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CONSTRAINTS ON NON-FLAT COSMOLOGIES WITH MASSIVE NEUTRINOS AFTER *PLANCK* 2015

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ABSTRACT

We investigate two dark energy cosmological models (i.e., the ΛCDM and ϕCDM models) with massive neutrinos assuming two different neutrino mass hierarchies in both the spatially flat and non-flat scenarios, where in the ϕCDM model the scalar field possesses an inverse power-law potential, $V(\phi) \propto \phi^{-\alpha}$ ($\alpha > 0$). Cosmic microwave background data from *Planck* 2015, baryon acoustic oscillation data from 6dFGS, SDSS-MGS, BOSS-LOWZ and BOSS CMASS-DR11, the joint light-curve analysis compilation of SNe Ia apparent magnitude observations, and the *Hubble Space Telescope* H_0 prior, are jointly employed to constrain the model parameters. We first determine constraints assuming three species of degenerate massive neutrinos. In the spatially flat (non-flat) ΛCDM model, the sum of neutrino masses is bounded as $\Sigma m_\nu < 0.165(0.299)$ eV at 95% confidence level (CL). Correspondingly, in the flat (non-flat) ϕCDM model, we find $\Sigma m_\nu < 0.164(0.301)$ eV at 95% CL. The inclusion of spatial curvature as a free parameter results in a significant broadening of confidence regions for Σm_ν and other parameters. In the scenario where the total neutrino mass is dominated by the heaviest neutrino mass eigenstate, we obtain similar conclusions to those obtained in the degenerate neutrino mass scenario. In addition, the results show that the bounds on Σm_ν based on two different neutrino mass hierarchies have insignificant differences in the spatially flat case for both the ΛCDM and ϕCDM models; however, the corresponding differences are larger in the non-flat case.

Key words: cosmology: miscellaneous – cosmology: theory – dark energy

1. INTRODUCTION

To date, there is firm evidence for neutrino oscillations (see the reviews of Maltoni et al. 2004; Fogli et al. 2006; Balantekin & Haxton 2013) from measurements on solar (Ahmad et al. 2001), atmospheric (Fukuda et al. 1998), reactor (Ahn et al. 2012; An et al. 2012) and accelerator beam (Agafonova et al. 2010) neutrinos. These measurements imply that neutrinos have small but non-zero masses, with at least two species being non-relativistic today. Experiments have placed restrictive limits on differences of two squared neutrino masses, such as $\Delta m_{21}^2 = m_2^2 - m_1^2 \sim 8 \times 10^{-5}\,\mathrm{eV}^2$ (Abe et al. 2008) and $\Delta m_{32}^2 = m_3^2 - m_2^2 \sim 3 \times 10^{-3}\,\mathrm{eV}^2$ (Ashie et al. 2005), but give no constraint on their absolute mass scales. Here m_1, m_2, and m_3 denote the masses of neutrino mass eigenstates. The measurement of the absolute neutrino mass scale remains a big challenge for both experimental particle physics and observational cosmology. Fortunately, a variety of cosmological probes can provide the crucial complementary information on absolute neutrino mass scale. Current cosmological data can provide an upper limit on the total neutrino mass $\sum m_\nu$ (summed over the three neutrino families) of order 1 eV or less (Lesgourgues & Pastor 2012), though they are not very sensitive to the neutrino mass hierarchy.

Massive neutrinos are the only particles that have undergone the transition from radiation to matter as the universe expanded and cooled (Lesgourgues & Pastor 2006). Before the non-relativistic transition the neutrinos behave like radiation. Thus, when the total neutrino mass Σm_ν increases, there is more relativistic matter at early times and the matter–radiation equality occurs later, so the scale factor at the epoch of matter–radiation equality $a_{\rm eq}$ increases (i.e., $z_{\rm eq}$ gets lower). The cosmic microwave background (CMB) radiation and large-scale structure (LSS) distributions are very sensitive to $a_{\rm eq}$, which provides potential ways to constrain Σm_ν through CMB and LSS observations. In addition, the massive neutrinos are non-relativistic today, so they contribute to the recent expansion rate of the universe as cold dark matter. Moreover, after thermal decoupling the massive neutrinos freely stream a distance called the free-streaming length. This disrupts the structure formation on scales below this length. Because of the above effects, massive neutrinos can leave imprints on cosmological observables. This is why a variety of cosmological tests are sensitive to the absolute scale of neutrino mass, such as the CMB anisotropy, galaxy, and Lyα forest distributions as well as the distance information from baryon acoustic oscillations (BAOs) and SNe Ia measurements.

