_LevelRatioDeltaM

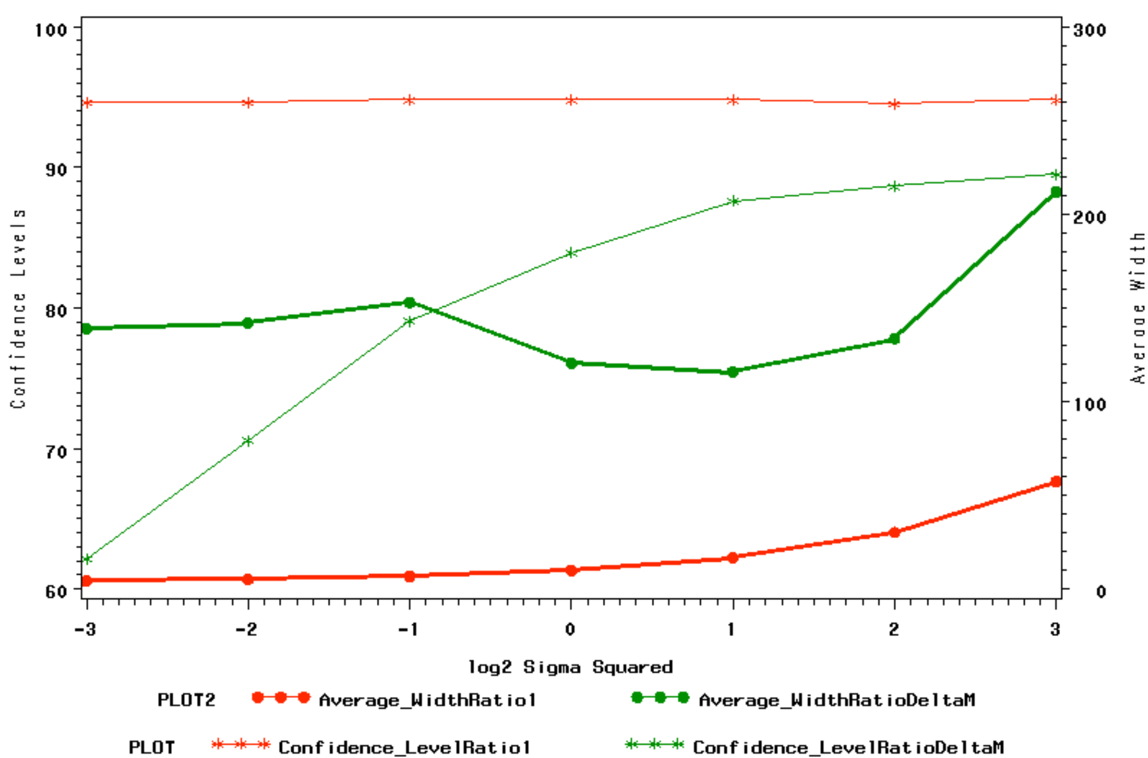
N=6 K=5

Confidence Intervals for the Ratio sigmaA^2/sigmaE^2

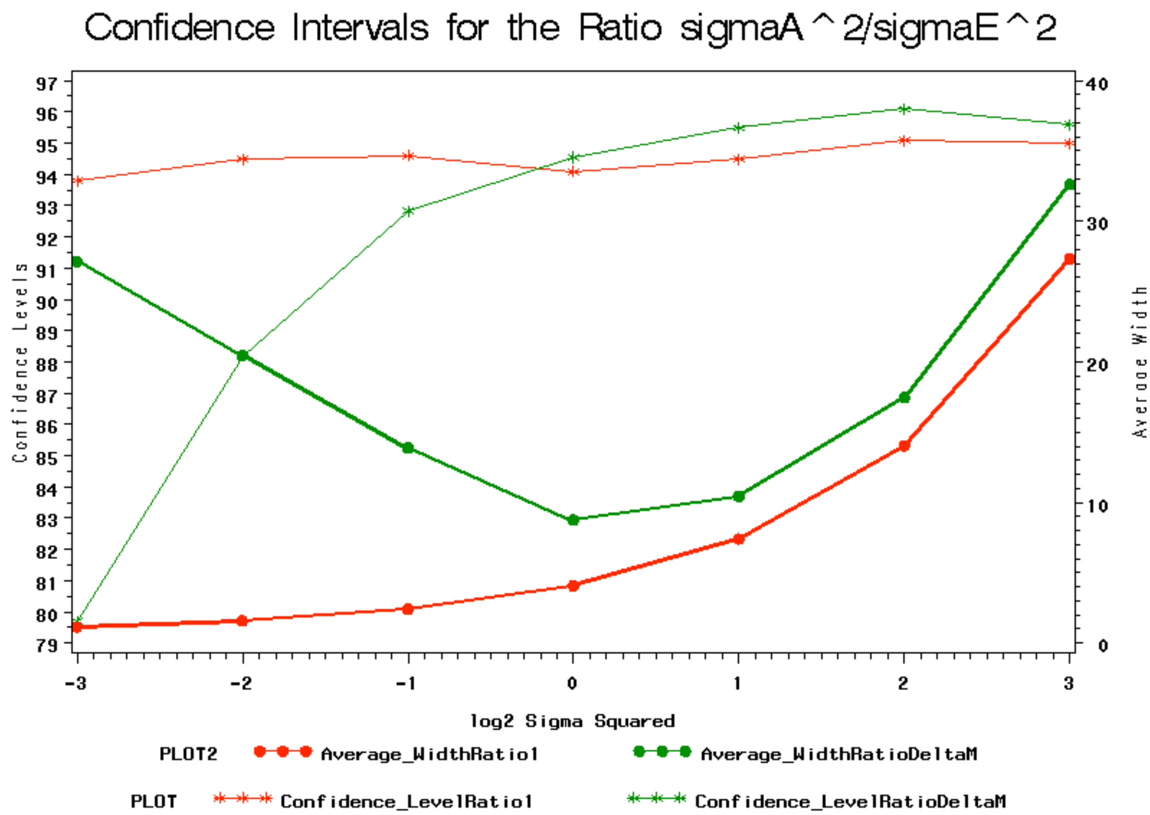


N=10 K=5

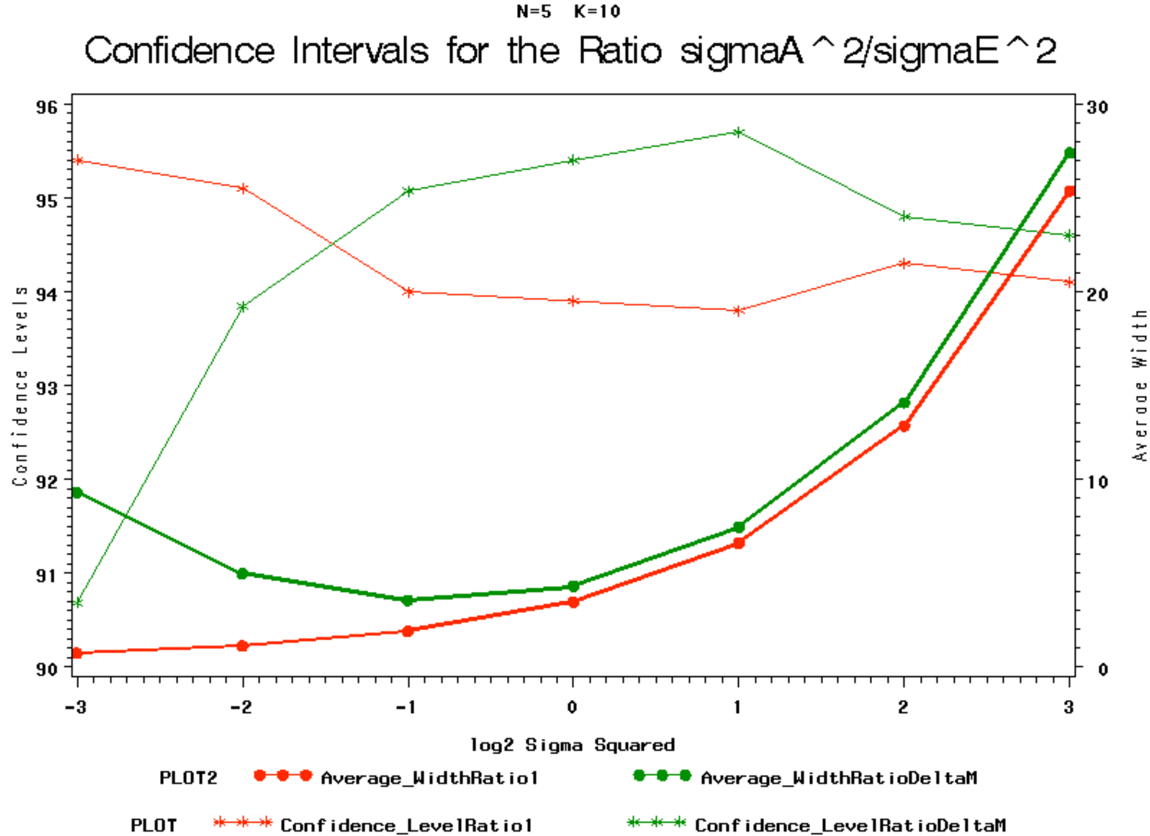
Confidence Intervals for the Ratio sigmaA^2/sigmaE^2



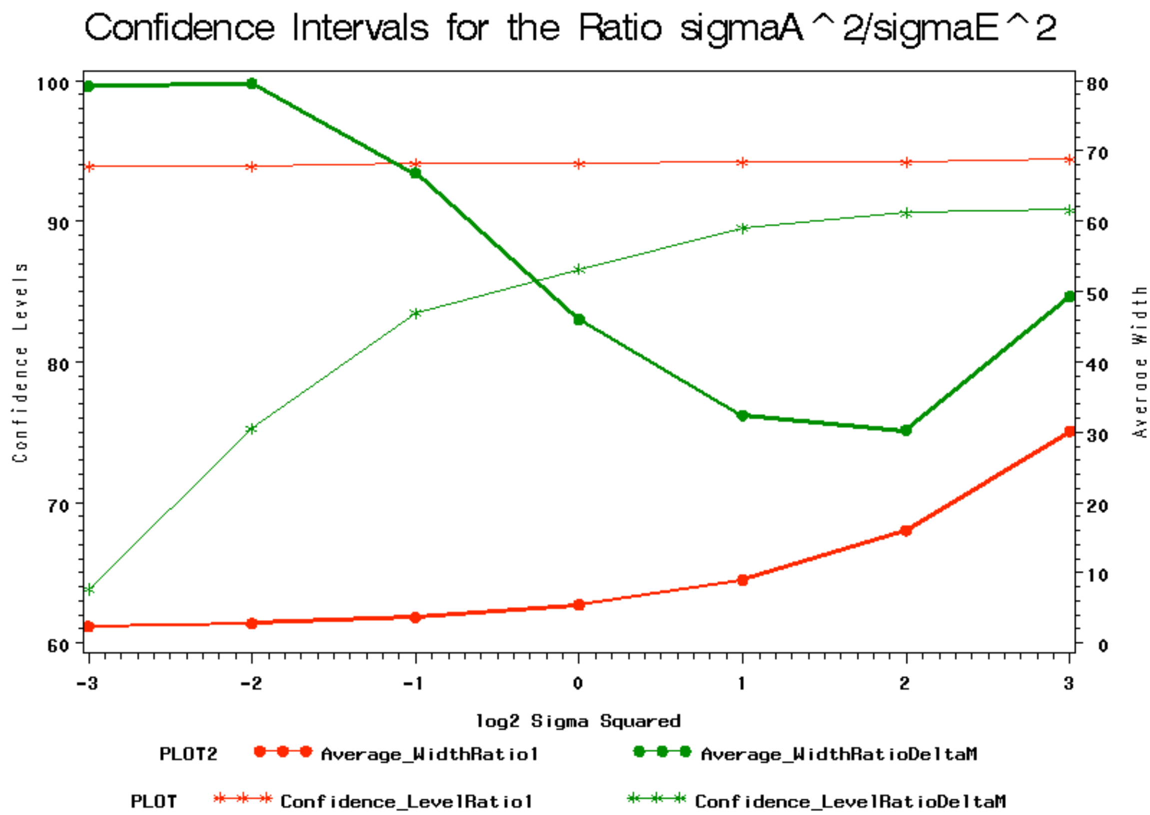
N=2 K=10



N=5 K=10

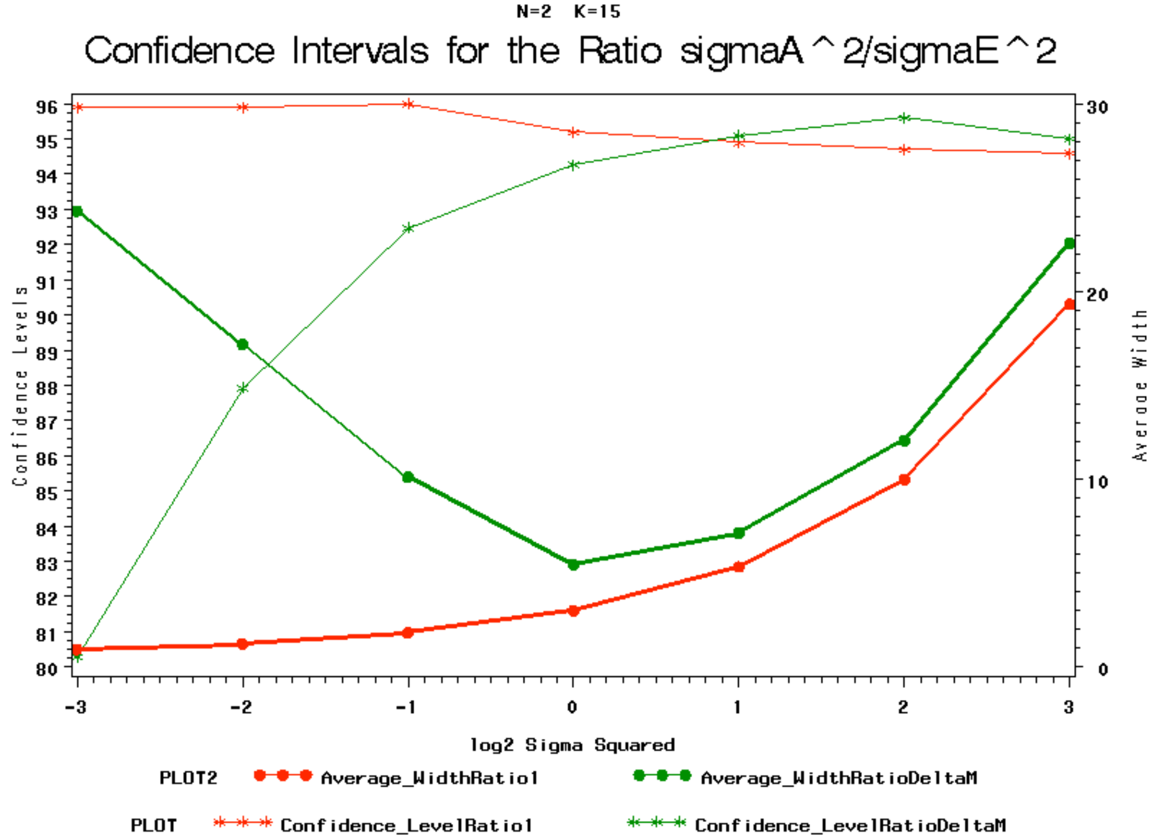


N=10 K=10



Confidence Intervals for the Ratio sigmaA^2/sigmaE^2
Confidence Levels
Average Width
log2 Sigma Squared
PLOT2 Average_WidthRatio1 Average_WidthRatioDeltaM
PLOT Confidence_LevelRatio1 Confidence_LevelRatioDeltaM

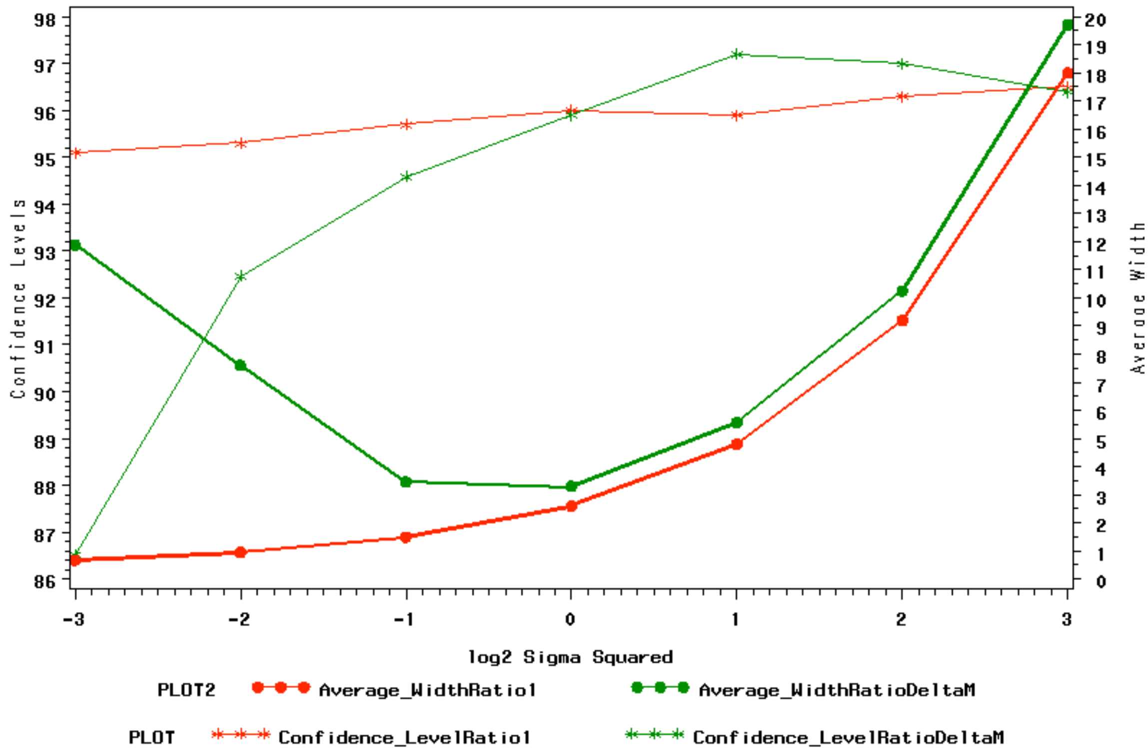
N=2 K=15



Confidence Intervals for the Ratio sigmaA^2/sigmaE^2
Confidence Levels
Average Width
log2 Sigma Squared
PLOT2 Average_WidthRatio1 Average_WidthRatioDeltaM
PLOT Confidence_LevelRatio1 Confidence_LevelRatioDeltaM