The limits on $\sum m_\nu$ obtained from cosmology, so far, are rather model dependent and vary strongly with the data combination adopted. In Hannestad (2005), it was found that when the dark energy equation of state (EoS) is taken as a free (but constant) parameter, the cosmological bound on $\sum m_\nu$ is relaxed by more than a factor of two, to $\sum m_\nu < 1.48$ eV (95% confidence level; CL), compared with $\sum m_\nu < 0.65$ eV (95% CL) in the ΛCDM model. The above results were obtained from a combination of CMB measurements from the first-year *Wilkinson Microwave Anisotropy Probe* (*WMAP*) observations (Bennett et al. 2003), the galaxy power spectrum based on the Sloan Digital Sky Survey (SDSS) Data Release 2 (Tegmark et al. 2004), the SNe Ia data from Riess et al. (2004), and the H_0 prior from the *Hubble Space Telescope* (*HST*) Key Project with $H_0 = 72 \pm 8\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (Freedman et al. 2001). The two models studied in Hannestad (2005) were also constrained in Wang et al. (2012) with updated cosmological data, where the

corresponding results turned out to be $\sum m_\nu < 0.627$ (95% CL) for an arbitrary (but constant) EoS and $\sum m_\nu < 0.476$ eV (95% CL) for the ΛCDM model. Based on the benefits of the more precise cosmological data, the bound on $\sum m_\nu$ is much more restrictive for each individual model, and the difference of the bounds on $\sum m_\nu$ from the two models is also reduced. The bound on $\sum m_\nu$ in the framework of time evolving EoS, $\omega(z) = \omega_0 + \omega_1 * z/(1+z)$, was also investigated in the literature (Xia et al. 2007, 2008; Li & Xia 2012), and revealed the degeneracy between $\sum m_\nu$ and the EoS ω parameters. In Smith et al. (2012), it was found that with non-vanishing curvature density parameter $\Omega_k \neq 0$ the 95% upper limit on $\sum m_\nu$ was more than double with respect to the case of a flat universe. This implies the strong degeneracy between curvature and $\sum m_\nu$.

In this paper, we present constraints on the total mass of ordinary (active) neutrinos $\sum m_\nu$ assuming no extra relics. Current cosmological data are not yet sensitive to the mass of individual neutrino species, i.e., the mass hierarchy. Under this situation, two scenarios for the mass splitting of the standard three-flavor neutrinos are often used in cosmology: (i) assuming three species of degenerate massive neutrinos, neglecting the small differences in mass expected from the observed mass splittings; and (ii) assuming the total neutrino mass dominated by the heaviest neutrino mass eigenstate (i.e., two massless and one massive neutrino). We will analyze and compare the constraints based on both the ΛCDM and ϕCDM models in both the spatially flat ($\Omega_k = 0$) and non-flat ($\Omega_k \neq 0$) cases taking into account two different mass hierarchies. The ϕCDM model—in which dark energy is modeled as a scalar field ϕ with a gradually decreasing (as a function of ϕ) potential $V(\phi)$—is a simple dynamical model with dark energy density slowly decreasing in time. This model could resolve some of the puzzles of the ΛCDM model, such as the coincidence and fine-tuning problems (Peebles & Ratra 1988; Ratra & Peebles 1988). Here we focus on an inverse power-law potential $V(\phi) \propto \phi^{-\alpha}$, where α is a nonnegative constant (Peebles & Ratra 1988; Ratra & Peebles 1988). When $\alpha = 0$ the ϕCDM model is reduced to the corresponding ΛCDM scenario. The ϕCDM model with this kind of $V(\phi)$ has been extensively investigated, mostly in the spatially flat case (Podariu & Ratra 2000; Chae et al. 2004; Chen & Ratra 2004; Samushia et al. 2007; Samushia & Ratra 2010; Chen & Ratra 2011a, 2011b; Farooq & Ratra 2013; Farooq et al. 2013a, 2013b; Avsajanishvili et al. 2014; Pavlov et al. 2014; Chen et al. 2015; Lima et al. 2015), and only limited attention has been paid to the non-flat scenario (Pavlov et al. 2013; Farooq et al. 2015; Gosenca & Coles 2015). However, the above-mentioned literature on the ϕCDM model did not consider massive neutrinos. In our previous work the ϕCDM model with massive neutrinos has been studied under the assumption of spatial flatness (Chen & Xu 2016) using a combination of CMB data from *Planck* 2013 and other data sets. In this work, the ϕCDM model with massive neutrinos will be further investigated in both flat and non-flat scenarios by using a combination of the CMB data from *Planck* 2015, BAO data from 6dFGS, SDSS-MGS, BOSS-LOWZ and CMASS-DR11, the joint light-curve analysis (JLA) compilation of SNe Ia observations, and two different H_0 priors.

The rest of the paper is organized as follows. Constraints from the cosmological data are derived in Section 2, and the results for the ϕCDM model are compared with those for the ΛCDM model in both the spatially flat and non-flat scenarios. We summarize our main conclusions in Section 3.

2. OBSERVATIONAL CONSTRAINTS

We consider four cosmological models with massive neutrinos in this paper, i.e., (i) the spatially flat ΛCDM model, (ii) the spatially non-flat ΛCDM model, (iii) the spatially flat ϕCDM model, and (iv) the spatially non-flat ϕCDM model. For each of the four models, we take into account two different scenarios for the neutrino mass hierarchy as mentioned above. Evolution of the background and perturbations are both considered within the linear perturbation theory. Appropriate formulae for the ΛCDM and ϕCDM models in the spatially flat scenario are presented in Section 2 of Chen & Xu (2016). It is easy to generalize them to the non-flat scenario by inclusion of the curvature term Ω_k. The parameter spaces of the models under consideration are as follows:

$$\boldsymbol{P}_1 \equiv \{\Omega_b h^2,\ \Omega_c h^2,\ 100\theta_{\rm MC},\ \tau,\ \ln(10^{10}A_s),\ n_s,\ \Sigma m_\nu\}, \quad (1)$$

$$\boldsymbol{P}_2 \equiv \{\Omega_b h^2,\ \Omega_c h^2,\ 100\theta_{\rm MC},\ \tau,\ \ln(10^{10}A_s),\ n_s,\ \Sigma m_\nu,\ \Omega_k\}, \quad (2)$$

$$\boldsymbol{P}_3 \equiv \{\Omega_b h^2,\ \Omega_c h^2,\ 100\theta_{\rm MC},\ \tau,\ \ln(10^{10}A_s),\ n_s,\ \Sigma m_\nu,\ \alpha\}, \quad (3)$$

$$\boldsymbol{P}_4 \equiv \{\Omega_b h^2,\ \Omega_c h^2,\ 100\theta_{\rm MC},\ \tau,\ \ln(10^{10}A_s),\ n_s,\ \Sigma m_\nu,\ \alpha,\ \Omega_k\}, \quad (4)$$

where $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ are the parameter spaces of the ΛCDM model in the spatially flat and non-flat scenarios, respectively; $\boldsymbol{P}_3$ and $\boldsymbol{P}_4$ are the corresponding ones for ϕCDM model in the flat and non-flat scenarios. Present-day densities of baryons and cold dark matter are denoted by $\Omega_b h^2$ and $\Omega_c h^2$, respectively, $\theta_{\rm MC}$ is an approximation to the angular size of the sound horizon at the time of decoupling $\theta_* = r_s(z_*)/D_A(z_*)$ built in the CosmoMC package which is based on fitting formulae given in Hu & Sugiyama (1996), τ refers to the Thomson scattering optical depth due to reionization, n_s and A_s are the power-law index and amplitude of the power-law scalar primordial power spectrum of scalar perturbations, Σm_ν is the sum of neutrino masses, Ω_k is the dimensionless spatial curvature density today, and α determines the steepness of the scalar field potential in the framework of ϕCDM model.

2.1. Cosmological Data Sets

According to the constraints from the current cosmological observations the value of $\Sigma m_\nu \lesssim 1$ eV. This is below the limit to which the CMB power spectrum (excluding the late-time gravitational lensing effect on the power spectrum) alone can be sensitive (Komatsu et al. 2009). In other words, the massive neutrinos are relativistic at the decoupling epoch, so the effect of the massive neutrinos in the primary CMB power spectrum is very small. The main effect is around the first acoustic peak and is due to the early integrated Sachs–Wolfe effect. After the relativistic-to-non-relativistic transition, the massive neutrinos behave like cold matter. However, the non-relativistic massive neutrinos can suppress the CMB lensing potential on scales smaller than the horizon size. Thus CMB lensing is a useful probe for massive neutrinos. The CMB data set adopted here is a combination of the low multipoles ($l = 2$–29) joint TT, EE, BB and TE likelihood, and high multipoles joint TT ($l = 30$–2508), TE ($l = 30$–1996), and EE ($l = 30$–1996)