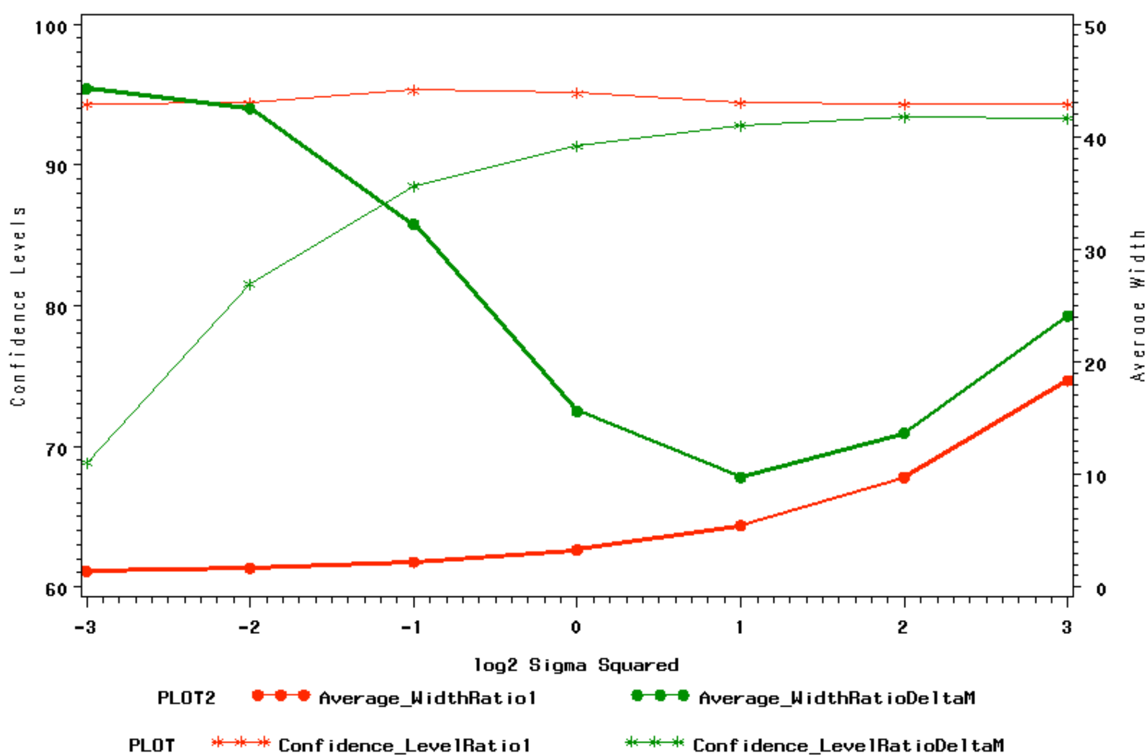
N=4 K=15

Confidence Intervals for the Ratio sigmaA^2/sigmaE^2

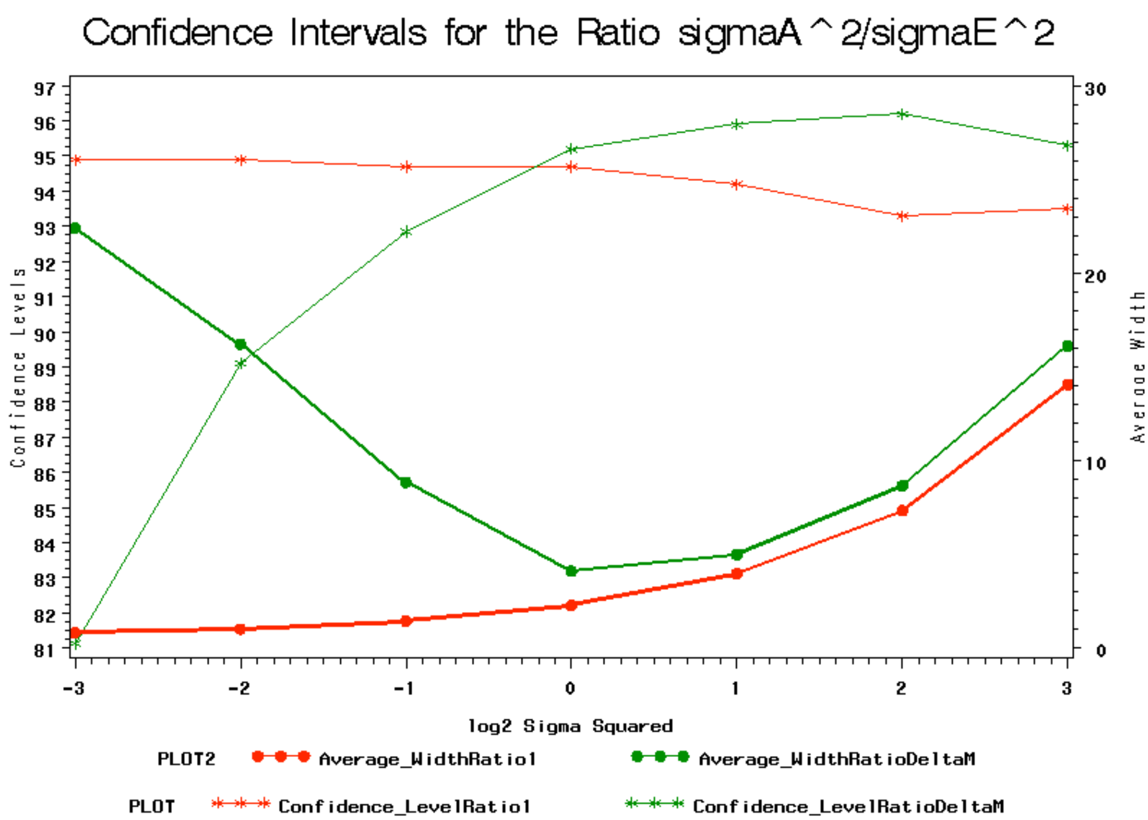


N=6 K=15

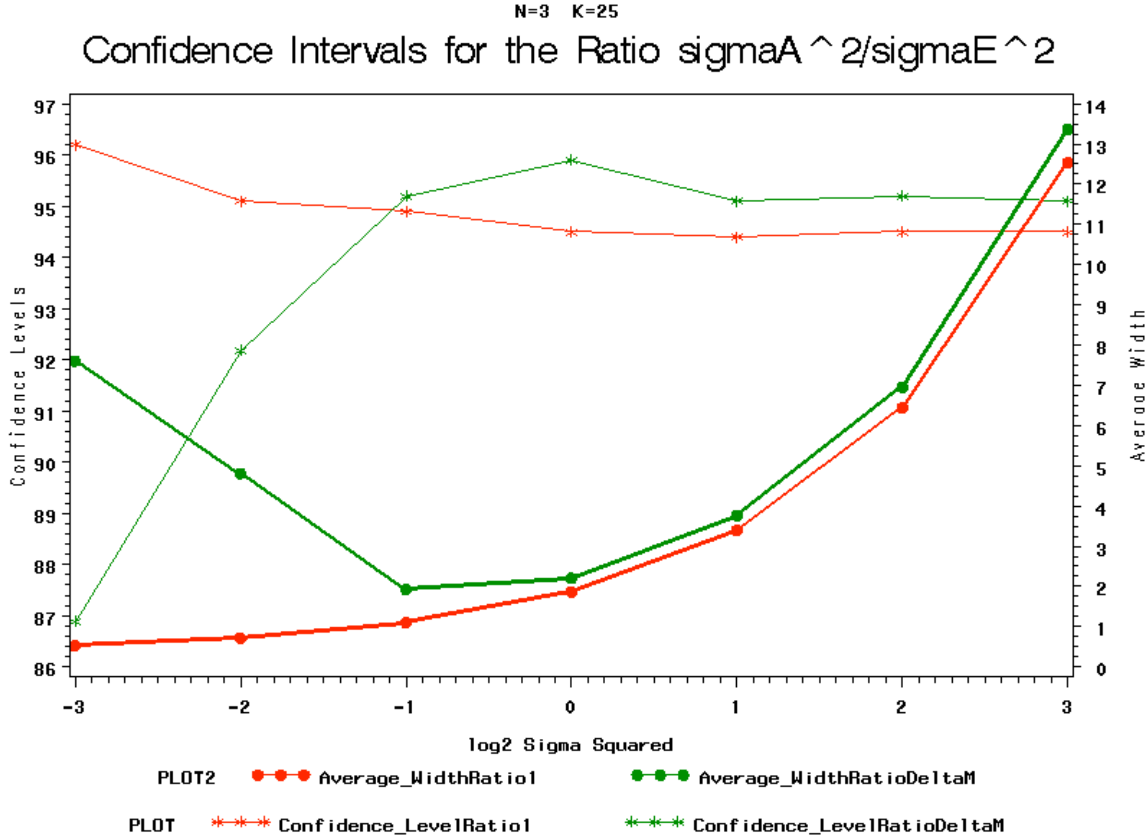
Confidence Intervals for the Ratio sigmaA^2/sigmaE^2



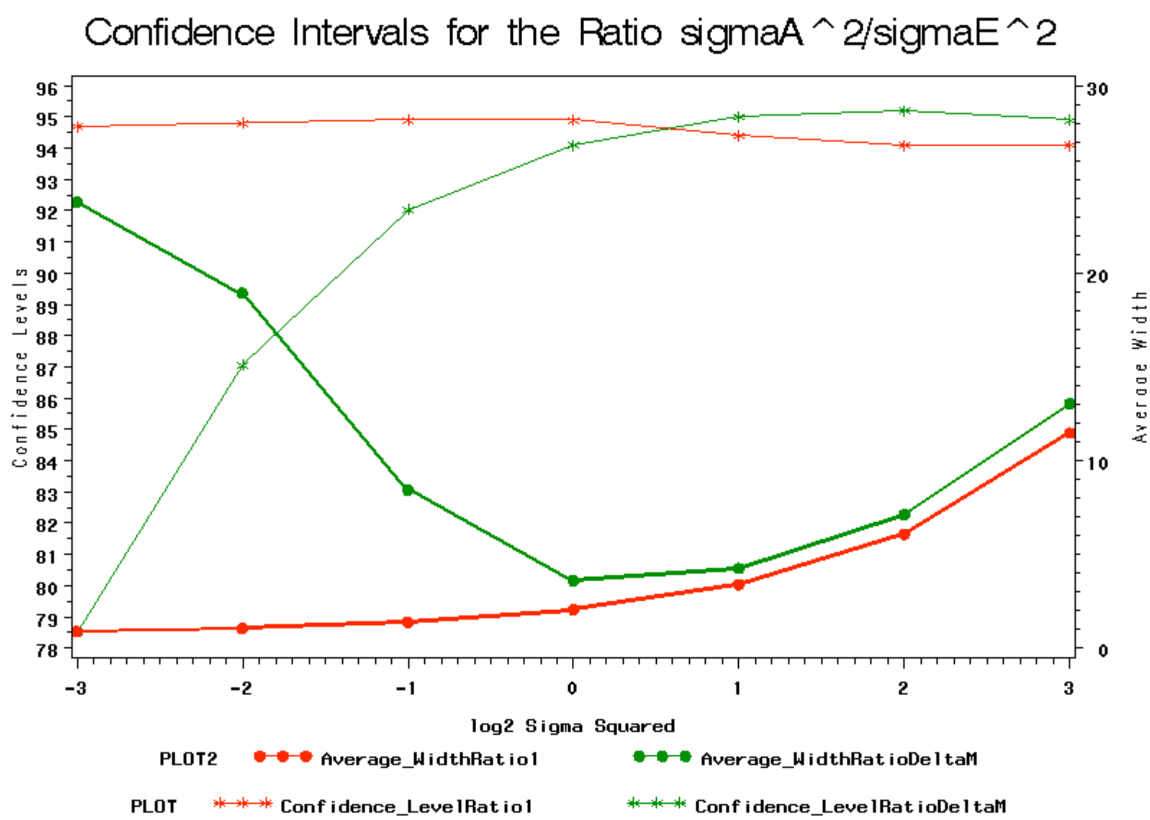
N=2 K=25


Confidence Intervals for the Ratio sigmaA^2/sigmaE^2
Confidence Levels
Average Width
log2 Sigma Squared
PLOT2 Average_WidthRatio1 Average_WidthRatioDeltaM
PLOT Confidence_LevelRatio1 Confidence_LevelRatioDeltaM

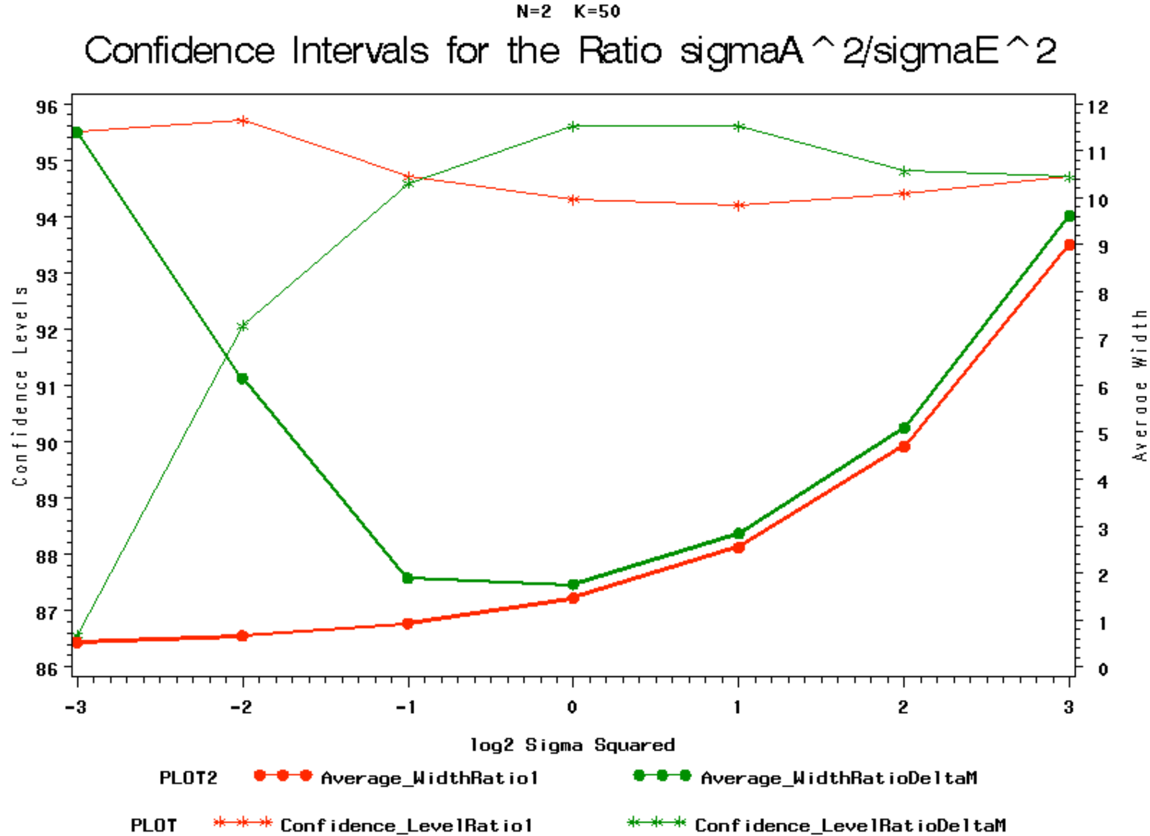
N=3 K=25


Confidence Intervals for the Ratio sigmaA^2/sigmaE^2
Confidence Levels
Average Width
log2 Sigma Squared
PLOT2 Average_WidthRatio1 Average_WidthRatioDeltaM
PLOT Confidence_LevelRatio1 Confidence_LevelRatioDeltaM

N=5 K=25

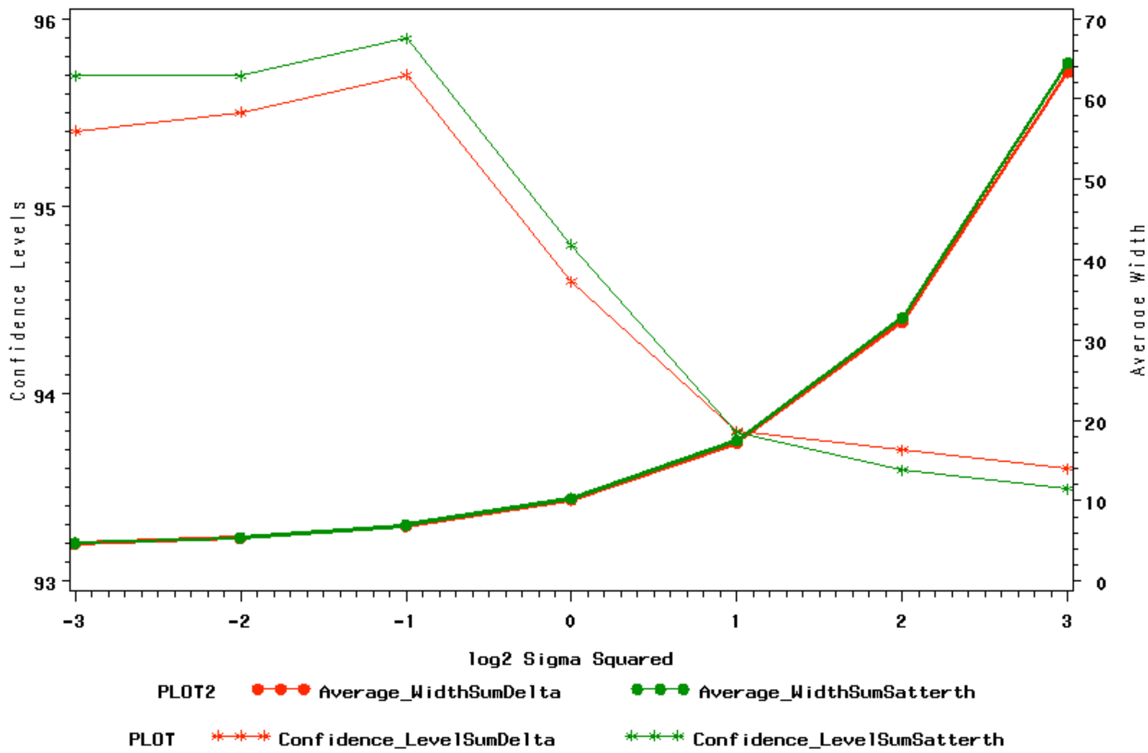

Confidence Intervals for the Ratio sigmaA^2/sigmaE^2
Confidence Levels
Average Width
log2 Sigma Squared
PLOT2 Average_WidthRatio1 Average_WidthRatioDeltaM
PLOT Confidence_LevelRatio1 Confidence_LevelRatioDeltaM

N=2 K=50


Confidence Intervals for the Ratio sigmaA^2/sigmaE^2
Confidence Levels
Average Width
log2 Sigma Squared
PLOT2 Average_WidthRatio1 Average_WidthRatioDeltaM
PLOT Confidence_LevelRatio1 Confidence_LevelRatioDeltaM