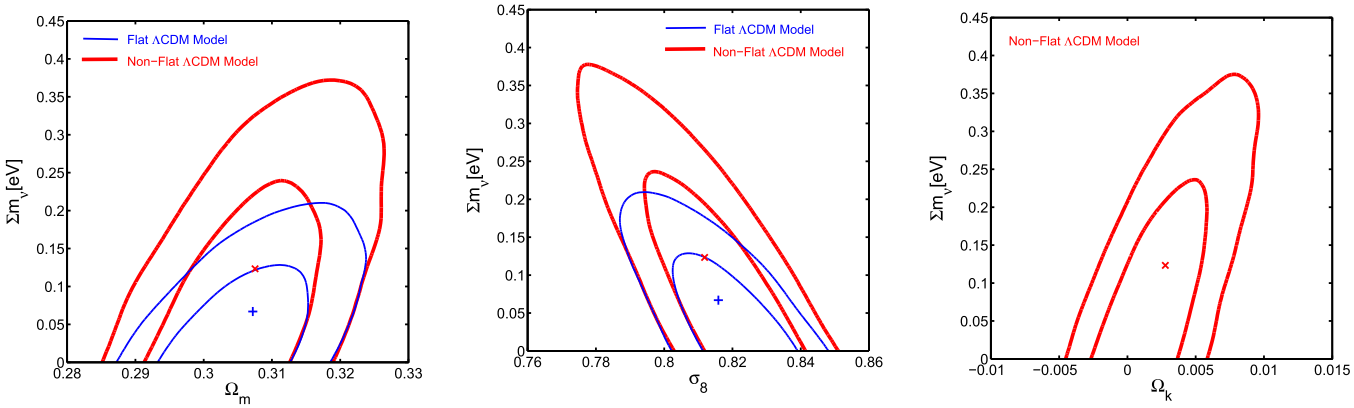

Figure 1. Contours refer to the marginalized likelihoods at 68% and 95% confidence levels constrained from the joint analysis using the *HST* H_0 prior for the ΛCDM model in the scenario assuming three species of degenerate massive neutrinos. Left and middle panels: contours in the (Ω_m, Σm_ν) and (σ_8, Σm_ν) planes, where the thin blue (thick red) lines correspond to constraints in the flat (non-flat) scenario. The "+" ("x") marks the mean values of the pair in the flat (non-flat) scenario. Right panel: contours in the (Ω_k, Σm_ν) plane for the non-flat scenario. The "x" marks the mean values of the (Ω_k, Σm_ν) pair.

likelihood, along with CMB lensing ($l = 40$–400) likelihood from *Planck* 2015 (Adam et al. 2015; Ade et al. 2015). BAO data from galaxy redshift surveys are a powerful cosmological probe, that can supply the Hubble expansion rate and angular diameter distance at different redshifts. The BAO data set employed here is a combination of measurements from the 6dFGS at $z_{\rm eff} = 0.1$ (Beutler et al. 2011), the SDSS Main Galaxy Sample (MGS) at $z_{\rm eff} = 0.15$ (Ross et al. 2014), the Baryon Oscillation Spectroscopic Survey (BOSS) "LOWZ" sample at $z_{\rm eff} = 0.32$ and BOSS CMASS-DR11 anisotropic BAO measurements at $z_{\rm eff} = 0.57$ (Anderson et al. 2014). Another important cosmological probe is offered by SNe Ia, which provided the first direct evidence for cosmic acceleration. The SNe Ia sample used here is the "JLA" compilation of SNe Ia (Betoule et al. 2014), which is a joint analysis of SNe Ia observations including several low-redshift samples ($z < 0.1$), all three seasons from the SDSS-II ($0.05 < z < 0.4$), three years from SNLS ($0.2 < z < 1$), and 14 very high redshift ($0.7 < z < 1.4$) from the *HST* observations. It totals 740 spectroscopically confirmed SNe Ia with high-quality light curves. The Riess et al. (2011) *HST* Cepheid + SNe Ia based estimate of $H_0 = (73.8 \pm 2.4)\,{\rm km\,s^{-1}\,Mpc^{-1}}$ is also used as a supplementary "H_0-prior." Another prior is the median statistics estimate of $H_0 = (68 \pm 2.8)\,{\rm km\,s^{-1}\,Mpc^{-1}}$ of Chen & Ratra (2011), which is more consistent with H_0 values estimated using CMB and BAO data (e.g., Sievers et al. 2013; Aubourg et al. 2015; see also Calabrese et al. 2012).

2.2. Results and Analysis

In our analysis, the likelihood is assumed to be Gaussian, thus we have the total likelihood

$$\mathcal{L} \propto e^{-\chi^2_{\rm tot}/2}, \tag{5}$$

where $\chi^2_{\rm tot}$ is constructed as

$$\chi^2_{\rm tot} = \chi^2_{\rm CMB} + \chi^2_{\rm BAO} + \chi^2_{\rm SNe} + \chi^2_{H_0}, \tag{6}$$

with $\chi^2_{\rm CMB}$, $\chi^2_{\rm BAO}$, $\chi^2_{\rm SNe}$ and $\chi^2_{H_0}$ denoting the contributions from CMB, BAO, SNe Ia and *HST* or median statistics H_0 prior data sets described above, respectively. We derive the posterior probability distributions of parameters with Markov Chain Monte Carlo (MCMC) exploration using the 2015 July version of CosmoMC (Lewis & Bridle 2002).

First, we give constraints assuming three species of degenerate massive neutrinos. Two-dimensional contours for the cosmological parameters of interest are shown in Figure 1 for the flat and non-flat ΛCDM models and in Figure 2 for the flat and non-flat ϕCDM models. In these two figures the *HST* value of H_0 was assumed as a prior. One can see that constraints from the joint data sample are quite restrictive, though there are degeneracies between some parameters. Moreover, it turns out that with Ω_k as a free parameter the ranges of allowed values for other parameters (except $\Omega_b h^2$ and $100\theta_{\rm MC}$) are all significantly broadened for both ΛCDM and ϕCDM models.

In order to investigate the impact of the neutrino mass hierarchy, we compare the constraint results based on two different scenarios of the neutrino mass hierarchy as mentioned previously. Hereafter, the scenario of assuming three species of degenerate massive neutrinos will be quoted as "Scenario I" for short, and the scenario of assuming the total neutrino mass dominated by the heaviest neutrino mass eigenstate will be quoted as "Scenario II." Corresponding mean values of the parameters of interest together with their 95% confidence limits constrained from the joint analysis using the *HST* H_0 prior are presented in Table 1 for the flat and non-flat ΛCDM models and in Table 2 for the flat and non-flat ϕCDM models. It turns out that the constraints on $\Omega_b h^2$, $\Omega_c h^2$, $100\theta_{\rm MC}$, τ, $\ln(10^{10}A_s)$, n_s, Ω_m, σ_8 and H_0 in the four models with different neutrino mass scenarios are consistent with each other at 95% CL. In the spatially flat case, we have $\Sigma m_\nu < 0.165(0.166)$ eV at 95% CL in "Scenario I" ("Scenario II") for the ΛCDM model, and $\Sigma m_\nu < 0.164(0.164)$ eV at 95% CL in "Scenario I" ("Scenario II") for the ϕCDM model. In the spatially non-flat case, we have $\Sigma m_\nu < 0.299(0.354)$ eV at 95% CL in "Scenario I" ("Scenario II") for the ΛCDM model, and $\Sigma m_\nu < 0.301$ (0.364) eV at 95% CL in "Scenario I" ("Scenario II") for the ϕCDM model. The results show that different neutrino mass scenarios just result in insignificant differences between the bounds on Σm_ν for both the ΛCDM and ϕCDM models in the spatially flat case; however, in the spatially non-flat case, the corresponding differences are larger than those in the spatially flat case, and the allowed scale of Σm_ν in "Scenario II" is a little larger than that in "Scenario I."

Let us focus on the constraints on Σm_ν and Ω_k. In "Scenario I," the limits at 95% CL on the sum of neutrino