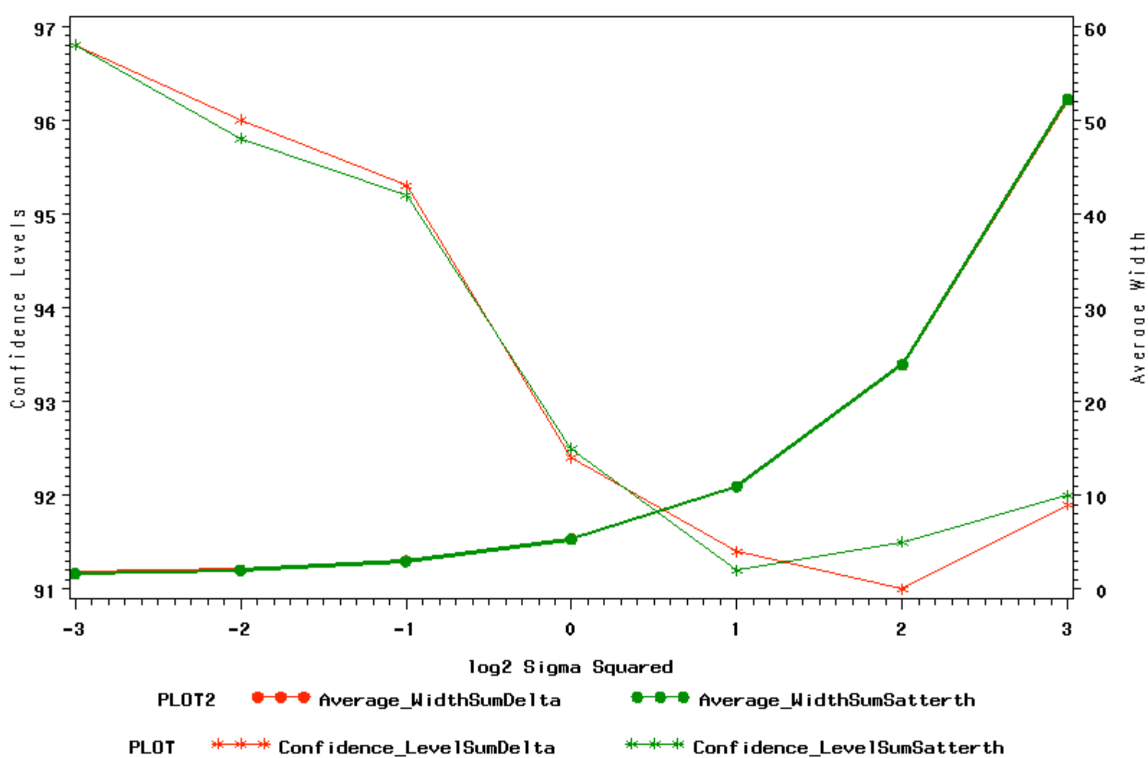
N=3 K=50

Confidence Intervals for the Sum of SigmaA^2 and SigmaE^2



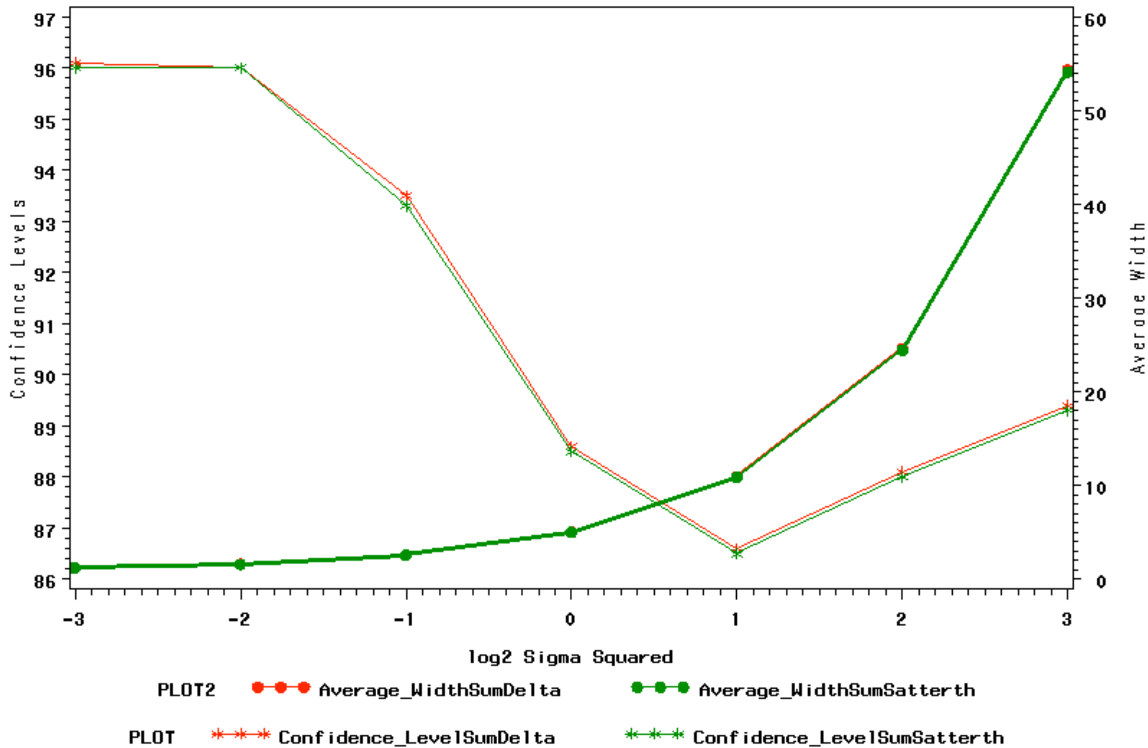
N=2 K=5

Confidence Intervals for the Sum of SigmaA^2 and SigmaE^2



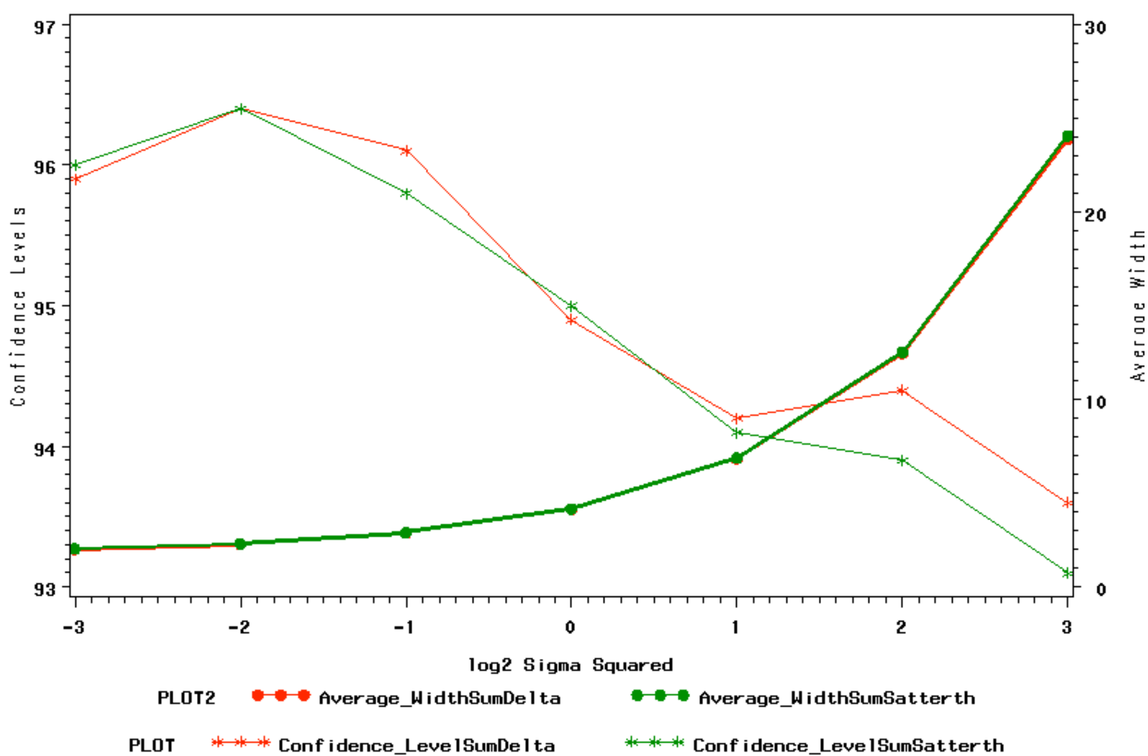
N=6 K=5

Confidence Intervals for the Sum of SigmaA^2 and SigmaE^2



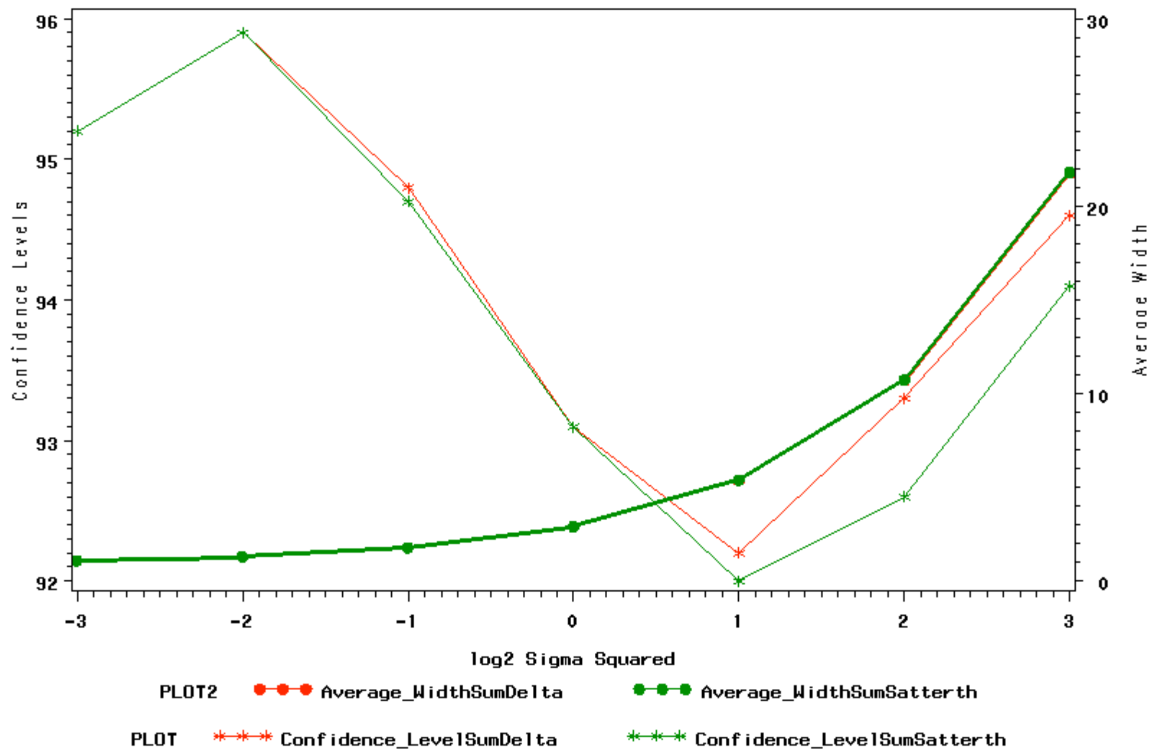
N=10 K=5

Confidence Intervals for the Sum of SigmaA^2 and SigmaE^2



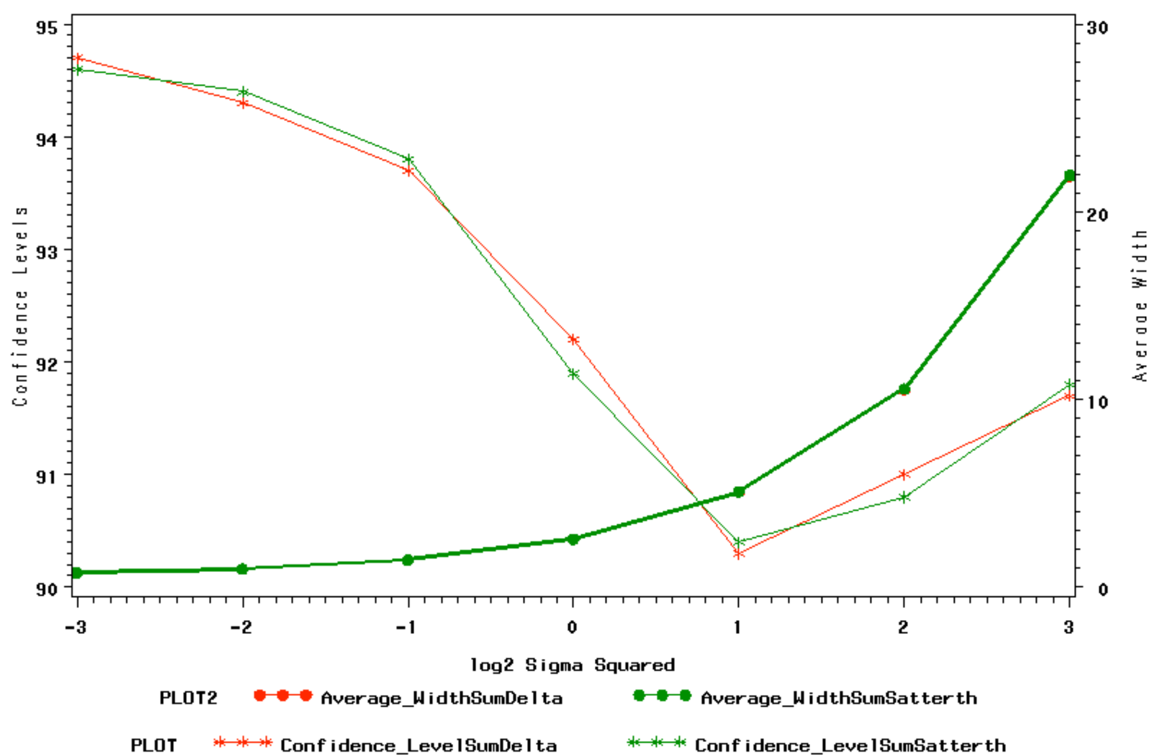
N=2 K=10

Confidence Intervals for the Sum of SigmaA ^2 and SigmaE ^2



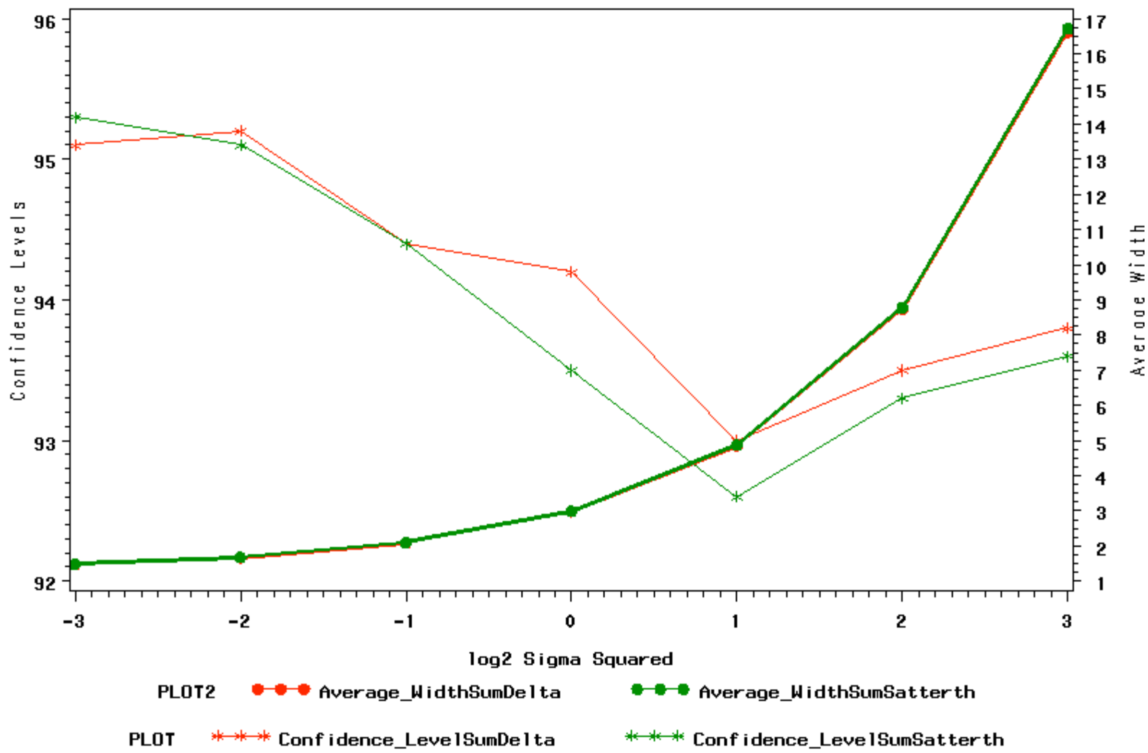
N=5 K=10

Confidence Intervals for the Sum of SigmaA ^2 and SigmaE ^2



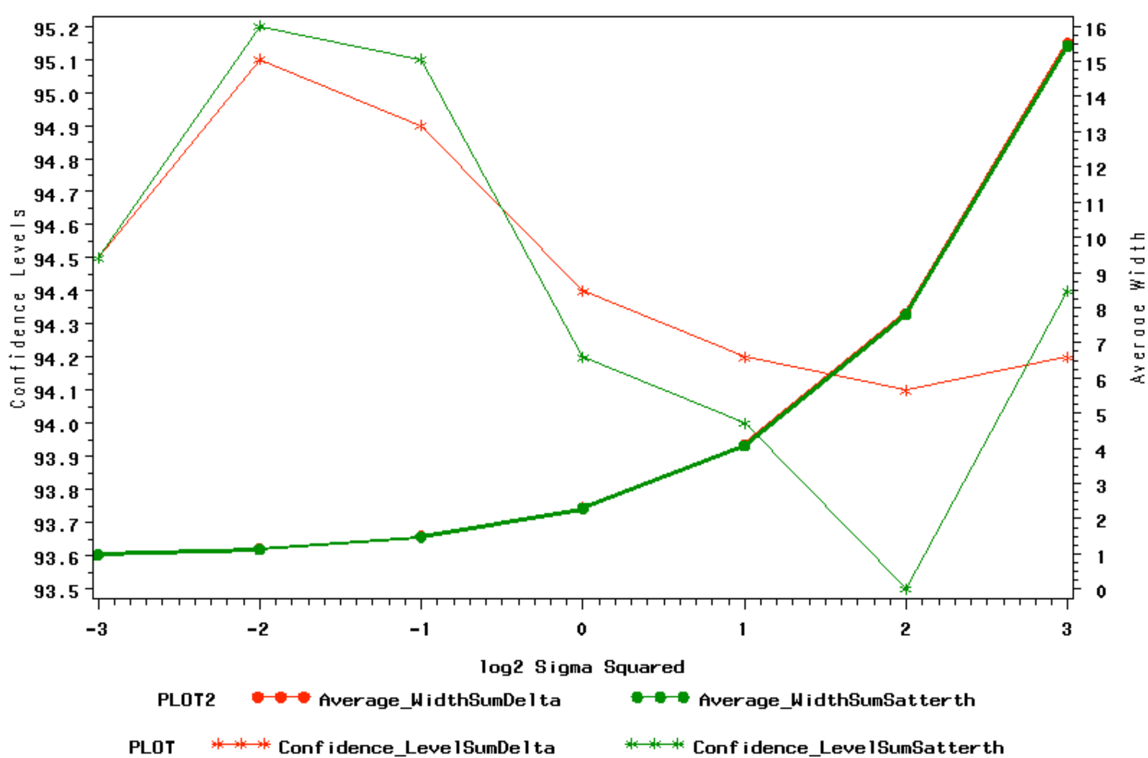
N=10 K=10

Confidence Intervals for the Sum of SigmaA^2 and SigmaE^2

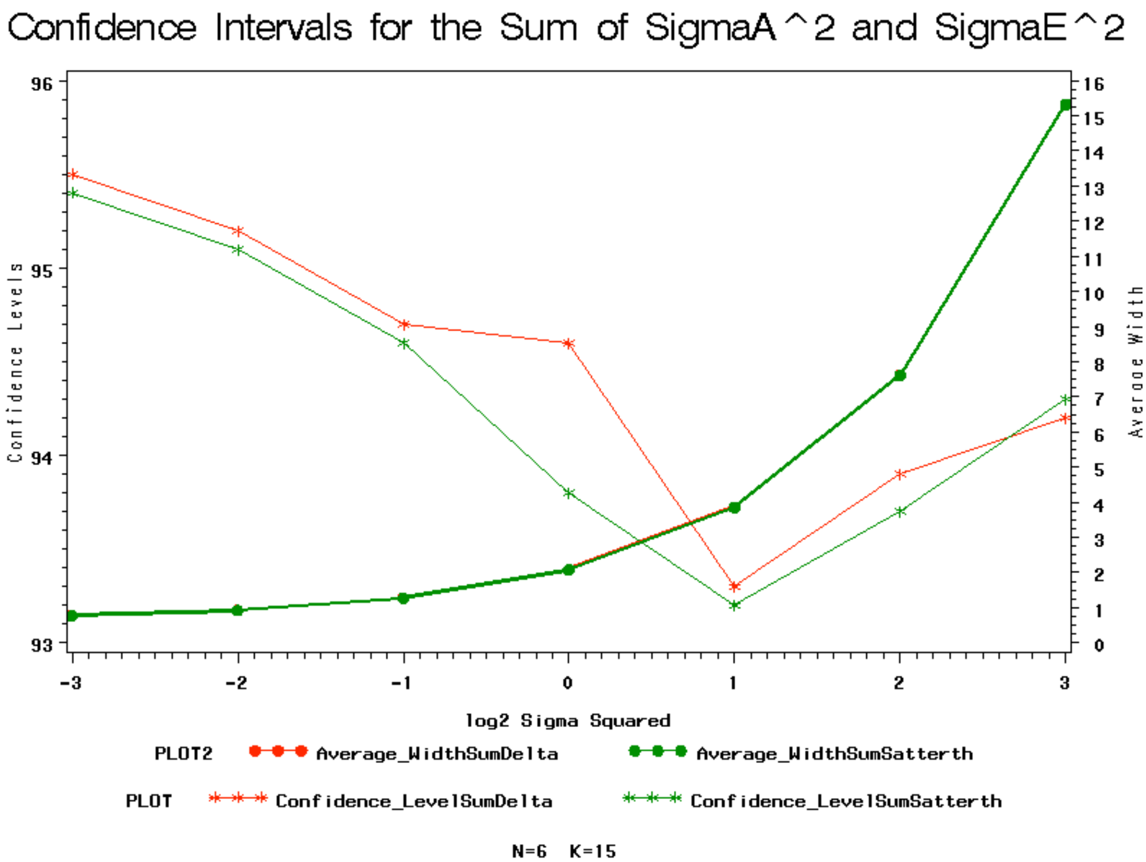


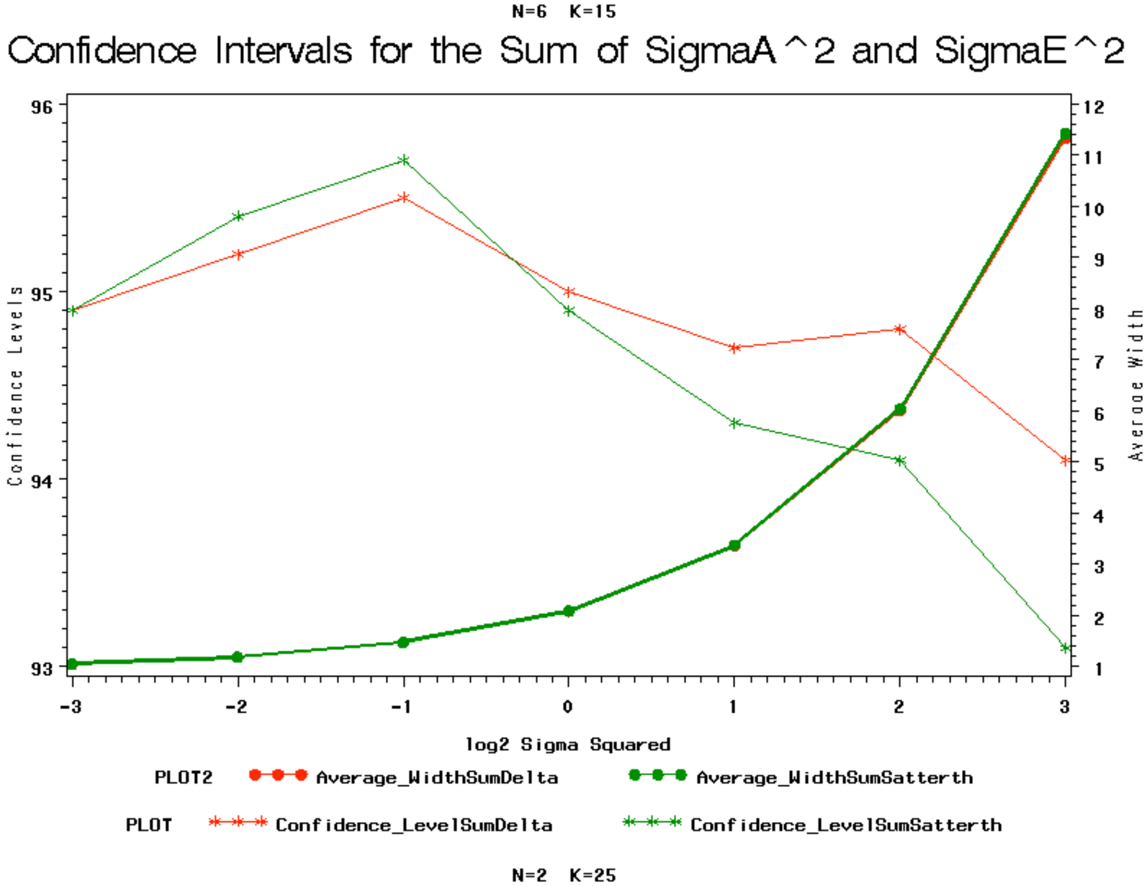
N=2 K=15

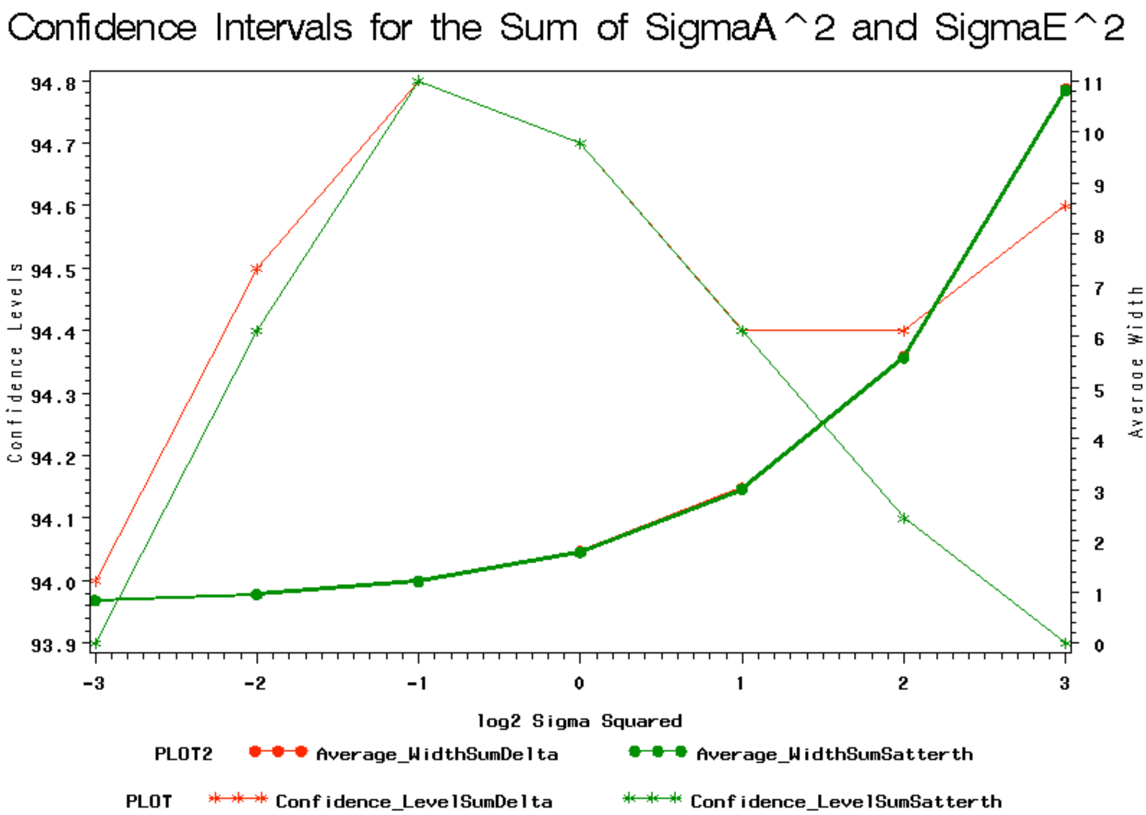
Confidence Intervals for the Sum of SigmaA^2 and SigmaE^2



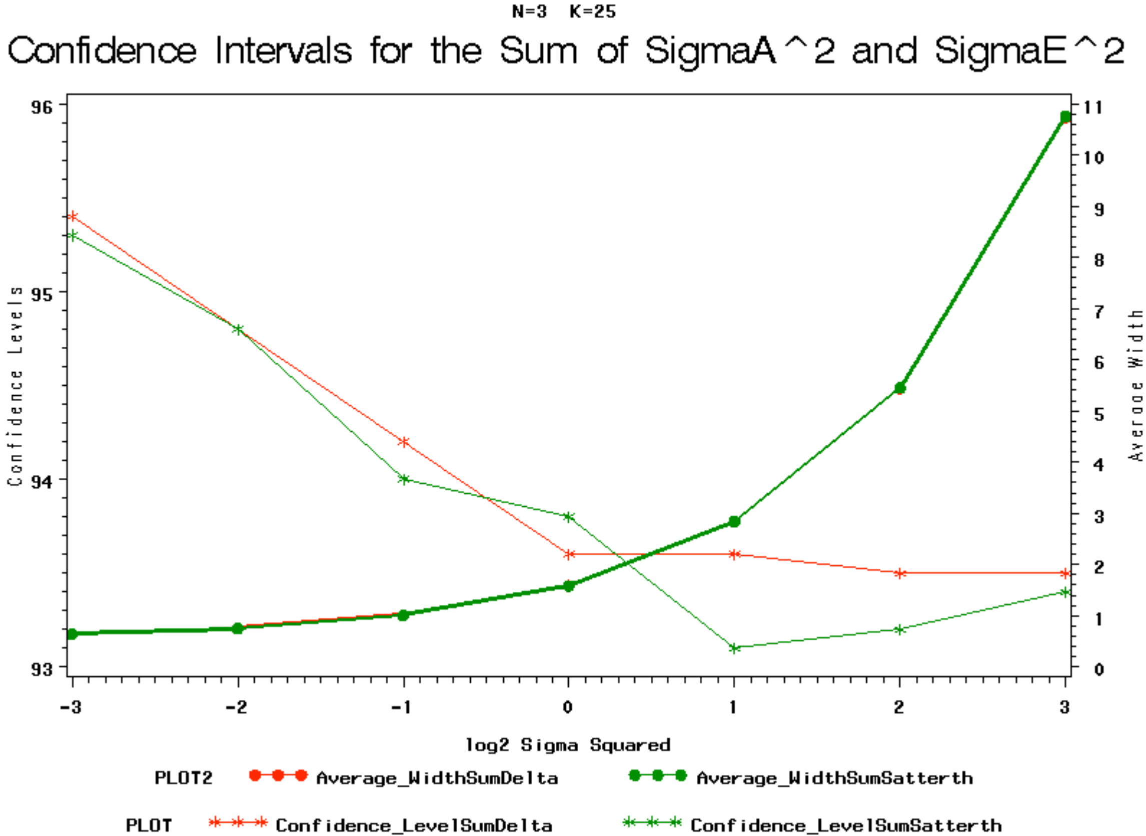
N=4 K=15


Confidence Intervals for the Sum of SigmaA^2 and SigmaE^2