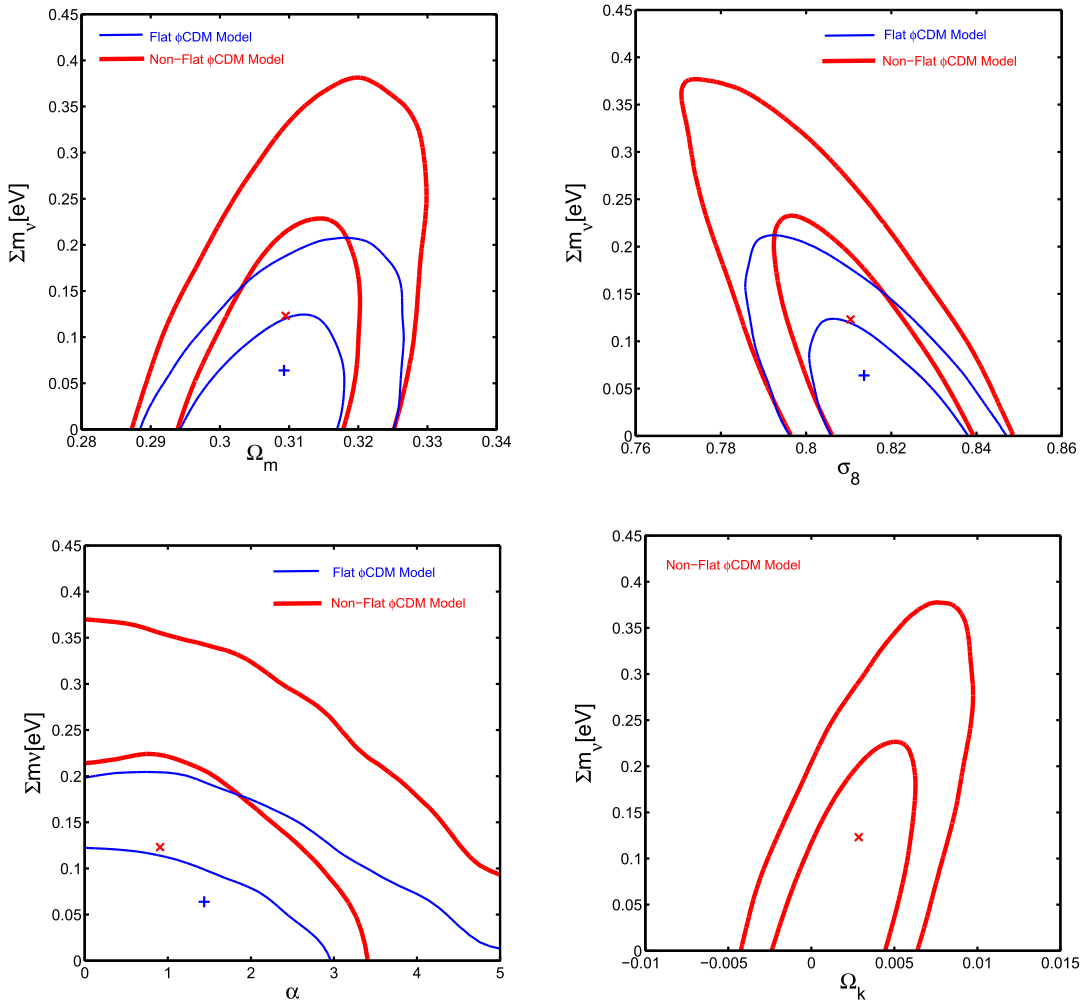

Figure 2. Contours refer to the marginalized likelihoods at 68% and 95% confidence levels constrained from the joint analysis using the *HST* H_0 prior for the ϕCDM model in the scenario assuming three species of degenerate massive neutrinos. Upper left, upper right and lower left panels: contours in the (Ω_m, Σm_ν), (σ_8, Σm_ν) and (α, Σm_ν) planes, where the thin blue (thick red) lines correspond to constraints in the flat (non-flat) scenario. The "+" ("x") marks the mean values of the pair in the flat (non-flat) scenario. Lower right panel: contours in the (Ω_k, Σm_ν) plane for the non-flat scenario. The "x" marks the mean values of the (Ω_k, Σm_ν) pair.

Table 1
Constraints from the Joint Analysis Using the *HST* H_0 Prior, for the ΛCDM Model in Spatially Flat and Non-flat Cases with Two Different Scenarios for the Neutrino Mass Hierarchy

Parameters	ΛCDM model			
	Scenario I		Scenario II	
	Flat	Non-flat	Flat	Non-flat
$\Omega_b h^2$	0.0223 ± 0.0003	0.0222 ± 0.0003	0.0223 ± 0.0003	0.0222 ± 0.0003
$\Omega_c h^2$	0.1184 ± 0.0021	$0.1195^{+0.0030}_{-0.0029}$	0.1184 ± 0.0021	0.1196 ± 0.0030
$100\theta_{\rm MC}$	1.0410 ± 0.0006	1.0408 ± 0.0006	1.0410 ± 0.0006	1.0408 ± 0.0006
τ	$0.0676^{+0.0289}_{-0.0260}$	$0.0715^{+0.0326}_{-0.0287}$	$0.0685^{+0.0279}_{-0.0260}$	$0.0739^{+0.0322}_{-0.0309}$
$\ln(10^{10}A_s)$	$3.0664^{+0.0537}_{-0.0488}$	$3.0767^{+0.0643}_{-0.0557}$	$3.0679^{+0.0520}_{-0.0486}$	$3.0812^{+0.0621}_{-0.0594}$
n_s	$0.9675^{+0.0082}_{-0.0080}$	0.9650 ± 0.0095	$0.9674^{+0.0079}_{-0.0078}$	$0.9642^{+0.0097}_{-0.0100}$
Ω_k	...	$0.0028^{+0.0055}_{-0.0051}$	...	$0.0033^{+0.0058}_{-0.0051}$
Σm_ν (eV)	<0.165	<0.299	<0.166	<0.354
Ω_m	$0.307^{+0.014}_{-0.013}$	$0.308^{+0.016}_{-0.015}$	$0.308^{+0.014}_{-0.013}$	0.309 ± 0.016
σ_8	$0.816^{+0.022}_{-0.024}$	$0.812^{+0.028}_{-0.030}$	$0.815^{+0.023}_{-0.024}$	$0.807^{+0.032}_{-0.037}$
H_0 (km s^{-1} Mpc^{-1})	$67.87^{+1.05}_{-1.11}$	$68.22^{+1.43}_{-1.38}$	$67.83^{+1.03}_{-1.12}$	$68.18^{+1.36}_{-1.38}$

Note. "Scenario I" and "Scenario II" denote two different scenarios of the neutrino mass hierarchy, the implications of which are described in Section 2.2. We present the mean values with 95% confidence limits for the parameters of interest. The top block contains parameters with uniform priors that are varied in the MCMC chains. The lower block shows various derived parameters.

Table 2
Constraints from the Joint Analysis Using the *HST* H_0 Prior, for the ϕCDM Model in Spatially Flat and Non-flat Cases with Two Different Scenarios for the Neutrino Mass Hierarchy

Parameters	ϕCDM model			
	Scenario I		Scenario II	
	Flat	Non-flat	Flat	Non-flat
$\Omega_b h^2$	0.0223 ± 0.0003	0.0222 ± 0.0003	0.0223 ± 0.0003	0.0222 ± 0.0003
$\Omega_c h^2$	0.1183 ± 0.0021	0.1196 ± 0.0030	$0.1183^{+0.0021}_{-0.0022}$	0.1196 ± 0.0030
$100\theta_{\rm MC}$	1.0410 ± 0.0006	1.0408 ± 0.0007	1.0410 ± 0.0006	1.0408 ± 0.0007
τ	$0.0685^{+0.0283}_{-0.0263}$	$0.0722^{+0.0330}_{-0.0313}$	$0.0699^{+0.0283}_{-0.0262}$	$0.0748^{+0.0319}_{-0.0298}$
$\ln(10^{10}A_s)$	$3.0679^{+0.0533}_{-0.0492}$	$3.0782^{+0.0642}_{-0.0601}$	$3.0703^{+0.0526}_{-0.0488}$	$3.0831^{+0.0616}_{-0.0567}$
n_s	$0.9680^{+0.0081}_{-0.0080}$	$0.9647^{+0.0096}_{-0.0092}$	$0.9678^{+0.0083}_{-0.0081}$	0.9643 ± 0.0097
Ω_k	...	$0.0031^{+0.0056}_{-0.0049}$	...	$0.0036^{+0.0059}_{-0.0055}$
Σm_ν (eV)	<0.164	<0.301	<0.164	<0.364
α	<3.494	<3.938	<3.425	<3.941
Ω_m	0.309 ± 0.015	$0.311^{+0.017}_{-0.015}$	$0.310^{+0.015}_{-0.014}$	0.311 ± 0.017
σ_8	$0.814^{+0.023}_{-0.024}$	$0.809^{+0.028}_{-0.031}$	$0.813^{+0.023}_{-0.025}$	$0.805^{+0.033}_{-0.038}$
H_0 (km s^{-1} Mpc^{-1})	$67.61^{+1.24}_{-1.34}$	$67.89^{+1.49}_{-1.50}$	$67.57^{+1.20}_{-1.33}$	$67.91^{+1.45}_{-1.50}$

Note. The mean values with 95% confidence limits for the parameters of interest are displayed. The top block contains parameters with uniform priors that are varied in the MCMC chains. The lower block shows various derived parameters. The implications of "Scenario I" and "Scenario II" are the same as those in Table 1.

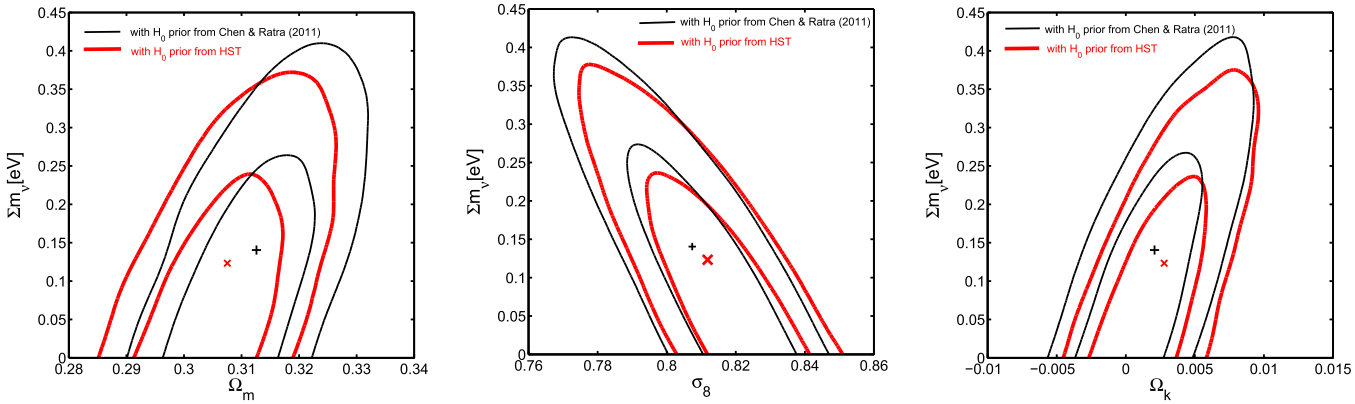

Figure 3. Contours refer to the marginalized likelihoods at 68% and 95% confidence levels in the non-flat ΛCDM model assuming three species of degenerate massive neutrinos constrained from the joint sample with two different H_0 priors. From left to right, contours in the (Ω_m, Σm_ν), (σ_8, Σm_ν) and (Ω_k, Σm_ν) planes are presented, respectively. The thin black lines correspond to constraints from the joint sample with the $H_0 = (68 \pm 2.8)$ km s^{-1} Mpc^{-1} prior from Chen & Ratra (2011). The thick red lines correspond to constraints from the joint sample with the $H_0 = (73.8 \pm 2.4)$ km s^{-1} Mpc^{-1} (Riess et al. 2011) prior from *HST* observations. The "+" marks the mean values of the corresponding pair with H_0 prior from Chen & Ratra (2011). The "x" marks the mean values with H_0 prior from Riess et al. (2011).

masses are $\Sigma m_\nu < 0.165(0.299)$ eV for the flat (non-flat) ΛCDM model, and $\Sigma m_\nu < 0.164(0.301)$ eV for the flat (non-flat) ϕCDM model. It shows that with Ω_k as a free parameter the 95% upper limit on Σm_ν is about double that in the flat case for both the ΛCDM and ϕCDM models. One can obtain the same conclusion in "Scenario II." The strong correlation between Ω_k and Σm_ν is because that the massive neutrinos are still relativistic until recombination so they act as an additional radiative component, and the constraint results also demonstrate that the spatially flat universe is still highly preferred.