Confidence Levels
Average Width
93
94
95
96
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
-3 -2 -1 0 1 2 3
log2 Sigma Squared
PLOT2 Average_WidthSumDelta Average_WidthSumSatterth
PLOT Confidence_LevelSumDelta Confidence_LevelSumSatterth
N=6 K=15


Confidence Intervals for the Sum of SigmaA^2 and SigmaE^2
Confidence Levels
Average Width
93
94
95
96
1 2 3 4 5 6 7 8 9 10 11 12
-3 -2 -1 0 1 2 3
log2 Sigma Squared
PLOT2 Average_WidthSumDelta Average_WidthSumSatterth
PLOT Confidence_LevelSumDelta Confidence_LevelSumSatterth
N=2 K=25

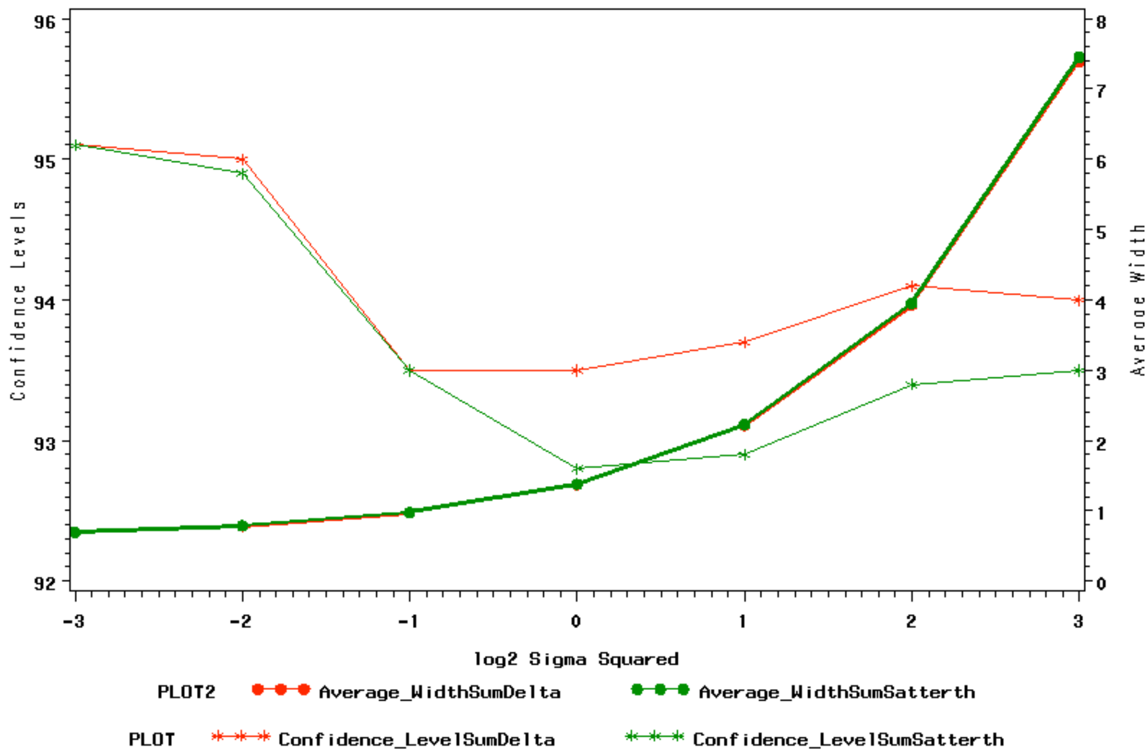

Confidence Intervals for the Sum of SigmaA^2 and SigmaE^2
Confidence Levels
Average Width
log2 Sigma Squared
PLOT2 Average_WidthSumDelta Average_WidthSumSatterth
PLOT Confidence_LevelSumDelta Confidence_LevelSumSatterth

N=3 K=25


Confidence Intervals for the Sum of SigmaA^2 and SigmaE^2
Confidence Levels
Average Width
log2 Sigma Squared
PLOT2 Average_WidthSumDelta Average_WidthSumSatterth
PLOT Confidence_LevelSumDelta Confidence_LevelSumSatterth

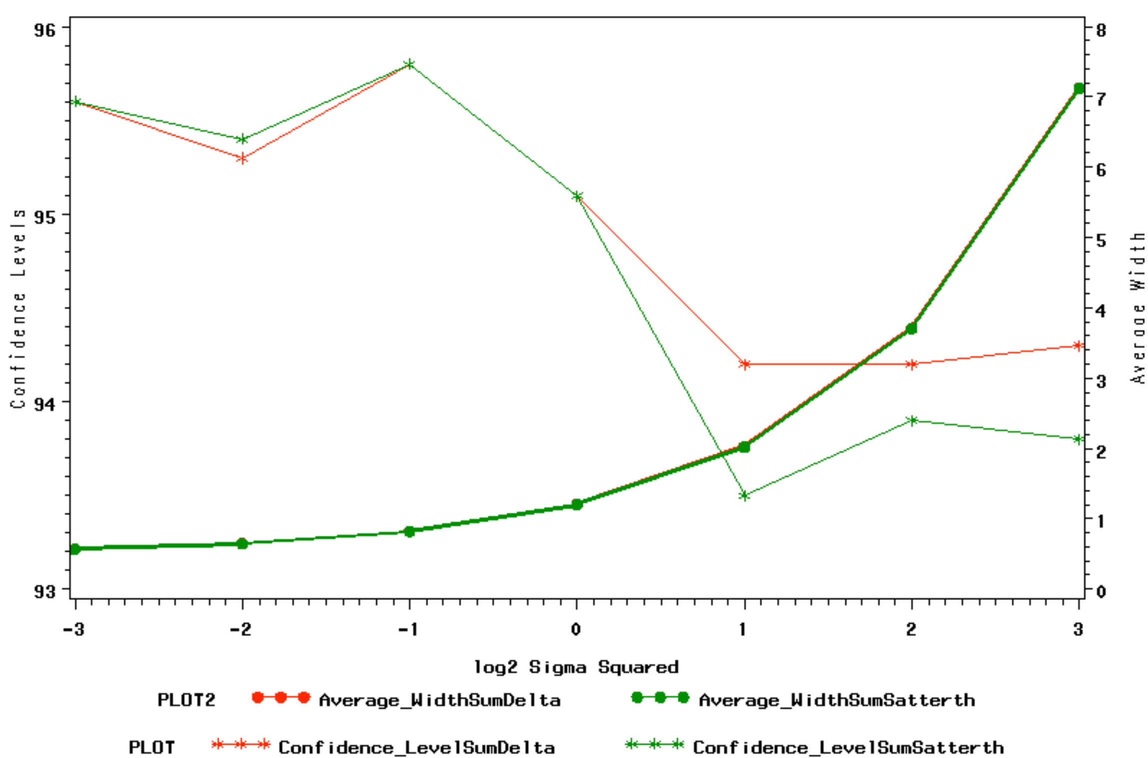
N=5 K=25

Confidence Intervals for the Sum of SigmaA^2 and SigmaE^2



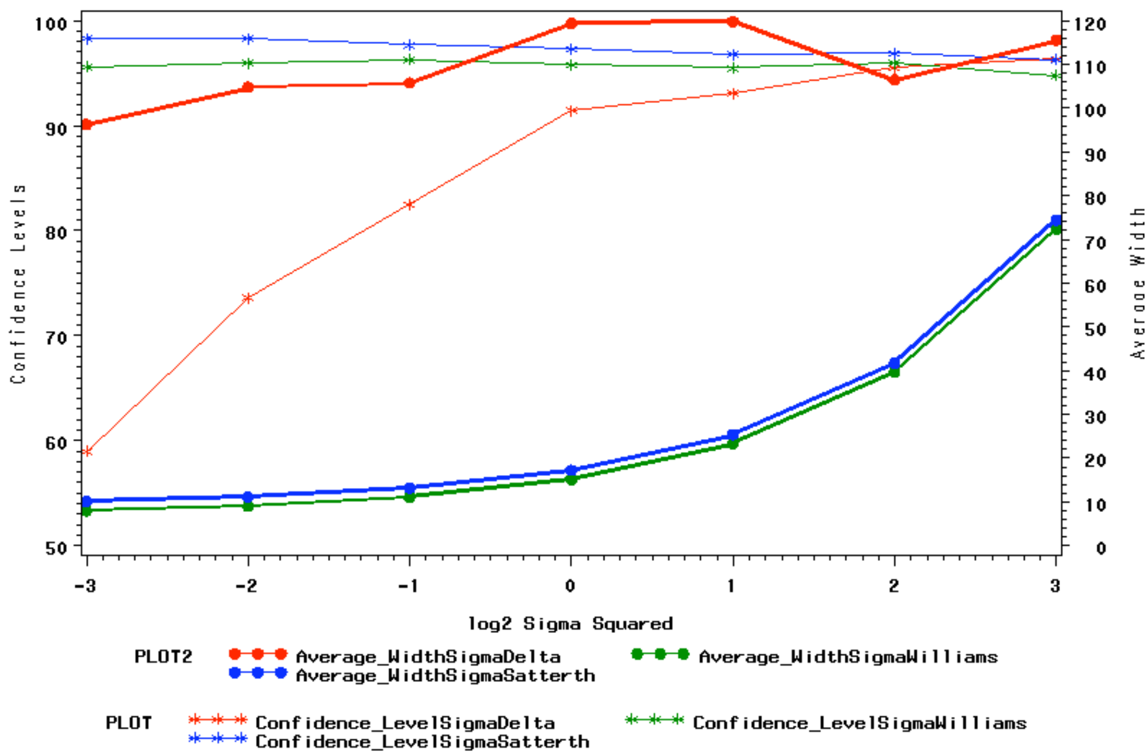
N=2 K=50

Confidence Intervals for the Sum of SigmaA^2 and SigmaE^2



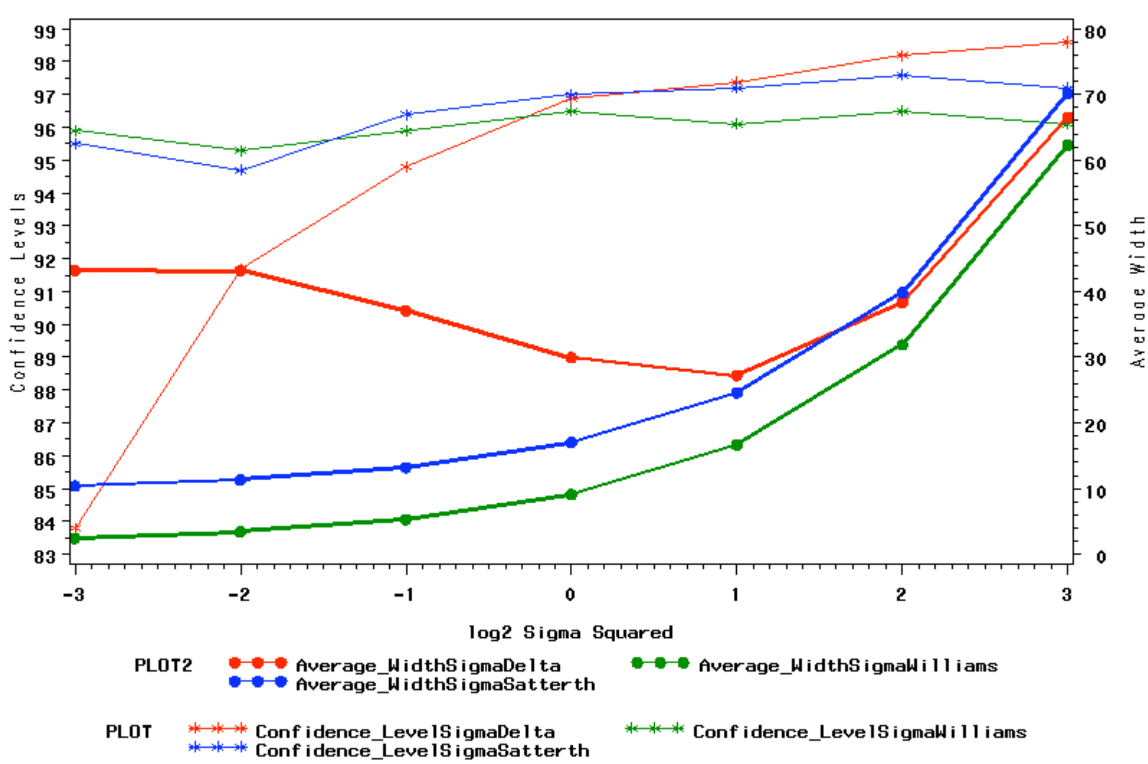
N=3 K=50

Confidence Intervals for sigmaA^2

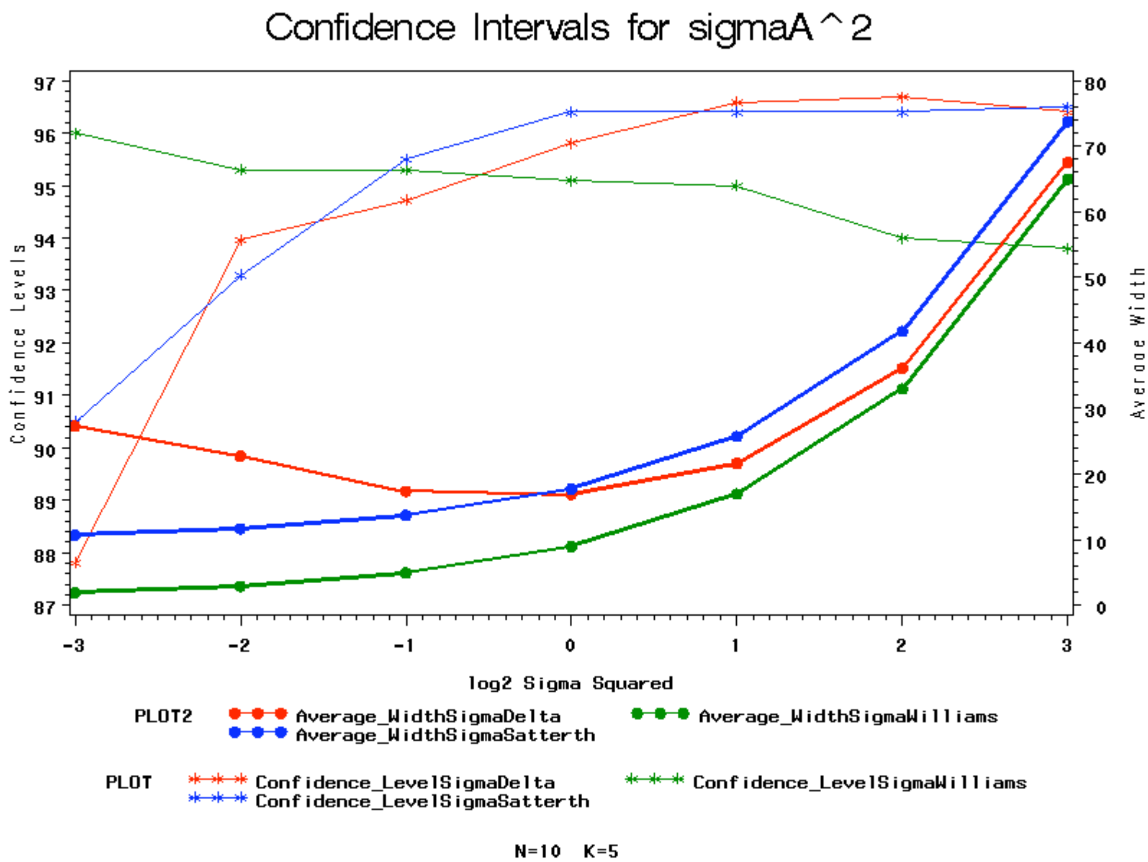


N=2 K=5

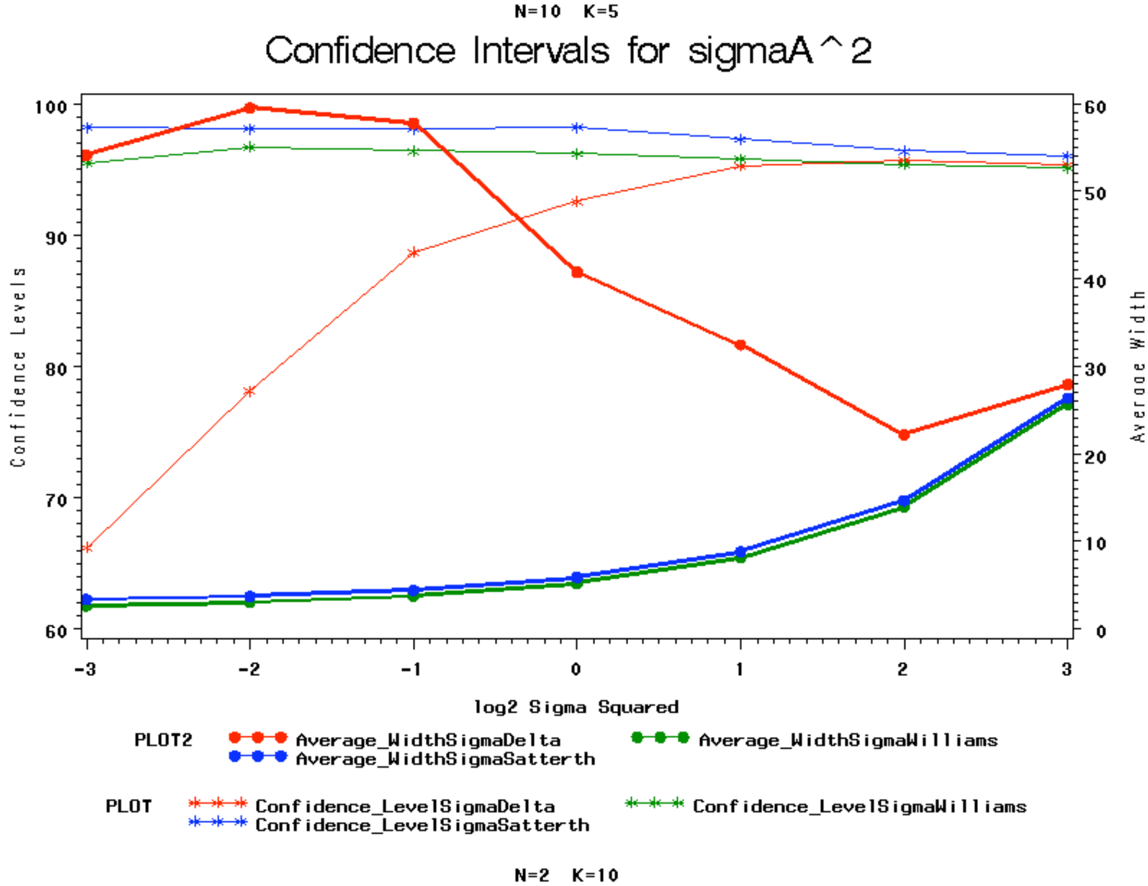
Confidence Intervals for sigmaA^2



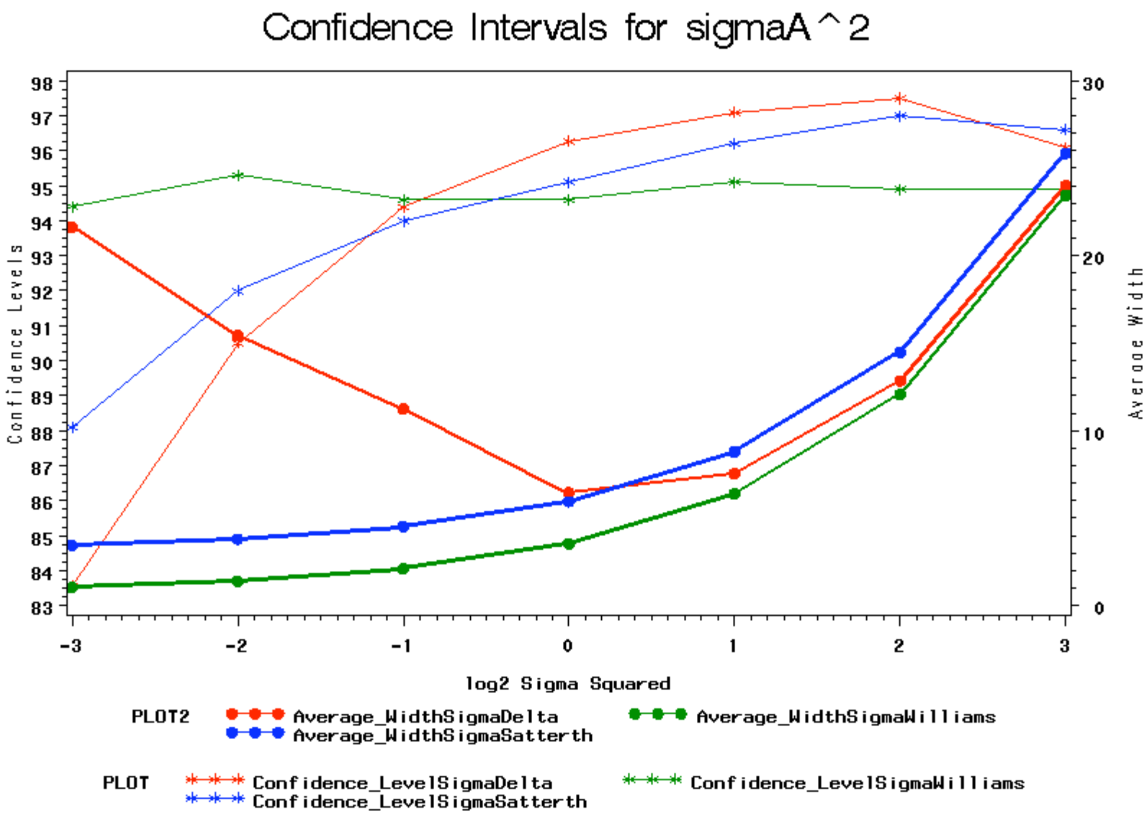
N=6 K=5



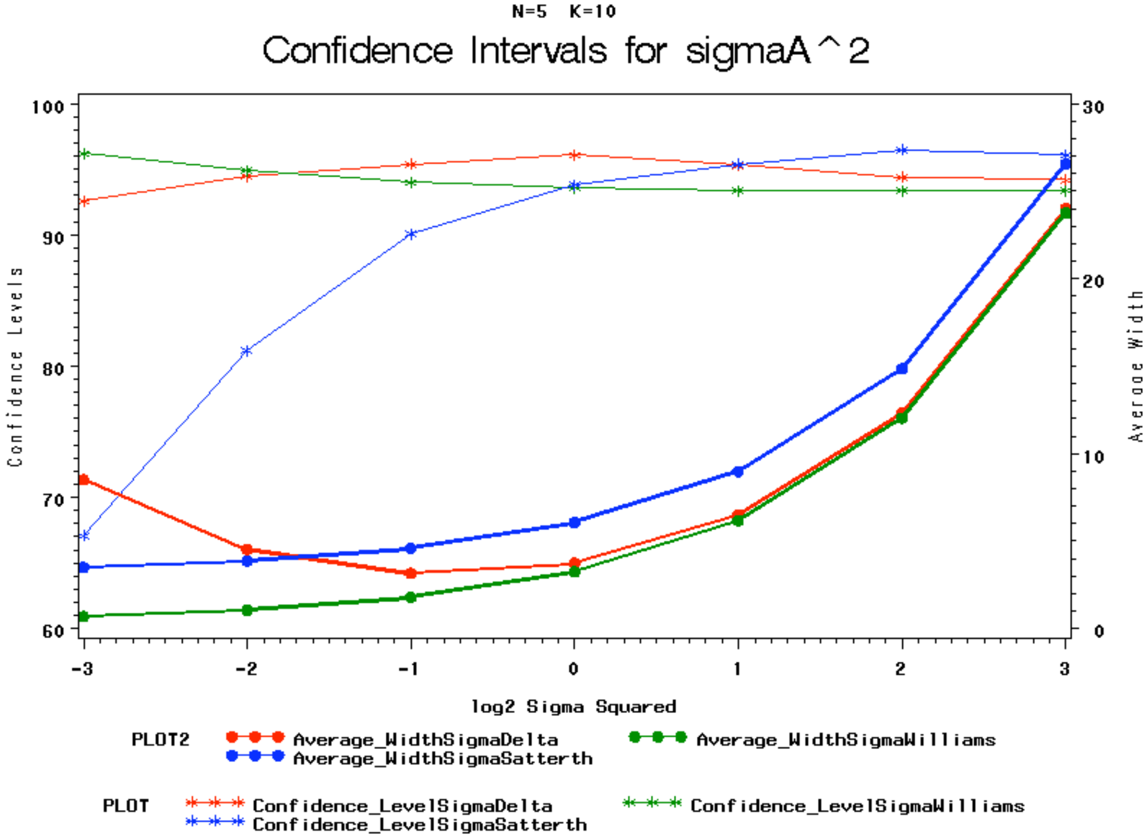
Confidence Intervals for sigmaA^2
Confidence Levels
Average Width
log2 Sigma Squared
97
96
95
94
93
92
91
90
89
88
87
80
70
60
50
40
30
20
10
0
-3
-2
-1
0
1
2
3
PLOT2
Average_WidthSigmaDelta
Average_WidthSigmaSatterth
Average_WidthSigmaWilliams
PLOT
Confidence_LevelSigmaDelta
Confidence_LevelSigmaSatterth
Confidence_LevelSigmaWilliams
N=10 K=5



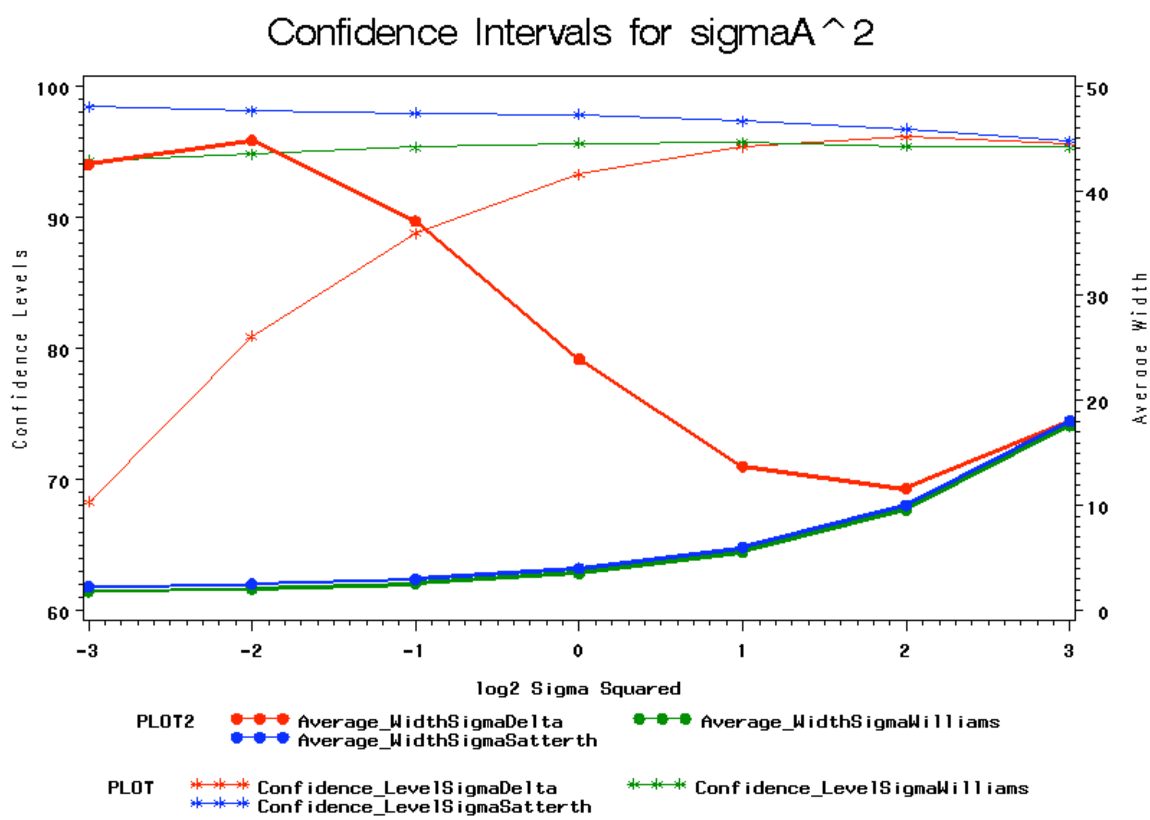
Confidence Intervals for sigmaA^2
Confidence Levels
Average Width
log2 Sigma Squared
100
90
80
70
60
60
50
40
30
20
10
0
-3
-2
-1
0
1
2
3
PLOT2
Average_WidthSigmaDelta
Average_WidthSigmaSatterth
Average_WidthSigmaWilliams
PLOT
Confidence_LevelSigmaDelta
Confidence_LevelSigmaSatterth
Confidence_LevelSigmaWilliams
N=2 K=10