In order to explore the impact of the prior value of the Hubble constant H_0 on the cosmological parameter estimation, we compare the constraints resulting from the joint data sample with two different H_0 priors in the non-flat ΛCDM model assuming three species of degenerate massive neutrinos. One is from *HST* observation with $H_0 = (73.8 \pm 2.4)$ km s^{-1} Mpc^{-1} (Riess et al. 2011) which is used above, and another is from the median statistics analysis of Chen & Ratra (2011) with $H_0 = (68 \pm 2.8)$ km s^{-1} Mpc^{-1}. Two-dimensional confidence contours for the cosmological parameters of interest are shown in Figure 3 for the non-flat ΛCDM model with the two different H_0 priors. One can see that the prior value of the Hubble constant H_0 affects cosmological parameter estimation, but not very significantly. In our combined analysis it is because of the weight of the other data used. However, one can notice a certain trend, namely with smaller values of the H_0 prior, the upper limit on Σm_ν gets larger. This implies that the parameters H_0 and Σm_ν are negatively correlated (Komatsu et al. 2009; Chen & Xu 2016). Our result is consistent with that of Di Valentino et al. (2016) who conclude that the bounds on the neutrino parameters may differ appreciably depending on the prior values of low redshift quantities, such as the Hubble constant, the cluster mass bias, and the reionization optical depth.

3. CONCLUSION

We have studied the ΛCDM and ϕCDM models with massive neutrinos assuming two different neutrino mass

hierarchies in both the spatially flat and non-flat scenarios. In the ϕCDM model under consideration, the dark energy scalar field ϕ with an inverse power-law potential $V(\phi) \propto \phi^{-\alpha}$ ($\alpha > 0$) powers the late-time accelerated cosmological expansion. In order to constrain model parameters, we performed a joint analysis on the data including *Planck* 2015 data comprising temperature and polarization of CMB anisotropies as well as CMB lensing, BAO data from 6dFGS, SDSS-MGS, BOSS-LOWZ and CMASS-DR11, the JLA compilation of SNe Ia observations, and the H_0 prior according to *HST* or median statistics. The results indicate that constraints on the cosmological parameters from this combination of data are quite restrictive. We find that the constraints on the parameters are much tighter than those in the previous literature (Chen & Xu 2016), which made use of a combination of the CMB temperature power spectrum likelihoods from *Planck* 2013 and the CMB polarization power spectrum likelihoods from nine-year *WMAP* (*WMAP*9), the galaxy clustering data from WiggleZ and BOSS DR11, and the JLA compilation of SNe Ia observations. A more recent paper by Chen & Xu (2016) studying the ΛCDM and ϕCDM models with massive neutrinos assumed only the spatially flat case.

The results of our paper clearly show that cosmological bounds on the total neutrino mass Σm_ν are very tight; however, they are significantly correlated with the curvature term. It turns out that with Ω_k as a free parameter the 95% upper limit on Σm_ν is relaxed by more than a factor of two with respect to that in the flat case for both the ΛCDM and ϕCDM scenarios. Furthermore, the bounds on Σm_ν based on two different neutrino mass hierarchies have insignificant differences in the spatially flat case for both the ΛCDM and ϕCDM models; however, the corresponding differences are larger in the non-flat case. Moreover, for a given neutrino mass hierarchy, the bounds on Σm_ν in ΛCDM and ϕCDM scenarios have small differences, irrespective of whether Ω_k is fixed at zero or is taken as a free parameter. For example, in the scenario of assuming three species of degenerate massive neutrinos, when $\Omega_k = 0$, we have $\Sigma m_\nu < 0.165(0.164)$ eV at 95% CL for the ΛCDM (ϕCDM) model; when $\Omega_k \neq 0$, we have $\Sigma m_\nu < 0.299$ (0.301) eV at 95% CL for the ΛCDM (ϕCDM) model. Additionally, in the scenario assuming three species of degenerate massive neutrinos, we find $\alpha < 3.494\,(3.938)$ at 95% CL for the flat (non-flat) ϕCDM model, while the ΛCDM scenario corresponding to $\alpha = 0$ is not ruled out at this CL. One can obtain the same conclusion in the scenario assuming the total neutrino mass dominated by the heaviest neutrino mass eigenstate. In general, the constraints on the cosmological parameters are similar in the ΛCDM and ϕCDM models, and the bounds on the total neutrino mass Σm_ν are not particularly sensitive to the underlying cosmological models under consideration. Massive neutrinos mainly affect the redshift of matter–radiation equality $z_{\rm eq}$ (and also being relativistic at the $z_{\rm eq}$ they are counted as non-relativistic now, thus being entangled with $\Omega_c h^2$). At this epoch neither Λ nor ϕ contributes significantly to the background expansion. Consequently, these results imply that the observational data that we have employed here still cannot distinguish whether dark energy is a time-independent cosmological constant or varies mildly in space and slowly in time.

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