N=5 K=10

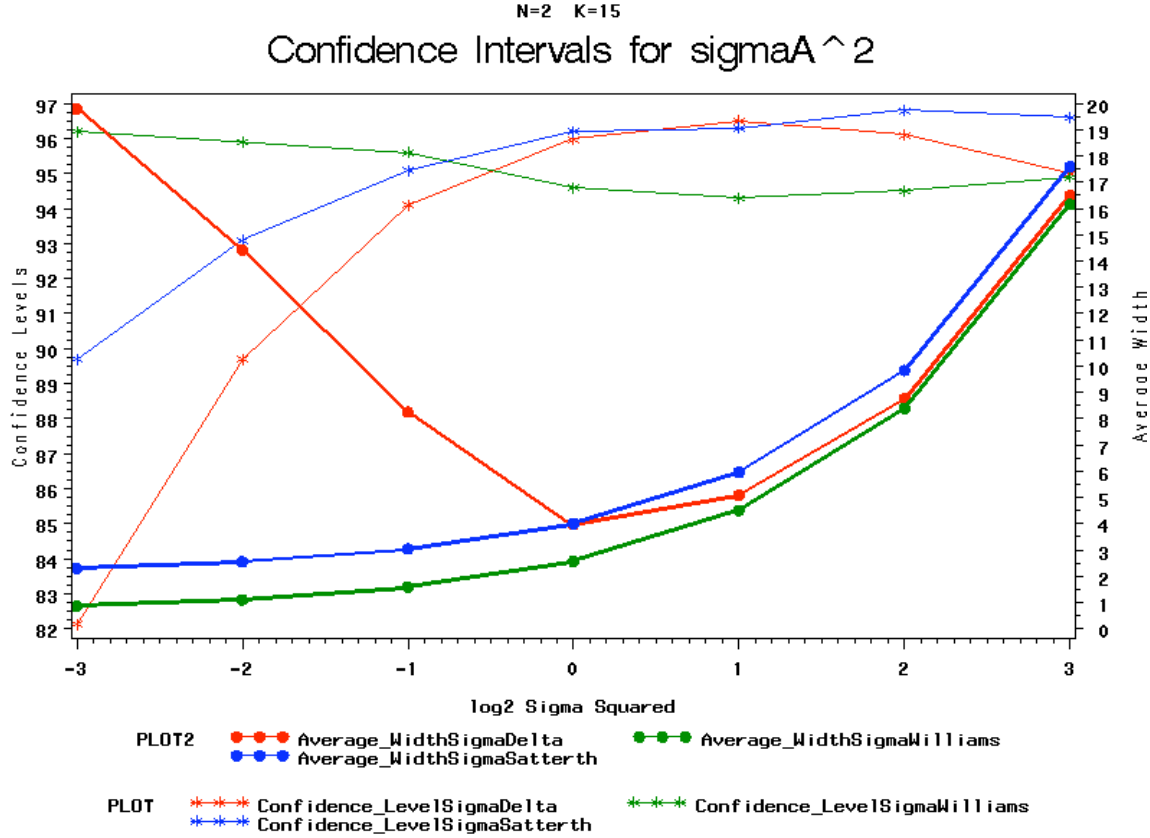


N=10 K=10



Confidence Intervals for sigmaA^2
Confidence Levels
Average Width
log2 Sigma Squared
PLOT2 Average_WidthSigmaDelta Average_WidthSigmaWilliams Average_WidthSigmaSatterth
PLOT Confidence_LevelSigmaDelta Confidence_LevelSigmaWilliams Confidence_LevelSigmaSatterth

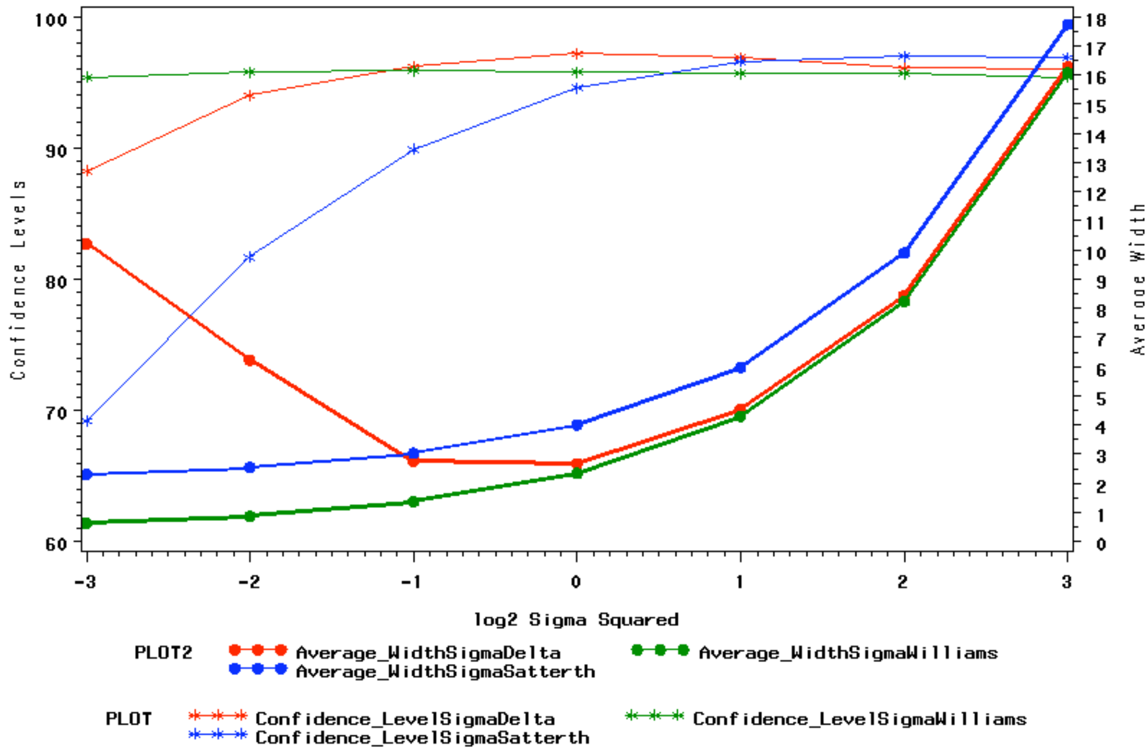
N=2 K=15



Confidence Intervals for sigmaA^2
Confidence Levels
Average Width
log2 Sigma Squared
PLOT2 Average_WidthSigmaDelta Average_WidthSigmaWilliams Average_WidthSigmaSatterth
PLOT Confidence_LevelSigmaDelta Confidence_LevelSigmaWilliams Confidence_LevelSigmaSatterth

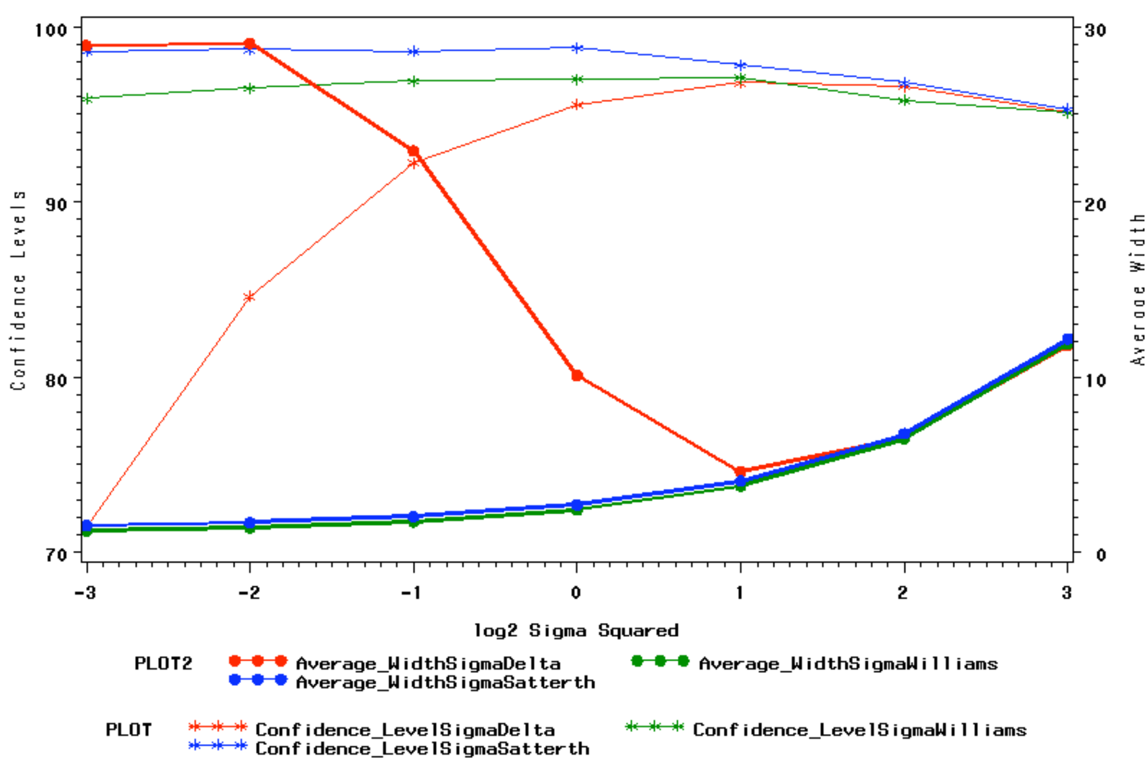
N=4 K=15

Confidence Intervals for sigmaA^2

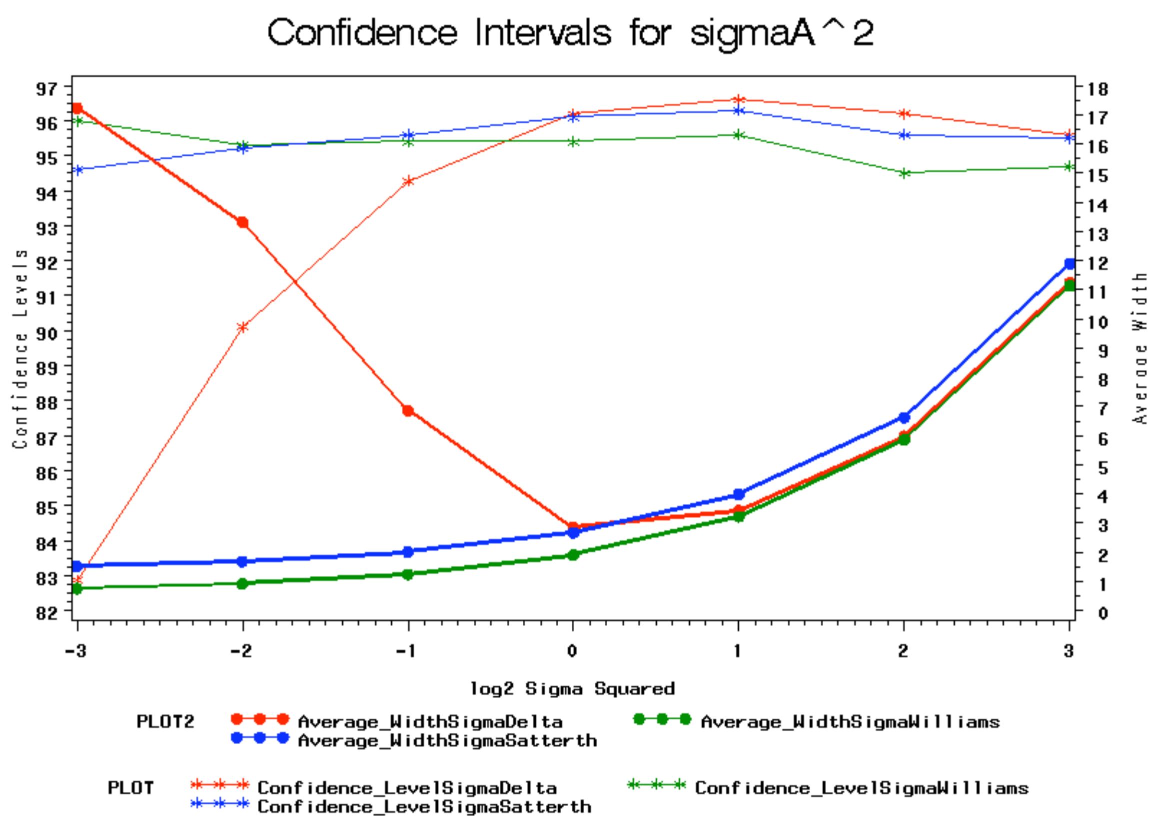


N=6 K=15

Confidence Intervals for sigmaA^2

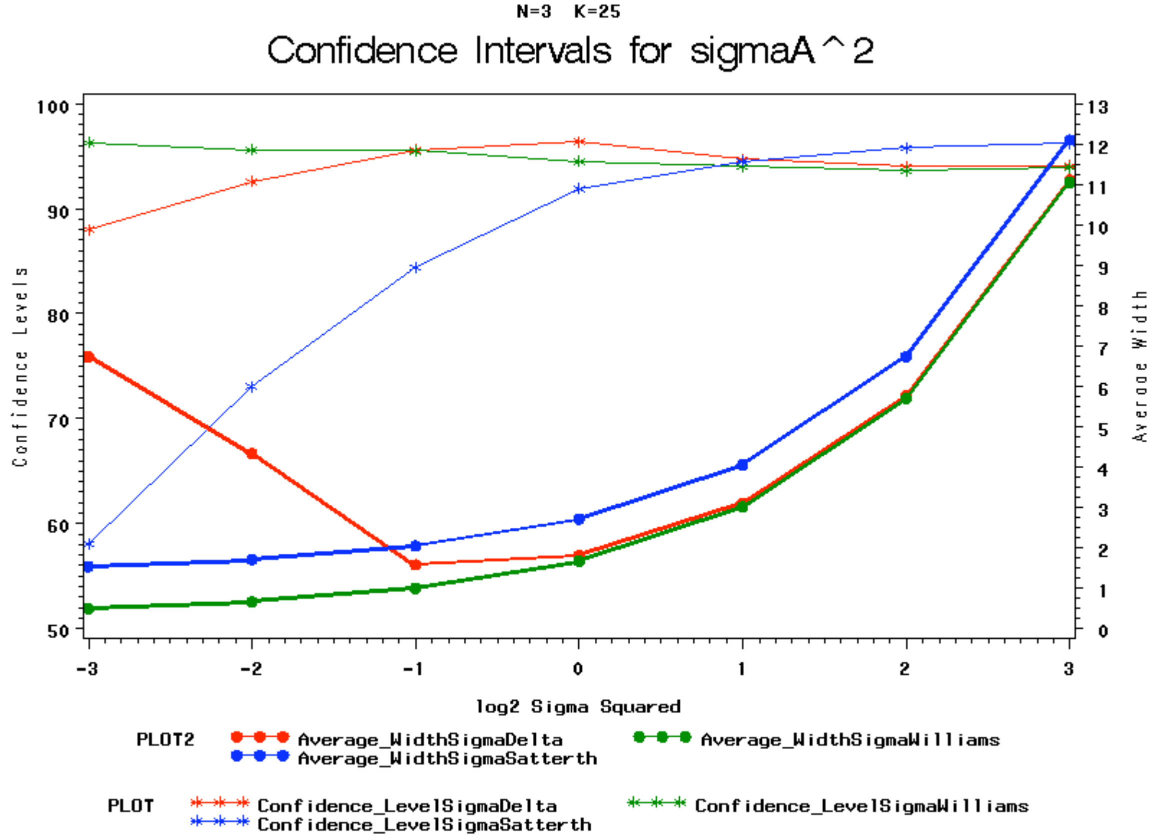


N=2 K=25



Confidence Intervals for sigmaA^2
Confidence Levels
Average Width
log2 Sigma Squared
PLOT2 Average_WidthSigmaDelta Average_WidthSigmaWilliams
Average_WidthSigmaSatterth
PLOT Confidence_LevelSigmaDelta Confidence_LevelSigmaWilliams
Confidence_LevelSigmaSatterth

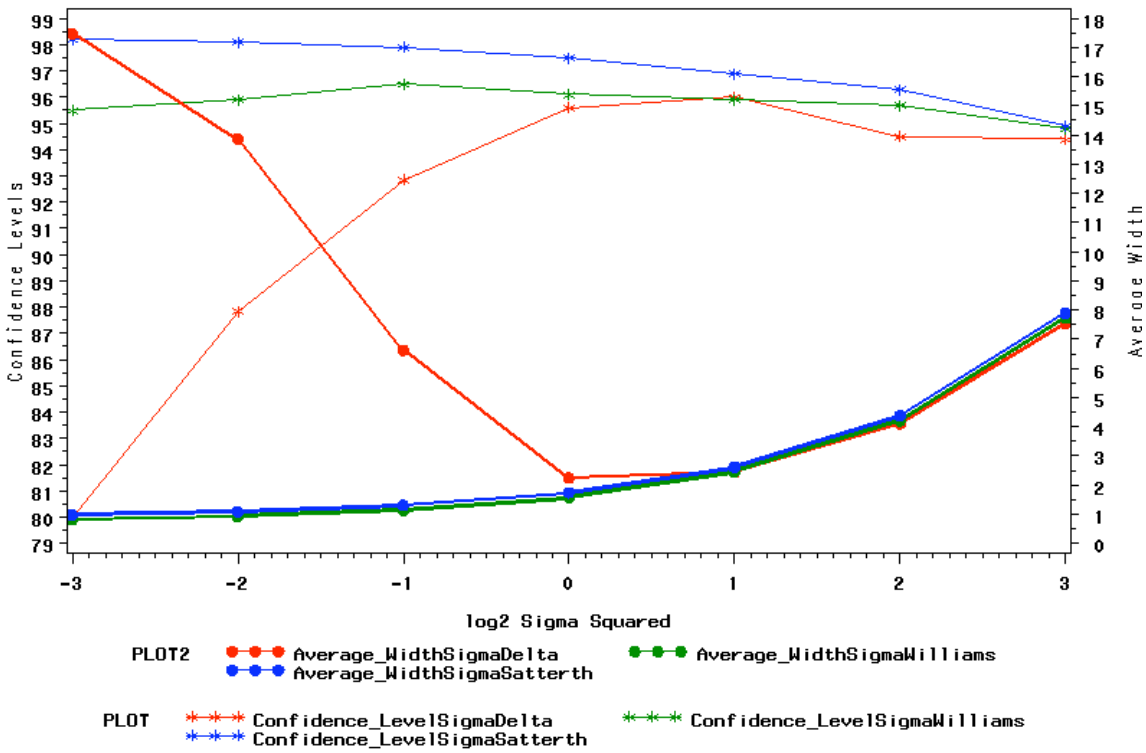
N=3 K=25



Confidence Intervals for sigmaA^2
Confidence Levels
Average Width
log2 Sigma Squared
PLOT2 Average_WidthSigmaDelta Average_WidthSigmaWilliams
Average_WidthSigmaSatterth
PLOT Confidence_LevelSigmaDelta Confidence_LevelSigmaWilliams
Confidence_LevelSigmaSatterth

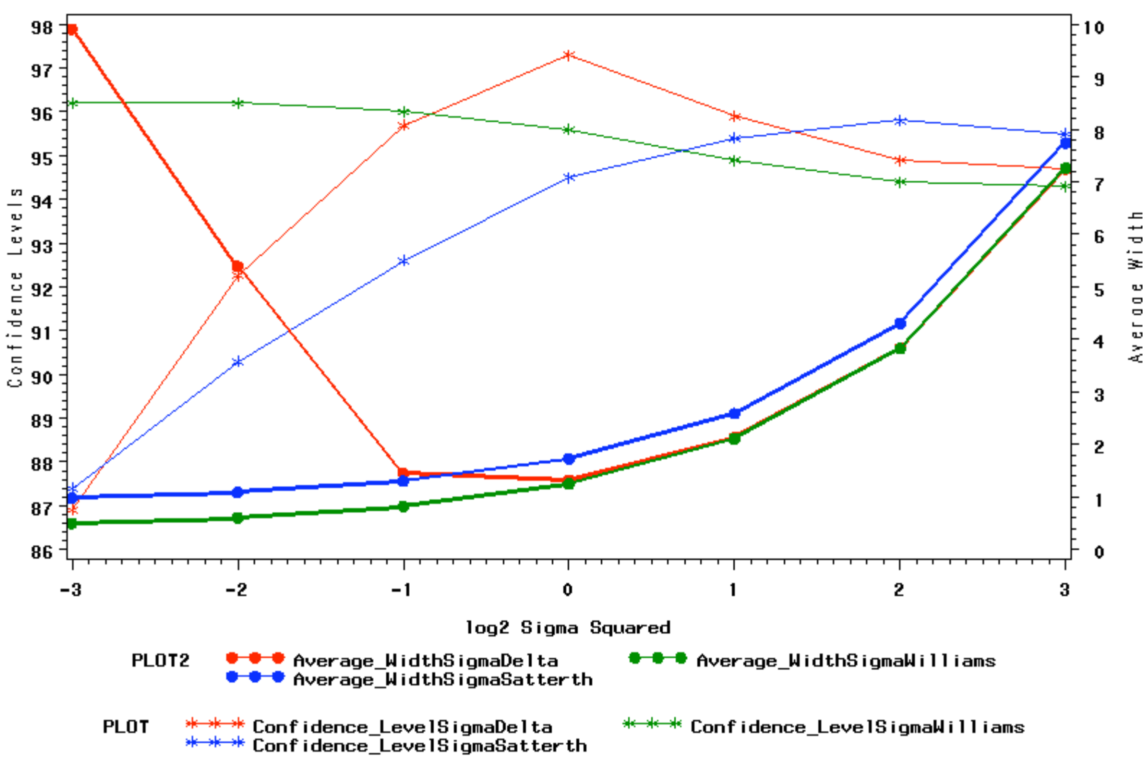
N=5 K=25

Confidence Intervals for sigmaA^2



N=2 K=50

Confidence Intervals for sigmaA^2



N=3 K=50

Appendix 3. SAS code.

```
libname project "C:\Confidence IntervalsREML\";
%macro usun(plik); *deletes input file;
proc datasets library=work; delete &plik; run;quit;
%mend;
%macro licz(numer,plik,fn);
proc sql; delete from &plik where success=-1; run; quit;
proc sql; delete from &plik where width<0; run; quit;
proc sql; create table plik1 as select *, count (*) as num_of_obs from &plik; run;quit;
proc sql; create table plik2 as select distinct num_of_obs, k from plik1; run; quit;
proc sql; create table pczas&numer as select distinct k, count (*) as sukces from &plik
where success ne 0;run;quit;
proc sql; create table pczas as select a.num_of_obs, b.sukces from plik2 a inner join
pczas&numer b on a.k=b.k;run;quit;
%let filename="&plik";
data pczas&numer; set pczas;
FileName=&filename;
Newname="&fn";
confidence_level=sukces/num_of_obs*100;
drop sukces;
run;
proc sql;
create table ppczas&numer as select k, n, sigmaA2, avg(width) as Average_Width from
&plik; run; quit;
data ppczas&numer; set ppczas&numer;
Filename=&filename;
run;
proc sql;
create table czas&numer as select a.*, b.* from ppczas&numer a inner join pczas&numer
b on a.Filename=b.Filename;
run;quit;
```

```
%usun(pczas);%usun(plikp); %usun(plikp1); %usun(plikpp); %usun(plik); %usun(plik2);
%mend;
%macro components(stdevA,groups,n,serial);
data one&serial;
do sim=1 to 1000;
do k=1 to &groups;
        a=&stdevA*normal(12456);
        do i=1 to &n;
                y=7+a+normal(0); ran=uniform(1243000); if ran<0.10 then y=.;
output;
end;end;end;run;
/*INFO sigmaA is actually sigmaA squared, and sigmaE is sigmaE squared
        bms=MSA, ems=MSE
Calculations needed for Smith Confidence Interval for intraclass correlation*/
proc sql; create table two&serial as select sim, k, count(case when y ne . then "count me"
end) "nix" from one&serial group by sim, k;run;quit;
proc sql; create table subtest1 as select sim, sum(_TEMA001) as N, uss(_TEMA001) as
Ni_sq from two&serial group by sim;run;quit;;
proc sql; create table CapZeroSq as select sim, N, Ni_sq, (N-(Ni_sq/N))/(&groups-1) as
N_o from subtest1 group by sim; run; quit;
proc sql; create table test1a1 as select sim,_TEMA001, k, _TEMA001**3 as cube,
1/_TEMA001 as nrec from two&serial group by sim, k;run; quit;
proc sql; create table NnCube as select sim, sum(cube) as Nncube, sum(nrec) as sumrec
from test1a1 group by sim; run; quit;
proc sql; create table Nhat as select sim, &groups/sumrec as nhat from NnCube group by
sim; run; quit;
proc sql; create table testY1 as select sim, k, mean(y) as mean_y from one&serial group
by sim, k; run; quit;
proc sql; create table testY2a as select sim, sum(mean_y) as y_sum from testY1 group by
sim; run; quit;
```

```
proc sql; create table testY2b as select sim, k, mean_y*mean_y as y_sq from testY1
group by sim, k; run; quit;
proc sql; create table testY2c as select sim, sum(y_sq) as y_ss from testY2b group by
sim; run; quit;
proc sql; create table testY2d as select sim, y_sum*y_sum as sq_y_sum from testY2a
group by sim; run; quit;
proc sql; create table testY3 as select a.*, b.* from testY2d a inner join testY2c b on
a.sim=b.sim; run; quit;
proc datasets library=work; delete testY1 testY2a testY2b testY2c testY2d subtest1
test1a1; run;
ods listing close;
proc mixed data=one&serial asycov method=reml cl; by sim;
class k;
model y=;
random k;
ods output asycov=ascov covparms=covparms;
run;
ods listing;
ods listing close;
proc mixed data=one&serial asycov method=type1 cl; by sim;
class k;
model y=;
random k;
ods output asycov=ascovtype1 covparms=covparmtype1;
run;
ods listing;
ods listing close;
ods output overallanova=anovatable;
proc glm data=one&serial; by sim;
class k; model y=k;
random k/test; run;
```

```
ods output close;
ods listing;
proc sql;
create table sigma1 as select distinct sim,Estimate as sigmaA from covparms where
Covparm="k";run;quit;
proc sql;
create table sigma2 as select distinct sim,Estimate as sigmaE from covparms where
Covparm="Residual";run; quit;
proc sql;
create table CovarianceParameters as select a.sim, a.sigmaA, b.sigmaE from sigma1 a
inner join sigma2 b on a.sim=b.sim;run;quit;
proc sql;
create table varcovar1 as select distinct sim,CovP1 as vsig11 from ascov where
CovParm="k";run;quit;
proc sql;
create table varcovar2 as select distinct sim,CovP1 as vsig12 from ascov where
CovParm="Residual";run;quit;
proc sql;
create table varcovar3 as select distinct sim,CovP2 as vsig22 from ascov where
CovParm="Residual";run;quit;
proc sql;
create table lowerlimit as select distinct sim,Lower as llimit from covparmtype1 where
CovParm="k";run;quit;
proc sql;
create table upperlimit as select distinct sim,Upper as ulimit from covparmtype1 where
CovParm="k";run;quit;
proc sql;
create table WaldInta as select a.sim, a.llimit, b.ulimit from lowerlimit a inner join
upperlimit b on a.sim=b.sim;
run;quit;
```

```
proc sql; create table WaldInt as select a.sim, a.*, b.sigmaA from WaldInta a inner join
covarianceparameters b on a.sim=b.sim;
run;quit;
data WaldInterval&serial; set WaldInt;
trueSigmaA=&stdevA**2;
if sigmaA<0 then sigmaA=0;
if LLimit<trueSigmaA<ULimit and sigmaA>0 then success=1; if (trueSigmaA<LLimit
or trueSigmaA>ULimit) and sigmaA>0 then success=0;
if sigmaA=0 then success=-1;
ULimit=max(0,ULimit);LLimit=max(LLimit,0);
width=ULimit-LLimit;
k=&groups; N=&n; sigmaA2=&stdevA*&stdevA;
run;
%licz(15&serial,WaldInterval&serial,Wald);
proc sql;
create table temp1 as select a.sim, a.vsig11, b.vsig12 from varcovar1 a inner join
varcovar2 b on a.sim=b.sim;
run;quit;
proc sql;
create table VarianceCovariance as select a.sim, a.*, b.* from temp1 a inner join
varcovar3 b on a.sim=b.sim;
run;quit;
proc sql;
create table model as select distinct sim,ms as bms, df as bdf, ss as ssa from anovatable
where source="Model";
run;quit;
proc sql;
create table error as select distinct sim,ms as ems, df as edf, ss as sse from anovatable
where source="Error";
run;quit;
proc sql;
```

```
create table temp2 as select a.sim,a.*, b.*, a.bms/b.ems as F from model a inner join error
b on a.sim=b.sim;
run;quit;
proc sql;
create table halfanova as select a.sim, a.*, b.* from temp2 a inner join
VarianceCovariance b on a.sim=b.sim;
run;quit;
proc sql;
create table temp3 as select a.sim, a.*, b.* from CapZeroSq a inner join halfanova b on
a.sim=b.sim;
run;quit;
/* Table with info needed for first confidence interval*/
proc sql;
create table maintableCI1 as select a.sim, a.*, b.*, a.sigmaA/(a.sigmaA+a.sigmaE) as
rho_est from covarianceparameters a inner join temp3 b on a.sim=b.sim;
run;quit;
/* DELETE TEMPORARY FILES
proc datasets library=work; delete covparms sigma1 sigma2 variancecovariance
covarianceparameters temp3 temp2 halfanova model error anovatable temp1 varcovar3
varcovar2 varcovar1; run;
/*------------------------*/
/*
data FtableCI1&serial;
set maintableCI1;
Fl=finv(.025,bdf,edf);
Fu=finv(.975,bdf,edf);
trueRho=&stdevA*&stdevA/(&stdevA*&stdevA+1);
LowerL=(F/Fu-1)/((N/&groups)+F/Fu-1);
UpperL=(F/Fl-1)/((N/&groups)+F/Fl-1);
if sigmaA<0 then sigmaA=0;
```

```
if LowerL<trueRho<UpperL and sigmaA>0 then success=1; if (trueRho<LowerL or
trueRho>UpperL) and sigmaA>0 then success=0;
if sigmaA=0 then success=-1;
UpperL=min(1,UpperL);LowerL=max(LowerL,0);
width=UpperL-LowerL;
k=&groups; n=&n; sigmaA2=&stdevA*&stdevA;
keep sim sigmaA sigmaA2 width rho_est trueRho k n LowerL UpperL success;
run;
%licz(1&serial,FtableCI1&serial,CI1);*/
/* Table with info needed for second confidence interval
proc sql;
create table maintableCI2 as select a.sim, a.*, b.* from maintableCI1 a inner join
CapZeroSq b on a.sim=b.sim;
run; quit;*/
data BalCI&serial;
set maintableCI1;
Fl=finv(.025,bdf,edf);
Fu=finv(.975,bdf,edf);
trueRho=&stdevA*&stdevA/(&stdevA*&stdevA+1);
LowerL=(F/Fu-1)/(N_o+F/Fu-1);
UpperL=(F/Fl-1)/(N_o+F/Fl-1);
if sigmaA<0 then sigmaA=0;
if LowerL<trueRho<UpperL and sigmaA>0 then success=1; if (trueRho<LowerL or
trueRho>UpperL) and sigmaA>0 then success=0;
if sigmaA=0 then success=-1;
UpperL=min(1,UpperL);LowerL=max(LowerL,0);
width=UpperL-LowerL;
k=&groups; n=&n; sigmaA2=&stdevA*&stdevA;
keep sim sigmaA width rho_est trueRho sigmaA2 k n LowerL UpperL success;
run;
```

```
%licz(2&serial,BalCI&serial,Bal);
/* Table with info needed for third confidence interval*/
proc sql;
create table temp4 as select a.sim, a.*, b.* from maintableCI1 a inner join Nhat b on
a.sim=b.sim; run; quit;
proc sql;
create table maintableCI3 as select a.*, b.*, (a.nhat*(b.y_ss-
b.sq_y_sum/&groups))/((&groups-1)*a.ems) as Fstar from temp4 a inner join testY3 b on
a.sim=b.sim;
run; quit;
data THCI&serial;
set maintableCI3;
Fl=finv(.025,bdf,edf);
Fu=finv(.975,bdf,edf);
trueRho=&stdevA*&stdevA/(&stdevA*&stdevA+1);
LowerL=(Fstar/Fu-1)/(Nhat+Fstar/Fu-1);
UpperL=(Fstar/Fl-1)/(Nhat+Fstar/Fl-1);
if sigmaA<0 then sigmaA=0;
if LowerL<trueRho<UpperL and sigmaA>0 then success=1; if (trueRho<LowerL or
trueRho>UpperL) and sigmaA>0 then success=0;
if sigmaA=0 then success=-1;
UpperL=min(1,UpperL);LowerL=max(LowerL,0);
width=UpperL-LowerL;
k=&groups; n=&n; sigmaA2=&stdevA*&stdevA;
keep sim sigmaA width rho_est trueRho sigmaA2 k n Fstar LowerL UpperL success;
run;
%licz(3&serial,THCI&serial,TH);
/*Confidence Interval using Delta Method*/
data DeltaMethodCI&serial;
set maintableCI1;
if sigmaA ne 0 then f1=(sigmaE)/(sigmaA+sigmaE)**2; else f1=.;
```

```
if sigmaA ne 0 then f2=(-sigmaA)/(sigmaA+sigmaE)**2; else f2=.;
VarrhoHat=f1*(f1*vsig11+f2*vsig12)+f2*(f1*vsig12+f2*vsig22);
rhat=2*(rho_est**2)/VarrhoHat;
Chil=cinv(0.025,max(rhat,1));
ChiU=cinv(0.975,max(rhat,1));
trueRho=&stdevA*&stdevA/(&stdevA*&stdevA+1);
LowerL=rhat*rho_est/Chiu;
UpperL=rhat*rho_est/Chil;
if sigmaA<0 then sigmaA=0;
if LowerL<trueRho<UpperL and sigmaA>0 then success=1; if (trueRho<LowerL or
trueRho>UpperL) and sigmaA>0 then success=0;
if sigmaA=0 then success=-1;
UpperL=min(1,UpperL); LowerL=max(LowerL,0);
width=UpperL-LowerL;
k=&groups; n=&n; sigmaA2=&stdevA*&stdevA;
keep sim sigmaA width rho_est trueRho sigmaA2 k n LowerL UpperL success;
run;
%licz(4&serial,DeltaMethodCI&serial,DeltaMethod);
%*%frequencyWidth(4&serial,FtableCI4&serial);
/* Table with info needed for Donner's confidence interval*/
proc sql;
create table maintableCIDon as select a.sim, a.*, b.*, (a.bms-a.ems)/(a.bms+(N_o-
1)*a.ems) as rA from maintableCI1 a inner join NnCube b on a.sim=b.sim;
run; quit;
data SmithCI&serial;
set maintableCIDon;
if sigmaA ne 0 then VarrAhat=((2*(1-rho_est)**2)/(N_o**2))*(((1+rho_est*(N_o-
1))**2)/(N-&groups)+((&groups-1)*(1-rho_est)*(1+rho_est*(2*N_o-
1))+(rho_est**2)*(Ni_sq-2/N*NnCube+(1/N**2)*((Ni_sq)**2)))/(&groups-1)**2);
else VarrAhat=.;
trueRho=&stdevA*&stdevA/(&stdevA*&stdevA+1);
```

```
x=probit(.975);
LowerL=rA-probit(.975)*sqrt(VarrAhat);
UpperL=rA+probit(.975)*sqrt(VarrAhat);
if sigmaA<0 then sigmaA=0;
if LowerL<trueRho<UpperL and sigmaA>0 then success=1; if (trueRho<LowerL or
trueRho>UpperL) and sigmaA>0 then success=0;
if sigmaA=0 then success=-1;
UpperL=min(1,UpperL);LowerL=max(LowerL,0);
width=UpperL-LowerL;
k=&groups; n=&n; sigmaA2=&stdevA*&stdevA;
keep sim sigmaA width rho_est trueRho sigmaA2 k n LowerL UpperL success;
run;
%licz(5&serial,SmithCI&serial,Smith);
data SwigerCI&serial;
set maintableCIDon;
if sigmaA ne 0 then VarrAhatprime=2*(N-1)*((1-rho_est)**2)*((1+(N_o-
1)*rho_est)**2)/((N_o**2)*(N-&groups)*(&groups-1));
else VarrAhatprime=.;
trueRho=&stdevA*&stdevA/(&stdevA*&stdevA+1);
x=probit(.975);
LowerL=rA-probit(.975)*sqrt(VarrAhatprime);
UpperL=rA+probit(.975)*sqrt(VarrAhatprime);
if sigmaA<0 then sigmaA=0;
if LowerL<trueRho<UpperL and sigmaA>0 then success=1; if (trueRho<LowerL or
trueRho>UpperL) and sigmaA>0 then success=0;
if sigmaA=0 then success=-1;
UpperL=min(1,UpperL);LowerL=max(LowerL,0);
width=UpperL-LowerL;
k=&groups; n=&n; sigmaA2=&stdevA*&stdevA;
keep sim sigmaA width rho_est trueRho sigmaA2 k n LowerL UpperL success;
run;
```

```
%licz(6&serial,SwigerCI&serial,Swiger);
/* A procedure based on Fisher's Transformation*/
data FisherCI&serial;
set maintableCIDon;
trueRho=&stdevA*&stdevA/(&stdevA*&stdevA+1);
z0=probit(.975);
Vz=0.5*(1/(&groups-1)+1/(N-&groups));
*Zf=1/2*log(F);
Zf=1/2*log((1+(N_o-1)*rA)/(1-rA));
LowerL=(exp(2*(Zf-z0*sqrt(Vz)))-1)/(exp(2*(Zf-z0*sqrt(Vz)))+N_o-1);
UpperL=(exp(2*(Zf+z0*sqrt(Vz)))-1)/(exp(2*(Zf+z0*sqrt(Vz)))+N_o-1);
if sigmaA<0 then sigmaA=0;
if LowerL<trueRho<UpperL and sigmaA>0 then success=1; if (trueRho<LowerL or
trueRho>UpperL) and sigmaA>0 then success=0;
if sigmaA=0 then success=-1;
UpperL=min(1,UpperL);LowerL=max(LowerL,0);
width=UpperL-LowerL;
k=&groups; n=&n; sigmaA2=&stdevA*&stdevA;
keep sim sigmaA width rho_est trueRho sigmaA2 k n LowerL UpperL success;
run;
%licz(7&serial,FisherCI&serial,Fisher);
/*----------------------------------------------------------------------------------------------------*/
/* CONFIDENCE INTERVALS FOR RATIO SIGMAA^2/SIGMAE^2 ni=n*/
data Ratio1CI&serial;
set maintableCI1;
ratio_est=sigmaA/sigmaE;
trueRatio=&stdevA**2/1;
Fl=finv(.025,bdf,edf);
Fu=finv(.975,bdf,edf);
LowerL=(F/Fu-1)/(N_o); /*for equal number of observations per group*/
UpperL=(F/Fl-1)/(N_o);
```

```
if LowerL<trueRatio<UpperL then success=1; else success=0;
width=UpperL-LowerL;
k=&groups; n=&n; sigmaA2=&stdevA*&stdevA;
keep sim sigmaA width ratio_est trueRatio sigmaA2 k n LowerL UpperL success;
run;
%licz(8&serial,Ratio1CI&serial,Ratio1);
data RatioDeltaMCI&serial;
set maintableCI1;
f1=1/sigmaE;
f2=-sigmaA/sigmaE**2;
Varratiohat=f1*(f1*vsig11+f2*vsig12)+f2*(f1*vsig12+f2*vsig22);
ratio_est=sigmaA/sigmaE;
rhat=2*(ratio_est**2)/Varratiohat;
Chil=cinv(0.025, max(rhat,1));
ChiU=cinv(0.975,max(rhat,1));
trueRatio=&stdevA**2/1;
LowerL=rhat*ratio_est/Chiu;
UpperL=rhat*ratio_est/Chil;
if sigmaA<0 then sigmaA=0;
if LowerL<trueRatio<UpperL and sigmaA>0 then success=1; if (trueRatio<LowerL or
trueRatio>UpperL) and sigmaA>0 then success=0;
if sigmaA=0 then success=-1;
width=UpperL-LowerL;
k=&groups; n=&n; sigmaA2=&stdevA*&stdevA;
keep sim sigmaA width Chil ChiU ratio_est trueRatio sigmaA2 k n LowerL UpperL
success;run;
%licz(9&serial,RatioDeltaMCI&serial,RatioDeltaM);
/*-----------------------------------------------------------------------------------------------------*/
/* CONFIDENCE INTERVALS FOR SUM OF SIGMAA^2 AND SIGMAE^2 */
/*Delta Method*/
data SumDeltaMCI&serial;
```

```
set maintableCI1;
Varsumhat=vsig11+vsig22+2*vsig12;
sum_est=sigmaA+sigmaE;
rhat=2*(sum_est**2)/Varsumhat;
Chil=cinv(0.025,max(rhat,1));
ChiU=cinv(0.975,max(rhat,1));
trueSum=&stdevA**2+1;
LowerL=rhat*sum_est/Chiu;
UpperL=rhat*sum_est/Chil;
if LowerL<trueSum<UpperL then success=1; else success=0;
width=UpperL-LowerL;
k=&groups; n=&n; sigmaA2=&stdevA*&stdevA;
keep sim sigmaA width Chil ChiU sum_est trueSum sigmaA2 k n LowerL UpperL
success;
run;
%licz(10&serial,SumDeltaMCI&serial,SumDelta);
/*Satterthwaite's Approximation method*/
data SumSatterthwaiteCI&serial;
set maintableCI1;
q1=1/N_o; q2=1-1/N_o;
Q=q1*bms+q2*ems;
vhat=(Q**2)/(((q1**2)*(bms**2)/bdf)+((q2**2)*(ems**2)/edf));
Chil=cinv(0.025, max(vhat,1));
ChiU=cinv(0.975,max(vhat,1));
trueSum=&stdevA**2+1;
LowerL=vhat*Q/ChiU;
UpperL=vhat*Q/ChiL;
if LowerL<trueSum<UpperL then success=1; else success=0;
width=UpperL-LowerL;
k=&groups; n=&n; sigmaA2=&stdevA*&stdevA;
```

```
keep sim sigmaA width Chil ChiU sum_est trueSum sigmaA2 k n LowerL UpperL
success;
run;
%licz(11&serial,SumSatterthwaiteCI&serial,SumSatterth);
/*-----------------------------------------------------------------------------------------------------*/
/* CONFIDENCE INTERVALS FOR SIGMAA SQUARED */
/* Delta Method */
data SigmaDeltaMCI&serial;
set maintableCI1;
trueSigmaA=&stdevA**2;
rhat=2*(sigmaA**2)/vsig11;
Chil=cinv(0.025,max(rhat,1));
ChiU=cinv(0.975,max(rhat,1));
LowerL=rhat*sigmaA/ChiU;
UpperL=rhat*sigmaA/ChiL;
if sigmaA<0 then sigmaA=0;
if LowerL<trueSigmaA<UpperL and sigmaA>0 then success=1; if (trueSigmaA<LowerL
or trueSigmaA>UpperL) and sigmaA>0 then success=0;
if sigmaA=0 then success=-1;
UpperL=max(0,UpperL);LowerL=max(LowerL,0);
width=UpperL-LowerL;
k=&groups; n=&n; sigmaA2=&stdevA*&stdevA;
keep sim sigmaA width trueSigmaA sigmaA2 k n LowerL UpperL success;
run;
%licz(12&serial,SigmaDeltaMCI&serial,SigmaDelta);
/*%frequencyWidth(12&serial,SigmaSatterthwaiteCI&serial);
/*William's Method*/
data SigmaWilliamsCI&serial;
set maintableCI1;
trueSigmaA=&stdevA**2;
Chil=cinv(.025,bdf);
```

```
ChiU=cinv(.975,bdf);
Fl=finv(.025,bdf,edf);
Fu=finv(.975,bdf,edf);
LowerL=(SSA*(1-Fu/F))/(N_o*ChiU);
UpperL=(SSA*(1-Fl/F))/(N_o*ChiL);
if LowerL<trueSigmaA<UpperL then success=1; else success=0;
width=UpperL-LowerL;
k=&groups; n=&n; sigmaA2=&stdevA*&stdevA;
keep sim sigmaA width trueSigmaA sigmaA2 k n LowerL UpperL success;
run;
%licz(13&serial,SigmaWilliamsCI&serial,SigmaWilliams);
/* From Simultaneous Confidence Interval */
data SigmaSCI&serial;
set maintableCI1;
trueSigmaA=&stdevA**2;
Q1=ems; Q2=bms; a=N_o; u1=edf; u2=bdf;
Chil=cinv(.025,u2);
ChiU=cinv(.975,u2);
c=(u2*Q2/ChiU-u1*Q1/Chil)/a;
d=(u2*Q2/Chil-u1*Q1/ChiU)/a;
if c<trueSigmaA<d then success=1; else success=0;
width=d-c;
k=&groups; n=&n; sigmaA2=&stdevA*&stdevA;
keep sim sigmaA width trueSigmaA sigmaA2 k n c d success;
run;
%licz(14&serial,SigmaSCI&serial,SigmaSatterth);
proc datasets library=work; delete testY3 temp4 One&serial Two&serial NnCube Nhat
MaintableCI3 MainTableCIDon Ascov Capzerosq; run;
%usun(MaintableCI1);
/*Final table contains info for each combination n&k and each stdev*/
```

```
proc sql;
create table final&serial as select * from czas2&serial
union select * from czas3&serial
union select * from czas4&serial
union select * from czas5&serial
union select * from czas6&serial
union select * from czas7&serial
union select * from czas8&serial
union select * from czas9&serial
union select * from czas10&serial
union select * from czas11&serial
union select * from czas12&serial
union select * from czas13&serial
union select * from czas14&serial
union select * from czas15&serial;
run; quit;
/*data project.RatioDeltaMCI&serial; set RatioDeltaMCI&serial; run;*/
*%usun(czas1&serial);%usun(czas2&serial);%usun(czas3&serial);%usun(czas4&serial);
%usun(czas5&serial);%usun(czas6&serial);%usun(czas7&serial);%usun(czas8&serial);
%usun(czas9&serial);%usun(czas10&serial);%usun(czas11&serial);%usun(czas12&seria
l);%usun(czas13&serial);%usun(czas14&serial);%usun(pczas1&serial);%usun(pczas2&s
erial);%usun(pczas3&serial);%usun(pczas4&serial);%usun(pczas5&serial);%usun(pczas
6&serial);%usun(pczas7&serial);%usun(pczas8&serial);%usun(pczas9&serial);%usun(p
czas10&serial);%usun(pczas11&serial);%usun(pczas12&serial);%usun(pczas13&serial);
%usun(pczas14&serial);%usun(ppczas1&serial);%usun(ppczas2&serial);%usun(ppczas3
&serial);%usun(ppczas4&serial);%usun(ppczas5&serial);%usun(ppczas6&serial);
%usun(ppczas7&serial);%usun(ppczas8&serial);%usun(ppczas9&serial);%usun(ppczas1
0&serial);%usun(ppczas11&serial);%usun(ppczas12&serial);%usun(ppczas13&serial);%
usun(ppczas14&serial);*%usun(FtableCI1&serial);%usun(BalCI&serial);%usun(THCI&
serial);%usun(DeltaMethodCI&serial);%usun(SmithCI&serial);%usun(SwigerCI&serial)
```

```
;%usun(FisherCI&serial);%usun(Ratio1CI&serial);%usun(RatioDeltaMCI&serial);%usu
n(SumDeltaMCI&serial);%usun(SumDeltaMCI&serial);%usun(SigmaSatterthwaiteCI&s
erial);%usun(SigmaWilliamsCI&serial);
%usun(SigmaSCI&serial);
%mend;
*Macro below macro creates table joining all stdev, but it separates for each n&k;
%macro variance(k,n,add);
%let jvar0=%eval(1+%eval(&add));
%components(sqrt(1/8),&k,&n,%eval(1+%eval(&add)));
%let jvar1=%eval(1+%eval(&add));
%components(sqrt(1/4),&k,&n,%eval(2+%eval(&add)));
%let jvar2=%eval(2+%eval(&add));
%components(sqrt(1/2),&k,&n,%eval(3+%eval(&add)));
%let jvar3=%eval(3+%eval(&add));
%components(sqrt(1),&k,&n,%eval(4+%eval(&add)));
%let jvar4=%eval(4+%eval(&add));
%components(sqrt(2),&k,&n,%eval(5+%eval(&add)));
%let jvar5=%eval(5+%eval(&add));
%components(sqrt(4),&k,&n,%eval(6+%eval(&add)));
%let jvar6=%eval(6+%eval(&add));
%components(sqrt(8),&k,&n,%eval(7+%eval(&add)));
%let jvar7=%eval(7+%eval(&add));
%put(%eval(1+%eval(&add));
proc sql; create table project.Last&jvar0 as select * from Final&jvar1
union select * from Final&jvar2
union select * from Final&jvar3
union select * from Final&jvar4
union select * from Final&jvar5
union select * from Final&jvar6
union select * from Final&jvar7 order by sigmaA2;run; quit;
/*
```

```
%usun(Final&jvar1);%usun(Final&jvar2);%usun(Final&jvar3);%usun(Final&jvar4);%us
un(Final&jvar5);
%usun(Final&jvar6);%usun(Final&jvar7);*/
%mend;
%variance(5,2,0);
%variance(5,6,7);
%variance(5,10,14);
%variance(10,2,21);
%variance(10,5,28);
%variance(10,10,35);
%variance(15,2,42);
%variance(15,4,49);
%variance(15,6,56);
%variance(25,2,63);
%variance(25,3,70);
%variance(25,5,77);
%variance(50,2,84);
%variance(50,3,91);
```

```
options mprint mlogic;
libname project "C:\Confidence IntervalsREML\";
%macro method(_data_,name,number);
data &number; set &dataname; where Newname="&name";run;
data &number;set &number;
rename Average_Width=Average_Width%trim(&name);
rename confidence_level=Confidence_Level%trim(&name);
logsigmaA2=log2(sigmaA2);
run;
%mend;
symbol1 color=red i=join v=dot w=2;
symbol2 color=green i=join v=dot w=2;
symbol3 color=blue i=join v=dot w=2;
symbol4 color=black i=join v=dot w=2;
symbol5 color=cyan i=join v=dot w=2;
symbol6 color=yellow i=join v=dot w=2;
symbol7 color=red i=join v=star w=1.5;
symbol8 color=green i=join v=star w=1.5;
symbol9 color=blue i=join v=star w=1.5;
symbol10 color=black i=join v=star w=1.5;
symbol11 color=cyan i=join v=star w=1.5;
symbol12 color=yellow i=join v=star w=1.5;
%macro plot(dataname,num);
*%method(&dataname,CI1,four);
%method(&dataname,Bal,five);
%method(&dataname,TH,three);
%method(&dataname,DeltaMethod,one);
%method(&dataname,Fisher,two);
%method(&dataname,Swiger,six);
%method(&dataname,Smith,seven);
%method(&dataname,SumSatterth,eight);
```

```
%method(&dataname,SigmaDelta,nine);
%method(&dataname,SigmaWilliams,ten);
%method(&dataname,SigmaSatterth,eleven);
%method(&dataname,Ratio1,twelve);
%method(&dataname,RatioDeltaM,thirteen);
%method(&dataname,SumDelta,fourteen);
%method(&dataname,Wald,fifteen);
data plot;merge
one
two
three
five
six
seven
eight
nine
ten
eleven
twelve
thirteen
fourteen
fifteen;
;by sigmaA2;
run;
%let p_f=_t;
%let path=C:\Confidence IntervalsREML\Excel Reports\;
%let out_file=&path&num..xls;
data &p_f&num ;set plot;run;
PROC EXPORT DATA= &p_f&num
OUTFILE= "&out_file"
DBMS=EXCEL REPLACE;
```

```
SHEET="data";
RUN;
proc sql noprint; select k into :k from &dataname;run;quit;
proc sql noprint; select n into :n from &dataname;run;quit;
%let sp=_;
%let tablename=%trim(&k)&sp%trim(&n);
%let l0=Confidence Intervals for Intraclass Correlation Coefficient  ;
%let line0=&l0&tablename;
%let plotname=1_&TABLENAME..GIF;
goptions device=gif gsfname=graph gsfmode=replace;
%let loc="C:\Confidence IntervalsREML\Plotslog\&plotname";
%PUT &LOC &PLOTNAME;
%let footnote=N=%trim(&n)  K=%trim(&k);
filename graph &loc ;
axis1 label=(angle=90 "Confidence Levels");
axis2 label=("log2 Sigma Squared");
axis3 label=(angle=90 "Average Width");
proc gplot data=plot;
footnote j=center "&footnote";
title &l0;
plot
confidence_levelBal*logsigmaA2=7
confidence_levelTH*logsigmaA2=8
confidence_levelDeltaMethod*logsigmaA2=9
confidence_levelFisher*logsigmaA2=10
confidence_levelSwiger*logsigmaA2=11
confidence_levelSmith*logsigmaA2=12
/overlay vaxis=axis1 haxis=axis2 legend;
plot2
Average_WidthBal*logSigmaA2=1
Average_WidthTH*logSigmaA2=2
```

```
Average_WidthDeltaMethod*logSigmaA2=3
Average_WidthFisher*logSigmaA2=4
Average_WidthSwiger*logSigmaA2=5
Average_WidthSmith*logSigmaA2=6
/overlay vaxis=axis3 legend;
run;quit;
%let plotname=2_&TABLENAME..GIF;
goptions device=gif gsfname=graph gsfmode=replace;
%let loc="C:\Confidence IntervalsREML\Plotslog\&plotname";
%let l_0=Confidence Intervals for the Ratio sigmaA^2/sigmaE^2  ;
filename graph &loc ;
axis1 label=(angle=90 "Confidence Levels");
axis2 label=("log2 Sigma Squared");
axis3 label=(angle=90 "Average Width");
proc gplot data=plot;
title &l_0;
%let footnote=N=%trim(&n)  K=%trim(&k);
plot confidence_levelRatio1*logsigmaA2=7
confidence_levelRatioDeltaM*logsigmaA2=8 /overlay vaxis=axis1 haxis=axis2 legend;
plot2 Average_WidthRatio1*logSigmaA2=1
Average_WidthRatioDeltaM*logSigmaA2=2 /overlay legend vaxis=axis3;
run; quit;
%let plotname=3_&TABLENAME..GIF;
goptions device=gif gsfname=graph gsfmode=replace;
%let loc="C:\Confidence IntervalsREML\Plotslog\&plotname";
%let l_0=Confidence Intervals for the Ratio sigmaA^2/sigmaE^2  ;
filename graph &loc ;
axis1 label=(angle=90 "Confidence Levels");
axis2 label=("log2 Sigma Squared");
axis3 label=(angle=90 "Average Width");
%let l1=Confidence Intervals for the Sum of SigmaA^2 and SigmaE^2  ;
```

```
proc gplot data=plot;
title &l1;
plot confidence_levelSumDelta*logsigmaA2=7
confidence_levelSumSatterth*logsigmaA2=8 /overlay legend haxis=axis2 vaxis=axis1;
plot2 Average_WidthSumDelta*logSigmaA2=1
Average_WidthSumSatterth*logSigmaA2=2 /overlay legend vaxis=axis3;
run; quit;
%let plotname=4_&TABLENAME..GIF;
goptions device=gif gsfname=graph gsfmode=replace;
%let loc="C:\Confidence IntervalsREML\Plotslog\&plotname";
filename graph &loc ;
axis1 label=(angle=90 "Confidence Levels");
axis2 label=("log2 Sigma Squared");
axis3 label=(angle=90 "Average Width");
%let l2=Confidence Intervals for sigmaA^2  ;
proc gplot data=plot;
title &l2;
plot confidence_levelSigmaDelta*logsigmaA2=7
confidence_levelSigmaWilliams*logsigmaA2=8
confidence_levelSigmaSatterth*logsigmaA2=9/overlay legend vaxis=axis1 haxis=axis2;
plot2 Average_WidthSigmaDelta*logSigmaA2=1
Average_WidthSigmaWilliams*logSigmaA2=2
Average_WidthSigmaSatterth*logsigmaA2=3/overlay legend vaxis=axis3;
run; quit;
%mend;
%plot(project.last1,1);
%plot(project.last8,2);
%plot(project.last15,3);
%plot(project.last22,4);
%plot(project.last29,5);
%plot(project.last36,6);
```

```
%plot(project.last43,7);
%plot(project.last50,8);
%plot(project.last57,9);
%plot(project.last64,10);
%plot(project.last71,11);
%plot(project.last78,12);
%plot(project.last85,13);
%plot(project.last92,14);
data master_table;
set _t1 _t2 _t3
_t4 _t5 _t6 _t7 _t8 _t9
_t10 _t11 _t12 _t13 _t14 ;
RHO=sigmaA2/(sigmaA2+1);
run;
proc sort;by K N RHO;run;
```